Algorithm Foundations of Data Science and Engineering
Welcome Tutorial :-)

Tutorial 4

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Tutorial 4

1. Given an input stream $< 4, 1, 3, 5, 1, 3, 2, 6, 7, 0, 9 >$ and hash functions in the form of $h(x) = (ax + b) \bmod 8$, where a and b are two arbitrary integers. If there are three following hash functions:
   (1) $h(x) = (3x + 2) \bmod 8$;
   (2) $h(x) = (7x + 5) \bmod 8$;
   (3) $h(x) = (5x + 3) \bmod 8$;

   Please address the following questions:
   a. Find the frequency count of every item given by Count-Min sketch;
   b. Analyze the accurate of counting result in *a*.;
   c. If we try to find the (ε, δ)-approximations of the frequency count, how to modify the algorithm;
2. Let the largest and second largest eigenvalues of matrix A be 2 and 1.7, respectively. Is it possible to find the largest eigenvalue via using the power iteration approach? Please explain how fast the power method converges?

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3. Given a matrix

$$A = \begin{pmatrix} 2 & 1 \\ 4 & 5 \end{pmatrix}$$

   a. Compute the eigenvalues and eigenvectors of A;
   b. Given a starting vector $v = (1,1)^T$, approximate the largest eigenvalue and eigenvector of A via using the power method.

4. Let $\lambda_1, \lambda_2, \cdots, \lambda_n$ be eigenvalues of matrix $A \in R^{n \times n}$. Prove that
   a. The matrix $A - \sigma I$ has eigenvalue $\lambda_i - \sigma$ for $i = 1, 2, \cdots, n$;
   b. The invertible matrix $(A - \sigma I)^{-1}$ has eigenvalue $(\lambda_i - \sigma)^{-1}$ for $i = 1, 2, \cdots, n$.