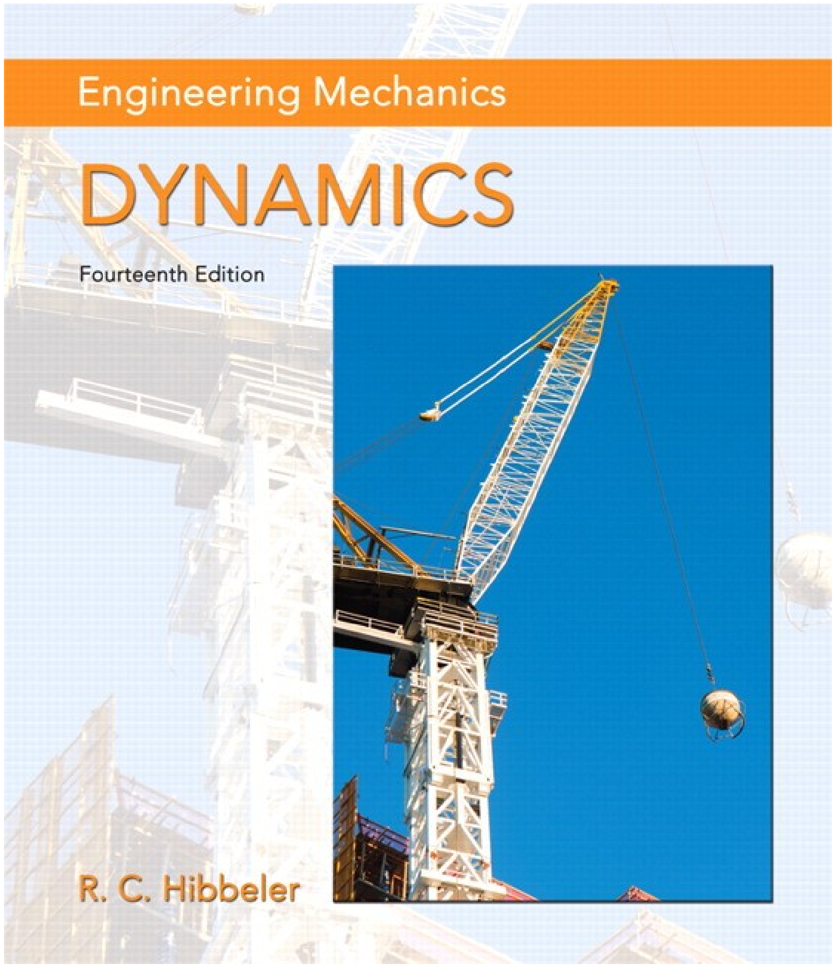
# Engineering Mechanics: Dynamics

Fourteenth Edition




## Chapter 13

Kinetics of a Particle:
Force and Acceleration

Pearson

# Equations of Motion: Rectangular Coordinates (1 of 2)

**Today's Objectives:**
Students will be able to:

1. Apply Newton's second law to determine forces and accelerations for particles in rectilinear motion.

# Equations of Motion: Rectangular Coordinates (2 of 2)

- In-Class Activities:
- Check Homework
- Reading Quiz
- Applications
- **Equations of Motion using Rectangular (Cartesian) Coordinates**
- Concept Quiz
- Group Problem Solving
- Attention Quiz

# Reading Quiz

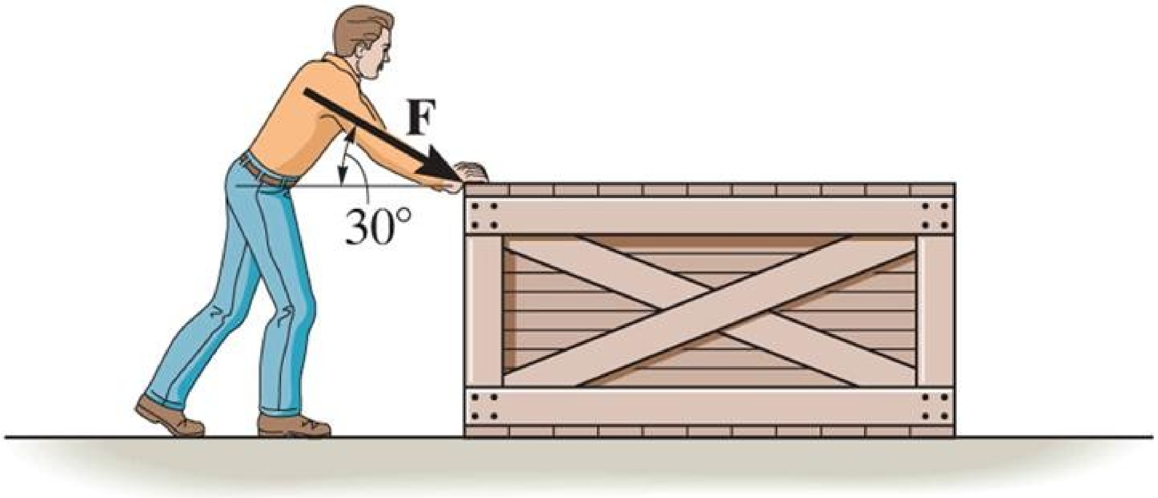
1. In dynamics, the friction force acting on a moving object is always __________

   A) in the direction of its motion.
   B) a kinetic friction.
   C) a static friction.
   D) zero.

2. If a particle is connected to a spring, the elastic spring force is expressed by $F = ks$ The "s" in this equation is the

   A) spring constant.
   B) un-deformed length of the spring.
   C) difference between deformed length and un-deformed length.
   D) deformed length of the spring.

# Applications (1 of 2)

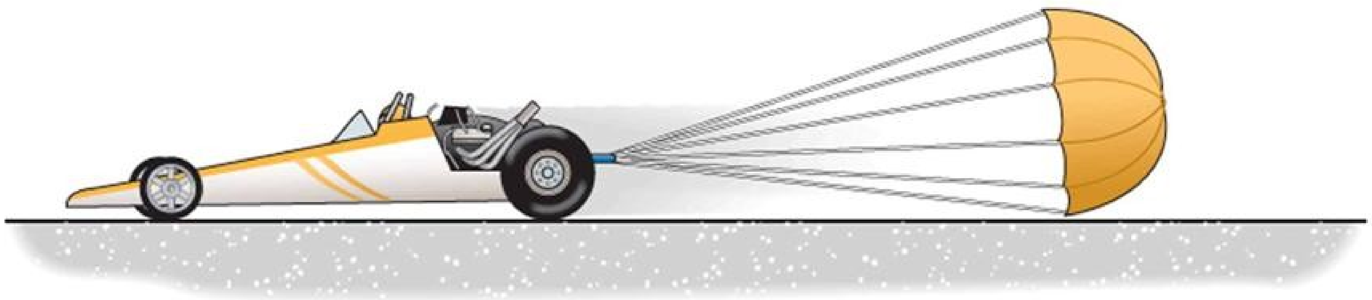



If a man is trying to move a 100 lb crate, how large a force F must he exert to start moving the crate? What factors influence how large this force must be to start moving the crate?

If the crate starts moving, is there acceleration present?

What would you have to know before you could find these answers?

# Applications (2 of 2)



Objects that move in air (or other fluid) have a drag force acting on them. This drag force is a function of velocity.

If the dragster is traveling with a known velocity and the magnitude of the opposing drag force at any instant is given as a function of velocity, can we determine the time and distance required for dragster to come to a stop if its engine is shut off? How ?

# Section 13.4

Rectangular Coordinates

# Rectangular Coordinates

The equation of motion, $F = ma$ is best used when the problem requires finding forces (especially forces perpendicular to the path), accelerations, velocities, or mass. **Remember, unbalanced forces cause acceleration!**

**Three scalar equations** can be written from this **vector equation**. The equation of motion, being a vector equation, may be expressed in terms of three components in the **Cartesian (rectangular) coordinate system** as

$$\Sigma F = ma \text{ or } \Sigma F_X i + \Sigma F_Y j + \Sigma F_Z k = m(a_x i + a_y j + a_z k)$$

or, as scalar equations, $\Sigma F_X = ma_X$, $\Sigma F_Y = ma_Y$ and $\Sigma F_Z = ma_Z$

# Procedure For Analysis (1 of 3)

- **Free Body Diagram (is always critical!!)**

  Establish your coordinate system and draw the particle's free body diagram showing **only external forces**. These external forces usually include the weight, normal forces, friction forces, and applied forces. Show the 'm**a**' vector (sometimes called the inertial force) on a separate kinetic diagram.

  Make sure any friction forces act opposite to the direction of motion! If the particle is connected to an elastic linear spring, a spring force equal to '**k s**' should be included on the FBD.

# Procedure For Analysis (2 of 3)

- **Equations of Motion**

If the forces can be resolved directly from the free-body diagram (often the case in 2-D problems), use the **scalar form** of the equation of motion. In more complex cases (usually 3-D), a Cartesian vector is written for every force and a **vector analysis** is often the best approach.

A Cartesian vector formulation of the second law is

$$\Sigma \boldsymbol{F} = m\boldsymbol{a} \text{ or}$$

$$\Sigma F_X \boldsymbol{i} + \Sigma F_Y \boldsymbol{j} + \Sigma F_Z \boldsymbol{k} = m(a_X \boldsymbol{i} + a_Y \boldsymbol{j} + a_Z \boldsymbol{k})$$

Three scalar equations can be written from this vector equation. You may only need two equations if the motion is in 2-D.

# Procedure For Analysis (3 of 3)

- **Kinematics**

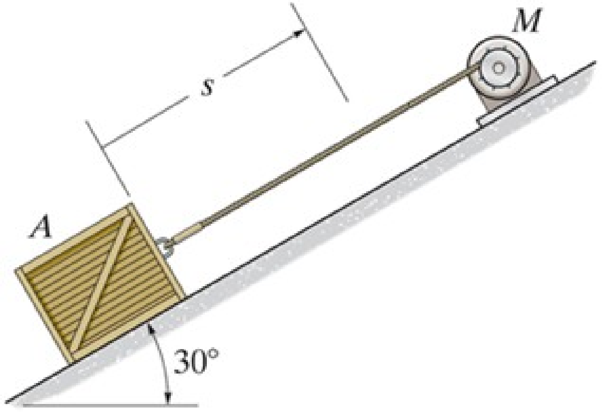
  The second law only provides solutions for forces and accelerations. If velocity or position have to be found, kinematics equations are used once the acceleration is found from the equation of motion.

  Any of the kinematics tools learned in Chapter 12 may be needed to solve a problem.

  Make sure you use **consistent** positive coordinate directions as used in the equation of motion part of the problem!

# Example (1 of 3)

**Given:** The motor winds in the cable with a constant acceleration such that the 20-kg crate moves a distance s = 6 m in 3 s, starting from rest. $\mu_k = 0.3$.




**Find:** The tension developed in the cable.

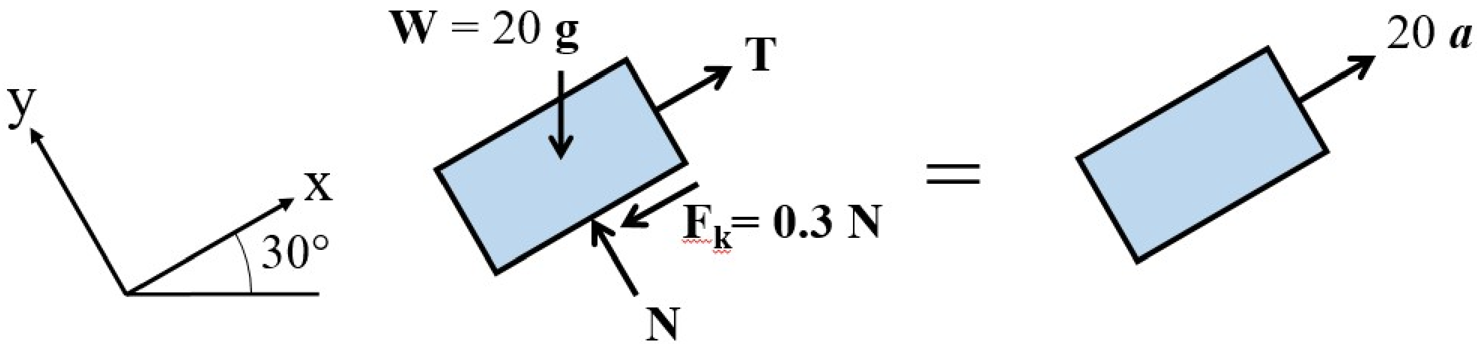
**Plan:**

1. Draw the free-body and kinetic diagrams of the crate.
2. Using a kinematic equation, determine the acceleration of the crate.
3. Apply the equation of motion to determine the cable tension.

# Example (2 of 3)

**Solution:**

1. Draw the free-body and kinetic diagrams of the crate.




Since the motion is up the incline, rotate the x-y axes so the x-axis aligns with the incline. Then, motion occurs only in the x-direction.

There is a friction force acting between the surface and the crate. Why is it in the direction shown on the FBD?

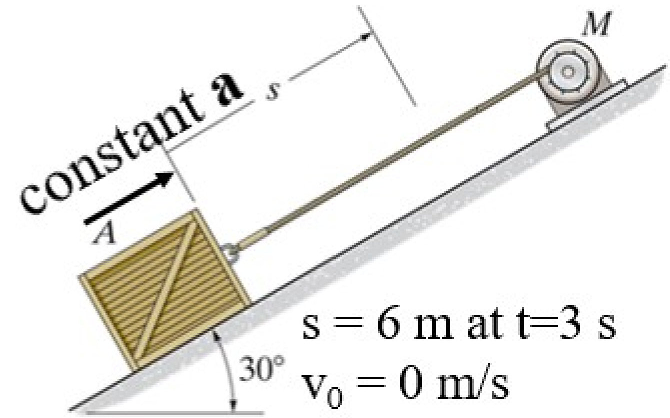
# Example (3 of 3)

2. Using kinematic equation

$$s = v_0 t + \tfrac{1}{2} a t^2$$

$$\Rightarrow 6 = (0)\,3 + \tfrac{1}{2} a (3^2)$$

$$\Rightarrow a = 1.333 \text{ m/s}^2$$




3. Apply the equations of motion

$$+\nwarrow \Sigma F_y = 0 \Rightarrow -20\,g(\cos 30°) + N = 0$$

$$\Rightarrow N = 169.9 N$$

$$+\nearrow \Sigma F_x = ma \Rightarrow T - 20\,g(\sin 30°) - 0.3N = 20\,a$$

$$\Rightarrow T = 20\,(981)\,(\sin 30°) + 0.3(169.9) + 20\,(1.333)$$

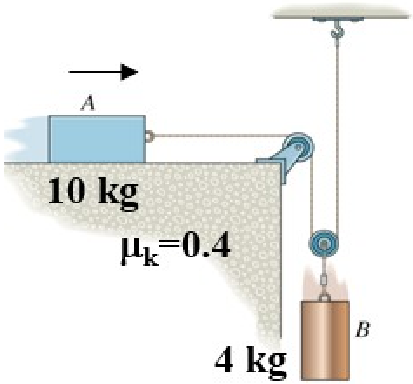
$$\Rightarrow T = 176 N$$

# Concept Quiz

1. If the cable has a tension of 3 N, determine the acceleration of block B.

   A) 4.26 m/s$^2$ ↑  B) 4.26 m/s$^2$ ↓

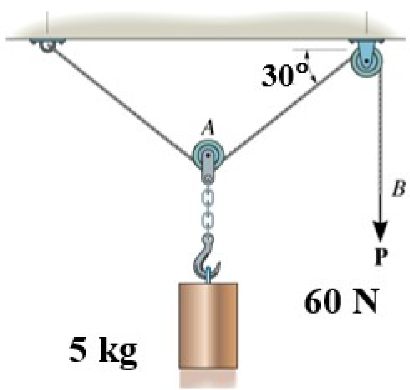
   C) 8.31 m/s$^2$ ↑  D) 8.31 m/s$^2$ ↓




2. Determine the acceleration of the block.

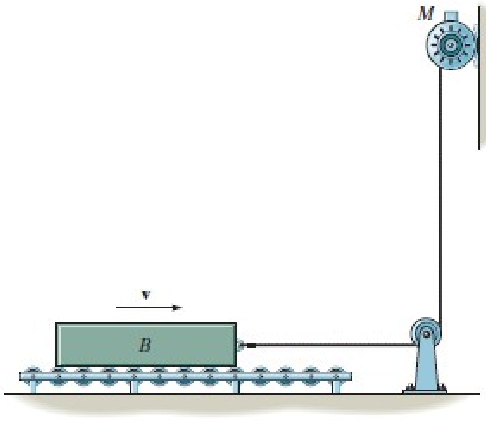
   A) 2.20 m/s$^2$ ↑  B) 3.17 m/s$^2$ ↑

   C) 11.0 m/s$^2$ ↑  D) 4.26 m/s$^2$ ↑



# Group Problem Solving (1 of 3)

**Given:** The 300-kg bar *B*, originally at rest, is towed over a series of small rollers. The motor *M* is drawing in the cable at a rate of $v = (0.4\,t^2)$ m/s, where **t** is in seconds.




**Find:** Force in the cable and distance s when t = 5 s

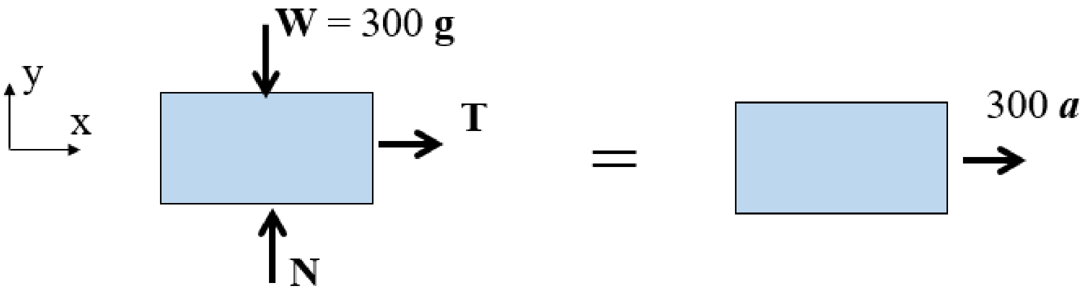
**Plan:** Since both forces and velocity are involved, this problem requires both kinematics and the equation of motion.

1. Draw the free-body and kinetic diagrams of the bar.
2. Apply the equation of motion to determine the acceleration and force.
3. Using a kinematic equation, determine distance.

# Group Problem Solving (2 of 3)

**Solution:**

1. Free-body and kinetic diagrams of the bar:




Note that the bar is moving along the x-axis.

2. Apply the scalar equation of motion in the x-direction

$$+\rightarrow \Sigma F_x = 300\,a \Rightarrow T = 300\,a$$

Since $v = 0.4t^2$, $a = (dv/dt) = 0.8t$

$T = 240t \Rightarrow T = 1200\,N$ when $t = 5s$.

# Group Problem Solving (3 of 3)

3. Using kinematic equation to determine distance;

Since $v = (0.4t^2)$ m/s

$$s = s_0 + \int v\, dt = 0 + \int_0^t (0.4t^2)\, dt$$

$$\Rightarrow \; s = \frac{0.4}{3} t^3$$

At t = 5 s,

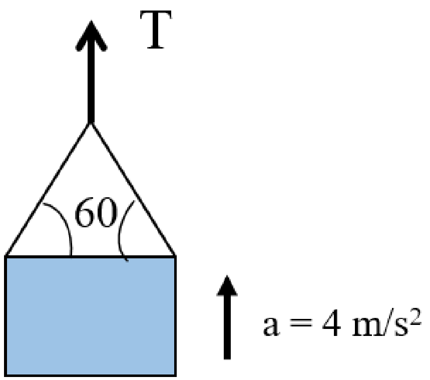
$$s = \frac{0.4}{3} 5^3 = 16.7\, m$$

# Attention Quiz

1. Determine the tension in the cable when the 400 kg box is moving upward with a $4 \text{ m/s}^2$ acceleration.

   A) 2265 N    B) 3365 N

   C) 5524 N    D) 6543 N




2. A 10 lb particle has forces of $F1 = (3i + 5j)$ lb and $F2 = (-7i + 9j)$ lb acting on it. Determine the acceleration of the particle.

   A) $(-0.4i + 1.4j)$ ft/s$^2$    B) $(-4i + 14j)$ ft/s$^2$

   C) $(-12.9i + 45j)$ ft/s$^2$    D) $(13i + 4j)$ ft/s$^2$

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