On-Manifold MPC

连续系统动力学离散化:

$$\dot{\boldsymbol{p}} = \boldsymbol{v}$$

$$\dot{\boldsymbol{v}} = -a_T \boldsymbol{R}\boldsymbol{e}_3 - \frac{1}{m}\boldsymbol{R}\boldsymbol{D}\boldsymbol{R}^T\boldsymbol{v} + \boldsymbol{g}$$

$$\dot{\boldsymbol{R}} = \boldsymbol{R}\lfloor \boldsymbol{\omega} \rfloor$$

$$(\boldsymbol{e}_3 = [0, 0, -1]^T)$$

$$\lfloor \boldsymbol{\omega} \rfloor = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$\boxplus : \mathcal{M} \times \mathbb{R}^n \to \mathcal{M}; \quad \boxminus : \mathcal{M} \times \mathcal{M} \to \mathbb{R}^n$$

$$\mathcal{M} = SO(3) : \boldsymbol{R} \boxplus \boldsymbol{r} = \boldsymbol{R}\mathrm{Exp}(\boldsymbol{r});$$

$$\boldsymbol{R}_1 \boxminus \boldsymbol{R}_2 = \mathrm{Log}(\boldsymbol{R}_2^{\mathrm{T}} \boldsymbol{R}_1)$$

$$\mathcal{M} = \mathbb{R}^n : \quad \boldsymbol{a} \boxplus \boldsymbol{b} = \boldsymbol{a} + \boldsymbol{b};$$

$$\boldsymbol{a} \boxminus \boldsymbol{b} = \boldsymbol{a} - \boldsymbol{b}$$

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$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k \boxplus (\Delta t \boldsymbol{f}(\boldsymbol{x}_k, \boldsymbol{u}_k))$$

其中

$$\mathcal{M} = \mathbb{R}^3 \times \mathbb{R}^3 \times SO(3), \quad \dim(\mathcal{M}) = 9$$

$$\boldsymbol{x} \doteq \begin{bmatrix} \boldsymbol{p} & \boldsymbol{v} & \boldsymbol{R} \end{bmatrix} \in \mathcal{M}$$

$$\boldsymbol{u} \doteq \begin{bmatrix} a_T & \boldsymbol{\omega} \end{bmatrix}^T \in \mathbb{R}^4$$

$$\boldsymbol{f}(\boldsymbol{x}_k, \boldsymbol{u}_k) = \begin{bmatrix} \boldsymbol{v}_k \\ \boldsymbol{g} - a_{Tk}\boldsymbol{R}_k\boldsymbol{e}_3 - \frac{1}{m}\boldsymbol{R}_k\boldsymbol{D}\left(\boldsymbol{R}_k^d\right)^T \boldsymbol{v}_k \\ \boldsymbol{\omega}_k \end{bmatrix} \in \mathbb{R}^9$$

[1] G. Lu, W. Xu and F. Zhang, "On-Manifold Model Predictive Control for Trajectory Tracking on Robotic Systems," in IEEE Transactions on Industrial Electronics, vol. 70, no. 9, pp. 9192-9202, Sept. 2023, doi: 10.1109/TIE.2022.3212397.
[2] Yunfan Ren et al. ,Safety-assured high-speed navigation for MAVs.Sci. Robot.10,eado6187(2025). https://github.com/hku-mars/SUPER，控制器部分未开源

On-Manifold MPC

定义误差状态：

$$\delta \boldsymbol{x} \triangleq \boldsymbol{x}_d \boxminus \boldsymbol{x} = \begin{bmatrix} \delta \boldsymbol{p}^{\mathrm{T}} & \delta \boldsymbol{v}^{\mathrm{T}} & \delta \boldsymbol{R}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^9,$$

$$\delta \boldsymbol{p} \triangleq \boldsymbol{p}_d \boxminus \boldsymbol{p} = \boldsymbol{p}_d - \boldsymbol{p} \in \mathbb{R}^3,$$

$$\delta \boldsymbol{v} \triangleq \boldsymbol{v}_d \boxminus \boldsymbol{v} = \boldsymbol{v}_d - \boldsymbol{v} \in \mathbb{R}^3,$$

$$\delta \boldsymbol{R} \triangleq \boldsymbol{R}_d \boxminus \boldsymbol{R} = \mathrm{Log}(\boldsymbol{R}^{\mathrm{T}} \boldsymbol{R}_d) \in \mathbb{R}^3,$$

$$\delta \boldsymbol{u} \triangleq \boldsymbol{u}_d - \boldsymbol{u} = \begin{bmatrix} \delta a_T & \delta \boldsymbol{\omega}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^4,$$

$$\delta a_T \triangleq a_{T_d} - a_T \in \mathbb{R},$$

$$\delta \boldsymbol{\omega} \triangleq \boldsymbol{\omega}_d - \boldsymbol{\omega} \in \mathbb{R}^3,$$

误差传播：

$$\begin{aligned} \delta \boldsymbol{x}_{k+1} &= \boldsymbol{x}_{k+1}^d \boxminus \boldsymbol{x}_{k+1} \\ &= \left(\boldsymbol{x}_k^d \boxplus \Delta t \boldsymbol{f}(\boldsymbol{x}_k^d, \boldsymbol{u}_k^d) \right) \boxminus \left(\boldsymbol{x}_k \boxplus \Delta t \boldsymbol{f}(\boldsymbol{x}_k, \boldsymbol{u}_k) \right) \\ &= \left(\boldsymbol{x}_k^d \boxplus \Delta t \boldsymbol{f}(\boldsymbol{x}_k^d, \boldsymbol{u}_k^d) \right) \\ &\quad \boxminus \left(\left(\boldsymbol{x}_k^d \boxplus (-\delta \boldsymbol{x}_k) \right) \boxplus \Delta t \boldsymbol{f}\left(\boldsymbol{x}_k^d \boxplus (-\delta \boldsymbol{x}_k), \boldsymbol{u}_k^d - \delta \boldsymbol{u}_k \right) \right) \end{aligned}$$

定义 $\boldsymbol{g}(\delta \boldsymbol{x}_k, \delta \boldsymbol{u}_k) = \Delta t \boldsymbol{f}\left(\boldsymbol{x}_k^d \boxplus (-\delta \boldsymbol{x}_k), \boldsymbol{u}_k^d - \delta \boldsymbol{u}_k \right)$

则有

$$\delta \boldsymbol{x}_{k+1} = \underbrace{\left(\boldsymbol{x}_k^d \boxplus \boldsymbol{g}(\mathbf{0}, \mathbf{0}) \right) \boxminus \left(\left(\boldsymbol{x}_k^d \boxplus (-\delta \boldsymbol{x}_k) \right) \boxplus \boldsymbol{g}(\delta \boldsymbol{x}_k, \delta \boldsymbol{u}_k) \right)}_{\boldsymbol{G}(\delta \boldsymbol{x}_k, \boldsymbol{g}(\delta \boldsymbol{x}_k, \delta \boldsymbol{u}_k))}$$

On-Manifold MPC

线性化: $$\delta \mathbf{x}_{k+1} \simeq \boldsymbol{F}_{\mathbf{x}} \delta \mathbf{x}_k + \boldsymbol{F}_{\boldsymbol{u}} \delta \boldsymbol{u}_k$$

线性化于 $\delta \mathbf{x}_k = \mathbf{0}, \delta \boldsymbol{u}_k = \mathbf{0}$

其中 $$\boldsymbol{F}_{\mathbf{x}} = \left(\frac{\partial \boldsymbol{G}(\delta \mathbf{x}_k, \boldsymbol{g}(\mathbf{0},\mathbf{0}))}{\partial \delta \mathbf{x}_k} + \frac{\partial \boldsymbol{G}(\mathbf{0}, \boldsymbol{g}(\delta \mathbf{x}_k, \mathbf{0}))}{\partial \boldsymbol{g}(\delta \mathbf{x}_k, \mathbf{0})} \frac{\partial \boldsymbol{g}(\delta \mathbf{x}_k, \mathbf{0})}{\partial \delta \mathbf{x}_k} \right) \Bigg|_{\delta \mathbf{x}_k = \mathbf{0}} \quad \boldsymbol{F}_{\boldsymbol{u}} = \left(\frac{\partial \boldsymbol{G}(\mathbf{0}, \boldsymbol{g}(\mathbf{0}, \delta \boldsymbol{u}_k))}{\partial \boldsymbol{g}(\mathbf{0}, \delta \boldsymbol{u}_k)} \frac{\partial \boldsymbol{g}(\mathbf{0}, \delta \boldsymbol{u}_k)}{\partial \delta \boldsymbol{u}_k} \right) \Bigg|_{\delta \boldsymbol{u}_k = \mathbf{0}}$$

$$\boldsymbol{G}(\delta \mathbf{x}_k, \boldsymbol{g}(\delta \mathbf{x}_k, \delta \boldsymbol{u}_k)) = \left(\mathbf{x}_k^d \boxplus \boldsymbol{g}(\mathbf{0},\mathbf{0}) \right) \boxminus \left(\left(\mathbf{x}_k^d \boxplus (-\delta \mathbf{x}_k) \right) \boxplus \boldsymbol{g}(\delta \mathbf{x}_k, \delta \boldsymbol{u}_k) \right)$$

$$:= \boldsymbol{d} \boxminus \left((\boldsymbol{a} \boxplus \boldsymbol{b}) \boxplus \boldsymbol{c} \right), \quad \boldsymbol{G}, \boldsymbol{b}, \boldsymbol{c} \in \mathbb{R}^9, \boldsymbol{a}, \boldsymbol{d} \in \mathbb{R}^6 \times SO(3)$$

情况 1: $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d} \in \mathbb{R}^n$ $$\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{b}} = \frac{\partial \boldsymbol{G}}{\partial \boldsymbol{c}} = -\boldsymbol{I}_n$$

情况 2: $\boldsymbol{G}, \boldsymbol{b}, \boldsymbol{c} \in \mathbb{R}^3, \boldsymbol{a}, \boldsymbol{d} \in SO(3)$ $$\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{b}} = -\boldsymbol{A}(\boldsymbol{G})^{-1} \operatorname{Exp}(-\boldsymbol{c}) \boldsymbol{A}(\boldsymbol{b})^T, \quad \frac{\partial \boldsymbol{G}}{\partial \boldsymbol{c}} = -\boldsymbol{A}(\boldsymbol{G})^{-1} \boldsymbol{A}(\boldsymbol{c})^T$$

其中 $$\boldsymbol{A}(\boldsymbol{u}) = \boldsymbol{I} + \frac{1 - \cos\|\boldsymbol{u}\|}{\|\boldsymbol{u}\|} \frac{\lfloor \boldsymbol{u} \rfloor}{\|\boldsymbol{u}\|} + \left(1 - \frac{\sin\|\boldsymbol{u}\|}{\|\boldsymbol{u}\|} \right) \frac{\lfloor \boldsymbol{u} \rfloor^2}{\|\boldsymbol{u}\|^2}$$

(左雅可比矩阵，源于 Baker-Campbell-Hausdorff (BCH) 公式)

On-Manifold MPC

线性化: $\delta \boldsymbol{x}_{k+1} \simeq \boldsymbol{F}_{\boldsymbol{x}} \delta \boldsymbol{x}_k + \boldsymbol{F}_{\boldsymbol{u}} \delta \boldsymbol{u}_k$

情况 2 推导:

$\frac{\partial G}{\partial b}$: $G = \mathrm{Log}\left(((a \boxplus b) \boxplus c)^T d\right)$ $\quad \mathrm{Exp}(G) = ((a \boxplus b) \boxplus c)^T d = \mathrm{Exp}(-c)\,\mathrm{Exp}(-b)\, a^{-1} d$

对G和b施加扰动: $\mathrm{Exp}(G + \Delta G) = \mathrm{Exp}(-c)\, \mathrm{Exp}[-(b + \Delta b)]\, a^{-1} d$

$$\mathrm{Exp}(\underset{\text{左扰动}}{A(G)\Delta G})\, \mathrm{Exp}(G) = \mathrm{Exp}(-c) \left(\mathrm{Exp}(b)\, \mathrm{Exp}(\underset{\text{右扰动}}{A^T(b)\Delta b})\right)^T a^{-1} d$$

← Lie 群上的BCH 公式

A为左雅克比阵

$$A(\theta) = I_3 + \left(\frac{1-\cos\|\theta\|}{\|\theta\|}\right)\frac{\lfloor\theta\rfloor}{\|\theta\|} + \left(1 - \frac{\sin\|\theta\|}{\|\theta\|}\right)\frac{\lfloor\theta\rfloor^2}{\|\theta\|^2}$$

$$\mathrm{Exp}(A(G)\Delta G) = \mathrm{Exp}(-c)(\cdots)^T a^{-1} d\, d\, a\, \mathrm{Exp}(b)\, \mathrm{Exp}(c)$$

$$= \mathrm{Exp}(-c)\, \mathrm{Exp}^T(A^T(b)\Delta b)\, \mathrm{Exp}(-b)\, a^{-1} d\, d\, a\, \mathrm{Exp}(b)\, \mathrm{Exp}(c)$$

$$= \underset{R}{\mathrm{Exp}(-c)}\, \underset{\lfloor p \rfloor}{\mathrm{Exp}^T(A^T(b)\Delta b)}\, \underset{R^T}{\mathrm{Exp}(c)}$$

$R\lfloor p\rfloor R^T = \lfloor Rp \rfloor$

$\mathrm{Exp}(-c) = \mathrm{Exp}^T(c)$

更常见的形式是

$$A(\theta\vec{a}) = \frac{\sin\theta}{\theta} I_3 + \left(1 - \frac{\sin\theta}{\theta}\right) a a^T + \left(\frac{1-\cos\theta}{\theta}\right)\lfloor a\rfloor$$

$$\mathrm{Exp}(A(G)\Delta G) = \mathrm{Exp}\left(\mathrm{Exp}(-c)(-A^T(b)\Delta b)\right)$$

$$A(G)\Delta G = -\mathrm{Exp}(-c)\, A^T(b)\Delta b$$

$$\frac{\partial G}{\partial b} = -A(G)^{-1}\, \mathrm{Exp}(-c)\, A^T(b)$$

$\frac{\partial G}{\partial c}$: $G = \mathrm{Log}\left(((a \boxplus b) \boxplus c)^T d\right)$ $\quad \mathrm{Exp}(G) = ((a \boxplus b) \boxplus c)^T d = \mathrm{Exp}(-c)\,\mathrm{Exp}(-b)\, a^{-1} d$

$$\mathrm{Exp}(G + \Delta G) = \mathrm{Exp}(-(c + \Delta c))\, \mathrm{Exp}(-b)\, a^{-1} d$$

$$\mathrm{Exp}(A(G)\Delta G)\, \mathrm{Exp}(G) = \left(\mathrm{Exp}(c)\, \mathrm{Exp}(A^T(c)\Delta c)\right)^T \mathrm{Exp}(-b)\, a^{-1} d$$

← Lie 群上的BCH 公式

$$\mathrm{Exp}(A(G)\Delta G) = \mathrm{Exp}^T(A^T(c)\Delta c)\, \cancel{\mathrm{Exp}^T(c)}\, \cancel{\mathrm{Exp}(-b)\, a^{-1} d} \times \cancel{d\, a}\, \mathrm{Exp}(b)\, \cancel{\mathrm{Exp}(c)}$$

$$\mathrm{Exp}(A(G)\Delta G) = \mathrm{Exp}(-A^T(c)\Delta c)$$

$$\frac{\Delta G}{\Delta c} = -A(G)^{-1} A^T(c)$$

On-Manifold MPC

线性化: $\delta \boldsymbol{x}_{k+1} \simeq \boldsymbol{F}_{\boldsymbol{x}} \delta \boldsymbol{x}_k + \boldsymbol{F}_{\boldsymbol{u}} \delta \boldsymbol{u}_k$

$$\left.\frac{\partial \boldsymbol{G}(\delta \boldsymbol{x}_k, \boldsymbol{g}(\boldsymbol{0},\boldsymbol{0}))}{\partial \delta \boldsymbol{x}_k}\right|_{\delta \boldsymbol{x}_k=\boldsymbol{0}} = (-1)\left.\frac{\partial \boldsymbol{G}(\delta \boldsymbol{x}_k, \boldsymbol{g}(\boldsymbol{0},\boldsymbol{0}))}{\partial (-\delta \boldsymbol{x}_k)}\right|_{\delta \boldsymbol{x}_k=\boldsymbol{0}} = \begin{bmatrix} \boldsymbol{I}_6 & \\ & \boldsymbol{A}(\boldsymbol{0})^{-1}\mathrm{Exp}\left(-\boldsymbol{g}(\boldsymbol{0},\boldsymbol{0})\right)\boldsymbol{A}(\boldsymbol{0})^T \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_6 & \\ & \mathrm{Exp}\left(-\boldsymbol{\omega}_k^d \Delta t\right) \end{bmatrix}$$

$$\left.\frac{\partial \boldsymbol{G}(\boldsymbol{0}, \boldsymbol{g}(\delta \boldsymbol{x}_k,\boldsymbol{0}))}{\partial \boldsymbol{g}(\delta \boldsymbol{x}_k,\boldsymbol{0})}\right|_{\delta \boldsymbol{x}_k=\boldsymbol{0}} = \begin{bmatrix} -\boldsymbol{I}_6 & \\ & -\boldsymbol{A}(\boldsymbol{0})^{-1}\boldsymbol{A}\left(\boldsymbol{g}(\boldsymbol{0},\boldsymbol{0})\right)^T \end{bmatrix} = \begin{bmatrix} -\boldsymbol{I}_6 & \\ & -\boldsymbol{A}\left(\boldsymbol{\omega}_k^d \Delta t\right)^T \end{bmatrix}, \quad \left.\frac{\partial \boldsymbol{G}(\boldsymbol{0}, \boldsymbol{g}(\boldsymbol{0},\delta \boldsymbol{u}_k))}{\partial \boldsymbol{g}(\boldsymbol{0},\delta \boldsymbol{u}_k)}\right|_{\delta \boldsymbol{u}_k=\boldsymbol{0}} = -\boldsymbol{I}_4$$

$$\boldsymbol{g}(\delta \boldsymbol{x}_k, \delta \boldsymbol{u}_k) = \Delta t \begin{bmatrix} \boldsymbol{v}_k^d - \delta \boldsymbol{v}_k \\ \boldsymbol{g} - \left(a_{Tk}^d - \delta a_{Tk}\right)\boldsymbol{R}_k^d \mathrm{Exp}\left(-\delta \boldsymbol{R}_k\right)\boldsymbol{e}_3 - \frac{1}{m}\boldsymbol{R}_k^d \mathrm{Exp}\left(-\delta \boldsymbol{R}_k\right)\boldsymbol{D}\left(\boldsymbol{R}_k^d \mathrm{Exp}\left(-\delta \boldsymbol{R}_k\right)\right)^T\left(\boldsymbol{v}_k^d - \delta \boldsymbol{v}_k\right) \\ \boldsymbol{\omega}_k^d - \delta \boldsymbol{\omega}_k \end{bmatrix}$$

$$\left.\frac{\partial \boldsymbol{g}(\delta \boldsymbol{x}_k,\boldsymbol{0})}{\partial \delta \boldsymbol{x}_k}\right|_{\delta \boldsymbol{x}_k=\boldsymbol{0}} = \begin{array}{c} \begin{array}{ccc} \delta \boldsymbol{p} & \delta \boldsymbol{v} & \delta \boldsymbol{R} \end{array} \\ \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{I}_3 \Delta t & \boldsymbol{0} \\ \boldsymbol{0} & \frac{\Delta t}{m}\boldsymbol{R}_k^d \boldsymbol{D}\left(\boldsymbol{R}_k^d\right)^T & \star \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \end{array}, \quad \left.\frac{\partial \boldsymbol{g}(\boldsymbol{0},\delta \boldsymbol{u}_k)}{\partial \delta \boldsymbol{u}_k}\right|_{\delta \boldsymbol{u}_k=\boldsymbol{0}} = \begin{array}{c} \begin{array}{cc} \delta a & \delta \boldsymbol{\omega} \end{array} \\ \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{R}_k^d \boldsymbol{e}_3 \Delta t & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{I}_3 \Delta t \end{bmatrix} \end{array}$$

On-Manifold MPC

线性化: $$\delta \mathbf{x}_{k+1} \simeq \boldsymbol{F}_{\boldsymbol{x}} \delta \mathbf{x}_k + \boldsymbol{F}_{\boldsymbol{u}} \delta \boldsymbol{u}_k$$

★ 的推导:

*的求取: 为求简便, 略去重力项、Δt 和下标, 记 $G = -aRe_3 - \frac{1}{m}RDR^Tv$

对 G 施加扰动有 $R' = R\exp(-\lfloor\delta\rfloor) \approx R(I - \lfloor\delta\rfloor)$

则 $G(R') = -aR'e_3 - \frac{1}{m}R'DR'^Tv$

展开至一阶有 $R'e_3 = Re_3 - R\lfloor\delta\rfloor e_3$

$R'DR'^Tv = R(I - \lfloor\delta\rfloor)D(I - \lfloor\delta\rfloor)R^Tv$

$\approx RDR^Tv + R(-\lfloor\delta\rfloor D + D\lfloor\delta\rfloor)R^Tv$ (忽略高阶项)

因此 $G(R') = -a(Re_3 - R\lfloor\delta\rfloor e_3) - \frac{1}{m}\left[RDR^Tv + R(-\lfloor\delta\rfloor D + D\lfloor\delta\rfloor)R^Tv\right]$

$\Rightarrow \Delta G = G(R') - G(R) = aR\lfloor\delta\rfloor e_3 - \frac{1}{m}\left[R(-\lfloor\delta\rfloor D + D\lfloor\delta\rfloor)R^Tv\right]$

利用性质 $\lfloor\delta\rfloor u = \delta \times u = -u \times \delta = -\lfloor u\rfloor \delta$ 有

$\Delta G = -aR\lfloor e_3\rfloor\delta + \frac{1}{m}R\lfloor\delta\rfloor DR^Tv - \frac{1}{m}RD\lfloor\delta\rfloor R^Tv$

$= -aR\lfloor e_3\rfloor\delta - \frac{1}{m}R\lfloor DR^Tv\rfloor\delta + \frac{1}{m}RD\lfloor R^Tv\rfloor\delta$

$\Rightarrow \frac{\Delta G}{\delta} = -aR\lfloor e_3\rfloor - \frac{1}{m}R\lfloor DR^Tv\rfloor + \frac{1}{m}RD\lfloor R^Tv\rfloor$

$$\boldsymbol{F}_{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{I}_3 & \boldsymbol{I}_3\Delta t & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_3 - \frac{\Delta t}{m}\boldsymbol{R}_k^d\boldsymbol{D}\left(\boldsymbol{R}_k^d\right)^T & \frac{\Delta t}{m}\boldsymbol{R}_k^d\left(a_{Tk}^d m\lfloor\boldsymbol{e}_3\rfloor + \boldsymbol{R}_k^d\left\lfloor\boldsymbol{D}\left(\boldsymbol{R}_k^d\right)^T\boldsymbol{v}_k^d\right\rfloor - \boldsymbol{D}\left\lfloor\left(\boldsymbol{R}_k^d\right)^T\boldsymbol{v}_k^d\right\rfloor\right) \\ \mathbf{0} & \mathbf{0} & \mathrm{Exp}(-\boldsymbol{\omega}_k^d\Delta t) \end{bmatrix}, \quad \boldsymbol{F}_{\boldsymbol{u}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\boldsymbol{R}_k^d\boldsymbol{e}_3 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{A}\left(\boldsymbol{\omega}_k^d\Delta t\right)^T \end{bmatrix}\Delta t$$

$$\xrightarrow{\boldsymbol{D}=\mathbf{0}} \boldsymbol{F}_{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{I}_3 & \boldsymbol{I}_3\Delta t & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_3 & \Delta t\boldsymbol{R}_k^d a_{Tk}^d\lfloor\boldsymbol{e}_3\rfloor \\ \mathbf{0} & \mathbf{0} & \mathrm{Exp}(-\boldsymbol{\omega}_k^d\Delta t) \end{bmatrix},$$

On-Manifold MPC

优化问题构建:

$$
\begin{aligned}
\min_{\delta \mathbf{u}_k} \sum_{k=0}^{N-1} & \left(\|\delta \mathbf{x}_k\|_{\mathbf{Q}}^2 + \|\delta \mathbf{u}_k\|_{\mathbf{R}}^2\right) + \|\delta \mathbf{x}_N\|_{\mathbf{P}}^2, \\
\text{s.t.} \quad & \delta \mathbf{x}_{k+1} = \mathbf{F}_{\mathbf{x}_k} \delta \mathbf{x}_k + \mathbf{F}_{\mathbf{u}_k} \delta \mathbf{u}_k, \\
& \delta \mathbf{u}_k \in \delta \mathbb{U}_k, \quad k = 0, \cdots, N-1,
\end{aligned}
$$

与 FAST_LIO的比较:

G.T. estimation

$x_k \boxminus \hat{x}_k$

disturbance

Linearized Error Dynamics of

LIO $\tilde{x}_{k+1} = F_{\tilde{x}} \tilde{x}_k + F_w w_i$

Linearize at (0,0)

OMMPC $\delta x_{k+1} = F_x \delta x_k + F_u \delta u_k$

$x_k^d \boxminus x_k$ deviation from desired state

$u_k^d \boxminus u_k$ deviation from desired control input