Tutorial 3

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Tutorial 3: Motion in Two Dimensions

Part A: The Position, Velocity and Acceleration Vectors

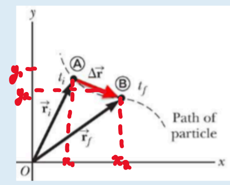
Displacement Vector: $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

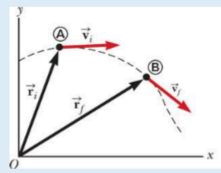
Average Velocity: $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$

Instantaneous Velocity: $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

Average Acceleration: $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$

Instantaneous Acceleration: $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$



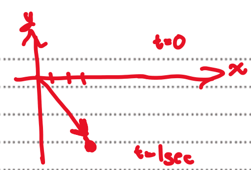


1) The vector position of a particle varies in time according to the expression $\vec{r}(t) = 3.00\hat{\imath} - 6.00t^2\hat{\jmath}\ m$, where $\vec{r}$ is in meters and t is in seconds.

a) Find an expression for the velocity of the particle as a function of time.

$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{d}{dt}(3.00\hat{\imath} - 6.00\hat{\jmath})$

$= (-12.00t\hat{\jmath})\ m/s$



b) Determine the acceleration of the particle as a function of time.

$\vec{a}(t) = \frac{d\vec{v}(t)}{dt}(-12.00t\hat{\jmath}) = (-12.00\hat{\jmath})\ m/s^2$

c) Calculate the particle's position and velocity at $t = 1.00\ s$.

$\vec{r}(t) = 3.00\hat{\imath} - 6.00t^2\hat{\jmath} \rightarrow \vec{r}(1.00s) = 3.00\hat{\imath} - 6.00\hat{\jmath}$

$\vec{v}(t) = (-12.00t\hat{\jmath}) \rightarrow \vec{v}(1.00s) = (-12.00\hat{\jmath})\ m/s$

2) Suppose the position vector for a particle is given as a function of time by $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$, with $x(t) = at + b$ and $y(t) = ct^2 + d$, where $a = 1.00\ m/s$, $b = 1.00\ m$, $c = 0.125\ m/s^2$, and $d = 1.00\ m$.

a) Calculate the average velocity during the time interval from $t = 2.00\ s$ to $t = 4.00\ s$.

$V_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = \frac{(5\hat{\imath} + 3\hat{\jmath}) - (3\hat{\imath} + 1.5\hat{\jmath})}{4-2} = \frac{2\hat{\imath} + 1.5\hat{\jmath}}{2} = (1\hat{\imath} + 0.75\hat{\jmath})\ m/s$

$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} = (at+b)\hat{\imath} + (ct^2+d)\hat{\jmath} = \underbrace{(1.00t + 1.00)}\hat{\imath} + \underbrace{(0.125t^2 + 1)}\hat{\jmath}$

$x(t)$ $y(t)$

at $t=2s \rightarrow \vec{r}_i = (3\hat{i} + 1.5\hat{j})\,m$
at $t=4s \rightarrow \vec{r}_f = (5\hat{i} + 3\hat{j})\,m$

magnitude of $V_{avg} = \sqrt{(1.0)^2 + (0.75)^2} = 1.25\,m/s$

b) Determine the velocity and the speed of the particle at $t = 2.00\ s$.

$\vec{V}(t) = \frac{d\vec{r}(t)}{dt} = \frac{d}{dt}\left[\underbrace{(1t+1)}_{x(t)}\hat{i} + \underbrace{(0.125t^2+1)}_{y(t)}\hat{j}\right]$

$= (1\hat{i} + 0.25t\,\hat{j})\,m/s$

at $t=2s \rightarrow \vec{V}(2s) = (1\hat{i} + 0.5\hat{j})\,m/s$

speed at $t=2s \rightarrow V = \sqrt{1^2 + (0.5)^2} = 1.12\,m/s$

Part B: Two-Dimensional Motion with Constant Acceleration

Particle moves in xy plane with a constant acceleration of $\vec{a}$ (a_x and a_y are both constant)

Velocity Vector: $\vec{v}_f = \vec{v}_i + \vec{a}t \quad \rightarrow \quad \begin{cases} \text{In the } x-\text{direction} \rightarrow \ v_{xf} = v_{xi} + a_x t \\ \text{In the } y-\text{direction} \rightarrow \ v_{yf} = v_{yi} + a_y t \end{cases}$

Position Vector: $\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \quad \rightarrow \quad \begin{cases} \text{In the } x-\text{direction} \rightarrow \ x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \\ \text{In the } y-\text{direction} \rightarrow \ y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \end{cases}$

1) A particle initially located at the origin has an acceleration of $\vec{a} = 3.00\hat{j}\ m/s^2$ and an initial velocity of $\vec{v}_i = 5.00\hat{i}\ m/s$.

a) Find the vector position of the particle at any time t,

$\vec{r}_f = \vec{r}_i + \vec{V}_i t + \frac{1}{2}\vec{a}\,t^2$

$= \vec{0} + (5\hat{i})t + \frac{1}{2}(3\hat{j})t^2 \rightarrow \vec{r}_f = (5t\hat{i} + 1.5t^2\hat{j})\,m$

b) Find the velocity of the particle at any time t,

$\vec{V}_f = \vec{V}_i + \vec{a}t = 5\hat{i} + 3\hat{j}t = 5\hat{i} + 3t\hat{j}\ m/s$

c) Find the coordinates of the particle at t = 2.00 s,

at $t=2s \rightarrow \vec{r}_f(2s) = (10\hat{i} + 6\hat{j})\,m$

d) Find the speed of the particle at t = 2.00 s,

$5\hat{i} + 3(2s)\hat{j}$ m/s

$5\hat{i} + 6\hat{j}$ m/s²

$v = \sqrt{5^2 + 6^2} = 7.81$ m/s

Part C: Projectile Motion

Projectile motion is a specific type of ....two.... dimensional motion.

If the projectile is launched at an upward angle from the horizontal, it will follow a path described mathematically as a ..parabola..

Projectile motion is considered as a combination of two analysis models:

1) Particle under ..constant velocity.. model in the x direction
2) Particle under ..constant acceleration.. model in the y direction with a_y = ..$-9.8 m/s^2$.. downward.

Constant Velocity Motion in x-direction: $x_f = x_i + v_{xi}t$

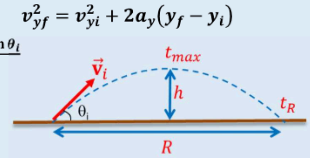
Constant Acceleration Motion in y-direction: $v_{yf} = v_{yi} + a_y t$, $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

Maximum height and its time: $h_{max} = \frac{v_i{}^2 \sin^2\theta_i}{2g}$, $t_{max} = \frac{v_i \sin\theta_i}{g}$

Range and its time: $R = \frac{v_i{}^2 \sin 2\theta_i}{g}$, $t_R = 2t_{max} = \frac{2v_i \sin\theta_i}{g}$

Maximum Range at $\theta_i = 45°$: $R_{max} = \frac{v_i{}^2}{g}$



1) A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?

$R = 3h_{max} \rightarrow \frac{v_i^2 \sin 2\theta_i}{g} = 3\frac{v_i^2 \sin^2\theta_i}{2g} \rightarrow \sin 2\theta_i = \frac{3}{2}\sin^2\theta_i$

$2\sin\theta_i \cos\theta_i = \frac{3}{2}\sin^2\theta_i \rightarrow 2\cos\theta_i = \frac{3}{2}\sin\theta_i \rightarrow \frac{4}{3} = \frac{\sin\theta_i}{\cos\theta_i} \rightarrow \frac{4}{3} = \tan\theta_i$ $\quad \theta_i = \tan^{-1}(\frac{4}{3})$ $= 53.1°$

2) A firefighter, a distance d = 100 m from a burning building, directs a stream of water from a fire hose at angle $\theta_i = 75°$ above the horizontal. If the initial speed of the stream is $v_i = 5\ m/s$, at what height h does the water strike the building?

$y_f = y_i + V_{yi}t + \frac{1}{2}a_j t^2$

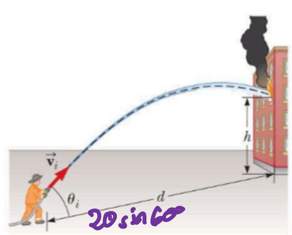
$h = 0 + (20\sin 60°)t + \frac{1}{2}(-9.8 m/s^2)t^2$

$h = 0 + (20\sin 60)$

$h = 15.04m$

$x_f = x_i + V_{xi}t \rightarrow 20m = 0 + (20\cos 60°)t$

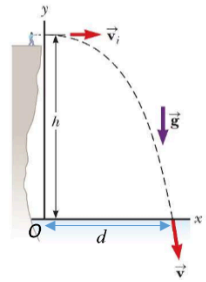
$t = \frac{20}{20\cos 60°} = 2s$



$20\sin 60$

$20\cos 60°$

3) A student stands at the edge of a cliff and throws a stone horizontally over the edge with a speed of v_i = 18.0 m/s. The cliff is h = 50.0 m above a body of water as shown in the figure. Assume the point O as the origin of the coordinate.



a) What are the coordinates of the initial position of the stone?

$(x_i, y_i) = (0, 50\,m)$

b) What are the components of the initial velocity of the stone?

$V_{yi} = V_i \sin\theta_i = 0$ $\quad \theta_i = 0$

$V_{xi} = V_i \cos\theta_i = V_i = 18.0\,m/s$

c) What is the appropriate analysis model for the vertical motion and horizontal motion of the stone?

d) How long after being released does the stone strike the water below the cliff?

$y_f = y_i + V_{yi} t + \frac{1}{2} a_y t^2$

$0 = 50 + 0 \times t + \frac{1}{2}(-9.8\,m/s^2)t^2 \rightarrow t^2 = \frac{2 \times 50}{9.8}$ $\quad t = 3.2\,s$

e) Find the horizonal distance of the stone when it strikes the water below the cliff?

$x_f = x_i + V_x t$

$d = 0 + (18\,m/s)(3.2\,s)$

$d = 57.93\,m$

f) With what speed and angle of impact does the stone land?

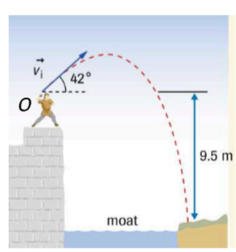
$V_{xf} = V_{xi} = V_i = 18\,m/s$

$V_{yf} = V_{yi} + a_y t = 0 + (-9.8\,m/s^2)(3.2\,s) = -31.3\,m/s$

$V_f = \sqrt{V_{xf}^2 + V_{yf}^2} = \sqrt{(18)^2 + (-31.2)^2} = 36.1\,m/s$

$\theta_f = \tan^{-1}\left(\frac{V_{yf}}{V_{xf}}\right) = \tan^{-1}\left(\frac{-31.3\,m/s}{18\,m/s}\right) = -60.1°$

4) A man throws a rock from the top of a wall with an initial velocity of 12.0 m/s at 42.0° above the horizontal line. The rock lands just on the far side of the moat, at a level of 9.5 m below the initial level. Assume the point O as the origin of the coordinate.



a) Determine the rock's time of flight

b) Determine the width of the moat

c) Determine the speed of the impact

Part D: Relative Velocity

The velocity $\vec{v}_{PA}$ of a particle measured in a fixed frame of reference S_A can be related to the velocity $\vec{v}_{PB}$ of the same particle measured in a moving frame of reference S_B as below

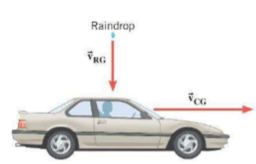
$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

where $\vec{v}_{BA}$ is the velocity of S_B relative to S_A.

1) The pilot of an airplane notes that the compass indicates a heading due west. The airplane's speed relative to the air is 150 km/h. The air is moving in a wind at 30.0 km/h toward the north. Find the velocity of the airplane relative to the ground.

2) A car travels due east with a speed of 50.0 km/h. Raindrops are falling at a constant speed vertically with respect to the Ground. The traces of the rain on the side windows of the car make an angle of 60.0° with the vertical.

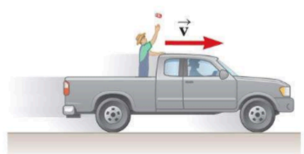
a) Find the velocity of the rain with respect to the car.



b) Find the velocity of the rain with respect to the Ground.

3) A farm truck moves due east with a constant velocity of 9.50 m/s on a limitless, horizontal stretch of road. A boy riding on the back of the truck throws a can of soda upward and catches the projectile at the same location on the truck bed, but 16.0 m farther down the road.

a) In the frame of reference of the truck, at what angle to the vertical does the boy throw the can?



b) What is the initial speed of the can relative to the truck?

c) What is the shape of the can's trajectory as seen by the boy?

d) An observer on the ground watches the boy throw the can and catch it. In this observer's frame of reference, describe the shape of the can's path.

e) Determine the magnitude and direction of the initial velocity of the can in the observer's frame of reference.