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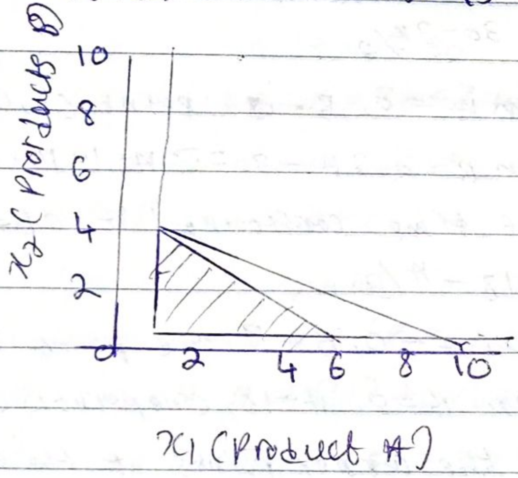
VUG/CSC/22/18458
CSC 333

# Assignment

1 $x_1$ Number units of product A produced daily $x_2$ number of units of product B produced daily objective function maximize profit. $Z = 3x_1 + 4x_2$ constraints i) machine time: $2x_1 + 3x_2 \le 12 : 2x_1 + 3x_2 = 12$
ii) Raw materials: $x_1 + 2x_2 \le 8 : x_1 + 2x_2 = 8$
Non-negativity: $x_1 \ge 0$. $x_2 \ge 0$
find intercepts for each line
- For $2x_1 + 3x_2 = 12$: when $x_1 = 0$: $3x_2 = 12$
$x_2 = 4$
when $x_2 = 0$: $2x_1 = 12$
$x_1 = 6$

Points (6, 0) and (0, 4)




identify the corner points of the feasible region (0,0) intersection of $x_1 = 0$ and $x_2 = 0$ (6,0) - intersection of $2x_1 + 3x_2 = 12$ on $x_1$ axis (0,4) - intersection of both lines on $x_1$ axis
Solve for intersection
$2x_1 + 3x_2 = 12$ - eq i
$\cancel{2}x_1 + 2x_2 = 8$ - eq ii
$x_1 = 8 - 2x_2$ - sub into eq i
$2(8 - 2x_2) + 3x_2 = 12$
$16 - 4x_2 + 3x_2 = 12$
$x_2 = -4 \Rightarrow x_2 = 4$
Sub $x_2 = 4$ into eq ii $= x_1 + 2(4) = 4 \Rightarrow x_1 = 0$

Objective function of each point $Z = 3x_1 + 4x_2$

| Corner point | $x_1$ | $x_2$ | $Z = 3x_1 + 4x_2$ |
|---|---|---|---|
| (0,0) | 0 | 0 | Z = 0 |
| (6,0) | 6 | 0 | Z = 18 |
| (0,4) | 0 | 4 | Z = 16 |
| (2,3) | 2 | 3 | Z = 18 |

max profit is Z = 18 achieved at:
i) (6,0) - produce 6 units of product A and 0 units of product B.
ii) (2,3) produce 2 units of product A and 3 units of product B.

2

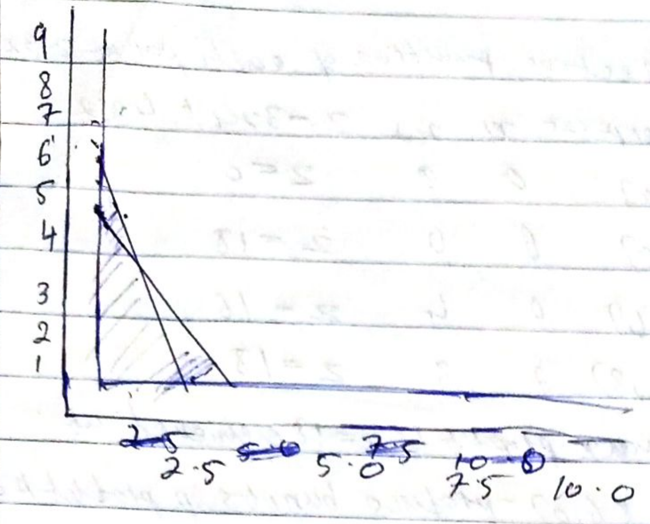
| | labour hours | units of materials | cost per units |
|---|---|---|---|
| Product x | 1 | 2 | ₦2 |
| Product y | 2 | 1 | ₦5 |

Total labor - 6hrs, total units of material - 5 units.

Objective function: minimize cost $C = 2x + 5y$
constraints.
i) $x + 2y \le 6$ (labor constraint)
ii) $2x + y \le 8$ (material constraint)
iii) $x \ge 0$, $y \ge 0$ (Non-negativity constraint)

Labor constraint
$x + 2y = 6 \Rightarrow y = 3 - x/2$
when $x = 0$: $y = 3 - 0/2 = 3 \Rightarrow$ Point A (0,3)
when $y = 0$: $0 = 3 - x/2 \Rightarrow x/2 = 3 \Rightarrow x = 6$
Point B (6, 0)

material constraint
$2x + y = 5 \Rightarrow y = 5 - 2x$
when $x = 0$: $y = 5 - 2(0) = 5$. Point (0,5)
when $y = 0$: $0 = 5 - 2x = 5/2 = 2.5$
Point (2.5, 0)



intesection of 2 lines

$x+2y=6$ = eq(i)

$2x+y=5$ -- eq (ii)

solu for y: $y=5-2x$ = equ(iii)

$x+2(5-2x)=6$

$x+10-4x=6 \Rightarrow -3x=4$

$x=4/3$ sub into equ(iii)

$y=5-2(4/3)=5-8/3=7/3$

intersection point is $(4/3, 7/3)$

to minimize cost: $C=2x+5y$

Evaluate C at each corner point:

At $(0,3)$: $C=2(0)+5(3)=15$

At $(2.5,0)$: $C=2(2.5)+5(0)=5$

At $(4/3; 7/3)$: $C=2(4/3)+5(7/3)=8/3+35/3$

$=43/3 = 14.33$

minimum cost occurs at $(2.5,0)$

3

| | Hours of labour | unit of material | Hours of machine | Profit per unit |
|---|---|---|---|---|
| Product A | 2 | 3 | 1 | 5 |
| Product B | 1 | 2 | 2 | 4 |

Resources available

20hrs of labor - 30 unit of materials

- 18 hrs of machine time.

objective function - max total profit

: $P=5A+4B$ let A and B represent number of unit products A & B

Labour constraint: $2A\ B \leq 20$

material constraint: $3A\ 2B \leq 30$

machine-time constraint: $A+2B \leq 18$

non-negetivity constraint: $A \geq 0$, $B \geq 0$

labour constraint:

$2A+B=20$

$B=20-2A$

when $A=0$, $B=20$ $[0,20]$

when $B=0$, $2A=20 \Rightarrow A=10$ $[10,0]$

materials constraint: $3A+2B=30$

$B=30-3A/2$

- when $A=0$, $B=15$ point $(0,15)$

- when $B=0$, $3A=30 \Rightarrow A=10$ point $(10,0)$

machine time constraint: $A+2B=18$

$B=18-A/2$

- when $A=0$, $B=9$, one point is $(0,9)$

- when $B=0$, $A=18$, one point is $(18,0)$

Find the corner points at the intersection

Between $2A+B=20$ and $3A+2B=30$

$2A+B=20$ - equ 1

$3A+2B=30$ -- equ 2

$B=20-2A$ - sub into equ 2

$3A+2(20-2A)=30$

$3A+40-4A=30 \Rightarrow -A=-10 \Rightarrow A=10$

sub $A=10$ into $B=20-2A$

$B=20-2(10)=0$

intersection of $2A+B=20$ and $A+2B=18$

$2A+B=20$ -- equ 1

$A+2B=18$ - equ 3

Sub $B = 20 - 2A$ into ~~eq~~ equ 3

$A + 2(20 - 2A) = 18$

$A + 40 - 4A = 18 \Rightarrow 3A - 22 = A = 22/3$

Sub $A = 22/3$ into $B = 20 - 2A$

$B = 20 - 2(22/3) = 20 - 44/3 = 60/3 - 44/3$

$= 16/3 = 5.33$

intersection point is $(22/3, 16/3)$

$= (7.33, 5.33)$

intersection of $3A + 2B = 30$ and $A + 2B = 18$

$3A + 2B = 30$ -- equ 2

$A + 2B = 18$ -- equ 3

$A = 18 - 2B$ sub into equ 2

$3(18 - 2B) + 2B = 30$

$54 - 6B + 2B = 30 \Rightarrow -4B = -24 \Rightarrow B = 6$

Sub $B = 6$ into equ 3

$A = 18 - 2(6) = 6$

intersection point $(6, 6)$

objective function: $P = 5A + 4B$

At $(10, 0)$: $P = 5(10) + 4(0) = 50$

At $(7.33, 5.33)$: $P = 5(7.33) + 4(5.33) = 57.97$

At $(6, 6)$: $P = 5(6) + 4(6) = 54$

∴ optimal solution: max profit occurs at $(7.33, 5.33)$ with profit of ₦58

(4)

| | Revenue | Advertisment budget | Production capacity |
|---|---|---|---|
| Product A | 4 | 1 | 1 |
| Product B | 5 | 2 | 2 |

Available resources

| | |
|---|---|
| Advertising budget | ₦20 total |
| production capacity | 15 |

objective function

maximize total revenue: $R = 4A + ₦5B$

let A and B represent the number of units of products A and B

constraints

Advertising budget constraint: $A + 2B \leq 20$

Production capacity constraint: $A + 2B \leq 15$

non-negativity constraint: $A \geq 0$ $B \geq 0$

Graphical solution

Advertising budget constraint: $A + 2B \leq 20$

$B = \frac{20 - A}{2}$

plot points

$A = 0, B = 10$

$A = 20, B = 0$

Production capacity: $A + 2B \leq 15$

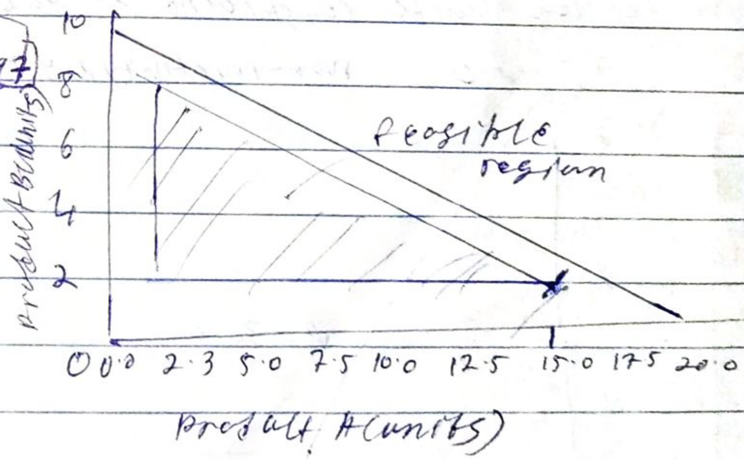
$B = \frac{15 - A}{2}$

Non-negativity $A \geq 0, B \geq 0$

$A = 0, B = 7.5$

$A = 15, B = 0$

Graphical ~~represent~~ representation




Corner points

1) intersection of $A + 2B = 20$ and $A + 2B = 15$

the lines are parallel so no intersection.

ii) intersection of $A + 2B = 20$ with $B = 0$: $A = 20$

$B = 0$

iii intersection of $A+2B=15$ with $B=0$: $A=15$, $B=0$ the revenue function is $R=4A+5B$

At $(0,10)$: $R=4(0)+5(10)=50$

At $(13,0)$: $R=4(15)+5(0)=60$

At $(0,7.5)$: $R=4(0)+5(7.5)=375$

Optimal solution

maximum revenue occurs at $(15,0)$ where maximum revenue $=$ ₦60

At $(4,0)$: $2=8(4)+7(0)=32$

At $(2,2)$: $2=8(2)+8(2)=30$

optimal solution

max profit $2=42$ is achieved at corner point $(0,6)$. company should allocate all resources to ~~pp~~ project P2 and produce 6 unit of it

5

| | labour hours | capital | Profit per units |
|---|---|---|---|
| P1 | 3 | 2 | 8 |
| P2 | 4 | 1 | 7 |

Total labor hours $=12$

Available ~~of~~ capital $-6$

Constraints

let $x$ represent units of project P1

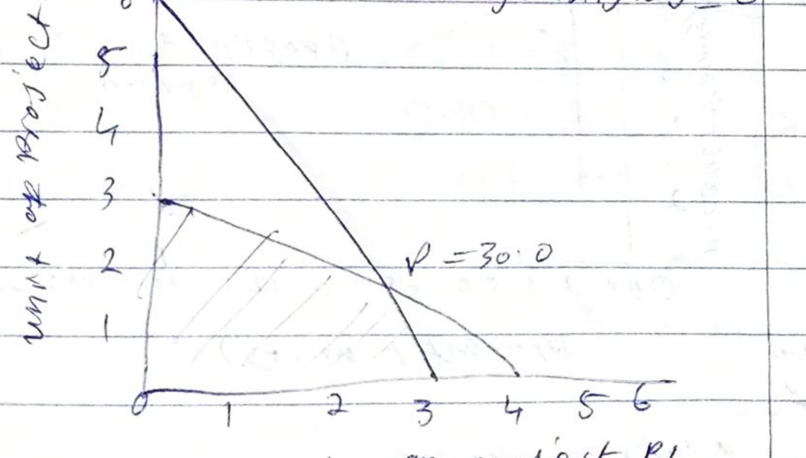
let $y$ represent units of project P2

labour hours constraint

$3x+4y \le 12$

capital units constraint: $2x+y \le 6$

Non-negativity $x, y \ge 0$




~~Gr~~ Graphical Solution for resources

Allocation corner points using $2=8x+7y$

At $(0,0)$: profit $=0$

At $(0,6)$: $2=8(0)+7(6)=42$

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Assignment

6 Bakery production planning

let C = number of chocolate cakes produced ~~let~~ ~~V = numb~~

let V = number of vanilla cakes produced

objective function

maximize profit $P = 5C + 3V$ P: total profit

Constraints

Baking time: $C + 2V \le 8$

Flour: $3C + 2V \le 12$

Non-negativity: $C \ge 0, V \ge 0$

| | Baking time | units of hours | Propt |
|---|---|---|---|
| cholodate | 1 | 3 | 5 |
| Vanilla | 2 | 2 | 3 |

for $C + 2V \le 8$

when $C = 0$, $V = 4$ point $(0,4)$

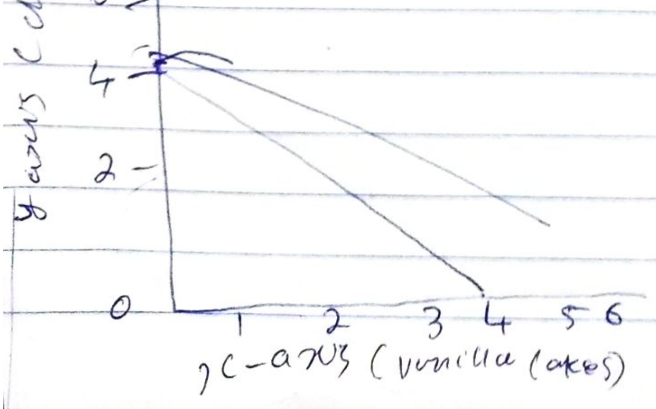
when $V = 0$, $C = 8$ Point $(8,0)$

for $3C + 2V \le 12$:

when $C = 0$, $V = 6$. Point $(0,6)$

when $C = 4$, $V = 0$ Point $(4,0)$

non-negativity ($C \ge 0, V \ge 0$)




corner points

i) intersection of $C + 2V = 8$ and $3C + 2V = 12$

$C + 2V = 8$ --- equ 1

$3C + 2V = 12$ --- equ 2

Subtract equ 1 from equ 2

$2C = 4 \Rightarrow C = 2$

Substitute $C = 2$ into equ 1

$2 + 2V = 8 \Rightarrow V = 3$ Points $(2,3)$

ii) intersection of $C + 2V = 8$ with $V = 0$:

$C = 8$ point: $(8,0)$

iii) intersection of $3C + 2V = 12$ with $V = 0$

$3C = 12 \Rightarrow C = 4$

iv) intersection $C = 0$ with $3C + 2V = 12$

$2V = 12 \Rightarrow V = 6$ point: $(0,6)$

objective function at corner point

$P = 5C + 3V$

i) At point $(2,3)$: $P = 5(2) + 3(3) = 19$

ii) At $(8,0)$: $P = 5(8) + 3(0) = 40$

iii) At $(4,0)$: $P = 5(4) + 3(0) = 20$

iv) At $(0,6)$: $P = 3(0) + 3(6) = 18$

Optimal solution

max profit is $P = 40$ at $(8,0)$ produce 8 chocolate cake and 0 vanilla cake