Algorithm Foundations of Data Science and Engineering

Welcome Tutorial :-)

Tutorial 8–Suggested Answers

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1. Given a Markov chain determined by the transition marix $P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$ and $\pi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T$.

a. Compute πP, πP^2, and πP^3, πP^4;

答：$\pi P = (0.5, 0.5)$, $\pi P^2 = (0.375, 0.625)$, $\pi P^3 = (0.34375, 0.65625)$, $\pi P^4 = (0.3359375, 0.6640625)$

b. Show that the results are approaches a constant vector.

答：常数向量为$(\frac{1}{3}, \frac{2}{3})$。设P的平稳分布为$\tilde{\pi} = (\tilde{\pi}(1), \tilde{\pi}(2))$，由细致平稳条件，满足$\tilde{\pi}(1)P_{12} = \tilde{\pi}(2)P_{21}$，并且$\tilde{\pi}(1) + \tilde{\pi}(2) = 1$，解得$\tilde{\pi} = (\frac{1}{3}, \frac{2}{3})$

2. Given a Markov chain determined by the transition matrix P. Prove that P and $(1/n)((n-1)I + P)$ have the same stationary distribution, where I is an identity matrix.

答：设P的平稳分布为π_1,则$\pi_1 P = \pi_1$。$\pi_1(1/n((n-1)I + P) =$

$(1/n)((n-1)\pi_1+\pi_1 P)=(1/n)((n-1)\pi_1+\pi_1)=\pi_1$;

设$(1/n)((n-1)I+P)$的平稳分布为π_2,则$\pi_2[(1/n)((n-1)I+P)]=\pi_2$，$\pi_2[(n-1)I+P]=n\pi_2$，$(n-1)\pi_2+\pi_2 P=n\pi_2$, 则有$\pi_2 P=\pi_2$。

所以P 和$(1/n)((n-1)I+P)$ 有相同的平稳分布。

3. A certain experiment is believed to be described by a two-state Markov chain with the transition matrix P, where $P=\begin{pmatrix} 0.5 & 0.5 \\ p & 1-p \end{pmatrix}$ and the parameter p is unknown. When the experiment is performed many times, the chain ends in state one approximately 20 percent of the time and in state two approximately 80 percent of the time

a. Compute a sensible estimate for the unknown parameter p and explain how you found it;

答：可知平稳分布为$\pi=(0.2,0.8)$，由于$\pi P=\pi$，

即$(0.2,0.8)*\begin{pmatrix} 0.5 & 0.5 \\ p & 1-p \end{pmatrix}=(0.2,0.8)$. 所以$p=\frac{1}{8}$

b. Whether is the Markov chain irreducible and aperiodic, or not? Why?

答：是反周期的，因为每个状态的周期为1；是不可约减的，因为每个状态都是连通的。

4. Given a Markov chain determined by the transition marix

$$P=\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

a. Show that $\pi=(0.4,0.6)$ is a stationary distribution of this chain;

答：$(0.4,0.6)*\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}=(0.4,0.6)$. 所以$\pi$是$P$的平稳分布。

b. Show that $\pi=(0.4,0.6)$ is also a stationary distribution of the Markov chain with the transition matrix $\frac{1}{2}(I+P)$, where I is an identity

matrix.

答：$(0.4,0.6)*\frac{1}{2}(I+P)=(0.4,0.6)$ 所以π是$\frac{1}{2}(I+P)$的平稳分布。

c. If P has a stationary distribution π. Prove that P and $\frac{1}{2}(I+P)$ have the same stationary distribution.

答：如同题2。

5. Given a Markov chain determined by the transition matrix $\begin{pmatrix} a & 1-a \\ 1-b & b \end{pmatrix}$, where $a,b\in[0,1]$.

a. If the Markov chain is periodic, what are the values of a and b?

答：因为是周期的，所以每个状态的周期不等于1，所以$a=0,b=0$

b.In this case, what is the period?

答：周期为2

c. In this case, is the Markov chain irreducible? (Hint: a Markov chain is irreducible if it is possible to go from every state to every state

(not necessarily in one move).)

答：不可约，因为每个状态都是连通的。