Algorithm Foundations of Data Science and Engineering
Welcome Tutorial :-)

Tutorial 9

GAO Ming

DaSE @ ECNU

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Tutorial 9

1. Let A be the adjacency matrix of graph G, where

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

   a. If there are two communities: $C_1 = \{1,3\}$ and $C_2 = \{2\}$, please using formula $Q = \frac{1}{2m}\sum_{c\in C}\sum_{i\in c}\sum_{j\in c}(A_{ij} - \frac{k_i k_j}{2m})$ to compute the modularity of the partition;
   b. Given the community structure in [a.], please using formula $Q = \sum_{c\in C}\left[\frac{\sum_{in}^{c}}{2m} - \left(\frac{\sum_{tot}^{c}}{2m}\right)^2\right]$ to compute the modularity of the partition;
   c. Given the community structure in [a.], please using formula $Q = \frac{1}{4m}\mathbf{s}^T B\mathbf{s}$ to compute the modularity of the partition, where $B_{ij} = A_{ij} - \frac{k_i k_j}{2m}$;
   d. How well is the community structure? Is there any way to increase the value of modularity via adjusting the community structure?

Tutorial 9 Cont'd

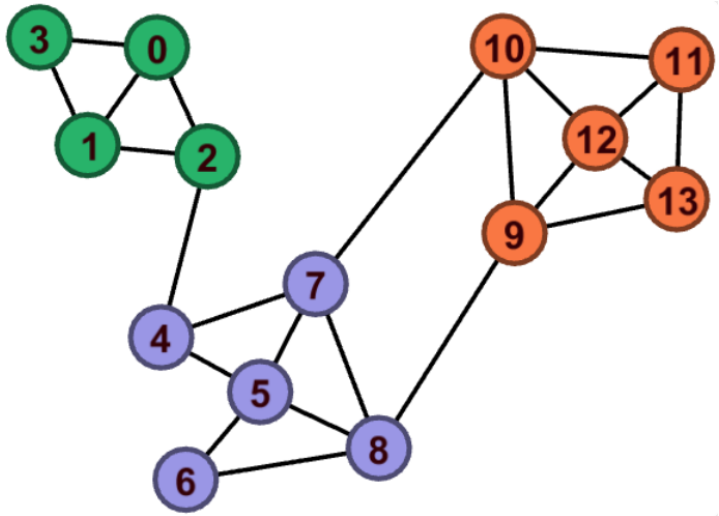
2. Let W be the weighted matrix of weighted graph G, where

$$W = \begin{pmatrix} 0 & 2 & 3 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix}$$

   a. If there are two communities: $C_1 = \{1,3\}$ and $C_2 = \{2\}$, please using formula $Q = \frac{1}{2m}\sum_{c\in C}\sum_{i\in c}\sum_{j\in c}(W_{ij} - \frac{k_i k_j}{2m})$ to compute the modularity of the partition;
   b. Given the community structure in [a.], please using formula $Q = \sum_{c\in C}\left[\frac{\sum_{in}^{c}}{2m} - \left(\frac{\sum_{tot}^{c}}{2m}\right)^2\right]$ to compute the modularity of the partition;
   c. Given the community structure in [a.], please using formula $Q = \frac{1}{4m}\mathbf{s}^T B\mathbf{s}$ to compute the modularity of the partition, where $B_{ij} = W_{ij} - \frac{k_i k_j}{2m}$;

Tutorial 9 Cont'd

3. Given a graph shown in the following figure, vertices locate in the same partition if they have the same color.



   a. Compute the modularity of community via selecting reasonable formula;
   b. Is it possible to increase the value of modularity by merging the communities given in the figure?