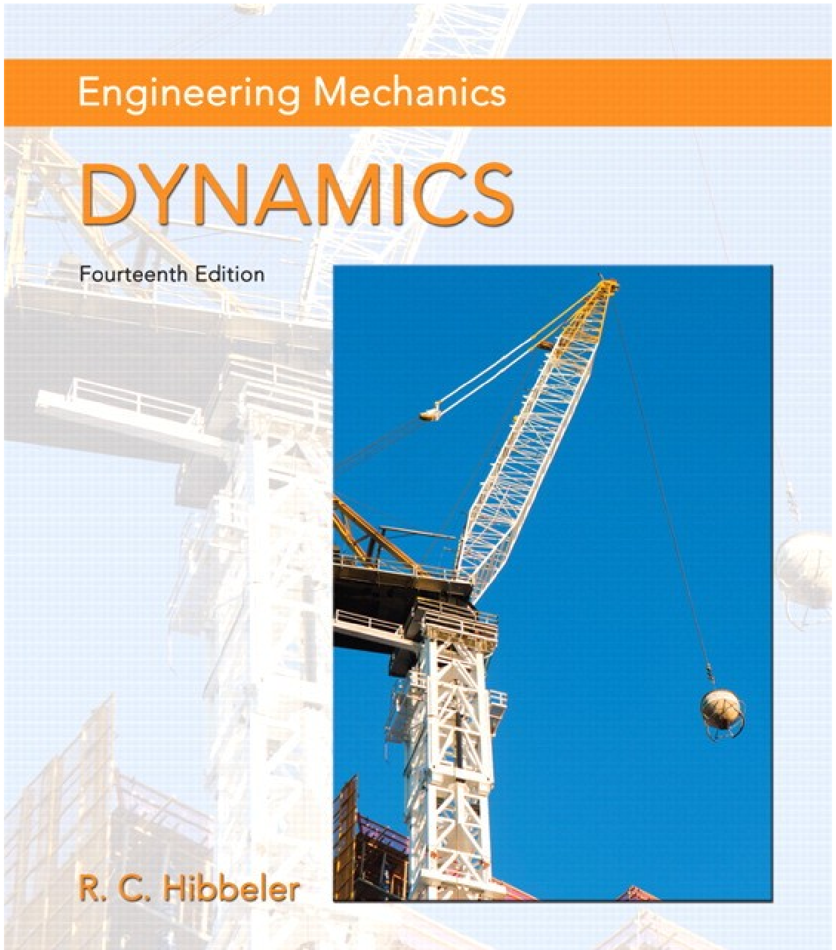
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 16

Planar Kinematics of a Rigid Body

Relative Motion Analysis: Acceleration
(1 of 2)

Today's Objective:

Students will be able to:

1. Resolve the acceleration of a point on a body into components of translation and rotation.
2. Determine the acceleration of a point on a body by using a relative acceleration analysis.

Relative Motion Analysis: Acceleration
(2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Translation and Rotation Components of Acceleration
- Relative Acceleration Analysis
- Roll-Without-Slip Motion
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1. If two bodies contact one another without slipping, and the points in contact move along different paths, the tangential components of acceleration will be ______ and the normal components of acceleration will be _________.

   A) the same, the same

   B) the same, different

   C) different, the same

   D) different, different

2. When considering a point on a rigid body in general plane motion,

   A) It's total acceleration consists of both absolute acceleration and relative acceleration components.

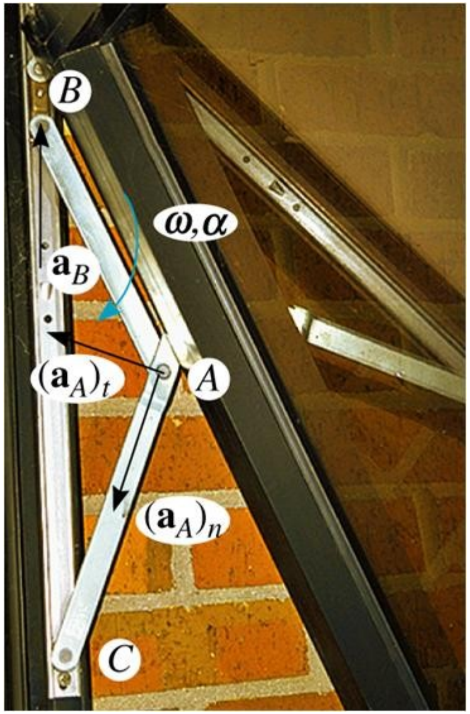
   B) It's total acceleration consists of only absolute acceleration components.

   C) It's relative acceleration component is always normal to the path.

   D) None of the above.

Applications (1 of 2)

In the mechanism for a window, link AC rotates about a fixed axis through C, and AB undergoes general plane motion. Since point A moves along a curved path, it has two components of acceleration while point B, sliding in a straight track, has only one.
The components of acceleration of these points can be inferred since their motions are known.

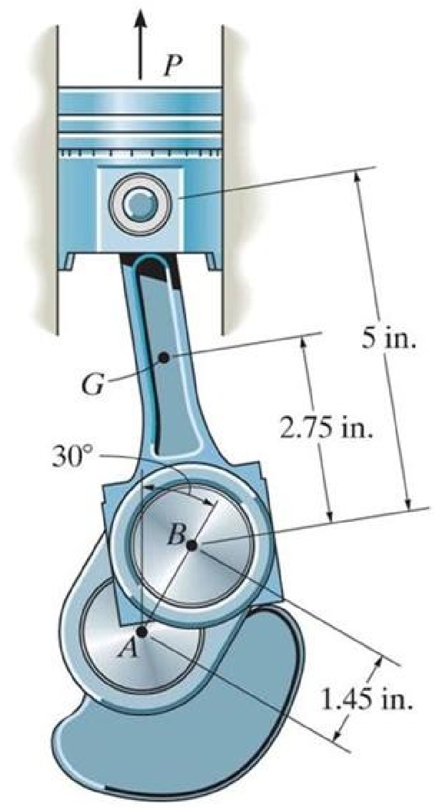


How can we determine the accelerations of the links in the mechanism?

Applications (2 of 2)

In an automotive engine, the forces delivered to the crankshaft, and the angular acceleration of the crankshaft, depend on the speed and acceleration of the piston.

How can we relate the accelerations of the piston, connection rod, and crankshaft to each other?



Section 16.7

RELATIVE MOTION ANALYSIS: ACCELERATION

Relative Motion Analysis: Acceleration

(1 of 3)

The equation relating the accelerations of two points on the body is determined by differentiating the velocity equation with respect to time.

$$\frac{dv}{dt} = \frac{dv_A}{dt} + \frac{dv_{B/A}}{dt}$$

These are absolute accelerations of points A and B. They are measured from a set of fixed x,y axes.

This term is the acceleration of B with respect to A and includes both **tangential** and **normal** components.

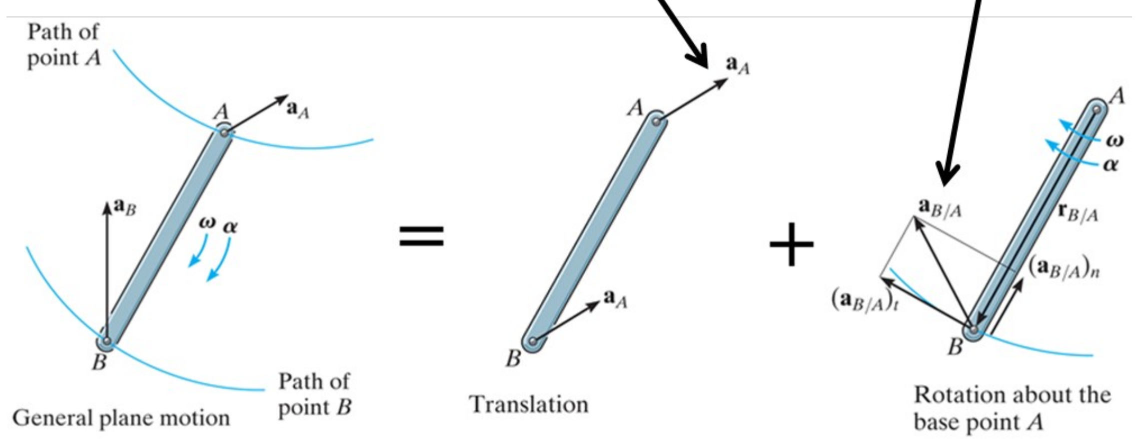
The result is $a_B = a_A + (a_{B/A})_t + (a_{B/A})_n$

Relative Motion Analysis: Acceleration

(2 of 3)

Graphically:

$$a_B = a_A + (a_{B/A})_t + (a_{B/A})_n$$



The relative tangential acceleration component $(a_{B/A})_t$ **is** $(a \times r_{B/A})_n$ and perpendicular to

The relative normal acceleration component $(a_{B/A})_n$ **is** $(-\omega^2 r_{B/A})_n$ and **the direction is always from B towards A**.

Relative Motion Analysis: Acceleration

Since the relative acceleration components can be expressed **as $(a_{B/A})_t = \alpha \times r_{B/A}$ and $(a_{B/A})_n = -\omega^2 r_{B/A}$, the relative** acceleration equation becomes

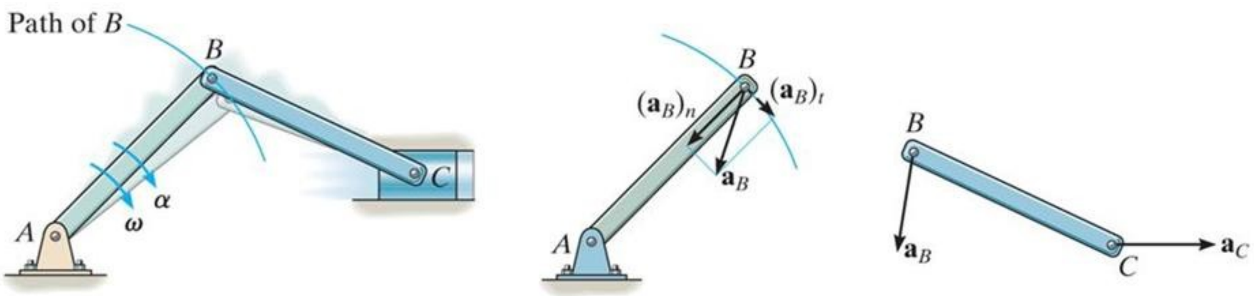
$$a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A}$$

Note that the **last term** in the relative acceleration equation is **not** a cross product. It is the product of a scalar (square of the magnitude of angular velocity,) and the relative position vector, $r_{B/A}$

ω^2 = **Square of the magnitude of angular velocity**

Application of the Relative Acceleration Equation

In applying the relative acceleration equation, the two points used in the analysis (A and B) should generally be selected as points which have a **known motion**, such as **pin connections** with other bodies.



In this mechanism, point B is known to travel along a **circular path**, so a_B can be expressed in terms of its normal and tangential components. Note that point B on link BC will have the **same acceleration** as point B on link AB.

Point C, connecting link BC and the piston, moves along **a straight-line path**. Hence, a_C is directed horizontally.

Procedure for Analysis

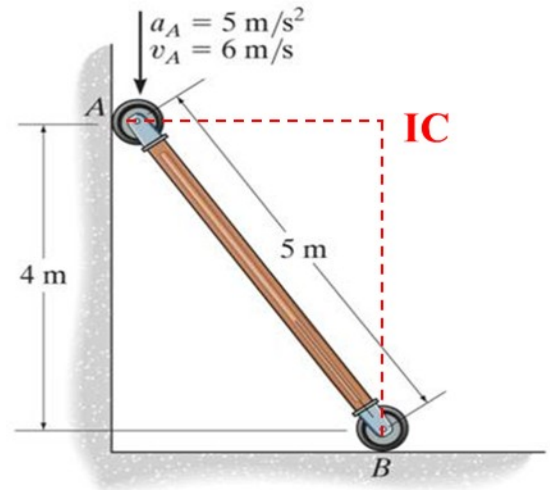
1. Establish a fixed coordinate system.
2. Draw the kinematic diagram of the body.
3. Indicate on it a_A, a_B, ω, α, and $r_{B/A}$. If the points A and B move along curved paths, then their accelerations should be indicated in terms of their tangential and normal components, i.e., $a_A = (a_A)_t + (a_A)_n$ and $a_B = (a_B)_t + (a_B)_n$.
4. Apply the relative acceleration equation:

$$a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A}$$

5. If the solution yields a negative answer for an unknown magnitude, this indicates that the sense of direction of the vector is opposite to that shown on the diagram.

Example (1 of 3)

Given: Point A on rod AB has an acceleration of 5 m/s^2 and a velocity of 6 m/s at the instant shown.



Find: The angular acceleration of the rod and the acceleration at B at this instant.

Plan: Follow the problem solving procedure!

Solution: First, we need to find the angular velocity of the rod at this instant. Locating the instant center (IC) for rod AB, we can determine ω

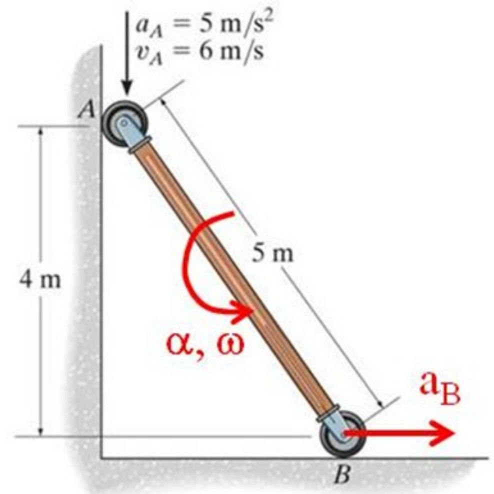
$$\omega = v_A / r_{A/IC} = 6/(3) = 2 \text{ rad/s}$$

Example (2 of 3)

Since points A and B both move along straight-line paths,

$$a_A = -5\,j \text{ m/s}^2$$

$$a_B = a_B i \text{ m/s}^2$$



Applying the relative acceleration equation

$$a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A}$$

$$a_B\, i = -5\, j + \alpha\, k \times (3\, i - 4\, j) - 2^2 (3\, i - 4\, j)$$

$$a_B\, i = -5\, j + 4\, \alpha\, i + 3\, \alpha\, j + (-12\, i + 16\, j)$$

$$= (4\, \alpha - 12)\, i + (3\, \alpha + 11)\, j$$

Example (3 of 3)

So with $a_B \, i = = (4\alpha - 12)\, i + (3a + 11)\, j$, we can solve for a_B and α.

By comparing the i, j components;

$$a_B = 4\alpha - 12$$

$$0 = 3\alpha + 11$$

Solving:

$$a_B = -26.7 \text{ m/s}^2$$

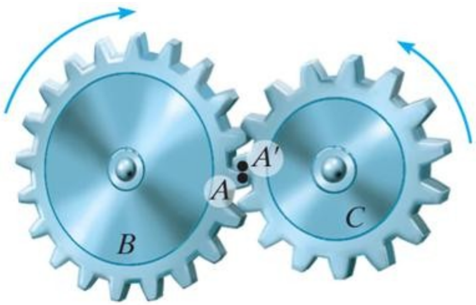
$$= -26.7 \text{ m/s}^2 \leftarrow$$

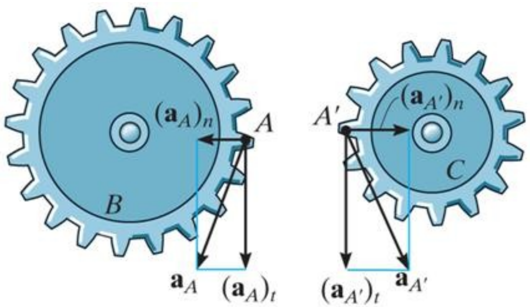
$$\alpha = -3.67 \text{ rad/s}^2$$

$$= -3.67 \text{ rad/s}^2$$

Bodies in Contact

Consider two bodies in contact with one another **without slipping,** where the points in contact move along **different paths.**





In this case, the **tangential components** of acceleration will be the **same**, i. e.,

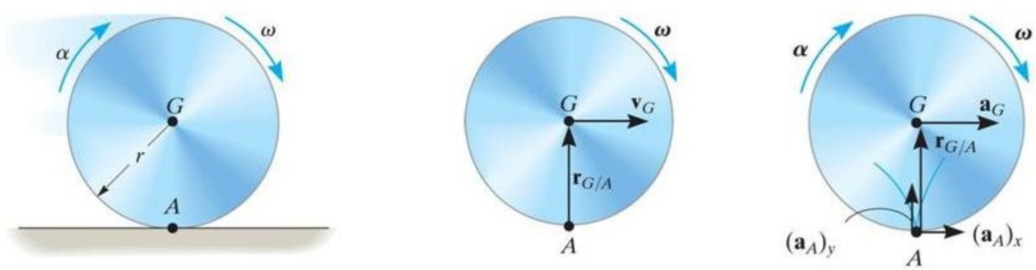
$$(a_A)_t = (a_{A'})_t \left(\text{which implies } \alpha_B\, r_B = \alpha_C\, r_C \right)$$

The **normal components** of acceleration will **not** be the same.

$$(a_A)_n \neq (a_{A'})_n \text{ so } a_A \neq a_{A'}$$

Rolling Motion (1 of 2)

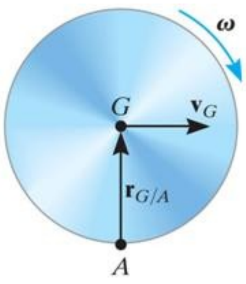
Another common type of problem encountered in dynamics involves **rolling motion without slip**; e.g., a ball, cylinder, or disk rolling without slipping. This situation can be analyzed using relative velocity and acceleration equations.



As the cylinder rolls, point G (center) moves along a **straight line.** If ω and α are known, the relative velocity and acceleration equations can be applied to A, at the instant A is in **contact** with the ground. The point A is the instantaneous center of zero velocity, however it **is not a point of zero acceleration.**

Rolling Motion (2 of 2)

- **Velocity:** Since no slip occurs, $v_A = 0$ when A is in contact with ground. From the kinematic diagram:

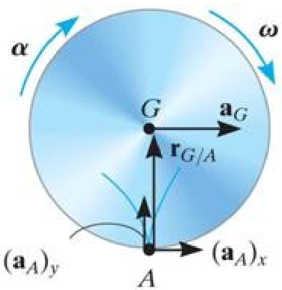


$$v_G = v_A + \omega \times r_{G/A}$$

$$v_G\, i = 0 + (-\omega\, k) \times (r\, j)$$

$$v_G = \omega r \text{ or } v_G = \omega r\, i$$

- **Acceleration:** Since G moves along a straight-line path, horizontal. Just **before** A touches ground, its velocity is directed **downward**, and just **after** contact, its velocity is directed **upward.** Thus, point A **accelerates upward** as it leaves the ground.



$$a_G = a_A + \alpha \times r_{G/A} - \omega^2 r_{G/A} \Rightarrow a_G\, i = a_A\, j + (-\alpha\, k) \times (r\, j) - \omega^2 (r\, j)$$

Evaluating and equating i and j components:

$$a_G = \alpha r \text{ and } a_A = \omega^2 r \text{ or } a_G = \alpha r\, i \text{ and } a_A = \omega^2 r\, j$$

Reading Quiz 2

1. If a ball rolls without slipping, select the tangential and normal components of the relative acceleration of point **A with respect to G.**

   A) $\alpha r\, i + \omega^2 r\, j$

   B) $-\alpha r\, i + \omega^2 r\, j$

   C) $\omega^2 r\, i - \alpha r\, i$

   D) Zero

2. What are the tangential and normal components of the relative acceleration of point **B with respect to G.**

   A) $-\omega^2 r\, i - \alpha r\, j$

   B) $-\alpha r\, i + \omega^2 r\, j$

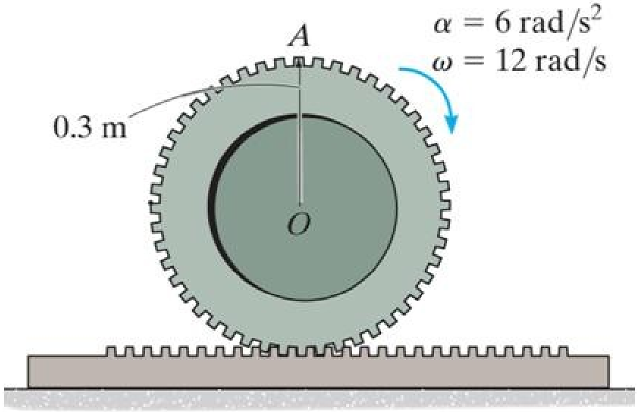
   C) $\omega^2 r\, i - \alpha r\, i$

   D) Zero

Example (1 of 2)

Given: The gear with a center at O rolls on the fixed rack.

Find: The acceleration of point A at this instant.



Plan: Follow the problem solving procedure!

Solution: Since the gear rolls on the fixed rack without slip, is directed to the right with a magnitude of

$$a_o = \alpha r = (6\text{ rad/s}^2)(0.3\text{ m}) = 1.8\text{ m/s}^2$$

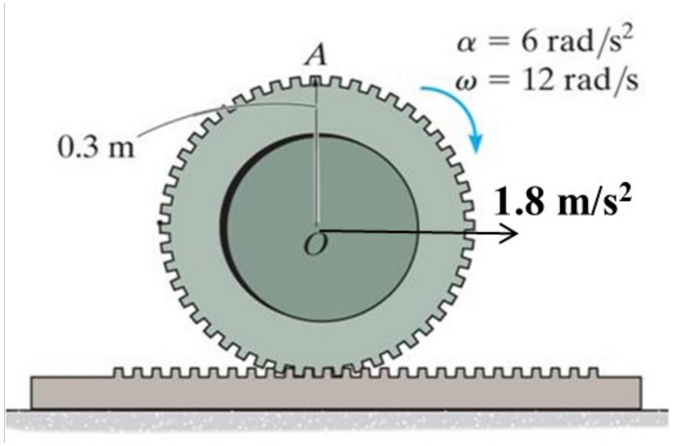
Example (2 of 2)

So now with $a_o = 1.8$ m/s², we can apply the relative acceleration equation between points O and A.

$$a_A = a_o + \alpha \times r_{A/O} - \omega^2 r_{A/O}$$

$$a_A = 1.8\,i + (-6\,k) \times (0.3\,j) - 12^2 (0.3\,j)$$

$$a_A = (3.6\,i - 43.2\,j)\,\text{m/s}^2$$



Group Problem Solving (1 of 5)

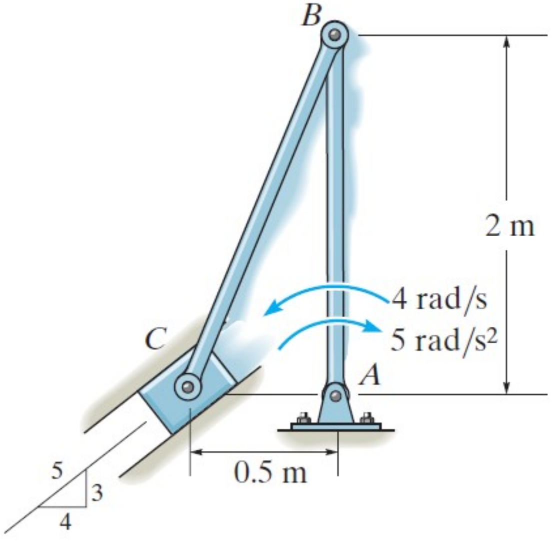
Given: Member AB is rotating with

$\omega_{AB} = 4$ rad/s, $\alpha_{AB} = 5$ rad/s^2

at this instant.

Find: The velocity and acceleration of the slider block C.

Plan: Follow the solution procedure!



Note that Point B is rotating about A. So what components of acceleration will it be experiencing?

Group Problem Solving (2 of 5)

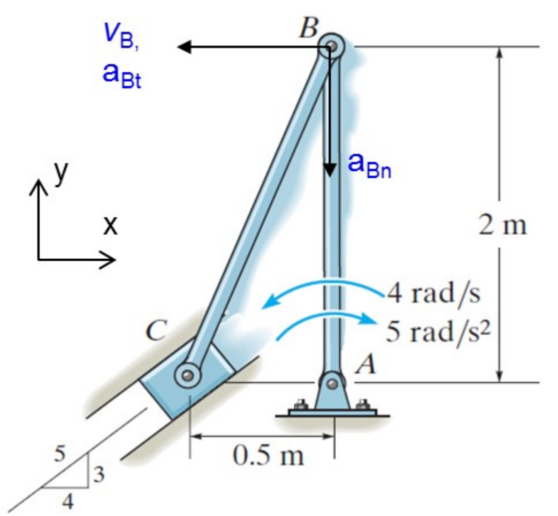
Solution:

Since Point B is rotating, its velocity and acceleration will be:

$$v_B = (\omega_{AB}) r_{B/A} = (4)2 = 8 \text{ m/s}$$

$$a_B = (\omega_{AB})^2 r_{B/A} = (4)^2\, 2 = 32 \text{ m/s}^2$$

$$a_{Bt} = (\alpha_{AB}) r_{B/A} = (-5)2 = -10 \text{ m/s}^2$$



Thus:

$$v_B = (-8\,i) \text{m/s}$$

$$a_B = (10\,i - 32\,j) \text{m/s}^2$$

Group Problem Solving (3 of 5)

Now apply the **relative velocity equation** between points B and C to find the angular velocity of link BC.

$$v_C = v_B + \omega_{BC} \times r_{C/B}$$

$$(-0.8\,i - 0.6\,j)\,v_C = (-8\,i) + \omega_{BC}\,k \times$$

$$(-0.8\,i - 0.6\,j)\,v_C = (-8 + 2\,\omega_{BC})\,i$$

$$-\,0.5\,\omega_{BC}\,j$$

By **comparing** the i, j components:

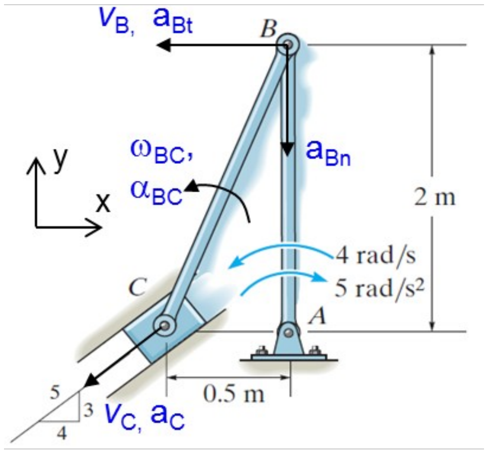
$$-0.8\,v_C = -8 + 2\,\omega_{BC}$$

$$-0.6\,v_C = -0.5\,\omega_{BC}$$

Solving:

$$\omega_{BC} = 3 \text{ rad/s}$$

$$v_C = 2.50 \text{ m/s}$$



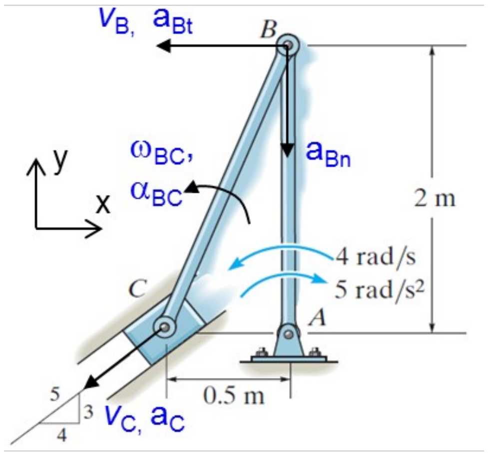
Group Problem Solving (4 of 5)

Now apply the **relative velocity equation** between points B and C to find the angular velocity of link BC.

$$a_C = a_B + \alpha_B \times r_{C/B} - \omega^2_{BC}\, r_{C/B}$$

$$(-0.8\,i - 0.6\,j)\,a_C = (10\,i - 32\,j) + \alpha_{BC}\,k \times (-0.5\,i - 2\,j) - (3^2)(-0.5\,i - 2\,j)$$

$$(-0.8\,i - 0.6\,j)\,a_C = (10\,i - 32\,j) + (2\alpha_{BC}\,i - 0.5\alpha_{BC}\,j) + (4.5\,i - 18\,j)$$



By **comparing** the i, j components:

$$-0.8a_C = 14.5 + 2\alpha_{BC}$$

$$-0.6a_C = -14 - 0.5\alpha_{BC}$$

Group Problem Solving (5 of 5)

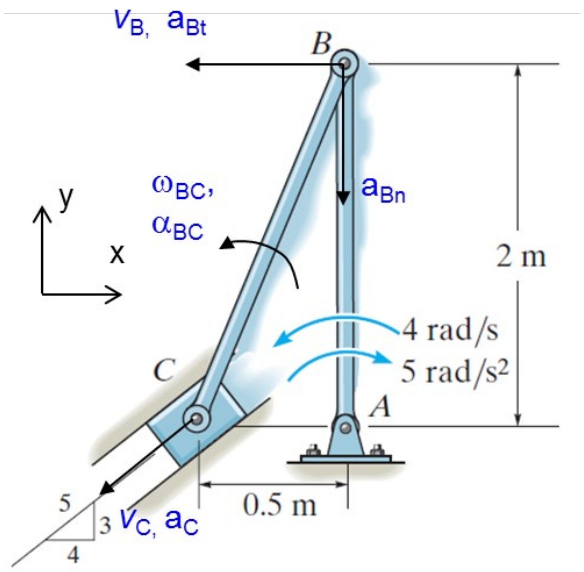
Solving these two i, j component equations for a_A and α_{AB} yields:

$$-0.8a_C = 14.5 + 2\alpha_{BC}$$

$$-0.6a_C = -14 - 0.5\alpha_{BC}$$

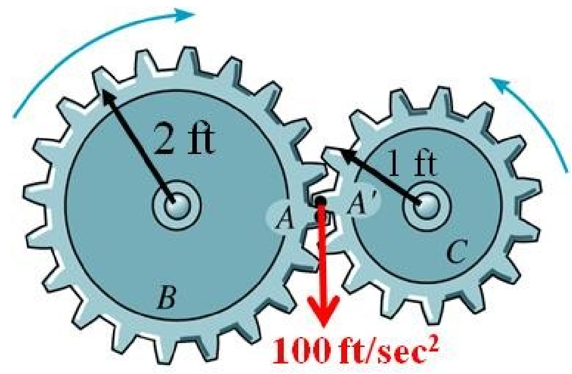
$$\alpha_{BC} = -12.4 \text{ rad/s}^2 = 12.4 \text{ rad/s}^2$$

$$a_C = 13.0 \text{ m/s}^2$$



Reading Quiz 3

1. Two bodies contact one another without slipping. If the tangential component of the acceleration of point A on gear B is 100 feet per second square. determine the tangential component of the acceleration of point A' on gear C.



A) $50\ ft/sec^2$ B) $100\ ft/sec^2$

C) $200\ ft/sec^2$ D) None of above.

2. What are the tangential and normal components of the relative acceleration of point **B with respect to G.**

A) $50\ rad/sec^2$ B) $100\ rad/sec^2$

C) $200\ rad/sec^2$ D) None of above.

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