MOTION OF A PROJECTILE

Today's Objectives:

Students will be able to:

1. Analyze the free-flight motion of a projectile.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Kinematic Equations for Projectile Motion
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

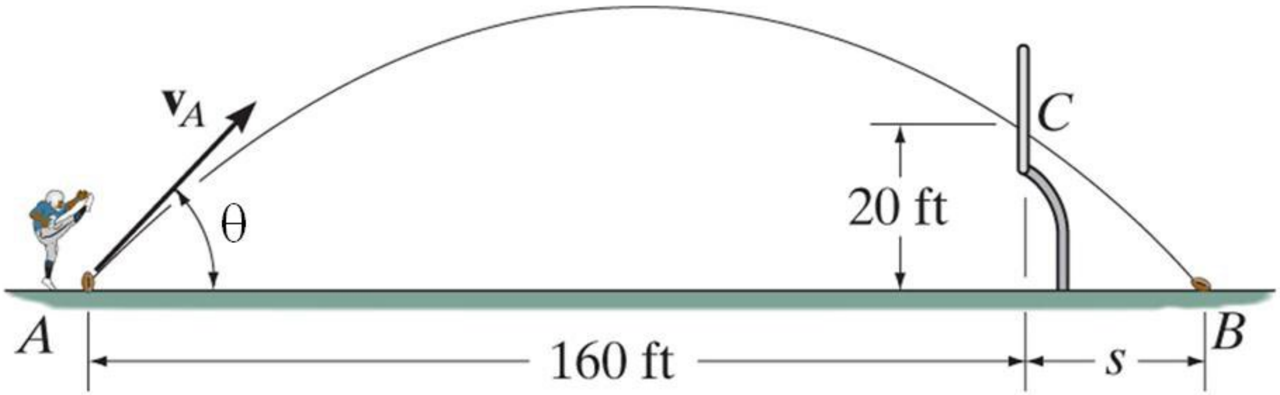
1. The downward acceleration of an object in free-flight motion is

   A) zero.
   B) increasing with time.
   C) 9.81 m/s^2.
   D) 9.81 ft/s^2.

2. The horizontal component of velocity remains __________ during a free-flight motion.

   A) zero
   B) constant
   C) at 9.81 m/s^2
   D) at 32.2 ft/s^2

APPLICATIONS



A good kicker instinctively knows at what angle, θ, and initial velocity, $\mathbf{v}_A$, he must kick the ball to make a field goal.

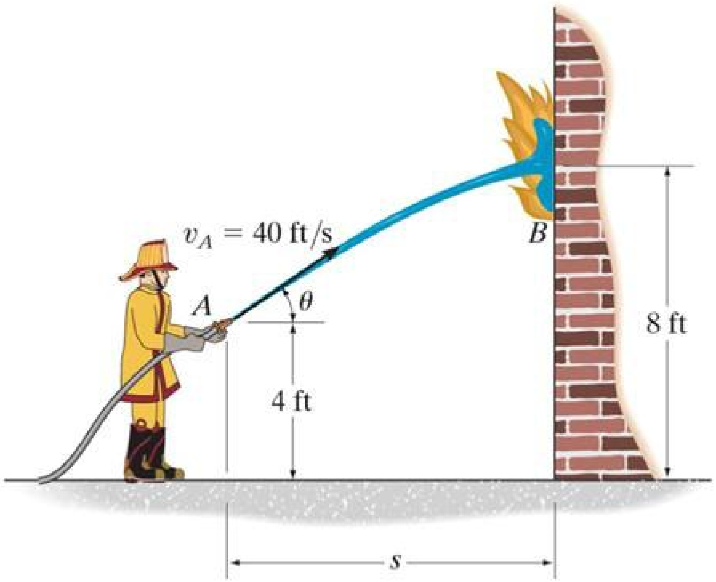
For a given kick "strength", at what angle should the ball be kicked to get the maximum distance?

APPLICATIONS (continued)

A basketball is shot at a certain angle. What parameters should the shooter consider in order for the basketball to pass through the basket?

Distance, speed, the basket location, … anything else?

APPLICATIONS (continued)

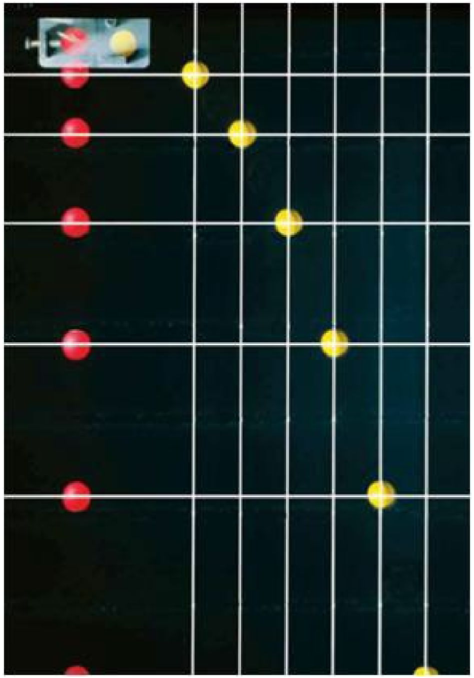


A firefighter needs to know the maximum height on the wall she can project water from the hose. What parameters would you program into a wrist computer to find the angle, θ, that she should use to hold the hose?

MOTION OF A PROJECTILE (Section 12.6)

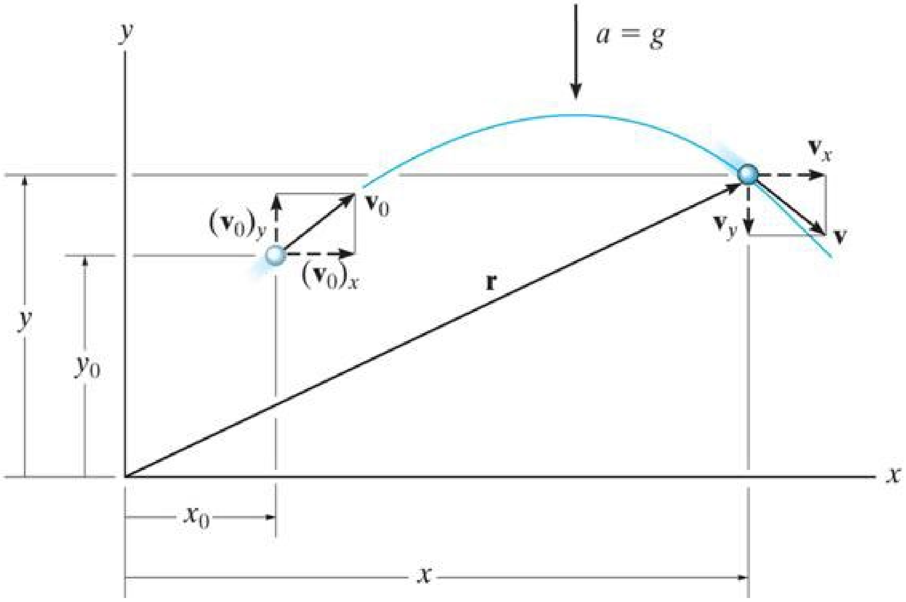
Projectile motion can be treated as two rectilinear motions, one in the horizontal direction experiencing zero acceleration and the other in the vertical direction experiencing constant acceleration (i.e., from gravity).

MOTION OF A PROJECTILE (Section 12.6)



For illustration, consider the two balls on the left. The red ball falls from rest, whereas the yellow ball is given a horizontal velocity. Each picture in this sequence is taken after the same time interval. Notice both balls are subjected to the same downward acceleration since they remain at the same elevation at any instant. Also, note that the horizontal distance between successive photos of the yellow ball is constant since the velocity in the horizontal direction is constant.

KINEMATIC EQUATIONS: HORIZONTAL MOTION



Since $a_x = 0$, the velocity in the horizontal direction remains constant ($v_x = v_{ox}$) and the position in the x direction can be determined by:

$$x = x_o + (v_{ox})\, t$$

Why is a_x equal to zero (what assumption must be made if the movement is through the air)?

KINEMATIC EQUATIONS: VERTICAL MOTION

Since the positive y-axis is directed upward, $a_y = -g$. Application of the constant acceleration equations yields:

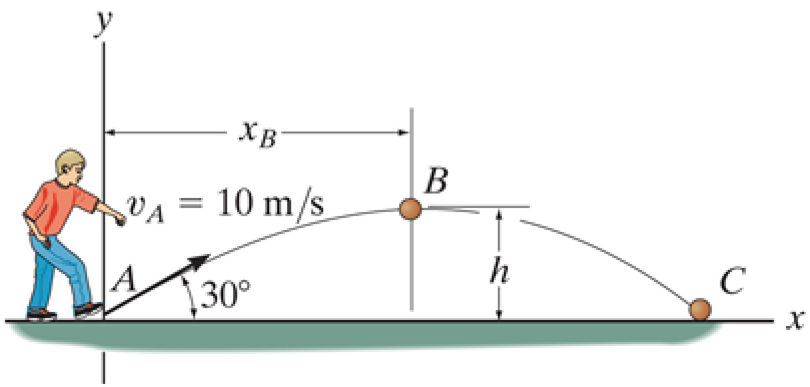
$$v_y = v_{oy} - g\,t$$

$$y = y_o + (v_{oy})\,t - \tfrac{1}{2}\,g\,t^2$$

$$v_y^{\,2} = v_{oy}^{\,2} - 2\,g\,(y - y_o)$$

For any given problem, only two of these three equations can be used. Why?

EXAMPLE I



Given: v_A and θ

Find: Horizontal distance it travels and v_C.

Plan: Apply the kinematic relations in x- and y- directions.

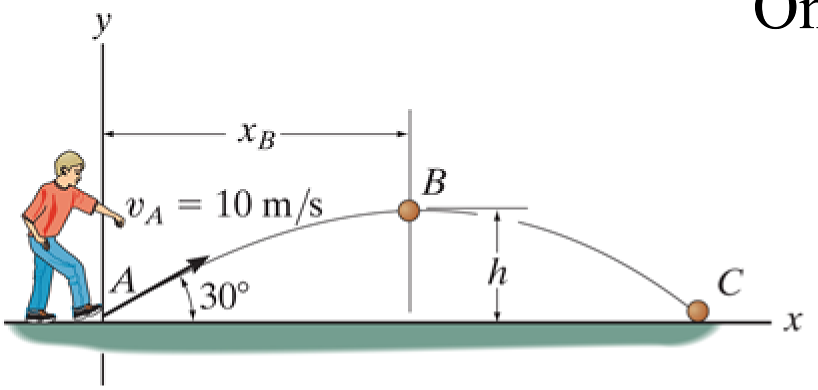
Solution: Using $v_{Ax} = 10 \cos 30$ and $v_{Ay} = 10 \sin 30$

We can write
$$v_x = 10 \cos 30$$
$$v_y = 10 \sin 30 - (9.81)\, t$$
$$x = (10 \cos 30)\, t$$
$$y = (10 \sin 30)\, t - \tfrac{1}{2}(9.81)\, t^2$$

Since y = 0 at C

$$0 = (10 \sin 30)\, t - \tfrac{1}{2}(9.81)\, t^2 \Rightarrow t = 0,\ 1.019 \text{ s}$$

EXAMPLE I (continued)



Only the time of 1.019 s makes sense!

Velocity components at C are;

v_{Cx} = 10 cos 30

= <u>8.66 m/s</u> →

v_{Cy} = 10 sin 30 – (9.81) (1.019)

= -5 m/s = <u>5 m/s</u> ↓

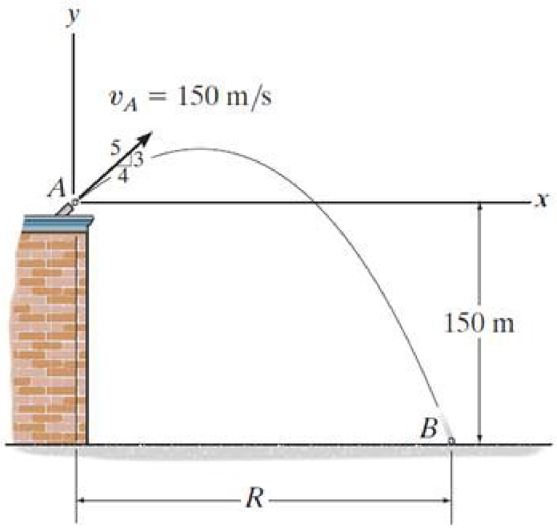
$$v_C = \sqrt{8.66^2 + (-5)^2} = \underline{10\ \text{m/s}}$$

Horizontal distance the ball travels is;

x = (10 cos 30) t

x = (10 cos 30) 1.019 = <u>8.83 m</u>

EXAMPLE II

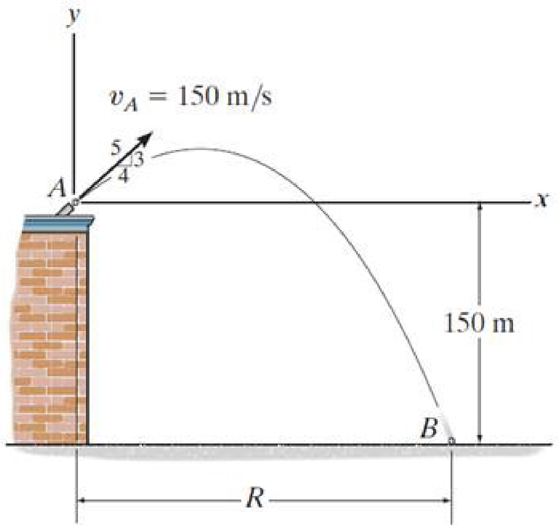


Given: Projectile is fired with v_A=150 m/s at point A.

Find: The horizontal distance it travels (R) and the time in the air.

Plan: How will you proceed?

EXAMPLE II



Given: Projectile is fired with v_A=150 m/s at point A.

Find: The horizontal distance it travels (R) and the time in the air.

Plan: Establish a fixed x, y coordinate system (in this solution, the origin of the coordinate system is placed at A). Apply the kinematic relations in x- and y-directions.

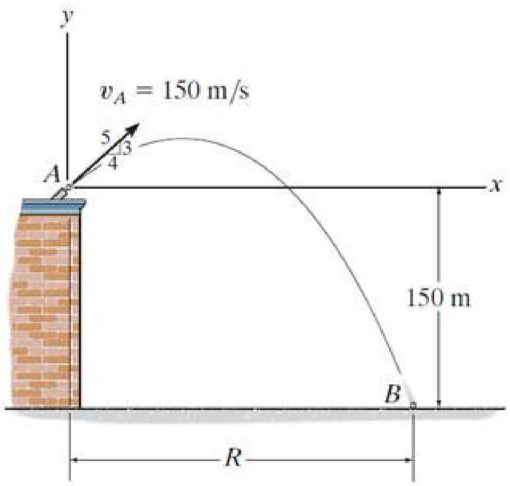
EXAMPLE II (continued)

Solution:

1) Place the coordinate system at point A.

Then, write the equation for horizontal motion.

$+\rightarrow \quad x_B = x_A + v_{Ax}\, t_{AB}$



where $x_B = R$, $x_A = 0$, $v_{Ax} = 150\ (4/5)$ m/s

Range, R, will be $R = 120\ t_{AB}$

2) Now write a vertical motion equation. Use the distance equation.

$+\uparrow \quad y_B = y_A + v_{Ay}\, t_{AB} - 0.5\, g\, t_{AB}^2$

where $y_B = -150$, $y_A = 0$, and $v_{Ay} = 150(3/5)$ m/s

We get the following equation: $-150 = 90\ t_{AB} + 0.5\ (-9.81)\ t_{AB}^2$

Solving for t_{AB} first, $t_{AB} = \underline{19.89\ \text{s}}$.

Then, $R = 120\ t_{AB} = 120\ (19.89) = \underline{2387\ \text{m}}$

PEARSON

CONCEPT QUIZ

1. In a projectile motion problem, what is the maximum number of unknowns that can be solved?

   A) 1 B) 2

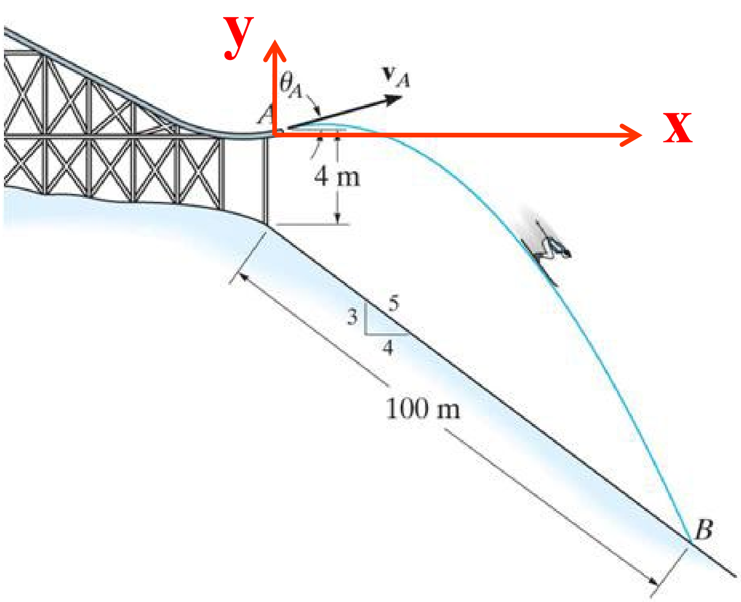
   C) 3 D) 4

2. The time of flight of a projectile, fired over level ground, with initial velocity V_o at angle θ, is equal to?

   A) $(v_o \sin \theta)/g$ B) $(2v_o \sin \theta)/g$

   C) $(v_o \cos \theta)/g$ D) $(2v_o \cos \theta)/g$

GROUP PROBLEM SOLVING I

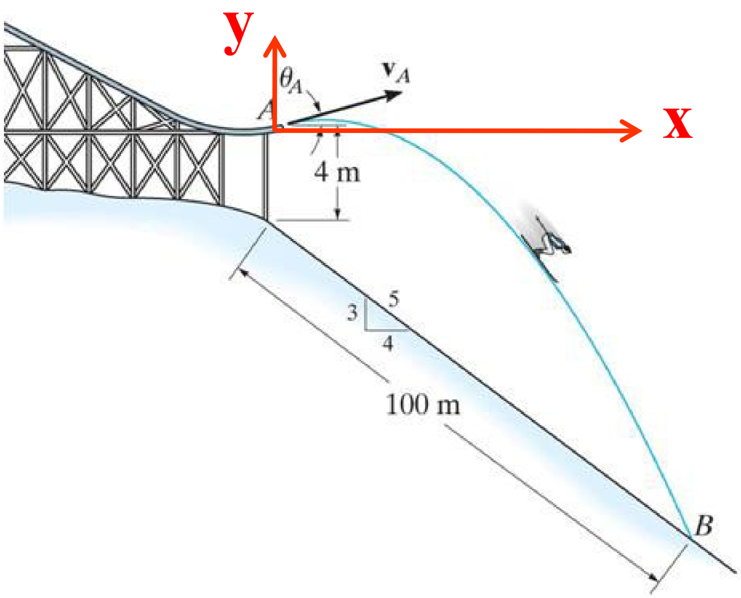


Given: A skier leaves the ski jump ramp at $\theta_A = 25^o$ and hits the slope at B.

Find: The skier's initial speed v_A.

Plan:

GROUP PROBLEM SOLVING I



Given: A skier leaves the ski jump ramp at $\theta_A = 25^o$ and hits the slope at B.

Find: The skier's initial speed v_A.

Plan: Establish a fixed x,y coordinate system (in this solution, the origin of the coordinate system is placed at A). Apply the kinematic relations in x- and y-directions.

GROUP PROBLEM SOLVING I (continued)

Solution:

Motion in x-direction:

Using $x_B = x_A + v_{ox}(t_{AB}) \Rightarrow (4/5)100 = 0 + v_A (\cos 25°)\, t_{AB}$

$$t_{AB} = \frac{80}{v_A (\cos 25°)} = \frac{88.27}{v_A}$$

Motion in y-direction:

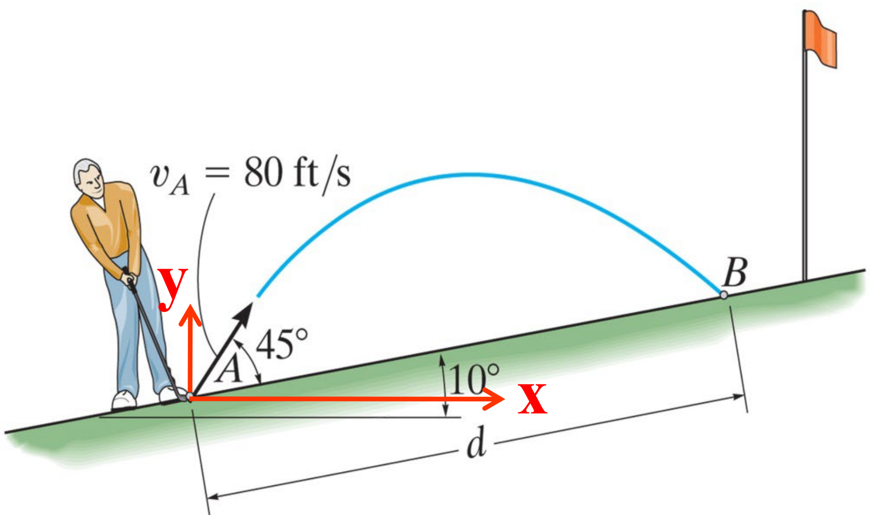
Using $y_B = y_A + v_{oy}(t_{AB}) - ½\, g(t_{AB})^2$

$$-64 = 0 + v_A(\sin 25°) \left\{\frac{88.27}{v_A}\right\} - ½\,(9.81) \left\{\frac{88.27}{v_A}\right\}^2$$

v_A = 19.42 m/s

t_{AB} = (88.27 / 19.42) = 4.54 s

GROUP PROBLEM SOLVING II

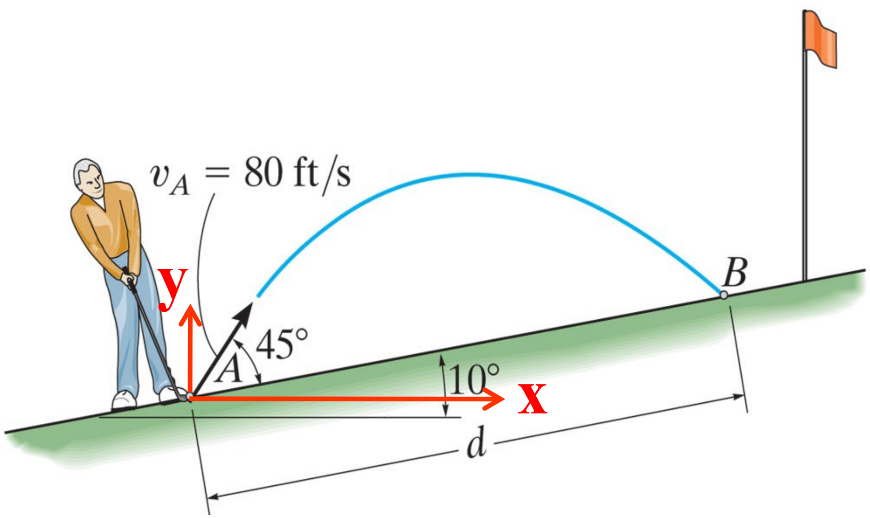


Given: The golf ball is struck with a velocity of 80 ft/s as shown.

Find: Distance d to where it will land.

Plan:

GROUP PROBLEM SOLVING II



Given: The golf ball is struck with a velocity of 80 ft/s as shown.

Find: Distance d to where it will land.

Plan: Establish a fixed x, y coordinate system (in this solution, the origin of the coordinate system is placed at A). Apply the kinematic relations in x- and y-directions.

GROUP PROBLEM SOLVING II (continued)

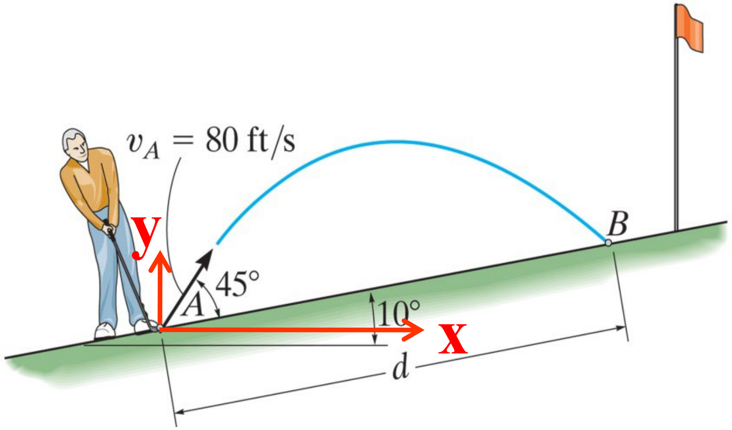
Solution:

Motion in x-direction:

Using $x_B = x_A + v_{ox}(t_{AB})$

$\Rightarrow d \cos 10 = 0 + 80 (\cos 55)\, t_{AB}$

$t_{AB} = 0.02146\ d$



Motion in y-direction:

Using $y_B = y_A + v_{oy}(t_{AB}) - ½\, g(t_{AB})^2$

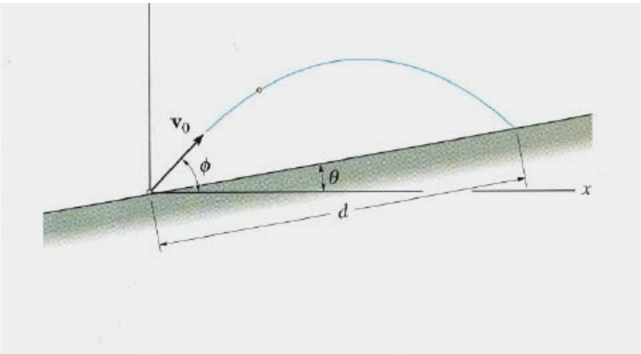
$\Rightarrow\ d \sin 10 = 0 + 80(\sin 55)(0.02146\ d) - ½\ 32.2\ (0.02146\ d)^2$

$\Rightarrow\ 0 = 1.233\ d - 0.007415\ d^2$

$d = 0,$ 166 ft   Only the non-zero answer is meaningful.

ATTENTION QUIZ

1. A projectile is given an initial velocity v_o at an angle ϕ above the horizontal. The velocity of the projectile when it hits the slope is ____________ the initial velocity v_o.



A) less than B) equal to
C) greater than D) None of the above.

2. A particle has an initial velocity v_o at angle ϕ with respect to the horizontal. The maximum height it can reach is when

A) $\phi = 30°$ B) $\phi = 45°$

C) $\phi = 60°$ D) $\phi = 90°$

End of the Lecture

Let Learning Continue