Assignment 2 Solutions

Exercise 1:

We need to show that the length of a free vector remains unchanged under rotation:

$\| v \| = \| Rv \|$.

Since the norm of a vector is given by:

$\| v \| = \sqrt{v^T v}$.

Applying the rotation matrix R:

$\| Rv \| = \sqrt{(Rv)^T (Rv)}$.

Using the property of orthogonal matrices: $R^T R = I$, we get:

$(Rv)^T (Rv) = v^T R^T R v = v^T I v = v^T v$.

Thus, $\| Rv \| = \| v \|$, proving that rotation preserves vector length.

Exercise 2:

The dot product of two vectors is:

$v1 \cdot v2 = v1^T v2$.

Applying a rotation matrix R:

$(Rv1) \cdot (Rv2) = (Rv1)^T (Rv2) = v1^T R^T R v2$.

Since $R^T R = I$, we obtain:

$v1^T v2$.

Thus, the dot product remains unchanged, proving independence from the choice of coordinate frame.

Exercise 3:

The sequence of rotations leads to the rotation matrix:

$R = R_z(\alpha) R_x(\psi) R_z(\theta) R_x(\phi)$. −5

Exercise 4:

Given matrices:

$R1^2 = [[1, 0, 0], [0, 1/2, -\sqrt{3}/2], [0, \sqrt{3}/2, 1/2]]$,

$R1^3 = [[0, 0, -1], [0, 1, 0], [1, 0, 0]]$.

We find $R2^3$ using:

$R2^3 = (R1^2)^T R1^3$.

Exercise 5:

For axis-angle rotation with unit vector:

k = (1/sqrt(3)) [1,1,1]^T, theta = 90 degrees.

Using Rodrigues' rotation formula:

Rk,theta = I + sin(theta) [k]_x + (1 - cos(theta)) k k^T.

Exercise 6:

Homogeneous transformation matrix:

R = Rotx(alpha) Transx(b) Transz(d) Rotz(theta).

Identifying commutative pairs:

- Rotations about different axes generally do not commute.
- Translations along different axes commute.
- A rotation about an axis does not commute with a translation along a different axis unless aligned.

Finding valid permutations requires verifying order preservation in transformation sequences.

-5

Permutations!