What is Pattern Recognition?

"The field of pattern recognition is concerned

- with the automatic discovery of regularities in data through the use of computer algorithms and
- with the use of these regularities to take actions such as classifying the data into different categories"

An Example

- Flip a coin
- Choose a ball
- Call the coin-toss

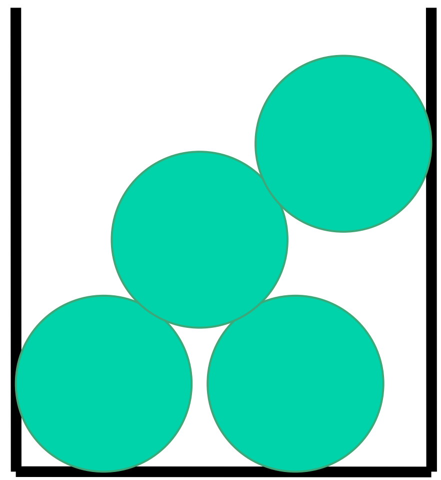
Flip a Coin

“Head”

“Tail”

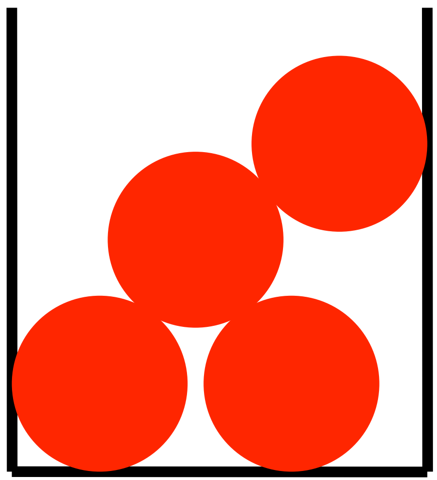
If (coin==“head”)

Choose one of these balls:



If (coin=="tail")

Choose one of these balls:

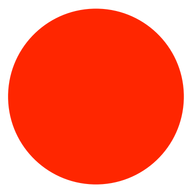


The Challenge

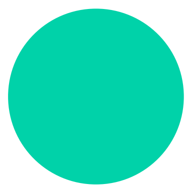
- You cannot see the coin toss outcome
- You can only see the drawn ball

- Given the observation of the drawn ball, decide the outcome of the coin toss

The Challenge

- Give it a try…
- Observe 
- Was it “head” or “tail”?

The Challenge

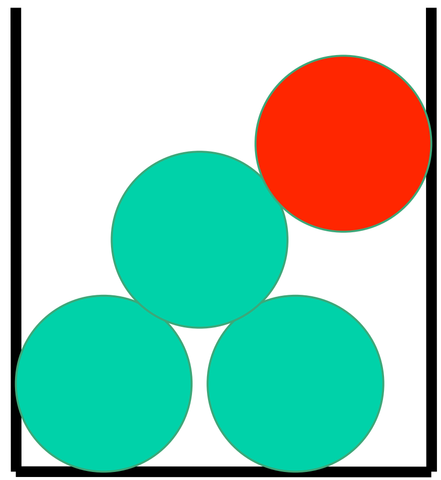
- Another one…
- Observe 
- Was it “head” or “tail”?

The Challenge

- Make it slightly more interesting

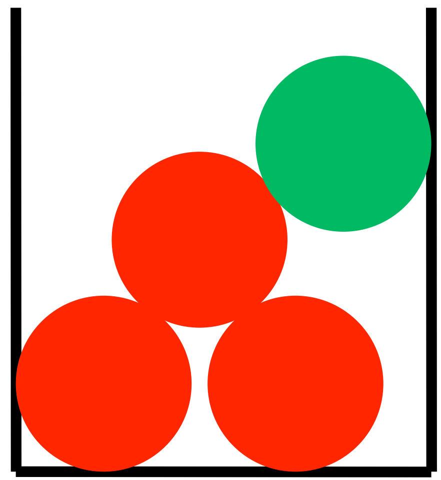
If (coin=="head")

Choose one of these balls:

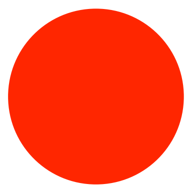


If (coin==“tail”)

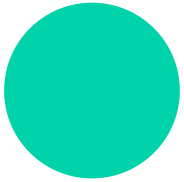
Choose one of these balls:



The Challenge

- Try again…
- Observe 
- Was it “head” or “tail”?

The Challenge

- And again…
- Observe 
- Was it “head” or “tail”?

Keeping Score

- Suppose I pay you $1 for every correct call, and you pay me $1 for every incorrect call.

- After, say, 100 calls, how much money do you think you will have?

Optimum Decision

- What is the maximum amount that you can expect to have, given the set up of the game?

Probability Theory Concepts

- Event
  - A subset of the set of all possible outcomes
- Probability
  - A measure defined for each event

- Random variable
- Expectation, Expected value

Our Example

- Events
  - Head, tail
  - Red ball, green ball

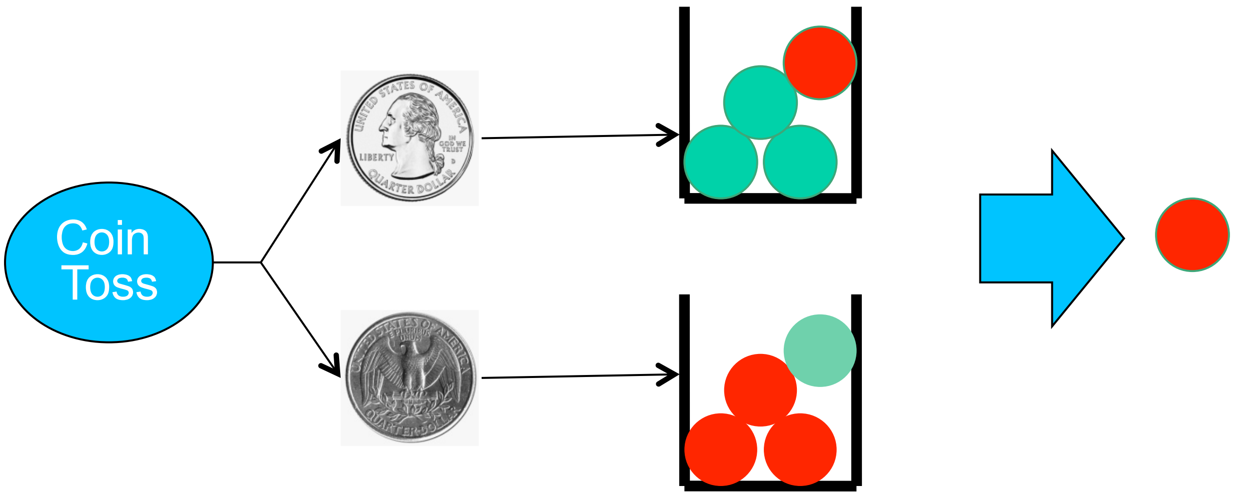
Assume the coin is fair,

$$P(\text{head}) = P(\text{tail}) = \frac{1}{2}.$$

Our Example

- Events
  - Red ball, green ball

Defining the probability of drawing a red ball or green ball is not so straightforward.



Random Variable

- A real-valued function defined on an event
  - assign a real-value to an event
  - has no relationship with how likely an event is to occur
  - assignment of the value has to do with the problem

Example

- Event: “Coin toss results in a tail”
- Random variable: Assign “1” if the coin toss results in a tail

Expected Value

- Expected value of a random variable is the sum of all values taken on by the random variable weighted by the probability of the associated event

- There is no expected value of an event; one only talks about the expected value of a random variable

Coin Toss Example

- Events:
  - “head”
  - “tail”
- Probabilities

Coin Toss Example

- Note that we will not ask “What is the expected value of the coin toss?”
- Random variable x defined as follows

  x(“head”) = 1

  x(“tail”) = 0
- What is the expected value of x?

Expected Value

- Expected value of a random variable is the sum of all values taken on by the random variable weighted by the probability of the associated event

Coin Toss Example:

$$
\begin{aligned}
E[x] &= 1 \times P(x=1) + 0 \times P(x=0) \\
&= 1 \times P(\text{Head}) \\
&= 1 \times \frac{1}{2} = \frac{1}{2}.
\end{aligned}
$$

Our Previous Example

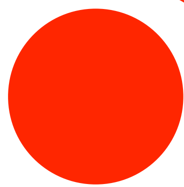
- What are the events?
- What are the probabilities?
- What is the random variable?
- Why do we care about the expected value?

Flip a Coin

“Head”

“Tail”

The Challenge

- Try again…
- Observe 
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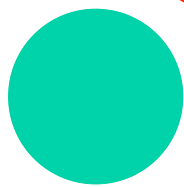
The Challenge

- And again…
- Observe 
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Our Previous Example

- What are the events?
- What are the probabilities?
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Our Previous Example

- What are the events?
- What are the probabilities?
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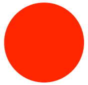
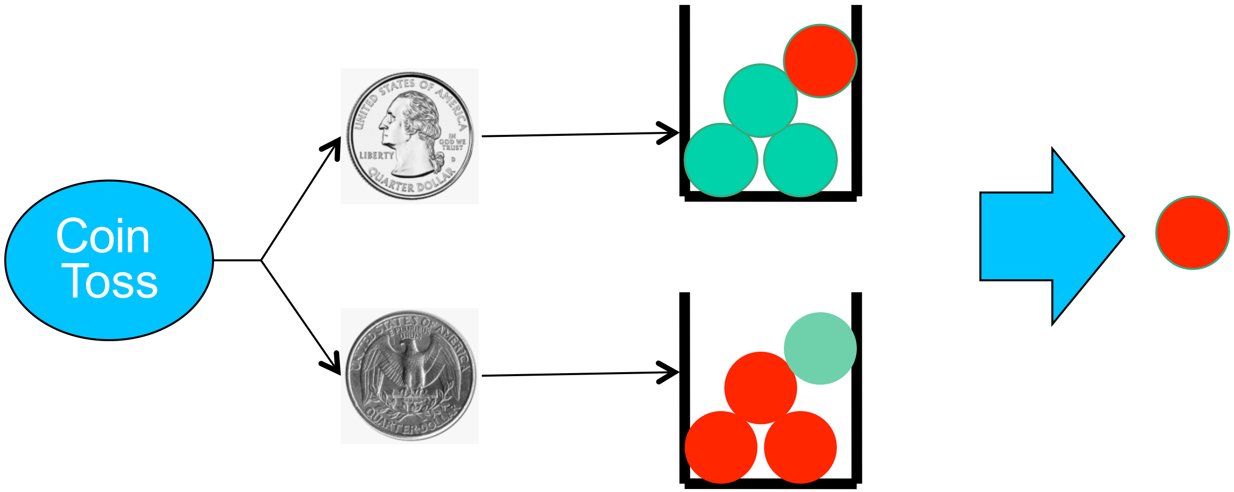
Probabilities

- What is the probability of observing  ?

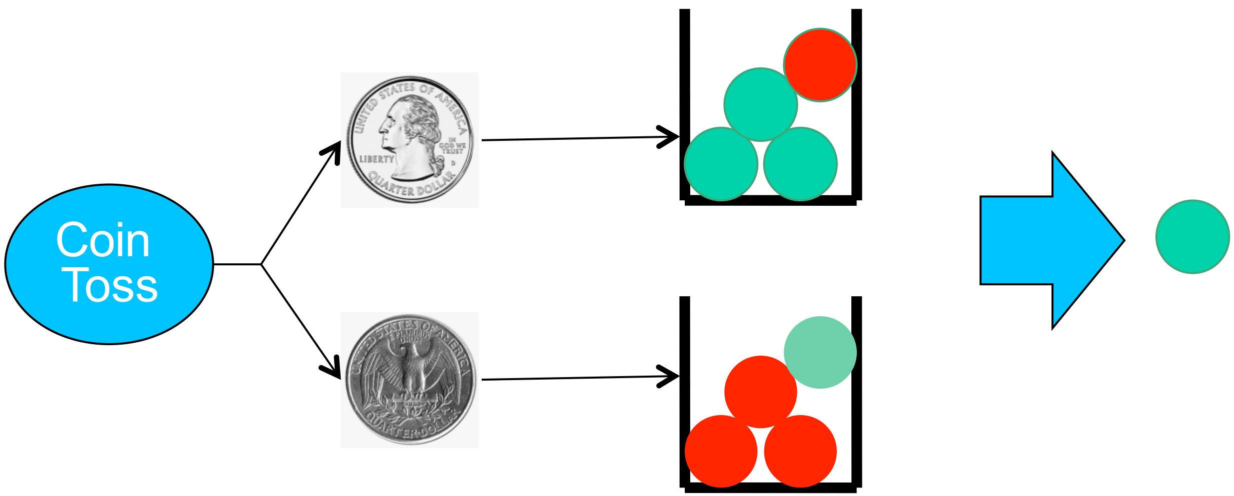


$$P(\text{red}) = P(\text{red} \mid \text{head})P(\text{head}) + P(\text{red} \mid \text{tail})P(\text{tail})$$

$$= \frac{1}{4}\frac{1}{2} + \frac{3}{4}\frac{1}{2} = \frac{1}{2}.$$

Probability (observed)

Similarly,



$$P(\text{green}) = \frac{1}{4}\frac{1}{2} + \frac{3}{4}\frac{1}{2} = \frac{1}{2}$$

Our Previous Example

- What are the events?
- What are the probabilities?
- What is the random variable?
- Why do we care about the expected value?

Keeping Score

- Suppose *A* pays *B* \$1 for every correct call, and *B* pays *A* \$1 for every incorrect call.

- After, say, 100 calls by *B*, how much money do you think *B* will have?

Expected Value

"Suppose *A* pays *B* \$1 for every correct call, and *B* pays *A* \$1 for every incorrect call."

Define x as the amount of money that B has.

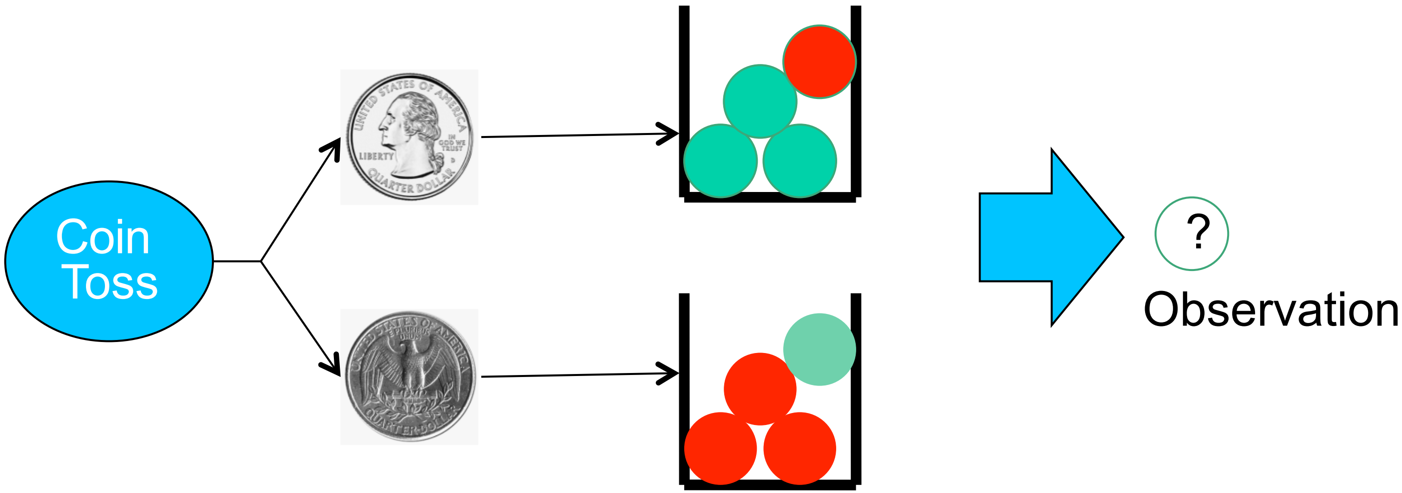
From the rule, x is given by

$$x = \begin{cases} +1 & \text{if the decision is correct} \\ -1 & \text{if the decision is incorrect} \end{cases}.$$

What is $E[x]$?

When is a decision correct?

- Need a decision rule



- Let us make up a decision rule

$$d(\text{observation}) = \begin{cases} \text{head} & \text{if observation is green} \\ \text{tail} & \text{if observation is red} \end{cases}$$

When is a decision correct?

$$d(\text{observation}) = \begin{cases} \text{head} & \text{if observation is green} \\ \text{tail} & \text{if observation is red} \end{cases}$$

The decision d is correct

if (the coin is head and the call is head)

or (the coin is tail and the call is tail);

i.e.,

if (the coin is head and the observation is green)

or (the coin is tail and the observation is red)

The probability of a correct decision is

$$P(\text{correct decision}) = \left(\frac{1}{2}\frac{3}{4}\right) + \left(\frac{1}{2}\frac{3}{4}\right) = \frac{3}{4}.$$

Expected Value

"Suppose *A* pays *B* \$1 for every correct call, and *B* pays *A* \$1 for every incorrect call."

Define x as the amount of money that B has.

From the rule, x is given by

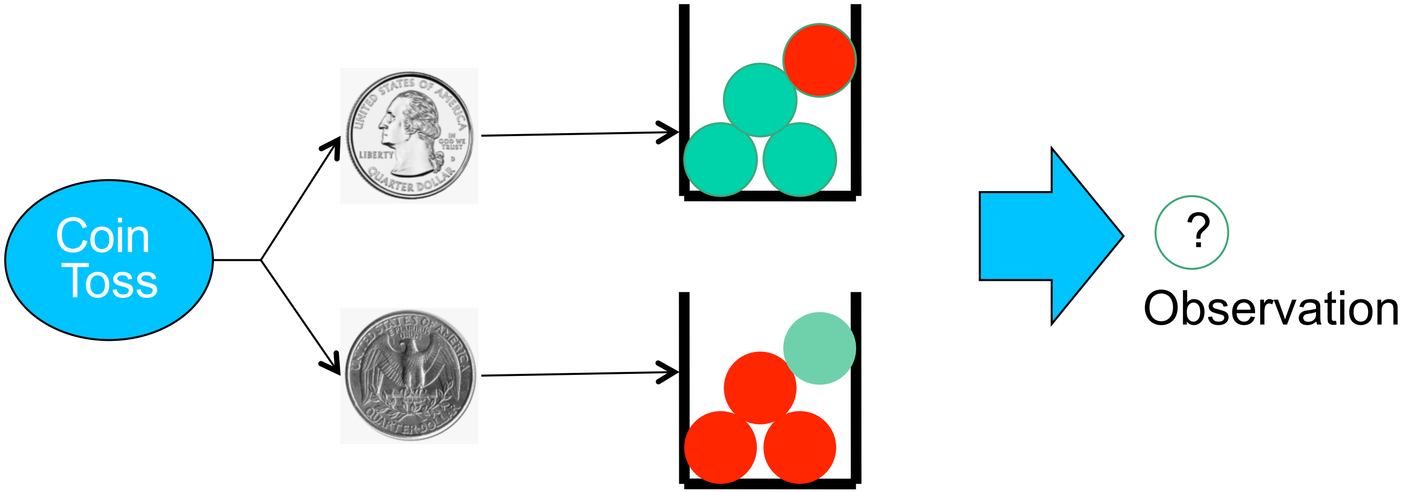
$$x = \begin{cases} +1 & \text{if the decision is correct} \\ -1 & \text{if the decision is incorrect} \end{cases}.$$

The expected value of x is

$$E[x] = +1 \times P(\text{correct decision}) - 1 \times P(\text{incorrect decision})$$

$$= +1 \times \frac{3}{4} - 1 \times \frac{1}{4} = \frac{2}{4} = \frac{1}{2}.$$

Pattern Recognition



Pattern Recognition Problem

Based on the observed color of the drawn ball, and knowing the probability of the coin toss and composition of each of the buckets,

decide the outcome of the coin toss.

In practice, we do not know the exact composition of each of the buckets. That is why we need to do learning, supervised or unsupervised.