Numerical Methods for Ordinary Differential Equations

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Physically Based Cloth Modeling

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1 Introduction

Modeling the motion of cloth has gained an extensive research in Computer Graphics Community. Mainly, the cloth is represented as a collection of particles connected to each other in some manner. The main approaches to cloth simulation are based either on geometric approaches or physical. The geometric approaches are usually applied for stationary models. Besides, they are not general enough to account for the physical properties of the cloth such as stretch (tension), stiffness, and masses of the particles.

2 Cloth Representation and Space discretization

Cloth is represented as a uniformly spaced mesh with distance **d** of M by N particles , where every point in a grid is a mass linked to its neighbors by massless springs with non-zero length. For this project, M=N=20 and the spacing between the springs is equal to 0.1. The are 3 different linkage types inbetween the neighboring springs: structural, shear and flexion, each representing a certain physical property and pictured below. Flexion springs are introduced to control the bending of the cloth.

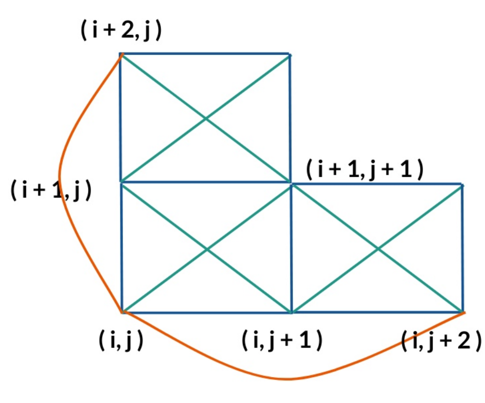

Figure 1: Springs
Structural, shear and flexion springs are coloured blue, green and orange correspondingly

3 Setting up the differential equations

In this report, it is assumed that the only forces acting on the system of the particles are gravitational, spring and damping forces. Since both of those forces model the relation between the acceleration and the position of the particles, the motion can be modeled as a second order ordinary differential equation. To make it first order, the velocity of particles are also considered. So, particle (i,j) is represented by its:

- Position x_{ij}
- Velocity v_{ij}
- Mass m_{ij}

For simplicity, it is assumed that masses of particles are set to 1. Thus, particle goes from $\mathbb{R}^2 \to \mathbb{R}^6$. The equation of forces are :

- **Spring Force:** Given spring connecting two particles located at x_p and x_q, stiffness K, rest length L_0.
$$\mathbf{F}_{\text{spring}} = K(L_0 - \|\mathrm{x_p} - \mathrm{x_q}\|)\frac{\mathrm{x_p} - \mathrm{x_q}}{\|\mathrm{x_p} - \mathrm{x_q}\|}.$$
Note that for the rest length L_0 is equal to $\mathbf{d}, \mathbf{d}\sqrt{2}, \mathbf{d+1}$ correspondingly for structural, shear and flexion springs

- **Damping Force:** Given spring connecting two particles located at x_p and x_q, with corresponding velocities v_p and v_q, and damping D
$$\mathbf{F}_{\text{damping}} = D(v_p - v_q)\|\mathrm{x_p} - \mathrm{x_q}\| = D(\dot{x_p} - \dot{x_q})\|\mathrm{x_p} - \mathrm{x_q}\|$$

- **Gravitational force**
$$\mathbf{F}_{\text{gravity}} = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix}$$

Finally,
$$\ddot{x_p} = \mathbf{F}_{\text{damping}} + \mathbf{F}_{\text{spring}} + \mathbf{F}_{\text{gravity}}$$
Also, at some specific locations x_p, the cloth is not changing its position and velocity. So, $x_p = 0, \dot{x_p} = 0$

4 Time discretization

Usually, for numerical stability, the choice of the length for the timestep is dependent on the choice of spring constant K. The greater the stiffness of the spring, the smaller the timestep should be. That is why, for simplicity, the timestep is fixed to 0.01 and spring stiffness is adjusted if needed.

5 Initial Conditions and Constraints

The initial position of the cloth is assumed to be 0 in z-direction for all the particles. Regarding the initial direction for initial velocity and the particles which are not changing their position in time, user can choose themselves. But, for this report, it is assumed that the cloth is hang on its four corners. In addition, initial velocity is 20 metre per second.

6 Numerical Methods used and Results

The numeric simulations have been done for Implicit, Explicit Euler's Methods and Runge-Kutta of 4s order based on the following scheme.

$$
\begin{array}{c|cccc}
0 & 0 & 0 & 0 & 0 \\
1/2 & 1/2 & 0 & 0 & 0 \\
1/2 & 0 & 1/2 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 \\
\hline
 & 1/6 & 1/3 & 1/3 & 1/6
\end{array}
$$

6.1 Simulation Results

All of the methods are very sensitive to increasing the stiffness of the spring, especially when the damping is set to 0. However, for Explicit Euler's method, it is clearly visible how large the accumulation of the errors are from iteration to iteration. Even by decreasing the

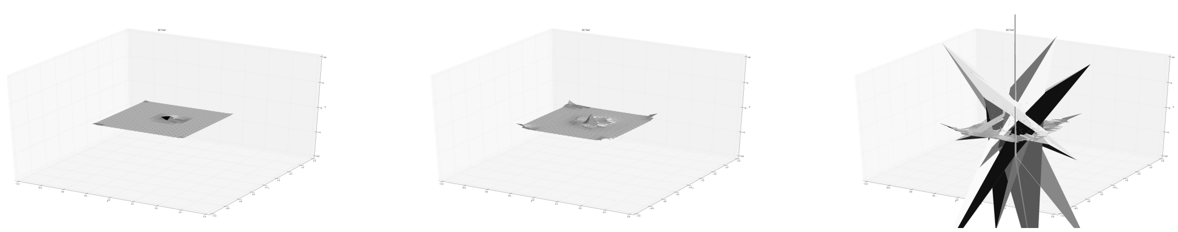

Figure 2: Euler's method with K, stiffness=200 and damping=0

stiffness of the spring, the simulation does not get more realistic, the changes of the position are still very rapid. Only adding damping factor makes the cloth simulation with Explicit

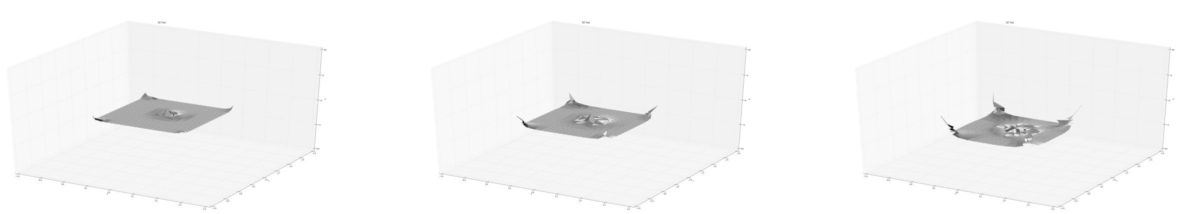

Figure 3: Explicit Euler's method with K, stiffness=50 and damping=0

Euler's method to look more realistic when the spring constants are very high.

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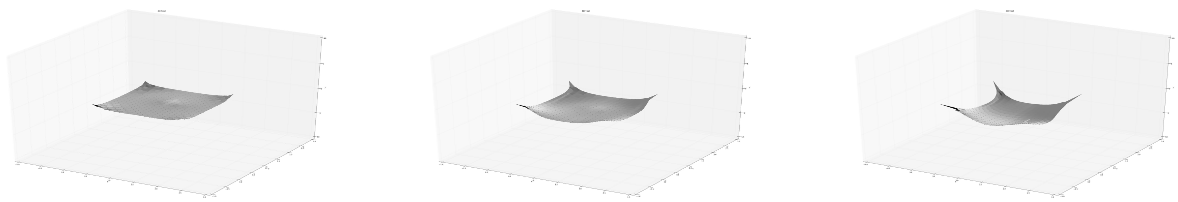

Figure 4: Explicit Euler's method with K, stiffness=200 and damping=20

For the Runge-Kutta Method basically the same conclusions can be drawn, the only difference is that it is more stable when considering large spring constants compared to Explicit Euler's method. The most realistic results were obtained by Implicit Euler's

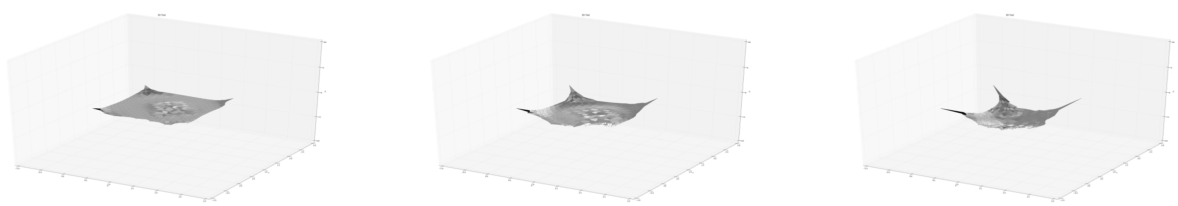

Figure 5: Runge-Kutta's method with K, stiffness=200 and damping=0

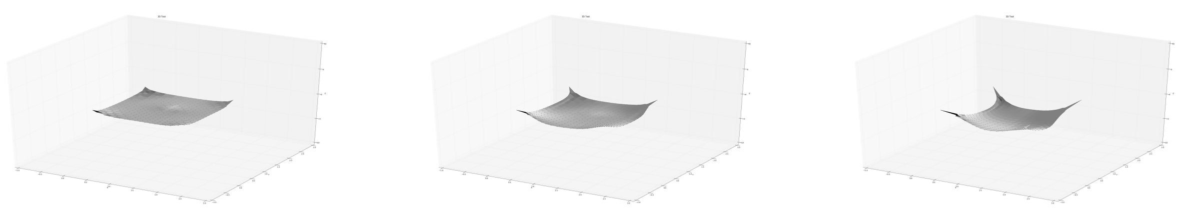

Figure 6: Runge-Kutta's method with K, stiffness=200 and damping=20

method for spring constant equal to 50.

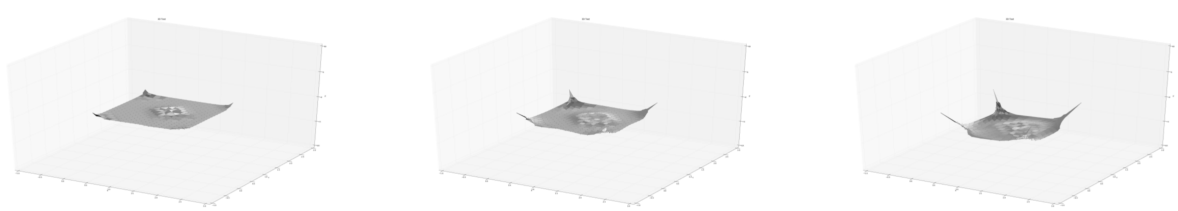

Figure 7: Implicit Euler's method with with K, stiffness=50 and damping=0

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