HUMBER ENGINEERING

MENG 3020
SYSTEMS MODELING & SIMULATION

LECTURE 1

HUMBER

WE ARE HUMBER

LECTURE 1
Introduction to Systems & System Modeling

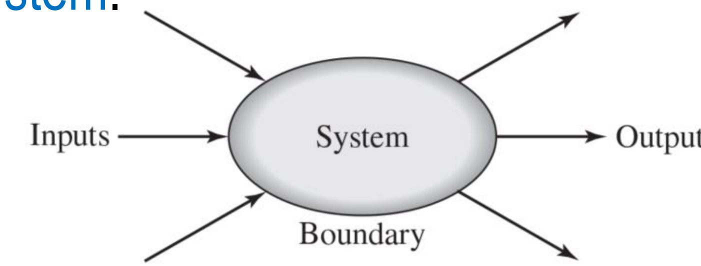
- Basic Definitions and Terminologies
- Introduction to Systems Modeling
- System Decomposition & Model Complexity
- Mathematical Modeling of Dynamic Systems
- System Modeling Procedure
- Simulation of Dynamic Systems

Basic Definitions and Terminologies

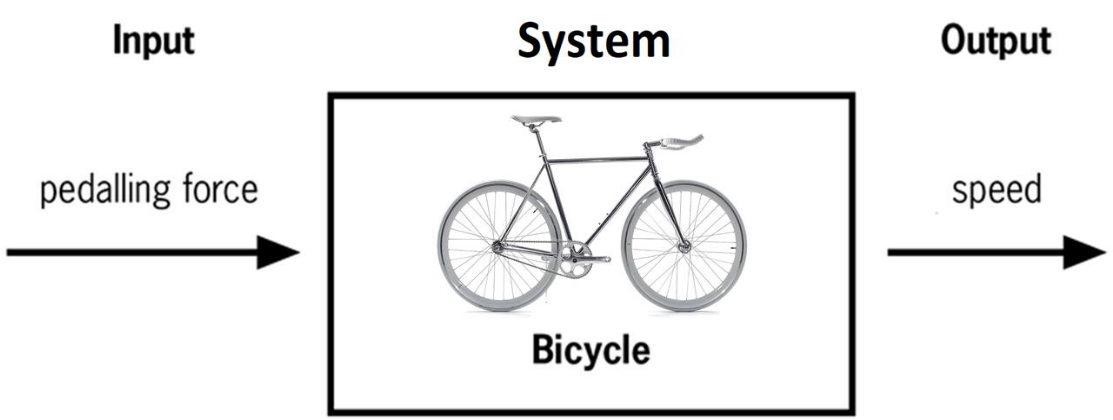
- A **system** is a combination of components or subsystems acting together to perform a specific objective.
- The **component** is a single function unit of a system.
  - Home Heating System
  - Industrial Robot
  - Laptop / Cell Phone
  - Manufacturing Process
  - Aircraft / Spacecraft
  - Automobile / Bicycle
  - Washing Machine
  - Road Traffic System
  - Solar System
  - Stock Market
  - ....
- There is a cause-and-effect (input-output) relationship between the system components and the systems and the world external.

Basic Definitions and Terminologies

- **Inputs:** Any causes or excitations acting on the system from the world external to drive the system.
  - Force or torque applied to a mass
  - Voltage or current source applied to an electrical circuit
  - Pressure source applied to liquid in a pipe

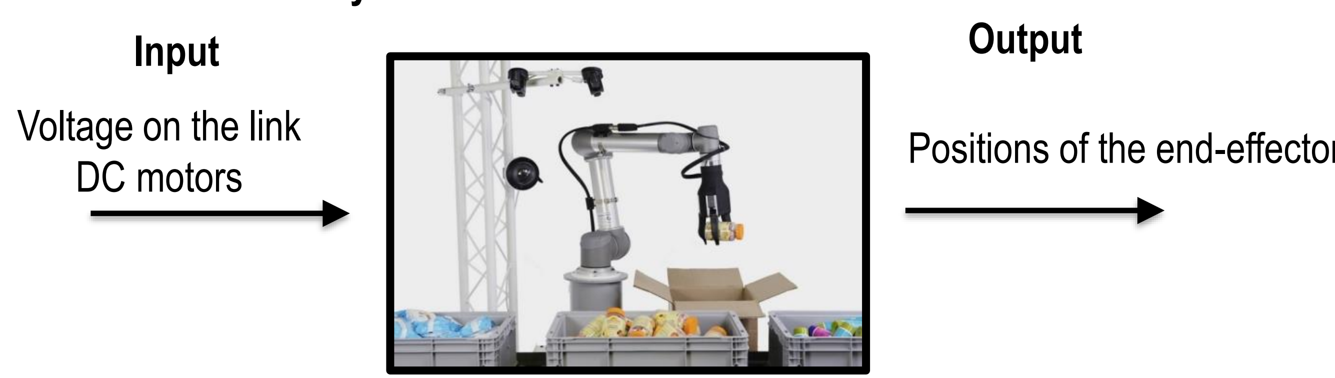


- **Outputs**: Variables of interest to be observed or measured by sensors to assess the dynamic condition of the system.
  - Speed of a car
  - Voltage across a resistor
  - Rate at which a liquid flows through a pipe
- **State Variables**: Variables to represent the internal status or memory of the system, that are used to mathematically model the dynamic behavior of the system.



State Variables: Handlebars angle, Angular acceleration

System: Pick-and-Place Robot Arm

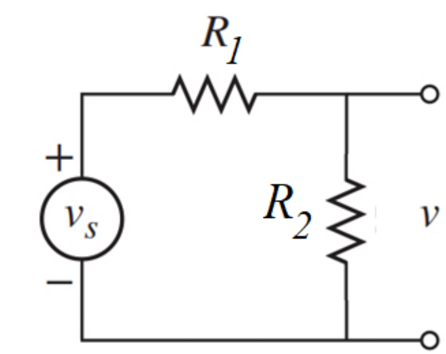


State Variables: DC Motor speed, Arm rotation angle

Basic Definitions and Terminologies

❑ Static System / Dynamic System

- **Static System:** The relationship between the input and output is **fixed** and does not change over time.
- ***Static systems have no memory (no energy storage element)***
- ***These systems are modeled by algebraic equations.***

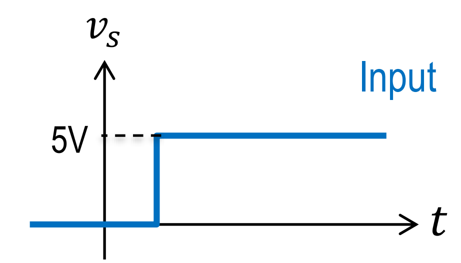


  - For example, a purely resistive circuit is a static system

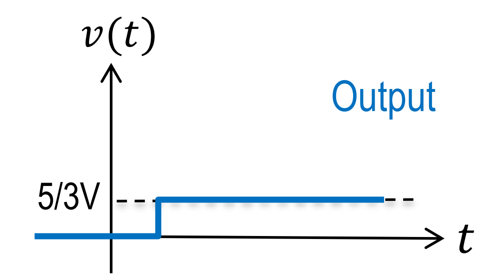
    Assume that the supplied voltage v_s is the input and voltage across resistor R_2 is the output.

    The input-output relationship is:

$$\boxed{v(t) = \frac{R_2}{R_1 + R_2} v_s(t)}$$



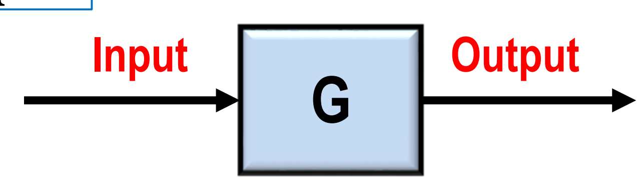
Assume $R_1 = 100k\Omega, \quad R_2 = 50k\Omega \quad \rightarrow \quad v(t) = \frac{1}{3} v_s(t)$



- In general, for such a system where the output is directly proportional to the input, we can write

$$\boxed{\text{Output} = G \times \text{Input}}$$

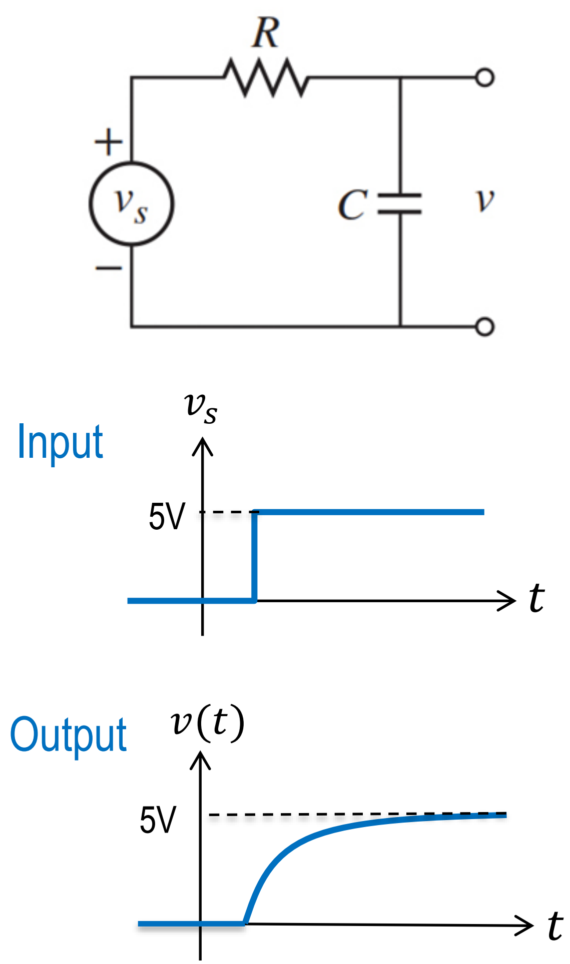
with G being a constant called the **Gain**.

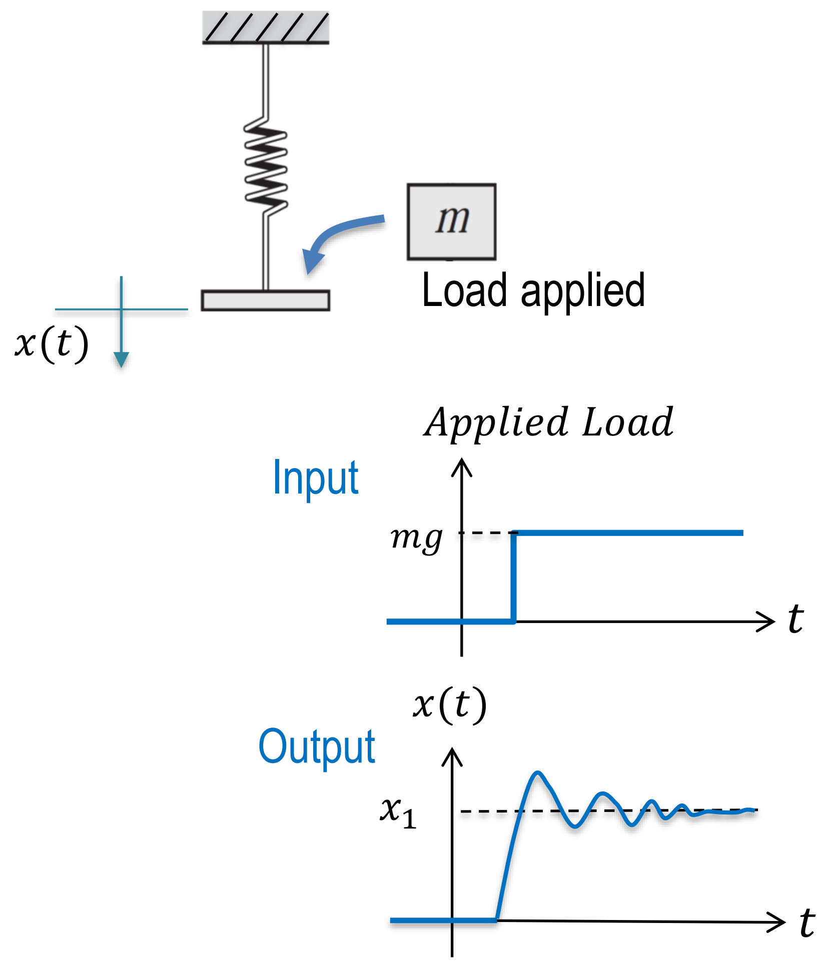


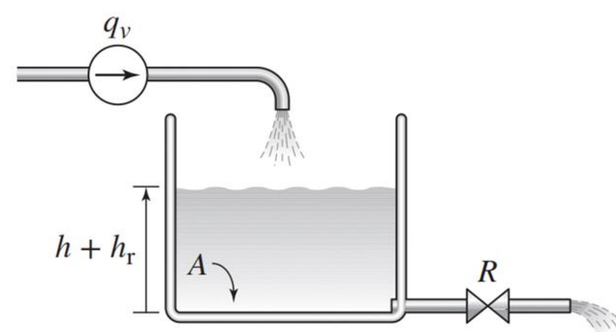
Basic Definitions and Terminologies

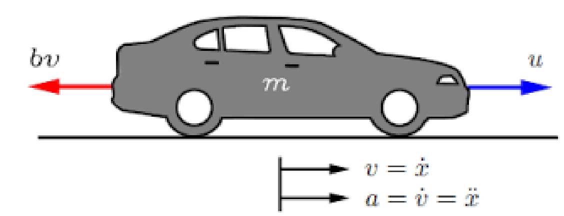
❑ Static System / Dynamic System

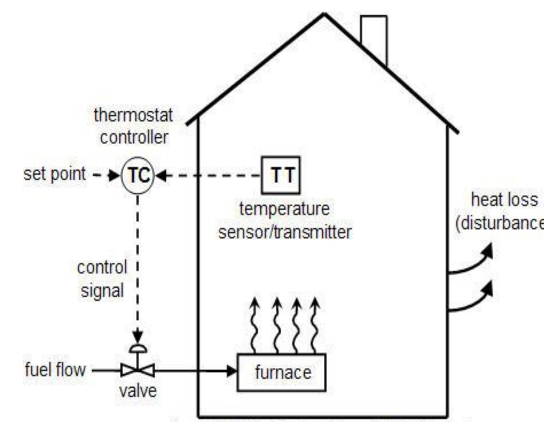
- **Dynamic System:** The relationship between the <u>input</u> and <u>output</u> is **not fixed** and changes over time.
- ***Dynamic systems have memory (energy storage element)***
- ***These systems are modeled by differential equations not just a simple gain.***
  - RLC circuits, an aircraft, an automobile, a robot, home heating system, mass-spring balanced system, …









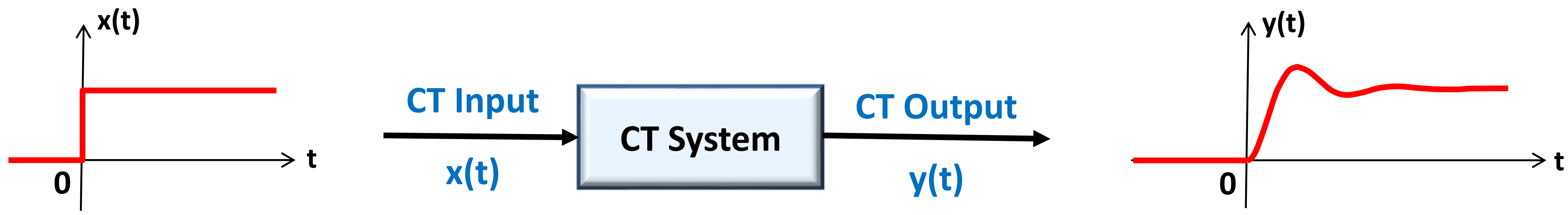


Home-heating System

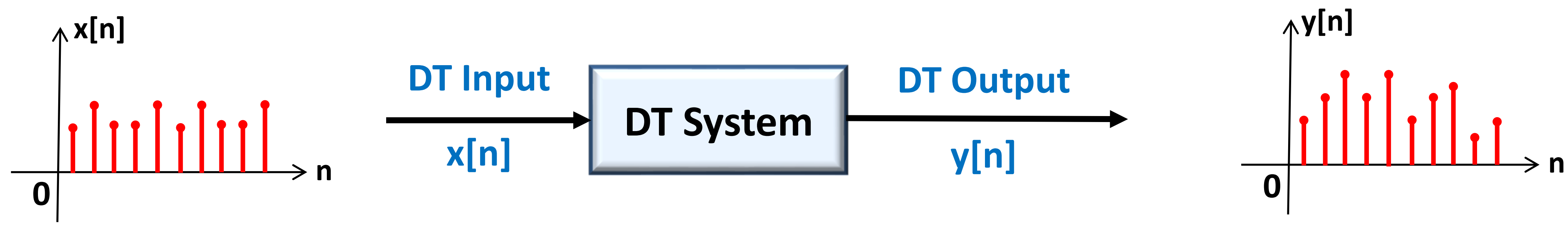
Basic Definitions and Terminologies

❑ Continuous-Time Systems / Discrete-Time Systems

- **Continuous-Time Systems:** The inputs, outputs, and state variables are defined over some continuous range of time. CT systems are described by *differential equations*.
  - Magnitudes like temperature, position, speed, pressure, etc. all of them are CT variables.



- **Discrete-Time Systems:** The system inputs, outputs, and state variables only have a value for discrete times. DT systems are described by *difference equations*.

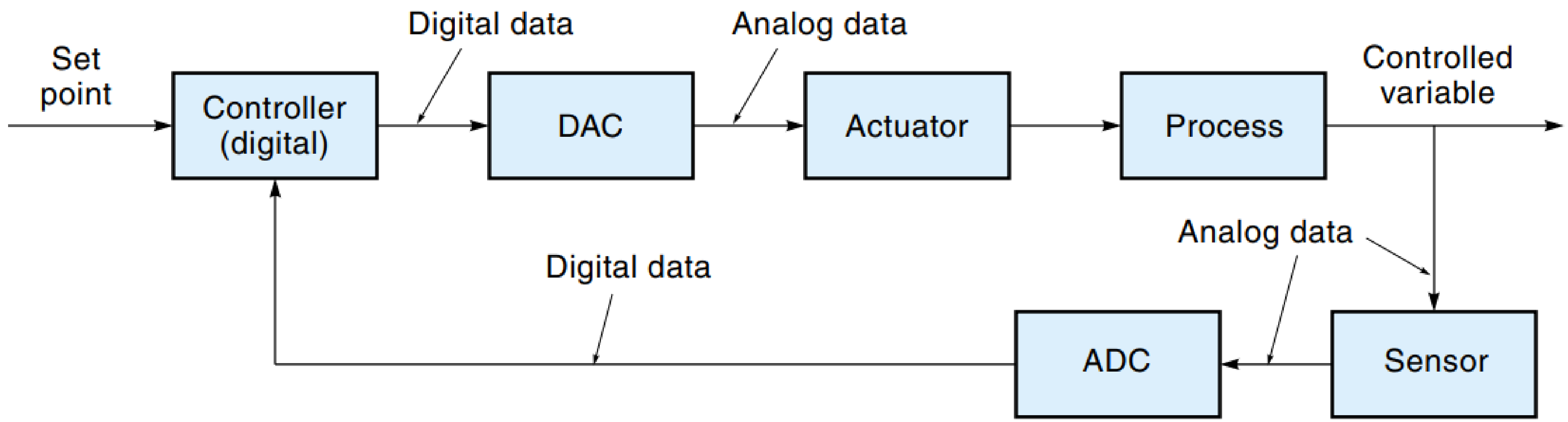


- Discrete-time variables can be obtained by taking the values (samples) of CT variables at instants separated by a sample time.

Basic Definitions and Terminologies

❑ Continuous-Time Systems / Discrete-Time Systems

- Usually, the DT systems are implemented in a digital computer to process CT signals.
- This can be achieved by converting a CT Analog signal into DT Digital signal through an Analog-To-Digital Converter cat the front end and through a Digital-To-Analog Converter at the output stage.



- Many modern control systems contain a computer as a subsystem.
- The variables that are associated with the computer are discrete in time, but the variables in the physical system are continuous in time.

Basic Definitions and Terminologies

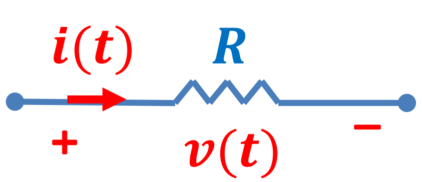
❑ Linear Systems / Non-linear Systems

- The system is called a Linear System if the system satisfies the Homogeneity and Superposition properties:

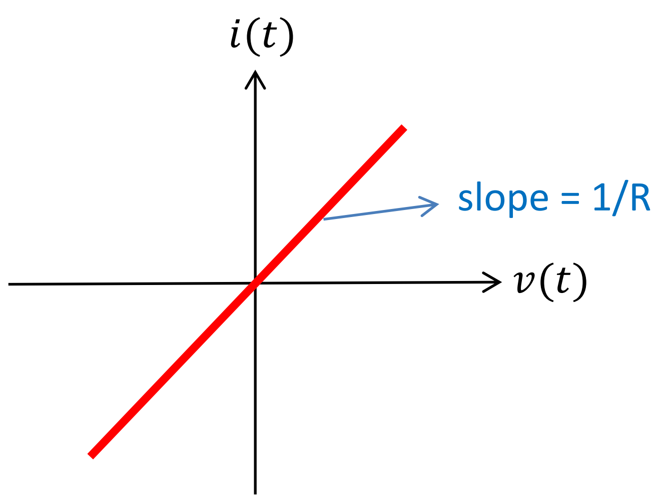
$x_1(t) \rightarrow \boxed{S} \rightarrow y_1(t)$ $\qquad$ $ax_1(t) \rightarrow \boxed{S} \rightarrow ay_1(t)$ $\qquad$ **Homogeneity**

$x_2(t) \rightarrow \boxed{S} \rightarrow y_2(t)$ $\qquad$ $x_1(t) + x_2(t) \rightarrow \boxed{S} \rightarrow y_1(t) + y_1(t)$ $\qquad$ **Superposition**

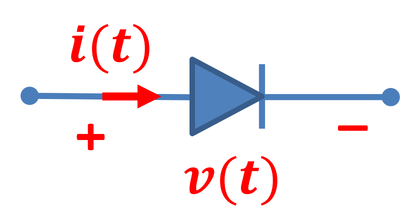
- For example, in electric systems resistor is a <u>linear</u> component, but diode is a <u>non-linear</u> component.



input $v(t) \rightarrow \boxed{\text{Resistor}} \rightarrow$ output $i(t)$

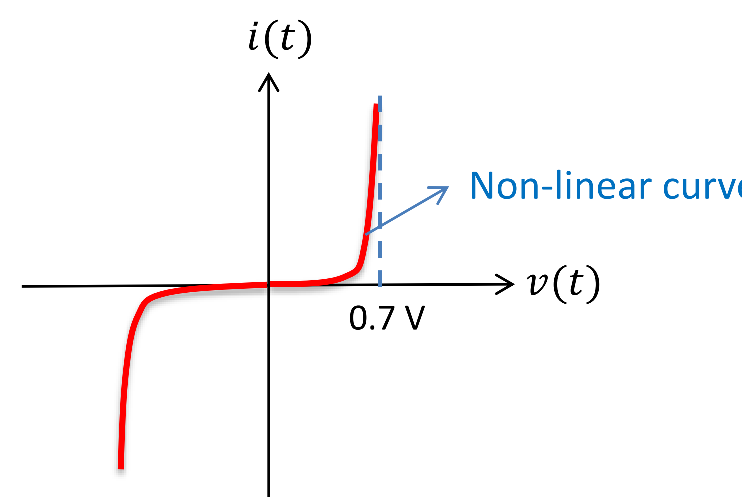


$$\boxed{i(t) = \frac{v(t)}{R}}$$



input $v(t) \rightarrow \boxed{\text{Diode}} \rightarrow$ output $i(t)$

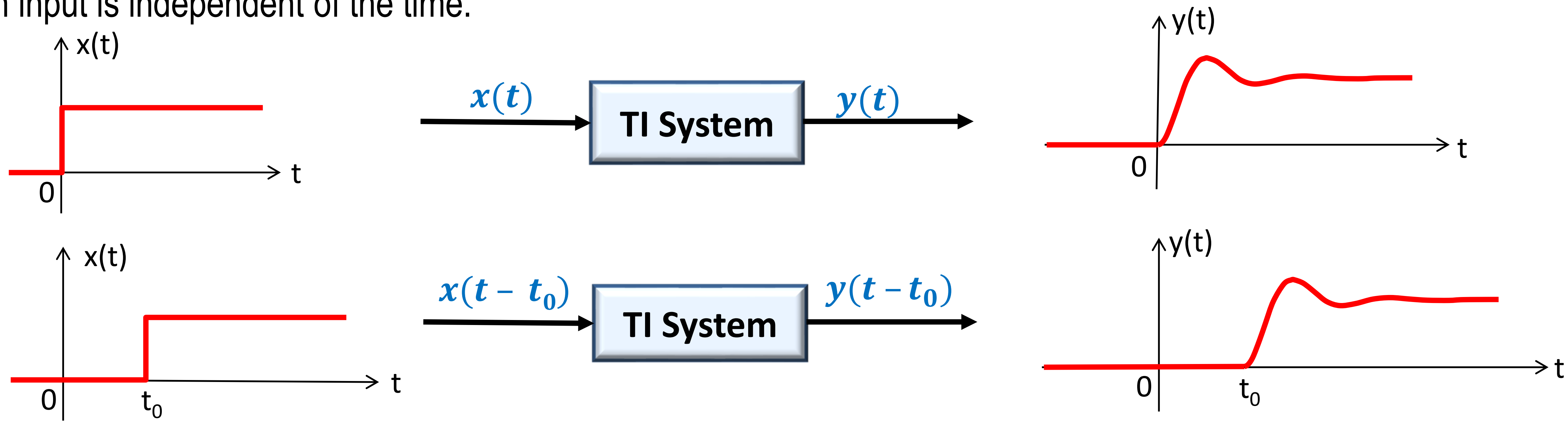
$$\boxed{i(t) = I_s(e^{kv(t)} - 1)}$$



Basic Definitions and Terminologies

❑ Time-Varying Systems / Time-Invariant Systems

- **Time-Invariant Systems:** The system parameters remain constant in time, which means the system response of a certain input is independent of the time.



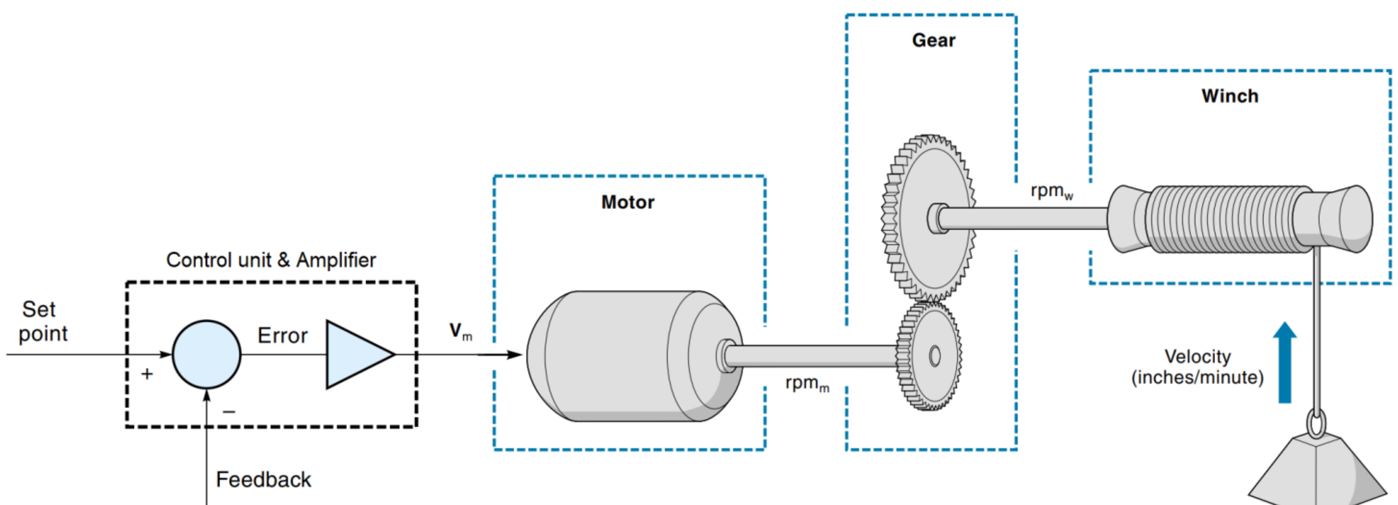
- **Time-Varying Systems:** The system parameters vary with time.
  - For example, during the course of a given flight, the mass of the aircraft will change. As the aircraft burns fuel, its <u>mass decreases</u>, and that changes the dynamics of the aircraft.
  - If the mass change is small, the change in the dynamics is small enough that we might call the system time invariant. For example, car can be considered as a <u>time-invariant</u> system.

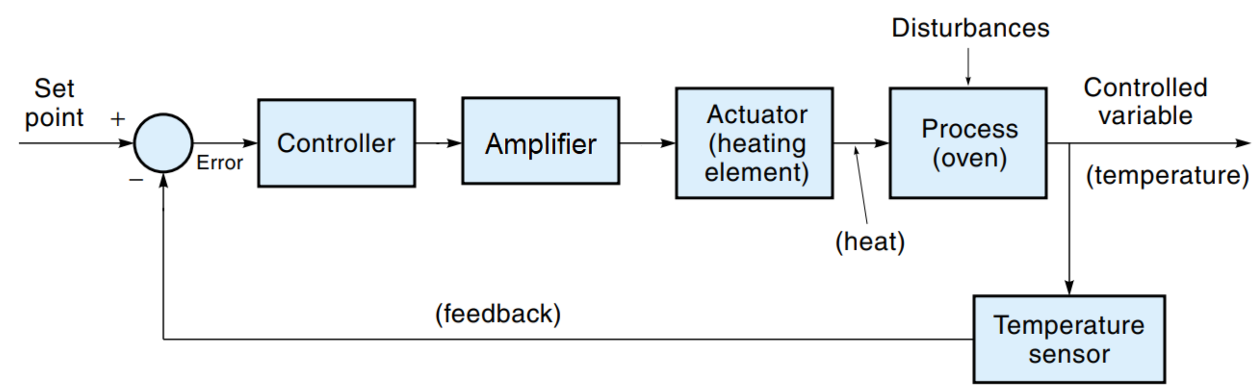
Introduction to Systems Modeling

- **System Modeling** is finding a mathematical representation to show how the <u>output</u> of the system is related to its <u>input</u>.

❑ Why do we need the system model?

- Physical dynamic systems are a collection of components and subsystems connected together to perform a useful function
- Each element in the system converts energy from one form to another.
    - For example, a temperature sensor as converting <u>degrees</u> to <u>volts</u> or a motor as converting <u>volts</u> to <u>rpm</u>.
- In order to design, analysis and control of dynamic systems, we need to first understand the input-output relationship of the system elements and find a **model** for them. This helps us:
    - To <u>simulate</u> and <u>predict</u> the dynamic response and behavior of the components and subsystems
    - To <u>analyze</u> how components affect each other and the overall system, that helps fault detection and maintenance
    - To <u>design</u> a control system to improve and enhance the dynamic response as desired





Introduction to Systems Modeling

- **System Modeling** is finding a mathematical representation to show how the output of the system is related to its input.

  - **How to find the system model?**

    - Here are a series of questions to consider when modeling a system:

      - Where do you get started?
      - What aspects of the system must you consider?
      - What tools or information will you need?
      - What metrics do you use to measure the system's performance?
      - How do you design or optimize the system to ensure reasonable performance?
      - How do you automate or control a system?

    - In this course, you will evolve an understanding of tools and skills employed in examining dynamic system to obtain the dynamic system model.

Introduction to Systems Modeling

❑ Lump-Parametric Modeling

- Find the input-output relationships for systems by considering them to be composed of just a few simple basic elements and applying the physical laws from first-principles.
- Mainly used to model electrical, mechanical, and electromechanical systems.
- The model is obtained as a differential equation, then shown in any standard form of, Transfer Function model, State-Space model or Block diagram model.
  - **Electrical systems** can be considered as compose of basic elements, which can be represented by resistors, capacitors, inductors and op-amps.
    - These are characterized by voltage –current relationships for components and the laws of interconnection Kirchhoff's Voltage Law (KVL) and Current Law (KCL)
  - **Mechanical systems** can be considered as compose of basic elements, which can be represented by springs, dampers and masses, and are characterized by Newton's Laws of motion.

❑ Empirical Modeling

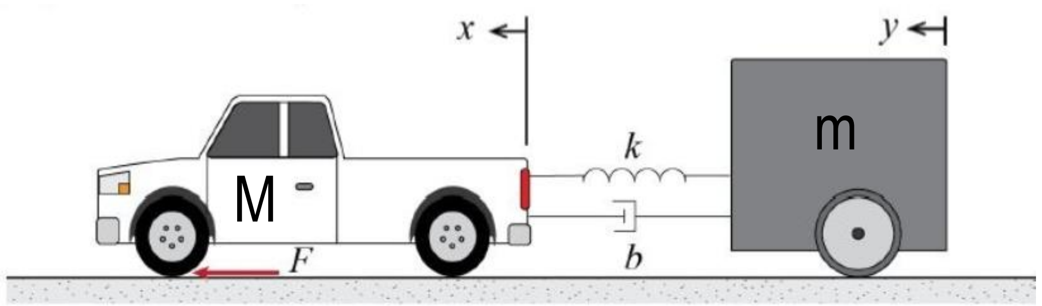
- An experimental approach, which mainly used to model thermal and fluid (hydraulic and pneumatic) processes.
- Some experiments are performed on the system to collect input-output data, a model is then fitted to the collected data by assigning suitable numerical values to its parameters.

System Decomposition & Model Complexity

- One of the essential skills in dynamic systems modeling is the ability to effectively dissect a system into subsystem and more basic components to better facilitate mathematical representations of the physics entailed.

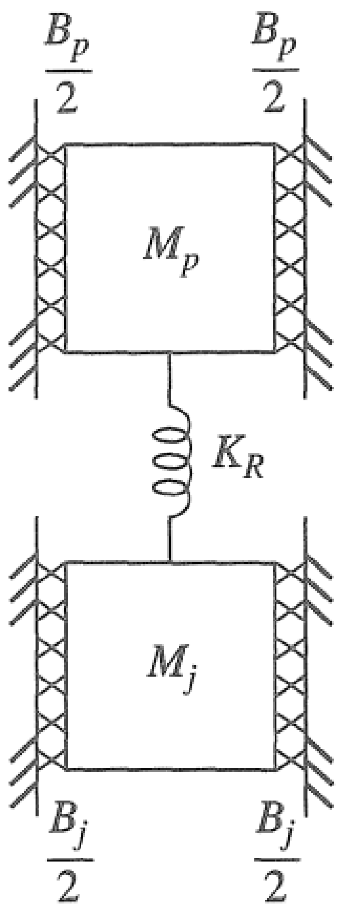
○ Truck Pulling a Cart

Input: Applied engine force
Outputs: Displacement of truck and cart



○ A Parachute Jumper

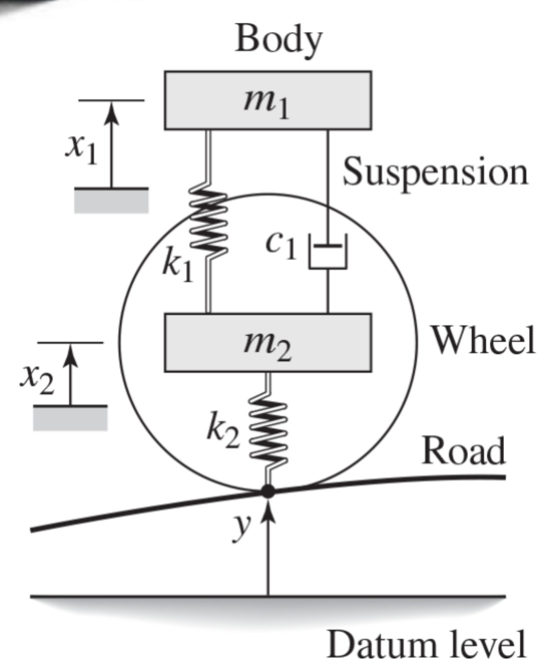
Inputs: Gravity forces
Outputs: Displacement of jumper and parachute



System Decomposition & Model Complexity

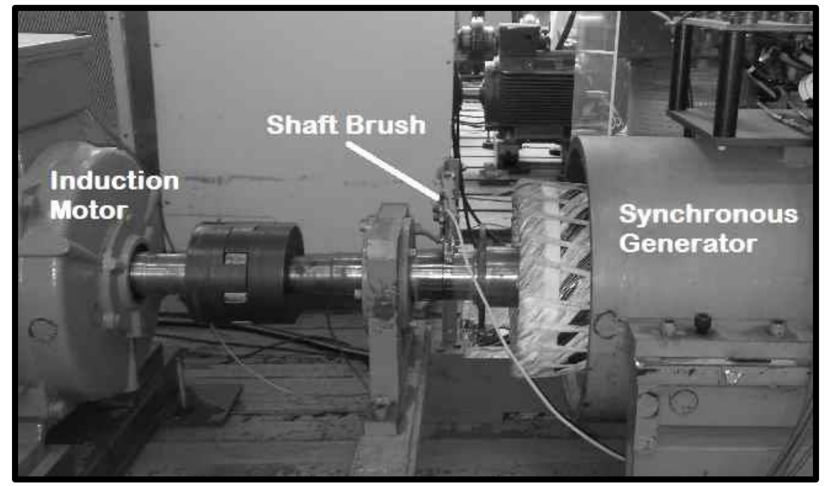
- One of the essential skills in dynamic systems modeling is the ability to effectively dissect a system into subsystem and more basic components to better facilitate mathematical representations of the physics entailed.

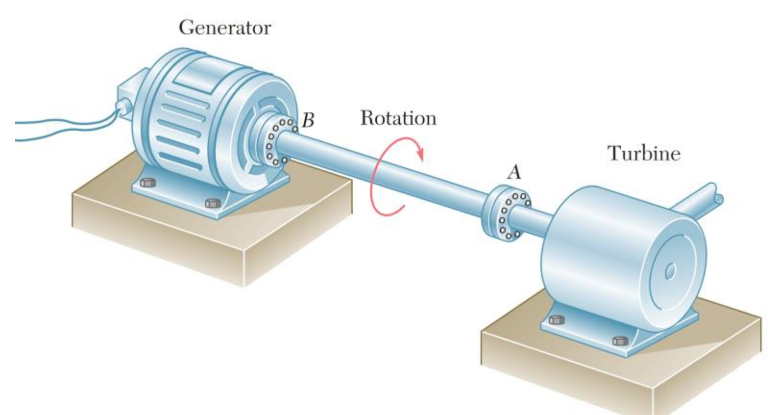
○ Vehicle Suspension System

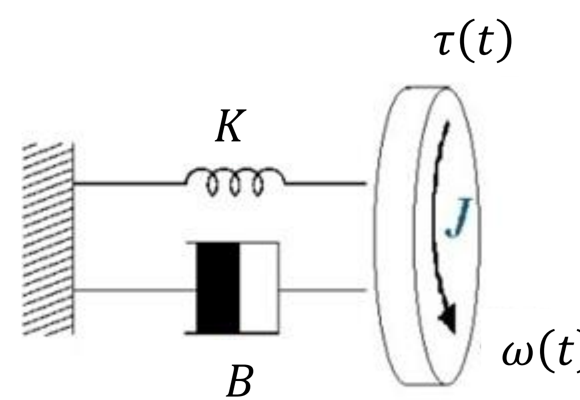


Input: Vertical displacement due to road bumps
Output: Vertical displacement of car

○ A Shaft under a Torsional Load System





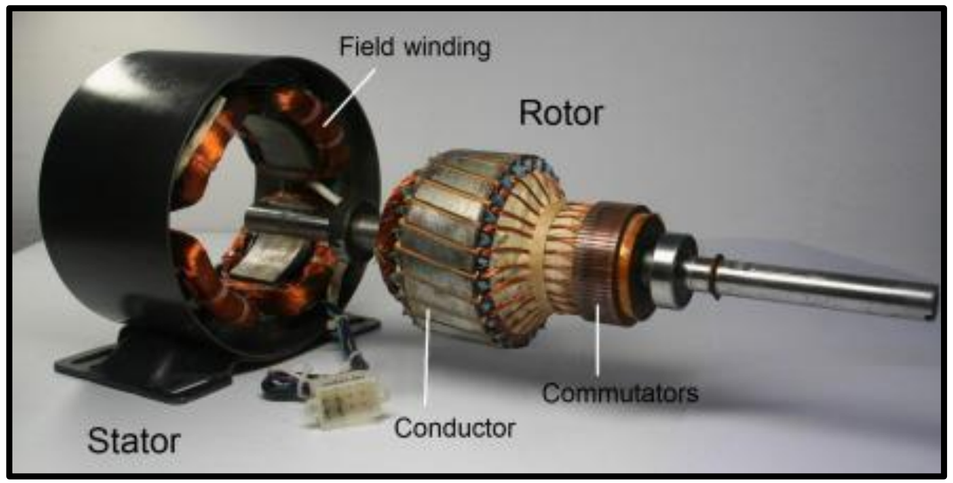


Input: Applied torque
Output: Angular speed of the shaft

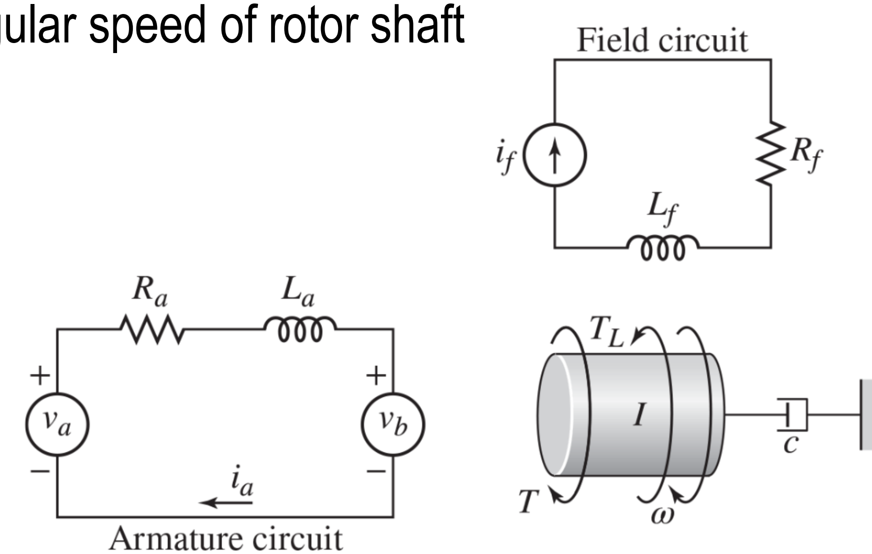
System Decomposition & Model Complexity

- One of the essential skills in dynamic systems modeling is the ability to effectively dissect a system into subsystem and more basic components to better facilitate mathematical representations of the physics entailed.

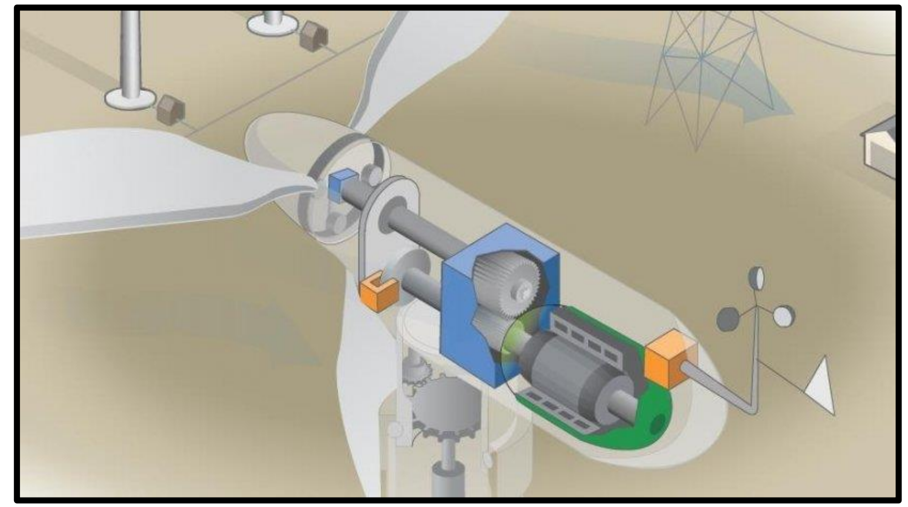
Armature Controlled DC Motor



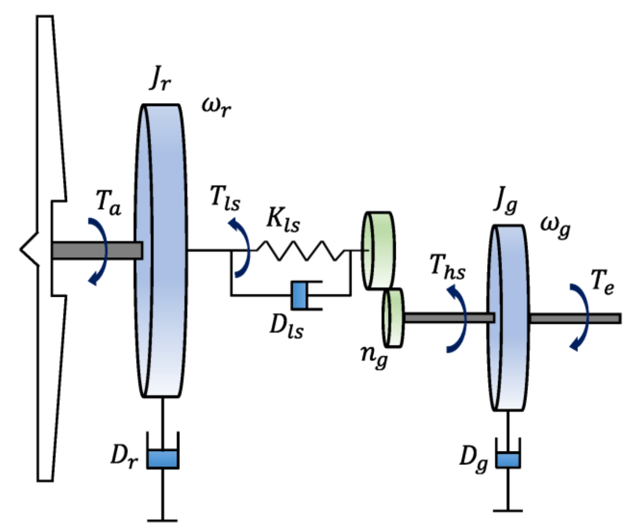
Input: Applied armature voltage
Output: Angular speed of rotor shaft

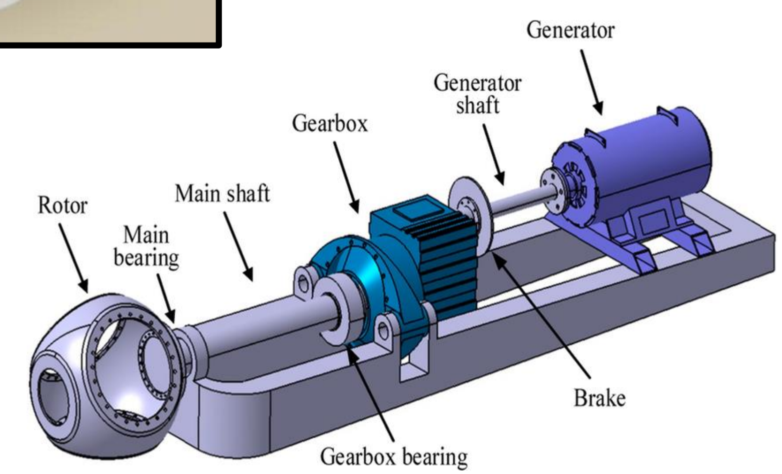


Wind Turbine Drivetrain



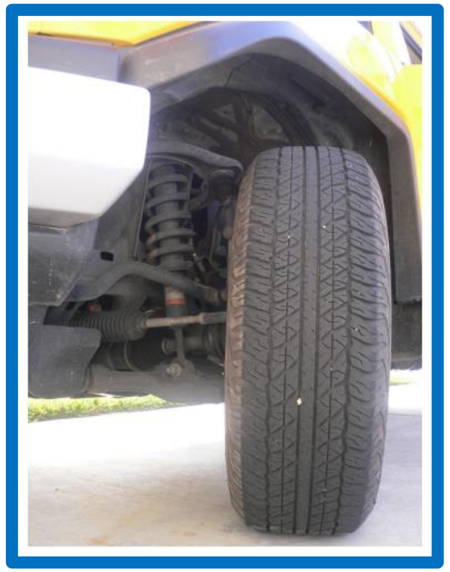
Input: Applied torque
Output: Speed of generator

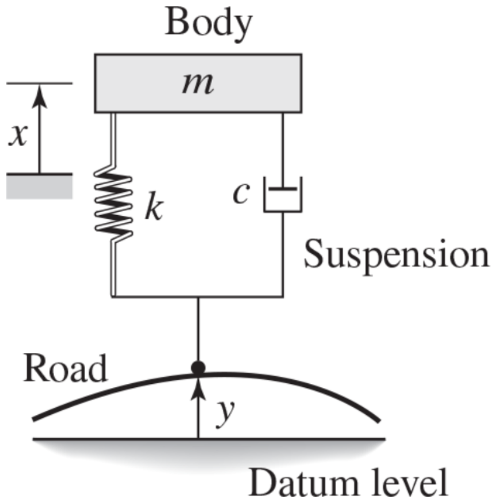




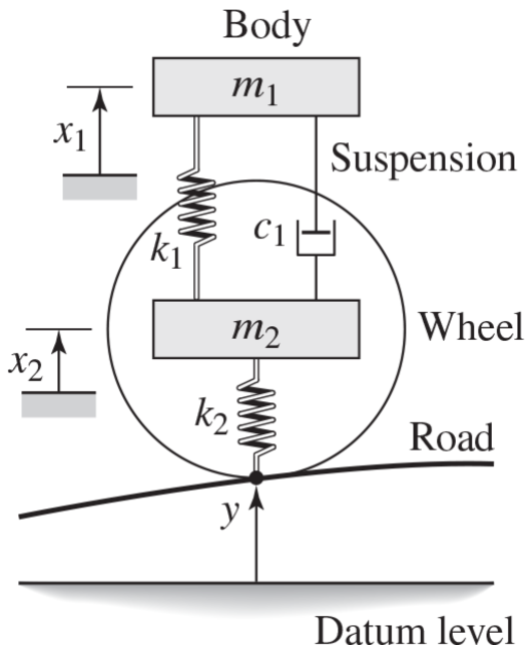
System Decomposition & Model Complexity

- The complexity of the model depends on its intended use.
- For example, consider the vehicle suspension system.
- The **quarter car suspension system** can be modeled as depicted in either model A or B.
- **Model A** is a simple model that includes the spring and damping due to the shock absorber, and mass of the front quarter of the vehicle.
  - This model is adequate for predicting the vertical motion of the front quarter for relatively slow (low frequency) inputs.
  - An example might be a series of regularly spaced speed bumps the vehicle travels over at slow to moderate speeds as it might do in a parking lot or residential area.
  - At such speeds, the suspension absorbs most of the energy and accounts for most of the dynamics.





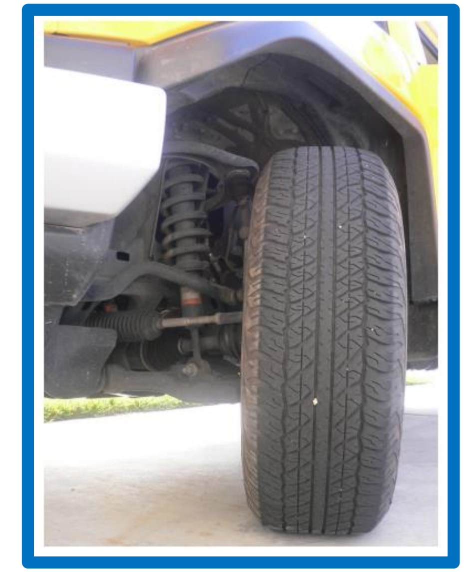
Model A

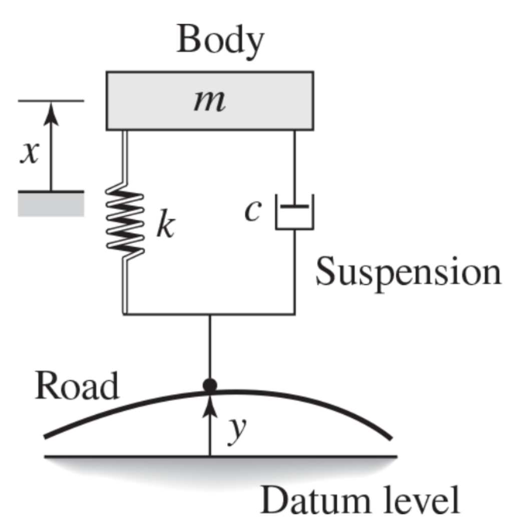


Model B

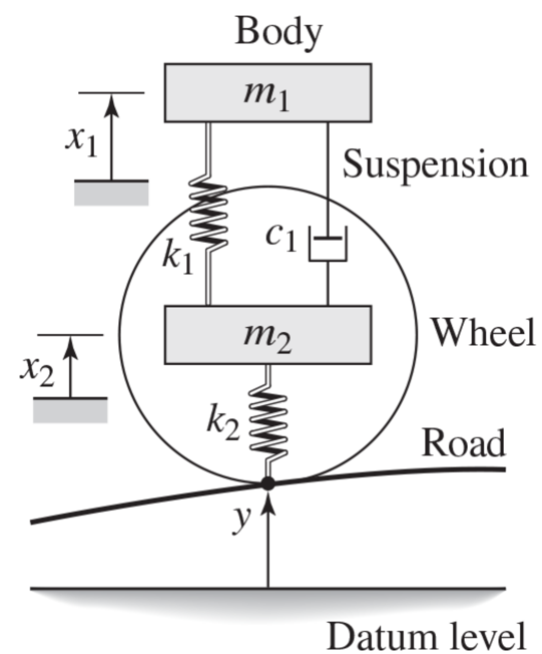
System Decomposition & Model Complexity

- The complexity of the model depends on its intended use.
- For example, consider the vehicle suspension system.
- The **quarter car suspension system** can be modeled as depicted in either model A or B.
- **Model B** is a more elaborate model that includes compliance of the tire due to the compressed air within, and the combined mass of the wheel, tire and brake assembly, as well.
    - This model can be used for predicting the vertical motion of the front quarter for high frequency inputs.
    - For example, at highway speeds, the suspension receives more frequent vertical displacements with small amplitudes from the highway markers.
    - At such frequencies, the vehicle suspension does not have sufficient time to respond and absorb the energy. Instead, the tires absorb much of the energy and play a more critical role in predicting the dynamic response and induced vibrations.





Model A

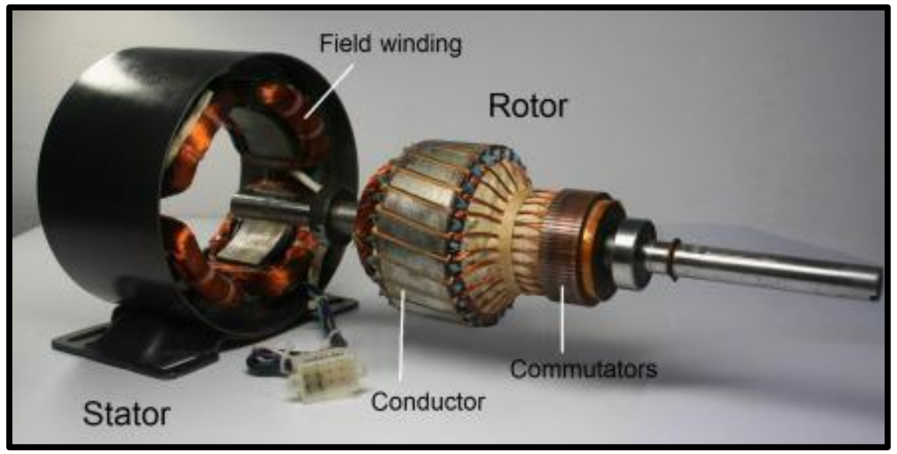


Model B

Mathematical Modeling of Dynamic Systems

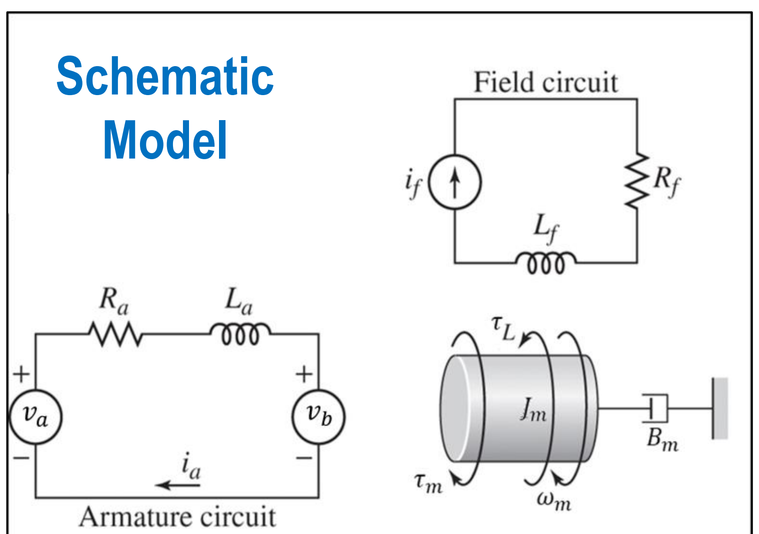
- The mathematical models are obtained as differential equations by applying the physical laws from first-principles.

○ Armature Controlled DC Motor



Input: Applied armature voltage
Output: Angular speed of rotor shaft

- Divide the system into idealized components
- Apply physical laws to the elements
- Apply interconnection laws between elements
- Combine the equations to obtain a standard form models
  - Transfer Function Model
  - State-Space Model
  - Block Diagram Model



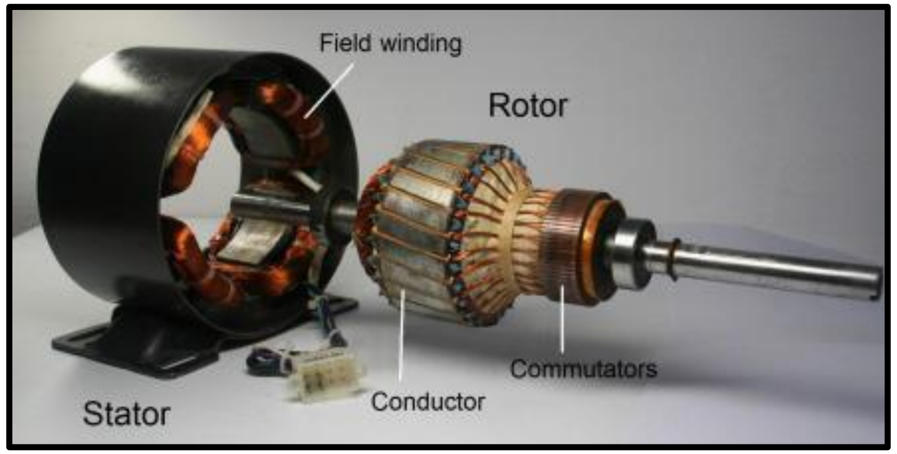
Differential Equations

$$\begin{cases} v_a(t) = R_a i_a(t) + L_a \dfrac{di_a(t)}{dt} + v_b(t) \\ \tau_m(t) = K_i i_a(t) \\ \tau_m(t) = J_m \dfrac{d\omega_m(t)}{dt} + B_m \omega_m(t) + \tau_L(t) \\ v_b(t) = K_b \omega_m(t) \end{cases}$$

Mathematical Modeling of Dynamic Systems

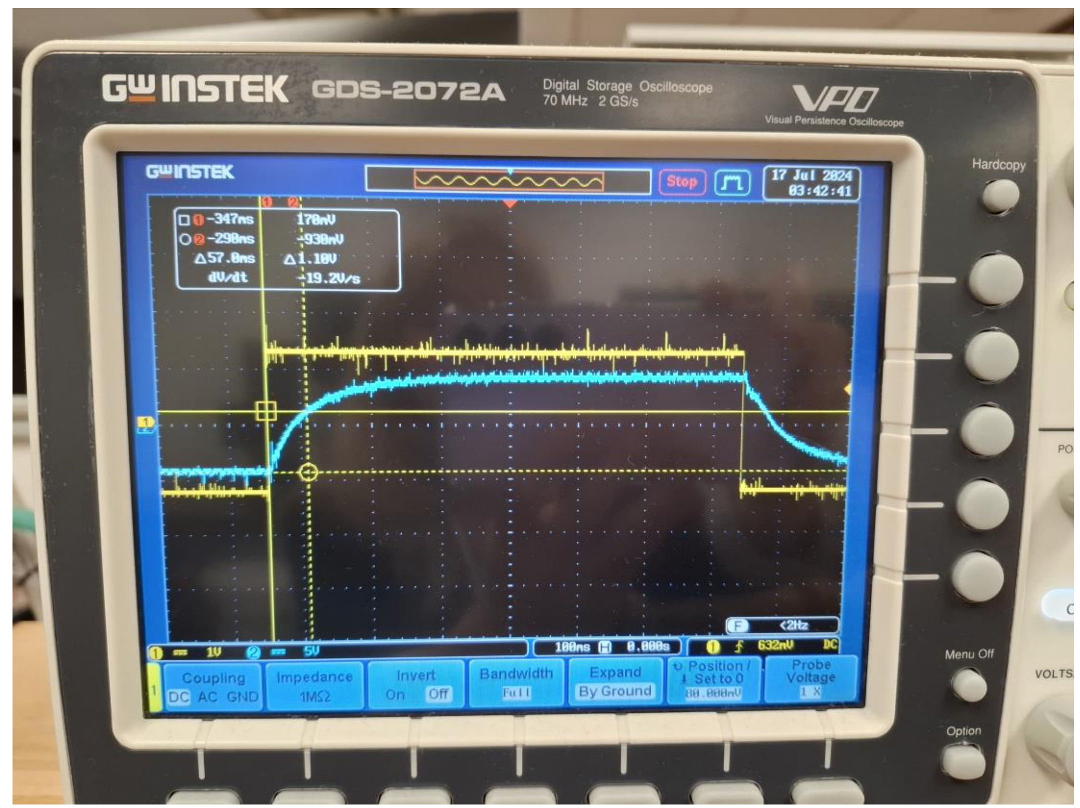
- The empirical models are obtained by collecting input-output data and fitting the model and estimating the parameters.

○ Armature Controlled DC Motor



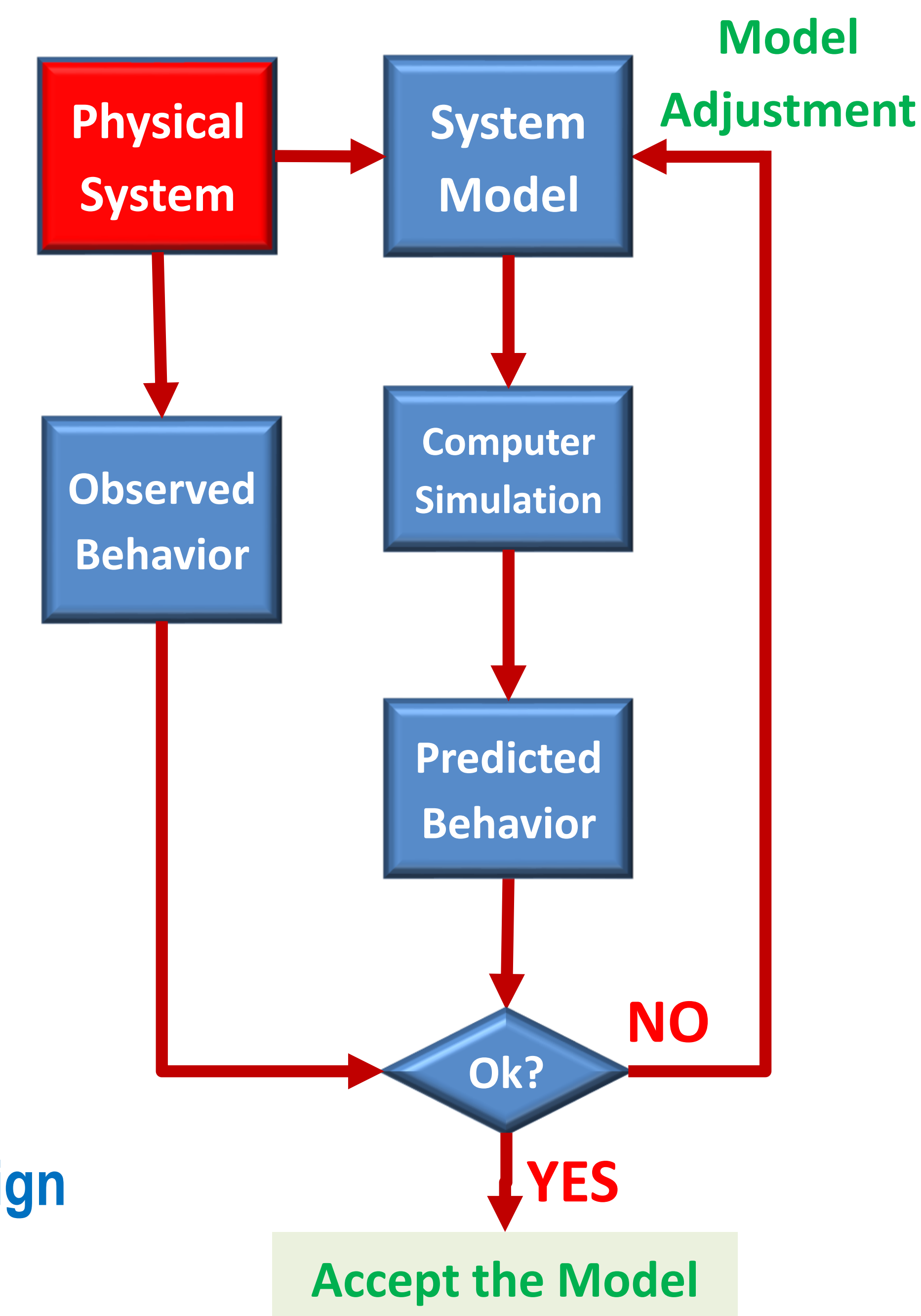
Input: Applied armature voltage
Output: Angular speed of rotor shaft

- Apply the appropriate input data.
- Collect the output data
- Match the output data with a standard model
    - Transfer Function Model
- Estimate the model parameters from the input-output data



System Modeling Procedure

- ✓ **Define the purpose or objective of the model**
  - Identify system boundaries, subsystems & components, interconnecting variables, inputs and outputs.
- ✓ **Determine the model for each component or subsystem**
  - Apply the known physical laws for obtaining <u>differential equations</u>, <u>transfer function</u> or <u>state-variable equations</u>, when possible, otherwise use <u>experimental data</u> to identify input-output relationships.
- ✓ **Integrate the subsystem models into an overall system model**
  - Combine equations, eliminate common variables, check for sufficient equations to solve the system.
- ✓ **Verify the model validity and accuracy**
  - Implement a <u>simulation</u> of the model equations and compared with experimental data for the same conditions.
- ✓ **Make simplifications to create an approximate model suitable for design**
  - <u>Linearization</u> of model equations
  - <u>Reduce the order</u> of the model by eliminating unimportant dynamics



Simulation of Dynamic Systems

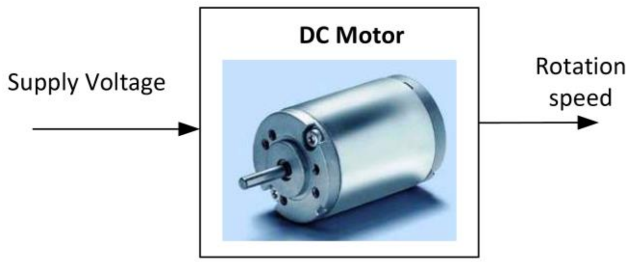
- The simulation technique includes an extensive collection of methods and applications aimed at reproducing the actual behavior of nonlinear systems on a digital computer with appropriate simulation software.

- We will particularize the computer simulation task by employing simulation software such as **MATLAB**, **Simulink** or **Simscape**, using ***codes***, ***block diagrams*** and ***physical elements*** interconnected to synthesize the model equations.

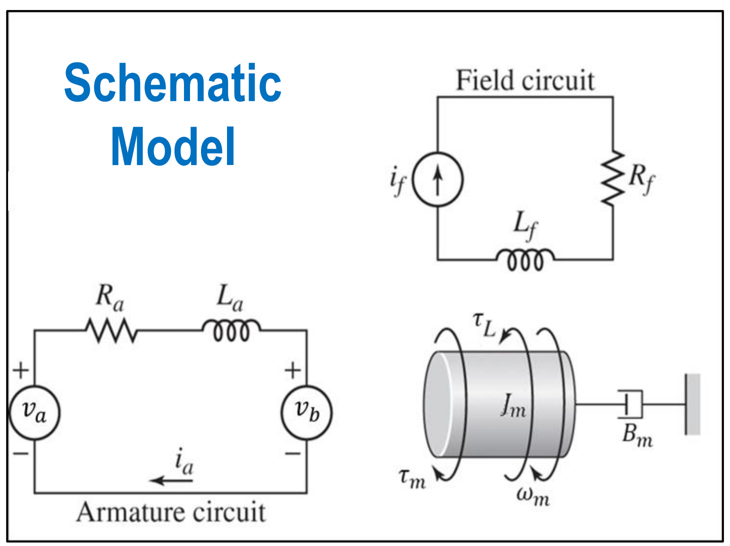
- A computer simulation must embody several components:
  - In first place, the structure of the dynamic model to be simulated must be known, as also the set of model parameters and initial conditions.
  - In second place, the set of input signals must also be embodied, and a set of output signals must be explicitly defined in order to follow the system evolution.
  - Finally, a simulation run time must be included in order to select the numerical integration method used and the value of its associated parameters, integration step, error tolerances, etc.

System Simulation Tools: MATLAB

- **MATLAB** is a high-level technical computing environment suitable for solving scientific and engineering problems.



Schematic Model



$$
\begin{cases}
v_a(t) = R_a i_a(t) + L_a \dfrac{di_a(t)}{dt} + v_b(t) \longrightarrow V_a(s) = (L_a s + R_a) I_a(s) + V_b(s) \\
\tau_m(t) = K_i i_a(t) \longrightarrow T_m(s) = K_i I_a(s) \\
\tau_m(t) = J_m \dfrac{d\omega_m(t)}{dt} + B_m \omega_m(t) + \tau_L(t) \longrightarrow T_m(s) = (J_m s + B_m)\Omega(s) + T_L(s) \\
v_b(t) = K_b \omega_m(t) \longrightarrow V(s) = K_b \Omega(s)
\end{cases}
$$

Differential Equations

$$
\boxed{\frac{\Omega(s)}{V_a(s)} = \frac{K_i}{L_a J_m s^2 + (L_a B_m + J_m R_a)s + R_a B_m + K_i K_b}}
$$

Transfer Function Model

```
Ki = 0.05; Kb = Ki;
La = 2e-3; Ra = 0.5;
Jm = 9e-5; Bm = 1e-4;

speed = tf(Ki,[La*Jm,La*Bm+Jm*Ra,Ra*Bm+Ki*Kb]);

step(speed);
```

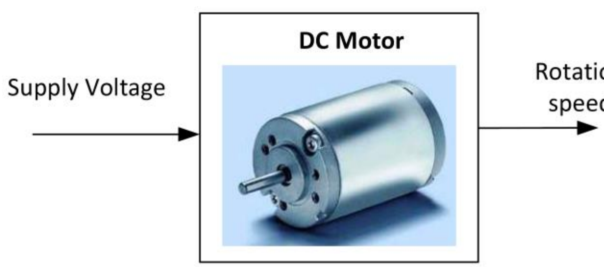
MATLAB Code

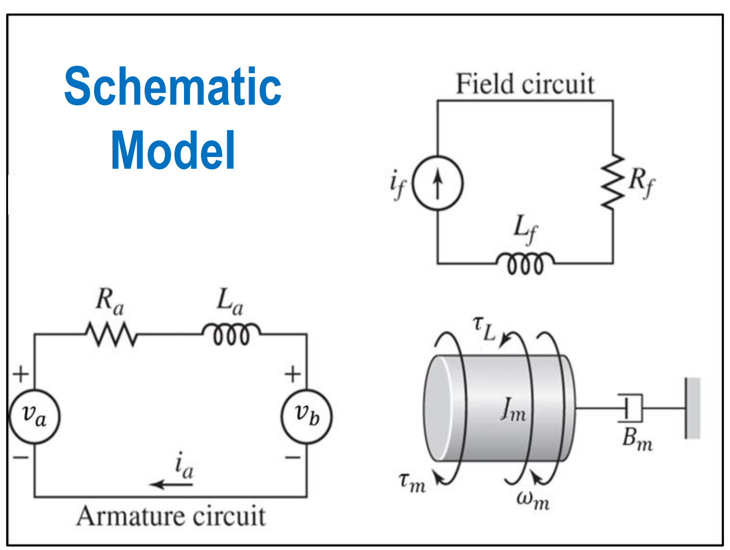
System Simulation Tools: Simulink

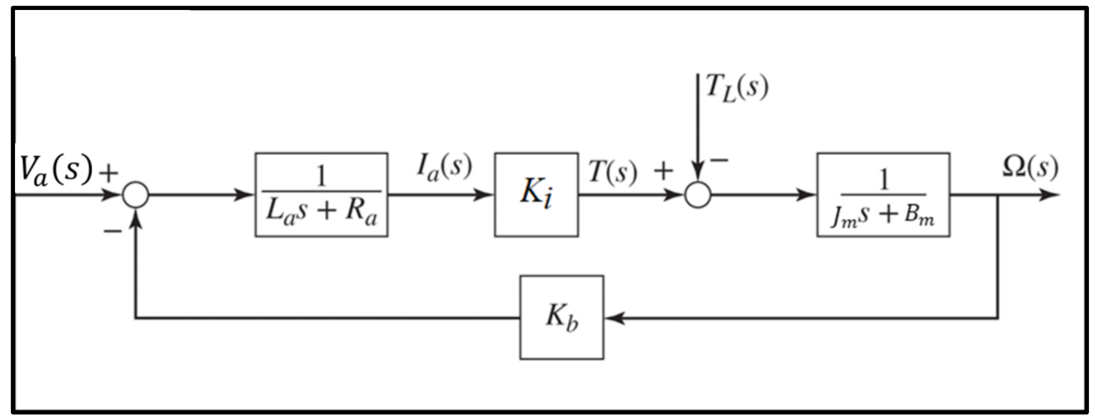
- **Simulink** is an extension to MATLAB that allows users to rapidly and accurately build computer models of dynamical systems using ***block diagrams***.

$$
\begin{cases}
v_a(t) = R_a i_a(t) + L_a \dfrac{di_a(t)}{dt} + v_b(t) \longrightarrow V_a(s) = (L_a s + R_a) I_a(s) + V_b(s) \\
\tau_m(t) = K_i i_a(t) \longrightarrow T_m(s) = K_i I_a(s) \\
\tau_m(t) = J_m \dfrac{d\omega_m(t)}{dt} + B_m \omega_m(t) + \tau_L(t) \longrightarrow T_m(s) = (J_m s + B_m)\Omega(s) + T_L(s) \\
v_b(t) = K_b \omega_m(t) \longrightarrow V_b(s) = K_b \Omega(s)
\end{cases}
$$

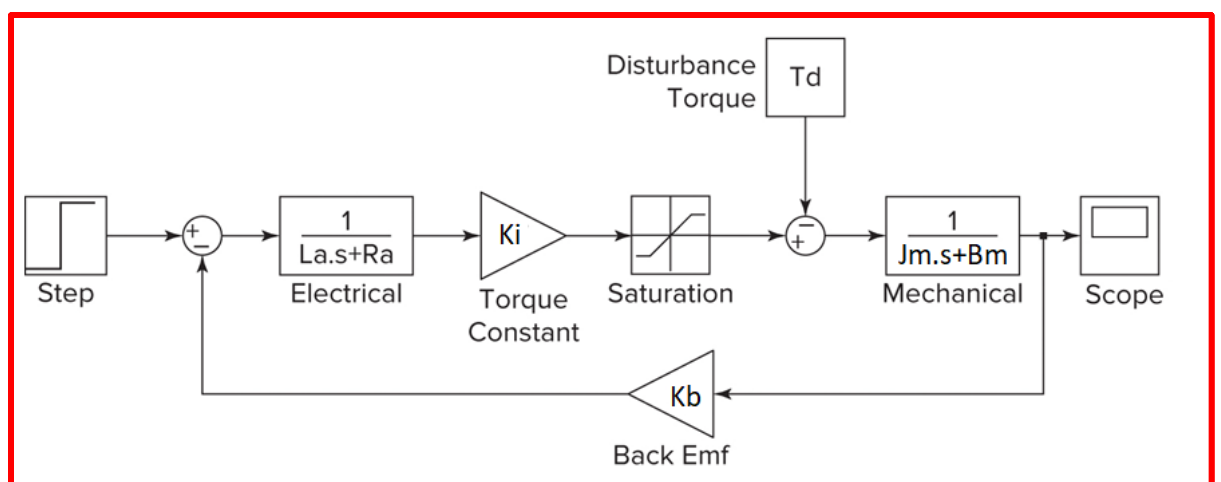
Differential Equations







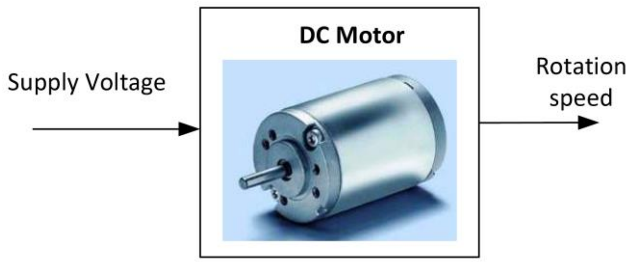
Block Diagram Model

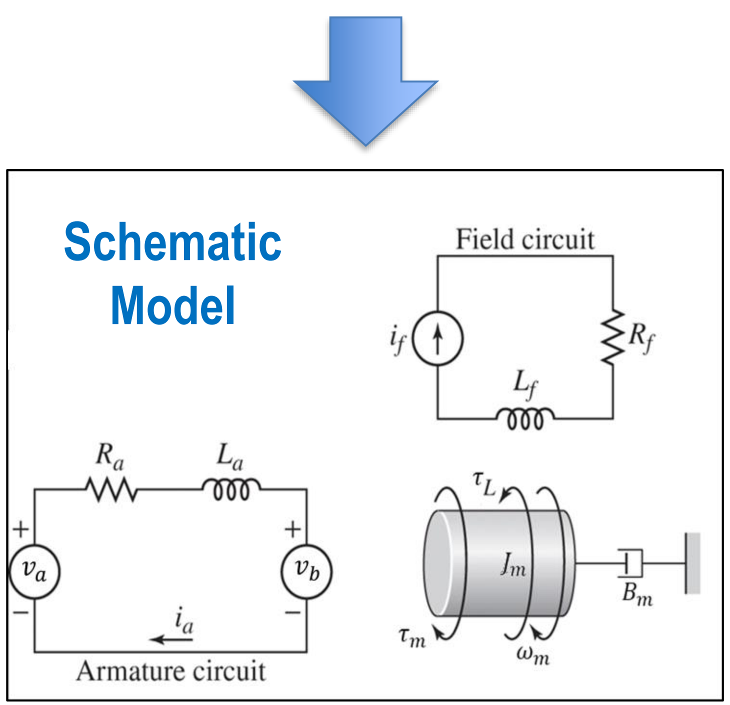


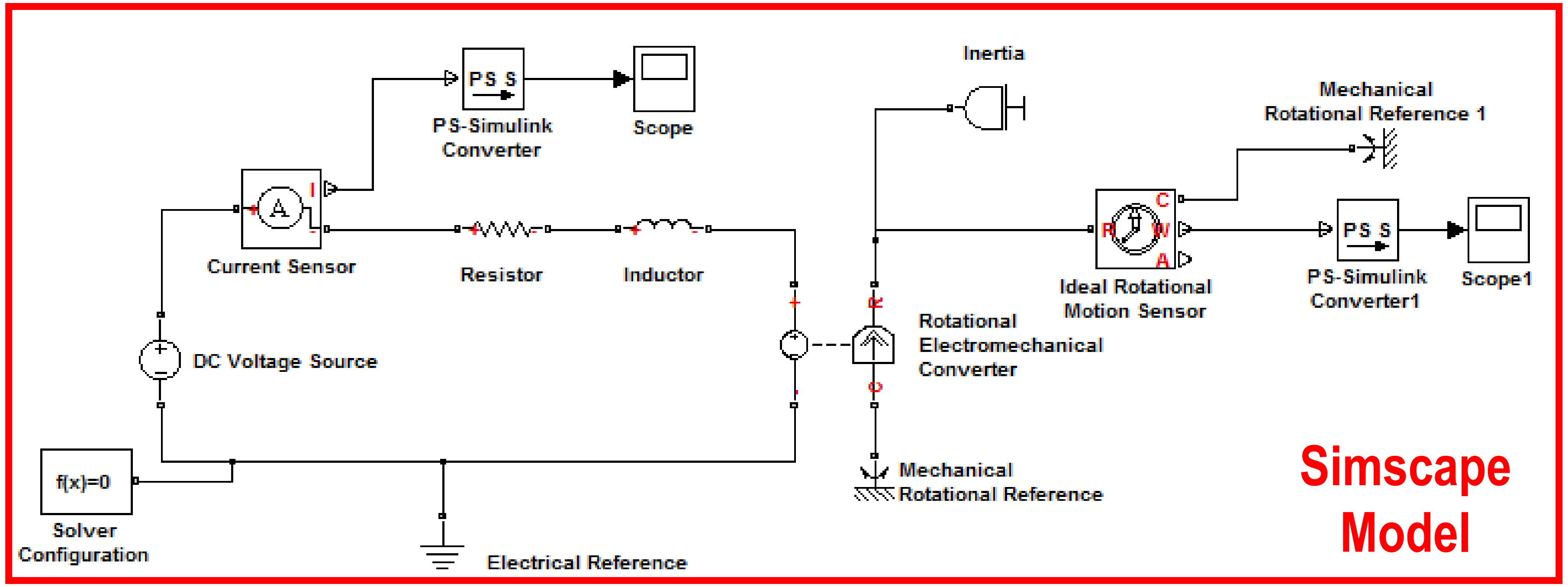
Simulink Model

System Simulation Tools: Simscape

- **Simcpace** is a MATLAB-based, **object-oriented *physical* modeling** language that enables the user to create models of ***physical components*** using an acausal modeling approach.
- This language is designed to use in MATLAB and Simulink environment, it can benefit MATLAB functions and Simulink blocks.







THANK YOU

HUMBER

WE ARE HUMBER