Additional Materials

E. Discussion on the Long-Tail Phenomenon of Access Curves

In the previous sections, we mentioned that access curves to UGC objects have a long-tail phenomenon which means that access curves show downward trends on the whole, but we did not give a formulaic definition about it because it is hard to give a general formulation. One reason is that access curves are uncertain even though historical information is known. The other is that although all access curves have a long-tail phenomenon, they have various trends among which there may be significant differences. In this subsection, we formulate an access curve with the well-known Pareto distribution [1] and show that it is well-designed of our algorithm for access curves having long-tail phenomenon. Suppose accesses to a UGC object with a lifetime of T follow the Pareto principle. That is, most accesses occur in the early 20% of its lifetime. Then we define the number of accesses per time slot occurred between time 1 and $0.2T$. The number of accesses at time t is denoted by $\mathcal{A}/x_t$, where $\mathcal{A}$ denotes the maximum number of accesses (i.e., $\mathcal{A} = \max_{1 \le t \le T} a(t)$) and x_t is a Pareto random variable following the probability density function of

$$f(x_t) = \begin{cases} \frac{5}{T}, & x_t = 1, \\ \frac{1}{x_t^2}, & x_t \in (1, \frac{T}{5}]. \end{cases}$$

Then Lemma 2 can be transformed as follows when Eq. (6) is applied.

Lemma 3. $\frac{t_E - t_o}{2} \le \frac{\mathcal{A}(R_c - R_h) + R_c V}{(e-1+\lambda/\alpha)(S_h - S_c)}$.

Proof. We get the following inequalities according to Lemma 2:

$$\begin{aligned} \frac{t_E - t_o}{2} &\le \max_{1 \le t < t_E} \mathbf{E}[z(t)] \\ &= \max_{1 \le t < t_E} \int_0^{\Delta(t)} \frac{e^{\frac{z(t)}{\Delta(t)}}}{\Delta(t)(e - 1 + \frac{\lambda}{\alpha})} z(t) dz(t) \\ &= \max_{1 \le t < t_E} \frac{\Delta(t)}{e - 1 + \lambda/\alpha} \\ &= \frac{\max_{1 \le t < t_E} a(t)(R_c - R_h) + R_h V}{(e - 1 + \lambda/\alpha)(S_h - S_c)} \\ &= \frac{\mathcal{A}(R_c - R_h) + R_h V}{(e - 1 + \lambda/\alpha)(S_h - S_c)}. \end{aligned}$$

□

The competitive ratio given by Proposition 1 also holds with Lemma 3.

References

[1] C. Barry, *Pareto Distributions.* International Co-Operative Publishing House, 1983.