Assignment 1

Exercise 1. Consider the following:

$$R_1 = \begin{bmatrix} 0.5 & -0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \tag{1}$$

Determine $w = R_1 v$, $w.v$ (dot product), and $w \times v$ (cross product) both by hand calculation and using MATLAB. (Show your calculation and attach screenshots of MATLAB). Calculate R_1^{-1} (inverse of R_1) and $\det(R_1)$ using only MATLAB and attach the screenshots.

Exercise 2. If v_1 and v_2 are free vectors, the expression

$$v_1 \ + v_2$$

makes sense under what conditions?

Exercise 3. If basic rotation matrix around z by θ is $R_{z,\theta}$ and basic rotation matrix around z by ϕ is $R_{z,\varnothing}$ prove the following:

$$R_{z,\theta} R_{z,\phi} = R_{z,\theta+\phi} \tag{2}$$

Exercise 4. Find $(R_{45^\circ})^2$ without multiplying it by itself.

Exercise 5. Prove the following matrix belongs to the rotation group of 2:

$$R = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \tag{3}$$

Exercise 6. If $R \in SO(3)$ determine a, b, and c:

$$R = \begin{bmatrix} 1 & a & 0 \\ 0 & b & -0.5 \\ 0 & c & 0.866 \end{bmatrix} \tag{4}$$

Exercise 1:

$$R1 = \begin{bmatrix} 0.5 & -0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$w = R1.v \rightarrow \begin{bmatrix} 0.5*1 & + & -0.866*2 & + & 0*3 \\ 0.866*1 & + & 0.5*2 & + & 0*3 \\ 0*1 & + & 0*2 & + & 1*3 \end{bmatrix}$$

$$w = \begin{bmatrix} 0.5 & + & -1.732 & + & 0 \\ 0.866 & + & 1 & + & 0 \\ 0 & + & 0 & + & 3 \end{bmatrix}$$

$$w = \begin{bmatrix} -1.232 \\ 1.866 \\ 3 \end{bmatrix}$$

Dot product = $[1(-1.232) + 2*1.866 + 3*3] \rightarrow 11.5$

$$\begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & 2 & 3 \\ -1.232 & 1.866 & 3 \end{vmatrix}$$

$$-\begin{vmatrix} 2 & 3 \\ 1.866 & 3 \end{vmatrix} \vec{\imath} + \begin{vmatrix} 1 & 3 \\ -1.232 & 3 \end{vmatrix} \vec{\jmath} - \begin{vmatrix} 1 & 2 \\ -1.232 & 1.866 \end{vmatrix} \vec{k}$$

$$[(2*3) - (3*1.866)]\vec{\imath} - [(1*3) - (3*-1.232)]\vec{\jmath} + [(1*1.866) - (2*-1.232)]\vec{k}$$

$$-0.402\vec{\imath} + 6.696\vec{\jmath} - 4.33\vec{k}$$

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R1 = [0.5 -0.866 0; 0.866 0.5 0; 0 0 1];
v = [1 2 3];
w = R1*v'

  w = 3×1
      -1.2320
       1.8660
       3.0000

dot(w,v)

 ans = 11.5000

cross(w,v)

  ans = 1×3
      -0.4020    6.6960   -4.3300

inv(R1)

  ans = 3×3
       0.5000    0.8660         0
      -0.8660    0.5000         0
            0         0    1.0000

det(R1)

 ans = 1.0000
```

Exercise 2:

Same Dimension, Field, Basis (if applicable)

Exercise 3

We need to prove: Rzθ, Rzϕ =Rz,θ+ϕ

This is a fundamental property of rotation matrices. The standard rotation matrix around the **z-axis** by an angle θ is:

$$R_{z,\theta} = \begin{bmatrix} cos\ \theta & -sin\ \theta & 0 \\ sin\ \theta & cos\ \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} R_{z,\phi} = \begin{bmatrix} cos\ \phi & -sin\ \phi & 0 \\ sin\ \phi & cos\ \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow R_{z,\theta}R_{z,\phi} \begin{bmatrix} cos\ \theta & -sin\ \theta & 0 \\ sin\ \theta & cos\ \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\ \phi & -sin\ \phi & 0 \\ sin\ \phi & cos\ \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} cos\ \theta\ cos\ \phi - sin\ \theta\ sin\ \phi & -(cos\ \theta\ sin\ \phi + sin\ \theta\ cos\ \phi) & 0 \\ sin\ \theta\ cos\ \phi + cos\ \theta\ sin\ \phi & cos\ \phi\ cos\ \phi - sin\ \theta\ sin\ \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow R_{z,\theta}R_{z,\phi} = \begin{bmatrix} \cos\ (\theta + \phi) & -sin(\theta + \phi) & 0 \\ sin\ (\theta + \phi) & \cos\ (\theta + \phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Exercise 4

rotation by 45° applied twice is the same as a **rotation by 90°**.

$$R_{90°} = \begin{bmatrix} cos\,90° & -sin\,90° \\ sin\,90° & cos\,90° \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \rightarrow (R_{45°})^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Exercise 5

$$R = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \rightarrow \det(R) = \left(\frac{\sqrt{3}}{2} * \frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2} * \frac{1}{2}\right) = \frac{3}{4} + \frac{1}{4} = 1$$

Det(R)=1

$$R^T R = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Exercise 6

$$R = \begin{bmatrix} 1 & a & 0 \\ 0 & b & -0.5 \\ 0 & c & 0.866 \end{bmatrix}$$

Column Dot Products

1*0+a*b+0*c=0 → ab =0

1*0+a*(-0.5) + 0*0.866 = 0 → -0.5a = 0 → a = 0

0*0+b*(-0.5)+c*0.866 =0 → -0.5b+0.866c = 0 → 0.5b = 0.866c

Column Norms

$0^2 + b^2 + c^2 = 1 \;\rightarrow\; 0^2 + (-0.5)^2 + (0.866)^2 = 0.25 + 0.75 = 1$

0.5b = 0.866c → b=$\frac{0.866}{0.5}$c=1.732c $\qquad b^2 + c^2 = 1$

$(1.732c)^2 + c^2 = 1 \rightarrow 3.9999c^2 = 1 \rightarrow \; c^2 \approx \frac{1}{4} = 0.25 \rightarrow c \pm 0.5$

$b = 1.732 * 0.5 \rightarrow b = 0.866, c = 0.5 \rightarrow \; b = -0.866, c = -0.5$