Mathematica HomeWork (共30 题，编号1~30)

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1. Basic Calculations

Read the *Mathematica* as a Calculator section of the Tour of *Mathematica* and then attempt the following exercises. If it is not already open, you may find the Basic Input palette helpful. For more information on palettes see Using Palettes.

1.1.1 Getting Started

You can use *Mathematica* as an enhanced scientific calculator. Let's start with a simple example.

45 + 78

123

The first line here is what you type in. The second line is the result.

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1. Use the Basic Input palette to type in the expression 3^{100} *below* this cell. Hit SHIFT-RET together to evaluate this expression. Note that you should get the *exact* answer. [1 Mark]

⚠ You can position the cursor *anywhere* on the line for expression evaluation to work. Alternatively you can select the cell-bracket (or a range of cell brackets) that contains an expression (s) and select Evaluate Cells under the Kernel/Evaluation menu.

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2. Select your input line above (either by selecting its

cell bracket or by scrolling over the expression with the mouse clicked) and make a copy using Copy under the Edit menu. Paste your expression below this box. Change the 100 to 1000 and evaluate the result.[1 Mark]

You should have just evaluated 3^{1000}.

An easy way to select an expression is to click on it as many times as is necessary to select the entire expression (this is a type of *scoping*).

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3. To get the result in the form you might get on a calculator, try $N[\%]$. [1 Mark]

% indicates the previous expression. Similarly **%%** indicates the second-last expression and **%** ***n*** refers to **Out[*n*]**. Alternatively, expressions can be named using assignment (denoted using =).

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4. Enter pi = $N[\pi, 200]$ below this cell and evaluate it. [1 Mark]

You should get the value of π to two hundred decimal places.

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5. Enter $e^{\frac{\sqrt{163}\,\text{pi}}{3}}$ below this cell and evaluate it. What is special about this number? [2 Marks]

Answer:

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6. π has a run of six consecutive 9's in the first 1000 digits. Can you see where they are located? (One way to find the location is to convert the number to a

string [using **ToString**], drop the leading **3.** [using **StringDrop**] and locating the position of the string **"999999"** [using **StringPosition**]). [3 Marks]

Answer:

1.1.2 More Functions

Mathematica knows about a big collection of mathematical functions — nearly all those you will find in any book of mathematical tables.

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7. Compute sin(13 π), log(2.1), and $e^{i\pi}$ by copying these expressions below this cell and entering them.[3 Marks]

⚠ Note that e denotes the exponential e and i denotes $\sqrt{-1}$. You can these objects using the Basic Input palette.

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8. Lookup cos in the on-line help and determine closed-form expressions for cos($\frac{\pi}{5}$). [2 Marks]

Answer:

Mathematica can do many kinds of exact computations with integers.

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9. Evaluate FactorInteger[70 612 139 395 722 186]. What is the meaning of this output? [2 Marks]

Answer: **FactorInteger** produces a set of pairs of numbers which are …

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10. Recombine the numbers in the above output to show how they form 70 612 139 395 722 186. [2 Marks]

1.1.3 Simple Expressions

Mathematica can deal with mathematical formulas in algebraic form.

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11. Enter the expression $(x^2 + 2\,x + 1)^{20}$. Expand this expression using **Expand[%]**. Factor the expanded result.[3 Marks]

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12. Re-do the steps in the above computation using the Basic Calculations palette. [3 Marks]

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13. Evaluate **TrigReduce[sin(*x*) cos(*y*) - cos(*x*) sin(*y*)]** and apply **TrigExpand** to the result. What can you say about the relationship between these functions? [2 Marks]

Answer:

1.1.4 Basic Plots

The above sections have guided you through each calculation. Now you are expected to use palettes such as the Basic Calculations palette or on-line help to work out how to achieve each of the following tasks.

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14. Produce a plot of $\sin(x^2)$ over $x \in [0, 5]$. [1 Mark]

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15. Produce a phase-space plot of $\cos(x)$ versus $\sin(3\,x)$ for over $x \in [0, 2\,\pi]$. *Hint*: lookup **ParametricPlot**. Do you recognise this figure? [2 Marks]

Answer:

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16. Produce a contour plot and density plot of $x\,e^{-x^2-y^2}$

for $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$. [2 Marks]

1.1.5 Calculus

You can do calculus. Try a simple integral:

17. Compute $\int \frac{x}{x^3-1}\, dx$. Check by differentiating the resulting expression with respect to x and then using **Simplify**. [3 Marks]

You can also get exact solutions to many definite integrals.

18. Compute $\int_0^\infty \frac{\sin^2(x)\cos^3(x)}{x}\, dx$. [1 Mark]

Many integrals do not have a simple closed form. If you try such a definite integral it will be returned unevaluated. You can still use N to get a numerical answer.

19. Try $\int_0^1 \sin(\sin(x))\, dx$. Compute the numerical value of this expression using $N[\%]$. [2 Marks]

⚠ Note that *Mathematica* can find a closed form for $\int_0^\pi \sin(\sin(x))\, dx$. Try it.

1.1.6 Solving Equations

20. Use **Solve** to obtain the roots of the quadratic equation $a\,x^2 + b\,x + c == 0$, and call the set of solutions s. Check the solutions by back-substitution using the syntax **$a\,x^2 + b\,x + c$ /. s**. You will need to **Simplify** the result. [3 Marks]

⚠ Note that == denotes *equality*, not assignment (which is =).

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21. Solve the cubic equation $x^3 - x + 1 == 0$. Evaluate this expression numerically using **N**. Does your answer satisfy the requirement that roots of equations with real coefficients must come in *conjugate pairs* (a conjugate pair is a set of two numbers of the form $a \pm b\,i$)? [3 Marks]

Answer:

1.1.7 Matrices

Entering matrices

Matrices can be entered using List brackets ({}). Alternatively, they can be entered using the Basic Input palette or from the Create Table/Matrix/Palette... entry under the Input menu.

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22. In a new cell, enter the 2×2 matrix, **mat** = $\begin{pmatrix} \mathbf{3.5} & \mathbf{7.2} \\ \mathbf{-2.4} & \mathbf{6.4} \end{pmatrix}$. Compute the inverse and eigenvalues of **mat** — you should be able to guess the names of the appropriate *Mathematica* commands. [2 Marks]

Linear algebra

Mathematica also does linear algebra on symbolic matrices.

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23. Enter **mat** = $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$. This overwrites your

previous value for the matrix **mat**. Compute the inverse of **mat** and assign (using =) the result to **minv**. Compute **minv.mat** and simplify the result. [3 Marks]

⚠ The **.** (*i.e.*, **Dot**) in ***a.b*** indicates that the usual dot product and is used both for vectors *and* matrices. This makes sense because each element in the resulting matrix is formed by dot product multiplication of rows of ***a*** with columns of ***b***. Without the **Dot** one gets the direct (*i.e.*, element-by-element) product. The direct product is useful for general matrix operations such as *masking* and *convolution*.

24. Generate a random real 3×3 matrix and call it **rand** (*Hint*: use **Random** and **Table**). Replace the $\{2, 2\}$ entry of **rand** with the symbolic parameter x (*Hint*: look-up **Part**). Compute the (symbolic) inverse of **rand** and determine the numerical value of the inverse when $x \to 1$. [4 Marks]

1.1.8 Curve Fitting

25. Use the command **Table[*i*!, {*i*, 1, 10}]** to compute the first 10 values of the factorial function and assign the answer to the variable called **is**. [2 Marks]

We could plot this table but the dynamic range is so large that it would be difficult to produce a meaningful plot. Let us take the logarithm of these values.

26. Take the log of values using **log(is)** and

numerically evaluate the result. Save this result as a named variable called **data**.[1 Mark]

Now that we have a table of values we can plot it using the **ListPlot** command.

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27. Use **ListPlot** to plot **data** and save your graphic as a named variable called, e.g., p_1. [1 Mark]

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28. Compute the best (least-squares) third-order polynomial fit to this data using **Fit**. Obtain a plot of this fit and call it p_2. You need to choose and appropriate range for the **Plot** command (see **PlotRange**). Show p_1 and p_2 superimposed using **Show**. [3 Marks]

⚠ You can, in fact, do fits using any linear combination of functions but **Fit** presently does not allow exponent optimization — you need to use the `Statistics`NonlinearFit`` package for this.

29. Plot animations of 2 *D* Curve and 3 *D* Surface.

30. Processing an image.