

Tutorial 10

1. 将条件转换为 LP constraints.

$$x_1 + x_2 + x_3 + x_4 \leq 4 + M(1-w)$$

$$3x_1 + x_2 - x_3 + x_4 \leq 3 + Mw$$

$$w \in \{0, 1\}.$$

2. Let
$$f(x) = \begin{cases} 10x, & \text{if } 0 \leq x \leq 50 \\ 500, & \text{if } 51 \leq x \leq 100 \\ 5x, & \text{if } x \geq 101 \end{cases}$$

Rewrite the following non-linear programming problem as an integer program

$$0 \leq x_1 \leq 50w_1, \quad w_1 \in \{0, 1\}$$

$$51w_2 \leq x_2 \leq 100w_2, \quad w_2 \in \{0, 1\}$$

$$x_3 \geq 101w_3, \quad w_3 \in \{0, 1\}.$$

$$w_1 + w_2 + w_3 = 1.$$

$$\text{则 } f(x) = 10x_1 + 500w_2 + 5x_3.$$

3. x, y are integers. a. How would ensure that $x+y \leq 3$, $2x+5y \leq 12$.
or both are satisfied by x and y ?

a). $x + y \leq 3 + M(1-w)$

$2x + 5y \leq 12 + Mw$

M 为很大的数, $w \in \{0, 1\}$.

b) $w = \begin{cases} 1, & x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$ M 为很大的数.

$x \leq 2 + M(1-w)$

$y \leq 3 + M(1-w)$

4. IP(1): Maximize: $10x_1 + 4x_2 + 9x_3$

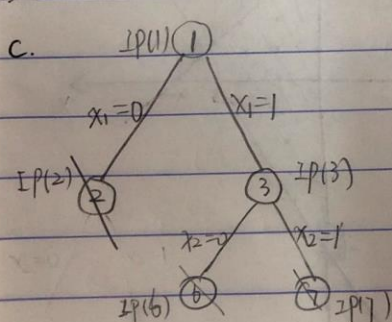
s.t. $5x_1 + 4x_2 + 3x_3 \leq 9$

$0 \leq x_i \leq 1$ and $x_i \in \mathbb{Z}$ for $i=1$ to 3

a. $\begin{cases} 5x_1 + 4x_2 + 3x_3 \leq 9 \\ 0 \leq x_i \leq 1, \text{ for } i=1 \text{ to } 3. \end{cases}$

b. max $10x_1 + 4x_2 + 9x_3$.

s.t. $5x_1 + 4x_2 + 3x_3 \leq 9$, $x_1 = 1$, $x_i \in \{0, 1\}$ $2 \leq i \leq 3$.



~~optimal~~ optimal solution for LP(2) is:

$x_1 = 0, x_2 = 1, x_3 = 1$

$Z_{LP(2)} = 13$. $Z_L = 13$.

optimal solution for LP(3) is:

$x_1 = 1, x_2 = \frac{1}{4}, x_3 = 1$

$Z_{LP(3)} = 20$

$Z_{LP(6)} = 19$. $Z_L = 19$. pruned.

$Z_{LP(7)} = 14$. pruned.

\therefore maximize value is 19. 4个 node 被 prune.

5. 记 $X_i = \begin{cases} 1, & S_i \text{ 被选择} \\ 0, & \text{otherwise} \end{cases}$

$Y_{ij} = \begin{cases} 1, & S_i \text{ 中包含 } U_j \\ 0, & \text{otherwise.} \end{cases} \quad \begin{matrix} i=1,2,\dots,m \\ j=1,2,\dots,n \end{matrix}$

$$\max \sum_{i=1}^m C_i X_i$$

s.t. $\sum_{i=1}^m X_i Y_{ij} \leq 1, \quad \forall j=1,2,\dots,n$

$X_i \in \{0,1\}, Y_{ij} \in \{0,1\}.$

$i=1,2,\dots,m, j=1,2,\dots,n.$

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6. maximize: $Z = 3x_1 + x_2$
 s.t. $2x_1 - x_2 \leq 6$
 $x_1 + x_2 \leq 4$
 $0 \leq x_i$ and $x_i \in \mathbb{Z}$ for $i=1$ to 2 .

a. $x_1 \leq 3, -6 \leq x_2 \leq 4.$

b. 如图

c. $x_1 \leq 3.$

d. (3,1) 点最优. $Z_{\max} = 10.$

