Algorithm Foundations of Data Science and Engineering
Welcome Tutorial :-)

Tutorial 10

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Tutorial 10

1. Write the following condition as integer programming constraints. At least one of the following two inequalities hold:

$$x_1 + x_2 + x_3 + x_4 \leq 4, 3x_1 - x_2 - x_3 + x_4 \leq 3$$

   Write the equivalent IP constraints and define any new variables. Assume that $x_j \geq 0$ for each $j = 1$ to 4, and that each variable is required to be integer.

2. Let

$$f(x) = \begin{cases} 10x, & \text{if } 0 \leq x \leq 50, \\ 500, & \text{if } 51 \leq x \leq 100, \\ 5x, & \text{if } x \geq 101, \end{cases}$$

   Rewrite the following non-linear programming problem as an integer program.

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3. If x and y are integers,
   a. How would ensure that $x+y \leq 3$, $2x+5y \leq 12$, or both are satisfied by x and y?
   b. How would ensure when $x \leq 2$, then $y \leq 3$?
4. Given the following IP problem

   IP(1)

   Maximize: $10x_1 + 4x_2 + 9x_3$

   Subject to: $5x_1 + 4x_2 + 3x_3 \leq 9$

   $0 \leq x_i \leq 1$ and $x_i \in Z$ for i = 1 to 3

   a. Write down the LP relaxation of IP(1);
   b. If $x_1 = 1$, please find the upper bound of the corresponding IP;
   c. Please use the Branch and Bound algorithm to solve IP(1), and determine how many nodes are pruned.

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5. The set packing problem is the problem of selecting the maximum cost sub-collection of sets, no two of which share a common element.

   Input: Universe set $U = \{u_1, u_2, \cdots, u_n\}$

   Subsets $S = \{s_i | s_i \subset U, 1 \leq i \leq m\}$

   Costs $C = \{c_1, c_2, \cdots, c_m\}$

   Please formulate the set packing problem as an IP problem.

6. Given the following IP problem

   Minimize: $z = 3x_1 + x_2$

   Subject to: $2x_1 - x_2 \leq 6$

   $x_1 + x_2 \leq 4$

   $0 \leq x_i$ and $x_i \in Z$ for i = 1 to 2

   a. Please find two valid inequalities of the IP;
   b. Determine the feasible region of the IP;
   c. Determine the convex hull of the IP;
   c. Please use LP to solve the IP.