## Algorithm Foundations of Data Science and Engineering Welcome Tutorial :-)

Tutorial 2

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## Tutorial 2

- 1. Compute the Jaccard similarities of each pair of the following three sets:  $\{1,2,3,4\}$ ,  $\{2,3,5,7\}$ , and  $\{2,4,6\}$ .
- 2. Prove that if the Jaccard similarity of two columns is 0, then minhashing always gives a correct estimate of the Jaccard similarity.
- 3. a. Compute the Jaccard similarity of each of the pairs of columns.
  - b. Compute the minhash signature for each column if we use the following three hash functions:  $h_1(x) = 7x + 1 \mod 6$ ;  $h_2(x) = 11x + 2 \mod 6$ ;  $h_3(x) = 5x + 2 \mod 6$ .

Element	$S_1$	$S_2$	$S_3$
0	1	1	0
1	0	1	0
2	1	0	0
3	0	0	1
4	1	0	1
5	0	0	0

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- 4. For LSH, please to determine the similarity threshold t, i.e., the value of similarity t at which the probability of becoming a candidate is 1/2, which can be a function of b and r.
- 5. Let two sets  $S_1$  and  $S_2$  be presented in the form of binary vectors,  $\{h_1, \cdots, h_k\}$  be k random permutations, and  $h_i(S)$  record the first 1 in each column after permutation. Please prove that  $\widehat{JS}(S_1, S_2) = \frac{1}{k} \sum_{i=1}^k X_i$  is within  $\varepsilon$  error with probability at leat  $1 \delta$  if  $k = \frac{2\ln(1/\delta)}{\varepsilon^2}$ , where  $JS(A, B) = \frac{|A \cap B|}{|A \cup B|}$ , and  $X_i = \begin{cases} 1, & \text{if } h_i(S_1) = h_i(S_2); \\ 0, & \text{otherwise.} \end{cases}$