NUMM 2500 | Humber College | Quiz 3 (5%) | Quiz Period: 50 min.
March 20, 2024

Student Name:

Student Number:

1. Fit a **quadratic polynomial** to the following data:

x	3	4	5	7	8	9	11	12
y	1.6	3.6	4.4	3.4	2.2	2.8	3.8	4.6

Along with the coefficients, determine r^2 and $S_{\frac{y}{x}}$.

$\hat{y} = ax^2 + bx + c$

x_i	y_i	x_i^2	x_i^3	x_i^4	$y_i x_i$	$y_i x_i^2$	y_i^2
3	1.6	9	27	81	4.8	14.4	2.56
4	3.6	16	64	256	14.4	57.6	12.96
5	4.4	25	125	625	22	110	19.36
7	3.4	49	343	2401	23.8	166.6	11.56
8	2.2	64	512	4096	17.6	140.8	4.84
9	2.8	81	729	6561	25.2	226.8	7.84
11	3.8	121	1331	14641	41.8	459.8	14.44
12	4.6	144	1728	20736	55.2	662.4	21.16
$\Sigma =$ 59	26.4	509	4859	49397	204.8	1838.4	94.72

$$\begin{cases} 8c + b(59) + a(509) = 26.4 \\ 26.4c + 509b + 4859a = 204.8 \\ 509c + 4859b + 49397a = 1838.4 \end{cases} \longmapsto \begin{cases} a = -0.0437 \\ b = 0.8168 \\ c = 0.0586 \end{cases}$$

$$\hat{y} = -0.0437\,x^2 + 0.8168\,x + 0.0586$$

$$r^2 = \left[\frac{(8)(204.8) - (59)(26.4)}{\sqrt{\left[(8)(509) - (59)^2\right]\left[(8)(94.72) - (26.4)^2\right]}} \right]^2 = 0.182$$

$$\hat{y} = -0.0437x^2 + 0.8168x + 0.0586$$

$S_{y/x}$

x_i	y_i	$\hat{y}_i$	$(y_i - \hat{y}_i)^2$
3	1.6	2.1157	0.2559
4	3.6	2.6266	0.9475
5	4.4	3.0501	1.8222
7	3.4	3.6349	0.0552
8	2.2	3.7962	2.5479
9	2.8	3.8701	1.1451
11	3.8	3.7557	0.0020
12	4.6	3.5674	1.0663
$\Sigma =$ 59	26.4	26.4167	7.8521 = SSE

$$S_{y/x} = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{7.8521}{6}} = 1.1440$$

2. For the function, the divided differences are given by:

$x_0 = 0.0$	$f[x_0]$		
		$f[x_0, x_1]$	
$x_1 = 0.4$	$f[x_1]$		$f[x_0, x_1, x_2] = \frac{50}{7}$
		$f[x_1, x_2] = 10$	
$x_2 = 0.7$	$f[x_2] = 6$		

Determine the missing entries in the table. (Show your work for full mark!)

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \Rightarrow \frac{50}{7} = \frac{10 - f[x_0, x_1]}{0.7}$$

$$\rightarrow \boxed{f[x_0, x_1] = 5}$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} \Rightarrow 10 = \frac{6 - f[x_1]}{0.3} \rightarrow \boxed{f[x_1] = 3}$$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} \Rightarrow 5 = \frac{3 - f[x_0]}{0.4} \rightarrow \boxed{f[x_0] = 1}$$

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3. Consider the following data values.

x	0.1	0.2	0.4	0.6	0.9	1.3
y	0.75	1.25	1.45	1.25	0.85	0.55

A function of the form

$$y = axe^{bx}$$

can be a good fit for the data set. Divide both sides by x and take the natural logarithm to transform the model to a linear model. Use this model to estimate coefficients a and b.

$$y = axe^{bx} \to \frac{y}{x} = ae^{bx} \to \ln\left(\frac{y}{x}\right) = \ln(a e^{bx})$$

$$\to \ln\left(\frac{y}{x}\right) = \ln a + bx \Rightarrow \ln\left(\frac{y}{x}\right) = A + Bx$$

where $\begin{cases} A = \ln a \\ B = b. \end{cases}$

To find A and B, we use the linear regression formula for the variables x and $Y = \ln(y/x)$

x	$Y = \ln(y/x)$	$x \cdot Y$	x^2
0.1	2.0149	0.2015	0.01
0.2	1.8326	0.3665	0.04
0.4	1.2879	0.5151	0.16
0.6	0.7340	0.4404	0.36
0.9	−0.0572	−0.0514	0.81
1.3	−0.8602	−1.1183	1.69
$\Sigma =$ 3.5	4.9519	0.3538	3.07

$$\text{Slope} = B = \frac{(6)(0.3538) - (3.5)(4.9519)}{(6)(3.07) - (3.5)^2} \simeq -2.465$$

$$\text{Y-intercept} = A = \bar{Y} - B\bar{x} = \frac{4.9519}{6} - (-2.465)\frac{3.5}{6} \simeq 2.2632$$

$$\begin{cases} \boxed{b = B = -2.465} \\ \ln a = A = 2.2632 \mapsto a = e^{2.2632} = 9.614 \end{cases} \mapsto \hat{y} = 9.614\, x\, e^{-2.465x}$$