COMP 250
INTRODUCTION TO COMPUTER SCIENCE

32 - Maps

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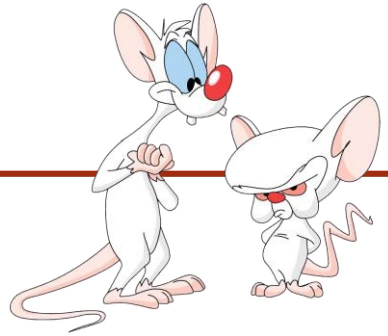
Slides adapted from Michael Langer's

WHAT ARE WE GOING TO DO IN THIS VIDEO?



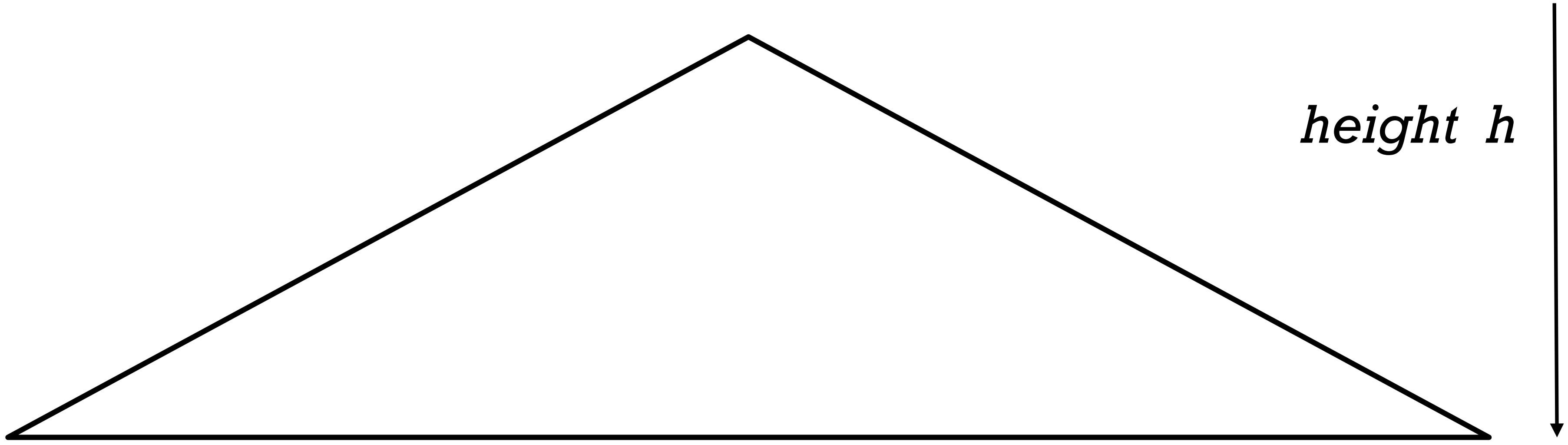
- Recap on heaps
- Maps

HOW TO BUILD A HEAP

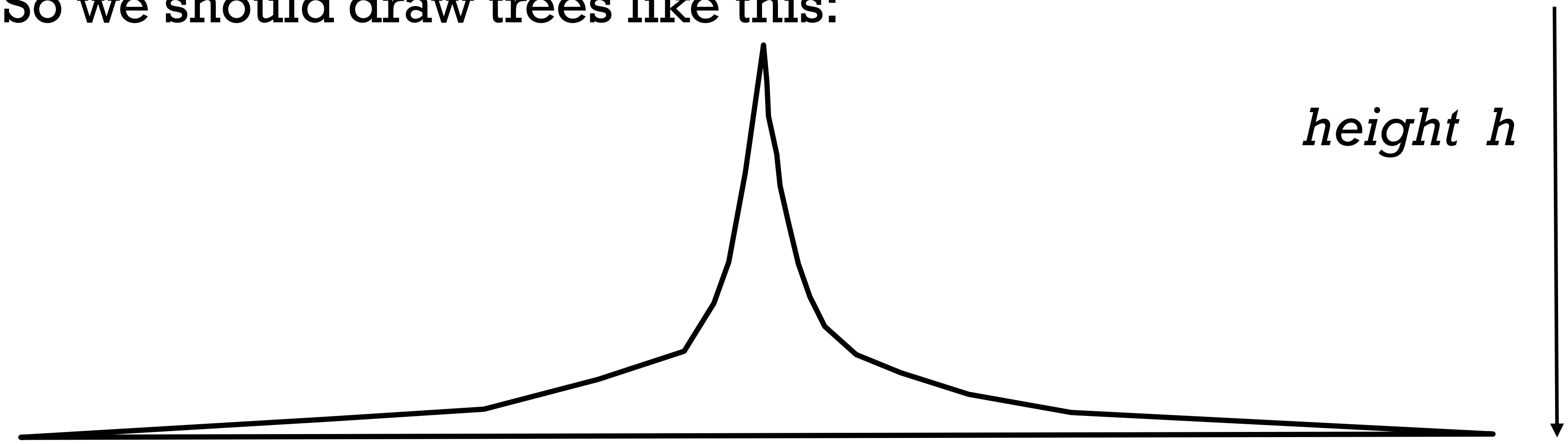
```
buildHeap(list){
    create new heap array
    for (k = 0; k < list.size(); k++)
        add( list[k] )
}
```

```
buildHeapFast(list){
    // copy elements from list to heap array
    for (k = size/2; k >= 1; k--)
        downHeap( k, size )
}
```

We tends to draw binary trees like this:



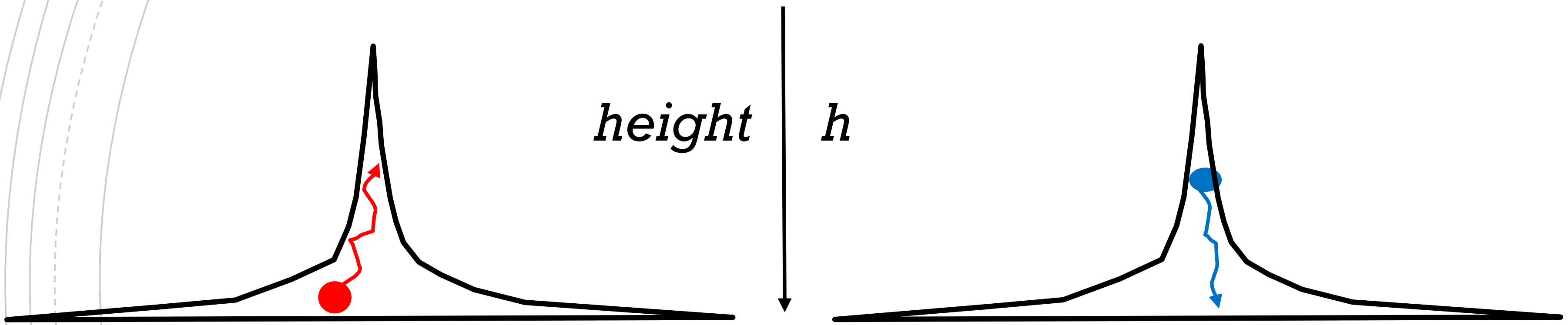
But the number of nodes doubles at each level.
So we should draw trees like this:



BUILDHEAP ALGORITHMS

naive

fast



Most nodes swap ~*h* times in worst case.

Few nodes swap ~h times in worst case.

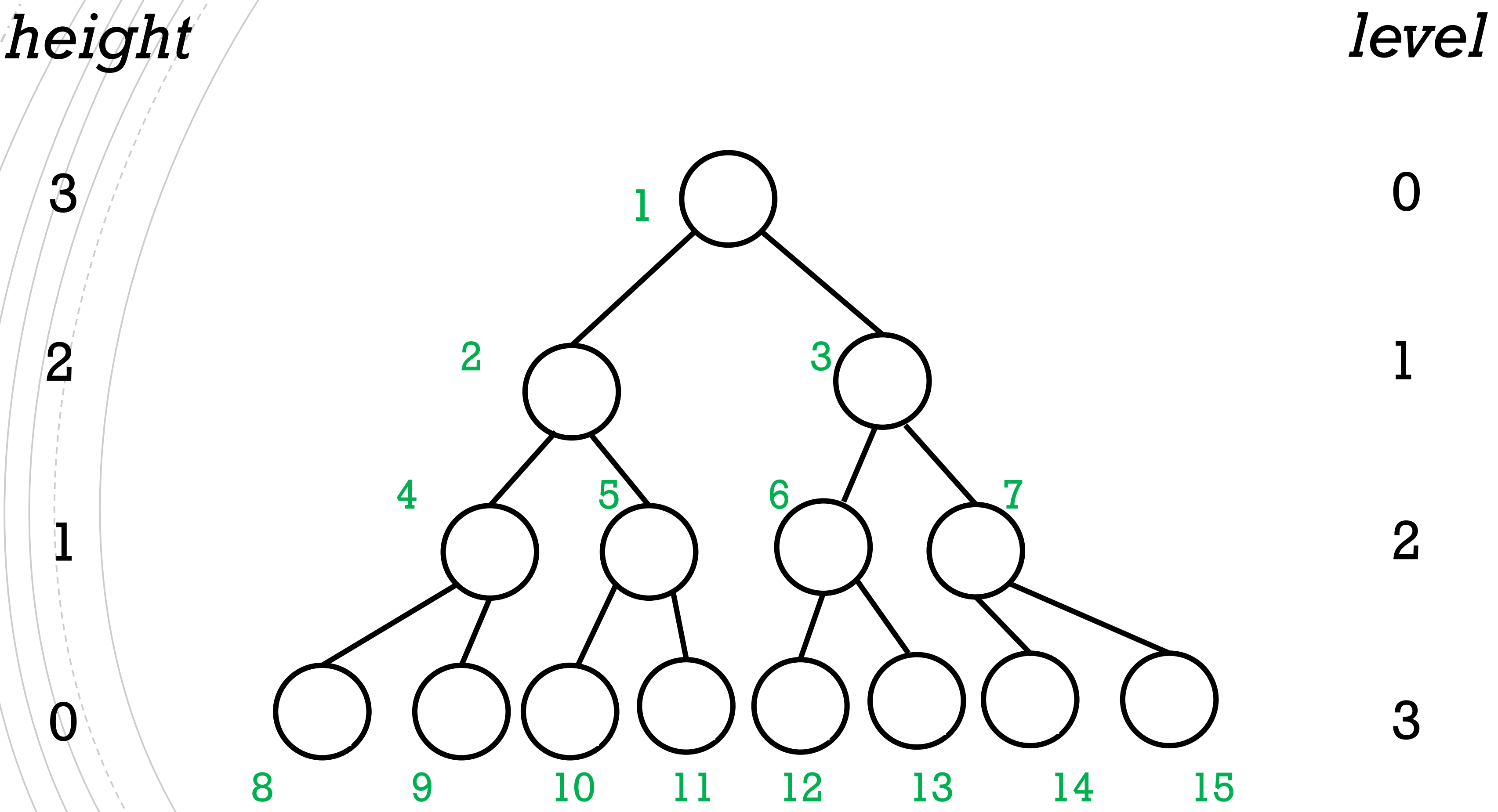
HOW TO SHOW BUILDHEAPFAST IS $\boldsymbol{O(n)}$?

The worst case number of swaps needed to downHeap node i is the height of that node.

$$t(n) = \sum_{i=1}^{n} height\ of\ node\ i$$

½ of the nodes do no swaps.
¼ of the nodes do at most one swap.
1/8 of the nodes do at most two swaps….

ASSUME THE LAST LEVEL IS FULL



WORSE CASE OF BUILDHEAPFAST ?

- How many elements at $level\ \ l$? $(l \in \ 0, \ldots, \ h)$
- What is the height of each $level\ \ l$ node?

WORSE CASE OF BUILDHEAPFAST ?

- How many elements at $level\ \ l$? $(l \in \ 0,\ldots,\ h)$
  - 2^l
- What is the height of each $level\ \ l$ node?
  - $h - l$

$$t(n) = \sum_{i=1}^{n} \ height\ of\ node\ i$$

$$= \quad ?$$

WORSE CASE OF BUILDHEAPFAST ?

- How many elements at $level\ \ l$? $(l \in\ 0, \ldots,\ h)$
  - 2^l
- What is the height of each $level\ \ l$ node?
  - $h - l$

$$t(n) = \sum_{i=1}^{n} \ height\ of\ node\ i$$

$$= \sum_{l=0}^{h} (h - l)\ 2^l$$

$$
\begin{aligned}
t_{worstcase}(h) &= \sum_{l=0}^{h} (h-l)\, 2^l \\
&= h \sum_{l=0}^{h} 2^l - \sum_{l=0}^{h} l\, 2^l
\end{aligned}
$$

↑ Easy (number of nodes)

↑ Difficult (sum of node levels)

$$
\begin{aligned}
t_{worstcase}(h) &= \sum_{l=0}^{h} (h-l)\ 2^l \\
&= h \sum_{l=0}^{h} 2^l - \boxed{\sum_{l=0}^{h} l\ 2^l} \\
&= h(2^{h+1}-1) - \boxed{(h-1)2^{h+1} - 2}
\end{aligned}
$$

(See next slide)

$$
\begin{aligned}
\sum_{l=0}^{h} l\ 2^l &= \sum_{l=0}^{h} l\ (2^{l+1} - 2^l) \qquad\qquad \text{(trick)} \\
&= \sum_{l=0}^{h} l\ 2^{l+1} - \sum_{l=0}^{h} l\ 2^l \\
&= \sum_{l=0}^{h} l\ 2^{l+1} - \sum_{l=0}^{h-1} (l+1)\ 2^{l+1} \\
&= h\ 2^{h+1} + 2\sum_{l=0}^{h-1} (l - (l+1))\ 2^l \\
&= h\ 2^{h+1} - 2\sum_{l=0}^{h-1} 2^l \\
&= h\ 2^{h+1} - 2(2^h - 1) \\
&= (h-1)2^{h+1} + 2
\end{aligned}
$$

Second term index goes to h-1 only

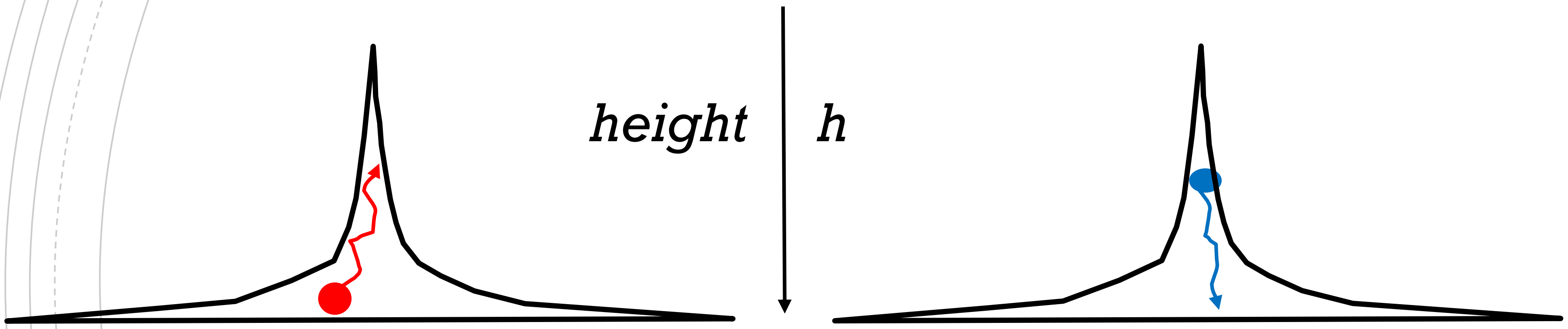
$$
\begin{aligned}
t_{worstcase}(h) &= \sum_{l=0}^{h} (h-l)\, 2^l \\
&= h \sum_{l=0}^{h} 2^l - \sum_{l=0}^{h} l\, 2^l \\
&= h(2^{h+1}-1) - (h-1)2^{h+1} - 2 \qquad \text{from above} \\
&= 2^{h+1} - h - 2
\end{aligned}
$$

Since $n = 2^{h+1} - 1$, we get :

$$t_{worstcase}(n) = n - \log(n+1)$$

SUMMARY: BUILDHEAP ALGORITHMS

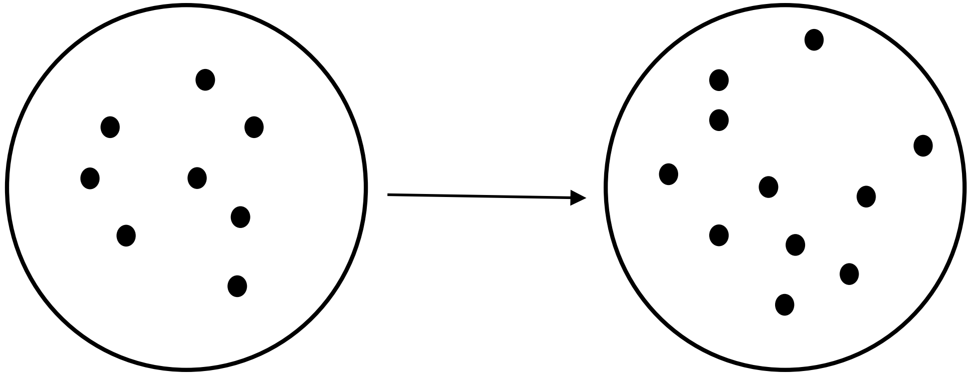
naive **fast**



$O(n\ \ log_2\ n)$ $O(n)$

MAPS

MAP (MATHEMATICS)



"domain"　　"codomain"

A map is a set of pairs $\{ (x, f(x)) \}$.

Each x in domain maps to exactly one $f(x)$ in codomain, but it can happen that $f(x1) = f(x2)$ for different $x1, x2$, i.e. many-to-one.

FAMILIAR EXAMPLES

Calculus 1 and 2 ("functions"):

$$f : \mathbb{R}^n \to \mathbb{R}^n$$

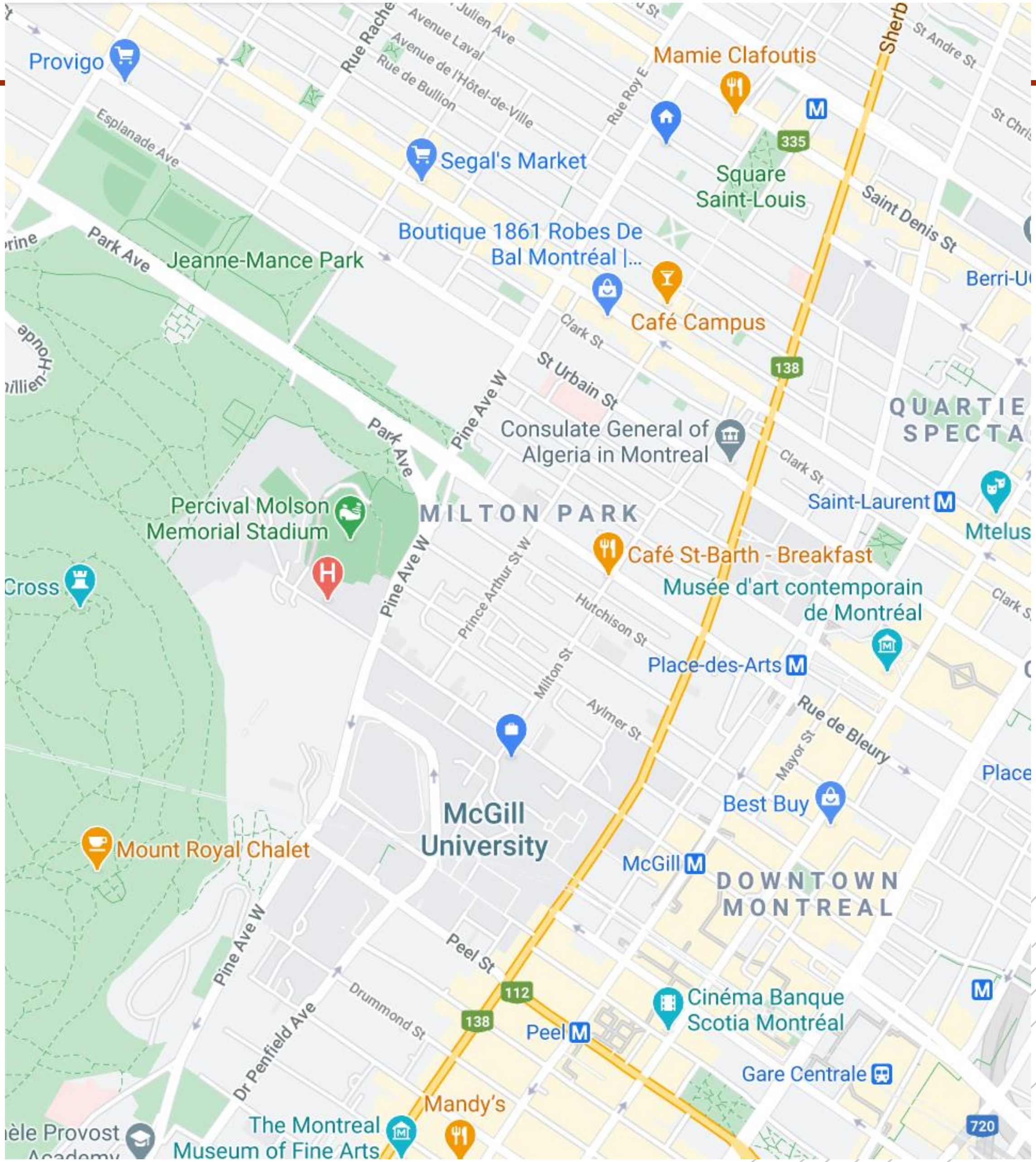
Asymptotic complexity in CS:

t : input size → number of steps in a algorithm.

MAPS IN EVERYDAY LIFE

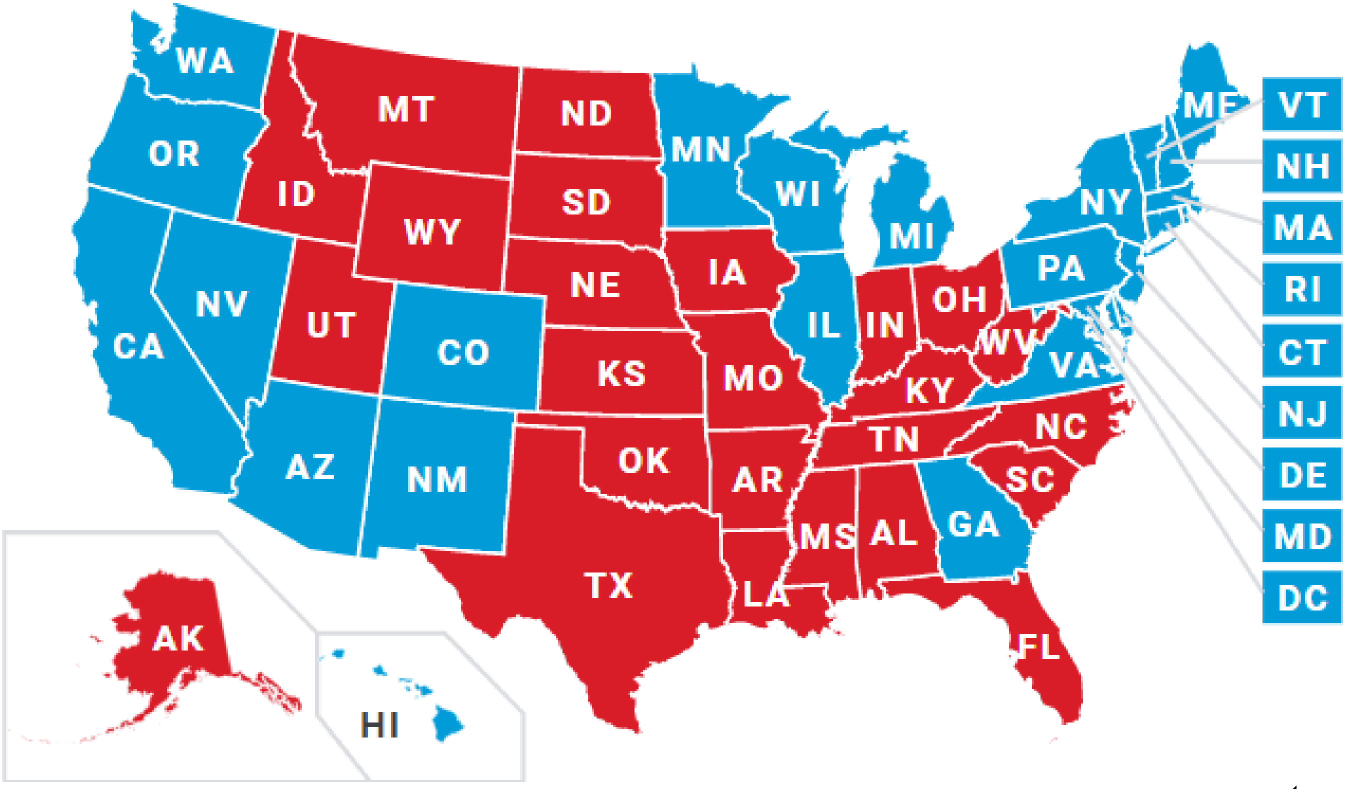
The term "map" commonly refers to a 2D spatial representation of a region of the earth's surface.

map(x, y) : position in image → position in 2D Montreal



COLOR MAP

The color map representing the USA election results in 2020.



vote_result : US_state → {D, R}

RESTAURANT MENU

menu : dish_name → price

PLATS

SAUMON POÊLÉ
caponata, yogourt, rattes confites, épinards, citron
27

"THE" POUTINE
canard confit, champignons, oignons sautés au Jack Daniel's, Tomme du Haut Richelieu, fromage en grains
19.5

POULET AU BABEURRE
esquites de maïs , cotija, coriandre, lime, salade verte
27

STEAKS

servis avec deux accompagnements

FILET MIGNON (7 OZ)
BLACK ANGUS "1855"
beurre miso/truffe
38

BAVETTE(8 OZ)
BLACK ANGUS "1855"
sauce au poivre
33

ONGLET(8 OZ)
BLACK ANGUS "1855"
mariné au chimichurri
33

BURGERS

servis avec salade & choix de frites régulières ou de patates douces

LE DOUBLE CHEESE
Classique
2 boulettes de boeuf 4oz, fromage orange, sauce secrète du H, oignon rouge, pickle, bacon
16

GUÉDILLE DE HOMARD
1/2 HOMARD
céleri, persil, oignons verts, mayo
22

LE BANH MI
haut de cuisse de poulet frits, légumes marinés, basilic thaï, mayo Sriracha,
18
aussi offert en version végétarienne (tofu skin)

LE MONTIGNAC 2.0
boulette cerf 8oz, oignons caramélisés au Jack Daniel, bacon, Gruyère suisse, sauce BBQ, mayo moutarde à l'ancienne, rondelles d'oignons du H
19

ACCOMPAGNEMENTS

POMME DE TERRE ALIGOT
purée, crème, cheddar vieilli
9

FRITES PATATES DOUCE & MAYO
7

FRITES & MAYO
6

POUTINE
sauce et fromage en grain
10

CHAMPIGNONS
9

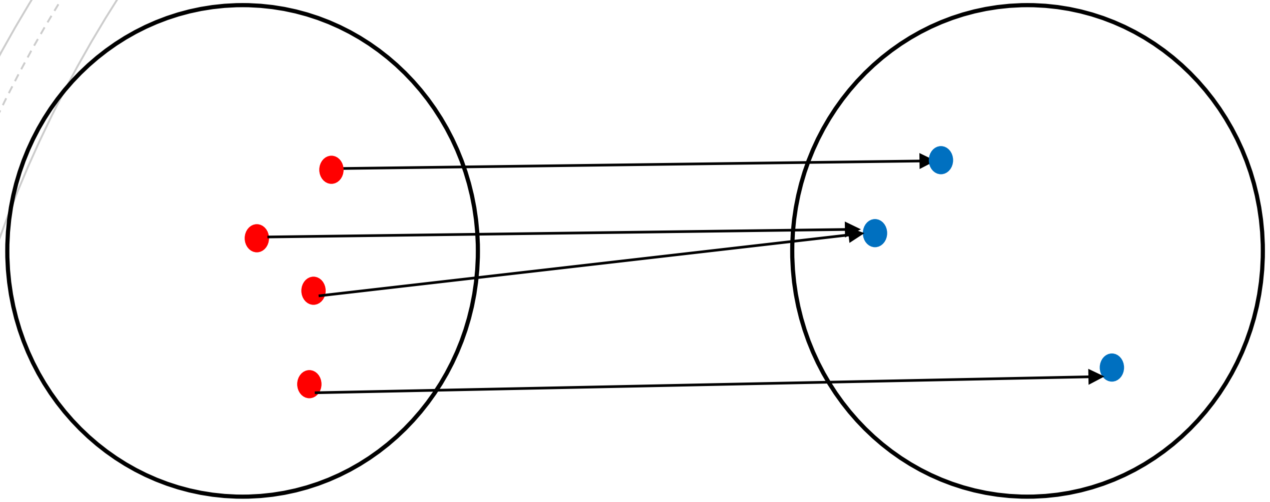
SALADE VERTE
vinaigrette au gingembre
7

RAPINIS AIL ET CITRON
9

INDEX IN A BOOK

index : term → list of pages

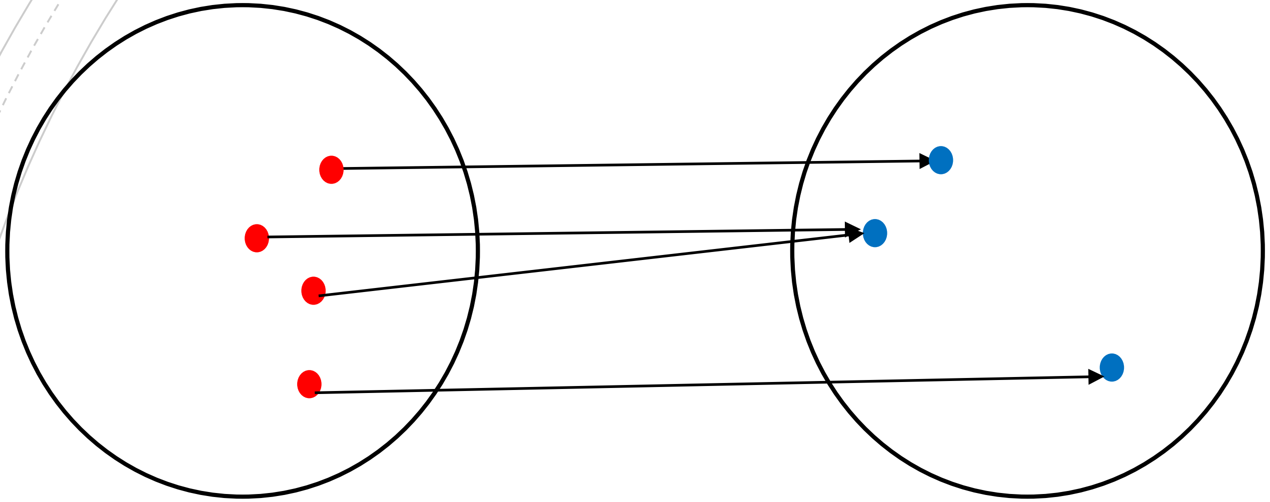
MAP (ADT)



keys (type K) values (type V)

A map is a set of (key, value) pairs.
For each key, there is at most one value.

MAP (ADT)

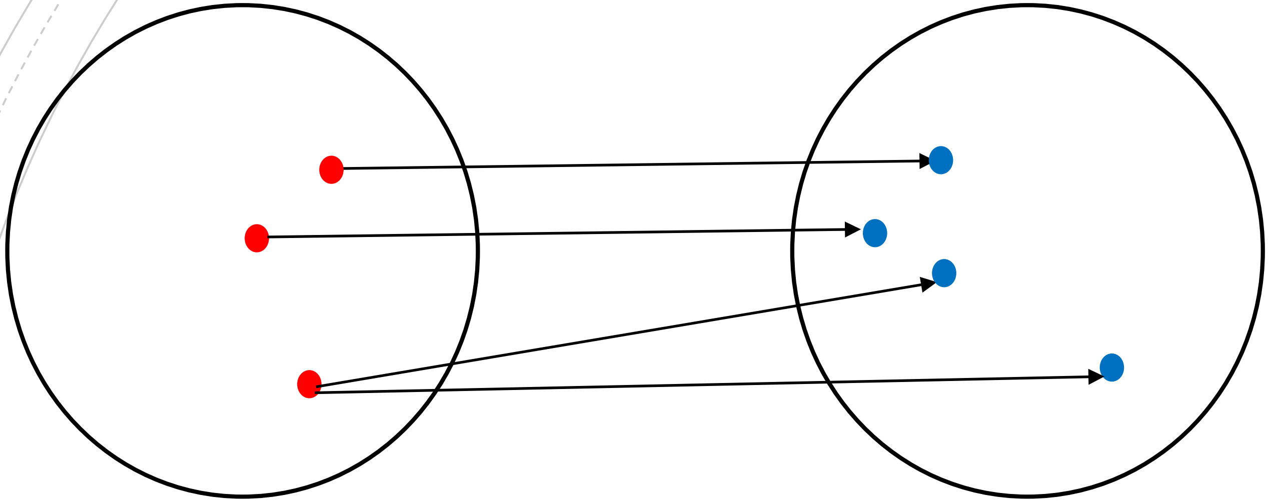


keys (type K)

values (type V)

Note that it is possible for two keys to map to the same value.

MAP (ADT)

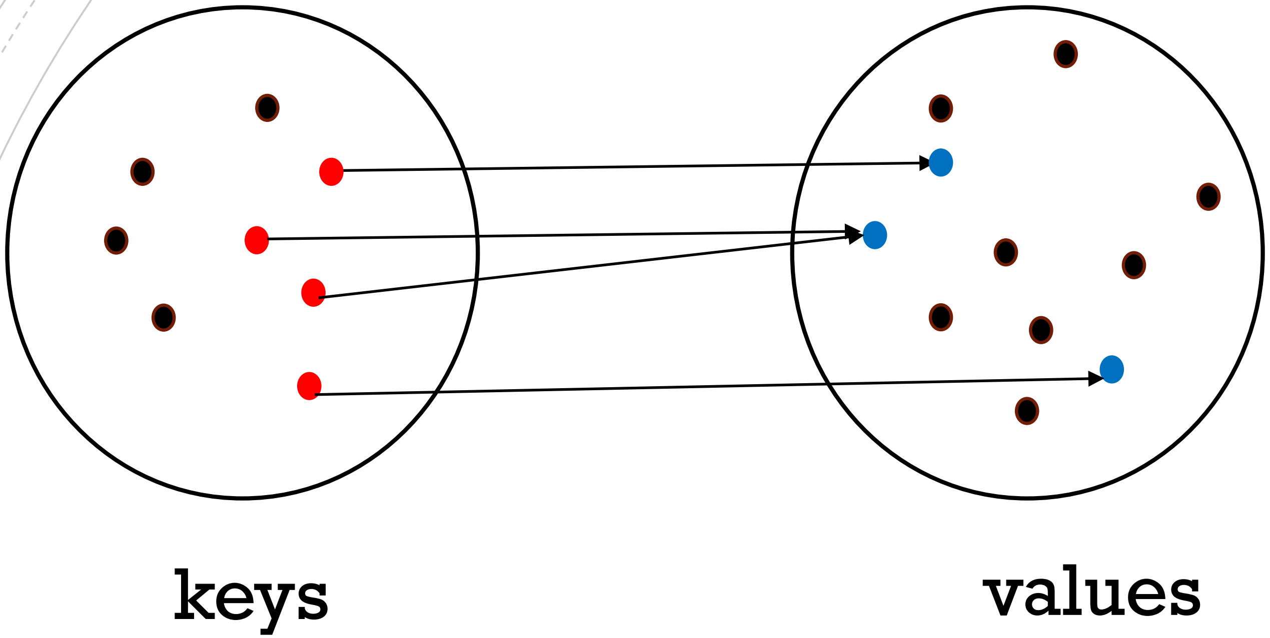


keys (type K)

values (type V)

It is NOT allowed for one key to map to two different values! The example above is NOT a map.

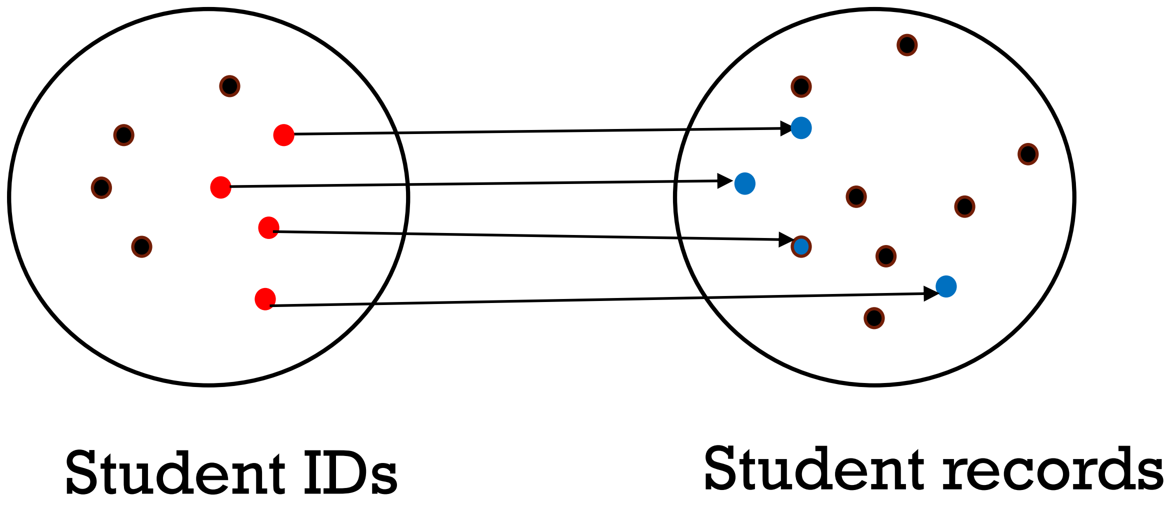
MAP ENTRIES



The black dots here indicate objects (or potential objects) of type K or V that are *not* in the map.

Each (key, value) pair is called an *entry*. In this example, there are four entries.

EXAMPLE



In COMP 250 this semester, the above mapping has ~650 entries.

Most McGill students are not taking COMP 250 this semester.

Student ID also happens to be part of the student record.

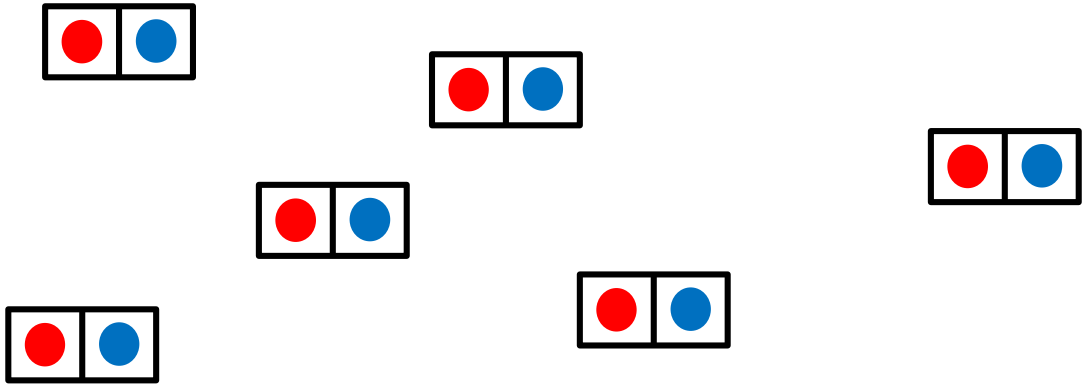
MAP ADT

put(key, value) // Add the entry (key, value) to the map. If the map previously contained an entry with key, the old value is replaced by the specified value.

get(key) // Returns the value to which the specified key is mapped. Why not get(key, value) ?

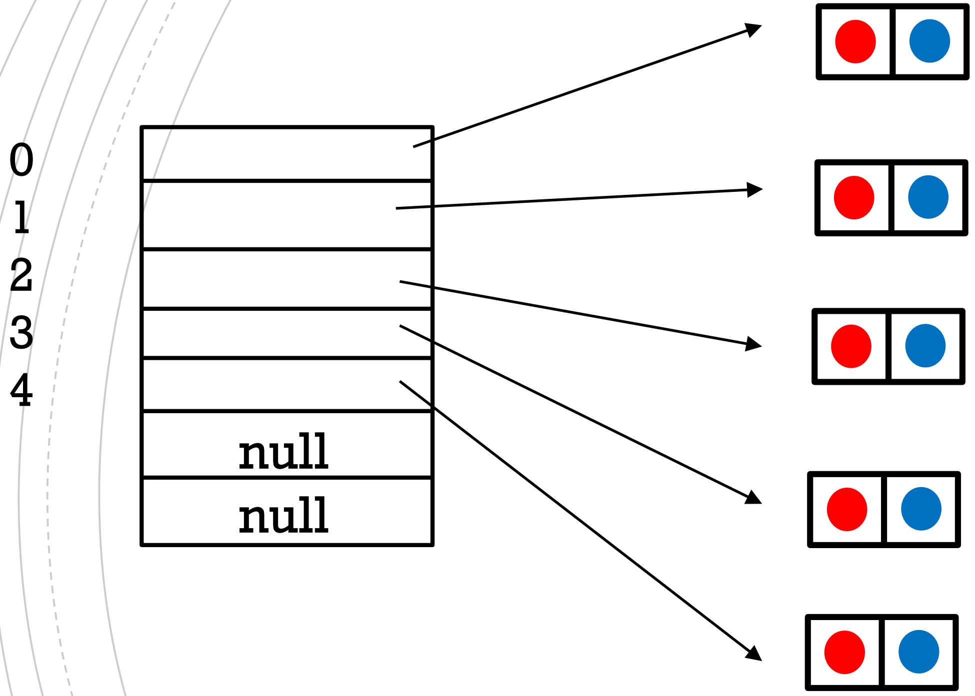
remove(key) // Removes the entry with the specified key. Returns true if the entry was removed, false otherwise.

DATA STRUCTURES FOR MAPS



How to organize a set of (key, value) pairs, i.e. entries ?

ARRAY LIST

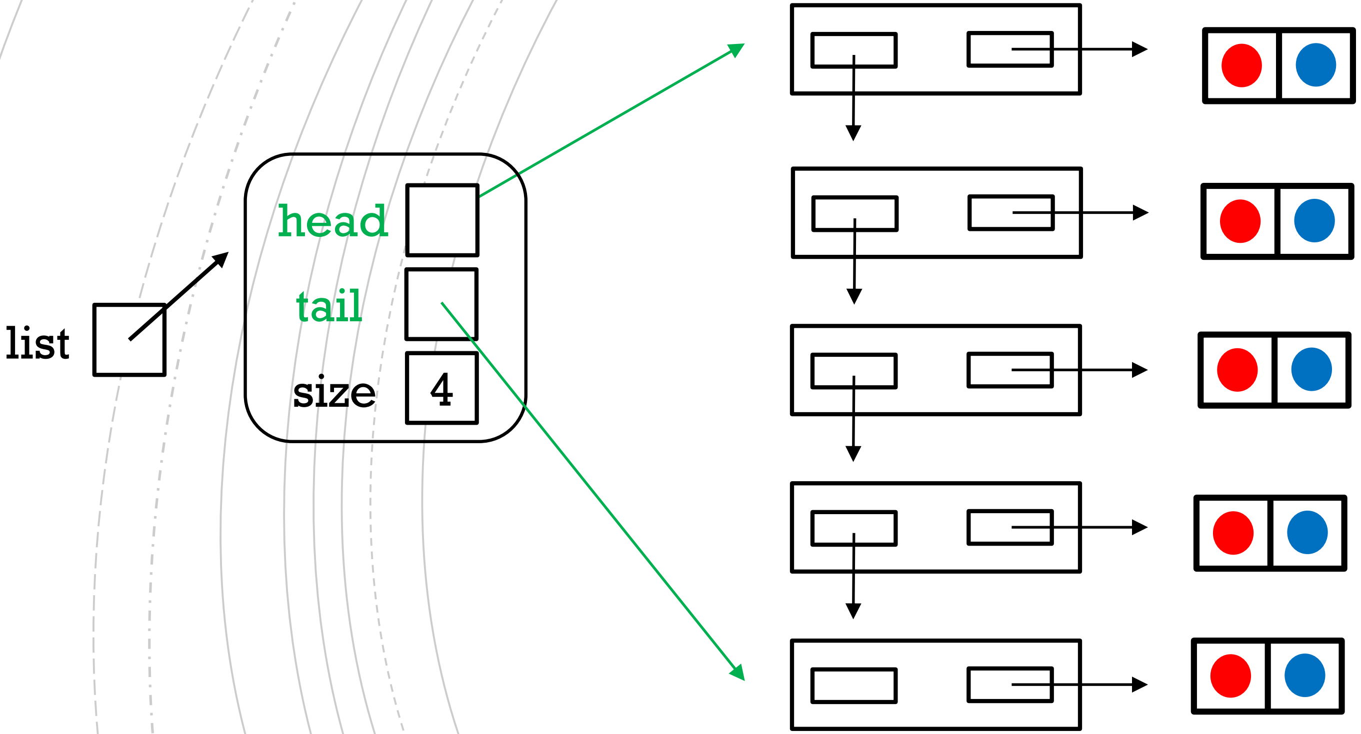


How can we implement the following methods?

put(key, value)
get(key)
remove(key)

put(), get() and remove() would be O(n)

SINGLY (OR DOUBLY) LINKED LIST



How can we implement the following methods?

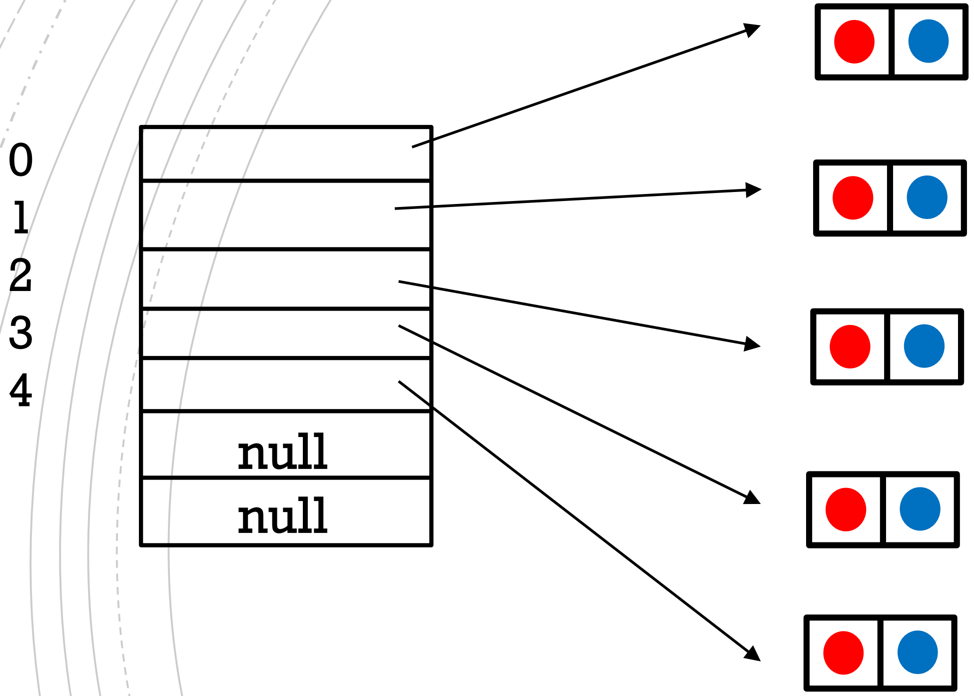
put(key, value)
get(key)
remove(key)

put(), get() and remove() would be O(n)

LET'S ADD ASSUMPTIONS

- Special case #1: what if keys are *comparable* ?

ARRAY LIST (SORTED BY KEY)



How can we implement the following methods?

put(key, value)
get(key)
remove(key)

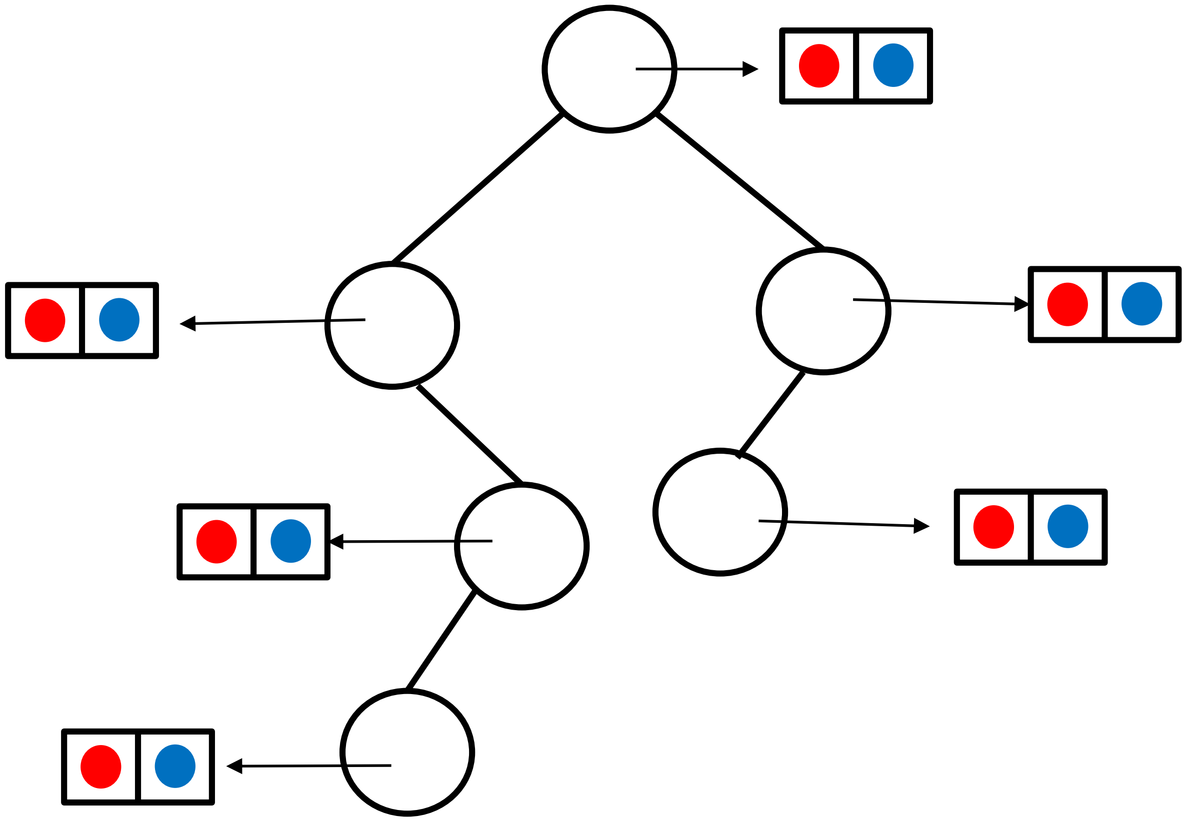
put() and remove() would be O(n), while get() could be performs in time O(log n) using binary search

BINARY SEARCH TREE (SORTED BY KEY)

How can we implement the following methods?

put(key, value)
get(key)
remove(key)

The performance of put(), get() and remove() depends on the tree. If we have a balances tree, then these operations would all take time O(log n) in worst case. You will learn more about balanced tree in COMP 251.

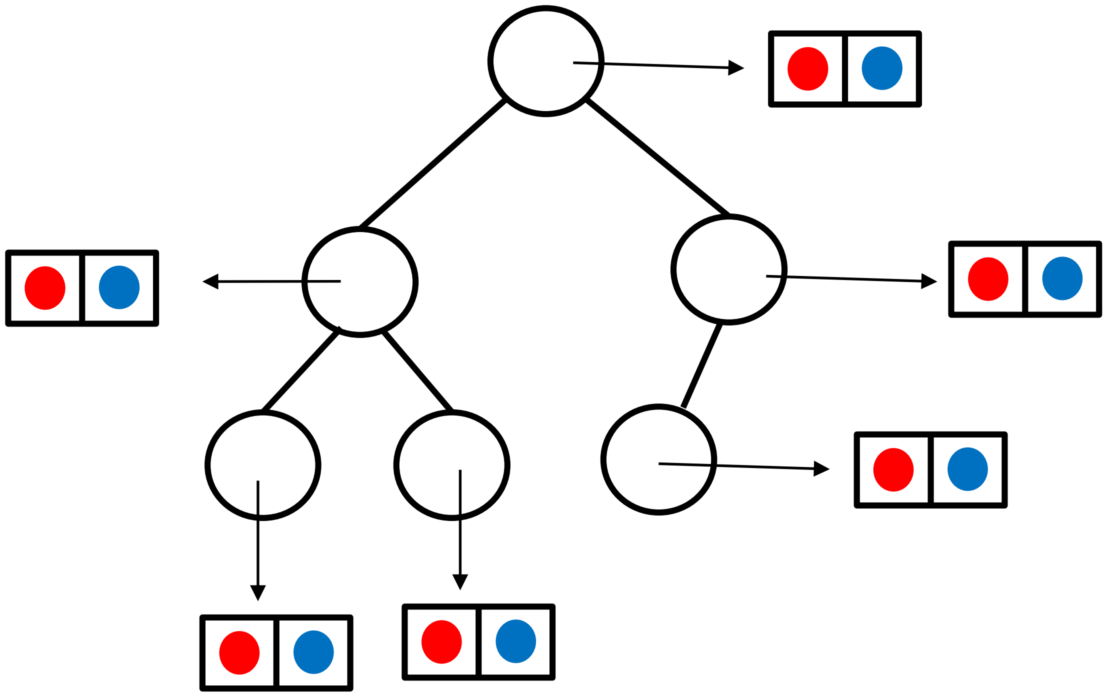


MINHEAP (PRIORITY DEFINED BY KEY)

How can we implement the following methods?

put(key, value)
get(key)
remove(key)

The performance of put() would be O(log n). Implementing get() would require traversing the tree, so it would be O(n). Implementing remove() would be a little weird for heaps…



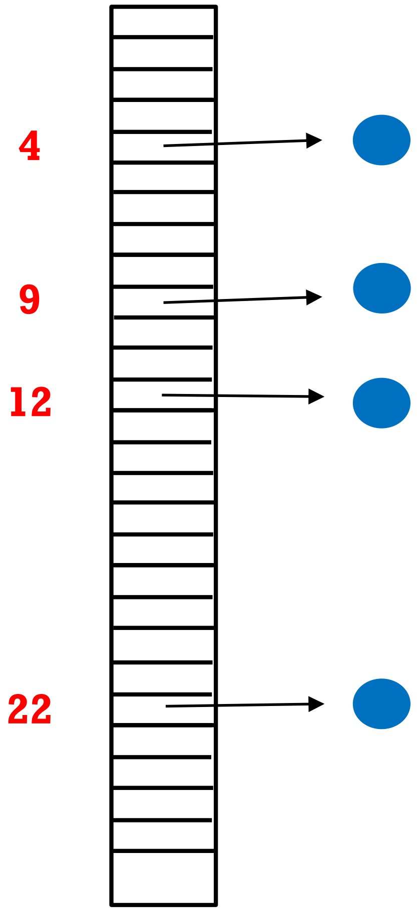
LET'S ADD ASSUMPTIONS

- ~~Special case #1: what if keys are *comparable* ?~~
- Special case #2: what if keys are unique positive integers in small range ?

ARRAYS OF VALUES

Then, we could use an array of type V (value) and have O(1) access.

This would not work well if keys are 9 digit student IDs.

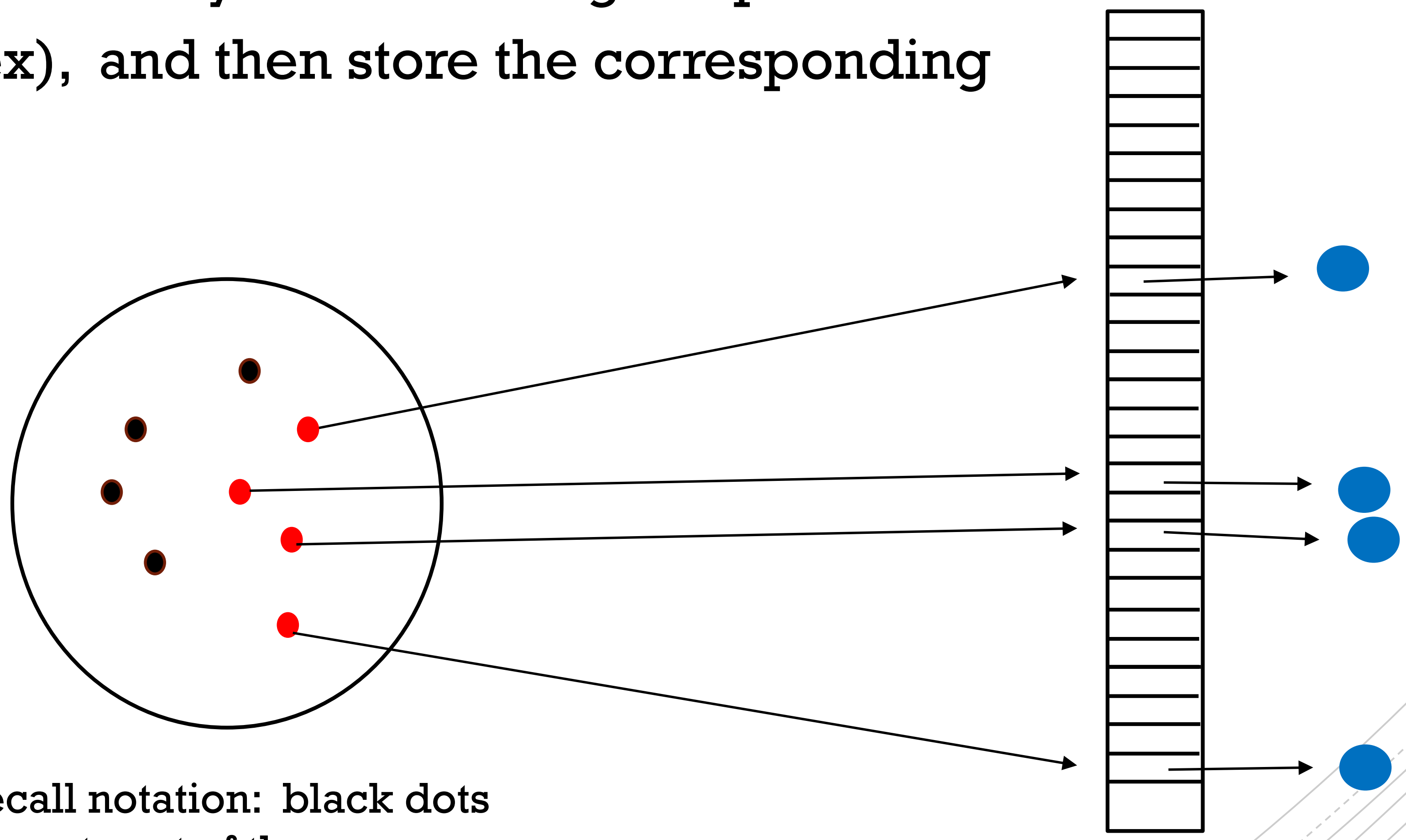


IN GENERAL

- Keys might not be comparable.
- Keys might be not be positive integers.
  e.g. Keys might be strings or some other type.

STRATEGY IN THE GENERAL CASE

Try to define a map from keys to *small* range of positive integers (array index), and then store the corresponding values in the array.



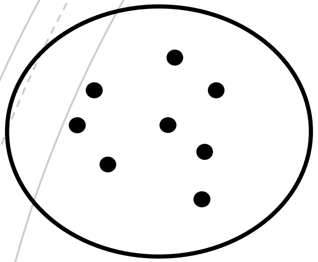
Recall notation: black dots are not part of the map.

TODAY

Define a map from keys to *large* range of positive integers

Such map is called *hash code*.

JAVA'S `Object.hashcode()`



1-to-1

$\longrightarrow$ $\{0, 1, 2, \ldots, 2^{24} - 1\}$

(not many-to-1)

objects in a Java program (runtime)

object's *address* in JVM memory (24 bits)

JAVA'S `String.hashcode()`

hashCode

```
public int hashCode()
```

Returns a hash code for this string. The hash code for a `String` object is computed as

```
s[0]*31^(n-1) + s[1]*31^(n-2) + ... + s[n-1]
```

using `int` arithmetic, where `s[i]` is the *i*th character of the string, `n` is the length of the string, and `^` indicates exponentiation. (The hash value of the empty string is zero.)

Overrides:

`hashCode` in class `Object`

Returns:

a hash code value for this object.

https://docs.oracle.com/javase/7/docs/api/java/lang/String.html

EXAMPLE HASH CODE FOR STRINGS

(not used in Java)

$$h(s) \equiv \sum_{i=0}^{s.length-1} s[i]$$

e.g.

$$h("eat") = h("ate") = h("tea")$$

ASCII values of 'a', 'e', 't' are 97, 101, 116.

JAVA'S `String.hashcode()`

$$\texttt{s.hashCode()} \equiv \sum_{i=0}^{s.length-1} s[i] * x^{s.length-1-i}$$

where $x = 31$.

JAVA'S `String.hashcode()`

$$\text{s.hashCode()} \equiv \sum_{i=0}^{s.length-1} s[i]\, x^{s.length\,-1\,-\,i}$$

where $x = 31$.

e.g. `s = "eat"` **then** `s.hashcode()` $= 101 * 31^2 + 97 * 31 + 116$

'e'	'a'	't'
s[0]	s[1]	s[2]

JAVA'S `String.hashcode()`

$$\text{s.hashCode()} \equiv \sum_{i=0}^{s.length-1} s[i]\, x^{s.length-1-i}$$

where $x = 31$.

e.g. `s = "ate"` **then** `s.hashcode()` $= 97 * 31^2 + 116 * 31 + 101$

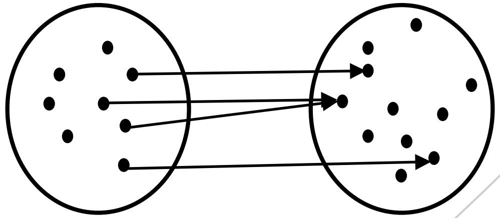
`'a'`	`'t'`	`'e'`
`s[0]`	`s[1]`	`s[2]`

JAVA'S `String.hashcode()`

$$\texttt{s.hashCode()} \equiv \sum_{i=0}^{s.length-1} s[i] * (31)^{s.length-1-i}$$

If `s1.hashCode() == s2.hashCode()` **then what can we conclude about** `s1.equals(s2)` **?**

`s1` *may or may not* be the same string as `s2`.



JAVA'S `String.hashcode()`

$$\texttt{s.hashCode()} \equiv \sum_{i=0}^{s.length-1} s[i] * (31)^{s.length - 1 - i}$$

If `s1.hashCode() != s2.hashCode()` **then what can we conclude about** `s1.equals(s2)` **?**

`s1` **is a different string then** `s2`**.**

Coming next:

- Hash Maps