Algorithm Foundations of Data Science and Engineering
Welcome Tutorial :-)
Tutorial 8

GAO Ming

DaSE @ ECNU

15 Apr., 2019

Tutorial 8

1. Given a Markov chain determined by the transition marix
$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} \text{ and } \pi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T.$$
   a. Compute πP, πP^2, πP^3 and πP^4;
   b. Show that the results are approaches a constant vector.
2. Given a Markov chain determined by the transition matrix P. Prove that P and $(1/n)((n-1)I+P)$ have the same stationary distribution, where I is an identity matrix.
3. A certain experiment is believed to be described by a two-state Markov chain with the transition matrix P, where $P = \begin{pmatrix} 0.5 & 0.5 \\ p & 1-p \end{pmatrix}$ and the parameter p is unknown. When the experiment is performed many times, the chain ends in state one approximately 20 percent of the time and in state two approximately 80 percent of the time.
   a. Compute a sensible estimate for the unknown parameter p and explain how you found it;
   b. Whether is the Markov chain irreducible and aperiodic, or not? Why?

Tutorial 8

4. Given a Markov chain determined by the transition marix

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

   a. Show that $\pi = (0.4, 0.6)$ is a stationary distribution of this chain;
   b. Show that $\pi = (0.4, 0.6)$ is also a stationary distribution of the Markov chain with the transition matrix $\frac{1}{2}(I + P)$, where I is an identity matrix.
   c. If P has a stationary distribution π. Prove that P and $\frac{1}{2}(I + P)$ have the same stationary distribution.

5. Given a Markov chain determined by the transition matrix $\begin{pmatrix} a & 1-a \\ 1-b & b \end{pmatrix}$, where $a, b \in [0, 1]$.
   a. If the Markov chain is periodic, what are the values of a and b?
   b. In this case, what is the period?
   c. In this case, is the Markov chain irreducible? (Hint: a Markov chain is irreducible if it is possible to go from every state to every state (not necessarily in one move).)