TOPIC 2 – Forces as Vectors

ENGI 1510 - ENGINEERING DESIGN

WINTER 2023

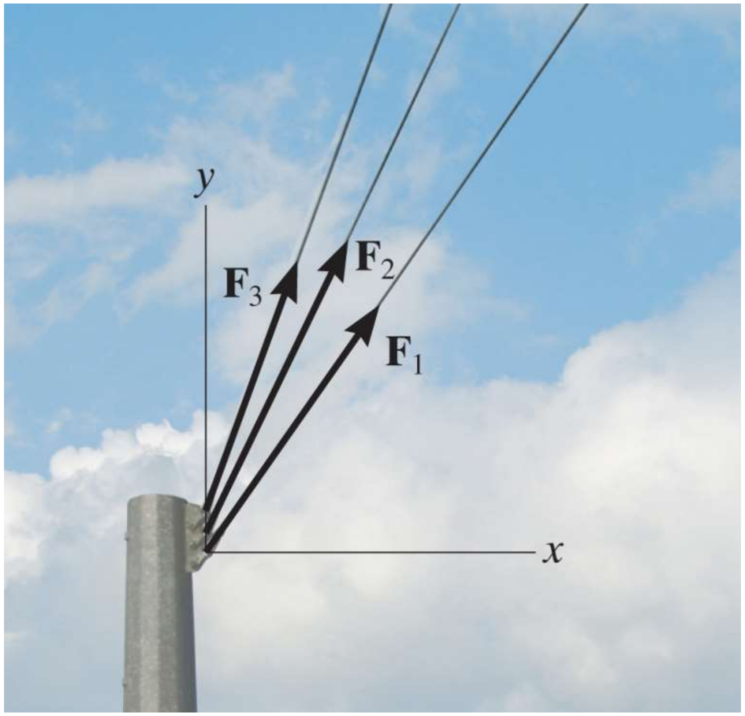
FORCE VECTORS, VECTOR OPERATIONS & ADDITION COPLANAR FORCES

Today's Objective:

Students will be able to :

a) Resolve a 2-D vector into components.

b) Add 2-D vectors using Cartesian vector notations.



In-Class activities:

- Check Homework
- Reading Quiz
- Application of Adding Forces
- Parallelogram Law
- Resolution of a Vector Using Cartesian Vector Notation (CVN)
- Addition Using CVN
- Example Problem
- Concept Quiz
- Group Problem
- Attention Quiz

READING QUIZ

1. Which one of the following is a scalar quantity?

A) Force   B) Position

C) Mass   D) Velocity

2. For vector addition, you have to use ______ law.

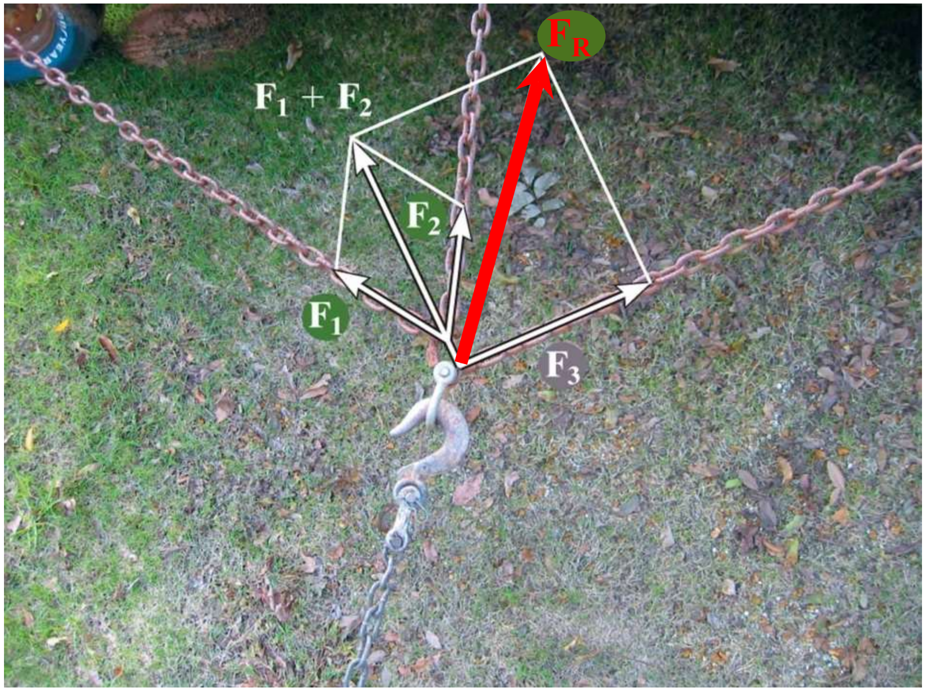
A) Newton's Second

B) the arithmetic

C) Pascal's

D) the parallelogram

APPLICATION OF VECTOR ADDITION



There are three concurrent forces acting on the hook due to the chains.

We need to decide if the hook will fail (bend or break).

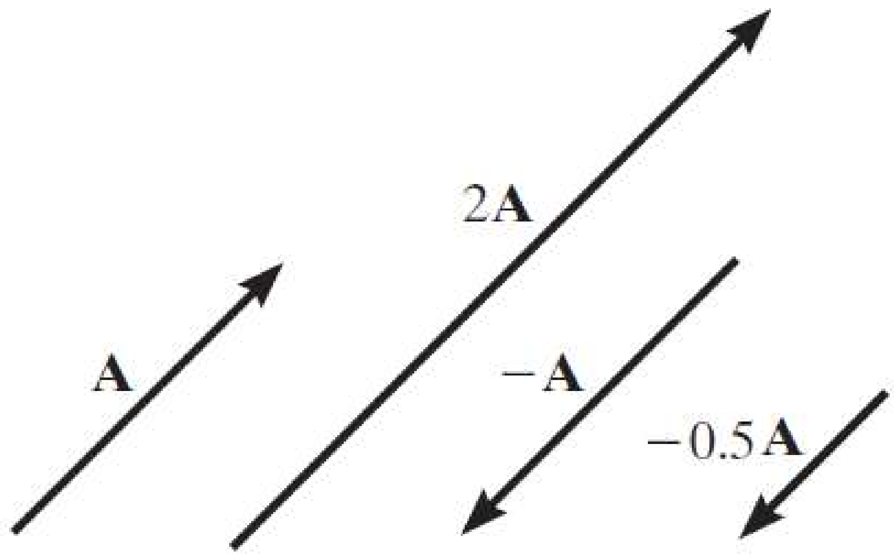
To do this, we need to know the resultant or total force acting on the hook as a result of the three chains.

SCALARS AND VECTORS
(Section 2.1)

	Scalars	***Vectors***
Examples:	Mass, Volume	Force, Velocity
Characteristics:	It has a magnitude (positive or negative)	It has a magnitude and direction
Addition rule:	Simple arithmetic	Parallelogram law
Special Notation:	None	Bold font, a line, an arrow or a "carrot"

In these PowerPoint presentations, a vector quantity is represented ***like this*** (in **bold**, *italics*, and red).

VECTOR OPERATIONS (Section 2.2)

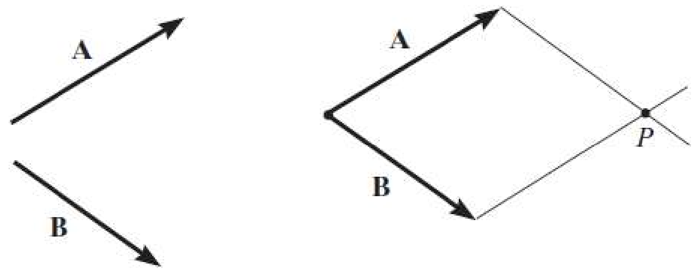


Scalar Multiplication

and Division

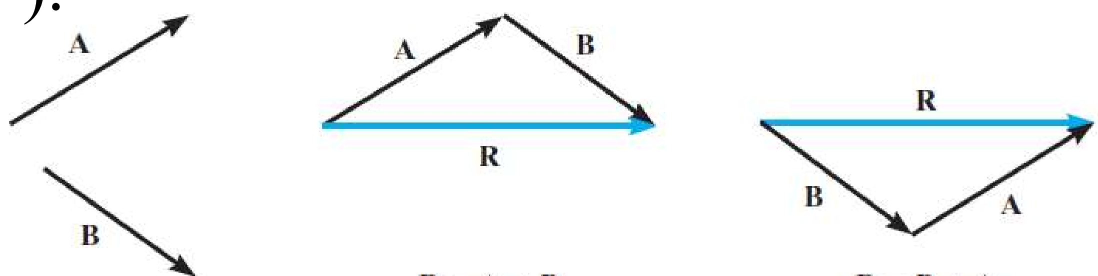
VECTOR ADDITION USING EITHER THE PARALLELOGRAM LAW OR TRIANGLE

Parallelogram Law:



R = A + B
Parallelogram law

Triangle method
(always 'tip to tail'):



R = A + B
Triangle rule

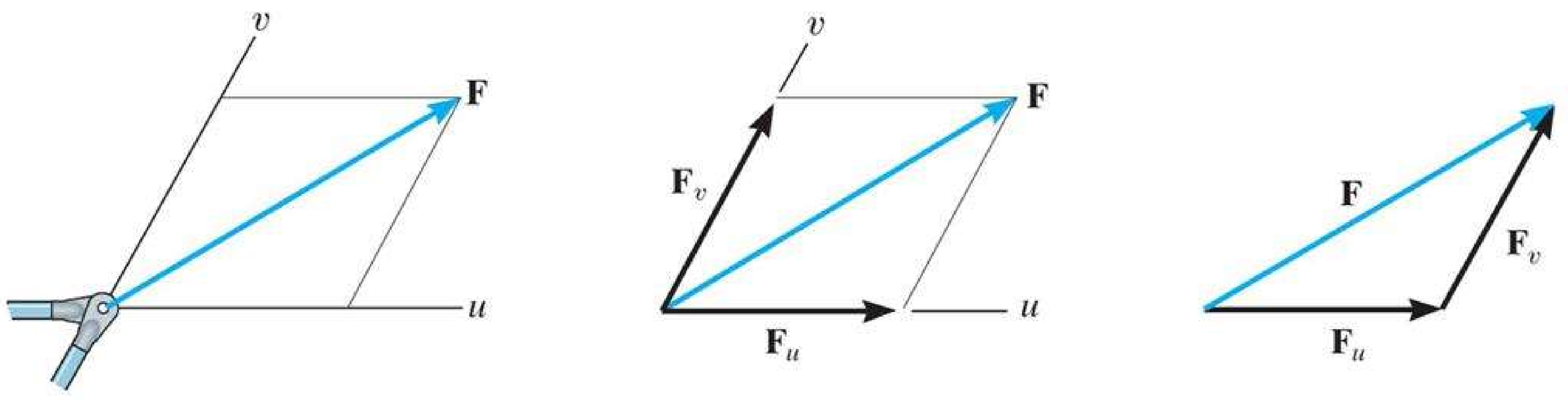
R = B + A
Triangle rule

How do you subtract a vector?

How can you add more than two concurrent vectors graphically?

RESOLUTION OF A VECTOR

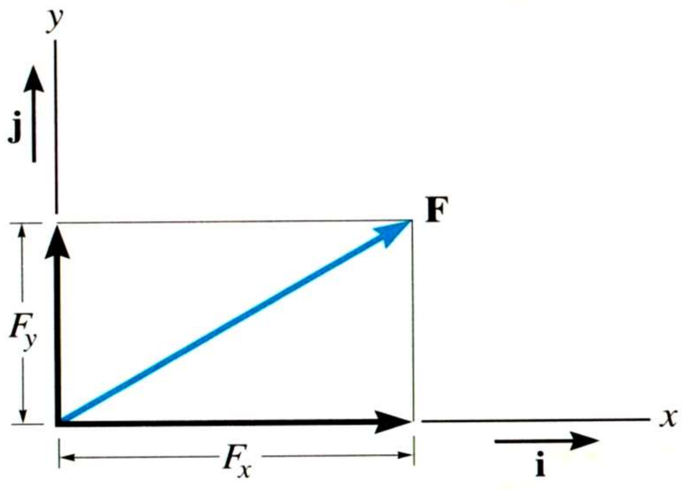
"Resolution" of a vector is breaking up a vector into components.



(a) (b) (c)

It is kind of like using the parallelogram law in reverse.

ADDITION OF A SYSTEM OF COPLANAR FORCES (Section 2.4)



(a)

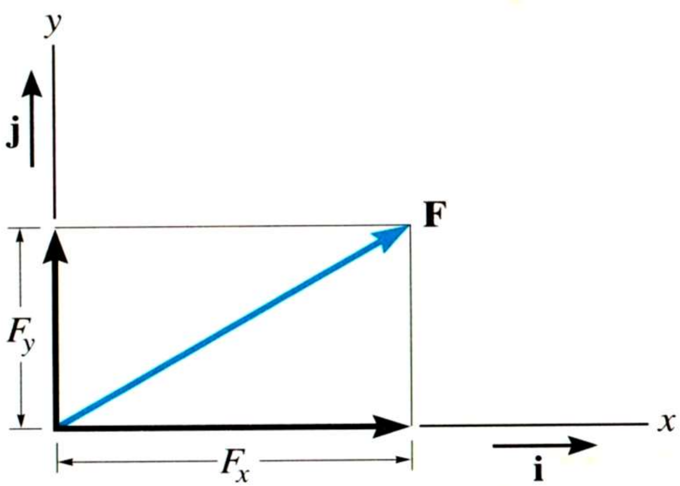
- We 'resolve' vectors into components using the x and y-axis coordinate system.

- Each component of the vector is shown as a magnitude and a direction.

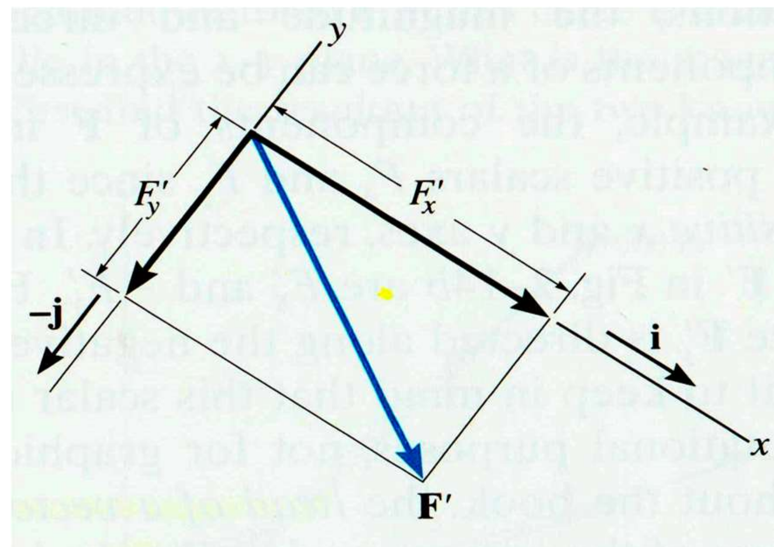
- The directions are based on the x and y axes. We use the "unit vectors" ***i*** and ***j*** to designate the x and y-axes.

For example,

$$\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j} \qquad \textbf{\textit{or}} \qquad \boldsymbol{F'} = F'_x\,\boldsymbol{i} + (-\;F'_y)\,\boldsymbol{j}$$



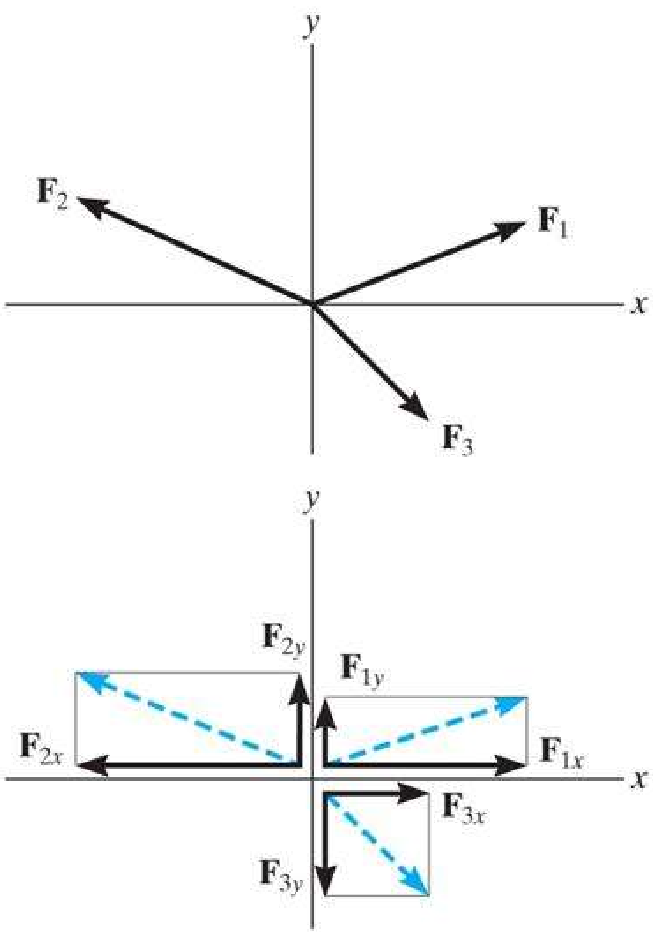
(a)



(b)

The x and y-axis are always perpendicular to each other. Together, they can be "set" at any inclination.

ADDITION OF SEVERAL VECTORS

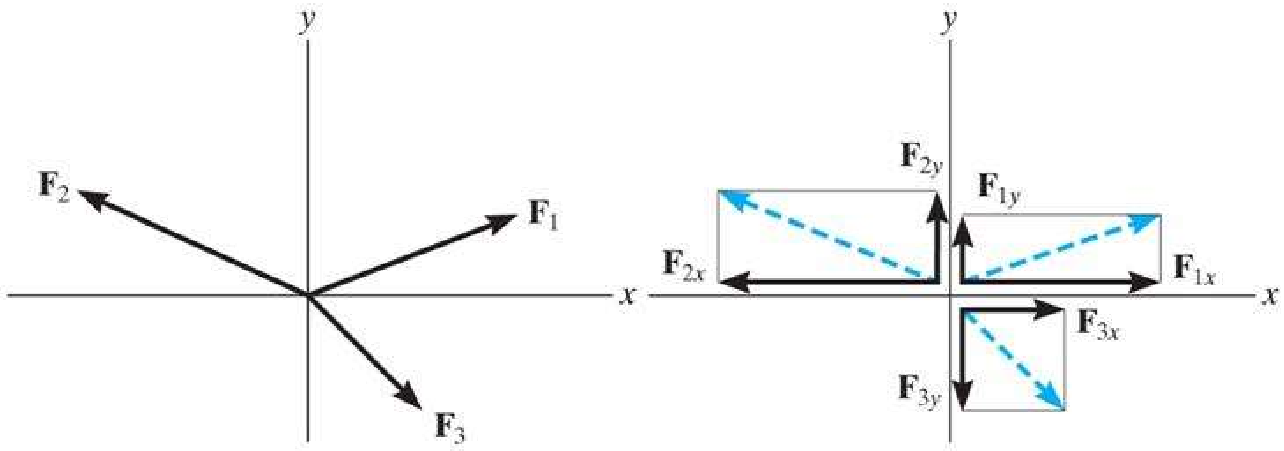


- Step 1 is to resolve each force into its components.

- Step 2 is to add all the x-components together, followed by adding all the y-components together. These two totals are the x and y-components of the resultant vector.

- Step 3 is to find the magnitude and angle of the resultant vector.

An example of the process:



Break the three vectors into components, then add them.

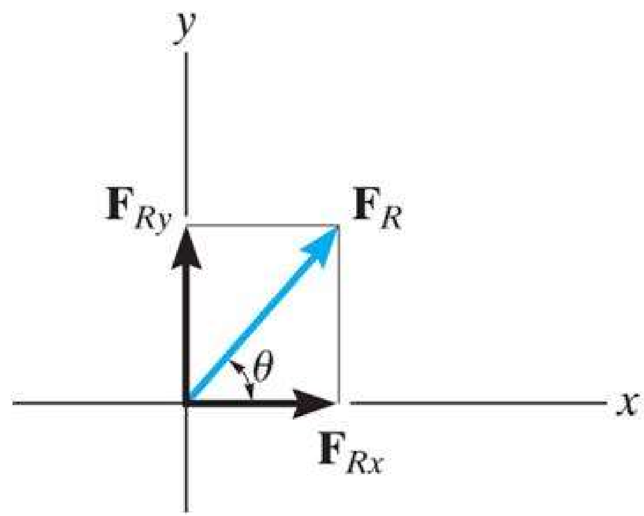
$$\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$$

$$= F_{1x}\,\boldsymbol{i} + F_{1y}\,\boldsymbol{j} - F_{2x}\,\boldsymbol{i} + F_{2y}\,\boldsymbol{j} + F_{3x}\,\boldsymbol{i} - F_{3y}\,\boldsymbol{j}$$

$$= (F_{1x} - F_{2x} + F_{3x})\,\boldsymbol{i} + (F_{1y} + F_{2y} - F_{3y})\,\boldsymbol{j}$$

$$= (F_{Rx})\,\boldsymbol{i} + (F_{Ry})\,\boldsymbol{j}$$

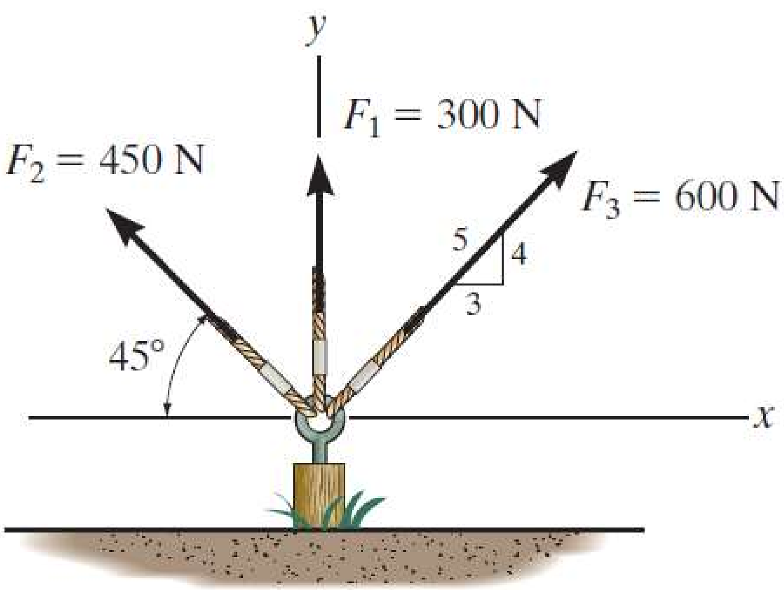
You can also represent a 2-D vector with a magnitude and angle.



$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

EXAMPLE I



Given: Three concurrent forces acting on a tent post.

Find: The magnitude and angle of the resultant force.

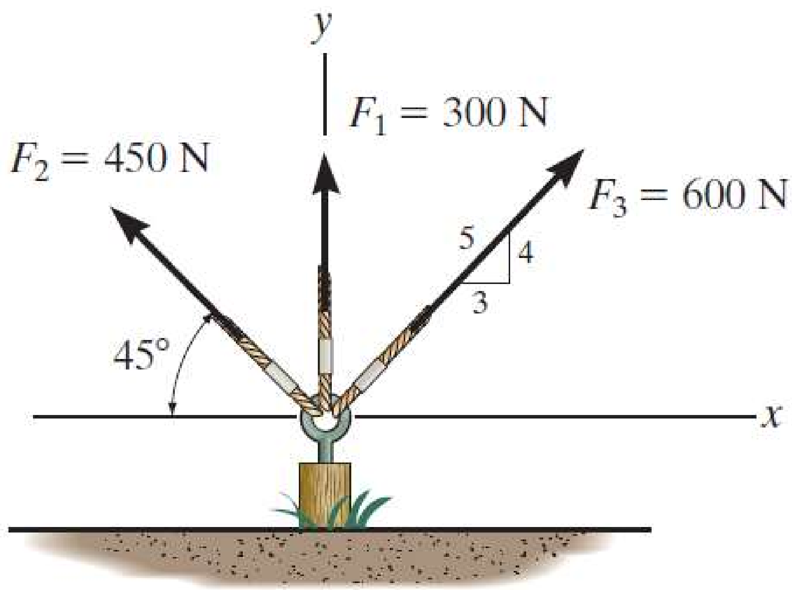
Plan:

a) Resolve the forces into their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

EXAMPLE I (continued)



$\boldsymbol{F_1} = \{0\ \boldsymbol{i} + 300\ \boldsymbol{j}\ \}$ N

$\boldsymbol{F_2} = \{-\ 450\ \cos\ (45°)\ \boldsymbol{i} + 450\ \sin\ (45°)\ \boldsymbol{j}\ \}$ N

$= \{-\ 318.2\ \boldsymbol{i} + 318.2\ \boldsymbol{j}\ \}$ N

$\boldsymbol{F_3} = \{\ (3/5)\ 600\ \boldsymbol{i} + (4/5)\ 600\ \boldsymbol{j}\ \}$ N

$= \{\ 360\ \boldsymbol{i} + 480\ \boldsymbol{j}\ \}$ N

EXAMPLE I (continued)

Summing up all the ***i*** and ***j*** components respectively, we get,

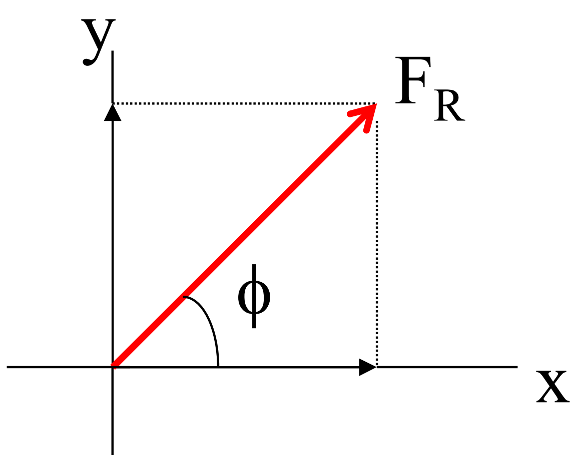
$$\boldsymbol{F_R} = \{ (0 - 318.2 + 360)\, \boldsymbol{i} + (300 + 318.2 + 480)\, \boldsymbol{j} \}\ \text{N}$$

$$= \{ 41.80\, \boldsymbol{i} + 1098\, \boldsymbol{j} \}\ \text{N}$$

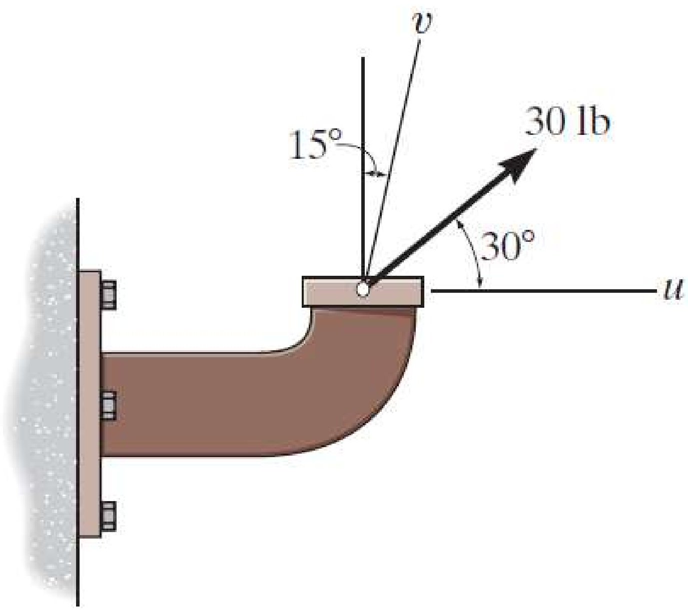
Using magnitude and direction:

$F_R = ((41.80)^2 + (1098)^2)^{1/2} = \underline{1099\ \text{N}}$

$\phi = \tan^{-1}(1098/41.80) = \underline{87.8°}$



EXAMPLE II



Given: A force acting on a pipe.

Find: Resolve the force into components along the u and v-axes, and determine the magnitude of each of these components.

Plan:

a) Construct lines parallel to the u and v-axes, and form a parallelogram.

b) Resolve the forces into their u-v components.

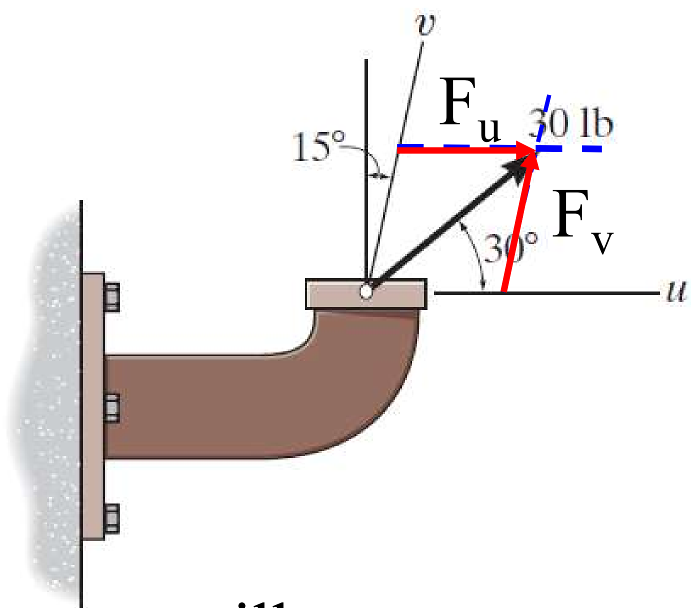
c) Find magnitude of the components from the law of sines.

EXAMPLE II (continued)

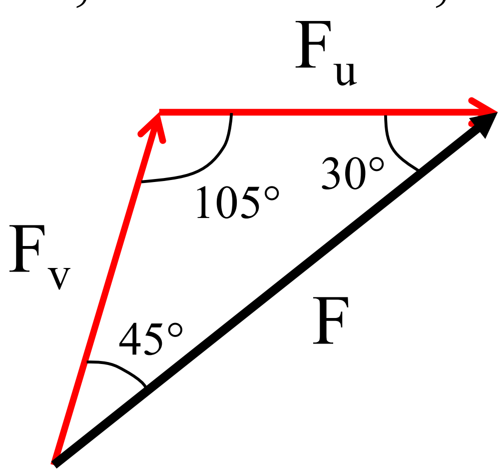
Solution:

Draw lines parallel to the u and v-axes.

And resolve the forces into the u-v components.

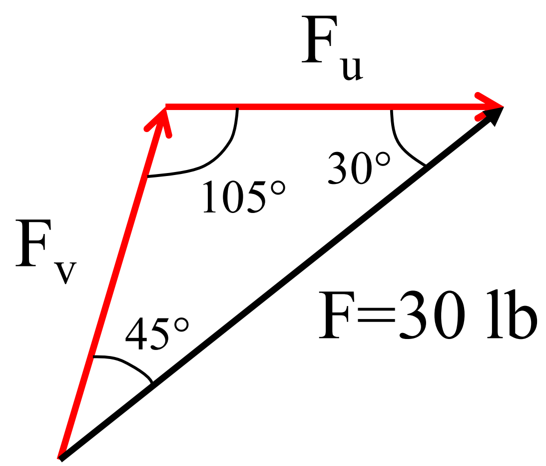


Redraw the top portion of the parallelogram to illustrate a Triangular, head-to-tail, addition of the components.



EXAMPLE II (continued)

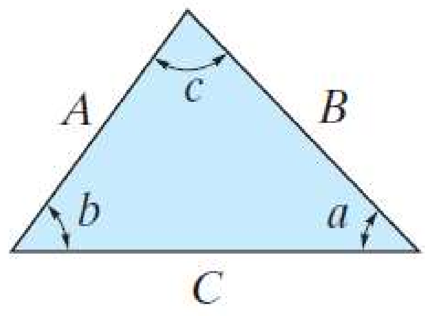
The magnitudes of two force components are determined from the law of sines. The formulas are given in Fig. 2–10*c*.



$$\frac{30}{\sin 105^\circ} = \frac{F_u}{\sin 45^\circ} = \frac{F_v}{\sin 30^\circ}$$

$F_u = (30/\sin 105^\circ) \sin 45^\circ = \underline{22.0 \text{ lb}}$

$F_v = (30/\sin 105^\circ) \sin 30^\circ = \underline{15.5 \text{ lb}}$



Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Fig. 2–10

CONCEPT QUIZ

1. Can you resolve a 2-D vector along two directions, which are not at 90° to each other?

   A) Yes, but not uniquely.

   B) No.

   C) Yes, uniquely.

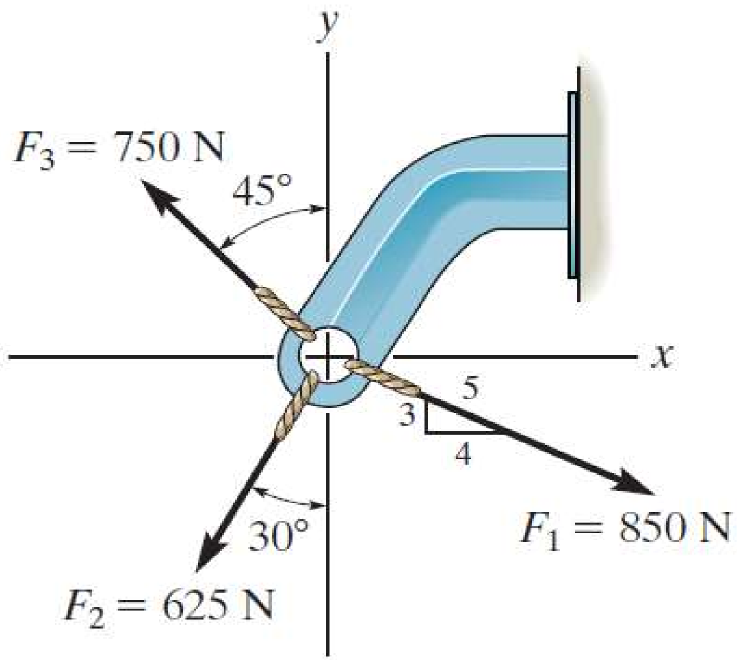
2. Can you resolve a 2-D vector along three directions (say at 0, 60, and 120°)?

   A) Yes, but not uniquely.

   B) No.

   C) Yes, uniquely.

GROUP PROBLEM SOLVING



Given: Three concurrent forces acting on a bracket.

Find: The magnitude and angle of the resultant force. Show the resultant in a sketch.

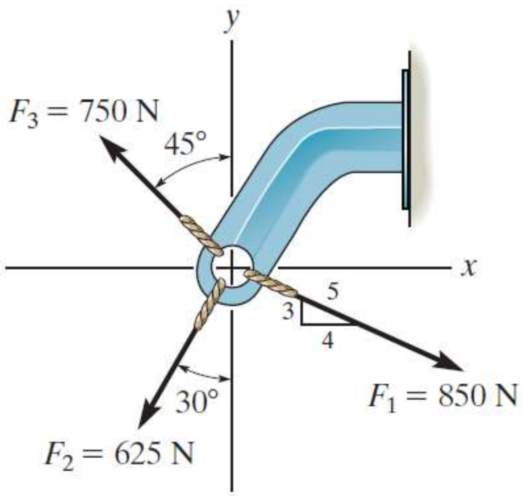
Plan:

a) Resolve the forces into their x and y-components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

GROUP PROBLEM SOLVING (continued)



$$\boldsymbol{F_1} = \{850\ (4/5)\ \boldsymbol{i} - 850\ (3/5)\ \boldsymbol{j}\ \}\ \text{N}$$

$$= \{\ 680\ \boldsymbol{i} - 510\ \boldsymbol{j}\ \}\ \text{N}$$

$$\boldsymbol{F_2} = \{-\ 625\ \sin(30°)\ \boldsymbol{i} - 625\ \cos(30°)\ \boldsymbol{j}\ \}\ \text{N}$$

$$= \{-\ 312.5\ \boldsymbol{i} - 541.3\ \boldsymbol{j}\ \}\ \text{N}$$

$$\boldsymbol{F_3} = \{-750\ \sin(45°)\ \boldsymbol{i} + 750\ \cos(45°)\ \boldsymbol{j}\ \}\ \text{N}$$

$$\{-\ 530.3\ \boldsymbol{i} + 530.3\ \boldsymbol{j}\ \}\ \text{N}$$

GROUP PROBLEM SOLVING (continued)

Summing all the ***i*** and ***j*** components, respectively, we get,

$\boldsymbol{F}_{\mathbf{R}} = \{ (680 - 312.5 - 530.3)\, \boldsymbol{i} + (-510 - 541.3 + 530.3)\, \boldsymbol{j} \}\text{N}$

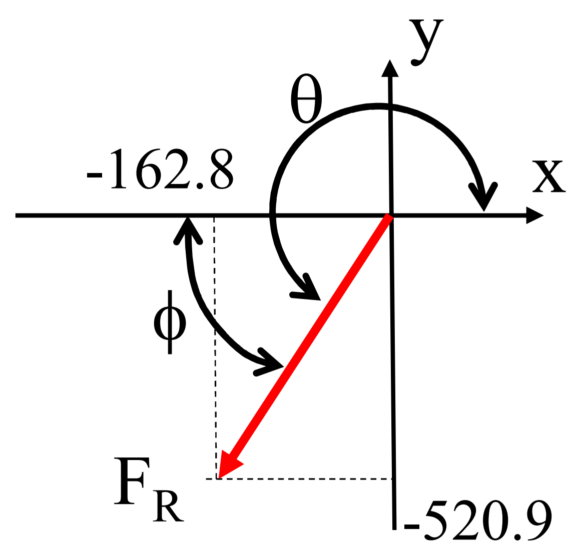
$= \{- 162.8\, \boldsymbol{i} - 520.9\, \boldsymbol{j} \}\ \text{N}$

Now find the magnitude and angle,

$F_R = ((- 162.8)^2 + (- 520.9)^2)^{½} =$ <u>546 N</u>

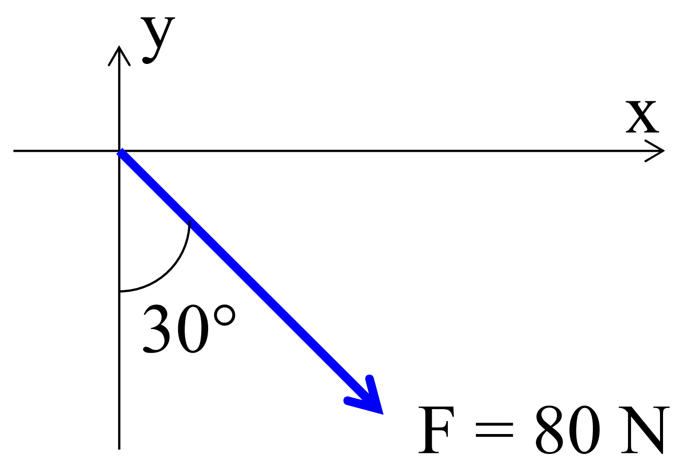
$\phi = \tan^{-1}(520.9 / 162.8) =$ <u>72.6°</u>

<u>From the positive x-axis, $\theta = 253°$</u>



ATTENTION QUIZ

1. Resolve ***F*** along x and y axes and write it in vector form. ***F*** = { _______________ } N



A) 80 cos (30°) ***i*** – 80 sin (30°) ***j***

B) 80 sin (30°) ***i*** + 80 cos (30°) ***j***

C) 80 sin (30°) ***i*** – 80 cos (30°) ***j***

D) 80 cos (30°) ***i*** + 80 sin (30°) ***j***

2. Determine the magnitude of the resultant ($\boldsymbol{F_1} + \boldsymbol{F_2}$) force in N when $\boldsymbol{F_1}$ = { 10 ***i*** + 20 ***j*** } N and $\boldsymbol{F_2}$ = { 20 ***i*** + 20 ***j*** } N .

A) 30 N

B) 40 N

C) 50 N

D) 60 N

E) 70 N

CARTESIAN VECTORS AND THEIR ADDITION & SUBTRACTION

<u>Objectives</u>**:**

Students will be able to:

a) Represent a 3-D vector in a Cartesian coordinate system.

b) Find the magnitude and coordinate angles of a 3-D vector

c) Add vectors (forces) in 3-D space

<u>In-Class Activities</u>**:**

- Reading Quiz
- Applications/Relevance
- A Unit Vector
- 3-D Vector Terms
- Adding Vectors
- Concept Quiz
- Examples
- Attention Quiz

READING QUIZ

1. Vector algebra, as we are going to use it, is based on a ___________ coordinate system.

   A) Euclidean B) Left-handed

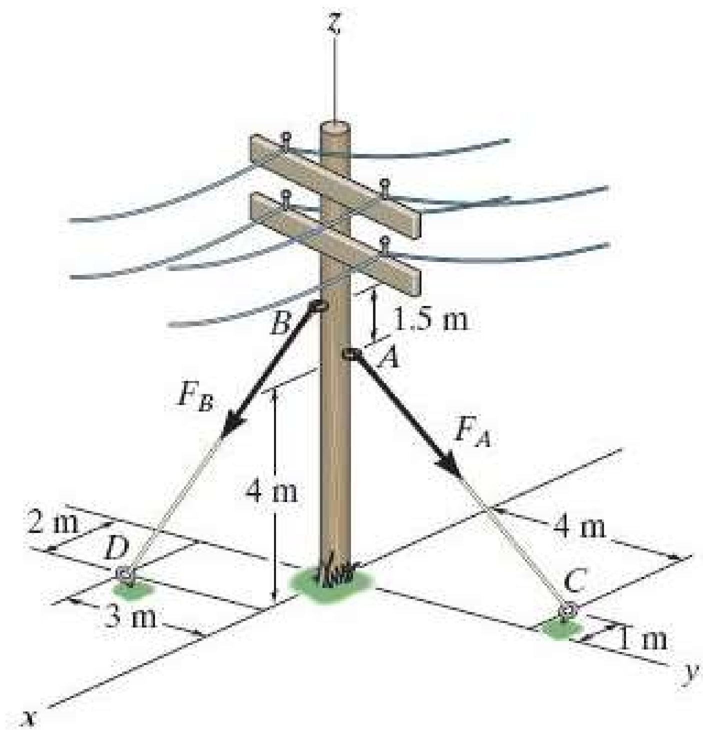
   C) Greek D) Right-handed E) Cartesian

2. The symbols α, β, and γ designate the __________ of a 3-D Cartesian vector.

   A) Unit vectors B) Coordinate direction angles

   C) Greek societies D) X, Y and Z components

APPLICATIONS

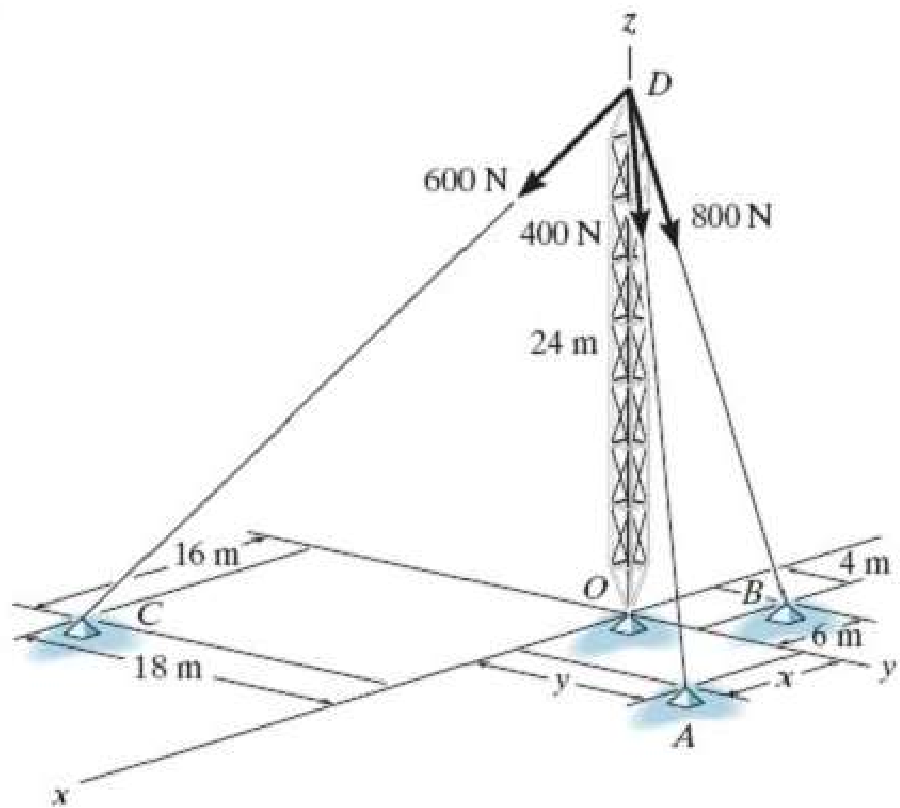


Many structures and machines involve 3-dimensional space.

In this case, the power pole has guy wires helping to keep it upright in high winds. How would you represent the forces in the cables using Cartesian vector form?

APPLICATIONS (continued)

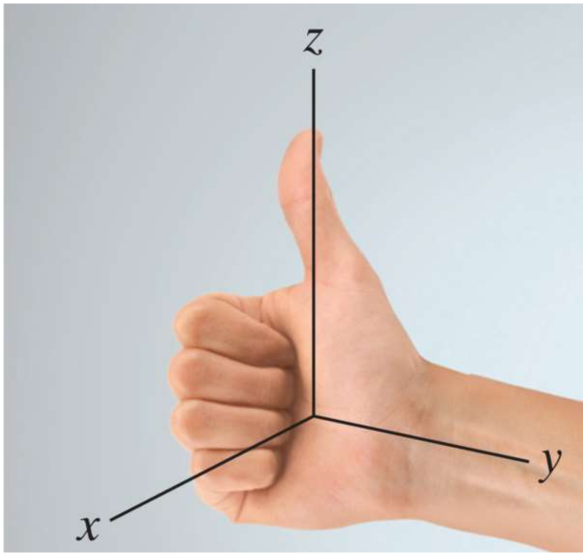
In the case of this radio tower, if you know the forces in the three cables, how would you determine the resultant force acting at D, the top of the tower?



CARTESIAN UNIT VECTORS

For a vector $\boldsymbol{A}$, with a magnitude of A, an unit vector is defined as

$$\boldsymbol{u}A = \boldsymbol{A} / A .$$

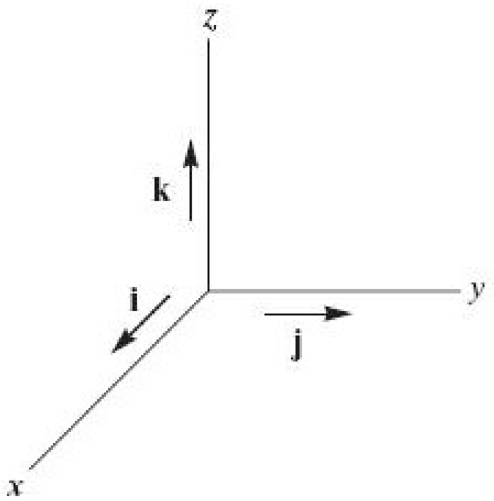


Characteristics of a unit vector **:**

a) Its magnitude is 1.

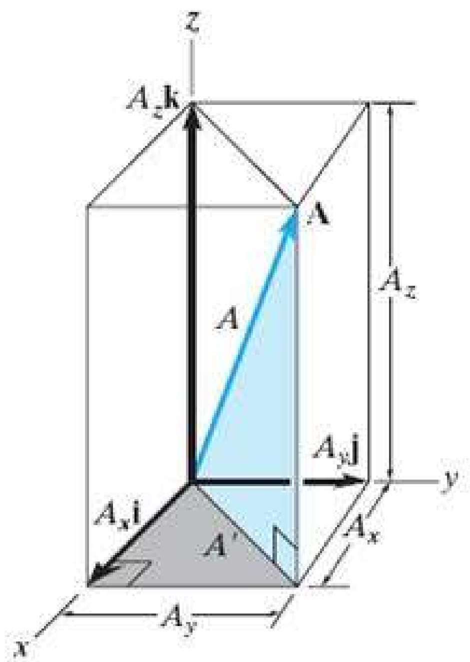
b) It is dimensionless (has no units).

c) It points in the same direction as the original vector ($\boldsymbol{A}$).



The unit vectors in the Cartesian axis system are $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$. They are unit vectors along the positive x, y, and z axes respectively.

CARTESIAN VECTOR REPRESENTATION



Consider a box with sides AX, AY, and AZ meters long.

The vector ***A*** can be defined as

$$\boldsymbol{A} = (AX\ \boldsymbol{i} + AY\ \boldsymbol{j} + AZ\ \boldsymbol{k})\ \text{m}$$

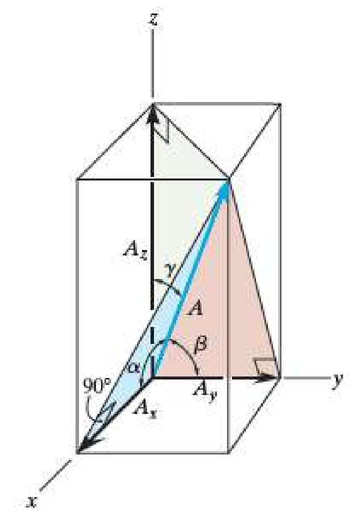
The projection of vector ***A*** in the x-y plane is A′. The magnitude of A′ is found by using the same approach as a 2-D vector: $A' = (AX^2 + AY^2)^{1/2}$.

The magnitude of the position vector ***A*** can now be obtained as

$$A = ((A')^2 + AZ^2)^{½} = (AX^2 + AY^2 + AZ^2)^{½}$$

DIRECTION OF A CARTESIAN VECTOR

The direction or orientation of vector **A** is defined by the angles α, β, and γ.



These angles are measured between the vector and the positive X, Y and Z axes, respectively. Their range of values are from 0° to 180°

Using trigonometry, "direction cosines" are found using

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

These angles are not independent. They must satisfy the following equation.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

This result can be derived from the definition of a coordinate direction angles and the unit vector. Recall, the formula for finding the unit vector of any position vector:

$$\boxed{\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}}$$

or written another way, $uA = \cos\alpha\, \boldsymbol{i} + \cos\beta\, \boldsymbol{j} + \cos\gamma\, \boldsymbol{k}$.

ADDITION OF CARTESIAN VECTORS (Section 2.6)

Once individual vectors are written in Cartesian form, it is easy to add or subtract them. The process is essentially the same as when 2-D vectors are added.

$$\boxed{\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}}$$

For example, if

$\boldsymbol{A} = \mathrm{AX}\ \boldsymbol{i} + \mathrm{AY}\ \boldsymbol{j} + \mathrm{AZ}\ \boldsymbol{k}$ and

$\boldsymbol{B} = \mathrm{BX}\ \boldsymbol{i} + \mathrm{BY}\ \boldsymbol{j} + \mathrm{BZ}\ \boldsymbol{k}$, then

$$\boldsymbol{A} + \boldsymbol{B} = (\mathrm{AX} + \mathrm{BX})\,\boldsymbol{i} + (\mathrm{AY} + \mathrm{BY})\,\boldsymbol{j} + (\mathrm{AZ} + \mathrm{BZ})\,\boldsymbol{k}$$

or

$$\boldsymbol{A} - \boldsymbol{B} = (\mathrm{AX} - \mathrm{BX})\,\boldsymbol{i} + (\mathrm{AY} - \mathrm{BY})\,\boldsymbol{j} + (\mathrm{AZ} - \mathrm{BZ})\,\boldsymbol{k}\,.$$

IMPORTANT NOTES

Sometimes 3-D vector information is given as:

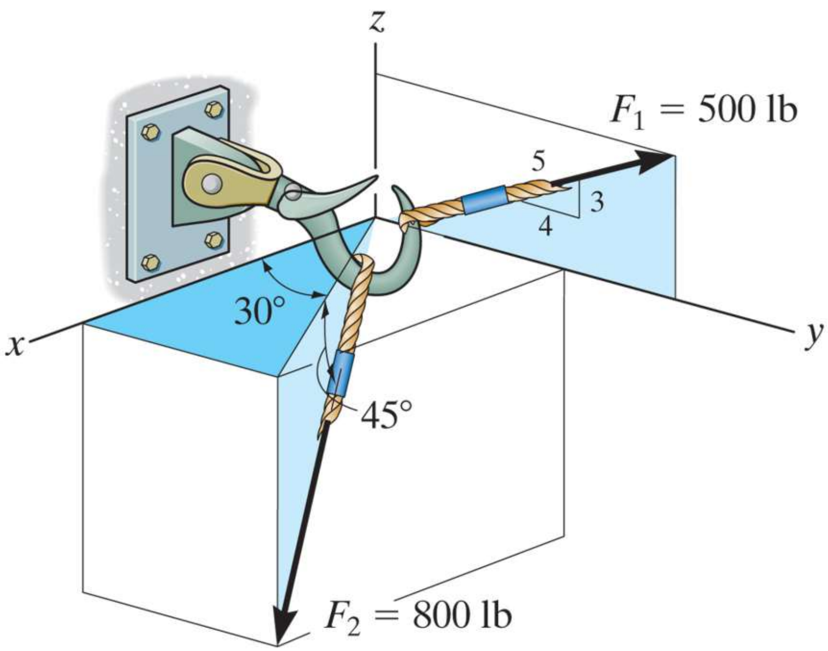
a) Magnitude and the coordinate direction angles, or,

b) Magnitude and projection angles.

You should be able to use both these sets of information to change the representation of the vector into the Cartesian form, i.e.,

$\boldsymbol{F} = \{10\,\boldsymbol{i} - 20\,\boldsymbol{j} + 30\,\boldsymbol{k}\}$ N .

EXAMPLE



Given: Two forces ***F1*** and ***F2*** are applied to a hook.

Find: The resultant force in Cartesian vector form.

Plan:

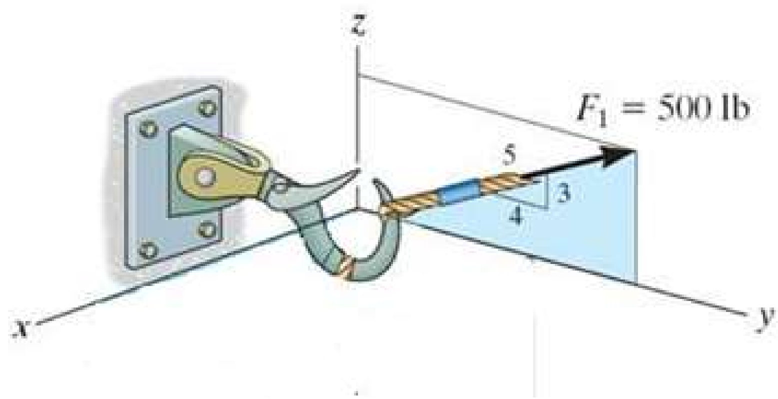
1) Using geometry and trigonometry, write ***F1*** and ***F2*** in Cartesian vector form.

2) Then add the two forces (by adding x and y-components).

EXAMPLE (continued)

Solution:

First, resolve force ***F1***.



Fx = 0 = 0 lb

Fy = 500 (4/5) = 400 lb

Fz = 500 (3/5) = 300 lb

Now, write ***F1*** in Cartesian vector form (don't forget the units!).

F1 = {0 ***i*** + 400 ***j*** + 300 ***k***} lb

EXAMPLE (continued)

Now, resolve force ***F2***.

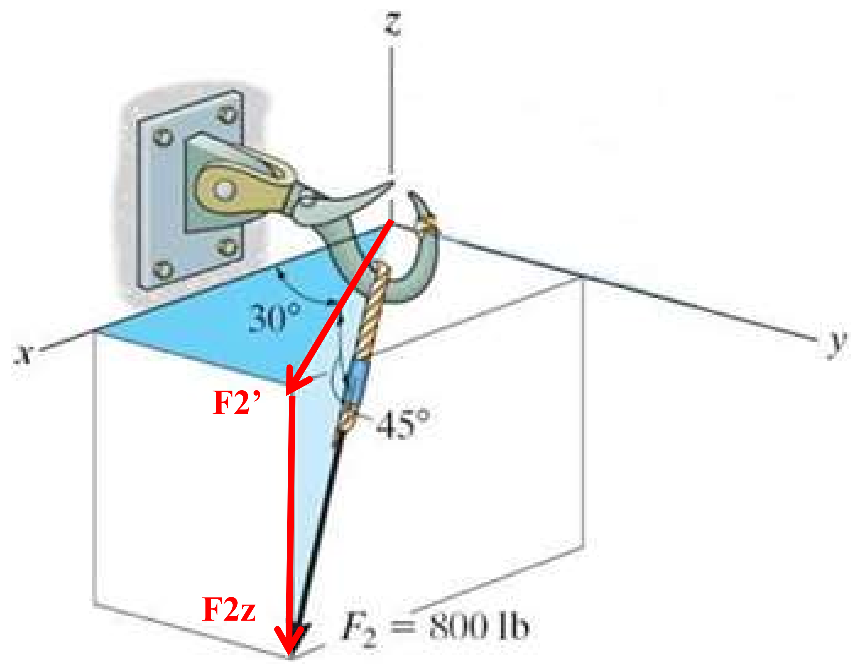
F2z = -800 sin 45° = – 565.7 lb

F2' = 800 cos 45° = 565.7 lb

F2' can be further resolved as,

F2x = 565.7 cos 30° = 489.9 lb

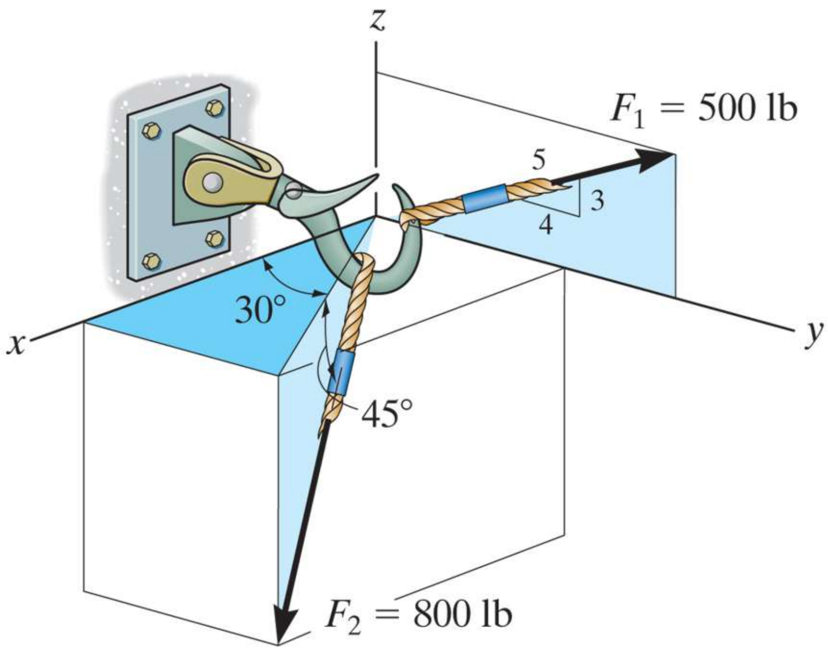
F2y = 565.7 sin 30° = 282.8 lb



Thus, we can write:

F2 = {489.9 ***i*** + 282.8 ***j*** – 565.7 ***k*** } lb

EXAMPLE (continued)



So ***FR*** = ***F1*** + ***F2*** and

F1 = {0 ***i*** + 400 ***j*** + 300 ***k***} lb

F2 = {489.9 ***i*** + 282.8 ***j*** – 565.7 ***k*** } lb

FR = { 490 ***i*** + 683 ***j*** – 266 ***k*** } lb

CONCEPT QUIZ

1. If you know only ***uA***, you can determine the ________ of ***A*** uniquely.

   A) magnitude

   B) angles (α, β and γ)

   C) components (AX, AY, & AZ)

   D) All of the above.

2. For a force vector, the following parameters are randomly generated. The magnitude is 0.9 N, $\alpha = 30^\circ$, $\beta = 70^\circ$, $\gamma = 100^\circ$. What is wrong with this 3-D vector ?

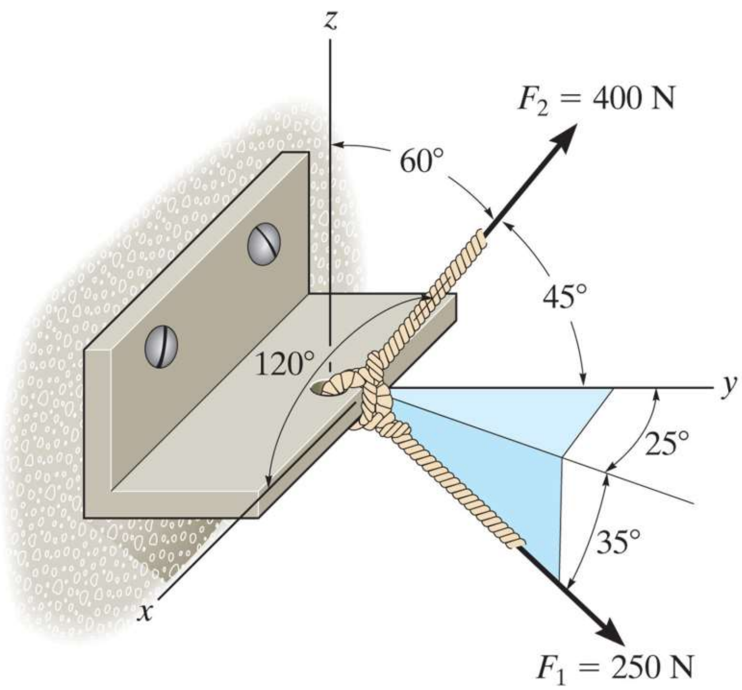
   A) Magnitude is too small.

   B) Angles are too large.

   C) All three angles are arbitrarily picked.

   D) All three angles are between 0° to 180°.

GROUP PROBLEM SOLVING



Given: The screw eye is subjected to two forces, ***F1*** and ***F2***.

Find: The magnitude and the coordinate direction angles of the resultant force.

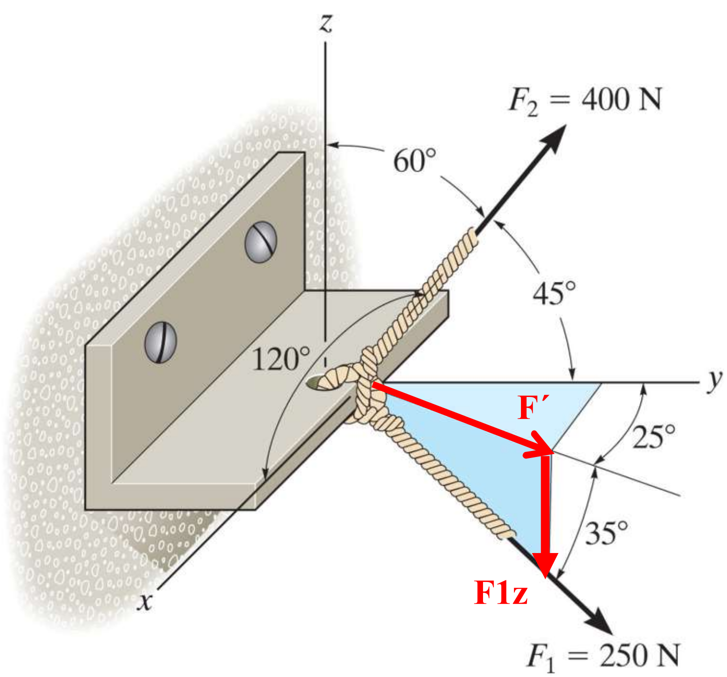
Plan:

1) Using the geometry and trigonometry, resolve and write ***F1*** and ***F2*** in the Cartesian vector form.

2) Add ***F1*** and ***F2*** to get ***FR***.

3) Determine the magnitude and angles α, β, γ.

GROUP PROBLEM SOLVING (continued)



First resolve the force ***F1***.

F1z = - 250 sin 35° = - 143.4 N

F´ = 250 cos 35° = 204.8 N

F´ can be further resolved as,

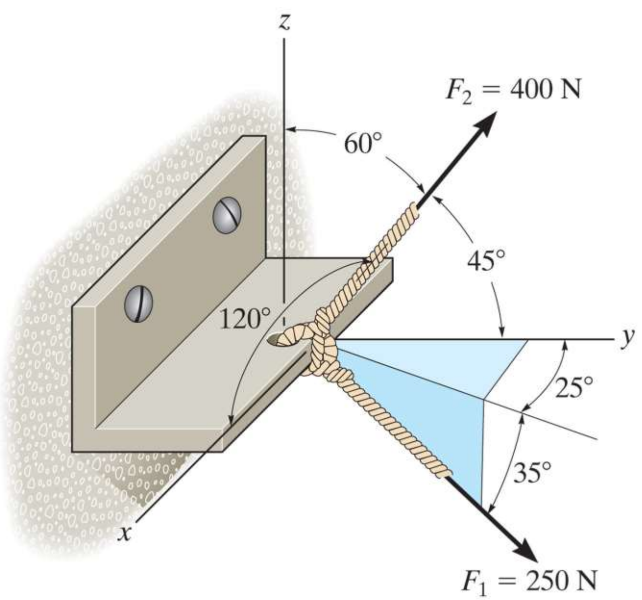
F1x = 204.8 sin 25° = 86.6 N

F1y = 204.8 cos 25° = 185.6 N

Now we can write:

F1 = {86.6 ***i*** + 185.6 ***j*** – 143.4 ***k*** } N

GROUP PROBLEM SOLVING (continued)



Now, resolve force ***F2***.

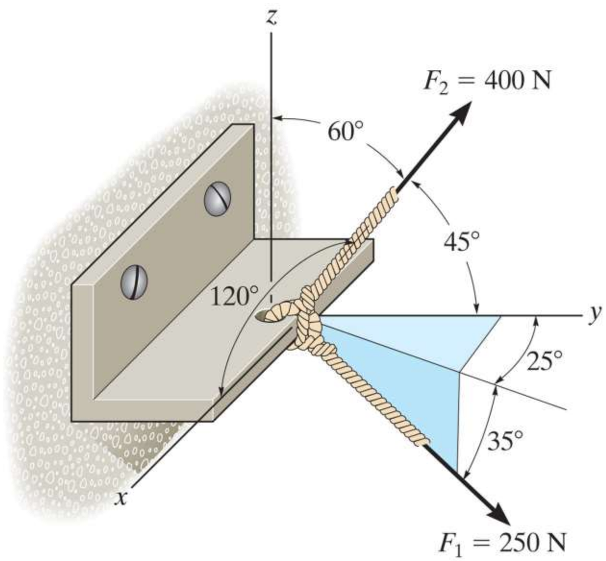
The force ***F2*** can be represented in the Cartesian vector form as:

F2 = 400{ cos 120° ***i*** + cos 45° ***j*** + cos 60° ***k*** } N

= { -200 ***i*** + 282.8 ***j*** + 200 ***k*** } N

F2 = { -200 ***i*** + 282.8 ***j*** +200 ***k*** } N

GROUP PROBLEM SOLVING (continued)



So ***FR*** = ***F1*** + ***F2*** and

F1 = { 86.6 ***i*** + 185.6 ***j*** – 143.4 ***k***} N

F2 = { -200 ***i*** + 282.8 ***j*** + 200 ***k***} N

FR = { -113.4 ***i*** + 468.4 ***j*** + 56.6 ***k***} N

Now find the magnitude and direction angles for the vector.

$FR = \{(-113.4)^2 + 468.4^2 + 56.6^2\}^{1/2} = 485.2 = \underline{485\ N}$

$\alpha = \cos^{-1}(FRx / FR) = \cos^{-1}(-113.4 / 485.2) = \underline{104°}$

$\beta = \cos^{-1}(FRy / FR) = \cos^{-1}(468.4 / 485.2) = \underline{15.1°}$

$\gamma = \cos^{-1}(FRz / FR) = \cos^{-1}(56.6 / 485.2) = \underline{83.3°}$

ATTENTION QUIZ

1. What is not true about an unit vector, e.g., ***uA*?**

   A) It is dimensionless.

   B) Its magnitude is one.

   C) It always points in the direction of positive X- axis.

   D) It always points in the direction of vector ***A***.

2. If ***F*** = {10 ***i*** + 10 ***j*** + 10 ***k***} N and

   G = {20 ***i*** + 20 ***j*** + 20 ***k*** } N, then ***F*** + ***G*** = { ____ } **N**

   A) 10 ***i*** + 10 ***j*** + 10 ***k***

   B) 30 ***i*** + 20 ***j*** + 30 ***k***

   C) – 10 ***i*** – 10 ***j*** – 10 ***k***

   D) 30 ***i*** + 30 ***j*** + 30 ***k***

POSITION VECTORS & FORCE VECTORS

Objectives:

Students will be able to :

a) Represent a position vector in Cartesian coordinate form, from given geometry.

b) Represent a force vector directed along a line.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications / Relevance
- Write Position Vectors
- Write a Force Vector along a line
- Concept Quiz
- Group Problem
- Attention Quiz

READING QUIZ

1. The position vector ***rPQ*** is obtained by
   A) Coordinates of Q minus coordinates of the origin
   B) Coordinates of P minus coordinates of Q
   C) Coordinates of Q minus coordinates of P
   D) Coordinates of the origin minus coordinates of P

2. A force of magnitude F, directed along a unit vector ***U***, is given by ***F*** = _______ .
   A) F (***U***)
   B) ***U*** / F
   C) F / ***U***
   D) F + ***U***
   E) F – ***U***

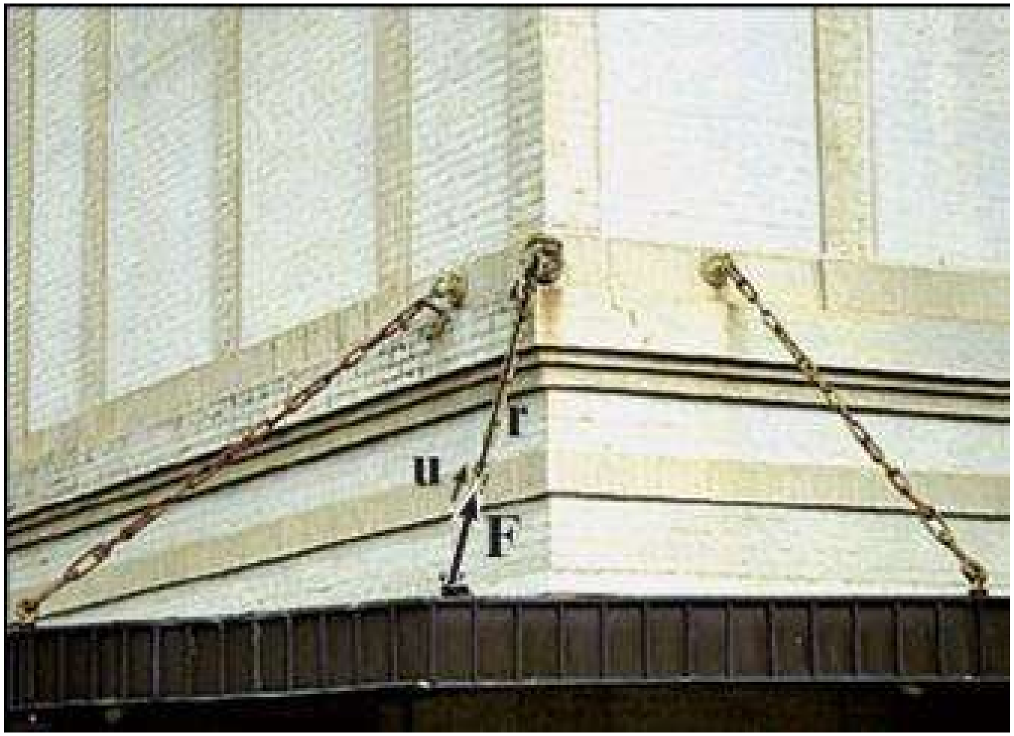
APPLICATIONS

This ship's mooring line, connected to the bow, can be represented as a Cartesian vector.

What are the forces in the mooring line and how do we find their directions?

Why would we want to know these things?

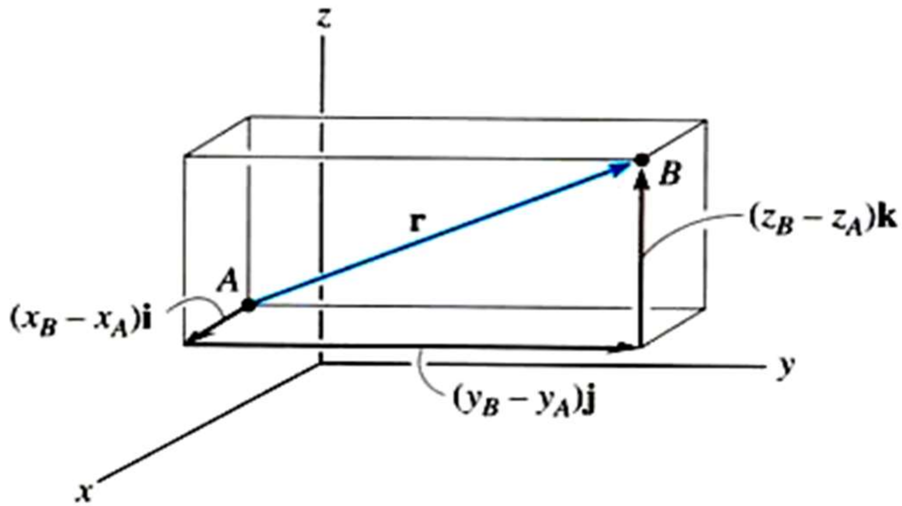
APPLICATIONS (continued)



This awning is held up by three chains. What are the forces in the chains and how do we find their directions? Why would we want to know these things?

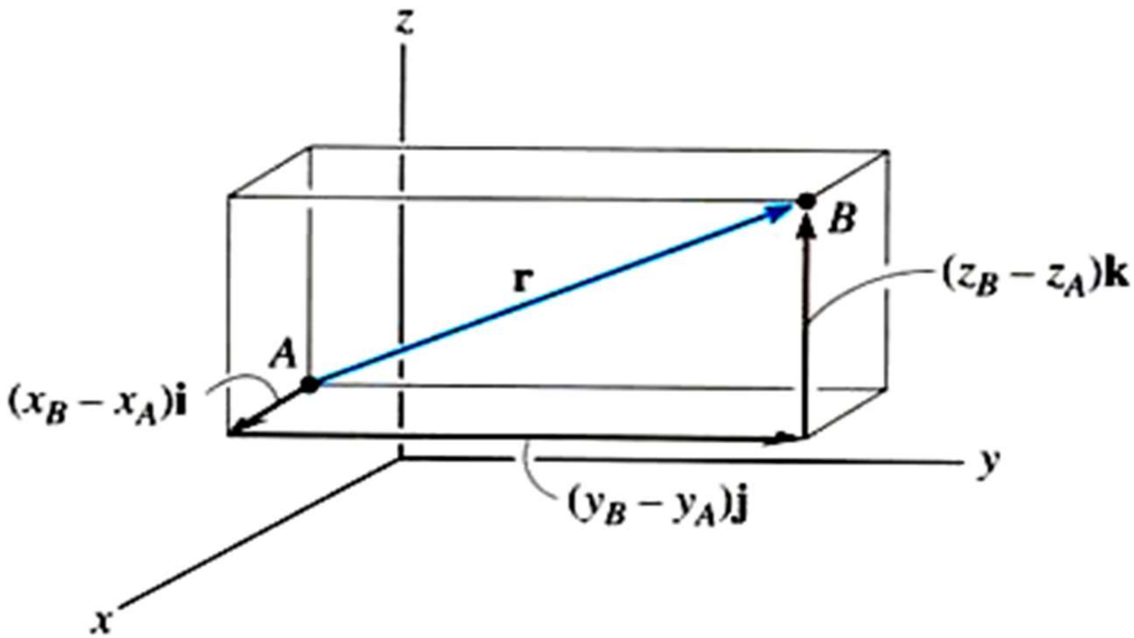
POSITION VECTOR

A position vector is defined as a fixed vector that locates a point in space relative to another point.



Consider two points, A and B, in 3-D space.
Let their coordinates be (XA, YA, ZA) and (XB, YB, ZB), respectively.

POSITION VECTOR (continued)

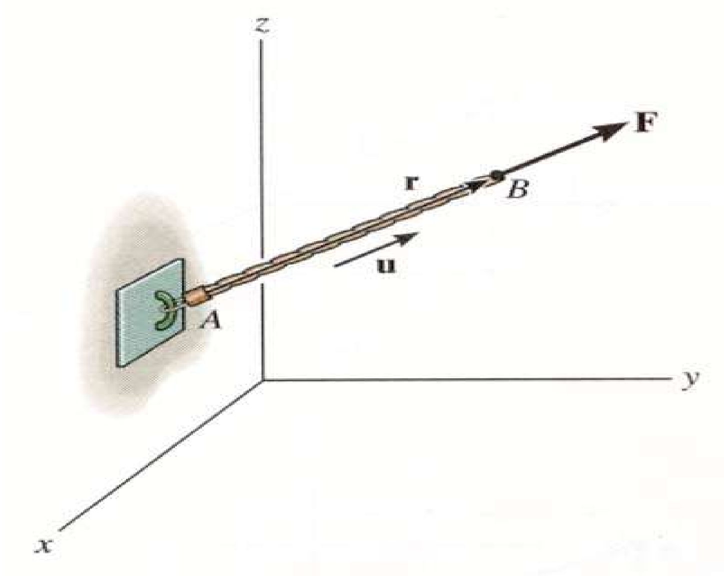


The position vector directed from A to B, ***rAB*** , is defined as

rAB = {(XB – XA) ***i*** + (YB – YA) ***j*** + (ZB – ZA) ***k*** }m

Please note that B is the ending point and A is the starting point. ALWAYS subtract the "tail" coordinates from the "tip" coordinates!

FORCE VECTOR DIRECTED ALONG A LINE (Section 2.8)



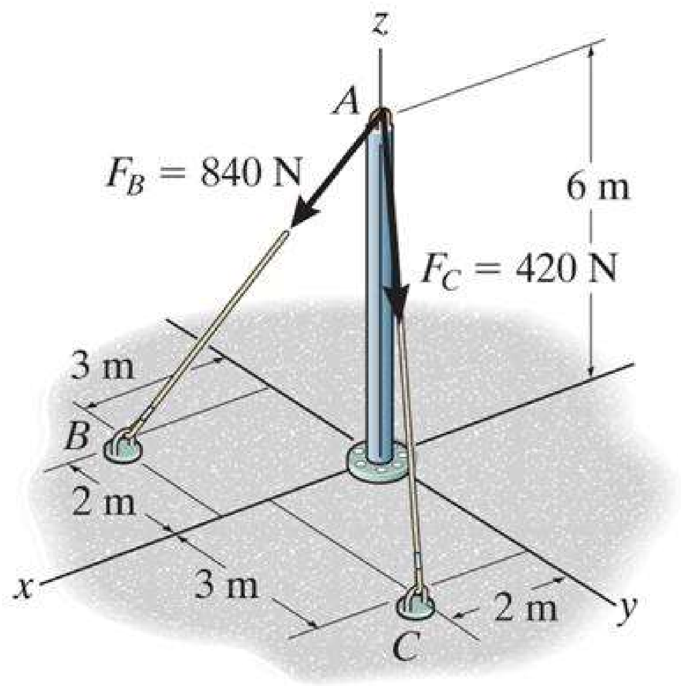
If a force is directed along a line, then we can represent the force vector in Cartesian coordinates by using a unit vector and the force's magnitude. So we need to:

a) Find the position vector, ***rAB*** , along two points on that line.

b) Find the unit vector describing the line's direction, ***uAB*** = (***rAB***/rAB).

c) Multiply the unit vector by the magnitude of the force, ***F*** = F ***uAB*** .

EXAMPLE



Given: The 420 N force along the cable AC.

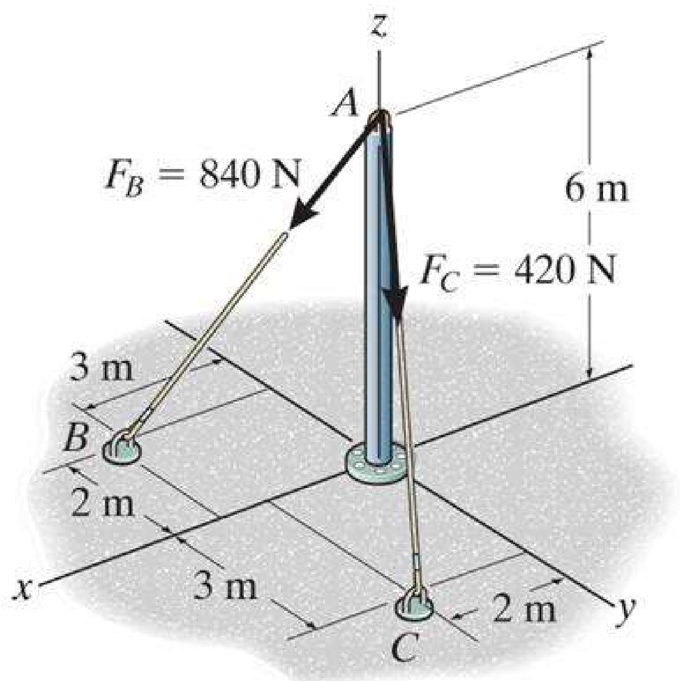
Find: The force ***FAC*** in the Cartesian vector form.

Plan:

1. Find the position vector ***rAC*** and its unit vector ***uAC***.

2. Obtain the force vector as ***FAC*** = 420 N ***uAC*** .

EXAMPLE (continued)



As per the figure, when relating A to C, we will have to go 2 m in the x-direction, 3 m in the y-direction, and -6 m in the z-direction. Hence,

rAC = {2 ***i*** + 3 ***j*** – 6 ***k***} m.

(We can also find ***rAC*** by subtracting the coordinates of A from the coordinates of C.)

rAC = $\{2^2 + 3^2 + (-6)^2\}^{1/2}$ = 7 m

Now ***uAC*** = ***rAC***/rAC and ***FAC*** = 420 ***uAC*** = 420 (***rAC***/rAC)

So ***FAC*** = 420{ (2 ***i*** + 3 ***j*** – 6 ***k***) / 7 } N

= {<u>120</u> ***i*** + <u>180</u> ***j*** <u>– 360</u> ***k*** } <u>N</u>

CONCEPT QUIZ

1. **P** and **Q** are two points in a 3-D space. How are the position vectors ***rPQ*** and ***rQP*** related?

   A) ***rPQ*** **=** ***rQP*** B) ***rPQ*** **= -** ***rQP***

   C) ***rPQ*** = 1/***rQP*** D) ***rPQ*** = *2* ***rQP***

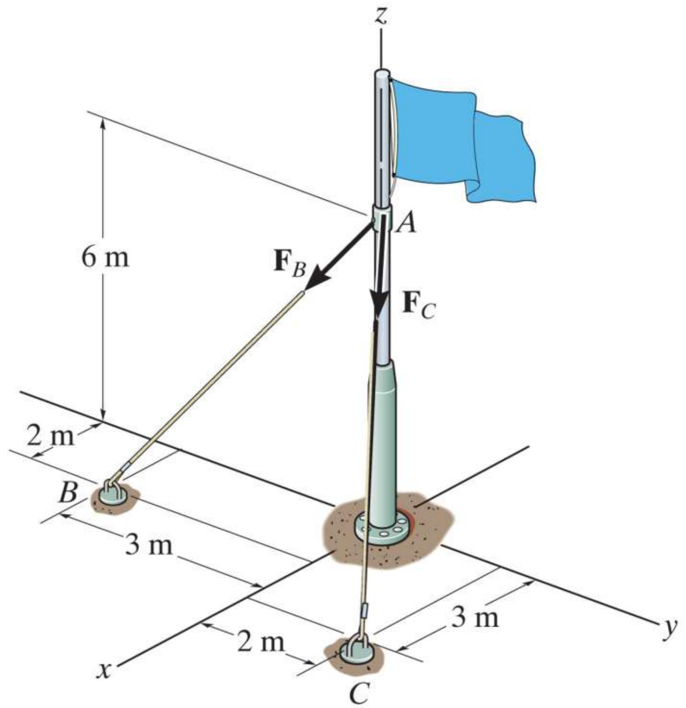
2. If ***F*** and ***r*** are force and position vectors, respectively, in SI units, what are the units of the expression (r * (***F*** / F)) ?

   A) Newton B) Dimensionless

   C) Meter D) Newton - Meter

   E) The expression is algebraically illegal.

GROUP PROBLEM SOLVING



Given: Two forces are acting on a flag pole as shown in the figure. FB = 560 N and FC = 700 N

Find: The magnitude and the coordinate direction angles of the resultant force.

Plan:

1) Find the forces along AB and AC in the Cartesian vector form.

2) Add the two forces to get the resultant force, ***FR***.

3) Determine the magnitude and the coordinate angles of ***FR***.

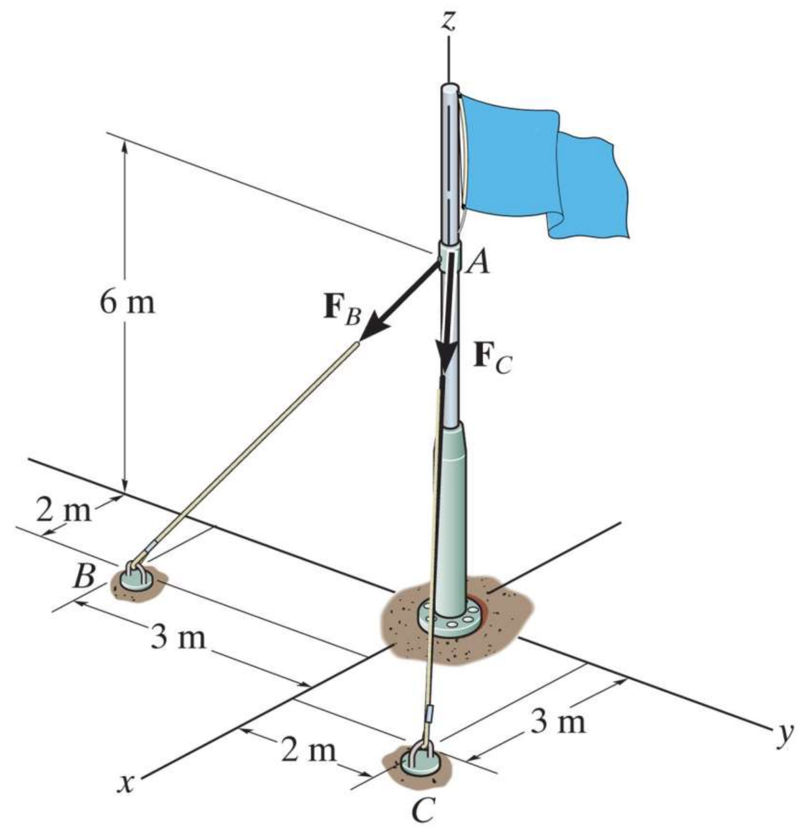
GROUP PROBLEM SOLVING (continued)

$\boldsymbol{r_{AB}} = \{2\,\boldsymbol{i} - 3\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ m

$\boldsymbol{r_{AC}} = \{3\,\boldsymbol{i} + 2\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ m

$r_{AB} = \{2^2 + (-3)^2 + (-6)^2\}^{1/2} = 7$ m

$r_{AC} = \{3^2 + 2^2 + (-6)^2\}^{1/2} = 7$ m



$\boldsymbol{F_{AB}} = 560\,(\boldsymbol{r_{AB}} / r_{AB})$ N

$\boldsymbol{F_{AB}} = 560\,(2\,\boldsymbol{i} - 3\,\boldsymbol{j} - 6\,\boldsymbol{k}) / 7$ N

$\boldsymbol{F_{AB}} = (160\,\boldsymbol{i} - 240\,\boldsymbol{j} - 480\,\boldsymbol{k})$ N

$\boldsymbol{F_{AC}} = 700\,(\boldsymbol{r_{AC}} / r_{AC})$ N

$\boldsymbol{F_{AC}} = 700\,(3\,\boldsymbol{i} + 2\,\boldsymbol{j} - 6\,\boldsymbol{k}) / 7$ N

$\boldsymbol{F_{AC}} = \{300\,\boldsymbol{i} + 200\,\boldsymbol{j} - 600\,\boldsymbol{k}\}$ N

GROUP PROBLEM SOLVING (continued)

$\boldsymbol{F_R} = \boldsymbol{F_{AB}} + \boldsymbol{F_{AC}}$

$= \{460\, \boldsymbol{i} - 40\, \boldsymbol{j} - 1080\, \boldsymbol{k}\}$ N

$F_R = \{460^2 + (-40)^2 + (-1080)^2\}^{1/2}$

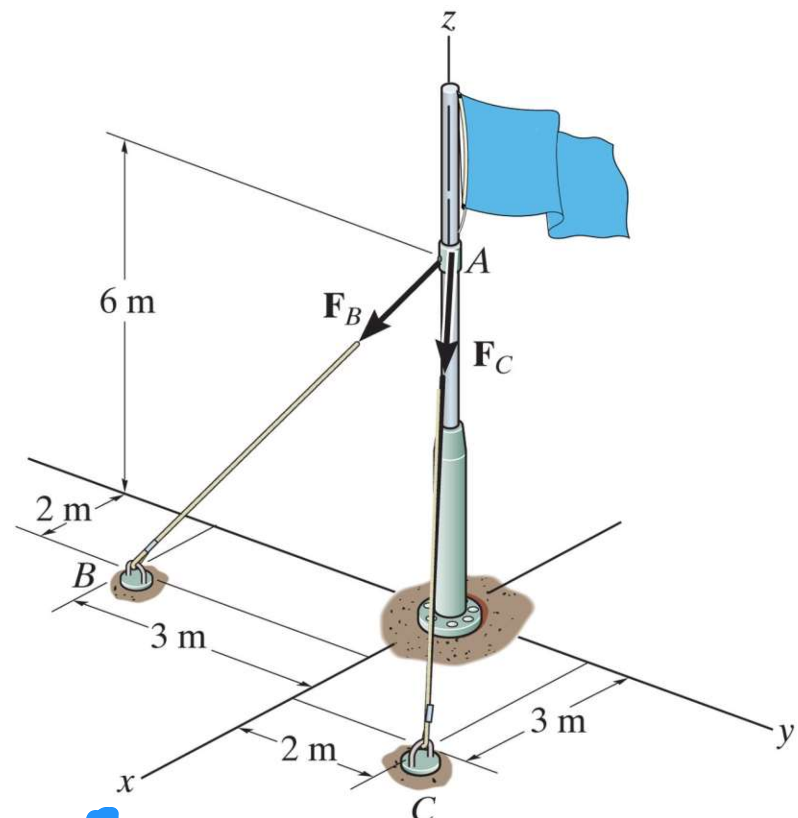
$= 1174.6$ N

$F_R =$ 1175 N

$\alpha = \cos^{-1}(460/1175) =$ 66.9°

$\beta = \cos^{-1}(-40/1175) =$ 92.0°

$\gamma = \cos^{-1}(-1080/1175) =$ 157°



$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

ATTENTION QUIZ

1. Two points in 3–D space have coordinates of P (1, 2, 3) and Q (4, 5, 6) meters. The position vector ***rQP*** is given by

   A) {3 ***i*** + 3 ***j*** + 3 ***k***} m

   B) {– 3 ***i*** – 3 ***j*** – 3 ***k***} m

   C) {5 ***i*** + 7 ***j*** + 9 ***k***} m

   D) {– 3 ***i*** + 3 ***j*** + 3 ***k***} m

   E) {4 ***i*** + 5 ***j*** + 6 ***k***} m

2. A force vector, ***F,*** directed along a line defined by PQ is given by

   A) (***F***/ F) ***rPQ*** B) ***rPQ***/rPQ

   C) F(***rPQ***/rPQ) D) F(rPQ/***rPQ***)

DOT PRODUCT

Today's Objective:

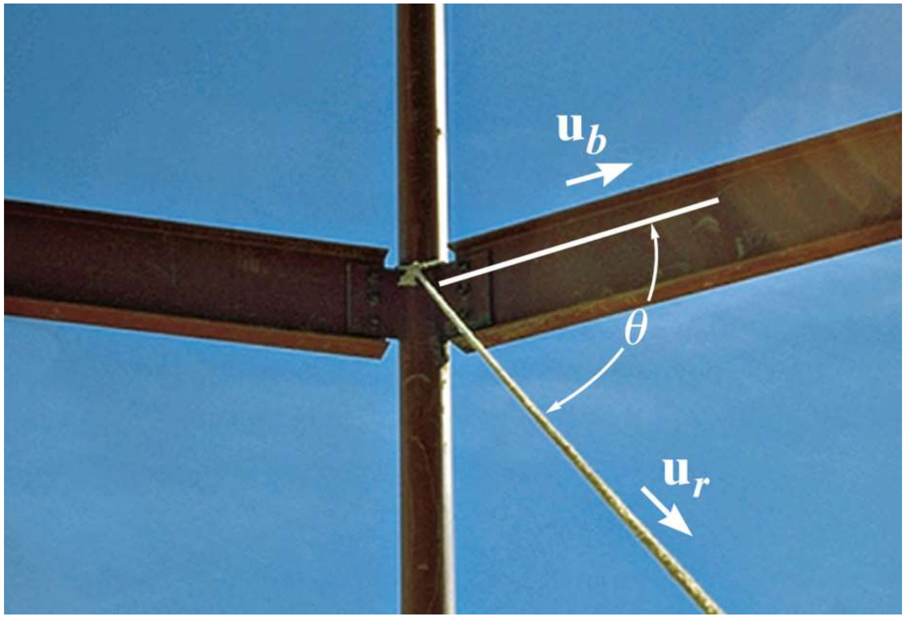
Students will be able to use the vector dot product to:

a) determine an angle between two vectors and,

b) determine the projection of a vector along a specified line.

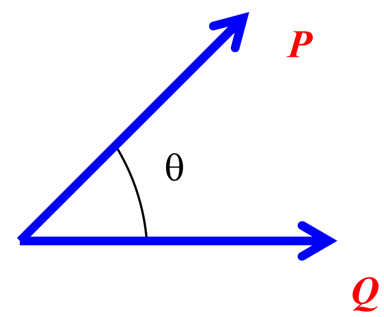
In-Class Activities:

- Check Homework
- Reading Quiz
- Applications / Relevance
- Dot product - Definition
- Angle Determination
- Determining the Projection
- Concept Quiz
- Group Problem Solving
- Attention Quiz



READING QUIZ

1. The dot product of two vectors ***P*** and ***Q*** is defined as



A) P Q sin θ B) P Q cos θ

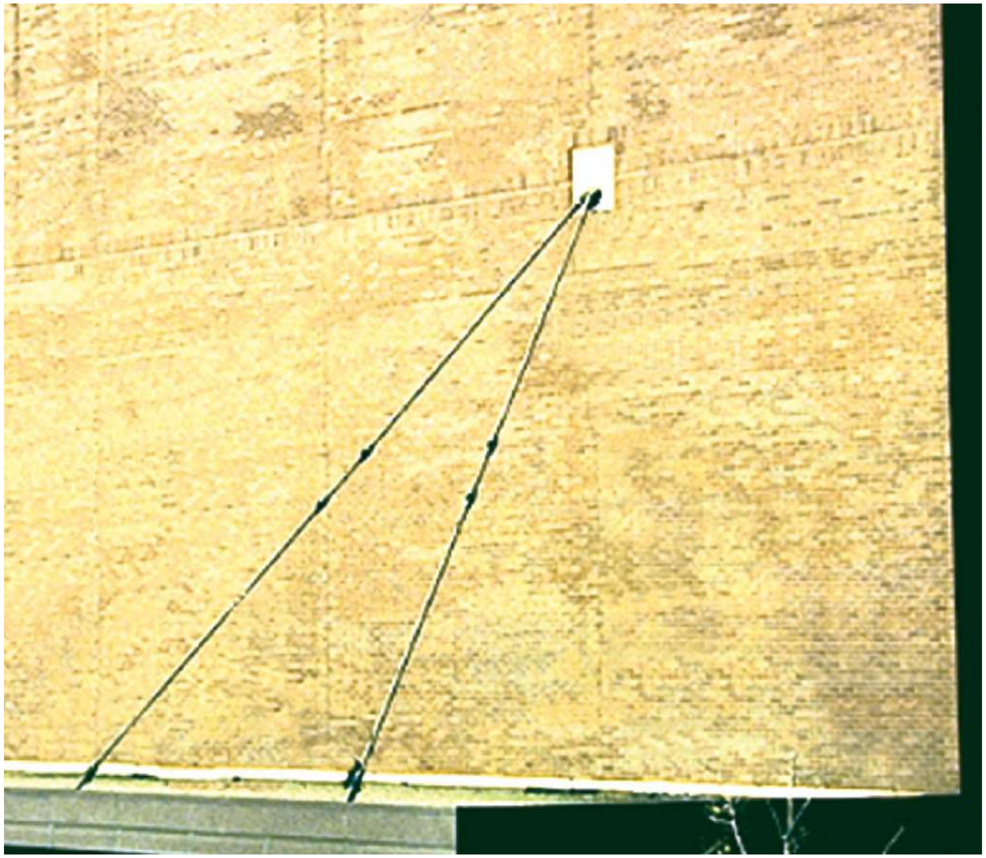
C) P Q tan θ D) P Q sec θ

2. The dot product of two vectors results in a _________ quantity.

A) Scalar B) Vector

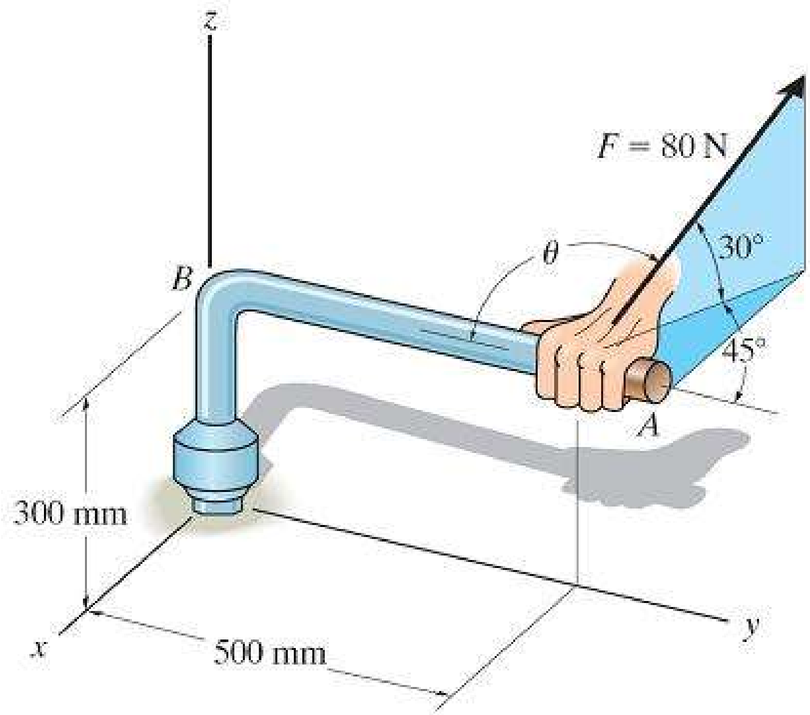
C) Complex D) Zero

APPLICATIONS



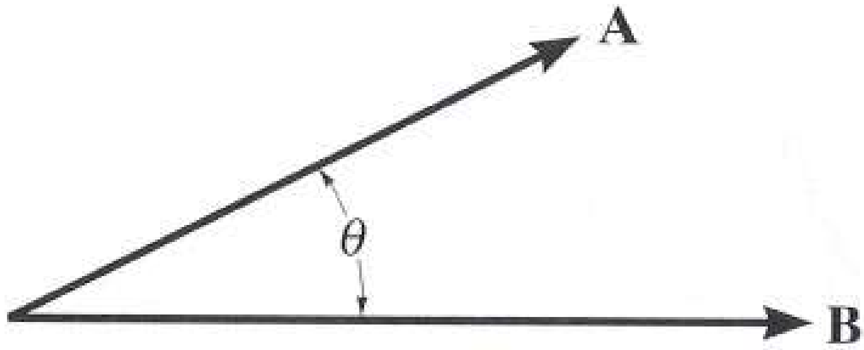
If you know the physical locations of the four cable ends, how could you calculate the angle between the cables at the common anchor?

APPLICATIONS (continued)



For the force ***F*** applied to the wrench at Point A, what component of it actually helps turn the bolt (i.e., the force component acting perpendicular to arm AB of the pipe)?

DEFINITION



The dot product of vectors ***A*** and ***B*** is defined as ***A***•***B*** = A B cos θ.

The angle θ is the smallest angle between the two vectors and is always in a range of 0^{o} to 180^{o}.

Dot Product Characteristics:

1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the ***A*** and ***B*** vectors.

DOT PRODUCT DEFINITON (continued)

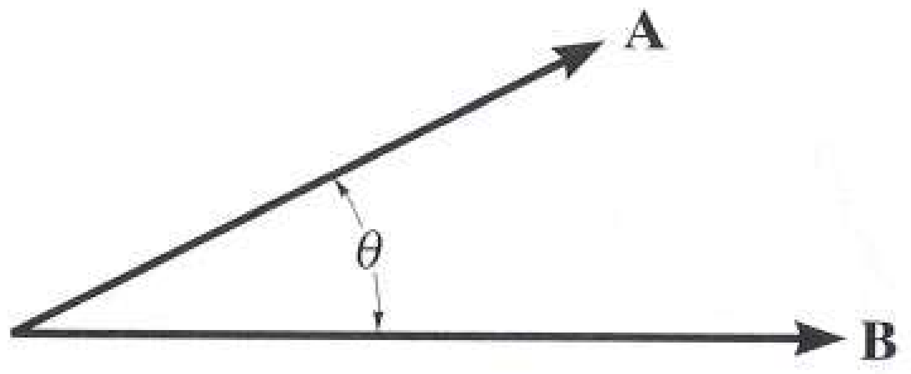
Examples: By definition, $\boldsymbol{i} \bullet \boldsymbol{j} = 0$

$$\boldsymbol{i} \bullet \boldsymbol{i} = 1$$

$$\boldsymbol{A} \bullet \boldsymbol{B} = (Ax\,\boldsymbol{i} + Ay\,\boldsymbol{j} + Az\,\boldsymbol{k}) \bullet (Bx\,\boldsymbol{i} + By\,\boldsymbol{j} + Bz\,\boldsymbol{k})$$

$$= Ax\,Bx + AyBy + AzBz$$

USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS



For these two vectors in Cartesian form, one can find the angle by

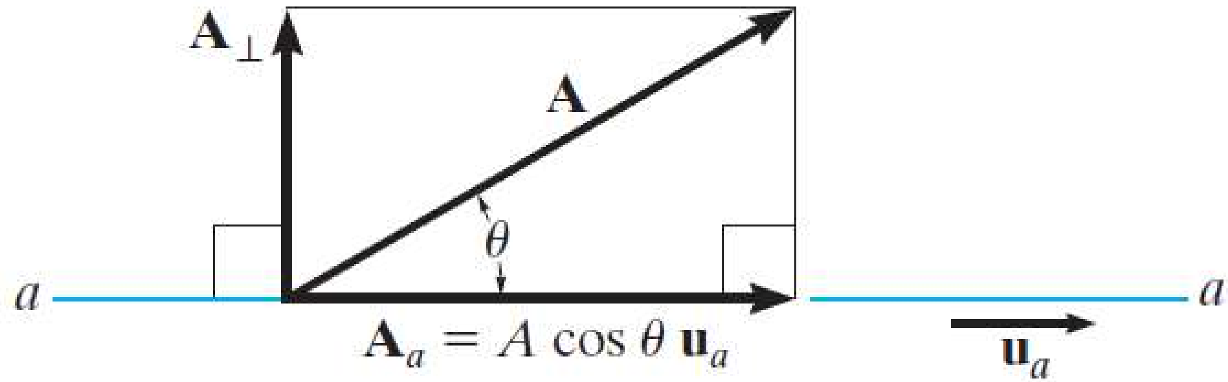
a) Find the dot product, ***A*** • ***B*** = ($A_x B_x + A_y B_y + A_z B_z$),

b) Find the magnitudes (A & B) of the vectors ***A*** & ***B***, and

c) Use the definition of dot product and solve for θ, i.e.,

$\theta = \cos^{-1}[(\mathbf{A} \bullet \mathbf{B})/(A\,B)]$, where $0^{o} \le \theta \le 180^{o}$.

DETERMINING THE PROJECTION OF A VECTOR



You can determine the components of a vector parallel and perpendicular to a line using the dot product.

Steps:

1. Find the unit vector, ***ua*** along line aa

2. Find the scalar projection of ***A*** along line aa by

$$A_{||} = \boldsymbol{A} \cdot \boldsymbol{u_a} = A_x u_x + A_y u_y + A_z u_z$$

DETERMINING THE PROJECTION OF A VECTOR (continued)

3. If needed, the projection can be written as a vector, $\boldsymbol{A}_{||}$, by using the unit vector $\boldsymbol{u_a}$ and the magnitude found in step 2.

$$\boldsymbol{A}_{||} = A_{||}\,\boldsymbol{u_a}$$

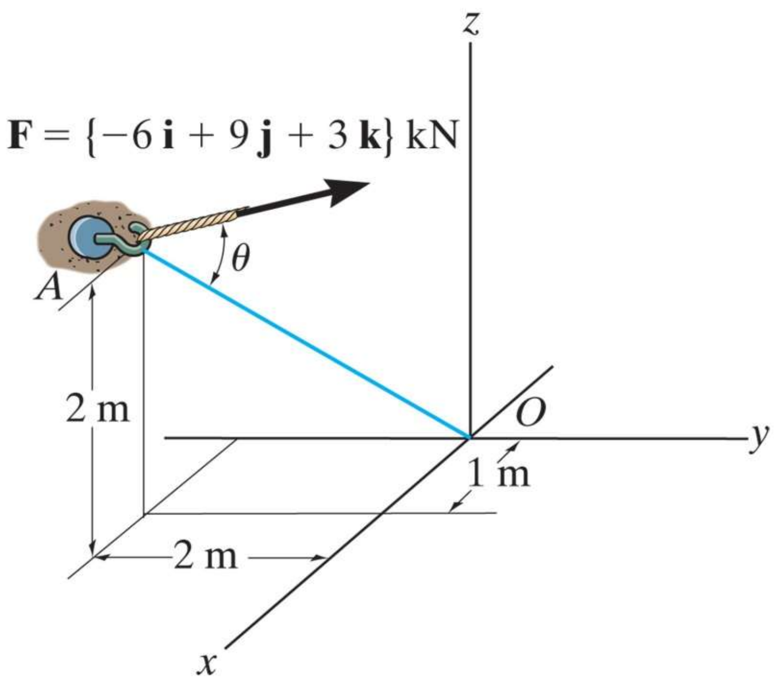
4. The scalar and vector forms of the perpendicular component can easily be obtained by

$$A_{\perp} = (A^2 - A_{||}^2)^{½} \quad \text{and}$$

$$\boldsymbol{A}_{\perp} = \boldsymbol{A} - \boldsymbol{A}_{||}$$

(rearranging the vector sum of $\boldsymbol{A} = \boldsymbol{A}_{\perp} + \boldsymbol{A}_{||}$)

EXAMPLE I



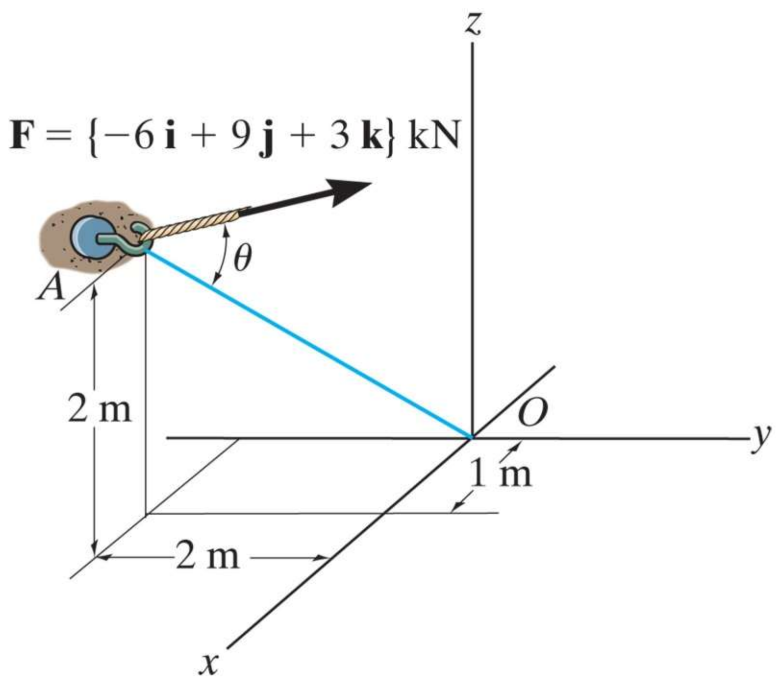
Given: The force acting on the hook at point A.

Find: The angle between the force vector and the line AO, and the magnitude of the projection of the force along the line AO.

Plan:

1. Find ***rAO***
2. Find the angle $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}AO)/(F\ \ rAO)\}$
3. Find the projection via $FAO = \boldsymbol{F} \bullet \boldsymbol{u}AO$ (or $F \cos\theta$)

EXAMPLE I (continued)



$\boldsymbol{r}_{AO} = \{-1\,\boldsymbol{i} + 2\,\boldsymbol{j} - 2\,\boldsymbol{k}\}$ m

$r_{AO} = \{(-1)^2 + 2^2 + (-2)^2\}^{1/2} = 3$ m

$\boldsymbol{F} = \{-6\,\boldsymbol{i} + 9\,\boldsymbol{j} + 3\,\boldsymbol{k}\}$ kN

$F = \{(-6)^2 + 9^2 + 3^2\}^{1/2} = 11.22$ kN

$\boldsymbol{F} \bullet \boldsymbol{r}_{AO} = (-6)(-1) + (9)(2) + (3)(-2) = 18$ kN·m

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}_{AO})/(F\, r_{AO})\}$

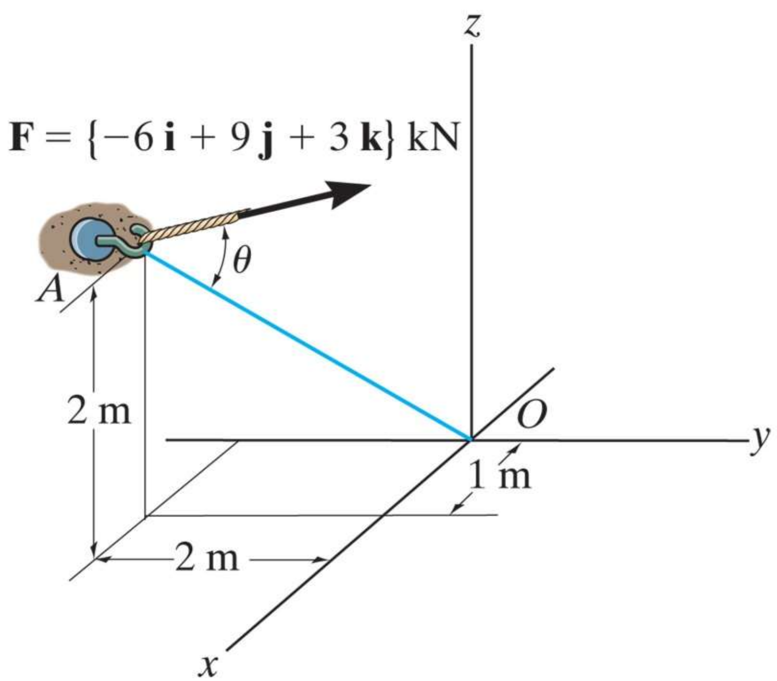
$\theta = \cos^{-1}\{18 / (11.22 \times 3)\} = \underline{57.67°}$

EXAMPLE I(continued)

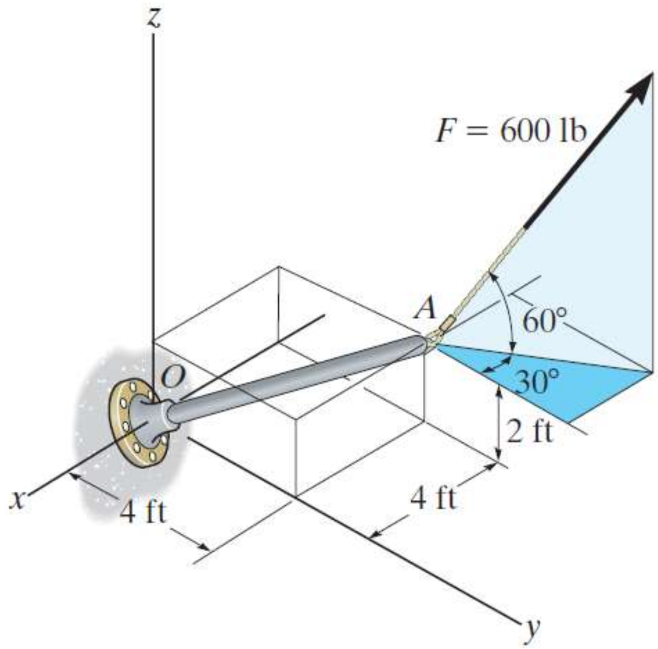
$\boldsymbol{u}_{AO} = \boldsymbol{r}_{AO} / r_{AO} = (-1/3)\,\boldsymbol{i} + (2/3)\,\boldsymbol{j} + (-2/3)\,\boldsymbol{k}$

$F_{AO} = \boldsymbol{F} \bullet \boldsymbol{u}_{AO} = (-6)(-1/3) + (9)(2/3) + (3)(-2/3) =$ 6.00 kN

Or: $F_{AO} = F \cos\theta = 11.22 \cos(57.67°) =$ 6.00 kN



EXAMPLE II



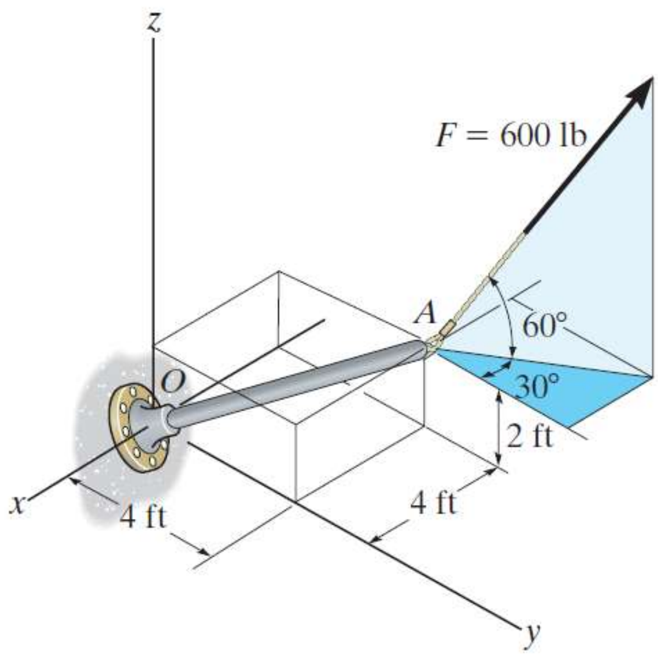
Given: The force acting on the pole at point A.

Find: The components of the force acting parallel and perpendicular to the axis of the pole.

Plan:

1. Find ***F***, ***rOA*** and ***uOA***
2. Determine the parallel component of ***F*** using $F_{||} = \mathbf{F} \bullet \mathbf{u}_{OA}$
3. The perpendicular component of ***F*** is $F_{\perp} = (F^2 - F_{||}^2)^{½}$

EXAMPLE II (continued)



$\boldsymbol{rOA} = \{-4\,\boldsymbol{i} + 4\,\boldsymbol{j} + 2\,\boldsymbol{k}\}$ m

$rOA = \{(-4)^2 + 4^2 + 2^2\}^{1/2} = 6$ m

$\boldsymbol{uOA} = -2/3\,\boldsymbol{i} + 2/3\,\boldsymbol{j} + 1/3\,\boldsymbol{k}$

$$\boldsymbol{F} = \{-(600 \cos 60°) \sin 30°\,\boldsymbol{i} + (600 \cos 60°) \cos 30°\,\boldsymbol{j} + (600 \sin 60°)\,\boldsymbol{k}\} \text{ lb}$$

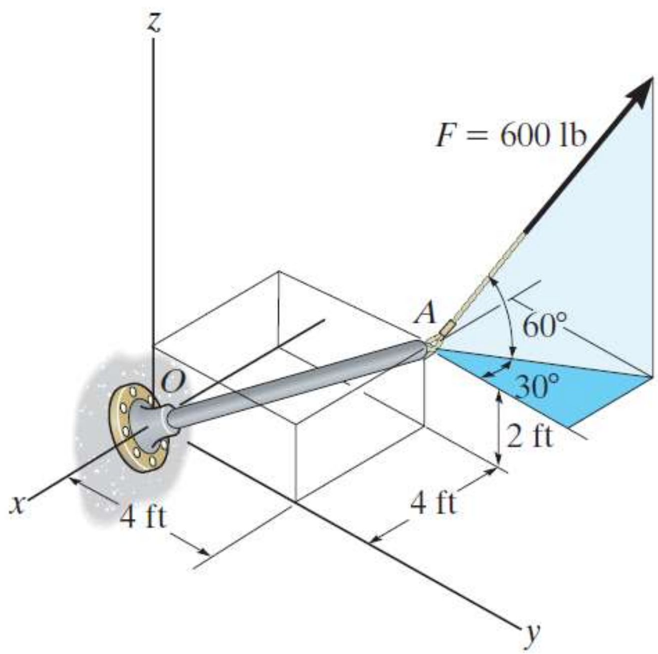
$\boldsymbol{F} = \{-150\,\boldsymbol{i} + 259.8\,\boldsymbol{j} + 519.6\,\boldsymbol{k}\}$ lb

The parallel component of ***F*** :

$F_{||} = \boldsymbol{F} \bullet \boldsymbol{uOA} = \{-150\,\boldsymbol{i} + 259.8\,\boldsymbol{j} + 519.6\,\boldsymbol{k}\} \bullet \{-2/3\,\boldsymbol{i} + 2/3\,\boldsymbol{j} + 1/3\,\boldsymbol{k}\}$

$F_{||} = (-150) \times (-2/3) + 259.8 \times (2/3) + 519.6 \times (1/3) =$ 446 lb

EXAMPLE II (continued)



F = 600 lb

F|| = 446 lb

The perpendicular component of ***F*** :

$F_{\perp} = (F^2 - F_{||}^2)^{½} = (600^2 - 446^2)^{½} = \underline{401 \text{ lb}}$

CONCEPT QUIZ

1. If a dot product of two non-zero vectors is 0, then the two vectors must be ______________ to each other.

   A) Parallel (pointing in the same direction)

   B) Parallel (pointing in the opposite direction)

   C) Perpendicular

   D) Cannot be determined.

2. If a dot product of two non-zero vectors equals -1, then the vectors must be ________ to each other.

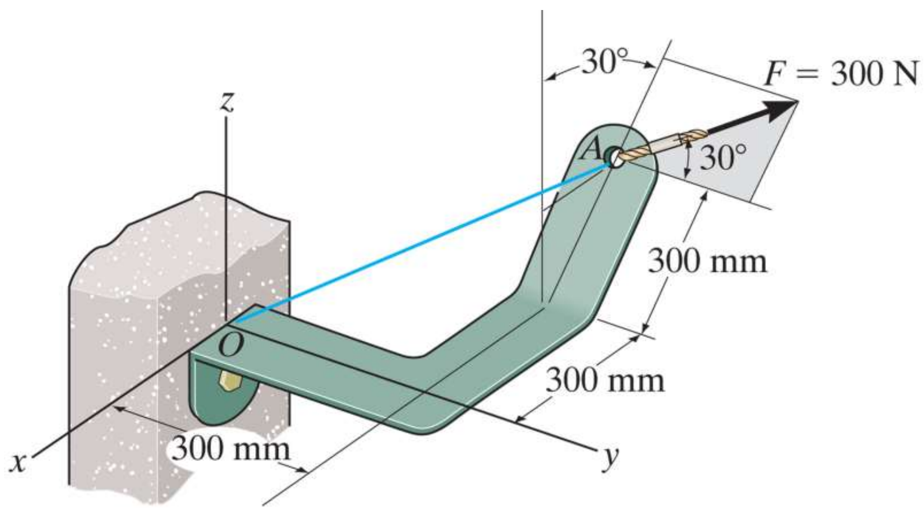
   A) Collinear but pointing in the opposite direction

   B) Parallel (pointing in the opposite direction)

   C) Perpendicular

   D) Cannot be determined.

GROUP PROBLEM SOLVING



Given: The 300 N force acting on the bracket.

Find: The magnitude of the projected component of this force acting along line OA

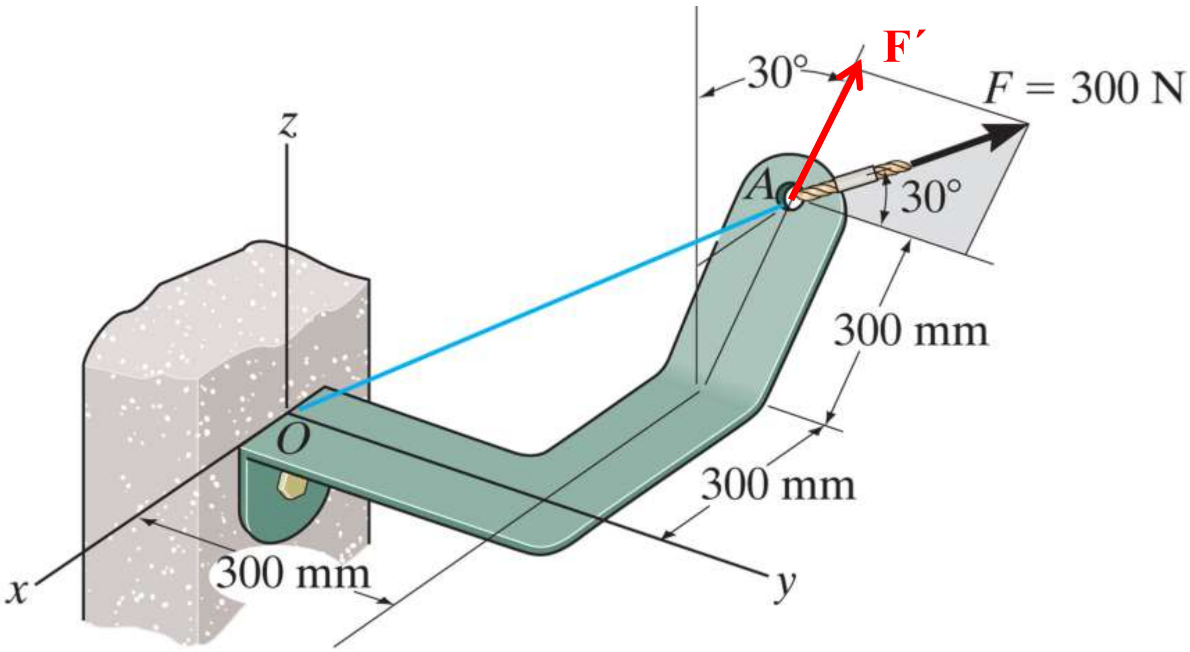
Plan:

1. Find ***r****OA* and ***u****OA*

2. Find the angle $\theta = \cos^{-1}$ {(***F*** • ***r****OA*)/(F × rOA)}

3. Then find the projection via FOA = ***F*** • ***u****OA* or F (1) cos θ

GROUP PROBLEM SOLVING (continued)

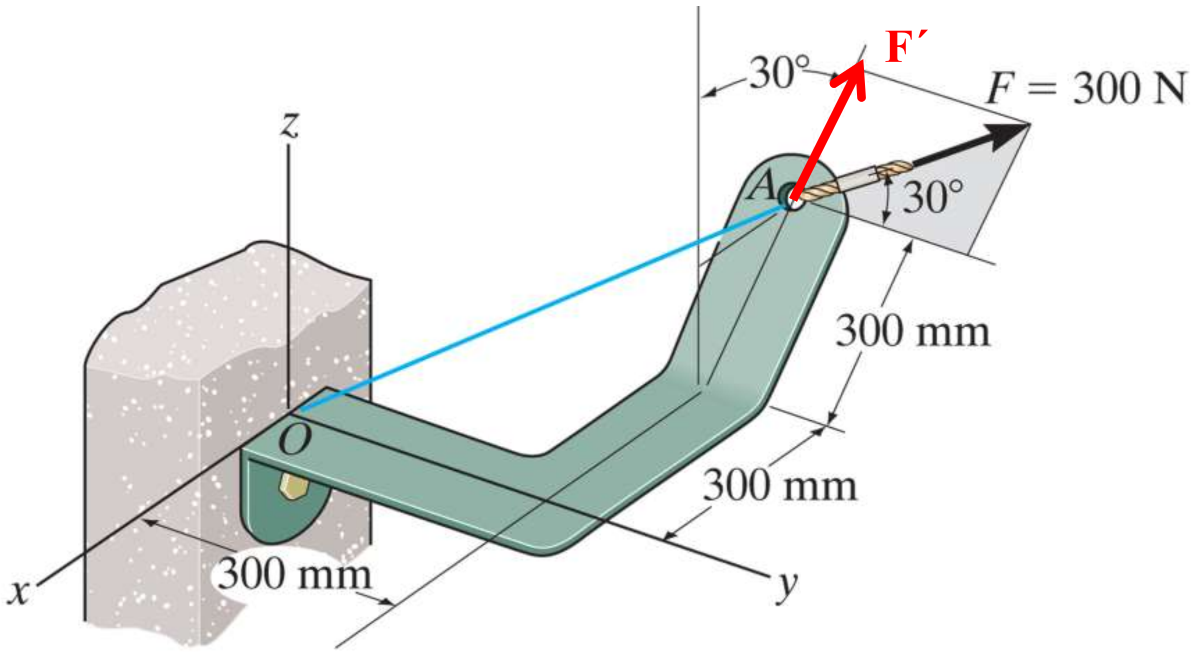


$\mathbf{r}_{OA}$ = {–0.450 $\mathbf{i}$ + 0.300 $\mathbf{j}$ + 0.260 $\mathbf{k}$} m

$r_{OA} = \{(-0.450)^2 + 0.300^2 + 0.260^2\}^{1/2} = 0.60$ m

$\mathbf{u}_{OA} = \mathbf{r}_{OA} / r_{OA}$ = {-0.75 $\mathbf{i}$ + 0.50 $\mathbf{j}$ + 0.433 $\mathbf{k}$ }

GROUP PROBLEM SOLVING (continued)



F´ = 300 sin 30° = 150 N

$\boldsymbol{F} = \{-150 \sin 30°\boldsymbol{i} + 300 \cos 30°\boldsymbol{j} + 150 \cos 30°\boldsymbol{k}\}$ N

$\boldsymbol{F} = \{-75\,\boldsymbol{i} + 259.8\,\boldsymbol{j} + 129.9\,\boldsymbol{k}\}$ N

$F = \{(-75)^2 + 259.8^2 + 129.9^2\}^{1/2} = 300$ N

GROUP PROBLEM SOLVING (continued)

$\boldsymbol{F} \bullet \boldsymbol{rOA} = (-75)(-0.45) + (259.8)(0.30) + (129.9)(0.26)$

$= 145.5 \text{ N·m}$

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{rOA})/(F \times rOA)\}$

$\theta = \cos^{-1}\{145.5/(300 \times 0.60)\} = \underline{36.1°}$

The magnitude of the projected component of ***F*** along line OA will be

$FOA = \boldsymbol{F} \bullet \boldsymbol{uOA}$

$= (-75)(-0.75) + (259.8)(0.50) + (129.9)(0.433)$

$= \underline{242 \text{ N}}$

Or

$FOA = F \cos\theta = 300 \cos 36.1° = \underline{242 \text{ N}}$

ATTENTION QUIZ

1. The dot product can be used to find all of the following except ____ .

A) sum of two vectors

B) angle between two vectors

C) component of a vector parallel to another line

D) component of a vector perpendicular to another line

2. Find the dot product of the two vectors ***P*** and ***Q***.

$\boldsymbol{P} = \{5\,\boldsymbol{i} + 2\,\boldsymbol{j} + 3\,\boldsymbol{k}\}$ m

$\boldsymbol{Q} = \{-2\,\boldsymbol{i} + 5\,\boldsymbol{j} + 4\,\boldsymbol{k}\}$ m

A) -12 m B) 12 m C) 12 m^2

D) -12 m^2 E) 10 m^2