Root-Locus for $K \in [0, +\infty)$

Magnitude Condition: $\lvert KG(s)H(s) \rvert = 1$	**Angle Condition:** $\angle\big(KG(s)H(s)\big) = \pm(2i+1)180^\circ$
Number of asymptote lines: *Relative degree =* $n - m$	**Angle of asymptote lines:** $\varphi_i = \dfrac{180^\circ}{n-m}(2i+1) \quad , \quad i = 0, 1, 2, \cdots$
Intersection of asymptote lines with real axis: $\alpha = \dfrac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m}$	**Intersection of root locus with imaginary axis:** Assume $s = j\omega$ and compute the cross points and K value from characteristic equation
Break-away (or break-in) points: $1 + KG(s)H(s) = 0 \quad \rightarrow \quad K = \dfrac{-1}{G(s)H(s)} \quad \rightarrow \quad \dfrac{dK}{ds} = 0 \quad \rightarrow \quad \text{solve for } s$	
Angle of departure from the complex pole: $180^\circ - (\mathit{sum\ of\ the\ angles\ of\ vectors\ drawn\ to\ this\ pole\ from\ other\ poles}) + (\mathit{sum\ of\ the\ angles\ of\ vectors\ drawn\ to\ this\ pole\ from\ zeros})$	
Angle of arrival to the complex zero: $180^\circ - (\mathit{sum\ of\ the\ angles\ of\ vectors\ drawn\ to\ this\ zero\ from\ other\ zeros}) + (\mathit{sum\ of\ the\ angles\ of\ vectors\ drawn\ to\ this\ zero\ from\ poles})$	

$G(s) = \dfrac{K_{dc}\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	**Overshoot:** $O.S. = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$
Settling-time (2%): $t_s = \dfrac{4}{\zeta\omega_n}$	**Damping ratio from overshoot:** $\zeta = \dfrac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}}$
Rise-time: $t_r \cong \dfrac{0.8 + 2.5\zeta}{\omega_n}$	**Step-error-constant:** $k_p = \lim\limits_{s\to 0} G(s) \quad \rightarrow \quad e_{ss} = \dfrac{R}{1 + k_p}$
Peak-time: $t_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$	**Ramp-error-constant:** $k_v = \lim\limits_{s\to 0} sG(s) \quad \rightarrow \quad e_{ss} = \dfrac{R}{k_v}$
Resonant peak: $M_r = K_{dc}\dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$	**Resonant frequency:** $\omega_r = \omega_n\sqrt{1-2\zeta^2}$
Initial-value Theorem: $f(0^+) = \lim\limits_{t\to 0^+} f(t) = \lim\limits_{s\to\infty} sF(s)$	**Final-value Theorem:** $f(\infty) = \lim\limits_{t\to\infty} f(t) = \lim\limits_{s\to 0} sF(s)$

PD Controller (Frequency Domain): $G_c(s) = K_P\left(1 + \dfrac{T_d s}{\frac{T_d}{\beta}s + 1}\right)$, $0 < \beta < 1$ $\omega_m = \dfrac{\sqrt{\beta}}{T_d}$, $\sin(\phi_m) = \dfrac{\beta - 1}{\beta + 1}$	**PI Controller:** $G_c(s) = K_p\left(1 + \dfrac{1}{T_i s}\right)$ **PD Controller:** $G_c(s) = K_p(1 + T_d s)$ **PID Controller:** $G_c(s) = K_p\left(1 + \dfrac{1}{T_i s} + T_d s\right)$
Lag Compensator: $G_c(s) = K_c\dfrac{s+z}{s+p}$, $z > p > 0$	**Lead Compensator:** $G_c(s) = K_c\dfrac{s+z}{s+p}$, $p > z > 0$

State space equations: $\begin{cases}\dot{\mathrm{x}}(t) = \mathrm{Ax}(t) + \mathrm{Bu}(t)\\ \mathrm{y}(t) = \mathrm{Cx}(t) + \mathrm{Du}(t)\end{cases}$	**Controllability matrix:** $\mathrm{Q_c} = [\mathrm{B} \quad \mathrm{AB} \quad \mathrm{A^2B} \quad \mathrm{A^3B} \quad \cdots \quad \mathrm{A^{n-1}B}]$
Characteristics Equation: $\det(s\mathrm{I} - \mathrm{A}) = 0$	**Transfer function formula:** $G(s) = \mathrm{C}(s\mathrm{I} - \mathrm{A})^{-1}\mathrm{B} + \mathrm{D}$
Closed-loop system with state feedback control: $\begin{cases}\dot{\mathrm{x}}(t) = (\mathrm{A} - \mathrm{BK})\mathrm{x}(t) + \mathrm{B}r(t)\\ y(t) = (\mathrm{C} - \mathrm{DK})\mathrm{x}(t) + \mathrm{D}r(t)\end{cases}$	**Closed-loop system with state feedback with integrator control:** $\begin{cases}\begin{bmatrix}\dot{\mathrm{x}}(t)\\ \dot{q}(t)\end{bmatrix} = \begin{bmatrix}\mathrm{A} - \mathrm{BK} & \mathrm{B}k_i\\ -\mathrm{C} + \mathrm{DK} & -\mathrm{D}k_i\end{bmatrix}\begin{bmatrix}\mathrm{x}(t)\\ q(t)\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}r(t)\\ y(t) = [\mathrm{C} - \mathrm{DK} \quad \mathrm{D}k_i]\begin{bmatrix}\mathrm{x}(t)\\ q(t)\end{bmatrix}\end{cases}$
Matrix inversion for 2×2 matrix: $\mathbf{A} = \begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} \rightarrow \mathbf{A}^{-1} = \dfrac{1}{a_{11}a_{22} - a_{21}a_{12}}\begin{bmatrix}a_{22} & -a_{12}\\ -a_{21} & a_{11}\end{bmatrix}$	

Mason's Formula: $\dfrac{Y_{out}}{Y_{in}} = \dfrac{1}{\Delta}\sum_{k=1}^{N} M_k\Delta_k$	N = Total number of forward paths between Y_{in} and Y_{out} M_k = Gain of the kth forward path between Y_{in} and Y_{out} Δ = 1 – (sum of all loop gains) + (sum of products of all combinations of two non-touching loops) - (sum of products of all combinations of three non-touching loops) + (sum of products of all combinations of four non-touching loops) - ……………… Δ_k = the Δ of the SFG non-touching with the forward path M_k when M_k has been removed