A SAT-Based Approach For PSPACE Modal Logics

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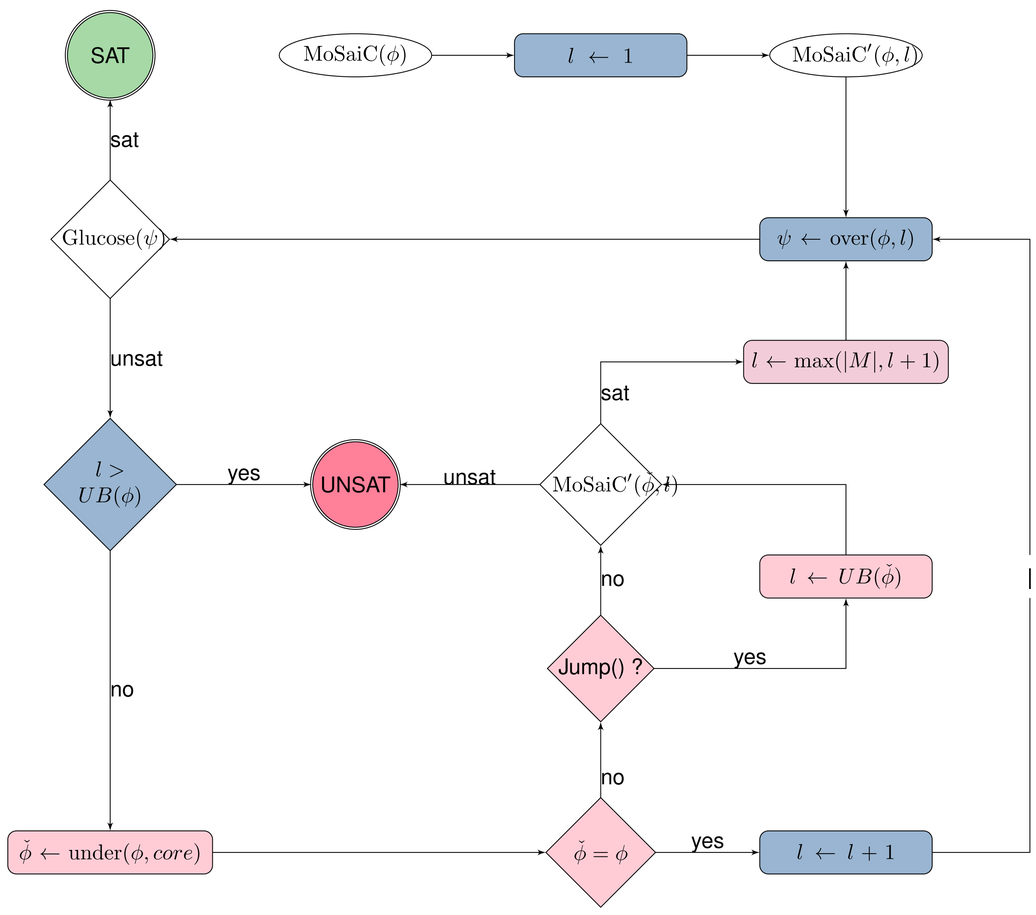
Motivation and Contributions

- Modal Logic K$\star$ Satisfiability Problems are **PSPACE complete**.
- MoSaiC already deal with modal logic K, but how to extend it to other modal logics?
- Contribution: MoSaiC is able to deal with many different modal logics.

MoSaiC: How the solver works



SAT Translation

From Modal Logic to Propositional Logic

$$\begin{aligned}
\text{over}(\phi, n) &= \text{over}'(\text{nnf}(\phi), 0, n) \\
\text{over}'(p, i, n) &= p_i \quad \text{over}'(\neg p, i, n) = \neg p_i \\
\text{over}'(\phi \wedge \psi, i, n) &= \text{over}'(\phi, i, n) \wedge \text{over}'(\psi, i, n) \\
\text{over}'(\phi \vee \psi), i, n) &= \text{over}'(\phi, i, n) \vee \text{over}'(\psi, i, n) \\
\text{over}'(\Box_a \phi, i, n) &= \bigwedge_{j=0}^{n} (r_{i,j}^a \rightarrow \text{over}'(\phi, j, n)) \\
\text{over}'(\Diamond_a \phi, i, n) &= \bigvee_{j=0}^{n} (r_{i,j}^a \wedge \text{over}'(\phi, j, n))
\end{aligned}$$

Translation of Axioms

$$\text{over}((T), n) = \bigwedge_{a=0}^{m} \bigwedge_{i=0}^{n} (r_{i,i}^a) \quad \text{over}((D), n) = \bigwedge_{a=0}^{m} \bigwedge_{i=0}^{n} \bigvee_{j=0}^{n} (r_{i,j}^a)$$

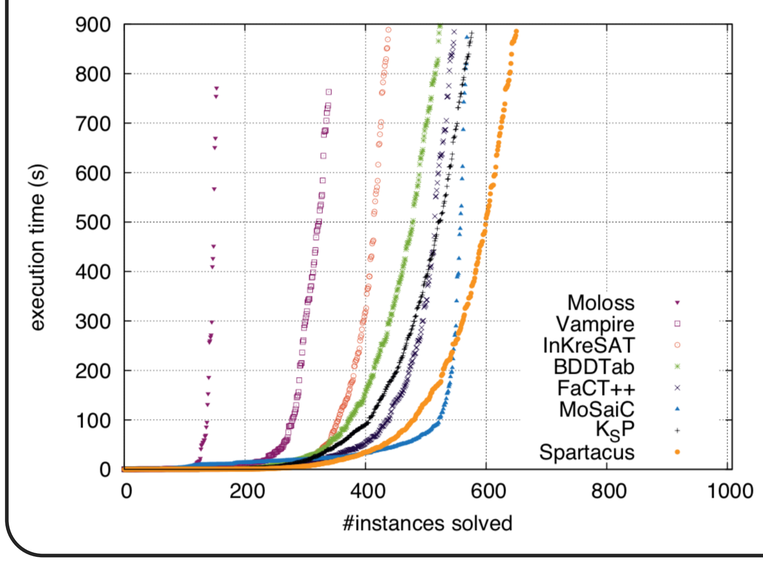
$$\text{over}((B), n) = \bigwedge_{a=0}^{m} \bigwedge_{i=0}^{n} \bigwedge_{j=0}^{n} (r_{i,j}^a \rightarrow r_{j,i}^a)$$

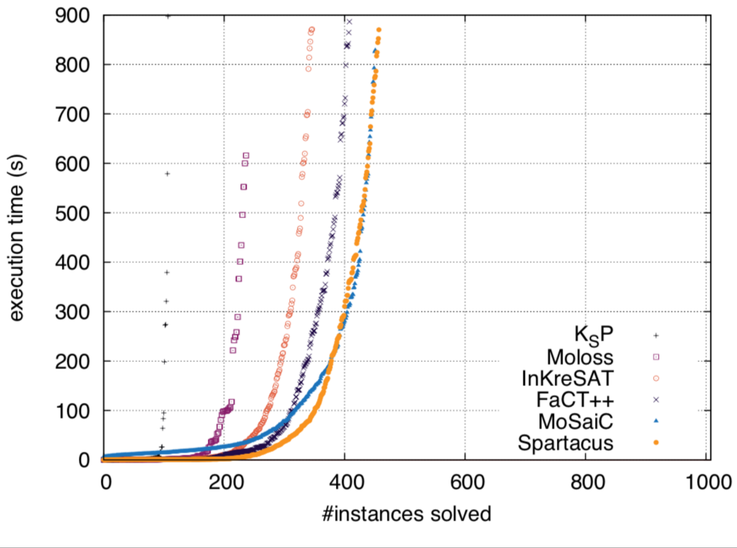
$$\text{over}((4), n) = \bigwedge_{a=0}^{m} \bigwedge_{i=0}^{n} \bigwedge_{j=0}^{n} \bigwedge_{k=0}^{n} ((r_{i,j}^a \wedge r_{j,k}^a) \rightarrow r_{i,k}^a)$$

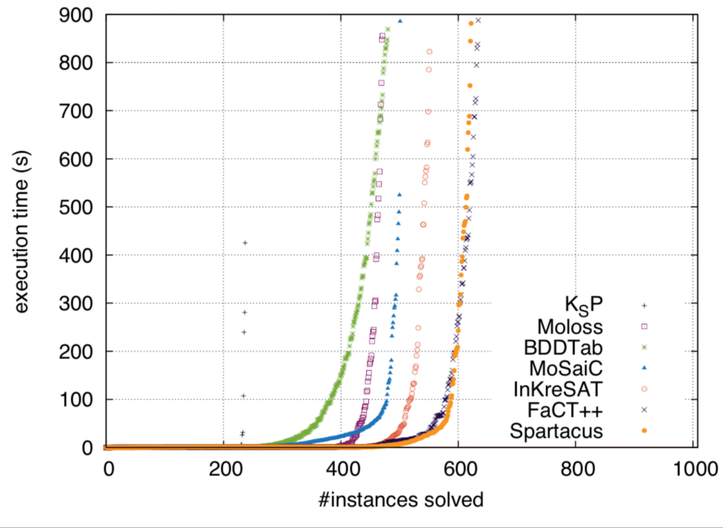
$$\text{over}((5), n) = \bigwedge_{a=0}^{m} \bigwedge_{i=0}^{n} \bigwedge_{j=0}^{n} \bigwedge_{k=0}^{n} ((r_{i,j}^a \wedge r_{i,k}^a) \rightarrow r_{j,k}^a)$$

Experimental results: MoSaiC against state-of-the-art solvers

Solver #Instances	**LWB$_K$ SAT** 504	**LWB$_K$ UNSAT** 504	**Total$_K$** 1008	**LWB$_{KT}$ SAT** 504	**LWB$_{KT}$ UNSAT** 504	**Total$_{KT}$** 1008	**LWB$_{S4}$ SAT** 504	**LWB$_{S4}$ UNSAT** 504	**Total$_{S4}$** 1008
Moloss	71 (0)	83 (0)	154 (0)	68 (0)	170 (0)	238 (0)	269 (0)	203 (0)	472 (0)
InKreSAT	192 (24)	247 (0)	439 (24)	155 (9)	193 (0)	348 (9)	248 (0)	304 (0)	552 (0)
BDDTab	248 (5)	277 (4)	525 (9)	–	–	–	211 (0)	270 (0)	481 (0)
FaCT++	264 (10)	284 (19)	548 (29)	184 (30)	226 (59)	410 (89)	298 (42)	338 (25)	636 (67)
MoSaiC	263 (241)	306 (198)	569 (439)	**230** (251)	222 (253)	452 (504)	**277** (229)	225 (277)	502 (506)
K$_S$P	249 (4)	**328** (3)	577 (7)	130 (2)	93 (0)	223 (2)	223 (0)	205 (0)	428 (0)
Spartacus	**331** (33)	320 (10)	**651** (43)	207 (74)	**251** (59)	**458** (133)	273 (17)	**350** (13)	**623** (30)
VBS	340	328	668	230	251	481	277	352	629







Experimental Settings

Tested on the LWB benchmarks for modal logic K, KT, and S4

Against many SOTA solvers for modal logic satisfiability problems

CentOS 6.0, bi-proc. XEON, 4 cores, 3.3 GHz, 32GB, 900 seconds.

Conclusion and Future Work

* MoSaiC is generic and able to deal with modal logic axioms

* MoSaiC is competitive in K and KT.

\- Improve MoSaiC to make it the most efficient approach