Algorithm Foundations of Data Science and Engineering
Welcome Tutorial :-)
Tutorial 11

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7 Jun., 2019

Tutorial 11

1. A functions $f : 2^V \to R$ is a submodular if and only if for all $S \subseteq T \subseteq V$, and $C \subseteq V \setminus T$,

$$f(S \cup C) - f(S) \geq f(T \cup C) - f(T).$$

2. Given the following ground set and collection of subsets, please find the $3-$max cover via using the Hill-climbing algorithm.

Ground set	$\{a, b, c, d, e, f, g, h, i, j, k, l\}$
Subsets	$A_1 = \{b, c, d\}, A_2 = \{e, f, g\}, A_3 = \{i, j, k, l\}$ $A_4 = \{a, e, i\}, A_5 = \{i, b, g\}, A_6 = \{c, d, g, h, k, l\}$ $A_7 = \{a, l\}, A_8 = \{a, e, i\}$

Tutorial 11 Cont'd

3. Given a set V, let $f(A)$ be a submodular function for $A \subseteq V$.
   a. Prove that $\overline{f}(A) = f(A^c)$ is also a submodular;
   b. Prove that $g(A) = f(A \cap S)$ is also a submodular for a fixed set $S \subset V$;
4. Let $w : N \to R$ denote the weights of the elements of a finite set N. Consider the linear function defined by

   $$f(S) = \sum_{i \in S} w_i, \text{ for } \forall S \subseteq N.$$

   Prove that the linear function $f(S)$ is a submodular.