HUMBER ENGINEERING

ENGI 1000 - Physics 1

WEEK 5 - MODULE 4

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Module 4

Applications of Newton’s Laws

- Analysis Models Using Newton’s Second Law
  - The Particle in Equilibrium
  - The Particle Under a Net Force
- Force of Friction
  - Force of Static Friction
  - Force of Kinetic Friction
  - Coefficient of Friction

What We Already Know?

- Displacement, velocity, and acceleration are **vector** quantities.
- **Force** is also a vector quantity
- The **gravitational force** exerted on any object is its mass multiplied by the acceleration due to gravity.

$$\boxed{F_g = mg}$$

- Its magnitude is called the object's Weight.
- **Weight** on Earth is different from on the moon and on other planets.
- The **Mass** of an object is the **same everywhere**.

What We Already Know?

- **Newton's Laws of Motion:**

1. In the absence of external forces and when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).

2. When viewed from an inertial reference frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass:

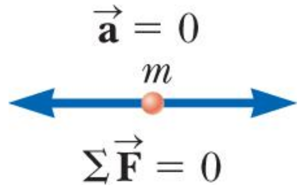
$$|\vec{\mathbf{a}}| \propto \frac{\sum \vec{\mathbf{F}}}{m} \qquad \boxed{\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}}$$

3. If two object interacts, the force $\vec{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\vec{F}_{21}$ exerted by object 2 on object 1

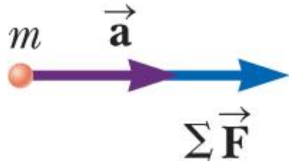
$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$

Analysis Models using Newton's Second Law

- When analysis models using Newton's second law, we can consider two scenarios:
  - **The object is in equilibrium**
    - The applied forces are balanced, the net force is zero: $\sum \vec{F} = 0$
    - No acceleration: $\vec{a} = 0$



  - **The object is accelerating under the effect of constant external forces**
    - The applied forces are unbalanced, there is a net force on the object: $\sum \vec{F} = m\vec{a}$
    - The object is accelerating in the direction of the net force: $\vec{a} \neq 0$



- In this part we analyze the models under the following assumptions
  - The objects are modeled as particles, no rotational motion
  - Neglect the effects of friction
  - Neglect the mass of any ropes, strings or cables

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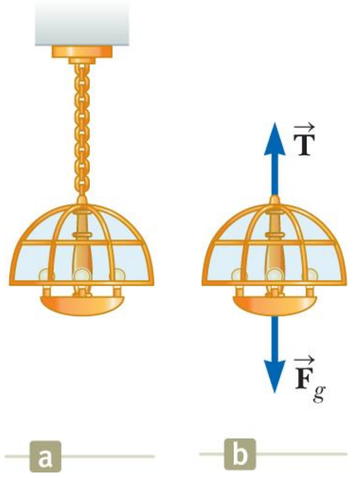
Analysis Model: The Particle in Equilibrium

- The object is treated with the particle in equilibrium model, if the acceleration of the object is zero.
- In this model the net force on the object is zero;

$$\sum \vec{F} = 0$$

- Consider a lamp suspended from a light chain fastened to the ceiling.
- The force diagram shows the forces acting on the lamp:
    - The gravitational force $\vec{F}_g$
    - The chain tension force $\vec{T}$
- Since the lamp is at rest ($a_x = 0, a_y = 0$) the forces are balanced in both x and y directions:

$$\sum F_x = 0 \qquad \sum F_y = 0 \;\rightarrow\; T - F_g = 0 \;\rightarrow\; T = F_g$$



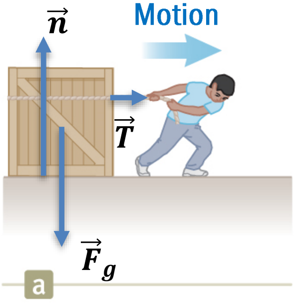
Force Diagram

Analysis Model: The Particle Under a Net Force

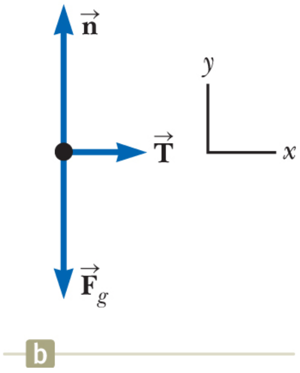
- If an object has acceleration, its motion can be analyzed with the particle under a net force model.
- Newton's second law is applied for this model as:

$$\sum \vec{F} = m\vec{a}$$

- Consider a crate being pulled to the right on a horizontal frictionless floor.
- The free-body diagram shows the forces acting on the crate:
  - The gravitational force $\vec{F}_g$
  - The normal force $\vec{n}$
  - The rope tension force $\vec{T}$
- Note that the magnitude of the applied force is equal to the tension in the rope.



Free-body Diagram



Analysis Model: The Particle Under a Net Force

- If an object has acceleration, its motion can be analyzed with the particle under a net force model.
- Newton's second law is applied for this model as:

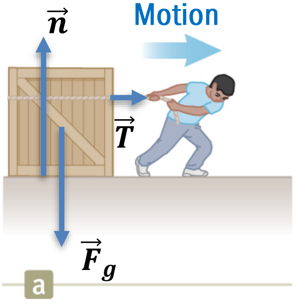
$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$$

- Newton's second law can be applied in x and y directions:
  - The rope tension force is acting in x direction:

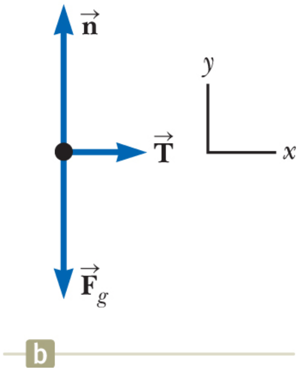
$$\sum F_x = ma_x \quad \rightarrow \quad T = ma_x \quad \rightarrow \quad a_x = \frac{T}{m}$$

  - No movement in y direction ($a_y = 0$):

$$\sum F_y = 0 \quad \rightarrow \quad n - F_g = 0 \quad \rightarrow \quad n = F_g$$



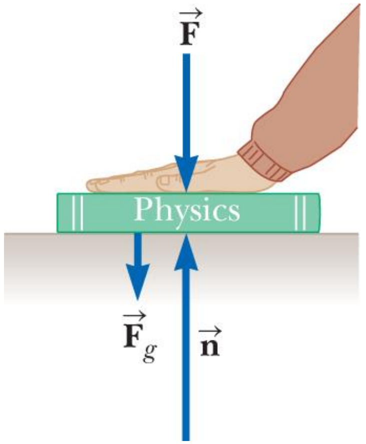
Free-body Diagram



Analysis Model: The Particle Under a Net Force

- In the previous scenario, the magnitude of the normal force $\vec{\boldsymbol{n}}$ was equal to the magnitude of the gravitational force $\vec{\boldsymbol{F}}_{\boldsymbol{g}}$, but this not always the case.

- For example, suppose a book is lying on a table and you push down the book with a force $\vec{\boldsymbol{F}}$.



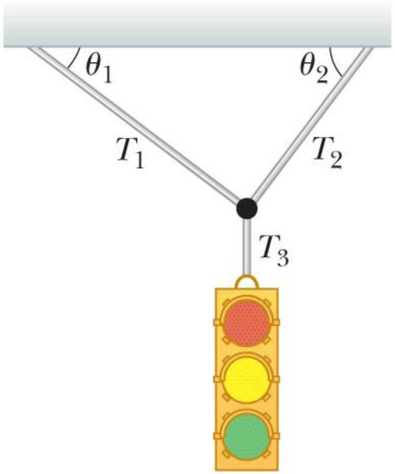
- Newton's second law can be applied in x and y directions.
- Since the lamp is at rest ($a_x = 0, a_y = 0$) the forces are balanced in both x and y directions:

$$\sum F_x = 0 \qquad\qquad \sum F_y = 0 \quad \rightarrow \quad n - F_g - F = 0 \quad \rightarrow \quad n = F_g + F$$

- In this situation, the normal force is greater than the gravitational force.

Analysis Model Examples

Example 1 (A Traffic Light at Rest): A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as the figure. The upper cables make angles of θ_1 = 37.0° and θ_2 = 53.0° with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N.



(a) Does the traffic light remain hanging in this situation, or will one of the cables break?

Since there is no motion and acceleration, we can model the object as a Particle in Equilibrium.

The given information:

$$F_g = 122\ N$$
$$\theta_1 = 37.0°, \qquad \theta_2 = 53.0°$$
$$T_{1\max} = T_{2max} = 100\ N$$

Analysis Model Examples

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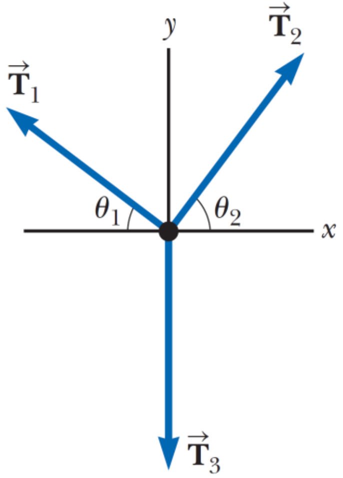
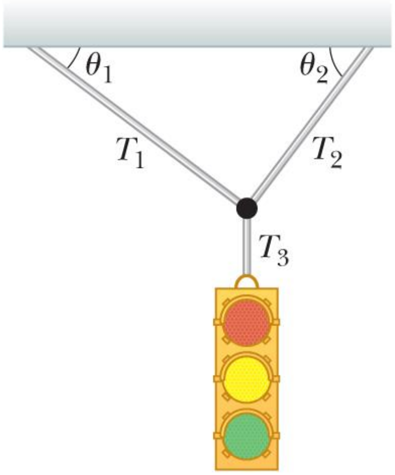
The forces acting on the traffic light are:

- Gravitational force: $\vec{F}_g$
- Tension force of vertical cable: $\vec{T}_3$

Free-body diagram for the knot shows the tension forces of cables: $\vec{T}_1$, $\vec{T}_2$ and $\vec{T}_3$





Analysis Model Examples

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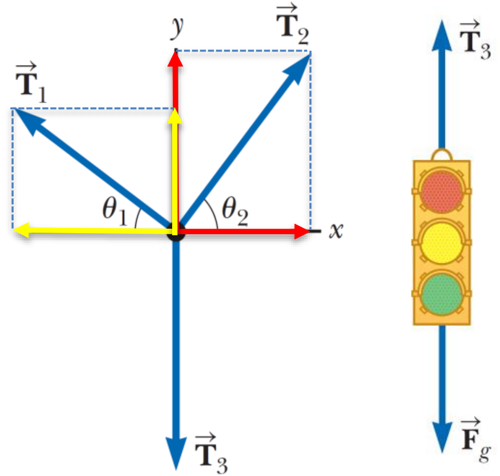
The tension forces are in equilibrium at the knob location:

$$\sum F_x = 0 \quad \rightarrow \quad -T_1 \cos\theta_1 + T_2 \cos\theta_2 = 0 \qquad \text{Equation (1)}$$

$$\sum F_y = 0 \quad \rightarrow \quad T_1 \sin\theta_1 + T_2 \sin\theta_2 - T_3 = 0 \qquad \text{Equation (2)}$$

The object is in equilibrium:

$$\sum F_y = 0 \quad \rightarrow \quad T_3 - F_g = 0 \quad \rightarrow \quad T_3 = F_g \qquad \text{Equation (3)}$$



Analysis Model Examples

$$-T_1 \cos\theta_1 + T_2 \cos\theta_2 = 0$$

$$T_1 \sin\theta_1 + T_2 \sin\theta_2 - T_3 = 0$$

$$T_3 = F_g$$

Example 1 (A Traffic Light at Rest):

Solve equations (1), (2) and (3) for T_1 and T_2:

$$-T_1 \cos\theta_1 + T_2 \cos\theta_2 = 0 \quad \rightarrow \quad T_2 = T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right)$$

$$T_1 \sin\theta_1 + T_2 \sin\theta_2 - T_3 = 0 \quad \rightarrow \quad T_1 \sin\theta_1 + T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right)(\sin\theta_2) - F_g = 0$$

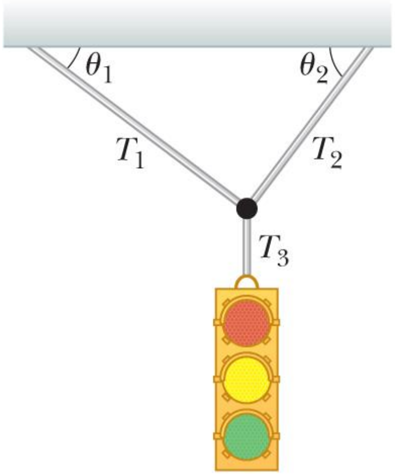
$$T_1 = \frac{F_g}{\sin\theta_1 + \cos\theta_1 \tan\theta_2} = \frac{122\ \mathrm{N}}{\sin 37.0\,° + \cos 37.0\,° \tan 53.0\,°} = \boxed{73.4\ \mathrm{N}}$$

$$T_2 = (73.4\ \mathrm{N})\left(\frac{\cos 37.0\,°}{\cos 53.0\,°}\right) = \boxed{97.4\ \mathrm{N}}$$

Since, both T_1 and T_2 are less than the maximum limit of 100 N, the cables will not break.

Analysis Model Examples

Example 1 (A Traffic Light at Rest): A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as the figure. The upper cables make angles of θ_1 = 37.0° and θ_2 = 53.0° with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N.



(b) Suppose the two angles in the figure are equal. What would be the relationship between T_1 and T_2?

From equation (1) we have:

$$T_2 = T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right) \xrightarrow{if\quad \theta_1=\theta_2} \boxed{T_2 = T_1}$$

Analysis Model Examples

Example 2 (The Runaway Car): A car of mass m is on an icy driveway inclined at an angle θ.

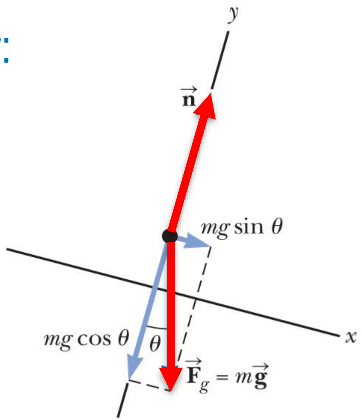
(a) Find the acceleration of the car, assuming the driveway is frictionless.

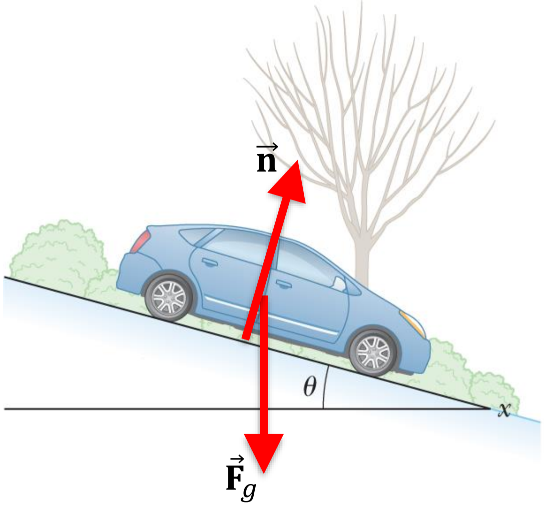
Since the car accelerates, we can model the car as a <u>Particle under Net Force.</u>

We should determine the forces acting on the car:

- Gravitational force: $\vec{\mathbf{F}}_g$
- Normal force: $\vec{\mathbf{n}}$

Then, draw the free-body diagram of the car.





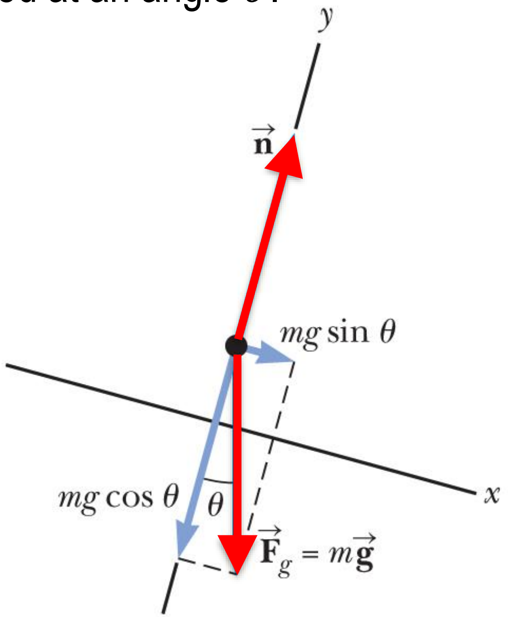
Analysis Model Examples

Example 2 (The Runaway Car): A car of mass m is on an icy driveway inclined at an angle θ.

(a) Find the acceleration of the car, assuming the driveway is frictionless.

Draw the free-body diagram of the car and apply the Newton's second law in x and y directions.

$$\sum F_x = ma_x \quad \rightarrow \quad mg\sin\theta = ma_x \quad \rightarrow \quad \boxed{a_x = g\sin\theta}$$



There is no motion in y direction ($a_y = 0$), the forces will be balanced in y direction.

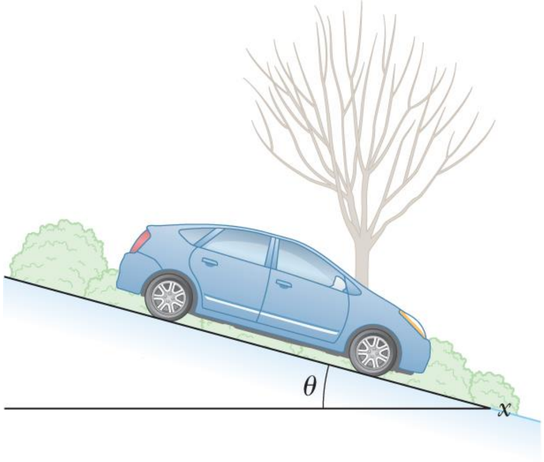
$$\sum F_y = 0 \quad \rightarrow \quad n - mg\cos\theta = 0 \quad \rightarrow \quad n = mg\cos\theta$$

Analysis Model Examples

Example 2 (The Runaway Car): A car of mass m is on an icy driveway inclined at an angle θ.

(b) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is *d*.

How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?



Imagine the car is sliding down the hill and you use a stopwatch to measure the entire time interval until it reaches the bottom.

This part of the problem belongs to <u>kinematics</u> rather than to dynamics, and the acceleration a_x is constant. Therefore, you should categorize the car in this part of the problem as a <u>particle under constant acceleration</u>.

Analysis Model Examples

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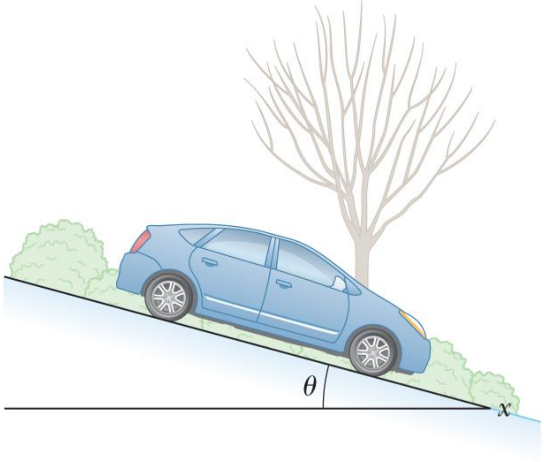


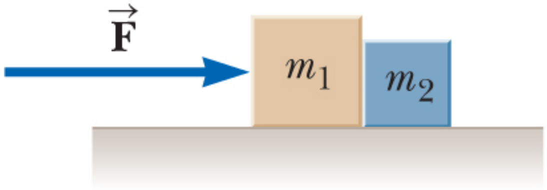
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \quad \rightarrow \quad d = \frac{1}{2}a_x t^2 \quad \rightarrow \quad t = \sqrt{\frac{2d}{a_x}} = \boxed{\sqrt{\frac{2d}{g\sin\theta}}}$$

From part (a): $a_x = g\sin\theta$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \quad \rightarrow \quad v_{xf} = \sqrt{2a_x d} = \boxed{\sqrt{2gd\sin\theta}}$$

Analysis Model Examples

Example 3 (One Block Pushes Another): Two blocks of masses m_1 and m_2 with $m_1 > m_2$, are placed in contact with each other on frictionless, horizontal surface. A constant horizontal force $\vec{\mathbf{F}}$ is applied to m_1 as shown.



(a) Find the magnitude of the acceleration of the system.

Since the boxes accelerate, we can model the boxes as a Particle under Net Force.

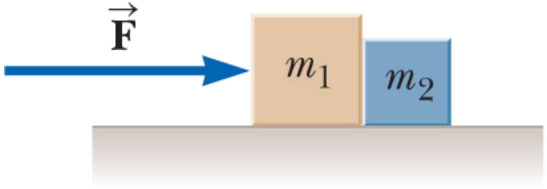
The motion is only on x direction, so we can apply the Newton's second law in x direction to find the acceleration.

$$\sum F_x = ma_x \quad \rightarrow \quad F = (m_1 + m_2)a_x \quad \rightarrow \quad \boxed{a_x = \frac{F}{m_1 + m_2}}$$

Analysis Model Examples

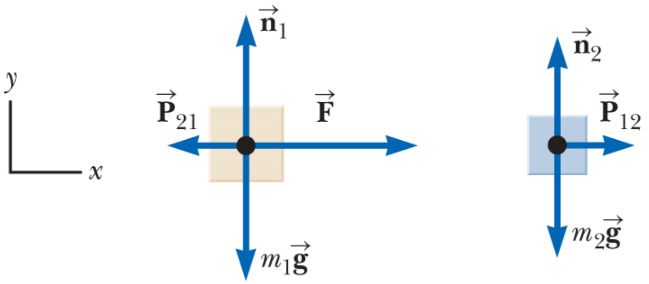
Example 3 (One Block Pushes Another): Two blocks of masses m_1 and m_2 with $m_1 > m_2$, are placed in contact with each other on frictionless, horizontal surface. A constant horizontal force $\vec{\mathbf{F}}$ is applied to m_1 as shown.

(b) Determine the magnitude of the contact force between the two blocks.



We should determine the forces acting on the boxes:

- Gravitational forces: $\vec{\mathbf{F}}_{g1}, \vec{\mathbf{F}}_{g2}$
- Normal forces: $\vec{\mathbf{n}}_1, \vec{\mathbf{n}}_2$
- Contact forces: $\vec{\mathbf{P}}_{12}, \vec{\mathbf{P}}_{21}$

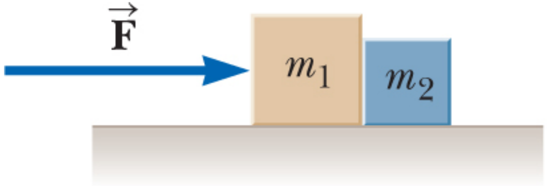


Then, draw the free-body diagram of the boxes.

Analysis Model Examples

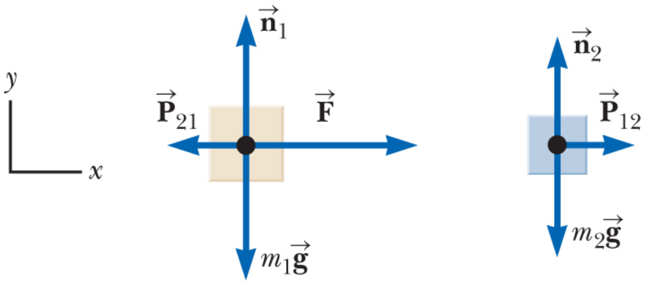
Example 3 (One Block Pushes Another): Two blocks of masses m_1 and m_2 with $m_1 > m_2$, are placed in contact with each other on frictionless, horizontal surface. A constant horizontal force $\vec{\mathbf{F}}$ is applied to m_1 as shown.

(b) Determine the magnitude of the contact force between the two blocks.



Apply the Newton's second law on the box m_2 in x direction.

$$\sum F_x = ma_x \quad \rightarrow \quad P_{12} = m_2 a_x$$



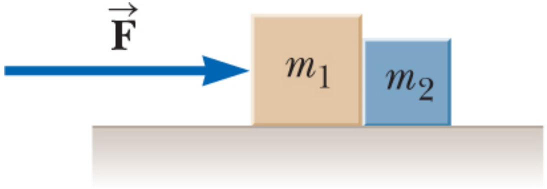
From Part (a): $a_x = \frac{F}{m_1+m_2}$

$$P_{12} = m_2 a_x = \left(\frac{m_2}{m_1+m_2}\right)F$$

Analysis Model Examples

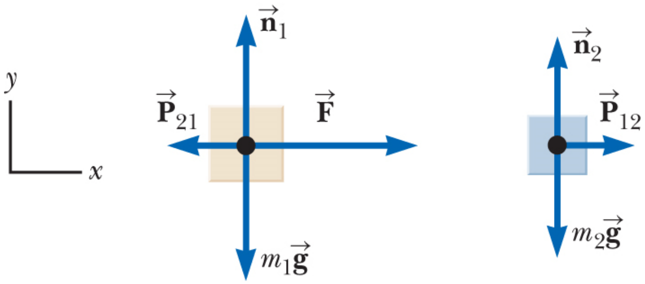
Example 3 (One Block Pushes Another): Two blocks of masses m_1 and m_2 with $m_1 > m_2$, are placed in contact with each other on frictionless, horizontal surface. A constant horizontal force $\vec{\mathbf{F}}$ is applied to m_1 as shown.

(b) Determine the magnitude of the contact force between the two blocks.



Apply the Newton's second law on the box m_1 in x direction.

$$\sum F_x = ma_x \;\rightarrow\; F - P_{21} = m_1 a_x \;\rightarrow\; P_{21} = F - m_1 a_x$$



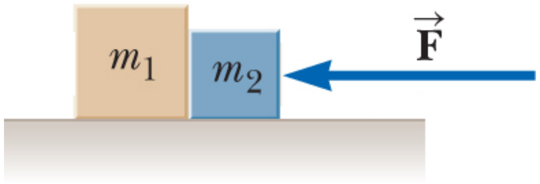
From Part (a): $a_x = \frac{F}{m_1 + m_2}$

$$P_{21} = F - m_1\left(\frac{F}{m_1 + m_2}\right) = \left(\frac{m_2}{m_1 + m_2}\right)F$$

Analysis Model Examples

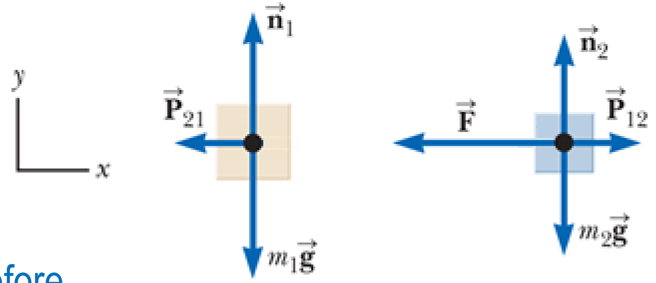
Example 3 (One Block Pushes Another): Two blocks of masses m_1 and m_2 with $m_1 > m_2$, are placed in contact with each other on frictionless, horizontal surface.

(c) Imagine that the force $\vec{\mathbf{F}}$ is applied toward the left on the mass m_2 as shown. Is the magnitude of the contact force $\vec{P}_{12}$ the same as it was when the force was applied toward the right on m_1?



When the force $\vec{\mathbf{F}}$ is applied from the right, for box m_2 we have:

$$\sum F_x = ma_x \rightarrow \quad -F + P_{12} = m_2(-a_x) \quad \rightarrow \quad P_{12} = F - m_2 a_x$$



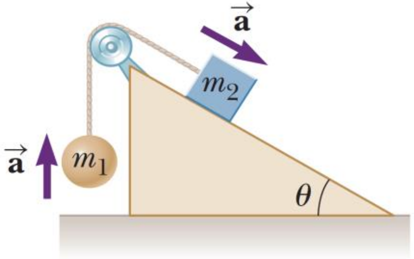
From Part (a): $a_x = \frac{F}{m_1+m_2}$

$$P_{12} = F - m_2\left(\frac{F}{m_1 + m_2}\right) = \left(\frac{m_1}{m_1 + m_2}\right)F$$

This is greater than before because $m_1 > m_2$.

Analysis Model Examples

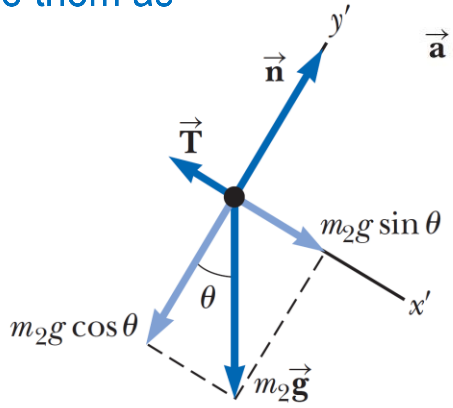
Example 4 (Acceleration of Two Objects Connected by a Cord): A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects and the tension in the cord.

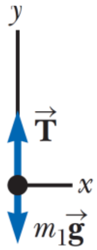


Since the ball and the box accelerate, we can model the them as a Particle under Net Force.

Draw the forces acting on each object.

Draw their free-body diagram





Analysis Model Examples

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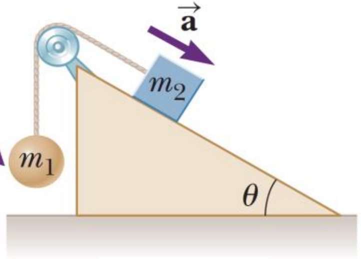


Apply Newton's second law for each object in x and y directions.

$$\sum F_y = ma_y \quad \rightarrow \quad T - m_1 g = m_1 a \quad \rightarrow \quad \boxed{T = m_1(g + a)}$$

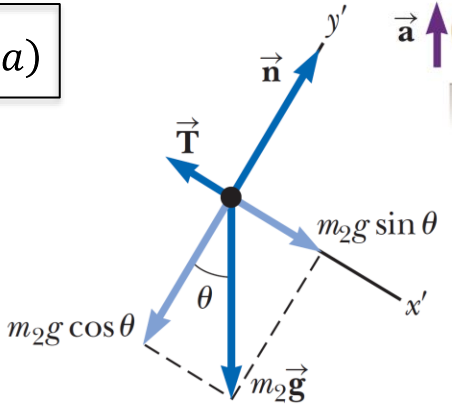
Equation (1)

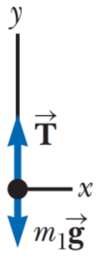
$$\sum F_{x'} = ma_{x'} \quad \rightarrow \quad m_2 g \sin\theta - T = m_2 a$$

$$\rightarrow \quad \boxed{T = m_2 g \sin\theta - m_2 a}$$

Equation (2)

$$\sum F_{y'} = n - m_2 g \cos\theta = 0$$





Analysis Model Examples

Example 4 (Acceleration of Two Objects Connected by a Cord): A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects and the tension in the cord.

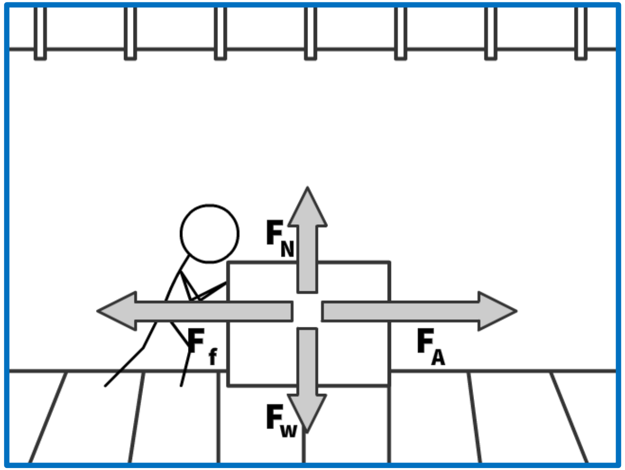
Solve equations (1) and (2) for a and T:

$$\left.\begin{aligned} T &= m_1(g+a) \\ T &= m_2 g\sin\theta - m_2 a \end{aligned}\right\} \quad m_2 g\sin\theta - m_1(g+a) = m_2 a \quad \rightarrow \quad \boxed{a = \left(\frac{m_2\sin\theta - m_1}{m_1 + m_2}\right)g}$$

Substitute the acceleration in Equation (1): $\rightarrow$ $\boxed{T = \left(\frac{m_1 m_2(\sin\theta + 1)}{m_1 + m_2}\right)g}$

Forces of Friction

- Have you ever noticed that it is more difficult to get an object to start sliding on a surface that it is to keep it moving once it starts?
- **Force of Friction f or F_f:** A force that opposes sliding motion between surfaces.
  - Friction always acts against the intended motion.
  - Direction of friction force is always opposite the object's sliding direction and is parallel to the contact surface.
  - Magnitude of friction force is proportional to the normal force magnitude n and to the roughness factor between the sliding surfaces.



- There are two types of Friction Force:
  - Force of Static Friction
  - Force of Kinetic Friction

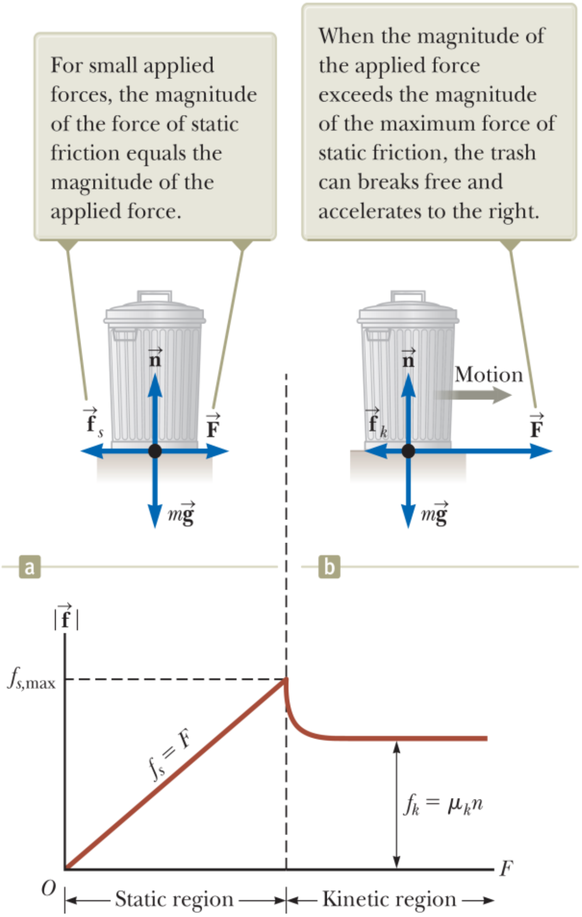
Forces of Static Friction

- **Force of Static Friction f_s or F_{f_s}:** The force of friction that acts on stationary objects preventing them from moving.
  - It opposes any applied force up to its maximum limit.
  - Any applied force greater than the maximum value, $f_{s,max}$, will result in the object beginning to slide or slip.

$$\boxed{0 \le f_s \le f_{s,max}}$$

$$\boxed{\underbrace{f_{s,max}}_{\text{Force of Maximum Static Friction}} = \underbrace{\mu_s}_{\text{Static Friction Coefficient}} \underbrace{n}_{\text{Normal Force}}}$$

- To determine the normal force n, look at the all forces perpendicular to the surface.



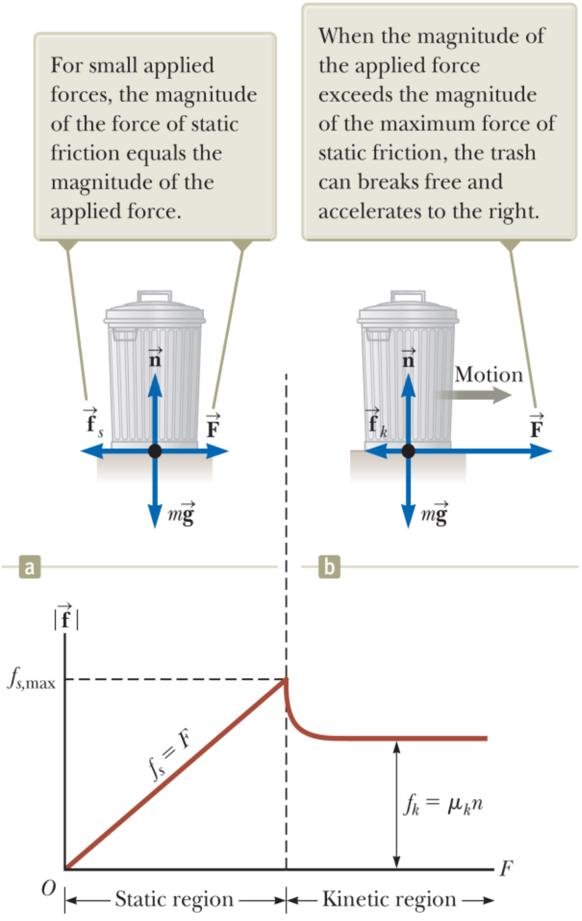
Forces of Kinetic Friction

- **Force of Kinetic Friction f_k or F_{f_k}:** The force of friction that resist the motion of moving objects along a surface.

$$\boxed{f_k = \mu_k\, n}$$

Force of Kinetic Friction (f_k) — Kinetic Friction Coefficient (μ_k) — Normal Force (n)

- Following graph shows magnitude of force of friction versus the magnitude of applied force for an object initially at rest.
  - When the object is at rest, the magnitude of f will always grow to match any pull until a maximum is reached.
  - After moving, the magnitude of f is reduced due to the change in the friction coefficient.



Coefficient of Friction

- **Coefficient of friction force** is a number that describes the interactions between the surfaces
  - It is a dimensionless scalar value, and symbolized by the Greek letter μ
  - It describes the ratio of the force of friction between two bodies and the force pressing them together.

$$\boxed{\mu_s = \frac{n}{f_{s,max}}}$$

Coefficient of Static Friction

$$\boxed{\mu_k = \frac{n}{f_k}}$$

Coefficient of Kinetic Friction

- For the same materials, the kinetic friction coefficient μ_k is <u>always</u> less than the static friction coefficient μ_s.

TABLE 5.1 Coefficients of Friction

	μ_s	μ_k
Rubber on concrete	1.0	0.8
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Glass on glass	0.94	0.4
Copper on steel	0.53	0.36
Wood on wood	0.25–0.5	0.2
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Teflon on Teflon	0.04	0.04
Ice on ice	0.1	0.03
Synovial joints in humans	0.01	0.003

Note: All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

Quick Quiz 1

The manager of a department store is pushing horizontally with a force of magnitude 200 N on a box of shirts. The box is sliding across the horizontal floor with a forward acceleration. Nothing else touches the box.

What must be true about the magnitude of the force of kinetic friction acting on the box?

a) It is greater than 200 N
b) It is less than 200 N
c) It is equal to 200 N
d) None of these statements is true.

Quick Quiz 2

- You press your physics textbook flat against a vertical wall with your hand. What is the direction of the friction force exerted by the wall on the book?

  a) downward
  b) upward
  c) out from the wall
  d) into the wall

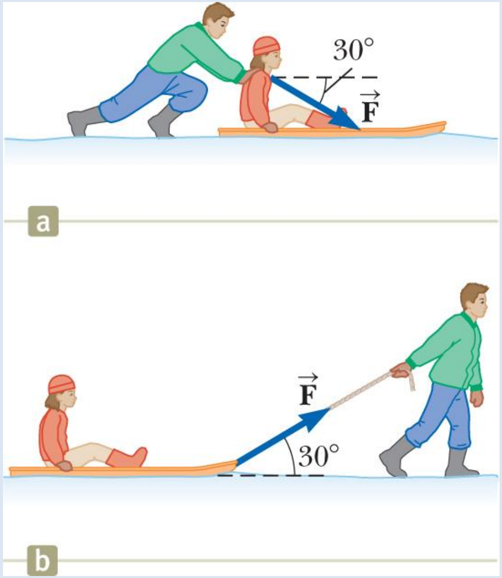
Quick Quiz 3

- Charlie is playing with his daughter Torrey in the snow. She sits on a sled and asks him to slide her across a flat, horizontal field. Charlie has following choices:

  a) pushing her from behind by applying a force downward on her shoulders at 30° below the horizontal or

  b) attaching a rope to the front of the sled and pulling with a force at 30° above the horizontal.

- Which would be easier for him and why?

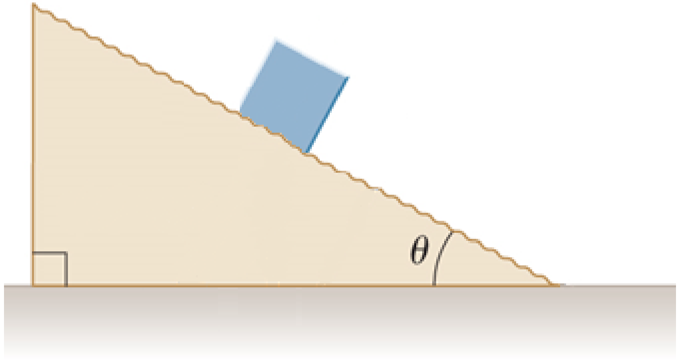


Force of Friction

Example 5 (Experimental Determination of μ_s): The following is a simple method of measuring coefficients of friction. Suppose a block is placed on a rough surface inclined relative to the horizontal as shown in the figure. The incline angle is increased until the block starts to move.

Show that you can obtain μ_s by measuring the critical angle θ_c at which this slipping just occurs.

Since we are raising the plane to the angle at which the block is just ready to begin to move but is not moving, we can categorize the block as a Particle in Equilibrium.



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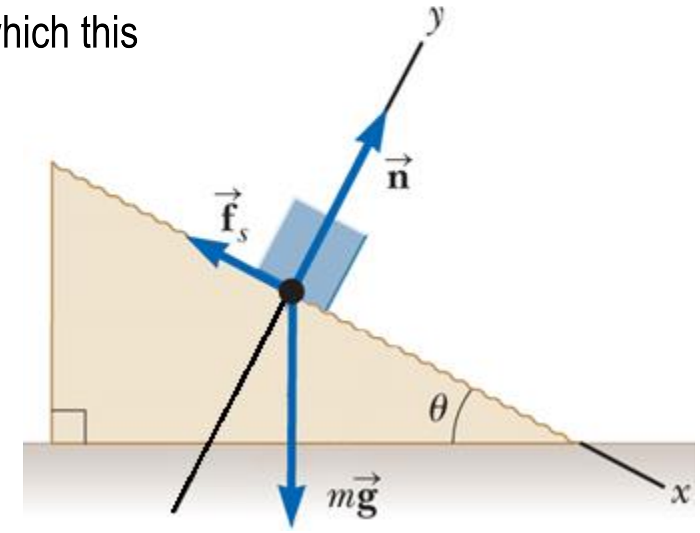

The forces acting on the block are:

- The gravitational force: $\vec{F}_g$
- The normal force: $\vec{n}$
- The force of static friction: $\vec{f}_s$

Draw the free-body diagram of the block.

Force of Friction

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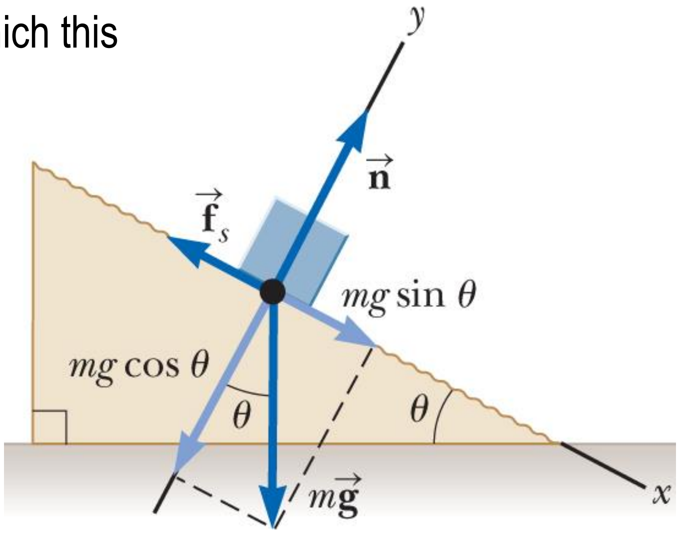


Apply Newton's second law for each object in x and y directions.

$$\sum F_x = 0 \quad \rightarrow \quad mg \sin \theta - f_s = 0 \quad \rightarrow \quad f_s = mg \sin \theta$$

$$\sum F_y = 0 \quad \rightarrow \quad n - mg \cos \theta = 0 \quad \rightarrow \quad n = mg \cos \theta$$

$$f_s = mg \sin \theta = \left(\frac{n}{\cos \theta}\right) \sin \theta = n \tan \theta \quad \rightarrow \quad \mu_s n = n \tan \theta_c$$

$$\boxed{\mu_s = \tan \theta_c}$$

Force of Friction

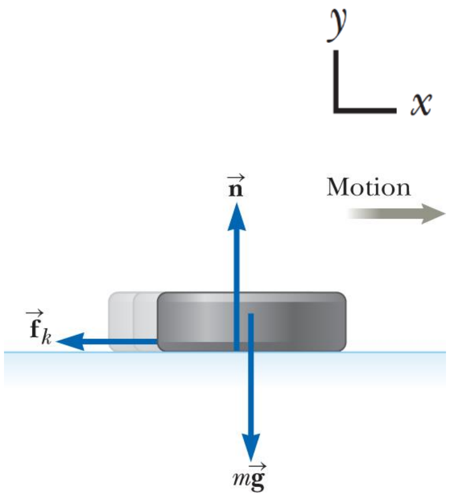
Example 6 (Sliding Hockey Puck): A hockey puck on a frozen pond is given an initial speed of $20.0\ m/s$. If the puck always remains on the ice and slides $115\ m$ before coming to rest. Determine the coefficient of kinetic friction between the puck and ice.

In the horizontal direction the hockey puck is modeled as a Particle Under a Net Force.

In the vertical direction the hockey puck is modeled as a Particle in Equilibrium.

The forces acting on the hockey puck are:

- The gravitational force: $\vec{\boldsymbol{F}}_g$
- The normal force: $\vec{n}$
- The force of kinetic friction: $\vec{f}_k$



Force of Friction

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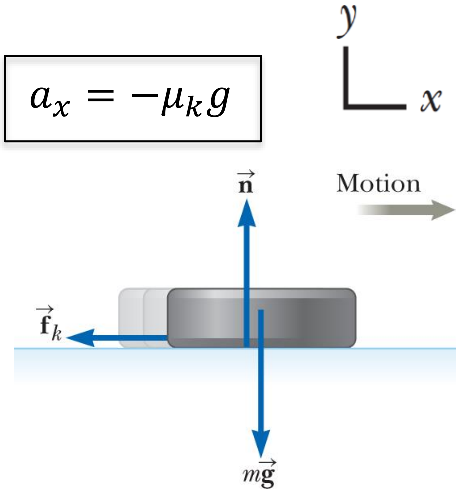
Apply Newton's second law for each object in x and y directions.

$$\sum F_x = ma_x \rightarrow -f_k = ma_x \rightarrow -\mu_k n = ma_x \rightarrow -\mu_k mg = ma_x \rightarrow \boxed{a_x = -\mu_k g}$$

$$\sum F_y = 0 \rightarrow n - mg = 0 \rightarrow n = mg$$



$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$0 = v_{xi}^2 + 2a_x x_f$$

$$0 = v_{xi}^2 - 2\mu_k g x_f$$

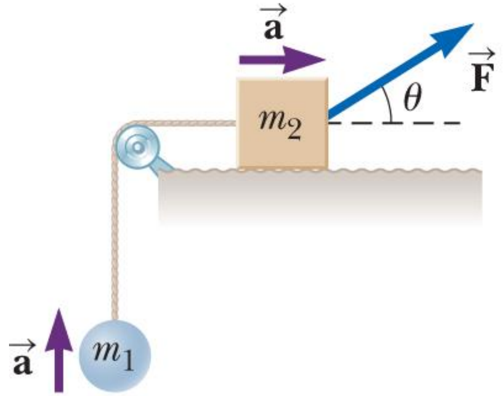
$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

$$\mu_k = \frac{(20.0\ \mathrm{m/s})^2}{2(9.80\ \mathrm{m/s^2})(115\ \mathrm{m})} = \boxed{0.177}$$

Force of Friction

Example 7 (Acceleration of Two Connected Objects when Friction is Present): A block of mass m_2 on a rough, horizontal surface is connected to a ball of mass m_1 by a lightweight cord over a lightweight, frictionless pulley. A force of magnitude F at an angle θ with the horizontal is applied to the block, and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.

Since the ball and the box accelerate, we can model the them as a Particle under Net Force.



Draw the forces acting on each object.

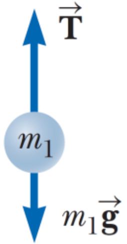
Draw their free-body diagram

Force of Friction

Example 7 (Acceleration of Two Connected Objects when Friction is Present):

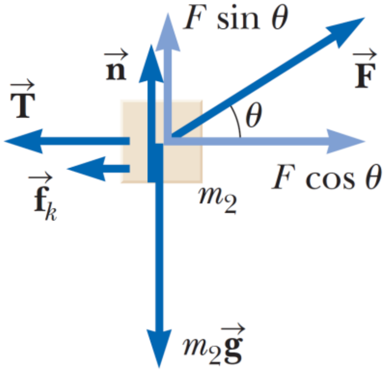
The forces acting on the ball m_1 are:

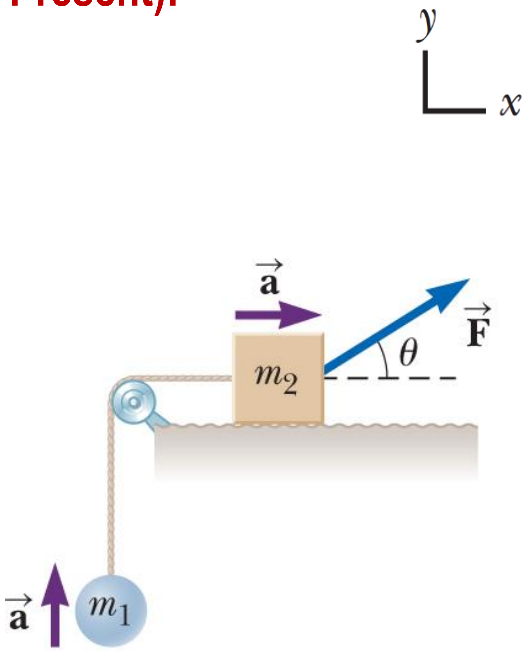
- The gravitational force: $\vec{F}_g$
- The tension force: $\vec{T}$



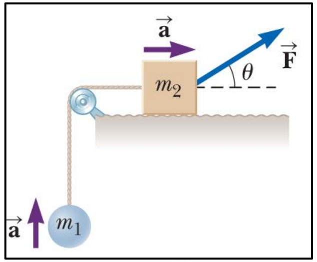
The forces acting on the block m_2 are:

- The gravitational force: $\vec{F}_g$
- The normal force: $\vec{n}$
- The tenson force: $\vec{T}$
- The force of kinetic friction: $\vec{f}_k$



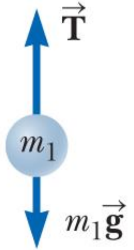


Force of Friction



Example 7 (Acceleration of Two Connected Objects when Friction is Present):

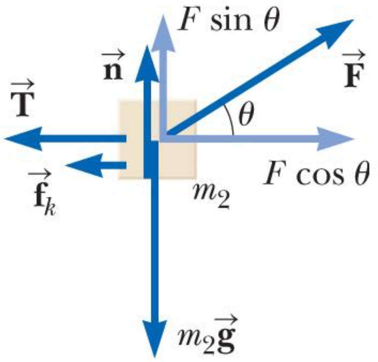
Apply Newton's second law for each object in x and y directions.



$$\sum F_y = ma_y \;\rightarrow\; T - m_1 g = m_1 a \;\rightarrow\; T = m_1(g + a)$$

$$\sum F_y = 0 \;\rightarrow\; n + F\sin\theta - m_2 g = 0 \;\rightarrow\; n = m_2 g - F\sin\theta$$

$$\sum F_x = ma_x \;\rightarrow\; F\cos\theta - f_k - T = m_2 a \;\rightarrow\; F\cos\theta - \mu_k n - T = m_2 a$$



$$F\cos\theta - \mu_k(m_2 g - F\sin\theta) - m_1(a + g) = m_2 a$$

$$\boxed{a = \frac{F(\cos\theta + \mu_k \sin\theta) - (m_1 + \mu_k m_2)g}{m_1 + m_2}}$$

Quick Quiz 4

A crate remains stationary after it has been placed on a ramp inclined at an angle with the horizontal.

Which of the following statements is correct about the magnitude of the friction force that acts on the crate?

a) It is larger than the weight of the crate
b) It is greater than the component of the gravitational force acting down the ramp.
c) It is equal to the component of the gravitational force acting down the ramp.
d) It is less than the component of the gravitational force acting down the ramp.

Quick Quiz 5

An object of mass m moves with acceleration $\vec{\mathbf{a}}$ down a rough incline.

Which of the following forces should appear in a free-body diagram of the object?
Choose all correct answers.

a) the gravitational force exerted by the planet

b) $m\vec{\mathbf{a}}$ in the direction of motion

c) the normal force exerted by the incline

d) the friction force exerted by the incline

e) the force exerted by the object on the incline

THANK YOU

HUMBER

WE ARE HUMBER