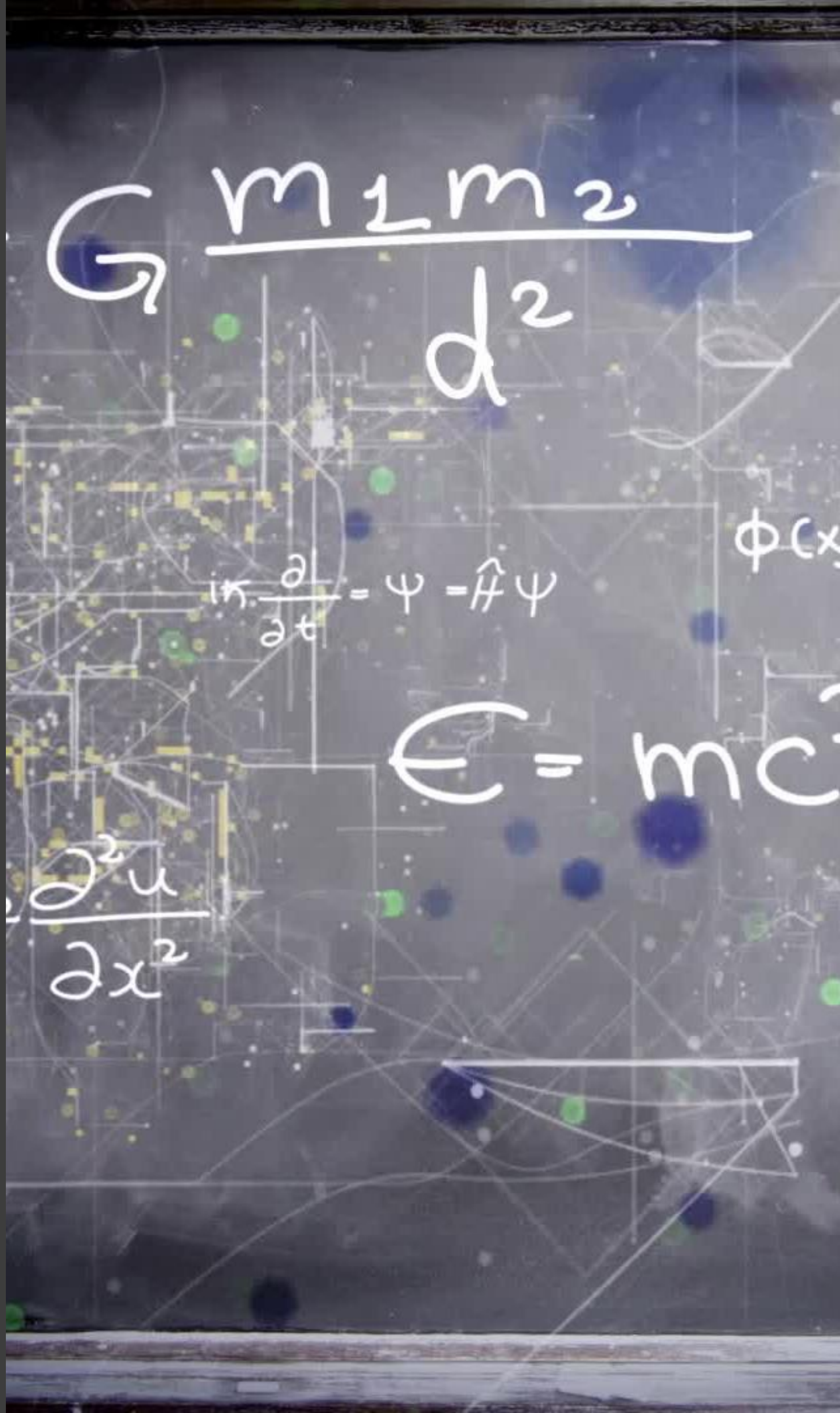
4.4 THE FUNDAMENTAL THEOREM OF CALCULUS

PART 1

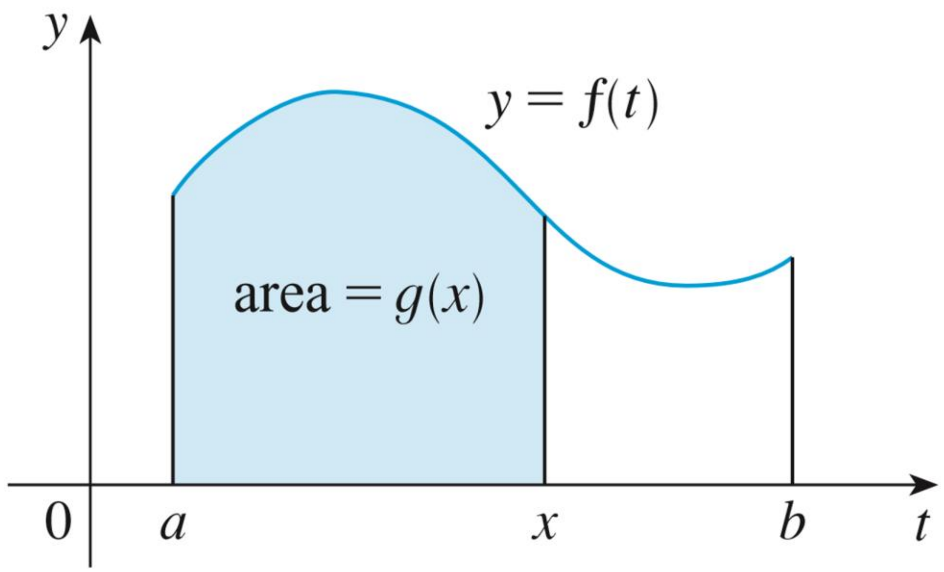


The Fundamental Theorem of Calculus (FTC) is appropriately named because it establishes a connection between the two branches of calculus: differential calculus and integral calculus.

Differential Calculus $\overset{F.T.C}{\Longleftrightarrow}$ Integral Calculus

FTC establishes the precise inverse relationship between the derivative and the integral

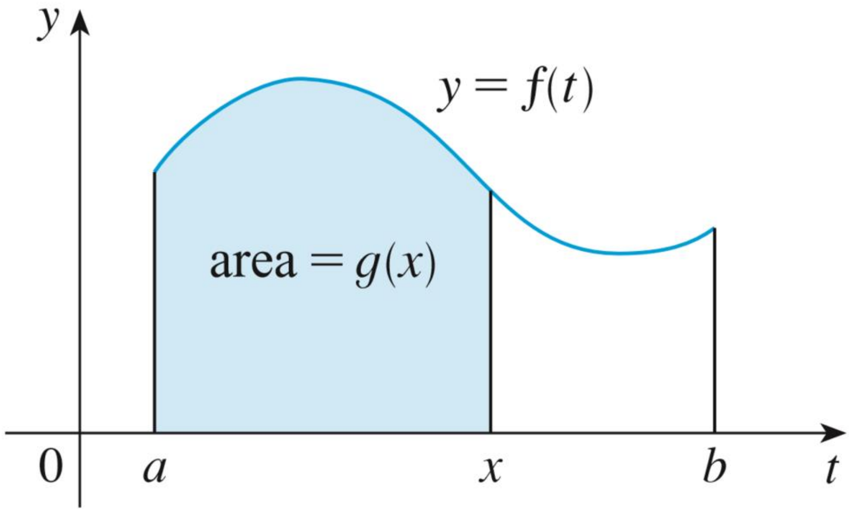
Let's define a new function $g(x) = \int_a^x f(t)dt$



- g depends only on x, the variable upper limit in the integral.
- If $f(x)$ is positive, then $g(x)$ can be interpreted as the area under the graph of f from a to x, where can vary from a to b. (Think of g as the "area so far" function)

Let's define a new function $g(x) = \int_a^x f(t)dt$

Consider $g(x) = \int_0^x t dt = \frac{x^2}{2}$



$$\int_0^x t\,dt = \frac{t^2}{2}\Big|_{t=0}^{t=x} = \frac{x^2}{2} - \frac{0^2}{2} = \frac{x^2}{2}$$

- g depends only on x, the variable upper limit in the integral.
- If $f(x)$ is positive, then $g(x)$ can be interpreted as the area under the graph of f from a to x, where can vary from a to b. (Think of g as the "area so far" function)

FUNDAMENTAL THEOREM OF CALCULUS (FTC), PART I

If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)dt\,, \text{for a} \leq x \leq b$$

- is continuous on $[a, b]$ and differentiable on (a, b),
- and $g'(x) = f(x)$

$$\frac{d}{dx} g(x) = \frac{d}{dx} \int_0^x f(t)dt$$

A MORE GENERAL FORM OF THE FTC

$$\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)dt = f\big(g(x)\big)\cdot\frac{d}{dx}g(x) - f(h(x))\cdot\frac{d}{dx}h(x)$$

Functional substitution:

$f(t) = \sin t$, $g(x) = x^3$, $f(g(x)) = \sin(x^3)$

$f(t) = \sqrt{t}$ $g(x) = \tan x$, $f(g(x)) = \sqrt{\tan x}$