# INTRODUCTION & RECTILINEAR KINEMATICS: CONTINUOUS MOTION

**Today's Objectives:**

Students will be able to:

1. Find the kinematic quantities (position, displacement, velocity, and acceleration) of a particle traveling along a straight path.

**In-Class Activities:**

- Check Homework
- Reading Quiz
- Applications
- Relations between s(t), v(t), and a(t) for general rectilinear motion.
- Relations between s(t), v(t), and a(t) when acceleration is constant.
- Concept Quiz
- Group Problem Solving
- Attention Quiz

## READING QUIZ

1. In dynamics, a particle is assumed to have __________.

   A) both translation and rotational motions

   B) only a mass

   C) a mass but the size and shape cannot be neglected

   D) no mass or size or shape, it is just a point

2. The average speed is defined as __________.

   A) $\Delta r/\Delta t$

   B) $\Delta s/\Delta t$

   C) $s_T/\Delta t$

   D) None of the above.

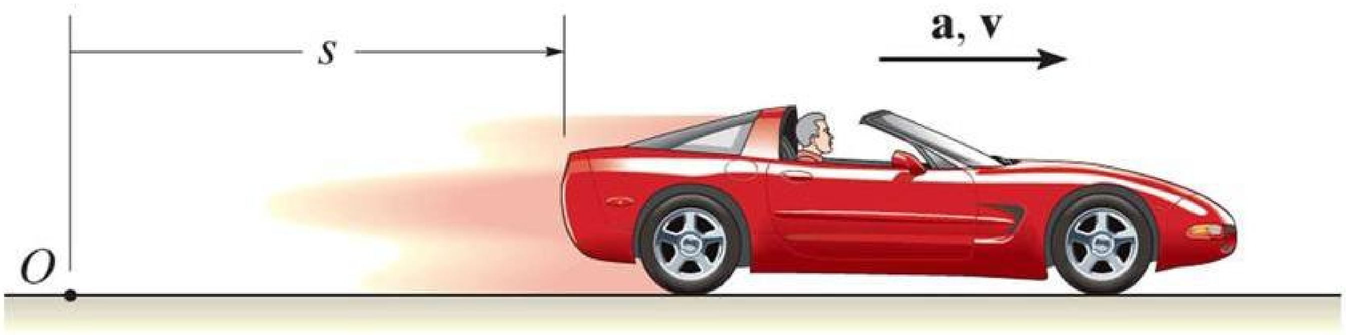
# APPLICATIONS


The motion of large objects, such as rockets, airplanes, or cars, can often be analyzed as if they were particles.

Why?

If we measure the altitude of this rocket as a function of time, how can we determine its velocity and acceleration?

# APPLICATIONS (continued)




A sports car travels along a straight road.

Can we treat the car as a particle?

If the car accelerates at a constant rate, how can we determine its position and velocity at some instant?

# An Overview of Mechanics

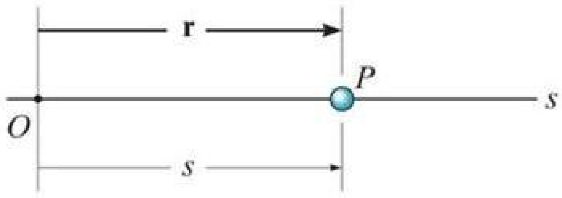
**Mechanics:** The study of how bodies react to the forces acting on them.

**Statics:** The study of bodies in equilibrium.

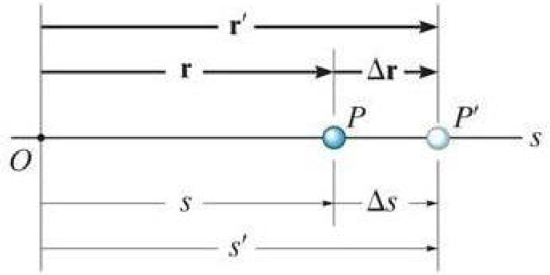
***Dynamics:***

1. **Kinematics** – concerned with the geometric aspects of motion
2. **Kinetics** - concerned with the forces causing the motion

# RECTILINEAR KINEMATICS: CONTINIOUS MOTION (Section 12.2)




Position




Displacement

A particle travels along a straight-line path defined by the coordinate axis *s*.

The position of the particle at any instant, relative to the origin, O, is defined by the position vector ***r***, or the scalar s. Scalar s can be positive or negative. Typical units for ***r*** and s are meters (m) or feet (ft).

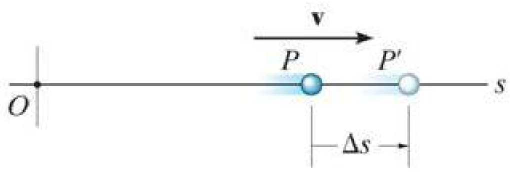
The displacement of the particle is defined as its change in position.

Vector form: $\Delta \boldsymbol{r} = \boldsymbol{r}' - \boldsymbol{r}$ Scalar form: $\Delta s = s' - s$

The total distance traveled by the particle, $s_T$, is a positive scalar that represents the total length of the path over which the particle travels.

# VELOCITY

Velocity is a measure of the rate of change in the position of a particle. It is a vector quantity (it has both magnitude and direction). The magnitude of the velocity is called speed, with units of m/s or ft/s.



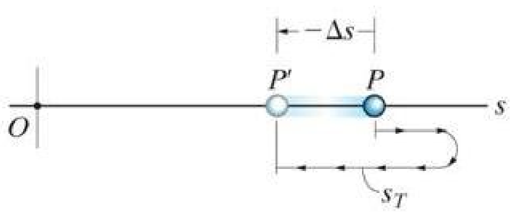
Velocity

The average velocity of a particle during a time interval Δt is

$$\boldsymbol{v}_{avg} = \Delta\boldsymbol{r} / \Delta t$$

The instantaneous velocity is the time-derivative of position.

$$\boldsymbol{v} = d\boldsymbol{r} / dt$$



Average velocity and
Average speed

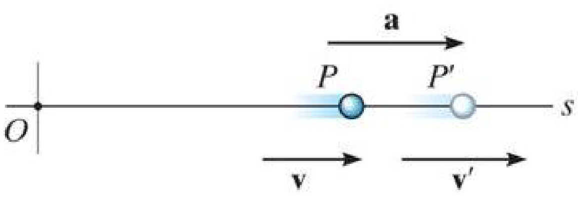
Speed is the magnitude of velocity: $v = ds / dt$

Average speed is the total distance traveled divided by elapsed time:

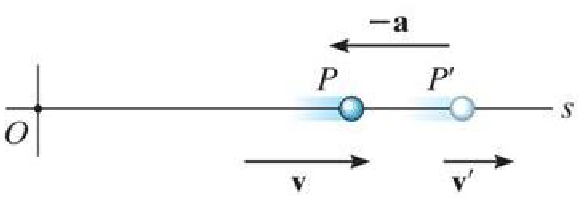
$$(v_{sp})_{avg} = s_T / \Delta t$$

# ACCELERATION

Acceleration is the rate of change in the velocity of a particle. It is a vector quantity. Typical units are $m/s^2$ or $ft/s^2$.




Acceleration




Deceleration

The instantaneous acceleration is the time derivative of velocity.

Vector form: $\mathbf{a} = d\boldsymbol{v} / dt$

Scalar form: $a = dv / dt = d^2s / dt^2$

Acceleration can be positive (speed increasing) or negative (speed decreasing).

As the text shows, the derivative equations for velocity and acceleration can be manipulated to get $a\ ds = v\ dv$

# SUMMARY OF KINEMATIC RELATIONS: RECTILINEAR MOTION

- Differentiate position to get velocity and acceleration.

$$v = ds/dt \; ; \qquad a = dv/dt \quad \text{or} \quad a = v\, dv/ds$$

- Integrate acceleration for velocity and position.

Velocity:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a\, dt \quad \text{or} \quad \int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a\, ds$$

Position:

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\, dt$$

- Note that $s_o$ and $v_o$ represent the initial position and velocity of the particle at $t = 0$.

# CONSTANT ACCELERATION

The three kinematic equations can be integrated for the special case when acceleration is constant ($a = a_c$) to obtain very useful equations. A common example of constant acceleration is gravity; i.e., a body freely falling toward earth. In this case, $a_c = g = 9.81\ m/s^2 = 32.2\ ft/s^2$ downward. These equations are:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a_c\, dt \quad \text{yields} \quad v = v_o + a_c t$$

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\, dt \quad \text{yields} \quad s = s_o + v_o t + (1/2)\, a_c\, t^2$$

$$\int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a_c\, ds \quad \text{yields} \quad v^2 = (v_o)^2 + 2a_c(s - s_o)$$

# EXAMPLE

**Given:** A particle travels along a straight line to the right with a velocity of $v = (4t - 3t^2)$ m/s where t is in seconds. Also, s = 0 when t = 0.

**Find:** The position and acceleration of the particle when t = 4 s.

**Plan:** Establish the positive coordinate, s, in the direction the particle is traveling. Since the velocity is given as a function of time, take a derivative of it to calculate the acceleration. Conversely, integrate the velocity function to calculate the position.

# EXAMPLE (continued)

**Solution:**

1) Take a derivative of the velocity to determine the acceleration.

$$a = dv / dt = d(4\,t - 3\,t^2) / dt = 4 - 6\,t$$

$$\Rightarrow \quad a = \underline{-20 \text{ m/s}^2} \text{ (or in the} \leftarrow \text{direction) when } t = 4 \text{ s}$$

2) Calculate the distance traveled in 4s by integrating the velocity using $s_o = 0$:

$$v = ds / dt \Rightarrow ds = v\,dt \Rightarrow \int_{s_o}^{s} ds = \int_{o}^{t} (4\,t - 3\,t^2)\,dt$$

$$\Rightarrow \quad s - s_o = 2\,t^2 - t^3$$

$$\Rightarrow \quad s - 0 = 2(4)^2 - (4)^3 \Rightarrow s = \underline{-32 \text{ m}} \text{ (or} \leftarrow\text{)}$$

# CONCEPT QUIZ


1. A particle moves along a horizontal path with its velocity varying with time as shown. The average acceleration of the particle is _________.

   A) 0.4 $m/s^2$ $\rightarrow$ B) 0.4 $m/s^2$ $\leftarrow$
   C) 1.6 $m/s^2$ $\rightarrow$ D) 1.6 $m/s^2$ $\leftarrow$

2. A particle has an initial velocity of 30 ft/s to the left. If it then passes through the same location 5 seconds later with a velocity of 50 ft/s to the right, the average velocity of the particle during the 5 s time interval is _______.

   A) 10 ft/s $\rightarrow$ B) 40 ft/s $\rightarrow$
   C) 16 m/s $\rightarrow$ D) 0 ft/s

## GROUP PROBLEM SOLVING

**Given:** A sandbag is dropped from a balloon ascending vertically at a constant speed of 6 m/s.
The bag is released with the same upward velocity of 6 m/s at t = 0 s and hits the ground when t = 8 s.

**Find:** The speed of the bag as it hits the ground and the altitude of the balloon at this instant.

**Plan:** The sandbag is experiencing a constant downward acceleration of 9.81 $m/s^2$ due to gravity. Apply the formulas for constant acceleration, with $a_c = -9.81\ m/s^2$.

# GROUP PROBLEM SOLVING (continued)

**Solution:**

The bag is released when $t = 0$ s and hits the ground when $t = 8$ s.

Calculate the distance using a position equation.

$$\downarrow + \; s_{bag} = (s_{bag})_o + (v_{bag})_o\, t + (1/2)\, a_c\, t^2$$

$$s_{bag} = 0 + (-6)\,(8) + 0.5\,(9.81)\,(8)^2 = 265.9 \text{ m}$$

During $t = 8$ s, the balloon rises

$$\uparrow + \; s_{balloon} = (v_{balloon})\, t = 6\,(8) = 48 \text{ m}$$

Therefore, altitude is of the balloon is $(s_{bag} + s_{balloon})$.

Altitude = 265.9 + 48 = 313.9 = 314 m.

# GROUP PROBLEM SOLVING (continued)

Calculate the velocity when t = 8 s, by applying a velocity equation.

$$\downarrow + \quad v_{bag} = (v_{bag})_o + a_c t$$

$$v_{bag} = -6 + (9.81)\,8 = \underline{72.5 \text{ m/s}} \downarrow$$

# ATTENTION QUIZ

1. A particle has an initial velocity of 3 ft/s to the left at $s_0 = 0$ ft. Determine its position when t = 3 s if the acceleration is 2 ft/s$^2$ to the right.

   A) 0.0 ft  B) 6.0 ft $\leftarrow$
   C) 18.0 ft $\rightarrow$  D) 9.0 ft $\rightarrow$

2. A particle is moving with an initial velocity of v = 12 ft/s and constant acceleration of 3.78 ft/s$^2$ in the same direction as the velocity. Determine the distance the particle has traveled when the velocity reaches 30 ft/s.

   A) 50 ft  B) 100 ft
   C) 150 ft  D) 200 ft

End of the Lecture

Let Learning Continue