Un raccourci récursif pour CEGAR

Application au problème de satisfiabilité en logique modale K

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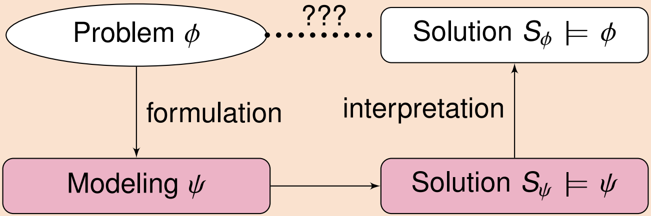
Abstraction: Idea & Motivation

- Comes from: **Mathematical Modeling**

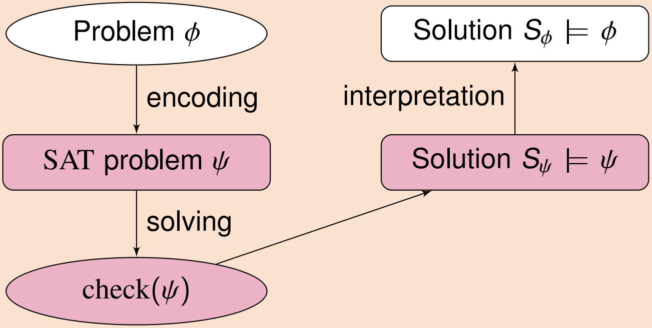


- Works for theoretical problems
- But what about practice?

Introduction: Abstraction via SAT

Modeling: Propositional Formula

- For many **NP** problems: Encoding into SAT



SAT solver

SAT solver

- Extremely efficient software
- Based on CDCL approach [SS99, MMZ+01]
- One of the current best is: Glucose [ES03, AS09] ☺
- Able to solve efficiently problems with $\approx 10^8$ clauses
- Able to give a “reason” why a formula is UNSAT [ES03]

SAT solver: One limitation

- What happen when the encoding of the problem is too big ?
- Could be solved ‘easily’ but will not because of memory...
- We need a SAT solver in a more complex procedure: CEGAR

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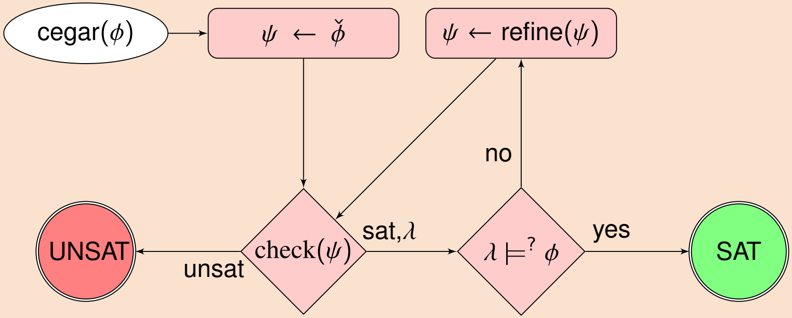
CEGAR: **C**ounter**E**xample **G**uided **A**bstraction **R**efinement

To solve a problem, consider only a part of it [CGJ+03]

- To abstract problems: hoping it will be easier to solve
- Two variants of abstraction:
  - Under-abstraction: abstraction has **more** solutions
  - Over-abstraction: abstraction has **less** solutions
- CEGAR-over: CEGAR approach using over-abstractions
- CEGAR-under: CEGAR approach using under-abstractions

CEGAR using under-abstractions

CEGAR-under



Example

SAT problem, by increasing step by step, the number of clauses

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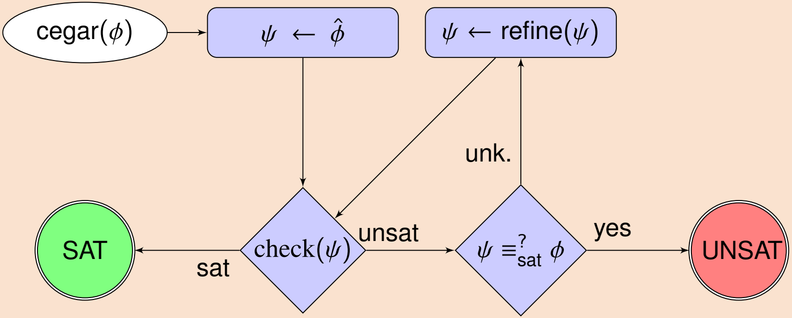
CEGAR using over-abstractions

CEGAR-over



Example

Planification problem, by increasing step by step, the horizon

CounterExample Guided Abstraction Refinement

Advantages

- If problem mainly satisfiable: CEGAR-over
- If problem mainly unsatisfiable: CEGAR-under
- Everytime check improves, CEGAR improves
- Many applications already use CEGAR

Drawbacks

- Not efficient when 50/50 chances of being SAT/UNSAT
- Not efficient when we need many refinement steps

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Recursive Explore and Check Abstraction Refinement

Recursive **E**xplore and **C**heck **A**bstraction **R**efinement

- Called *RECAR* [LLdLM17]
- Inspired by CEGAR [CGJ+03]
- Rely on 5 very important assumptions

RECAR Assumptions

1. Function ‘check’ is sound, complete and terminates
2. $isSAT(\hat{\phi})$ implies $isSAT(\text{refine}(\hat{\phi}))$
3. $\exists . n \in \mathbb{N}$ s.t. $\text{refine}^n(\hat{\phi}) \equiv^?_{\text{sat}} \phi$.
4. $\text{isUNSAT}(\check{\phi})$ implies $\text{isUNSAT}(\phi)$
5. $\exists n \in \mathbb{N}$ s.t. $RC(under^n(\phi), under^{n+1}(\phi))$ is false.

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Recursive Explore and Check Abstraction Refinement

$\exists n \in \mathbb{N}$ s.t. $RC(under^n(\phi), under^{n+1}(\phi))$ is false.

RC function

- 'true' if we can do a recursive call, 'false' otherwise
- It compares $under^i(\phi)$ and $under^{i+1}(\phi)$
- It checks if $under^{i+1}(\phi)$ will be "easier to solve" than $under^i(\phi)$

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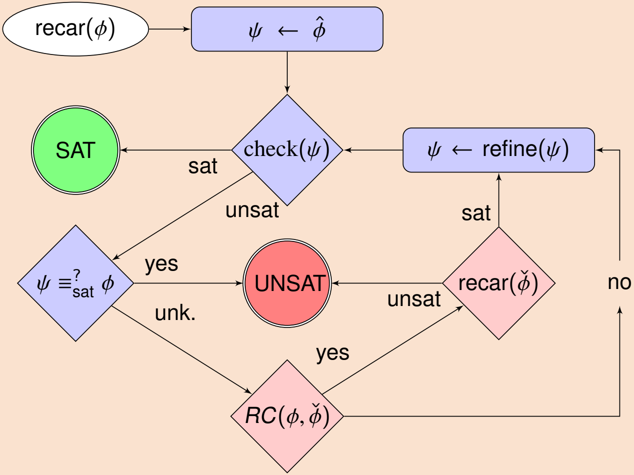
Recursive Explore and Check Abstraction Refinement

RECAR



Recursive Explore and Check Abstraction Refinement

RECAR

- 2 levels of abstractions
  - One at the Oracle level $(\text{check}(\psi))$
  - One at the Domain level (recursive call)
- Efficient even when 50/50 chance of being SAT/UNSAT
- Everytime check improves, RECAR improves
- The return of the recursive call can reduce the number of refinement
- Totally generic, can change SAT solver $\rightarrow$ FO solver?

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RECAR: Instanciation for Modal Logic K

RECAR for Modal Logic K

- Modal Logic K is **PSPACE**-complete [Lad77, Hal95]
- What is Modal Logic K?
- How we over-approximate a formula ϕ?
- How we under-approximate a formula ϕ?
- Is it competitive against a CEGAR approach?
- Is it competitive against the state-of-the-art approaches?

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Preliminaries: Modal Logic

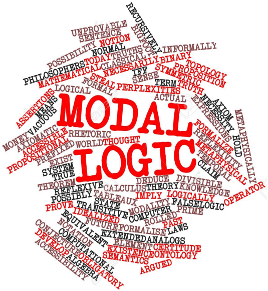
Modal Logic = Propositional Logic + $\Box$ and $\Diamond$

Modal Logic

- $\Box\phi$ means ϕ is necessarily true
- $\Diamond\phi$ means ϕ is possibly true

$$\Diamond\phi \leftrightarrow \neg\Box\neg\phi$$

$$\Box\phi \leftrightarrow \neg\Diamond\neg\phi$$



Preliminaries: Kripke Structure

- $\mathbb{P}$ finite non-empty set of propositional variables

Kripke Structure [Kri59]

$M = \langle W, R, V \rangle$ with:

- W, a non-empty set of possible worlds
- R, a binary relation on W
- V, a function that associate to each $p \in \mathbb{P}$, the set of possible worlds where p is true

Pointed Kripke Structure: $\langle \mathcal{K}, w \rangle$

- $\mathcal{K}$: Kripke Structure
- w: a possible world in W

Preliminaries: Satisfaction Relation

Definition (Satisfaction Relation)

The relation $\models$ between Kripke Structures and formulae is recursively defined as follows:

$$
\begin{array}{lll}
\langle \mathcal{K}, w \rangle \models p & \text{iff} & w \in V(p) \\
\langle \mathcal{K}, w \rangle \models \neg\phi & \text{iff} & \langle \mathcal{K}, w \rangle \not\models \phi \\
\langle \mathcal{K}, w \rangle \models \phi_1 \wedge \phi_2 & \text{iff} & \langle \mathcal{K}, w \rangle \models \phi_1 \text{ and } \langle \mathcal{K}, w \rangle \models \phi_2 \\
\langle \mathcal{K}, w \rangle \models \phi_1 \vee \phi_2 & \text{iff} & \langle \mathcal{K}, w \rangle \models \phi_1 \text{ or } \langle \mathcal{K}, w \rangle \models \phi_2 \\
\langle \mathcal{K}, w \rangle \models \Box\phi & \text{iff} & (w, w') \in R \text{ implies } \langle \mathcal{K}, w' \rangle \models \phi \\
\langle \mathcal{K}, w \rangle \models \Diamond\phi & \text{iff} & (w, w') \in R \text{ and } \langle \mathcal{K}, w' \rangle \models \phi
\end{array}
$$

$\mathcal{K}$ that satisfied a formula ϕ will be called “Kripke model of ϕ”

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MoSaiC

MoSaiC

- Open-Source Modal Logic K solver
- Uses Glucose as internal SAT solver
- Uses a RECAR approach

RECAR Assumptions: Reminder

1. Function 'check' is sound, complete and terminates
2. $isSAT(\hat{\phi})$ implies $isSAT(\text{refine}(\hat{\phi}))$
3. $\exists.n \in \mathbb{N}$ s.t. $\text{refine}^n(\hat{\phi}) \equiv^?_{\text{sat}} \phi$
4. $\text{isUNSAT}(\check{\phi})$ implies $\text{isUNSAT}(\phi)$
5. $\exists n \in \mathbb{N}$ s.t. $RC(under^n(\phi), under^{n+1}(\phi))$ is false

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MoSaiC: Over-Approximation

ϕ always in NNF and over(ϕ, i) in CNF thanks to Tseitin

$$\begin{aligned}
\text{over}(\phi, n) &= \text{over}'(\phi, 0, n) \\
\text{over}'(p_k, i, n) &= p_{k,i} \\
\text{over}'(\neg p_k, i, n) &= \neg p_{k,i} \\
\text{over}'(\Box\phi, i, n) &= \bigwedge_{j=0}^{n} (r_{i,j} \rightarrow \text{over}'(\phi, j, n)) \\
\text{over}'(\Diamond\phi, i, n) &= \bigvee_{j=0}^{n} (r_{i,j} \wedge \text{over}'(\phi, j, n))
\end{aligned}$$

- $p_{k,i}$ means p_k is true in the world w_i
- $r_{i,j}$ means that there is a relation between worlds w_i and w_j

RECAR Assumptions: Reminder

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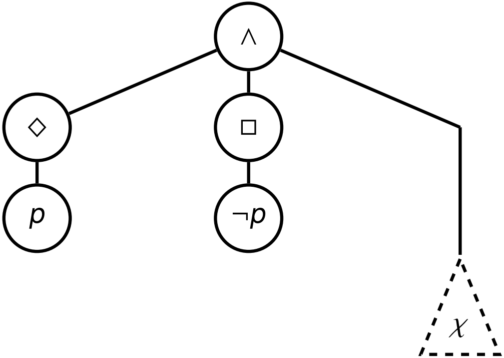
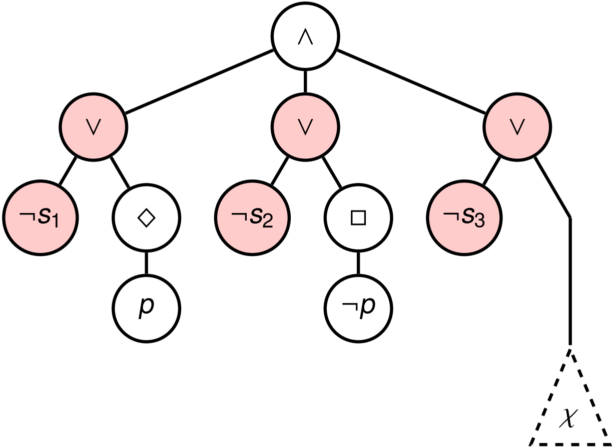
MoSaiC: Under-Approximation

Let's take an example, with χ huge but satisfiable...



Worst case for CEGAR using our 'over' function

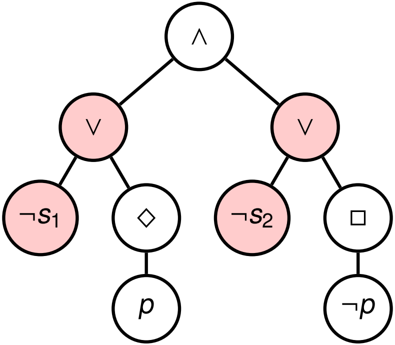
MoSaiC: Under-Approximation



Modern SAT solvers returns 'the reason' why a formula with n worlds is unsatisfiable ($core = \{s_1, s_2\}$)

MoSaiC: Under-Approximation

We want to cut what is not part of the 'unsatisfiability' ($s_i \notin core$)



We just create $\check{\phi}$ smaller than ϕ and easier to solve.
The function *RC* from RECAR just says here: did we cut something ?

RECAR Assumptions: Reminder

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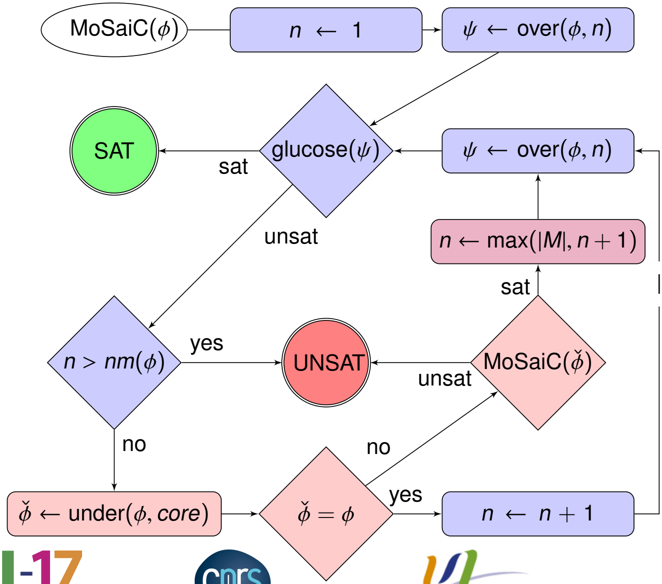
MoSaiC: RECAR for Modal Logic K



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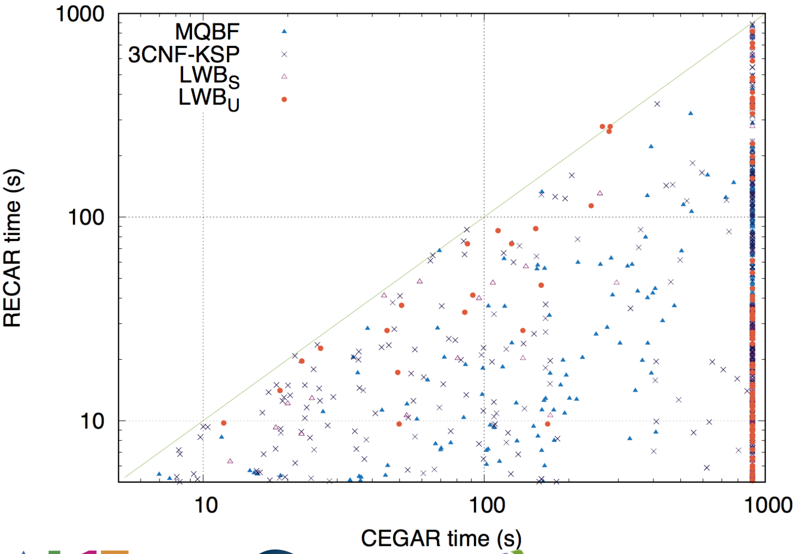
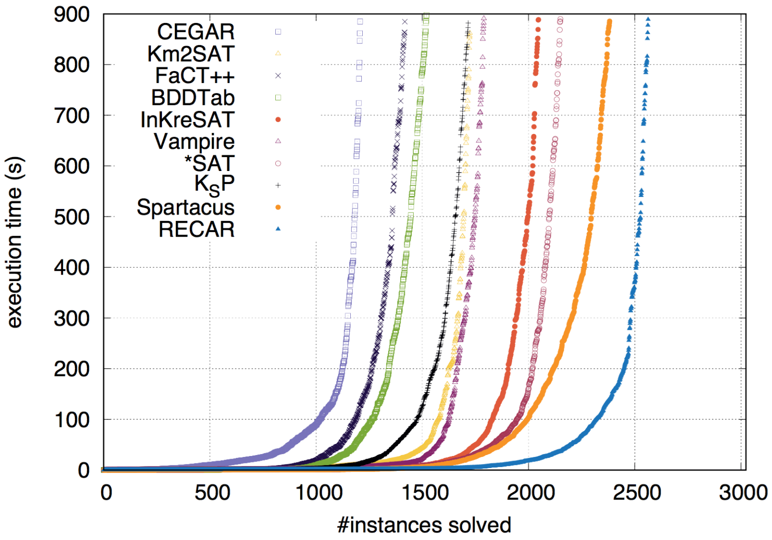
MoSaiC: RECAR for Modal Logic K



MoSaiC: RECAR for Modal Logic K



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Conclusion

Perspective: Other modal logics

NP
K5
K45
KB45
KD5
KD45
KT5

PSPACE
K
KT
KT4
KB
KD4
KD
K4
KDB
KBT

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