Une approche basée sur SAT
pour le problème de satisfiabilité en logique modale S5

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Preliminaries: Modal Logic

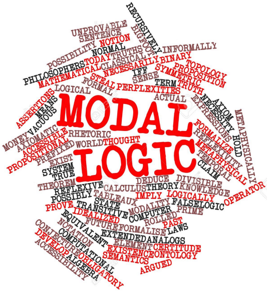
Modal Logic = Propositional Logic + $\Box$ and $\Diamond$

Modal Logic

- $\Box\phi$ means ϕ is necessarily true
- $\Diamond\phi$ means ϕ is possibly true

$$\Diamond\phi \leftrightarrow \neg\Box\neg\phi$$

$$\Box\phi \leftrightarrow \neg\Diamond\neg\phi$$



Preliminaries: Different logics → Different complexities

Sum-up of complexities

NP
K5
K45
KB45
KD5
KD45
S5

PSPACE
K
KT
S4
KB
KD4
KD
K4
KDB
KBT

Proofs of complexities are in [Lad77, HR07]

Preliminaries: S5-Structure

- $\mathbb{P}$ finite non-empty set of propositional variables

S5-Structure [Kri59]

$M = \langle W, R, V \rangle$ with:

- W, a non-empty set of possible worlds
- R, a binary relation on W (which is total: $\forall .w \forall .v \ (w, v) \in R$)
- V, a function that associate to each $p \in \mathbb{P}$, the set of possible worlds where p is true

Pointed S5 Structure: $\langle \mathcal{K}, w \rangle$

- $\mathcal{K}$: S5 Structure
- w is a possible world in W

Preliminaries: Satisfaction Relation

Definition (Satisfaction Relation)

The relation $\models$ between S5 Structures and formulae is recursively defined as follows:

$$
\begin{array}{lll}
\langle \mathcal{K}, w \rangle \models p & \text{iff} & w \in V(p) \\
\langle \mathcal{K}, w \rangle \models \neg\phi & \text{iff} & \langle \mathcal{K}, w \rangle \not\models \phi \\
\langle \mathcal{K}, w \rangle \models \phi_1 \wedge \phi_2 & \text{iff} & \langle \mathcal{K}, w \rangle \models \phi_1 \text{ and } \langle \mathcal{K}, w \rangle \models \phi_2 \\
\langle \mathcal{K}, w \rangle \models \phi_1 \vee \phi_2 & \text{iff} & \langle \mathcal{K}, w \rangle \models \phi_1 \text{ or } \langle \mathcal{K}, w \rangle \models \phi_2 \\
\langle \mathcal{K}, w \rangle \models \Box\phi & \text{iff} & \forall v \in R \text{ we have } \langle \mathcal{K}, v \rangle \models \phi \\
\langle \mathcal{K}, w \rangle \models \Diamond\phi & \text{iff} & \exists v \in R \text{ such that } \langle \mathcal{K}, v \rangle \models \phi
\end{array}
$$

$\mathcal{K}$ that satisfied a formula ϕ will be called "Kripke model of ϕ"

Diamond-Degree: Strictly better than nm

Comparison dd(ϕ) vs nm(ϕ)

$\mathrm{nm}(\phi) = \mathrm{nm}'(\mathrm{nnf}(\phi))$	$\mathrm{dd}(\phi) = \mathrm{dd}'(\mathrm{nnf}(\phi))$
$\mathrm{nm}'(p) = \mathrm{nm}'(\neg p) = 0$	$\mathrm{dd}'(p) = \mathrm{dd}'(\neg p) = 0$
$\mathrm{nm}'(\phi \wedge \psi) = \mathrm{nm}'(\phi) + \mathrm{nm}'(\psi)$	$\mathrm{dd}'(\phi \wedge \psi) = \mathrm{dd}'(\phi) + \mathrm{dd}'(\psi)$
$\mathrm{nm}'(\phi \vee \psi) = \mathrm{nm}'(\phi) + \mathrm{nm}'(\psi)$	$\mathrm{dd}'(\phi \vee \psi) = \max(\mathrm{dd}'(\phi), \mathrm{dd}'(\psi))$
$\mathrm{nm}'(\Box\phi) = 1 + \mathrm{nm}'(\phi)$	$\mathrm{dd}'(\Box\phi) = \mathrm{dd}'(\phi)$
$\mathrm{nm}'(\Diamond\phi) = 1 + \mathrm{nm}'(\phi)$	$\mathrm{dd}'(\Diamond\phi) = 1 + \mathrm{dd}'(\phi)$

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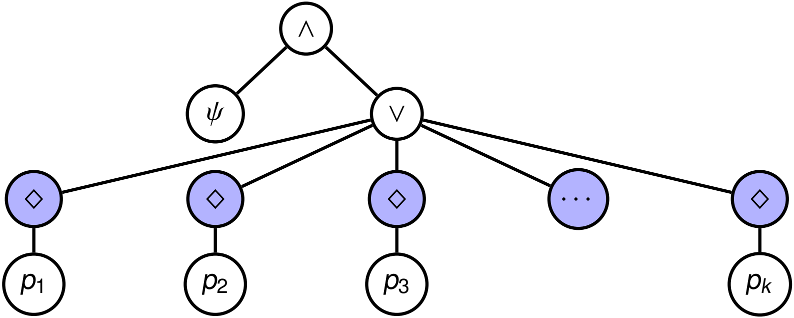
Diamond-Degree: Strictly better than nm



- nm(φ) equals k
- dd(φ) equals 1

- We just need to satisfy 'one' diamond, not all of them
- The entire formula needs only dd(φ) $+ 1$ worlds

Modal Logic S5 solver: S52SAT

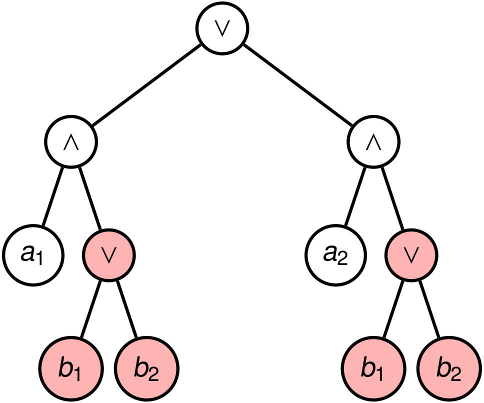
- Translation from S5-SAT to SAT.
- Polynomial reduction: S5-SAT is NP-complete [Lad77]

Translation from S5 to SAT

$$\mathrm{tr}(\phi, n) = \mathrm{tr}(\mathrm{nnf}(\phi), 1, n)$$
$$\mathrm{tr}(p, i, n) = p_i$$
$$\mathrm{tr}(\Box\phi, i, n) = \bigwedge_{j=1}^{n}((\mathrm{tr}(\phi, j, n))$$
$$\mathrm{tr}(\Diamond\phi, i, n) = \bigvee_{j=1}^{n}((\mathrm{tr}(\phi, j, n))$$

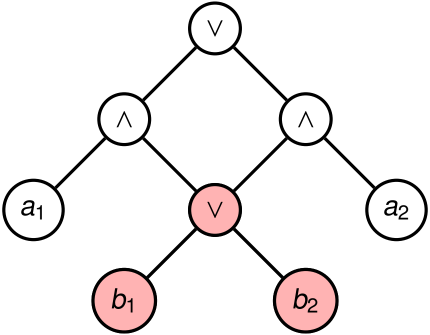
Modal Logic S5 solver: S52SAT

Let $\phi = \Diamond(a \wedge \Diamond b)$ as example (with n = 2).



Modal Logic S5 solver: S52SAT - structural caching

$\phi = \Diamond(a \land \Diamond b)$, with caching.



Modal Logic S5 solver: S52SAT - with/without caching

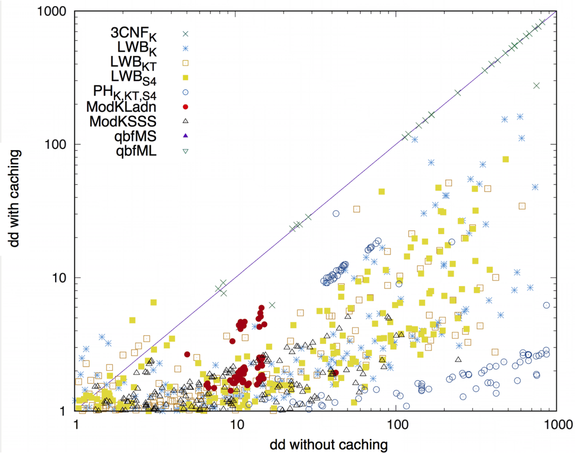


Figure: Scatter plot with/without caching

Modal Logic S5 solver: S52SAT - against SotA solvers

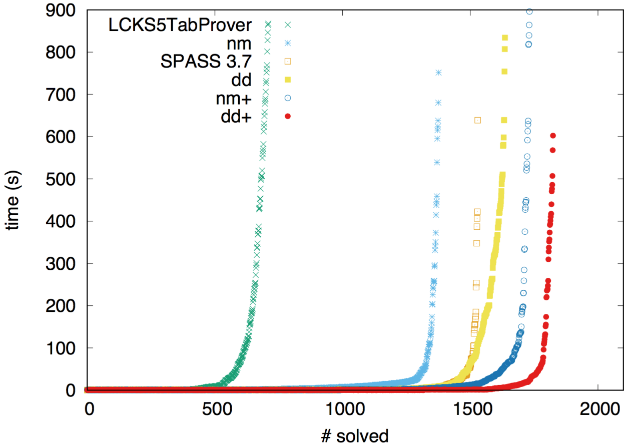


Figure: Cactus-Plot of the runtime distributions

Conclusion

- S52SAT the most efficient approach on benchmarks considered
- Benchmarks considered are not “real” problems
- Modal Logic S5 is more expressive than SAT
- S52SAT returns S5-model when it found one

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Bibliographie I

Joseph Y. Halpern and Leandro Chaves Rêgo.
Characterizing the NP-PSPACE gap in the satisfiability problem for modal logic.
J. Log. Comput., 17(4):795–806, 2007.

Saul Kripke.
A completeness theorem in modal logic.
J. Symb. Log., 24(1):1–14, 1959.

Richard E. Ladner.
The Computational Complexity of Provability in Systems of Modal Propositional Logic.
SIAM J. Comput., 6(3):467–480, 1977.