

算法 12

No: \_\_\_\_\_ Date: \_\_\_\_\_

1. 1)  $L(p|X) = \prod_{i=1}^n P(X_i = x_i) = \prod_{i=1}^n p(1-p)^{x_i-1} = p^n (1-p)^{\sum_{i=1}^n x_i}$

2)  $\log L(p|X) = n \log p + (\sum_{i=1}^n x_i - n) \log(1-p)$

$\frac{\partial}{\partial p} \log L(p|X) = \frac{n}{p} - \frac{\sum_{i=1}^n x_i - n}{1-p} = 0$

$\hat{p} = \frac{n}{\sum_{i=1}^n x_i}$

2.  $L(p|X, \alpha) = \prod_{i=1}^n \frac{\beta^\alpha}{\Gamma(\alpha)} e^{-\beta x_i} x_i^{\alpha-1} = \left( \frac{\beta^\alpha}{\Gamma(\alpha)} \right)^n e^{-\beta \sum_{i=1}^n x_i} \left( \prod_{i=1}^n x_i \right)^{\alpha-1}$

$\ln L = n\alpha \ln \beta - n \ln \Gamma(\alpha) - \beta \sum_{i=1}^n x_i + (\alpha-1) \sum_{i=1}^n \ln x_i$

$\frac{\partial}{\partial \beta} \ln L = \frac{n\alpha}{\beta} - \sum_{i=1}^n x_i = 0$

$\hat{\beta} = \frac{\alpha}{\bar{x}}$