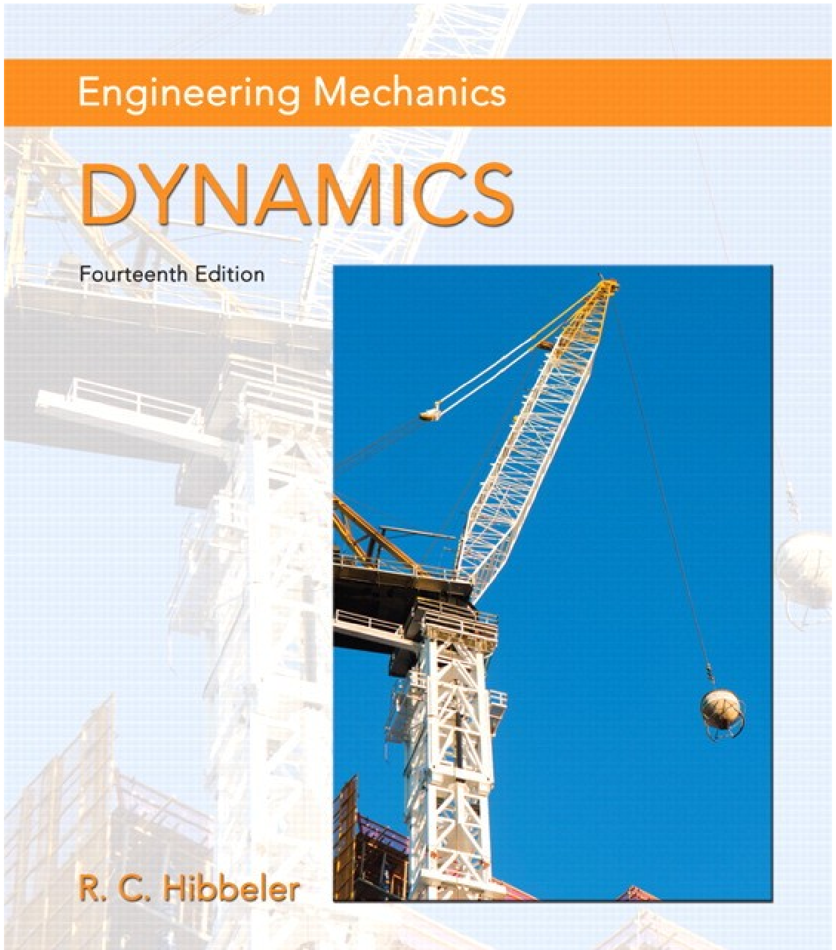
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 13

Kinetics of a Particle:
Force and Acceleration

Newton's Laws of Motion, Equations of Motion, & Equations of Motion for a System of Particles (1 of 2)

Today's Objectives:
Students will be able to:

1. Write the equation of motion for an accelerating body.
2. Draw the free-body and kinetic diagrams for an accelerating body.

Newton's Laws of Motion, Equations of Motion, & Equations of Motion for a System of Particles (2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Newton’s Laws of Motion
- Newton’s Law of Gravitational Attraction
- Equation of Motion for a Particle or System of Particles
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1. Newton's second law can be written in mathematical form as $\Sigma \boldsymbol{F} = m\boldsymbol{a}$. Within the summation of forces, $\Sigma \boldsymbol{F}$, ________ are(is) not included.

   A) external forces
   B) weight
   C) internal forces
   D) All of the above.

2. The equation of motion for a system of n-particles can be written as $\Sigma F_i = \Sigma m_i a_i = m a_G$, where a_G indicates ______.

   A) summation of each particle's acceleration
   B) acceleration of the center of mass of the system
   C) acceleration of the largest particle
   D) None of the above.

Applications (1 of 3)

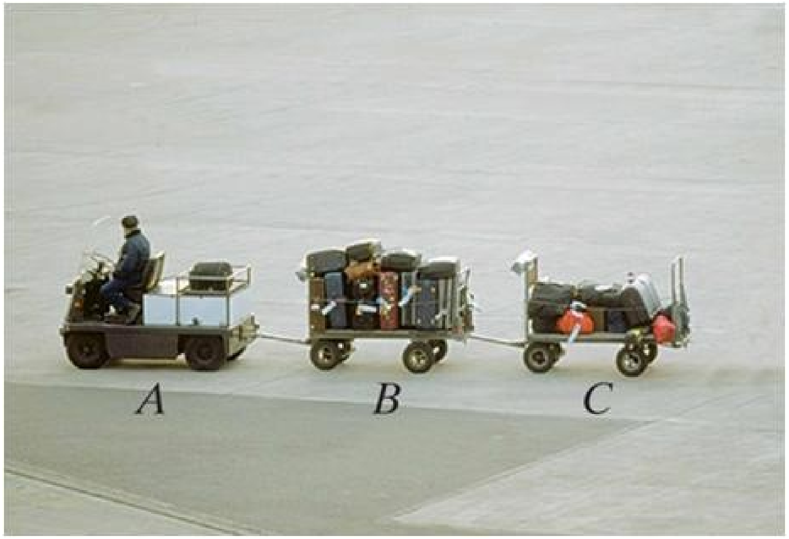
The motion of an object depends on the forces acting on it.

A parachutist relies on the atmospheric drag resistance force generated by her parachute to limit her velocity.

Knowing the drag force, how can we determine the acceleration or velocity of the parachutist at any point in time? This has some importance when landing!

Applications (2 of 3)

The baggage truck A tows a cart B, and a cart C.



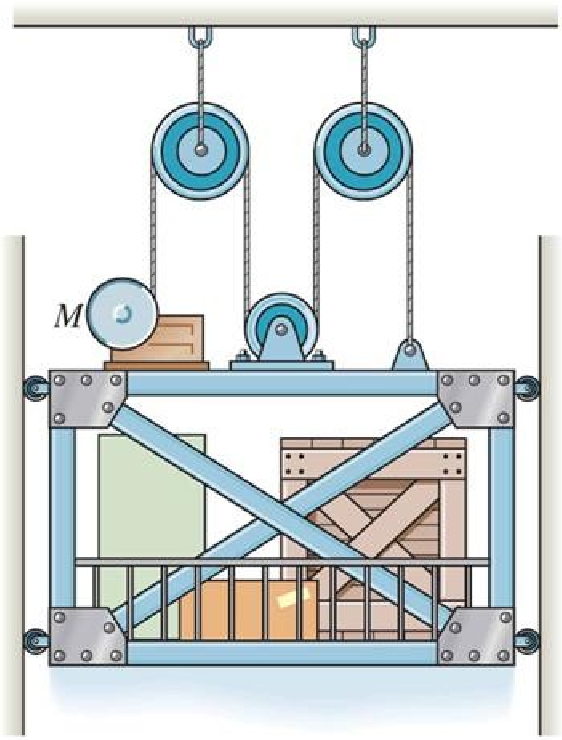
If we know the frictional force developed at the driving wheels of the truck, could we determine the acceleration of the truck?

How?

Can we also determine the horizontal force acting on the coupling between the truck and cart **B?** This is needed when designing the coupling (or understanding why it failed).

Applications (3 of 3)

A freight elevator is lifted using a motor attached to a cable and pulley system as shown.



How can we determine the tension force in the cable required to lift the elevator and load at a given acceleration? This is needed to decide the size of the cable that should be used.

Is the tension force in the cable greater than the weight of the elevator and its load?

Section 13.1

Newton’s Laws of Motion

Newton’s Laws of Motion (1 of 2)

The motion of a particle is governed by **Newton’s three laws of motion**.

First Law: A particle originally at rest, or moving in a straight line at constant velocity, will remain in this state if the resultant force acting on the particle is zero.

Second Law: If the resultant force on the particle is not zero, the particle experiences an acceleration in the same direction as the resultant force. This acceleration has a magnitude proportional to the resultant force.

Third Law: Mutual forces of action and reaction between two particles are equal, opposite, and collinear.

Newton's Laws of Motion (2 of 2)

The first and third laws were used in developing the concepts of statics. Newton's **second law** forms the basis of the study of dynamics.

Mathematically, Newton's second law of motion can be written

$$F = ma$$

where **F** is the **resultant unbalanced force** acting on the particle, and ***a*** is the **acceleration** of the particle. The positive scalar m is the **mass** of the particle.

Newton's second law cannot be used when the particle's speed approaches the speed of light, or if the size of the particle is extremely small (~ size of an atom).

Newton's Law of Gravitational Attraction

Any two particles or bodies have a **mutually attractive gravitational force** acting between them. Newton postulated the law governing this gravitational force as

$$F = G\frac{m_1 m_2}{r^2}$$

where F = force of attraction between the two bodies,

G = universal constant of gravitation ,

m_1, m_2 = mass of each body, and

r = distance between centers of the two bodies.

When near the surface of the earth, the only gravitational force having any sizable magnitude is that between the earth and the body. This force is called the **weight** of the body.

Mass and Weight

It is important to understand the difference between the mass and weight of a body!

Mass is an **absolute property** of a body. It is independent of the gravitational field in which it is measured. The mass provides a measure of the **resistance of a body to a change in velocity**, as defined by Newton's second law of motion (m = **F**/**a**).

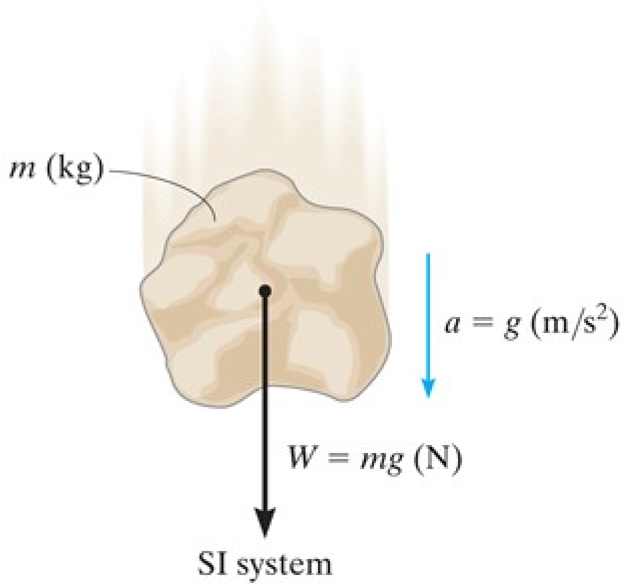
The weight of a body is not absolute, since it depends on the gravitational field in which it is measured. **Weight** is defined as

$$W = mg$$

where g is the **acceleration due to gravity**.

Units: SI System Versus FPS System (1 of 2)

SI system: In the SI system of units, **mass** is a **base unit** and **weight** is a **derived unit**. Typically, mass is specified in **kilograms** (kg), and weight is calculated from $W = mg$



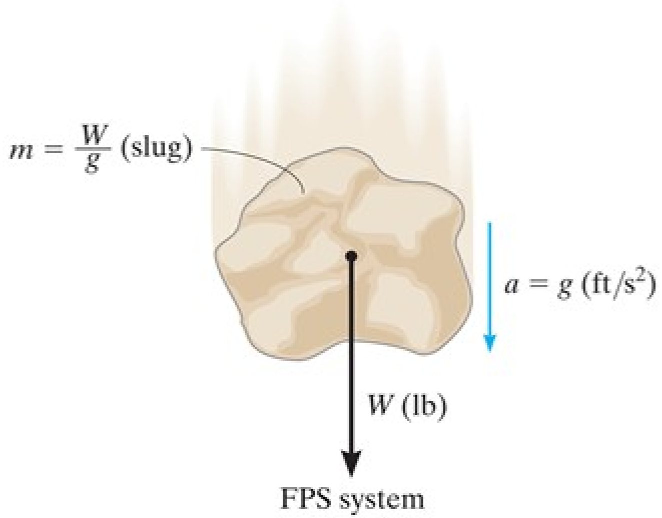
If the gravitational acceleration (g) is specified in units of m/s^2 then the weight is expressed in **newtons** (N).

On the earth's surface, g can be taken as $g = 9.81 m/s2.$

$$W\ (N) = m\ (kg)\ g\ (m/s^2) \Rightarrow N = kg \bullet m/s^2$$

Units: SI System Versus FPS System (2 of 2)

FPS System: In the FPS system of units, **weight** is a **base unit** and **mass** is a **derived unit.** Weight is typically specified in **pounds** (lb), and mass is calculated from $m = W/g$



If g is specified in units of ft/s^2 then the mass is expressed in **slugs**.

On the earth's surface, g is approximately **32.2 ft/s^2**.

$$\mathbf{m\ (slugs) = W\ (lb)/g\ (ft/s^2) \Rightarrow slug = lb \bullet s^2/ft}$$

Section 13.2

Equation of Motion

Equation of Motion

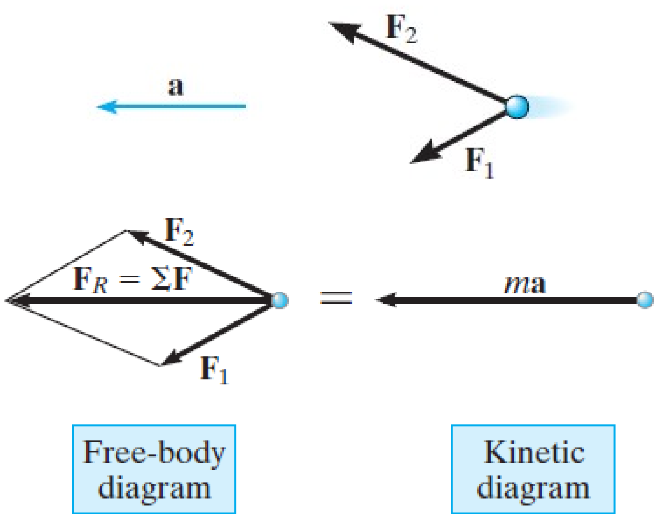
The motion of a particle is governed by Newton's second law, relating the unbalanced forces on a particle to its acceleration. If more than one force acts on the particle, the equation of motion can be written

$$\sum F = F_R = ma$$

where F_R is the **resultant force**, which is a **vector summation** of all the forces.

To illustrate the equation, consider a particle acted on by two forces.

First, draw the particle's **free-body diagram**, showing all forces acting on the particle. Next, draw the **kinetic diagram**, showing the **inertial force** m***a*** acting in the same direction as the resultant force F_R



Inertial Frame of Reference

This equation of motion is only valid if the acceleration is measured in a **Newtonian** or **inertial frame of reference.** What does this mean?

For problems concerned with motions at or near the earth's surface, we typically assume our "inertial frame" to be **fixed to the earth.** We neglect any acceleration effects from the earth's rotation.

For problems involving satellites or rockets, the inertial frame of reference is often **fixed to the stars.**

Section 13.3

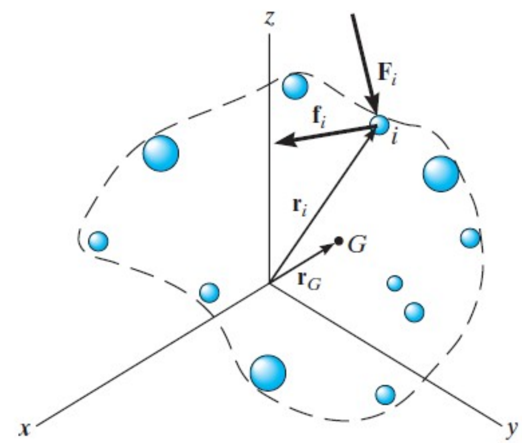
Equation of Motion for a System of Particles

Equation of Motion for a System of Particles

The equation of motion can be extended to include **systems of particles.** This includes the motion of solids, liquids, or gas systems.

As in statics, there are **internal forces** and **external forces** acting on the system. What is the difference between them?

Using the definitions of $m = \Sigma m_i$ as the total mass of all particles and a_G as the acceleration of the **center of mass** G of the particles, then $m\,a_G = \Sigma m_i a_i$.



Inertial coordinate system

The text shows the details, but for a system of particles:

$$\Sigma F = m a_G$$

where ΣF is the sum of the **external forces** acting on the entire system.

Key Points

1. **Newton's second law** is a "law of nature"--experimentally proven, not the result of an analytical proof.
2. **Mass** (a property of an object) is a measure of **the resistance to a change in velocity** of the object.
3. **Weight** (a force) depends on the **local gravitational field**. Calculating the weight of an object is an application of $F = ma$, i.e., $W = mg$.
4. **Unbalanced** forces cause the **acceleration** of objects. This condition is fundamental to all dynamics problems!

Procedure for the Application of the Equation of Motion

1. Select a convenient **inertial coordinate system.** Rectangular, normal/tangential, or cylindrical coordinates may be used.

2. Draw a **free-body diagram** showing **all external forces** applied to the particle. Resolve forces into their appropriate components.

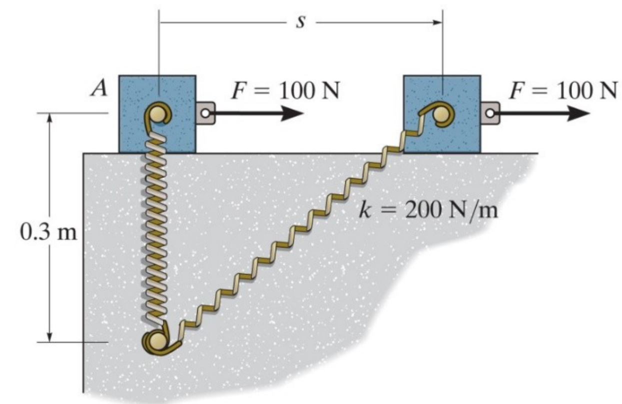
3. Draw the **kinetic diagram**, showing the particle's inertial force, m***a***. Resolve this vector into its appropriate components.

4. Apply the **equations of motion** in their scalar component form and solve these equations for the unknowns.

5. It may be necessary to apply the proper **kinematic relations** to generate additional equations.

Example (1 of 3)

Given: A 25-kg block is subjected to the force F=100 N. The spring has a stiffness of k = 200 N/m and is unstretched when the block is at A. The contact surface is smooth.



Find: Draw the free-body and kinetic diagrams of the block when s=0.4 m.

Plan:

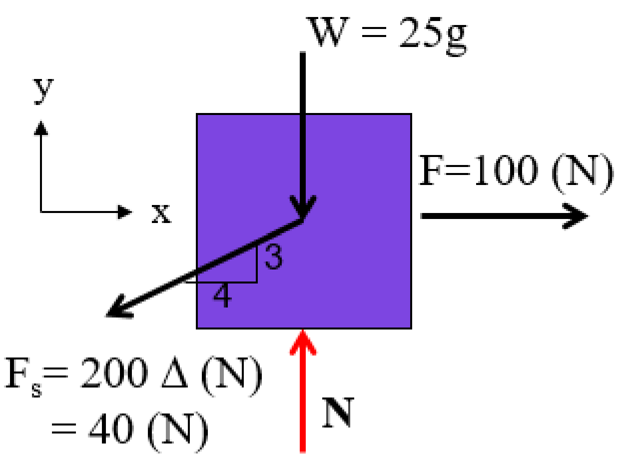
1. Define an inertial coordinate system.
2. Draw the block's free-body diagram, showing all external forces.
3. Draw the block's kinetic diagram, showing the inertial force vector in the proper direction.

Example (2 of 3)

1. An inertial x-y frame can be defined as fixed to the ground.
2. Draw the free-body diagram of the block:

The weight force (W) acts through the block's center of mass. F is the applied load and **$F_s = 200\Delta$** is the spring force, where Δ is the spring deformation. When s = 0.4,

$$\Delta = 0.5 - 0.3 = 0.2$$

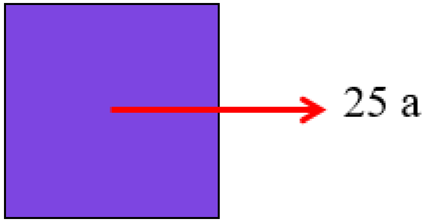


The normal force (N) is perpendicular to the surface. There is no friction force since the contact surface is smooth.

Example (3 of 3)

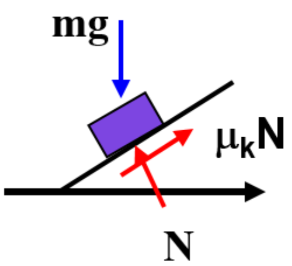
3. Draw the kinetic diagram of the block.

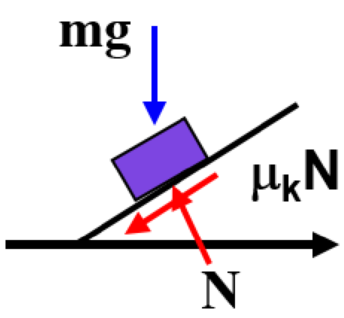
The block will be moved to the right. The acceleration can be directed to the right if the block is speeding up or to the left if it is slowing down.

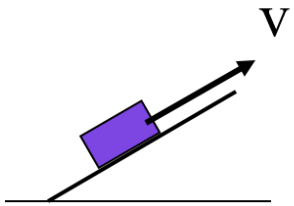


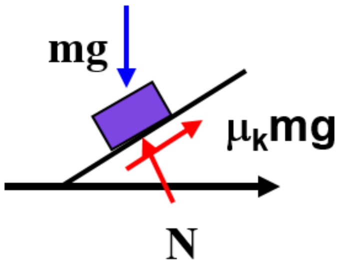
Concept Quiz (1 of 2)

1. The block (mass = m) is moving upward with a speed v. Draw the FBD if the kinetic friction coefficient is μ_k

A) 

B) 

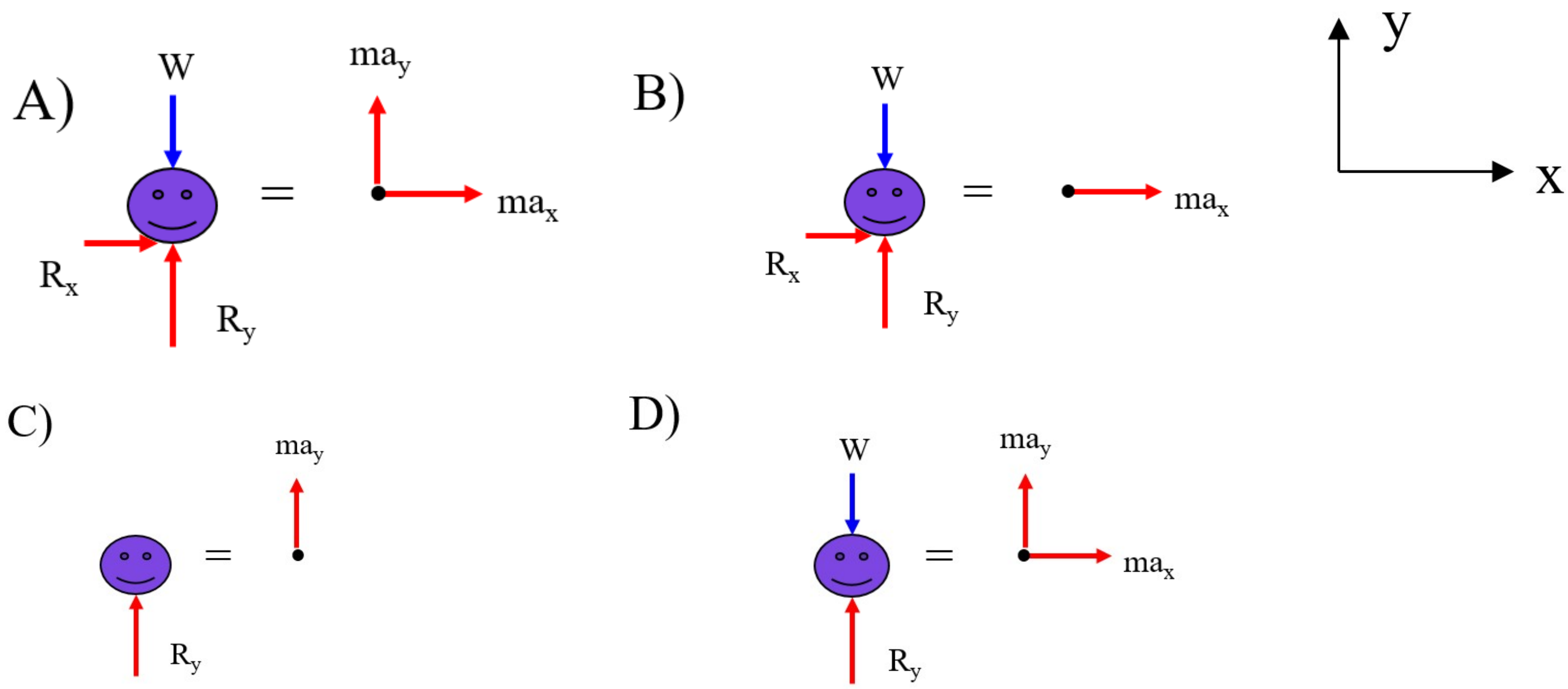


C) 

D) None of these

Concept Quiz (2 of 2)

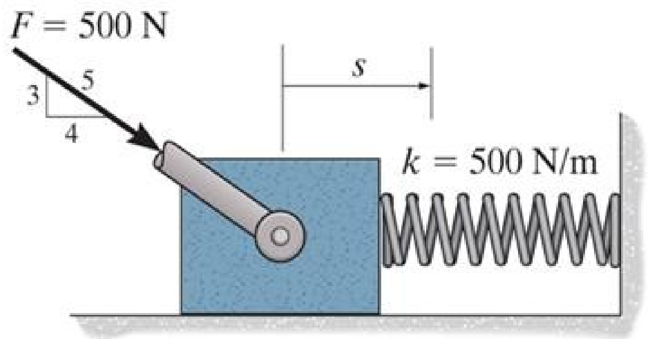
2. Packaging for oranges is tested using a machine that exerts $a_y = 20\ m/s^2$ and $a_x = 3\ m/s^2$, simultaneously. Select the correct FBD and kinetic diagram for this condition.



Pearson

Group Problem Solving (1 of 2)

Given: A 10-kg block is subjected to a force F=500 N. A spring of stiffness k=500 N/m is mounted against the block. When s = 0, the block is at rest and the spring is uncompressed. The contact surface is smooth.



Find: Draw the free-body and kinetic diagrams of the block.

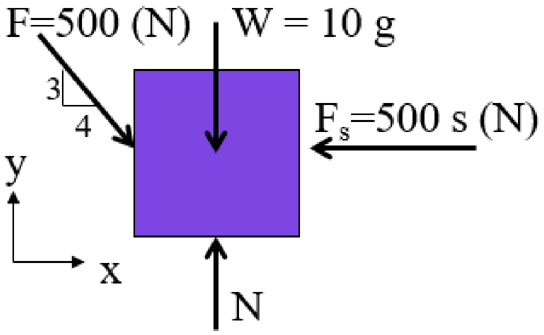
Plan:
1. Define an inertial coordinate system.
2. Draw the block's free-body diagram, showing all external forces applied to the block in the proper directions.
3. Draw the block's kinetic diagram, showing the inertial force vector m***a*** in the proper direction.

Group Problem Solving (2 of 2)

Solution:

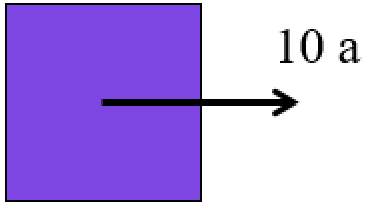
1. An inertial x-y frame can be defined as fixed to the ground.

2. Draw the free-body diagram of the block:



The weight force (W) acts through the block's center of mass. F is the applied load and $F_s = 500\,s\,(N)$ is the spring force, where s is the spring deformation. The normal force (N) is perpendicular to the surface. There is no friction force since the contact surface is smooth.

3. Draw the kinetic diagram of the block:



The block will be moved to the right. The acceleration can be directed to the right if the block is speeding up or to the left if it is slowing down.

Concept Quiz

1. Internal forces are not included in an equation of motion analysis because the internal forces are_____

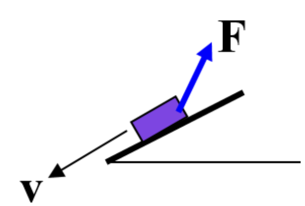
   A) equal to zero.

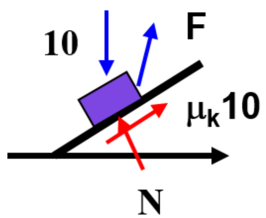
   B) equal and opposite and do not affect the calculations.

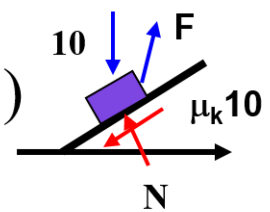
   C) negligibly small.

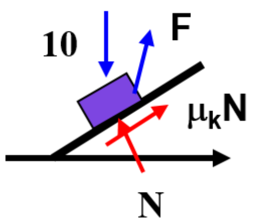
   D) not important.

2. A 10 lb block is initially **moving down** a ramp with a velocity of **v**. The force **F** is applied to bring the block to rest. Select the correct FBD.



A) 

B) 

C) 

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