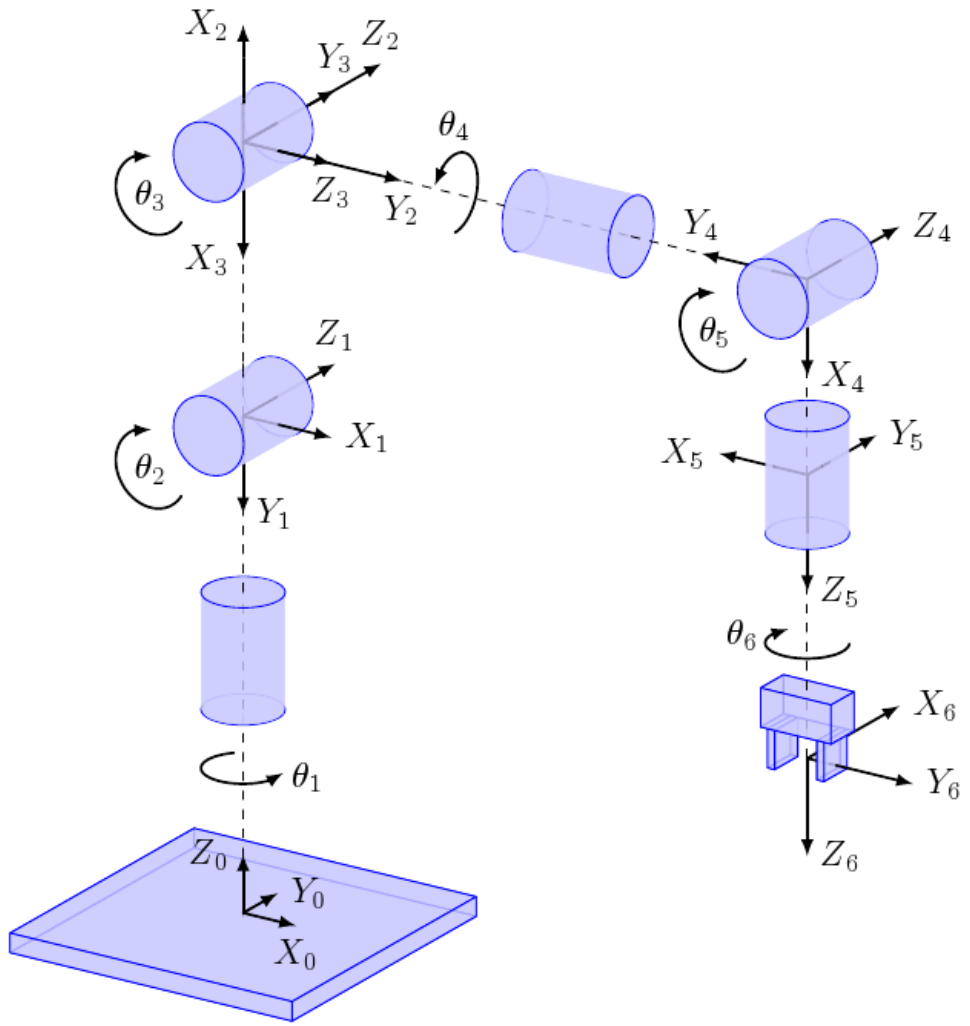
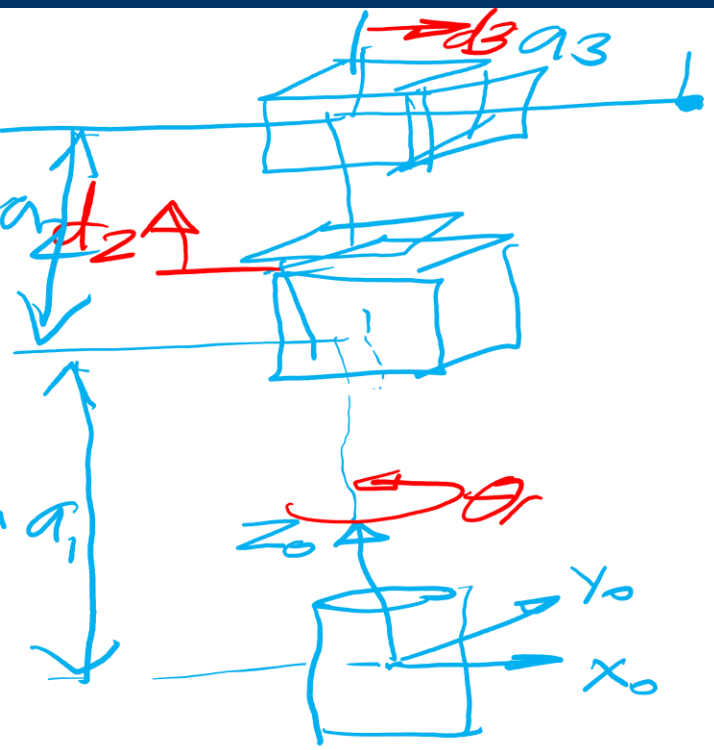
Kinematics and Dynamics of Robots

Module 11

Inverse Kinematics 2

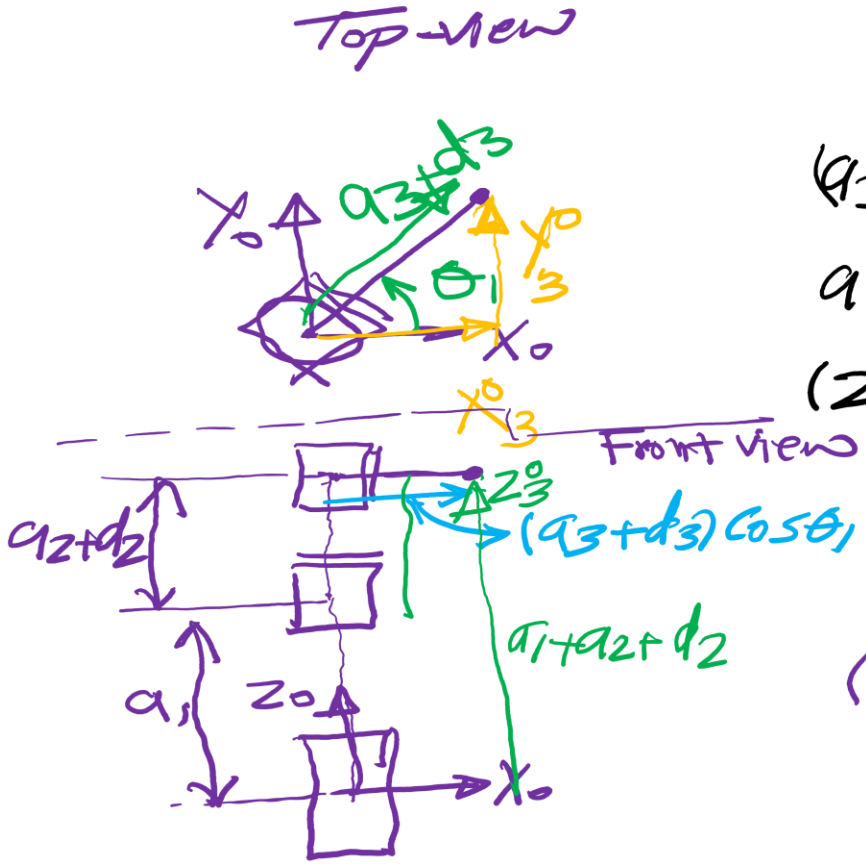


Inverse Kinematics 3D



Known : X_3^0, Y_3^0, Z_3^0

unknown : θ_1, d_2, d_3



$$\theta_1 = \tan^{-1}\left(\frac{Y_3^0}{X_3^0}\right) \quad (1)$$

$$(q_3+d_3)^2 = X_3^{0^2} + Y_3^{0^2}$$

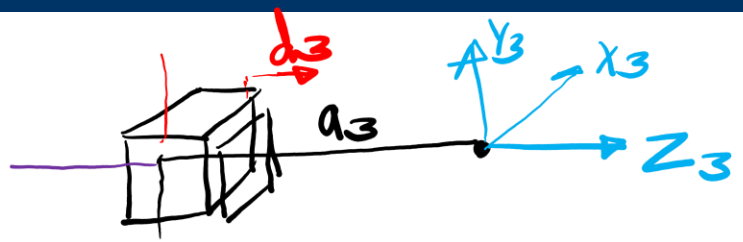
$$q_3+d_3 = \sqrt{X_3^{0^2}+Y_3^{0^2}}$$

$$(2)\quad d_3 = \sqrt{X_3^{0^2}+Y_3^{0^2}} - q_3$$

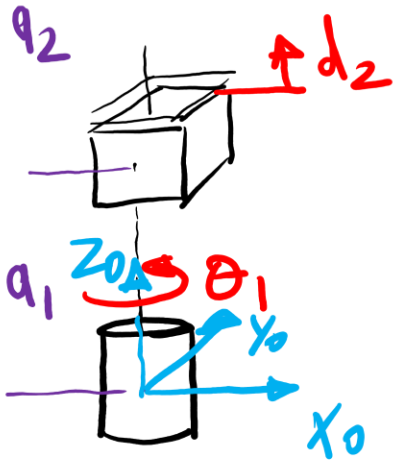
$$Z_3^0 = q_1 + q_2 + d_2$$

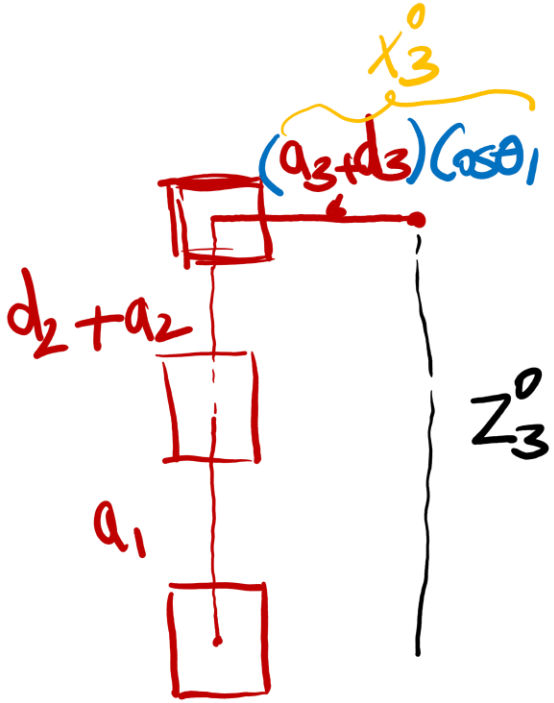
$$(3)\quad d_2 = Z_3^0 - q_1 - q_2$$

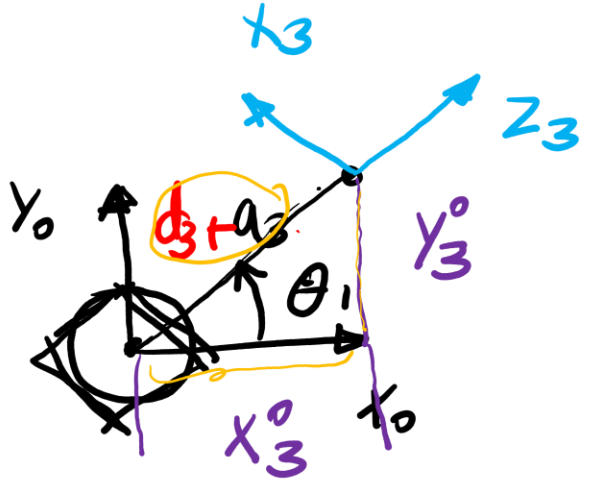
Inverse Kinematics 3D



Unknow
θ_1, d_2, d_3
Known: x_3^0, y_3^0, z_3^0







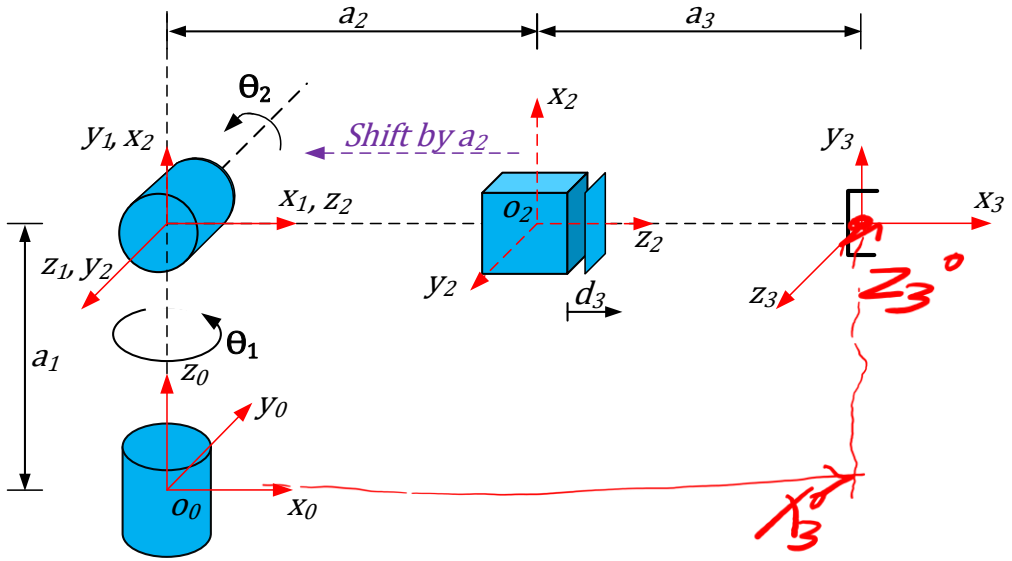
$$\theta_1 = \tan^{-1}\frac{y_3^0}{x_3^0}$$

$$(d_3+a_3)^2 = x_3^{0\,2} + y_3^{0\,2}$$

$$d_3 = \sqrt{x_3^{0\,2} + y_3^{0\,2}} - a_3$$

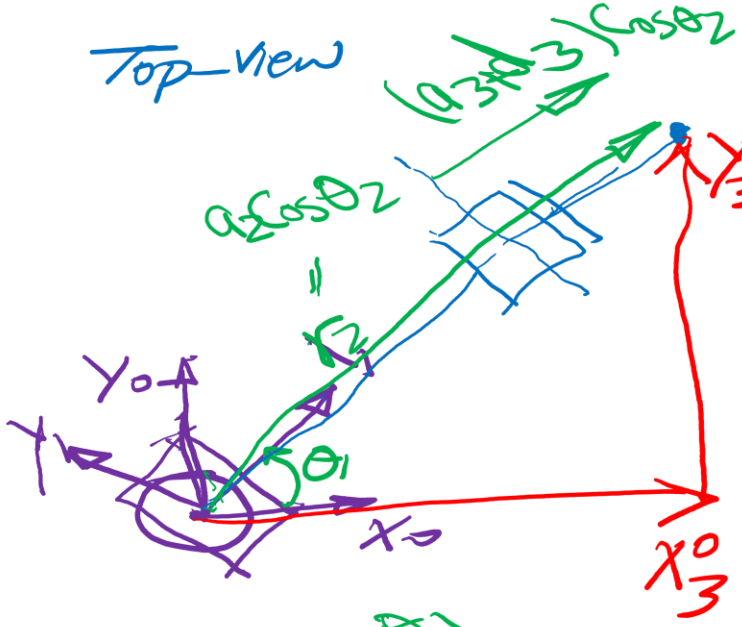
$$z_3^0 = a_1 + a_2 + d_2 \Rightarrow d_2 = z_3^0 - a_1 - a_2$$

Inverse Kinematics 3D



Known: x_3^0, y_3^0, z_3^0

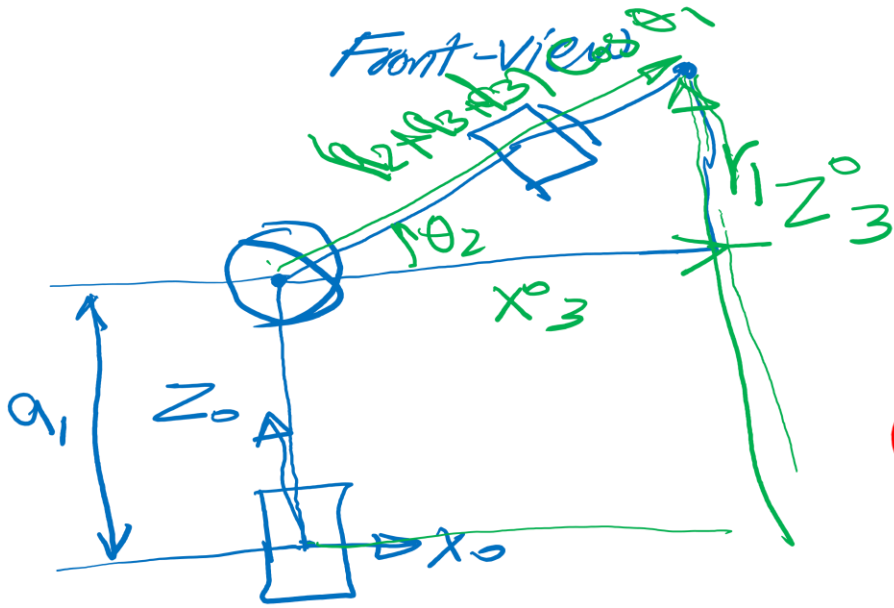
Unknown: θ_1, θ_2, d_3



(1) $\theta_1 = \tan^{-1}\left(\frac{y_3^0}{x_3^0}\right)$

$r_2 = \sqrt{x_3^{0\,2} + y_3^{0\,2}}$

$r_2 = (a_2 + a_3 + d_3)\cos\theta_2$

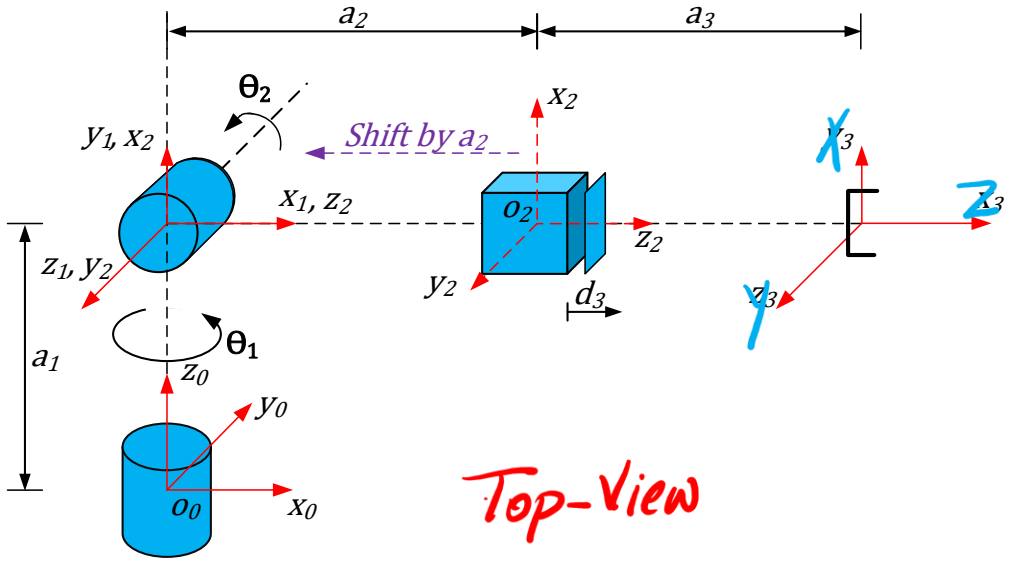


(3) $\theta_2 = \tan^{-1}\left(\frac{r_1}{x_0^3}\right)$

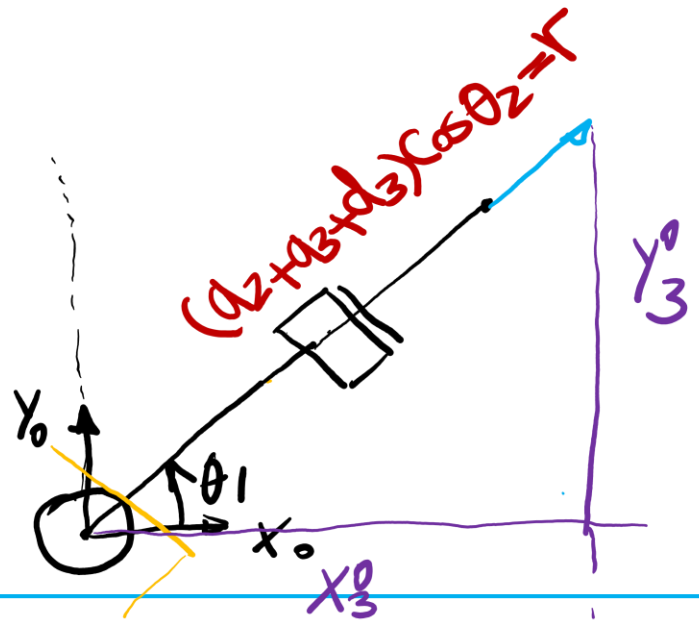
(2) $r_1 = z_3^0 - a_1$

$d_3 = \frac{r_2}{\cos\theta_2} - a_2 - a_3$

(4) $d_3 = \frac{\sqrt{x_3^2 + y_3^2}}{\cos\theta_2} - a_2 - a_3$

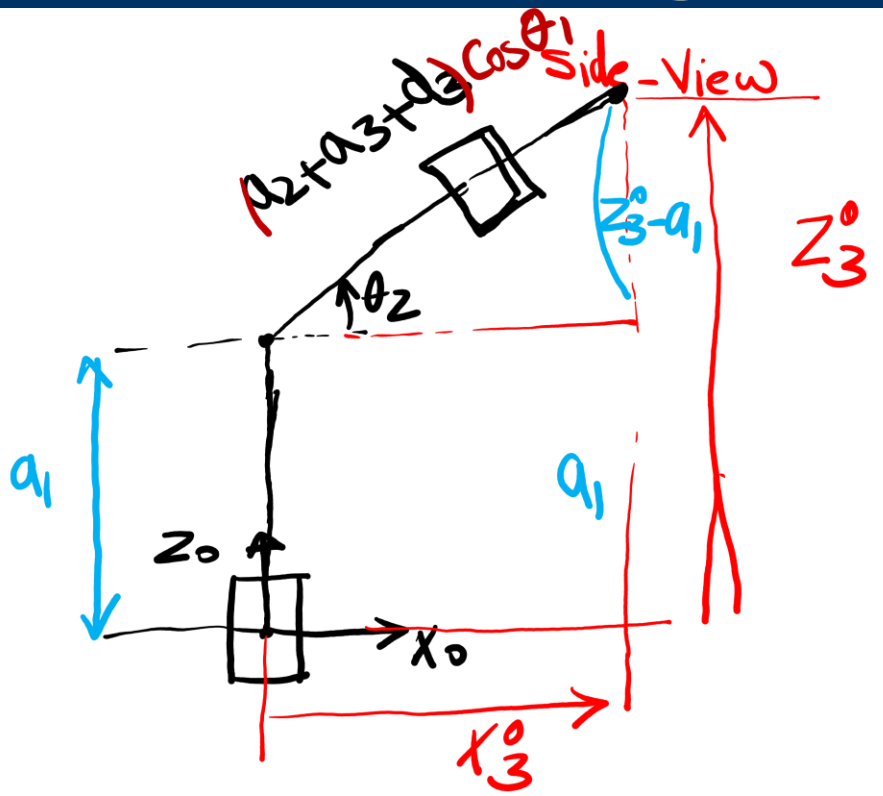


Top-View



$$r = \sqrt{x_3^{0\,2} + y_3^{0\,2}}$$

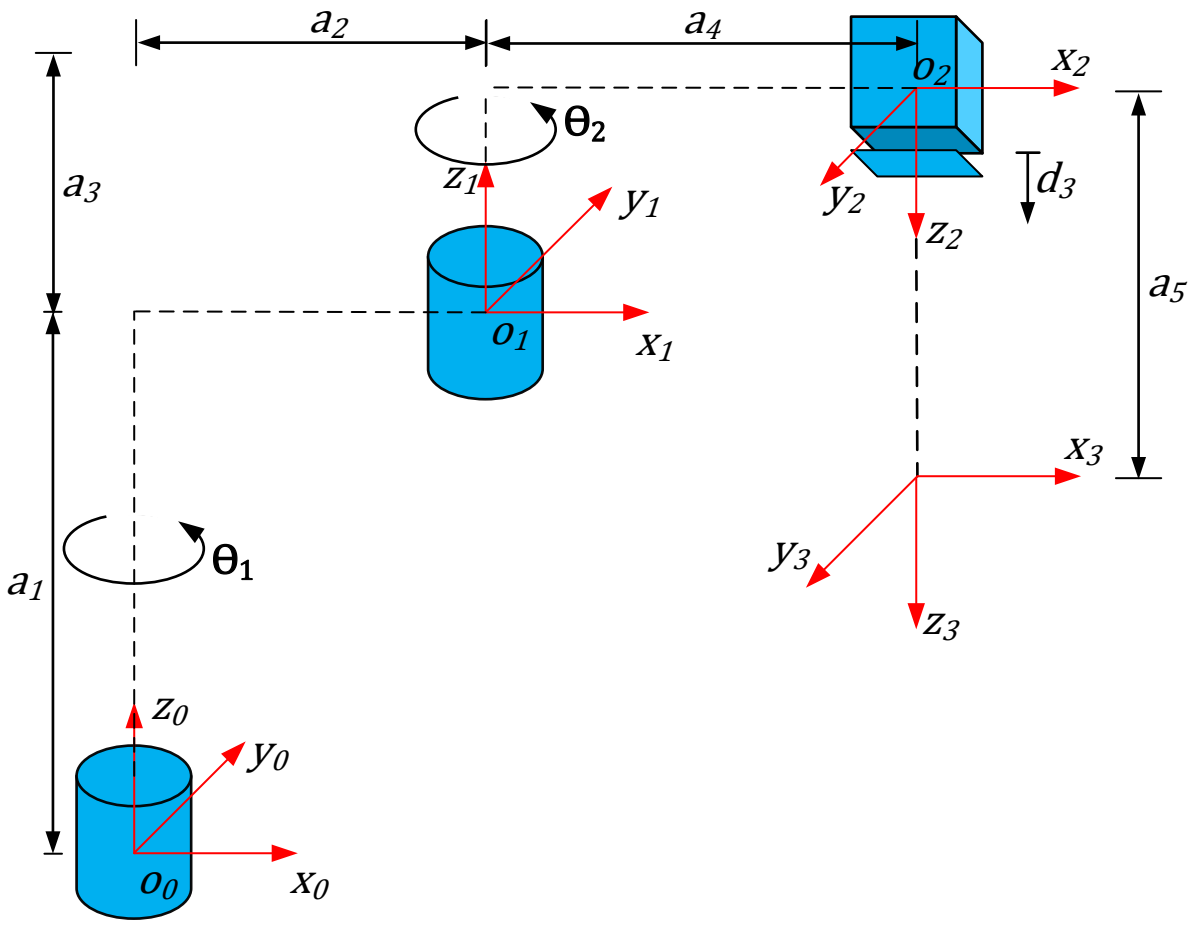
$$\theta_1 = \tan^{-1}\left(\frac{y_3^0}{x_3^0}\right)$$



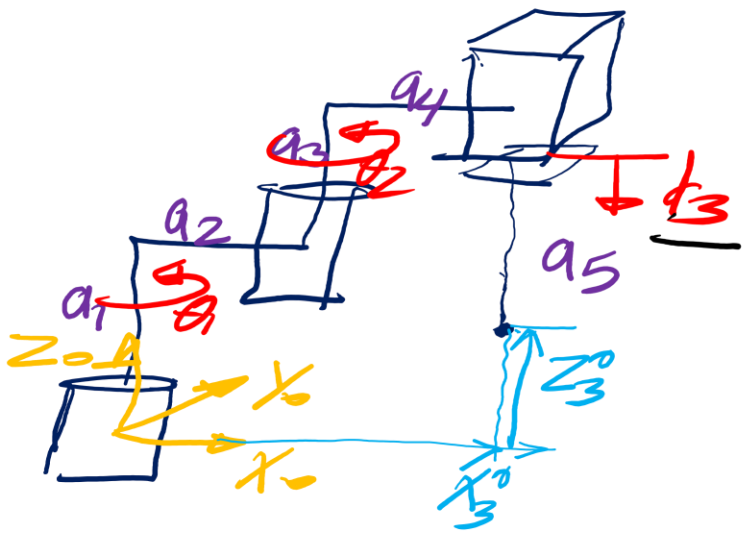
$$\theta_2 = \tan^{-1}\frac{z_3^0 - a_1}{x_3^0}$$

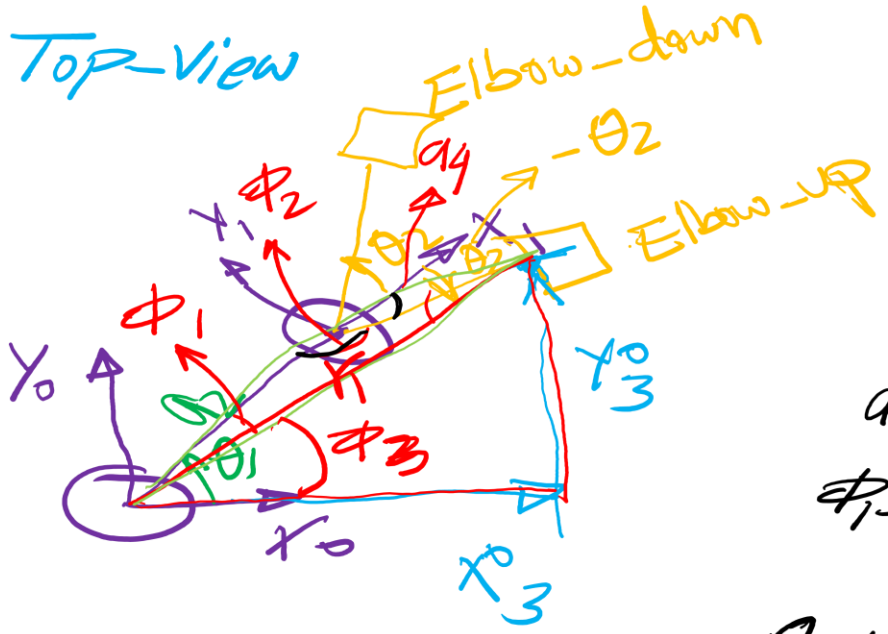
$$x_3^0 = \left[(a_2+a_3+d_3)\cos\theta_1\right]\cos\theta_2$$

$$d_2 = \frac{x_3^0}{\cos\theta_1\cos\theta_2} - a_2 - a_3$$


a_2
a_4
a_3
a_1
a_5
d_3
θ_1
θ_2
z_0
y_0
x_0
o_0
z_1
y_1
x_1
o_1
z_2
y_2
x_2
o_2
z_3
y_3
x_3

Inverse Kinematics 3D





$\phi_1 + \phi_3 = \theta_1$

$\phi_3 = \tan^{-1}\left(\frac{y_3^0}{x_3^0}\right)$

$r_1 = \sqrt{x_3^{0\,2} + y_3^{0\,2}}$

$a_4^2 = a_2^2 + r_1^2 - 2a_2 r_1 \cos\phi_1$

$\phi_1 = \cos^{-1}\left(\frac{a_4^2 + a_2^2 + r_1^2}{2a_2 r_1}\right)$

$-\theta_2 + \phi_2 = 180°$

$\theta_2 = \phi_2 - 180°$

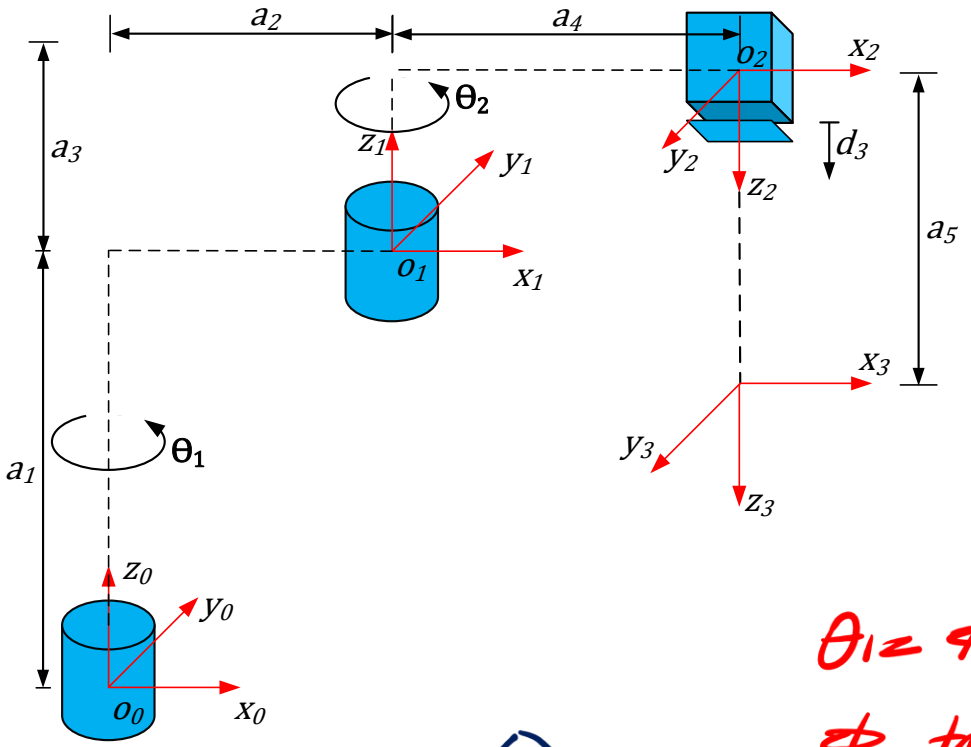
$r_1^2 = a_2^2 + a_4^2 - 2a_2 a_4 \cos\phi_2$

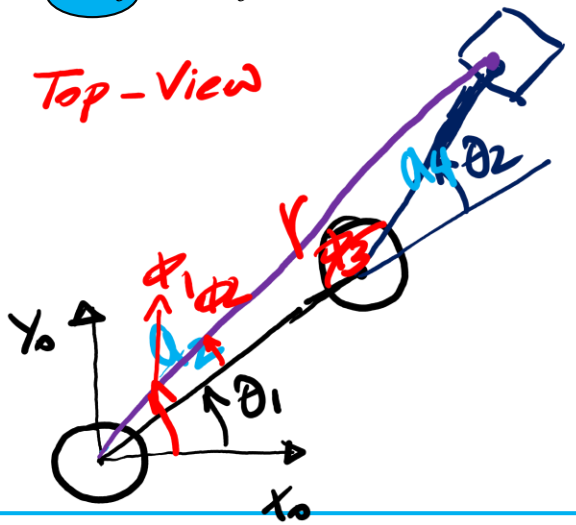
$\phi_2 = \cos^{-1}\left(\frac{a_2^2 - a_4^2 - r_1^2}{2a_2 a_4}\right)$

$z_3^0 = a_1 + a_3 - a_5 - d_3$

$d_3 = a_1 + a_3 - a_5 - z_3^0$

Inverse Kinematics 3D





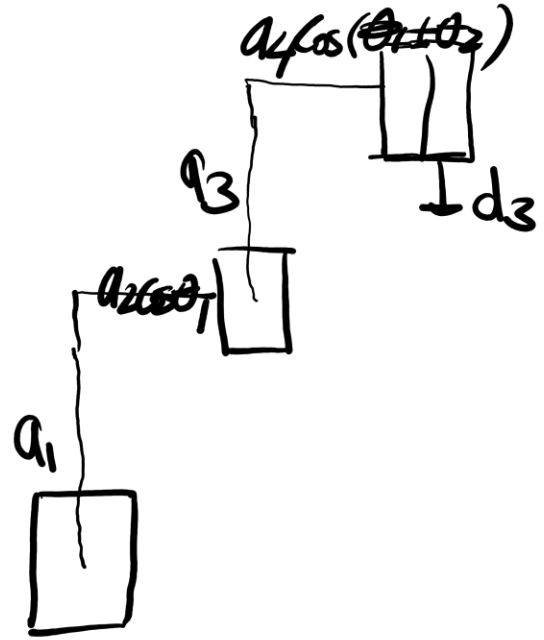
$\theta_1 = \phi_1 - \phi_2 \qquad r = \sqrt{x_3^{02} + y_3^{02}}$

$\phi_1 = \tan^{-1} \frac{Y_3^0}{X_3^0}$

$\phi_2 = \cos^{-1} \frac{a_2^2 + r^2 - a_4^2}{2 r a_2}$

$\phi_3 = \cos^{-1} \frac{a_2^2 + a_4^2 - r^2}{2 a_2 a_4}$

$\theta_2 = 180 - \phi_3$



$Z_3^0 = q_1 + q_3 - d_3$

$d_3 = q_1 + q_2 - Z_3^0$