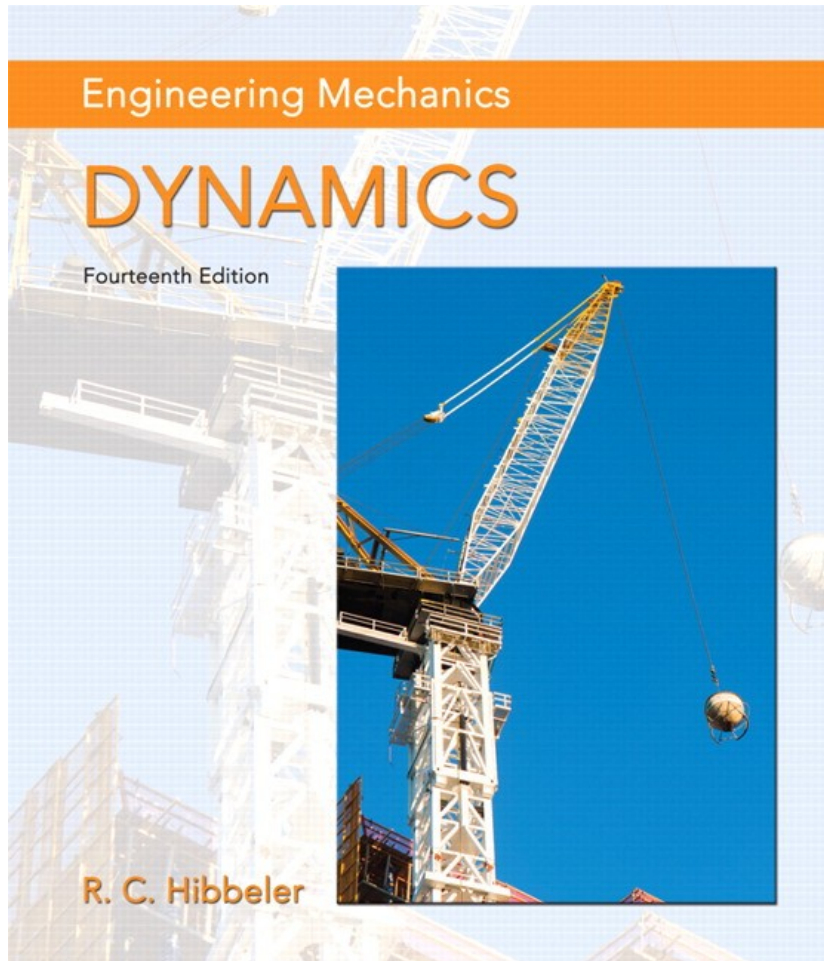


Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 16

Planar Kinematics of a Rigid Body

Instantaneous Center of Zero Velocity (1 of 2)

Today's Objectives:

Students will be able to:

1. Locate the instantaneous center of zero velocity.
2. Use the instantaneous center to determine the velocity of any point on a rigid body in general plane motion.



Instantaneous Center of Zero Velocity (2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Location of the Instantaneous Center
- Velocity Analysis
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1. If applicable, the method of instantaneous center can be used to determine the _____ of any point on a rigid body.

A) velocity	B) acceleration
C) Velocity and acceleration	D) force

2. The velocity of any point on a rigid body is _____ to the relative position vector extending from the IC to the point.

A) always parallel	B) always perpendicular
C) in the opposite direction	D) in the same direction

Applications (1 of 2)

The instantaneous center (IC) of zero velocity for this bicycle wheel is at the point in contact with ground. The velocity direction at any point on the rim is perpendicular to the line connecting the point to the IC.



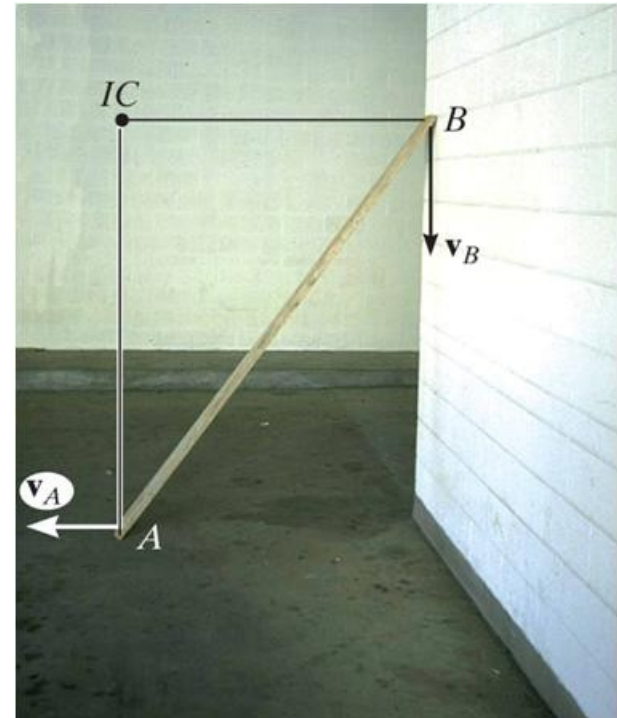
Which point on the wheel has the maximum velocity?
Does a larger wheel mean the bike will go faster for the same rider effort in pedaling than a smaller wheel?

Applications (2 of 2)

As the board slides down the wall (to the left), it is subjected to general plane motion (both translation and rotation).

Since the directions of the velocities of ends A and B are known, the IC is located as shown.

How can this result help you analyze other situations?



What is the direction of the velocity of the center of gravity of the board?

Section 16.6

Instantaneous Center of Zero Velocity

Instantaneous Center of Zero Velocity

For any body undergoing planar motion, there always exists a point in the plane of motion at which the **velocity is instantaneously zero** (if it is rigidly connected to the body).

This point is called the instantaneous center (IC) of zero velocity. **It may or may not lie on the body!**

If the location of this point can be determined, the velocity analysis can be simplified because the body appears to rotate about this point at that instant.

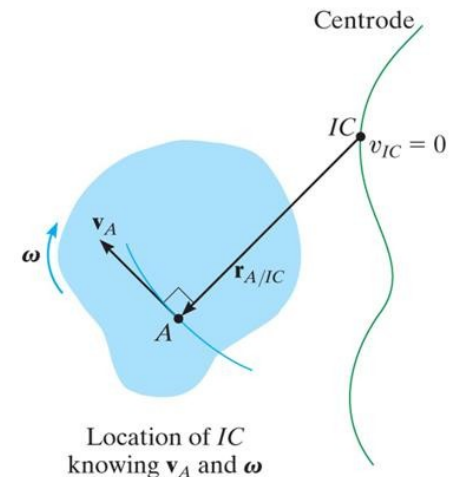
Location of The Instantaneous Center (1 of 3)

To locate the IC, we use the fact that the velocity of a point on a body is always perpendicular to the relative position vector from the IC to the point. Several possibilities exist.

First, consider the case when velocity \mathbf{v}_A of a point A on the body and the angular velocity ω of the body are known.

In this case, the IC is located along the line drawn perpendicular to \mathbf{v}_A at A, a distance $r_{A/IC} = v_A/\omega$ from A.

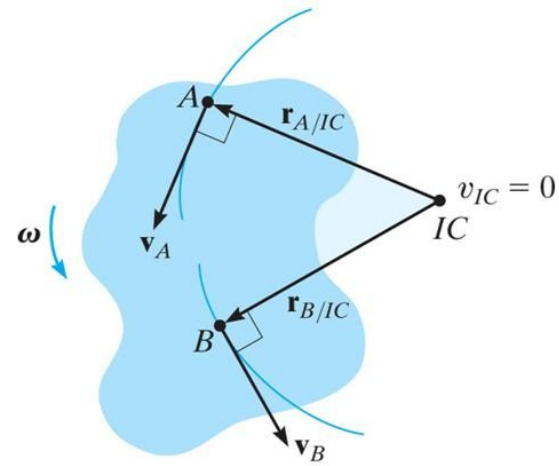
Note that the IC lies up and to the right of A since \mathbf{v}_A must cause a clockwise angular velocity ω about the IC.



Location of The Instantaneous Center (2 of 3)

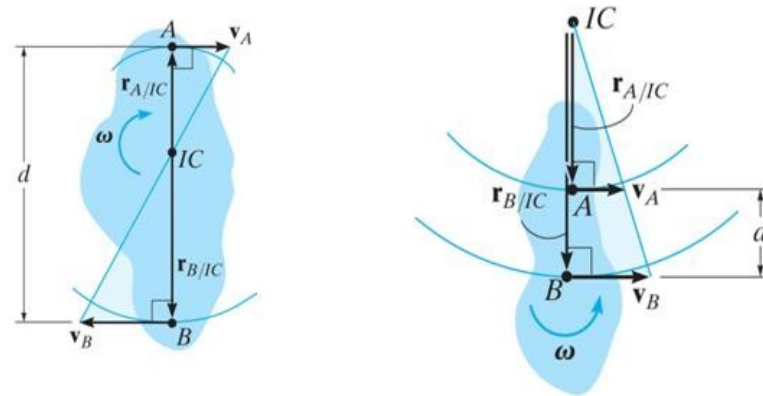
A second case occurs when the **lines of action of two non-parallel velocities, v_A and v_B** are known.

First, construct line segments from A and B perpendicular to **v_A and v_B** . The point of intersection of these two line segments locates the IC of the body.



Location of IC
knowing the directions
of v_A and v_B

Location of The Instantaneous Center (3 of 3)



Location of IC
knowing \mathbf{v}_A and \mathbf{v}_B

A third case is when the **magnitude and direction of two parallel velocities** at A and B are known. Here the location of the IC is determined by proportional triangles.

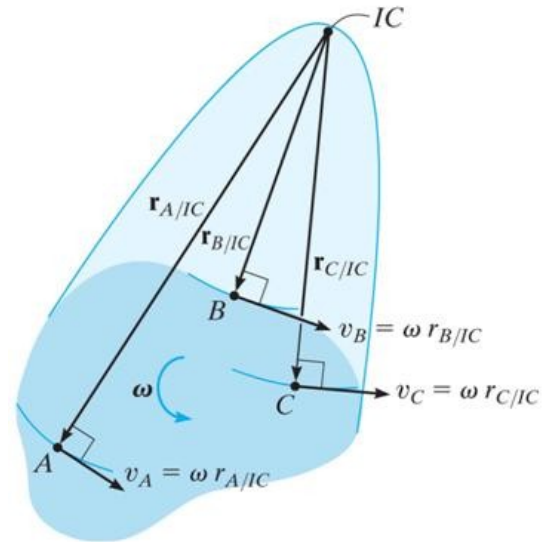
As a special case, note that if the body is translating only ($\mathbf{v}_A = \mathbf{v}_B$), then the IC would be located at infinity. Then ω equals zero, as expected.

Velocity Analysis

The velocity of any point on a body undergoing general plane motion can be determined easily, often with a scalar approach, once the instantaneous center of zero velocity of the body is located.

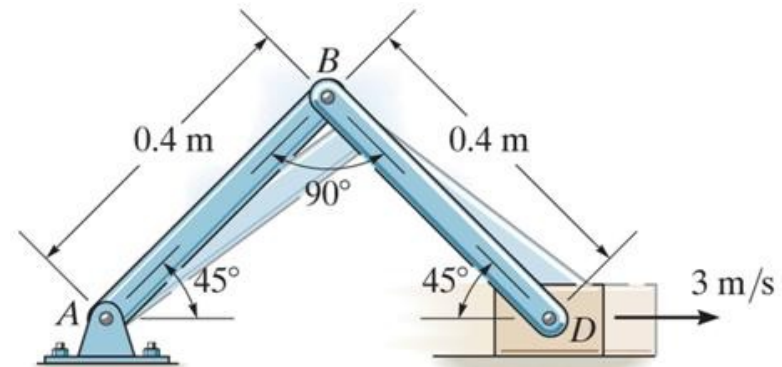
Since **the body seems to rotate about the IC at any instant**, as shown in this kinematic diagram, the magnitude of velocity of any arbitrary point is $\mathbf{v} = \omega \mathbf{r}$ where r is the radial distance from the IC to the point.

The velocity's line of action is perpendicular to its associated radial line.



Example 1 (1 of 3)

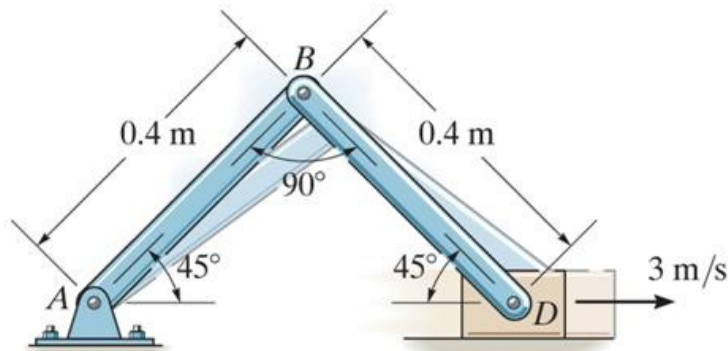
Given: A linkage undergoing motion as shown. The velocity of the block, v_D is 3 m/s



Find: The angular velocities of links AB and BD.

Plan: Locate the instantaneous center of zero velocity of link BD and then solve for the angular velocities.

Example 1 (2 of 3)



Solution: Since D moves to the right, it causes link AB to rotate clockwise about point A. The instantaneous center of velocity for BD is located at the intersection of the line segments drawn perpendicular to v_B and v_D . Note that v_B is perpendicular to link AB. Therefore we can see that the IC is located along the extension of link AB.

Example 1 (3 of 3)

Using these facts,

$$r_{B/IC} = 0.4 \tan 45^\circ = 0.4 \text{ m}$$

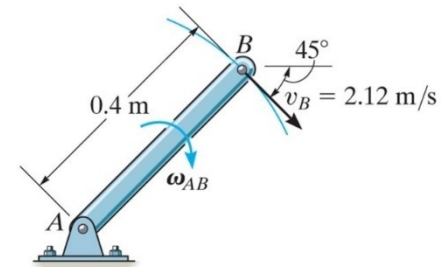
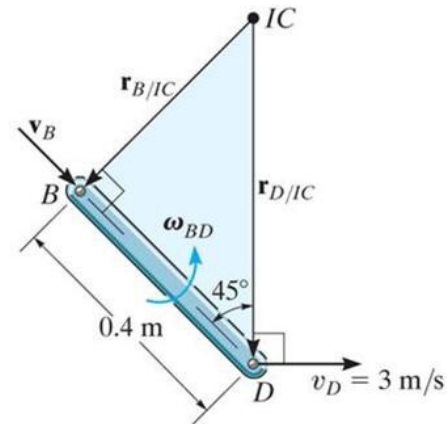
$$r_{D/IC} = 0.4 \cos 45^\circ = 0.4 \text{ m}$$

Since the magnitude of v_D is known the angular velocity of link BD can be found from $v_D = \omega_{BD} r_{D/IC}$.

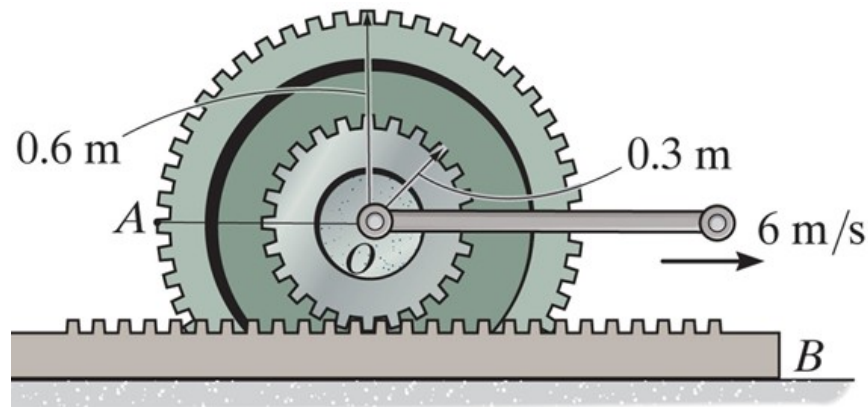
$$\omega_{BD} = v_D / r_{D/IC} = 3 / 0.566 = 5.3 \text{ rad/s}$$

Link AB is subjected to rotation about A.

$$\omega_{AB} = v_B / r_{B/A} = (r_{B/IC}) \omega_{BD} / r_{B/A} = 0.4(5.3) / 0.4 = 5.3 \text{ rad/s}$$



Example 2 (1 of 2)



Given: The center O of the gear set rolls with $v_O = 6\text{ m/s}$
The gear rack B is fixed.

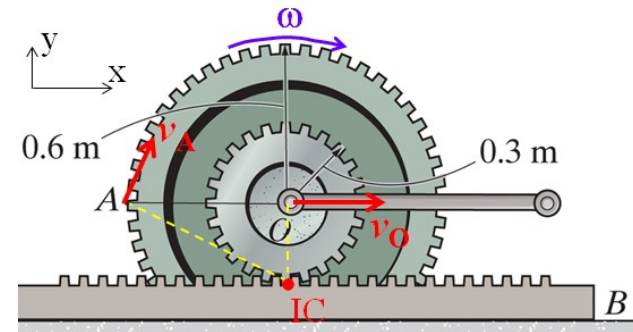
Find: The velocity of point A on the outer gear.

Plan: Locate the IC of the smaller gear. Then calculate the velocities at A .

Example 2 (2 of 2)

Solution:

Note that the gear rolls without slipping. Thus, the IC is at the contact point with the gear rack B.



The angular velocity of the wheel can be found from

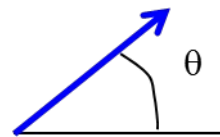
$$\omega = v_O / r_{O/IC} = 6 / 0.3 = 20 \text{ rad/s}$$

The velocity at A will be

$$v_D = \omega_{BD} \times r_{A/IC} = (-20)\mathbf{k} \times (-0.6\mathbf{i} + 0.3\mathbf{j}) = (6\mathbf{i} + 12\mathbf{j}) \text{ m/s}$$

$$v_A = \sqrt{6^2 + 12^2} = 13.4$$

$$\theta = \tan^{-1}(12/6) = 63.4^\circ$$



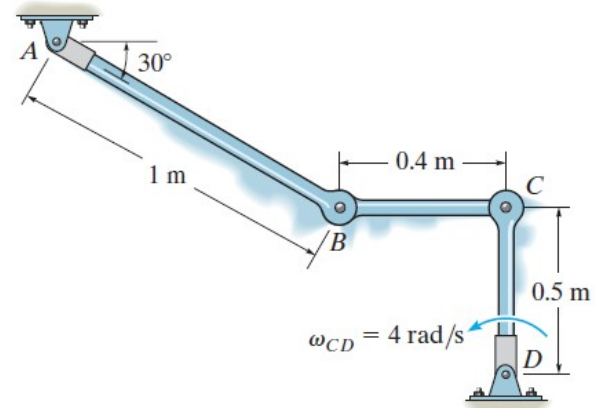
Concept Quiz

1. When the velocities of two points on a body are equal in magnitude and parallel but in opposite directions, the IC is located at
 - A) infinity.
 - B) one of the two points.
 - C) the midpoint of the line connecting the two points.
 - D) None of the above.
2. When the direction of velocities of two points on a body are perpendicular to each other, the IC is located at
 - A) infinity.
 - B) one of the two points.
 - C) the midpoint of the line connecting the two points.
 - D) None of the above.

Group Problem Solving (1 of 3)

Given: Rod CD is rotating with an angular velocity $\omega_{CD} = 4 \text{ rad/s}$ CCW

Find: The angular velocities of rods AB and BC .



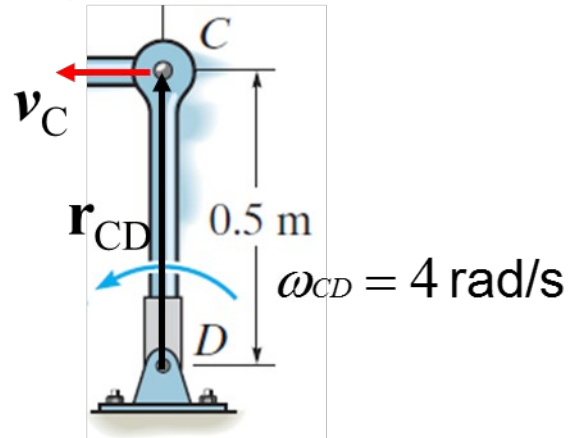
Plan: This is an example of the second case in the lecture notes. Since the direction of Point B 's velocity must be perpendicular to AB , and Point C 's velocity must be perpendicular to CD , the location of the instantaneous center, I , for link BC can be found.

Group Problem Solving (2 of 3)

Solution:

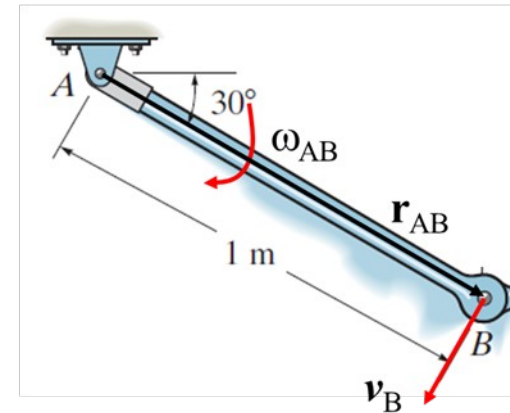
Draw kinematic diagrams for Link CD and Link AB:

Link CD:



$$\begin{aligned} v_C &= \omega_{CD}(r_{CD}) \\ &= 4(0.5) \\ &= 2 \text{ m/s} \end{aligned}$$

Link AB:



$$v_B = \omega_{AB}(r_{AB}) \text{ m/s}$$

Group Problem Solving (3 of 3)

With the results of v_B and v_D the IC for link BC can be located.

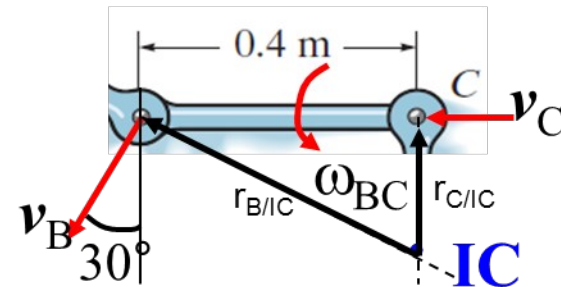
$$r_{C/IC} = (0.4) \tan 30^\circ$$

$$r_{C/IC} = 0.2309 \text{ m}$$

$$r_{B/IC} = 0.4 / \cos 30^\circ$$

$$r_{B/IC} = 0.4619 \text{ m}$$

Kinematic diagram for Link BC:



Then, the kinematics gives:

$$v_C = \omega_{BC}(r_{C/IC}) \Rightarrow 2.0 = \omega_{BC}(0.2309)$$

$$\omega_{BC} = 8.66 \text{ rad/s}$$

$$v_B = \omega_{BC}(r_{B/IC}) = \omega_{BC}(r_{AB}) \Rightarrow 8.66(0.4619) = \omega_{AB}(1)$$

$$\omega_{AB} = 4.0 \text{ rad/s}$$

Attention Quiz

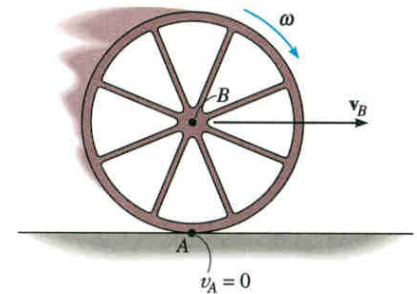
1. The wheel shown has a radius of 15 in and rotates clockwise at a rate of $\omega_{CD} = 3 \text{ rad/s}$. What is v_B ?

A) 5 in/s

B) 15 in/s

C) 0 in/s

D) 45 in/s



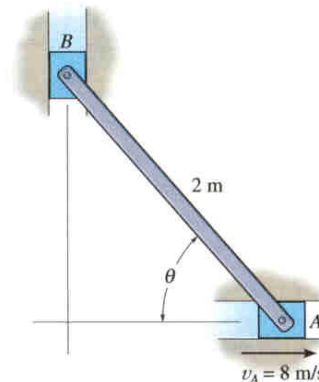
2. Point A on the rod has a velocity of 8 meters/second to the right. Where is the IC for the rod?

A) Point A.

B) Point B.

C) Point C.

D) Point D.



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