

Resolution	Error	Order of Accuracy
25 x 30	0.000583961	0
50 x 60	0.000278453	1.06844
100 x 120	0.000136207	1.03163

CS 111 Final

Problem 1

Introduction

The goal of this first problem is to find the optimal time it takes to boil a potato. In order to do this, we will approximate the shape of a potato as a two dimensional 4cm by 5cm rectangle. At time $t_{start} = 0$ a pan is filled with water and contains the potato; it is placed on the stove top. The initial temperature of the potato and water is $T_{room} = 20^\circ C$. The water temperature then quickly rises from $20^\circ C$ to $100^\circ C$ in 60 seconds and then stays at $100^\circ C$. This is modeled by the equation $T_{water} = \min(20 + 80\frac{t}{60}, 100)$. Due to the thermal conduction heat propagates from water inside the potato. At the temperature of $T_{cooking} = 65^\circ C$, the cellular structure of the potato begins to change and the starch starts to gelatinize. We will assume that it takes 5 minutes (300 seconds) for the potato to fully cook once the temperature of the potato has reached the temperature $T_{cooking}$.

The temperature $T = T(t, x, y)$ inside the potato satisfies the heat equation (diffusion equation) shown below:

$$\frac{\partial T}{\partial t} = \lambda \Delta T, \quad (x, y) \in \Omega$$

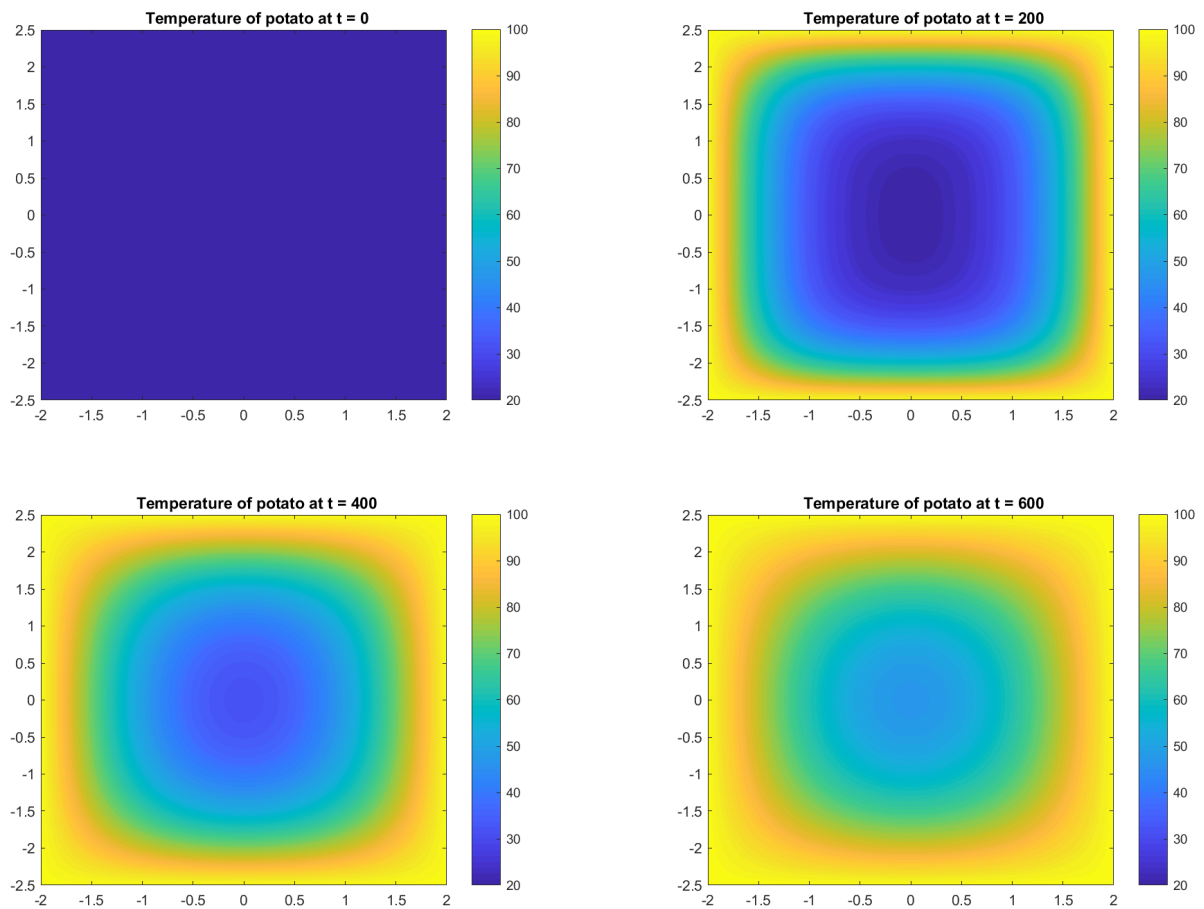
with the boundary conditions:

$$T(t, x, y) = T_{water}(t), \quad (x, y) \in \partial\Omega$$

and the initial conditions:

$$T(0, x, y) = T_{room}(t), \quad (x, y) \in \partial\Omega$$

where λ is the thermal diffusivity constant of the potato's material. Ω and $\partial\Omega$ denote both the domain occupied in space by the potato and the boundary of this domain.



Temperature ($^{\circ}\text{C}$) throughout the potato at various times

Part A

Part B

Part C

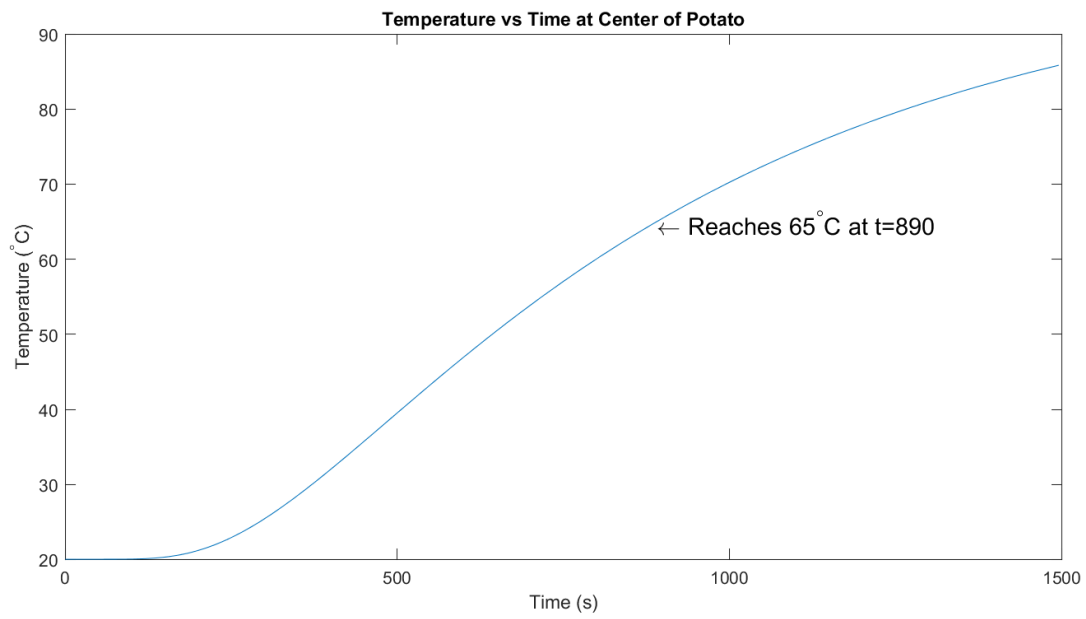
Problem 2

Introduction

Part A

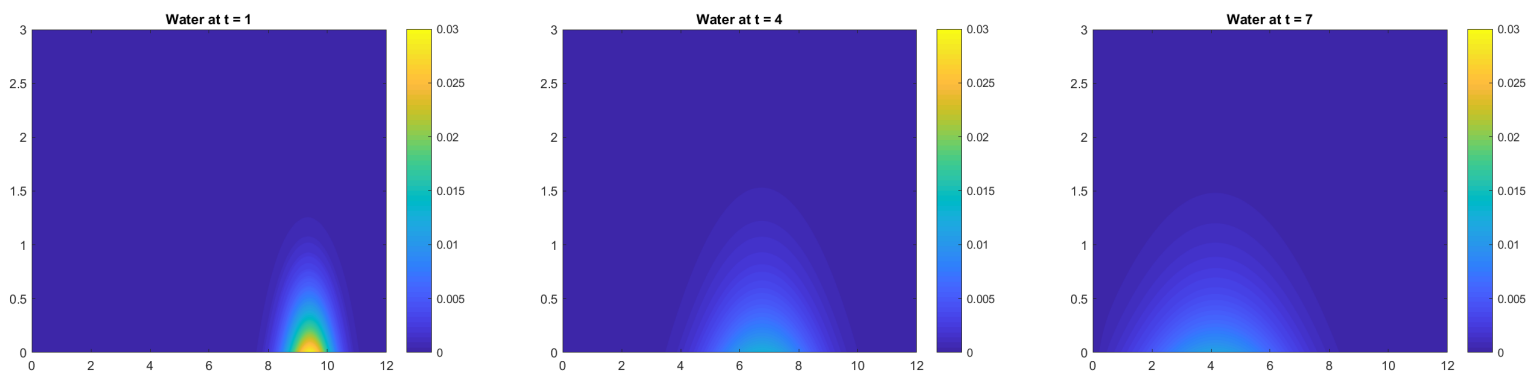
Part B

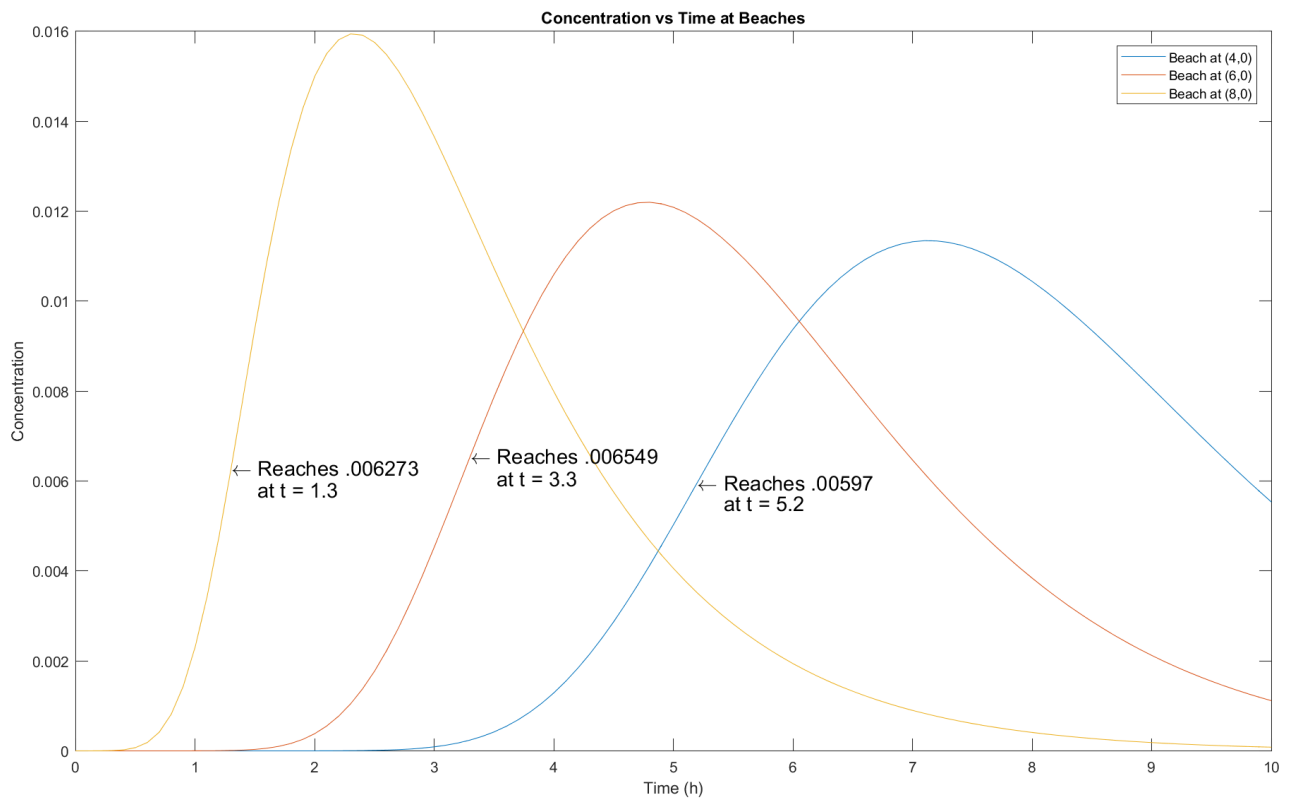
Part C



Temperature (°C) at the center of potato vs time

Resolution	Error	Order of Accuracy
20 x 15	0.0240409	0
40 x 30	0.0119852	1.00424
80 x 60	0.00598461	1.00193
160 x 120	0.00299154	1.00037





Concentration of oil at each beach vs time

References

- 1 Daniil Bochkov, CS 111 - Introduction to Computational Science - Midterm, Fall 2017
- 2 Daniil Bochkov, CS 111 - Introduction to Computational Science - Lecture 4 - High-Order Methods, Fall 2017
- 3 Daniil Bochkov, CS 111 - Introduction to Computational Science - Lecture 6 - Systems of ODEs, Fall 2017
- 4 Daniil Bochkov, CS 111 - Introduction to Computational Science - Lecture 7 - High-Order ODEs, Fall 2017