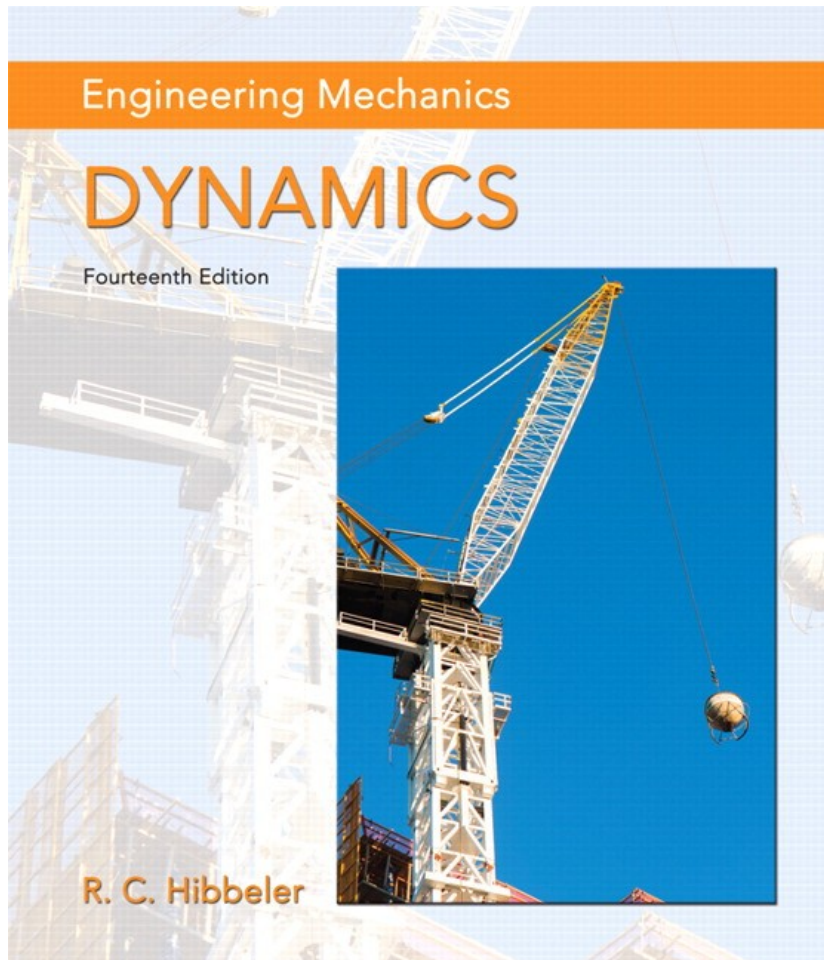


Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 15

Kinetics of a Particle:
Impulse and
Momentum

Angular Momentum, Moment of a Force and Principle of Angular Impulse and Momentum (1 of 2)

Today's Objectives:

Students will be able to:

1. Determine the angular momentum of a particle and apply the principle of angular impulse & momentum.
2. Use conservation of angular momentum to solve problems.



Angular Momentum, Moment of a Force and Principle of Angular Impulse and Momentum (2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Angular Momentum
- Angular Impulse and Momentum Principle
- Conservation of Angular Momentum
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1. Select the correct expression for the angular momentum of a particle about a point

A) $r \times v$

B) $r \times (mv)$

C) $v \times r$

D) $(mv) \times r$

2. The sum of the moments of all external forces acting on a particle is equal to

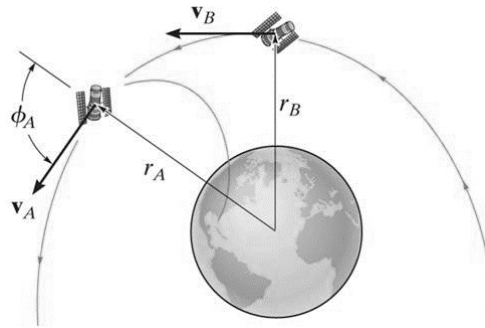
A) angular momentum of the particle.

B) linear momentum of the particle.

C) time rate of change of angular momentum.

D) time rate of change of linear momentum.

Applications (1 of 2)



Planets and most satellites move in elliptical orbits. This motion is caused by gravitational attraction forces. Since these forces act in pairs, the sum of the moments of the forces acting on the system will be zero. This means that angular momentum is conserved.

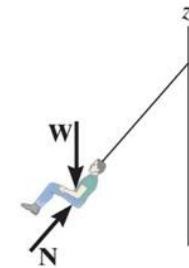
If the angular momentum is constant, does it mean the linear momentum is also constant? Why or why not?

Applications (2 of 2)

The passengers on the amusement-park ride experience conservation of angular momentum about the axis of rotation (the z -axis). As shown on the free body diagram, the line of action of the normal force, N , passes through the z -axis and the weight's line of action is parallel to it.

Therefore, the sum of moments of these two forces about the z -axis is zero.

If the passenger moves away from the z -axis, will his speed increase or decrease? Why?

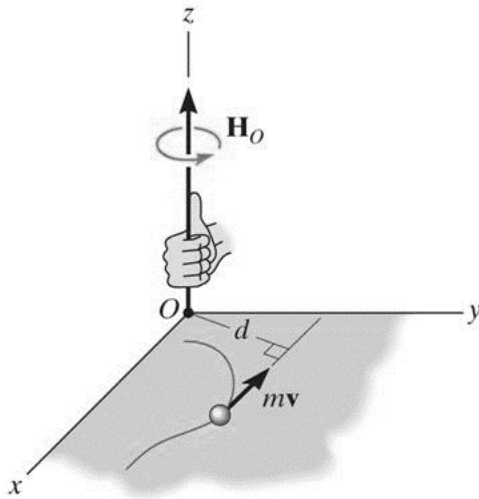


Section 15.5

Angular Momentum

Angular Momentum

The angular momentum of a particle about point O is defined as the “moment” of the particle’s linear momentum about O.



$$H_O = r \times mv = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

The magnitude of H_O is $(H_O)_z = mvd$

Section 15.6

Relationship Between Moment of a Force and Angular Momentum

Relationship Between Moment of a Force and Angular Momentum

The resultant force acting on the particle is equal to the time rate of change of the particle's linear momentum. Showing the time derivative using the familiar “dot” notation results in the equation

$$\Sigma F = \dot{L} = m \dot{v}$$

We can prove that the resultant moment acting on the particle about point O is equal to the time rate of change of the particle's angular momentum about point O or

$$\Sigma M_O = r \times F = \dot{H}_O$$

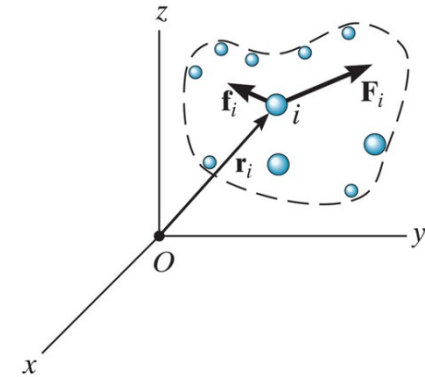
Moment and Angular Momentum of a System of Particles

$$\sum M_o = r \times F = \dot{H}_o$$

The same form of the equation can be derived for the system of particles.

The forces acting on the i-th particle of the system consist of a resultant external force

F_i and a resultant internal force f_i



Then, the moments of these forces for the particles can be written as

$$\sum (r_i \times F_i) + \sum (r_i \times f_i)$$

The second term is zero since the internal forces occur in equal but opposite collinear pairs. Thus,

$$\sum M_o = \sum (r_i \times F_i) = \sum \left(\dot{H}_i \right)_o$$

Section 15.7

Principle of Angular Impulse and Momentum

Principle of Angular Impulse and Momentum

Considering the relationship between moment and time rate of change of angular momentum

$$\sum M_o = \dot{H}_o = dH_o / dt$$

By integrating between the time interval

$$\sum \int_{t_1}^{t_2} M_o dt = (H_o)_2 - (H_o)_1 \quad \text{or} \quad (H_o)_1 + \sum \int_{t_1}^{t_2} M_o dt = (H_o)_2$$

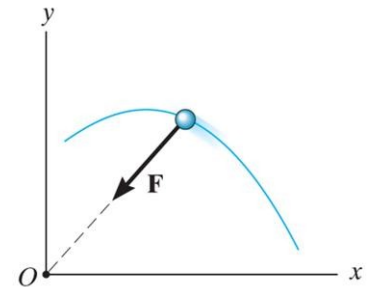
This equation is referred to as the **principle of angular impulse and momentum**. The second term on the left side $\sum \int M_o dt$, is called the **angular impulse**. In cases of 2D motion, it can be applied as a scalar equation using components about the z-axis.

Conservation of Angular Momentum

When the sum of angular impulses acting on a particle or a system of particles is zero during the time t_1 to t_2 the angular momentum is conserved. Thus,

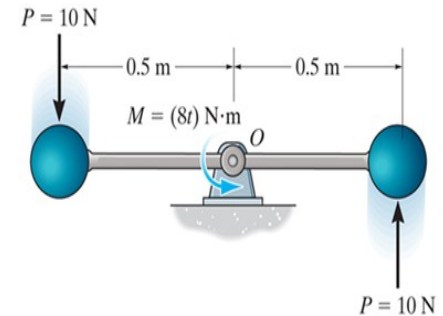
$$(H_o)_1 = (H_o)_2$$

An example of this condition occurs when a particle is subjected only to a central force. In the figure, the force \mathbf{F} is always directed toward point O. Thus, the angular impulse of \mathbf{F} about O is always zero, and angular momentum of the particle about O is conserved.



Example (1 of 2)

Given: Two identical 10-kg spheres are attached to the rod, which rotates in the plane of the page. The spheres are subjected to tangential forces of $P = 10\text{ N}$, and the rod is subjected to a couple moment $M = (8t)\text{ N}\cdot\text{m}$, where t is in seconds.



Find: The speed of the spheres at $t = 4\text{ s}$, if the system starts from rest.

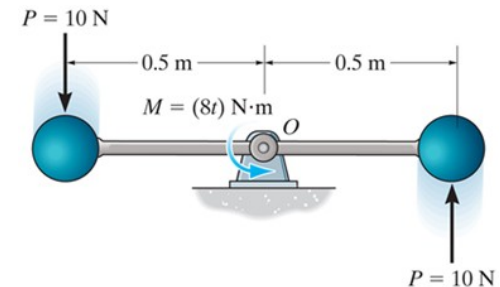
Plan: Apply the principles of conservation of energy and conservation of angular momentum to the system.

Example (2 of 2)

Solution:

Conservation of angular momentum :

$$\Sigma (H_0)_1 + \Sigma \int_{t_1}^{t_2} M_0 dt = \Sigma (H_0)_2$$



The above equation about the axis of rotation (z-axis) through O can be written as

$$0 + \int_0^4 8t \, dt + \int_0^4 [2(10)(0.5)] dt = 2[10v(0.5)]$$

$$\Rightarrow 4(4)^2 + 2(5)4 = 10v$$

$$\Rightarrow 104 = 10v$$

$$v = 10.4 \text{ m/s}$$

Concept Quiz

1. If a particle moves in the $x - y$ plane, its angular momentum vector is in the

A) x direction

B) y direction

C) z direction

D) $x - y$ direction

2. The sum of the moments of all external forces acting on a particle is equal to

A) only linear momentum is conserved.

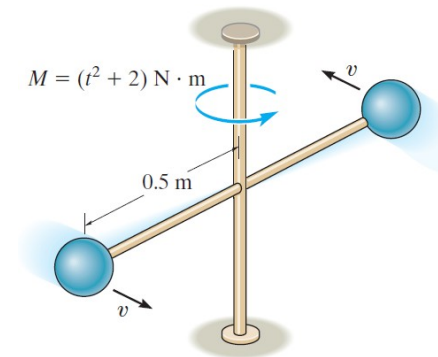
B) only angular momentum is conserved.

C) both linear momentum & angular momentum are conserved.

D) neither linear momentum nor angular momentum are conserved.

Group Problem Solving (1 of 2)

Given: The two 10 kg balls are attached to the end of a rod of negligible weight. A torque acts on the rod as shown, $M = (t^2 + 2) \text{ N} \cdot \text{m}$.



Find: The velocity of each ball after 3 seconds, if each ball has a speed $v = 2 \text{ m/s}$ when $t = 0$

Plan: Apply the principle of angular impulse and momentum about the axis of rotation (z-axis).

Group Problem Solving (2 of 2)

Solution

Angular momentum: $H_z = r \times mv$ reduces to a scalar equation

$$(H_z)_1 = 2 \times \{0.5(10)2\} = 20 (\text{kg} \cdot \text{m}^2 / \text{s}) \text{ and}$$

$$(H_z)_2 = 2 \times \{0.5(10)v\} = 10v (\text{kg} \cdot \text{m}^2 / \text{s})$$

Angular impulse:

$$\int_{t_1}^{t_2} M dt = \int_{t_1}^{t_2} (t^2 + 2) dt = \left[(1/3)t^3 + 2t \right]_0^3 = 15 \text{ N.m.s}$$

Apply the principle of angular impulse and momentum.

$$(H_0)_1 + \sum \int_{t_1}^{t_2} M_0 dt = (H_0)_2$$

$$20 + 15 = 10v \Rightarrow v = 3.5 \text{ m/s}$$

Attention Quiz

1. A ball is traveling on a smooth surface in a 3 feet radius circle with a speed of 6 ft/s . If the attached cord is pulled down with a constant speed of 2 ft/s which of the following principles can be applied to solve for the velocity of the ball when $r = 2$ feet?
A) Conservation of energy
B) Conservation of angular momentum
C) Conservation of linear momentum
D) Conservation of mass
2. If a particle moves in the $z - y$ plane, its angular momentum vector is in the
A) x direction. B) y direction.
C) z direction. D) $z - y$ direction.

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