Control Systems Design Project

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Student Name	Signature*
Joshua Mongal	
Mikaeel Khanzada	M. K
Michael McCorkell	

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Project Grading Sheet

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Mikaeel Khanzada</td><td>Third Student Name:
Micahel McCorkell</td></tr>
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Executive Summary:

When it comes to this project, an investigation was done regarding the design and control of a Magnetic Levitation train. The main objective of this project was focused on designing a feedback close loop system that stabilizes the airgap between the train and the rail track through the suspension system maintaining a response with an overshoot that does not deviate more than 10% and settles within 1 seconds to the reference air gap which is 0.08 m.

In the first portion of this project, we modelled the electromagnetic levitation system as a transfer function using the differential equation provided. The result of this calculation was a third order transfer function that related the input voltage to the air gap.

With the derivation of the 3rd order transfer function, an analysis was started for the open-loop system to identify how stable the system is and how well it performs. The initial analysis which found the poles shown in figure 1 had shown that the open-loop system was inherently unstable due to the presence of a pole on the right half plane. With this we evaluated the open loop system using the root locus method in figure 3 and the time-domain analysis in figure 2 which both supported the conclusion that the system was inherently unstable. With that being found, there was no point in evaluating the system with the Bode and Nyquist plot as they both rely on a stable open loop system.

The next step in the project was to now stabilize the system and meet the performance requirements, which can only be done with a properly designed controller. In this stage, several controller design methods were evaluated in which the second order approximation method was ultimately selected due to accuracy in PID tuning when compared to the other available methods. The PID controller was designed by comparing the second order with the desired characteristic equation that is based on performance requirements to the system response. The close loop system was designed in Simulink with proportional control as shown in figure 4. The response for the system with only the proportional control was shown in figure 5 where there was a large steady state error and no overshoot. Then the derivative and integral value were found and added to the system as seen in figure 7 which gave a more reasonable time-domain response as seen in figure 9.

With all these steps in building the close loop maglev system, it was found that it successfully met the main design criteria where the overshoot remained below 10%, and the train's air gap stabilized close to the desired 0.08 m. However, the settling time slightly exceeded 1 second, though this deviation from the desired performance requirement can be overlooked as it does not present a significant safety issue to the public if implemented. Overall, the PID controller designed using second-order methods was found to provide a very practical and robust solution for real-world implementation of the maglev control systems.

Introduction:

The objective of this project would be to design a feedback control system for a Magnetic Levitation train with the aim of achieving stable levitation of the train and proper control of the train's height above the tracks by continuous adjustment of the airgap. This type of control system would be very critical to manage as it is very non-linear and has an unstable nature when it comes to the electromagnetic suspension system used in the Maglev trains. The technical objectives of this project would be to maintain an air gap of 80 mm while undergoing different external forces that will allow the system to respond within a settling time of less than 1 second. The deviation in terms of overshoot should also be less than 10%.

Maglev trains are trains where conventional wheels that are used for its mobility are replaced with magnetic forces for the purpose of getting rid of friction and allowing for a smooth ride at high speeds in transportation of people especially [3]. When it comes to the systems that allow for this levitation, there are two main types being the electromagnetic suspension system and the electrodynamic suspension system. The electro dynamic system which is the one that is being analyzed in this project would suspend the train using magnetic attraction forces which would be generated by the electromagnets beneath the train that would interact with the ferro magnetic rails [3]. This method of suspending the train would allow for continuous and active control over the train's position by altering the current being supplied to the coils as it directly affects the force of levitation.

When it comes to the applications of the Maglev systems, they are widespread across the world with the most developments relating to them being done in Japan and Germany. In these countries, it demonstrates how effective these trains are as they move at high speed in cities, or any urban environment for that matter [4]. This technology would offer the advantages of having less maintenance to do, less noise than traditional trains and greater efficiency due to the ability to precisely control them. They have very competitive speeds when it comes to air travel. With less friction occurring, there is a significant reduction in mechanical wear and an extension in the system's longevity.

When it comes to the control of the trains stability and movement, there are many system components involved which include sensors and actuators. The sensors that would be included in this system would be the displacement sensor which detects the position of the train's suspension system's electromagnetic from the rail. There would also be a current sensor to detect how much current is being fed into the system [5]. Lastly there should be a weight sensor as the train would have the maximum capacity it is able to carry to ensure safety and stability of the train's movement. These sensors would serve as feedback information gathers to keep the train stable as it moves [5]. The actuators involved in this system would be electromagnets as they would serve to alter the magnetic field for the purpose of generating the lifting force to keep the train levitated above the tracks [4]. These would be amplified with current if more force is needed and decreased if less force is needed

System Description & Mathematical Modeling:

The system parameters are as follows where we have

m - This is the mass of the bogey which needs to be considered as it is its own force that is being applied unto the

$z(t)$- This would be the air gap that is measured between the rail and the electromagnet over time.

F - This would be the load that is caused by the train in newtons due to what is in the carriages and an external disturbance force like people or wind resistance.

f_e - This is the electromagnetic suspension force that is acting to keep the bogie system up.

$i(t)$- This is the current running through the magnetic coil over time which changes depending on the feedback from airgap distance information.

A - This is the area of the coil where the electromagnetic force is exerted.

N - This is the number of turns in the coil which can show how strong the magnetic force can be generated from the coil when energized.

μ_0 - This is the permeability of free space in the bogie system.

R - This is the resistance of the coil towards current which can affect the amount of force generated.

L_0- This would be the inductance of the coil at equilibrium.

$V(t)$- This is the voltage applied to the system which changes overtime based on the varying amount of force needed from the electromagnet.

The system input is as follows where we have

$V(t)$- Voltage would be applied to the system as the input because it adjusts the amount of force needed from the electromagnets to keep the train stable over time when there are varying forces acting on it. For example, it would decrease its input if there were less force disturbance or increase its input when there is more disturbance as there would be more force.

$z_0\,(t)$- This serves as the input for the reference air gap which the system must maintain for the train. The trains control system will constantly look at what the air gap distance needed to be maintained and adjust the voltage accordingly to meet that requirement.

$F(s)$- Force would serve as an input variable because the system would change the amount of voltage supply based on this input. As people get on or off the train, that is a force(weight) which is considered as the system would have to take that data and adjust the voltage supply to provide enough levitation force to handle that weight as it moves.

The system outputs are as follows where we have

$z(t)$- This would be the current air gap that the maglev train has for its levitation. This air gap value is updated over time as it takes different inputs to formulate the output. This system's role is to ensure that this output matches that of the desired input, which is 0.08 m.

The linearized differential equations for the levitation system are as follows:

- $m \cdot \frac{d^2 z(t)}{dt^2} = \left(\frac{\mu_0 \cdot A \cdot N^2 \cdot i_0^2}{2 \cdot z_0^3}\right) \cdot z(t) - \left(\frac{\mu_0 \cdot A \cdot N^2 \cdot i_0}{2 \cdot z_0^2}\right) i(t) + F$
- $L_0 \cdot \frac{di(t)}{dt} = V(t) - R \cdot i(t) + \left(\frac{\mu_0 \cdot A \cdot N^2 \cdot i_0}{2 \cdot z_0^2}\right) \cdot \frac{dz(t)}{dt}$

With equilibrium points:

$\mu_0 = 4 \cdot \pi x 10^{-7}\, N.A^{-2}$ $m = 700kg$ $N = 450$ $A = 0.024 \pm 0.0024\, m^2$ $V(t) = 400V$$R = 1.2 \pm 0.12\Omega$ $F = 27440 \pm 5488\, N$ $z_0 = 0.08m$ $i_0 = 34A$ $L_0 = \frac{\mu_0 \cdot A \cdot N^2}{2 \cdot z_0}$

To simplify the equation:

$$k2 = \left(\frac{\mu_0 \cdot A \cdot N^2 \cdot i_0^2}{2 \cdot z_0^3}\right) and\, k1 = \left(\frac{\mu_0 \cdot A \cdot N^2 \cdot i_0}{2 \cdot z_0^2}\right)$$

Since this is a 2 input one output system, first we found the overall transfer function using the linearized differential equations. To do that first we find the Laplace transform of both equations:

$$m \cdot s^2 \cdot Z(s) = k2 \cdot Z(s) - k1 \cdot I(s) + F(s)$$

$$L_0 \cdot s \cdot I(t) = V(s) - R \cdot I(s) + k1 \cdot s \cdot Z(s)$$

We then rearrange the equation to isolate Z(s) and I(s):

$$(m \cdot s^2 - k2) \cdot Z(s) = -k1 \cdot I(s) + F(s)$$

$$I(s) = \frac{\big(V(s) + k1 \cdot s \cdot Z(s)\big)}{(L_0 \cdot s - R)}$$

Sub equation 2 into 1:

$$(m \cdot s^2 - k2) \cdot Z(s) = -k1 \cdot \frac{\big(V(s) + k1 \cdot s \cdot Z(s)\big)}{(L_0 \cdot s - R)} + F(s)$$

Multiply the equation by $(L_0 \cdot s - R)$:

$$(L_0 \cdot s - R) \cdot (m \cdot s^2 - k2) \cdot Z(s) = -k1 \cdot \big(V(s) + k1 \cdot s \cdot Z(s)\big) + (L_0 \cdot s - R) \cdot F(s)$$

Isolate for Z(s):

$$\big((L_0 \cdot s - R) \cdot (m \cdot s^2 - k2) + k1^2 \cdot s\big) \cdot Z(s) = -k1 \cdot V(s) + (L_0 \cdot s - R) \cdot F(s)$$

Create 2 transfer function in terms of $\frac{Z(s)}{V(s)}$ and $\frac{Z(s)}{F(s)}$:

$$\frac{Z(s)}{V(s)} = \frac{-k1}{\left((L_0 \cdot s - R) \cdot (m \cdot s^2 - k2) + k1^2 \cdot s\right)}$$

$$\frac{Z(s)}{F(s)} = \frac{(L_0 \cdot s - R)}{\left((L_0 \cdot s - R) \cdot (m \cdot s^2 - k2) + k1^2 \cdot s\right)}$$

Then to find the transfer function take equation 1 and expand and collect like terms:

$$\frac{Z(s)}{V(s)} = \frac{-\frac{k1}{m \cdot L_0}}{s^3 + \frac{R}{L_0}s^2 + \frac{k1^2 - k2 \cdot L_0}{m \cdot L_0}s - \frac{R \cdot k2}{m \cdot L_0}}$$

Then we substitute the values:

$$\frac{Z(s)}{V(s)} = \frac{-0.6071}{s^3 + 31.44s^2 + 0s - 309.6}$$

<u>System's Stability and Performance Analysis:</u>

To assess the stability and performance of the system, we looked at the characteristics of the levitation system in the open loop system where analysis in the frequency domain and the time domain was found. With this information, the inherent stability and responsiveness of the system can be captured.

When evaluating the systems stability, the characteristic equation was derived from the differential equations open loop transfer function was taken, and the poles were found. This is as shown below.

```
G =

          -0.6071
  -----------------------
  s^3 + 31.44 s^2 - 309.6

Continuous-time transfer function.
Model Properties

ans = 3×1
    -31.1182
     -3.3184
      2.9986
```

Figure 1: Image showing poles evaluated from the characteristic function using MATLAB

From these pole locations on these figures, it can be determined that the system is inherently unstable as the result presents with a positive pole being 2.9986.

When looking at the Time-Domain analysis, we can examine this by simulating the step response of the transfer function Z(s)/V(s). When observed in the scope, it presents the result shown below.

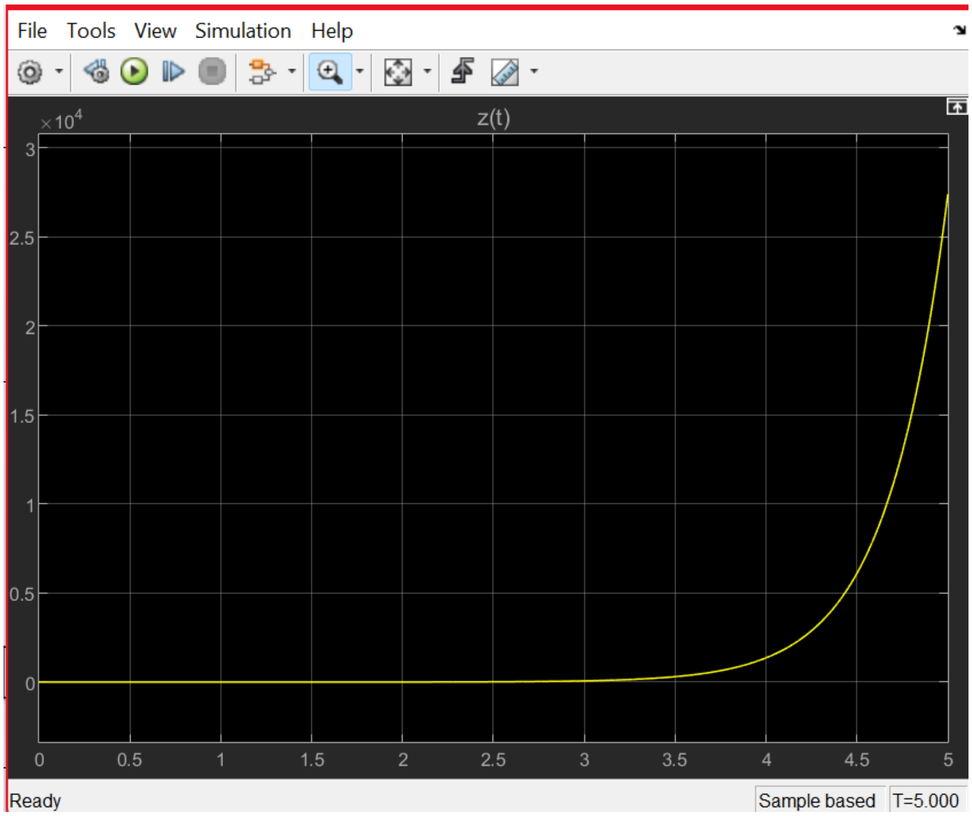

Figure 2: Graph showing time-domain response of Maglev open loop system

From this response in the time domain, we observe that the system approaches positive infinity. This shows that the open-loop maglev system is unstable. This instability can be said to be so due to the non-linear dependence of the electromagnetic force. There is a lack of natural damping in the system and no control to keep it in check so the magnetic force will just keep increasing as shown resulting in a failure for the train system.

When it comes to the Frequency domain, we can observe the system response using the Bode and Nyquist plots. Since we have previously established with the other two ways that the system is unstable, the Bode and Nyquist plots would not be applicable as they a primarily used for stable systems.

When it comes to the root locust analysis, we are also able to analyze the stability characteristic of the open-loop electromagnetic suspension system and check whether stabilization is possible through feedback control. The root locus plot as shown below

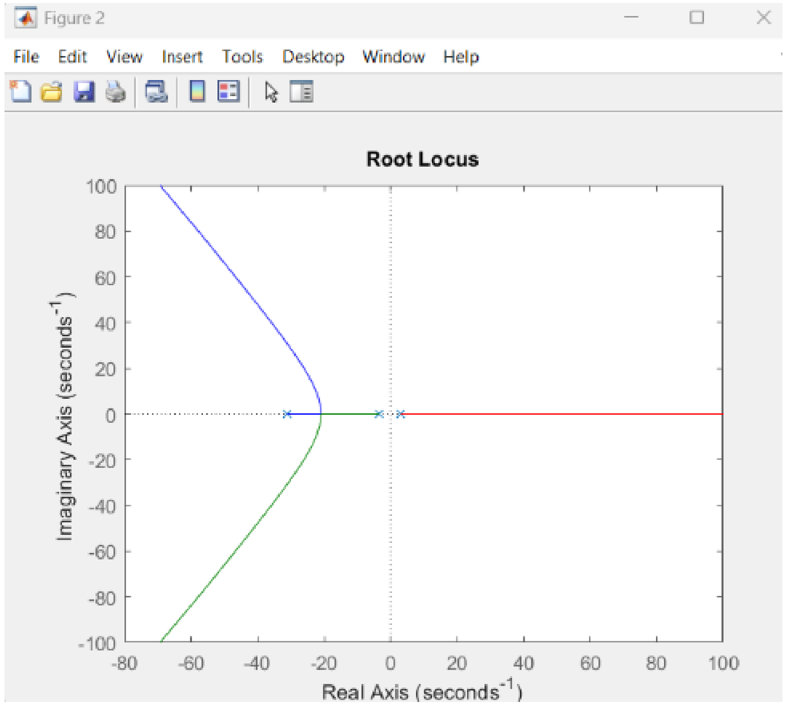

Figure 3: Graph showing root locus graph of maglev open loop system

Here we have the poles as established before where two of them are on the left half plane while one pole is on the right half plane. This establishes the idea that stabilization is achievable though it is unstable now. The root locus plot shows that with the appropriate gain adjustment; it is possible to reposition the poles so that all of them are in the stable region. This also indicates the need for a proportional gain.

Control System Design:

When it comes to the selection of an appropriate controller, there were four options that were being looked at. These options included basing the controller design off the Bode plot, Nyquist plot, Root locust or using the second order approximation method.

The Bode plot is useful when it comes to frequency domain design. However, as mentioned before, the open loop maglev system was found to be inherently unstable. This makes the analysis in the frequency domain to design a PID controller very complicated. Bode plots would rely on an open loop being stable to accurately calculate margins. The open loop being unstable also serves as a reason or not using the Nyquist plots as it also relies on the open loop system being stable to be more effective in the design of PID controllers.

When it comes to the root locus, it does provide information that would help in designing a PID controller for the system. However, this did not serve as the base for the design of the controller as fine tuning the PID based on root locus inspection is more trial and error which is much less efficient than using parameter-based tuning which is used in the second order approach. Hence the second order method was chosen to be the most appropriate for this application.

From transfer function

$$G(s) = \frac{Z(s)}{V(s)} = \frac{-0.6071}{s^3 + 31.44s^2 + 0s - 309.6}$$

Since the poles found were

- -31.1182
- -3.3184
- 2.9986

The dominant poles are -3.3184 and 2.9986, and we use them to create a 2^{nd} order transfer function:

$$G(s) = \frac{numerator}{(s + 3.3184)(s - 2.9986)} = \frac{numerator}{s^2 + 0.319s - 9.95}$$

But to be able to use this transfer function we need to make sure the DC gain is the same between the 3^{rd} and 2^{nd} order function by first making s=0 and solving the 3^{rd} order function:

$$DC\ gain = Lim_{s \to 0} \frac{-0.6071}{s^3 + 31.44s^2 + 0s - 309.6}$$

$$DC\ gain = \frac{-0.6071}{0^3 + 31.44 \cdot 0^2 + 0s - 309.6}$$

$$DC\ gain = 0.00196$$

Therefore, the DC gain = 0.0019. now to make sure that the 2^{nd} order transfer function has the same DC gain we also set s=0:

$$DC\ gain = Lim_{s \to 0} \frac{numerator}{s^2 + 0.319s - 9.95}$$

$$DC\ gain = \frac{numerator}{0^2 + 0.319 \cdot 0 - 9.95}$$

$$numerator\ = DC\ gain\ \cdot -9.95$$

$$numerator\ = 0.0019\ \cdot -9.95$$

$$numerator\ = -0.0195$$

Therefore the 2nd order transfer function is as follows:

$$G(s) = \frac{-0.0195}{s^2 + 0.319s - 9.95}$$

Now that we have the 2nd order transfer function, we can find the desired 2nd order characteristic equation using our conditions:

For this we used:

Overshoot: 0.3%

Settling time 0.5s

We did this to compensate for the variation in our initial parameters.

First, we find the dampening ratio using overshoot:

$$\xi = \frac{-\ln(O.s)}{\sqrt{\pi^2 + \ln^2(O.s)}}$$

$$\xi = \frac{-\ln(0.003)}{\sqrt{\pi^2 + \ln^2(0.003)}}$$

$$\xi = 0.8796$$

Now using the dampening ratio and desired settling time, we can find natural frequency:

$$w_n = \frac{4}{\xi \cdot ts}$$

$$w_n = \frac{4}{0.8796 \cdot 0.5}$$

$$w_n = 9.0949$$

Now with both settling time and dampening ratio we can find the desired characteristic equation:

$$Gd(s) = s^2 + 2 * w_n * \zeta \cdot s + w_n^2$$

$$Gd(s) = s^2 + 16s + 82.7178$$

How that we have the desired characteristic equation we can now find PD controller gain value Kp and Td:

$$Kp = \frac{w_n^2 - (-9.95)}{-0.0195} = -4753.2$$

$$Td = \frac{2 \cdot w_n \cdot \xi - 0.319}{Kp \cdot -0.0195} = 0.1692$$

Since we are using a rate feedback system:

$$Gc(s) = Kp$$
$$H(s) \ = \ Tds + 1$$

Now we find the overall transfer function of the system including the Gc(s), G(s) and H(s)

$$G(s) = \frac{Gc(s) * G(s)}{1 + Gc(s) * G(s) * H(s)}$$

After substituting the transfer function in and expanding and collecting like terms:

$$G(s) \ = \ \frac{Kp \cdot -0.0195}{s^2 + (0.319 + Td \cdot Kp \cdot -0.0195)s + (Kp \cdot -0.0195 - 9.95)}$$

$$G(s) \ = \ \frac{92.67}{s^2 + 16s + 82.72}$$

Now with these values we can put them into our PD controller on the Simulink model that uses the original 3rd order transfer function.

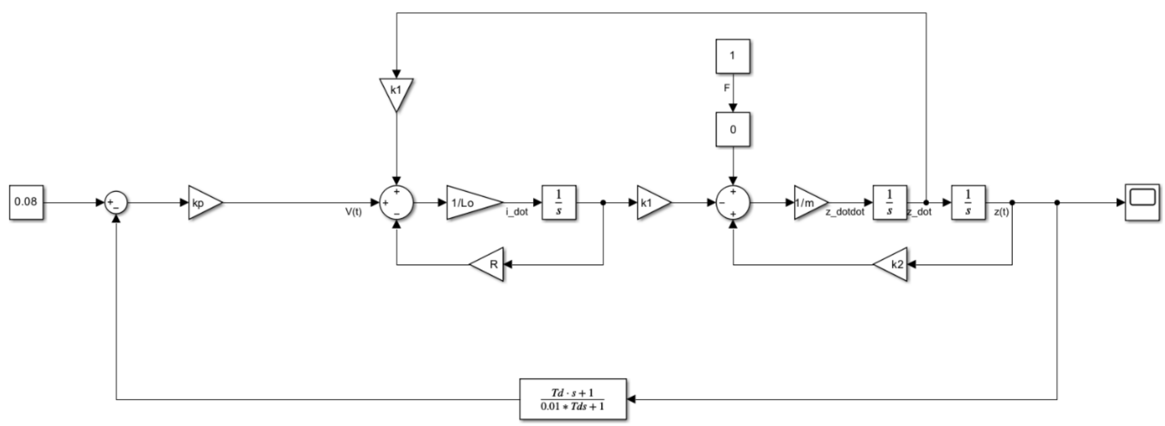

Figure 4: Image showing Block diagram of Maglev close loop system with proportional control

From the transfer function we get the following step response:

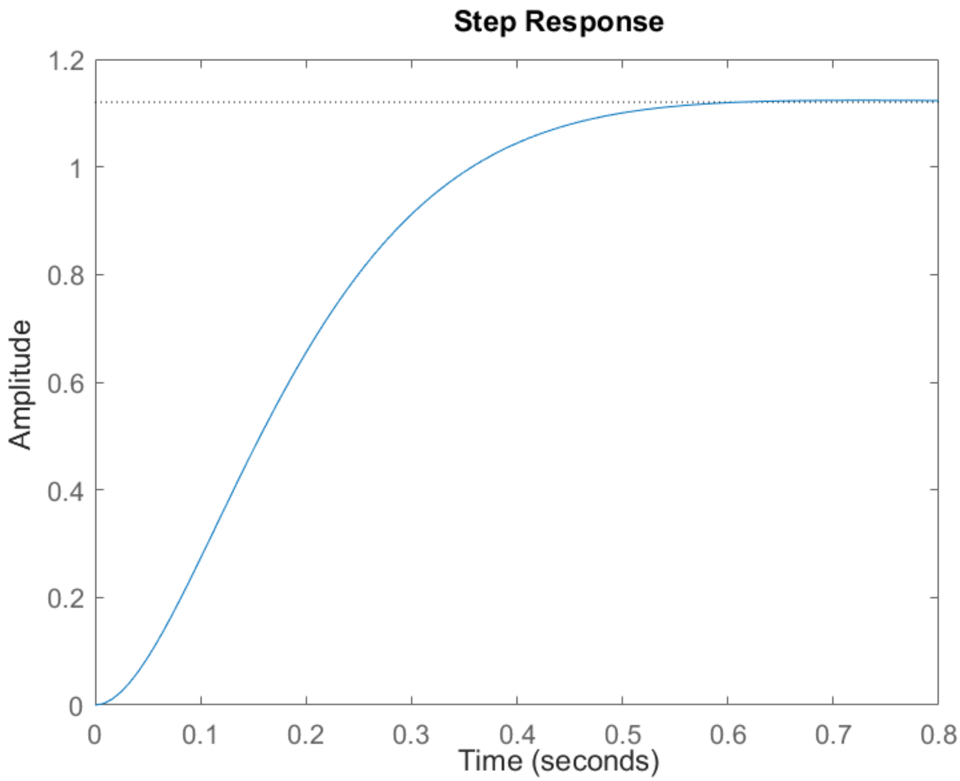

Figure 5: Graph showing step time-domain response of Maglev close loop system with proportional control

Once we confirm that the PD controller is working, we can see there is some steady state error for that we need to find the integral, to calculate for that we take the poles of the G(s):

```
p = 2×1 complex
    -8.0000 + 4.3264i
    -8.0000 - 4.3264i
```

Figure 6: Image showing poles of G(s) evaluated through MATLAB

Using the real part of the pole we find the value for Ti:

$$Ti = \frac{6}{|Re(p)|}$$

$$Ti = \frac{6}{|Re(-8)|}$$

$$Ti = 0.75$$

We selected 6 from trial and error as it gave the best resulting graph:

Using the values of Kp, Ti and Td we can create the final overall transfer function using the following:

$$PI = \frac{Kp \cdot Ti \cdot s + Kp}{Ti \cdot s}$$

$$PI = \frac{-3564s - 4752}{0.75s}$$

$$FD = \frac{Tds + 1}{1}$$

$$FD = 0.1692s + 1$$

Using these transfer functions we can find the final G(s) of this system:

$$G_f(s) = \frac{2164s + 2885}{0.75s^4 + 2358s^3 + 366.2s^2 + 2420s + 2885}$$

Performance and Stability Analysis of Feedback Control System:

When it came to evaluating the effectiveness of the feedback controller in the system, the close loop system was analyzed both in the time domain and the frequency domain, This was to ensure that the control system was able to reach the desired performance criteria of a settling time less than 1 second and a deviation that is less than 10% of the reference air gap which is 0.08 m in this case.

The close loop control block diagram with a PID architecture is as shown below.

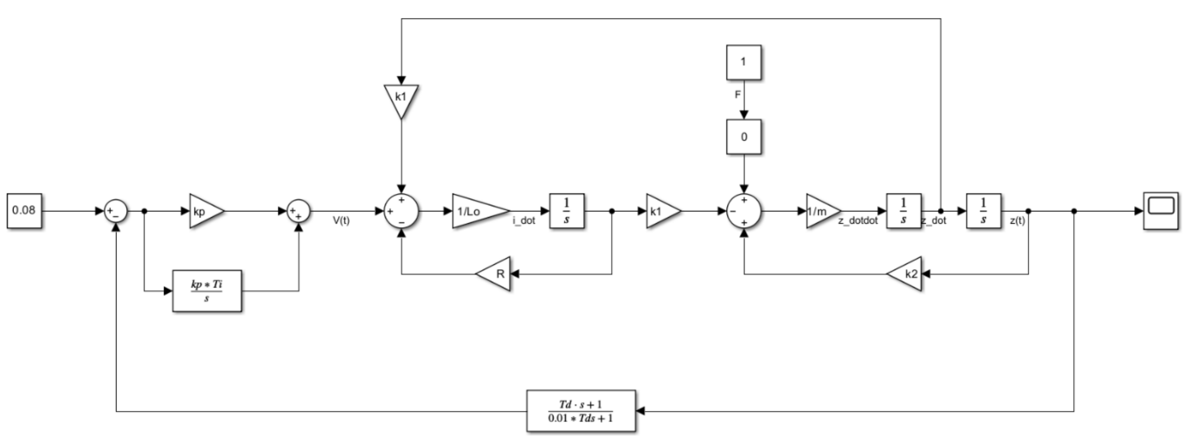

Figure 7: Image showing Block diagram of Maglev close loop system with PID controller

This diagram first consists of the input where the reference airgap information is taken, this is then passed through the controller where it is designed to check what comes from the feedback to make the correct adjustments for the appropriate output. There is then a component where the levitation system is modeled using the differential linear equations. This would serve as the constant internal force the system is familiar with which is then summed up with the force disturbance component and output into the scope on the right to show the response of the system. The current air gap value is taken and fed back into the controller so it can make dynamic adjustments for the trains to stay stable.

The confirm the stability of the system, the poles of final transfer function are as shown below:

```
gfinal =

                   2164 s + 2885
  ----------------------------------------------
  0.75 s^4 + 23.58 s^3 + 366.2 s^2 + 2420 s + 2885

Continuous-time transfer function.
Model Properties

finalpoles = 4×1 complex
    -9.5487 +12.0427i
    -9.5487 -12.0427i
   -10.8403 + 0.0000i
    -1.5023 + 0.0000i
```

Figure 8: Image showing poles of final transfer function evaluated through MATLAB

As you can see the resulting poles are all in the real plane and as such, we can confirm that the final closed loop transfer function is stable.

With the system being set, a step response of the close loop system was simulated to see how well it reacts to the change in the desired gap from 0.016 m to the value of 0.08 m. This can be seen in Figure 7 as the system reached the setpoint.

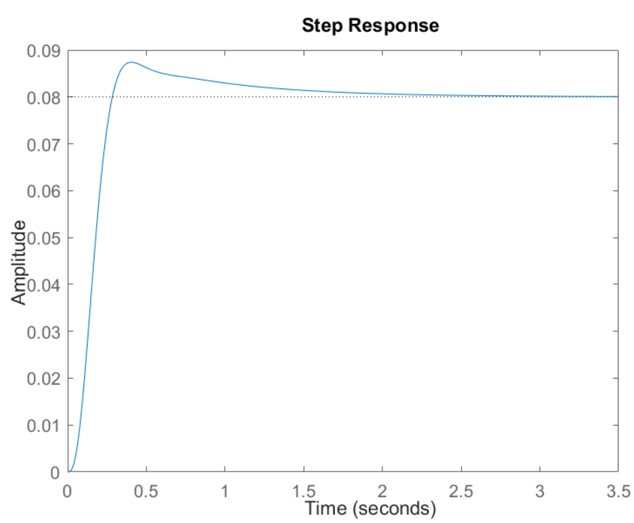

Figure 9: Graph showing time-domain response of Magnetic levitation close loop PID controller system

As you can see in the above step response of the system, the overshoot present in the system is less than 10% but the settling time is over 1 second. The overshoot, which is the essential criteria as the design of the bogie leaves no room for error greater than 10% of the setpoint which i s achieved. The settling time being greater than 1 second is acceptable as this criterion has more tolerance allotted to it as there are no possible damages present if the criteria is not met therefore the parameters that produced the above result are the final parameters of this system.

Conclusion and Discussion:

The Maglev train levitation control system project was successful in demonstrating how theoretical control systems can be applied to real world applications and in our case a Maglev train. Modelling the electromagnetic systems, analyzing system stability had helped us to design a PID controller capable of stabilizing the unstable open loop system using a rate Feeback controller which was designed by converting the 3rd order system to second order and using a 2nd order desired characteristic equation to find the proportional, integral and derivative gains for the systems. This system managed to achieve the desired overshoot but was slightly out of spec for the settling time, which was deemed acceptable. As such, the final PID controller step response in figure 7 confirmed that the approach used was acceptable. There were challenges regarding the fine tuning of the system in trying to balance the overshoot with the settling time as adjusting the Integral gain would affect both criteria which led to the result as seen in figure 7. The final result illustrates practical control of a maglev train which could be applied to a real-world environment.

Considering that this system is modeled after real world characteristics, the system inevitably exhibited nonlinear characteristics. Electromagnetic force generated by the coil is nonlinear since the effective area of the bogie is relatively small, the slight variations in current, resistance or voltage can have a big impact in the overall system as well as inductance which is a possible source of error as it is not considered in this system. Depending on the sensors used in the system to measure the air gap, some nonlinearity is introduced due to the sensor having limited resolution or delays in transmission of data due to noisy conditions. In this system some observations were made about the physical limitations present in this project. One being the system can only operate within a range for the air gap around 0.08m as it being too high leading to loss of levitation and too low leading to collision with the tracks. To operate the system, continuous current is required to go through the coil which can cause heat and without a proper cooling system the heat can damage components or increase characteristics like resistance which would cause the system to fail. Throughout this project many assumptions were made which led to sources of error such as the system being linearized about the setpoint of 0.08m air gap which could lead to the system to not performing properly if the system deviates too much from the setpoint due to large force disturbances. During this project, values such as mass, voltage, current or resistance were assumed and as such may vary in a real-world environment, where a controller built on these assumed values may fail. Finally, the open loop model for which we created a PID controller for is missing many real-world variables which could drastically affect the system performance by adding unexpected poles and zeros to the transfer function. Some of these variables could be rail flexibility, electrical delay or chassis resonance.

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[2] Dorf, R. C., & Bishop, R. H. (2016). *Modern Control Systems*. Pearson.

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[5] Author links open overlay panel Wenzhong Huang, Highlights This study provides new ideas for maglev train positioning. The positioning technology comprises both absolute positioning and relative positioning. Designed a noncontact sensing method and avoids the discreteness of FBG measurement., Abstract The positioning system is crucial in operating, Li, F., Lee, H.-W., Peng, Z., He, Y., Meng, C., Sun, Y., Zhang, D., Jiang, X., Nai, W., Dou, F., Yuan, Y., Dai, C., Hong, X., You, Ch., Li, L., Xu, S., … Gill, E. (2024, June 22). *Maglev train high-precision positioning technology based on FBG Array Time Division Multiplexing*. Measurement. https://www.sciencedirect.com/science/article/abs/pii/S0263224124010455#:~:text=The%20onboard%20Halbach%20permanent%20magnet,ideas%20about%20maglev%20train%20positioning.