ENGI-1500
Physics -2

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Humber Institute of Technology and Advanced Learning
Winter 2023

Week 9 / Class 7

Faraday’s Law (Ch. 30)

Outline of Week 9 / Class 7

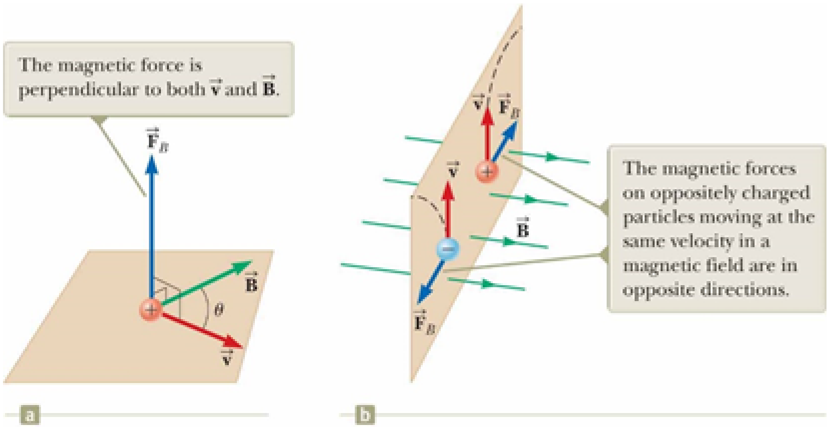
- Reminder of the previous week

- Faraday’s Law (Ch. 30)
  - Faraday’s Law of Induction
  - Motional EMF
  - Lenz’s Law
  - The General Form of Faraday’s Law
  - Generators and Motors
  - Eddy Currents

- Examples
- Next week’s topic

Reminder of the previous week – Magnetic Fields

Particle in a magnetic field



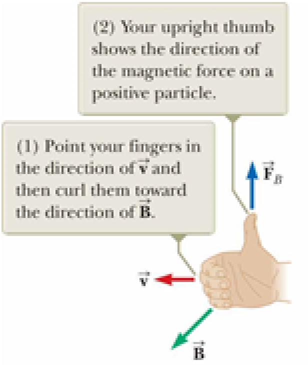
$$\vec{F}_B = q\vec{v}\times\vec{B}$$

$$1\ \mathrm{T} = 1\frac{\mathrm{N}}{\mathrm{C}\cdot\mathrm{m/s}}$$

$$1\ \mathrm{T} = 1\frac{\mathrm{N}}{\mathrm{A}\cdot\mathrm{m}}$$

$$1\ \mathrm{T} = 10^4\ \mathrm{G}$$

Right Hand Rule



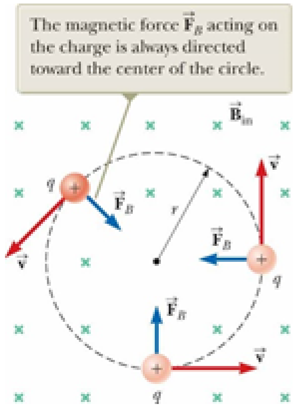
Motion of a charged particle

$$\sum F = F_B = ma$$

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB} \qquad \omega = \frac{v}{r} = \frac{qB}{m}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

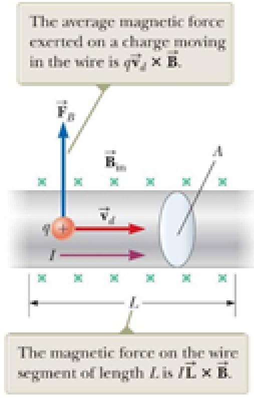


Magnetic force on current

$$\vec{F}_B = I\vec{L}\times\vec{B}$$

$$d\vec{F}_B = I\,d\vec{s}\times\vec{B}$$

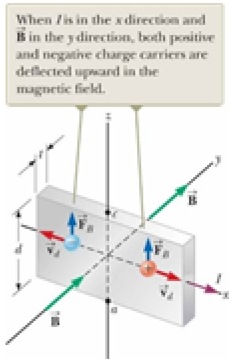
$$\vec{F}_B = I\int_a^b d\vec{s}\times\vec{B}$$



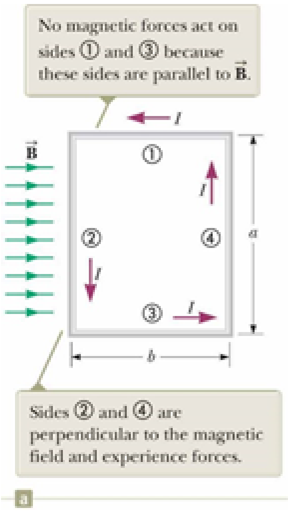
The Hall Effect

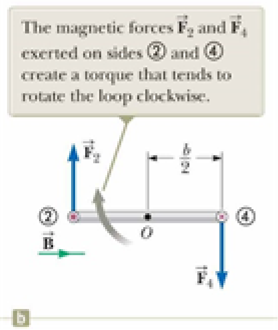
$$\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$$

$$R_H = \frac{1}{nq} \quad \text{(Hall coefficient)}$$



Torque on a current loop





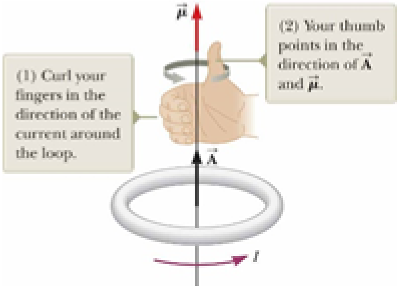
$$\tau_{max} = F_2\frac{b}{2} + F_4\frac{b}{2} = (IaB)\frac{b}{2} + (IaB)\frac{b}{2} = IabB$$

$$\vec{\tau} = I\vec{A}\times\vec{B}$$

$$\vec{\mu} \equiv I\vec{A}$$

$$\vec{\mu}_{coil} = NI\vec{A}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$



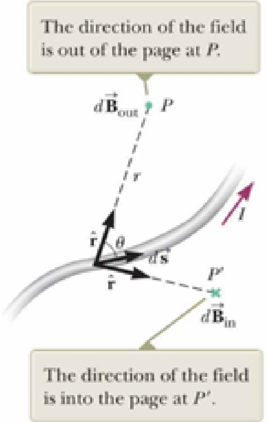
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Reminder of the previous week – Sources of Magnetic Fields

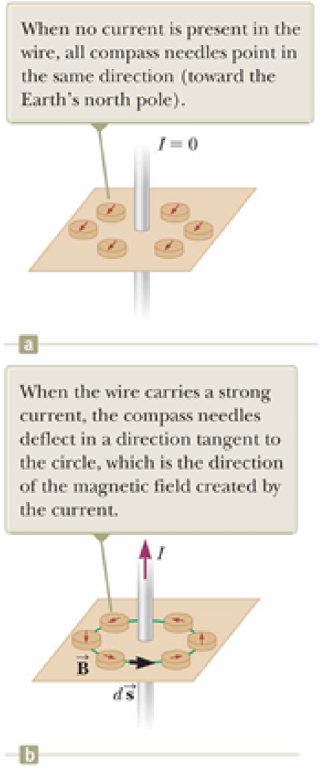
The Biot – Savart Law

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

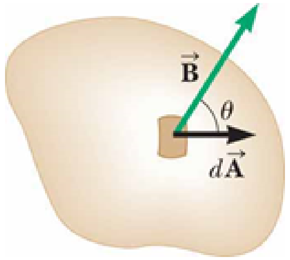


Ampère's Law

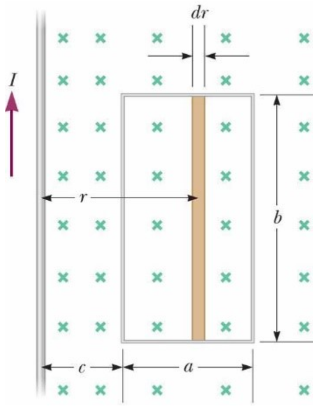


$$\oint\vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

Gauss's Law in Magnetism



$$\Phi_B \equiv \int\vec{\mathbf{B}}\cdot d\vec{\mathbf{A}}$$



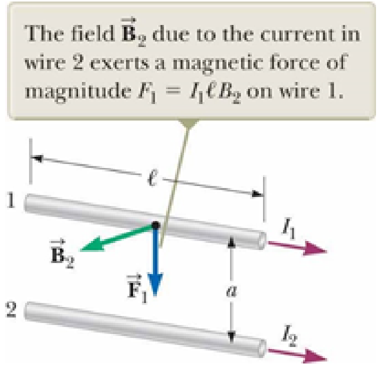
$$\Phi_B = \int\vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = \int B\,dA = \int\frac{\mu_0 I}{2\pi r}\,dA$$

$$\Phi_B = \int\frac{\mu_0 I}{2\pi r}\,b\,dr = \frac{\mu_0 I b}{2\pi}\int\frac{dr}{r}$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi}\int_c^{a+c}\frac{dr}{r} = \frac{\mu_0 I b}{2\pi}\ln r\Big|_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi}\ln\left(\frac{a+c}{c}\right) = \frac{\mu_0 I b}{2\pi}\ln\left(1+\frac{a}{c}\right)$$

The Magnetic Force Between Two Parallel Conductors

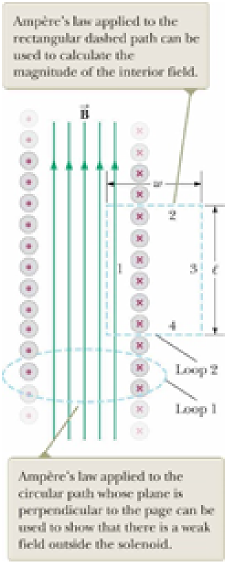


$$F_1 = I_1\ell B_2 = I_1\ell\left(\frac{\mu_0 I_2}{2\pi a}\right) = \frac{\mu_0 I_1 I_2}{2\pi a}\ell$$

The Magnetic Field of a Solenoid

$$\oint\vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = B\ell = \mu_0 NI$$

$$B = \mu_0\frac{N}{\ell}I = \mu_0 nI$$



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Faraday's Law (Ch. 30)
→ **Faraday's Law of Induction**
Motional EMF
Lenz's Law
The General Form of Faraday's Law
Generators and Motors
Eddy Currents

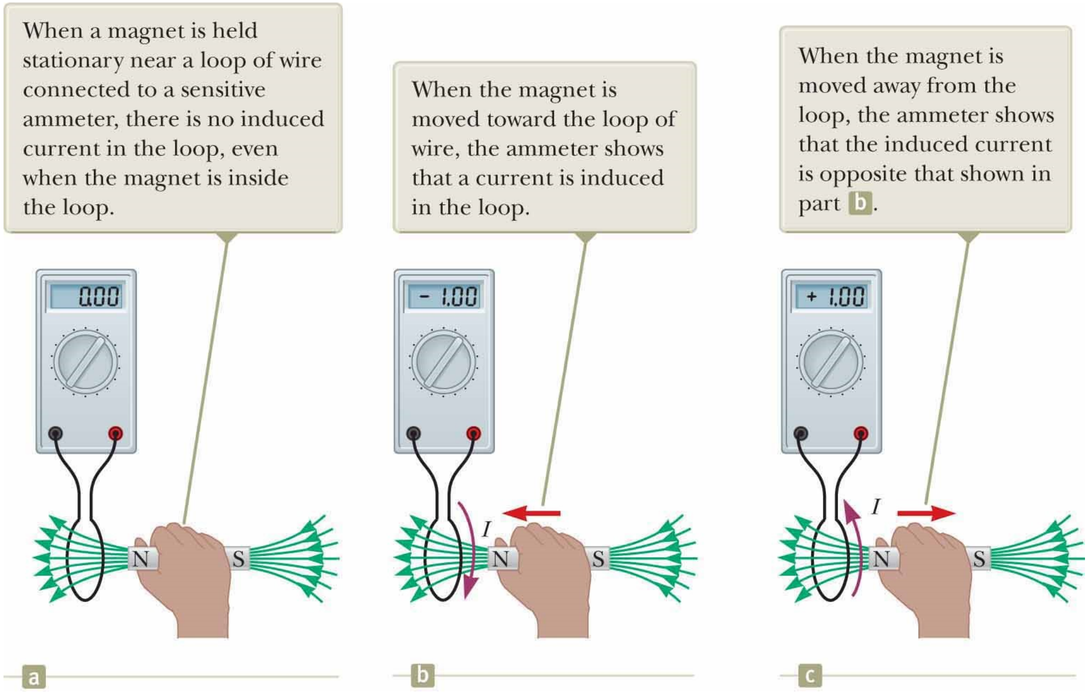
Faraday's Law (Ch. 30)

Faraday's Law of Induction

Faraday’s Law of Induction

- Consider the experiment illustrated in the figures:
- When the magnet is brought to rest and held ***stationary*** relative to the loop, a reading of ***zero*** is observed
- When a magnet is ***moved toward*** the loop, the reading on the ammeter changes from zero to a nonzero value, arbitrarily shown as ***negative***
- When the magnet is ***moved away*** from the loop, the reading on the ammeter changes to a ***positive*** value
- These results are quite remarkable because a current is set up even though no batteries are present in the circuit. We will call such a current an ***induced current*** and say that it is produced by an ***induced emf***.

Video:
https://www.youtube.com/watch?v=KUihEkvabpo [2:10]



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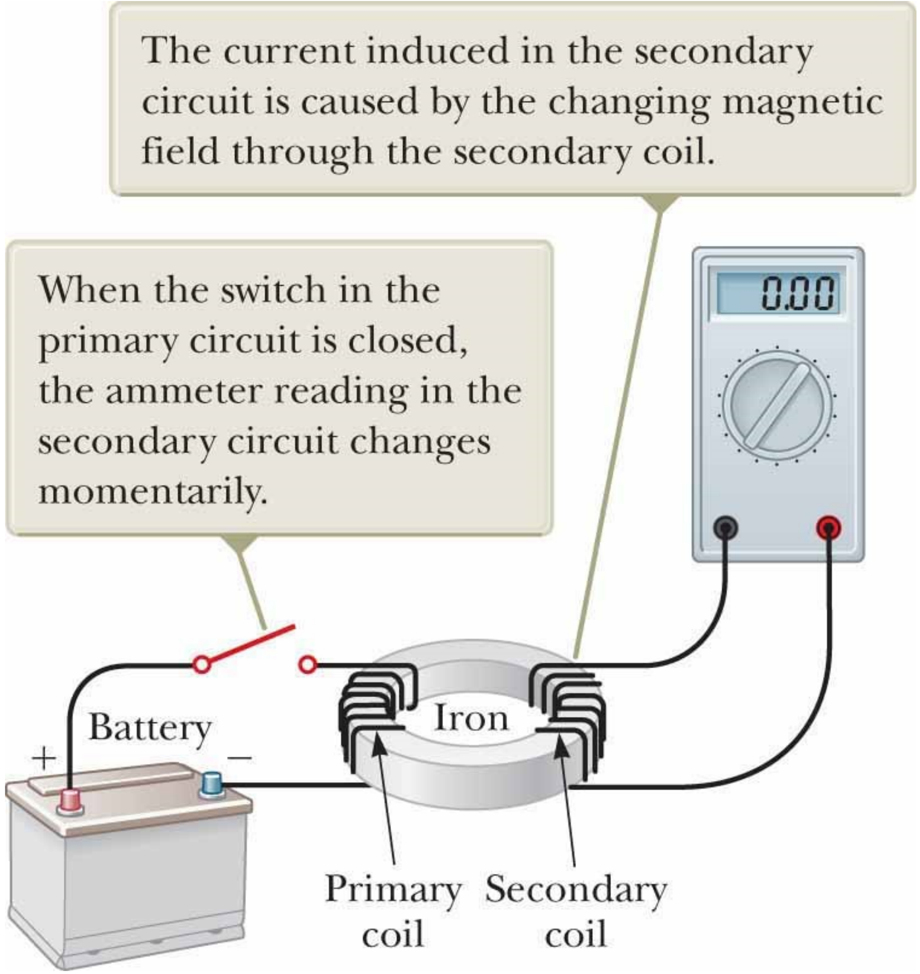
Faraday’s Law of Induction

Faraday’s Experiment

- A primary coil is wrapped around an iron ring and connected to a switch and a battery.
- A current in the coil produces a magnetic field when the switch is closed.
- A secondary coil also is wrapped around the ring and is connected to a sensitive ammeter.
- No battery is present in the secondary circuit, and the secondary coil is not electrically connected to the primary coil.
- Any current detected in the secondary circuit must be induced by some external agent.



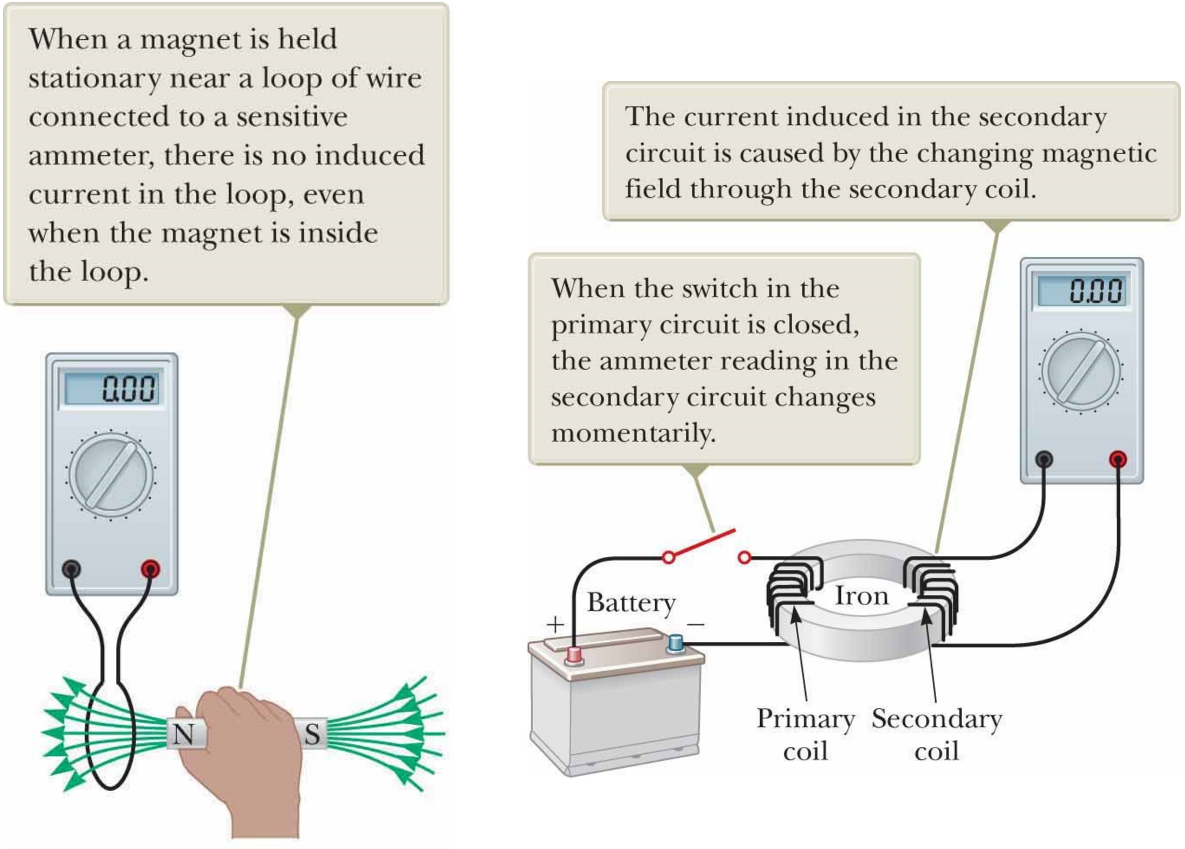
<u>Source:</u> Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Faraday's Law of Induction

Faraday's Experiment

- The experiments have one thing in common: in each case, an ***emf is induced*** in a loop when the magnetic flux through the loop ***changes with time***.
- In general, this emf is directly proportional to the ***time rate of change of the magnetic flux*** through the loop.
- This statement can be written mathematically as ***Faraday's law of induction***:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$



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Faraday’s Law of Induction

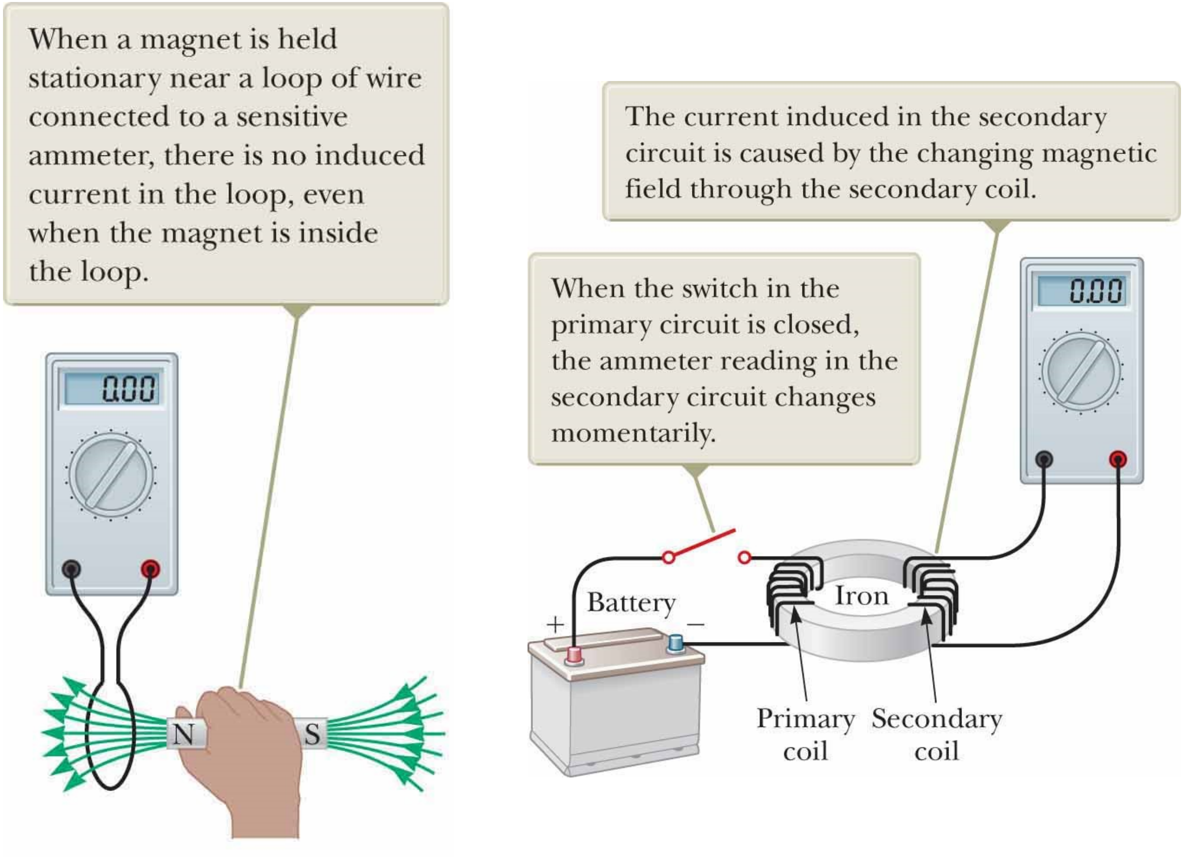
Faraday’s Experiment

- This statement can be written mathematically as ***Faraday’s law of induction***:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

- If a coil consists of N loops with the same area and $\boldsymbol{\Phi_B}$ is the magnetic flux through one loop, an ***emf*** is induced in every loop. The loops are in series, so their emfs add; therefore, the ***total induced emf*** in the coil is given by:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$



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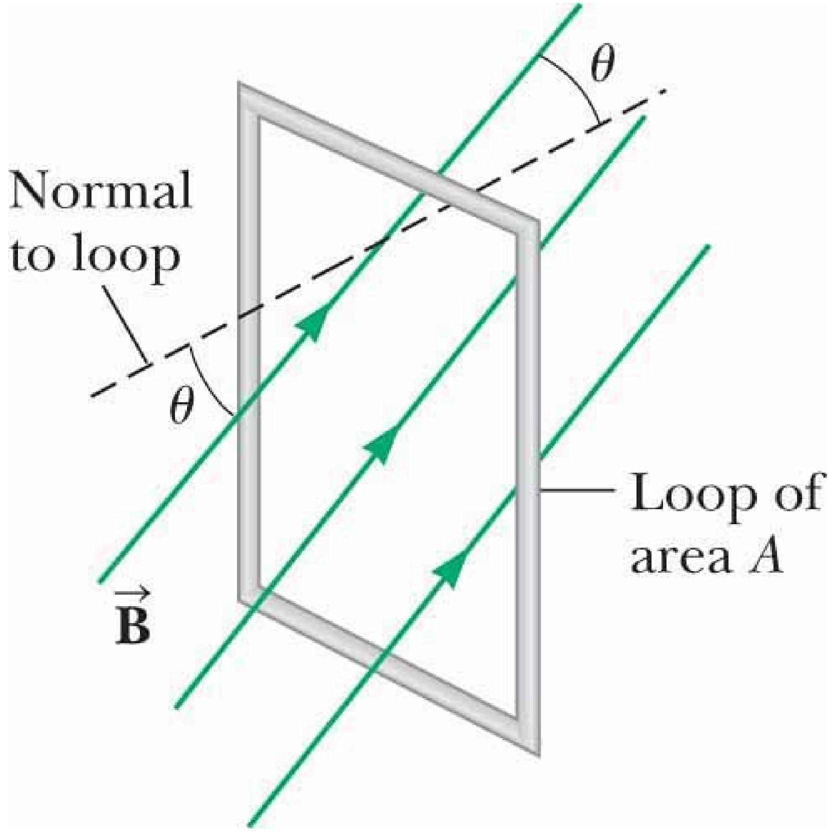
Faraday’s Law of Induction

Faraday’s Experiment

- If the magnetic flux through the loop is equal to ***BAcosΘ***, where ***Θ*** is the angle between the magnetic field and the normal to the loop; the induced emf can be expressed as:

$$\varepsilon = -\frac{d}{dt}\left(BA\cos\theta\right)$$



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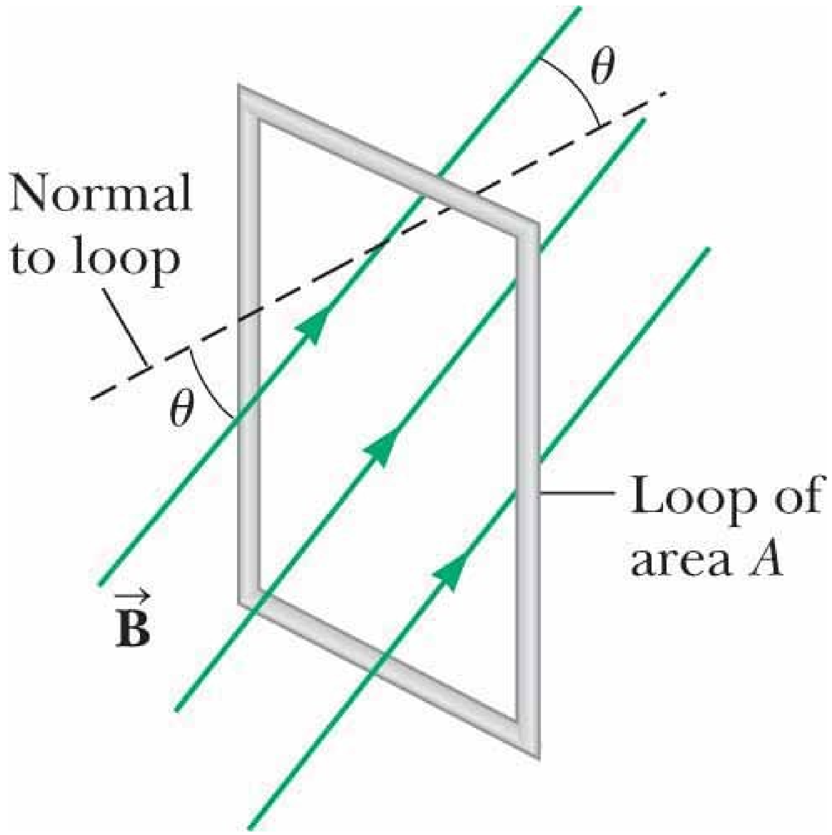
Faraday's Law of Induction

Faraday's Experiment

- If the magnetic flux through the loop is equal to ***BAcosϴ***, where ***ϴ*** is the angle between the magnetic field and the normal to the loop; the induced emf can be expressed as:

$$\varepsilon = -\frac{d}{dt}\left(BA\cos\theta\right)$$

- The ***emf*** can be induced in several ways:
  - The magnitude of $\vec{\mathbf{B}}$ can change with time.
  - The area enclosed by the loop (**A**) can change with time.
  - The angle ***ϴ*** between $\vec{\mathbf{B}}$ and the normal to the loop can change with time.
  - Any combination of the above.



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Faraday’s Law of Induction

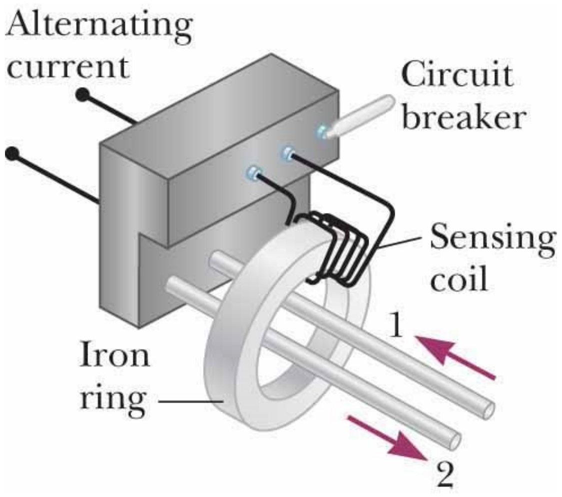
Ground Fault Circuit Interrupter (GFCI)

- In ***GFCI*** outlets, wire 1 leads from the wall outlet to the appliance to be protected and wire 2 leads from the appliance back to the wall outlet. An iron ring surrounds the two wires, and a sensing coil is wrapped around part of the ring.

- Under normal circumstances, because the currents in the wires are in opposite directions and of equal magnitude, there is ***zero net current*** flowing through the ring and the net magnetic flux through the sensing coil is zero.

- Now suppose the return current in wire 2 changes so that the two currents are not equal in magnitude. (That can happen if, for example, the appliance becomes wet, enabling ***current to leak*** to ground.) Then the net current through the ring is not zero and the magnetic flux through the sensing coil is no longer zero.

- With alternating household current, the magnetic flux through the sensing coil changes with time, ***inducing an emf*** in the coil. This induced emf is used to trigger a ***circuit breaker***, which stops the current before it is able to reach a harmful level.



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Magnetic Flux Through a Rectangular Loop

Example 30.1

A coil consists of ***N=200*** turns of wire. Each turn is a square of side ***d=18 cm,*** and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from ***B = 0 to 0.50 T*** in ***t=0.80 s***, what is the ***magnitude of the induced emf*** in the coil while the field is changing?

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Solution

Conceptualize
From the description in the problem, imagine magnetic field lines passing through the coil. Because the magnetic field is changing in magnitude, an emf is induced in the coil.

Categorize
We will evaluate the emf using ***Faraday's law*** from this section, so we categorize this example as a substitution problem.

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Solution

Let's start by writing Faraday's law of induction for a coil with N turns:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

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Solution

Let's start by writing Faraday's law of induction for a coil with N turns:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

$$|\varepsilon| = N\frac{\Delta\Phi_B}{\Delta t} = N\frac{\Delta(BA)}{\Delta t} = NA\frac{\Delta B}{\Delta t} = Nd^2\,\frac{B_f - B_i}{\Delta t}$$

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$$|\varepsilon| = N\frac{\Delta\Phi_B}{\Delta t} = N\frac{\Delta(BA)}{\Delta t} = NA\frac{\Delta B}{\Delta t} = Nd^2\frac{B_f - B_i}{\Delta t}$$

Substitute numerical values

$$|\boldsymbol{\varepsilon}| = (200)(0.18\ \mathrm{m})^2\frac{(0.50\ \mathrm{T} - 0)}{0.80\ \mathrm{s}} = \boxed{4.0\ \mathrm{V}}$$

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Faraday’s Law (Ch. 30)
- Faraday’s Law of Induction
- **Motional EMF**
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Faraday’s Law (Ch. 30)

Motional EMF

Motional EMF

- Earlier we analyzed a situation in which a coil of wire was stationary and the magnetic field changed in time – resulting in an induced current.

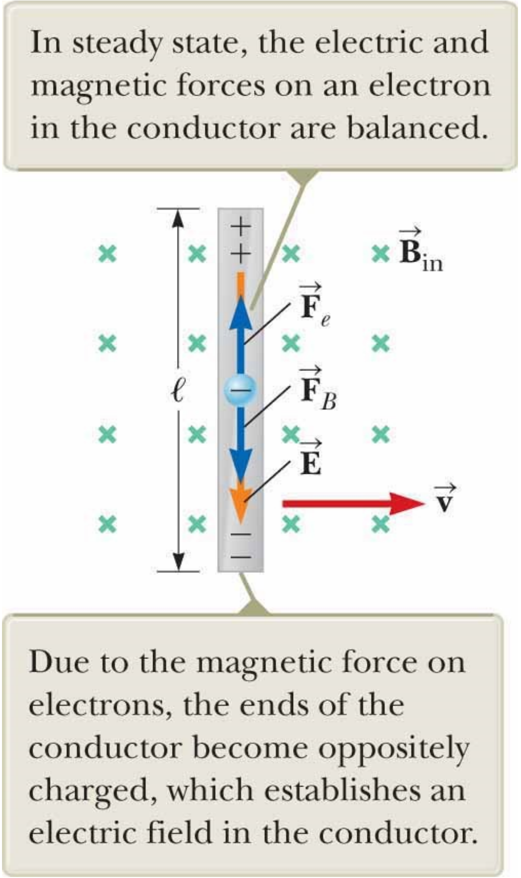
- Let's look at another case where the magnetic field is constant and uniform, and we move a conductor in the field. We find that there is an emf induced in the conductor. We will call this a ***motional emf***.

- The straight conductor of length ***l*** shown in the figure is moving through a uniform magnetic field directed into the page. For simplicity, let's assume the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent.

- The electrons in the conductor experience a force: $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

- Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end. As a result of this charge separation, an electric field $\vec{E}$ is produced inside the conductor.

- The charges accumulate at both ends until the downward magnetic force ***qvB*** on charges remaining in the conductor is balanced by the upward electric force ***qE***.



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Motional EMF

- Electrons in equilibrium state:

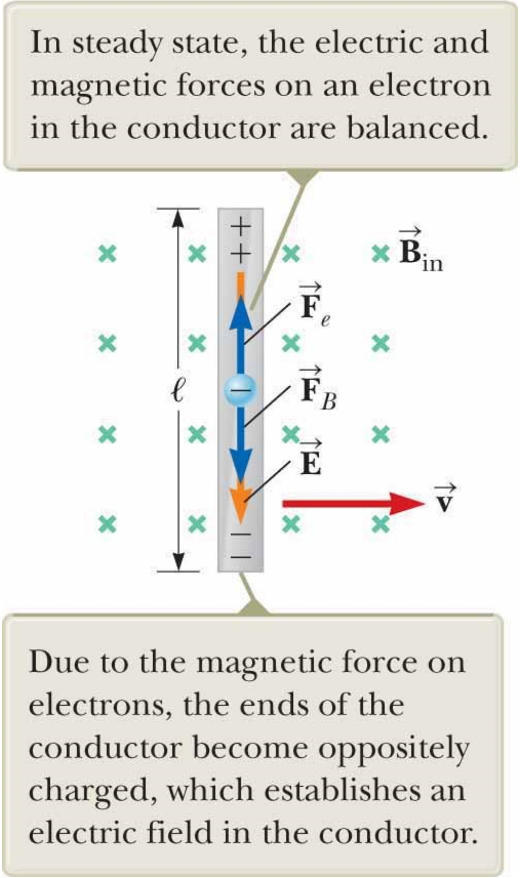
$$\sum F = 0 \Rightarrow qE - qvB = 0$$

$$\Rightarrow E = vB$$

- Potential difference across ends of conductor:

$$\Delta V = E\ell \qquad \ell$$

- Upper end of the conductor is at higher electric potential than the lower end.
- Polarity of the potential difference would be reversed if the direction of motion is reversed.



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Motional EMF

- Consider a circuit consisting of a conducting bar of length ***l*** sliding along two fixed, parallel conducting rails.
- As we move the bar, a potential difference is established between the ends (moving bar acts as a source of EMF).
- Area enclosed by the circuit at any instant = ***lx***
- Magnetic flux through this area:

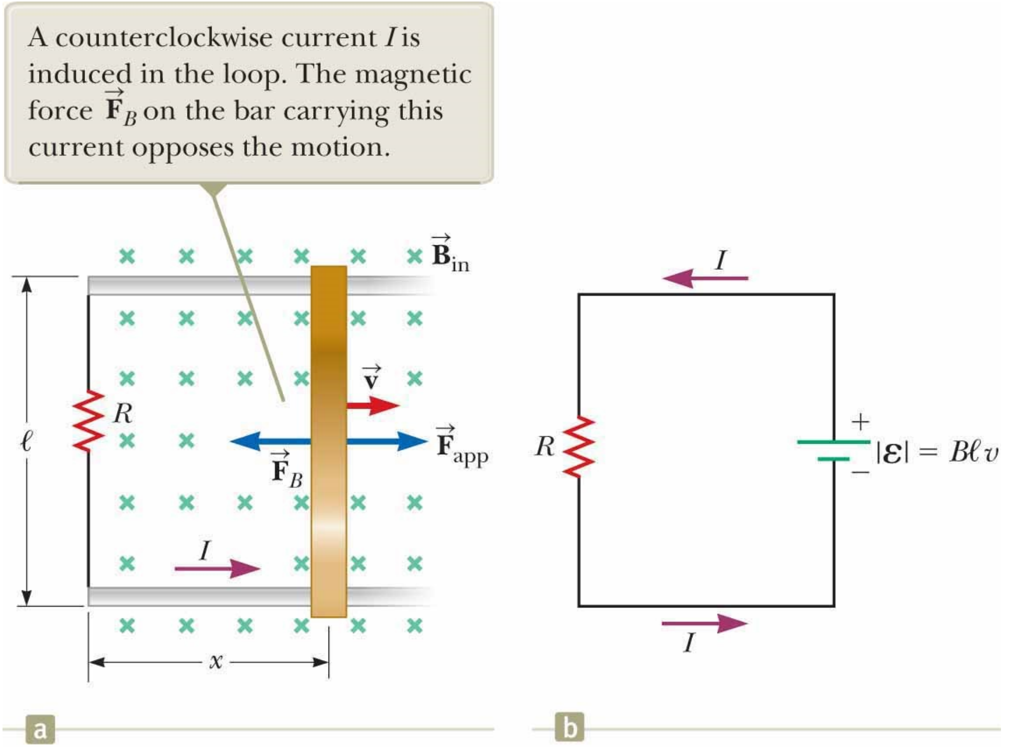
$$\Phi_B = B\ell x$$

- Using Faraday's Law, the induced EMF can be calculated as:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(B\ell x\right) = -B\ell\frac{dx}{dt} \Rightarrow \varepsilon = -B\ell v$$

- The current through the circuit can be calculated as:

$$I = \frac{|\varepsilon|}{R} = \frac{B\ell v}{R}$$



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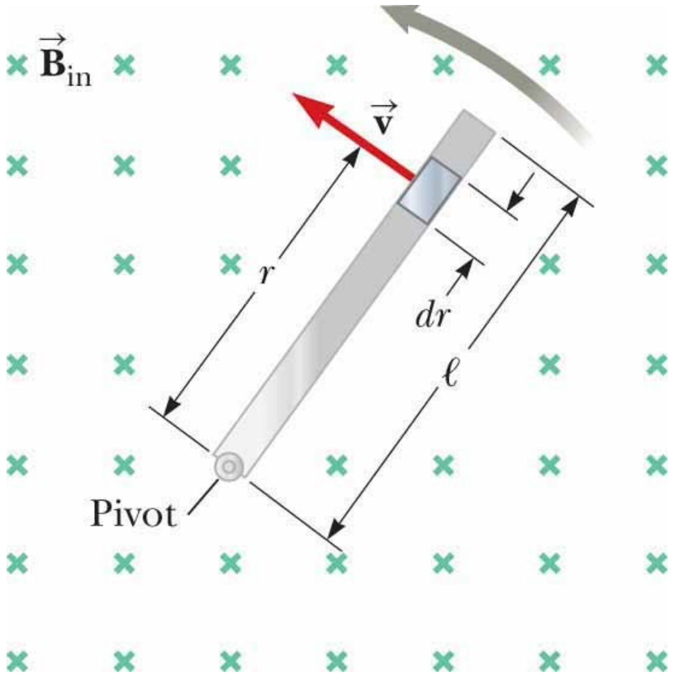
Motional EMF induced in a Rotating Bar

Example 30.3

A conducting bar of length ***l***, rotates with a constant angular speed ***ω*** about a pivot at one end. A uniform magnetic field $\vec{B}$ is directed perpendicular to the plane of rotation as shown in the figure. Find the ***motional emf*** induced between the ends of the bar.



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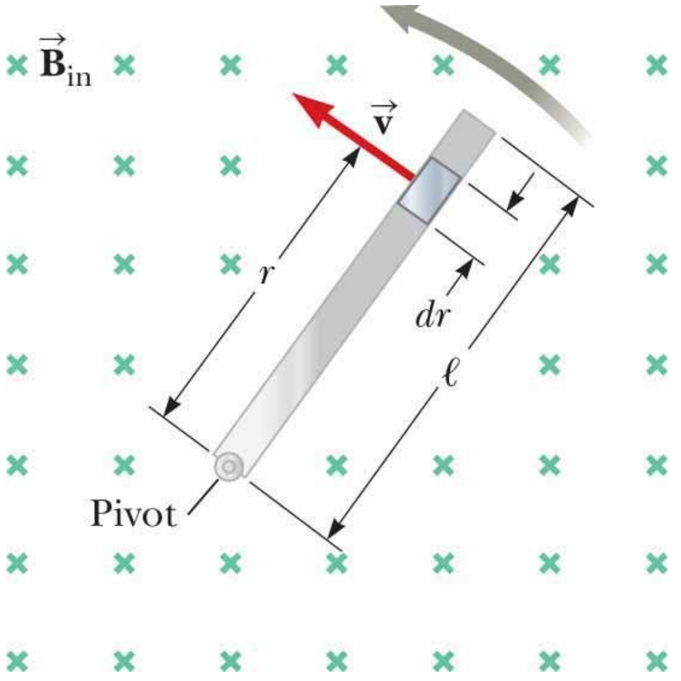
A conducting bar of length ***l***, rotates with a constant angular speed $\boldsymbol{\omega}$ about a pivot at one end. A uniform magnetic field $\vec{B}$ is directed perpendicular to the plane of rotation as shown in the figure. Find the ***motional emf*** induced between the ends of the bar.

Solution

The rotating bar is different in nature from a bar moving translationally. Consider a small segment of the bar, however. It is a short length of conductor moving in a magnetic field and has an emf generated in it like the moving bar. By thinking of each small segment as a source of emf, we see that all segments are in series and the emfs add over the length of the bar.



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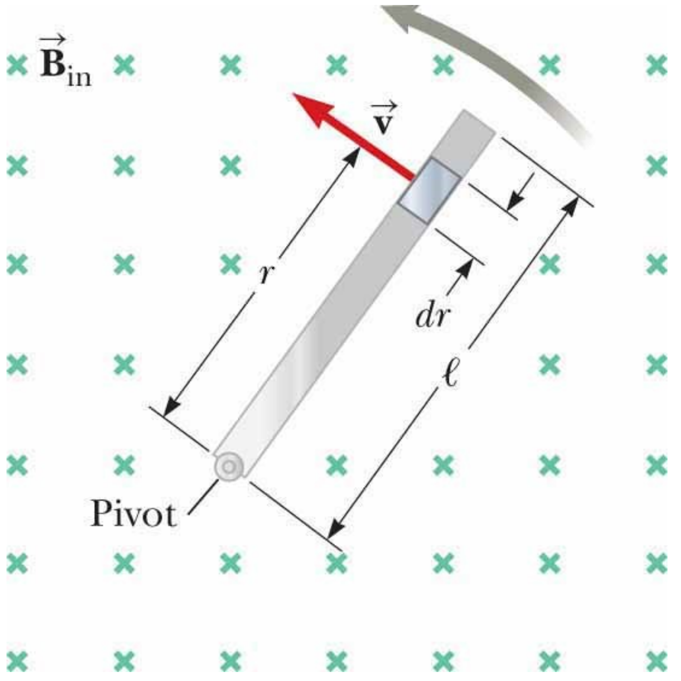
$$d\varepsilon = Bv\,dr \Rightarrow \varepsilon = \int Bv\,dr$$



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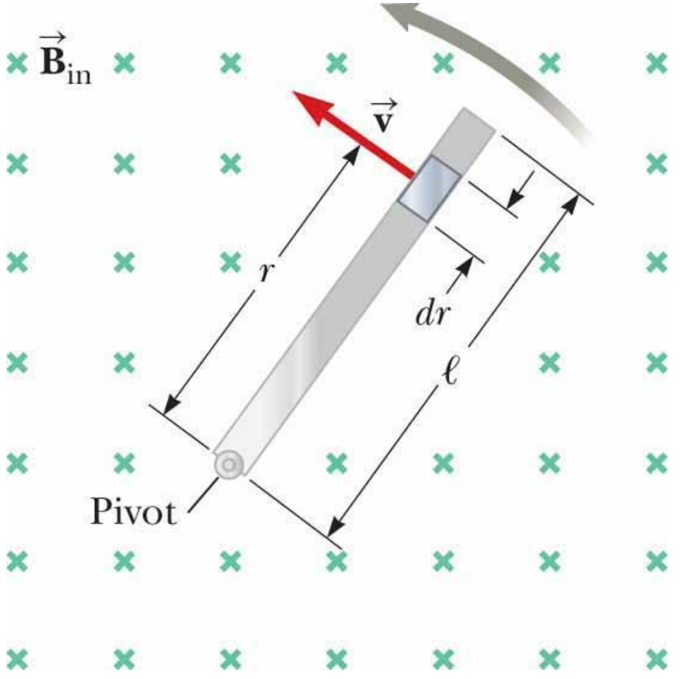
Example 30.3

A conducting bar of length ***l***, rotates with a constant angular speed ***ω*** about a pivot at one end. A uniform magnetic field $\vec{B}$ is directed perpendicular to the plane of rotation as shown in the figure. Find the ***motional emf*** induced between the ends of the bar.



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Solution

The rotating bar is different in nature from a bar moving translationally. Consider a small segment of the bar, however. It is a short length of conductor moving in a magnetic field and has an emf generated in it like the moving bar. By thinking of each small segment as a source of emf, we see that all segments are in series and the emfs add over the length of the bar.

$$d\varepsilon = Bv\,dr \Rightarrow \varepsilon = \int Bv\,dr$$

$$\varepsilon = B\int v\,dr = B\omega\int_0^{\ell} r\,dr$$

$$= \boxed{\frac{1}{2}B\omega\ell^2}$$

The tangential speed ***v*** of an element is related to the angular speed ***ω*** through the relationship ***v = rω***.

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Faraday’s Law (Ch. 30)

Lenz’s Law

Lenz's Law

- Faraday's law: indicates that induced emf and change in flux have opposite algebraic signs:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

- Physical interpretation of the negative sign → ***Lenz's law***:
  - Induced current tends to keep the original magnetic flux through the loop from changing
  - Consequence of law of conservation of energy

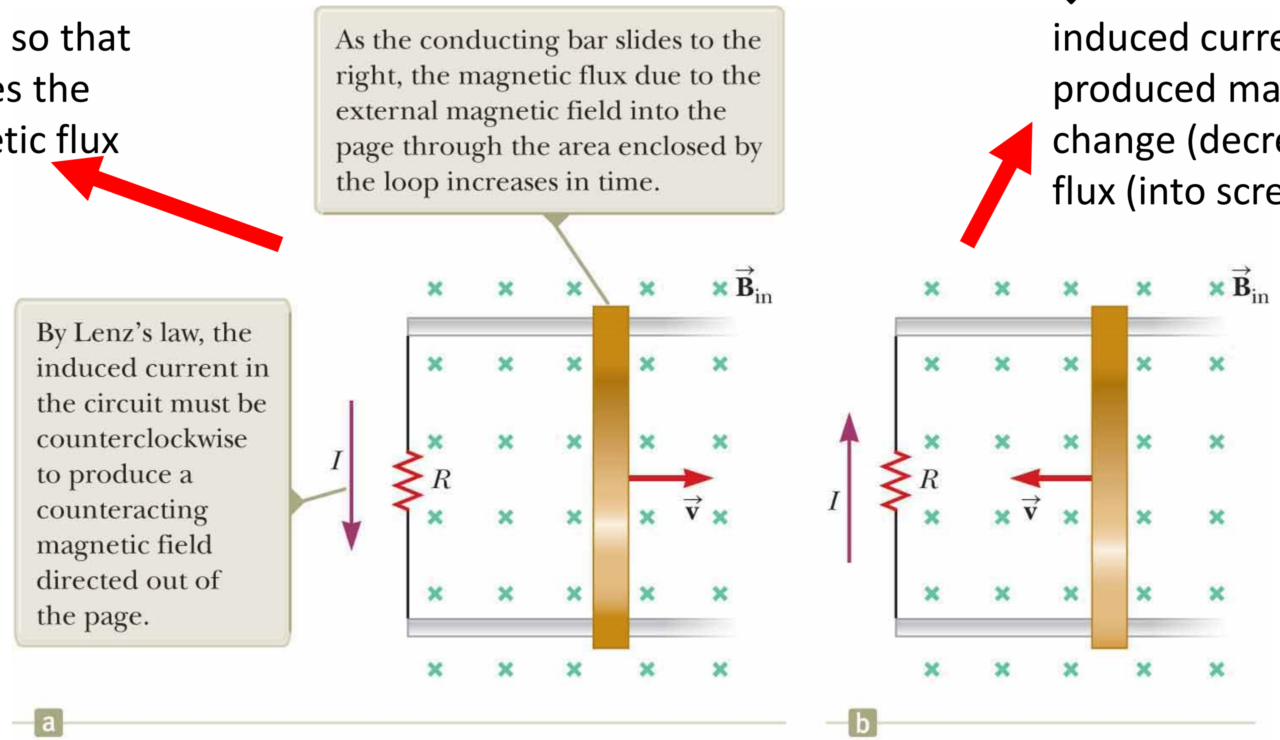
The induced current in a loop is in the direction that creates a magnetic field that **opposes the change** in magnetic flux through the area enclosed by the loop.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Lenz's Law

Magnetic flux through the area enclosed by the circuit increases with time
↓
counterclockwise current is induced so that the produced magnetic field opposes the change (increase) in external magnetic flux (into screen)

Magnetic flux through the area enclosed by the circuit decreases with time
↓
induced current is clockwise so that the produced magnetic field opposes the change (decrease) in external magnetic flux (into screen)



- **Otherwise the current directions will incur a system that acquires energy with no input of energy.**
  → Violates law of conservation of energy

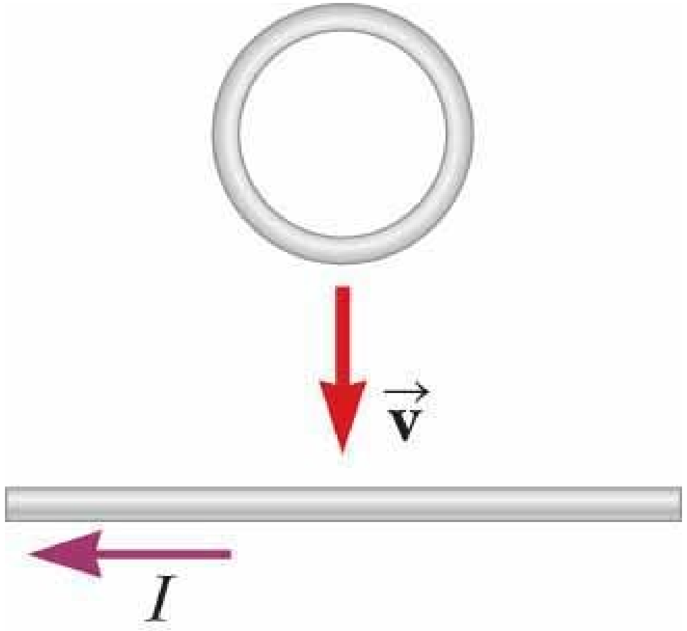
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Lenz’s Law

Quick Quiz

The figure shows a circular loop of wire falling toward a wire carrying a current to the left. What is the direction of the induced current in the loop of wire?

(a) clockwise
(b) counter-clockwise
(c) zero
(d) impossible to determine



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Lenz's Law

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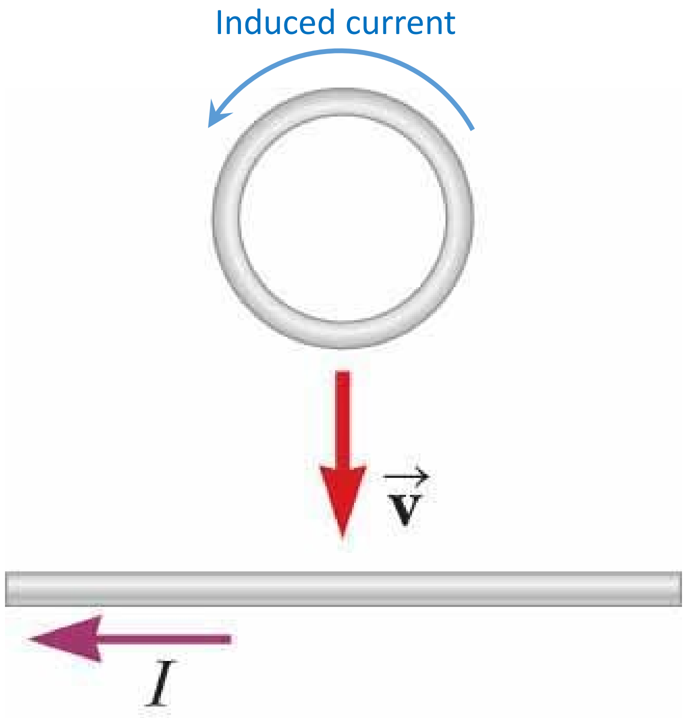


(a) clockwise

True → **(b) counter-clockwise**

(c) zero

(d) impossible to determine

- The magnetic field due to current carrying wire is into the page through the ring.
- As the ring gets closer to the wire, the magnetic field intensity increases, hence the flux through the ring increases.
- The induced current on the ring will try to oppose this change: hence, the induced current will be counter-clock wise direction (producing out of the page flux).

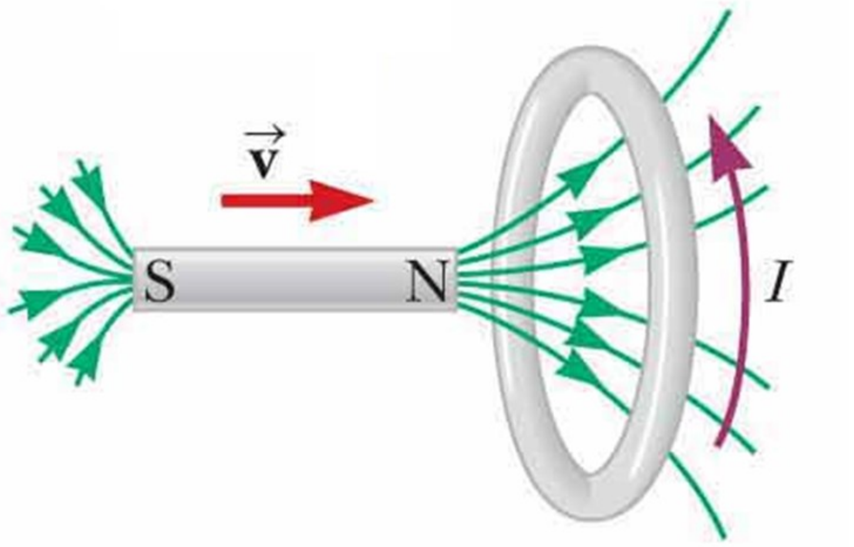
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Lenz’s Law

Conceptual Example 30.4

A magnet is placed near a metal loop as shown in the figure.

(A) Find the direction of the induced current in the loop when the magnet is pushed toward the loop.



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

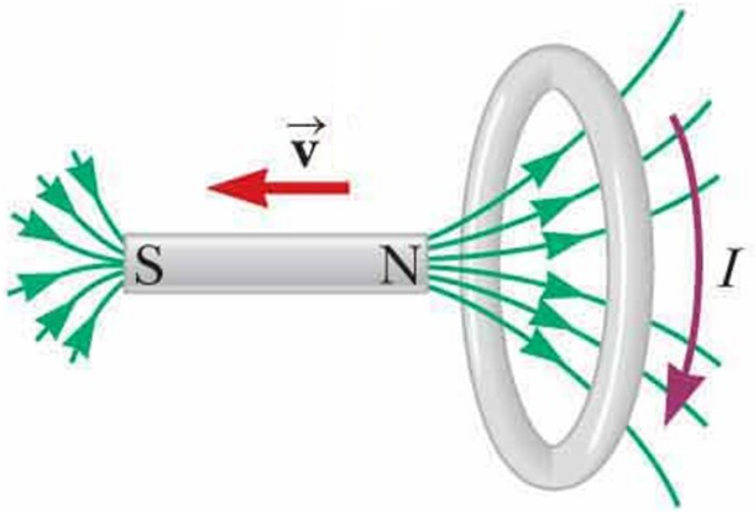
Lenz’s Law

Conceptual Example 30.4

A magnet is placed near a metal loop as shown in the figure.

(A) Find the direction of the induced current in the loop when the magnet is pushed toward the loop.

(B) Find the direction of the induced current in the loop when the magnet is pulled away from the loop.



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

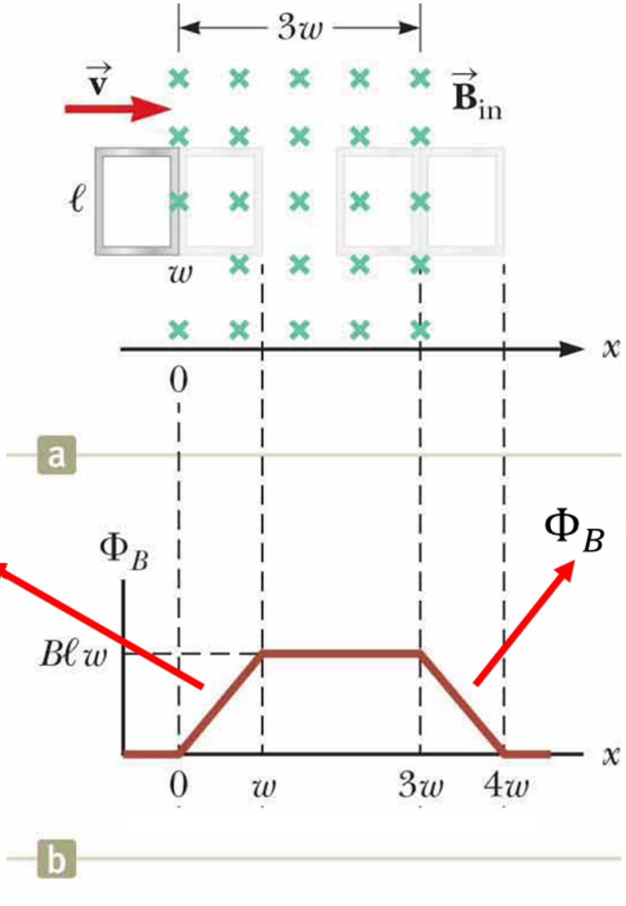
Lenz's Law

Conceptual Example 30.5

A rectangular metallic loop of dimensions ***l*** and ***w*** and resistance ***R*** moves with constant speed ***v*** to the right as shown in the figure. The loop passes through a uniform magnetic field ***B*** directed into the page and extending a distance ***3w*** along the x axis. Define x as the position of the right side of the loop along the x axis.

(a) Plot the ***magnetic flux*** through the area enclosed by the loop as a function of x.

$\Phi_B = B\ell x$



$\Phi_B = B\ell(w\text{-}(x\text{-}3w))$

$= B\ell(4w\text{-}x)$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Lenz’s Law

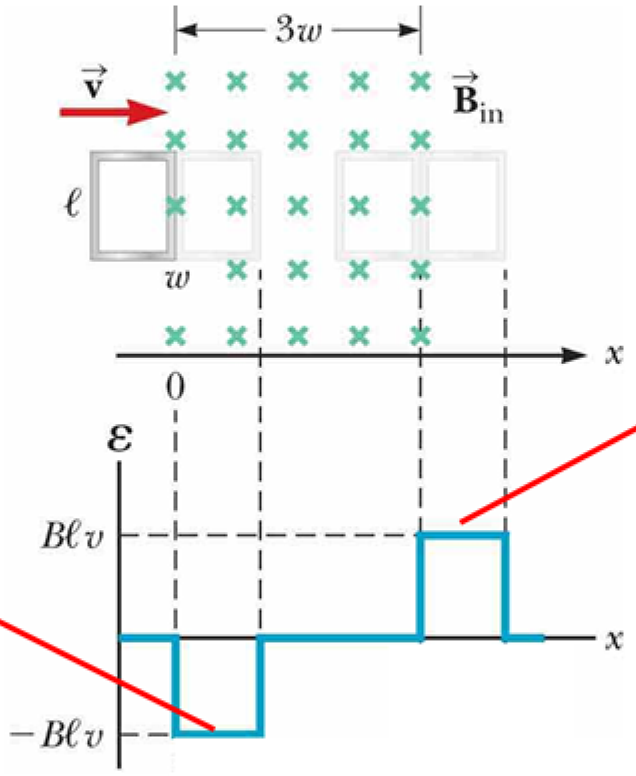
Conceptual Example 30.5

A rectangular metallic loop of dimensions ***l*** and ***w*** and resistance ***R*** moves with constant speed ***v*** to the right as shown in the figure. The loop passes through a uniform magnetic field ***B*** directed into the page and extending a distance ***3w*** along the x axis. Define x as the position of the right side of the loop along the x axis.

(a) Plot the ***magnetic flux*** through the area enclosed by the loop as a function of x.

(b) Plot the ***induced motional emf*** in the loop as a function of *x*.

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(Blx)}{dt} = -B\ell v$$



$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(Bl(4w-x))}{dt} = B\ell v$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

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Eddy Currents

Faraday’s Law (Ch. 30)

The General Form of Faraday’s Law

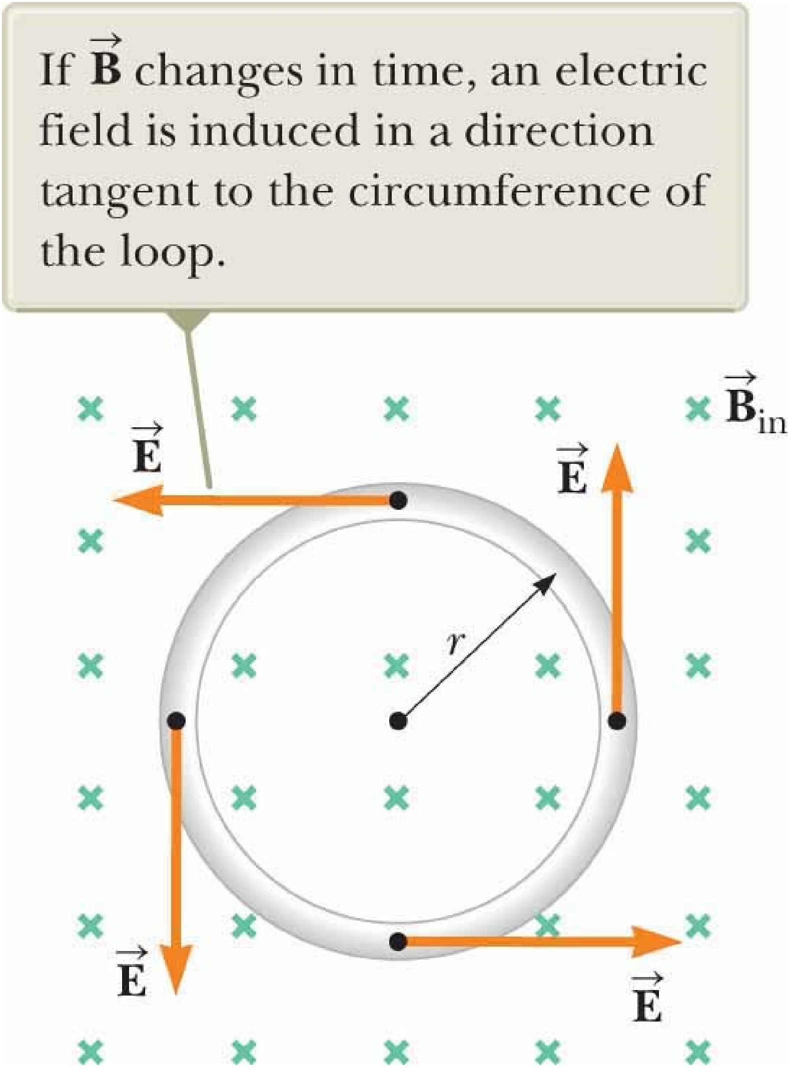
Optional

The General Form of Faraday's Law

Optional

Conducting loop in a magnetic field:

What if the magnetic field changes in time?



The General Form of Faraday's Law

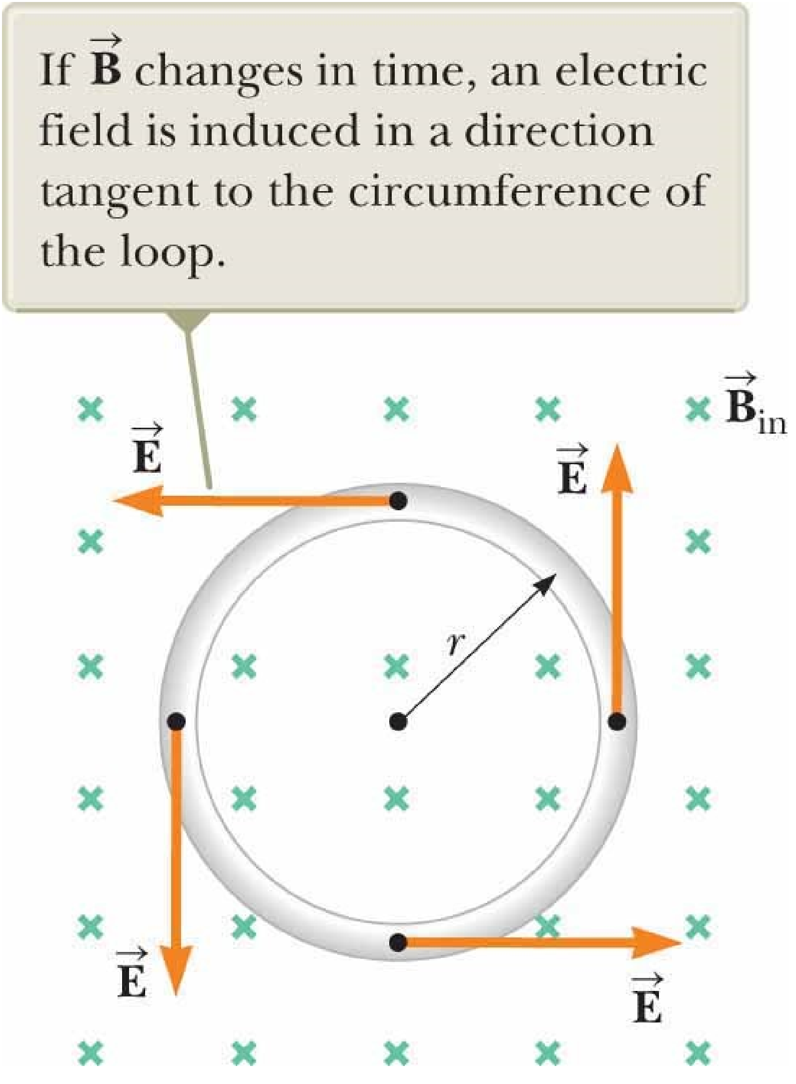
Optional

Conducting loop in a magnetic field:

What if the magnetic field changes in time?

$$\Delta V = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \quad \Rightarrow \quad \boxed{\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}}$$

- Represents all situations in which changing magnetic field generates electric field



The General Form of Faraday's Law

Optional

Conducting loop in a magnetic field:

What if the magnetic field changes in time?

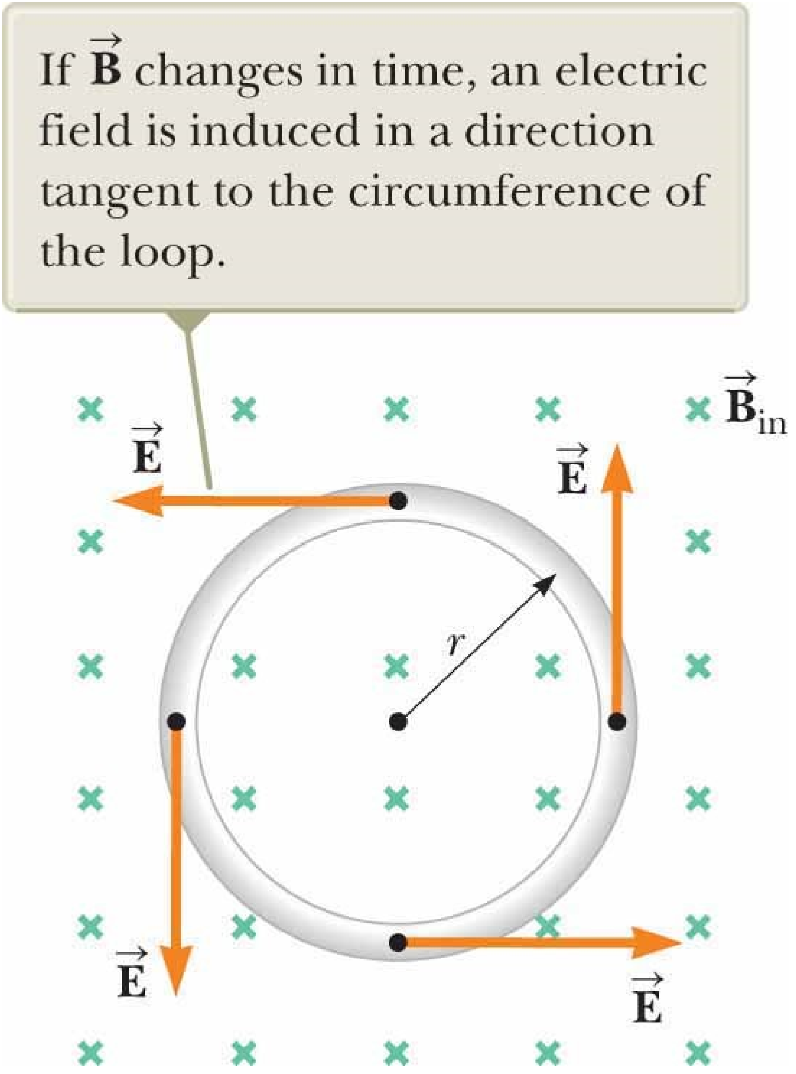
$$\Delta V = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \Rightarrow \boxed{\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}}$$

- Represents all situations in which changing magnetic field generates electric field
- When electric field everywhere is parallel to displacement vectors on loop: $\rightarrow$

$$E \cdot \oint ds = -\frac{d}{dt}(BA)$$

$$\Rightarrow E(2\pi r) = -\frac{dB}{dt}(\pi r^2)$$

$$\Rightarrow E = -\frac{r}{2}\frac{dB}{dt}$$



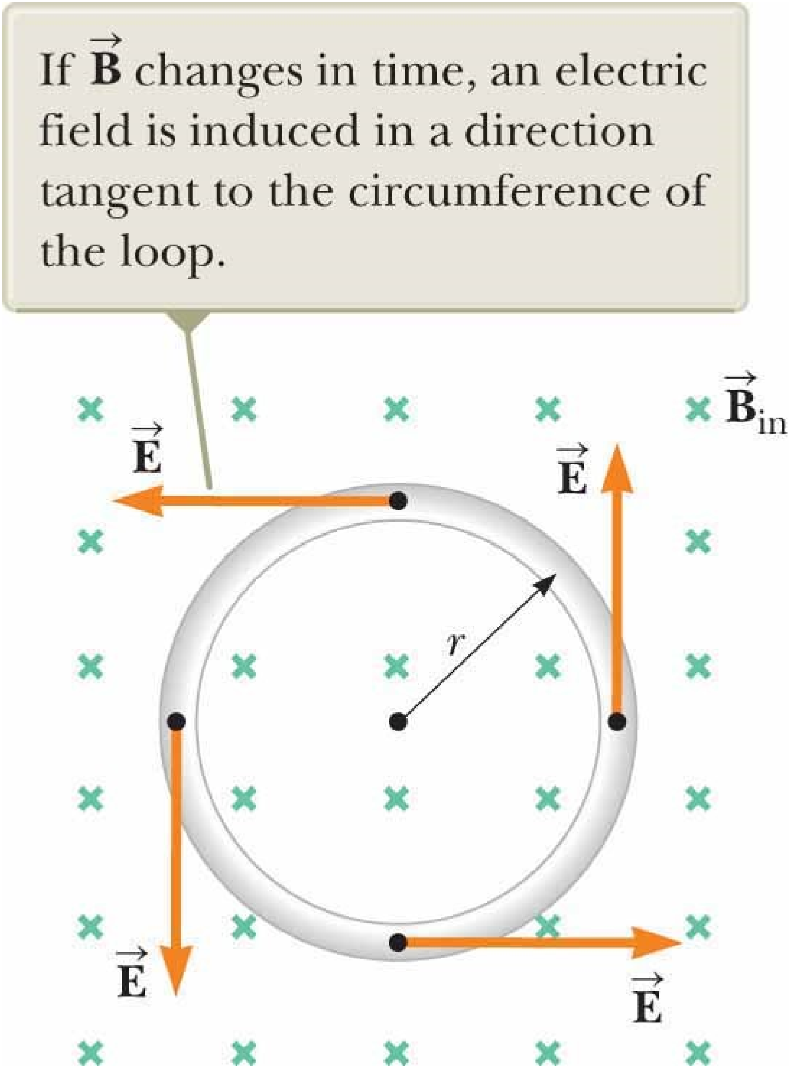
The General Form of Faraday's Law

Optional

- Potential difference between two points in space as an integral between those two points of electric field created by some source charges:

$$\Delta V = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- If two points are the same as in the circular loop: $\rightarrow$ integral reduces to zero
    - Integral for this situation ***not zero***!
    - Induced electric field: ***nonconservative***
- Induced electric field: many of the same properties as electric fields due to source charges
    - Example: induced electric field can apply forces on charged particles



Electric Field Induced in a Solenoid

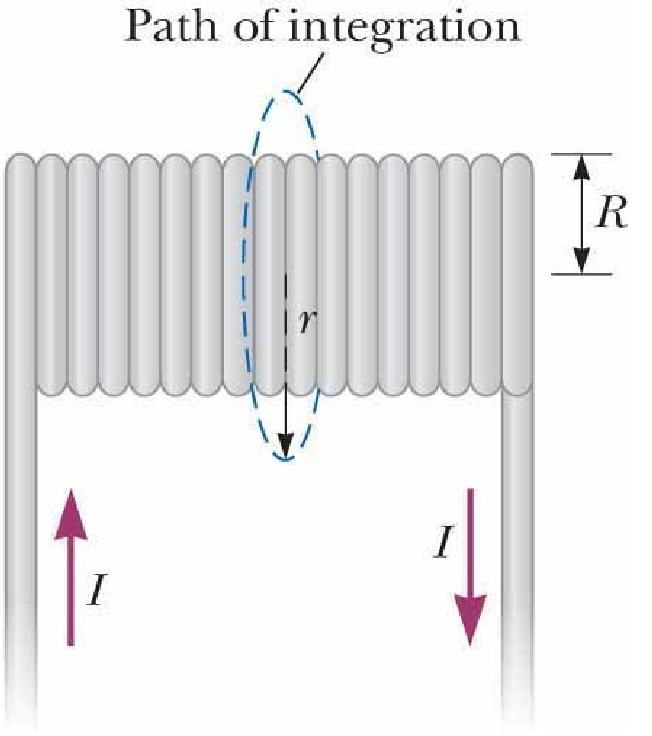
Optional

Example 30.6

A long solenoid of radius ***R*** has ***n*** turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I = I_{max} \cos \omega t$, where I_{max} is the maximum current and ω is the angular frequency of the alternating current source.

(a) Determine the magnitude of the induced electric field outside the solenoid at a distance $r > R$ from its long central axis.

(b) What is the magnitude of the induced electric field inside the solenoid, a distance ***r*** from its axis?



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Electric Field Induced in a Solenoid

Optional

Example 30.6

(a) induced electric field outside the solenoid at a distance ***r > R***

Induced emf: $-\dfrac{d\Phi_B}{dt} = -\dfrac{d}{dt}(B\pi R^2) = -\pi R^2 \dfrac{dB}{dt}$

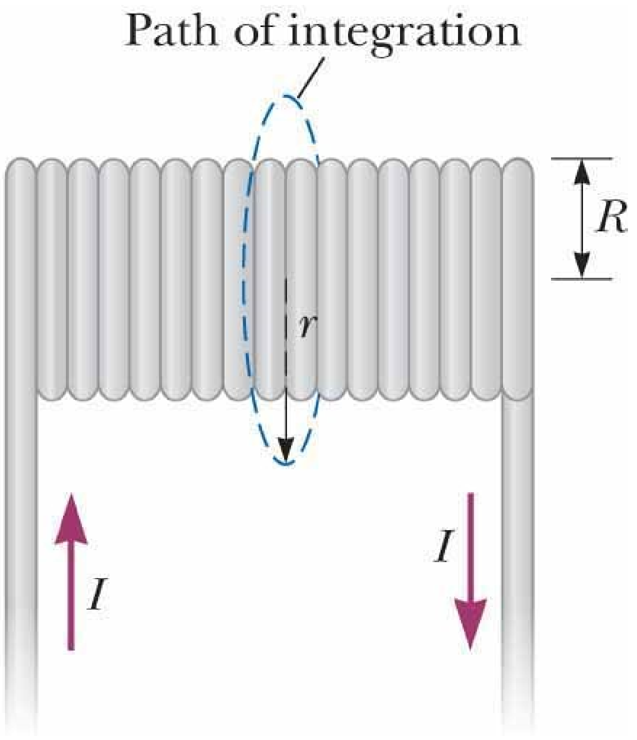
$B = \mu_0 n I = \mu_0 n I_{\text{max}} \cos\omega t$ **(*Ch. 29, Eq. 29.17)**

$$-\frac{d\Phi_B}{dt} = -\pi R^2 \mu_0 n I_{\text{max}} \frac{d}{dt}(\cos\omega t) = \pi R^2 \mu_0 n I_{\text{max}} \omega \sin\omega t$$

General Form of Faraday's Law : $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E(2\pi r)$

$$E(2\pi r) = \pi R^2 \mu_0 n I_{max} \sin\omega_{\text{max}}$$

$$E = \boxed{\frac{\mu_0 n I_{\max} \omega R^2}{2r} \sin\omega t} \quad (\text{for } r > R)$$



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Electric Field Induced in a Solenoid

Optional

Example 30.6

(b) induced electric field inside the solenoid

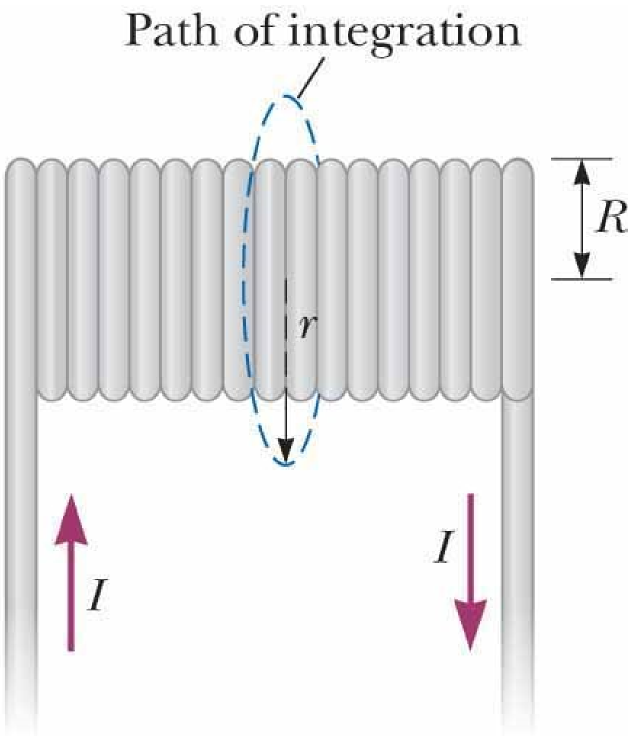
$$-\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi r^2) = -\pi r^2 \frac{dB}{dt}$$ **(r instead of R!)**

$$B = \mu_0 n I = \mu_0 n I_{\text{max}} \cos \omega t$$ (*Ch. 29)

$$-\frac{d\Phi_B}{dt} = -\pi r^2 \mu_0 n I_{\max} \frac{d}{dt}(\cos \omega t)$$
$$= \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E(2\pi r) = \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E = \boxed{\frac{\mu_0 n I_{\max} \omega}{2} r \sin \omega t} \quad (\text{for } r < R)$$



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Faraday’s Law (Ch. 30)

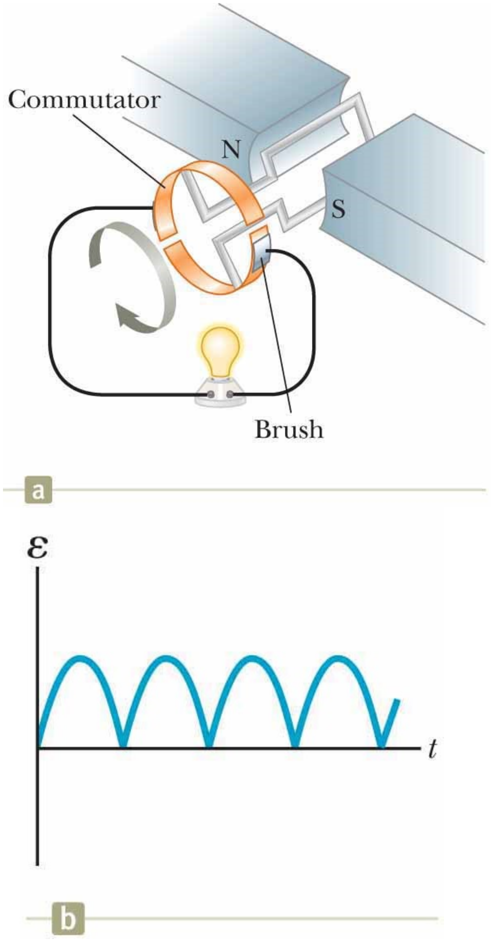
Generators and Motors

Direct Current (DC) Generator

- **Direct-current (DC) generator (**Figure (a))
- Simplest form: consists of loop of wire rotated by some external means in magnetic field
    - As loop rotates $\rightarrow$ magnetic flux through area enclosed by loop changes with time
        - Change induces emf and current in loop according to Faraday's law

- Ends of loop connected to split ring device (*commutator*):
    - Rotates with loop
    - Connections from commutator (acts as output terminals of generator) to external circuit made by stationary metallic brushes in contact with commutator

- Output voltage always has same polarity
    - Pulsates with time (figure (b))
- Note: contacts to split ring reverse roles every half cycle
- At the same time polarity of induced emf reverses $\rightarrow$
    - Polarity of split ring (which is same as polarity of output voltage) remains same

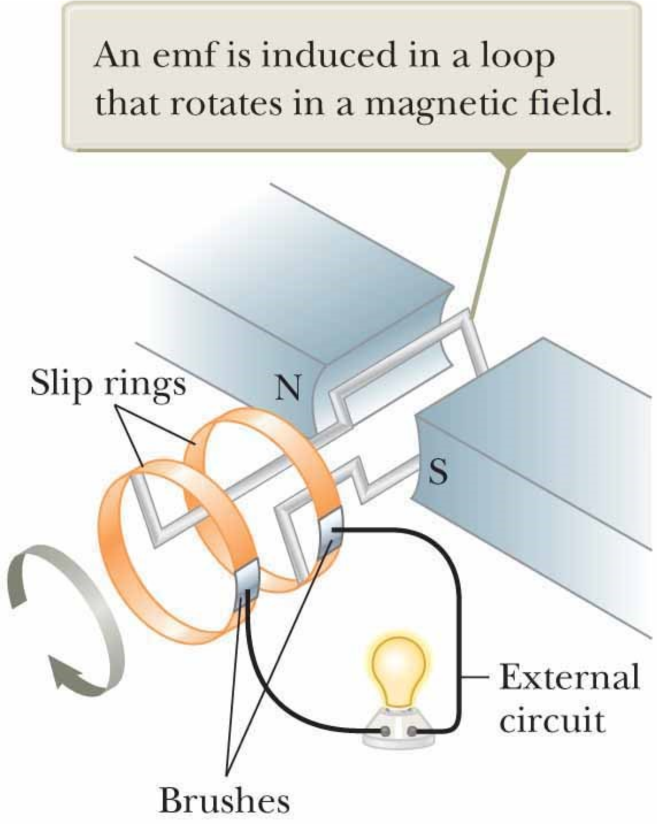


Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Alternating Current (AC) Generator

- **Alternating-current (AC) generator**
  - Consists of loop of wire rotated by some external means in a magnetic field
  - Ends of loop connected to two slip rings that rotate with loop
    - Connections from these slip rings (act as output terminals of generator) to external circuit made by stationary metallic brushes in contact with slip rings

- In commercial power plants:
  - Energy required to rotate loop can be derived from different sources such as:
    - Hydroelectric plant $\rightarrow$ falling water directed against blades of turbine produces rotary motion
    - Energy released by burning natural gas used to convert water to steam $\rightarrow$ steam directed against turbine blades



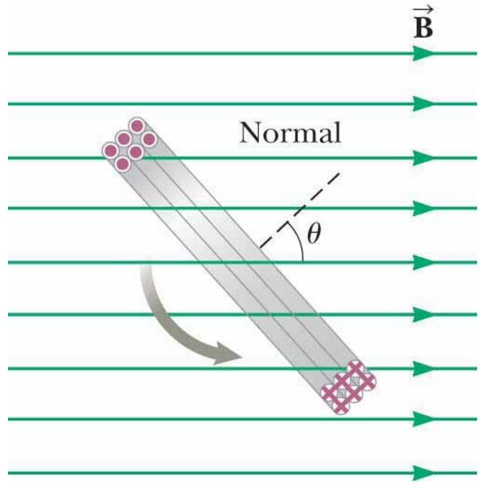
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Sinusoidal Nature of Induced emf

- Magnetic flux through a coil at any time *t:*

$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

θ: Angle between magnetic field and normal to plane of coil
A: Coil area within the magnetic field
ω: Rotating angular frequency of the coil

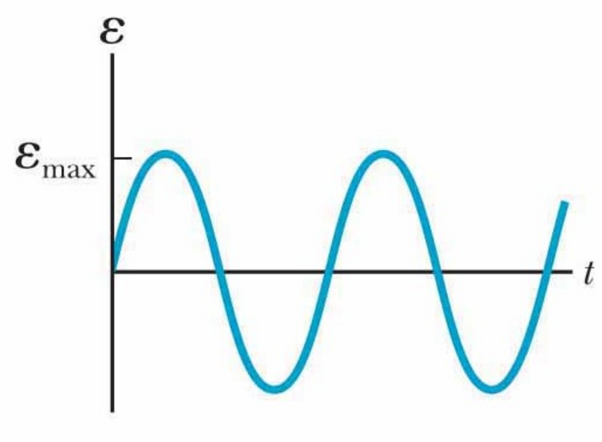


- Generated emf*:*

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NBA\frac{d}{dt}\left(\cos\omega t\right) = NBA\omega\sin\omega t$$

N: Number of coil turns

$$\varepsilon_{\max} = NBA\omega$$

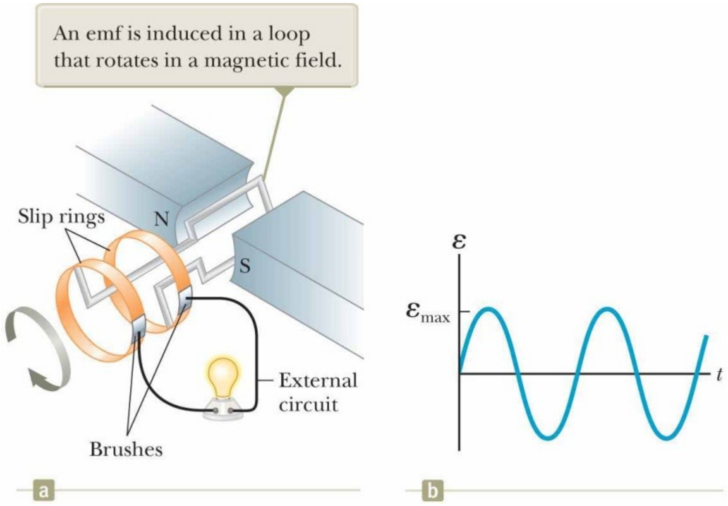


EMF Induced in a Generator

Example 30.7

The coil in an AC generator consists of ***8 turns*** of wire, each of area $\boldsymbol{A = 0.090\ m^2}$, and the total resistance of the wire is $\boldsymbol{R=12.0\Omega}$. The coil rotates in a $\boldsymbol{B=0.500\ T}$ magnetic field at a constant frequency of $\boldsymbol{f=60.0\ Hz}$.

(A) Find the maximum induced emf in the coil.



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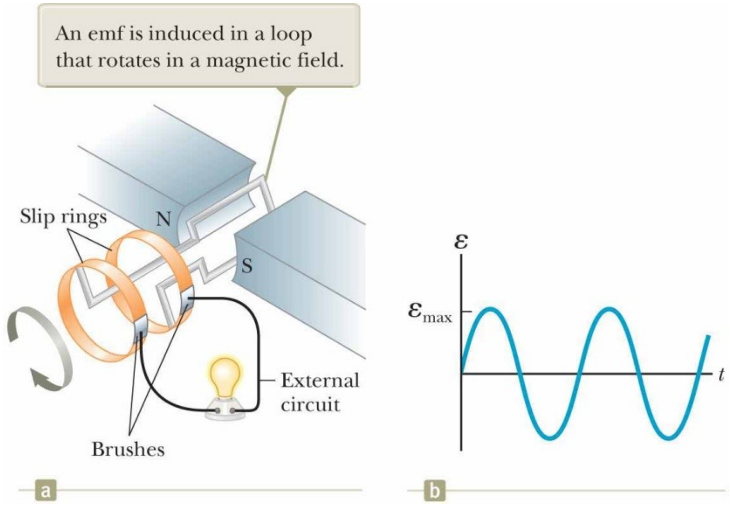
(A) Find the maximum induced emf in the coil.

$$\begin{aligned}\varepsilon_{\max} &= NBA\omega \\ &= NBA(2\pi f)\end{aligned}$$

$$\varepsilon_{\max} = 8(0.500\ \mathrm{T})(0.0900\ \mathrm{m}^2)(2\pi)(60.0\ \mathrm{Hz}) = \boxed{136\ \mathrm{V}}$$



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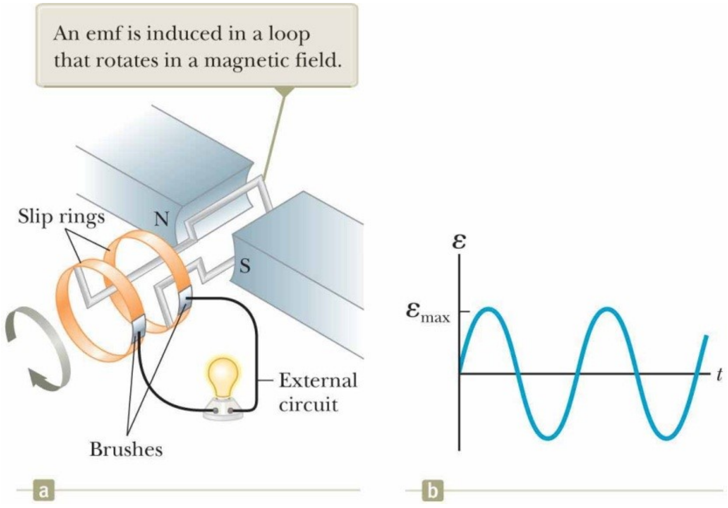
$$\varepsilon_{\max} = 8(0.500\ \mathrm{T})(0.0900\ \mathrm{m}^2)(2\pi)(60.0\ \mathrm{Hz}) = \boxed{136\ \mathrm{V}}$$

(B) What is the maximum induced current in the coil when the output terminals are connected to a low-resistance conductor?



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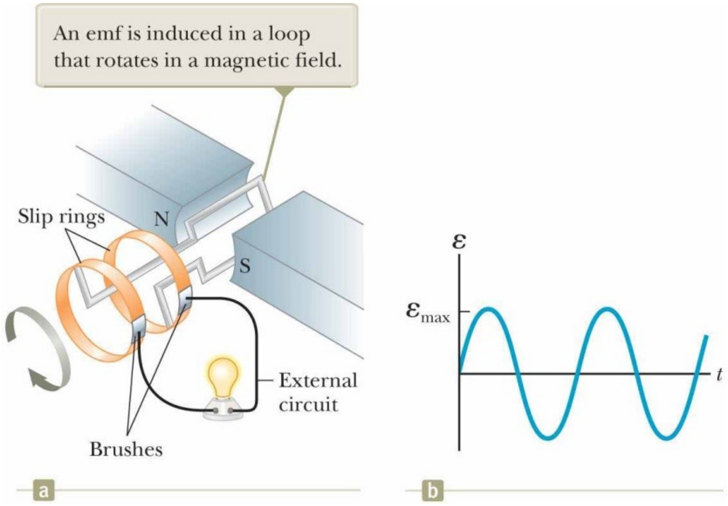
(B) What is the maximum induced current in the coil when the output terminals are connected to a low-resistance conductor?

$$\begin{aligned}I_{\max} &= \frac{\varepsilon_{\max}}{R} \\ &= \frac{136\ \mathrm{V}}{12.0\ \Omega} \\ &= \boxed{11.3\ \mathrm{A}}\end{aligned}$$



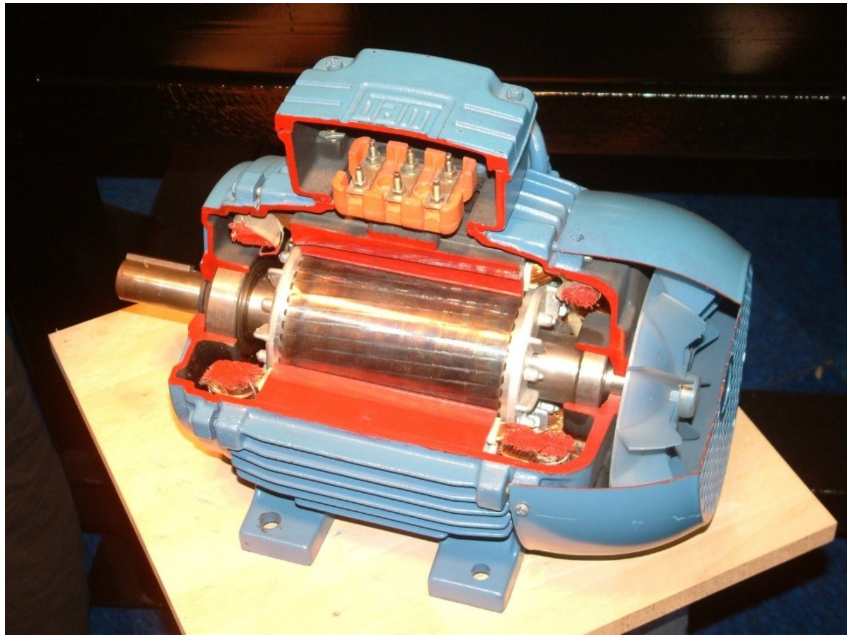
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Motors

- **Motor:** generator operating in reverse
  - Current supplied to coil by battery
  - Torque acting on current-carrying coil causes it to rotate
  - Energy transferred out by work

- As coil rotates in magnetic field $\rightarrow$
  - Changing magnetic flux induces emf in coil (Lenz’s law)
  - Induced emf acts to reduce current in coil
    - *Back emf*: indicates emf that tends to reduce supplied current

- Voltage available to supply current = difference between supply voltage and back emf:
  - Current in rotating coil limited by back emf



Source: https://commons.wikimedia.org/wiki/File%3ACut-away_version_of_an_electric_motor_(1).JPG
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Videos

Hydroelectric Power

https://www.youtube.com/watch?v=OC8Lbyeyh-E [2:10]
https://www.youtube.com/watch?v=Lx6UfiEU3Q0 [4:18]
https://www.youtube.com/watch?v=pa1ryuQV7Mk [2:50]

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Eddy Currents

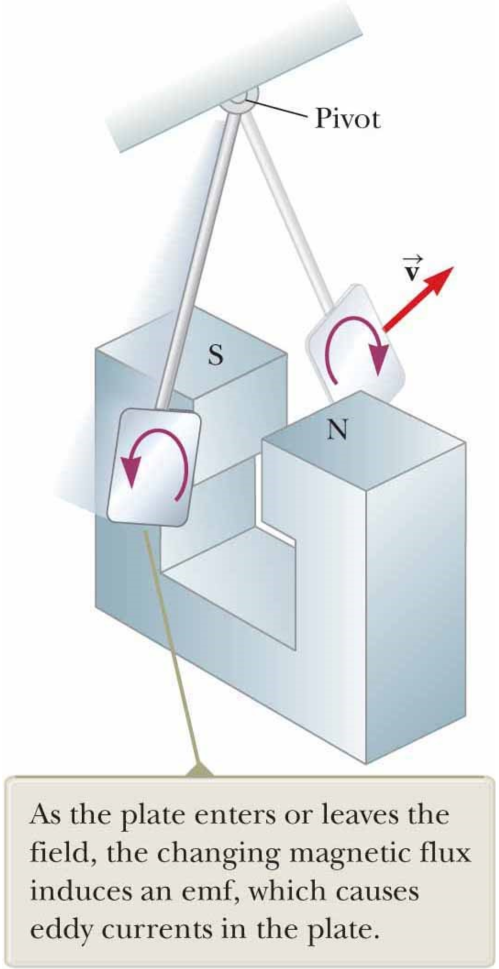
Eddy Currents

- Recall: emf and current induced in loop of wire by changing magnetic flux
- Imagine plate of metal (figure)
  - Consider plate as concentric circular conducting loops of various radii
    - Circulating currents (**eddy currents**) induced in bulk pieces of metal moving through magnetic field
- Imagine plate swings back and forth through magnetic field

- As plate enters field:
  - Changing magnetic flux induces emf in plate
    - Causes free electrons in plate to move
    - Producing swirling eddy currents

- Lenz's law: direction of eddy currents $\rightarrow$ they create magnetic fields that oppose change that causes currents
  - Eddy currents must produce effective magnetic poles on plate
    - Which are repelled by poles of magnet
  - Result: repulsive force that opposes motion of the plate

- If opposite were true: plate would accelerate and its energy would increase after each swing
  - Violation of law of conservation of energy



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Eddy Currents

Position 1: Induced eddy current is counterclockwise as swinging plate enters the field

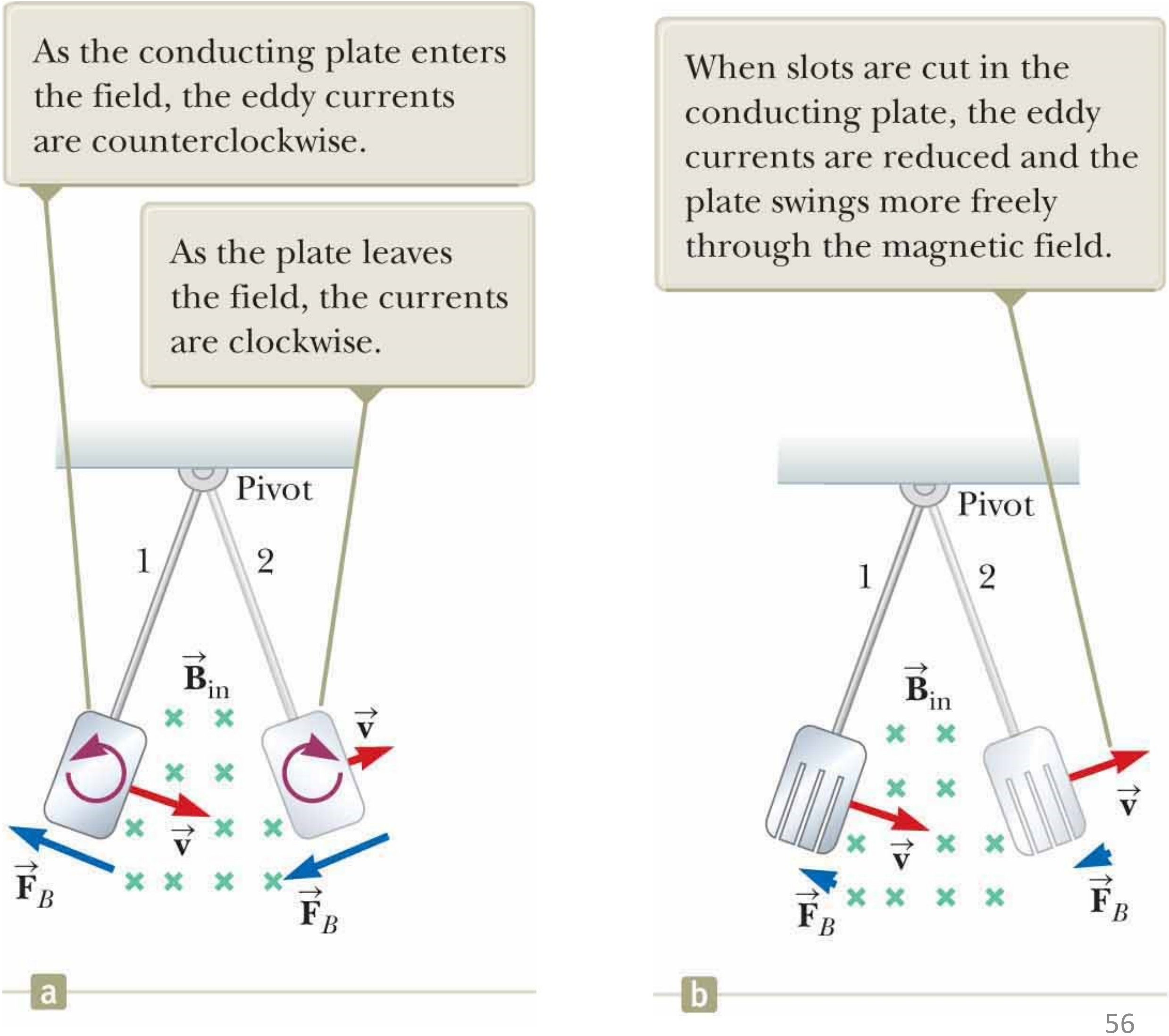
Position 2: Induced eddy current is clockwise as swinging plate leaves the field

Induced eddy current always produces magnetic slowing force $\mathbf{F}_B$

If slots are cut in the plate (figure (b)):

- Conducting loops in the plate are disrupted
- Eddy currents and the resulting slowing force are greatly reduced

Video:
https://www.youtube.com/watch?v=MglUIiBy2lQ [4:43]



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Eddy Currents - Applications

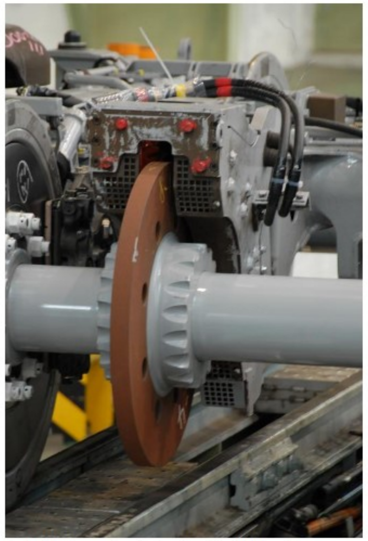
The braking systems on many subway and rapid-transit cars make use of electromagnetic induction and eddy currents.

An electromagnet attached to the train is positioned near the steel rails. (An electromagnet is essentially a solenoid with an iron core.) The braking action occurs when a large current is passed through the electromagnet. The relative motion of the magnet and rails induces eddy currents in the rails, and the direction of these currents produces a drag force on the moving train. Because the eddy currents decrease steadily in magnitude as the train slows down, the braking effect is quite smooth.

As a safety measure, some power tools use eddy currents to stop rapidly spinning blades once the device is turned off.

The Shanghai magnetic levitation train, the fastest commercial high-speed electric train in the world. Image by Andreas Krebs – Own work. Licensed under CC BY-SA 2.0, via Flickr Creative Commons.

Eddy current brakes on a rollercoaster track. Image by Stefan Scheer – Own work. Licensed under CC BY-SA 3.0, via Wikimedia Commons.



Linear eddy current brake in a German high-speed train (left). Image by Sebastian Terfloth – Own work. Licensed under CC BY-SA 3.0, via Wikimedia Commons. Closeup of an eddy current brake used on a high-speed Japanese train (right). Image by Take-y – Own work. Licensed under CC BY-SA 3.0, via Wikimedia Commons.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

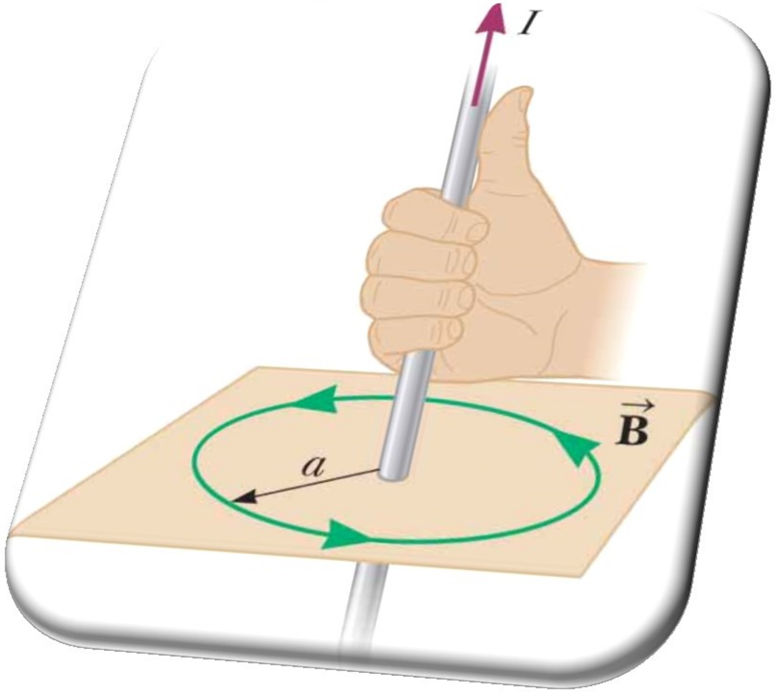
Source: https://www.comsol.com/blogs/how-eddy-current-braking-technology-is-freeing-us-from-friction/

Summary of Week 9, Class 7

- Reminder of the previous week

- Faraday's Law (Ch. 30)
  - Faraday's Law of Induction
  - Motional EMF
  - Lenz's Law
  - The General Form of Faraday's Law
  - Generators and Motors
  - Eddy Currents

- Examples
- Next week's topic



Reading / Preparation for Next Week

Topics for next week:

- Inductance (Ch. 31)
- Alternating Current Circuits (Ch. 32)