HUMBER ENGINEERING

ENGI 1000 - Physics 1

WEEK 6 - MODULE 5

HUMBER

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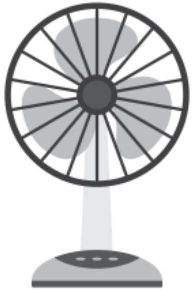
Module 5
Circular Motion

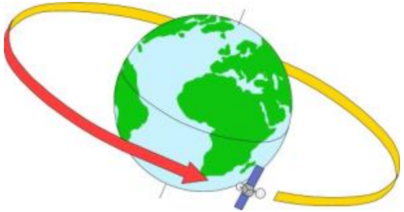
- Particle in Uniform Circular Motion
- Kinematics of Uniform Circular Motion
    - Centripetal Acceleration
- Dynamics of Uniform Circular Motion
    - Centripetal Force
- Nonuniform Circular Motion
- Motion in Accelerated Frames
    - Fictitious Forces

Uniform Circular Motion

- Uniform Circular Motion is a special kind of two-dimensional motion occurs when an object moves with constant speed on a circular path.

- There are many examples of uniform circular motion:
  - Electric fans and motors
  - Lawn mower blades
  - Wheels of a car
  - Rotating rides at amusement parks
  - Orbiting a satellite around Earth
  - Orbiting an electron around a nucleus

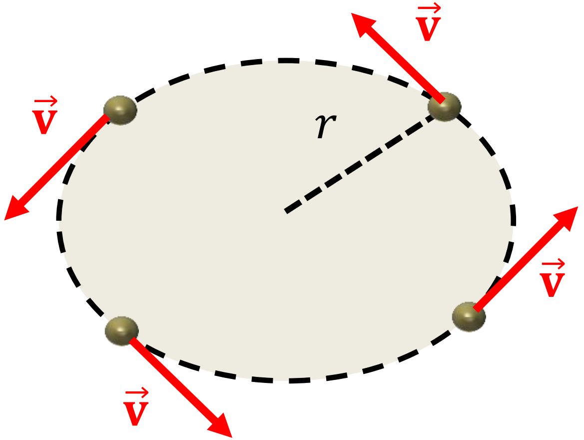




Uniform Circular Motion

- In uniform circular motion the velocity vector at any instant is tangent to the circle
  - The magnitude of velocity vector (speed) is constant
  - The direction of velocity vector is not constant
  - The speed is determined as

$$\boxed{v = \frac{d}{\Delta t} = \frac{2\pi r}{T}}$$

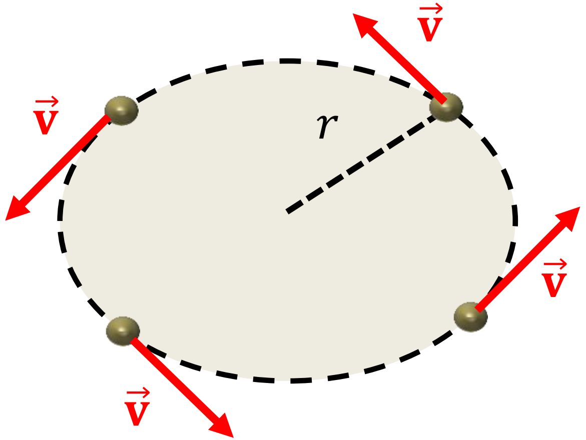


- The period of motion T is the time required to travel once around the circle.
- Relationship between angular speed ω and translational speed v:

$$\omega = \frac{2\pi}{T} \quad \rightarrow \quad \omega = 2\pi\left(\frac{v}{2\pi r}\right) = \frac{v}{r} \quad \rightarrow \quad \boxed{v = r\omega}$$

Uniform Circular Motion

- Any change in velocity (magnitude or direction) causes acceleration.
- The type of acceleration that occurs in uniform circular motion is called centripetal acceleration.
- Centripetal acceleration happens due to the change in the direction of velocity.

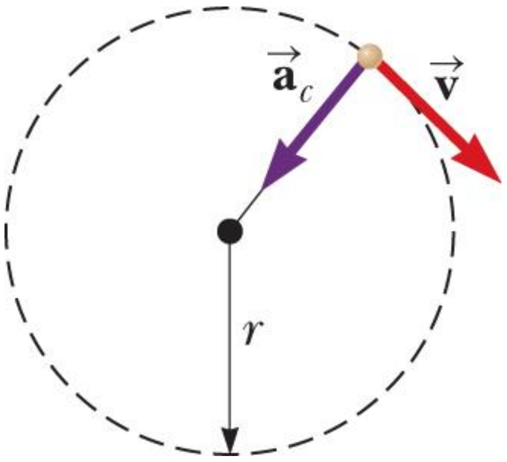


Velocity

	Magnitude	Direction
Linear Acceleration	Changing	Constant
Centripetal Acceleration	Constant	Changing

Centripetal Acceleration

- Centripetal acceleration $\vec{a}_c$ has following properties:
  - It is an instantaneous acceleration.
  - It is perpendicular to the instantaneous velocity.
  - Its direction is toward the center of the circle.
  - Its magnitude is determined as



$$\boxed{a_c = \frac{v^2}{r} \xrightarrow{v=\frac{2\pi r}{T}} a_c = \frac{4\pi^2 r}{T^2}}$$

  - Relationship between angular speed ω and centripetal acceleration a_c:

$$a_c = \frac{v^2}{r} \xrightarrow{v=r\omega} \boxed{a_c = r\omega^2}$$

Centripetal Acceleration

Example 1 (The Centripetal Acceleration of the Earth): What is the centripetal acceleration and the angular speed of the Earth as it moves in its orbit around the Sun? The radius of the Earth's orbit around the Sun is about 1.496×10^{11} m.

We know that the period of the Earth's orbit is one year.

$$a_c = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(1.496 \times 10^{11}\ m)}{(1\ yr)^2}\left(\frac{1\ yr}{3.156 \times 10^7\ s}\right)^2 = 5.93 \times 10^{-3}\ m/s^2$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1\ yr}\left(\frac{1\ yr}{3.156 \times 10^7\ s}\right) = 1.99 \times 10^{-7}\ rad/s$$

The angular speed of the Earth is very small because the Earth takes an entire year to go around the circular path once.

Quick Quiz 1

- A particle moves in a circular path of radius r with speed v. It then increases its speed to $2v$ while traveling along the same circular path.

The centripetal acceleration of the particle has changed by what factor?

a) 0.25
b) 0.5
c) 2
d) 4
e) Impossible to determine

Quick Quiz 2

- A particle moves in a circular path of radius r with speed v. It then increases its speed to $2v$ while traveling along the same circular path.

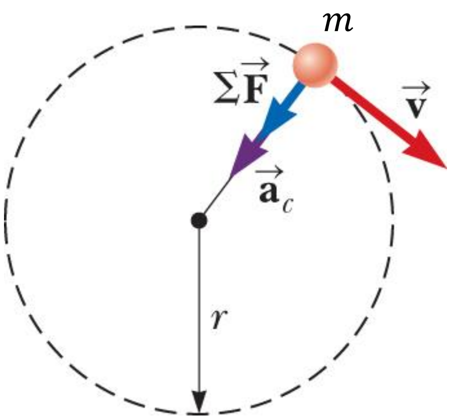
  By what factor has the period of the particle changed?

  a) 0.25
  b) 0.5
  c) 2
  d) 4
  e) Impossible to determine

Forces in Circular Motion

- We know that an object moving at a constant speed in a circle experiences centripetal acceleration $\vec{a}_c$ toward the center of circle.

- According to Newton's second law of motion, $\sum \vec{F} = m\vec{a}$, whenever an object accelerates, there must be a net force to crate the acceleration.
- Thus, in uniform circular motion there must be a net force to produce the centripetal acceleration.

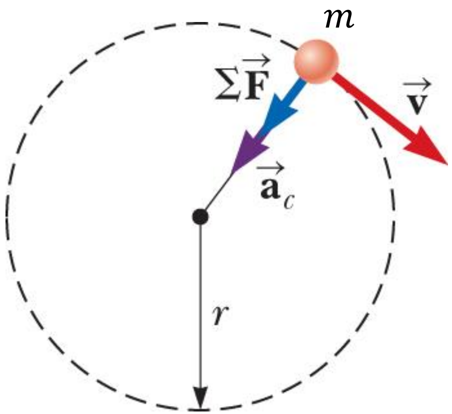


- The net force causing the centripetal acceleration is called the **centripetal force** and acting in the same direction as the centripetal acceleration.

- This net force might be gravity, friction, tension, a normal force, or a combination of two or more forces.

Centripetal Force

- Centripetal force has the following properties
  - It is the required net force to keep an object moves in the uniform circular motion
  - It acts perpendicular to the velocity
  - It directed inward towards the center of the circle.
  - Its magnitude is determined as



$$\sum F = ma_c \xrightarrow{a_c = \frac{v^2}{r}} \sum F = m\frac{v^2}{r}$$

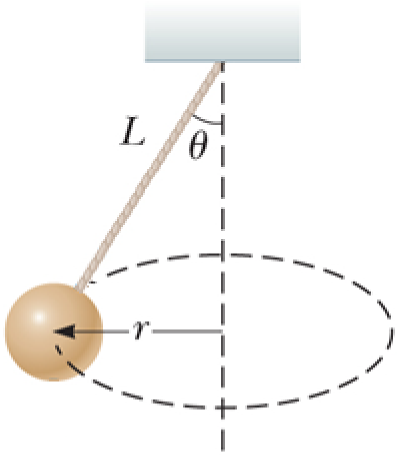
  - Relationship between angular speed ω and centripetal force:

$$\sum F = m\frac{v^2}{r} \xrightarrow{v = r\omega} \boxed{\sum F = mr\omega^2}$$

Uniform Circular Motion Examples

Example 2 (The Conical Pendulum): A small ball of mass m is suspended from a string of length L. The ball revolves with constant speed v in a horizontal circle of radius r. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum.*)

Find an expression for v in terms of the length of the string L and the angle it makes with the vertical θ in the figure.



No motion in vertical direction, the ball is in equilibrium in the vertical direction.

The ball experiences a centripetal acceleration in the horizontal direction, so it can be modeled as a Particle in Uniform Circular Motion in this direction.

Uniform Circular Motion Examples

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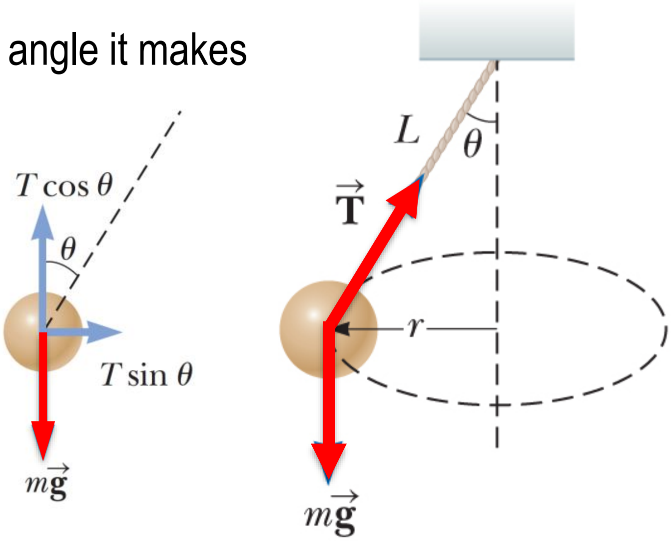
Find an expression for v in terms of the length of the string L and the angle it makes with the vertical θ in the figure.

Draw the forces acting on each object.

Draw their free-body diagram

The forces acting on the ball are:

- The gravitational force: $\vec{F}_g$
- The tension force: $\vec{T}$



Uniform Circular Motion Examples

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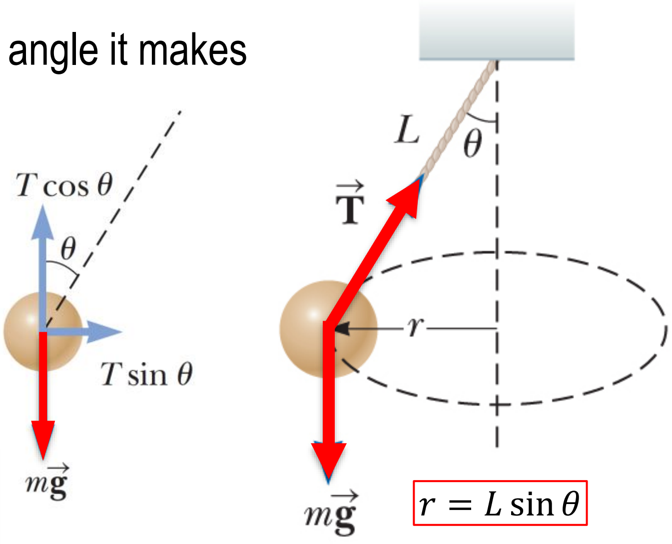
Find an expression for v in terms of the length of the string L and the angle it makes with the vertical θ in the figure.

Apply the Newton's second law on the ball in x and y directions.

$$\sum F_y = 0 \quad \rightarrow \quad T\cos\theta - mg = 0 \quad \rightarrow \quad T\cos\theta = mg$$

$$\sum F_x = ma \quad \rightarrow \quad T\sin\theta = ma_c \quad \rightarrow \quad T\sin\theta = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{rg} \quad \rightarrow \quad v = \sqrt{rg\tan\theta} \quad \Rightarrow \quad \boxed{v = \sqrt{Lg\sin\theta\tan\theta}}$$

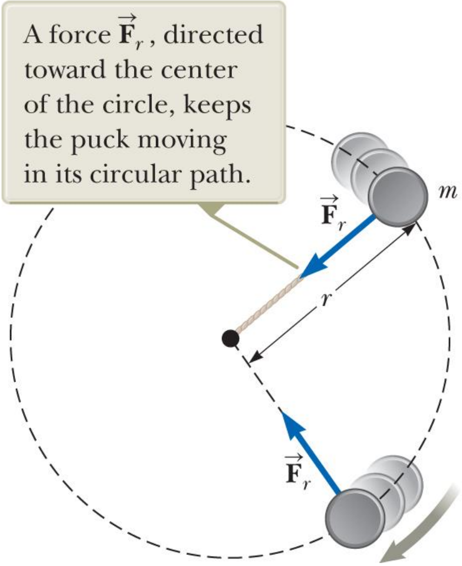


The speed is independent of the mass of the ball

Uniform Circular Motion Examples

Example 3 (How Fast Can It Spin): A puck of mass 0.500 kg is attached to the end of a cord 1.50 m long. The puck moves in a horizontal circle as shown in the figure. If the cord can withstand a maximum tension of 50.0 N.

(a) What is the maximum speed at which the puck can move before the cord breaks? Assume the string remains horizontal during the motion.



The punk moves in a circular path, so it can be modeled as a Particle in Uniform Circular Motion.

Uniform Circular Motion Examples

Example 3 (How Fast Can It Spin): A puck of mass 0.500 kg is attached to the end of a cord 1.50 m long. The puck moves in a horizontal circle as shown in the figure. If the cord can withstand a maximum tension of 50.0 N.

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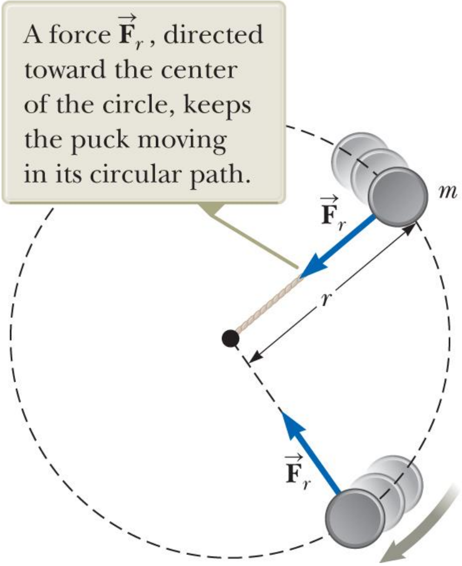


By incorporating the tension force and the centripetal acceleration into Newton's second law:

$$\sum F = ma \quad \rightarrow \quad T = ma_c \quad \rightarrow \quad T = m\frac{v^2}{r} \quad \rightarrow \quad v = \sqrt{\frac{Tr}{m}}$$

$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50.0\ \text{N})(1.50\ \text{m})}{0.500\ \text{kg}}} = \boxed{12.2\ \text{m/s}}$$

Uniform Circular Motion Examples

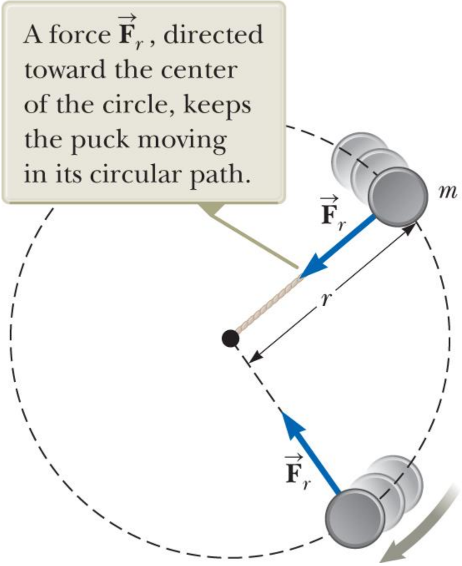
Example 3 (How Fast Can It Spin): A puck of mass 0.500 kg is attached to the end of a cord 1.50 m long. The puck moves in a horizontal circle as shown in the figure. If the cord can withstand a maximum tension of 50.0 N.

(b) Suppose the puck moves in a circle of larger radius at the same speed v. Is the cord more likely or less likely to break?

From $a_c = \frac{v^2}{r}$, the larger radius means the acceleration is smaller.

Since $T = ma_c$, then the required tension in the string is smaller.

The string is less likely to break



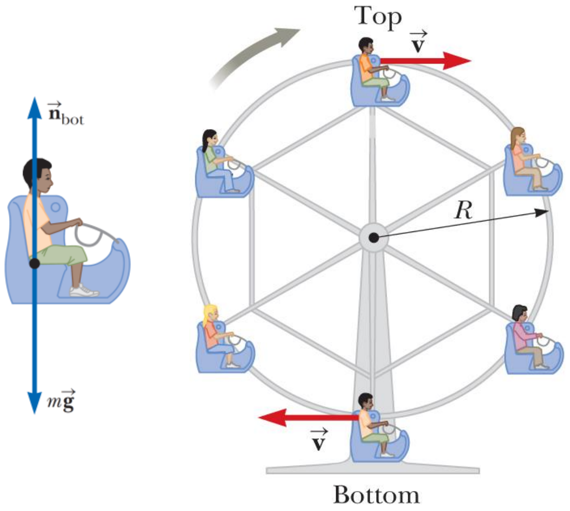
Uniform Circular Motion Examples

Example 4 (Riding the Ferris Wheel): A child of mass m rides on a Ferris wheel as shown in the Figure. The child moves in a vertical circle of radius $10.0\ m$ at a constant speed of $3.00\ m/s$.

(a) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child mg?

Draw the forces acting on the child at the bottom of the ride:

- The downward gravitational force: $\vec{\boldsymbol{F}}_g = m\vec{g}$
- The upward normal force: $\vec{\boldsymbol{n}}_{\boldsymbol{bot}}$



Uniform Circular Motion Examples

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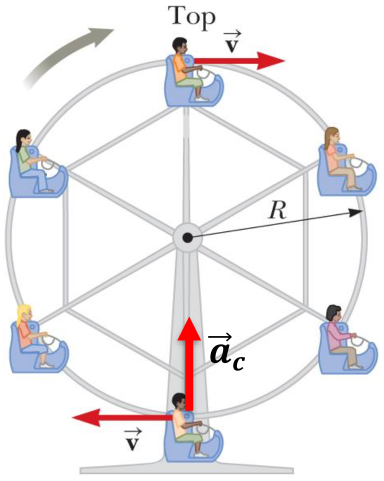
(a) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child mg?

Apply the Newton's second law on the child in the radial direction when the child is at the bottom of the ride:

$$\sum F_y = ma \;\rightarrow\; n_{bottom} - mg = ma_c \;\rightarrow\; n_{bottom} = mg + \frac{mv^2}{r}$$

$$n_{bottom} = mg\left(1 + \frac{v^2}{rg}\right) = mg\left(1 + \frac{(3.00\ m/s)^2}{(10.0\ m)(9.8\ m/s^2)}\right) = 1.09mg$$



The force exerted by the seat on the child is greater than the weight of the child. So, the child experiences an <u>apparent weight</u> that is greater than his <u>true weight</u> by a factor of 1.09

Uniform Circular Motion Examples

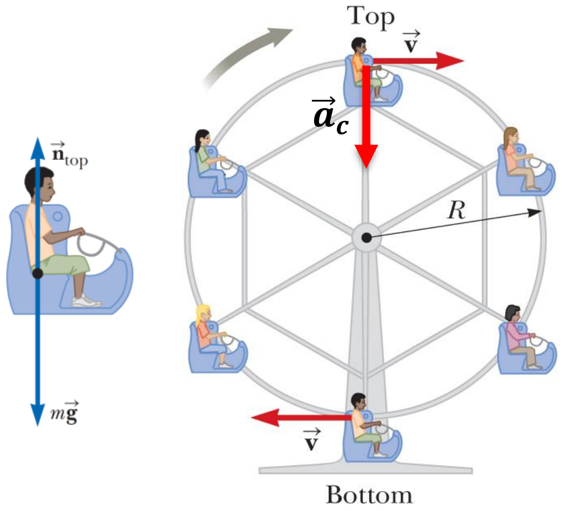
Example 4 (Riding the Ferris Wheel): A child of mass m rides on a Ferris wheel as shown in the Figure. The child moves in a vertical circle of radius $10.0\ m$ at a constant speed of $3.00\ m/s$.

(b) Determine the force exerted by the seat on the child at the top of the ride. Express your answer in terms of the weight of the child mg?

Apply the Newton's second law on the child in the radial direction when the child is at the top of the ride:

$$\sum F_y = ma \quad \rightarrow \quad n_{top} - mg = m(-a_c) \quad \rightarrow \quad n_{top} = mg - \frac{mv^2}{r}$$

$$n_{top} = mg\left(1 - \frac{v^2}{rg}\right) = mg\left(1 - \frac{(3.00\ m/s)^2}{(10.0\ m)(9.8\ m/s^2)}\right) = 0.908mg$$



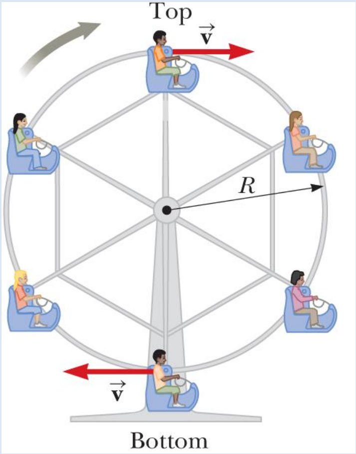
The force exerted by the seat on the child is less than the weight of the child. So, the child experiences an apparent weight that is less than his true weight by a factor of 0.908 and the child feels lighter.

Quick Quiz 3

- You are riding on a Ferris wheel that is rotating with constant speed. The car in which you are riding always maintains its correct upward orientation; it does not invert.

  What is the direction of the normal force on you from the seat when you are at the top of the wheel?

  a) upward
  b) downward
  c) impossible to determine



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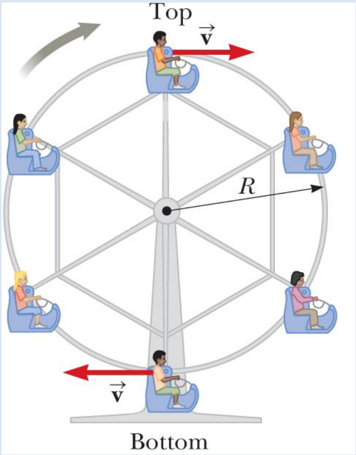
Quick Quiz 4

- You are riding on a Ferris wheel that is rotating with constant speed. The car in which you are riding always maintains its correct upward orientation; it does not invert.

  What is the direction of the net force on you when you are at the top of the wheel?

  a) upward
  b) downward
  c) impossible to determine



Centripetal Force and Safe Driving

- When a car travels around a curve it experiences centripetal acceleration.
- In general, the static friction between the road and the tires is enough to prevent the car from slipping.
- However, if the static friction force is insufficient, given the speed and the radius of the turn, the car will skid off the road.
- Therefore, engineers do not trust in friction, and they use banked curve to make the road safe.
- By banking the curve with respect to the horizontal, we can lessen the amount of friction necessary, because the normal force will help out.

Centripetal Force and Safe Driving

Unbanked Flat Curve

Apply the Newton's second law on the car in x and y directions.

1) The car does not accelerate in vertical direction

$$\sum F_y = 0 \quad \rightarrow \quad n - F_g = 0 \quad \rightarrow \quad n = mg$$

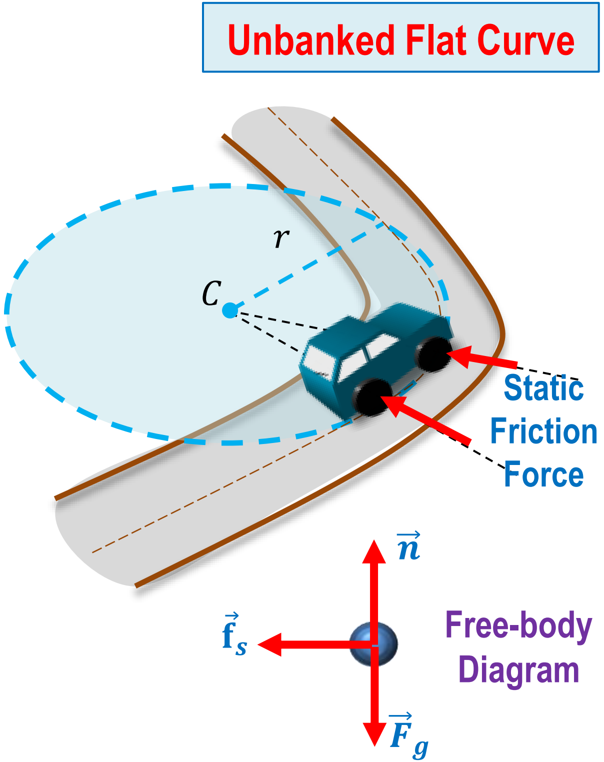
2) The static friction force acts as the centripetal force in horizontal direction

$$\sum F_x = ma_c \quad \rightarrow \quad f_s = m\frac{v^2}{r} \quad \rightarrow \quad \mu_s n = m\frac{v^2}{r}$$

Required minimum static friction coefficient to keep the car on the road, given the speed and the radius of the turn.

$$\boxed{\mu_{s_{min}} = \frac{v^2}{rg}}$$

- If the static friction force is insufficient, given the <u>speed</u> and the <u>radius of the turn</u>, the car will skid off the road.



Centripetal Force and Safe Driving

Banked Curve

Apply the Newton's second law on the car in x and y directions.

1) The car does not accelerate in vertical direction

$$\sum F_y = 0 \quad \rightarrow \quad n\cos\theta - F_g = 0 \quad \rightarrow \quad n\cos\theta = mg$$

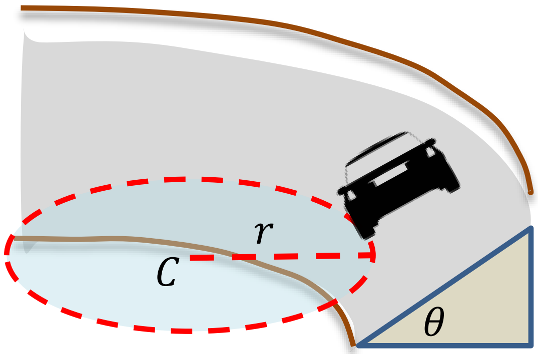
2) The horizontal component of the normal force acts as the centripetal force

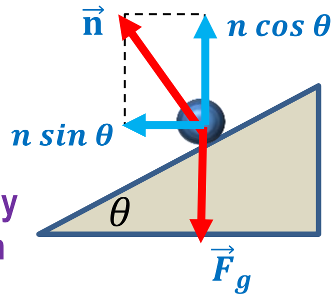
$$\sum F_x = ma_c \quad \rightarrow \quad n\sin\theta = \frac{mv^2}{r}$$

$$\frac{n\sin\theta}{n\cos\theta} = \frac{mv^2/r}{mg} \longrightarrow \boxed{\tan\theta = \frac{v^2}{rg}}$$

θ is the required angle of the roadbed to have a safe turn, given the speed and the radius of the turn, that is <u>independent of mass</u>.

Banked Curve



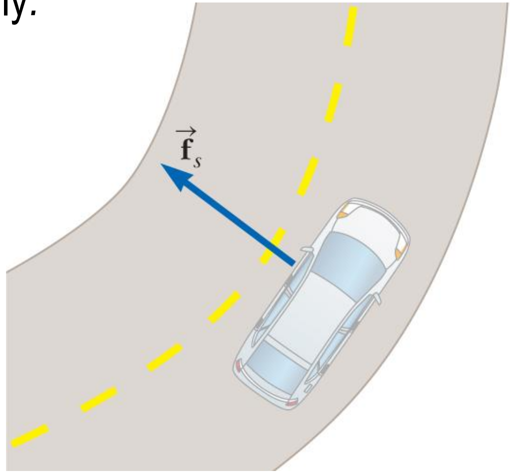


Free-body Diagram

Centripetal Force and Safe Driving

Example 5 (What is the Maximum Speed of the Car): A 1500 kg car moving on a flat, horizontal road negotiates a curve as shown in the overhead view in the figure. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523.

(a) Find the maximum speed the car can have and still make the turn successfully.



No motion in vertical direction, the car is in equilibrium in the vertical direction.

The car experiences a centripetal acceleration in the horizontal direction, so it can be modeled as a Particle in Uniform Circular Motion in this direction.

Centripetal Force and Safe Driving

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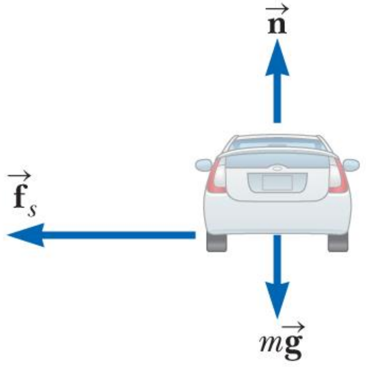
(a) Find the maximum speed the car can have and still make the turn successfully.

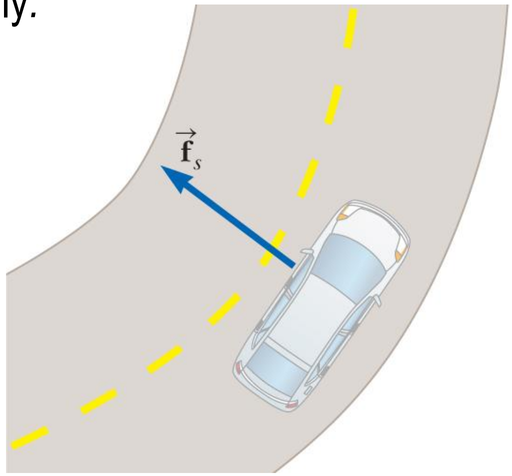
Draw the forces acting on each object.

Draw their free-body diagram

The forces acting on the ball are:

- The gravitational force: $\vec{F}_g$
- The normal force: $\vec{n}$
- The force of friction: $\vec{f}_s$





Centripetal Force and Safe Driving

Example 5 (What is the Maximum Speed of the Car): A 1500 kg car moving on a flat, horizontal road negotiates a curve as shown in the overhead view in the figure. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523.

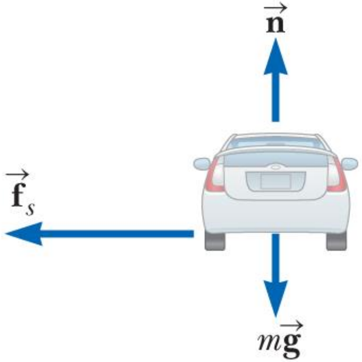
(a) Find the maximum speed the car can have and still make the turn successfully.

$$\sum F_y = 0 \quad \rightarrow \quad n - mg = 0 \quad \rightarrow \quad n = mg$$

$$\sum F_x = ma_c \quad \rightarrow \quad f_{s,max} = ma_{c,max} \quad \rightarrow \quad \mu_s n = m\frac{v_{max}^2}{r}$$

$$v_{max} = \sqrt{\frac{\mu_s n r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r}$$

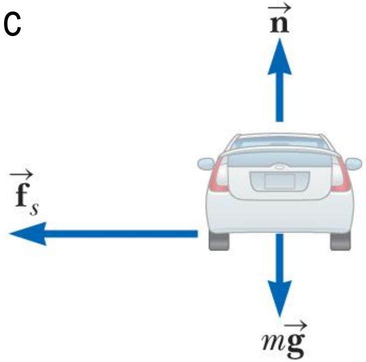
$$\boxed{v_{max} = \sqrt{(0.523)(9.8\,m/s^2)(35.0\,m)} = 13.4\,m/s}$$



Centripetal Force and Safe Driving

Example 5 (What is the Maximum Speed of the Car): A 1500 kg car moving on a flat, horizontal road negotiates a curve as shown in the overhead view in the figure. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523.

(b) Suppose a car travels this curve on a wet day and begins to skid on the curve when its speed reaches only 8.00 m/s. What can we say about the coefficient of static friction in this case?

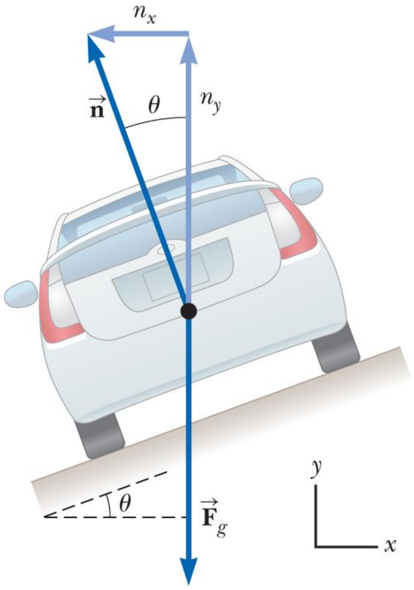


The coefficient of static friction between the tires and a wet road should be smaller than that between the tires and a dry road.

$$v_{max} = \sqrt{\mu_s g r} \quad \rightarrow \quad \mu_s = \frac{v_{max}^2}{gr} \quad \rightarrow \quad \mu_s = \frac{(8.00\ m/s)^2}{(9.8\ m/s^2)(35.0\ m)} = 0.187$$

Centripetal Force and Safe Driving

Example 6 (The Banked Roadway): You are a civil engineer who has been given the assignment to design a curved roadway in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designed speed can negotiate the curve even when the road is covered with ice.



Such a road is usually *banked*, which means that the roadway is tilted toward the inside of the curve.

Suppose the designated speed for the road is to be 13.4 m/s and the radius of the curve is 35.0 m.

You need to determine the angle at which the roadway on the curve should be banked.

Centripetal Force and Safe Driving

Example 6 (The Banked Roadway):

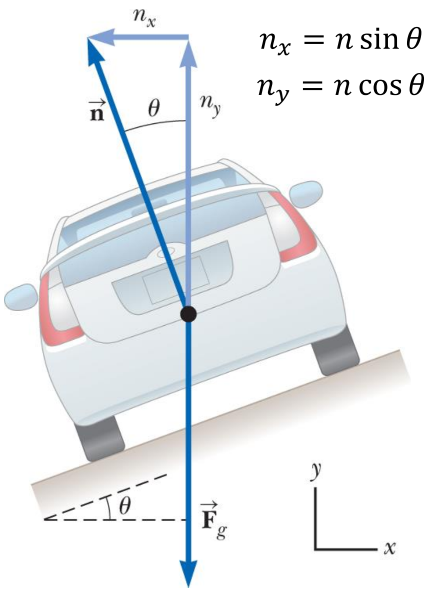
Apply Newton's second law for the car in the vertical and horizontal directions:

$$\sum F_y = 0 \quad \rightarrow \quad n_y - F_g = 0 \quad \rightarrow \quad n\cos\theta = mg$$

$$\sum F_x = ma_c \quad \rightarrow \quad n_x = ma_c \quad \rightarrow \quad n\sin\theta = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{rg} \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{\left(13.4\,\frac{m}{s}\right)^2}{(35.0\,m)(9.80\,m/s^2)}\right)$$

$$\boxed{\theta = 27.6°}$$



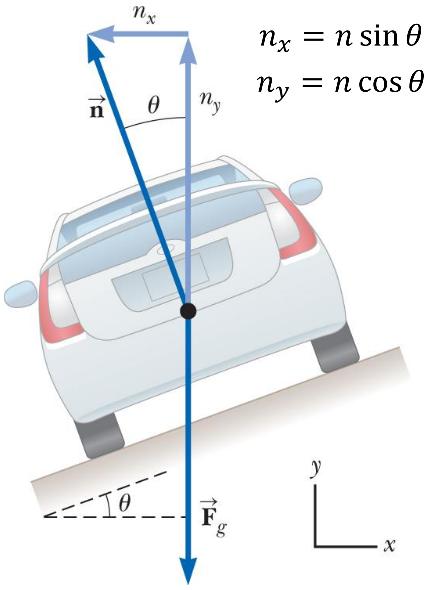
$n_x = n\sin\theta$

$n_y = n\cos\theta$

Centripetal Force and Safe Driving

Example 6 (The Banked Roadway): Imagine that this same roadway were built on Mars in the future to connect different colony centers.

Could it be traveled at the same speed?



$$n_x = n \sin\theta$$
$$n_y = n \cos\theta$$

No, v would be reduced.

$$\tan\theta = \frac{v^2}{rg} \quad \Rightarrow \quad v = \sqrt{rg\tan\theta}$$

The speed v is proportional to the square root of g for a roadway of fixed radius r banked at a fixed angle θ.

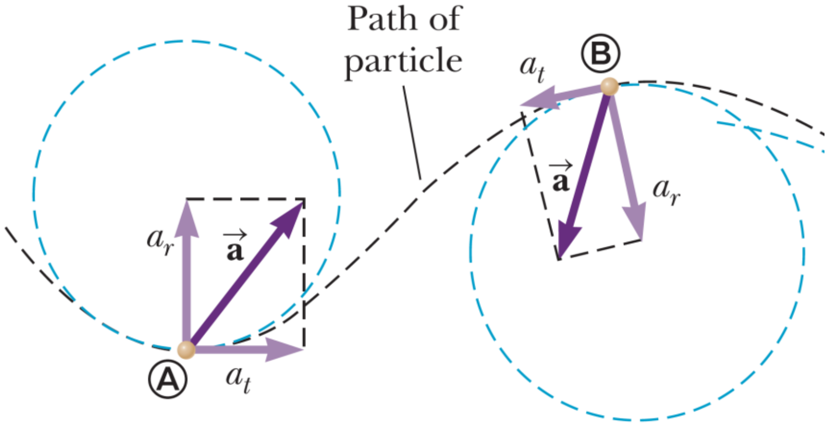
Therefore, if g is smaller, as it is on Mars, the speed v with which the roadway can be safely traveled is also smaller.

Nonuniform Circular Motion

Tangential and Radial Acceleration

- Consider a particle moves to right along curved path, and its velocity changes both in direction and in magnitude.
- As particle moves, the direction of the total acceleration vector $\vec{a}$ changes from point to point.
- At any instant, the total acceleration vector has two components:
  - Radial component: $\vec{a}_r$
  - Tangential component: $\vec{a}_t$
- The total acceleration vector can be written as the vector sum of the components:

$$\vec{a} = \vec{a}_r + \vec{a}_t \qquad a = \sqrt{{a_r}^2 + {a_t}^2}$$



Nonuniform Circular Motion

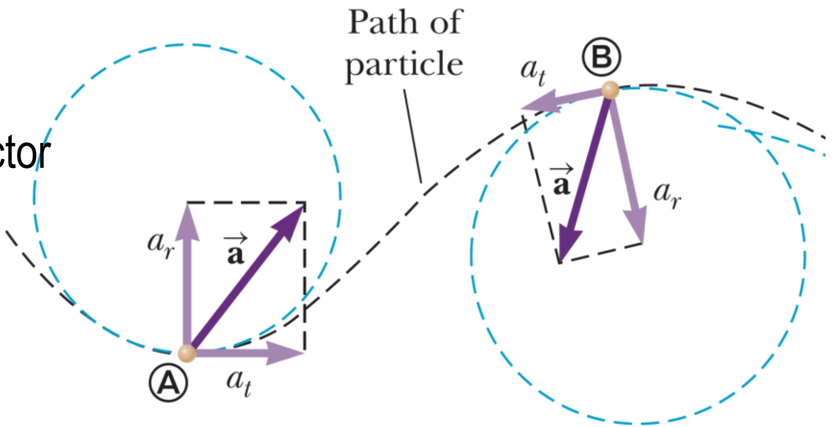
Tangential and Radial Acceleration

1) Properties of the tangential acceleration component $\vec{a}_t$:
   - Causes a change in the speed v of the particle
   - Parallel to the instantaneous velocity
   - Its magnitude is given by

$$a_t = \left|\frac{dv}{dt}\right|$$

2) Properties of the radial acceleration component $\vec{a}_r$:
   - Arise from a change in direction of the velocity vector
   - Perpendicular to the instantaneous velocity
   - Its magnitude is given by

$$a_r = a_c = \frac{v^2}{r}$$



Nonuniform Circular Motion

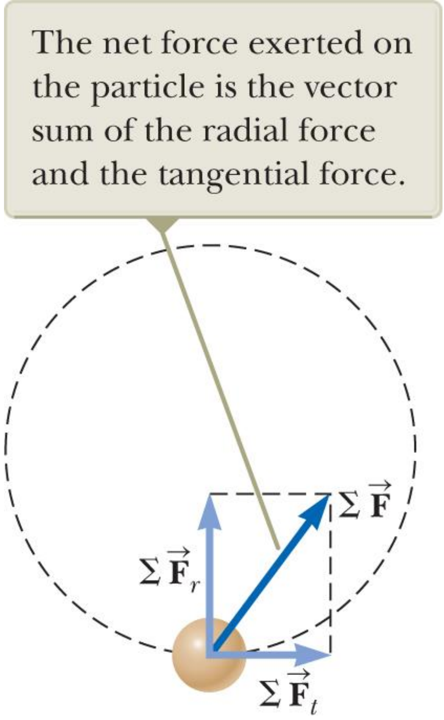
- If a particle moves with varying speed in a circular path, there is a tangential component in addition to the radial component of acceleration.

$$\vec{a} = \vec{a}_r + \vec{a}_t$$

- Therefore, the force acting on the particle must also have a tangential and a radial component.

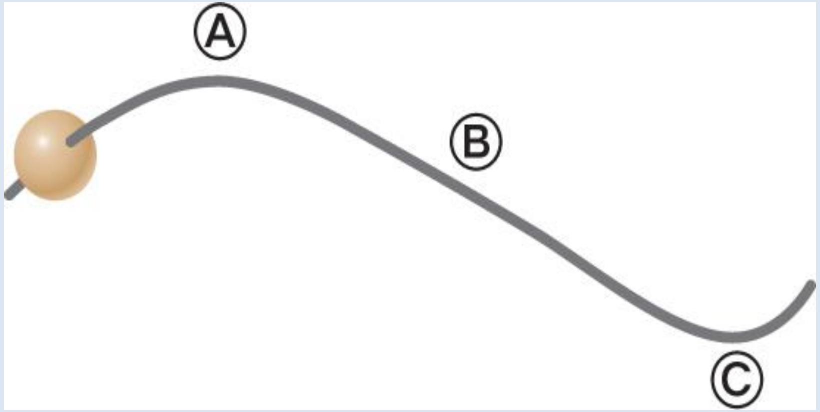
$$\sum \vec{F} = \sum \vec{F}_r + \sum \vec{F}_t$$

- The radial component $\sum \vec{F}_r$ is directed toward the center of the circle and is responsible for the centripetal acceleration.
- The tangential component $\sum \vec{F}_t$ is tangent to the circle and is responsible for the tangential acceleration, which changes the particle speed.



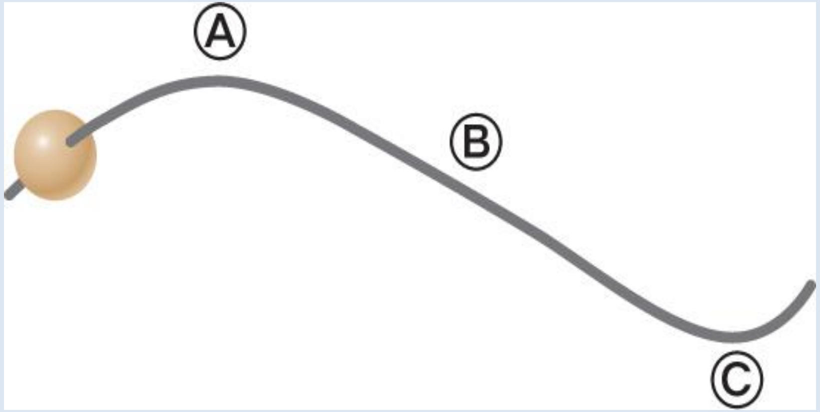
Quick Quiz 5

- A bead slides at constant speed along a curved wire lying on a horizontal surface as shown in figure.
- Draw the vectors representing the force exerted by the wire on the bead at points A, B, and C.



Quick Quiz 6

- A bead slides along a curved wire lying on a horizontal surface as shown in the figure.
- Suppose the bead speeds up with constant tangential acceleration as it moves toward the right. Draw the vectors representing the force on the bead at points A, B, and C.



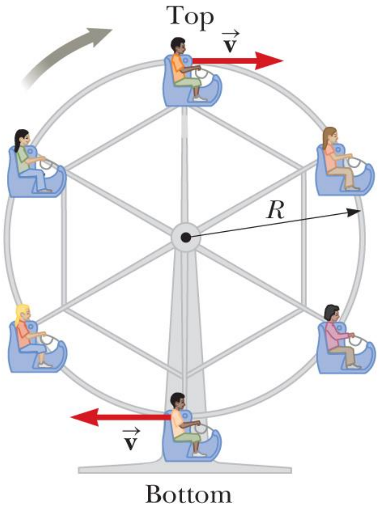
Nonuniform Circular Motion Examples

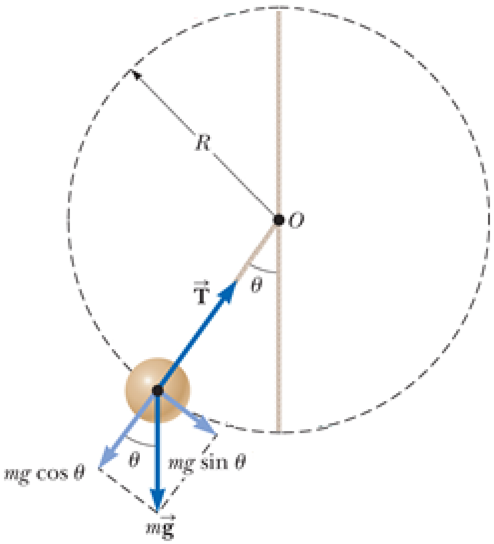
Example 7 (Keep Your Eye on the Ball): A small sphere of mass m is attached to the end of a cord of length R and set into motion in a *vertical* circle about a fixed-point O as illustrated in the figure.

(a) Determine the tangential acceleration of the sphere and the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

Compare the motion of the sphere with that of the child on the Ferris wheel.

Unlike the child, however, the speed of the sphere is *not* uniform in this example because, at most points along the path, a tangential component of acceleration arises from the gravitational force exerted on the sphere.





Nonuniform Circular Motion Examples

Example 7 (Keep Your Eye on the Ball): A small sphere of mass m is attached to the end of a cord of length R and set into motion in a *vertical* circle about a fixed-point O as illustrated in the figure.

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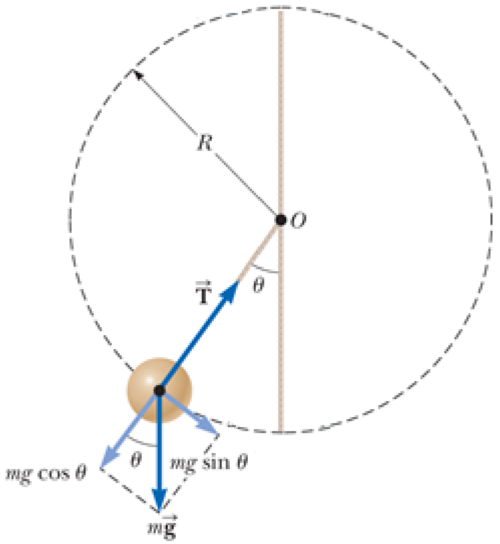


Apply the Newton's second law in tangential and radial directions:

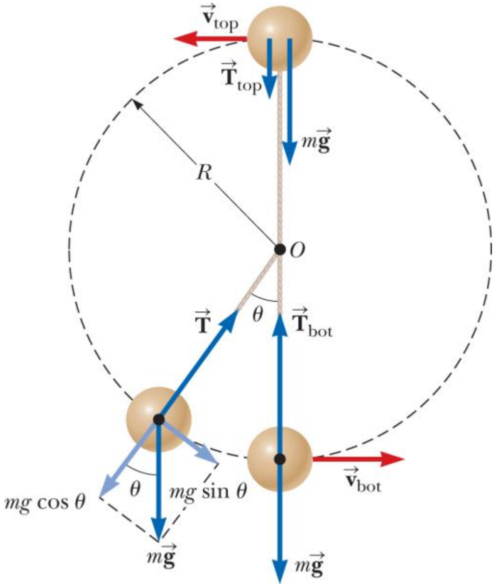
$$\sum F_t = ma_t \quad \rightarrow \quad mg\sin\theta = ma_t \quad \rightarrow \quad a_t = \boxed{g\sin\theta}$$

$$\sum F_r = ma_r \quad \rightarrow \quad T - mg\cos\theta = \frac{mv^2}{R} \quad \rightarrow \quad T = \boxed{mg\left(\frac{v^2}{Rg} + \cos\theta\right)}$$

Nonuniform Circular Motion Examples

Example 7 (Keep Your Eye on the Ball): A small sphere of mass m is attached to the end of a cord of length R and set into motion in a *vertical* circle about a fixed-point O as illustrated in the figure.

(b) Determine the tension in the cord when the sphere passes the vertical top and bottom of the vertical circle.



From part (a) we have: $T = mg\left(\frac{v^2}{Rg} + \cos\theta\right)$

At the top of the vertical circle $\rightarrow$ $\theta = 180°$

$$\boxed{T_{top} = mg\left(\frac{v_{top}^2}{Rg} - 1\right)}$$

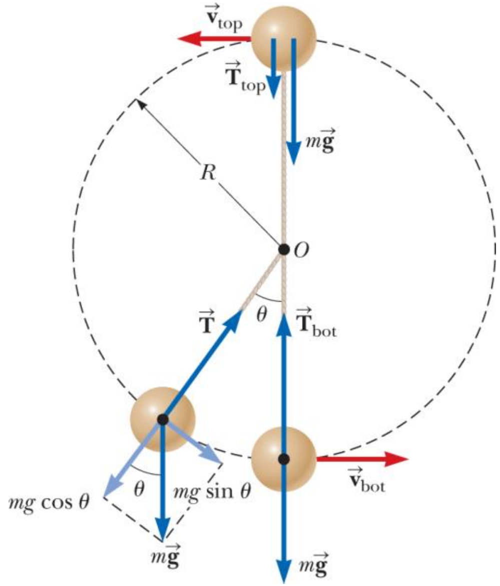
At the bottom of the vertical circle $\rightarrow$ $\theta = 0°$

$$\boxed{T_{bottom} = mg\left(\frac{v_{bottom}^2}{Rg} + 1\right)}$$

Nonuniform Circular Motion Examples

Example 7 (Keep Your Eye on the Ball): A small sphere of mass m is attached to the end of a cord of length R and set into motion in a *vertical* circle about a fixed-point O as illustrated in the figure.

(c) What speed would the sphere have if it passes over the top of the circle if the tension in the cord goes to zero instantaneously at this point?



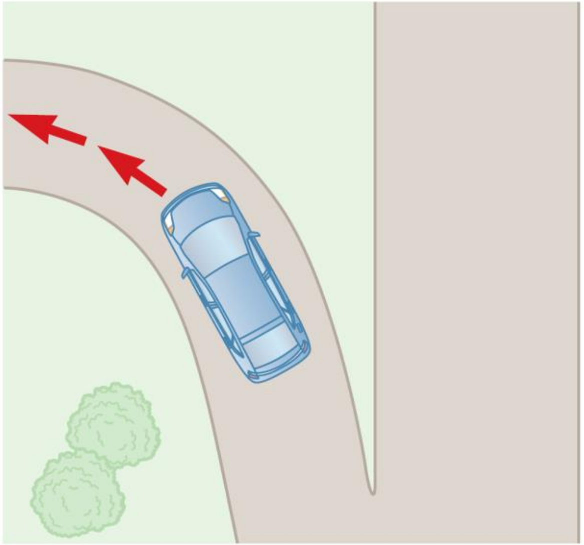
Set the tension equal to zero in the expression for top position

$$T_{top} = mg\left(\frac{v_{top}^2}{Rg} - 1\right)$$

$$0 = mg\left(\frac{v_{top}^2}{Rg} - 1\right) \quad \rightarrow \quad \boxed{v_{top} = \sqrt{gR}}$$

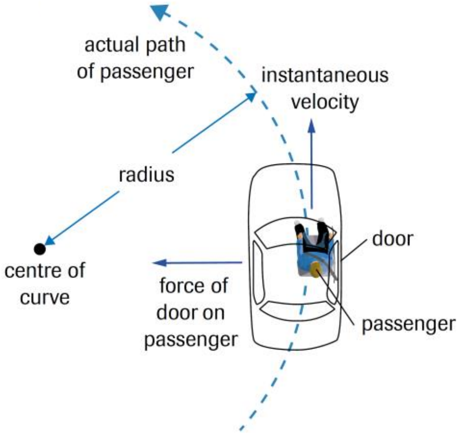
Motion in Accelerated Frames

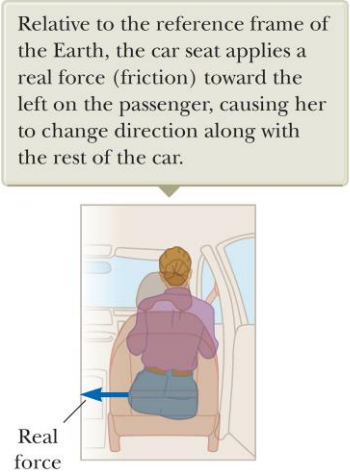
- Newton's laws of motion describe observation that are made in an inertial frame of reference.
- Since an object in circular motion is accelerating, any motion observed from that object must exhibit properties of a non-inertial frame of reference.
- Consider the forces you feel when you are the passenger in a car during the left turn.
- You fell as if your right shoulder is being pushed against the passenger-side door.
- We can analyze the case from two different frame of references:
  1) From Earth's frame of reference, which is an inertial frame.
  2) From the accelerating frame of reference of the car, which is non-inertial frame.



Motion in Accelerated Frames

1) From Earth's frame of reference, this force that you feel can be explained by Newton's first law of motion or law of inertia:
   - You tend to maintain your initial velocity (in both magnitude and direction).
   - When the car you are riding in goes left, you tend to go straight, but the car door pushes on you and causes you to go in a circular path along with the car.
   - Thus, there is a centripetal force to the left on your body.

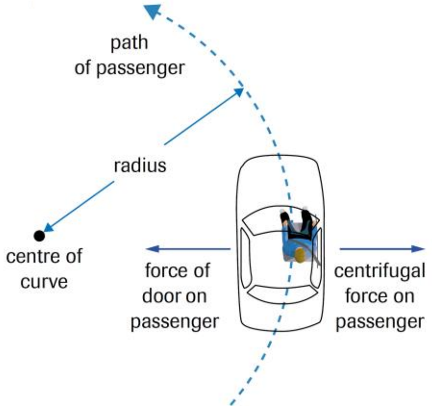


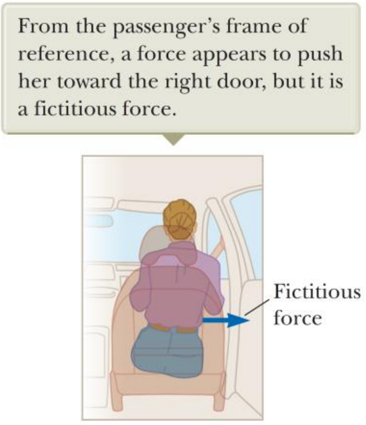


Motion in Accelerated Frames

2) From the accelerating frame of reference of the car:
   - You feel as if something is pushing you toward the outside of the circle.
   - This force away from the center is a fictitious force called the centrifugal force.
   - In this example the fictitious force caused by changing the *direction* of velocity vector.

- **Fictitious Force** is an *apparent* force that seems to act on any mass described in an accelerating (non-inertial) frame of reference.
- The law of inertia is <u>not true</u> in non-inertial reference frame unless the fictitious forces are introduced.

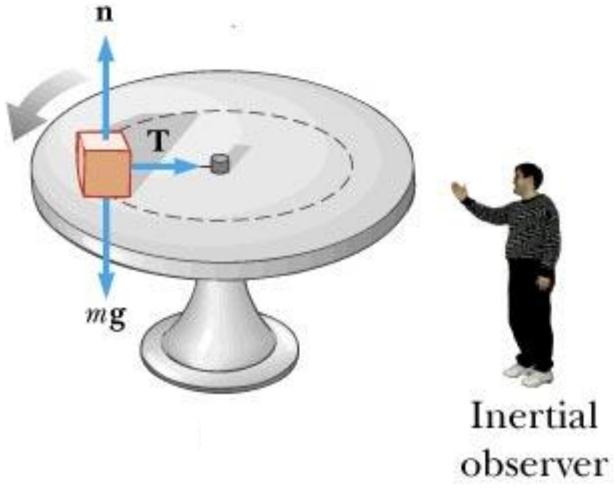




Fictitious Forces Examples

Example 8 (Fictitious Forces in Circular Motion): Consider a rotating turntable with a mass on it, held in place by a spring.

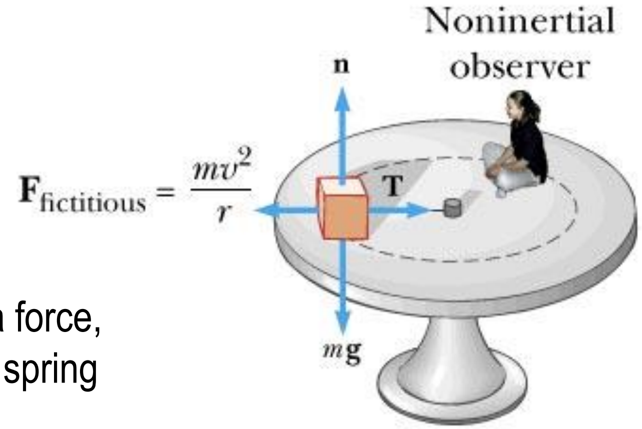
To an observer on the ground, who is an inertial observer, the stretched spring provides a centripetal force, which keeps the mass moving in a circle.



Fictitious Forces Examples

Example 8 (Fictitious Forces in Circular Motion): Consider a rotating turntable with a mass on it, held in place by a spring.

However, the mass is at rest to an onboard observer, who is a non-inertial observer sitting on the rotating turntable.



Why should the spring be stretched?

The onboard observer will see the spring stretched, which means it exerts a force, and conclude that there is some outward force acting it which the stretched spring just balances.

In this non-inertial frame of reference there is a fictitious force of mv^2/r acting outward, which is often called a centrifugal force.

Quick Quiz 7

- Consider the passenger in the car making a left turn.

  Which of the following is correct about forces in the horizontal direction if she is making contact with the right-hand door?

  a) The passenger is in equilibrium between real forces acting to the right and real forces acting to the left.
  b) The passenger is subject only to real forces acting to the right.
  c) The passenger is subject only to real forces acting to the left.
  d) None of those statements is true.

Fictitious Forces Examples

Example 9 (Fictitious Forces in Linear Motion): Consider the experiment that you are riding on an accelerating railroad car with a weight hanging inside it, suspended from the ceiling by a string. Now suppose your friend stands on solid ground beside the ride watching you.

Both the inertial observer (your friend) and the non-inertial observer (you) agree that the string makes an angle θ with respect to the vertical. You claim that a force, which we know to be fictitious, causes the observed deviation of the string from the vertical.

How is the magnitude of this force related to the weight's centripetal acceleration measured by the inertial observer?

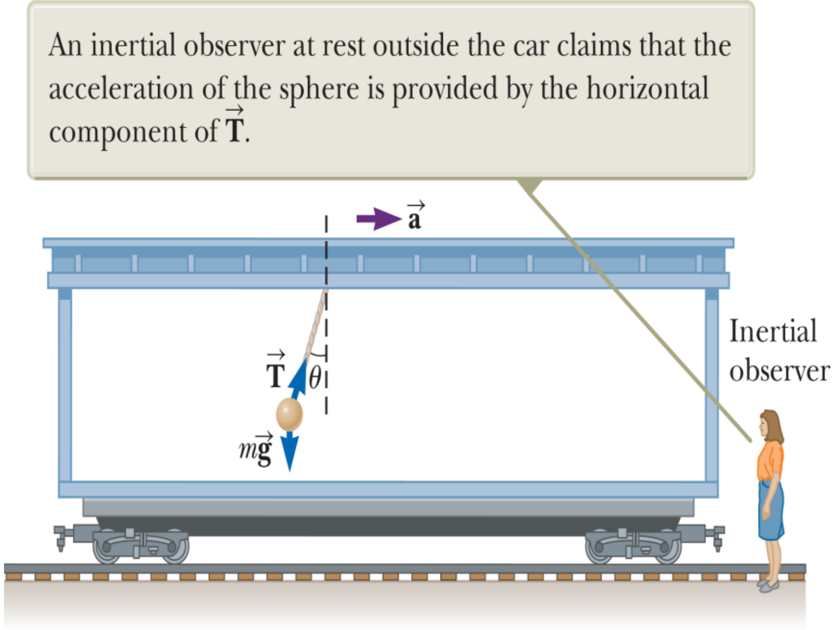
Fictitious Forces Examples

Example 9 (Fictitious Forces in Linear Motion):

1) The observer in the ground is an observer in an inertial reference frame, therefore the Newton's Laws of Motion is valid in her reference frame.

The inertial observer "sees" the forces we have sketched in the figure.
The suspended weight does not hang straight down because the tension needs to provide a horizontal component to give it an acceleration.

$$\begin{cases} \sum F_x = ma \quad \rightarrow \quad T\sin\theta = ma \\ \sum F_y = 0 \quad \rightarrow \quad T\cos\theta - mg = 0 \end{cases}$$

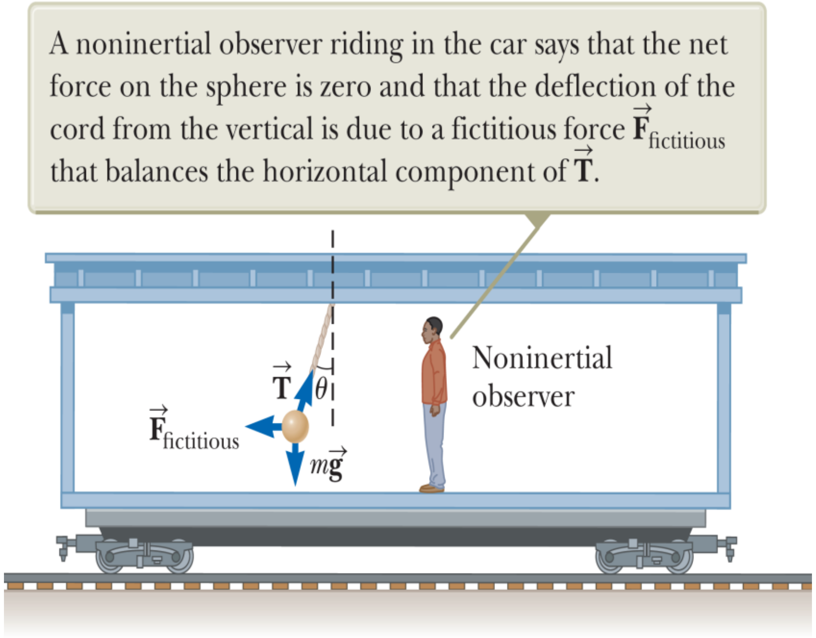


Fictitious Forces Examples

Example 9 (Fictitious Forces in Linear Motion):

2) The observer inside the accelerating railroad car is a non-inertial observer.

The non-inertial observer "sees" that the hanging weight still does not hang straight down. It is still suspended by the string at the angle of θ.
But the weight is at rest with respect to the onboard observer. If it is at rest, how can it hang suspended like that?



Fictitious Forces Examples

Example 9 (Fictitious Forces in Linear Motion):

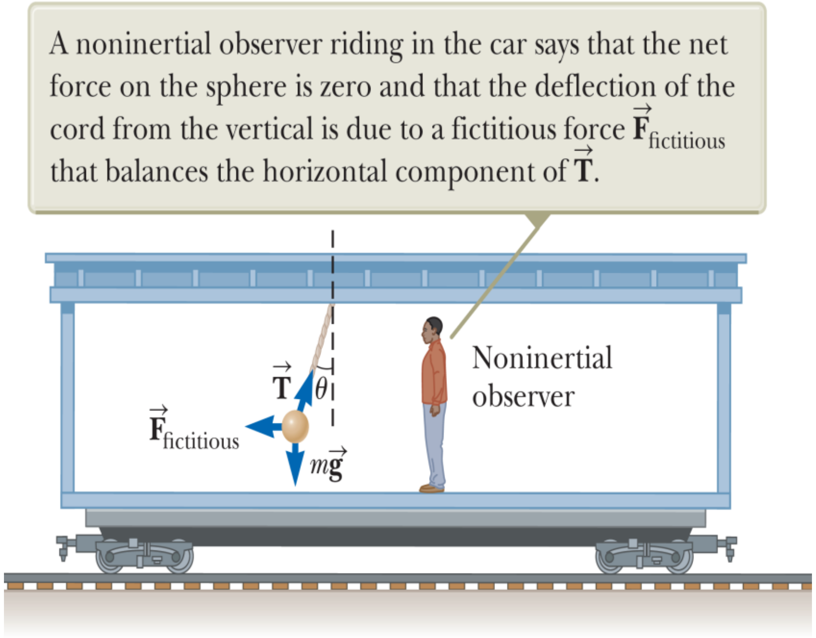
To keep the Law of Motion true, there must be some additional force pulling this weight out to one side.

We call this a Fictitious Force.

$$\begin{cases} \sum F_x' = 0 \quad \rightarrow \quad T\sin\theta - F_{\text{fictitious}} = 0 \\ \sum F_y' = 0 \quad \rightarrow \quad T\cos\theta - mg = 0 \end{cases}$$

This equations are equivalent to the inertial observer equations if:

$$\boxed{F_{\text{fictitious}} = ma}$$



THANK YOU

HUMBER

WE ARE HUMBER