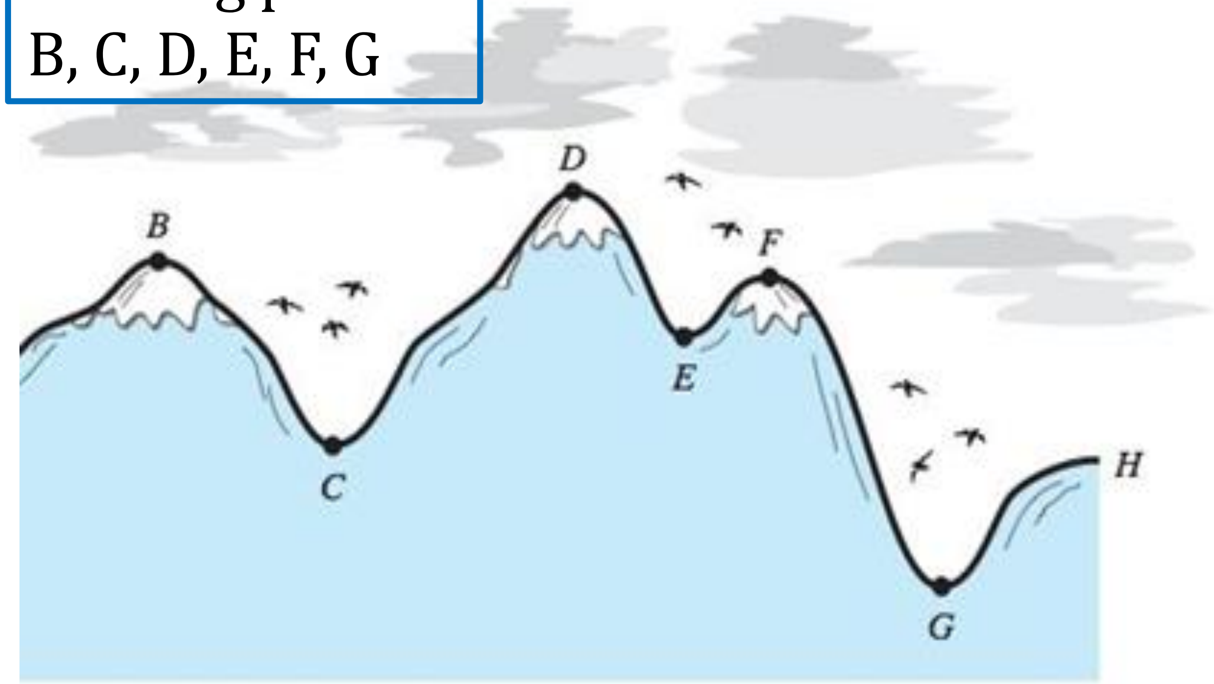
3.2 Extrema and Curve Sketching.

Extreme Values: Relative(Local) Maximum/Minimum;

Absolute Maximum/Minimum

Turning points:
B, C, D, E, F, G



E 28-5 Path over the mountains.

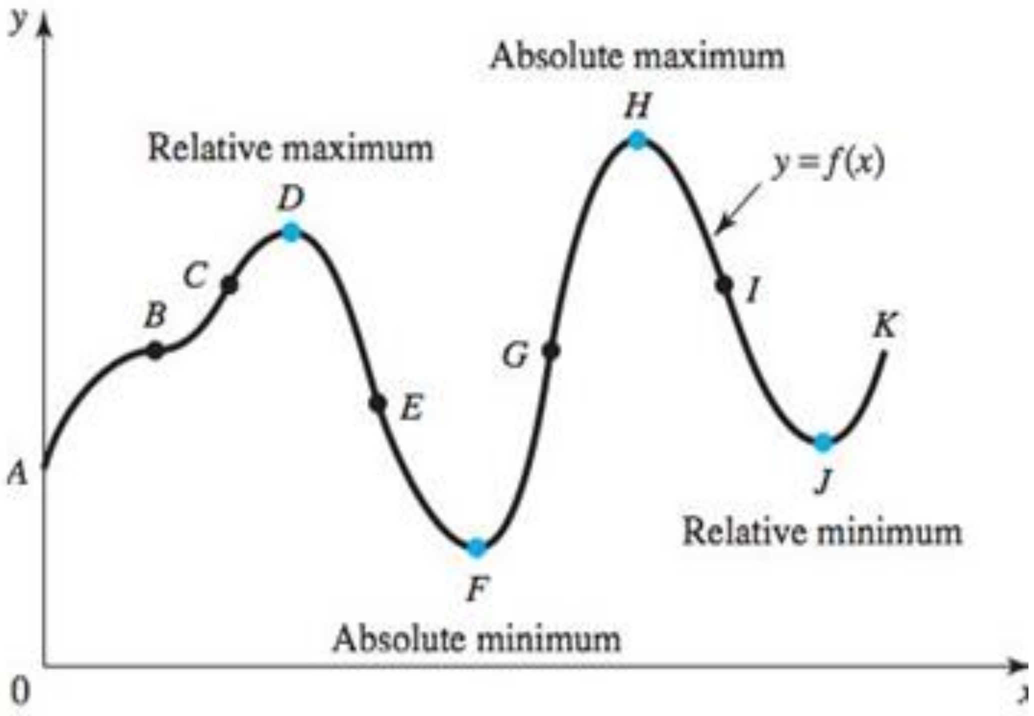


FIGURE 28-6 Maximum, minimum, and inflection points.

- **Relative Maxima** are *high* points in their immediate vicinity;
- **Relative Minima** are *low* points in their immediate vicinity;
- Note, that *tangents* are horizontal (slope m=0) for the extreme values, which implies that the *derivatives* are zeroes for such points.

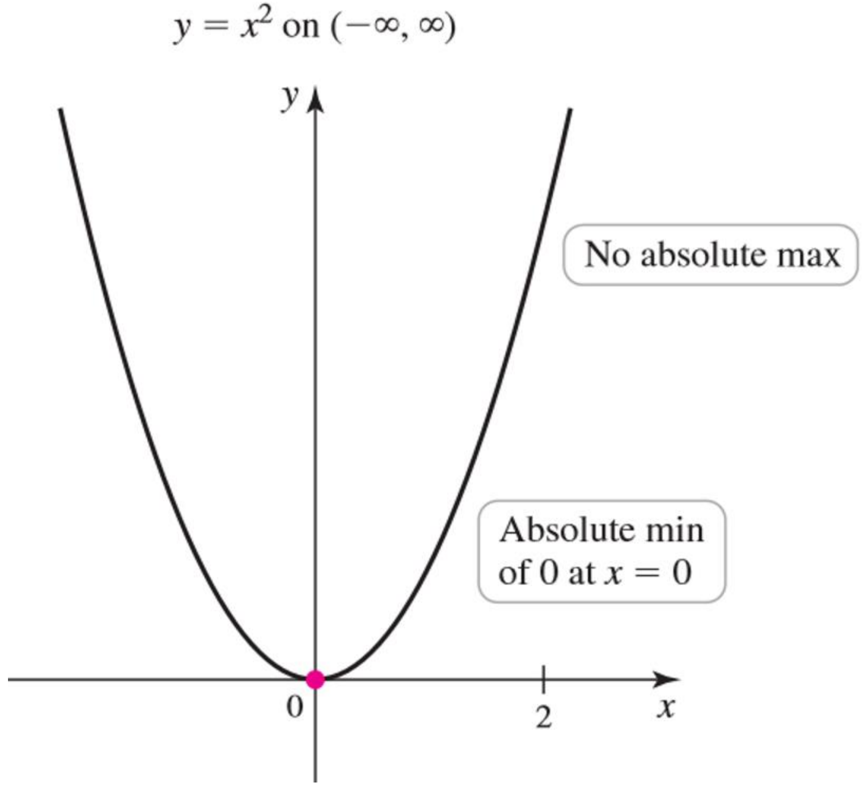
Definition: Absolute (Global) Maximum and Minimum

Let c be a number in the domain D of a function f. Then $f(c)$ is the:

Absolute (global) maximum value of f on D if $f(c) \geq f(x)$ for all $x \in D$

Absolute (global) minimum value of f on D if $f(c) \leq f(x)$ for all $x \in D$

Example: $f(x) = x^2$



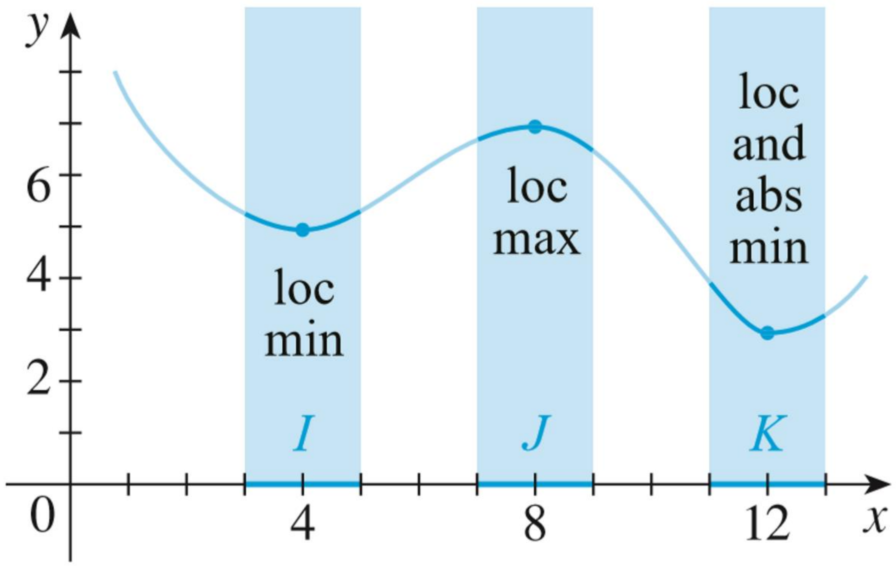
Solution: As $f(0) = 0$ and $f(x) = x^2 \geq 0$ for all $x \in \mathbb{R}$ then $f(0) = 0$ is an absolute minimum of f.

Definition: Local Maximum and Minimum

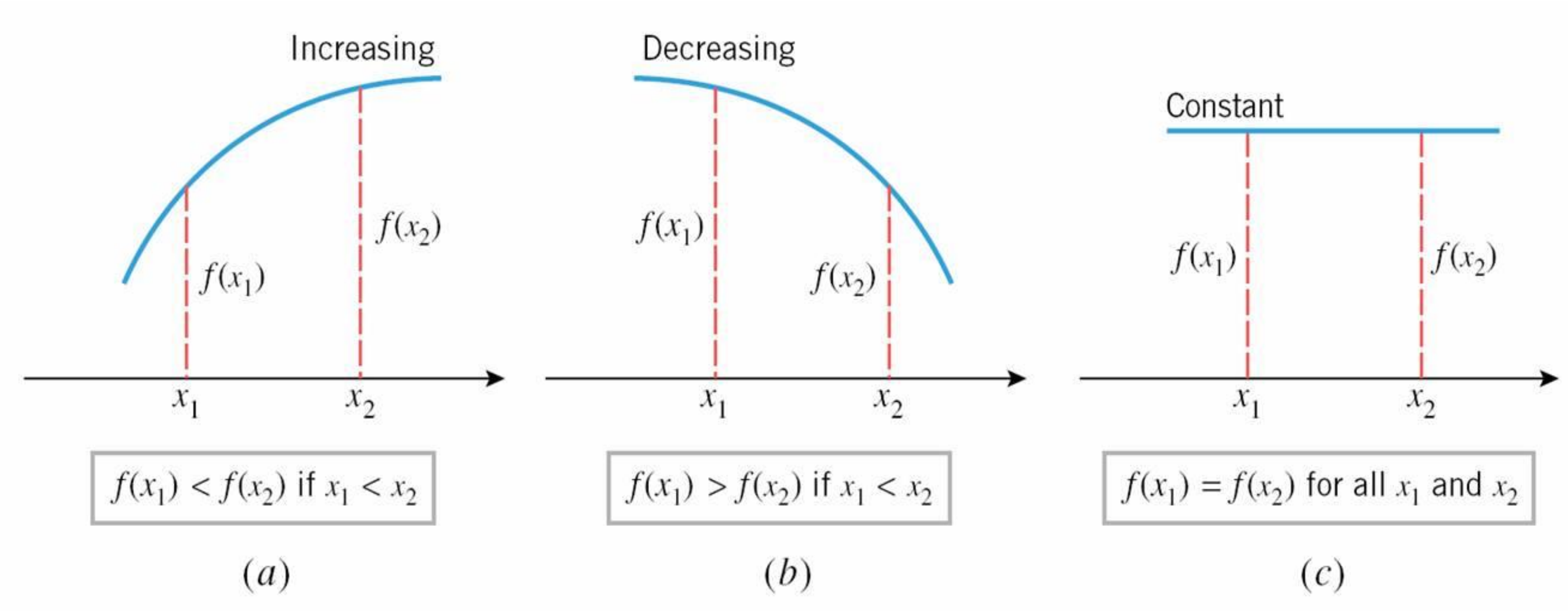
The number $f(c)$ is a:

Local maximum value of f if $f(c) \geq f(x)$ when x is near c

Local minimum value of f if $f(c) \leq f(x)$ when x is near c



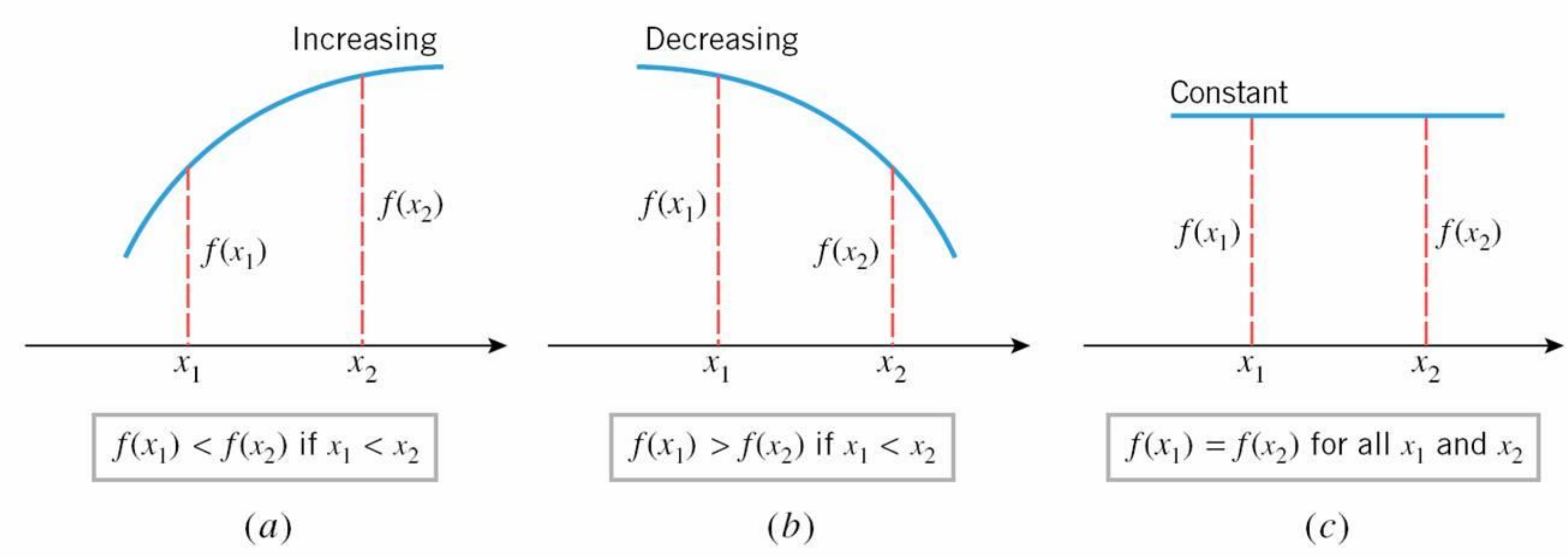
Increasing and Decreasing Functions



Describing the behavior of a function $y = f(x)$ in terms of increasing/decreasing/constant we always assume that we travel from **left to the right** for the independent variable x.

INCREASING/DECREASING **TEST**

based on the sign of a first derivative



(a) If $f'(x) > 0$ on an interval, then f increases on the interval (x_1, x_2)

(b) If $f'(x) < 0$ on an interval, then f decreases on the interval (x_1, x_2)

(c) If $f'(x) = 0$ on an interval, then f is constant on the interval (x_1, x_2)

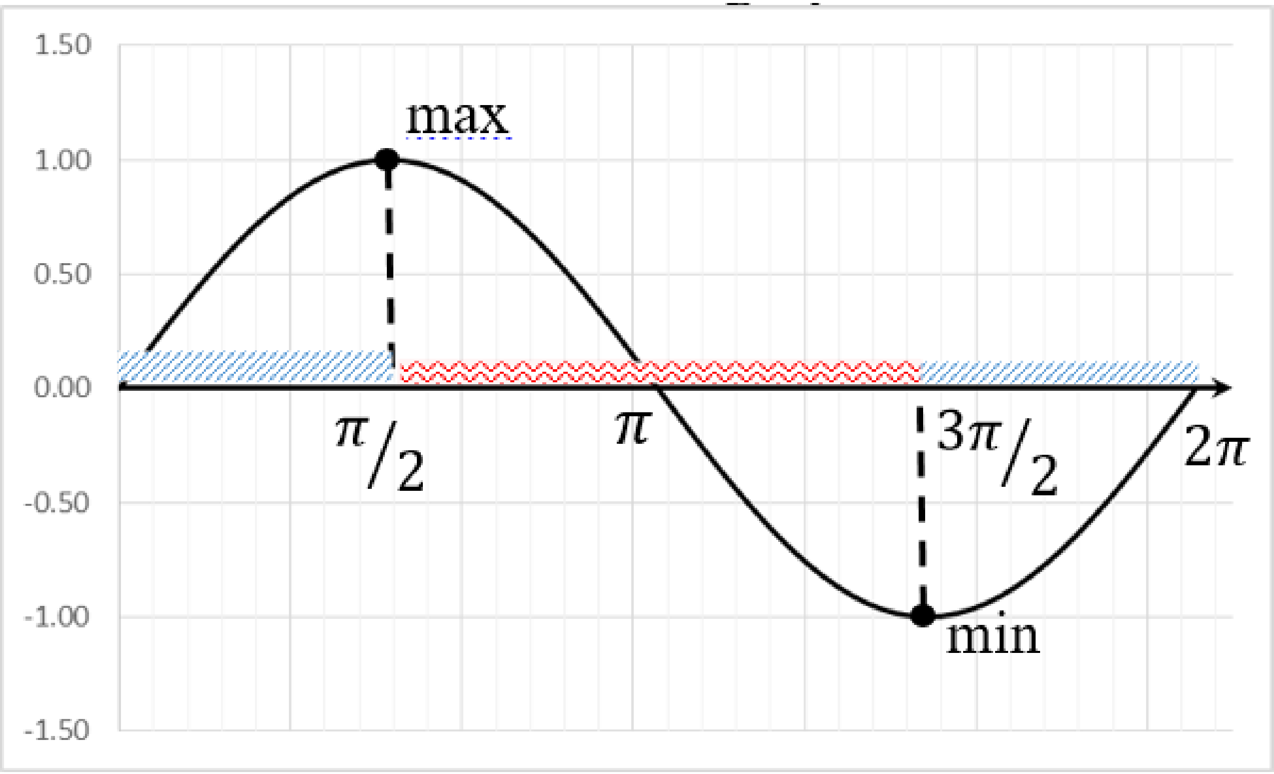
Example 1. For the function $y = sinx$ on a single period $0 \leq x \leq 2\pi$, identify the intervals of increase/decrease from the graph. State the relative max and min.

Solution:

$$max@\left(x = \frac{\pi}{2}, y = 1\right) \quad min@\left(x = \frac{3\pi}{2}, y = -1\right)$$

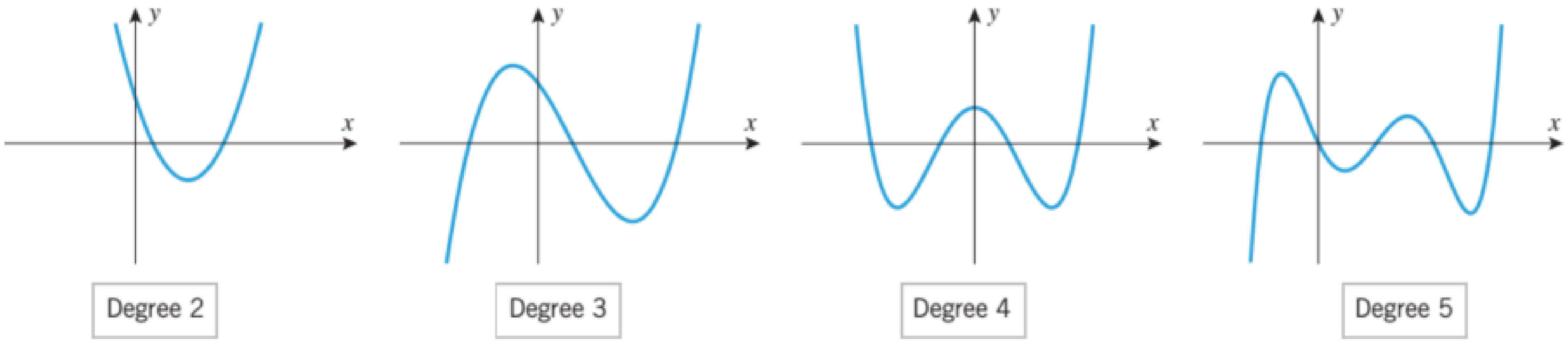
y increases on intervals of x: $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$.

y decreases on intervals of x: $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

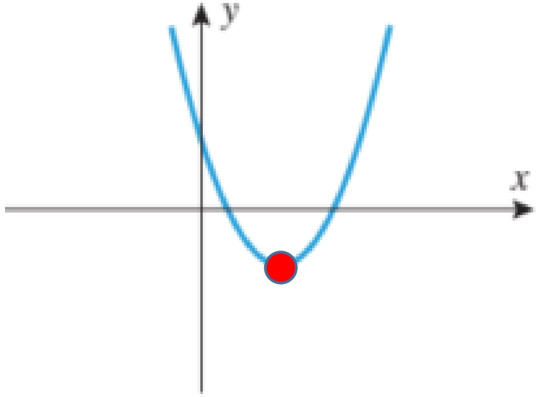


Graphing Polynomial Functions

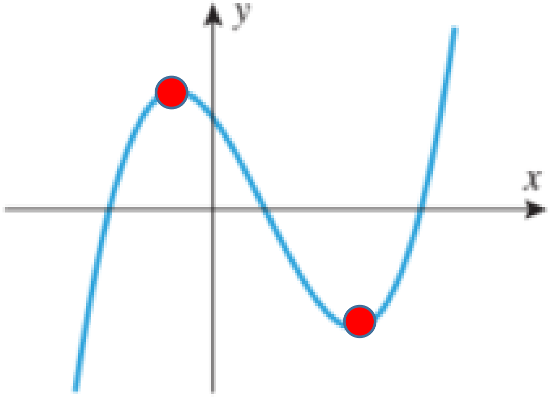
- Domain: all real number line $\mathbb{R} = (-\infty, +\infty)$
- All polynomial functions are <u>continuous</u> and <u>differentiable</u> on the domain.
- <u>Characteristic features</u>:
  - Y-and X- intercepts;
  - extreme values/turning points;
  - intervals of increase and decrease;
  - PI-points of inflection and intervals of concavity.



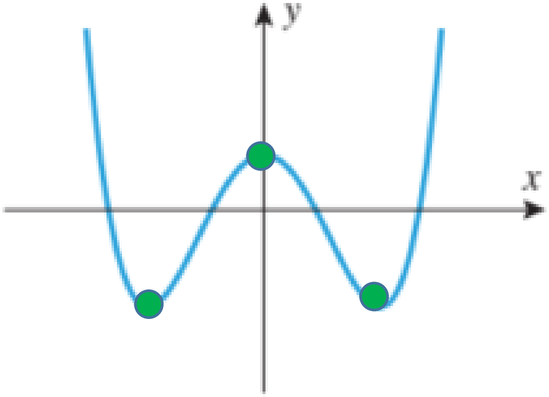
Graphing Polynomial Functions



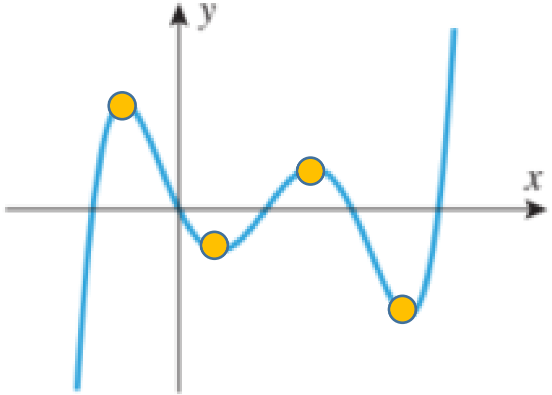
Degree 2



Degree 3



Degree 4



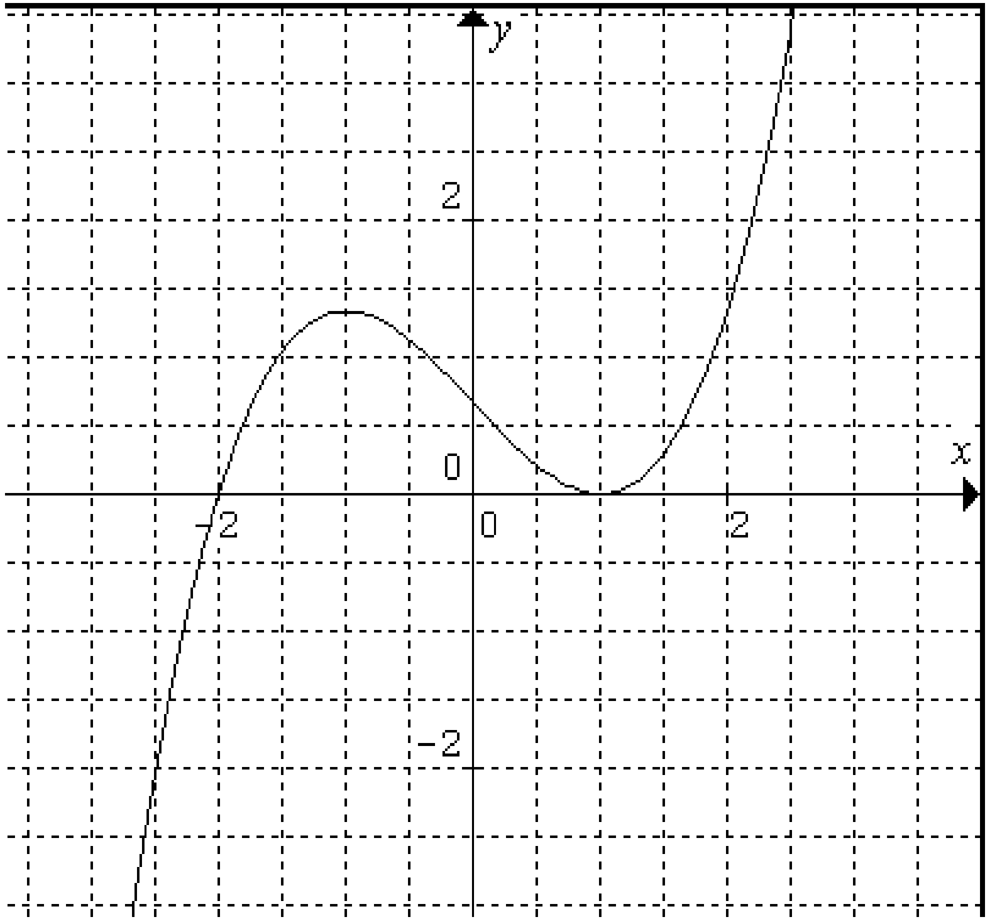
Degree 5

The expected number of extreme/turning points

- Degree 2/quadratic: one turning point
- Degree 3/ cubic: two turning points
- Degree 4: three turning points
- Degree 5: four turning points

Example 2. Graphical approach

$$y = \frac{1}{3}x^3 - x + \frac{2}{3}$$

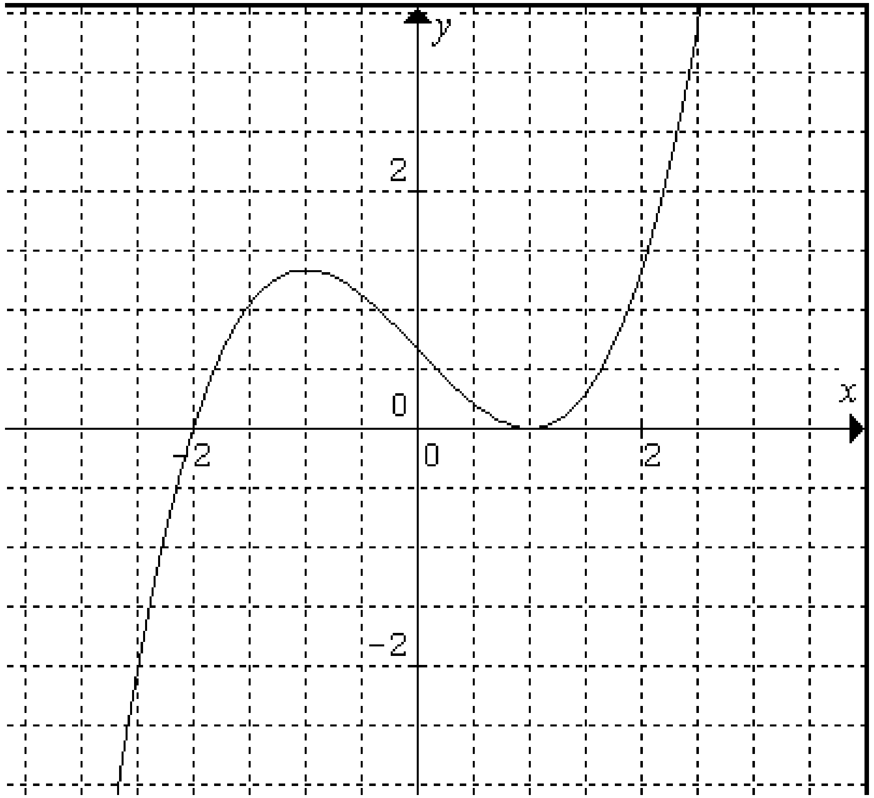


A. Intervals of increase/decrease

B. Relative max/min:

Example 2.

$$y = \frac{1}{3}x^3 - x + \frac{2}{3}$$



Solution: moving from left to the right along X-axis:

A. y increases for x on intervals: $(-\infty, -1)$ and $(1, +\infty)$.

y decreases for x on interval: $(-1, 1)$.

B. Relative (local) max and min:

$\boldsymbol{max}$:

$x_{max} = -1$; $y_{max} = f(-1) = \frac{1}{3}(-1)^3 - (-1) + \frac{2}{3} = \frac{4}{3}$;

$\boldsymbol{min}$:

$x_{min} = 1$; $y_{min} = f(1) = \frac{1}{3}(1)^3 - (1) + \frac{2}{3} = 0$;

$max@\left(-1, \frac{4}{3}\right) \qquad min@(1,\ 0)$

Observe that the zeroes of the derivative y' match the extreme values of the function y

Critical Values
and Their Use for Finding Relative Max and Min

The rule for identifying potential extremal points:

to find the critical values of a function, find the points at which the first derivative is equal to zero or doesn’t exist.

For the critical point set

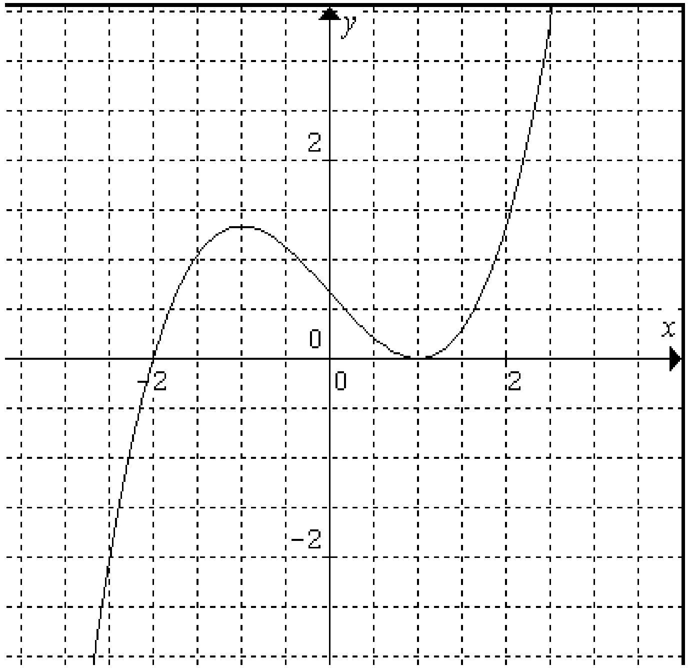
$f'(x) = 0$ or $f'(x)$ does not exist and solve for x

Find the critical values for the function $y = \frac{1}{3}x^3 - x + \frac{2}{3}$

- $y' = x^2 - 1 = \text{in factored form} = (x-1)(x+1)$
- **Critical values**:

$$y' = 0 \xrightarrow{\text{yields}} (x-1)(x+1) = 0$$

$$x = -1 \text{ and } x = 1$$



Critical values – potential extreme values – are: $x = \pm 1$.

Testing for Maximum and Minimum

- The 1st and 2nd derivative test
- The FIRST DERIVATIVE TEST is used to identify intervals of increase/decrease and local max/min

Suppose that ***c*** **is a critical value** of a continuous function f.

a) If f' changes from positive to negative at c, then f has a **relative max** at c

b) If f' changes from negative to positive at c, then f has a **relative min** at c

c) If f' does not change sign at c, then f has no relative extremum at c

Example 4

Find the intervals of increase/decrease and max and min for the function $y = \frac{1}{3}x^3 - x + \frac{2}{3}$.

Recall: CVs are $x = \pm 1$ and $y' = x^2 - 1 =$ in factored form $= (x-1)(x+1)$