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CS483

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HW#6

1.

Source code:

```
import numpy as np
from hmmlearn import hmm

states = ["box 1", "box 2", "box3"]
n_states = len(states)    # =3

observations = ["red", "green", "yellow"]
n_observations = len(observations)

start_probability = np.array([0.4, 0.35, 0.25])

transition_probability = np.array([
  [0.3, 0.2, 0.5],
  [0.1, 0.3, 0.6],
  [0.7, 0.25, 0.05]
])

emission_probability = np.array([
  [0.8, 0.1, 0.1],
  [0.2, 0.4, 0.4],
  [0.15, 0.25, 0.6]
])

model = hmm.MultinomialHMM(n_components=n_states)
# MultinomialHMM: observation distribution in Multinomial
model.startprob_=start_probability
```

```
model.transmat_=transition_probability
model.emissionprob_=emission_probability

seen = np.array([[0,0,2,1,2]]).T
logprob, box = model.decode(seen, algorithm="viterbi")
seen = [0,0,2,1,2]
print("The ball picked:", ", ".join(map(lambda x: observations[x], seen)))
print("The hidden box:", ", ".join(map(lambda x: states[x], box)))

"""
    Find the probability of observation sequence
"""
seen = np.array([[0,0,2,1,2]]).T
prob = np.exp(model.score(seen))
print("Probability of observation sequence: ", prob)
```

Run program & result:

```
The ball picked: red, red, yellow, green, yellow

The hidden box: box 1, box 1, box3, box 2, box3

Probability of observation sequence:  0.0047848335687500024
```

Bottles in the series most likely come from Box 1, Box 1, Box 3, Box 2, and Box 3. And the probability of observation sequence is around 0.5%

2.

(a)

We have:

Transition matrix A		
	H	C
H	0.7	0.3
C	0.4	0.6

Emission matrix B			
	s	m	l
H	0.1	0.4	0.5
C	0.7	0.2	0.1

Initial Prob. π
P(H) = 0.6
P(C) = 0.4

We know that:

P(smsl HHCC)

= P(sms HHCC) * P(l | sms HHCC)

= P(sms HHC) * P(C | sms HHC) * P(l | C)

= P(sm HHC) * P(s | sm HHC) * P(C | C) * P(l | C)

= P(sm HH) * P(C | sm HH) * P(s | C) * P(C | C) * P(l | C)

= P(s HH) * P(m | s HH) * P(C | H) * P(s | C) * P(C | C) * P(l | C)

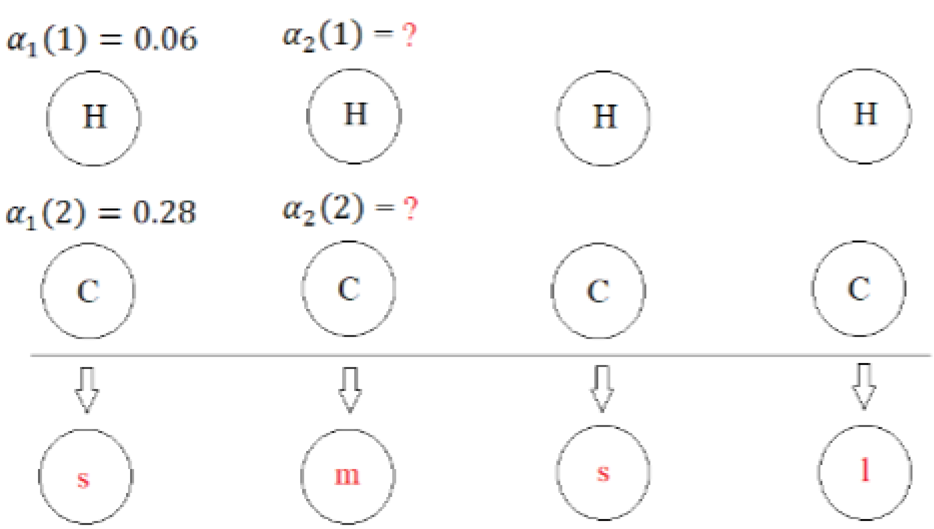
= P(s H) * P(H | s H) * P(m | H) * P(C | H) * P(s | C) * P(C | C) * P(l | C)

= P(H) * P(s | H) * P(H | H) * P(m | H) * P(C | H) * P(s | C) * P(C | C) * P(l | C)

= 0.6 * 0.1 * 0.7 * 0.4 * 0.3 * 0.7 * 0.6 * 0.1 = 2.1168 * 10^-4

(b)

We know that:



We have the recursion:

$$\alpha_2(1) = [\alpha_1(1) * P(H|H) + \alpha_1(2) * P(H|C)] * P(m|H)$$

$\alpha_2(1) = (0.06 * 0.7 + 0.28 * 0.4) * 0.4 = 0.3488$

We have the recursion:

$$\alpha_2(2) = [\alpha_1(1) * P(C|H) + \alpha_1(2) * P(C|C)] * P(m|C)$$

$\alpha_2(2) = (0.06 * 0.3 + 0.28 * 0.6) * 0.2 = 0.1056$