Name of the Candidate: **Reg. No.:**

19DS220

M.Sc.DEGREE – APRIL 2022– EXAMINATIONS

BRANCH:DATA SCIENCE

TRANSFORMS AND THEIR APPLICATIONS

Duration : 3Hours **Maximum: 100 Marks**

Answer All Questions

		PART – A (5 x 2 = 10)	**CO**	**Marks**
A1.	*What is mean by Linear Transformations? Give an example.*		CO1	**(2)**
A2.	*State and Prove First shifting Property of Laplace Transforms.*		CO2	**(2)**
A3.	*Find the Root Mean Square value of the function* $f(x)=1-x,\ 0<x<1.$		CO3	**(2)**
A4.	*Define Parseval's relation for Discrete Fourier Transforms.*		CO5	**(2)**
A5.	*Find* $Z[n\,a^{n-1}]$.		CO6	**(2)**

		PART – B (4 x 5 = 20)	**CO**	**Marks**
B1.	*Find i)* $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$ *ii)* $L^{-1}\left[\frac{s}{(s+2)^2+4}\right]$ *by using the Inverse properties.*		CO2	**(5)**
B2.	*Find the Fourier series of the function* $f(x)=\pi x, -l<x<l.$		CO3	**(5)**
B3.	*State and Prove Conjugation Property of Discrete Fourier Transform.*		CO5	**(5)**
B4.	*Obtain inverse Z transform of* $\frac{z^3}{(z-2)^2(z-3)}$ *using partial fraction method.*		CO6	**(5)**

PART – C (*70 Marks*) **CO** **Marks**

C1. ***a.*** *Determine the Laplace Transforms of the half sine wave rectifier function* — *CO2* — ***(7)***

$$f(t)=\begin{cases}\sin t, & 0<t<\pi\\ 0, & \pi<t<2\pi\end{cases}$$

b. *Apply Laplace transforms to Solve the differential equation,* — *CO2* — ***(10)***

$$y''+4y'+4y=e^{-t},\ y(0)=y'(0)=0.$$

(OR)

C2. ***a.*** *Apply Convolution theorem to evaluate* $L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right]$ — *CO2* — ***(7)***

b. *Solve the integral equation, using Laplace transforms* — *CO2* — ***(10)***

$$y'+4y+5\int_0^t y\,dt = e^{-t}, y(0)=0.$$

C3. ***a.*** *Find the Half Range Fourier Cosine series expansion for the function* — *CO3* — ***(8)***

$f(x)=(x-1)^2$, *in* $0\le x\le 1$. *Hence show that* $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots=\frac{\pi^2}{6}$.

b. *Express the function* $f(x)=x(2\pi-x)$ *as a Fourier series in* $0\le x\le 2\pi$. — *CO3* — ***(10)***

(OR)

CONTD....

C4. ***a.*** *Obtain the Fourier series expansion for the function* $f(x)=\begin{cases} l-x, & 0<x<l \\ 0, & l<x<2l \end{cases}$ *with* ***CO3*** ***(8)***

period 2l. Also deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots$.

b. *Using Parseval's identity theorem, to find the Fourier series for* $f(x)=x^2$ *in the interval* $-\pi \le x \le \pi$. *Hence deduce that* $\sum_{n=1}^{\infty}\frac{1}{n^4}$ ***CO3*** ***(10)***

C5. ***a.*** *Show that the* $e^{-x^2/2}$ *is self-reciprocal under Fourier Cosine transform of and also* $x\,e^{-x^2/2}$ *is self-reciprocal under Fourier Sine transform.* ***CO4*** ***(8)***

b. *Obtain the Fourier transform of* $f(x)=\begin{cases} a-|x|, & |x|\le a \\ 0, & |x|>a \end{cases}$. *And hence show that* ***CO4*** ***(10)***

i) $\int_0^{\infty}\left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$ *ii)* $\int_0^{\infty}\left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$

(OR)

C6. ***a.*** *Apply Fourier Integral Theorem, show that* ***CO4*** ***(8)***

$$e^{-ax}-e^{-bx}=\frac{2(b^2-a^2)}{\pi}\int_0^{\infty}\frac{\lambda\sin\lambda x}{(\lambda^2+a^2)(\lambda^2+b^2)}\,d\lambda,\quad a,b>0.$$

b. *Evaluate the Fourier cosine and Fourier sine transform of* e^{-ax}. *Also by using Parseval's identity, Evaluate* $\int_0^{\infty}\frac{dx}{(x^2+a^2)(x^2+b^2)}$ *and* $\int_0^{\infty}\frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}$. ***CO4*** ***(10)***

C7. ***a.*** *Apply the concept of Convolution Theorem, Obtain* $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$. ***CO6*** ***(7)***

b. *Solve* $y_{n+2}+4y_{n+1}+3y_n=3^n$ *with* $y(0)=0,\ y(1)=1$. ***CO6*** ***(10)***

(OR)

C8. ***a.*** *i) Form the differential equations by eliminating the constants from* $y_n=(A+Bn)\,2^n$ *, ii) Find* $Z[2^n\cos\frac{n\pi}{2}]$ *and* $z[3^n\sin\frac{n\pi}{2}]$ ***CO6*** ***(7)***

b. *Solve the given differential equation using Z-transform,* $y_{n+2}+y_n=2^n.n$ ***CO6*** ***(10)***

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