



中山大學
SUN YAT-SEN UNIVERSITY



国家超级计算广州中心
NATIONAL SUPERCOMPUTER CENTER IN GUANGZHOU

用MPI进行分布式内存编程

任课教师：吴迪

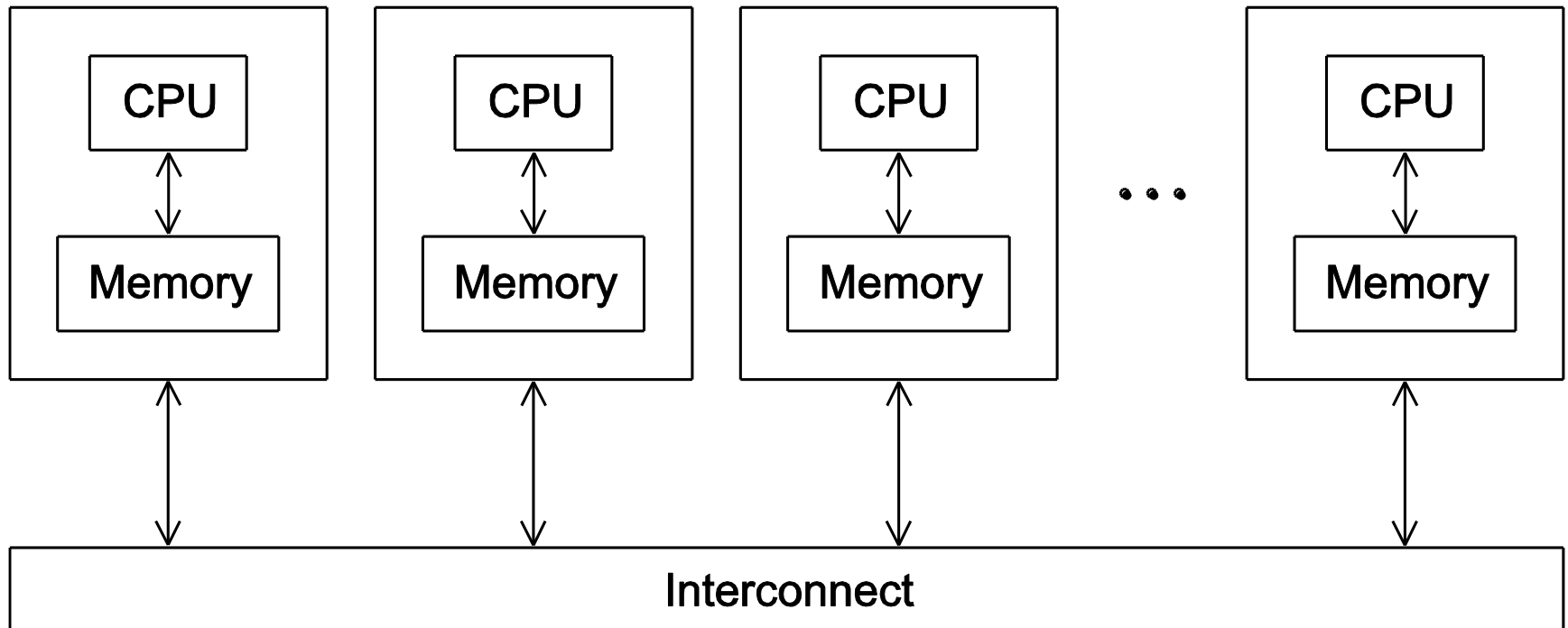
课程内容

- 参考资料：
 - 并行程序设计导论, Peter S Pacheco, 机械工业出版社, 2016
 - Chapter 3 Distributed-Memory Programming With MPI

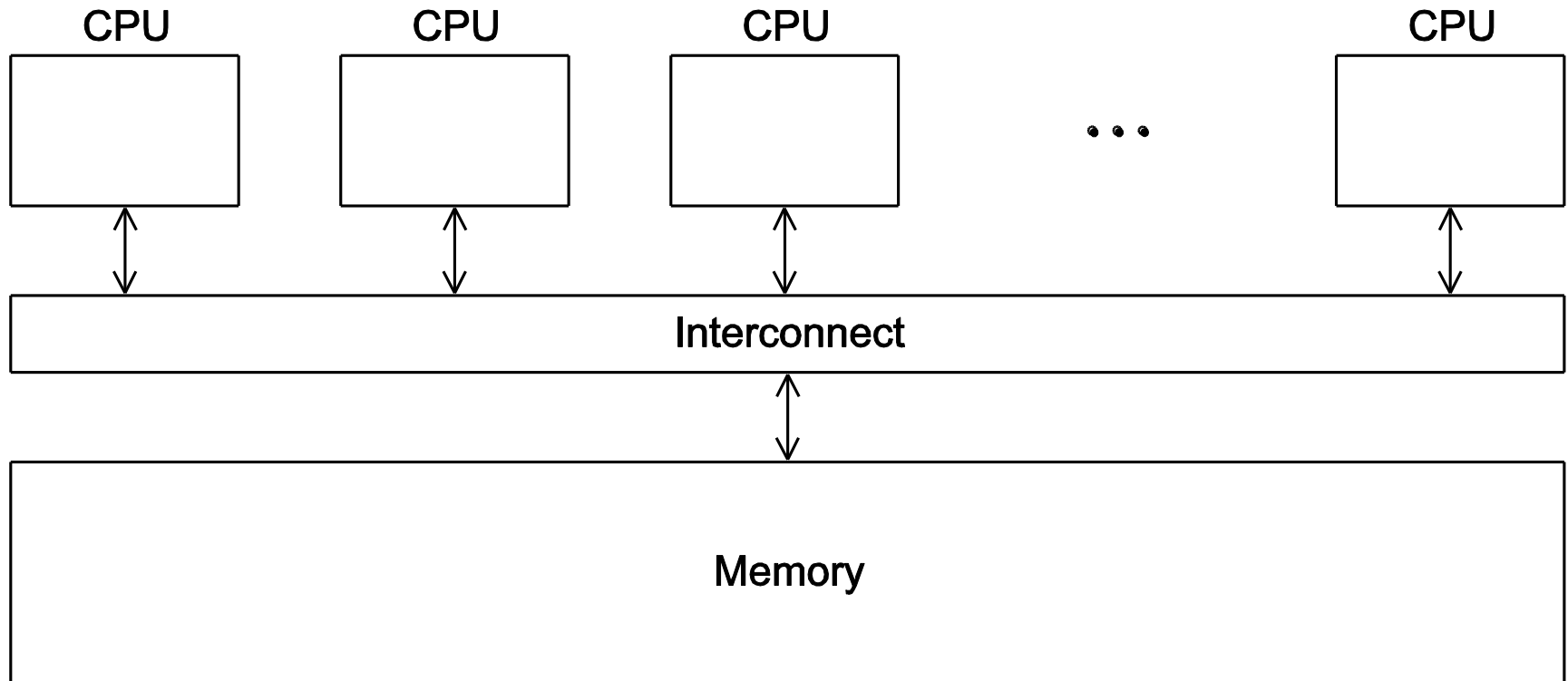
Roadmap

- Writing your first MPI program.
- Using the common MPI functions.
- The Trapezoidal Rule in MPI.
- Collective communication.
- MPI derived datatypes.
- Performance evaluation of MPI programs.
- Parallel sorting.
- Safety in MPI programs.

A distributed memory system



A shared memory system



Hello World!

```
#include <stdio.h>

int main(void) {
    printf("hello, world\n");

    return 0;
}
```

(a classic)



Identifying MPI processes

- Common practice to identify processes by nonnegative integer ranks.
- p processes are numbered $0, 1, 2, \dots, p-1$

Our first MPI program



```
1 #include <stdio.h>
2 #include <string.h> /* For strlen */
3 #include <mpi.h> /* For MPI functions , etc */
4
5 const int MAX_STRING = 100;
6
7 int main(void) {
8     char    greeting[MAX_STRING];
9     int      comm_sz; /* Number of processes */
10    int      my_rank; /* My process rank */
11
12    MPI_Init(NULL, NULL);
13    MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
14    MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
15
16    if (my_rank != 0) {
17        sprintf(greeting, "Greetings from process %d of %d!",
18                my_rank, comm_sz);
19        MPI_Send(greeting, strlen(greeting)+1, MPI_CHAR, 0, 0,
20                 MPI_COMM_WORLD);
21    } else {
22        printf("Greetings from process %d of %d!\n", my_rank, comm_sz);
23        for (int q = 1; q < comm_sz; q++) {
24            MPI_Recv(greeting, MAX_STRING, MPI_CHAR, q,
25                     0, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
26            printf("%s\n", greeting);
27        }
28    }
29
30    MPI_Finalize();
31    return 0;
32 } /* main */
```

Number of Processes

My Process Rank

Compilation

wrapper script to compile

source file

mpicc -g -Wall -o mpi_hello mpi_hello.c

produce debugging information

create this executable file name (as opposed to default a.out)

turns on all warnings

The diagram shows the command **mpicc -g -Wall -o mpi_hello mpi_hello.c** with blue arrows pointing from descriptive text to specific parts of the command. An arrow from 'wrapper script to compile' points to **mpicc**. An arrow from 'source file' points to **mpi_hello.c**. An arrow from 'produce debugging information' points to **-g**. An arrow from 'turns on all warnings' points to **-Wall**. An arrow from 'create this executable file name (as opposed to default a.out)' points to **-o mpi_hello**.

Execution

mpiexec -n <number of processes> <executable>

mpiexec -n 1 ./mpi_hello

 *run with 1 process*

mpiexec -n 4 ./mpi_hello

 *run with 4 processes*

Execution

```
mpiexec -n 1 ./mpi_hello
```

Greetings from process 0 of 1 !

```
mpiexec -n 4 ./mpi_hello
```

Greetings from process 0 of 4 !

Greetings from process 1 of 4 !

Greetings from process 2 of 4 !

Greetings from process 3 of 4 !

MPI Programs

- Written in C.
 - Has main.
 - Uses `stdio.h`, `string.h`, etc.
- Need to add `mpi.h` header file.
- Identifiers defined by MPI start with “`MPI_`”.
- First letter following `underscore` is `uppercase`.
 - For function names and MPI-defined types.
 - Helps to avoid confusion.

MPI Components

- MPI_Init
 - Tells MPI to do all the **necessary setup**.

```
int MPI_Init(  
    int*      argc_p  /* in/out */,  
    char***   argv_p  /* in/out */);
```

- MPI_Finalize
 - Tells MPI we're done, so **clean up anything** allocated for this program.

```
int MPI_Finalize(void);
```

Basic Outline

```
. . .
#include <mpi.h>
. . .
int main(int argc, char* argv[]) {
    . . .
    /* No MPI calls before this */
    MPI_Init(&argc, &argv);
    . . .
    MPI_Finalize();
    /* No MPI calls after this */
    . . .
    return 0;
}
```

Communicators

- A **collection of processes** that can send messages to each other.
- MPI_Init defines a communicator that consists of **all the processes** created when the program is started.
- Called **MPI_COMM_WORLD**.

Communicators



```
int MPI_Comm_size(  
    MPI_Comm comm      /* in */,  
    int* comm_sz_p     /* out */);
```

number of processes in the communicator

```
int MPI_Comm_rank(  
    MPI_Comm comm      /* in */,  
    int* my_rank_p     /* out */);
```

my rank (the process making this call)

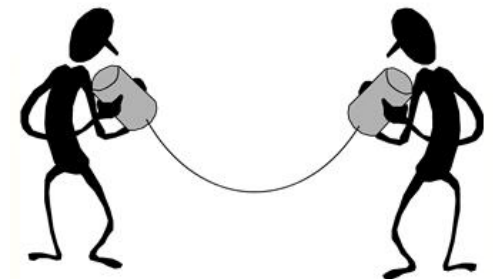
SPMD

- Single-Program Multiple-Data
- We compile one program.
- Process 0 does something different.
 - Receives messages and prints them while the other processes do the work.
- The if-else construct makes our program SPMD.

Communication

```
int MPI_Send(  
  
    void*      msg_buf_p      /* in */,  
    int        msg_size       /* in */,  
    MPI_Datatype msg_type      /* in */,  
    int        dest           /* in */,  
    int        tag            /* in */,  
    MPI_Comm   communicator    /* in */);
```

*Tag is used to distinguish
messages that are
otherwise identical*

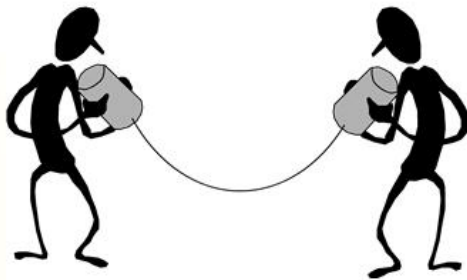


Data types

MPI datatype	C datatype
MPI_CHAR	signed char
MPI_SHORT	signed short int
MPI_INT	signed int
MPI_LONG	signed long int
MPI_LONG_LONG	signed long long int
MPI_UNSIGNED_CHAR	unsigned char
MPI_UNSIGNED_SHORT	unsigned short int
MPI_UNSIGNED	unsigned int
MPI_UNSIGNED_LONG	unsigned long int
MPI_FLOAT	float
MPI_DOUBLE	double
MPI_LONG_DOUBLE	long double
MPI_BYTE	
MPI_PACKED	

Communication

```
int MPI_Recv(  
    void*      msg_buf_p      /* out */,  
    int        buf_size       /* in  */,  
    MPI_Datatype buf_type      /* in  */,  
    int        source          /* in  */,  
    int        tag             /* in  */,  
  
    MPI_Comm    communicator   /* in  */,  
    MPI_Status* status_p       /* out */);
```



Message matching

```
MPI_Send(send_buf_p, send_buf_sz, send_type, dest, send_tag,  
send_comm);
```

MPI_Send

src = q



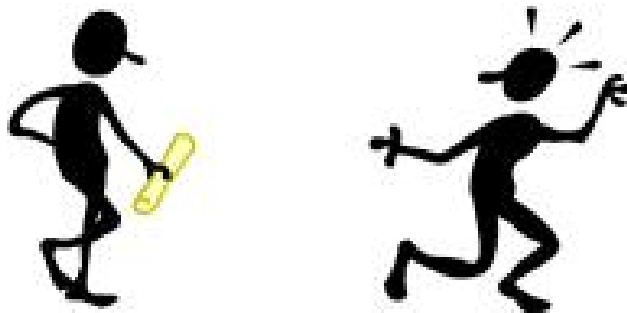
MPI_Recv

dest = r

```
MPI_Recv(recv_buf_p, recv_buf_sz, recv_type, src, recv_tag,  
recv_comm, &status);
```

Receiving messages

- A receiver can get a message **without knowing:**
 - the amount of data in the message,
 - the sender of the message,
 - or the tag of the message.



status_p argument

```
MPI_Recv(recv_buf_p, recv_buf_sz, recv_type, src, recv_tag,  
recv_comm, &status);
```

MPI_Status*



```
MPI_Status* status;
```

```
status.MPI_SOURCE
```

```
status.MPI_TAG
```

MPI_SOURCE

MPI_TAG

MPI_ERROR

How much data am I receiving?

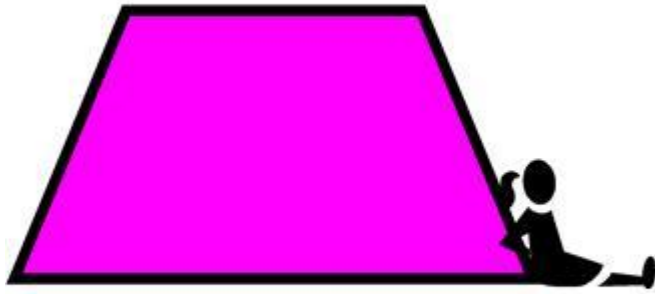
```
int MPI_Get_count(  
    MPI_Status* status_p /* in */,  
    MPI_Datatype type /* in */,  
    int* count_p /* out */);
```



Issues with send and receive

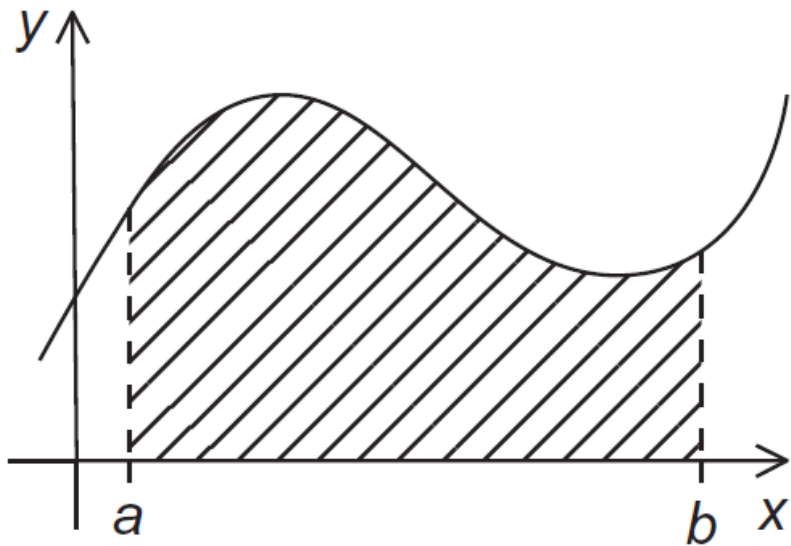
- Exact behavior is determined by the MPI implementation.
- **MPI_Send may behave differently** with regard to buffer size, cutoffs and blocking.
- **MPI_Recv always blocks** until a matching message is received.
- Know your implementation; don't make assumptions!



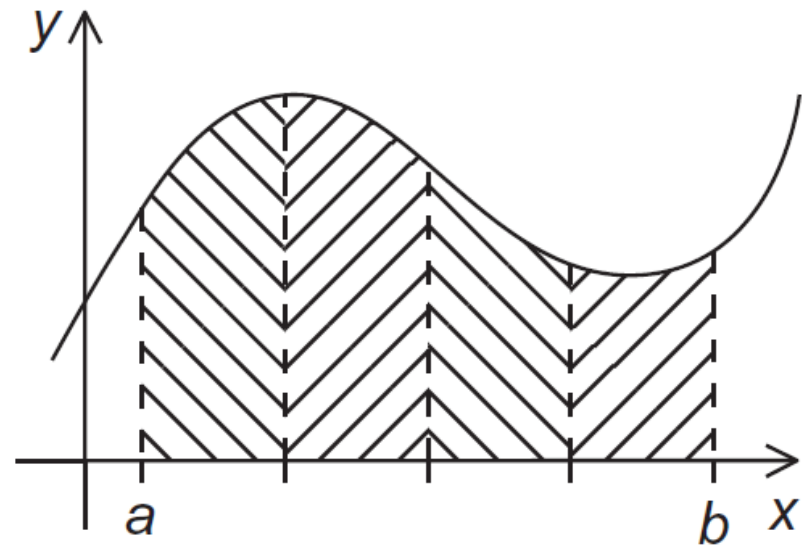


Trapezoidal (梯形) rule in
mpi

The Trapezoidal Rule

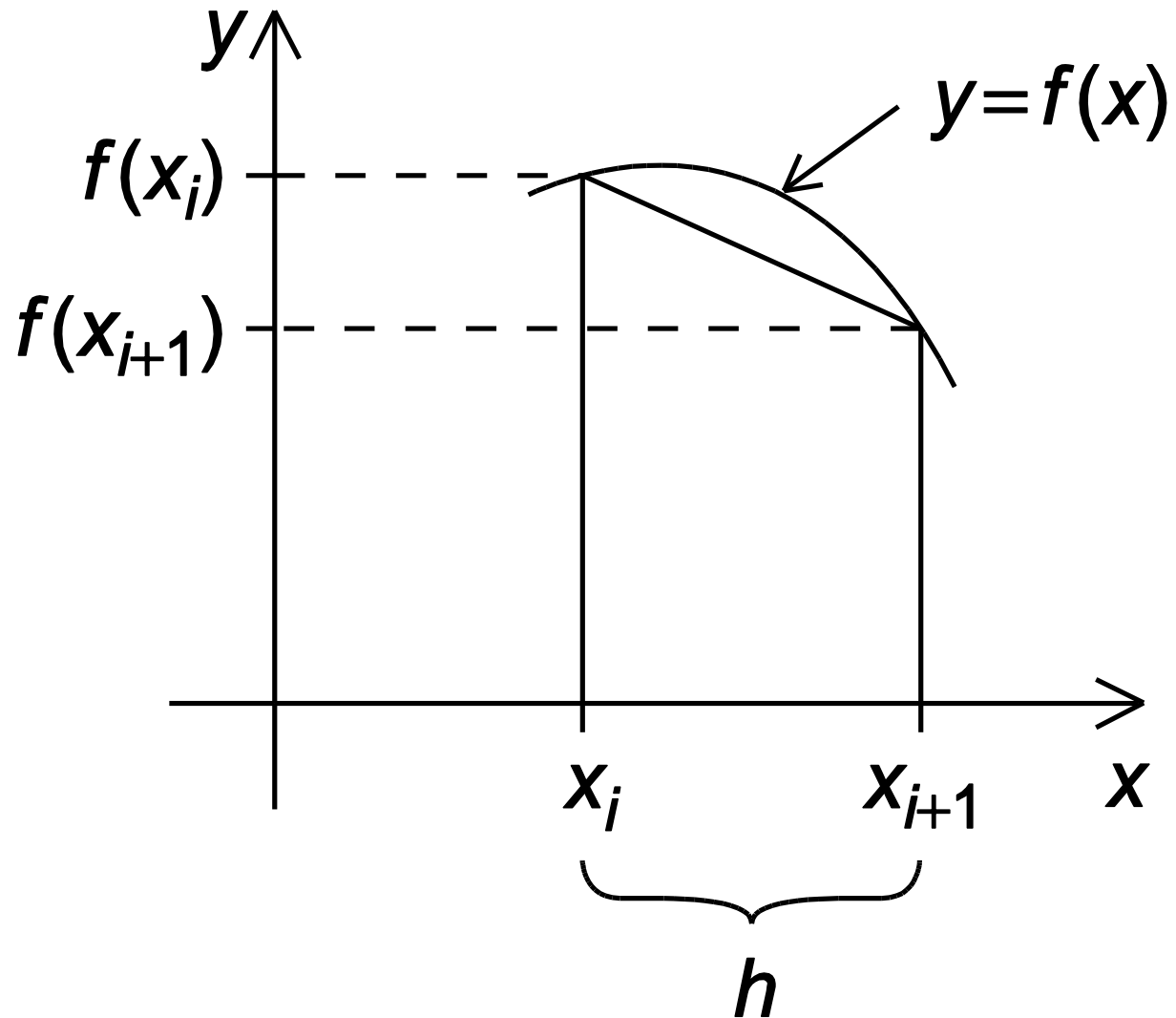


(a)



(b)

One trapezoid



The Trapezoidal Rule

$$\text{Area of one trapezoid} = \frac{h}{2}[f(x_i) + f(x_{i+1})]$$

$$h = \frac{b-a}{n}$$

$$x_0 = a, x_1 = a + h, x_2 = a + 2h, \dots, x_{n-1} = a + (n-1)h, x_n = b$$

$$\text{Sum of trapezoid areas} = h[f(x_0)/2 + f(x_1) + f(x_2) + \dots + f(x_{n-1}) + f(x_n)/2]$$

Pseudo-code for a serial program

```
/* Input:  a, b, n */  
h = (b-a)/n;  
approx = (f(a) + f(b))/2.0;  
for (i = 0, i <= n-1; i++) {  
    x_i = a + i*h;  
    approx += f(x_i);  
}  
approx = h*approx;
```

1

Any error?

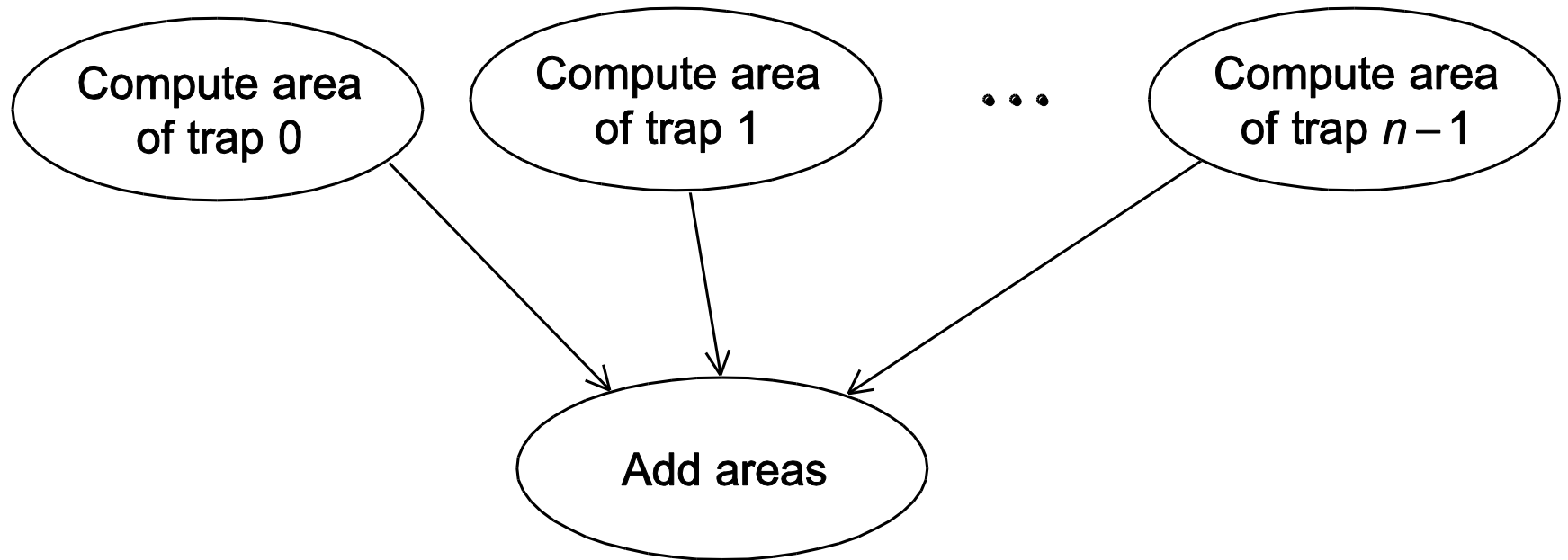
Parallelizing the Trapezoidal Rule

1. Partition problem solution into tasks.
2. Identify communication channels between tasks.
3. Aggregate tasks into composite tasks.
4. Map composite tasks to cores.

Parallel pseudo-code

```
1   Get a, b, n;
2   h = (b-a)/n;
3   local_n = n/comm_sz;
4   local_a = a + my_rank*local_n*h;
5   local_b = local_a + local_n*h;
6   local_integral = Trap(local_a, local_b, local_n, h);
7   if (my_rank != 0)
8       Send local_integral to process 0;
9   else /* my_rank == 0 */
10      total_integral = local_integral;
11      for (proc = 1; proc < comm_sz; proc++) {
12          Receive local_integral from proc;
13          total_integral += local_integral;
14      }
15  }
16  if (my_rank == 0)
17      print result;
```


Tasks and communications for Trapezoidal Rule



First version (1)

```
1  int main(void) {
2      int my_rank, comm_sz, n = 1024, local_n;
3      double a = 0.0, b = 3.0, h, local_a, local_b;
4      double local_int, total_int;
5      int source;
6
7      MPI_Init(NULL, NULL);
8      MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
9      MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
10
11     h = (b-a)/n;          /* h is the same for all processes */
12     local_n = n/comm_sz; /* So is the number of trapezoids */
13
14     local_a = a + my_rank*local_n*h;
15     local_b = local_a + local_n*h;
16     local_int = Trap(local_a, local_b, local_n, h);
17
18     if (my_rank != 0) {
19         MPI_Send(&local_int, 1, MPI_DOUBLE, 0, 0,
20                 MPI_COMM_WORLD);
```

First version (2)

```
21     } else {  
22         total_int = local_int;  
23         for (source = 1; source < comm_sz; source++) {  
24             MPI_Recv(&local_int, 1, MPI_DOUBLE, source, 0,  
25                     MPI_COMM_WORLD, MPI_STATUS_IGNORE);  
26             total_int += local_int;  
27         }  
28     }  
29  
30     if (my_rank == 0) {  
31         printf("With n = %d trapezoids, our estimate\n", n);  
32         printf("of the integral from %f to %f = %.15e\n",  
33             a, b, total_int);  
34     }  
35     MPI_Finalize();  
36     return 0;  
37 } /* main */
```

First version (3)

```
1  double Trap(  
2      double left_endpt  /* in */,  
3      double right_endpt /* in */,  
4      int    trap_count  /* in */,  
5      double base_len    /* in */) {  
6      double estimate, x;  
7      int i;  
8  
9      estimate = (f(left_endpt) + f(right_endpt))/2.0;  
10     for (i = 1; i <= trap_count-1; i++) {  
11         x = left_endpt + i*base_len;  
12         estimate += f(x);  
13     }  
14     estimate = estimate*base_len;  
15  
16     return estimate;  
17 } /* Trap */
```

Dealing with I/O

```
#include <stdio.h>
#include <mpi.h>

int main(void) {
    int my_rank, comm_sz;

    MPI_Init(NULL, NULL);
    MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
    MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);

    printf("Proc %d of %d > Does anyone have a toothpick?\n",
           my_rank, comm_sz);

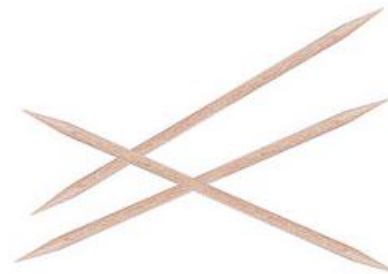
    MPI_Finalize();
    return 0;
} /* main */
```

*Each process just
prints a message.*

Running with 6 processes

```
Proc 0 of 6 > Does anyone have a toothpick?  
Proc 1 of 6 > Does anyone have a toothpick?  
Proc 2 of 6 > Does anyone have a toothpick?  
Proc 4 of 6 > Does anyone have a toothpick?  
Proc 3 of 6 > Does anyone have a toothpick?  
Proc 5 of 6 > Does anyone have a toothpick?
```

unpredictable output



Input

- Most MPI implementations only allow process 0 in MPI_COMM_WORLD access to [stdin](#).
- Process 0 must read the data (scanf) and send to the other processes.

```
. . .  
MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);  
MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);  
  
Get_data(my_rank, comm_sz, &a, &b, &n);  
  
h = (b-a)/n;  
. . .
```

Function for reading user input

```
void Get_input(  
    int      my_rank    /* in */,  
    int      comm_sz    /* in */,  
    double*  a_p        /* out */,  
    double*  b_p        /* out */,  
    int*     n_p        /* out */) {  
    int dest;  
  
    if (my_rank == 0) {  
        printf("Enter a, b, and n\n");  
        scanf("%lf %lf %d", a_p, b_p, n_p);  
        for (dest = 1; dest < comm_sz; dest++) {  
            MPI_Send(a_p, 1, MPI_DOUBLE, dest, 0, MPI_COMM_WORLD);  
            MPI_Send(b_p, 1, MPI_DOUBLE, dest, 0, MPI_COMM_WORLD);  
            MPI_Send(n_p, 1, MPI_INT, dest, 0, MPI_COMM_WORLD);  
        }  
    } else { /* my_rank != 0 */  
        MPI_Recv(a_p, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD,  
                 MPI_STATUS_IGNORE);  
        MPI_Recv(b_p, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD,  
                 MPI_STATUS_IGNORE);  
        MPI_Recv(n_p, 1, MPI_INT, 0, 0, MPI_COMM_WORLD,  
                 MPI_STATUS_IGNORE);  
    }  
} /* Get_input */
```

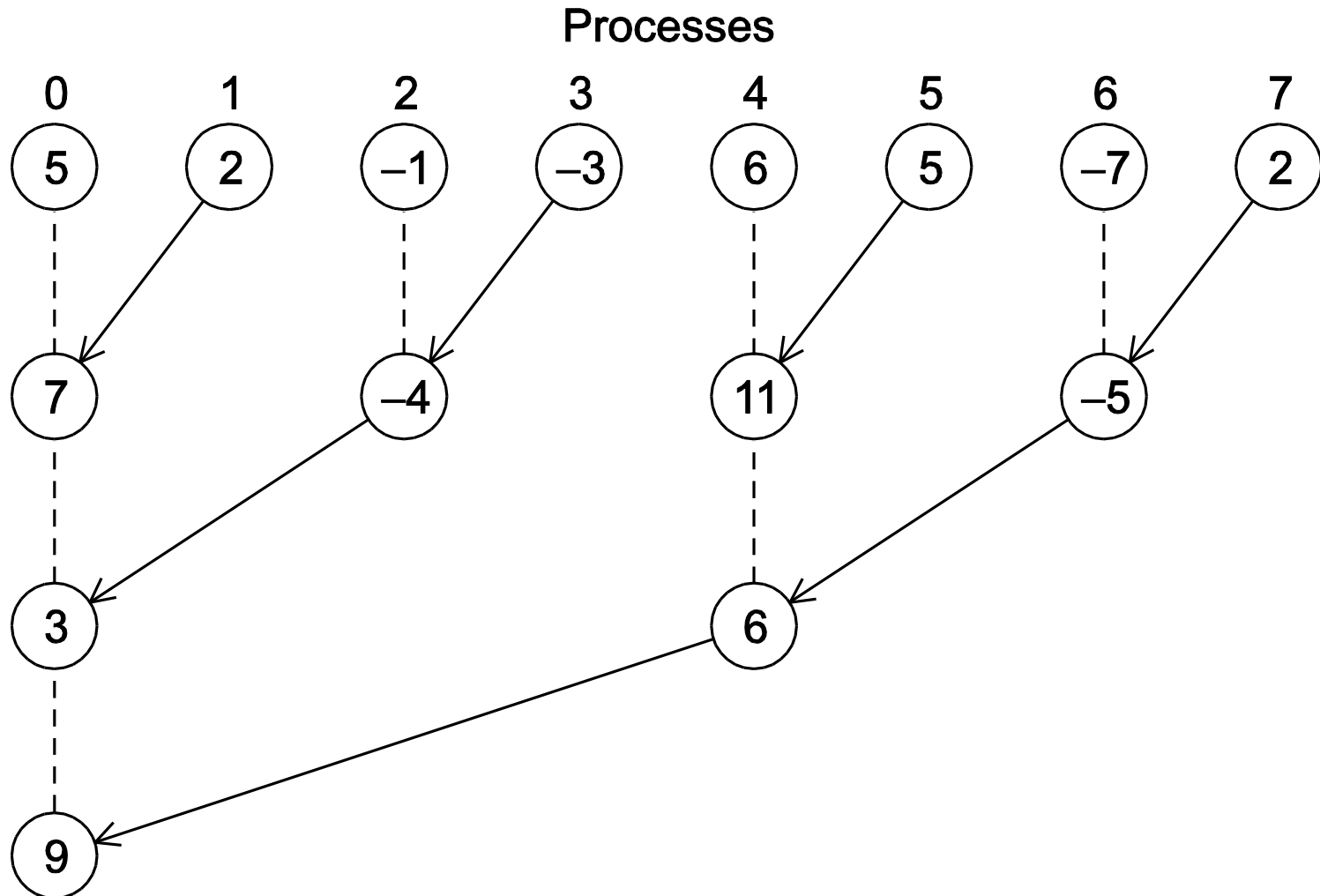



Collective communication

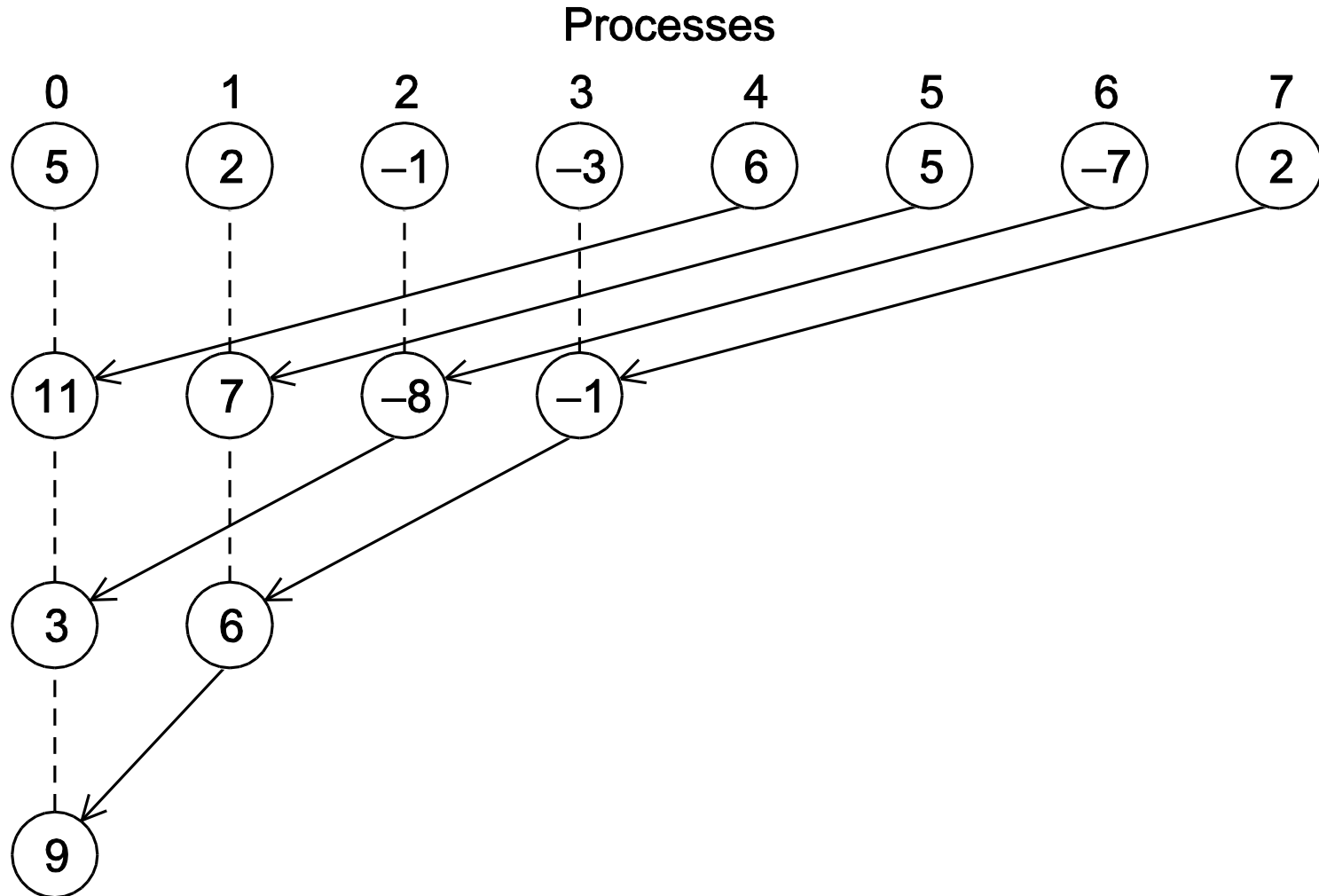
Tree-structured communication

1. In the first phase:
 - (a) Process 1 sends to 0, 3 sends to 2, 5 sends to 4, and 7 sends to 6.
 - (b) Processes 0, 2, 4, and 6 add in the received values.
 - (c) Processes 2 and 6 send their new values to processes 0 and 4, respectively.
 - (d) Processes 0 and 4 add the received values into their new values.
2.
 - (a) Process 4 sends its newest value to process 0.
 - (b) Process 0 adds the received value to its newest value.

A tree-structured global sum



An alternative tree-structured global sum



Collective Communications

- ***Collective Communication:*** communication functions that involve **all the processes** in a communicator, like MPI_Reduce
- ***Point-to-Point Communication:*** communication functions between point and point, like MPI_Send, MPI_Recv

MPI_Reduce

```
int MPI_Reduce(  
    void*          input_data_p    /* in */,  
    void*          output_data_p /* out */,  
    int           count            /* in */,  
    MPI_Datatype   datatype        /* in */,  
    MPI_Op         operator        /* in */,  
    int           dest_process     /* in */,  
    MPI_Comm       comm            /* in */);
```

```
MPI_Reduce(&local_int, &total_int, 1, MPI_DOUBLE, MPI_SUM, 0,  
MPI_COMM_WORLD);
```

```
double local_x[N], sum[N];  
...  
MPI_Reduce(local_x, sum, N, MPI_DOUBLE, MPI_SUM, 0,  
MPI_COMM_WORLD);
```

Predefined reduction operators in MPI

Operation Value	Meaning
<code>MPI_MAX</code>	Maximum
<code>MPI_MIN</code>	Minimum
<code>MPI_SUM</code>	Sum
<code>MPI_PROD</code>	Product
<code>MPI_LAND</code>	Logical and
<code>MPI_BAND</code>	Bitwise and
<code>MPI_LOR</code>	Logical or
<code>MPI_BOR</code>	Bitwise or
<code>MPI_LXOR</code>	Logical exclusive or
<code>MPI_BXOR</code>	Bitwise exclusive or
<code>MPI_MAXLOC</code>	Maximum and location of maximum
<code>MPI_MINLOC</code>	Minimum and location of minimum

Collective vs. Point-to-Point Communications

- All the processes in the communicator must call the **same collective function**.
- For example, a program that attempts to **match** a call to **MPI_Reduce** on one process with a call to **MPI_Recv** on another process is **erroneous**, and, in all likelihood, the program will hang or crash.

Collective vs. Point-to-Point Communications

- The arguments passed by each process to an MPI collective communication must be “compatible.”
- For example, if one process passes in 0 as the `dest_process` and another passes in 1, then the outcome of a call to `MPI_Reduce` is *erroneous*, and, once again, the program is likely to hang or crash.

Collective vs. Point-to-Point Communications

- The `output_data_p` argument is only used on `dest_process`.
- However, all of the processes still need to pass in an actual argument corresponding to `output_data_p`, even if it's just `NULL`.

Collective vs. Point-to-Point Communications

- **Point-to-point** communications are matched on the basis of **tags and communicators**.
- **Collective** communications **don't use tags**.
- They're matched solely on the basis of the **communicator** and **the order** in which they're called.

Example (1)

Time	Process 0	Process 1	Process 2
0	<code>a = 1; c = 2</code>	<code>a = 1; c = 2</code>	<code>a = 1; c = 2</code>
1	<code>MPI_Reduce (&a, &b, ...)</code>	<code>MPI_Reduce (&c, &d, ...)</code>	<code>MPI_Reduce (&a, &b, ...)</code>
2	<code>MPI_Reduce (&c, &d, ...)</code>	<code>MPI_Reduce (&a, &b, ...)</code>	<code>MPI_Reduce (&c, &d, ...)</code>

Multiple calls to MPI_Reduce

Example (2)

- Suppose that each process calls `MPI_Reduce` with operator `MPI_SUM`, and destination process 0.
- At first glance, it might seem that after the two calls to `MPI_Reduce`, the value of `b` will be 3, and the value of `d` will be 6.

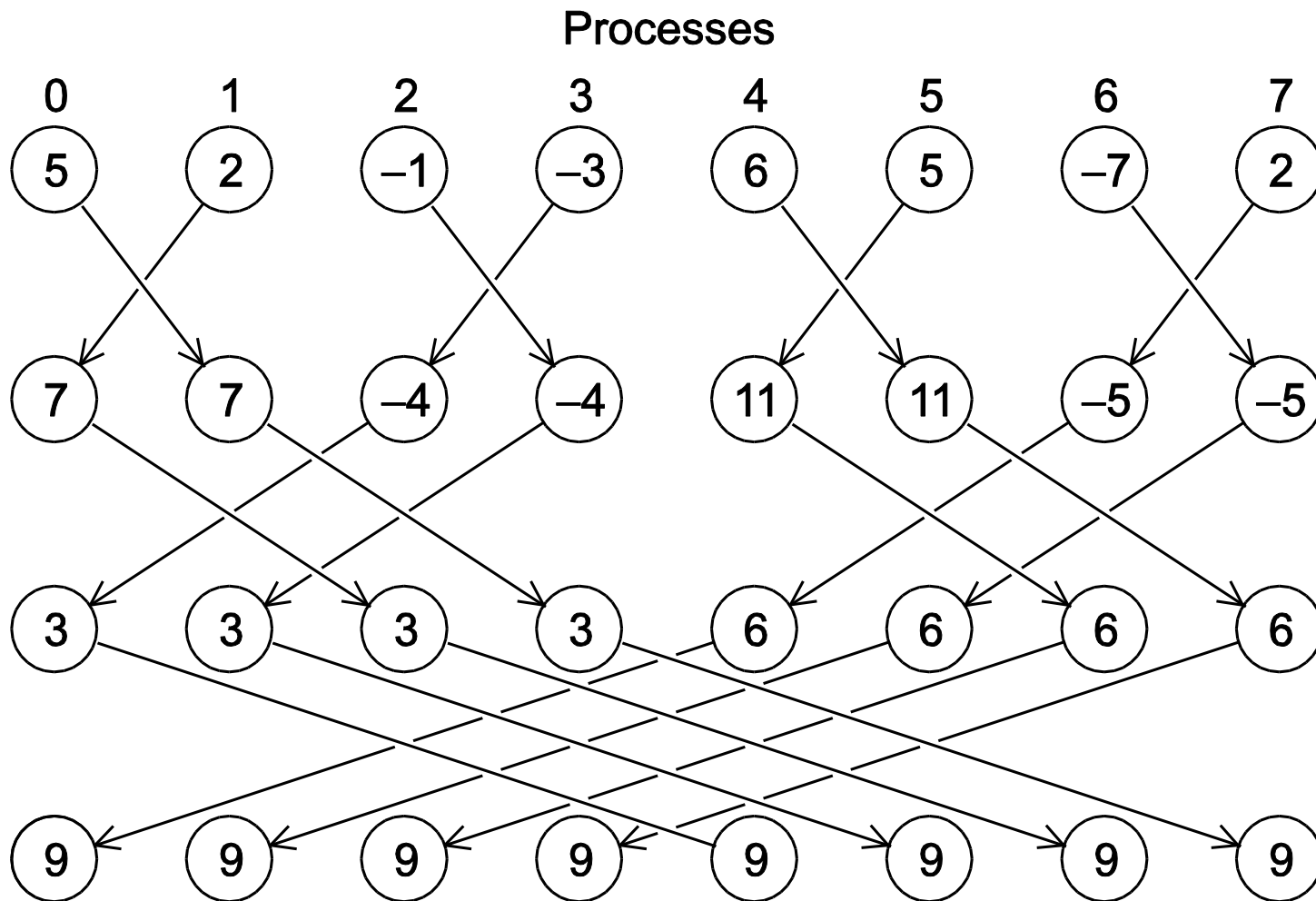
Example (3)

- However, the names of the memory locations are irrelevant to the matching of the calls to `MPI_Reduce`.
- The order of the calls will determine the matching so the value stored in b will be $1+2+1 = 4$, and the value stored in d will be $2+1+2 = 5$.

MPI_Allreduce

- Useful in a situation in which **all of the processes need the result** of a **global sum** in order to complete some larger computation.

```
int MPI_Allreduce(  
    void*      input_data_p    /* in */,  
    void*      output_data_p   /* out */,  
    int        count           /* in */,  
    MPI_Datatype datatype       /* in */,  
    MPI_Op      operator        /* in */,  
    MPI_Comm    comm           /* in */);
```

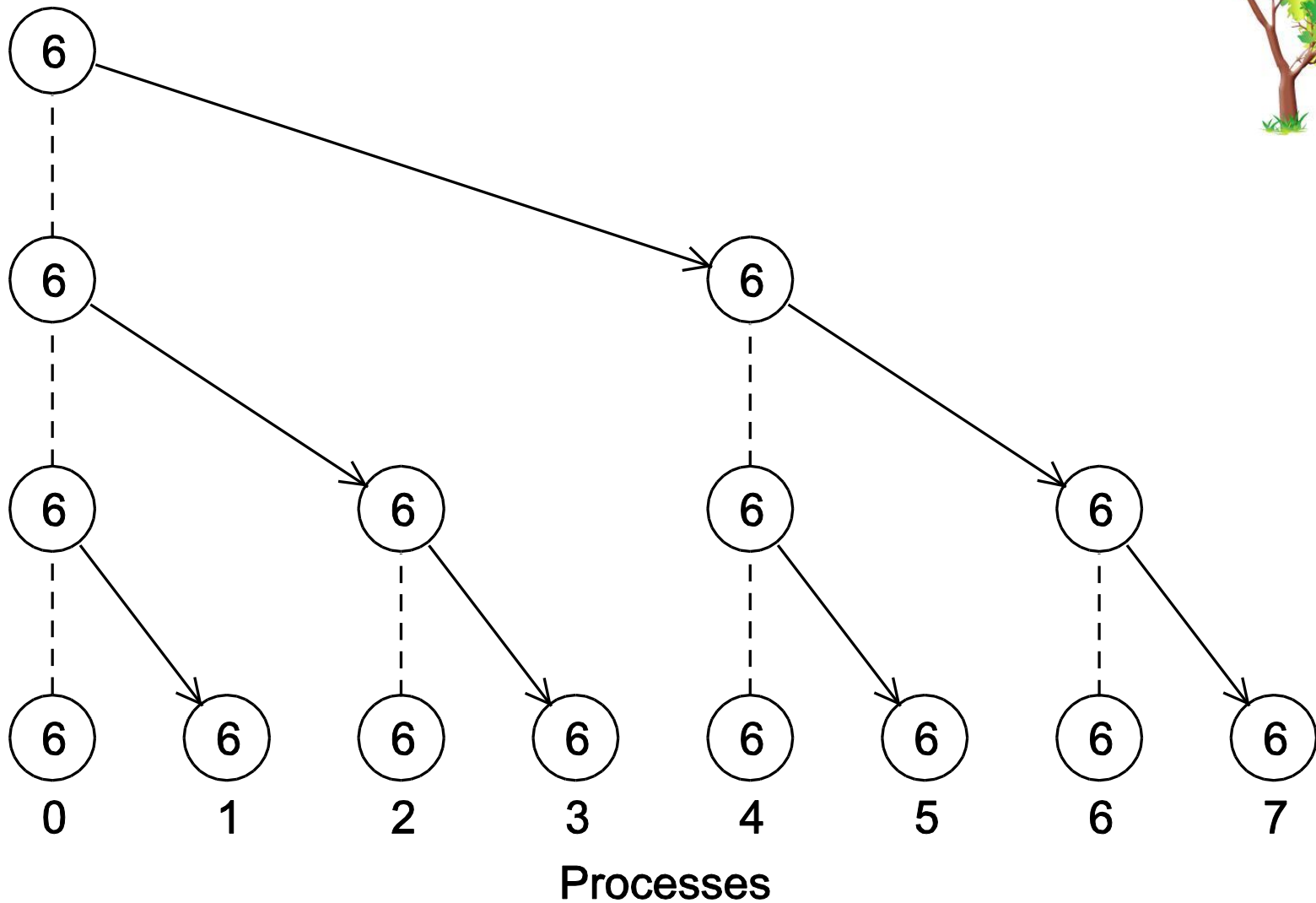
A butterfly-structured global sum.

Broadcast

- Data belonging to a single process is **sent to all of the processes** in the communicator.

```
int MPI_Bcast(  
    void*          data_p          /* in/out */ ,  
    int           count           /* in    */ ,  
    MPI_Datatype  datatype        /* in    */ ,  
    int           source_proc     /* in    */ ,  
    MPI_Comm      comm            /* in    */ );
```

A tree-structured broadcast.



A version of Get_input that uses MPI_Bcast

```
void Get_input(  
    int      my_rank    /* in */,  
    int      comm_sz    /* in */,  
    double*  a_p        /* out */,  
    double*  b_p        /* out */,  
    int*     n_p        /* out */) {  
  
    if (my_rank == 0) {  
        printf("Enter a, b, and n\n");  
        scanf("%lf %lf %d", a_p, b_p, n_p);  
    }  
  
    MPI_Bcast(a_p, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);  
    MPI_Bcast(b_p, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);  
    MPI_Bcast(n_p, 1, MPI_INT, 0, MPI_COMM_WORLD);  
} /* Get_input */
```

Data distributions

$$\begin{aligned}\mathbf{x} + \mathbf{y} &= (x_0, x_1, \dots, x_{n-1}) + (y_0, y_1, \dots, y_{n-1}) \\ &= (x_0 + y_0, x_1 + y_1, \dots, x_{n-1} + y_{n-1}) \\ &= (z_0, z_1, \dots, z_{n-1}) \\ &= \mathbf{z}\end{aligned}$$

Compute a vector sum.

Serial implementation of vector addition

```
void Vector_sum(double x[], double y[], double z[], int n) {  
    int i;  
  
    for (i = 0; i < n; i++)  
        z[i] = x[i] + y[i];  
} /* Vector_sum */
```

Different partitions of a 12-component vector among 3 processes

Process	Components											
	Block				Cyclic				Block-cyclic Blocksize = 2			
0	0	1	2	3	0	3	6	9	0	1	6	7
1	4	5	6	7	1	4	7	10	2	3	8	9
2	8	9	10	11	2	5	8	11	4	5	10	11

Partitioning options

- Block partitioning
 - Assign blocks of consecutive components to each process.
- Cyclic partitioning
 - Assign components in a round robin fashion.
- Block-cyclic partitioning
 - Use a cyclic distribution of blocks of components.

Parallel implementation of vector addition

```
void Parallel_vector_sum(  
    double local_x[] /* in */,  
    double local_y[] /* in */,  
    double local_z[] /* out */,  
    int local_n /* in */) {  
    int local_i;  
  
    for (local_i = 0; local_i < local_n; local_i++)  
        local_z[local_i] = local_x[local_i] + local_y[local_i];  
} /* Parallel_vector_sum */
```

Scatter

- MPI_Scatter can be used in a function that **reads in an entire vector on process 0** but only **sends the needed components to each** of the other processes.

```
int MPI_Scatter(  
    void*          send_buf_p    /* in */ ,  
    int           send_count    /* in */ ,  
    MPI_Datatype   send_type     /* in */ ,  
    void*          recv_buf_p   /* out */ ,  
    int           recv_count    /* in */ ,  
    MPI_Datatype   recv_type     /* in */ ,  
    int           src_proc       /* in */ ,  
    MPI_Comm       comm         /* in */ );
```

Reading and distributing a vector

```
void Read_vector(  
    double    local_a[]    /* out */,  
    int        local_n     /* in  */,  
    int        n           /* in  */,  
    char       vec_name[]  /* in  */,  
    int        my_rank     /* in  */,  
    MPI_Comm   comm        /* in  */) {  
  
    double* a = NULL;  
    int i;  
  
    if (my_rank == 0) {  
        a = malloc(n*sizeof(double));  
        printf("Enter the vector %s\n", vec_name);  
        for (i = 0; i < n; i++)  
            scanf("%lf", &a[i]);  
        MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n, MPI_DOUBLE,  
                    0, comm);  
        free(a);  
    } else {  
        MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n, MPI_DOUBLE,  
                    0, comm);  
    }  
} /* Read_vector */
```

Gather

- **Collect** all of the components of the vector onto **process 0**, and then process 0 can process all of the components.

```
int MPI_Gather(  
    void*          send_buf_p    /* in */ ,  
    int           send_count    /* in */ ,  
    MPI_Datatype   send_type    /* in */ ,  
    void*         recv_buf_p    /* out */ ,  
    int           recv_count    /* in */ ,  
    MPI_Datatype   recv_type    /* in */ ,  
    int           dest_proc     /* in */ ,  
    MPI_Comm       comm        /* in */ );
```

Print a distributed vector (1)

```
void Print_vector(  
    double    local_b[]    /* in */,  
    int      local_n      /* in */,  
    int      n             /* in */,  
    char     title[]      /* in */,  
    int      my_rank       /* in */,  
    MPI_Comm comm         /* in */) {  
  
    double* b = NULL;  
    int i;
```

Print a distributed vector (2)

```
if (my_rank == 0) {  
    b = malloc(n*sizeof(double));  
    MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE,  
               0, comm);  
    printf("%s\n", title);  
    for (i = 0; i < n; i++)  
        printf("%f ", b[i]);  
    printf("\n");  
    free(b);  
} else {  
    MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE,  
               0, comm);  
}  
} /* Print_vector */
```

Allgather

- Concatenates the contents of each process' `send_buf_p` and stores this in **each process'** `recv_buf_p`.
- As usual, `recv_count` is the amount of data being received from each process.

```
int MPI_Allgather(  
    void*      send_buf_p  /* in */,  
    int        send_count  /* in */,  
    MPI_Datatype send_type  /* in */,  
    void*      recv_buf_p /* out */,  
    int        recv_count  /* in */,  
    MPI_Datatype recv_type  /* in */,  
    MPI_Comm   comm        /* in */);
```

No param of destination process in MPI_Allgather

Matrix-vector multiplication

$A = (a_{ij})$ is an $m \times n$ matrix

\mathbf{x} is a vector with n components

$\mathbf{y} = A\mathbf{x}$ is a vector with m components

$$y_i = a_{i0}x_0 + a_{i1}x_1 + a_{i2}x_2 + \cdots a_{i,n-1}x_{n-1}$$

i -th component of y

*Dot product of the i th
row of A with x .*

Matrix-vector multiplication

a_{00}	a_{01}	\cdots	$a_{0,n-1}$
a_{10}	a_{11}	\cdots	$a_{1,n-1}$
\vdots	\vdots		\vdots
a_{i0}	a_{i1}	\cdots	$a_{i,n-1}$
\vdots	\vdots		\vdots
$a_{m-1,0}$	$a_{m-1,1}$	\cdots	$a_{m-1,n-1}$

x_0
x_1
\vdots
x_{n-1}

 $=$

y_0
y_1
\vdots
$y_i = a_{i0}x_0 + a_{i1}x_1 + \cdots a_{i,n-1}x_{n-1}$
\vdots
y_{m-1}

Multiply a matrix by a vector

```
/* For each row of A */  
for (i = 0; i < m; i++) {  
    /* Form dot product of ith row with x */  
    y[i] = 0.0;  
    for (j = 0; j < n; j++)  
        y[i] += A[i][j]*x[j];  
}
```

Serial pseudo-code

C style arrays

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \end{pmatrix}$$

stored as

0 1 2 3 4 5 6 7 8 9 10 11

Serial matrix-vector multiplication

```
void Mat_vect_mult(  
    double A[] /* in */,  
    double x[] /* in */,  
    double y[] /* out */,  
    int m /* in */,  
    int n /* in */) {  
    int i, j;  
  
    for (i = 0; i < m; i++) {  
        y[i] = 0.0;  
        for (j = 0; j < n; j++)  
            y[i] += A[i*n+j]*x[j];  
    }  
} /* Mat_vect_mult */
```

An MPI matrix-vector multiplication function (1)

```
void Mat_vect_mult(  
    double    local_A[]  /* in  */,  
    double    local_x[]  /* in  */,  
    double    local_y[]  /* out */,  
    int       local_m    /* in  */,  
    int       n          /* in  */,  
    int       local_n    /* in  */,  
    MPI_Comm  comm       /* in  */) {  
    double* x;  
    int local_i, j;  
    int local_ok = 1;
```

An MPI matrix-vector multiplication function (2)

```
x = malloc(n*sizeof(double));  
MPI_Allgather(local_x, local_n, MPI_DOUBLE,  
              x, local_n, MPI_DOUBLE, comm);  
  
for (local_i = 0; local_i < local_m; local_i++) {  
    local_y[local_i] = 0.0;  
    for (j = 0; j < n; j++)  
        local_y[local_i] += local_A[local_i*n+j]*x[j];  
}  
free(x);  
} /* Mat_vect_mult */
```



Mpi derived datatypes

Derived datatypes

- Used to represent **any collection of data items** in memory by storing both the **types** of the items and their **relative locations** in memory.
- The idea is that **if a function that sends data knows this information about a collection of data items, it can collect the items from memory before they are sent.**
- Similarly, a function that receives data can distribute the items into their correct destinations in memory when they're received.

Derived datatypes

- Formally, consists of a **sequence** of basic **MPI data types** together with a **displacement** for each of the data types.
- Trapezoidal Rule example:

Variable	Address
a	24
b	40
n	48

$\{(\text{MPI_DOUBLE}, 0), (\text{MPI_DOUBLE}, 16), (\text{MPI_INT}, 24)\}$

MPI_Type_create_struct

- Builds a derived datatype that consists of individual elements that have different basic types.

```
int MPI_Type_create_struct(  
    int          count          /* in */,  
    int          array_of_blocklengths[] /* in */,  
    MPI_Aint      array_of_displacements[] /* in */,  
    MPI_Datatype  array_of_types[] /* in */,  
    MPI_Datatype* new_type_p    /* out */);
```

MPI_Get_address

- Returns the address of the memory location referenced by `location_p`.
- The special type `MPI_Aint` is an integer type that is big enough to store an `address` on the system.

```
int MPI_Get_address(  
    void*      location_p  /* in */,  
    MPI_Aint*  address_p   /* out */);
```

MPI_Type_commit

- Allows the MPI implementation to optimize its internal representation of the datatype for use in communication functions.

```
int MPI_Type_commit(MPI_Datatype* new_mpi_t_p /* in/out */);
```

MPI_Type_free

- When we're finished with our new type, this **free**s any additional storage used.

```
int MPI_Type_free(MPI_Datatype* old_mpi_t_p /* in/out */);
```

Get input function with a derived datatype (1)

```
void Build_mpi_type(  
    double*      a_p      /* in */,  
    double*      b_p      /* in */,  
    int*         n_p      /* in */,  
    MPI_Datatype* input_mpi_t_p /* out */) {  
  
    int array_of_blocklengths[3] = {1, 1, 1};  
    MPI_Datatype array_of_types[3] = {MPI_DOUBLE, MPI_DOUBLE, MPI_INT};  
    MPI_Aint a_addr, b_addr, n_addr;  
    MPI_Aint array_of_displacements[3] = {0};
```

Get input function with a derived datatype (2)

```
MPI_Get_address(a_p, &a_addr);
MPI_Get_address(b_p, &b_addr);
MPI_Get_address(n_p, &n_addr);
array_of_displacements[1] = b_addr-a_addr;
array_of_displacements[2] = n_addr-a_addr;
MPI_Type_create_struct(3, array_of_blocklengths,
                      array_of_displacements, array_of_types,
                      input_mpi_t_p);
MPI_Type_commit(input_mpi_t_p);
} /* Build_mpi_type */
```

Get input function with a derived datatype (3)

```
void Get_input(int my_rank, int comm_sz, double* a_p, double* b_p,
               int* n_p) {
    MPI_Datatype input_mpi_t;

    Build_mpi_type(a_p, b_p, n_p, &input_mpi_t);

    if (my_rank == 0) {
        printf("Enter a, b, and n\n");
        scanf("%lf %lf %d", a_p, b_p, n_p);
    }
    MPI_Bcast(a_p, 1, input_mpi_t, 0, MPI_COMM_WORLD);

    MPI_Type_free(&input_mpi_t);
} /* Get_input */
```

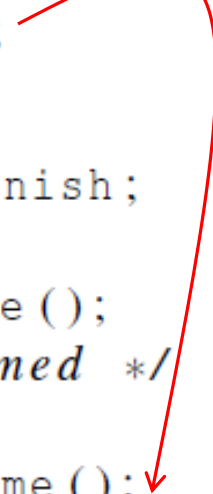



Performance evaluation

Elapsed parallel time

- Returns the number of seconds that have elapsed since some time in the past.

```
double MPI_Wtime(void);  
  
    double start, finish;  
    . . .  
    start = MPI_Wtime();  
    /* Code to be timed */  
    . . .  
    finish = MPI_Wtime();  
    printf("Proc %d > Elapsed time = %e seconds\n"  
           my_rank, finish-start);
```



Elapsed serial time

- In this case, you don't need to link in the MPI libraries.
- Returns time in microseconds elapsed from some point in the past.

```
#include "timer.h"  
.  
.  
.  
double now;  
.  
.  
.  
GET_TIME(now);
```



Elapsed serial time

```
#include "timer.h"
. . .
double start, finish;
. . .
GET_TIME(start);
/* Code to be timed */
. . .
GET_TIME(finish);
printf("Elapsed time = %e seconds\n", finish-start);
```

MPI_Barrier

- Ensures that no process will return from calling it until every process in the communicator has started calling it.

```
int MPI_Barrier(MPI_Comm comm /* in */);
```



MPI_Barrier

```
double local_start, local_finish, local_elapsed, elapsed;
. . .
MPI_Barrier(comm);
local_start = MPI_Wtime();
/* Code to be timed */
. . .

local_finish = MPI_Wtime();
local_elapsed = local_finish - local_start;
MPI_Reduce(&local_elapsed, &elapsed, 1, MPI_DOUBLE,
           MPI_MAX, 0, comm);

if (my_rank == 0)
    printf("Elapsed time = %e seconds\n", elapsed);
```

Run-times of serial and parallel matrix-vector multiplication

comm_sz	Order of Matrix				
	1024	2048	4096	8192	16,384
1	4.1	16.0	64.0	270	1100
2	2.3	8.5	33.0	140	560
4	2.0	5.1	18.0	70	280
8	1.7	3.3	9.8	36	140
16	1.7	2.6	5.9	19	71

(Seconds)

Speedup

$$S(n, p) = \frac{T_{\text{serial}}(n)}{T_{\text{parallel}}(n, p)}$$

Efficiency

$$E(n, p) = \frac{S(n, p)}{p} = \frac{T_{\text{serial}}(n)}{p \times T_{\text{parallel}}(n, p)}$$

Speedups of Parallel Matrix-Vector Multiplication

comm_sz	Order of Matrix				
	1024	2048	4096	8192	16,384
1	1.0	1.0	1.0	1.0	1.0
2	1.8	1.9	1.9	1.9	2.0
4	2.1	3.1	3.6	3.9	3.9
8	2.4	4.8	6.5	7.5	7.9
16	2.4	6.2	10.8	14.2	15.5

Efficiencies of Parallel Matrix-Vector Multiplication

comm_sz	Order of Matrix				
	1024	2048	4096	8192	16,384
1	1.00	1.00	1.00	1.00	1.00
2	0.89	0.94	0.97	0.96	0.98
4	0.51	0.78	0.89	0.96	0.98
8	0.30	0.61	0.82	0.94	0.98
16	0.15	0.39	0.68	0.89	0.97

Scalability

- A program is **scalable** if the problem size can be increased at a rate so that the efficiency doesn't decrease as the number of processes increase.



Scalability

- Programs that can maintain a constant efficiency without increasing the problem size are sometimes said to be **strongly scalable**.
- Programs that can maintain a constant efficiency if the problem size increases at the same rate as the number of processes are sometimes said to be **weakly scalable**.

A parallel sorting algorithm

Sorting

- n keys and $p = \text{comm_sz}$ processes.
- n/p keys assigned to each process.
- No restrictions on which keys are assigned to which processes.
- When the algorithm terminates:
 - The keys assigned to each process should be sorted in (say) increasing order.
 - If $0 \leq q < r < p$, then each key assigned to process q should be less than or equal to every key assigned to process r .

Serial bubble sort

```
void Bubble_sort(  
    int a[] /* in/out */,  
    int n /* in */) {  
    int list_length, i, temp;  
  
    for (list_length = n; list_length >= 2; list_length--)  
        for (i = 0; i < list_length-1; i++)  
            if (a[i] > a[i+1]) {  
                temp = a[i];  
                a[i] = a[i+1];  
                a[i+1] = temp;  
            }  
    }  
} /* Bubble_sort */
```



Odd-even transposition sort

- A sequence of phases.
- Even phases, compare swaps:

$$(a[0], a[1]), (a[2], a[3]), (a[4], a[5]), \dots$$

- Odd phases, compare swaps:

$$(a[1], a[2]), (a[3], a[4]), (a[5], a[6]), \dots$$

Example

Start: 5, 9, 4, 3

Even phase: compare-swap (5,9) and (4,3)
getting the list 5, 9, 3, 4

Odd phase: compare-swap (9,3)
getting the list 5, 3, 9, 4

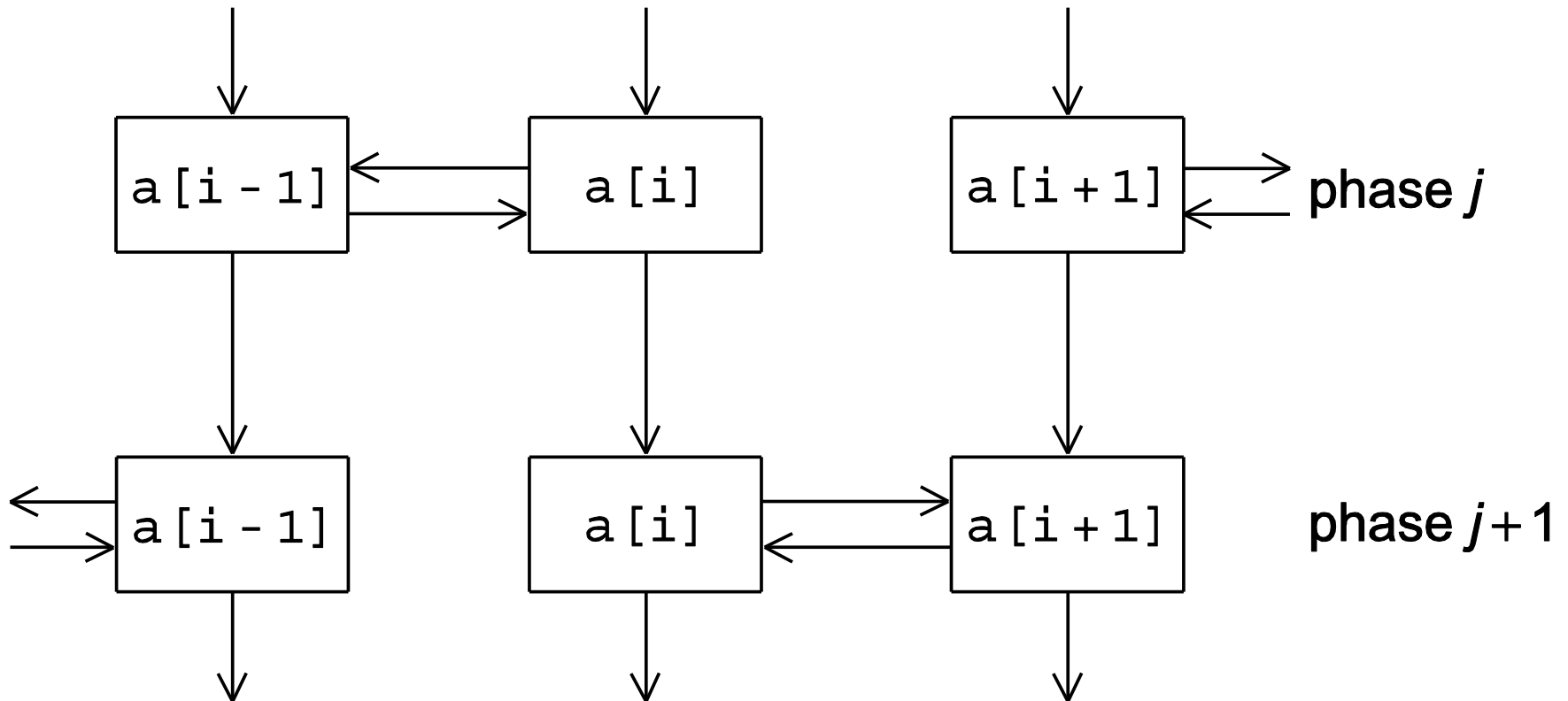
Even phase: compare-swap (5,3) and (9,4)
getting the list 3, 5, 4, 9

Odd phase: compare-swap (5,4)
getting the list 3, 4, 5, 9

Serial odd-even transposition sort

```
void Odd_even_sort(  
    int a[] /* in/out */,  
    int n /* in */) {  
    int phase, i, temp;  
  
    for (phase = 0; phase < n; phase++)  
        if (phase % 2 == 0) { /* Even phase */  
            for (i = 1; i < n; i += 2)  
                if (a[i-1] > a[i]) {  
                    temp = a[i];  
                    a[i] = a[i-1];  
                    a[i-1] = temp;  
                }  
        } else { /* Odd phase */  
            for (i = 1; i < n-1; i += 2)  
                if (a[i] > a[i+1]) {  
                    temp = a[i];  
                    a[i] = a[i+1];  
                    a[i+1] = temp;  
                }  
        }  
    }  
} /* Odd_even_sort */
```

Communications among tasks in odd-even sort



Tasks determining $a[i]$ are labeled with $a[i]$.

Parallel odd-even transposition sort

Time	Process			
	0	1	2	3
Start	15, 11, 9, 16	3, 14, 8, 7	4, 6, 12, 10	5, 2, 13, 1
After Local Sort	9, 11, 15, 16	3, 7, 8, 14	4, 6, 10, 12	1, 2, 5, 13
After Phase 0	3, 7, 8, 9	11, 14, 15, 16	1, 2, 4, 5	6, 10, 12, 13
After Phase 1	3, 7, 8, 9	1, 2, 4, 5	11, 14, 15, 16	6, 10, 12, 13
After Phase 2	1, 2, 3, 4	5, 7, 8, 9	6, 10, 11, 12	13, 14, 15, 16
After Phase 3	1, 2, 3, 4	5, 6, 7, 8	9, 10, 11, 12	13, 14, 15, 16

Pseudo-code

```
Sort local keys;
for (phase = 0; phase < comm_sz; phase++) {
    partner = Compute_partner(phase, my_rank);
    if (I'm not idle) {
        Send my keys to partner;
        Receive keys from partner;
        if (my_rank < partner)
            Keep smaller keys;
        else
            Keep larger keys;
    }
}
```

Compute_partner

```
if (phase % 2 == 0)           /* Even phase */
    if (my_rank % 2 != 0)       /* Odd rank */
        partner = my_rank - 1;
    else                         /* Even rank */
        partner = my_rank + 1;
else                           /* Odd phase */
    if (my_rank % 2 != 0)       /* Odd rank */
        partner = my_rank + 1;
    else                         /* Even rank */
        partner = my_rank - 1;
if (partner == -1 || partner == comm_sz)
    partner = MPI_PROC_NULL;
```

Safety in MPI programs

- The MPI standard allows MPI_Send to behave in two different ways:
 - it can simply **copy** the message into an MPI managed buffer and **return**,
 - or it can **block** until the matching call to MPI_Recv starts.

Safety in MPI programs

- Many implementations of MPI set a **threshold** at which the system switches from buffering to blocking.
- Relatively **small messages** will be **buffered** by MPI_Send.
- **Larger messages**, will cause it to **block**.

Safety in MPI programs

- If the MPI_Send executed by each process blocks, no process will be able to start executing a call to MPI_Recv, and the program will hang or **deadlock**.
- Each process is blocked waiting for an event that will never happen.

(see pseudo-code)

Safety in MPI programs

- A program that relies on MPI provided buffering is said to be **unsafe**.
- Such a program may run without problems for various sets of input, but it may hang or crash with other sets.

MPI_Ssend

- An alternative to MPI_Send defined by the MPI standard.
- The extra “s” stands for **synchronous** and MPI_Ssend is guaranteed to block until the matching receive starts.

```
int MPI_Ssend(  
    void*      msg_buf_p      /* in */,  
    int        msg_size       /* in */,  
    MPI_Datatype msg_type      /* in */,  
    int        dest           /* in */,  
    int        tag            /* in */,  
    MPI_Comm   communicator    /* in */);
```

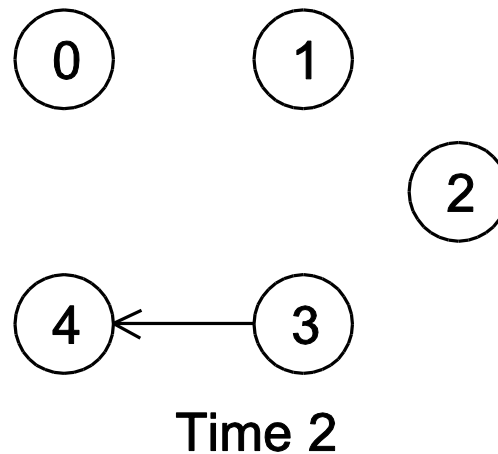
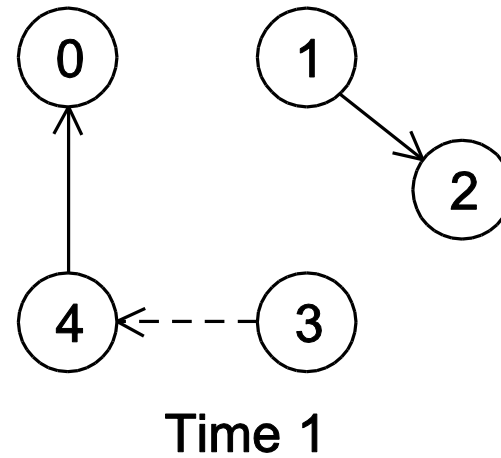
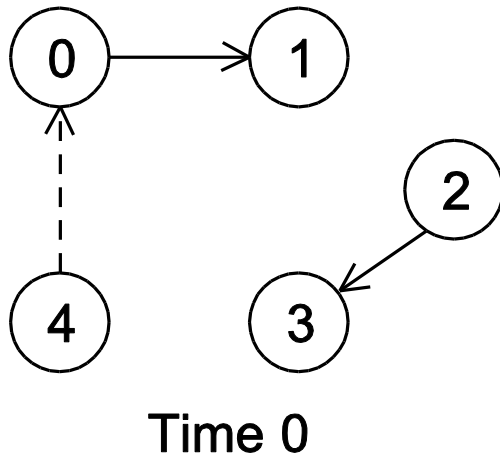
Restructuring communication

```
MPI_Send(msg, size, MPI_INT, (my_rank+1) % comm_sz, 0, comm);  
MPI_Recv(new_msg, size, MPI_INT, (my_rank+comm_sz-1) % comm_sz,  
         0, comm, MPI_STATUS_IGNORE.
```



```
if (my_rank % 2 == 0) {  
    MPI_Send(msg, size, MPI_INT, (my_rank+1) % comm_sz, 0, comm);  
    MPI_Recv(new_msg, size, MPI_INT, (my_rank+comm_sz-1) % comm_sz,  
            0, comm, MPI_STATUS_IGNORE.  
}  
else {  
    MPI_Recv(new_msg, size, MPI_INT, (my_rank+comm_sz-1) % comm_sz,  
            0, comm, MPI_STATUS_IGNORE.  
    MPI_Send(msg, size, MPI_INT, (my_rank+1) % comm_sz, 0, comm);  
}
```

Safe comm. with five processes



MPI_Sendrecv

- An alternative to scheduling the communications ourselves.
- Carries out a **blocking send** and a **receive** in a **single call**.
- The dest and the source can be the same or different.
- Especially useful because MPI schedules the communications **so that the program won't hang or crash**.

MPI_Sendrecv

```
int MPI_Sendrecv(  
    void*          send_buf_p      /* in */,  
    int           send_buf_size   /* in */,  
    MPI_Datatype   send_buf_type  /* in */,  
    int           dest             /* in */,  
    int           send_tag        /* in */,  
    void*          recv_buf_p      /* out */,  
    int           recv_buf_size   /* in */,  
    MPI_Datatype   recv_buf_type  /* in */,  
    int           source           /* in */,  
    int           recv_tag        /* in */,  
    MPI_Comm       communicator   /* in */,  
    MPI_Status*    status_p       /* in */);
```


Parallel odd-even transposition sort

```
void Merge_low(  
    int  my_keys[],      /* in/out      */  
    int  recv_keys[],  /* in       */  
    int  temp_keys[],    /* scratch   */  
    int  local_n        /* =  $n/p$ , in */) {  
    int  m_i, r_i, t_i;  
  
    m_i = r_i = t_i = 0;  
    while (t_i < local_n) {  
        if (my_keys[m_i] <= recv_keys[r_i]) {  
            temp_keys[t_i] = my_keys[m_i];  
            t_i++; m_i++;  
        } else {  
            temp_keys[t_i] = recv_keys[r_i];  
            t_i++; r_i++;  
        }  
    }  
  
    for (m_i = 0; m_i < local_n; m_i++)  
        my_keys[m_i] = temp_keys[m_i];  
} /* Merge_low */
```

Run-times of parallel odd-even sort

Processes	Number of Keys (in thousands)				
	200	400	800	1600	3200
1	88	190	390	830	1800
2	43	91	190	410	860
4	22	46	96	200	430
8	12	24	51	110	220
16	7.5	14	29	60	130

(times are in milliseconds)

Concluding Remarks (1)

- **MPI** or the Message-Passing Interface is a **library of functions** that can be called from C, C++, or Fortran programs.
- A **communicator** is a **collection of processes** that can send messages to each other.
- Many parallel programs use the single-program multiple data or **SPMD** approach.

Concluding Remarks (2)

- Most serial programs are deterministic: if we run the same program with the same input we'll get the same output.
- **Parallel** programs often **don't possess this property**.
- Collective communications involve all the processes in a communicator.

Concluding Remarks (3)

- When we time parallel programs, we're usually interested in elapsed time or “**wall clock time**”.
- **Speedup** is the ratio of the serial run-time to the parallel run-time.
- **Efficiency** is the speedup divided by the number of parallel processes.

Concluding Remarks (4)

- If it's possible to increase the problem size (n) so that the efficiency doesn't decrease as p is increased, a parallel program is said to be **scalable**.
- An MPI program is **unsafe** if its correct behavior depends on the fact that MPI_Send is buffering its input.