HUMBER ENGINEERING

ENGI 1000 - Physics 1

WEEK 14

HUMBER

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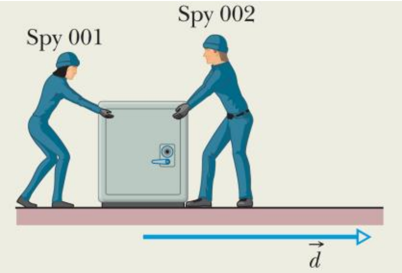
Final Exam Review

- Work and Energy
- Conservation of Energy
- Collision and Conservation of Momentum
- Rotational Motion
- Oscillatory Motion
- Wave Motion

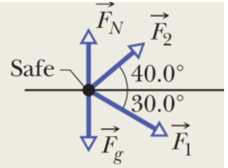
Final Exam Review

Example 1 (Work Done by two Constant Forces): Two spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m. The push $\vec{F}_1$ of spy 001 is 12.0 N at an angle 30.0° downward from the horizontal. The pull $\vec{F}_2$ of spy 002 is 10.0 N at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?



First draw the free body diagram of the safe



The work done by $\vec{F}_1$ and $\vec{F}_2$ are

$$W_1 = F_1 cos\theta_1 \, \Delta x = (12.0 \, N) \cos 30.0° \, (8.50 \, m) = 88.33 \, J$$

$$W_2 = F_2 cos\theta_2 \, \Delta x = (10.0 \, N) \cos 40.0° \, (8.50 \, m) = 65.11 \, J$$

The net external work done on the safe is

$$W_{ext} = W_1 + W_2 = 88.33 \, J + 65.11 \, J = 153.44 \, J$$

Final Exam Review

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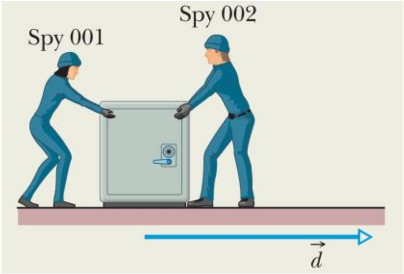
(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

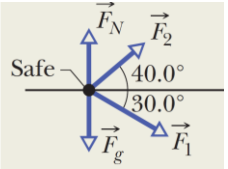


Since these forces are perpendicular to the displacement of the safe, they do zero work on the safe.

$$W_g = F_g cos\theta \, \Delta x = F_g \cos 90° \; d = 0$$

$$W_N = F_N cos\theta \, \Delta x = F_N \cos 90° \; d = 0$$

Only force components parallel to the displacement do work.



Final Exam Review

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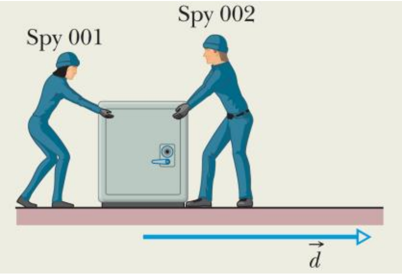
(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

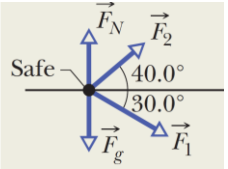


We can find the final speed from the Work-Kinetic Energy theorem

$$W_{ext} = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$153.44\,J = \frac{1}{2}(225\,kg)v_f^2 - 0 \quad \rightarrow \quad v_f = \sqrt{\frac{2(153.44\,J)}{225\,kg}} = 1.17\,m/s$$

Only force components parallel to the displacement do work.

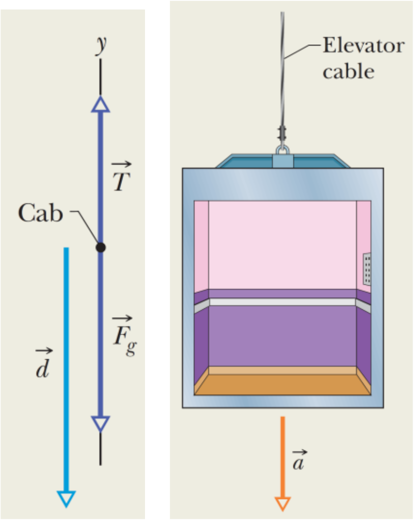


Final Exam Review

Example 2 (Work done on an accelerating elevator cab): An elevator cab of mass $m = 500\ kg$ is descending with speed $v_i = 4.0\ m/s$ when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$.

(a) During the fall through a distance $d = 12\ m$, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

First, draw the free body diagram of the cab



The work done by the gravitational force $\vec{F}_g$ is

$$W_g = F_g cos\theta\ \Delta x = mg\cos 0°\, d$$

$$= (500\ kg)(9.8\ m/s^2)(12\ m)$$

$$= 5.88 \times 10^4\ J$$

Since the gravitational force is in the same direction of the displacement, the work done by the gravitational force is positive.

Final Exam Review

Example 2 (Work done on an accelerating elevator cab): An elevator cab of mass $m = 500\ kg$ is descending with speed $v_i = 4.0\ m/s$ when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$.

(b) During the $d = 12\ m$ fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

First, apply the Newton's second law to find the tension force $\vec{T}$

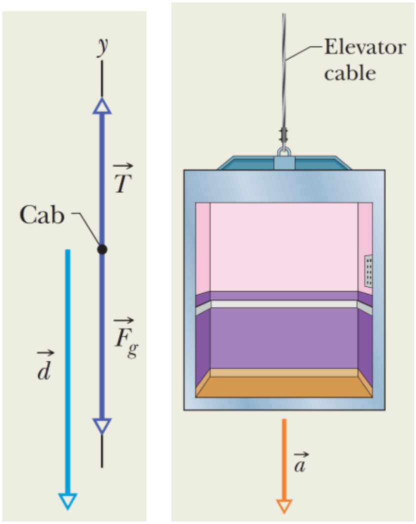
$$\sum F_y = ma \quad \rightarrow \quad T - F_g = ma \quad \rightarrow \quad T = m(g + a) = m\left(g - \frac{g}{5}\right) = \frac{4}{5}mg$$

The work done by the tension force $\vec{T}$ is

$$W_T = T cos\theta\ \Delta x = \left(\frac{4}{5}mg\right)\ \cos 180°\, d \ = -0.8(500\ kg)(9.8\ m/s^2)(12\ m)$$

$$= -4.70 \times 10^4\ J$$

Since the tension force is in the opposite direction of the displacement, the work done by the tension force is negative.



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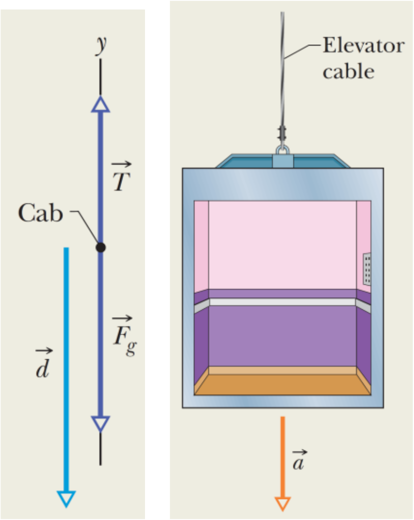
Example 2 (Work done on an accelerating elevator cab): An elevator cab of mass $m = 500\ kg$ is descending with speed $v_i = 4.0\ m/s$ when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$.

(c) What is the net external work done on the cab during the fall?

The net work is the sum of the works done by the forces acting on the cab:

$$W_{ext} = W_g + W_T \quad = 5.88 \times 10^4\ J + (-4.70 \times 10^4\ J) = 1.18 \times 10^4\ J$$



(d) What is the cab's kinetic energy at the end of the 12 m fall?

The kinetic energy changes due to the external work done on the cab,

$$W_{ext} = K_f - K_i \quad \rightarrow \quad K_f = W_{ext} + K_i = W_g + W_T + \frac{1}{2} m v_i^2$$

$$K_f = 1.18 \times 10^4\ J + \frac{1}{2}(500\ kg)(4.0 m/s)^2 \quad = 1.58 \times 10^4\ J$$

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Example 3 (Work done by a spring to change kinetic energy): A block of mass $m = 0.40\ kg$ slides across a horizontal frictionless counter with speed $v = 0.50\ m/s$. It then runs into and compresses a spring with spring constant of $k = 750\ N/m$.

When the block is momentarily stopped by the spring, by what distance d is the spring compressed?

From the Work-Kinetic Energy theorem we have

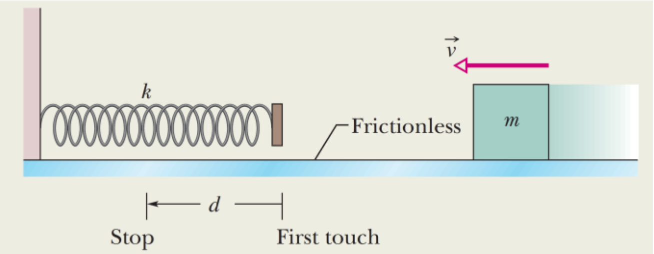
$$W_{ext} = K_f - K_i \ \rightarrow\ W_s = K_f - K_i$$

$$\frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$0 - \frac{1}{2}kd^2 = 0 - \frac{1}{2}mv_i^2 \quad \rightarrow \quad d = |v_i|\sqrt{\frac{m}{k}}$$

$$d = (0.50\ m/s)\sqrt{\frac{0.40\ kg}{750\ N/m}} = 1.2 \times 10^{-2}\ m$$

The spring force does negative work, decreasing speed and kinetic energy



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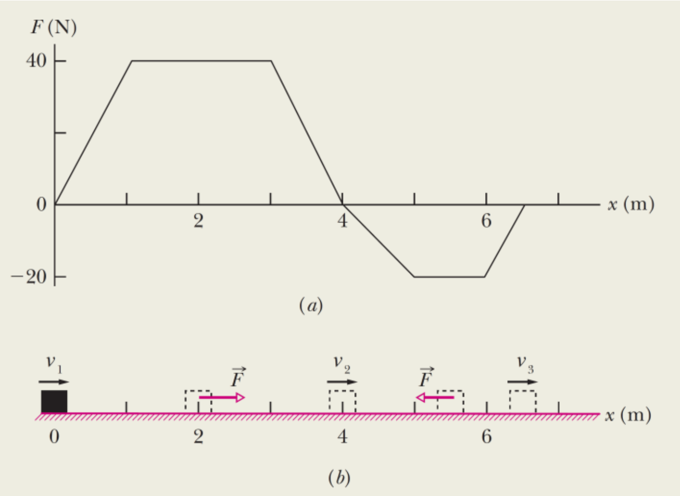
Example 4 (Work done by a variable force): An 8.0 kg block slides along a frictionless floor as a force acts on it, starting at $x_1 = 0$ and ending at $x_3 = 6.5\ m$. As the block moves, the magnitude and direction of the force varies according to the graph. If the block's kinetic energy at x_1 is $K_1 = 280\ J$.

What is the block's speed at $x_1 = 0, x_2 = 4.0\ m$ and $x_3 = 6.5\ m$?

The block speed at $x_1 = 0$ is

$$K_1 = \frac{1}{2}mv_1^2 \quad \rightarrow \quad v_1 = \sqrt{\frac{2K_1}{m}} = \sqrt{\frac{2(280\ J)}{8.0\ kg}} = 8.37\ m/s$$



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Final Exam Review

Example 4 (Work done by a variable force): An 8.0 kg block slides along a frictionless floor as a force acts on it, starting at $x_1 = 0$ and ending at $x_3 = 6.5\ m$. As the block moves, the magnitude and direction of the force varies according to the graph. If the block's kinetic energy at x_1 is $K_1 = 280\ J$.

What is the block's speed at $x_1 = 0, x_2 = 4.0\ m$ and $x_3 = 6.5\ m$?

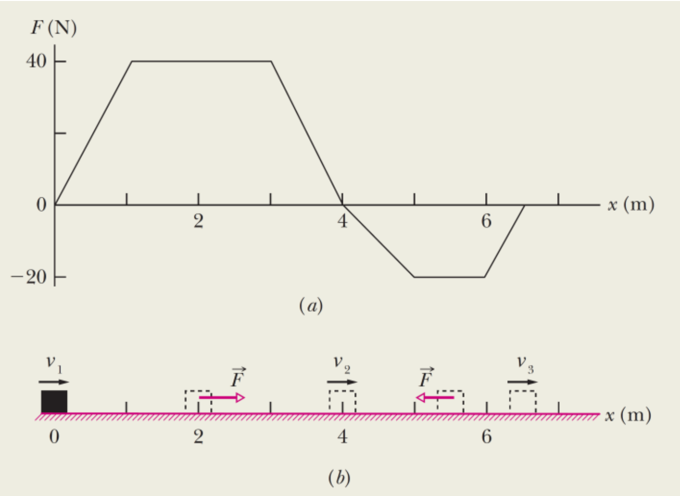
The block's kinetic energy at x_2 is obtained from the work-kinetic energy theorem

$$W_{ext} = K_f - K_i \quad \rightarrow \quad W_{ext} = K_2 - K_1$$

The work done by the external force from x_1 to x_2 is equal to the area under the curve from x_1 to x_2

$$W_{ext} = \frac{1}{2}(2\ m + 4\ m)(40\ N) = 120\ J$$

$$K_2 = W_{ext} + K_1 = 120\ J + 280\ J = 400\ J$$



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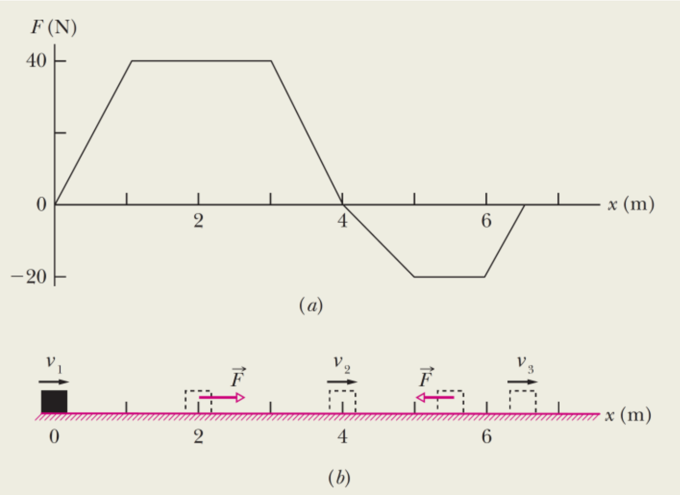
What is the block's speed at $x_1 = 0, x_2 = 4.0\ m$ and $x_3 = 6.5\ m$?

The block speed at $x_2 = 4.0\ m$ is obtained from kinetic energy of the block at x_2

$$K_2 = \frac{1}{2} m v_2^2 \quad \rightarrow \quad v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(400\ J)}{8.0\ kg}} = 10\ m/s$$



This is the block's greatest speed because from $x = 4.0\ m$ to $x = 6.5\ m$ the force is negative, meaning that it oppose the block's motion. Thus, decreasing the speed.

Final Exam Review

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What is the block's speed at $x_1 = 0, x_2 = 4.0\ m$ and $x_3 = 6.5\ m$?

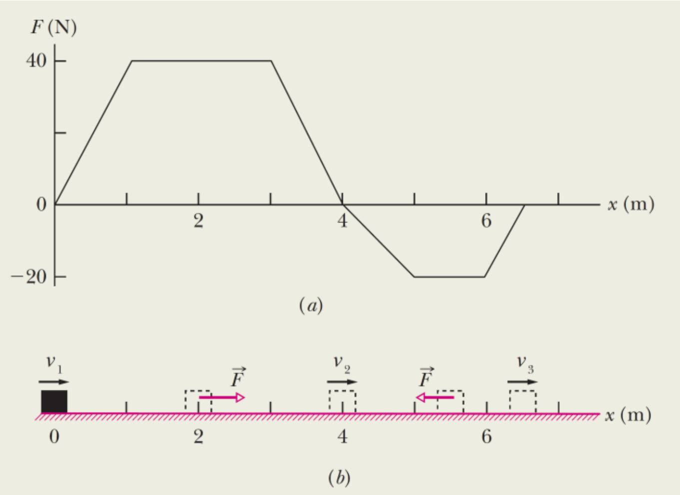
The block's kinetic energy at x_3 is obtained from the work-kinetic energy theorem

$$W_{ext} = K_f - K_i \quad \rightarrow \quad W_{ext} = K_3 - K_2$$

The work done by the external force from x_2 to x_3 is equal to the area under the curve from x_2 to x_3

$$W_{ext} = \frac{1}{2}(1\ m + 2.5\ m)(-20\ N) = -35\ J$$

$$K_3 = W_{ext} + K_2 = -35\ J + 400\ J = 365\ J$$



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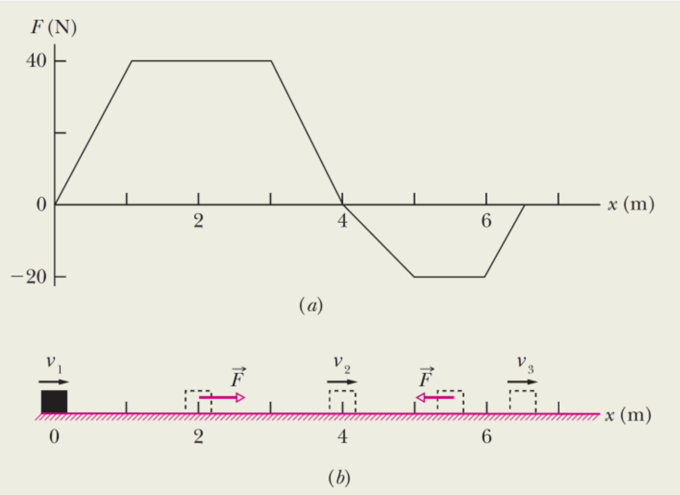
What is the block's speed at $x_1 = 0, x_2 = 4.0\ m$ and $x_3 = 6.5\ m$?

The block speed at $x_3 = 6.5\ m$ is obtained from kinetic energy of the block at x_3

$$K_3 = \frac{1}{2}mv_3^2 \quad \rightarrow \quad v_3 = \sqrt{\frac{2K_3}{m}} = \sqrt{\frac{2(365\ J)}{8.0\ kg}} = 9.55\ m/s$$

The block still moving in the positive direction of x-axis, a bit faster than the initially.



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Example 5 (Conservation of Mechanical Energy): A child of mass m is released from rest at the top of a water slide, at height $h = 8.5\ m$ above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.

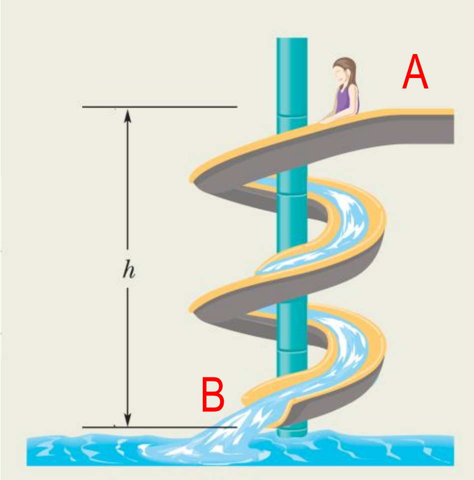
Since the water slide is frictionless, based on the conservation of mechanical energy from point A to B,

$$E_{mB} = E_{mA} \quad \rightarrow \quad K_B + U_B = K_A + U_A$$

$$\frac{1}{2}mv_B^2 + mgy_B = \frac{1}{2}mv_A^2 + mgy_A$$

$$\frac{1}{2}mv_B^2 + 0 = 0 + mgh$$

$$v_B = \sqrt{2gh} = \sqrt{2(9.8\ m/s^2)(8.5\ m)} = 13\ m/s$$



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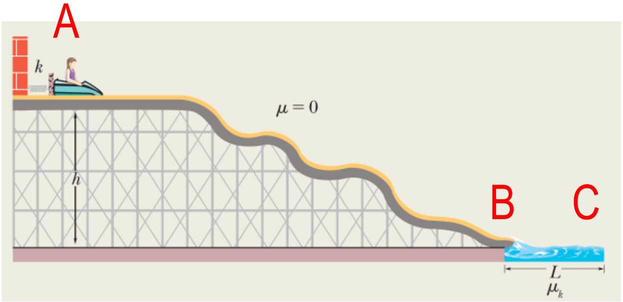
Example 6 (Conservation of Energy): In a spring-loaded water-slide ride a glider is shot by a spring along a frictionless water-drenched track that takes the glider from a horizontal section down to ground level. As the glider then moves along ground-level track, it is gradually brought to rest by friction.

The total mass of the glider and its rider is $m = 200\ kg$, the initial compression of the spring is $x = 5.00\ m$, the spring constant is $k = 3.20 \times 10^3\ N/m$, the initial height is $h = 35.0\ m$, and the coefficient of kinetic friction along the ground-level track is $\mu_k = 0.800$.

Trough what distance L does the glider slide along the ground-level track until it stops?



Since the water-drenched track is frictionless, based on the conservation of mechanical energy from point A to B,

$$E_{mB} = E_{mA} \quad \rightarrow \quad E_{mB} = \frac{1}{2}kx^2 + mgh$$

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Example 6 (Conservation of Energy):

The total mass of the glider and its rider is $m = 200\ kg$, the initial compression of the spring is $x = 5.00m$, the spring constant is $k = 3.20 \times 10^3\ N/m$, the initial height is $h = 35.0\ m$, and the coefficient of kinetic friction along the ground-level track is $\mu_k = 0.800$.

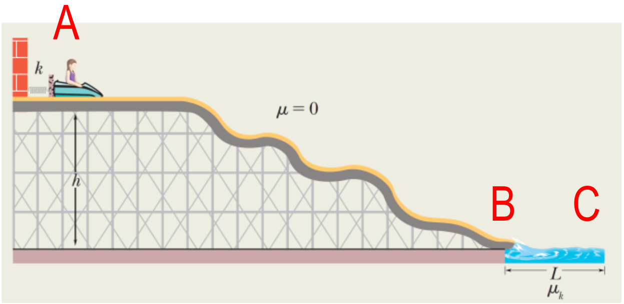
Trough what distance L does the glider slide along the ground-level track until it stops?

Since the ground-level track has friction, the work done by the non-conservative force from point B to C,

$$W_{nc} = E_{mC} - E_{mB} \quad \rightarrow \quad f_k \cos\theta\, L = 0 - \left(\frac{1}{2}kx^2 + mgh\right)$$

$$\mu_k mg \cos 180°\, L = 0 - \left(\frac{1}{2}kx^2 + mgh\right)$$

$$L = \frac{kx^2}{2\mu_k mg} + \frac{h}{\mu_k} = \frac{(3.20 \times 10^3 N/m)(5.00\ m)^2}{2(0.800)(200\ kg)(9.8\ m/s^2)} + \frac{35.0m}{0.800} = 69.3\ m$$



Final Exam Review

Example 7 (The Impulse-Momentum Theorem): A model rocket is constructed with a motor that can provide a total impulse of 29.0 N.s. The mass of the rocket is 0.175 kg.

(a) What is the speed that this rocket achieves when launched from rest? Neglect the effect of gravity and air resistance.

According to the impulse-momentum theorem.

$$\vec{\mathrm{I}} = \Delta\vec{\mathrm{p}} \quad \rightarrow \quad \vec{\mathrm{I}} = \vec{\mathrm{p}}_f - \vec{\mathrm{p}}_i = m\vec{\mathrm{v}}_f - m\vec{\mathrm{v}}_i = m\vec{\mathrm{v}}_f - 0 \quad \rightarrow \quad v_f = \frac{\mathrm{I}}{m} = \frac{29.0\ N.s}{0.175\ kg} = 165.7\ m/s$$

(b) If the total impulse impact time is 0.850 s, find the average net force applied to the model rocket.

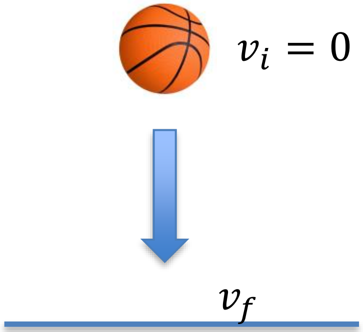
$$\left(\sum \vec{\mathrm{F}}\right)_{\mathrm{avg}} = \frac{\vec{\mathrm{I}}}{\Delta t} = \frac{29.0\ \mathrm{N.s}}{0.850\ \mathrm{s}} = 34.12\ \mathrm{N}$$

Final Exam Review

Example 8 (Momentum and Energy): A basketball with mass $m = 0.60\ kg$ is dropped from rest. Just before striking the floor, the ball has a momentum whose magnitude is 3.1 kg.m/s. At what height was the basketball dropped? Ignore the effect of air resistance.

Find the speed of the ball from its momentum just before hitting the floor

$$\vec{p}_f = m\vec{v}_f \quad \rightarrow \quad v_f = \frac{p_f}{m} = \frac{3.1\ kg.m/s}{0.60\ kg} = 5.17\ m/s$$



Based on the conservation of energy between the initial and the final points:

$$K_i + U_i = K_f + U_f \quad \rightarrow \quad 0 + mgh = \frac{1}{2}mv_f^2 + 0$$

$$h = \frac{v_f^2}{2g} = \frac{(5.17\ m/s)^2}{2(9.8\ m/s^2)} = 1.36\ m$$

Final Exam Review

Example 9 (Elastic Collision): A 3.0 kg ball moving 8.0 m/s in the positive x direction has a one-dimensional elastic collision with a ball mass = M that is initially at rest. After the collision, the ball of unknown mass has a velocity of 6.0 m/s in the positive x direction.

(a) What is the mass of the second ball M?

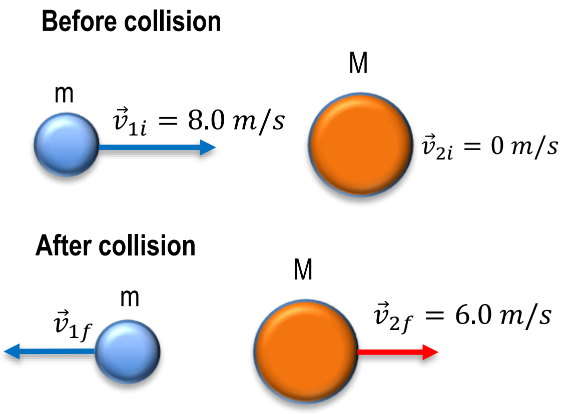
An elastic collision both kinetic energy and momentum are conserved:

$$\vec{p}_i = \vec{p}_f \quad \rightarrow \quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$K_i = K_f \quad \rightarrow \quad \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \qquad (1)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i} \qquad (2)$$



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Example 9 (Elastic Collision): A 3.0 kg ball moving 8.0 m/s in the positive x direction has a one-dimensional elastic collision with a ball mass = M that is initially at rest. After the collision, the ball of unknown mass has a velocity of 6.0 m/s in the positive x direction.

(a) What is the mass of the second ball M?

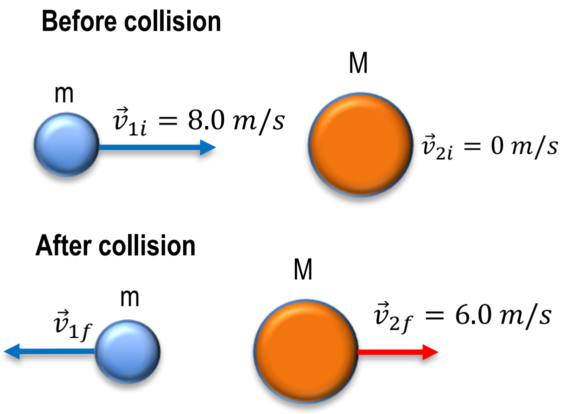
Mass of the second ball is obtained from equation (2):

$$(2) \qquad v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

Since the second ball is at rest before the collision:

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} \quad \rightarrow \quad m_2 = \frac{2m_1 v_{1i}}{v_{2f}} - m_1$$

$$M = \frac{2(3.0\,kg)(8.0\,m/s)}{(6.0\,m/s)} - (3.0\,kg) = 5.0\,kg$$



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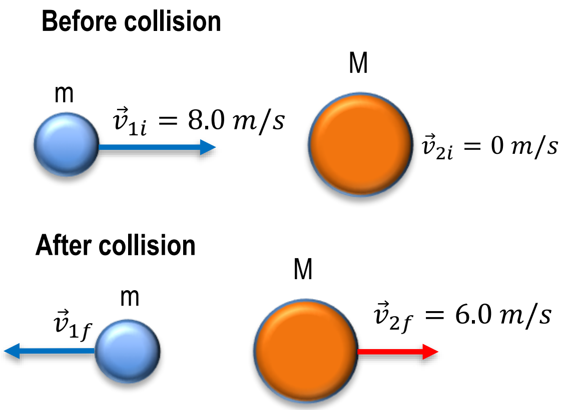
(b) What is the velocity (magnitude and direction) of the first ball just after the collision?

Velocity of the first ball is obtained from equation (1):

(1) $$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{1f} = \left(\frac{3.0\ kg - 5.0\ kg}{3.0\ kg + 5.0\ kg}\right)(8.0\ m/s) = -2\ m/s$$

The first ball has a speed of 2 m/s in the negative x direction



Final Exam Review

Example 9 (Elastic Collision): A 3.0 kg ball moving 8.0 m/s in the positive x direction has a one-dimensional elastic collision with a ball mass = M that is initially at rest. After the collision, the ball of unknown mass has a velocity of 6.0 m/s in the positive x direction.

(c) What is the total kinetic energy of the two-mass system before and after the collision?

The kinetic energy before the collision is:

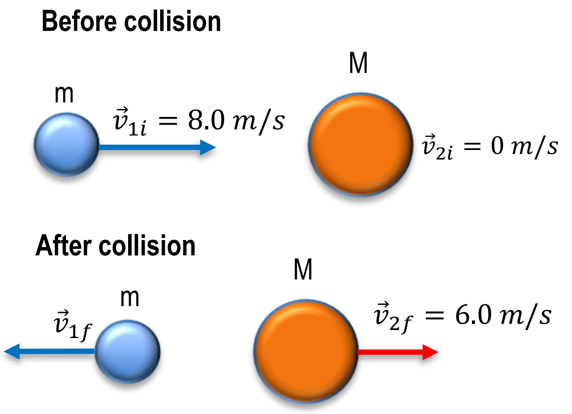
$$K_i = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2$$

$$K_i = \frac{1}{2}(3.0\ kg)(8\ m/s)^2\ + 0\ = 96\ J$$

The kinetic energy after the collision is:

$$K_f = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$K_f = \frac{1}{2}(3.0\ kg)(-2\ m/s)^2\ + \frac{1}{2}(5.0\ kg)(6.0\ m/s)^2\ = 96\ J$$



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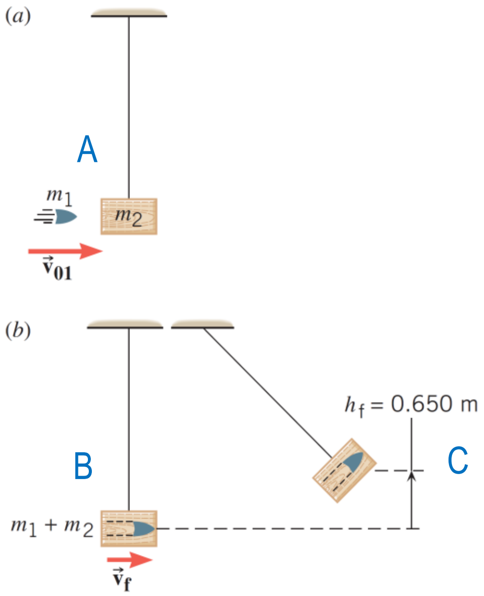
Example 10 (Perfectly Inelastic Collision): A ballistic pendulum consists of a stationary 2.50 kg block of wood suspended by a wire of negligible mass. A 0.010 kg bullet is fired into the block, and the block with the bullet in it swings to a maximum height of 0.650 m above the initial position.
Find the speed with which the bullet is fired.



A perfectly inelastic collision just after the bullet collides with the block:

$$\vec{p}_i = \vec{p}_f \quad \rightarrow \quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$m_1\vec{v}_{1A} + 0 = (m_1 + m_2)\vec{v}_B \quad \rightarrow \quad v_{1A} = \frac{(m_1 + m_2)v_B}{m_1}$$

Conservation of mechanical energy in the swing from position B to position C:

$$E_{mB} = E_{mC} \quad \rightarrow \quad K_B + U_B = K_C + U_C$$

$$\frac{1}{2}(m_1 + m_2)v_B^2 + 0 = 0 + (m_1 + m_2)gh \quad \rightarrow \quad v_B = \sqrt{2gh}$$

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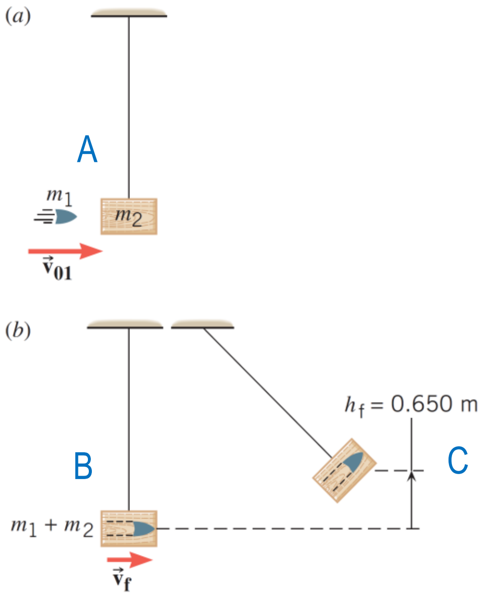
Find the speed with which the bullet is fired.

$$v_{1A} = \frac{(m_1 + m_2)v_B}{m_1}$$

$$v_B = \sqrt{2gh}$$

$$\Rightarrow \quad v_{1A} = \left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh}$$



$$v_{1A} = \left(\frac{0.010\,kg + 2.50\,kg}{0.010\,kg}\right)\sqrt{2(9.8\,m/s^2)(0.650\,m)} = +896\,m/s$$

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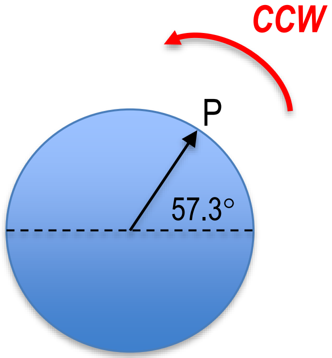
Example 11 (Angular and Translational Quantities): A wheel 2.00 m in diameter lies in a vertical plane and rotates about its central axis CCW with a constant angular acceleration of 4.00 rad/s^2 . The wheel starts at rest at t = 0, and the radius vector of a certain point P on the rim makes an angle of 57.3° with the horizontal at this time.

(a) Find the angular speed of the wheel at time t = 2.00s.

$$\omega_f = \omega_i + \alpha t \quad \rightarrow \quad \omega_f = 0 + (4.00\, rad/s^2)(2.00\, s) = 8.00\, rad/s$$



(b) Find the tangential speed for point P, at time t = 2.00s.

$$v = r\omega \quad \rightarrow \quad v = (1.00\, m)(8.00\, rad/s) = 8.00\, m/s$$

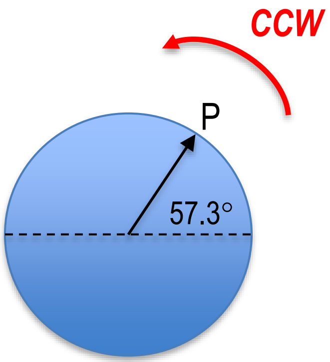
(c) Find the angular position for point P, at time t = 2.00s.

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \quad \rightarrow \quad \theta_f = (1\, rad) + 0 + \frac{1}{2}(4.00\, rad/s^2)(2.00\, s)^2 = 9.00\, rad$$

Final Exam Review

Example 11 (Angular and Translational Quantities): A wheel 2.00 m in diameter lies in a vertical plane and rotates about its central axis CCW with a constant angular acceleration of 4.00 rad/s^2 . The wheel starts at rest at t = 0, and the radius vector of a certain point P on the rim makes an angle of 57.3° with the horizontal at this time.

(d) Find the total acceleration for point P, at time t = 2.00s.



$$a = \sqrt{a_r^2 + a_t^2} \qquad \theta = \tan^{-1}\left(\frac{a_t}{a_r}\right)$$

$$a_r = a_c = r\omega^2 = (1.00\, m)(8.00\, rad/s)^2 = 64.0\, m/s^2$$

$$a_t = r\alpha = (1.00\, m)(4.00\, rad/s^2) = 4.00\, m/s^2$$

$$a = \sqrt{(64.0\, m/s^2)^2 + (4.00\, m/s^2)^2} = 64.1\, m/s^2 \qquad \theta = \tan^{-1}\left(\frac{4.00}{64.0}\right) = 3.58°$$

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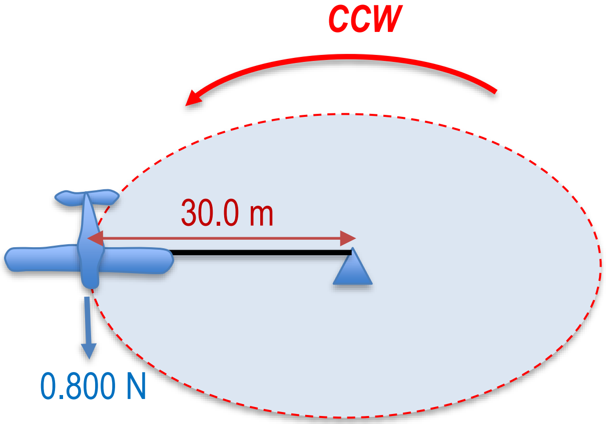
Example 12 (Rigid Object Under Net Torque): A model airplane with mass m = 0.750 kg is tethered to the ground by a wire so that it flies in a horizontal circle r = 30.0 m in radius. The airplane engine provides a net thrust of F = 0.800 N perpendicular to the tethering wire.



(a) Find the torque the net thrust produces about the center of the circle.

The net torque is obtained as:

$$\sum \tau_{net} = Fr = (0.800\,N)(30.0\,m) = 24.0\,N.m$$

(b) Find the angular and the translational acceleration of the airplane.

$$\sum \tau_{net} = I\alpha \quad \rightarrow \quad \alpha = \frac{Fr}{mr^2} = \frac{24.0\,N.m}{(0.750\,kg)(30.0\,m)^2} = 0.0356\,rad/s^2$$

$$a = r\alpha = (30.0\,m)(0.0356\,rad/s^2) = 1.07\,m/s^2$$

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Example 13 (Newton's Second Law, Rotation and Torque): A uniform disk, with mass M = 2.5 kg and radius R = 20 cm, mounted on a fixed horizontal axle. A block with mass m = 1.2 kg hangs from a massless cord that is wrapped around the rim of the disk. The cord does not slip, and there is no friction at the axle.

(a) Find the acceleration of the falling bock.

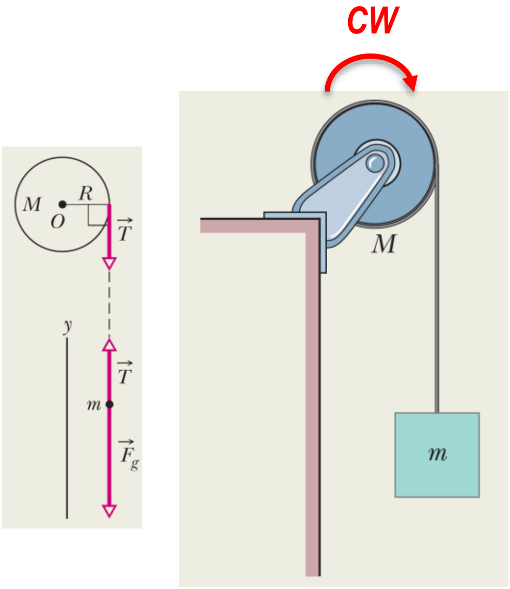
First draw the free body diagram of the objects:

Apply Newton's second law for motion of the object:

$$\sum F_y = ma \quad \rightarrow \quad T - F_g = m(-a) \quad \rightarrow \quad a = \frac{mg - T}{m} \qquad (1)$$

Tension can be found by modeling the disk as a rigid object under uniform torque:

$$\sum \tau_{\text{ext}} = I\alpha \quad \rightarrow \quad -TR = I(-\alpha) \quad \rightarrow \quad T = \frac{I\alpha}{R} = \frac{1}{2}MR\alpha \qquad (2)$$



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(a) Find the acceleration of the falling bock.

From equations (1), (2) and (3) we have:

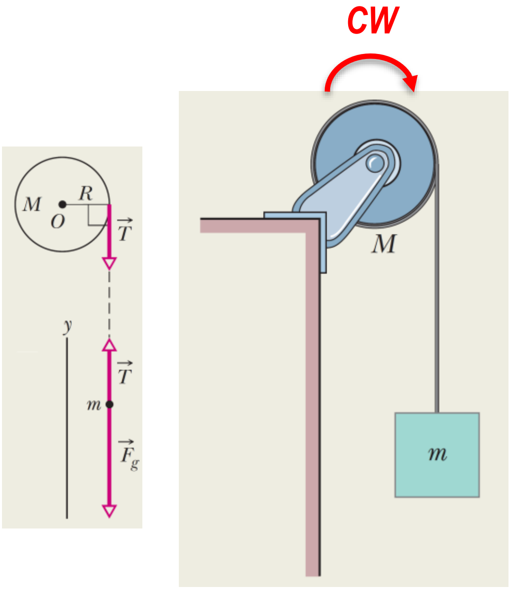
(1) $$a = \frac{mg - T}{m}$$

(2) $$T = \frac{1}{2} M R \alpha$$

(3) $$\alpha = \frac{a}{R}$$

$$a = \frac{mg - \frac{1}{2} M a}{m} \rightarrow a = \frac{mg}{m + \frac{1}{2} M}$$

$$a = \frac{(1.2\,kg)(9.8\,m/s^2)}{(1.2\,kg) + \frac{1}{2}(2.5\,kg)} = 4.8\,m/s^2$$



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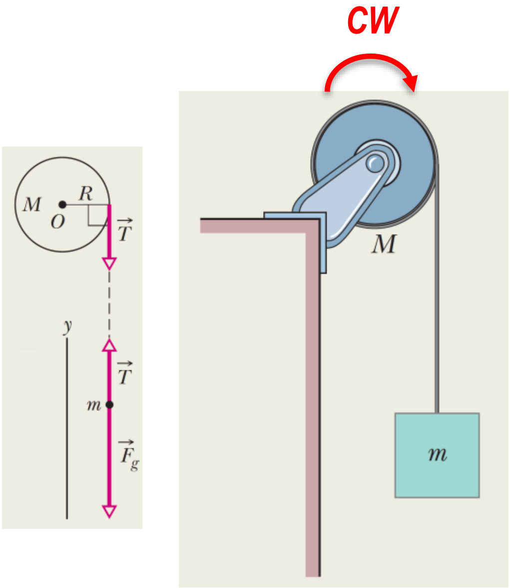
Example 13 (Newton's Second Law, Rotation and Torque): A uniform disk, with mass M = 2.5 kg and radius R = 20 cm, mounted on a fixed horizontal axle. A block with mass m = 1.2 kg hangs from a massless cord that is wrapped around the rim of the disk. The cord does not slip, and there is no friction at the axle.

(b) Find the angular acceleration of the disk.



The magnitude of the angular acceleration of the disk is obtained from equation (3):

$$\alpha = \frac{a}{R} = \frac{4.8\, m/s^2}{20 \times 10^{-2}\, m} = 24\, rad/s^2$$

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Example 13 (Newton's Second Law, Rotation and Torque): A uniform disk, with mass M = 2.5 kg and radius R = 20 cm, mounted on a fixed horizontal axle. A block with mass m = 1.2 kg hangs from a massless cord that is wrapped around the rim of the disk. The cord does not slip, and there is no friction at the axle.

(c) Find the tension in the cord and compare it with the weight of the hanging block.

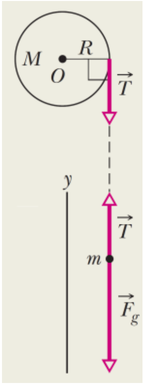
The magnitude of the tension force is obtained from equation (2):

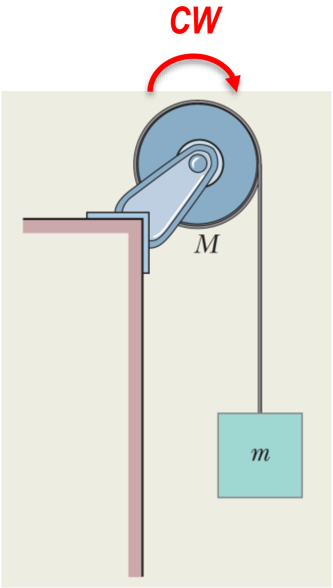
$$T = \frac{1}{2}MR\alpha = \frac{1}{2}(2.5\,kg)(0.20\,m)(24\,rad/s^2) = 6.0\,N$$

The weight of the hanging block is:

$$mg = (1.2\,kg)(9.8\,m/s^2) = 11.8\,N$$

The tension in the cord is less than the weight of the hanging block. Therefore, the block moves downward.





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Example 14 (Simple Harmonic Motion): A 1.00 kg glider attached to a spring with a spring constant of 25.0 N/m oscillates on a frictionless, horizontal air track. At t = 0, the glider is released from rest at x = -3.00 cm (that is the spring is compressed by 3.00 cm).

(a) Find the period of the glider's motion.

First, find the angular frequency of the system:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25.0\ \mathrm{N/m}}{1.00\ \mathrm{kg}}} = 5.00\ \mathrm{rad/s}$$

Then, find the period of the system:

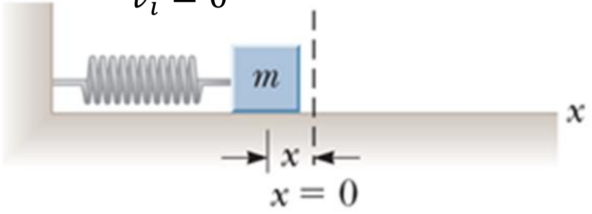
$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{5.00\ \mathrm{rad/s}} = \boxed{1.26\ \mathrm{s}}$$

$t = 0$

$x_i = -3.00\ cm$

$v_i = 0$



Final Exam Review

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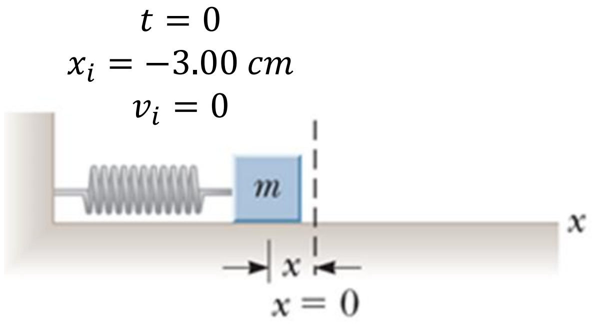
(b) Find the maximum values of its speed and acceleration.

The maximum speed of the glider:

$$v_{\text{max}} = \omega A = (5.00\ \text{rad/s})(3.00 \times 10^{-2}\ \text{m}) = \boxed{0.150\ \text{m/s}}$$

The maximum acceleration of the glider:

$$a_{\text{max}} = \omega^2 A = (5.00\ \text{rad/s})^2(3.00 \times 10^{-2}\ \text{m}) = \boxed{0.75\ \text{m/s}^2}$$



Final Exam Review

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(c) Find the position, velocity and acceleration as a function of time.

$$x(t) = A\cos(\omega t + \phi) \qquad v(t) = -\omega A\sin(\omega t + \phi) \qquad a(t) = -\omega^2 A\cos(\omega t + \phi)$$

Find the phase constant ϕ from the initial condition that $x = -A$ at $t = 0$

$$x(0) = -A = A\cos\phi \ \rightarrow \ \cos\phi = -1 \ \rightarrow \ \phi = 180° = \pi\ rad$$

$$x = A\cos(\omega t + \phi) = 0.03\cos(5.00t + \pi)$$

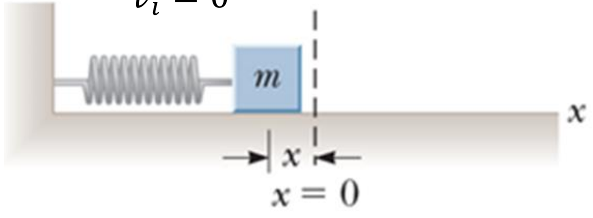
$$v = -\omega A\sin(\omega t + \phi) = -0.15\sin(5.00t + \pi)$$

$$a = -\omega^2 A\cos(\omega t + \phi) = -0.75\cos(5.00t + \pi)$$

$t = 0$

$x_i = -3.00\ cm$

$v_i = 0$



Final Exam Review

Example 15 (Energy of the Simple Harmonic Motion): A 326 g object is attached to a spring and executes simple harmonic motion with a period of 0.250 s on a horizontal frictionless surface. If the total energy of the system is 5.83 J,

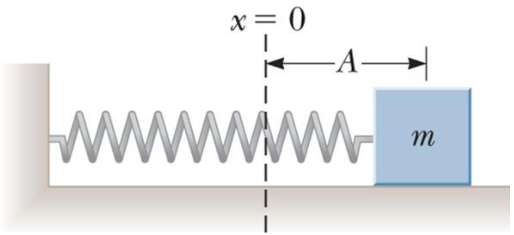
(a) Find the maximum speed of the object.

At the equilibrium position $x = 0$, the total energy of the system is in the form of kinetic energy. The maximum speed is:

$$E_m = \frac{1}{2} m v_{max}^2 \quad \rightarrow \quad v_{max} = \sqrt{\frac{2E_m}{m}} = \sqrt{\frac{2(5.83\ J)}{0.326\ kg}} = 5.98\ m/s$$

(b) Find the spring constant of the spring.

The spring constant can be obtained from the period formula:



$$T = 2\pi\sqrt{\frac{m}{k}} \quad \rightarrow \quad k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.326\ kg)}{(0.250\ s)^2} = 206\ N/m$$

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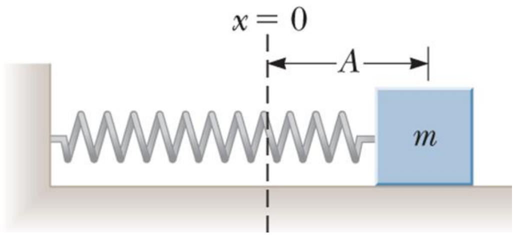
Example 15 (Energy of the Simple Harmonic Motion): A 326 g object is attached to a spring and executes simple harmonic motion with a period of 0.250 s on a horizontal frictionless surface. If the total energy of the system is 5.83 J,

(c) Find the amplitude of the motion.

At the turning points, $x = \pm A$, the total energy of the system is in the form of elastic potential energy:

$$E_m = \frac{1}{2}kx_{max}^2 \quad \rightarrow \quad x_{max} = A = \sqrt{\frac{2E_m}{k}} = \sqrt{\frac{2(5.83\ J)}{206\ N/m}} = 0.238\ m$$



Final Exam Review

Example 16 (A Simple Pendulum): Suppose that a simple pendulum consists of a small 60.0 g bob at the end of a cord of negligible mass. If the angle θ between the cord and the vertical is given by

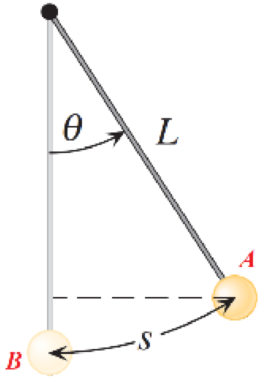
$$\theta = (0.0800\, rad)cos((4.43\, rad/s)t + \phi)$$

(a) What is the pendulum's length.

Angular position of a simple pendulum under the simple harmonic motion is: $\theta = \theta_{\max}\cos(\omega t + \phi)$

$$\omega = \frac{2\pi}{T} \quad \rightarrow \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{4.43\, rad/s} = 1.42\, s$$

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \rightarrow \quad L = \frac{T^2 g}{4\pi^2} = \frac{(1.42\, s)^2(9.8\, m/s^2)}{4\pi^2} = 0.50\, m$$



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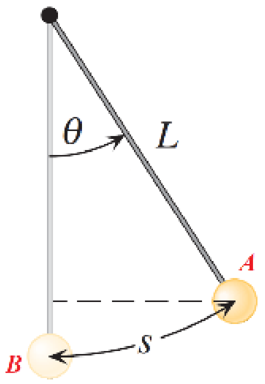
$$\theta = (0.0800\, rad) cos((4.43\, rad/s)t + \phi)$$

(b) What is the maximum kinetic energy.

The maximum kinetic energy is obtained at point B, where the bob passes the equilibrium point, and it has the maximum speed.

$$\theta = \theta_{\max} \cos(\omega t + \phi)$$



From the conservation of mechanical energy, total mechanical energy at point A and B are equal:

$$E_{MA} = E_{mB} \quad \rightarrow \quad K_A + U_A = K_B + U_B \quad \rightarrow \quad 0 + U_{max} = K_{max} + 0$$

$$K_{max} = mgh = mg(L - L\cos\theta_{max})$$

$$= (0.060\, kg)(9.8\, m/s^2)(0.50 - 0.50\cos 4.58°)$$
$$= 9.39 \times 10^{-4}\, J$$

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Example 17 (Traveling Wave): A transverse wave on a string is described by the following wave function, where the x and y are in meters and t is in seconds.

$$y(x,t) = 0.120 \sin\left(\frac{\pi}{8}x - 4\pi t\right)$$

Determine the wavelength, the period and the speed of propagation of this wave.

Compare with the general formula of the wave equation: $y(x,t) = A sin(kx - \omega t + \phi)$

$$k = \frac{2\pi}{\lambda} \quad \rightarrow \quad \lambda = \frac{2\pi}{k} = \frac{2\pi \text{ rad}}{\frac{\pi}{8}\ rad/m} = 16.0\ m$$

$$\omega = \frac{2\pi}{T} \quad \rightarrow \quad T = \frac{2\pi}{\omega} = \frac{2\pi\ rad}{4\pi\ rad/s} = 0.50\ s$$

$$v = \frac{\lambda}{T} \quad \rightarrow \quad v = \frac{16.0\ m}{0.50\ s} = 32.0\ m/s$$

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Example 18 (Traveling Wave): A sinusoidal wave traveling in the negative x direction (to the left) has an amplitude of 20.0 cm, a wavelength of 35.0 cm, and a frequency of 12.0 Hz. The vertical position of an element of the medium at t = 0, x = 0 is y = -3.00 cm, and the element has a positive velocity here.

Write an expression for the wave function y(x,t) and sketch the snapshot graph of the wave at t = 0.

The general formula of the wave equation is: $y(x,t) = Asin(kx - \omega t + \phi)$

$$k = \frac{2\pi}{\lambda} \quad \rightarrow \quad k = \frac{2\pi \text{ rad}}{0.350\ m} = 18.0\ rad/m$$

$$\omega = 2\pi f \quad \rightarrow \quad \omega = 2\pi(12.0\ Hz) = 75.4\ rad/s$$

$$\boxed{y(x,t) = (0.20\ m)sin(18.0x + 75.4t - 0.151)}$$

At $t = 0$ and $x = 0$ $\rightarrow$ $-3.00\ cm = (20.0\ cm)\sin\phi$

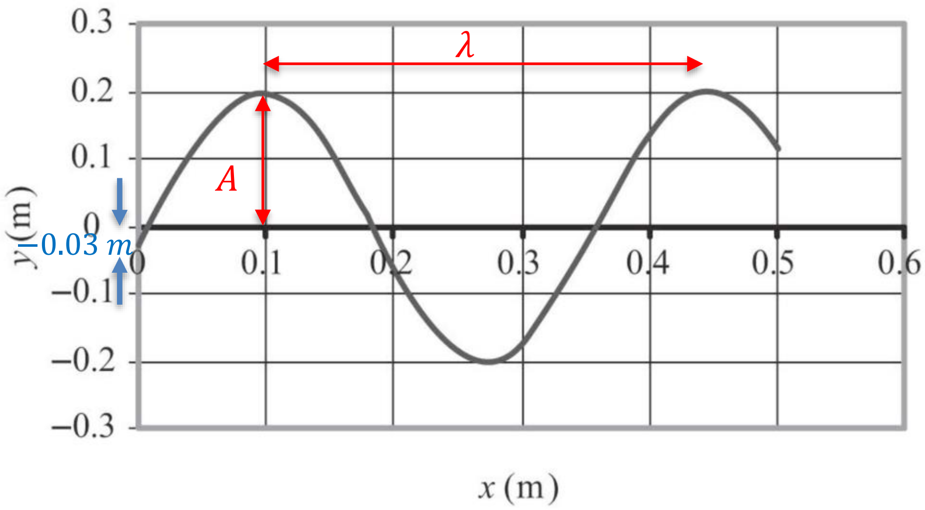
$$\sin\phi = \frac{-3.00\ cm}{20.0\ cm} \quad \rightarrow \quad \phi = -0.151 \text{ rad}$$

Final Exam Review

Example 18 (Traveling Wave): A sinusoidal wave traveling in the negative x direction (to the left) has an amplitude of 20.0 cm, a wavelength of 35.0 cm, and a frequency of 12.0 Hz. The vertical position of an element of the medium at t = 0, x = 0 is y = -3.00 cm, and the element has a positive velocity here.

Write an expression for the wave function y(x,t) and sketch the snapshot graph of the wave at t = 0.

The general formula of the wave equation is: $y(x,t) = Asin(kx - \omega t + \phi)$



$$\boxed{y(x,t) = (0.20\ m)sin(18.0x + 75.4t - 0.151)}$$

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Example 19 (Wave Speed on a Stretched String): Tension is maintained in a string. The observed wave speed is v = 24.0 m/s when the suspended mass is m = 3.00 kg.

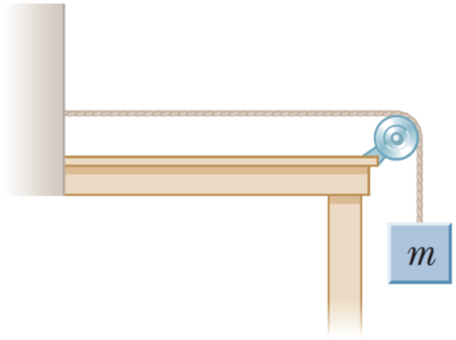
What is the mass per unit length of the string?

The tension in the string is equal to the weight of the block:

$$\sum F_y = 0 \;\rightarrow\; T - mg = 0 \;\rightarrow\; T = mg = (3.00\,kg)(9.8\,m/s^2) = 29.4\,N$$

The mass per unit length can be determined from the speed formula:

$$v = \sqrt{\frac{T}{\mu}} \;\rightarrow\; \mu = \frac{T}{v^2} = \frac{29.4\,N}{(24.0\,m/s)^2} = 0.0510\,kg/m$$



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Example 20 (Doppler Effect): A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1400 Hz. The speed of sound in the water is 1533 m/s. A second submarine (sub B) is located such that both submarines are traveling directly toward each other. The second submarine moving at 9.00 m/s.

(a) What frequency is detected by an observer riding on sub B as the subs approach each other?

Both the sound source and the observer to move with respect to the medium of sound

$$f_o = f_s \left(\frac{v \pm v_o}{v \mp v_s} \right)$$

- In the numerator:
  - The plus sign applies when the observer moves toward the source
  - The minus sign applies when the observer moves away from the source
- In the denominator,
  - The minus sign is used when the source moves toward the observer
  - The plus sign is used when the source moves away from the observer

Final Exam Review

Example 20 (Doppler Effect): A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1400 Hz. The speed of sound in the water is 1533 m/s. A second submarine (sub B) is located such that both submarines are traveling directly toward each other. The second submarine moving at 9.00 m/s.

(a) What frequency is detected by an observer riding on sub B as the subs approach each other?

Here, observer is on sub B and sound source is on sub A

$$f_o = f_s\left(\frac{v + v_o}{v - v_s}\right)$$

$$f_o = (1400\ Hz)\left(\frac{1533\ m/s + 9.00 m/s}{1533\ m/s - 8.00\ m/s}\right) = 1416\ Hz$$

Final Exam Review

Example 20 (Doppler Effect): A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1400 Hz. The speed of sound in the water is 1533 m/s. A second submarine (sub B) is located such that both submarines are traveling directly toward each other. The second submarine moving at 9.00 m/s.

(b) What frequency is detected by an observer riding on sub B as the subs recede from each other?

Here, observer is on sub B and sound source is on sub A

$$f_o = f_s\left(\frac{v - v_o}{v + v_s}\right)$$

$$f_o = (1400\ Hz)\left(\frac{1533\ m/s - 9.00 m/s}{1533\ m/s + 8.00\ m/s}\right) = 1385\ Hz$$

THANK YOU

HUMBER

WE ARE HUMBER