Additional Materials

E. An Example of the Long-Tail Phenomenon of Access Curves

In the previous sections, we mentioned that access curves to UGC objects have a long-tail phenomenon which means that access curves show downward trends on the whole, but we did not give a formulaic definition about it because it is hard to give a general formulation. One reason is that access curves are uncertain even though historical information is known. The other is that although all access curves have a long-tail phenomenon, they have various trends among which there may be significant differences. In this subsection, we formulate an access curve with the well-known Pareto distribution [1] and show that it is well-designed of our algorithm for access curves having long-tail phenomenon. Suppose accesses to a UGC object with a lifetime of T follow the Pareto principle. That is, most accesses occur in the early 20% of its lifetime. Then we define the number of accesses per time slot occurred between time 1 and $0.2T$. The number of accesses at time t is denoted by $\frac{\mathcal{A}x_t}{T}$, where $\mathcal{A}$ denotes the maximum number of accesses (i.e., $\mathcal{A} = \max_{1 \leq t \leq T} a(t)$) and x_t is a Pareto random variable following the probability density function of

$$f(x_t) = \begin{cases} \frac{1}{T}, & x_t = 1, \\ \frac{1}{x_t^2}, & x_t \in (1, T]. \end{cases}$$

Then Lemma 2 can be transformed as follows when Eq. (6) is applied.

Lemma 3. $\frac{t_E - t_o}{2} \leq \frac{\mathcal{A}(R_c - R_h) + R_c V}{(e-1+\lambda/\alpha)(S_h - S_c)}$.

Proof. We get the following inequalities according to Lemma 2:

$$\begin{aligned}
\frac{t_E - t_o}{2} &\leq \max_{1 \leq t < t_E} \mathbf{E}[z(t)] \\
&= \max_{1 \leq t < t_E} \int_0^{\Delta(t)} \frac{e^{\frac{z(t)}{\Delta(t)}}}{\Delta(t)(e - 1 + \frac{\lambda}{\alpha})} z(t) dz(t) \\
&= \max_{1 \leq t < t_E} \frac{\Delta(t)}{e - 1 + \lambda/\alpha} \\
&= \frac{\max_{1 \leq t < t_E} a(t)(R_c - R_h) + R_h V}{(e - 1 + \lambda/\alpha)(S_h - S_c)} \\
&= \frac{\mathcal{A}(R_c - R_h) + R_h V}{(e - 1 + \lambda/\alpha)(S_h - S_c)}.
\end{aligned}$$

□

We note that $z(t)$ is a compound probability distribution because $z(t)$ is distributed according to the parametrized distribution Eq. (5) or (6) with the parameter $a(t)$ that is again distributed according to some other distribution.

In the previous sections, we do not give a specific distribution of $a(t)$ because access curves which have long-tail phenomena may have various distributions. Therefore, we use the maximum expectation of $z(t)$ under the maximum value of $a(t)$ (i.e., $\max_{1 \leq t < t_E} \mathbf{E}[z(t)]$) to bound the competitive ratio. In this section, we give an example distribution of $a(t)$, so we can give the compound distribution's expectation as follows:

$$
\begin{aligned}
\mathbf{E}[z(t)] &= \mathbf{E}\left[\mathbf{E}[z(t)|a(t)]\right] \\
&= \mathbf{E}\left[\int_0^{\Delta(t)} \frac{e^{\frac{z(t)}{\Delta(t)}}}{\Delta(t)(e-1+\frac{\lambda}{\alpha})} z(t) dz(t)\right] \\
&= \mathbf{E}\left[\frac{\Delta(t)}{e-1+\lambda/\alpha}\right] \\
&= \mathbf{E}\left[\frac{a(t)(R_c-R_h)+R_h V}{(e-1+\lambda/\alpha)(S_h-S_c)}\right] \\
&= \mathbf{E}\left[\frac{\frac{\mathcal{A}x_t}{T}(R_c-R_h)+R_h V}{(e-1+\lambda/\alpha)(S_h-S_c)}\right] \\
&= \frac{\frac{\mathcal{A}\ln T}{T}(R_c-R_h)+R_h V}{(e-1+\lambda/\alpha)(S_h-S_c)}
\end{aligned}
$$

References

[1] C. Barry, *Pareto Distributions*. International Co-Operative Publishing House, 1983.