LAB 1: Dynamic Systems Modeling in MATLAB

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Michael McCorkell		43 / 50

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LAB 1 Grading Sheet

Student First Name and Last Name: Michael McCorkell	
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Total Mark	43 **/50**

LAB 1: Dynamic Systems Modeling in MATLAB

OBJECTIVES

- To learn how to model the differential equation of motion of mechanical systems in MATLAB
- To compute and analyze the time response of a dynamic system to the step input and ramp input

Modeling and Analysis of Mechanical Systems

Consider the mass-spring-damper system mounted on a massless cart as shown in Figure 1. A damper is a device that provides viscous friction, or damping. It consists of a piston and oil-filled cylinder. Any relative motion between the piston rod and the cylinder is resisted by the oil because the oil must flow around the piston (or through orifices provided in the piston) from one side of the piston to the other. The damper essentially absorbs energy, which is dissipated as heat. The damper does not store any kinetic or potential energy.

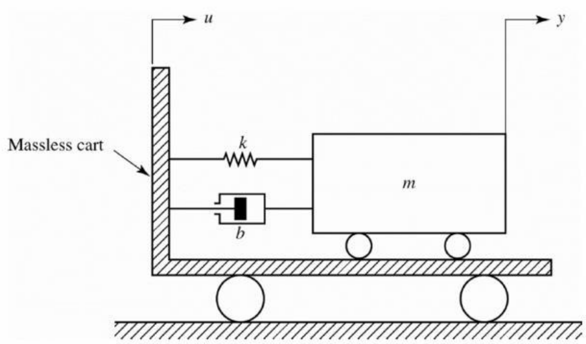


Figure 1: Mass-spring-damper system mounted on a cart.

Let us obtain a mathematical model of this system by assuming that both the cart and the mass-spring-damper system on it are standing still for $t < 0$. In this system,

$u(t)$ is the displacement of the cart and the **input** to the system,
$y(t)$ is the displacement of the mass relative to the ground is the **output** of the system,
m denotes the mass,
b denotes the viscous friction (damping) coefficient,
k denotes the spring constant.

We assume that the friction force of the damper is proportional to the relative velocity $\dot{y}(t) - \dot{u}(t)$ and that the spring is linear; that is, the spring force is proportional to relative displacement $y(t) - u(t)$.

For translational systems, Newton's second law states that:

$$\sum F = ma$$

where m is a mass, a is the acceleration of the mass, and $\sum F$ is the sum of the forces acting on the mass in the direction of the acceleration.

Applying Newton's second law to the present system and noting that the cart is massless, we obtain:

$$m\ddot{y}(t) = -b\big(\dot{y}(t) - \dot{u}(t)\big) - k\big(y(t) - u(t)\big)$$

or

$$m\ddot{y}(t) + b\dot{y}(t) + ky(t) = b\dot{u}(t) + ku(t) \qquad (1)$$

The latter equation represents a mathematical model of the system under consideration.

A.1. CREATE & SOLVE THE ODE DYNAMIC MODEL

Symbolic Math Toolbox in MATLAB enables the user to create and analytically solve the ordinary differential equations. This feature helps to define the equation of motion of a dynamic system and analyze the solution.

1. Consider the given dynamic model of the mass-spring-damper system from equation (1).
   Assume that $m = 10\ kg$, $b = 20\ Ns/m$, and $k = 100\ N/m$.
   Displacement of the cart as an input is constant 1 meter, $u(t) = 1$. The input is called **step input**.
   Rewrite the differential equation model having the numerical values and the given input.

   $10\ddot{y}(t) + 20\dot{y}(t) + 100y(t) = 1$ X [-1]

   Open a new script file and save it as **Lab1_PartA1**. Insert and run the following code in MATLAB to create the dynamic model.

```
m = 10;                    % mass (kg)
b = 20;                    % viscous friction coefficient (Ns/m)
k = 100;                   % spring constant (N/m)

syms t y(t);               % Define symbolic variables and symbolic function

%% Definition of successive derivatives of y(t)
D1y = diff(y,1);
D2y = diff(y,2);


eqn1 = m*D2y + b*D1y + k*y == k        % Define the dynamic model
```

   Provide the results here:

   eqn1(t) =

   100*y(t) + 20*diff(y(t), t) + 10*diff(y(t), t, t) == 100 ✓

2. The **`dsolve`** function in MATLAB enables the user to find an analytic solution of ordinary differential equations. Run the following code to obtain the response of the dynamic model to the applied input $u(t) = 1$, assume that the initial conditions are zero, $y(0) = 0,\ \dot{y}(0) = 0$.

```
sol1 = dsolve(eqn1, y(0)==0, D1y(0)==0)       % Response of the model
```

Provide the solution results:

sol1 =

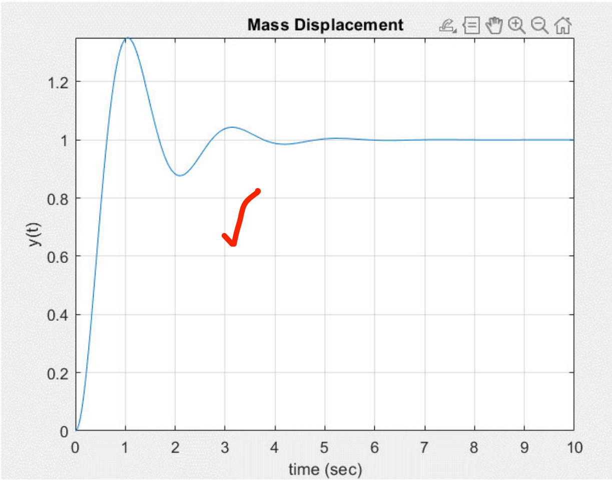
1 - (sin(3*t)*exp(-t))/3 - cos(3*t)*exp(-t)

3. We can plot the displacement of mass, using **fplot** function, and assign the time duration and add a title and axis labels to the graph.

   Run the following code in MATLAB to plot the displacement of mass as a function of time.

```
% Plot the solution y(t) as a function of t
figure;
fplot(sol1,[0,10]), grid on
xlabel('time (sec)'), ylabel('y(t)'), title('Mass Displacement')
```

Provide the graph:



4. Answer the following questions based on the graph.
   a) What is the final steady-state value of the mass displacement? Is it same as the cart displacement?

The final steady state value of mass displacement is 1m, it is the same as the cart Displacement.

b) Determine the maximum displacement of the mass from the steady-state value?

The maximum displacement of mass from steady state value would be 1.1 but reaching a maximum 1.3 in the miliseconds

c) Roughly determine the settling time, which is the time it takes for the mass to stop at the final position.

6 to 8 seconds, to fully stop. With friction and air probably after 11 seconds

A.2. EFFECT OF THE DAMPING ELEMENT

5. Modify the MATLAB code by changing the damping coefficient value first to $b = 5\ N.s/m$ and then to $b = 80\ N.s/m$. Run the code and find the solution for each case.

   Provide the results:

   Solution for $b = 5\ N.s/m$

```
sol1 =
1 - (159^(1/2)*exp(-t/4)*sin((159^(1/2)*t)/4))/159 - exp(-t/4)*cos((159^(1/2)*t)/4)
```

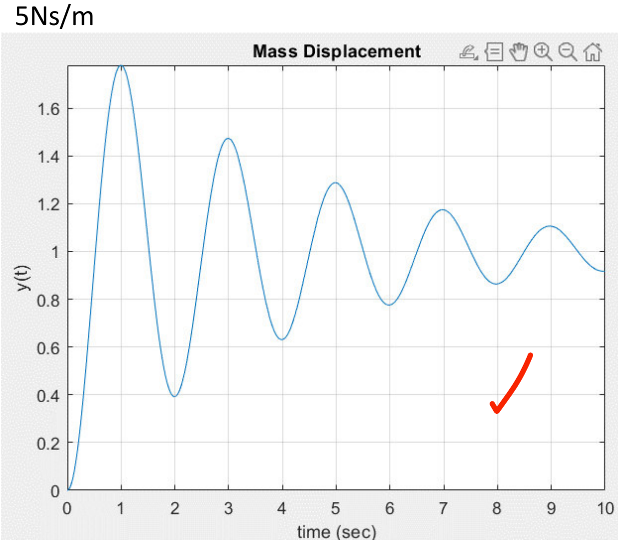
   Solution for $b = 80\ N.s/m$

```
sol1 =
exp(-t*(6^(1/2) + 4))*(6^(1/2)/3 - 1/2) - (6^(1/2)*exp(t*(6^(1/2) - 4))*(6^(1/2) + 4))/12 + 1
```

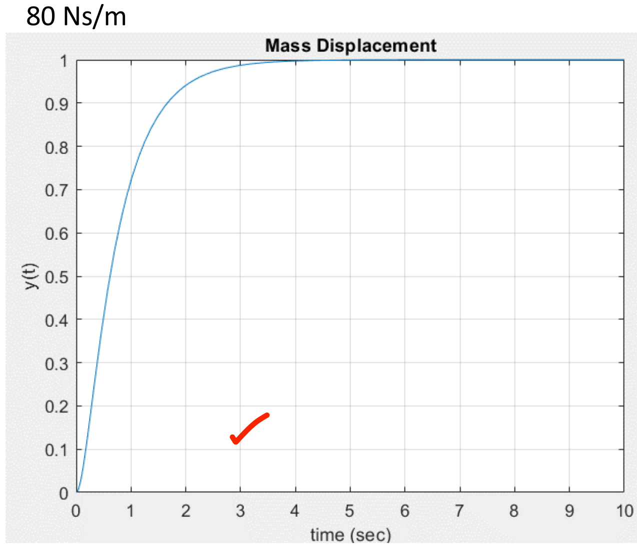
6. Plot the response graphs for each case. Adjust the time range to capture and appropriate response.

   Provide the graphs:

5Ns/m



80 Ns/m



7. Explain how changing the damping coefficient affects the transient response, oscillation and the settling time. Which system is underdamped, and which one is an over-damped system?

In a mass-spring-damper system, the damping coefficient b plays a crucial role determining how the system responds to a disturbance (like a step input) and how quickly it returns to steady-state.

Explain the effect of increasing and decreasing the damping coefficient.
Which system is overdamped and which one is underdamped system? [-2]

A.3. EFFECT OF THE APPLIED INPUT

8. Assume that the displacement of the cart (input) changes proportional to the time, $u(t) = t$. The input is called **ramp input**.

Rewrite the dynamic model of the system having the numerical values $m = 10\ kg, b = 20\ Ns/m$, and $k = 100\ N/m$ and the new input $u(t) = t$.

$$10\ddot{y}(t) + 20\dot{y}(t) + 100y(t) = t$$ X [-1]

9. Modify your MATLAB code to plot the new input, $u(t) = t$ and the response of the system to the new input.

Provide your code: Where is the code and plot of the input signal? [-2]

```
m = 10; % mass (kg)
b = 20; % viscous friction coefficient (Ns/m)
k = 100; % spring constant (N/m)
syms t y(t); % Define symbolic variables and symbolic function
%% Definition of successive derivatives of y(t)
D1y = diff(y,1);
D2y = diff(y,2);
eqn1 = m*D2y + b*D1y + k*y == t; % Define the dynamic model

sol1 = dsolve(eqn1, y(0)==0, D1y(0)==0);    % Response of the model
disp('Solution y(t): ');
disp(sol1);

% Plot the solution y(t) as a function of t
figure;
fplot(sol1,[0,10]), grid on;
xlabel('time (sec)'), ylabel('y(t)'), title('Mass Displacement');
```

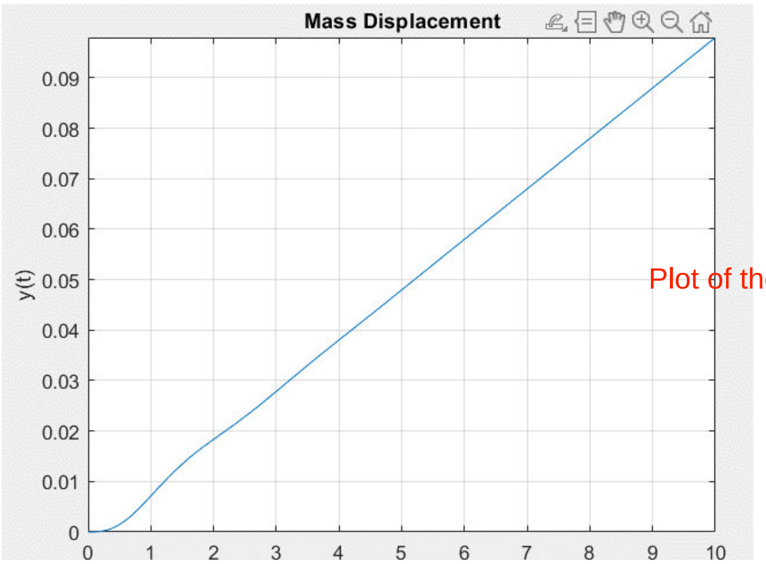
X [-1]

Run the code and provide the solution of the equation and the graph.
Solution of the model:

Solution y(t):
t/100 + (cos(3*t)*exp(-t))/500 - (sin(3*t)*exp(-t))/375 - 1/500

X

Graph of the mass displacement and input:



Plot of the input signal?

Post Lab Assignment

1) Consider the following differential equation represented a dynamic system with input of $f(t)$ and output of $x(t)$:

$$\ddot{x}(t) + 3\dot{x}(t) + 5x(t) = 10f(t), \qquad x(0) = 0, \quad \dot{x}(0) = 0$$

   a) Create the differential equation model using the symbolic math in MATLAB for step input $f(t) = 2$. Provide your code and results.

   b) Solve and plot the time response of the system for $f(t) = 2$, over the time interval $0 \le t \le 10$. Provide your code, results and the graph.

   c) Find the steady-state value and roughly estimate the settling time of the response from the graph.

   d) Solve and plot the time response of the system for $f(t) = 1.5t$, over the time interval $0 \le t \le 10$. Provide your code, results and the graph.

Post Lab Assignment

Consider the following differential equation represented a dynamic system with input of f(t) and output of x(t):

$\ddot{x}(t)+3\dot{x}(t)+5x(t)=10f(t),$ $x(0)=0,$ $\dot{x}(0)=0$

Create the differential equation model using the symbolic math in MATLAB for step input f(t)=2. Provide your code and results.

```
syms t x(t); % Define symbolic variable t and symbolic function x(t)
%% Definition of derivatives
D1x = diff(x,1);
D2x = diff(x,2);
%% Define the input f(t)
f_t = 2;
eqn = D2x + 3*D1x + 5*x == 10*f_t;
%% Solution
sol = dsolve(eqn, x(0) == 0, D1x(0) == 0);
disp('Solution = ');
disp(sol);
```

Solution =

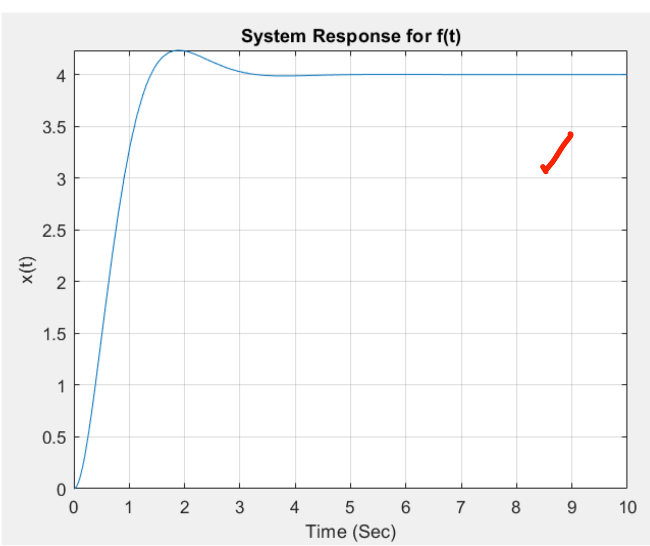
4 - (12*11^(1/2)*exp(-(3*t)/2)*sin((11^(1/2)*t)/2))/11 - 4*exp(-(3*t)/2)*cos((11^(1/2)*t)/2)

Solve and plot the time response of the system for f(t)=2, over the time interval 0≤t≤10. Provide your code, results and the graph.

```
%% Plot the Solution
figure;
fplot(sol,[0,10]), grid on;
xlabel('Time (Sec)'), ylabel('x(t)'), title('System Response for f(t)');
```

Solution =

4 - (12*11^(1/2)*exp(-(3*t)/2)*sin((11^(1/2)*t)/2))/11 - 4*exp(-(3*t)/2)*cos((11^(1/2)*t)/2)



Find the steady-state value and roughly estimate the settling time of the response from the graph.

```
The steady state values is 4 settling time around 3 to 5 seconds
```

Solve and plot the time response of the system for f(t)=1.5t, over the time interval 0≤t≤10. Provide your code, results and the graph.

```
syms t x(t); % Define symbolic variable t and symbolic function x(t)
%% Definition of derivatives
D1x = diff(x,1);
D2x = diff(x,2);
%% Define the input f(t)
f_t = 1.5*t;
eqn = D2x + 3*D1x + 5*x == 10*f_t;
%% Solution
sol = dsolve(eqn, x(0) == 0, D1x(0) == 0);
disp('Solution = ');
disp(sol);
%% Plot the Solution
figure;
fplot(sol,[0,10]), grid on;
xlabel('Time (Sec)'), ylabel('x(t)'), title('System Response for f(t)');
```

Solution =

4 - (12*11^(1/2)*exp(-(3*t)/2)*sin((11^(1/2)*t)/2))/11 - 4*exp(-(3*t)/2)*cos((11^(1/2)*t)/2)

Solution =

3*t + (9*exp(-(3*t)/2)*cos((11^(1/2)*t)/2))/5 - (3*11^(1/2)*exp(-(3*t)/2)*sin((11^(1/2)*t)/2))/55 - 9/5