Navigation solution – mechanization of the Eq.

$$\dot{\hat{\mathbf{R}}}_b^n = \hat{\mathbf{R}}_b^n \left(\hat{\boldsymbol{\Omega}}_{ib}^b - \hat{\boldsymbol{\Omega}}_{in}^b\right) \qquad \hat{\omega}_{in}^n = \begin{bmatrix} \omega_N \\ \omega_E \\ \omega_D \end{bmatrix} = \begin{bmatrix} \left(\dot{\hat{\lambda}} + \omega_{ie}\right)\cos(\hat{\phi}) \\ -\dot{\hat{\phi}} \\ -\left(\dot{\hat{\lambda}} + \omega_{ie}\right)\sin(\hat{\phi}) \end{bmatrix}$$

$$\begin{bmatrix} \dot{\hat{v}}_n \\ \dot{\hat{v}}_e \\ \dot{\hat{v}}_d \end{bmatrix} = \begin{bmatrix} \hat{f}_n \\ \hat{f}_e \\ \hat{f}_d \end{bmatrix} + \mathbf{g}^n - \begin{bmatrix} 0 & -\omega_d & \omega_e \\ \omega_d & 0 & -\omega_n \\ -\omega_e & \omega_n & 0 \end{bmatrix} \begin{bmatrix} \hat{v}_n \\ \hat{v}_e \\ \hat{v}_d \end{bmatrix}$$

where

$$\begin{bmatrix} \omega_n \\ \omega_e \\ \omega_d \end{bmatrix} = \left(\boldsymbol{\omega}_{en}^n + 2\boldsymbol{\omega}_{ie}^n\right) = \begin{bmatrix} \left(\dot{\hat{\lambda}} + 2\omega_{ie}\right)\cos(\hat{\phi}) \\ -\dot{\hat{\phi}} \\ -\left(\dot{\hat{\lambda}} + 2\omega_{ie}\right)\sin(\hat{\phi}) \end{bmatrix}$$

$$\begin{bmatrix} \hat{f}_n \\ \hat{f}_e \\ \hat{f}_d \end{bmatrix} = \hat{\mathbf{f}}^n = \hat{\mathbf{R}}_b^n \hat{\mathbf{f}}^b$$

Be aware of the centripetal force to be compensated!

$$\begin{bmatrix} \dot{\hat{\phi}} \\ \dot{\hat{\lambda}} \\ \dot{\hat{h}} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_M + \hat{h}} & 0 & 0 \\ 0 & \frac{1}{\cos(\hat{\phi})(R_N + \hat{h})} & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \hat{v}_n \\ \hat{v}_e \\ \hat{v}_d \end{bmatrix}$$

WGS84 defining parameters

Name	Sysmbol	Value	Units
Equatorial radius	a	6378137	m
Reciprocal flattening	$\frac{1}{f}$	298.257223563	
Angular rate	ω_{ie}	7.292115×10^{-5}	$\frac{rad}{s}$
Gravitational constant	GM	$3.986004418 \times 10^{14}$	$\frac{m^3}{s^2}$

$$R_M(\phi) = \frac{a(1-e^2)}{\left(1 - e^2\sin^2(\phi)\right)^{\frac{3}{2}}} \qquad R_N(\phi) = \frac{a}{\left(1 - e^2\sin^2(\phi)\right)^{\frac{1}{2}}}$$

Model

$$\delta\dot{\mathbf{x}}(t) = \mathbf{F}(t)\delta\mathbf{x}(t) + \mathbf{\Gamma q}$$

$$\begin{bmatrix} \delta\dot{\mathbf{p}} \\ \delta\dot{\mathbf{v}} \\ \dot{\boldsymbol{\rho}} \\ \delta\dot{\mathbf{x}}_a \\ \delta\dot{\mathbf{x}}_g \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{pp} & \mathbf{F}_{pv} & \mathbf{F}_{p\rho} & \mathbf{0} & \mathbf{0} \\ \mathbf{F}_{vp} & \mathbf{F}_{vv} & \mathbf{F}_{v\rho} & -\mathbf{R}_b^n\mathbf{F}_{va} & \mathbf{0} \\ \mathbf{F}_{\rho p} & \mathbf{F}_{\rho v} & \mathbf{F}_{\rho\rho} & \mathbf{0} & \mathbf{R}_b^n\mathbf{F}_{\rho g} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{aa} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{F}_{gg} \end{bmatrix} \begin{bmatrix} \delta\mathbf{p} \\ \delta\mathbf{v} \\ \boldsymbol{\rho} \\ \delta\mathbf{x}_a \\ \delta\mathbf{x}_g \end{bmatrix}$$
$$+ \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{R}_b^n & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_b^n & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \nu_a \\ \nu_g \\ \boldsymbol{\omega}_\mathbf{a} \\ \boldsymbol{\omega}_\mathbf{g} \end{bmatrix}.$$

Position

$$\dot{\mathbf{p}} = \mathbf{f}_p(\mathbf{p}, \mathbf{v})$$

$$\dot{\mathbf{p}} = \mathbf{f}_p(\hat{\mathbf{p}}, \hat{\mathbf{v}}) + \mathbf{F}_{pp}\delta\mathbf{p} + \mathbf{F}_{pv}\delta\mathbf{v} + \mathbf{F}_{p\rho}\boldsymbol{\rho}$$

$$\delta\dot{\mathbf{p}} = \cancel{\mathbf{F}_{pp}}\delta\mathbf{p} + \mathbf{F}_{pv}\delta\mathbf{v} + \cancel{\mathbf{F}_{p\rho}}\boldsymbol{\rho}$$

Can be simplified -> [0] 😊

$$\mathbf{F}_{pp} = \cancel{\begin{bmatrix} 0 & 0 & \frac{-\hat{v}_n}{(R_M+\hat{h})^2} \\ \frac{\hat{v}_e \sin(\hat{\phi})}{((R_N+\hat{h})\cos(\hat{\phi})^2)} & 0 & \frac{-\hat{v}_e}{\left((R_N+\hat{h})^2\cos(\hat{\phi})\right)} \\ 0 & 0 & 0 \end{bmatrix}}$$

$$\mathbf{F}_{pv} = \begin{bmatrix} \frac{1}{(R_M+\hat{h})} & 0 & 0 \\ 0 & \frac{1}{((R_N+\hat{h})\cos(\hat{\phi}))} & 0 \\ 0 & 0 & -1 \end{bmatrix}, \text{ and}$$

Can be simplified for NED 😊

$$\mathbf{F}_{p\rho} = \mathbf{0}$$

Velocity

$$\dot{\mathbf{v}}_e^n = \hat{\mathbf{f}}^n + \hat{\mathbf{g}}^n - (\cancel{\hat{\boldsymbol{\Omega}}_{en}^n} + 2\cancel{\hat{\boldsymbol{\Omega}}_{ie}^n})\hat{\mathbf{v}}^n$$

$$\delta\dot{\mathbf{v}} = \cancel{\mathbf{F}_{vp}}\delta\mathbf{p} + \cancel{\mathbf{F}_{vv}}\delta\mathbf{v} + \mathbf{F}_{v\rho}\boldsymbol{\rho} - \mathbf{R}_b^n\delta\mathbf{f}^b$$

Can be simplified -> [0] 😊

$$\mathbf{F}_{vp} = \begin{bmatrix} -2\Omega_N v_e - \frac{\rho_N v_e}{\cos^2(\phi)} & 0 & \rho_E k_D - \rho_N \rho_D \\ 2(\Omega_N v_n + \Omega_D v_d) + \frac{\rho_N v_n}{\cos(\phi)^2} & 0 & -\rho_E\rho_D - k_D\rho_N \\ -2\hat{v}_e\Omega_D & 0 & F_{63} \end{bmatrix},$$

$$\mathbf{F}_{vv} = \begin{bmatrix} k_D & 2\omega_D & -\rho_E \\ -(\omega_D + \Omega_D) & (k_D - \rho_E\tan(\phi)) & \omega_N + \Omega_N \\ 2\rho_E & -2\omega_N & 0 \end{bmatrix}, \text{ and}$$

Can be simplified -> [0]

$$\mathbf{F}_{v\rho} = \begin{bmatrix} 0 & f_D & -f_E \\ -f_D & 0 & f_N \\ f_E & -f_N & 0 \end{bmatrix}.$$

Attitude

$$\dot{\boldsymbol{\rho}} = \hat{\mathbf{R}}_b^n(\delta\boldsymbol{\omega}_{ib}^b - \delta\boldsymbol{\omega}_{in}^b)$$

$$\dot{\boldsymbol{\rho}} = \cancel{\mathbf{F}_{\rho p}}\delta\mathbf{p} + \cancel{\mathbf{F}_{\rho v}}\delta\mathbf{v} + \cancel{\mathbf{F}_{\rho\rho}}\boldsymbol{\rho} + \hat{\mathbf{R}}_b^n\delta\boldsymbol{\omega}_{ib}^b$$

Can be simplified -> [0]

$$\mathbf{F}_{\rho p} = -\frac{\partial\hat{\boldsymbol{\omega}}_{in}^n}{\partial\hat{\mathbf{p}}} = \begin{bmatrix} \omega_{ie}\sin(\hat{\phi}) & 0 & \frac{v_e}{(R_N+\hat{h})^2} \\ 0 & 0 & \frac{-\hat{v}_n}{(R_M+\hat{h})^2} \\ \omega_{ie}\cos(\hat{\phi}) + \frac{\hat{v}_e}{(R_N+\hat{h})\cos(\hat{\phi})^2} & 0 & \frac{-\hat{v}_e\tan(\hat{\phi})}{(R_N+\hat{h})^2} \end{bmatrix}$$

$$\mathbf{F}_{\rho v} = -\frac{\partial\hat{\boldsymbol{\omega}}_{in}^n}{\partial\hat{\mathbf{v}}} = \begin{bmatrix} 0 & \frac{-1}{R_N+\hat{h}} & 0 \\ \frac{1}{R_M+\hat{h}} & 0 & 0 \\ 0 & \frac{\tan(\hat{\phi})}{R_N+\hat{h}} & 0 \end{bmatrix}, \text{ and}$$

Can be simplified -> [0]

$$\mathbf{F}_{\rho\rho} = -\hat{\boldsymbol{\Omega}}_{in}^n = \begin{bmatrix} 0 & \omega_D & -\omega_E \\ -\omega_D & 0 & \omega_N \\ \omega_E & -\omega_N & 0 \end{bmatrix}$$

Can be simplified -> [0]

Matrices F_{aa} and F_{gg} are set with respect of the bias model, which can be modeled as exponentially correlated random process or random constant.