Lecture 3

November 9, 2023 8:09 PM

Nodal and Mesh Analysis and Circuit Theorems

Circuit Analysis

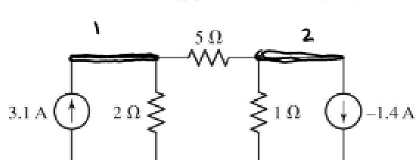
As circuits get more complicated, we need an organized method of applying KVL, KCL, and Ohm's Law

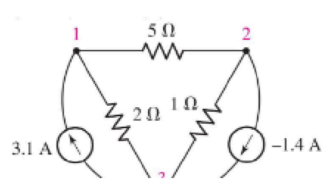
Nodal analysis assigns voltages to each node, and then we apply KCL

Mesh analysis assigns currents to each mesh, and then we apply KVL

The Nodal Analysis Method

Label all nodes in the circuit





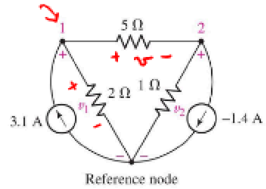
Choosing the Reference Node

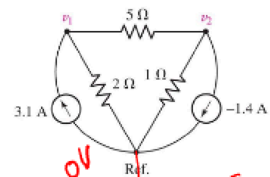
1. Make one of the node as a reference node

As the bottom node, or As the ground connection, if there is one, or A node with many connections

non-reference

$v_1 \rightarrow$ element voltage $= V_{10} = V_1 - 0 = V_1$

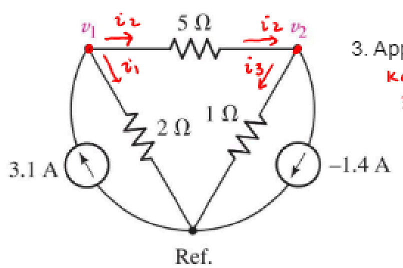




$v \rightarrow$ element voltage $= V_{12} = V_{10} - V_{20} = V_1 - V_2$

2. Assign voltages relative to the reference node

Apply KCL at each non-reference node to Find Voltages



3. Apply KCL at each non reference node

KCL @ node v_1

$3.1 - i_1 - i_2 = 0$ ---- (1)

KCL @ node v_2

$+i_2 - i_3 - (-1.4) = 0$

$i_2 - i_3 + 1.4 = 0$ --- (2)

V_a a, 10^V, V_b b

$v_{ab} = 10^V$

$10 = V_a - V_b$

4. Express the branch currents in terms of the node voltages

v_1, i_1, 2Ω, $+ v -$, 0^V

$i_1 = \frac{v}{2} = \frac{v^+ - v^-}{2} = \frac{V_1 - 0}{2} = \frac{V_1}{2}$

V_1 i_2 5^Ω i_2 V_2, $+ v_x -$

$i_2 = \frac{v_x}{5} = \frac{v_x^+ - v_x^-}{5} = \frac{V_1 - V_2}{5}$

V_2, i_3, 1Ω, 0^V

$i_3 = \frac{V_2 - 0}{1} = \frac{V_2}{1} = V_2$

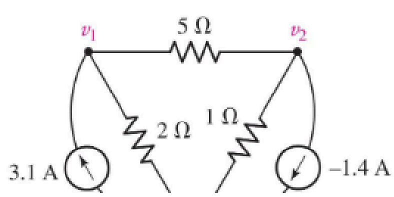
Apply KCL at each non-reference node to Find Voltages

$i_x = \frac{V_2 - V_1}{R}$ $\quad i = \frac{V_1 - V_2}{R}$

V_1 i R i_x V_2

i = (Voltage of the node which the current is leaving)

− (Voltage of the node which



the current is entering into)

R

① $3.1 - \frac{V_1}{2} - \frac{(V_1 - V_2)}{5} = 0$

$\left(-\frac{1}{2} - \frac{1}{5}\right)V_1 + \frac{V_2}{5} = -3.1 \Big] \times 10$

$(-5-2)V_1 + 2V_2 = -31$

$-7V_1 + 2V_2 = -31 \quad ---①$

$\frac{V_1 - V_2}{5} - \frac{V_2}{1} + 1.4 = 0$

$\frac{1}{5}V_1 + \left(-\frac{1}{5} - \frac{1}{1}\right)V_2 = -1.4 \Big\} \times 5$

$V_1 + (-1-5)V_2 = -7$

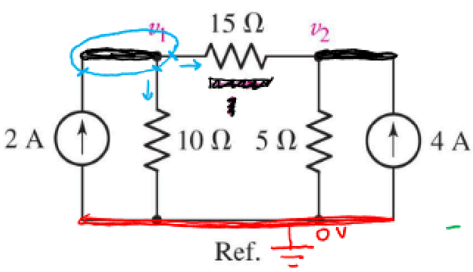
$V_1 - 6V_2 = -7 \quad ---②$

5. Solve the equations using the method of substitution or the matrices to calculate the node voltages

$V_1 = 5V \quad V_2 = 2V$

Example: Nodal Analysis

Find the current *i* in the circuit.



KCL @ node $v_1 \Rightarrow -2^A + \left(\frac{v_1 - 0}{10}\right) + \left(\frac{v_1 - v_2}{15}\right) = 0$

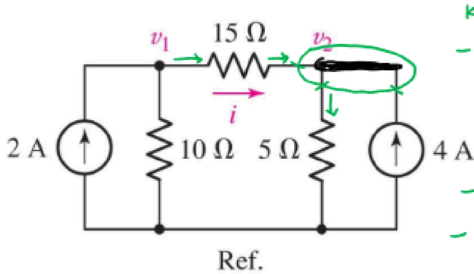
$\left(\frac{1}{10} + \frac{1}{15}\right)v_1 - \frac{1}{15}v_2 = 2 \Big\} \times 150$

$(15+10)v_1 - 10v_2 = 300$

$25v_1 - 10v_2 = 300 \quad ---①$

Example: Nodal Analysis

Find the current *i* in the circuit.



KCL @ node v_2

$-4^A + \left(\frac{v_2 - 0}{5}\right) - \left(\frac{v_1 - v_2}{15}\right) = 0$

$-\frac{1}{15}v_1 + \left(\frac{1}{5} + \frac{1}{15}\right)v_2 = 4 \Big\} \times 15$

$-v_1 + (3+1)v_2 = 60$

$-1v_1 + 4v_2 = 60 \quad ---②$

$\boxed{\begin{array}{l} v_1 = 20^V \\ v_2 = 20^V \end{array}}$

$i = \frac{v_1 - v_2}{15} = \frac{0}{15} = 0^A$

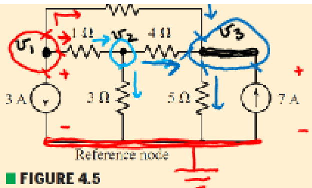
Example: Nodal Analysis

PRACTICE

4.2 For the circuit of Fig. 4.5, compute the voltage across each current source.

KCL @ node v_1

2 Ω



■ FIGURE 4.5

Ans: v_{3A} = 5.235 V; v_{7A} = 11.47 V.

$+3^A + \left(\frac{v_1-v_2}{1}\right)+\left(\frac{v_1-v_3}{2}\right)=0$

$\left(1+\frac{1}{2}\right)v_1 - v_2 - \frac{1}{2}v_3 = -3 \,]\times 2$

$3v_1 - 2v_2 - 1v_3 = -6$ --- (1)

KCL @ node v_2

$-\left(\frac{v_1-v_2}{1}\right)+\frac{v_2-v_3}{4}+\frac{v_2-0}{3}=0 \;\}\times 12$

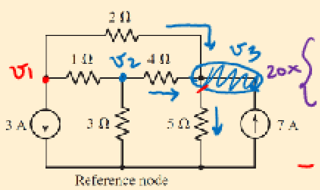
$-12(v_1-v_2)+3(v_2-v_3)+4v_2=0$

$-12v_1+19v_2-3v_3=0$ --- (2)

Example: Nodal Analysis

PRACTICE

4.2 For the circuit of Fig. 4.5, compute the voltage across each current source.



■ FIGURE 4.5

Ans: v_{3A} = 5.235 V; v_{7A} = 11.47 V.

KCL @ node v_3

$20\times\left\{-7^A - \left(\frac{v_1-v_3}{2}\right)-\left(\frac{v_2-v_3}{4}\right)+\frac{v_3}{5}=0\right.$

$-10(v_1-v_3)-5(v_2-v_3)+4v_3=140$

$-10v_1-5v_2+19v_3=140$ --- (3)

$v_1 = 5.24^V$

$v_2 = 5.11^V$

$v_3 = 11.47^V$

voltage across 3^A current source $= v_1 = 5.24^V$

Voltage across 7^A current source $v_3 = 11.47^V$

Mesh Analysis: Nodal Alternative

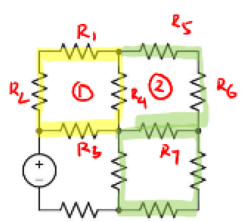
A mesh is a loop which does not contain any other loops within it

In mesh analysis, we assign currents and solve using KVL

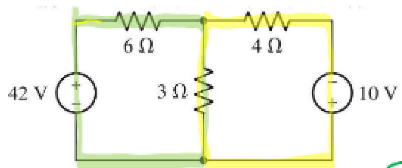
Assigning mesh currents automatically ensures KCL is followed

This circuit has four meshes:

closed path > 10



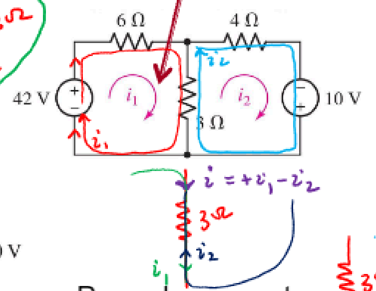
The Mesh Analysis Method

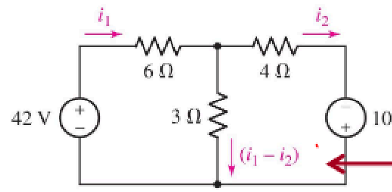


Mesh currents

mesh 1 elements: 42^V, 6Ω, 3Ω

mesh 2 elements: 10^V, 4Ω, 3Ω



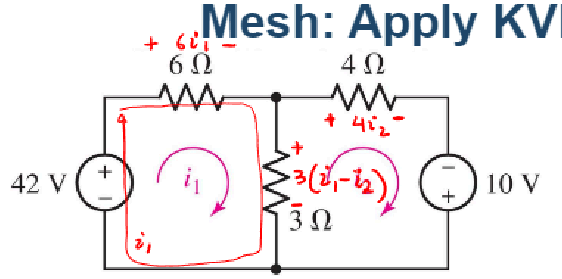


Branch currents

$-i_1 + i_2 = I$

Mesh: Apply KVL

Voltage rise → +
Voltage drop → −



KVL @ mesh 1

$+42^V - 6i_1 - 3(i_1 - i_2) = 0$

$-9i_1 + 3i_2 = -42$ --- (1)

KVL @ mesh 2

$-4i_2 + 10^V + 3(i_1 - i_2) = 0$

$3i_1 - 7i_2 = -10$ --- (2)

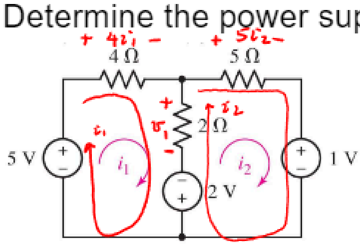
$i_1 = 6^A$

$i_2 = 4^A$

Solve the equations using the method of substitution or the matrices to calculate the ~~node voltages~~ mesh currents

Example: Mesh Analysis

Determine the power supplied by the 2 V source.



$v_1 = 2(i_1 - i_2)$

KVL @ mesh 2 :

$-1^V - 2^V + 2(i_1 - i_2) - 5i_2 = 0$

$2i_1 - 7i_2 = 3$ --- (2)

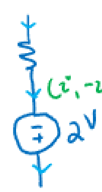
$i_1 = 1.131^A$

$i_2 = -0.105^A$

KVL @ mesh 1 :

$+5^V - 4i_1 - 2(i_1 - i_2) + 2^V = 0$

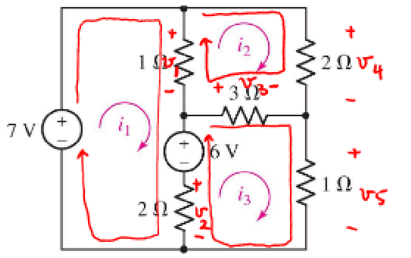
$-6i_1 + 2i_2 = -7$ --- (1)



$(i_1 - i_2) = (1.131 + 0.105)$
$= 1.236^A$

$P_{2V} = 2 \times 1.236 = -2.47^W$ (S)

Example - Mesh Analysis



$v_1 = 1(i_1 - i_2)$ $\quad v_2 = 2(i_1 - i_3)$

$v_3 = 3(i_3 - i_2)$ $\quad v_4 = 2i_2$

$v_5 = 1 i_3$

KVL @ mesh 3

$+6^V - 3(i_3 - i_2) - 1i_3 + 2(i_1 - i_3) = 0$

$2i_1 + 3i_2 - 6i_3 = -6$ -- 3

KVL @ mesh 1

$+7^V - 1(i_1 - i_2) - 6^V - 2(i_1 - i_3) = 0$

$-3i_1 + i_2 + 2i_3 = -1$ -- 1

KVL @ mesh 2:

$-2i_2 + 3(i_3 - i_2) + 1(i_1 - i_2) = 0$

$1i_1 - 6i_2 + 3i_3 = 0$ --- 2

$i_1 = 3A$

$i_2 = 2A$

$i_3 = 3A$

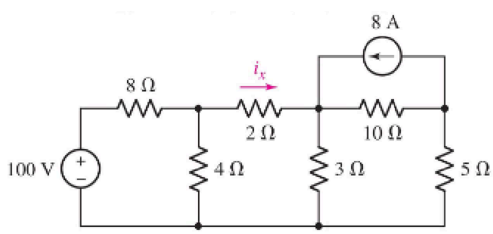
Node or Mesh: How to Choose?

Use the one with fewer equations, or

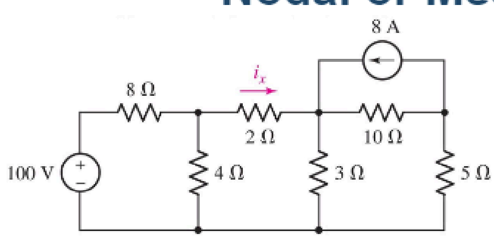
Use the method you like best, or

Use both (as a check), or

Use circuit simplifying methods from the next chapter



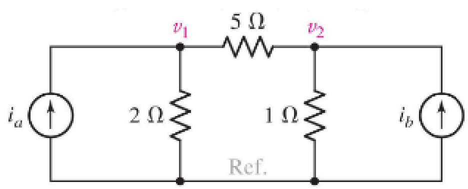
Nodal or Mesh



The Superposition Principle

Chapter 5 : Textbook

For the circuit shown, the equations can be written as:



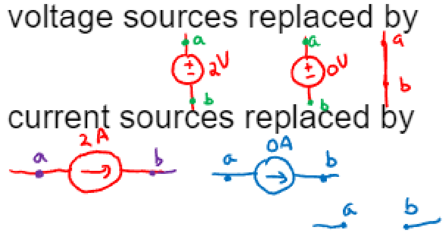
Question: How much of v_1 is due to source *a*, and how much is because of source *b*?

We use the superposition principle to answer.

The Superposition Theorem

In a linear network, the **voltage across** or the **current through** any element may be calculated by *adding algebraically* all the individual voltages or currents caused by the separate independent sources acting “alone”, that is, with

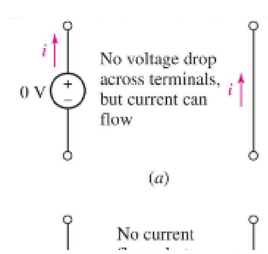
- all other independent voltage sources replaced by short circuits
- all other independent current sources replaced by open circuits



Applying Superposition

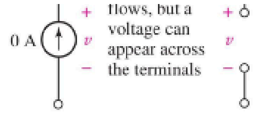
Leave one source ON and turn all other sources OFF:

- voltage sources: *set v*=0.
- These become *short circuits*.
- current sources: *set i*=0.
- These become *open circuits*.



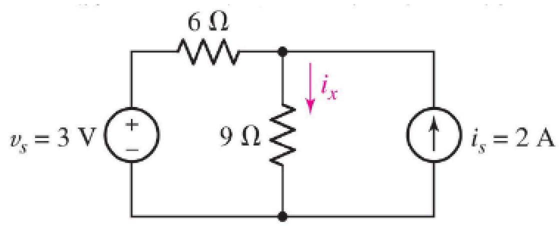
- *Find the response from this source.*

Add the resulting responses to find the total response.



Superposition Example

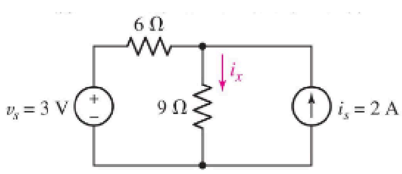
Use superposition to solve for the current i_x



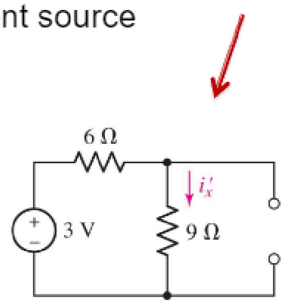
① consider only 3^V source. $2^A \to$ OFF $\to$ open

$i_{x_1} = \frac{3^V}{(6+9)} = \frac{1}{5}$ A

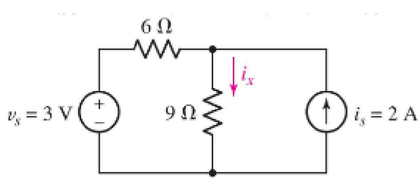
Superposition Example



First, only considering the voltage source and turn OFF the current source



Superposition Example



only consider the current source.

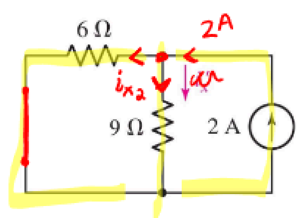
$3^V \to$ OFF $\to$ short

Then, only considering the current source and turn

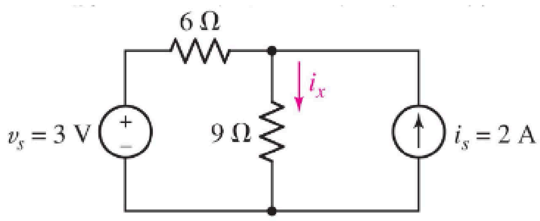
Then, only considering the current source and turn OFF the voltage source

current division

$$i_{x2} = \frac{6}{(6+9)}(2) = \frac{12}{15}\text{ A}$$



Superposition Example



Finally, combine the results:

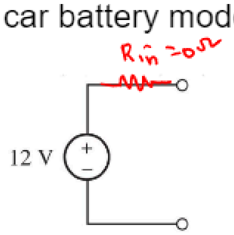
$$i_x = +i_{x1} + i_{x2} = \frac{1}{5} + \frac{12}{15} = \frac{1}{5} + \frac{4}{5} = \frac{5}{5} = 1\text{ A}$$

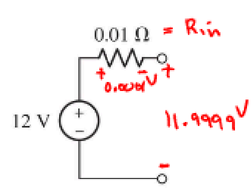
Practical Voltage Sources

Ideal voltage sources: a first approximation model for a battery.

Why do real batteries have a current limit and experience voltage drop as current increases?

Two car battery models:

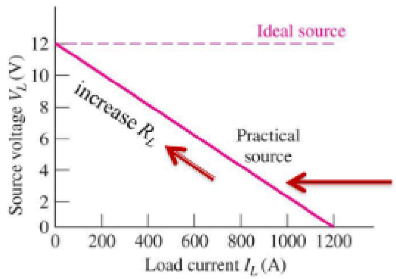


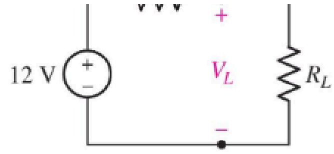


Practical Source: Effect of Connecting a Load

For the car battery example:

$$V_L = 12 - 0.01\, I_L$$

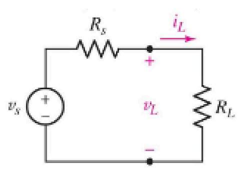


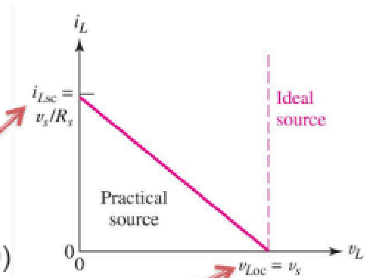


This line represents all possible R_L

Practical Voltage Source

The source has an internal resistance or output resistance, which is modeled as R_s



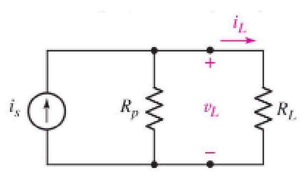


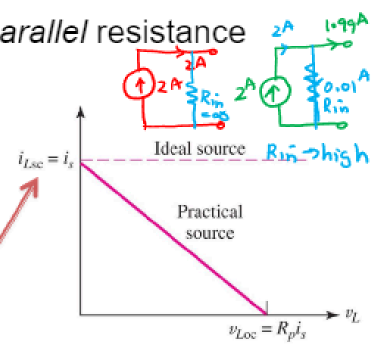
short circuit current (when R_L=0)

open circuit voltage (when R_L=∞)

Practical Current Source

The source has an internal *parallel* resistance which is modeled as R_p





short circuit current (when RL=0)

open circuit voltage (when R_L=∞)

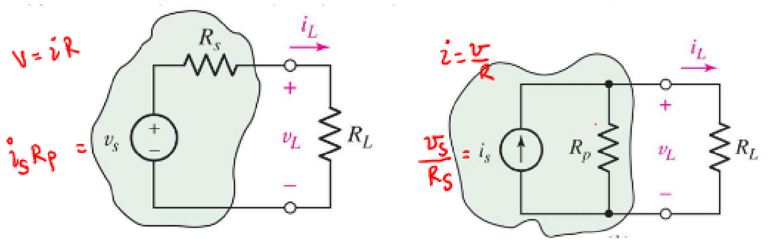
Source Transformation and

Equivalent Sources

The sources are equivalent if

$R_s = R_p$ and $v_s = i_s R_s$

$R_s = 0$
$R_p = \infty$
not possible



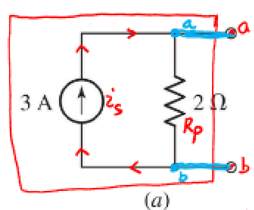
Source Transformation

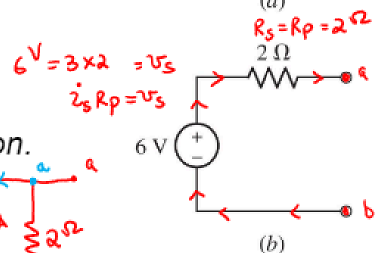
The circuits (a) and (b) are equivalent at the terminals.

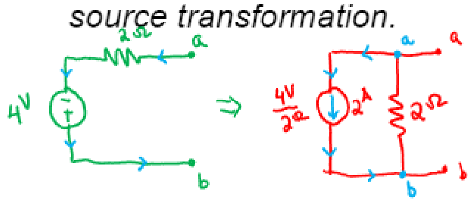
If given circuit (a), but circuit (b) is more convenient, switch them!

This process is called

source transformation.

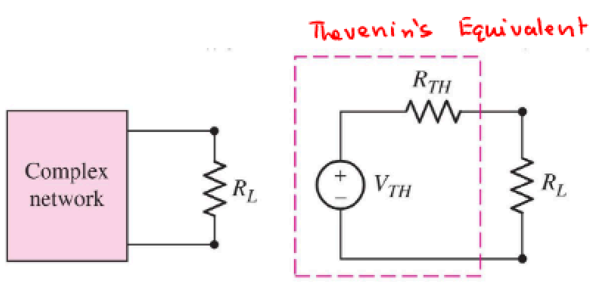






Thévenin' Theorem

Thévenin's theorem: a linear network can be replaced by its Thévenin equivalent circuit, as shown below:



Finding the Thévenin Equivalent

Disconnect the load.

Find the open circuit voltage v_{oc}

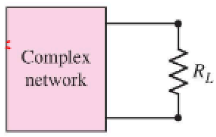
Find the equivalent resistance R_{eq} of the network with all independent sources turned off.

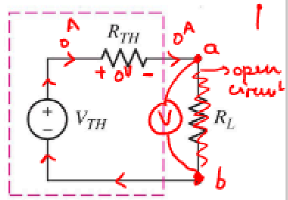
voltage source →
current source →

Then:

$$V_{TH} = v_{oc} \text{ and}$$

$$R_{TH} = R_{eq}$$

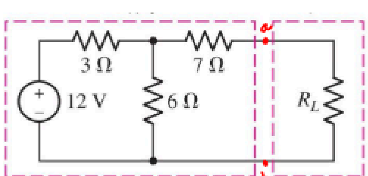




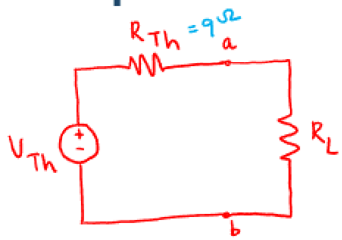
Voltmeter

$V_{oc} = V_{ab} = V_{th}$

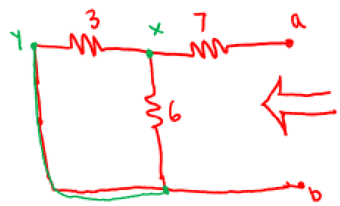
Thévenin Example



Network A Network B



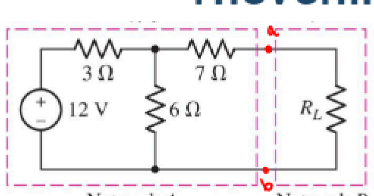
R_{Th} → Turn off 12V source



$R_{eq} = 7 + (3 // 6)$

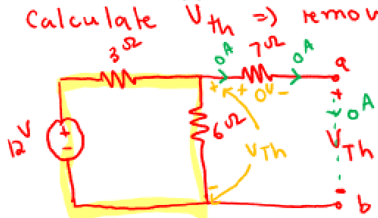
$= (3 // 6) + 7 = 2 + 7 = 9\Omega$

Thévenin Example

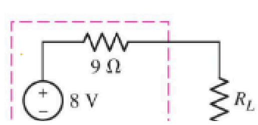


Network A Network B

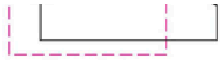
Calculate V_{th} => remove R_L (open a-b)



Voltage division



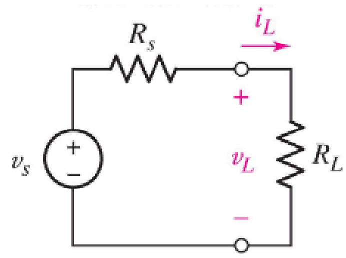
$V_{Th} = \frac{6}{(6+3)}(12) = \frac{6}{9} \times 12 = 8^V$



Network A

Maximum Power Transfer

What load resistor will allow the practical source to deliver the maximum power to the load?



Answer: $R_L = R_s$

$\left[\text{Solve } dp_L / dp_L = 0.\right]$

$\left[\text{Or: } p_L = i\left(v_s - iR_s\right), \text{set } dp_L / di = 0 \text{ to find } i_{max} = v_s / 2R_s. \text{ Hence } R_L = R_s\right]$