Optimization of the design of residential towers

The mathematical model

Sets, parameters and optimization variables

In this section, the key components of the model are descibed. Table 1 provides a detailed explanation of the sets used in the model, which include sectors, areas, owners, and floor configurations. Table 2 summarizes the floor parts and their corresponding surface area configurations, arranged by the different elements a, b, c, d, and e. Additionally, the parameters utilized in the model are described in Table 3, including profits. Lastly, Table 4 presents the decision variables that define the number of floors, apartments, and ownership structures across various configurations, sectors, and areas.

Table 1: Indices

index	**set**	**elements**
i, l	sectors	{social, middle, free}
j	areas	{36, 42, 48, 52, 60, 68, 70, 71, 96, 131}
h	owners	{corporation, investor, private}
v	floor configurations	{aa, ab, ac, bb, bc, cc, cd, ce, dd, de, ee}

Table 2: Floor parts with surface area configuration j

element	**description**	**surface area configurations (j)**
a	Floor part a	{36, 36, 42, 42, 48, 48}
b	Floor part b	{42, 42, 52, 52, 58}
c	Floor part c	{60, 60, 71, 71}
d	Floor part d	{70, 96, 96}
e	Floor part e	{131, 131}

Table 3: Parameters

parameter	**Description**
K	Total number of floors (e.g. K=23, 40, or 56)
R_{jv}	Number of apartments with floor area *j* and configuration *v*
O_{ijh}	Profit per apartment for owner *h* in sector *i* and floor area *j*
α_{ih}	Minimal floor area for sector *i* and owner *h*. Current case: $\alpha_{social,corporation} = 40$, $\alpha_{middle,corporation} = 50$, $\alpha_{free,investors} = 60$, the others are zero.
β_i	Minimum percentage apartments of sector *i* in total program. Current case: $\beta = (0.4, 0.4, 0)$.
γ_i	Minimum average floor area for sector *i*. Current case: $\gamma = (40, 50, 0)$.
δ_h	Minimal percentage apartments of owner *h* in total program. Current case: $\delta = (0, 0.7, 0)$. (So, 0.7 for the "investors").

Table 4: Decision variables

variable	**Description**	**variables**
X_v	Number of floors with configuration *v*	11
Y_{ijh}	Number of apartments in sector *i* with floor area *j* and owner *h*	90
W_{vh}	Number of floors of type *v* with owner h	33
Z_{jh}	Number of apartments with floor area *j* and owner *h*	30

The model

The objective of the model is to maximize the profit, i.e., the overall net result.

Objective:

$$\text{Max } \sum_{i,j,h} O_{ijh} Y_{ijh}$$

Subject to:

Design constraints: the design constraints below outline the rules for distributing apartments across different floor configurations, owners, and sectors in a building optimization problem. They ensure that the total number of apartments is correctly allocated, owners do not share floors. The constraints also guarantee minimum apartment allocations in certain sectors and maintain consistency between the planned distribution and the actual assignment of apartments. The goal is to create an efficient and organized layout that meets various allocation requirements while avoiding conflicts between different ownership and housing types.

1. Total number of floors:

$$\sum_{v} X_v = K$$

2. Total apartments consistency:

$$\sum_{v} R_{jv} X_v = \sum_{i,h} Y_{ijh} \qquad \forall j$$

   The sum of apartments across all configurations are equal the total number of apartments across all areas *j*, sectors *i*, and owners *h*. This ensures that the number of available apartments matches the actual number assigned to areas and owners.

3. Owner floor configuration**:**

$$\sum_{h} W_{vh} = X_v \quad \forall v$$

   The total number of apartments allocated to owners *h* for each configuration *v* must equal the total apartments available in that configuration. This ensures that all apartments in each configuration are assigned to owners.

4. No floors with multiple owners:

$$Z_{jh} = \sum_{v} W_{vh} \cdot R_{jv} \qquad \forall j, h$$

This constraint ensures that a specific floor can only have apartments from a single owner h. No floor should be split among multiple owners.

5. Apartments by owner and area:

$$Z_{jh} = \sum_{i} Y_{ijh} \qquad \forall j, h$$

The total number of apartments owned by a specific owner h in area j must equal the total apartments allocated to that owner in the optimization program.

Program constraints: ensure a balanced and fair housing distribution in the optimization model. They set minimum percentages for e.g. social housing and mid-rental housing (both must be at least 40% of the total program) and establish minimum average floor areas for these housing types (at least 40 square meters for social housing and 50 square meters for mid-rental housing). The goal is to provide sufficient quantities and quality of housing while ensuring both social and mid-rental units meet certain space standards. These constrains are set by municipalities:

6. Minimal percentage of a sector in total program:

$$\sum_{j,h} Y_{i,jh} \geq \beta_i \sum_{l,j,h} Y_{ljh} \qquad \forall i$$

7. Minimal average floor area of apartments in a sector:

$$\sum_{j,h} j \cdot Y_{ijh} \geq \gamma_i \sum_{j,h} Y_{ijh} \qquad \forall i$$

E.g.: The average floor area of social housing is at least 40 square meters. The average floor area of middle rental housing is at least 50 square meters.

Sale constraints regulate the acquisition of apartments by different parties based on apartment size and sector. The goal is to enforce specific ownership and acquisition criteria based on apartment size and sector, ensuring clear guidelines for property transactions.

Sales constraints housing corporations:

8. Minimal floor area for a sector and owner:

$$Y_{i,j,h} = 0 \qquad \forall i, h \, \forall j < \alpha_{ih}$$

E.g.: Social rental apartments acquired by the housing corporation must have a minimum floor area of 40 square meters. Any apartment smaller than this is excluded from the corporation's purchase. Free rental apartments for institutional investors must have a minimum floor area of 60 square meters. Apartments smaller than this are excluded from the investor's acquisition.

9. No free sector apartments for housing corporation:

$$Y_{free,j,corporation} = 0 \quad \forall j$$

The housing corporation is not allowed to acquire free sector apartments. This ensures the corporation only focuses on affordable rent.

Sales constraints institutional investors:

10. Sector ownership requirement: If we want to sell to a certain sector, then a certain minimal percentage of the whole building must be owned by that party:

$$\sum_{i,j} Y_{ijh} \geq \delta_h \sum_{i,j,p} Y_{ijp} \qquad \forall h$$

For example: If a building is to be sold to an institutional investor, that investor must own at least 70% of the building. This ensures that the investor has a significant stake in the building and thus control in the owners association.

General

11. All variables are natural numbers:

$$X_v, Y_{ijh}, W_{vh}, Z_{jh} \in N \quad \forall v, i, j, h$$