Development of a Horizontal Slinky Ground Heat Exchanger Model

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ABSTRACT

Slinky ground heat exchangers are increasingly considered among more traditional ground heat exchanger configurations. Although studied for many years as a research topic, the increase in efficiency and reduction in building energy demands has increased the viability of these specialized heat exchangers, especially in cases where limited space is available for burying horizontal heat exchanger tubing. By formulating an accurate simulation model, the feasibility and efficiency of such heat exchangers in different climates and building configurations can be studied. In addition, it will serve as a powerful tool for design and energy analysis of the ground source heat pump (GSHP) systems.

This paper describes the derivation of an analytic solution for modeling the thermal response of horizontal Slinky heat exchangers. The analytic results are presented in the form of temperature response factors. A computer module with fast algorithms is written to calculate temperature response factors of any specified Slinky ground heat exchanger. The factors then can be used as input for a buried pipe model to calculate hourly exit fluid temperature of the heat exchanger. Knowledge of exit fluid temperature is essential for simulation of a GSHP system.

INTRODUCTION

Serving as a major part of GSHP systems, ground heat exchangers (GHX) have a large impact on the systems' efficiency. The GHX exit fluid temperature will influence the coefficient of performance (COP) of the heat pump unit directly. One main category of GHX is the horizontal ground heat exchangers (HGHX), which usually have a lower installation cost compared with vertical ones. Slinky ground heat exchangers reduce trench length significantly compared to a two-pipe HGHX system. Slinky GHX use coil-like pipes, **as Figure 1 shows**. This application increases pipe length per trench length, which allows GHX to extract/inject more heat per trench length.

In the last decades, mathematical models of different kinds of GHX are developed to assist design and simulation of GSHP systems. However, for Slinky ground heat exchangers, an appropriate model is absent for simulation due to the complex geometry of Slinky ground heat exchangers. In most literature related to Slinky ground heat exchangers, the approach to study thermal performance of Slinky heat exchangers is either experimental testing or numerically modeling. Recently, an analytical model (Li et al., 2012) for Slinky heat exchangers was presented. The study is limited to a discussion of ground temperature variation while knowledge of fluid temperature is required for designing and simulating a GSHP system.

Aiming to develop a feasible simulation tool, an analytical solution for thermal response of horizontal Slinky ground heat exchanger is given in this study. Derived from the equation giving temperature perturbation caused by a point source in an infinite homogeneous medium (Ingersoll et al., 1954), a general formula is given to compute mean tube surface

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temperature of horizontal Slinky ground heat exchangers. The results are presented in the form of temperature response factors. A computer module is written to calculate the temperature response function for any specified horizontal Slinky ground heat exchanger. Several algorithms were applied to shorten the calculation time. By combining the calculated temperature response factors with a buried pipe model, hourly exit fluid temperature of the Slinky heat exchanger can be obtained.

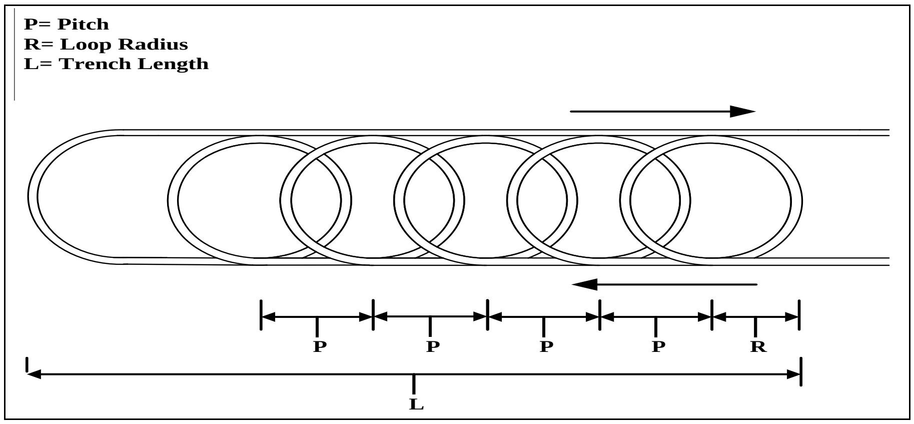

Figure 1 Schematic of A Typical Horizontal Slinky Ground Heat Exchanger

LITERATURE REVIEW

Different kinds of models of Ground Heat Exchangers (GHX) have been developed in the last decades. Several classic models are: Ingersoll et al. (1954), Carslaw and Jaeger (1959), Eskilson (1987). One representative category among these models is the analytical or semi-analytical models based on line source theory, which was proposed by Kelvin (1882). Based on Eskilson's (1987) approach, Yavuzturk and Spitler (1999) developed a short time step response factor model for vertical ground loop heat exchanger. An aggregation algorithm was developed for the ground loads to reduce the computational time of doing an hourly simulation for years.

In recent years, finite line-source models have been introduced to solve a wide range of problems associated with GHX (Zeng et al., 2002; Cui et al., 2006; Lamarche and Beauchamp, 2007; Marcotte and Pasquier, 2009). The finite line source solution is obtained by integrating the result of a point source over a specified line path. These models have an advantage over the infinite line source model by considering the axial effect. In those models, the ground is regarded as a homogeneous semi-infinite medium; the ground surface is treated as an isothermal surface by introducing a fictitious finite line source. Cui et al. (2006) use this approach to analyze inclined borehole vertical GHEs. Cui et al. (2006) discussed two kinds of representative temperature in the paper. The representative temperature of cross-section circle of borehole can be any point temperature on the circle with an error less than 0.001%. However, using the temperature of the middle section of the borehole as the representative temperature of the entire borehole wall can produce the error as large as 10%. In Lamarche and Beauchamp's (2007) paper, the expression of the finite line source solution is modified to eliminate the double integral, which will largely shorten the calculation time.

A ring source model for spiral ground heat exchangers, which we call Slinky GHX in this paper, was presented by Li et al. (2012). The ring source solution for spiral ground heat exchangers was verified by an experiment. The equipment that used in the experiment is a well-insulated stainless steel box, which creates adiabatic boundary conditions. While the boundary conditions are obviously different in the reality applications, the experiment acted as an examination of the applicability of the superposition algorithm of multiple ring source. A fast algorithm was created to reduce computation time largely. Since a double integral is included in the solution, the computation is slow, especially when numerous ring source units exist. When the distance between two rings is far enough compared with the ring's radius, the temperature variation of ring 1 due to ring 2 can be considered as the variation at a point, whose distance to ring 2's center is the average value of points on ring 1. The model will aid determining the size of a spiral ground heat exchanger by giving soil temperature variation and tube surface temperature response. However, the paper does not include the discussion of tube fluid temperature calculation, which is essential for system simulation.

ANALYTICAL SOLUTION

The finite line source method integrates the results of a point source solution through a specified path, e.g. vertical lines for vertical GHX, to obtain the temperature perturbation of the boreholes. The same idea may be applied to obtain the solution for the temperature response of horizontal Slinky ground heat exchangers. The configuration of a typical Slinky coil is simplified as a series of connected circles, **as Figure 2 shows**. In subsequent discussion, the circles will be labeled as circle *i* or circle *j*.

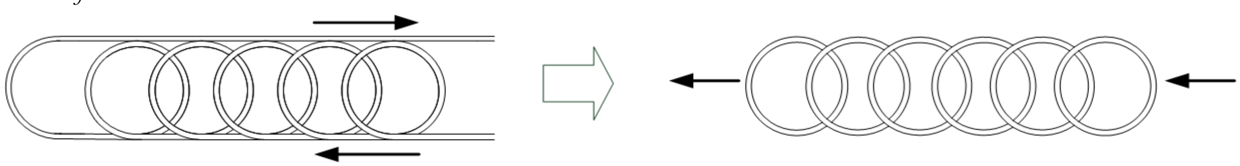

Figure 2 Simplification of A Slinky Coil

The point source solution is given by Ingersoll et al. (1954):

$$\Delta T(r,t) = \frac{Q\alpha}{8k(\alpha\pi t)^{3/2}} e^{-(\frac{r_d^2}{4\alpha t})} \quad (1)$$

The equation above gives the temperature variation at distance *r* after time *t*, due to a point source emitting instantaneously heat units *Q* at *t*=0. When the point source is emitting *q* heat units continuously, the temperature variation can be obtained by integrating Equation 1 along time:

$$\Delta T(r,t) = \frac{q\alpha}{8k(\alpha\pi)^{\frac{3}{2}}} \int_0^t e^{-\left(\frac{r_d^2}{4\alpha t'}\right)} (t')^{-\frac{3}{2}} dt' \quad (2)$$

By changing the variable to $\beta = r_d / (2(\alpha t')^{0.5})$, we may transform Equation 2 into the form below:

$$\Delta T(r,t) = \frac{q}{4\pi k r_d} erfc(\frac{r_d}{2\sqrt{\alpha t}}) \quad (3)$$

where: $\mathrm{erfc}(\beta) = (\frac{2}{\sqrt{\pi}}) \int_\beta^\infty e^{-\beta^2} d\beta$

Assuming q_l is the heat input rate per trench length, then for a point source P_j in an arbitrary circle *j*, the intensity of heat input is $(q_l \cdot L \cdot d\omega)/(N_{circle} \cdot 2\pi)$. A large number of such point sources located in circle *j* constitute a ring source. The effect of this ring source can be viewed as the combined action of these point sources. The temperature perturbation at point P_i caused by ring source *j* can be calculated by integrating the results of the point sources in circle *j*.

$$\Delta T(P_i,t) = \frac{q_l L}{8\pi^2 k N_{circle}} \int_0^{2\pi} \frac{erfc\left(\frac{d(P_j,P_i)}{2\sqrt{\alpha t}}\right)}{d(P_j,P_i)} d\omega \tag{4}$$

where: $d(P_i,P_j) = \frac{d(P_{ii},P_j)+d(P_{io},P_j)}{2}$

$$d(P_{ii},P_j) = \sqrt{(x_{0i} + (R-r)cos\varphi - x_{0j} - Rcos\omega)^2 + (y_{0i} + (R-r)sin\varphi - y_{0j} - Rsin\omega)^2}$$

$$d(P_{io},P_j) = \sqrt{(x_{0i} + (R+r)cos\varphi - x_{0j} - Rcos\omega)^2 + (y_{0i} + (R+r)sin\varphi - y_{0j} - Rsin\omega)^2}$$

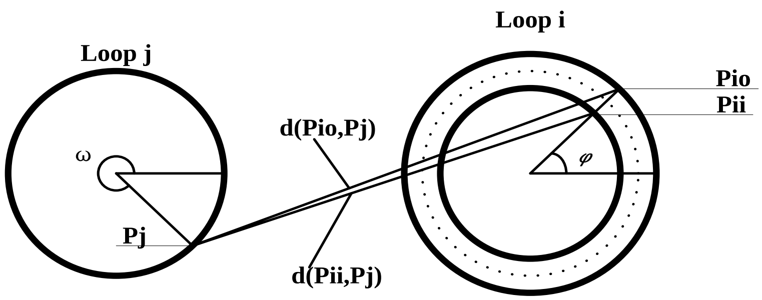

Figure 3 Distance Between Point P_i in Circle i and Point P_j in Ring Source j

As Figure 3 shows, point P_i is a fictitious representative point of a cross-section of tube *i* at angle φ. The distance between point P_j and any point P_j in the 2-dimensional plane is the average value of the distance between the outer point P_{io} and point P_j and the distance between the inner point P_{ii} and point P_j. Since only the distance is changing, the average temperature perturbation of the cross-section can use the temperature perturbation of the fictitious representative point. Specially, when *i* is equal to *j*, the dashed line **in Figure 3 shows** the ring source of circle *i* itself.

The formulas above are for the case of an infinite homogeneous medium; this is obviously not true in reality. For Slinky coil buried underground, the thermal interaction can be deemed as happening in a semi-infinite medium with isothermal boundary condition. The solution for this case can be obtained by viewing the ground surface as a mirror. A fictitious ring source *j'* is created for circle *j*, **as Figure 4 shows**. The fictitious ring source *j'* is *2h* higher than the circle *j* perpendicularly and with the same heat input rate, yet opposite in sign. Then Equation 4 should be revised to include the effect of fictitious ring source to point P_i

The distance between P_j' and P_i can always be expressed as $[(d(P_i, P_j))^2+4h^2]^{0.5}$. This approach allows us to calculate the effect of two ring sources using only one integral instead of two, which will improve calculation efficiency.

$$\Delta T(P_i,t) = \frac{q_l L}{8\pi^2 k N_{circle}} \int_0^{2\pi} \left(\frac{erfc\left(\frac{d(P_j,P_i)}{2\sqrt{\alpha t}}\right)}{d(P_j,P_i)} - \frac{erfc\left(\left(d(P_j,P_i)^2+4h^2\right)^{\frac{1}{2}}/(2\sqrt{\alpha t})\right)}{\left(d(P_j,P_i)^2+4h^2\right)^{1/2}} \right) d\omega \tag{5}$$

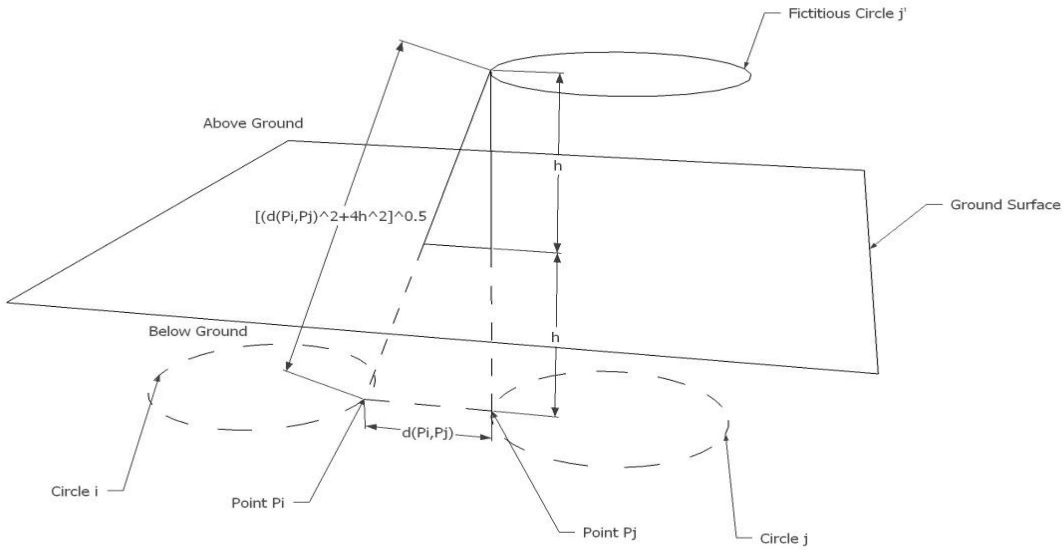

Figure 4 Three-Dimension View of Fictitious Ring Source of Circle j

While Equation 5 gives the temperature perturbation of any point in loop *i*, the average temperature perturbation of loop *i* caused by ring source *j* can be obtained by integrating Equation 5.

$$\Delta T_{j-i}(t) = \frac{q_l L}{16\pi^3 k N_{circle}} \int_0^{2\pi} \int_0^{2\pi} \left(\frac{erfc\left(\frac{d(P_j,P_i)}{2\sqrt{\alpha t}}\right)}{d(P_j,P_i)} - \frac{erfc\left(\left(d(P_j,P_i)^2 + 4h^2\right)^{\frac{1}{2}} / (2\sqrt{\alpha t})\right)}{\left(d(P_j,P_i)^2 + 4h^2\right)^{1/2}} \right) d\omega d\varphi \qquad (6)$$

When i is equal to j, $\Delta T_{j-i}(t)$ gives the temperature perturbation of the tube wall of the circle caused by the circle itself as a ring source. When i doesn't equal j, $\Delta T_{j-i}(t)$ counts the interaction between two circles. Regarding Slinky ground heat exchangers with multiple Slinky coils, the two circles can either in the same Slinky coil or in different Slinky coil. When summing up all the $\Delta T_{j-i}(t)$ for any two circles or one circle when j equals to i, the thermal effect of any Slinky coil, as a series of connected ring source, on its own coil and on other Slinky coils has been included.

The mean temperature variation of a Slinky coil can then be computed based on Equation 6.

$$\Delta\bar{T}(t) = \frac{1}{N_{circle}} \sum_{i=1}^{N_{circle}} \sum_{j=1}^{N_{circle}} \Delta T_{j-i}(t) \qquad (7)$$

The g-function is an approach proposed by Eskilson (1987) to calculate the borehole wall temperature. The temperature response of boreholes is converted to a set of non-dimensional temperature response factors, named g-function. The g-function approach is widely used in solving vertical borehole related problem. Mimicking the definition of the g-function, temperature response function for Slinky ground heat exchangers is defined as:

$$g(t) = \frac{2\pi k}{q_l} \Delta\bar{T}(t) \qquad (8)$$

By combining Equations 6 to 8, the analytical solution of the temperature response factors for a Slinky coil can be obtained:

$$g(t) = \sum_{i=1}^{N_{circle}} \sum_{j=1}^{N_{circle}} \left(\frac{L}{8\pi^2 {N_{circle}}^2} \int_0^{2\pi} \int_0^{2\pi} \left(\frac{erfc\left(\frac{d(P_j,P_i)}{2\sqrt{\alpha t}}\right)}{d(P_j,P_i)} - \frac{erfc\left(\left(d(P_j,P_i)^2+4h^2\right)^{\frac{1}{2}}/(2\sqrt{\alpha t})\right)}{\left(d(P_j,P_i)^2+4h^2\right)^{1/2}} \right) d\omega d\varphi \right) \quad (9)$$

For ground heat exchangers with multiple Slinky coils, more parameters need to be included in the calculation, e.g. the number of Slinky coils and the distance between Slinky coils. Still the same method could be applied to solve for the temperature response factors.

A computer module is created to calculate temperature response factors for Slinky ground heat exchangers, which can have one or multiple Slinky coils. Temperature response factors for a specified buried Slinky coil **are shown in Figure 6**.

CALCULATION OF FLUID TEMPERATURE

By using the temperature response factors, the mean tube wall temperature of a given Slinky ground heat exchanger at a given time is obtained. The fluid temperature in the coil is then calculated from the mean tube wall temperature. To do so, two assumptions have to be made:

1. The heat transfer between tube wall and fluid is at quasi-steady state
2. The mean fluid temperature is the average value of the inlet fluid temperature and outlet fluid temperature

According to heat balance principle, we may have the following equations:

$$T_{ave} = (T_{in} + T_{out})/2 \quad (10)$$

$$T_{ave} - T_{tw} = qR_{tw} \quad (11)$$

$$q = \dot{m}C_p(T_{in} - T_{out}) \quad (12)$$

By solving the three equations above simultaneously, we are able to calculate the outlet fluid temperature of the Slinky ground heat exchanger. R_{tw} is the resistance of the tube wall. This method is used to calculate the hourly exit fluid temperature of a Slinky ground heat exchanger, which has the parameters **as Table 1 shows**, during a five day running period. A constant heat load 4400W (15017 Btu/h) is applied to the exchanger. The ground temperature is assumed constant along the running period, which is 25.6°C (78 °F) at the buried depth. The hourly inlet and outlet water temperature **is shown in Figure 7**. The water temperature is within a reasonable range.

Table 1. Parameters of the Slinky Ground Heat Exchanger

Parameters	Value	Parameters	Value
Diameter of circles	0.8 m (2.62 ft)	Water flow rate	0.233 kg/s (30.82 lb/min)
Diameter of tube	0.012 m (0.04 ft)	Soil thermal conductivity	1.09 W/m·K (0.63 Btu/h·ft·°F)
Buried Depth	1.5 m (4.92 ft)	Tube thermal conductivity	0.35W/m·K (0.20 Btu/h·ft·°F)
Number of Slinky coil	2	Gap between tubes	2 m (6.56 ft)
Number of circles	100	Thickness of tube	5 mm (0.2 in.)
Pitch between circles	0.4 m (1.31 ft)		

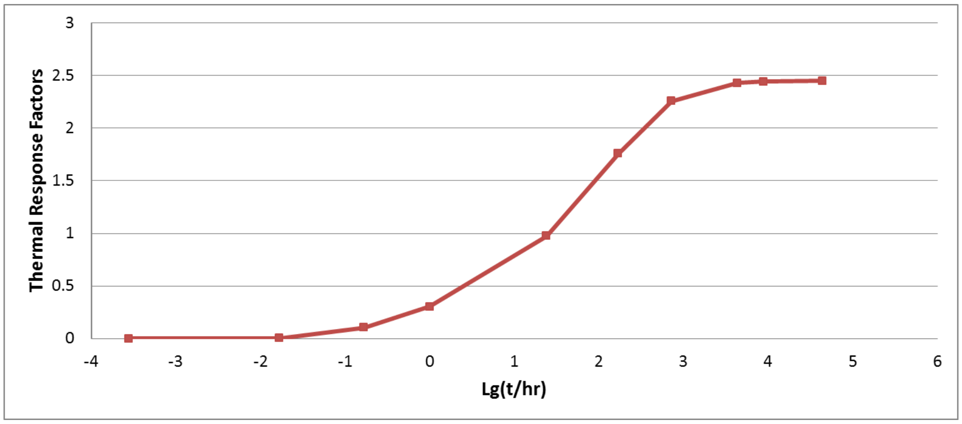

Figure 6 Thermal Response Factors for the Slinky Ground Heat Exchanger with One tube

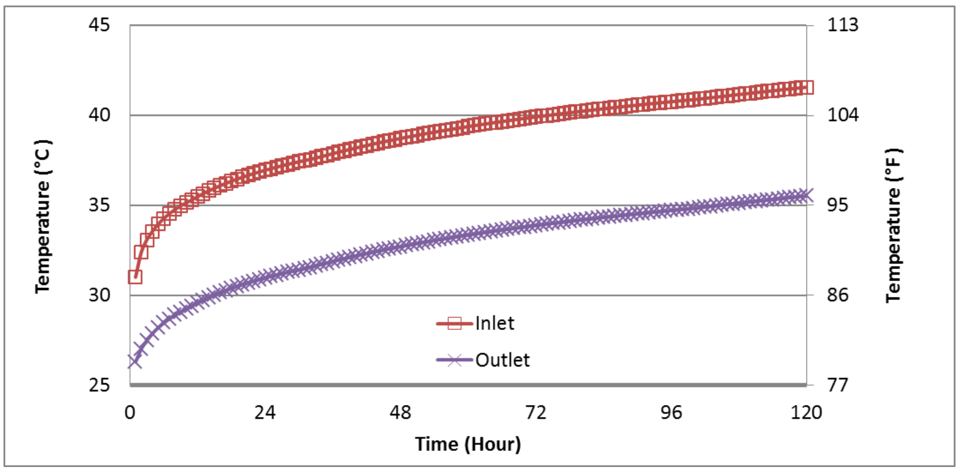

Figure 7 Hourly Inlet and Outlet Water Temperature of the Slinky Ground Heat Exchanger

CONCLUSIONS AND RECOMMENDATIONS

This paper presented an analytical solution for horizontal Slinky ground heat exchangers. Based on the solution, a computer module was written to calculate temperature response factors of any specified horizontal Slinky ground heat exchangers. These factors can be further used in a buried pipe model to calculate the exit fluid temperature of the Slinky ground heat exchanger. Since exit fluid temperature is essential for system design or simulation, this model will aid casting an accurate simulation tool for Slinky ground heat exchangers.

Several things are recommended for future work. Detailed ground heat balance is highly desirable to include in this model. Because for Slinky ground heat exchangers, ground heat balance will have a large impact on their thermal performance due to their shallow buried depth. In addition, a field test of a Slinky heat exchanger is recommended, which will help to validating the model.

NOMENCLATURE

α = thermal diffusivity (m^2/s or ft^2/h)

C_p = specific heat (J/kg·K or Btu/lbm·°F)

d = distance between two points (m or ft)

$erfc$ = complementary error function

h = buried depth (m or ft)

k = conductivity (W/m·K or Btu/h·ft·°F)

L = trench length (m or ft)

$\dot{m}$ = mass flow rate (kg/s or lb/min)

N_{circle} = number of circles

Q = heat impulse (J or ft ·lbf)

q = heat rate (W or Btu/h)

q_i = heat rate per trench length (W/m or Btu/h ·ft)

R = radius of circle (m or ft)

r = radius of coil (m or ft)

r_d = distance between two points (m or ft)

T = tmperature (°C or °F)

T_{ave} = mean fluid tmperature (°C or °F)

t, t' = time (s)

P = arbitrary point

φ,ω = angular coordinate

Subscripts

in = inlet

out = outlet

tw = tube wall

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