ENGI-1500
Physics -2

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Humber Institute of Technology and Advanced Learning
Winter 2023

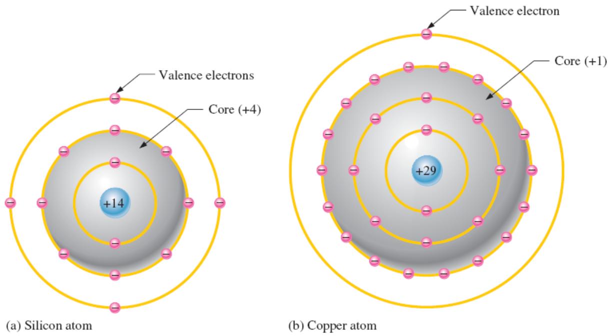
HUMBER

Reminder of the previous week

- Electrical charges
  - ***Positive*** and ***Negative*** charges
  - Same signs repel each other
  - Opposite signs attract each other

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

- The atom - Bohr model and valence electrons



Source: Floyd, Thomas L. Electronic devices: conventional current version. Pearson, 2012.

- Coulomb's Law

Electric force between charges

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

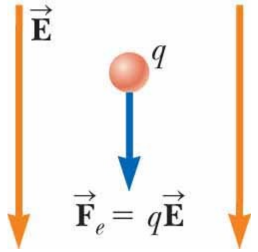
$$k_e = 8.9876 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$$

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

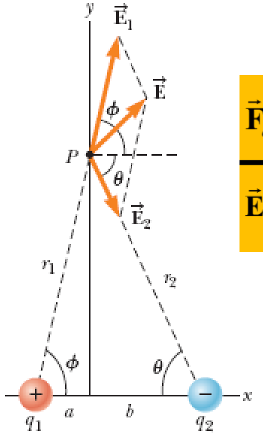
$$\varepsilon_0 = 8.8542 \times 10^{-12} \ \mathrm{C^2/N \cdot m^2}$$

$$e = 1.60218 \times 10^{-19} \ \mathrm{C}$$

- Electric Fields



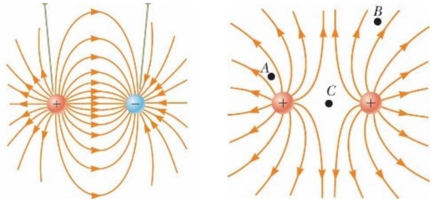
Electric field due to charges

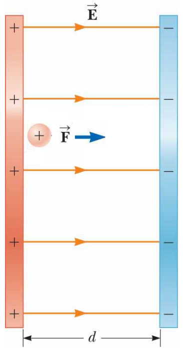


$$\vec{\mathbf{F}}_e = k_e \frac{q q_0}{r^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{E}} = k_e \sum_i \frac{q}{r^2}\hat{\mathbf{r}}$$

- Electric Field Lines



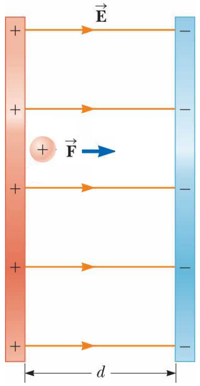


Charges in a Uniform Field

$$\sum \vec{\mathbf{F}} = q\vec{\mathbf{E}} = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} = \frac{q\vec{\mathbf{E}}}{m}$$

- Motion of a charged particle in uniform E field



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Week 2 / Class 2

- ❑ Continuous Charge Distribution & Gauss's Law (Ch. 23)
- ❑ Electric Potential (Ch. 24)

Outline of Week 2 / Class 2

- Reminder of the previous week

- Continuous Charge Distributions and Gauss's Law (Ch. 23)
  - Electric field of cont. charge dist.
  - Electric flux
  - Gauss's Law
  - Application of Gauss's law to various charge distributions

- Electrical Potential (Ch. 24)
  - Electric potential and potential difference
  - Potential difference in a uniform electric field
  - Electric potential and potential energy due to point charges
  - Obtaining the Electric Field from the Electric Potential
  - Electric potential due to continuous charge distributions
  - Conductors in Electrostatic Equilibrium

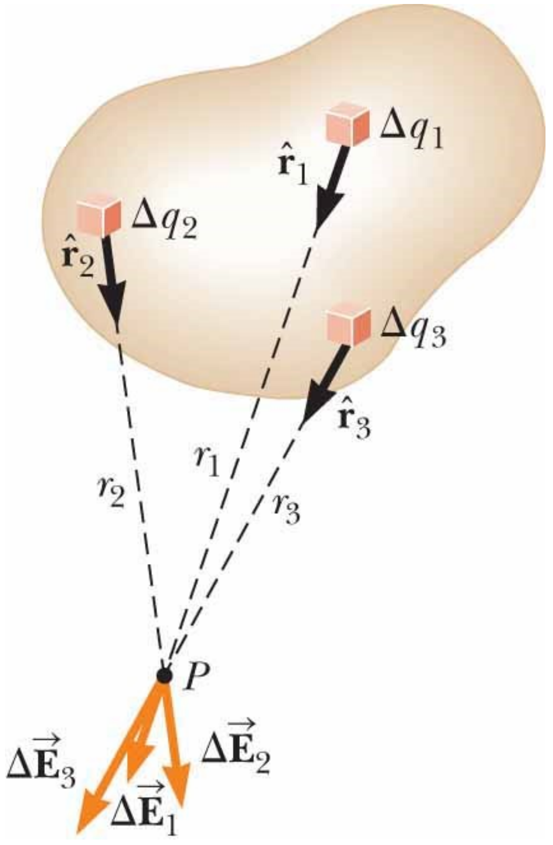
- Examples
- Next week's topic

Continuous Charge Distributions & Gauss’s Law (Ch. 23)

Electric Field of a Continuous Charge Distribution

Electric Field of a Cont. Charge Dist.

- Consider continuous distribution of charge
  - Can be described as continuously distributed along some line, over some surface, or throughout some volume
- Divide charge distribution into small elements
  - Each contains small charge ***Δq*** (figure)
- Calculate electric field due to one of these elements at point ***P***
- Evaluate total electric field at ***P*** due to charge distribution by summing contributions of all charge elements (superposition principle)



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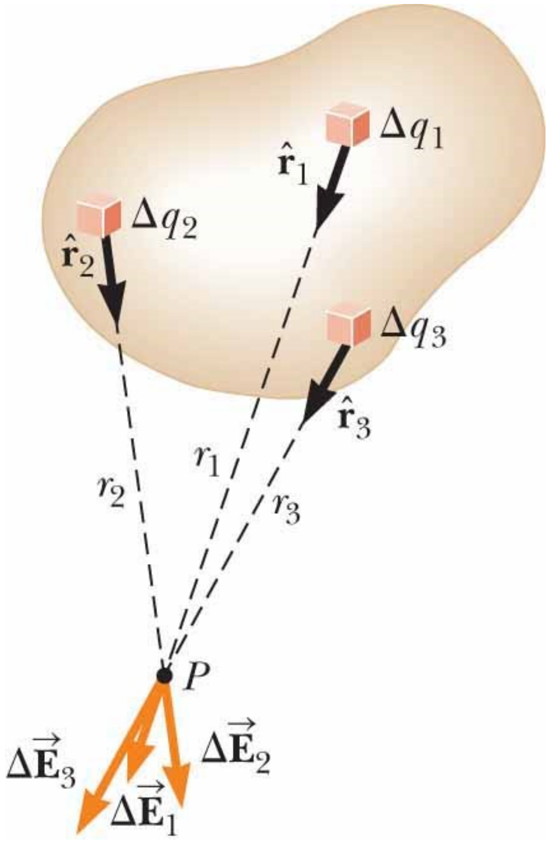
Electric Field of a Cont. Charge Dist.

- Electric field at P due to one charge element carrying charge Δq is:
  - r = distance from charge element to point P
  - $\hat{\mathbf{r}}$ = unit vector directed from element toward P



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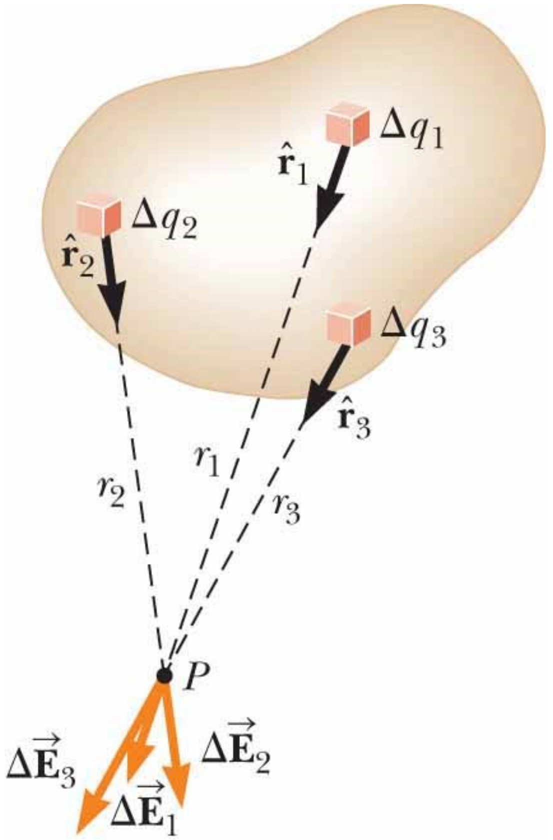
$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$



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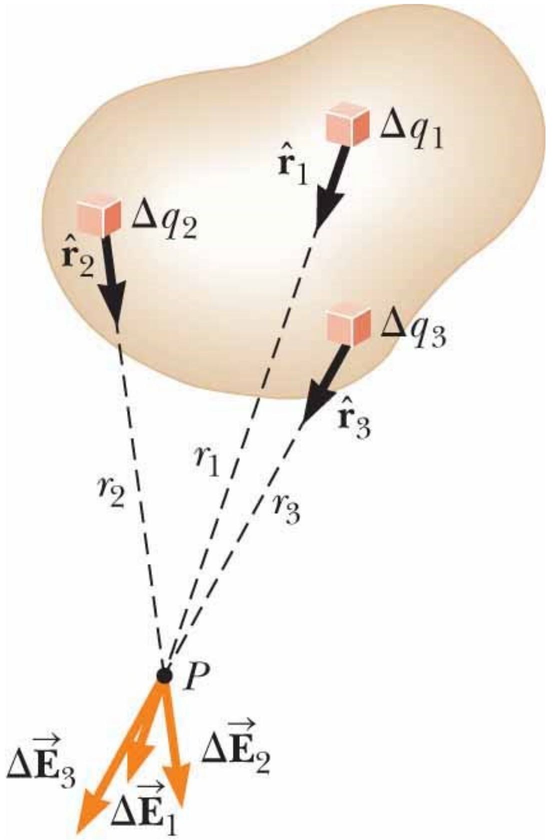
$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

- Total electric field at P due to all elements in charge distribution approximately:
  - Index i refers to i^{th} element in distribution

$$\vec{\mathbf{E}} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i$$



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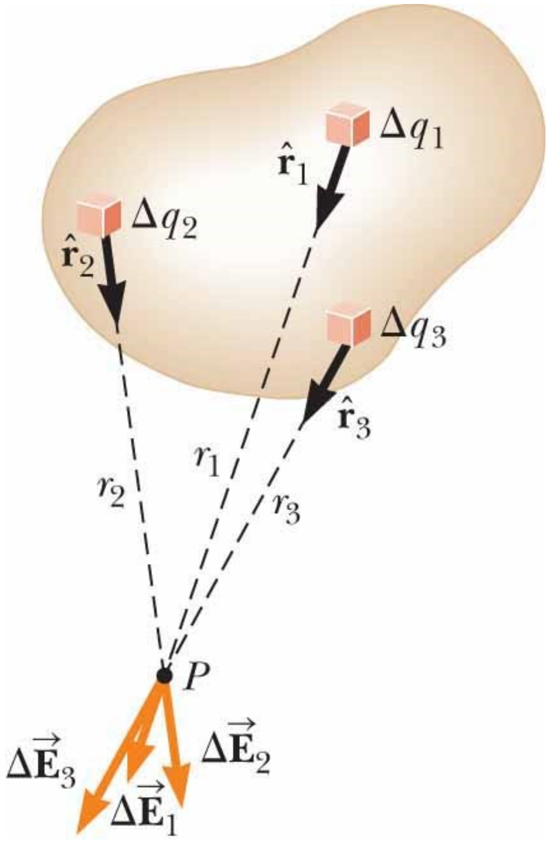
- Total electric field at P due to all elements in charge distribution approximately:
  - Index i refers to i^{th} element in distribution

$$\vec{\mathbf{E}} \approx k_e \sum_i \frac{\Delta q_i}{r_i^{\,2}}\hat{\mathbf{r}}_i$$

- Because number of elements very large and charge distribution modeled as continuous:
  - Total field at P in limit $\Delta q_i \rightarrow 0$ is:
  - Integration over entire charge distribution

$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

- Note: integration → vector operation



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Charge Density

- If charge $\boldsymbol{Q}$ is uniformly distributed throughout a volume $\boldsymbol{V}$ → volume charge density:
  - $\boldsymbol{\rho}$ has units of coulombs per cubic meter (C/m^3)

$$\rho \equiv \frac{Q}{V} \qquad dq = \rho\, dV$$

- If charge $\boldsymbol{Q}$ is uniformly distributed on a surface of area $\boldsymbol{A}$ → surface charge density:
  - $\boldsymbol{\sigma}$ has units of coulombs per square meter (C/m^2)

$$\sigma \equiv \frac{Q}{A} \qquad dq = \sigma\, dA$$

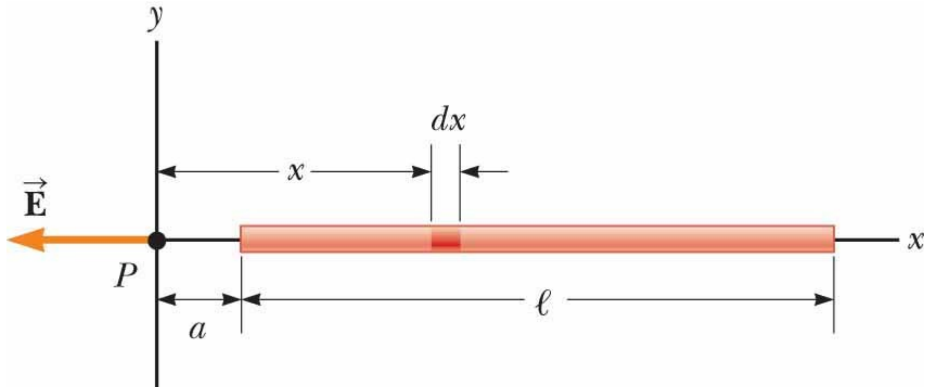
- If charge $\boldsymbol{Q}$ is uniformly distributed along a line of length $\boldsymbol{\ell}$ → linear charge density:
  - $\boldsymbol{\lambda}$: units of coulombs per meter (C/m)

$$\lambda \equiv \frac{Q}{\ell} \qquad dq = \lambda\, d\ell$$

Electric Forces between Charged Objects

Example 23-1: Electric Field Due to a Charged Rod

A rod of length ℓ has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.



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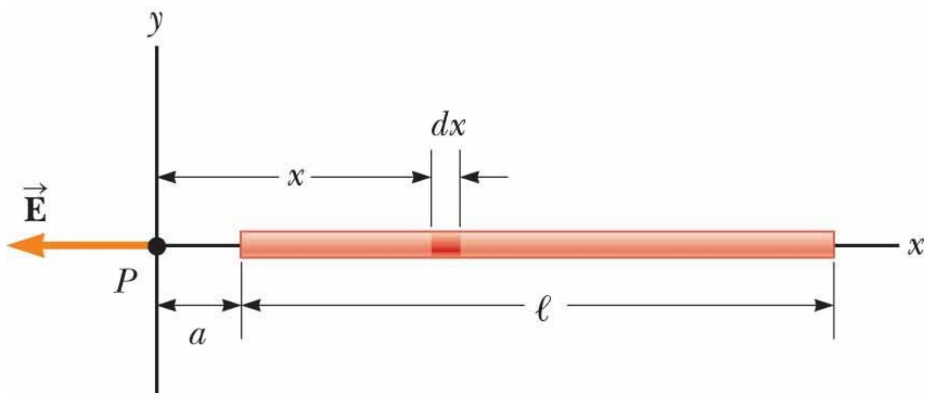

Conceptualize

The field d***E*** at ***P*** due to each segment of charge on the rod is in the negative *x* direction because every segment carries a positive charge. The figure shows the appropriate geometry. In our result, we expect the electric field to become smaller as the distance ***a*** becomes larger because point ***P*** is farther from the charge distribution.

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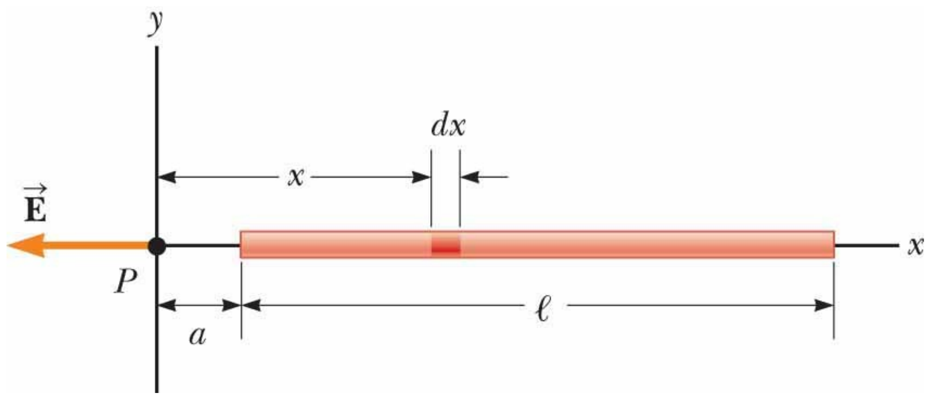

Categorize

Because the rod is continuous, we are evaluating the field due to a continuous charge distribution rather than a group of individual charges. Because every segment of the rod produces an electric field in the negative *x* direction, the sum of their contributions can be handled without the need to add vectors.

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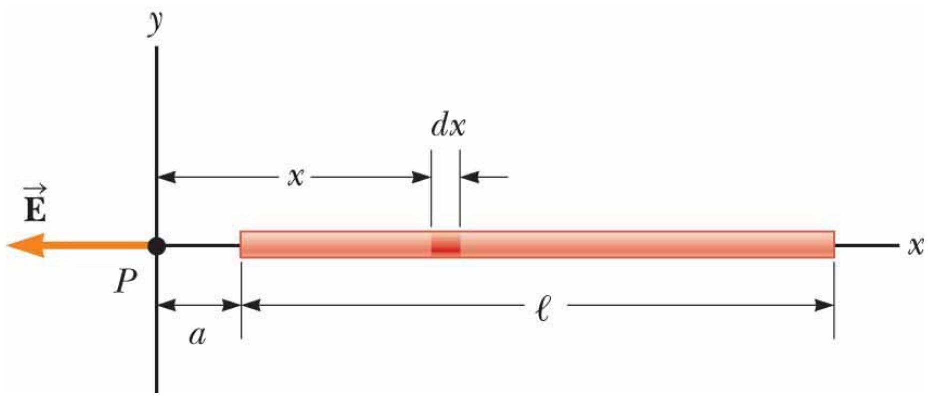

Analyze

Let's assume the rod is along the *x* axis, ***dx*** is the length of one small segment, and ***dq*** is the charge on that segment. Because the rod has a charge per unit length $\boldsymbol{\lambda}$, the charge ***dq*** on the small segment is $\boldsymbol{dq = \lambda dx}$.

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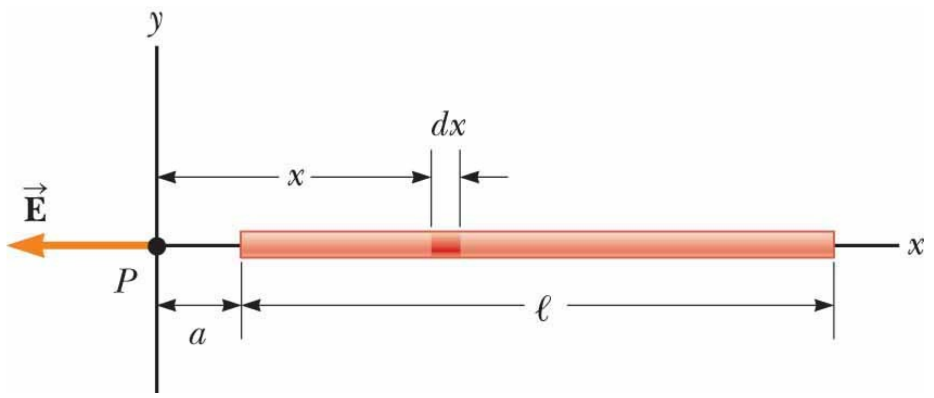

Analyze

$dq = \lambda dx$

Find the magnitude of the electric field at *P* due to one segment of the rod having a charge *dq*:

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda dx}{x^2}$$

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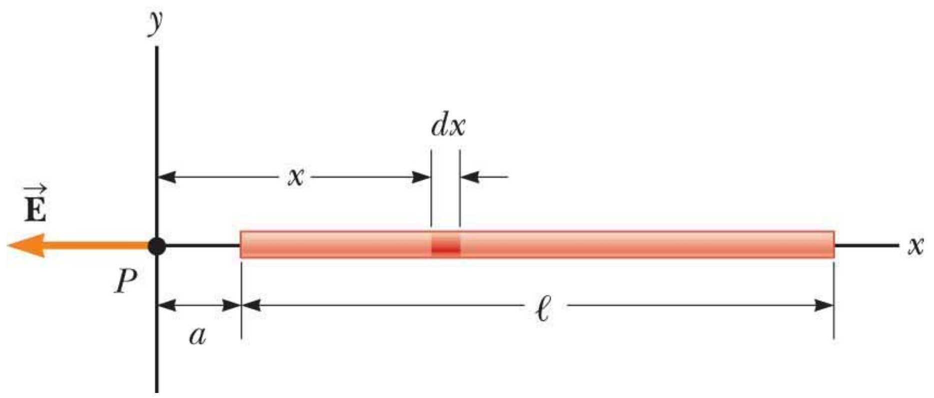

Analyze

$dq = \lambda dx$ → $$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda dx}{x^2}$$

Find the total field at *P*:

$$E = \int_a^{\ell+a} k_e \lambda \frac{dx}{x^2}$$

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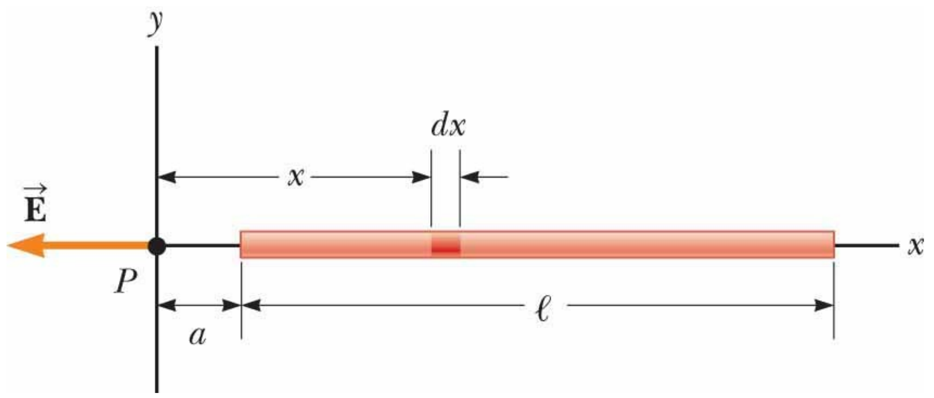

Analyze

$dq = \lambda dx$ → $dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda dx}{x^2}$ → $E = \int_a^{\ell+a} k_e \lambda \frac{dx}{x^2}$

Noting that k_e and $\lambda = Q/\ell$ are constants and can be removed from the integral, evaluate the integral:

$$E = k_e \lambda \int_a^{\ell+a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell+a}$$

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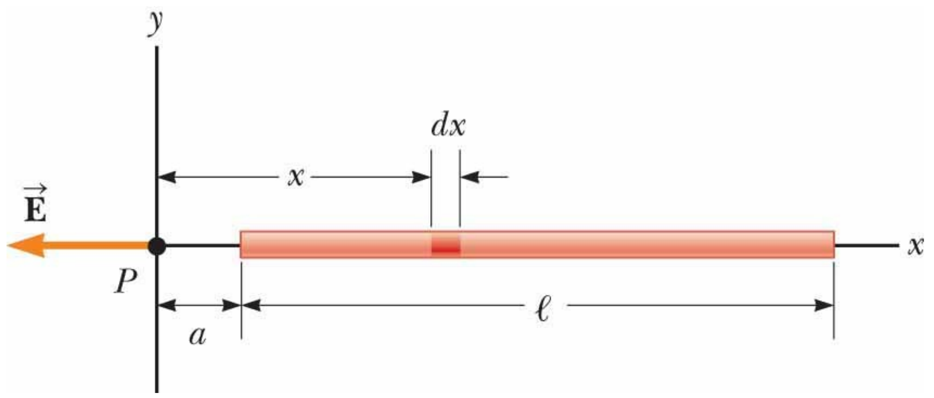

Finalize

$$dq = \lambda dx \rightarrow dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda dx}{x^2} \rightarrow E = \int_a^{\ell+a} k_e \lambda \frac{dx}{x^2} \rightarrow E = k_e \lambda \int_a^{\ell+a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell+a}$$

$$E = k_e \frac{Q}{\ell} \left(\frac{1}{a} - \frac{1}{\ell+a} \right) = \boxed{\frac{k_e Q}{a(\ell+a)}}$$

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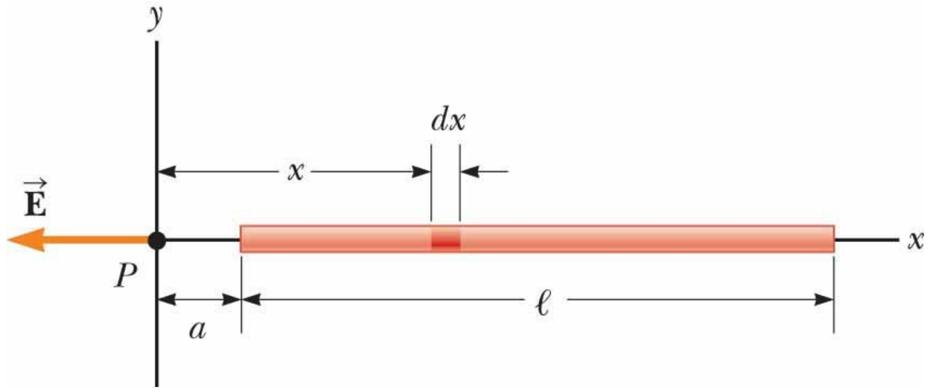


Observations

1. If ***a*** becomes larger, the denominator of the fraction grows larger, and ***E*** becomes smaller
2. If ***a*** → ***0***, which corresponds to sliding the bar to the left until its left end is at the origin, then ***E*** → ∞
3. If ***P*** is far from the rod (***a*** >> ***ℓ***), the charge distribution appears to be a point charge of magnitude ***Q***

$$E = k_e \frac{Q}{\ell}\left(\frac{1}{a} - \frac{1}{\ell + a}\right) = \boxed{\frac{k_e Q}{a(\ell + a)}}$$

$$E = \frac{k_e Q}{a(\ell + a)} \approx \frac{k_e Q}{a^2}$$

urce: Serway, Raymond A., and John W. vett. *Physics for scientists and engineers*. th Edition. Cengage learning, 2018.

Continuous Charge Distributions & Gauss’s Law (Ch. 23)

Electric Flux

Electric Flux

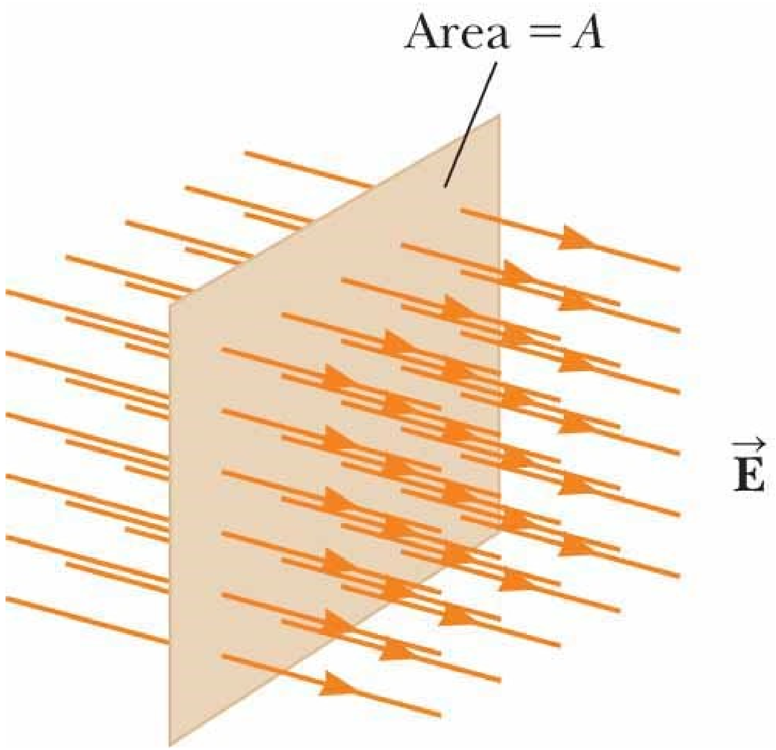
- Consider an electric field that's uniform in magnitude and direction

- Field lines penetrate a rectangular surface of area ***A***
  - Plane oriented perpendicular to the field

- **Recall**: the number of lines per unit area (line density) → magnitude of the electric field

- Total number of lines penetrating the surface → product ***EA*** (***electric flux*** $\boldsymbol{\Phi_E}$):
  - $\boldsymbol{\Phi_E}$: units of newton meters squared per coulomb (**N·m²/C**)

$$\Phi_E = EA$$



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Electric Flux

- Consider the figure with a surface area (***A***) at an angle (***Ө***)
- Area ***A*** = product of the length and the width of the surface

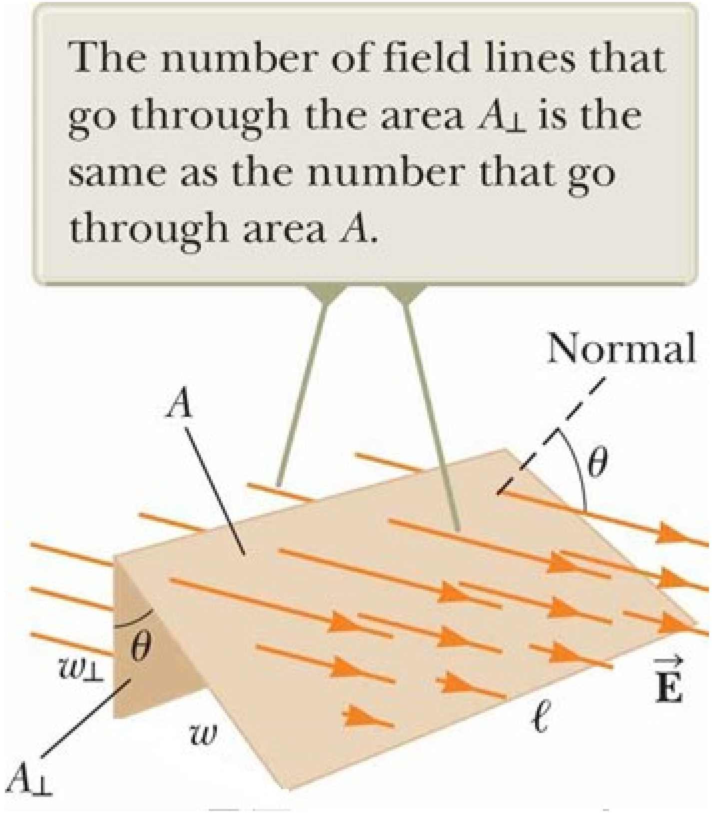
$$A = \ell w$$

- Perpendicular projection of the area A = A_T

$$w_\perp = w\cos\theta$$
$$A_\perp = \ell w_\perp = \ell w\cos\theta$$
$$A_\perp = A\cos\theta$$

- Flux through A is equal to flux through A_T
- Flux through the surface can be written in two forms:
    - E_n = component of electric field normal to surface

$$\Phi_E = EA_\perp = EA\cos\theta$$
$$\Phi_E = (E\cos\theta)A = E_n A$$



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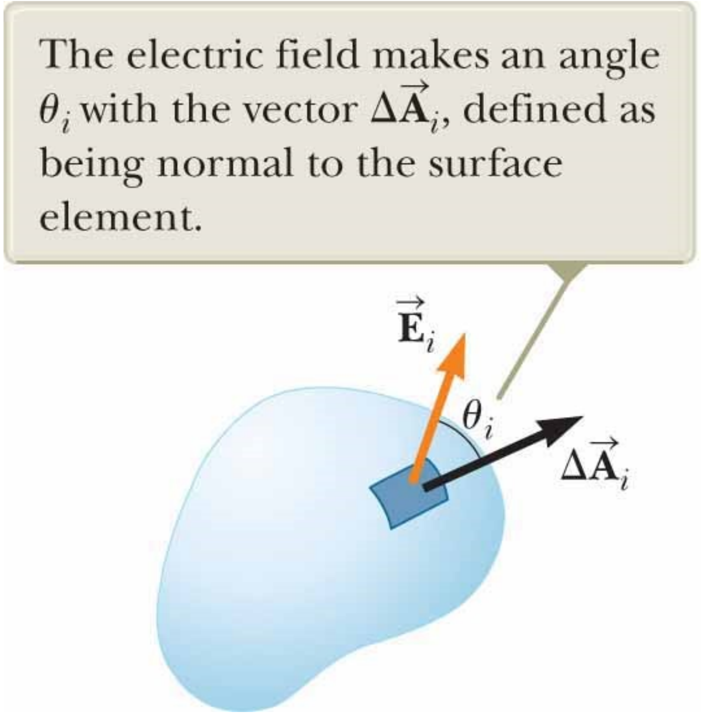
Electric Flux

- Consider a situation where the electric field varies over a large surface
- Define vector ΔA_i
  - Magnitude represents the area of i^{th} element of the large surface
  - Direction is perpendicular to the surface element (figure)
- Electric field $\mathbf{E_i}$ at the location of this element makes an angle Θ_i with the vector ΔA_i
- Electric flux $\Phi_{E,i}$ through this element is:

$$\Phi_{E,i} = E_i \Delta A_i \cos\theta_i = \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i$$

- Summing the contributions of all elements gives an approximation of total flux through the surface.
- If the sum is replaced by an integral, the general definition of electric flux can be written as:

$$\Phi_E \equiv \int_{\text{surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$



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Electric Flux

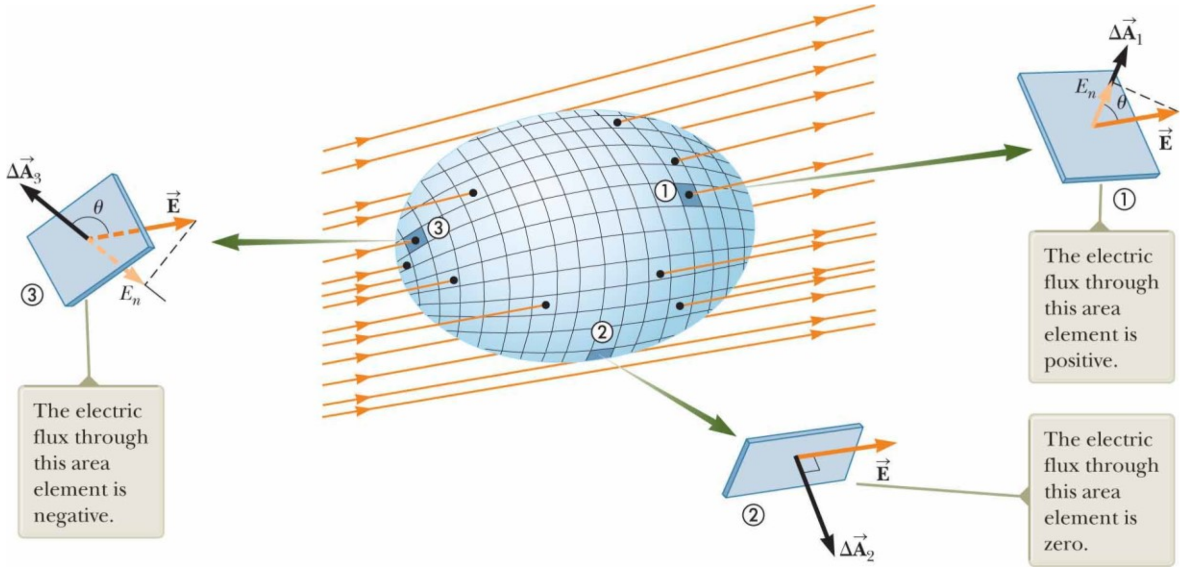
Closed Surface

- Closed surface: a surface that divides the space into ***inside*** and ***outside*** regions

- At the element labeled 1:
  - Field lines crossing the surface from inside to outside
    - $\Theta < 90°$
    - Flux $\Phi_{E,1} = E \cdot \Delta A_1$ through this element is positive

- At the element labeled 2:
  - Field lines graze the surface (perpendicular to ΔA_2)
    - $\Theta = 90°$
    - Flux $\Phi_{E,2} = E \cdot \Delta A_2 = 0$

- At the element labeled 3:
  - Field lines crossing the surface from outside to inside
    - $90° < \Theta < 180°$
    - Flux $\Phi_{E,1} = E \cdot \Delta A_3$ through this element is negative (cosΘ is negative)



- Using the symbol $\oint$ to represent integral over a closed surface
- Φ_E through a closed surface is:
  - E_n = component of electric field normal to surface

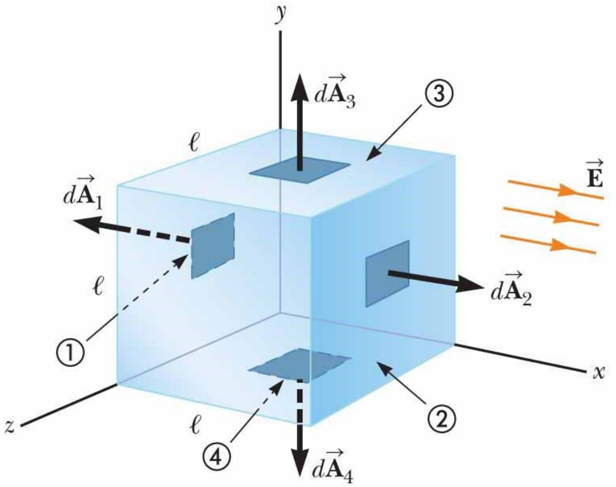
$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E_n dA$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Electric Field of a Uniformly Charged Ring

Example 23-4: Flux through a cube

Consider a uniform electric field $\bar{\boldsymbol{E}}$ oriented in the x direction in empty space. A cube of length ***l*** is placed in the field, oriented as shown in the figure. Find the net electric flux through the surface of the cube.



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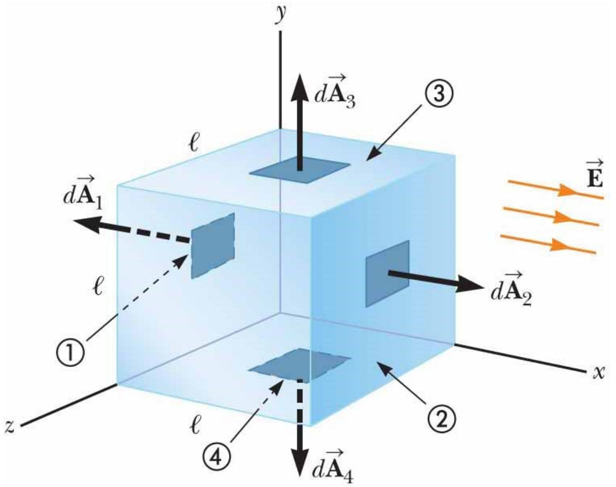

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Solution

Conceptualize: Examine the figure carefully. Notice that the electric field lines pass through two faces perpendicularly and are parallel to four other faces of the cube. The flux through four of the faces (3, 4, and the unnumbered faces) is zero because $\bar{\boldsymbol{E}}$ is parallel to the four faces and therefore perpendicular to $d\bar{\boldsymbol{A}}$ on these faces.

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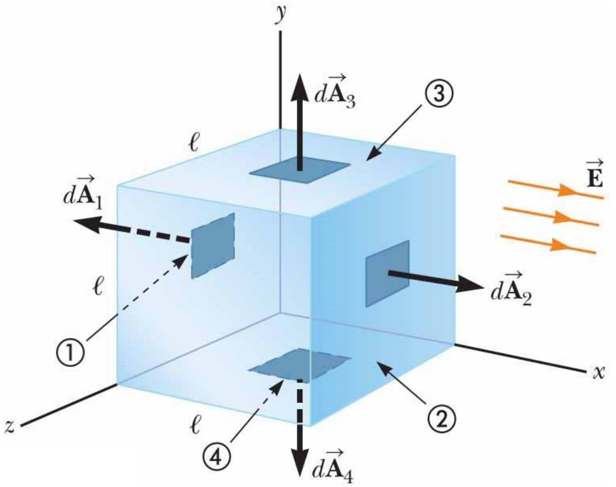

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Solution

Write the integrals for the net flux through faces ① and ②:

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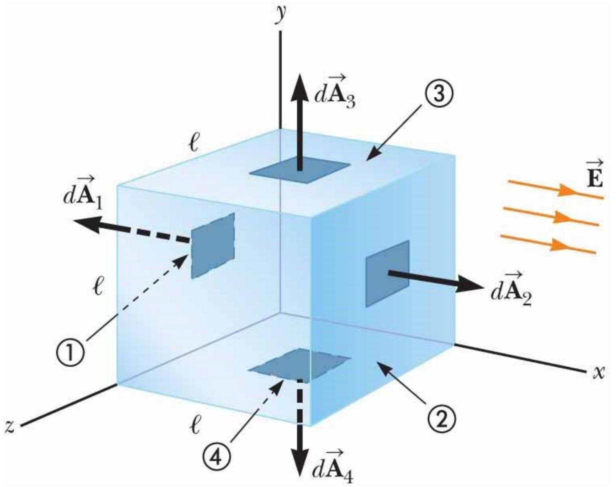

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Solution

Write the integrals for the net flux through faces ① and ②:

$$\Phi_E = \int_1 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_2 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

Electric Field of a Uniformly Charged Ring

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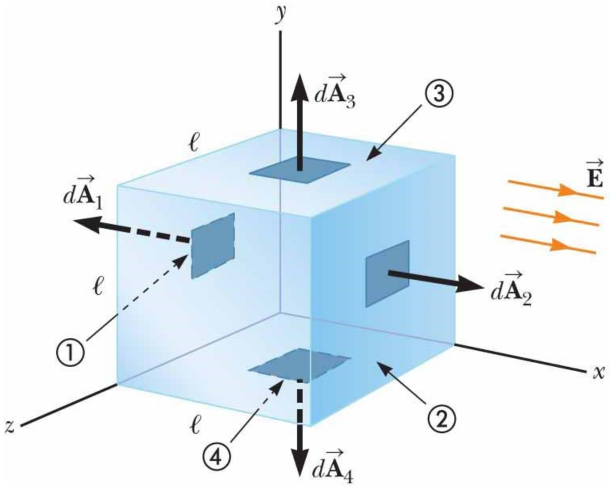

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For face ①, $\vec{\mathbf{E}}$ is constant and directed inward but $d\vec{\mathbf{A}}_1$ is directed outward ($\theta = 180°$). Find the flux through this face:

Electric Field of a Uniformly Charged Ring

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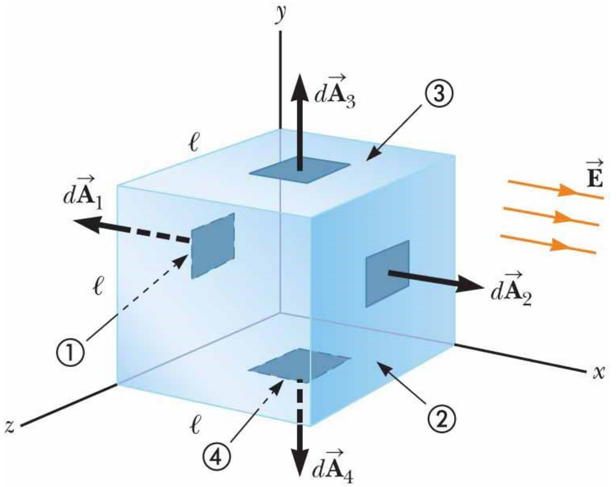

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$$\int_1 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_1 E(\cos 180°)\, dA = -E\int_1 dA = -EA = -E\ell^2$$

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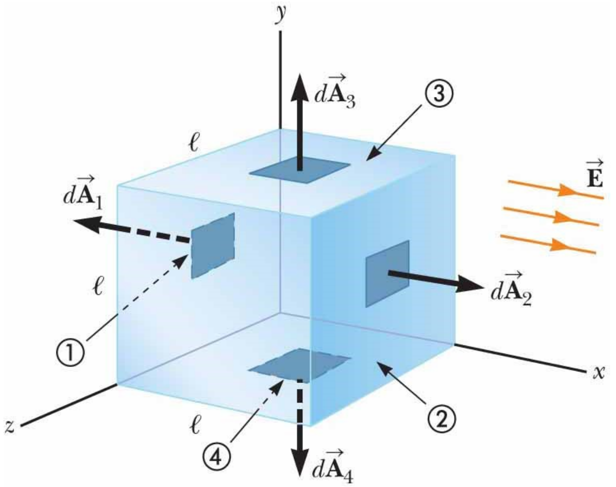

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For face ②, $\vec{\mathbf{E}}$ is constant and outward and in the same direction as $d\vec{\mathbf{A}}_2$ ($\theta = 0°$). Find the flux through this face:

Electric Field of a Uniformly Charged Ring

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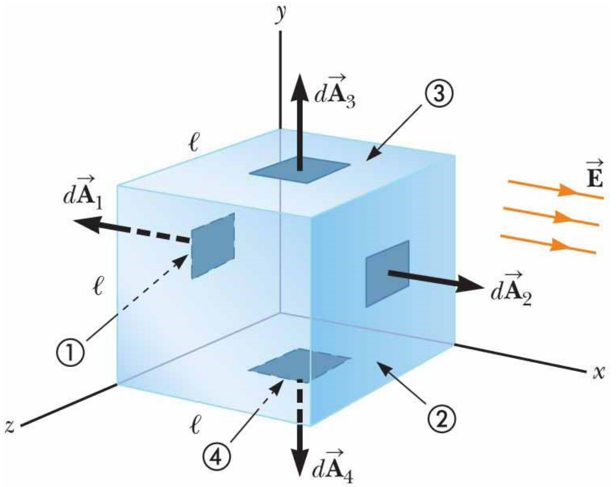

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$$\int_2 \vec{E} \cdot d\vec{A} = \int_2 E(\cos 0°)\, dA = E\int_2 dA = +EA = E\ell^2$$

Electric Field of a Uniformly Charged Ring

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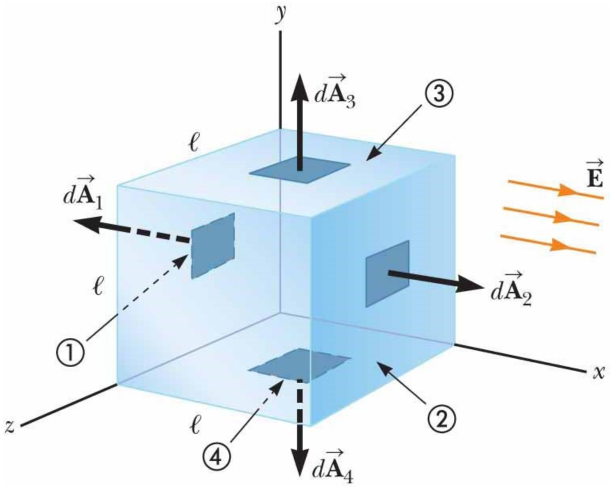

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$$\int_2 \vec{E} \cdot d\vec{A} = \int_2 E(\cos 0°)\, dA = E\int_2 dA = +EA = E\ell^2$$

Find the net flux by adding the flux over all six faces:

Electric Field of a Uniformly Charged Ring

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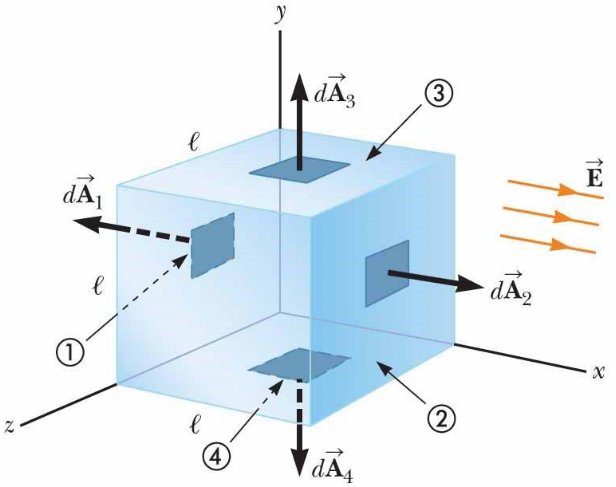

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Find the net flux by adding the flux over all six faces:

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = \boxed{0}$$

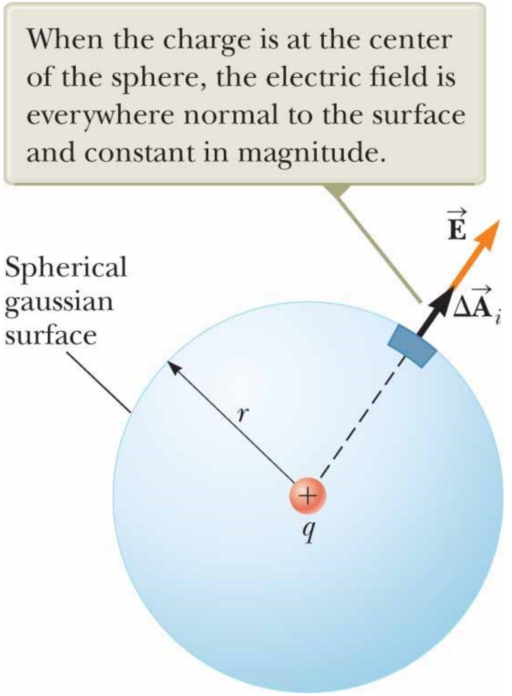
Continuous Charge Distributions & Gauss's Law (Ch. 23)

Gauss's Law

Gauss's Law

- Gauss's Law describes the general relationship between the net electric flux through a closed surface (gaussian surface) and the charge enclosed by a closed surface

- Consider a positive point charge ***q*** located at the center of a sphere of radius ***r*** (figure). Magnitude of the electric field everywhere on the surface of sphere is:



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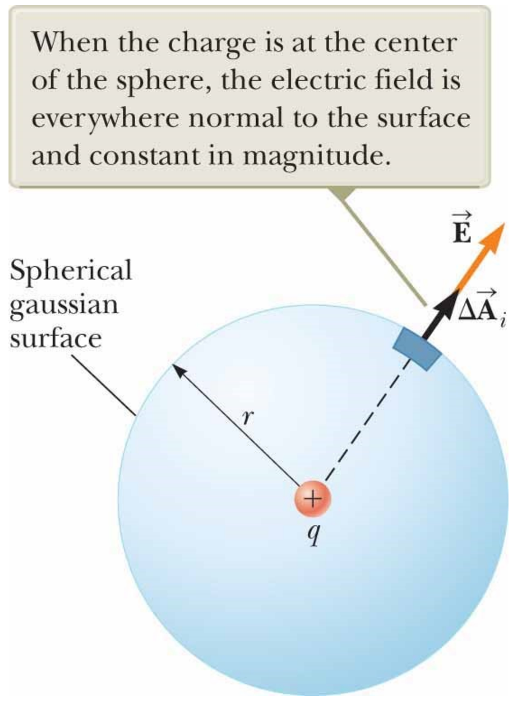
$$E = \frac{k_e q}{r^2}$$



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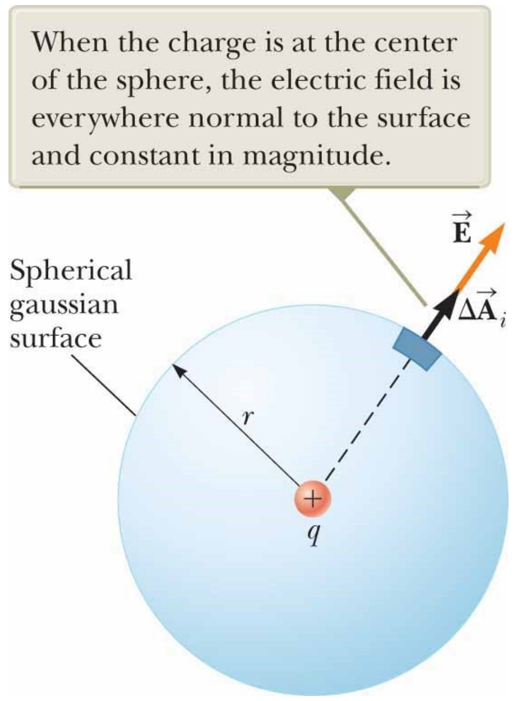

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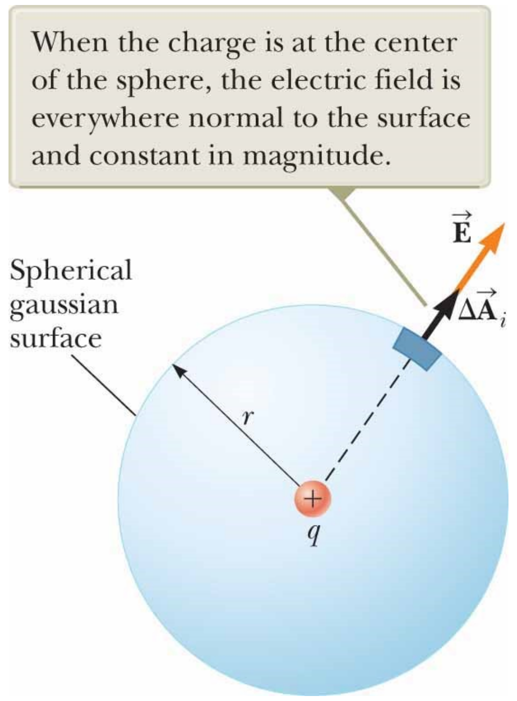
$$\vec{\mathbf{E}} \cdot \Delta\vec{\mathbf{A}}_i = E \Delta A_i \qquad \Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E dA = E \oint dA$$



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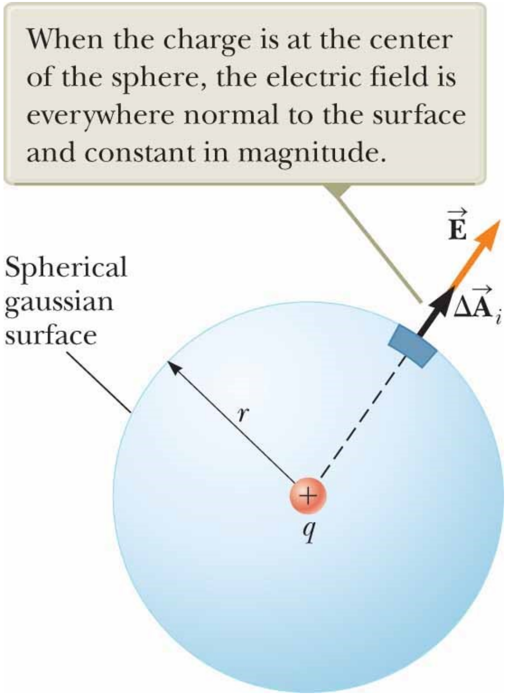
- We can move E outside of the integral (by symmetry, E is constant over the surface)
- Because the surface is spherical; $\oint dA = A = 4\pi r^2$.
- Net flux through the gaussian surface is:



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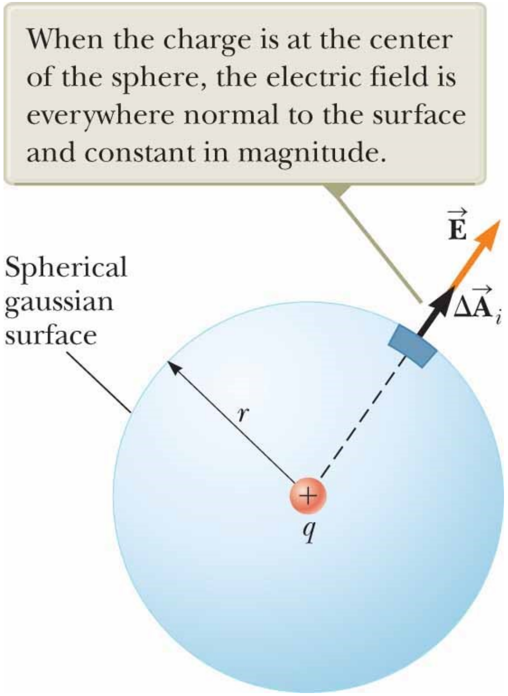
$$\Phi_E = \frac{k_e q}{r^2}\left(4\pi r^2\right) = 4\pi k_e q$$



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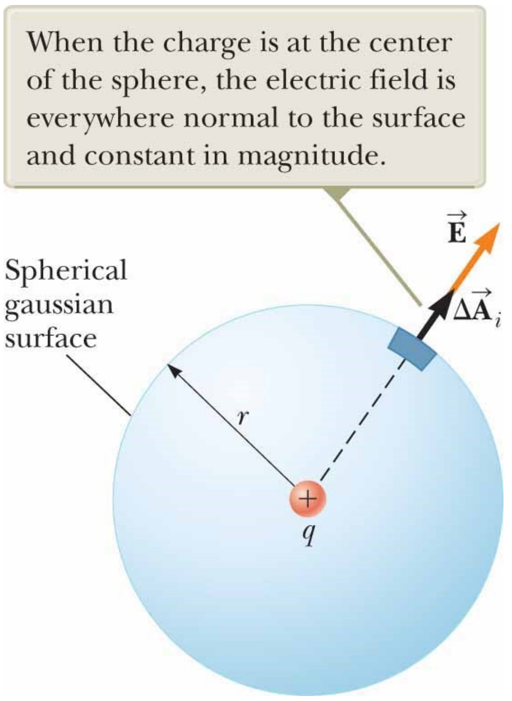
$$\boxed{k_e = 1/4\pi\varepsilon_0}$$

$$\Phi_E = \frac{k_e q}{r^2}\left(4\pi r^2\right) = 4\pi k_e q \longrightarrow \Phi_E = \frac{q}{\varepsilon_0}$$



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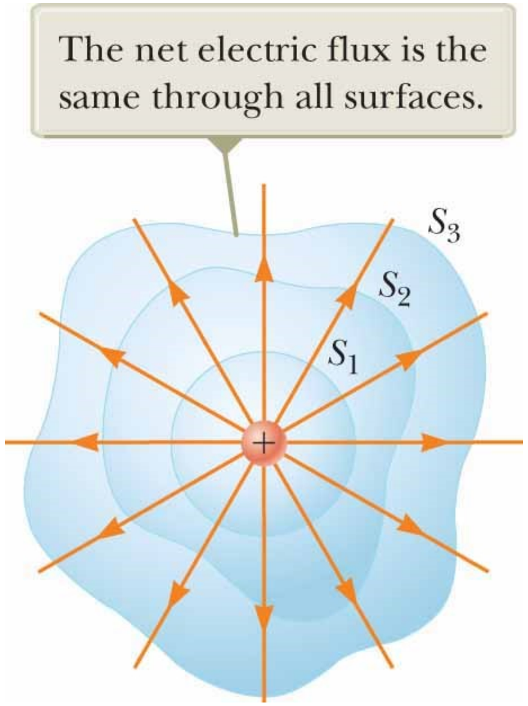
Gaussian Surface and Flux

- Consider several closed surfaces surrounding a charge ***q*** (figure)
  - Surface $\mathbf{S_1}$ is spherical, but surfaces $\mathbf{S_2}$ and $\mathbf{S_3}$ are not
- Flux that passes through $\mathbf{S_1}$ has value $\mathbf{q/\varepsilon_0}$
  - Recall: flux is ***proportional*** to the number of electric field lines passing through the surface
- In the Figure: every field line that passes through $\mathbf{S_1}$ also passes through the non-spherical surfaces $\mathbf{S_2}$ and $\mathbf{S_3}$



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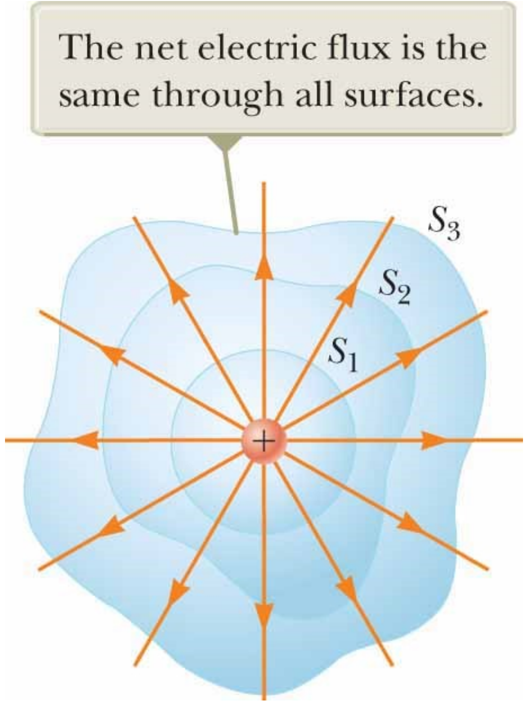
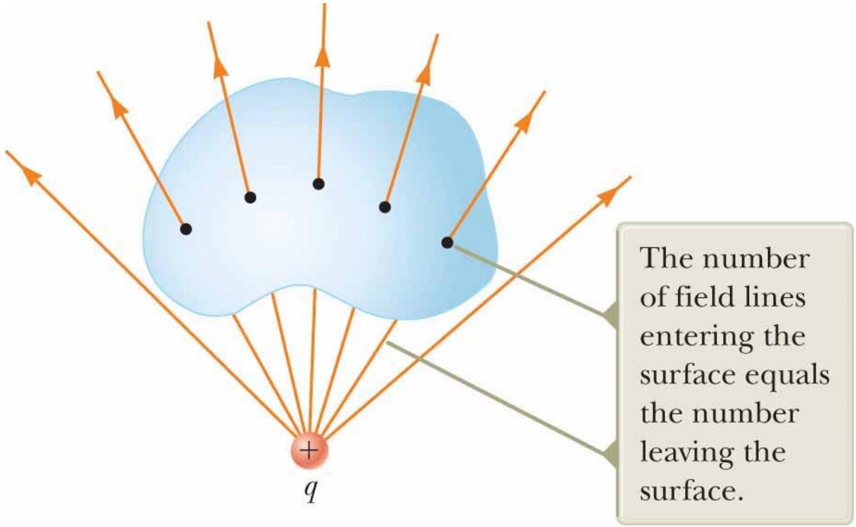


Net flux through any closed surface surrounding a point charge ***q*** is given by $\boldsymbol{q/\varepsilon_0}$ and is <u>independent of the shape</u> of that surface.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Gaussian Surface and Flux

- Consider a point charge located outside a closed surface of arbitrary shape (figure)
- Any electric field line entering the surface leaves the surface at another point
- Number of electric field lines entering the surface = number leaving the surface



Net electric flux through a closed surface that surrounds no charge = 0

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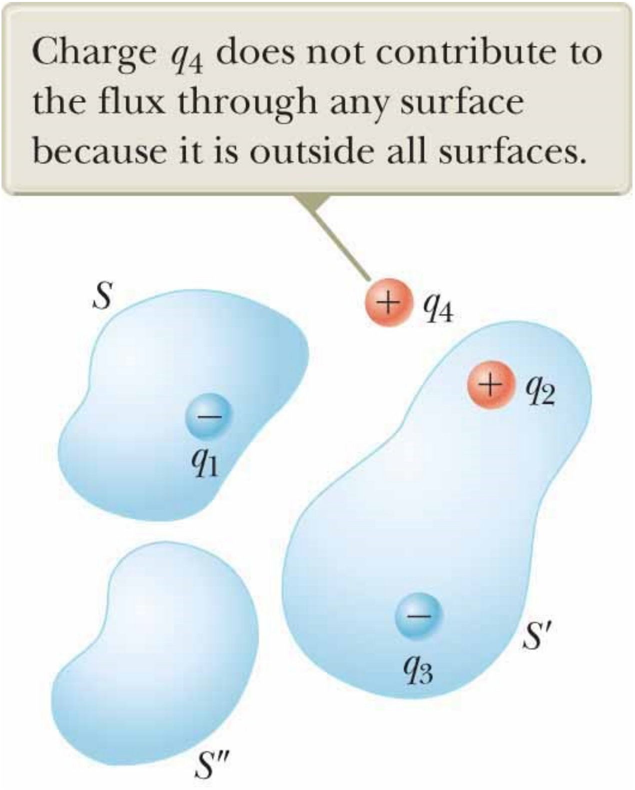
Gaussian Surface and Flux

Generalized Cases

- Generalized cases:
  - Many point charges
  - Continuous distribution of charge
- Use superposition principle →
  - Electric field due to many charges = vector sum of the electric fields produced by individual charges
- Flux through any closed surface:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint \left(\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \ldots \right) \cdot d\vec{\mathbf{A}}$$

- E = total electric field at any point on the surface produced by the vector addition of electric fields at that point due to individual charges



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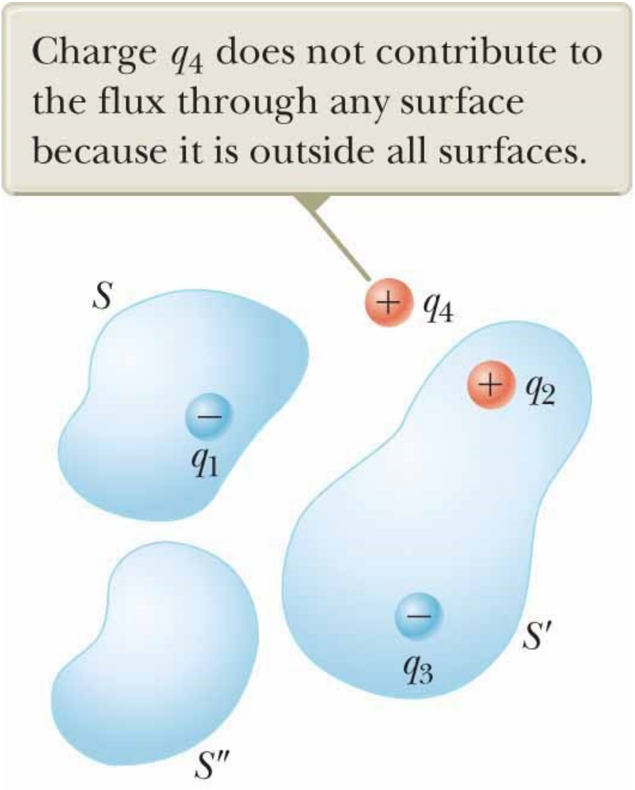
Gaussian Surface and Flux

Generalized Cases

- Surface S surrounds one charge: q_1
  - Net flux through S = q_1/ε_0
- Flux through S due to charges q_2, q_3, and q_4 outside = 0 →
  - Each electric field line from these charges that enters S at one point leaves it at another
- Surface S' surrounds charges q_2 and q_3 →
  - Net flux through it = $(q_2 + q_3)/\varepsilon_0$
- Net flux through the surface S'' = 0:
  - No charge inside surface
  - All electric field lines that enter S'' at one point leave at another
- Charge q_4 does not contribute to net flux through any of surfaces



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Gaussian Surface and Flux

Generalized Cases

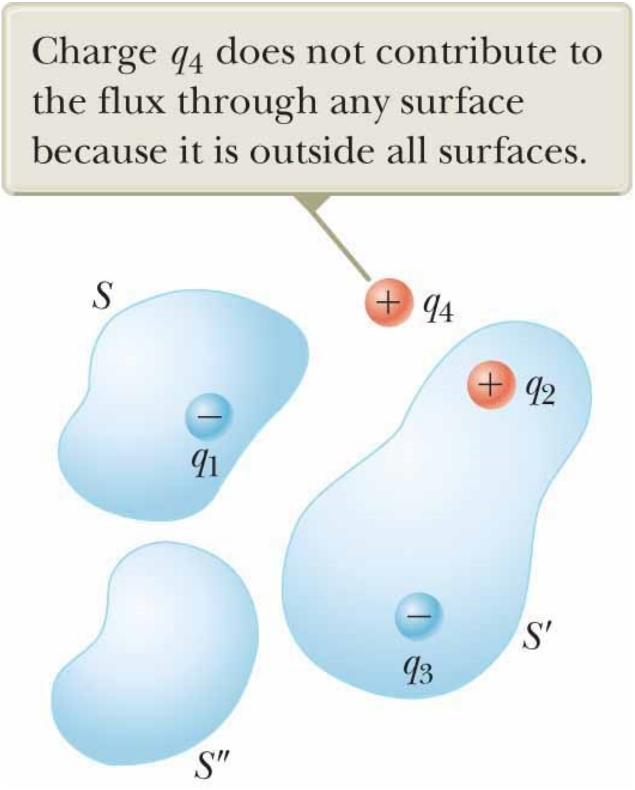
- Mathematical form of Gauss's law → net flux through any closed surface:

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\varepsilon_0}$$

  - $\bar{E}$ = electric field at any point on surface
  - q_{in} = net charge inside surface

- **Note**: although charge q_{in} = net charge inside the gaussian surface →
  - $\bar{E}$ = total electric field
  - Includes contributions from charges both inside and outside surface

- In principle, Gauss's law can be solved for $\bar{E}$ to determine the electric field due to a system of charges or continuous distribution of charge
- In practice: applicable only in limited number of highly symmetric situations



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Gaussian Surface and Flux

Quick Quiz

If the net flux through a gaussian surface is zero, the following four statements could be true. Which of the statements must be true?

(a) There are no charges inside the surface.

(b) The net charge inside the surface is zero.

(c) The electric field is zero everywhere on the surface.

(d) The number of electric field lines entering the surface equals the number leaving the surface.

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Continuous Charge Distributions & Gauss's Law (Ch. 23)

Applications of Gauss's Law to Various Charge Distributions

Applications of Gauss's Law

- Gauss's law → useful for determining the electric fields when charge distribution is highly symmetric

- Choosing a gaussian surface:
  - Always take advantage of the symmetry of charge distribution so that ***E*** can be removed from the integral

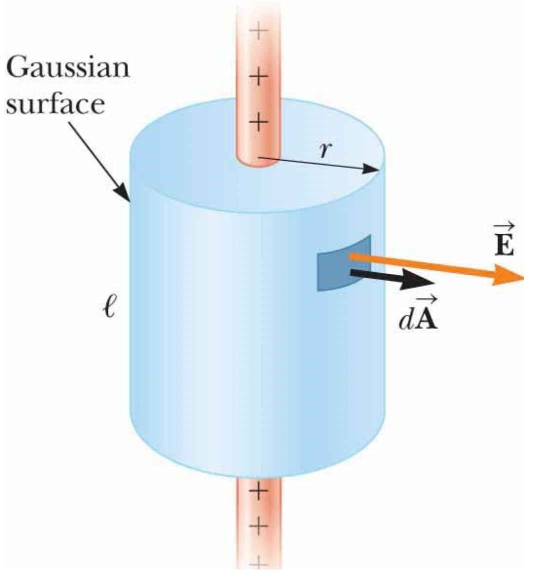
- Determine the surface to satisfy the following special conditions:
  - (1) Value of the electric field can be argued by symmetry to be constant over a portion of the surface
  - (2) Dot product can be expressed as a simple algebraic product ***E*dA*** because ***E*** and ***dA*** vectors are parallel
  - (3) Dot product = 0 because ***E*** and ***dA*** vectors are perpendicular
  - (4) Electric field = 0 over a portion of the surface
-
- Different portions of a gaussian surface can satisfy different conditions as long as every portion satisfies at least one condition

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10^{th} Edition. Cengage learning, 2018.

A Cylindrically Symmetric Charge Distribution

Example 23-7:

Find the electric field at a distance ***r*** from a line of positive charge of infinite length and constant charge per unit length $\boldsymbol{\lambda}$.



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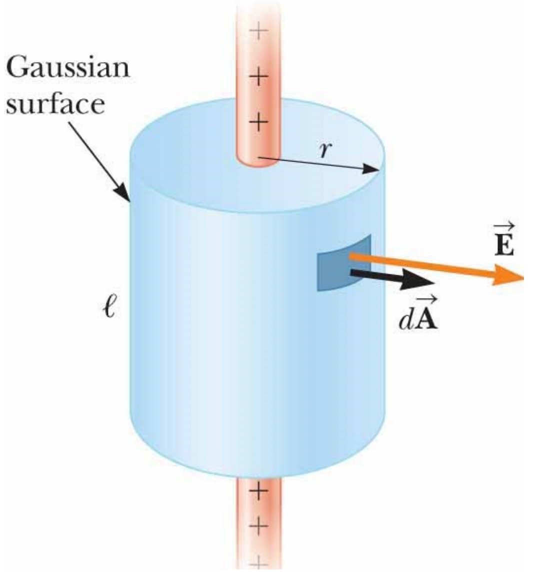

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Conceptualize The line of charge is *infinitely* long. Therefore, the field is the same at all points equidistant from the line, regardless of the vertical position of the point in the figure. We expect the field to become weaker as we move farther away radially from the line of charge.

Categorize Because the charge is distributed uniformly along the line, the charge distribution has cylindrical symmetry and we can apply Gauss's law to find the electric field

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Example 23-7:

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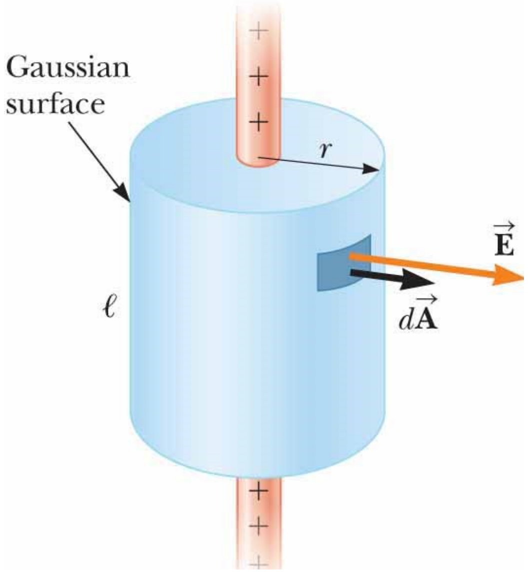

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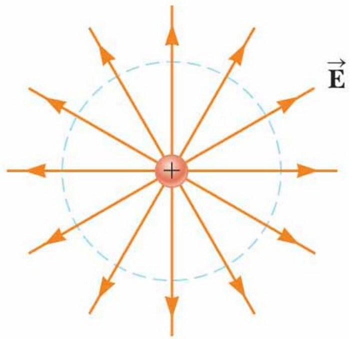


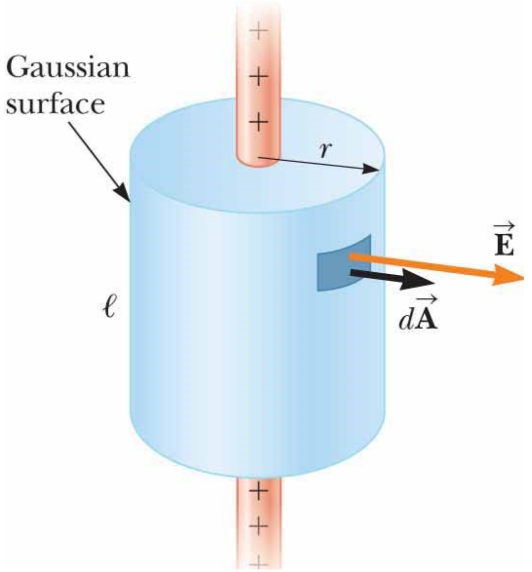
Analyze

- The symmetry of the charge distribution requires that ***E*** be perpendicular to the line charge and directed outward as shown in the figure on the right.
- To reflect the symmetry of the charge distribution, let's choose a cylindrical gaussian surface of radius ***r*** and length ***ℓ*** that is coaxial with the line charge. For the curved part of this surface, ***E*** is constant in magnitude and perpendicular to the surface at each point.
- The flux through the ends of the gaussian cylinder is zero because ***E*** is parallel to these surfaces.
- We must take the surface integral in Gauss's law over the entire gaussian surface. Because ***E·dA*** is zero for the flat ends of the cylinder, however, we restrict our attention to only the curved surface of the cylinder.

A Cylindrically Symmetric Charge Distribution

Example 23-7:

Find the electric field at a distance ***r*** from a line of positive charge of infinite length and constant charge per unit length $\boldsymbol{\lambda}$.



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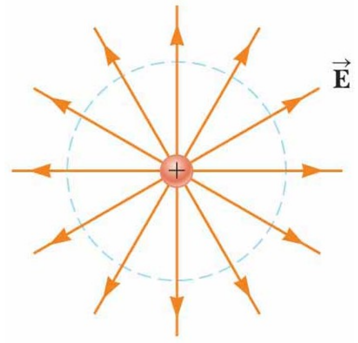
Solution

Apply Gauss's law and conditions (1) and (2) for the curved surface, noting that the total charge inside our gaussian surface is $\lambda\ell$:



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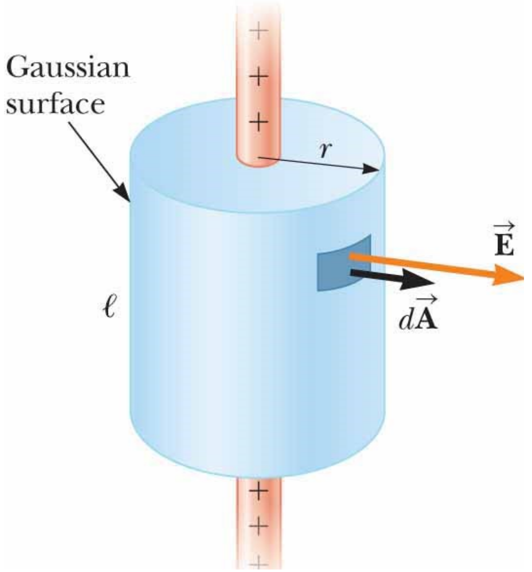

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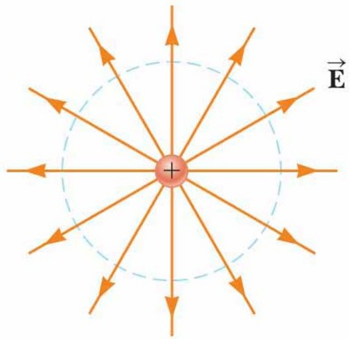
Solution

Apply Gauss's law and conditions (1) and (2) for the curved surface, noting that the total charge inside our gaussian surface is $\lambda\ell$:

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda\ell}{\epsilon_0}$$



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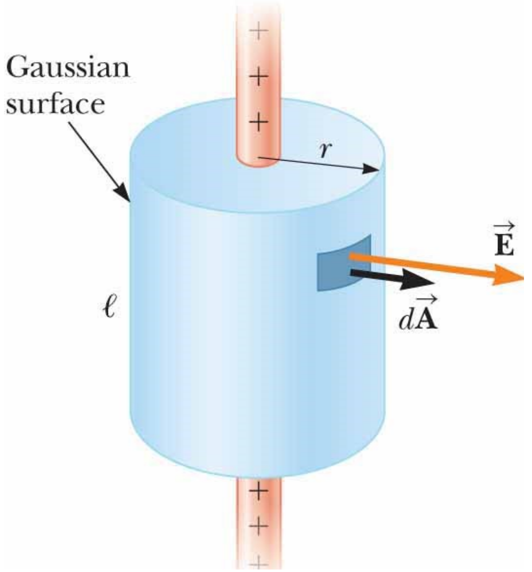

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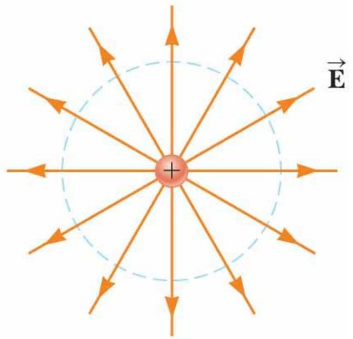
Substitute the area $A = 2\pi r\ell$ of the curved surface:



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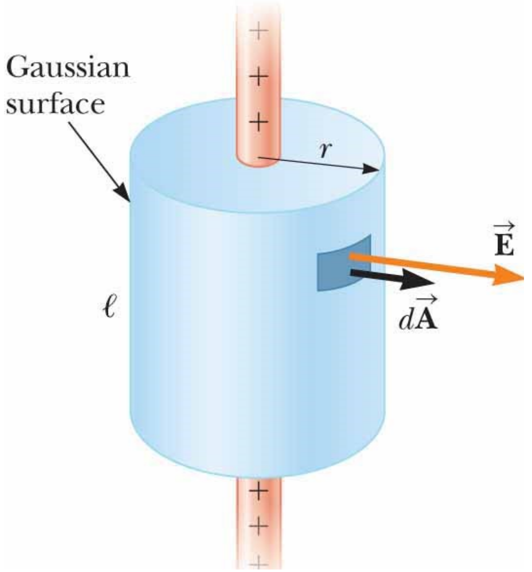

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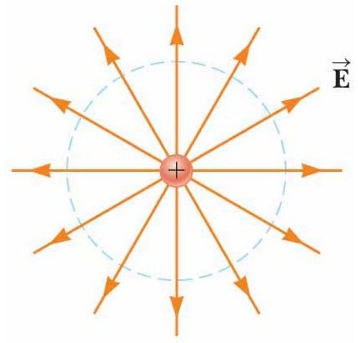
$$E(2\pi r\ell) = \frac{\lambda\ell}{\epsilon_0}$$



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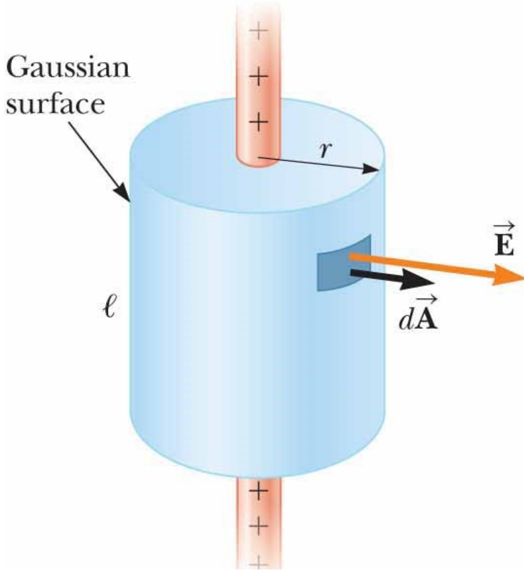

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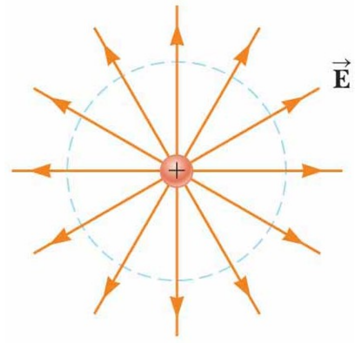

Solve for the magnitude of the electric field:

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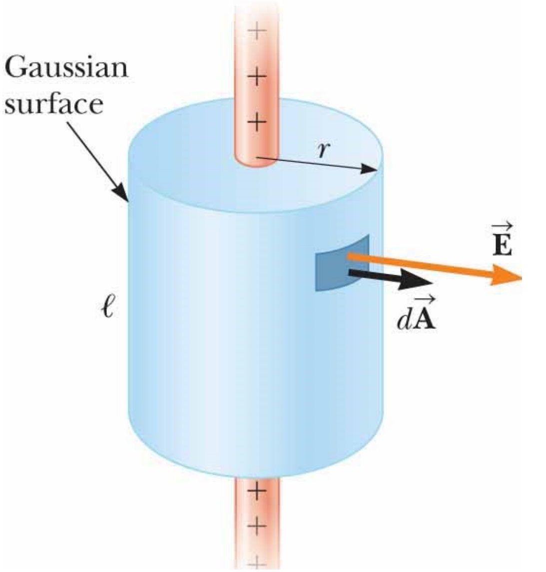

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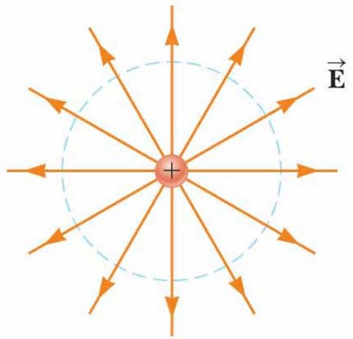
Substitute the area $A = 2\pi r\ell$ of the curved surface:

$$E(2\pi r\ell) = \frac{\lambda\ell}{\epsilon_0}$$

Solve for the magnitude of the electric field:

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2k_e\frac{\lambda}{r}$$

$$\boxed{k_e = 1/4\pi\varepsilon_0}$$



Electric Potential (Ch. 24)

Electric Potential and Potential Difference

Electric Potential

- Earlier, we linked our new study of ***electromagnetism*** to our earlier studies of ***force***. In this chapter, we make a link between electromagnetism and our earlier investigations into ***energy***.

- The concept of potential energy was studied in connection with conservative forces such as the ***gravitational force*** and the ***elastic force*** exerted by a spring. By using the ***law of conservation of energy***, we could solve various problems in mechanics that were not solvable with an approach using forces.

- The electrostatic force is conservative, too; therefore, electrostatic phenomena can be conveniently described in terms of an electric potential energy, which is of great value in the study of electricity. We will define a related quantity known as ***electric potential***.

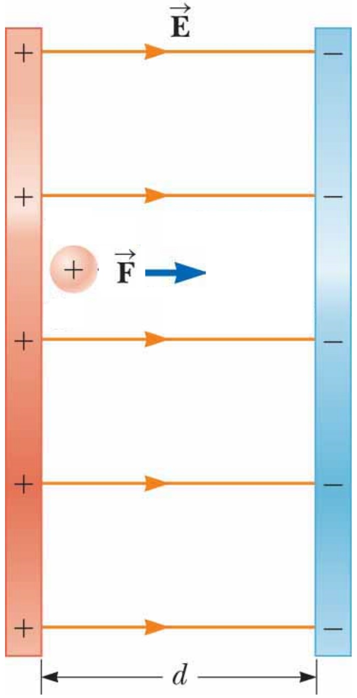
- The electric potential is a ***scalar quantity***; therefore easier to express compared to vector quantities.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Electric Potential and Potential Difference

- When charge ***q*** is placed in an electric field ***E*** →
  - There is an electric force ***qE*** acting on the charge (figure)
  - Force is conservative because the force between the charges described by Coulomb's law is conservative

- Identify the charge and the field as a system. If the charge is free to move → it will do so in response to the electric force
  - The electric field does work on the charge
  - Work is internal to the system



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Electric Potential and Potential Difference

- For an infinitesimal displacement ***ds*** of point charge ***q*** that is immersed in an electric field, the work done within the charge–field system is:

$$W_{\text{int}} = \vec{\mathbf{F}}_e \cdot d\vec{\mathbf{s}} = q\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- Identify the electric potential energy $\boldsymbol{U_E}$ for the charge–field system.
- As charge ***q*** is displaced → the electric potential energy of charge–field system is changed by an amount:

$$W_{\text{int}} = -\Delta U_E$$

$$dU_E = -W_{\text{int}} = -q\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

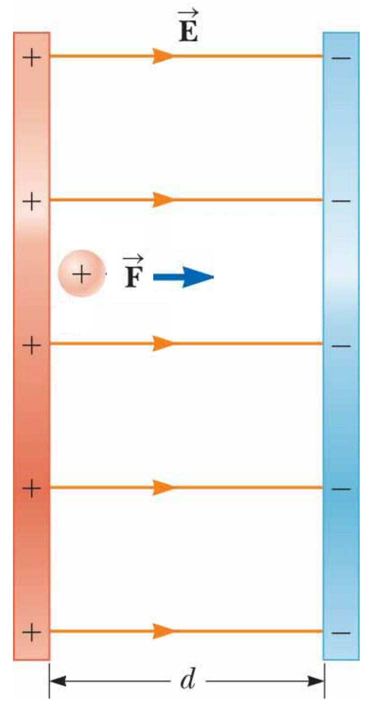
- For a finite displacement of charge from some point **A** to point **B** → the change in the electric potential energy of the system is:
    - Because the force ***qE*** is conservative, the line integral does not depend on the path taken from A to B

$$\Delta U_E = -q\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- Dividing the potential energy by the charge gives a physical quantity that has a value at every point in the electric field

    → called **electric potential V** (scalar quantity)

$$V = \frac{U_E}{q}$$



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Electric Potential and Potential Difference

- Potential difference $\Delta V = V_B - V_A$ between the two points A and B in an electric field is defined as:
  - Change in the electric potential energy of the system when charge **q** is moved between the points divided by the charge

$$\Delta V \equiv \frac{\Delta U_E}{q} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- Work done is equal to the change in potential energy:

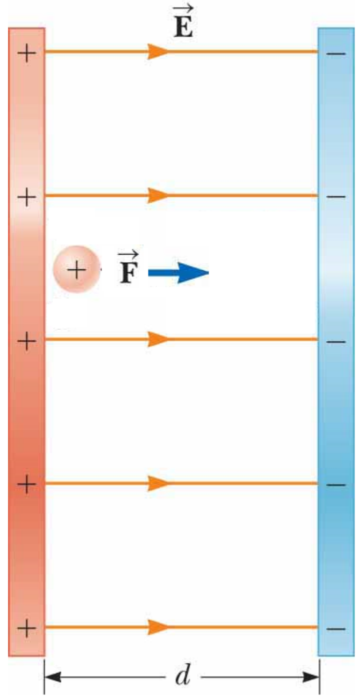
$$W = \Delta U_E = q\Delta V$$

- SI unit of electric potential is ***joules per coulomb***
- Also defined as a **volt (V)**:
- 1 J of work must be done to move 1-C charge through potential difference of 1 V

$$1\ \mathrm{V} = 1\ \mathrm{J/C}$$

- Potential difference can also be expressed as units of electric field times distance.

$$1\ \mathrm{V} = [\mathrm{N/C}]\cdot\mathrm{m}$$



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Electric Potential and Potential Difference

Quick Quiz

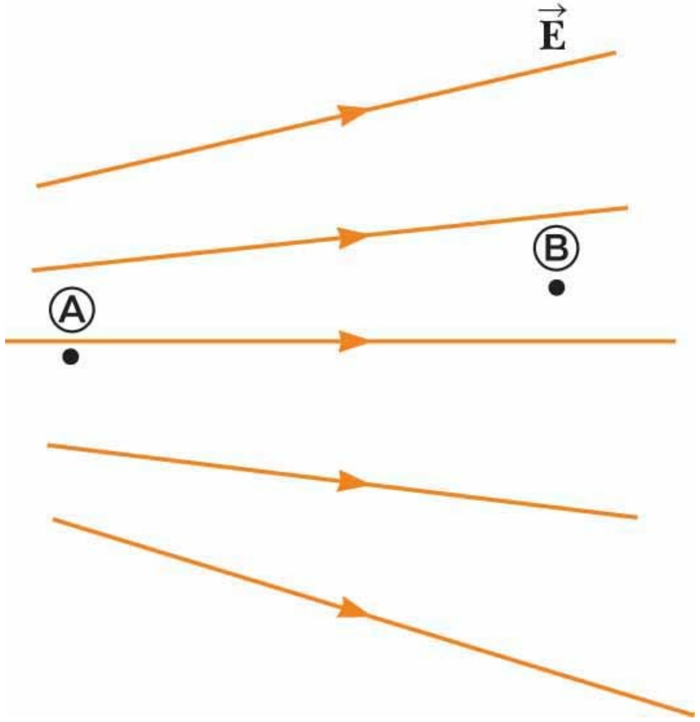
In the figure, two points A and B are located within a region in which there is an electric field. How would you describe the potential difference $\Delta V = V_B - V_A$?

(a) It is positive.

(b) It is negative.

(c) It is zero.

$$\Delta V \equiv \frac{\Delta U_E}{q} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$



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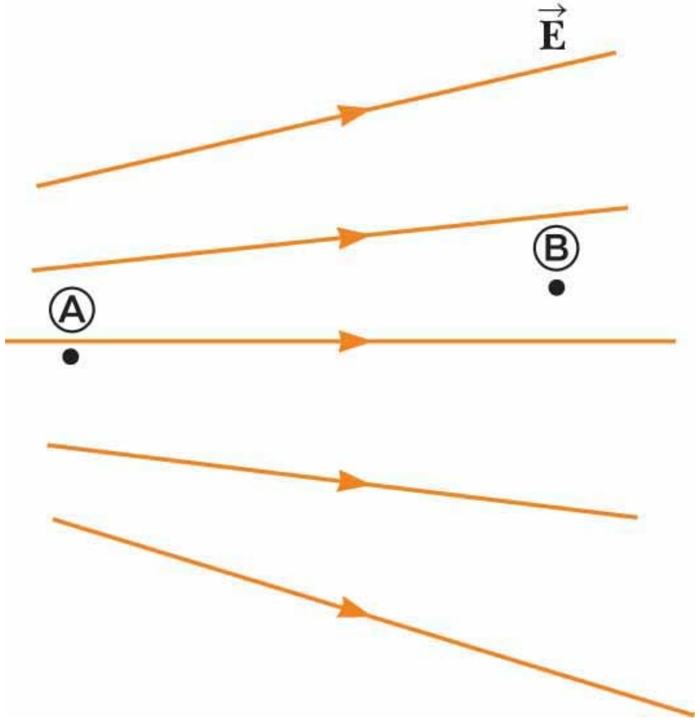
In the figure, two points A and B are located within a region in which there is an electric field. How would you describe the potential difference $\Delta V = V_B - V_A$?

(a) It is positive.

True → **(b) It is negative.**

(c) It is zero.

$$\Delta V \equiv \frac{\Delta U_E}{q} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$



When moving straight from A to B, $\vec{\mathbf{E}}$ and $\vec{\mathbf{ds}}$ both point toward the right. Therefore, the dot product $\vec{E}\cdot\vec{ds}$ in $\Delta v = -\int E\cdot ds$ is positive and ΔV is negative.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Electric Potential (Ch. 24)

Potential Difference in a Uniform Electric Field

Potential Difference in a Uniform Electric Field

- Let's consider a special case of uniform electric field directed along the negative y axis as shown on the figure. Let's calculate the potential difference between the two points **A** and **B** separated by a distance **d**.

$$V_B - V_A = \Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_A^B E\, ds\,(\cos 0°) = -\int_A^B E\, ds$$

- Displacement vector $\overrightarrow{\mathbf{ds}}$ is parallel to the field lines and points from A to B
- **E** is constant (uniform), so it can be removed from the integral sign

$$\Delta V = -E\int_A^B ds$$

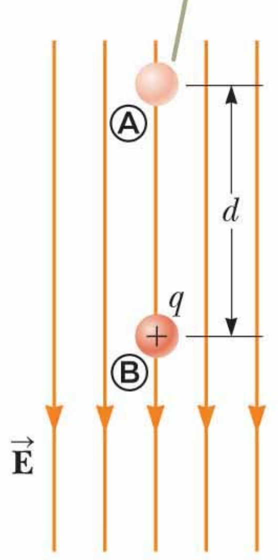
- Result → negative sign indicates that the electric potential at point B is lower than it is at point A:
- $V_B < V_A$ (Electric field lines point in the direction of decreasing electric potential)

$$\Delta V = -Ed$$

- We can calculate the change in potential energy of charge–field system
  - Electric potential energy of the system decreases when charge moves in the direction of the field
  - If positive charge is released from *rest* in this electric field → the charge experiences electric force ***qE*** in the direction of **E** (downward in figure)
  - ***q*** accelerates downward, gaining kinetic energy (by equal amount)

$$\Delta U_E = q\Delta V = -qEd$$

When a positive charge moves from point Ⓐ to point Ⓑ, the electric potential energy of the charge–field system decreases.



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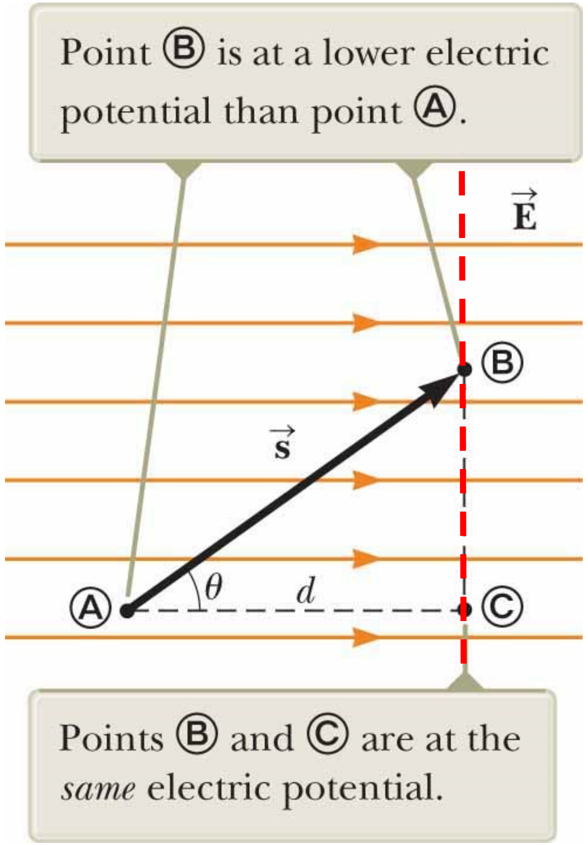
Equipotential Surfaces

- Let's consider a more general case: a charged particle that moves between A and B in a uniform electric field
  - Vector **s** is not parallel to field lines (figure)
- **E** can be removed from integral because it is constant
- Potential difference V_B - V_A = potential difference V_C - V_A
  - Therefore: $V_B = V_C$
- Equipotential surface: any surface consisting of continuous distribution of points having the same electric potential
- Equipotential surfaces are associated with uniform electric fields. They consist of family of parallel planes that are all perpendicular to the field.

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$
$$= -\vec{\mathbf{E}} \cdot \int_A^B d\vec{\mathbf{s}} = -\vec{\mathbf{E}} \cdot \vec{\mathbf{s}}$$

$$\Delta U_E = q\Delta V = -q\vec{\mathbf{E}} \cdot \vec{\mathbf{s}}$$



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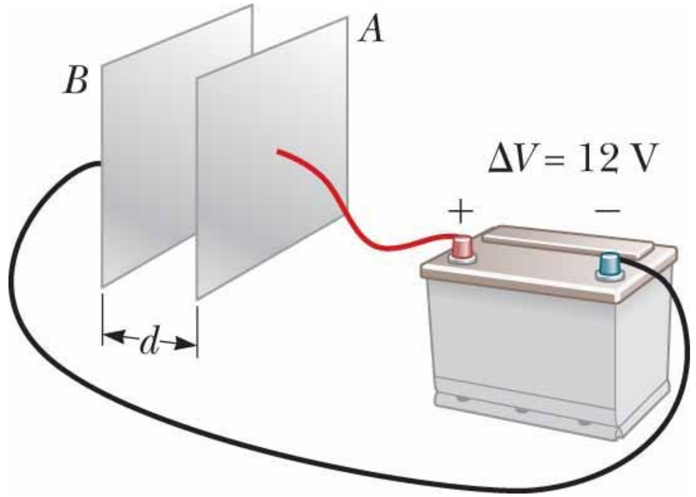
The Electric Field Between Two Parallel Plates

Example 24-1:

A ***12-V*** battery is connected between two parallel plates as shown in the figure. The separation between the plates is ***d = 0.30 cm***, and we assume the electric field between the plates to be uniform. Find the ***magnitude of the electric field*** between the plates.



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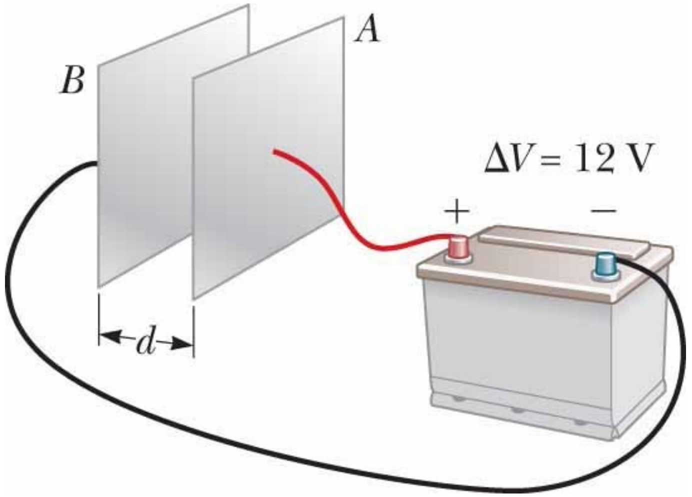

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Solution

Conceptualize: In this example, the electric field is related to the new concept of electric potential.

Categorize The electric field is evaluated from a relationship between the electric field and potential.

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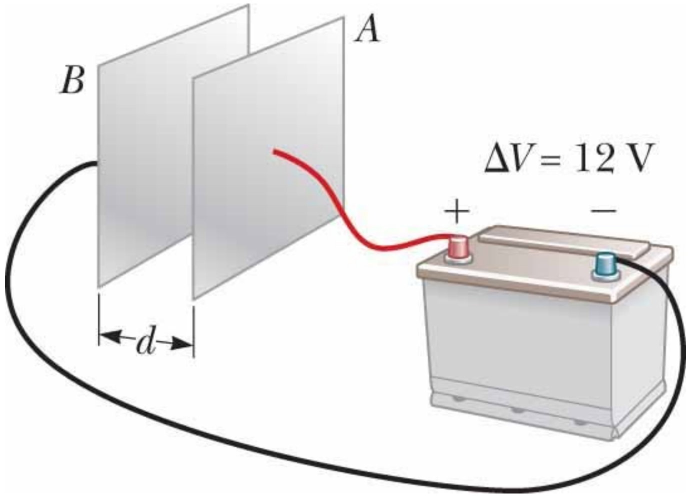

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Solution

Potential difference → electric field times distance.

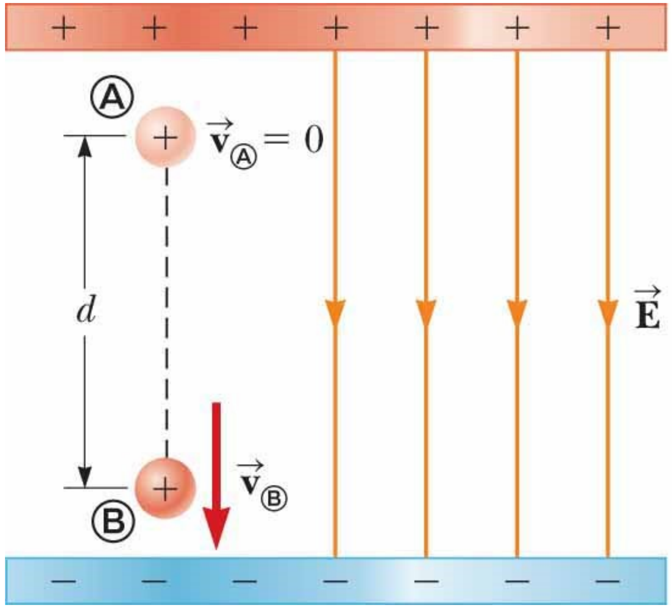
$$\Delta V \equiv \frac{\Delta U_E}{q} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$E = \frac{|V_B - V_A|}{d} = \frac{12\ \text{V}}{0.30 \times 10^{-2}\ \text{m}} = \boxed{4.0 \times 10^{3}\ \text{V/m}}$$

Motion of a Proton in a Uniform E Field

Example 24-2:

A proton is released from rest at point ***A*** in a uniform electric field that has a magnitude of $\boldsymbol{E=8.0x10^4\ V/m}$, as shown in the figure. The proton undergoes a displacement of magnitude ***d=0.50m*** to point ***B*** in the direction of $\vec{\mathbf{E}}$. Find the speed of the proton after completing the displacement.



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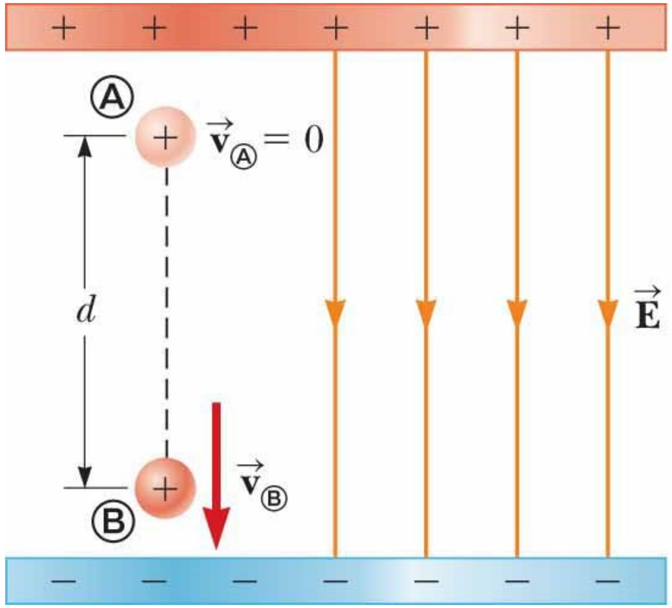

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Solution

Conceptualize: The situation is analogous to an object falling through a gravitational field. Also compare this to a previous example where a positive charge was moving in a uniform electric field. In that example, we applied the particle under constant acceleration and non isolated system models. Now that we have investigated electric potential energy, what model can we use here?

Categorize: The system of the proton and the two plates in the figure does not interact with the environment, so we model it as an isolated system for energy.

Motion of a Proton in a Uniform E Field

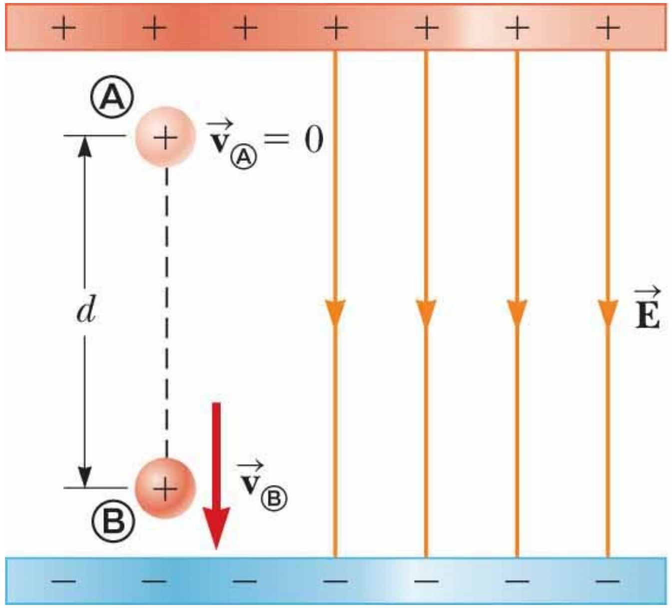
Example 24-2:

A proton is released from rest at point ***A*** in a uniform electric field that has a magnitude of ***E=8.0x10⁴ V/m***, as shown in the figure. The proton undergoes a displacement of magnitude ***d=0.50m*** to point ***B*** in the direction of $\vec{\mathbf{E}}$. Find the speed of the proton after completing the displacement.

Solution

We can solve this problem from the perspective of conservation of energy:

$$\Delta K + \Delta U_E = 0$$



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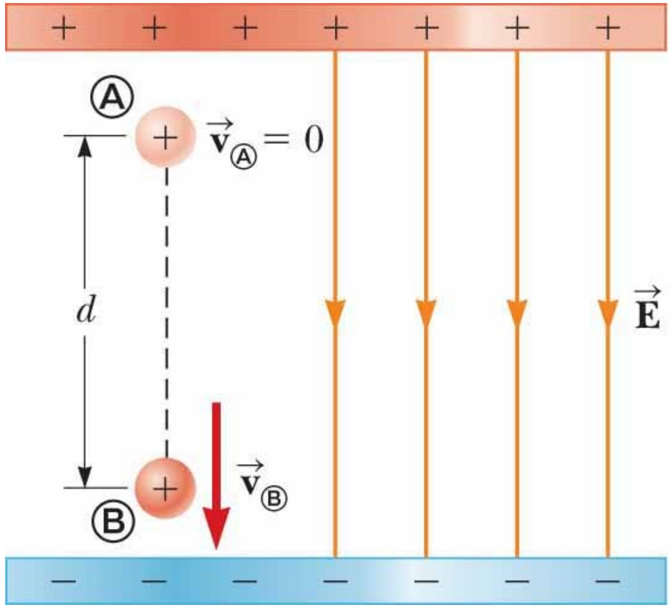
Solution

We can solve this problem from the perspective of conservation of energy:

$$\Delta K + \Delta U_E = 0$$

$$\Rightarrow \left(\frac{1}{2}mv^2 - 0\right) + e\overbrace{\Delta V}^{\Delta V = -Ed} = 0$$



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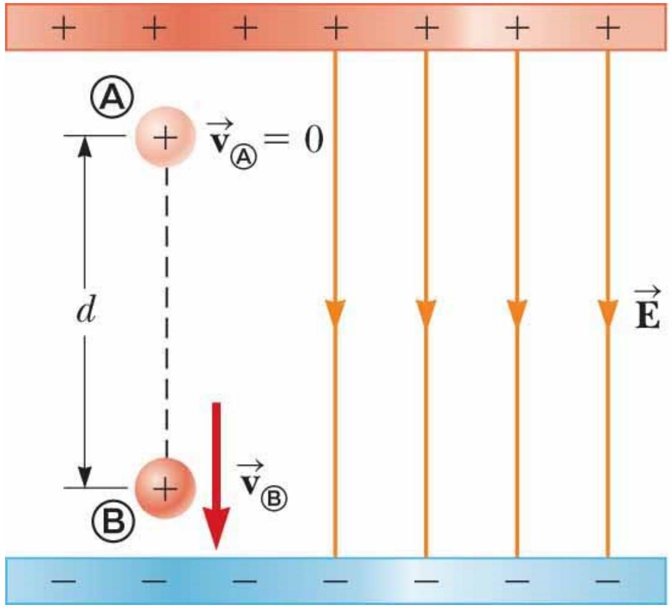

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Solution

We can solve this problem from the perspective of conservation of energy:

$$\Delta K + \Delta U_E = 0 \quad \Rightarrow \left(\frac{1}{2}mv^2 - 0\right) + e\overbrace{\Delta V}^{\Delta V = -Ed} = 0$$

Re-organizing to solve for final speed v:

$$v = \sqrt{\frac{-2e\Delta V}{m}} = \sqrt{\frac{-2e(-Ed)}{m}} = \sqrt{\frac{2eEd}{m}}$$

Motion of a Proton in a Uniform E Field

Example 24-2:

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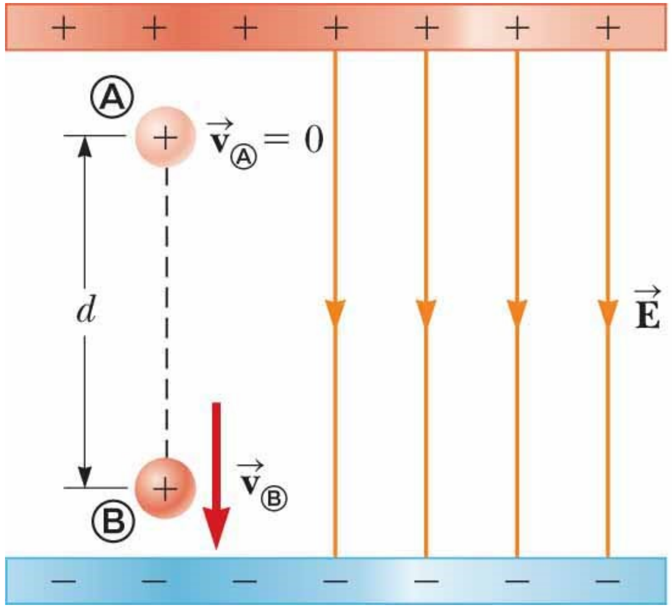

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$q_e=1.6\text{x}10^{-19}\text{C}$ $\quad$ $m_p=1.67\text{x}10^{-27}\text{kg}$ $\quad$ $E=8.0\text{x}10^{4}\text{V/m}$

Substituting numerical values:

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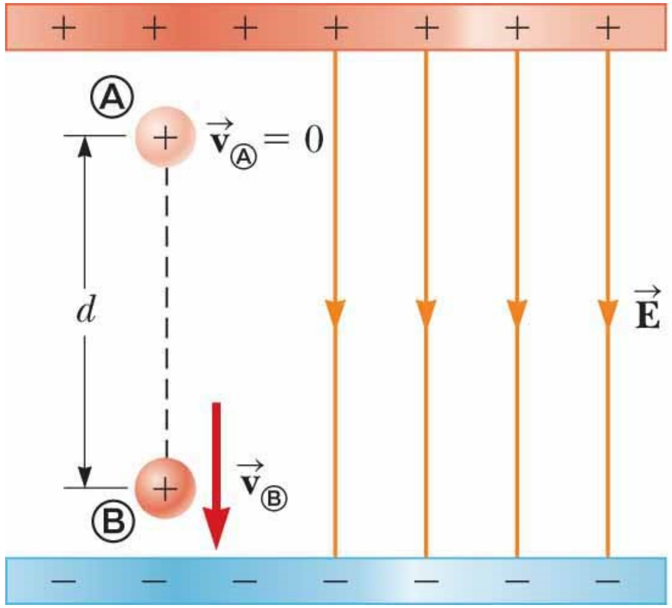

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q_e=1.6x10^{-19}C $\qquad$ m_p=1.67x10^{-27}kg $\qquad$ E=8.0x10^{4}V/m

Substituting numerical values:

$$v = \sqrt{\frac{2(1.6\times10^{-19}\ \text{C})(8.0\times10^{4}\ \text{V})(0.50\ \text{m})}{1.67\times10^{-27}\ \text{kg}}} = \boxed{2.8\times10^{6}\ \text{m/s}}$$

Electric Potential (Ch. 24)

Electric Potential and Potential Energy Due to Point Charges

Electric Potential Due to Point Charges

- Let's try to find the electrical potential at a point located at a distance ***r***:

- A and B: two arbitrary points as shown in the figure.
  - At any point in space → $\vec{E}$ due to a point charge is:
  - $\vec{r}$ = unit vector directed radially outward from the charge

- $\vec{E}\cdot\vec{ds}$ can be expressed as:
  - Magnitude of $\hat{r}$ = 1
  - Θ = angle between **r** and d**s**
  - ds·cosΘ is the projection of ds onto r
  - Any displacement ds along path from A to B produces change dr in direction of r

- Substitute to find:

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

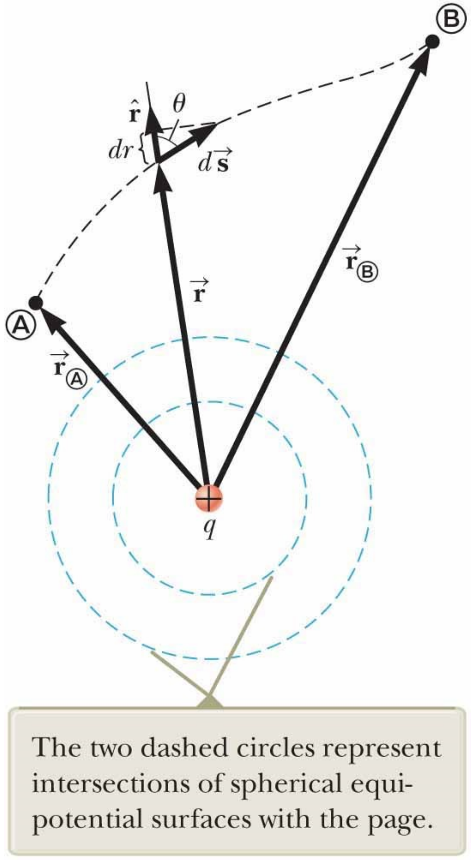
$$\vec{\mathbf{E}} = \frac{k_e q}{r^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = k_e \frac{q}{r^2}\hat{\mathbf{r}} \cdot d\vec{\mathbf{s}}$$

$$\hat{\mathbf{r}} \cdot d\vec{\mathbf{s}} = ds\cos\theta$$

$$ds\cos\theta = dr$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = k_e \frac{q}{r^2} dr$$



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Electric Potential Due to Point Charges

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = k_e \frac{q}{r^2} dr$$

- Potential difference becomes

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = k_e \frac{q}{r}\Bigg|_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

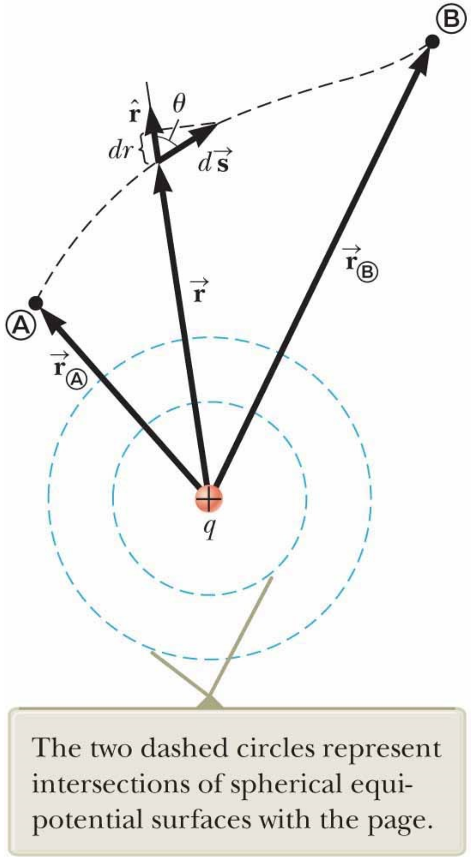
- Potential difference between any two points A and B in a field created by a point charge depends only on radial coordinates r_A and r_B

- If we bring a charge from infinity (V_A=0)

$$V_A = 0 \text{ at } r_A = \infty$$

- Electric potential due to a point charge at any distance ***r*** from the charge is:

$$\boxed{V = k_e \frac{q}{r}}$$



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Electric Potential Due to Point Charges

Multiple Point Charges

- For a group of point charges, we apply superposition principle:

$$V = k_e \sum_i \frac{q_i}{r_i}$$

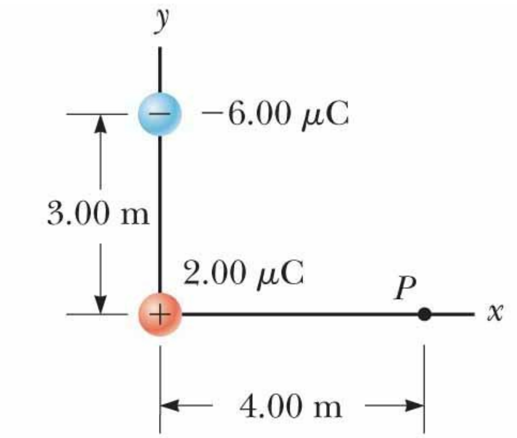
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The E Potential Due to Two Point Charges

Example 24-3:

As shown in the figure, a charge $\boldsymbol{q_1=2.00\mu C}$ is located at the origin and a charge $\boldsymbol{q_2=-6.00\mu C}$ is located at (0, 3.00) m. Find the **total electric potential** due to these charges at the point P, whose coordinates are (4.00, 0) m.

$$V = k_e \sum_i \frac{q_i}{r_i}$$



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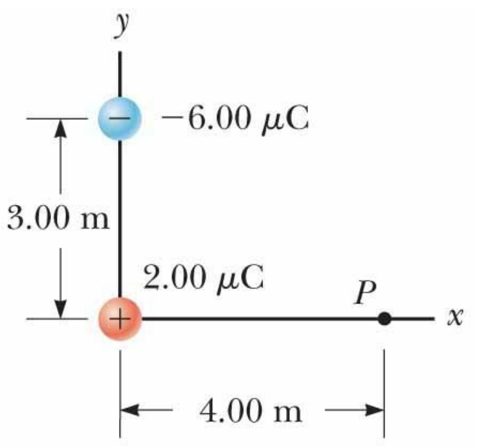
$$V = k_e \sum_i \frac{q_i}{r_i}$$



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Solution

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\times\left(\frac{2.00\times10^{-6}\ \mathrm{C}}{4.00\ \mathrm{m}} + \frac{-6.00\times10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right) = \boxed{-6.29\times10^3\ \mathrm{V}}$$

Electric Potential (Ch. 24)

Obtaining the Electric Field from the Electric Potential

Deriving Electric Field from Electric Potential

Uniform Electric Field

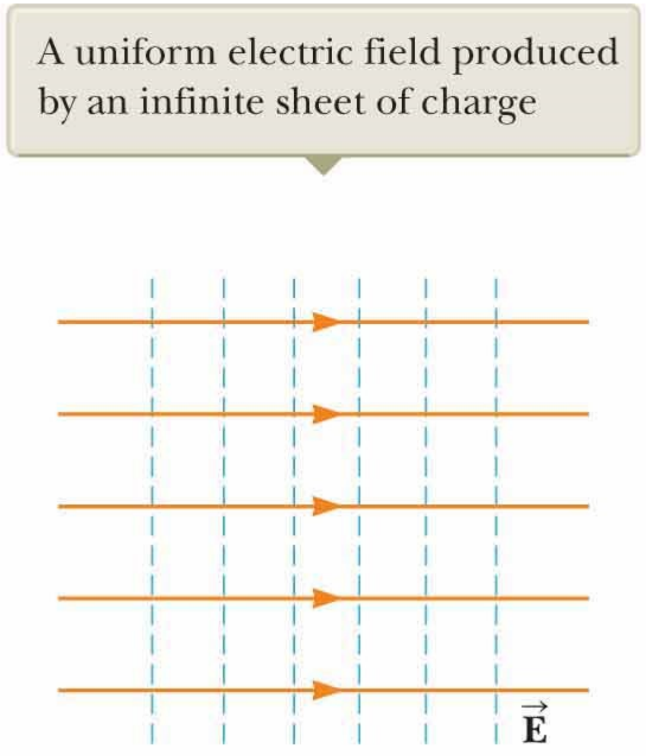
$$\Delta V = -\int_{A}^{B} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E_x dx$$

If electric field has only one component E_x

$$E_x = -\frac{dV}{dx}$$



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Deriving Electric Field from Electric Potential

Spherical Symmetric Electric Field

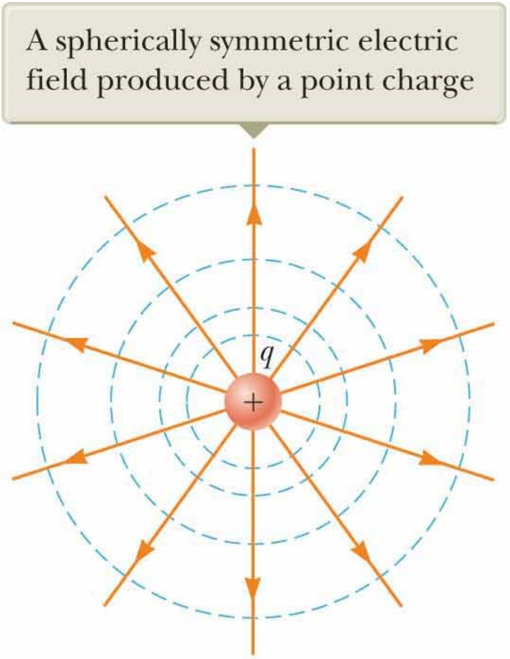
$$\Delta V = -\int_{A}^{B} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E_r dr$$

If electric field has only radial component E_r

$$E_r = -\frac{dV}{dr}$$



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Deriving Electric Field from Electric Potential

In General

- In general: electric field can be found with the derivative of the electric potential.

- If electric potential is a function of all three spatial coordinates, partial derivatives would be needed.
  - If V(x,y,z) is given in terms of Cartesian coordinates → Electric field components are partial derivatives:

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

Electric Potential (Ch. 24)

Electric Potential due to Cont. Charge Distributions

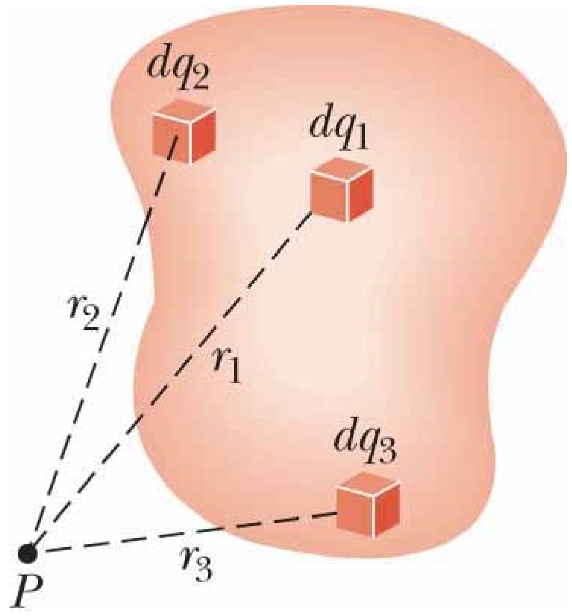
Electric Potential Due to Cont. Charge Dist.

Electric potential d***V*** at some point ***P*** due to a charge element d***q*** is:

$$dV = k_e \frac{dq}{r}$$

Total potential at point ***P*** can be found by integrating to include the contributions from all elements of charge distribution:

$$V = k_e \int \frac{dq}{r}$$



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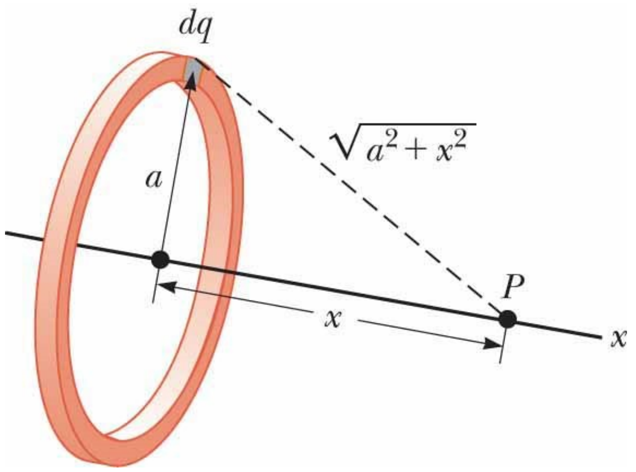
The E Potential Due to a Ring

Example 24-5:

(A) Find an expression for the electric potential at a point **P** located on the perpendicular central axis of a uniformly charged ring of radius ***a*** and total charge ***Q.***

$$V = k_e \int \frac{dq}{r}$$



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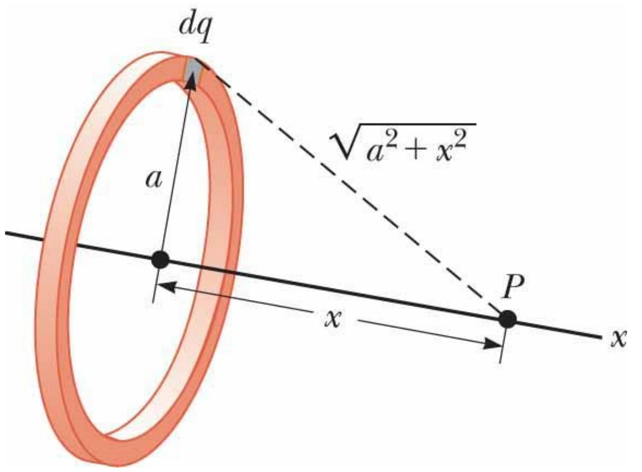

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Solution

Conceptualize: The center of the ring is at the origin, and it's on a plane perpendicular to the x-axis. Due to symmetry, all charges on the ring are at the same distance from point ***P***. And since the electric potential is scalar, no need for any vector considerations.

Categorize: Since we are working with a continuous charge distribution (as opposed to discreet charges), we need to use integration.

The E Potential Due to a Ring

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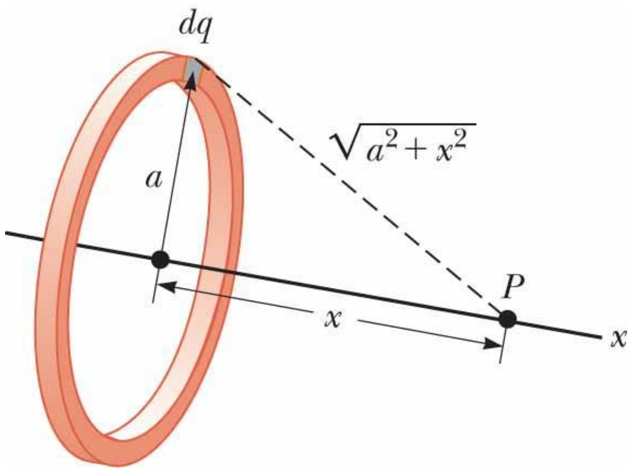
Solution

Express **V** in terms of the geometry:



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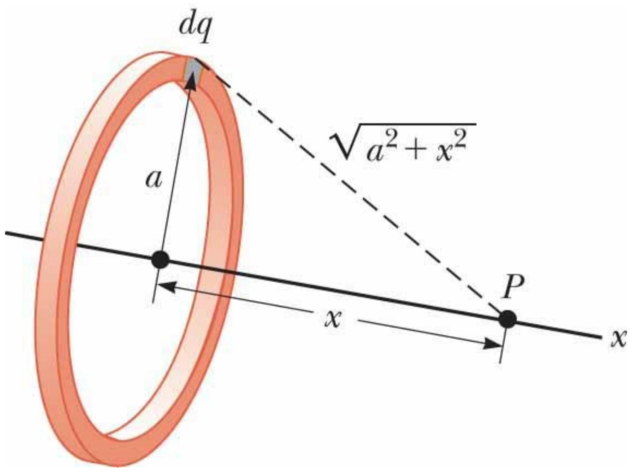
Solution

Express **V** in terms of the geometry:

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{a^2 + x^2}}$$



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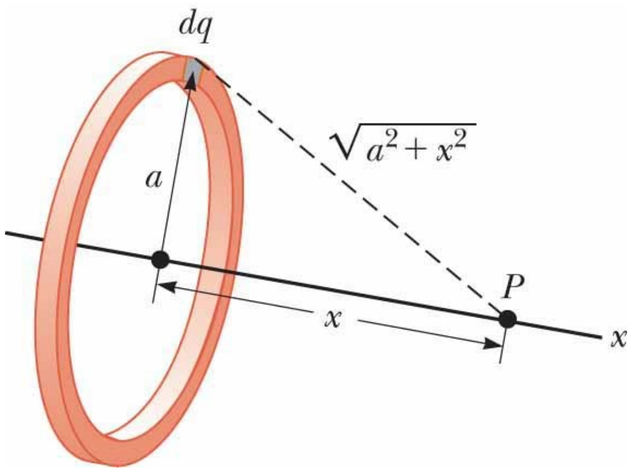

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$$V = k_e \int \frac{dq}{r}$$

Solution

Express **V** in terms of the geometry:

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{a^2 + x^2}}$$

a and ***x*** do not vary over the ring integration; therefore, they can come out of the integral:

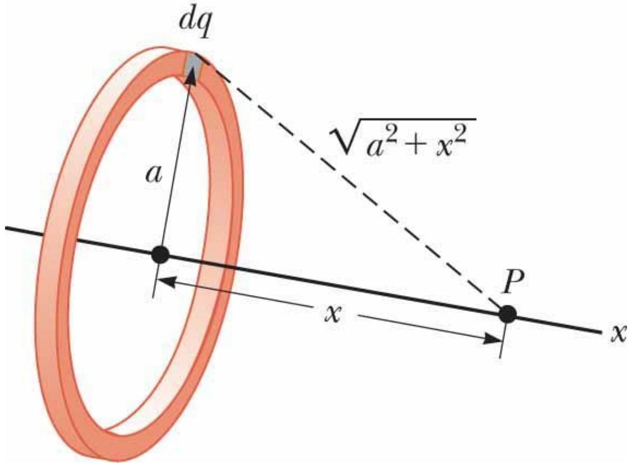
$$V = \frac{k_e}{\sqrt{a^2 + x^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{a^2 + x^2}}}$$

The E Potential Due to a Ring

Example 24-5:

(B) Find an expression for the magnitude of the electric field at point ***P.***

$$V = \boxed{\frac{k_e Q}{\sqrt{a^2 + x^2}}}$$



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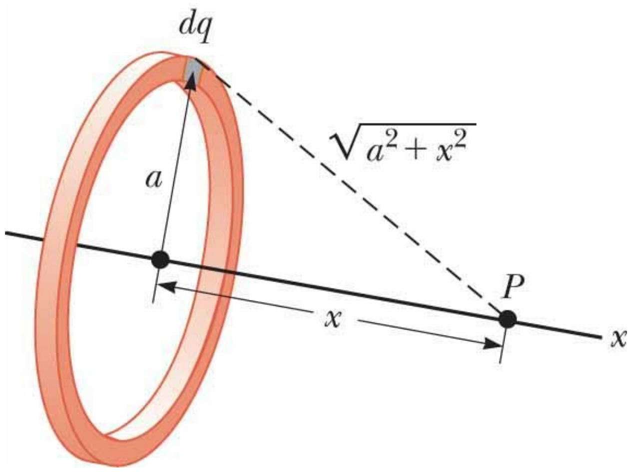

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Solution

From symmetry, ***E*** can only have an x component.

$$E_x = -\frac{dV}{dx}$$

The E Potential Due to a Ring

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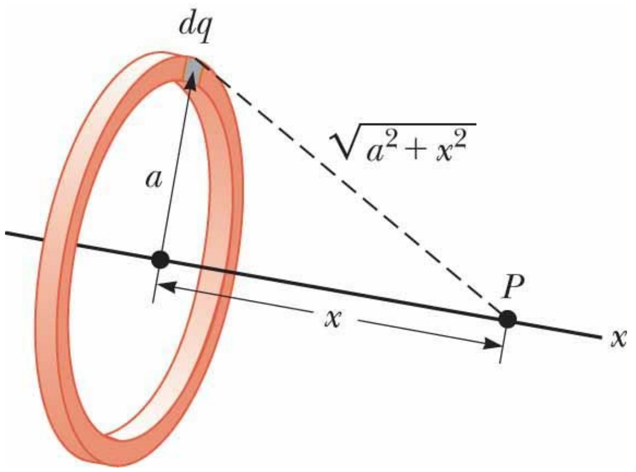
$$V = \boxed{\dfrac{k_e Q}{\sqrt{a^2 + x^2}}}$$



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Solution

From symmetry, ***E*** can only have an x component.

$$E_x = -\frac{dV}{dx}$$

$$E_x = -\frac{dV}{dx} = -k_e Q \frac{d}{dx}\left(a^2 + x^2\right)^{-1/2}$$

$$= -k_e Q\left(-\frac{1}{2}\right)\left(a^2 + x^2\right)^{-3/2}(2x)$$

$$E_x = \boxed{\frac{k_e x}{\left(a^2 + x^2\right)^{3/2}} Q}$$

Electric Potential (Ch. 24)

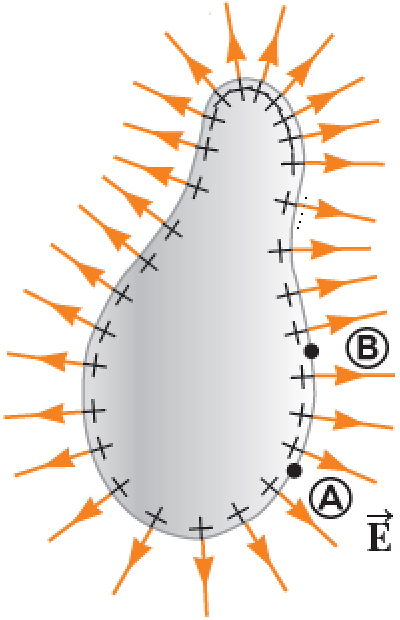
Conductors in Electrostatic Equilibrium

Conductors in Electrostatic Equilibrium

1. Good electrical conductors have charges (electrons) that are not bound to any atom and that can move freely within the material

2. If there is no net motion of charge within the conductor; the state is called ***electrostatic equilibrium.***

3. For conductors in electrostatic equilibrium:
   1. Electric field is zero everywhere inside the conductor, whether conductor is solid or hollow
   2. If conductor is isolated and carries charge → the charge resides on its surface
   3. Electric field at a point just outside the charged conductor is perpendicular to the surface of the conductor and has a magnitude σ/ε_0
      - σ = surface charge density at that point
   4. On irregularly shaped conductors: surface charge density is the greatest at locations where radius of the curvature of the surface is smallest
      - charge density is highest at sharp tips



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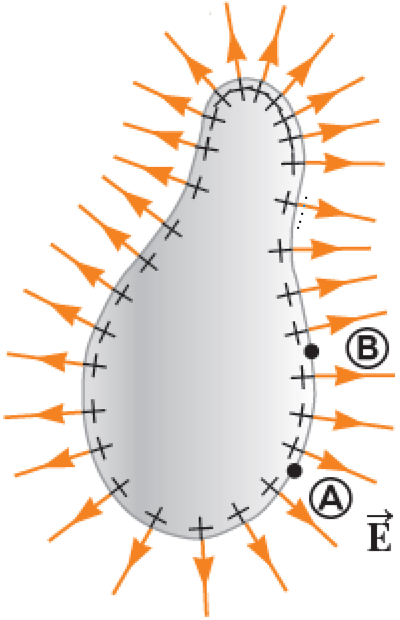
Conductors in Electrostatic Equilibrium

1. $\vec{E} = 0$ inside the conductor
2. If the conductor is charged, charges reside on the surface of the conductor.
3. $\vec{E}$ field at a point just outside the conductor is perpendicular to the surface, and has a magnitude of $\boldsymbol{\sigma/\varepsilon_0}$
4. For irregularly shaped conductors: **σ** is greatest where the radius of the curvature is smallest



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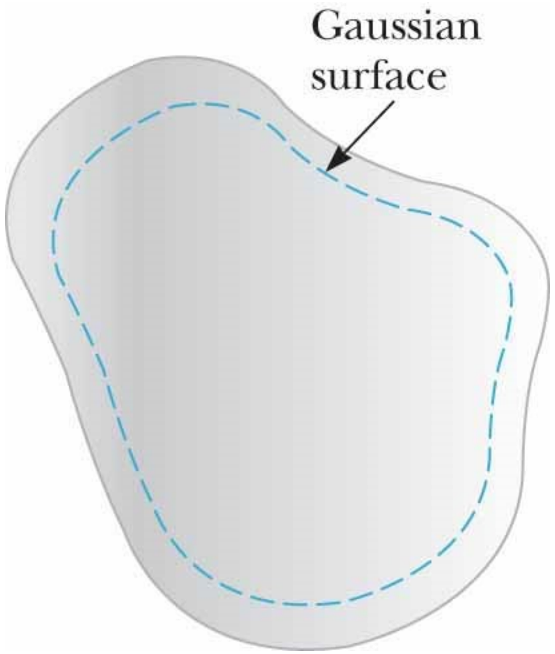
Conductors in Electrostatic Equilibrium

- Gaussian surface drawn inside arbitrarily-shaped conductor
- Electric field everywhere inside the conductor is 0 when in electrostatic equilibrium
- Electric field must be zero at every point on gaussian surface
  - Net flux through gaussian surface = 0
  - Conclusion: net charge inside gaussian surface = 0
- No net charge inside gaussian surface (which is arbitrarily close to conductor's surface) →
  - Any net charge on the conductor must reside on its surface
- Gauss's law does not indicate how excess charge is distributed on conductor's surface. Only that it resides exclusively on its surface



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Electric Fields and Charged Conductors

- $\vec{E}$ field at a point just outside the conductor, perpendicular to the surface, has a magnitude of **σ/ε_0**

If the field vector ***E*** had a component parallel to conductor's surface →

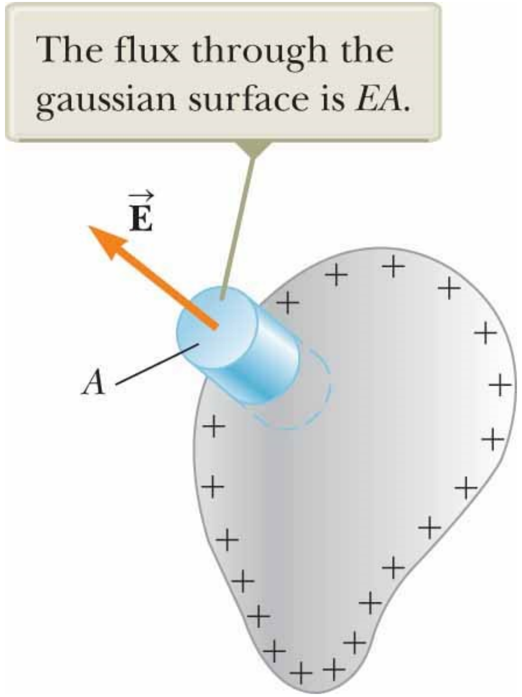
  - Free electrons would experience an electric force and move along the surface
  - Conductor would not be in equilibrium
  - Therefore, the field vector must be perpendicular to the surface

Flux = EA

  - E = electric field just outside conductor
  - A = area of cylinder's face
  - σ = surface charge density at that point

$$\Phi_E = \square \quad \frac{q_{\text{in}}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$



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E Potential and E Field of Charged Conductor

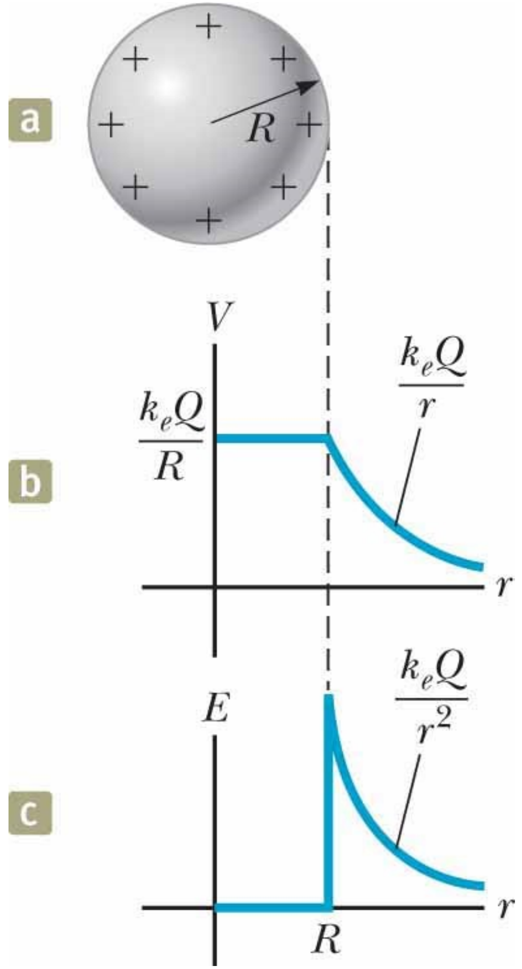
- Consider a solid metal conducting sphere of radius **R** and total positive charge **Q** (figure (a))

- Electric field outside sphere = k_eQ/r^2

- At the surface of the conducting sphere, the potential must be k_eQ/R

- Because the entire sphere must be at the same potential → potential at any point within the sphere must also be k_eQ/R

- Figure (b): plot of electric potential as function of r
- Figure (c): shows how electric field varies with r



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Summary of Week 2, Class 2

- Reminder of the previous week

- Continuous Charge Distributions and Gauss's Law (Ch. 23)
  - Electric field of cont. charge dist.
  - Electric flux
  - Gauss's Law
  - Application of Gauss's law to various charge distributions

- Electrical Potential (Ch. 24)
  - Electric potential and potential difference
  - Potential difference in a uniform electric field
  - Electric potential and potential energy due to point charges
  - Obtaining the Electric Field from the Electric Potential
  - Electric potential due to continuous charge distributions
  - Conductors in Electrostatic Equilibrium

- Examples
- Next week's topic

Reading / Preparation for Next Week

Topics for next week:

- Capacitance and Dielectrics (Ch. 25)
- Current and Resistance (Ch. 26)