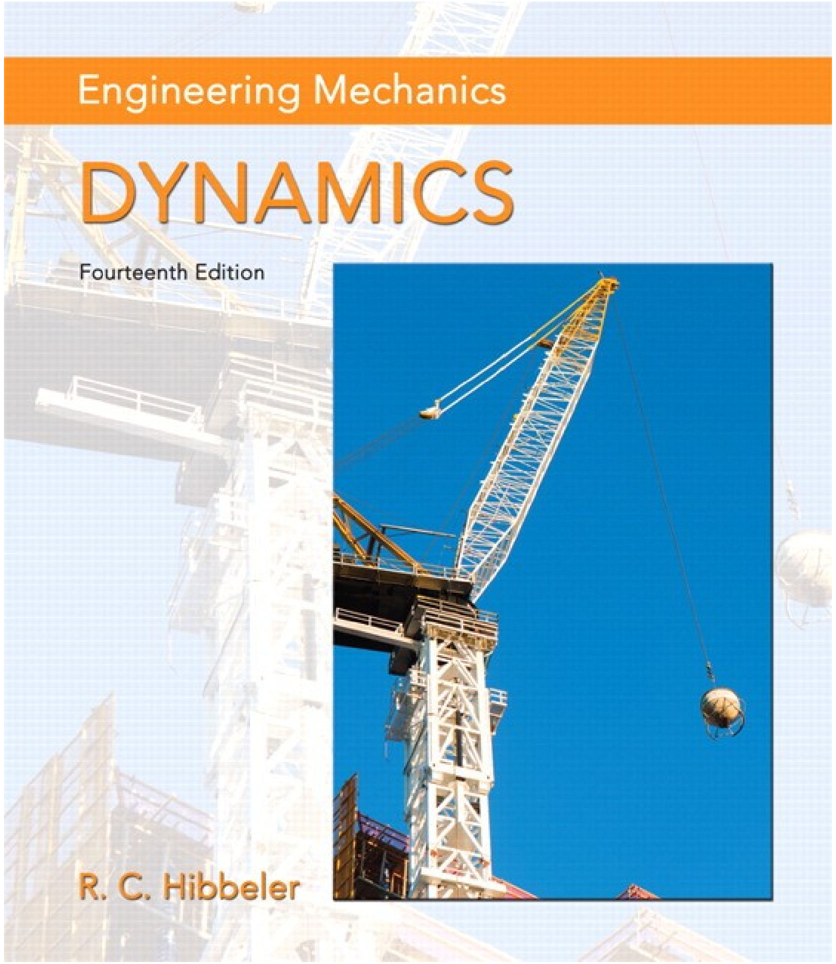
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 15

Kinetics of a Particle: Impulse and Momentum

Impact (1 of 2)

Today's Objectives:

Students will be able to:

1. Understand and analyze the mechanics of impact.
2. Analyze the motion of bodies undergoing a collision, in both central and oblique cases of impact.

Impact (2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Central Impact
- Coefficient of Restitution
- Oblique Impact
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1. When the motion of one or both of the particles is at an angle to the line of impact, the impact is said to be __________

   A) central impact. B) oblique impact.

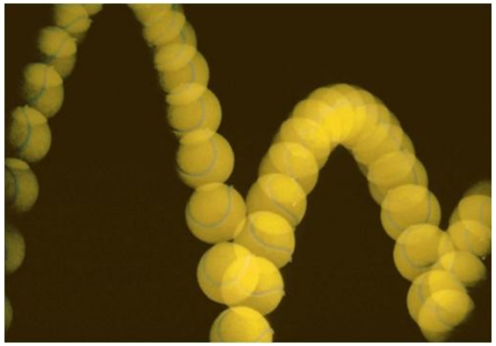
   C) major impact. D) None of the above.

2. The ratio of the restitution impulse to the deformation impulse is called ___________

   A) impulse ratio. B) restitution coefficient.

   C) energy ratio. D) mechanical efficiency.

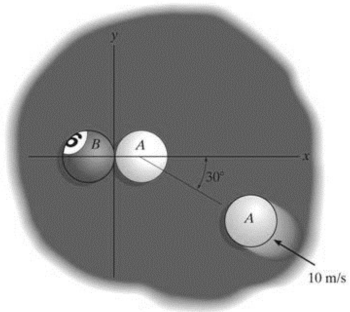
Applications (1 of 2)



The quality of a tennis ball is measured by the height of its bounce. This can be quantified by the **coefficient of restitution** of the ball.

If the height from which the ball is dropped and the height of its resulting bounce are known, how can we determine the coefficient of restitution of the ball?

Applications (2 of 2)



In the game of billiards, it is important to be able to predict the trajectory and speed of a ball after it is struck by another ball.

If we know the velocity of ball A before the impact, how can we determine the magnitude and direction of the velocity of ball B after the impact?

What parameters would we need to know to do this?

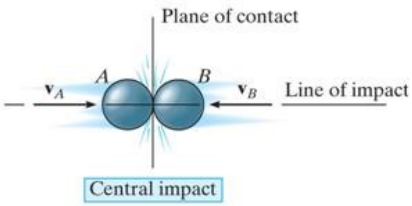
Section 15.4

Impact

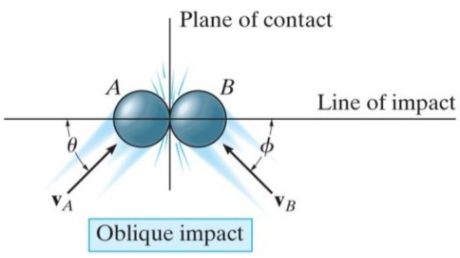
Impact

Impact occurs when two bodies collide during a very **short** time period, causing large impulsive forces to be exerted between the bodies. Common examples of impact are a hammer striking a nail or a bat striking a ball. The **line of impact** is a line through the **mass centers** of the colliding particles. In general, there are **two** types of impact:

Central impact occurs when the directions of motion of the two colliding particles are along the line of impact.

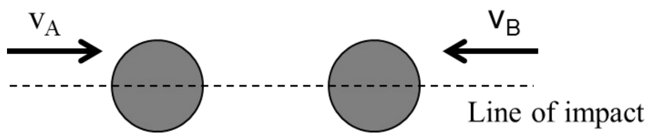


Oblique impact occurs when the direction of motion of one or both of the particles is at an angle to the line of impact.

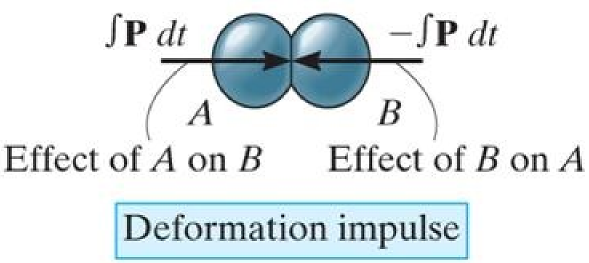


Central Impact (1 of 3)

Central impact happens when the velocities of the two objects are along the line of impact (recall that the line of impact is a line through the particles' mass centers).



Once the particles contact, they may **deform** if they are non-rigid. In any case, energy is transferred between the two particles.



There are two primary equations used when solving impact problems. The textbook provides extensive detail on their derivation.

Central Impact (2 of 3)

In most problems, the initial velocities of the particles, $(v_A)_1$ and $(v_B)_1$ are known, and it is necessary to determine the final velocities, $(v_A)_2$ and $(v_B)_2$ So the **first** equation used is the **conservation of linear momentum,** applied along the line of impact.

$$(m_A v_A)_1 + (m_B v_B)_1 = (m_A v_A)_2 + (m_B v_B)_2$$

This provides one equation, but there are usually two unknowns, $(v_A)_2$ and $(v_B)_2$ So another equation is needed. The **principle of impulse and momentum** is used to develop this equation, which involves the **coefficient of restitution, or e.**

Central Impact (3 of 3)

The **coefficient of restitution, e,** is the ratio of the particles ' **relative separation velocity** after impact, $(v_B)_2 - (v_A)_2$, to the particles' **relative approach velocity** before impact, $(v_A)_1 - (v_B)_1$. The coefficient of restitution is also an indicator of the energy lost during the impact.

The equation defining the coefficient of restitution, *e*, is

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

If a value for *e* is specified, this relation provides the second equation necessary to solve for $(v_A)_2$ and $(v_B)_2$

Coefficient of Restitution

In general, e has a value between zero and one. The two limiting conditions can be considered:

Elastic impact (e = 1): In a perfectly elastic collision, no energy is lost and the relative separation velocity equals the relative approach velocity of the particles. In practical situations, this condition cannot be achieve

Plastic impact (e = 0): In a plastic impact, the relative separation velocity is zero. The particles stick together and move with a common velocity after the impact.

Some typical values of e are:

Steel on steel: 0.5 - 0.8 Wood on wood: 0.4 - 0.6

Lead on lead: 0.12 - 0.18 Glass on glass: 0.93 - 0.95

Impact: Energy Losses

Once the particles' velocities before and after the collision have been determined, the **energy loss** during the collision can be calculated on the basis of the difference in the particles' **kinetic energy.** The energy loss is

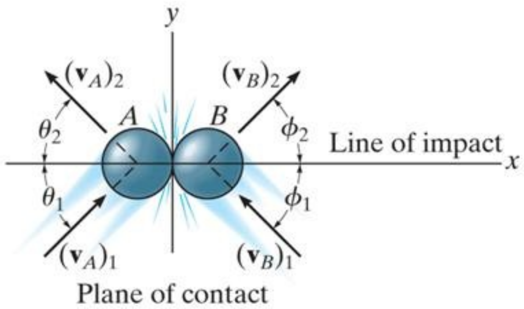
$$\sum U_{1-2} = \sum T_2 - \sum T_1 \text{ where } T_i = 0.5 m_i (v_j)^2$$

During a collision, some of the particles' initial kinetic energy will be lost in the form of heat, sound, or due to localized deformation.

In a **plastic collision** ($e = 0$), the energy lost is a maximum, although the energy of the combined masses does not necessarily go to zero. Why?

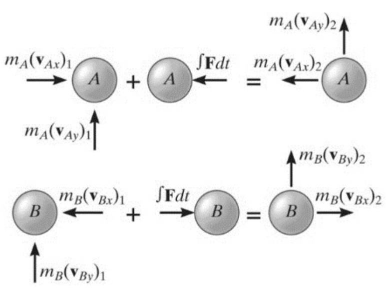
Oblique Impact

In an **oblique impact**, one or both of the particles' motion is at an angle to the line of impact. Typically, there will be four unknowns: the **magnitudes** and **directions** of the final velocities.



The four equations required to solve for the unknowns are:

Conservation of momentum and the coefficient of restitution equation are applied **along** the line of impact (x-axis):



$$m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$$

$$e = [(v_{Bx})_2 - (v_{Ax})_2]/[(v_{Ax})_1 - (v_{Bx})_1]$$

Momentum of each particle is conserved in the direction **perpendicular** to the line of impact (y-axis):

$$m_A(v_{Ay})_1 = m_A(v_{Ay})_2 \text{ and } m_B(v_{By})_1 = m_B(v_{By})_2$$

Procedure for Analysis

- In most impact problems, the initial velocities of the particles and the coefficient of restitution, *e*, are known, with the final velocities to be determined.
- Define the x-y axes. Typically, the **x-axis** is defined **along** the line of impact and the **y-axis** is in the plane of contact **perpendicular** to the x-axis
- For both **central and oblique** impact problems, the following equations apply **along** the line of impact (x-dir.):

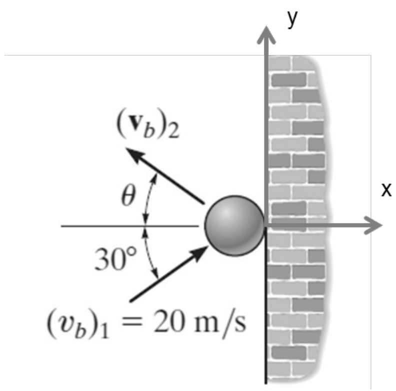
  $\sum m(v_x)_1 = \sum m(v_x)_2$ and $e = \left[(v_{Bx})_2 - (v_{Ax})_2\right] / \left[(v_{Ax})_1 - (v_{Bx})_1\right]$

- For **oblique** impact problems, the following equations are also required, applied **perpendicular** to the line of impact (y-dir.):

  $m_A(v_{Ay})_1 = m_A(v_{Ay})_2$ and $m_B(v_{By})_1 = m_B(v_{By})_2$

Example 1 (1 of 2)

Given: The ball strikes the smooth wall with a velocity $(v_b)_1 = 20\ m/s.$ The coefficient of restitution between the ball and the wall is e = 0.75.



Find: The velocity of the ball just after the impact.

Plan: The collision is an **oblique impact**, with the line of impact perpendicular to the plane (through the relative centers of mass).

Thus, the coefficient of restitution applies perpendicular to the wall and the momentum of the ball is conserved along the wall.

Example 1 (2 of 2)

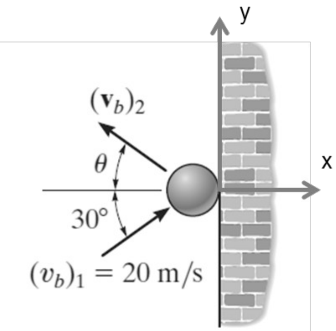
Solution:

Solve the impact problem by using x-y axes defined along and perpendicular to the line of impact, respectively:

The momentum of the ball is **conserved in the y-dir:**

$$m(v_b)_1 \sin 30^\circ = m(v_b)_2 \sin\theta$$

$$(v_b)_2 \sin\theta = 10 m/s$$



The coefficient of restitution applies in the **x-dir**:

$$e = [0 - (v_{bx})_2]/[(v_{bx})_1 - 0]$$

$$\Rightarrow 0.75 = [0 - (-v_b)_2 \cos\theta]/[20\cos 30^\circ - 0]$$

$$\Rightarrow (v_b)_2 \cos\theta = 12.99 m/s \quad (2)$$

Using Eqs. (1) and (2) and solving for the velocity and θ yields

$$(v_b)_2 = \sqrt{12.99^2 + 10^2} = 16.4 m/s$$

$$\theta = \tan^{-1}(10/12.99) = 37.6$$

Concept Quiz

1. Two balls impact with a coefficient of restitution of 0.79. Can one of the balls leave the impact with a kinetic energy greater than before the impact?

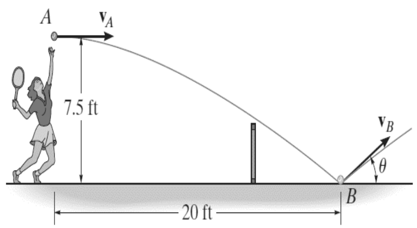
   A) Yes
   B) No
   C) Impossible to tell
   D) Don’t pick this one!

2. Under what condition is the energy lost during a collision maximum?

   A) $e = 1.0$
   B) $e = 0.0$
   C) $-e = 1.0$
   D) Collision is non-elastic.

Group Problem Solving (1 of 3)

Given: The tennis ball is served horizontally 7.5 feet above the ground and strikes the smooth ground at *B*. It is known that *e* = 0.8.



Find: v_A and v_B the speed of the ball just after it strikes the court.

Plan:

1) Using projectile motion analysis, determine v_A and the speed of the ball just before hitting the ground.

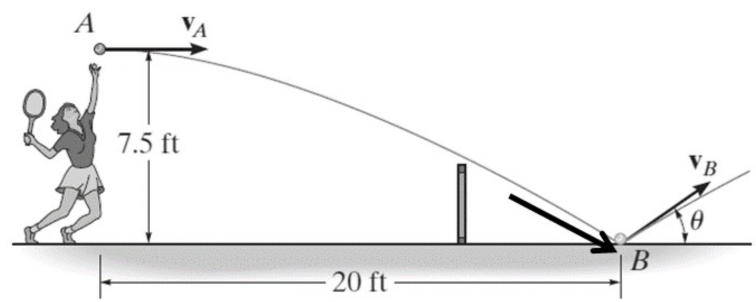
2) Apply the coefficient of restitution in the y-dir motion, and the conservation of momentum in the x-dir motion.

3) Find θ using the two components of v_B

Group Problem Solving (2 of 3)

Solution:

1) By considering the vertical motion of the falling ball, we have:



$$+\downarrow \; S_y = S_{0y} + v_{0y} t + 0.5 g\, t^2$$

$$7.5 = 0 + 0 + 0.5(32.2)t^2$$

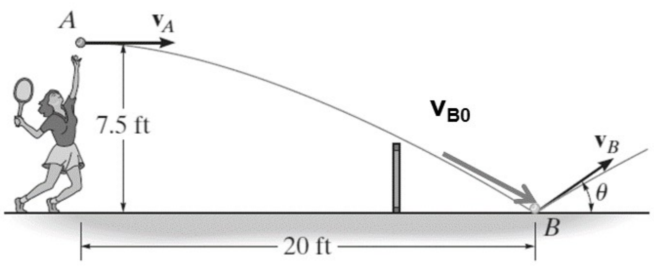
$$t = 0.6852 \text{ s}$$

$$+\downarrow \; V_{B0y} = V_{0y} + gt = 0 + 32.2(0.6825) = 21.98\, ft/s$$

$$+\rightarrow \; d = v_A t \Rightarrow v_A = 20/0.6825 = 29.3 \text{ ft/s}$$

Group Problem Solving (3 of 3)

2) Apply the coefficient of restitution in the *y*-dir to determine the velocity of the ball just after it rebounds from the ground



$$e = \frac{v_{By}}{v_{B0y}} \Rightarrow 0.7 = \frac{v_{By}}{21.98}$$

$$v_{By} = 15.38 \text{ ft/s} \uparrow$$

Apply **conservation of momentum** to the system in the x-dir :

$$+\rightarrow \; m(V_{B0x}) = m(V_{Bx}) \Rightarrow V_{Bx} = V_{B0x} = V_A = 29.3 \, ft/s \rightarrow$$

Speed and angle of the ball just after it strikes the court :

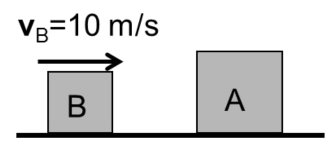
$$v_B = (29.3i + 15.4j) \, ft/s$$

$$\theta = \tan^{-1}(15.4/29.3) = 27.7^\circ$$

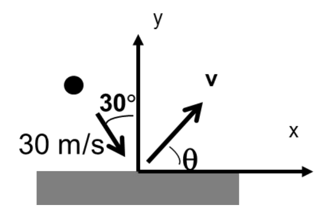
Attention Quiz

1. Block B (1 kg) is moving on the smooth surface at $10m/s$ when it squarely strikes block A (3 kg), which is at rest. If the velocity of block A after the collision is $4m/s$ to the right, $(v_B)_2$ is

   A) $2m/s \rightarrow$
   B) $7m/s \leftarrow$
   C) $7m/s \rightarrow$
   D) $2m/s \leftarrow$



2. A particle strikes the smooth surface with a velocity of $30m/s$ if $e = 0.8, (v_x)_2$ is ______ after the collision.



   A) zero
   B) equal to $(v_x)_1$
   C) less than $(v_x)_1$
   D) greater than $(v_x)_1$

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