

tutorial 7 参考答案

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2019/4/16

1、 For two-dimensional function $f(x, y) = 25x^2 + y^2$

a) Compute the gradient vector at initial point $(0.6, 4)$;

解： 梯度表达式 $g=(50x, 2y)$, 所以初始点梯度 $(30, 8)$

b) Given the normalized gradient vector c at point $(0.6, 4)$;

解： 标准化梯度向量 $c=g/\|g\|=(0.9662, 0.2577)$

c) If we decrease the initial point in the direction c by a step size of 0.5, what is value of the function at the new point?

解： $(0.6, 4) - 0.5c = (0.1169, 3.87115)$

d) If we decrease the initial point in the direction $(1, 0)^T$ by a step size of 0.5, what is value of the function at the new point?

解： $(0.6, 4) - 0.5(1, 0) = (0.1, 4)$

2、 As illustrated in following figure, is there any ideal to improve the optimization process for minimizing the function? [Hint: Robust gradient methods should re-scale the step size empirically depending on local properties of the function.]

解： No! 梯度的方向是可信的，但梯度的大小什么都说明不了，和步长没有一毛钱的关系，鲁棒的梯度方法应该根据局部的性质来选择步长。根据经验，步长的选择应遵循以下两个原则：

1) 如果走了这一步之后，函数值变大了，说明步子迈的太大，此时应该撤回这一步，并且减少步长。

2) 如果走了一步之后，函数值变小了，则该步是成功的，说明步长再大一点也可能是ok的，此时应该增大步长。

3、 1) 解：

$$\frac{\partial}{\partial p_{uj}} \mathcal{J}(R; P, Q, b, d) = - \sum_{i: (u, i) \in \mathcal{K}} e_{ui} q_{ji} + \lambda p_{uj}$$

$$\frac{\partial}{\partial q_{ji}} \mathcal{J}(R; P, Q, b, d) = - \sum_{u:(u,i) \in \mathcal{K}} e_{ui} p_{uj} + \lambda q_{ji}$$

2)解:

$$\frac{\partial}{\partial b_u} \mathcal{J}(R; P, Q, b, d) = - \sum_{i:(u,i) \in \mathcal{K}} e_{ui} + \lambda b_u$$

$$\frac{\partial}{\partial d_i} \mathcal{J}(R; P, Q, b, d) = - \sum_{u:(u,i) \in \mathcal{K}} e_{ui} + \lambda d_i$$

3)解: 更新规则如下

$$p_{uj} \leftarrow p_{uj} - t \left(- \sum_{i:(u,i) \in \mathcal{K}} e_{ui} q_{ji} + \lambda p_{uj} \right)$$

$$q_{ji} \leftarrow q_{ji} - t \left(- \sum_{u:(u,i) \in \mathcal{K}} e_{ui} p_{uj} + \lambda q_{ji} \right)$$

$$b_u \leftarrow b_u - t \left(- \sum_{i:(u,i) \in \mathcal{K}} e_{ui} + \lambda b_u \right)$$

$$d_i \leftarrow d_i - t \left(- \sum_{u:(u,i) \in \mathcal{K}} e_{ui} + \lambda d_i \right)$$

其中 $e_{ui} = (r_{ui} - \mu - b_u - d_i - p_u^T q_i)$