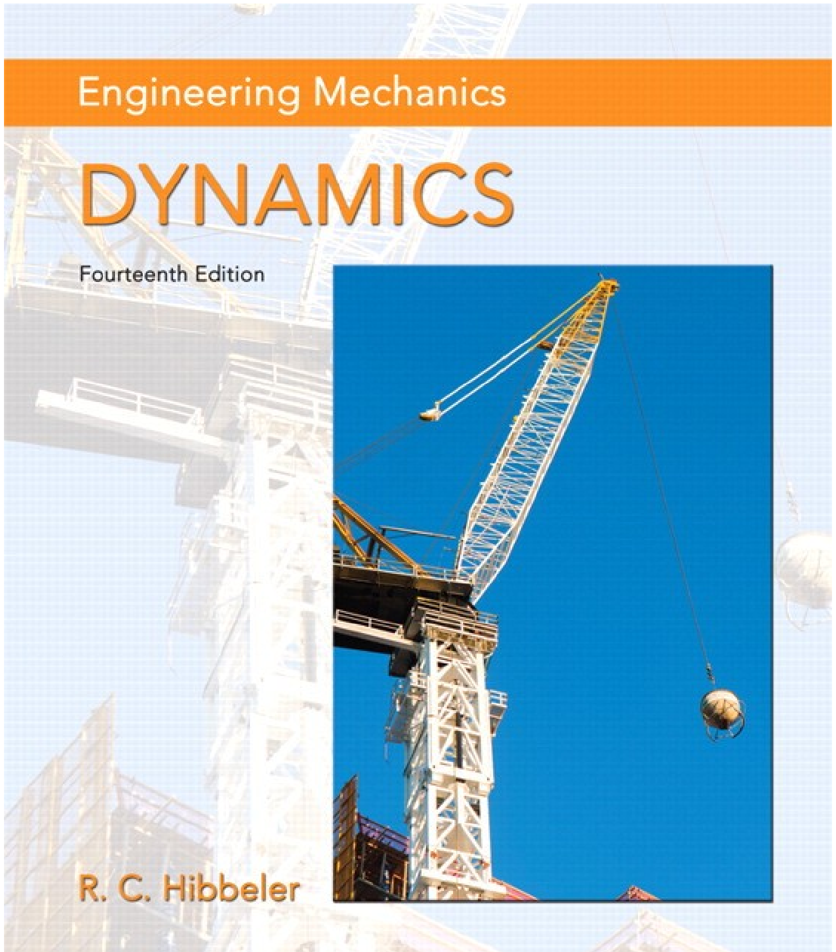
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 16

Planar Kinematics of a Rigid Body

ABSOLUTE MOTION ANALYSIS (1 of 2)

Today's Objective:

Students will be able to:

1. Determine the velocity and acceleration of a rigid body undergoing **general plane motion** using an absolute motion analysis.

ABSOLUTE MOTION ANALYSIS (2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- General Plane Motion
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. A body subjected to general plane motion undergoes a/an

A) translation.
B) rotation.
C) simultaneous translation and rotation.
D) out-of-plane movement.

2. In general plane motion, if the rigid body is represented by a slab, the slab rotates

A) about an axis perpendicular to the plane.
B) about an axis parallel to the plane.
C) about an axis lying in the plane.
D) None of the above.

Applications (1 of 3)

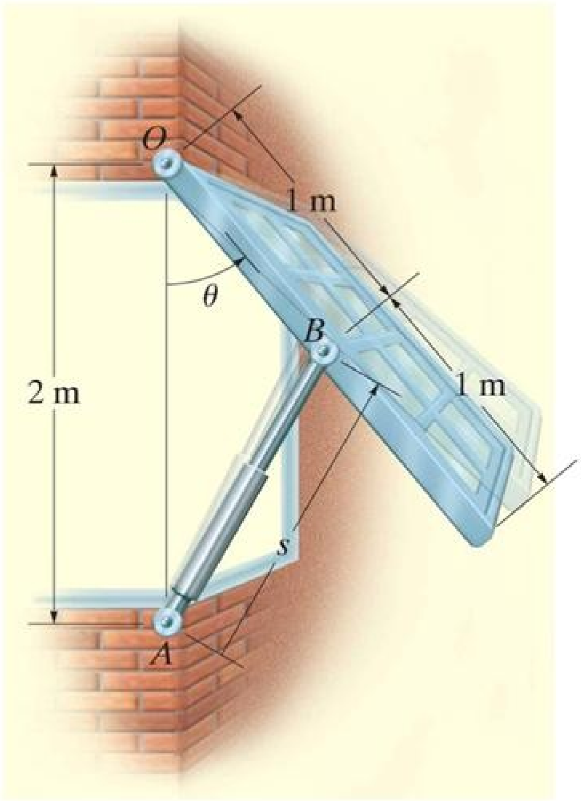
The dumping bin on the truck rotates about a fixed axis passing through the pin at A. It is operated by the extension of the hydraulic cylinder BC.

The angular position of the bin can be specified using the angular position coordinate θ and the position of point C on the bin is specified using the coordinate s.

As a part of the design process for the truck, an engineer had to relate the velocity at which the hydraulic cylinder extends and the resulting angular velocity of the bin.

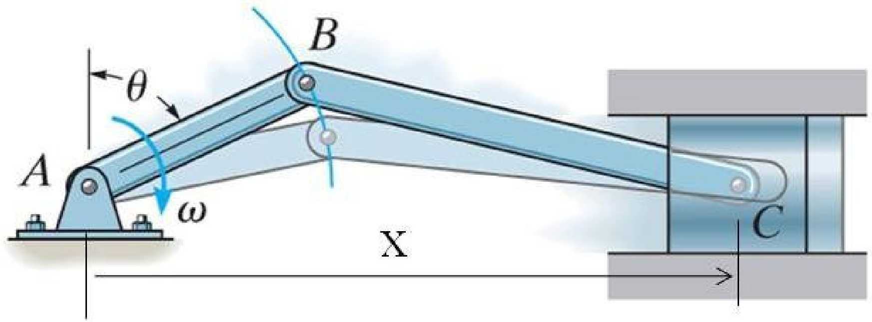
Applications (2 of 3)

The large window is opened using a hydraulic cylinder AB.
The position B of the hydraulic cylinder rod is related to the angular position, *q*, of the window.



A designer has to relate the translational velocity at B of the hydraulic cylinder and the angular velocity and acceleration of the window? How would you go about the task?

Applications (3 of 3)

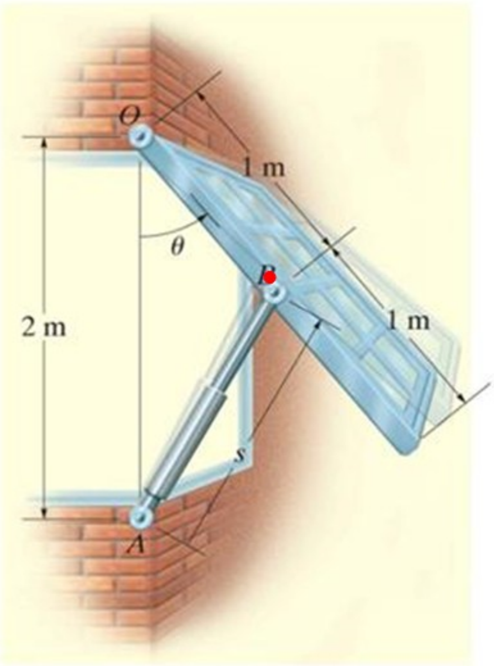


The position of the piston, x, can be defined as a function of the angular position of the crank, *q*. By differentiating x with respect to time, the velocity of the piston can be related to the angular velocity, w, of the crank. This is necessary when designing an engine.

The stroke of the piston is defined as the total distance moved by the piston as the crank angle varies from 0 to 180°. How does the length of crank AB affect the stroke?

Absolute Motion Analysis (Section 16.4)

The figure below shows the window using a hydraulic cylinder AB.



The **absolute motion analysis method** relates the position of a point, B, on a rigid body undergoing rectilinear motion to the angular position, *q*, of a line contained in the body.

Once a relationship in the form of $s_B = f(q)$

is established, the velocity and acceleration of point B are obtained in terms of the angular velocity and angular acceleration of the rigid body by taking the **first and second time derivatives** of the position function.

Usually the **chain rule** must be used when taking the derivatives of the position coordinate equation.

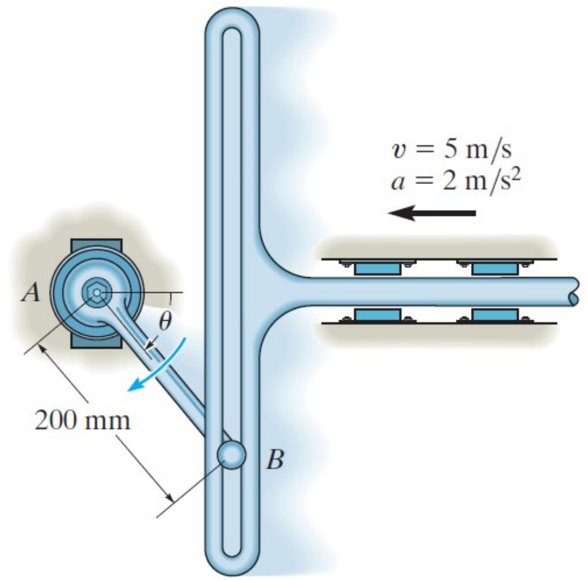
Procedure for Absolute Motion Analysis

The velocity and acceleration of a point undergoing rectilinear motion can be related to the angular velocity and angular acceleration of a line contained within a body using the following procedure.

1. Locate point on the body using position coordinate s, which is measured from a fixed origin.
2. From a fixed reference line, measure the angular position θ of a line lying in the body. Using the dimensions of the body, relate s to θ e.g., $s = f(\theta)$
3. Take the first time derivative of $s = f(\theta)$ to get a relationship between v and ω
4. Take the second time derivative to get a relationship between a and α

Example (1 of 3)

Given: Link AB is rotating clockwise direction. At the instant $\theta = 60^0$ the slotted guide rod is moving to the left with $a = 2m/s^2$ and $V = 5m/s$.



Find: The angular velocity and angular acceleration of link AB.

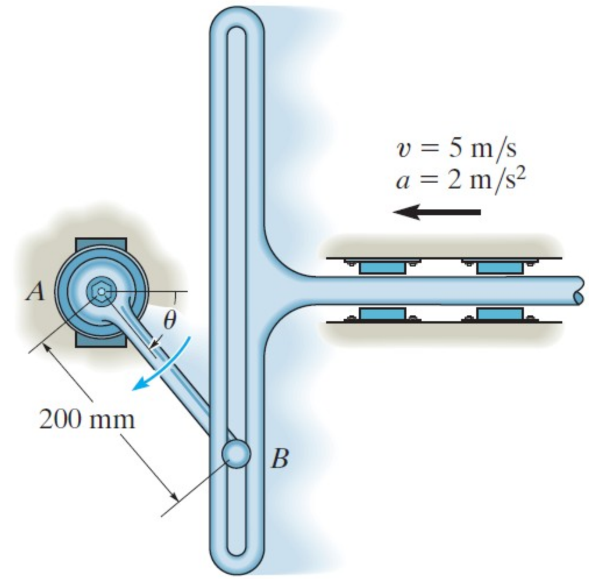
Plan: Set the coordinate x to be the distance between A and B. Relate x to the angular position, q. Then take time derivatives of the position equation to find the velocity and acceleration relationships.

Example (2 of 3)

Solution:

Relate x, the horizontal distance between A and B, to q.

$$x = 0.2\cos\theta (m)$$



Take time derivatives of the position to find the velocity and acceleration.

$$\dot{x} = 0.2(-\sin\theta)\dot{\theta}$$

$$\dot{x} = -0.2(\cos\theta)\dot{\theta}^2 - 0.2(\sin\theta)\ddot{\theta}$$

Notice that the velocity and acceleration of the guide rod are directed toward the negative sense of x.

When $\theta = 60°$, $\ddot{x} = -5$ m/s and $\ddot{x} = -2$ m/s^2.

Example (3 of 3)

Therefore,

$$-5 = 0.2(-\sin 60°)\dot{\theta}$$

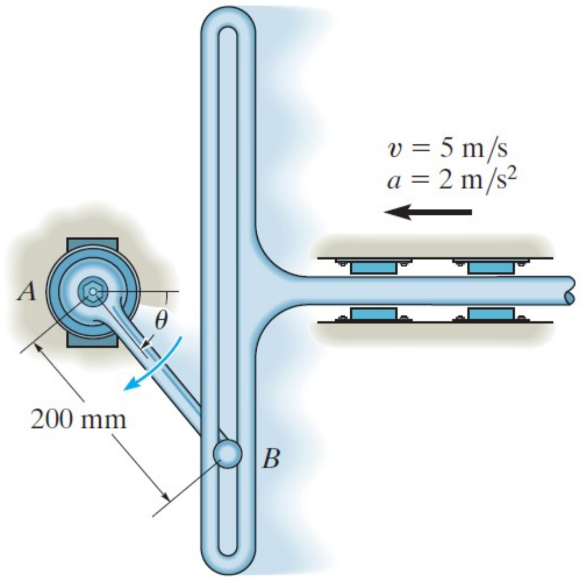
$$\dot{\theta} = \omega = 28.9 \text{ rad/s}$$

and

$$-2 = -0.2(\cos 60°)\dot{\theta}^2$$
$$-0.2(\sin 60°)\ddot{\theta}$$

$$-2 = -0.2(\cos 60°)(28.9)^2$$
$$-0.2(\sin 60°)\ddot{\theta}$$

$$\ddot{\theta} = \alpha = -470 \text{ rad/s}^2 = 470 \text{ rad/s}^2$$



Concept Quiz

1. The position, s, is given as a function of angular position, q, as $s = 10\sin 2q$. The velocity, v, is

   A) $20\cos 2\theta$ B) $20\sin 2\theta$

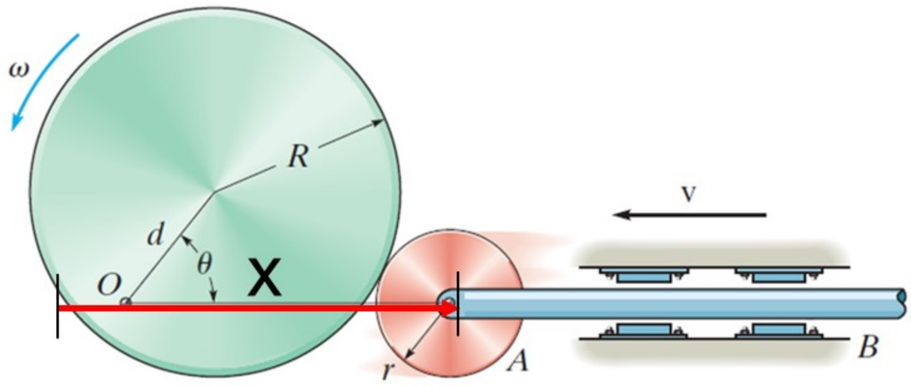
   C) $20\omega\cos 2\theta$ D) $20\omega\sin 2\theta$

2. If $s = 10\sin 2q$, the acceleration, a, is

   A) $20\alpha\sin 2\theta$ B) $20\alpha\cos 2\theta - 40\omega^2\sin 2\theta$

   C) $20\alpha\cos 2\theta$ D) $-40\alpha\sin 2\theta$

Group Problem Solving (1 of 3)



Given: The circular cam rotates about the fixed point *O* with a constant angular velocity ω

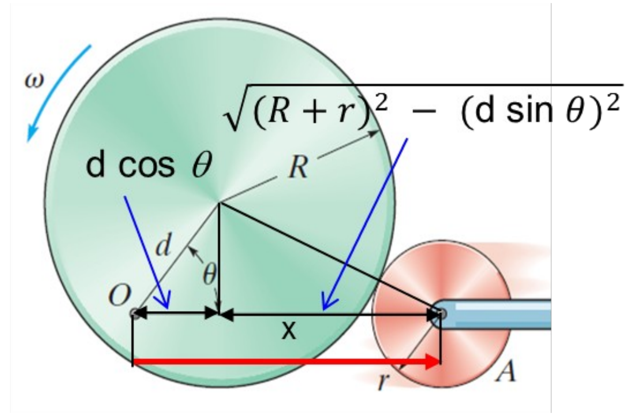
Find: The velocity **v** of the follower rod AB as a function of θ

Plan: Set the coordinate x to be distance OA. Then relate x to the angular position, q. Take time derivative of this position relationship to find the velocity.

Group Problem Solving (2 of 3)

1. Relate x, the distance OA, to θ

$$x = d\cos\theta$$
$$+\sqrt{(R+r)^2 - (d\sin\theta)^2}$$



2. Take time derivatives of **x** to find the velocity, v.

$$x = -d\sin\theta\dot{\theta} - \frac{d^2(2\sin\theta\cos\theta\dot{\theta})}{2\sqrt{(R+r)^2 - (d\sin\theta)^2}}$$

$$-d\sin\theta(\omega) - \frac{d^2(\sin 2\theta)\omega}{2\sqrt{(R+r)^2 - (d\sin\theta)^2}}$$

Group Problem Solving (3 of 3)

Note that the velocity of the rod AB is $-\dot{x}$

$$v = -\dot{x} = \omega \sin\theta + \frac{\omega d^2 (\sin 2\theta)}{2\sqrt{(R+r)^2 - (d \sin\theta)^2}}$$

Attention Quiz

1. The sliders shown below are confined to move in the horizontal and vertical slots. If $V_A = 10\, m/s$, determine the connecting bar's angular velocity when $\theta = 30^0$

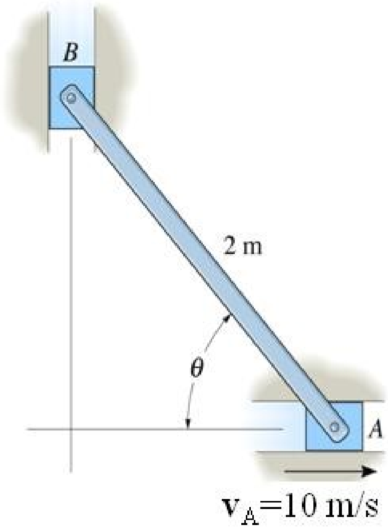
   A) 10 rad/s ↺ B) 10 rad/s ↻

   C) 8.7 rad/s ↻ D) 8.7 rad/s ↺

2. If $V_A = 10$ m/s, and $a_A = 10$ m/s, determine the angular acceleration, a, when

   A) 0 rad/s^2 B) - 50.2 rad/s^2

   C) - 112 rad/s^2 D) - 173 rad/s^2



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