Astronomy, Space Science and Astrophysics

Exp.4 Gyroscopes

PH520 - Stage 2

Physics Laboratory A

Date: 21st Oct - 11th Nov 2019

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Word Count: 2414

Contents

1 Abstract

This experiment proves the theoretical anticipations of the precession and the nutation of a gyroscope as its proved that the an external torque force directly affects how the pendulum/ gyroscope motion acts. It also shows that the angular frequency is directly related to the angular frequency of precession and nutation as the gyroscope is in motion and how the Sun and other celestial bodies affect the Earth's rotation on its axis and why the North pole rotates every 26,000 years and causes "ice ages". Throughout this experiment, many errors arise as the human error take precedent due to reaction time set at $+0.5$s and the human manipulation of the speed of the torque that is induced upon the gyroscopes.

2 Introduction

"As this experiment observes rotational motion, the utilisation of newtons laws of torque and angular momentum will take effect as it shows how gyroscopes are formed and affected by multiple factors. This can be scaled to large masses such as planets and celestial objects, such as the Earth. The Earth rotates on its axis while orbiting the Sun, so as the Sun exerts a force upon Earth, other celestial bodies exert a torque that affects the Earth's angular momentum so as it presses, it causes a nutating rotation of its Northern pole every 26,000 years which causes "ice ages" to appear. Precesison of a gyroscope occurs when a applied torque cause the gyroscope to rotate in the same direction due to the weight of the spinning gyroscope, the torque is perpendicular to the angular momentum of the gyroscope. Nutation of a gyroscope occurs when the gyroscopes experiences precession and another small rotation takes place which is directly related to another external pushing/ pulling force. " [2]

3 Methodology

3.1 Theory

Angular frequency [2]:

$$w = 2\pi f = \frac{2\pi}{\Delta t} n \tag{1}$$

Torque for a small angle [2]:

$$T = M_{wt}\ g\ sin\theta\ R_{wt} \approx M_{wt}\ g\ \theta\ R_{wt} = k_{wt}\ \theta\ [2] \tag{2}$$

Moment of Inertia of A1 Disk [2]:

$$I_{A1} = M_{wt}\ R_{wt}\ g\ \Delta t^2/4\pi^2) - (M_{wt}\ {R_{wt}}^2) \tag{3}$$

Useful Constants [2]:

Parameter	Symbol	Value
Standard gravitational acceleration	g	$9.80665 ms^{-2}$ [1]
Mass of the gyroscope (inc. spindle)	M_{Gyro}	(3.15 ±0.01)kg
Mass of A1 disk	M_{A1}	(0.72955 ±0.00002)kg

Table 1: Useful Constants. [2]

3.2 Procedure

3.2.1 Moment of inertia of an aluminium disk

Physical pendulum:

The apparatus was setup as shown in fig. 1, the use of a vertically rod connected to a stand was used to elevate the aluminium disk that hung vertically off a horizontal rod connected via a multi-clamp to the bench clamp. The disk was then exposed to a torque from attaching two weights approx 100g each, and the mass of the screw and nut (all measured with a set of scales with a $\pm 0.01g$ tolerance) at the base of the aluminium disk. Note that the horizontal rod suspending the disk and the weights attached to the disk are opposite each other at each opposite edge of the A1 disk at a distance R (to be measured with a ruler with a $\pm 0.01cm$ tolerance). By moving the weights thus creating a small angle, a torque was formed, thus the period was repeatedly measured and an average value was formed as the period Δt timed via a stopwatch with a $\pm 0.01s$ tolerance while factoring in a reaction time at $+0.5$, thus allowing the moment of inertia for the A1 disk eq. (3) to be determined.

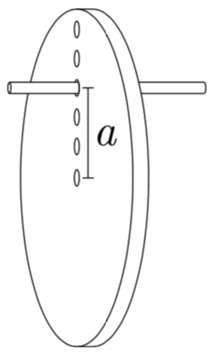

Figure 1: A1 disk setup for measuring moment of inertia. [2]

Steiner's law:

Utilizing the same setup in fig. 1, the d ≈ 4cm spaced holes on the aluminium disk were measured with the vernier with a $\pm 0.01cm$ tolerance, the same weights, screw and nut (all measured with a set of scales with a $\pm 0.01g$ tolerance) were added to the edge of the aluminium disk, the horizontal rod suspending the disk was moved into the centre of the disk. By applying the disk with a small angle by shifting the weights to one side to provide a torque, this was repeated 3 times and the distance between the horizontal support and the weights measured at 0cm to 16cm in of 4cm increments by moving the horizontal rod to each pre-cut hole. The period of these pendulum swings is measured by a stopwatch with a ±0.01s error while factoring in a reaction time at +0.5s, the physical data of the period can be compared with the theoretical data.

3.2.2 Precession of a gyroscope

Moment of inertia:

By setting the gyroscope vertically supported by a horizontal rod connected to a stand, a infrared sensor is supported via a bench clamp that is connected to a digital counter and is placed so that the gyroscope is in between the two sensors so that the sensors can count the spokes of the gyroscope as seen in fig. 2 so that the period of the spinning gyroscope can be determined. At the base of the the gyroscopes, two weights, nut and bolt (all measured with a set of scales with a $\pm 0.01g$ tolerance) are hung to provide a torque so when a small angle is induced, the gyroscope is subject to a pendulum motion. This motion will produce a period to which will be measured via the digital counter with a ±0.001 error, this is repeated 5 times and an average value is deduced.

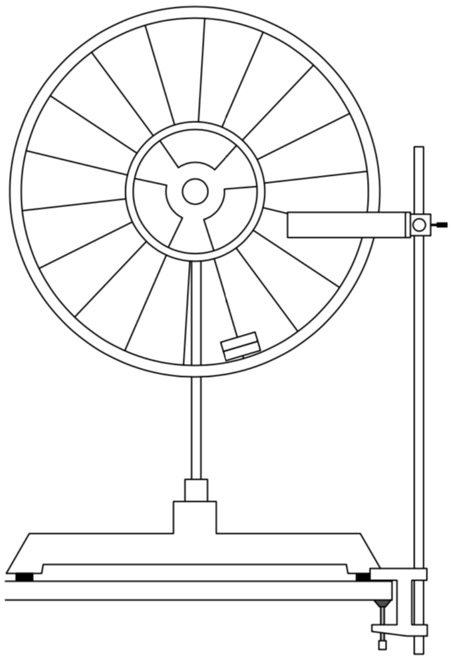

Figure 2: Setup of the gyroscope to form a physical pendulum. [2]

Centre of gravity:

Suspending the gyroscope horizontally on a vertical rod connected to a stand as shown in fig. 4 and thus changing the gyroscopes support pole height s as seen in fig. 3 by using an hex key to loosen and tighten a pin that secures the gyroscope to the support pole. The centre of gravity can be found when the gyroscope is perfectly balanced on the vertical rod and when subject to an external force will return to is original position. The use of the vernier caliper is to measure the distance $s(s_0 + d)$ shown on fig. 3 with a tolerance of $\pm 0.01cm$.

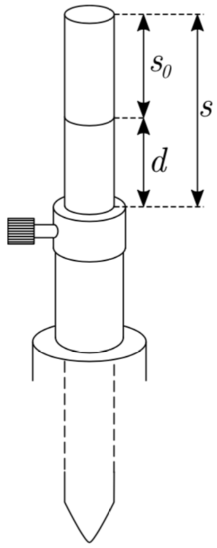

Figure 3: Changing the height of the gyroscope to find the centre of gravity [2]

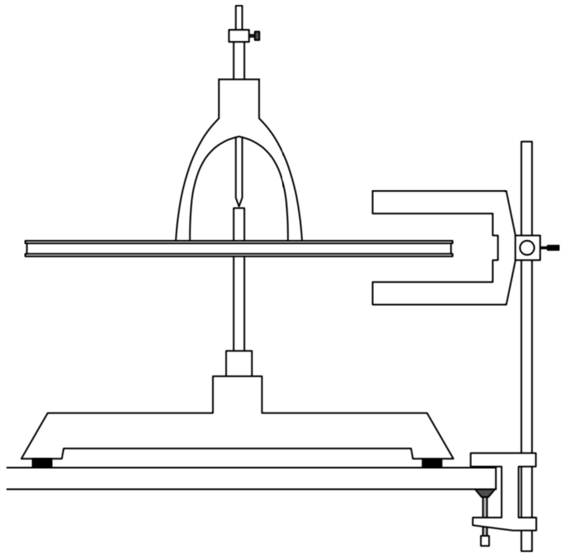

Figure 4: Setup of the gyroscope to find its centre of gravity. [2]

Precession:

By finding the centre of gravity of the gyroscope, by suspending the gyroscope via its support pole horizontally on a vertical rod shown in fig. 4, the gyroscope is in a state of equilibrium. To find the precession, the height of the support pole must be altered, by changing $s(s_0 + d)$ in fig. 3 by $\pm 3cm$ in 1cm increments (measured with vernier calipers with a $\pm 0.01cm$ tolerance) allows a variable rate of precession to take place.

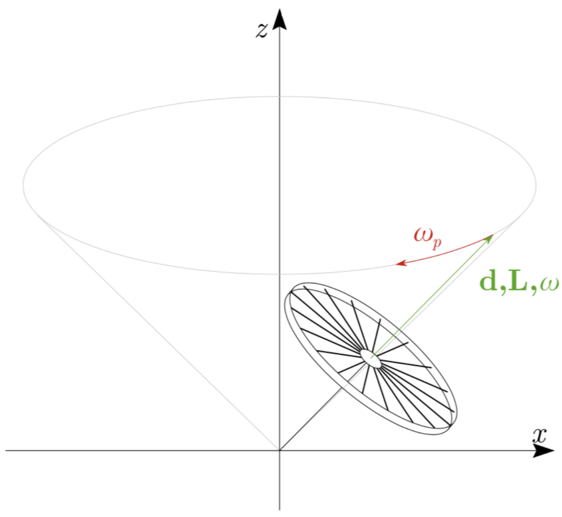

Figure 5: Diagram showing the precession of the gyroscope. [2]

By placing the infrared sensor so that it can count the spokes of the gyroscope and connect the infrared sensor to the digital counter, the precession can be measured. By spinning the gyroscope at the different heights, precession takes place upon the gyroscope where the digital counter counts the frequency of the gyroscopes time period (rotation) with a $\pm 0.001s$ tolerance and the stopwatch is used to measure the top of the gyroscope as it makes a complete rotation with a $\pm 0.01s$ tolerance as the gyroscope will now spin of an angled axis and will slowly rotate, this is precession. With the values recorded, the angular frequency and angular frequency of precession shown in fig. 5 can be determined.

3.2.3 Nutation of a gyroscope

By utilizing the set up shown in fig. 4 where the gyroscope sits horizontal suspending of its support pole on a vertical rod connected to a stand, with the infrared sensor positioned so which its sensors can measure the spokes of the gyroscope, with the infrared sensor connected to the digital counter. By changing the height of the gyroscopes support pole $s(s_0 + d)$ in fig. 3 to find the centre of gravity, once found, with the gyroscope spinning a small vertical external force is applied to allow nutation of the gyroscope to take place, where the gyroscope 3 rotational motions are taking place as shown in fig. 6.

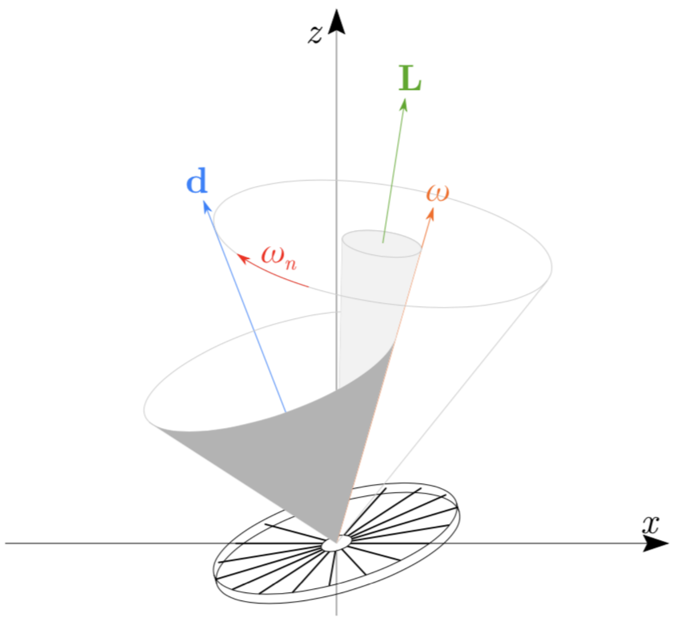

Figure 6: Diagram showing the nutation of the gyroscope. [2]

With the digital counter set to count the frequency of the gyroscopes time period (rotation) with a $\pm 0.001s$ tolerance and the stopwatch is used to measure the top of the gyroscope with a $\pm 0.01s$ tolerance as it makes a complete rotation w. w_n can then be calculate and compared with w and relationship can formed.

4 Results & Findings

4.1 Moment of inertia of an aluminium disk

4.1.1 Physical pendulum

Starting with a simple pendulum set up utilising the A1 disk shown in fig. 1, the total mass of the two weights, screw and nut measured to be 195.84g with the mass of the screw taken into effect shown in table 2. By setting the distance between the horizontal support point and the weights at 16.2cm $\pm 0.1cm$. There is no way to accurately measure the angle that was induced but the angle doesn't affect the period of the pendulum, the period of the pendulum is directly related to the length of the pendulum R_{wt} and the mass at the base of the pendulum M_{wt}.

Total mass of the weights and pendulum					
	Mass 1	Mass 2	Mass 3	Avg. Mass	Error (g)
Mass of Weight 1 (g)	96.36	96.37	96.37	96.37	±0.01g
Mass of Weight 2 (g)	95.84	95.85	95.84	95.84	±0.01g
Mass of Screw/Nut (g)	3.63	3.63	3.63	3.63	±0.01g
Total Mass (g)				195.84	±0.03g
Mass of A1 Disk (g)				729.55	±0.02g

Table 2: Values of the total mass of the weights and pendulum

Period of the physical pendulum							
	1	2	3	4	5	Avg. Time	Error (s)
ΔT (s)	1.19	1.15	1.10	1.13	1.21	1.156	0.01s

Table 3: Values of the period of the physical pendulum

Moment of Inertia of A1 Disk: By converting all measurements into SI units so that eq. (3) can be determined

$$I_{A1} = \frac{0.19584kg\, 0.162m\, 9.80665ms^{-2}\, 1.156s^2}{4\pi^2} - (0.19584kg\, 0.162m^2) = 0.005392kgm^{-2} \tag{4}$$

Errors Analysis:

Distance (r) =	±0.001m
Stopwatch (s) =	±0.01s
Weights (g) =	±0.01g

Sample SD : Δt: 0.044497
Error : Δt: ±0.0205
Total Error : Δt: 1.16 ±0.0205

Table 4: Physical Pendulum Error Analysis.

4.1.2 Steiner's Law

Utilizing the setup from fig. 1, it was found that increasing the length of the pendulum directly affects the period of the pendulum. The induced angle doesn't doesn't affect the period of the pendulum and thus is irrelevant, only the length and mass affects the period of the pendulum. As physics implies a heavier mass and longer length increases the period of the pendulum motion. In table 5, its seen that this is not the case, the increase of length slows the pendulum motion down not increases as theory would suggest. The only reasoning behind this is the reaction time of the human operators that are operating the stopwatch, which means the time can be $\pm 0.5s$ faster/ slower as the operator tried to for see the time the pendulum motion had completed its swing and thus creating small but vital errors in the calculations. From these values the moment of inertia is calculated as shown in table 6.

Precession of the physical pendulum							
		1	2	3	Avg. Time	Error (s)	Error (cm)
0cm	ΔT(s)	0	0	0	0	0	0
4cm	ΔT(s)	1.19	1.12	0.94	1.08	±0.01s	±0.01cm
8cm	ΔT(s)	0.87	0.78	0.90	0.85	±0.01s	±0.01cm
12cm	ΔT(s)	0.87	0.78	0.81	0.82	±0.01s	±0.01cm
16cm	ΔT(s)	0.94	0.81	0.87	0.87	±0.01s	±0.01cm

Table 5: Values of the period of the physical pendulum

Moment of Inertia Calculated vs Theoretical Results					
Distance (cm)	0	4	8	12	16
Actual Δt	0	0.00196	0.00158	0.00111	0.00088
Theoretical Δt	0	0.00729	0.00581	0.00412	0.00327

Table 6: Moment of Inertia Calculated vs Theoretical Results.

Errors Analysis:

Distance (r) =	±0.001m
Stopwatch (s) =	±0.01s
Weights (g) =	±0.01g

Table 7: Steiner's Law Error Analysis.

	0cm	4cm	8cm	12cm	16cm
Sample SD: ΔT(s):	0	0.12897	0.06245	0.045826	0.065064
Error: ΔT(s):	0	±0.07	±0.04	±0.03	±0.04
Total Error: ΔT(s):	0	1.08 ±0.07	0.85 ±0.04	0.82 ±0.03	0.87 ±0.04

Table 8: Steiner's Law Error Analysis.

4.1.3 Moment of Inertia of a Gyroscope

By using the gyroscope and placing the weights at the base of the gyroscope with the combined mass of 195.84g $\pm 0.01g$ proved in table 2 and the distance between the weights and the horizontal support point set at 23.9cm $\pm 0.011cm$. Using the the average time of the repeatedly measured results, the moment of inertia can be determined by using eq. (3).

Period of the gyroscope							
	1	2	3	4	5	Avg. Time	Error (s)
ΔT (s)	3.47	3.31	3.35	3.41	3.38	3.38	±0.01s

Table 9: Period of the gyroscope

Moment of Inertia of A1 Disk: By converting all measurements into SI units so that eq. (3) can be determined

$$I_{A1} = \frac{0.19584kg\ 0.239m\ 9.80665ms^{-2}\ 3.38s^2}{4\pi^2} - (0.19584kg\ 0.239m^2) = 0.122kgm^{-2} \quad (5)$$

Errors Analysis:

Distance (r) =	±0.001m
Stopwatch (s) =	±0.01s
Weights (g) =	±0.01g

Table 10: Moment of Inertia Error Analysis.

4.1.4 Centre of Gravity

By using the vernier callipers the centre of gravity was found by changing d in fig. 3, its found that the centre of gravity was at $s = 9.96cm \pm 0.01cm$. It was confirmed the be the centre of gravity by applying a small force on the gyroscope, the gyroscope then returned to a flat stationary position where $F = 0$.

4.1.5 Precession of a Gyroscope

As the gyroscope is at its centre of gravity, there cannot be any precession, to find the precession s is changed in fig. 3 in reference to the centre of gravity. The results shown in table 11 tell the further away from the centre of gravity the gyroscope sits, the more precession is obtained. Using this to explore how the Earth experiences precession around the Sun, as the Suns forces causes the Earth to rotate allowing the Earth to endure a daytime and nighttime per a 24hr clock system.

Precession of a gyroscope								
Distance (cm)	ΔT (s)	ΔT_p (s)	ΔT/18 (s)	w (Hz)	w_p (Hz)	Error (s)	Error (cm)	Error (Hz)
1cm	0.044	14.53	0.792	9.93	0.43	±0.01s	±0.01g	±0.02Hz
1cm	0.043	15.30	0.774	8.12	0.41	±0.01s	±0.01g	±0.02Hz
1cm	0.045	15.63	0.810	7.76	0.40	±0.01s	±0.01g	±0.02Hz
2cm	0.049	9.10	0.882	7.12	0.69	±0.01s	±0.01g	±0.02Hz
3cm	0.045	8.09	0.810	7.76	0.78	±0.01s	±0.01g	±0.02Hz
-1cm	0.040	22.44	0.720	8.73	0.28	±0.01s	±0.01g	±0.02Hz
-2cm	0.038	13.41	0.684	9.19	0.47	±0.01s	±0.01g	±0.02Hz
-3cm	0.038	8.84	0.702	8.95	0.71	±0.01s	±0.01g	±0.02Hz

Table 11: Precession of a gyroscope

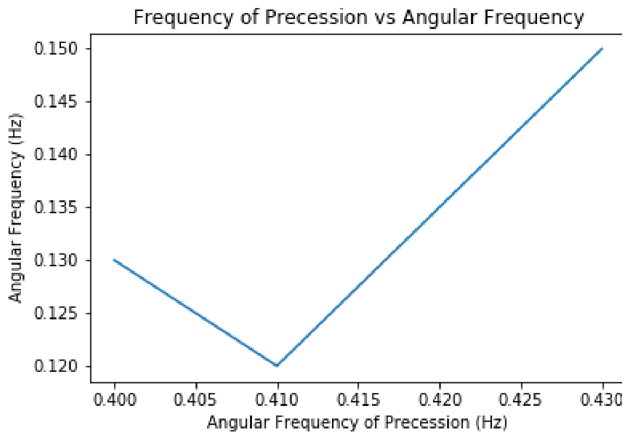

Figure 7: The relationship between angular frequency w and angular frequency of precession w_p.

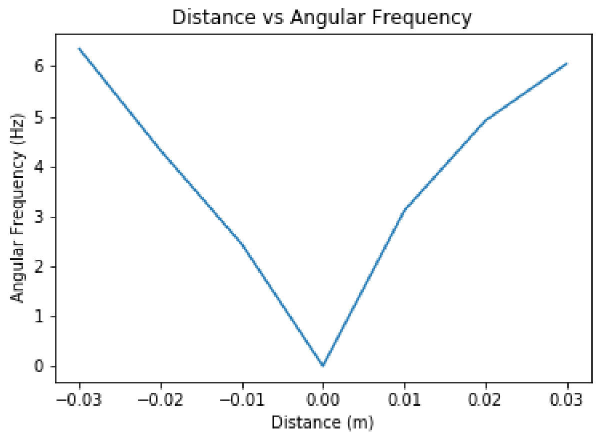

Figure 8: The relationship between angular frequency w and distance s.

By plotting the angular frequency against the angular frequency of precession in fig. 7 and distance fig. 8, relationships can be seen. In fig. 7 where the distance did not change over 3 measurements, it shows that as angular frequency rises so does the angular frequency of precession, the only error in this is in the second measurement, but the human error factor is taken into account. When changing the distance in fig. 8 no matter the increase of distance from the centre of gravity whether its direction is positive or negative, the angular frequency increases steadily according to the increase in length.

Exploring further, the precession of the gyroscope is subject to human error as the human operators reaction time is $+0.5s$ on the stopwatch time Δt this is consecutive across all of the measurements and is therefore equal throughout the results as every value experiences this.

Errors Analysis:

Distance (r) =	±0.001m
Stopwatch (s) =	±0.01s
Weights (g) =	±0.01g

Table 12: Precession of a Gyroscopes Error Analysis.

4.2 Nutation of a Gyroscope

By moving the gyroscope into its centre of gravity by changing s in fig. 3 so that $s = 4.0cm$ $\pm 0.01cm$ the nutation can be measured and w & w_n are calculated using eq. (1).

Nutation of a gyroscope								
	$\Delta t/18$ (s)	$s\Delta t_n$ (s)	Δt (s)	Δt_n (s)	Error (s)	w (Hz)	w_n (Hz)	Error (Hz)
1	0.051	3.00	0.918	0.600	±0.01s	6.84	10.47	±0.02Hz
2	0.035	2.06	0.630	0.412	±0.01s	9.97	15.25	±0.02Hz
3	0.027	1.53	0.306	0.306	±0.01s	12.93	20.53	±0.02Hz

Table 13: Nutation of a gyroscope

Taking all the relevant measurements shown in table 13 allows a relationship to be seen between w and w_n. It can be seen in fig. 9 that the angular frequency of nutation is always greater than the angular frequency and as the angular frequency increases the angular frequency of nutation steadily increases also proving that the angular frequency of nutation is directly related to the angular frequency. When applying a force down onto the rotating gyroscope to cause nutation to occur, that force is caused by human manipulation and cannot be accurately measured and directly affects the time period $s\Delta t_n$.

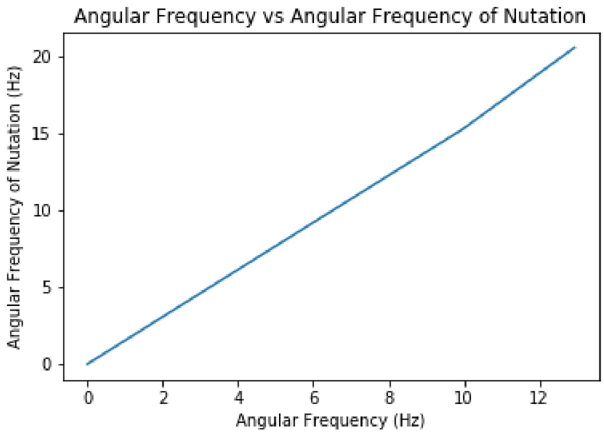

Figure 9: The relationship between angular frequency w and angular frequency of nutation w_n.

Errors Analysis:

Distance (r) =	±0.001m
Stopwatch (s) =	±0.01s
Weights (g) =	±0.01g

Table 14: Nutation of a Gyroscope Error Analysis.

5 Discussion

Within this experiment, its seen that the torque directly affects the angular momentum of the gyroscope but it encounters a variety of errors, human error plays throughout this experiment as all the time periods are measured with a stopwatch and the human operator must manually press the start and stop button, therefore a reaction time from when the stopwatch is started and stop to when the pendulum/ gyroscope is released and completed one measurable motion was set at $+0.5s$. It's also seen that the length of the pendulum affected the time period shown in section 4.1.1, when the theoretical data tells otherwise, when the only error that could be accountable for this is human error by the operators anticipating the end of the pendulums motion to early and stopping the stopwatch earlier than it needed to be.

6 Conclusion

Its clear to see that the precession and the nutation are affected by external forces such as a torque, this allows the exploration of how the Earth's rotation is affected by the Sun and other celestial objects. This is proved in section 4.1.5 where in fig. 8 where the distance from the centre of gravity increases, the angular frequency increases from the torque force that is enforced upon the gyroscope. Its also proved in section 4.2 where in fig. 9 shows that as the angular frequency increases so does the angular frequency of nutation, thus shows that as with Earth's radius, the radius also being the distance from the centre of gravity it shows that the Suns causes a torque force upon the Earth and causes it to precess whereas other celestial bodies also affect the Earths rotation by applying their external forces upon it and cause the Earth to undergo nutation thus explaining why the earths north pole rotates every 26,000 years and why "ice ages" occur.

References

[1] CODATA. Committee on data for science and technology data set, 2014.

[2] E. Pugh. Exp.4 gyroscopes. University of Kent Moodle 2019, September 2018. PH520 Physics Labs A.