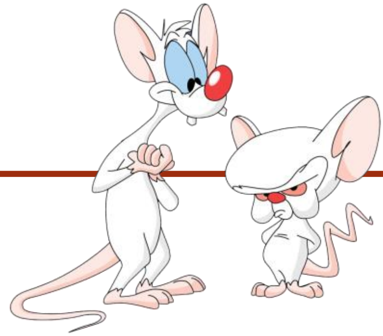
COMP 250
INTRODUCTION TO COMPUTER SCIENCE

34 - Graphs

Giulia Alberini, Fall 2022

Slides adapted from Michael Langer's

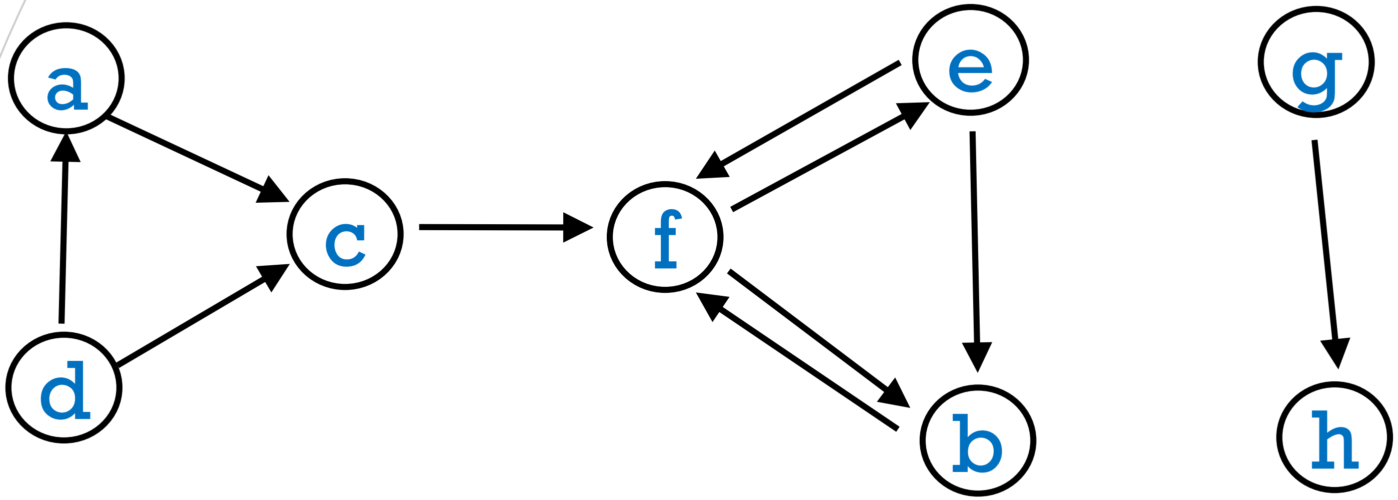
WHAT ARE WE GOING TO DO TODAY?



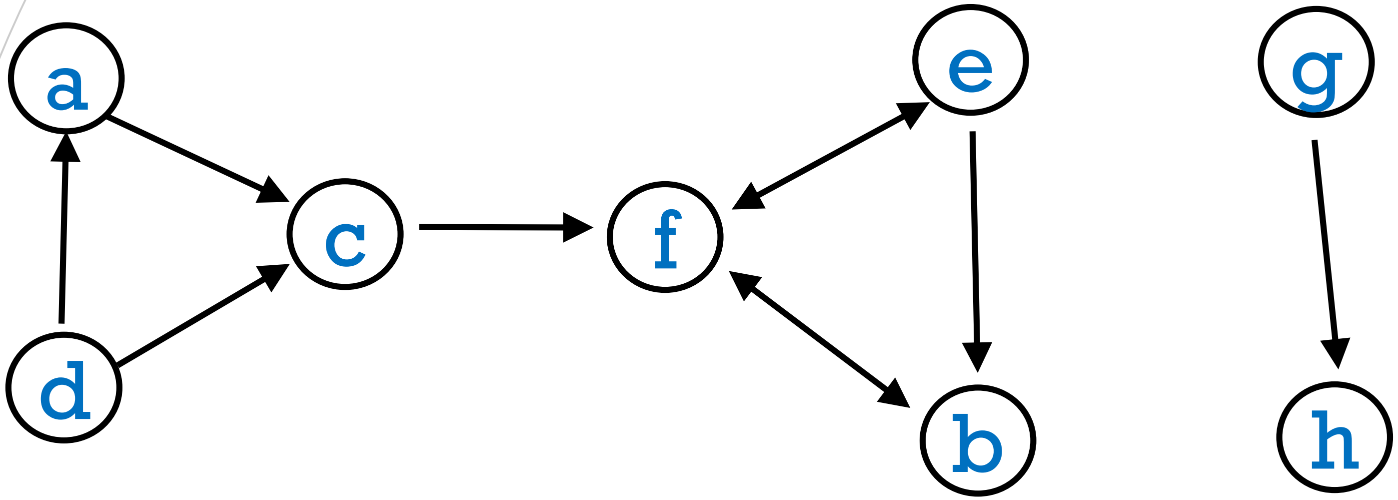
- Graphs
  - Definitions

- Recursive graph traversal
  - depth first

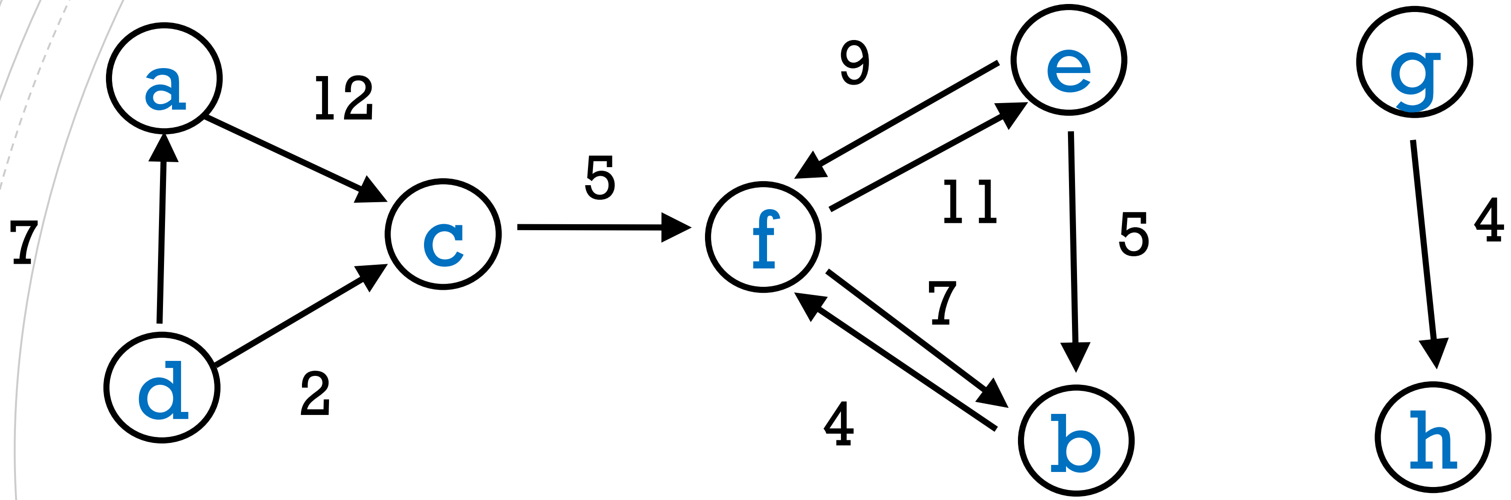
EXAMPLE



SAME EXAMPLE – DIFFERENT NOTATION



WEIGHTED GRAPH



DEFINITION

A *directed graph* is a set of *vertices*

$$V = \{v_i \ : \ i \in \{1, \dots, n\} \}$$

and set of ordered pairs of these vertices called *edges*.

$$E = \{ (v_i , v_j) \ : \ i,j \in \{1, \dots, n\} \}$$

In an *undirected* graph, the edges are *unordered* pairs.

$$E = \{ \{v_i, v_j\} \ : \ i,j \in \{1, \dots, n\} \}$$

EXAMPLES

Vertices

airports

web pages

Java objects

Edges

EXAMPLES

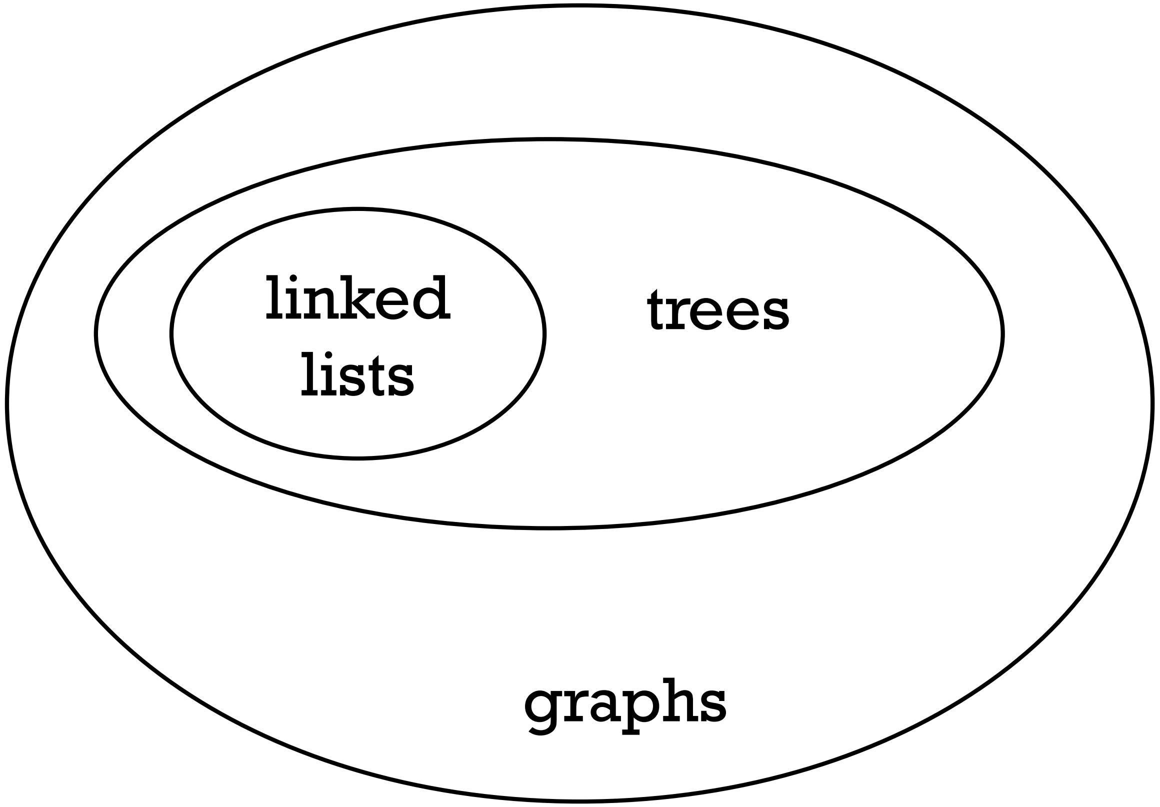
Vertices	Edges
airports	flights
web pages	
Java objects	

EXAMPLES

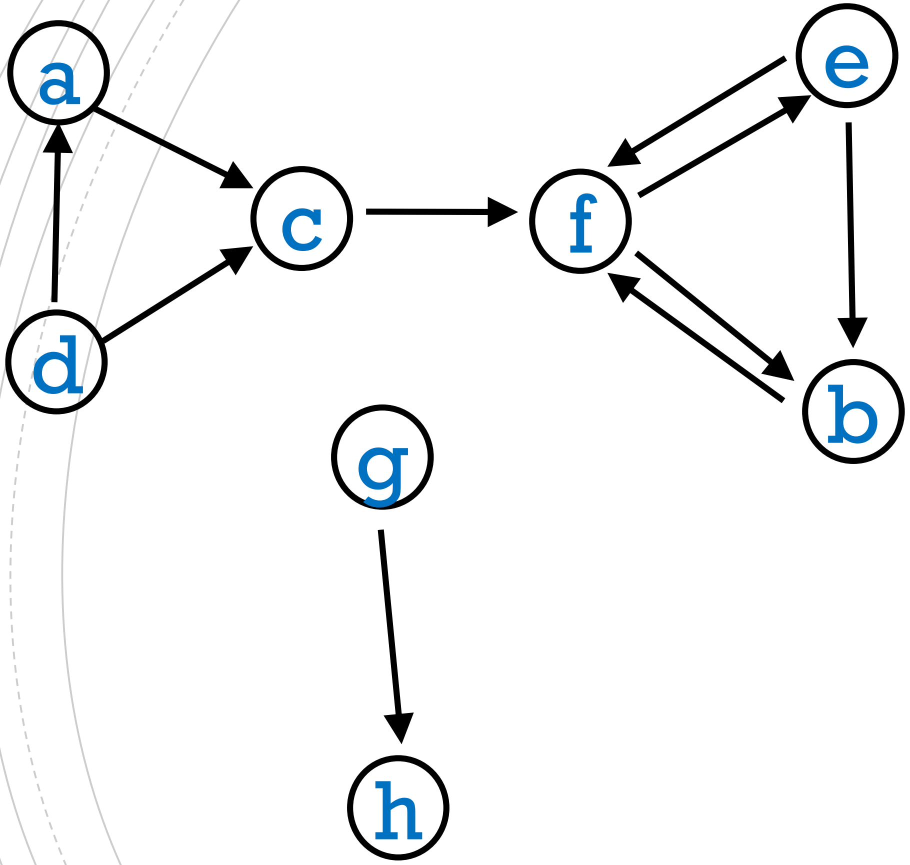
Vertices	Edges
airports	flights
web pages	links (URLs)
Java objects	

EXAMPLES

Vertices	Edges
airports	flights
web pages	links (URLs)
Java objects	references

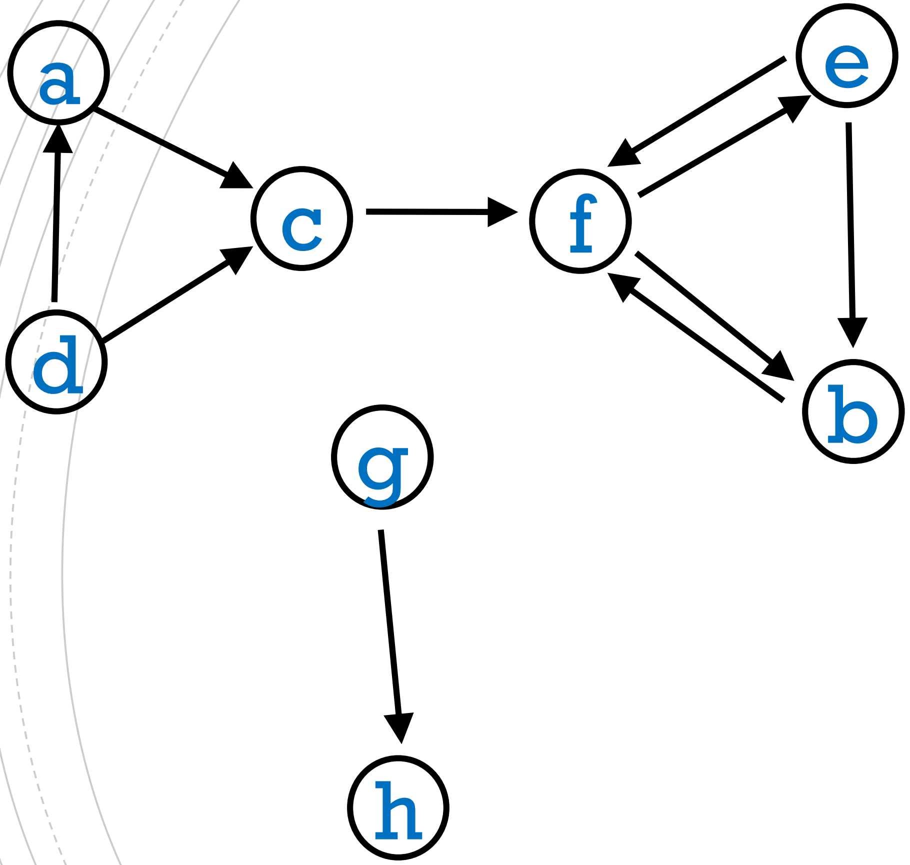

linked
lists
trees
graphs

TERMINOLOGY: "IN DEGREE"



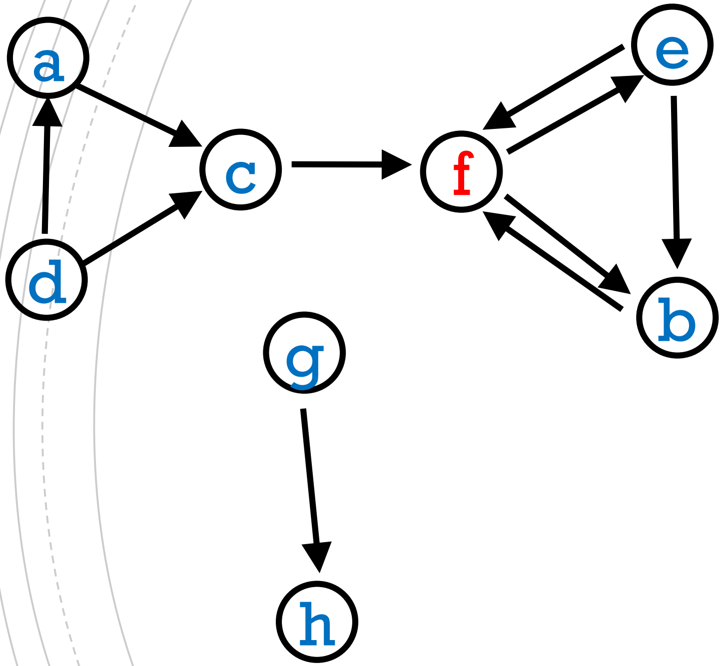
v	in degree
a	1
b	2
c	2
d	0
e	1
f	3
g	0
h	1

TERMINOLOGY: "OUT DEGREE"



v	out degree
a	1
b	1
c	1
d	2
e	2
f	2
g	1
h	0

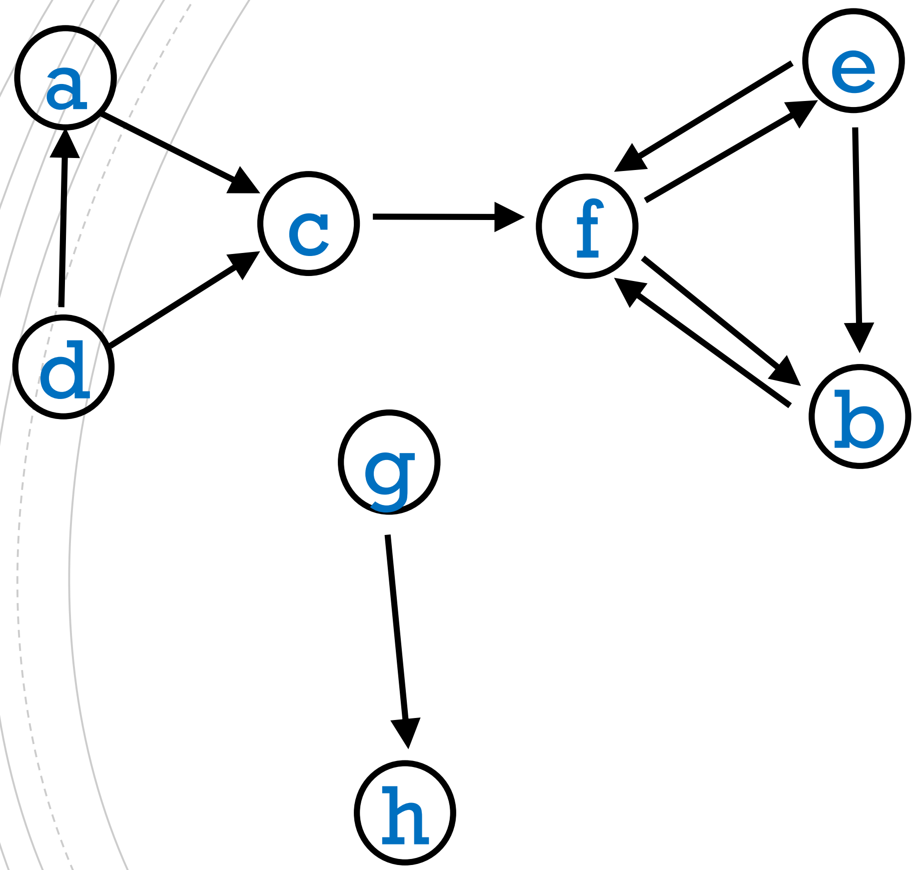
EXAMPLE: WEB PAGES



In degree: How many web pages link to some web page (e.g. f) ?

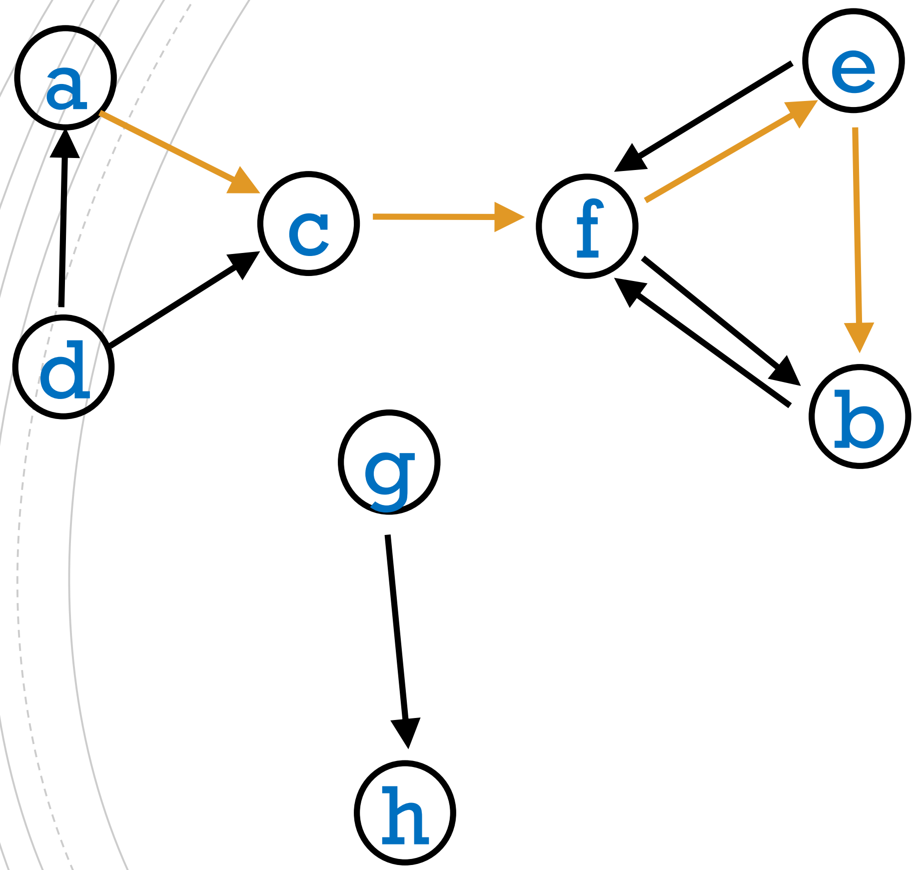
Out degree: How many web pages does some web page (e.g. f) link to ?

TERMINOLOGY: PATH



A *path* is a sequence of edges such that the end vertex of one edge is the start vertex of the next edge and no vertex is repeated except maybe first and last.

TERMINOLOGY: PATH



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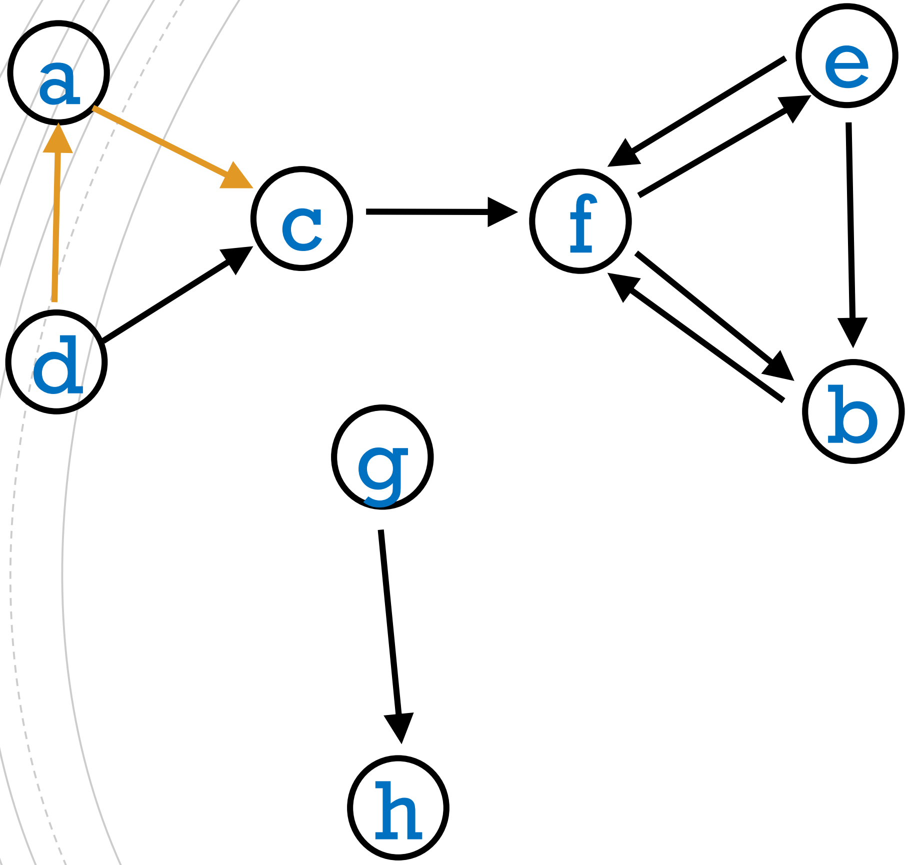
Examples

- acfeb

TERMINOLOGY: PATH

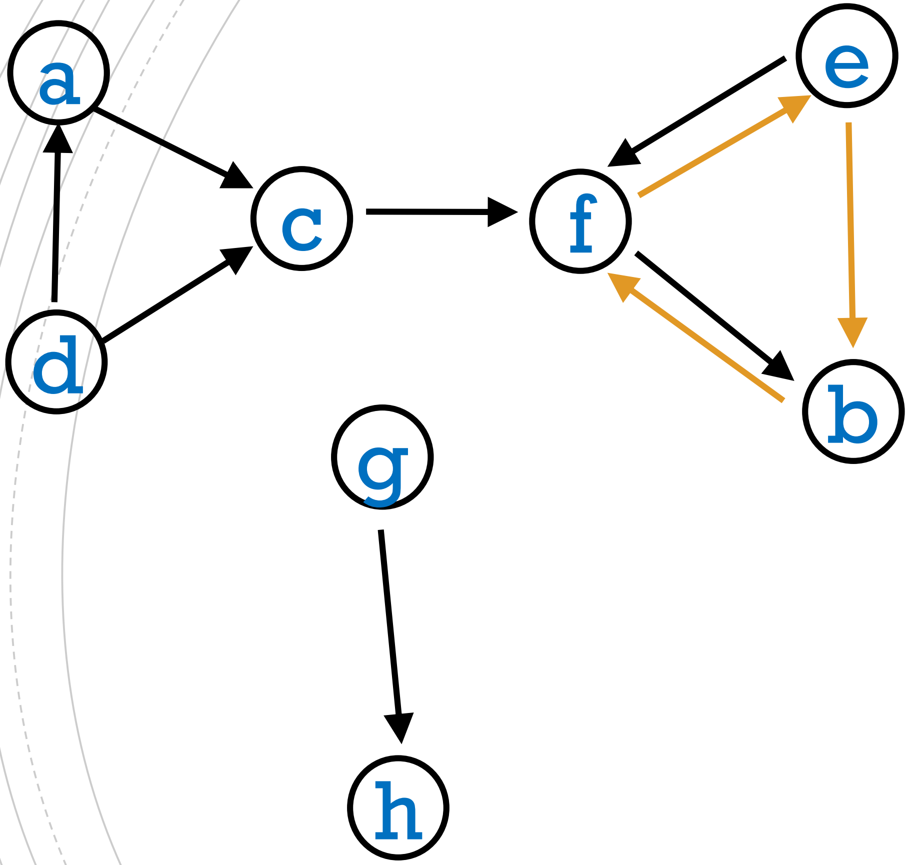


A *path* is a sequence of edges such that the end vertex of one edge is the start vertex of the next edge and no vertex is repeated except maybe first and last.

Examples

- acfeb
- dac

TERMINOLOGY: PATH



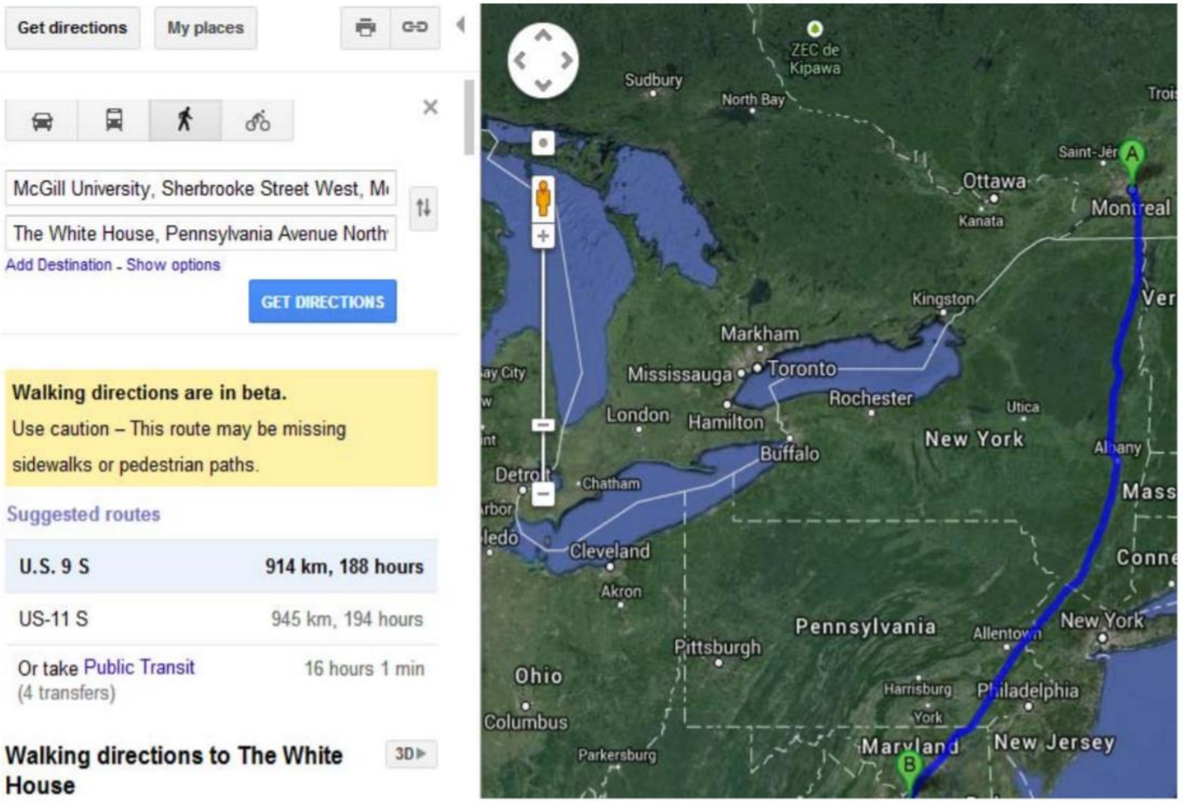
A *path* is a sequence of edges such that the end vertex of one edge is the start vertex of the next edge and no vertex is repeated except maybe first and last.

Examples

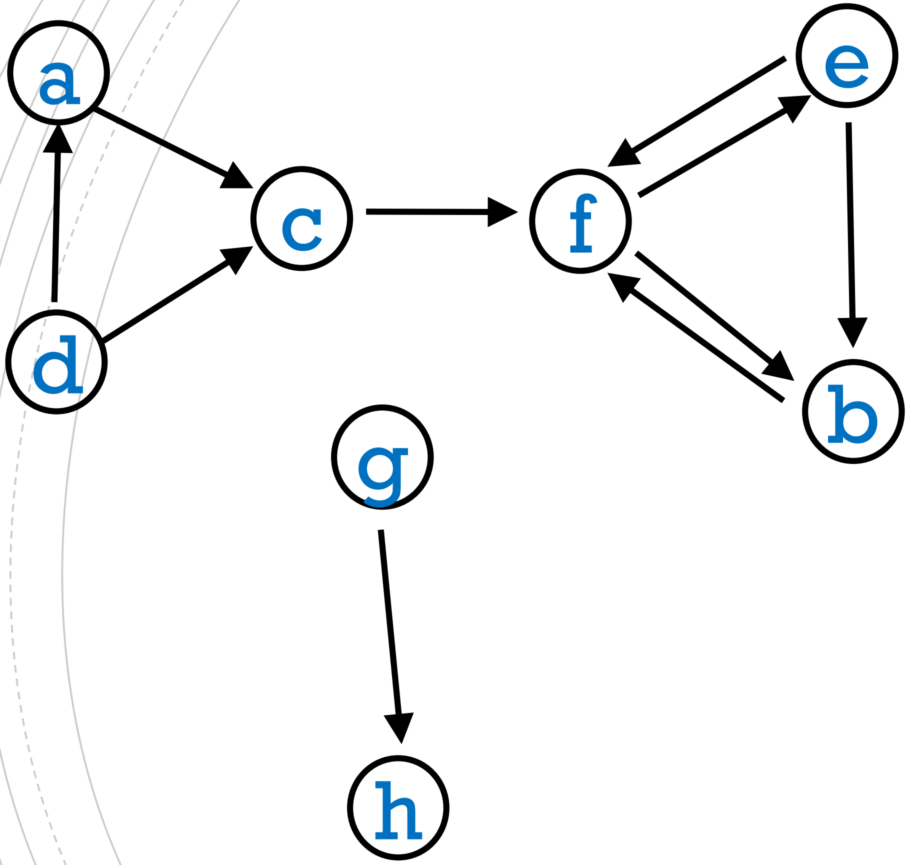
- acfeb
- dac
- febf
- .....

GRAPH ALGORITHMS IN COMP 251 (DIJKSTRA'S ALGORITHM)

Given a graph, what is the shortest (weighted) path between two vertices?

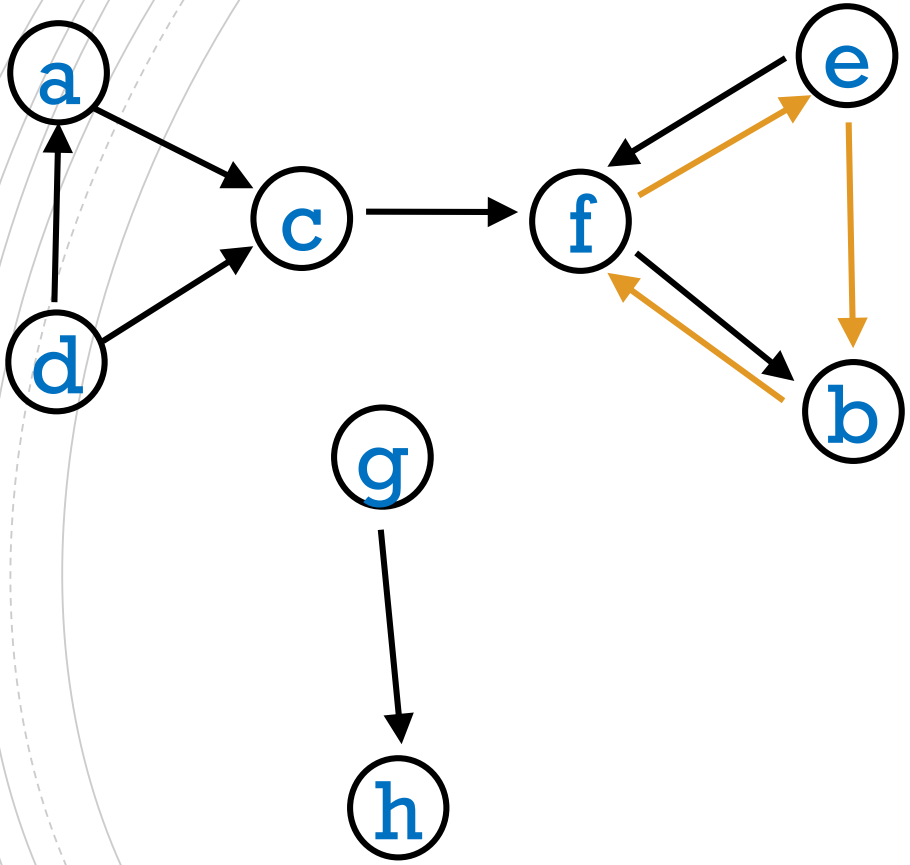


TERMINOLOGY: CYCLE



A *cycle* is a path such that the last vertex is the same as the first vertex.

TERMINOLOGY: CYCLE



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Examples

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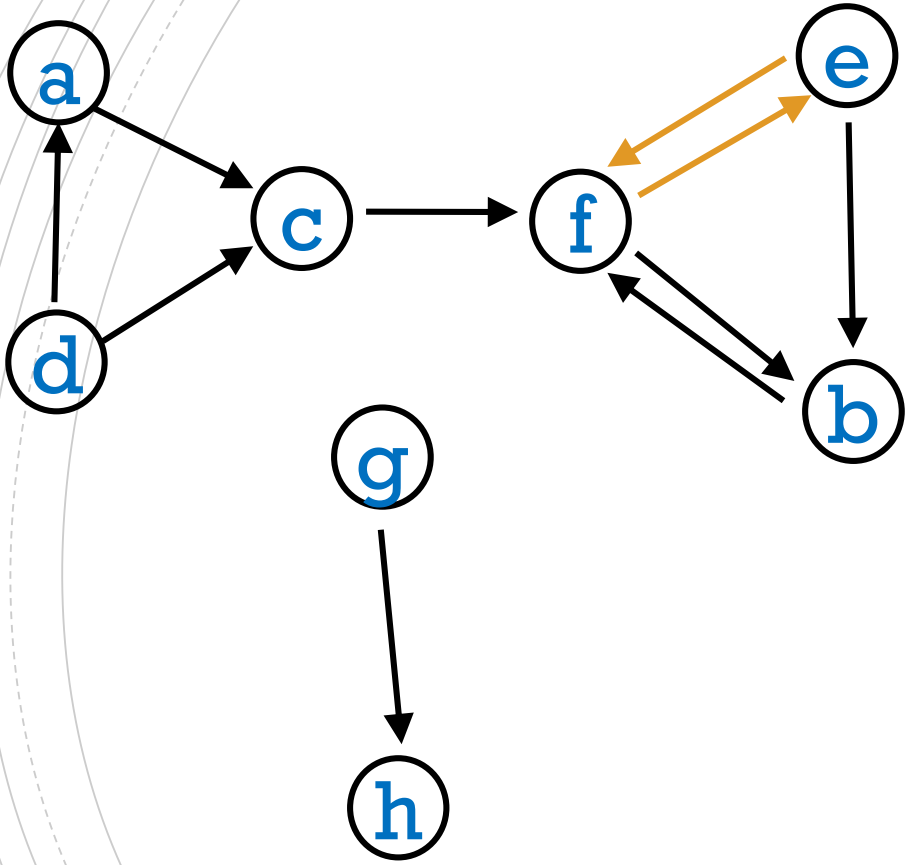
TERMINOLOGY: CYCLE



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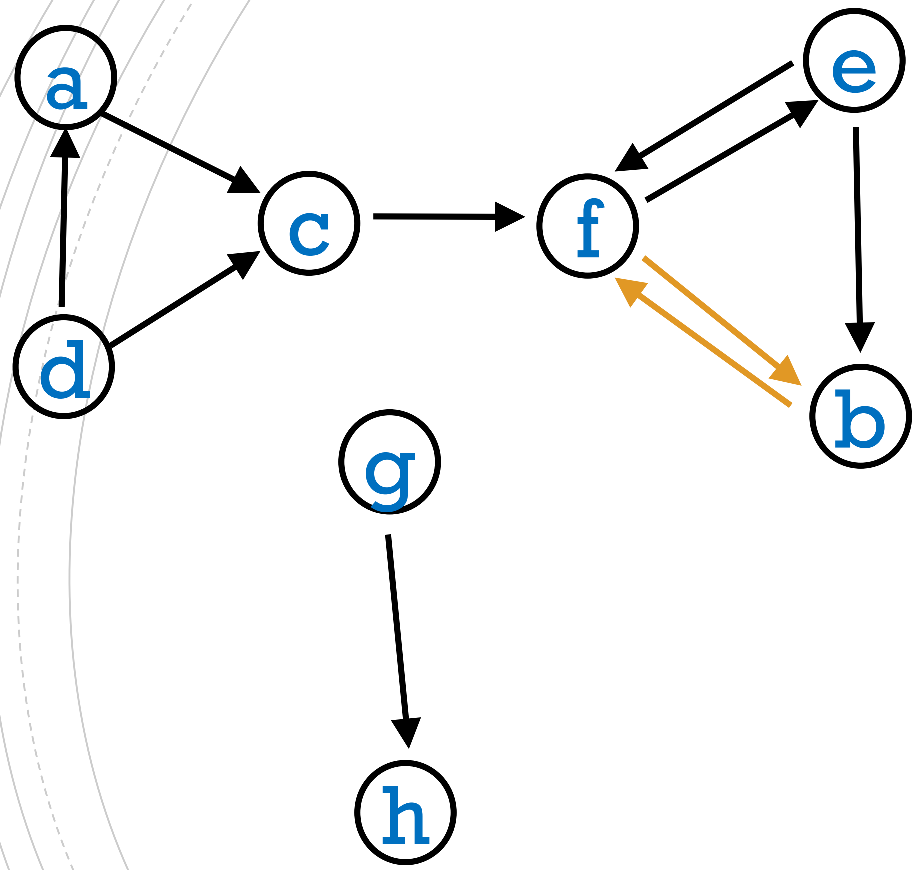
Examples

- febf
- efe

TERMINOLOGY: CYCLE

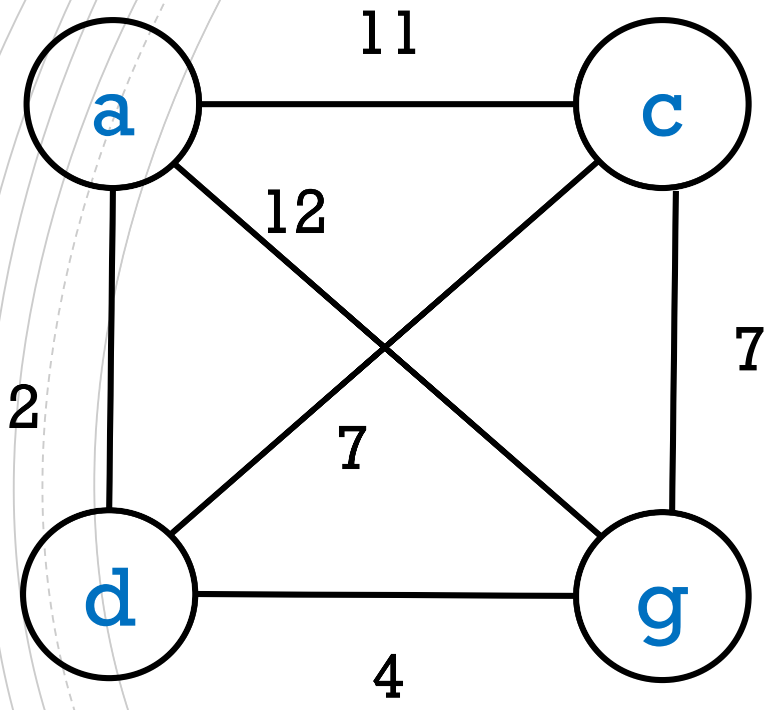


A *cycle* is a path such that the last vertex is the same as the first vertex.

Examples

- febf
- efe
- fbf
- …

"TRAVELLING SALESMAN" COMP 360 - (HAMILTONIAN CIRCUIT)

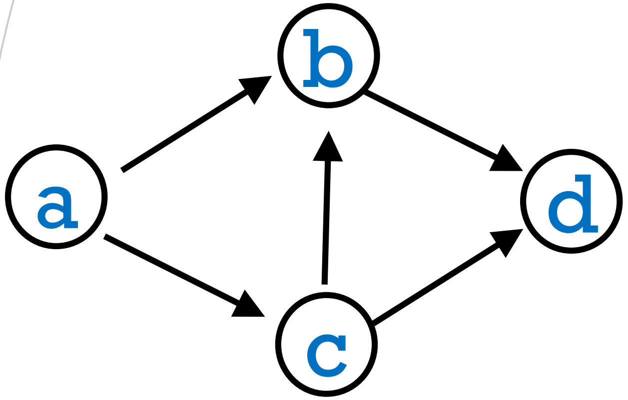


Find the shortest cycle that visits all vertices once.

How many potential cycles are there in a graph of n vertices ?

DIRECTED *ACYCLIC* GRAPH

no cycles

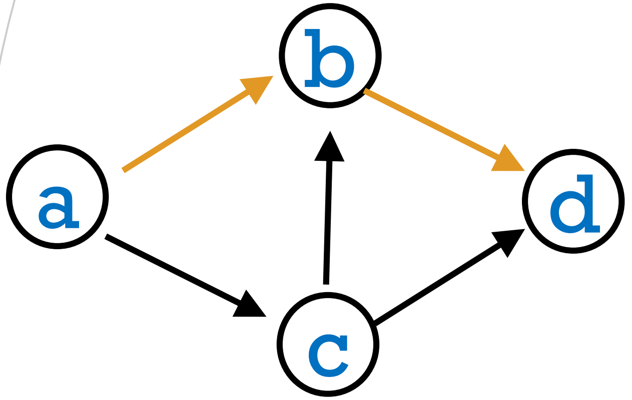


Used to capture dependencies.

There are three paths from a to d.

DIRECTED *ACYCLIC* GRAPH

no cycles

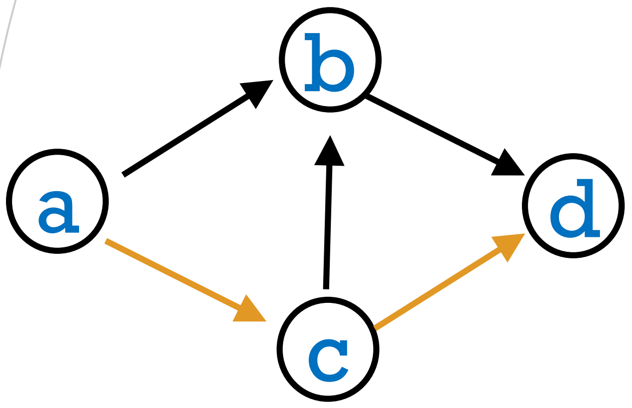


Used to capture dependencies.

There are three paths from a to d.

DIRECTED *ACYCLIC* GRAPH

no cycles

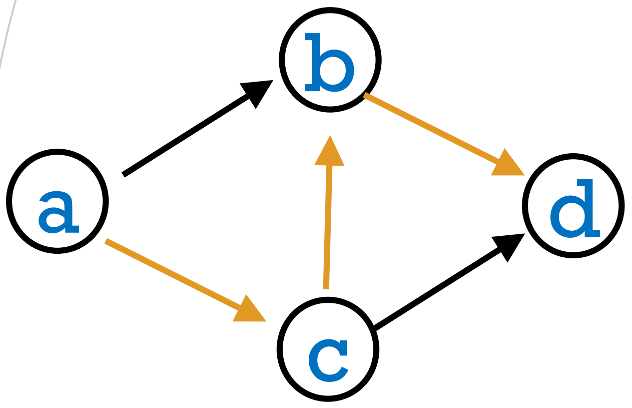


Used to capture dependencies.

There are three paths from a to d.

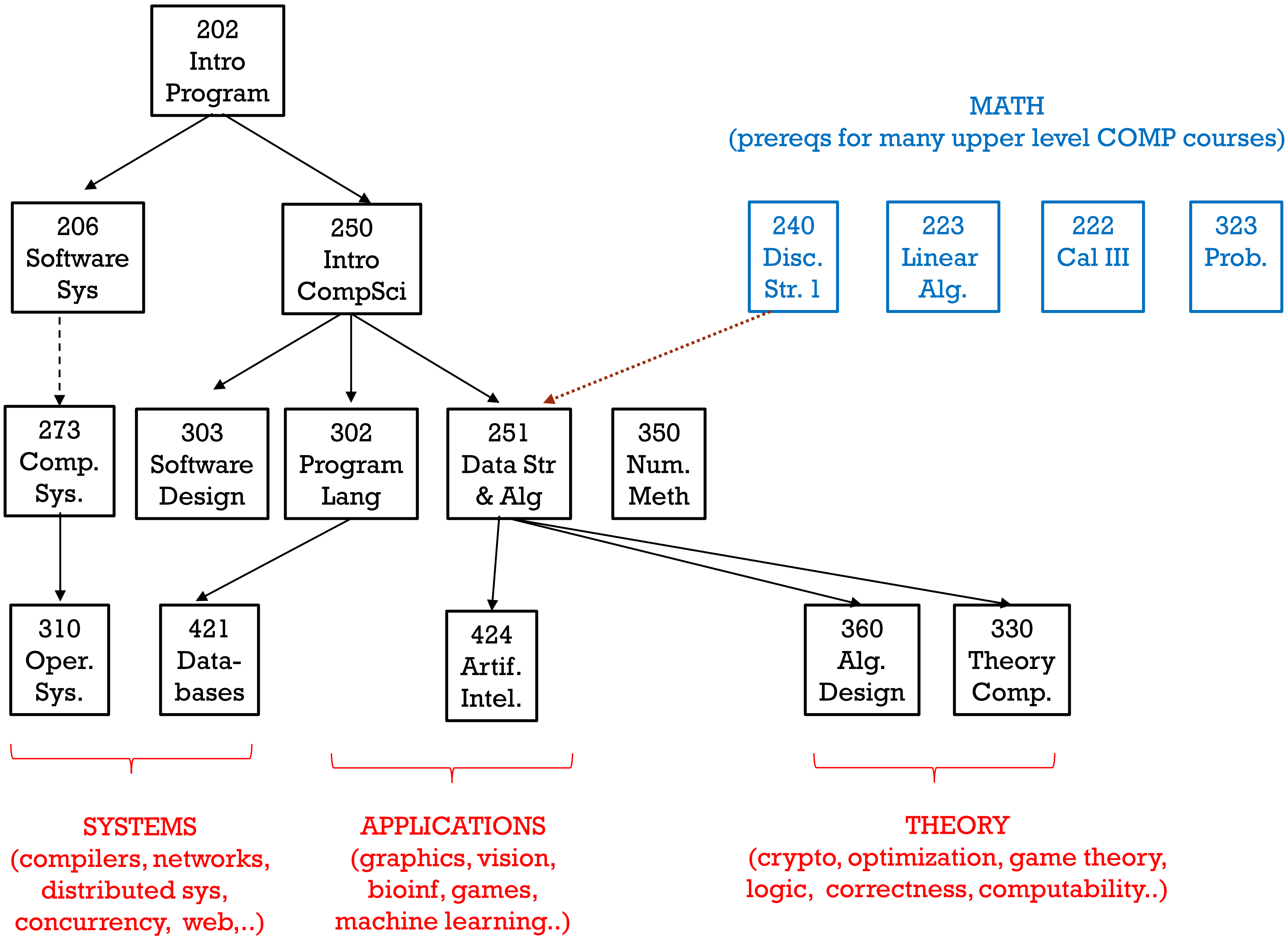
DIRECTED *ACYCLIC* GRAPH

no cycles



Used to capture dependencies.

There are three paths from a to d.


202
Intro
Program
206
Software
Sys
250
Intro
CompSci
MATH
(prereqs for many upper level COMP courses)
240
Disc.
Str. 1
223
Linear
Alg.
222
Cal III
323
Prob.
273
Comp.
Sys.
303
Software
Design
302
Program
Lang
251
Data Str
& Alg
350
Num.
Meth
310
Oper.
Sys.
421
Data-
bases
424
Artif.
Intel.
360
Alg.
Design
330
Theory
Comp.
SYSTEMS
(compilers, networks,
distributed sys,
concurrency, web,..)
APPLICATIONS
(graphics, vision,
bioinf, games,
machine learning..)
THEORY
(crypto, optimization, game theory,
logic, correctness, computability..)

GRAPH ADT

- addVertex( ), addEdge( )
- containsVertex( ), containsEdge( )
- getVertex( ), getEdge( )
- removeVertex( ), removeEdge( )
- numVertices( ), numEdges( )
- …

How to implement a Graph class?

A graph is a generalization of a tree, so …

RECALL: HOW TO IMPLEMENT A ROOTED TREE IN JAVA ?

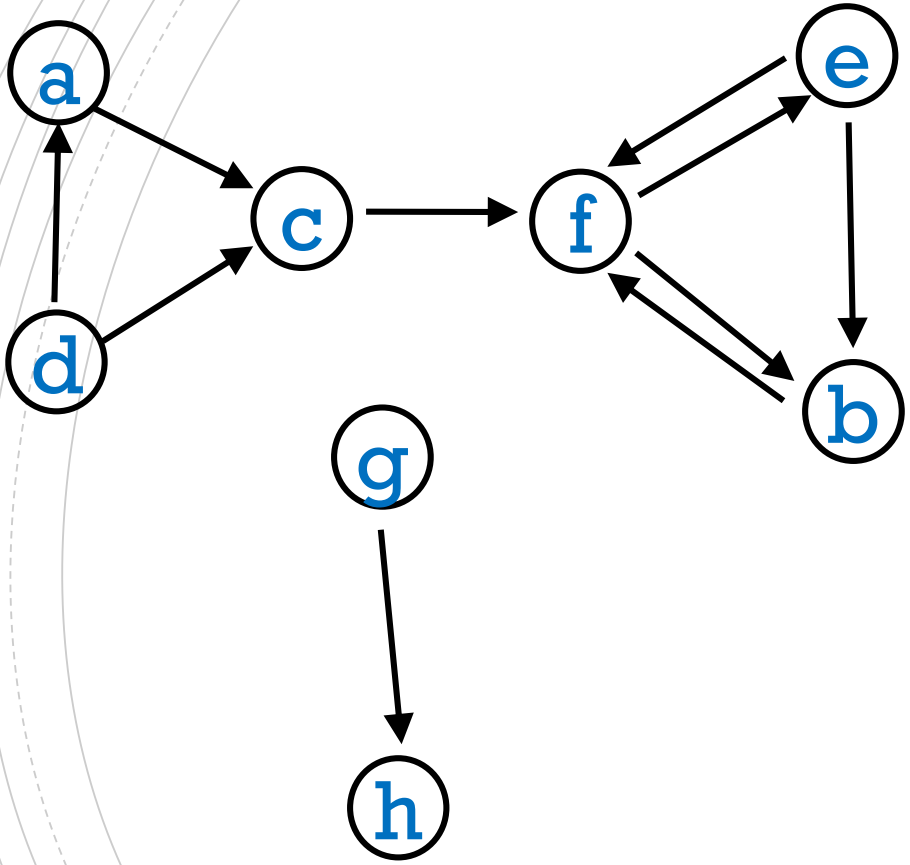
```
class Tree<T>{
   TreeNode<T> root;
      :

   class TreeNode<T>{
      T element;
      ArrayList<TreeNode<T>> children;
      TreeNode<T> parent; // optional
   }
}
```

// alternatively….

```
class Tree<T>{
   TreeNode<T> root;
      :

   class TreeNode<T>{
      T element;
      TreeNode<T> firstChild;
      TreeNode<T> nextSibling;
   }
}
```

ADJACENCY LIST

(GENERALIZATION OF CHILDREN FOR GRAPHS)



v	v.adjList
a	c
b	f
c	f
d	a, c
e	b, f
f	b, e
g	h
h	

Here each adjacency list is sorted, but that is not always possible (or necessary).

HOW TO IMPLEMENT A GRAPH CLASS IN JAVA?

A very basic Graph class:

```
class Graph<T> {
   class Vertex<T> {// We could have called it GNode
      ArrayList<Vertex> adjList;
      T element;
   }
}
```

HOW TO IMPLEMENT A GRAPH CLASS IN JAVA?

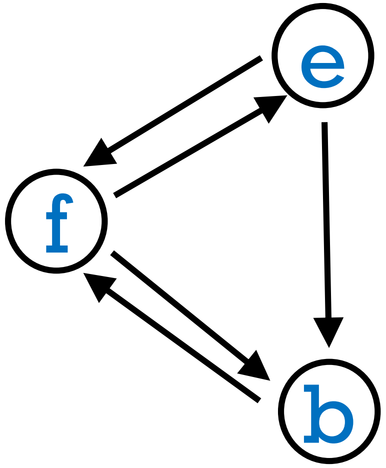
```
class Graph<T> {
   class Vertex<T> {
      ArrayList<Edge> adjList;
      T element;
      boolean visited;
   }

   class Edge {
      Vertex endVertex;
      double weight;
            :
   }
}
```

Note that, unlike a rooted tree, there is no notion of a root vertex in a graph.

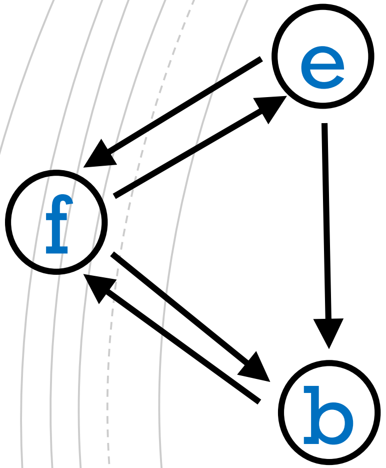
HOW TO REFERENCE VERTICES?

```
class  Graph<T> {
     ArrayList< Vertex<T> > vertexList;
         :
     class Vertex<T> { ... }
     class  Edge<T>  { ... }
}
```

HOW MANY OBJECTS ?



HOW MANY OBJECTS ?



Graph

ArrayList <Vertex>

Vertex

Vertex

Vertex

Edge

Edge

Edge

Edge

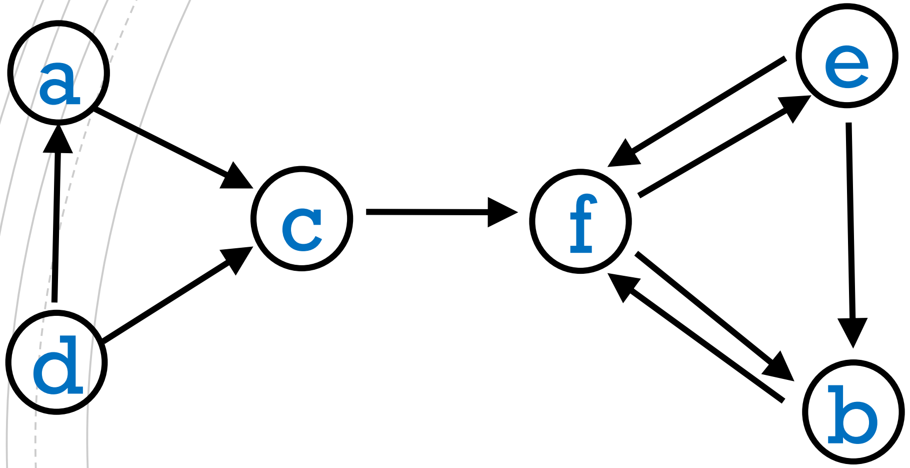
Edge

ArrayList<Edge>

ArrayList<Edge>

ArrayList<Edge>

ADJACENCY MATRIX

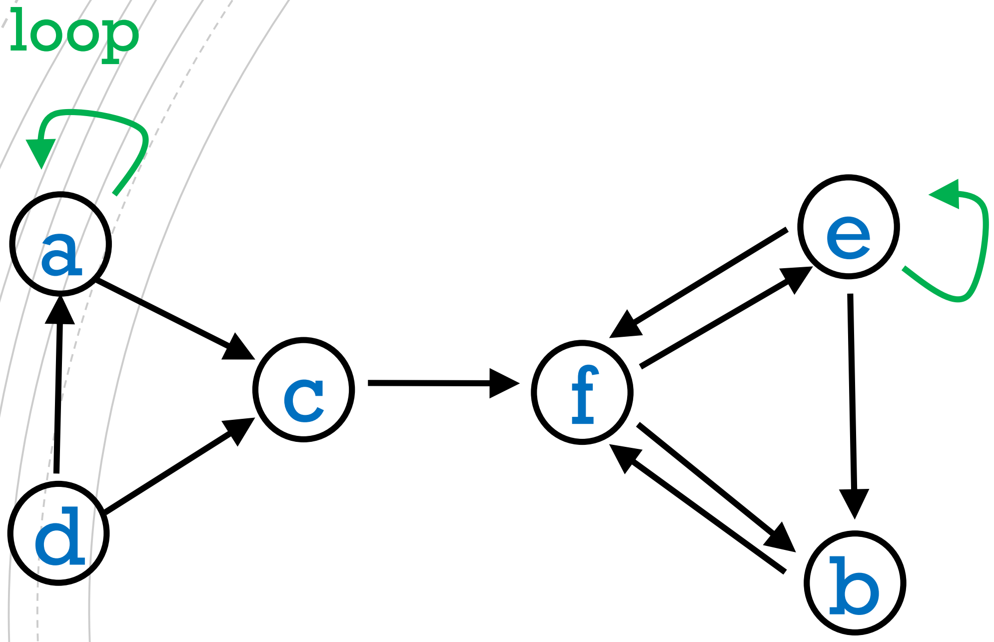


	a	b	c	d	e	f
a	0	0	1	0	0	0
b	0	0	0	0	0	1
c	0	0	0	0	0	1
d	1	0	1	0	0	0
e	0	1	0	0	0	1
f	0	1	0	0	1	0

Assume we have a mapping from vertex names to 0, 1, ... , n-1.

```
boolean[][] adjMatrix = new boolean[6][6]
```

ADJACENCY MATRIX



	a	b	c	d	e	f
a	1	0	1	0	0	0
b	0	0	0	0	0	1
c	0	0	0	0	0	1
d	1	0	1	0	0	0
e	0	1	0	0	1	1
f	0	1	0	0	1	0

Assume we have a mapping from vertex names to 0, 1, ... , n-1.

```
boolean[][] adjMatrix = new Boolean[6][6]
```

“DEFINITIONS”

Consider a graph with n vertices.

We say that the graph is *dense* if the number of edges is close to n^2.

We say that the graph is *sparse* if the number of edges is close to n.

(These are not formal definitions.)

EXERCISE

Would you use an *adjacency list* or *adjacency matrix* for each of the following scenarios?

- The graph is sparse e.g. 10,000 vertices and 20,000 edges and we want to use as little space as possible.
- The graph is dense e.g. 10,000 vertices and 20,000,000 edges, and we want to use as little space as possible. .
- Answer the query areAdjacent() as quickly as possible, no matter how much space you use.
- Perform operation insertVertex(v).
- Perform operation removeVertex(v).

GRAPH TRAVERSAL

(RECURSIVE)

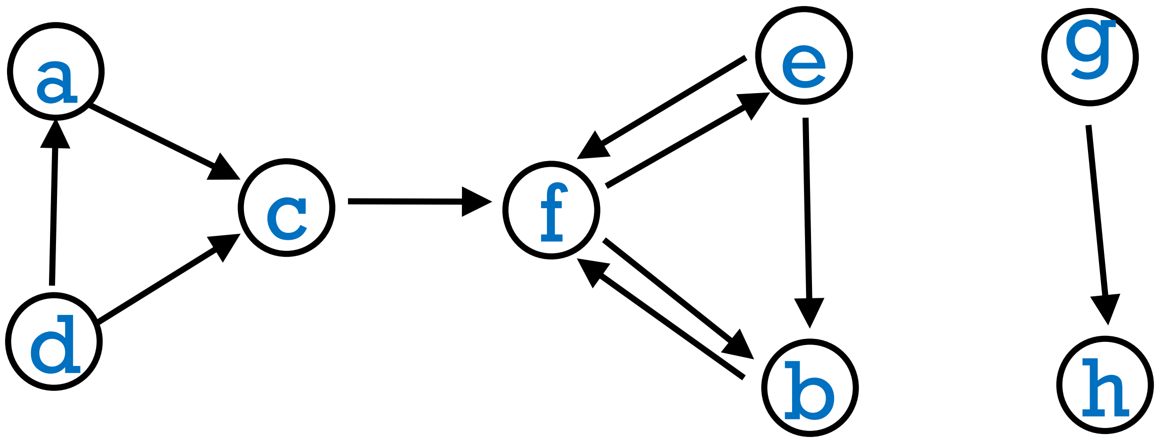
RECALL: TREE TRAVERSAL (RECURSIVE)

```
depthFirst_Tree (root){
   if (root is not empty){
      visit root // preorder
      for each child of root
         depthfirst_Tree( child )
   }
}
```

GRAPH TRAVERSAL (RECURSIVE)

Need to specify a starting vertex.

Visit all nodes that are “reachable” by a path from a starting vertex.



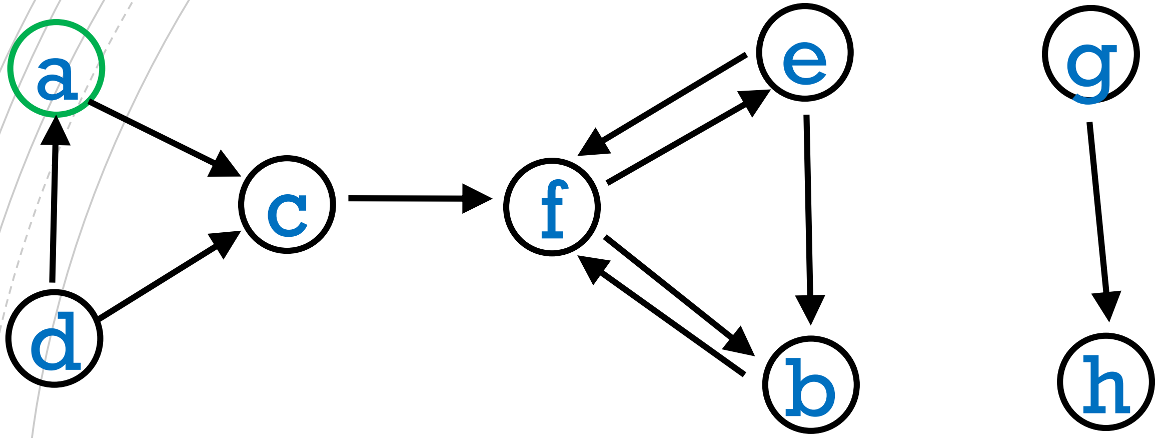
GRAPH TRAVERSAL (RECURSIVE)

```
depthFirst_Graph (v){
   v.visided = true
   for each w such that (v,w) is in E
   // i.e. v.adjList.contains(w) returns true

      ??
}
```

GRAPH TRAVERSAL (RECURSIVE)

```
depthFirst_Graph (v){
   v.visided = true
   visit v // do something with v
   for each w such that (v,w) is in E
   // i.e. for each w in v.adjList
      if !(w.visited) // avoid cycles!
         dephFirst_Graph(w)
}
```

CALL STACK FOR depthFirst(a)



a

```
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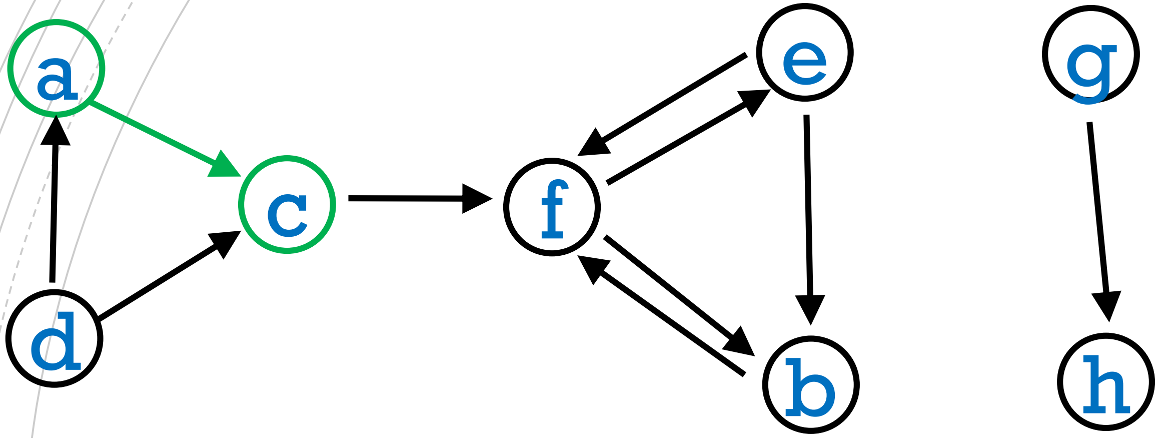

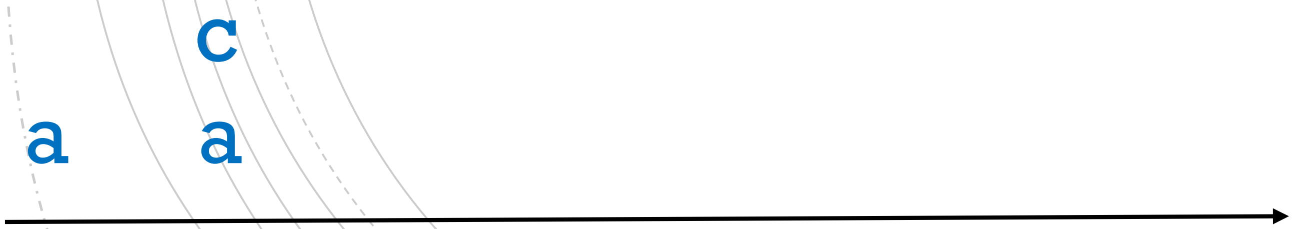


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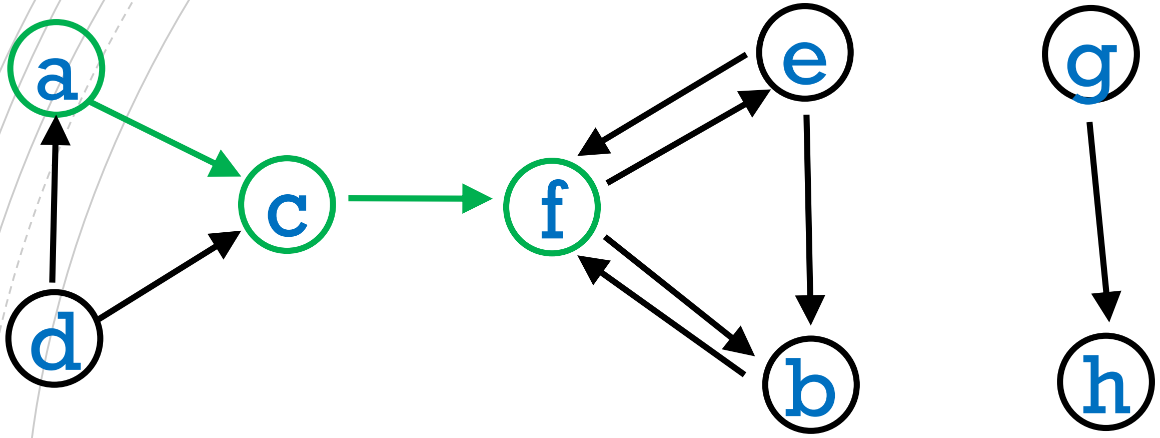

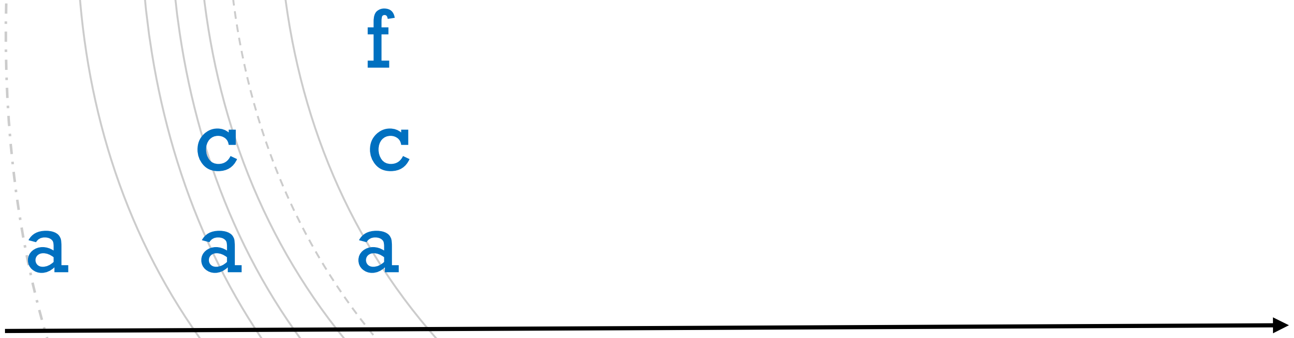


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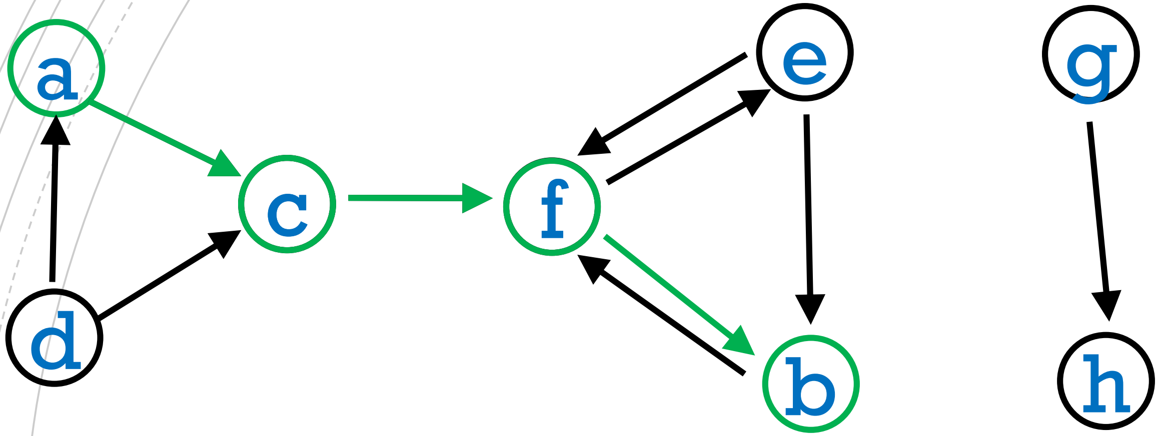

			b
		f	f
	c	c	c
a	a	a	a

```
depthFirst_Graph (v){
    v.visided = true
    for each w s.t. (v,w) is in E
        if !(w.visited)
            dephFirst_Graph(w)
}
```

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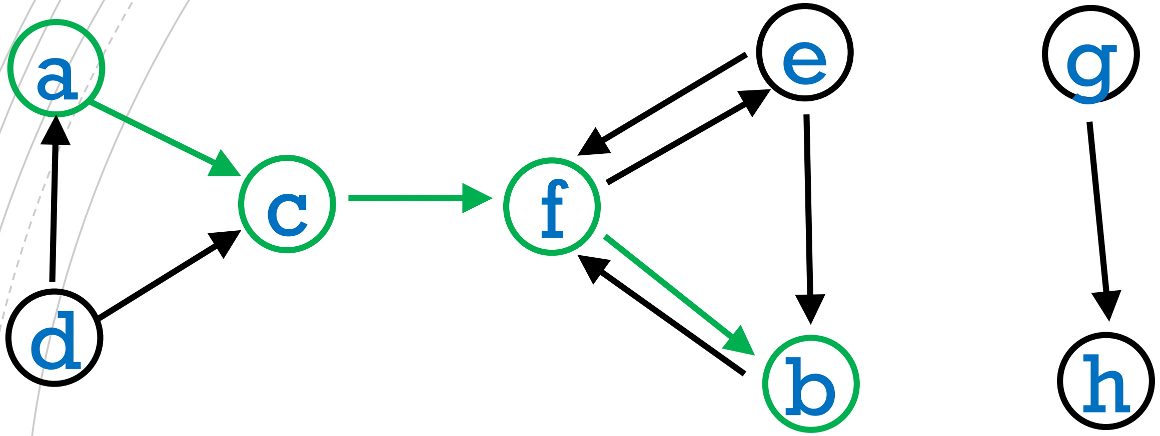

			b	
		f	f	f
	c	c	c	c
a	a	a	a	a

```
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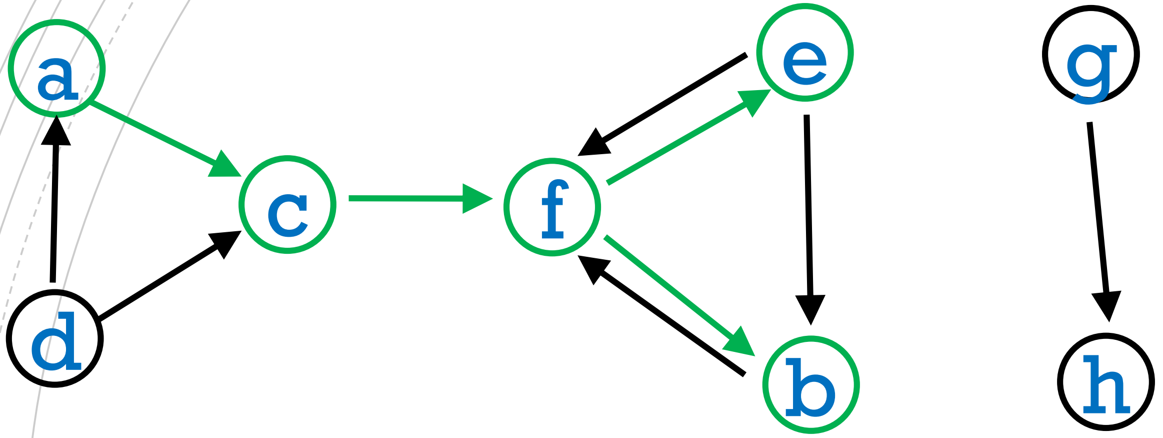

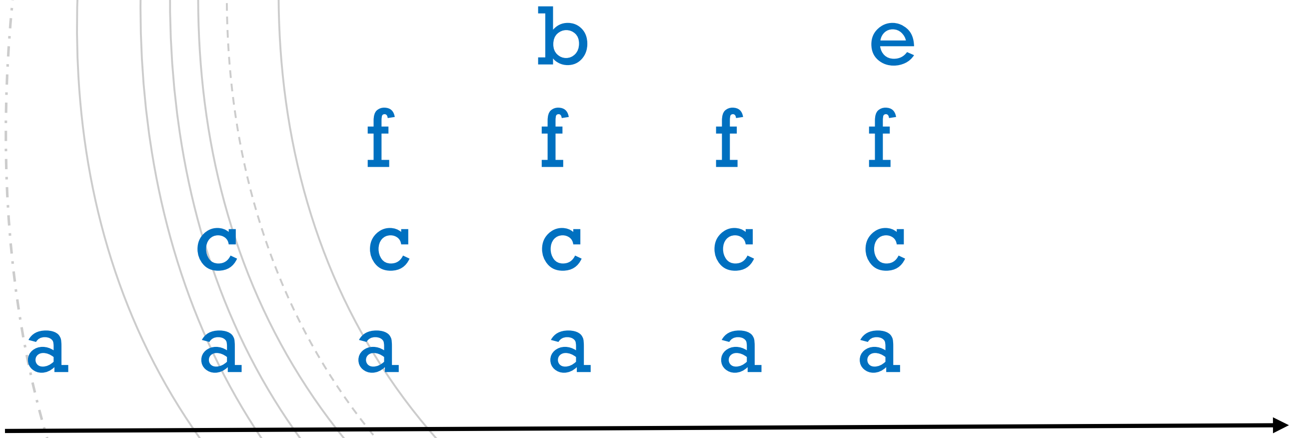


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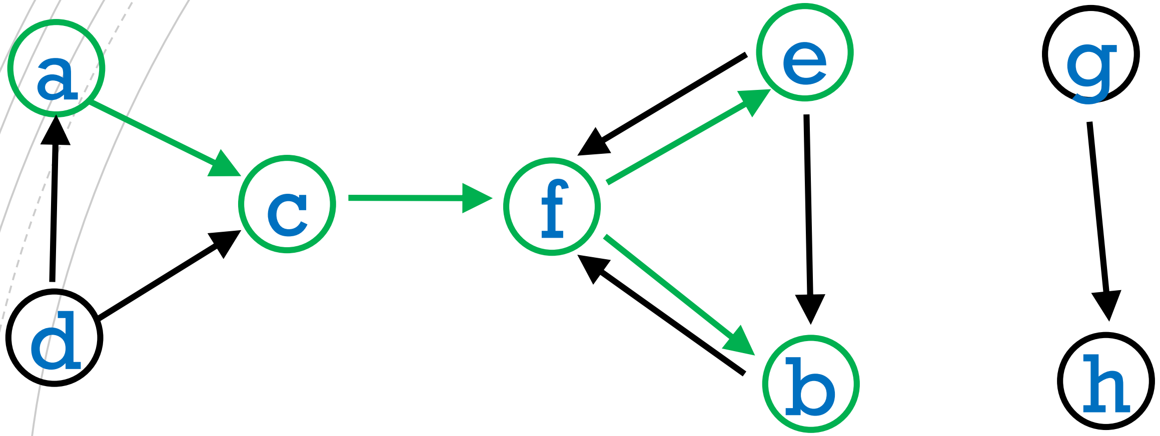

			b		e			
		f	f	f	f	f		
	c	c	c	c	c	c	c	
a	a	a	a	a	a	a	a	a

```
depthFirst_Graph (v){
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    for each w s.t. (v,w) is in E
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            dephFirst_Graph(w)
}
```
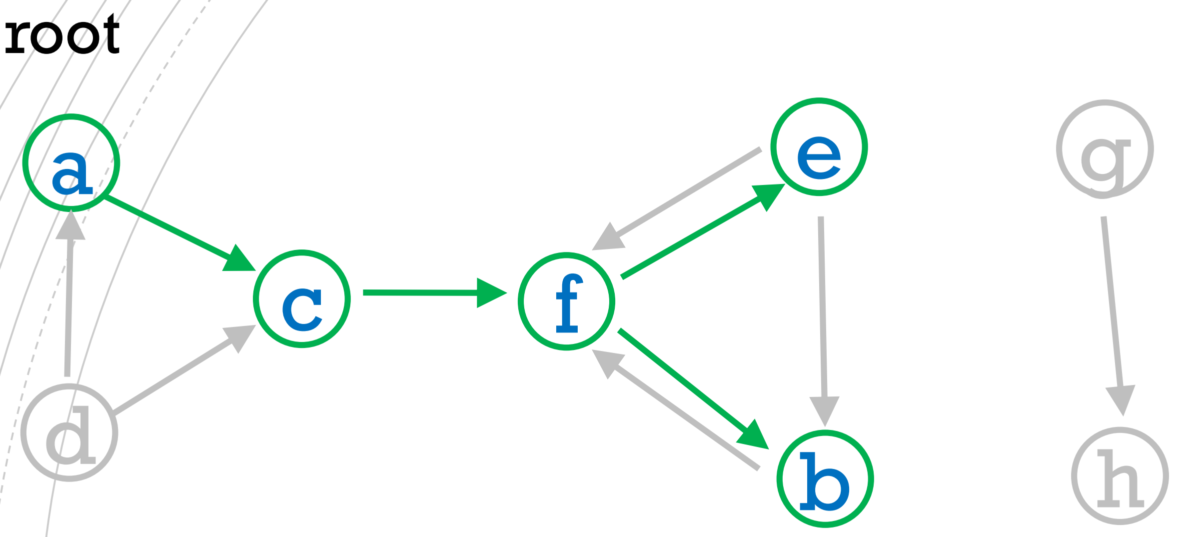

CALL TREE



			b		e			
		f	f	f	f	f		
	c	c	c	c	c	c	c	
a	a	a	a	a	a	a	a	a

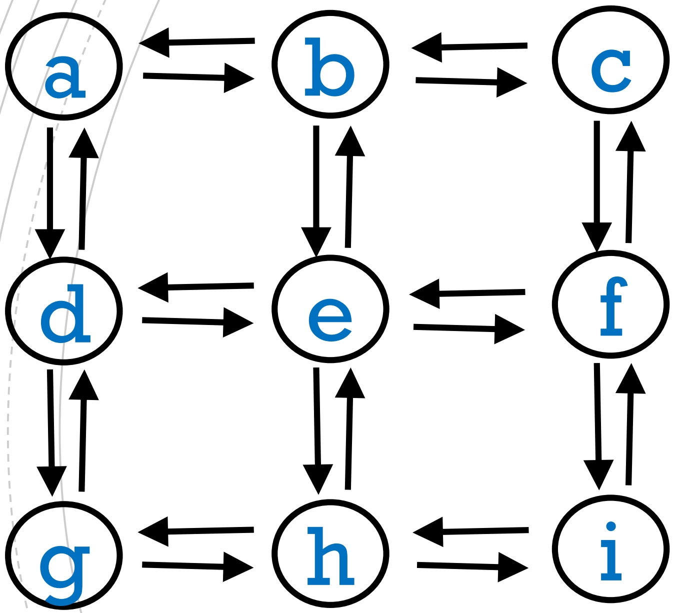
GRAPH TRAVERSALS

- Unlike tree traversal for rooted tree, a graph traversal started from some arbitrary vertex does not necessarily reach all other vertices.

- *Knowing which vertices can be reached by a path from some starting vertex is itself an important problem. You will learn about such graph `connectivity' problems in COMP 251.*

- The order of nodes visited depends on the order of nodes in the adjacency lists.

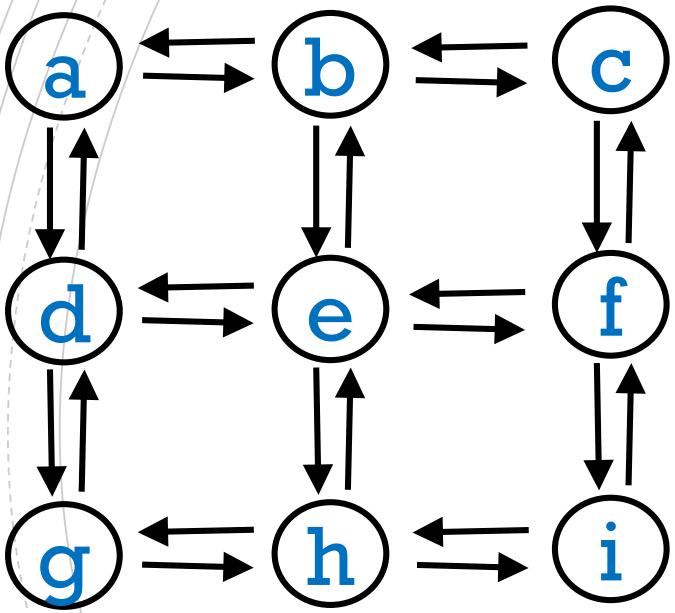
EXAMPLE 2



Adjacency List

a - (b,d)
b - (a,c,e)
c - (b,f)
d - (a,e,g)
e - (b,d,f,h)
f - (c,e,i)
g - (d,h)
h - (e,g,i)
i - (f,h)

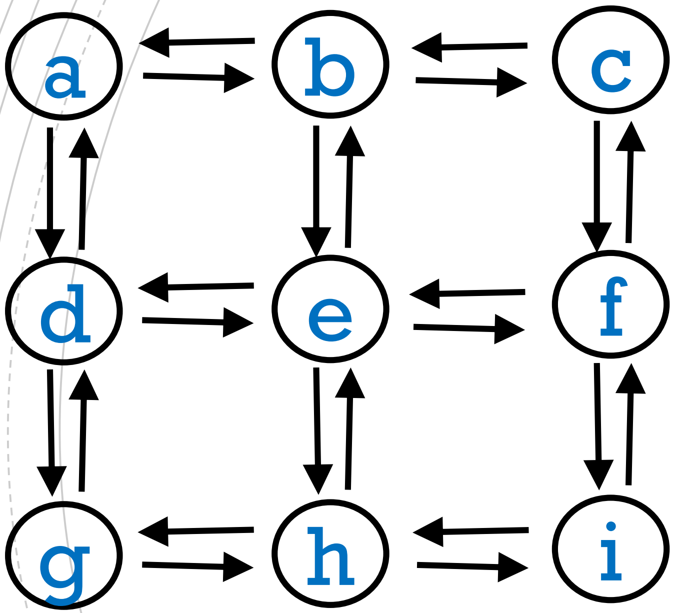
EXAMPLE 2

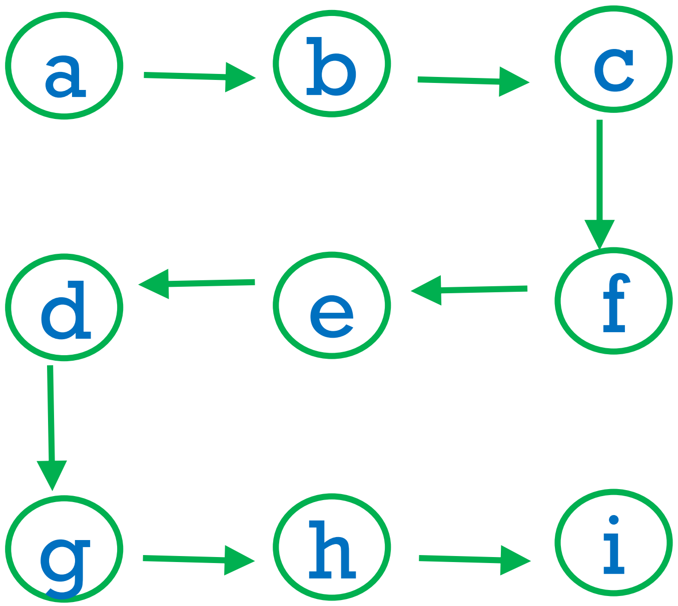


What is the call tree for `depthFirst(a)` ?

(Do it in your head)

EXAMPLE 2





call tree for **`depthFirst`**`(a)`

Coming next:

- Non-recursive graph traversal
  - depth first
  - breadth first
- Shortest path in a DAG
  - Dijkstra's Algorithm