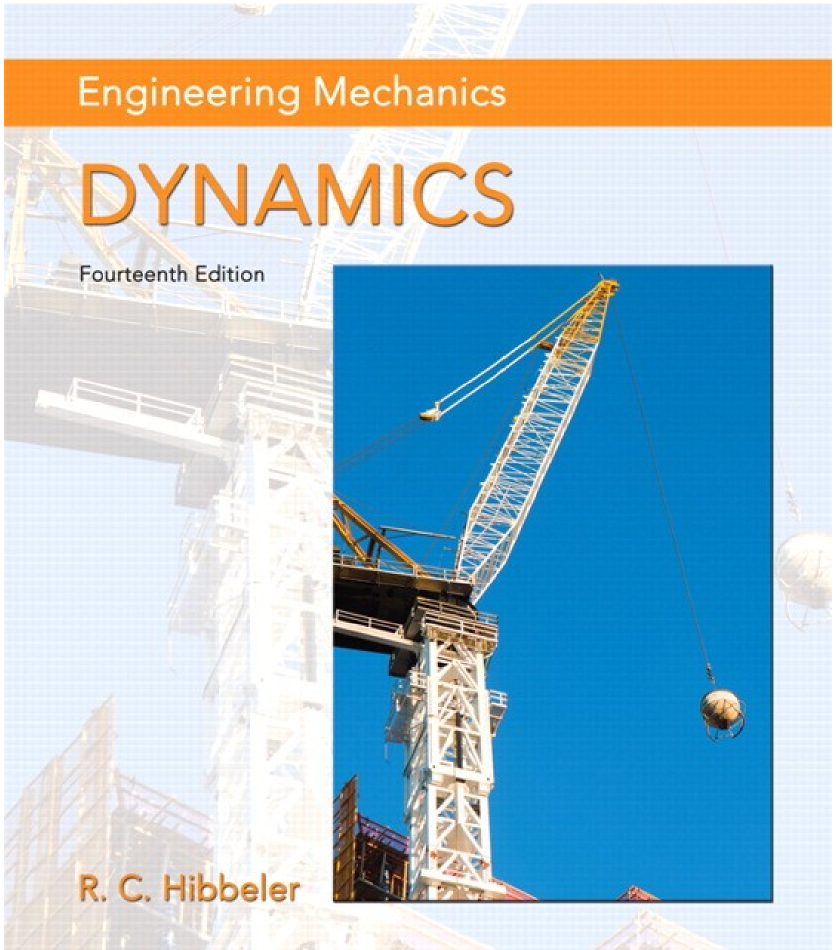
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 16

Planar Kinematics of a Rigid Body

Pearson

Planar Rigid Body Motion : Translation & Rotation (1 of 2)

Today's Objectives :

Students will be able to:

1. Analyze the kinematics of a rigid body undergoing planar translation or rotation about a fixed axis.

Planar Rigid Body Motion : Translation & Rotation (2 of 2)

- Check Homework
- Reading Quiz
- Applications
- Types of Rigid-Body Motion
- Planar Translation
- Rotation about a Fixed Axis
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1. If a rigid body is in translation only, the velocity at points A and B on the rigid body ________ .

A) are usually different
B) are always the same
C) depend on their position
D) depend on their relative position

2. If a rigid body is rotating with a constant angular velocity about a fixed axis, the velocity vector at point P is ________.

A) $\boldsymbol{w} \times \boldsymbol{r}_p$
B) $r_p \times w$
C) dr_p / dt
D) All of the above.

Application (1 of 2)

Passengers on this amusement ride are subjected to curvilinear translation since the vehicle moves in a circular path but they always remains upright.

If the angular motion of the rotating arms is known, how can we determine the velocity and acceleration experienced by the passengers? Why would we want to know these values?

Does each passenger feel the same acceleration?

Application (2 of 2)

Gears, pulleys and cams, which rotate about fixed axes, are often used in machinery to generate motion and transmit forces. The angular motion of these components must be understood to properly design the system.

To do this design, we need to relate the angular motions of contacting bodies that rotate about different fixed axes. How is this different than the analyses we did in earlier chapters?

Section 16.1

Rigid Body Motion

Rigid Body Motion

There are cases where an object **cannot** be treated as a particle. In these cases the **size** or **shape** of the body must be considered. **Rotation** of the body about its center of mass requires a different approach.

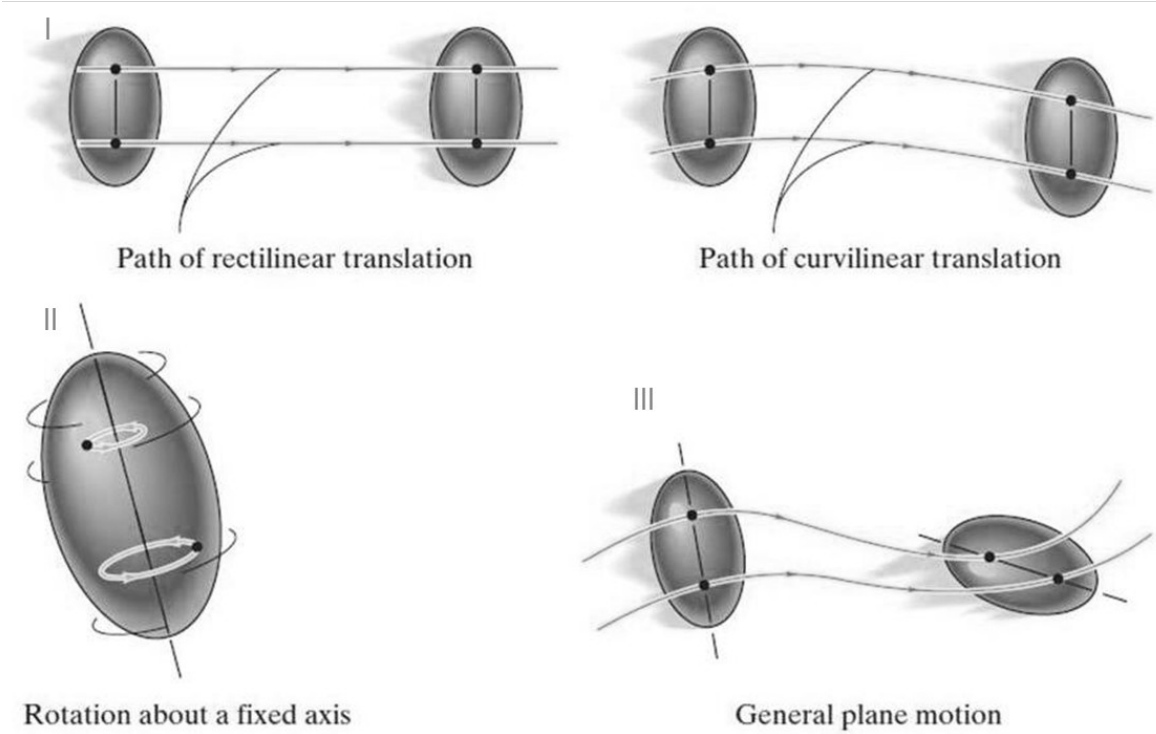
For example, in the design of gears, cams, and links in machinery or mechanisms, rotation of the body is an important aspect in the analysis of motion.

We will now start to study **rigid body motion**. The analysis will be limited to **planar motion**.

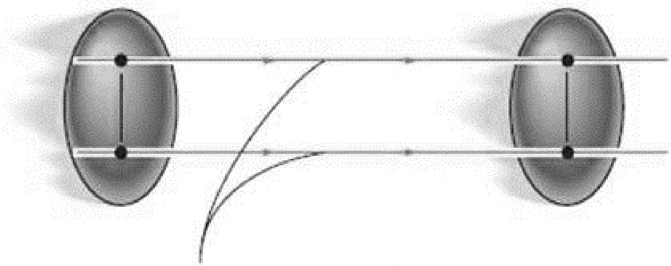
A body is said to undergo planar motion when all parts of the body move along paths equidistant from a fixed plane.

Planar Rigid Body Motion (1 of 4)

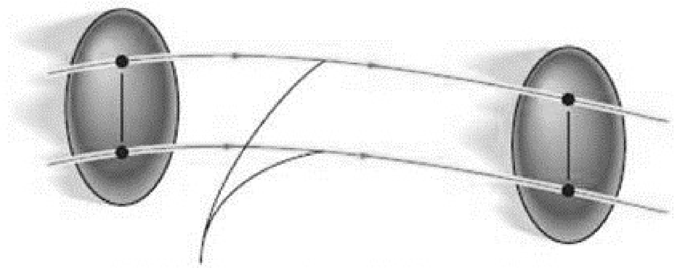
There are **three** types of planar rigid body motion.



Planar Rigid Body Motion (2 of 4)



Path of rectilinear translation

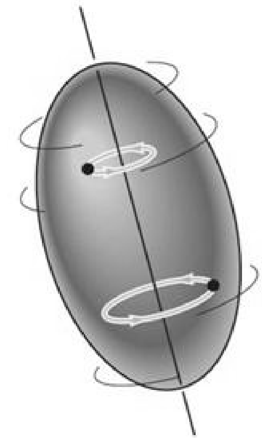


Path of curvilinear translation

Translation: Translation occurs if every line segment on the body remains parallel to its original direction during the motion. When all points move along straight lines, the motion is called **rectilinear** translation. When the paths of motion are curved lines, the motion is called **curvilinear** translation.

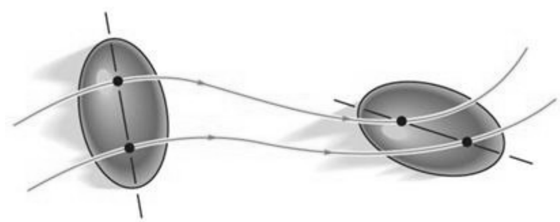
Planar Rigid Body Motion (3 of 4)

Rotation about a fixed axis: In this case, all the particles of the body, except those on the axis of rotation, move along **circular paths** in planes perpendicular to the axis of rotation.



Rotation about a fixed axis

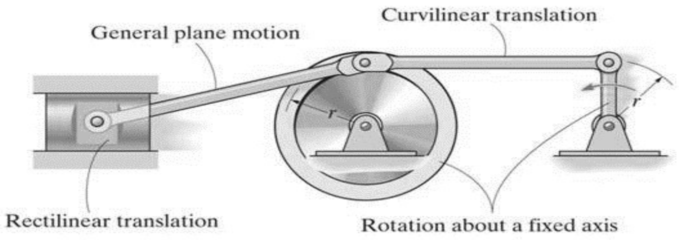
General plane motion: In this case, the body undergoes **both translation and rotation.** Translation occurs within a plane and rotation occurs about an axis perpendicular to this plane.



General plane motion

Planar Rigid Body Motion (4 of 4)

An example of bodies undergoing the three types of motion is shown in this mechanism.



The wheel and crank undergo **rotation about a fixed axis**. In this case, both axes of rotation are at the location of the pins and perpendicular to the plane of the figure.

The piston undergoes **rectilinear translation** since it is constrained to slide in a straight line.

The connecting rod undergoes **curvilinear translation**, since it will remain horizontal as it moves along a circular path.

The connecting rod undergoes **general plane motion**, as it will both translate and rotate.

Section 16.2

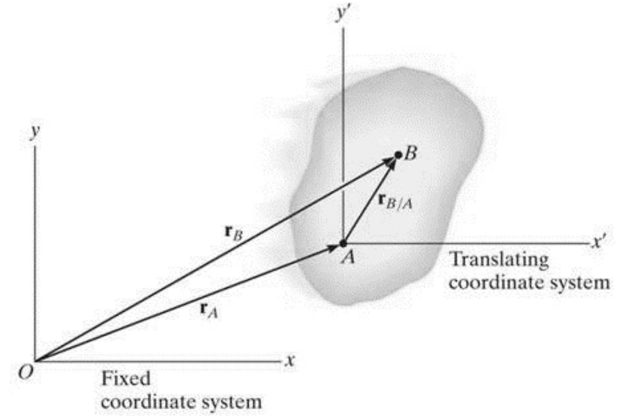
Rigid-body Motion: Translation

Pearson

Rigid-body Motion: Translation

The positions of two points A and B on a translating body can be related by

$$r_B = r_A + r_{B/A}$$



where r_A & r_B are the absolute position vectors defined from the fixed x-y coordinate system, and $r_{B/A}$ is the relative-position vector between B and A.

The **velocity** at B is $v_B = v_A + dr_{B/A}/dt$

Now $dr_{B/A}/dt = 0$ since $r_{B/A}$ is constant. So, $v_B = v_A$,

and by following similar logic, $a_B = a_A$.

Note, all points in a rigid body subjected to translation move with the **same velocity and acceleration.**

Section 16.3

Rigid-body Motion: Rotation About A Fixed Axis

Rigid-body Motion: Rotation About A Fixed Axis (1 of 2)

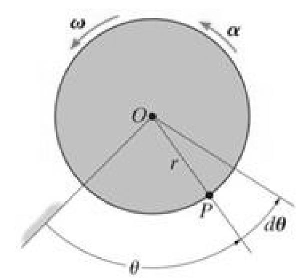
When a body rotates about a fixed axis, any point P in the body travels along a **circular path**. The angular position of P is defined by θ.

The change in angular position, $d\theta$, is called the angular displacement, with units of either radians or revolutions. They are related by

$$1 \text{ revolution} = (2\pi) \text{ radians}$$

Angular velocity, ω, is obtained by taking the time derivative of angular displacement:

$$\omega = d\theta / dt \, (rad/s) +$$



Similarly, **angular acceleration** is

$$\alpha = d^2\theta / dt^2 = d\omega / dt \text{ or } \alpha = \omega (d\omega / d\theta) \, rad/s^2 +$$

Rigid-body Motion: Rotation About A Fixed Axis (2 of 2)

If the angular acceleration of the body is **constant**,

$\alpha = \alpha_C$ the equations for angular velocity and

acceleration can be integrated to yield the set of **algebraic** equations below.

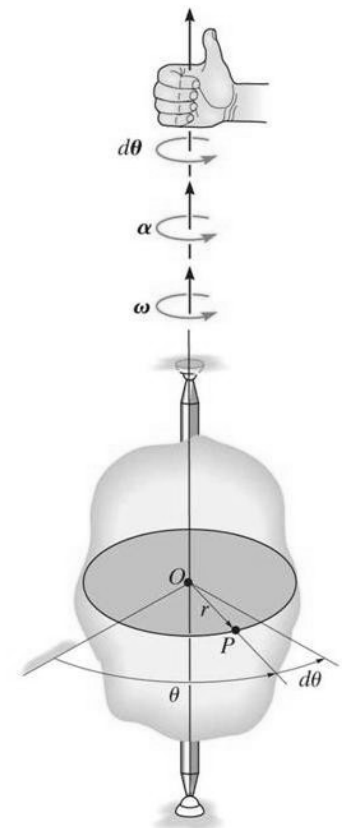
$$\omega = \omega_0 + \alpha_C t$$

$$\theta = \theta_0 + \omega_0 t + 0.5 \alpha_C t^2$$

$$\omega^2 = (\omega_0)^2 + 2\alpha_C(\theta - \theta_0)$$

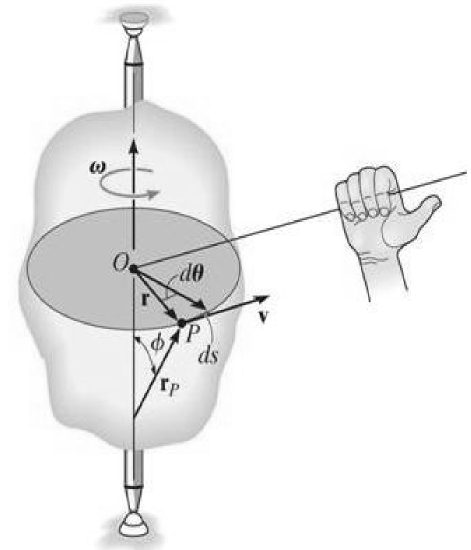
θ_0 and ω_0 are the initial values of the body's

angular position and angular velocity. Note these equations are very similar to the constant acceleration relations developed for the **rectilinear** motion of a particle.



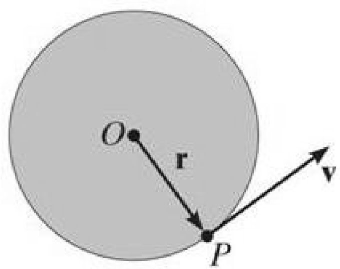
Rigid-body Rotation: Velocity of Point P (1 of 2)

The magnitude of the velocity of P is equal to ωr (the text provides the derivation). The velocity's direction is tangent to the circular path of P.



In the **vector** formulation, the magnitude and direction of *v* can be determined from the **cross product** of ω and r_p. here r_p is a vector from any point on the axis of rotation to P.

$$v = w \times r_p = w \times r$$



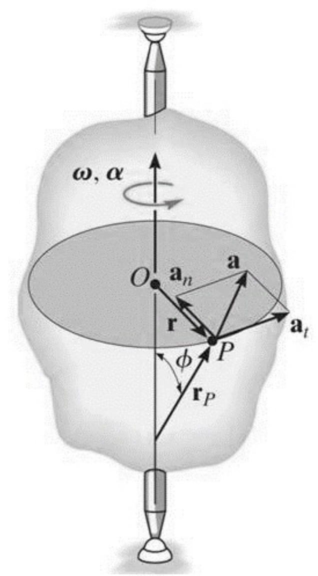
The direction of *v* is determined by the right-hand rule.

Rigid-body Rotation: Acceleration of Point P (1 of 2)

The acceleration of P is expressed in terms of its **normal** (a_n) and **tangential** (a_t)

In scalar form, these are $a_t = \alpha r$ and $a_n = \omega^2 r$.

The **tangential component,** (a_t) represents the time rate of change in the velocity's **magnitude.** It is directed **tangent** to the path of motion.

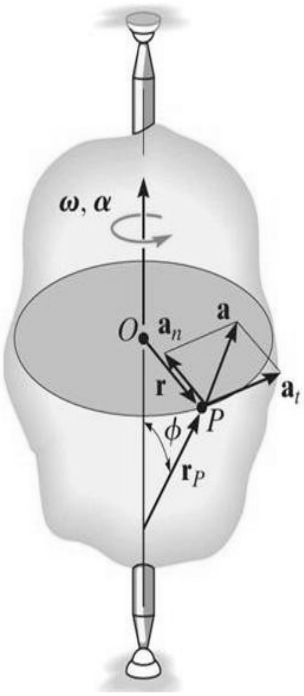


The **normal component,** (a_n), represents the time rate of change in the velocity's **direction.** It is directed **toward** the **center** of the circular path.

Rigid-body Rotation: Acceleration of Point P (2 of 2)

Using the **vector** formulation, the acceleration of P can also be defined by differentiating the velocity.

$$a = dv/dt = dw/dt \times r_p + \omega \times dr_p/dt$$
$$= \alpha \times r_p + \omega \times (\omega \times r_p)$$



It can be shown that this equation reduces to

$$a = \alpha \times r - \omega^2 r = a_t + a_n$$

The **magnitude** of the acceleration vector is

$$a = \sqrt{(a_n)^2 + (a_t)^2}$$

Rotation About A Fixed Axis : Procedure

- Establish a **sign convention** along the axis of rotation.
- If a relationship is known between any **two** of the variables (α,ω,θ, or t), the other variables can be determined from the equations:

$$\omega = d\theta / dt \quad \alpha = d\omega / dt \quad \alpha d\theta = \omega d\omega$$

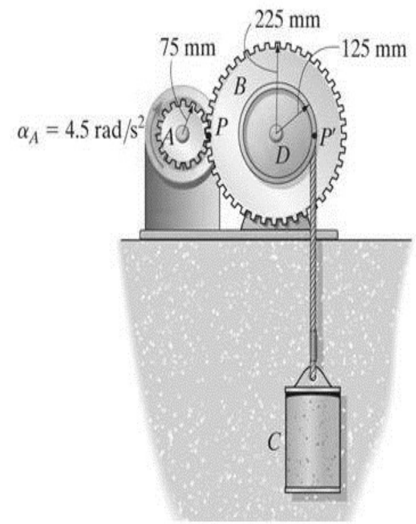
- If α is **constant**, use the equations for constant angular acceleration.
- To determine the **motion of a point**, the scalar equations $V = \omega r, a_t = \alpha r, a_n = \omega^2 r$ and $a = \sqrt{(a_n)^2 + (a_t)^2}$ can be used.
- Alternatively, the **vector** form of the equations can be used (with **i**, **j**, **k** components).

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}_p = \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{a} = \mathbf{a}_t + \mathbf{a}_n = \boldsymbol{\alpha} \times \mathbf{r}_p + \boldsymbol{\omega}(\boldsymbol{\omega} \times \mathbf{r}_p) = \boldsymbol{\alpha} \times \mathbf{r} - \omega^2 \mathbf{r}$$

Example (1 of 3)

Given: The motor turns gear A with a constant angular acceleration, $\alpha_A = 4.5\ rad/s^2$ starting from rest. The cord is wrapped around pulley D which is rigidly attached to gear B.



Find: The velocity of cylinder C and the. distance it travels in 3 seconds

Plan:

1) The angular acceleration of gear B (and pulley D) can be related to α_A.
2) The acceleration of cylinder C can be determined by using the equations of motion for a point on a rotating body since $(a_t)_D$ at point P is the same as a_C.
3) The velocity and distance of C can be found by using the constant acceleration equations.

Example (2 of 3)

Solution:

1) Gear A and B will have the **same** speed and tangential component of acceleration at the point where **they mesh.** Thus,

$$a_t = \alpha_A r_A = \alpha_B r_B \Rightarrow (4.5)(7.5) = \alpha_B (225) \Rightarrow \alpha_B = 1.5 rad/s^2$$

Since gear B and pulley D turn together, $\alpha_D = \alpha_B = 1.5 rad/s^2$

2) Assuming the cord attached to pulley D is inextensible and does not slip, the velocity and acceleration of cylinder C will be the same as the velocity and tangential component of acceleration along the pulley D.

$$a_C = (a_t)_D = \alpha_D r_D = (1.5)(0.125) = 0.1875 m/s^2 \uparrow$$

Example (3 of 3)

3) Since α_A is constant, α_D and α_C will be constant .The constant acceleration equation for rectilinear motion can be used to determine the velocity and displacement of cylinder C when $t = 3\ s\ (s_0 = v_0 = 0)$

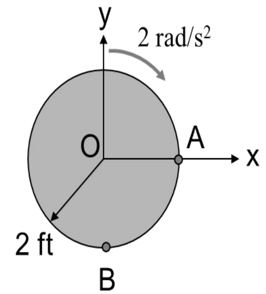
$$v_c = v_0 + a_C t = 0 + 0.1875(3) = 0.563\ m/s \uparrow$$

$$s_c = s_0 + v_0 t + (0.5) a_C t^2$$

$$= 0 + 0 + (0.5)0.1875(3)^2 = 0.844\ m/s \uparrow$$

Concept Quiz

1. A disk is rotating at $4 rad/s$ If it is subjected to constant angular acceleration of $2 rad/s^2$ determine the acceleration at B.



A) $(4i + 32j) ft/s^2$ B) $(4i - 32j) ft/s^2$

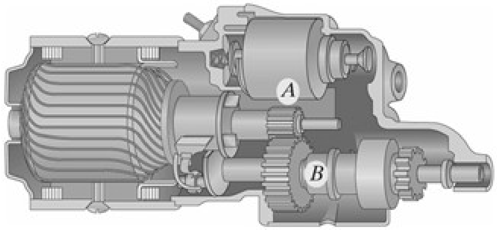
C) $(-4i + 32j) ft/s^2$ D) $(-4i - 32j) ft/s^2$

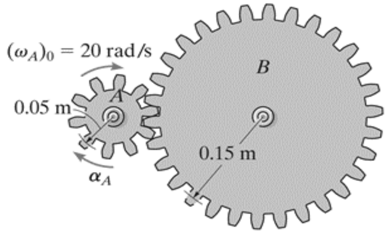
2. A Frisbee is thrown and curves to the right. It is experiencing

A) rectilinear translation. B) curvilinear translation.

C) pure rotation. D) general plane motion.

Group Problem Solving





Given: Gear A is given an angular acceleration $\alpha_A = 4t^3 rad/s^2$

where t is in seconds, and $(\omega_A)_0 = 20 rad/s$

Find: The angular velocity and angular displacement of

gear B when t = 2 s

Plan:

1) Apply the kinematic equation of variable angular acceleration to find the angular velocity of gear A.

2) Find the relationship of angular motion between gear A and gear B in terms of time and then use 2 s.

Group Problem Solving (2 of 3)

Solution

:

1) Motion of Gear A : Applying **the kinematic equation**

$$\int_{\omega_{A0}}^{\omega_A} d\omega_A = \int_0^t \alpha_A \, dt$$

$$\omega_A - 20 = \int_0^t 4t^3 \, dt = t^4 \Rightarrow \omega_A = t^4 + 20$$

$$\int_0^{\theta_A} d\theta_A = \int_0^t \omega_A \, dt$$

$$\theta_A = \int_0^t \left(t^4 + 20\right) dt = \frac{1}{5} t^5 + 20t$$

when $t = 2s, \omega_A = 36 rad/s$ and $\theta_A = 46.4 rad$

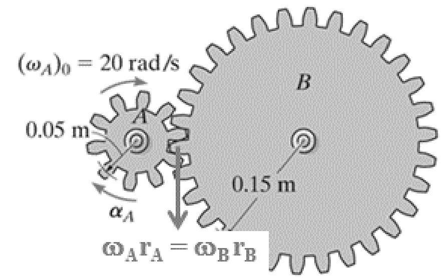
Group Problem Solving (3 of 3)

2) Since gear B meshes with gear A,

$$\omega_A r_A = \omega_B r_B$$

$$\Rightarrow \omega_B = \omega_A (r_A / r_B) = \omega_B = \omega_A (0.05/0.15)$$

Similarly $\theta_B = \theta_A (0.05/0.15)$



Since $\omega_A = 36 rad/s$ and $\theta_A = 46.4 rad$ at $t = 2\ s$,

$$\omega_A = 36(0.05/0.15) = 12 rad/s$$

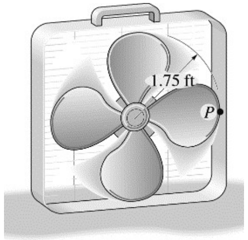
$$\theta_A = 46.4(0.05/0.15) = 12 rad/s$$

Attention Quiz

1. The fan blades suddenly experience an angular acceleration of $2rad/s^2$ If the blades are rotating with an initial angular velocity of $4rad/s$ determine the speed of point P when the blades have turned 2 revolutions (when $\omega = 8.14 rad/s$).

   A) $14.2 ft/s$ B) $17.7 ft/s$

   C) $23.1 ft/s$ D) $26.7 ft/s$



2. Determine the magnitude of the acceleration at P when the blades have turned the 2 revolutions

   A) $0 ft/s^2$ B) $3.5 ft/s^2$

   C) $115.95 ft/s^2$ D) $116 ft/s^2$

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