

(*This mathematica code solves the differential equation for the height of the interface between the almost colloid free liquid phase and the gel of particles. The differential equation can be found in the paper by Alexis Darras et al. named: "Erythrocyte sedimentation: Fracture and collapse of a high-volume-fraction soft-colloid gel".
 A permeability which depends on the fractal dimension is used.
 A scaled time and height are used. The results from the two-dimensional model are included for comparison.
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In[ ]:= Clear["Global`*"]
(*Variable values:*)
phi0 = 4/10; (*initial colloid volume fraction*)
h0 = 4; (*[cm] initial height of the interface*)
a = 0.00033; (*[cm] size of RBC*)
drho = 80 * 10^(-6);
(*density difference between the colloids and the suspending medium*)
g = 981; (*[cm/s^2] gravitational acceleration*)
mu = 0.000012; (*[kg/(cm s)] suspending medium viscosity*)
phim = 86/100; (*maximum colloid volume fraction*)
r = 3.3 * 10^(-4); (*cm, radius of RBC*)
tau = 6 * mu * h0 / (r^2 * drho * g); (*sedimentation time of a single RBC*)
hm = h0 * phi0 / phim;
(*minimum height of the interface*)

(*Integration:*)
tfinal = 10^16;
df1 = 1.7;
hsol1 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df1 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];
df2 = 1.8;
hsol2 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df2 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];
df3 = 1.9;
hsol3 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df3 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];
df4 = 2;
hsol4 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df4 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];
df5 = 2.1;
hsol5 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df5 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[1] == h0}, h, {t, 0, tfinal}];
df6 = 2.2;
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hsol6 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df6 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];
df7 = 2.3;
hsol7 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df7 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];
df8 = 2.4;
hsol8 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df8 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];
df9 = 2.5;
hsol9 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df9 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];
df10 = 2.6;
hsol10 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df10 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];
df11 = 2.7;
hsol11 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df11 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];
df12 = 2.8;
hsol12 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df12 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];
df13 = 2.9;
hsol13 = NDSolveValue[{h'[t] ==
  - ((drho * g * a^2) / mu) * (phim - phi0 * h0 / (h[t]))^3 *
  ((phi0 * h0 / (h[t]))^(1 + 2 / (df13 - 3))) / ((1 - phi0 * h0 / (h[t]))),
  h[0] == h0}, h, {t, 0, tfinal}];

In[ ]:= (*Plot:*)
tfplot = 3 * 10^4;
legend1 = {"1.7", "1.8", "1.9", "2.0", "2.1", "2.2", "2.3", "2.4", "2.5", "2.6", "2.7",
  "2.8", "2.9", FractionBox[Subscript[h, m], Subscript[h, 0]] // DisplayForm};
coll = Table[Hue[0.65 * i / Length[legend1]], {i, 1, Length[legend1]}];
p1 = Plot[{hsol1[t * tau] / h0, hsol2[t * tau] / h0, hsol3[t * tau] / h0,
  hsol4[t * tau] / h0, hsol5[t * tau] / h0, hsol6[t * tau] / h0, hsol7[t * tau] / h0,
  hsol8[t * tau] / h0, hsol9[t * tau] / h0, hsol10[t * tau] / h0,
  hsol11[t * tau] / h0, hsol12[t * tau] / h0, hsol13[t * tau] / h0, hm / h0},
{t, 0, tfplot / tau}, PlotRange -> {0, 1}, AxesLabel -> {t, h[t]}, PlotPoints -> 1000,
PlotLegends -> legend1, PlotStyle -> coll, LabelStyle -> Directive[FontSize -> 14]]

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In[ ]:= (*Export picture:*)
Export[NotebookDirectory[] <> "1D_h(t)_plot_df_scaled.png", p1, ImageResolution → 1000]

In[ ]:= (*LogLinearPlot:*)
tfplot = 10^7;
legend1 = {"1.7", "1.8", "1.9", "2.0", "2.1", "2.2", "2.3", "2.4", "2.5", "2.6", "2.7",
  "2.8", "2.9", FractionBox[Subscript[h, m], Subscript[h, 0]] // DisplayForm};
coll = Table[Hue[0.65 * i / Length[legend1]], {i, 1, Length[legend1]}];
p2 = LogLinearPlot[{hsol1[t * tau] / h0, hsol2[t * tau] / h0,
  hsol3[t * tau] / h0, hsol4[t * tau] / h0, hsol5[t * tau] / h0, hsol6[t * tau] / h0,
  hsol7[t * tau] / h0, hsol8[t * tau] / h0, hsol9[t * tau] / h0, hsol10[t * tau] / h0,
  hsol11[t * tau] / h0, hsol12[t * tau] / h0, hsol13[t * tau] / h0, hm / h0},
{t, 10^(-7) / tau, tfplot / tau}, PlotRange → {0, 1}, AxesLabel → {t, h[t]},
PlotPoints → 1000, (*PlotLabel → "Height of the interface", *)
PlotLegends → legend1, PlotStyle → coll, LabelStyle → Directive[FontSize → 14]]

In[ ]:= (*Export picture:*)
Export[NotebookDirectory[] <> "1D_h(t)_Logplot_df_scaled.png",
p2, ImageResolution → 1000]

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