Week 6 – Moments and Couples

ENGI 1510 - ENGINEERING DESIGN

MOMENT OF A FORCE (SCALAR FORMULATION), CROSS PRODUCT, MOMENT OF A FORCE (VECTOR FORMULATION), & PRINCIPLE OF MOMENTS

Today's Objectives :

Students will be able to:

a) understand and define moment, and,

b) determine moments of a force in 2-D and 3-D cases.

In-Class Activities :

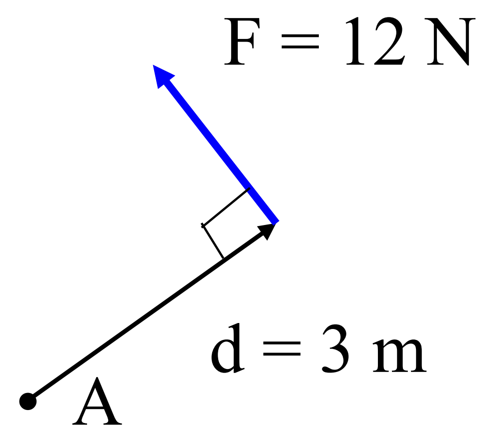
- Reading Quiz
- Applications
- Moment in 2-D
- Moment in 3-D
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. What is the moment of the 12 N force about point A (M_A)?

   A) 3 N·m B) 36 N·m C) 12 N·m

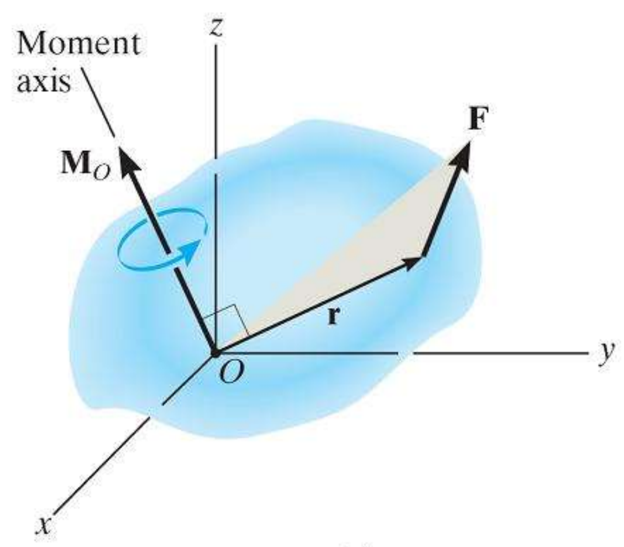
   D) (12/3) N·m E) 7 N·m



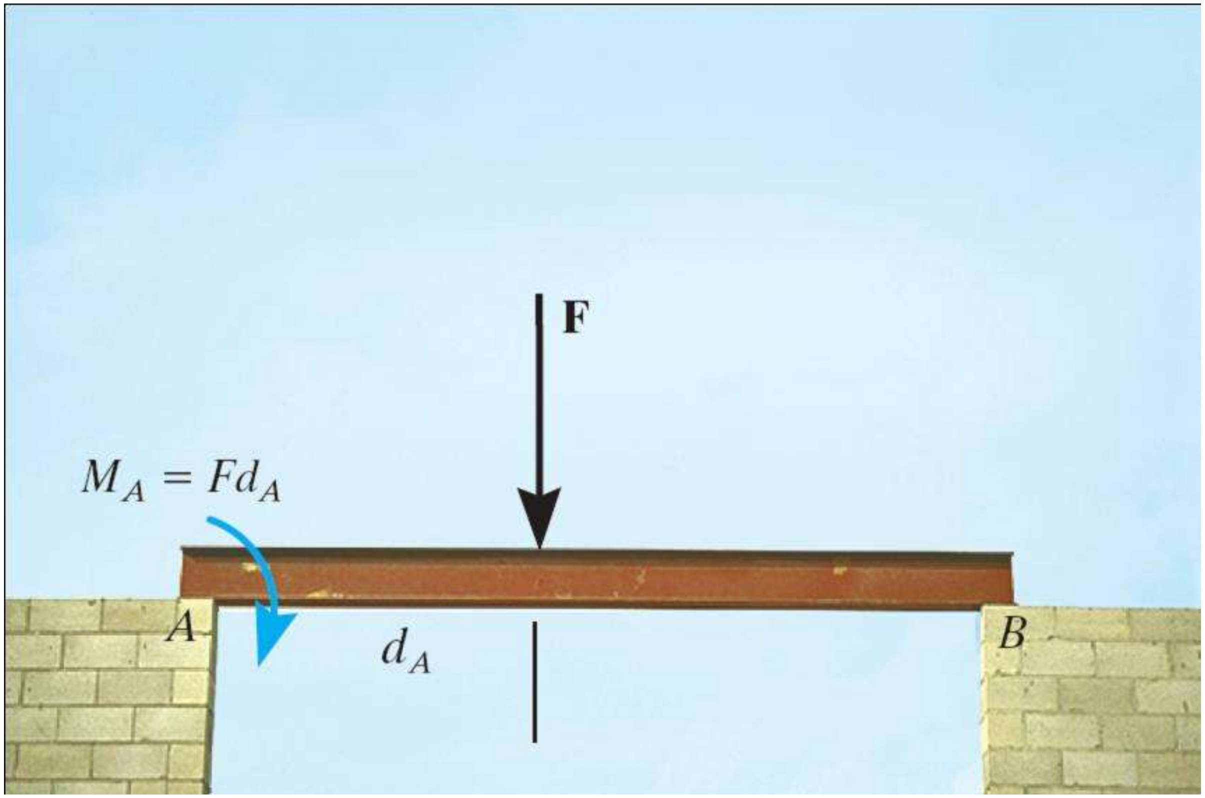
2. The moment of force ***F*** about point O is defined as $\boldsymbol{M_O}$ = ____________ .

   A) ***r*** × ***F*** B) ***F*** × ***r***

   C) ***r*** • ***F*** D) r * F



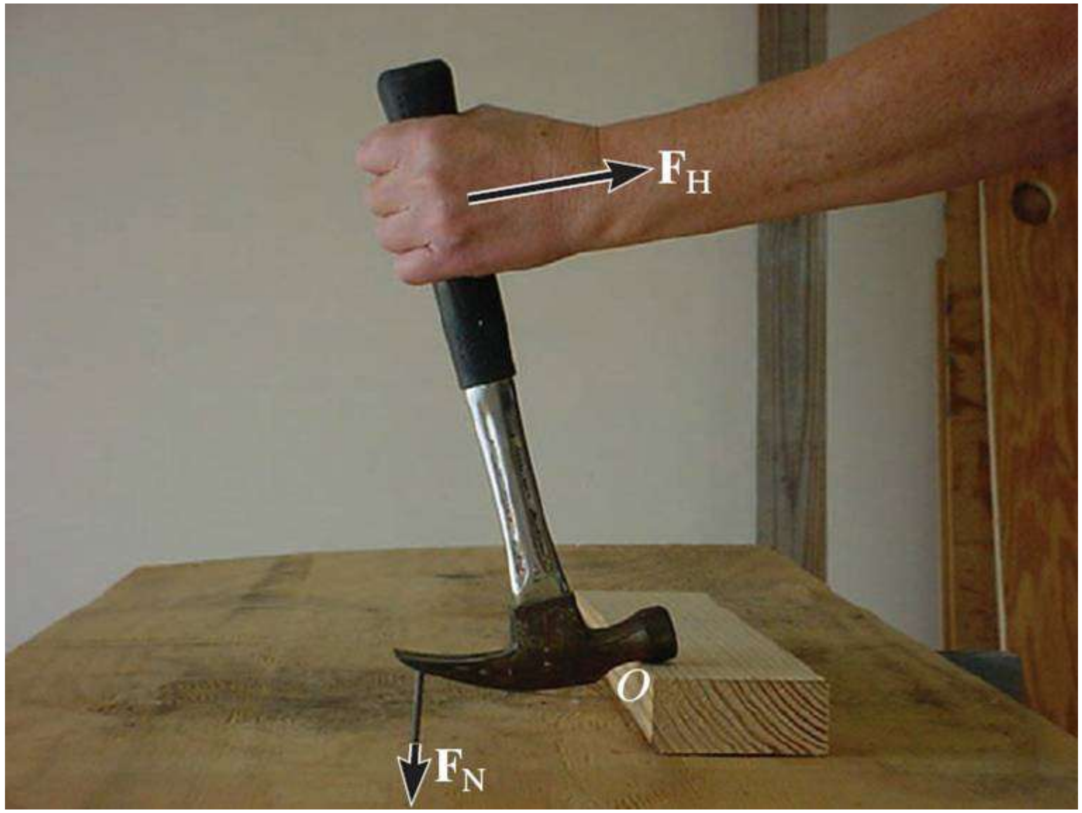
APPLICATIONS



Beams are often used to bridge gaps in walls. We have to know what the effect of the force on the beam will have on the supports of the beam.

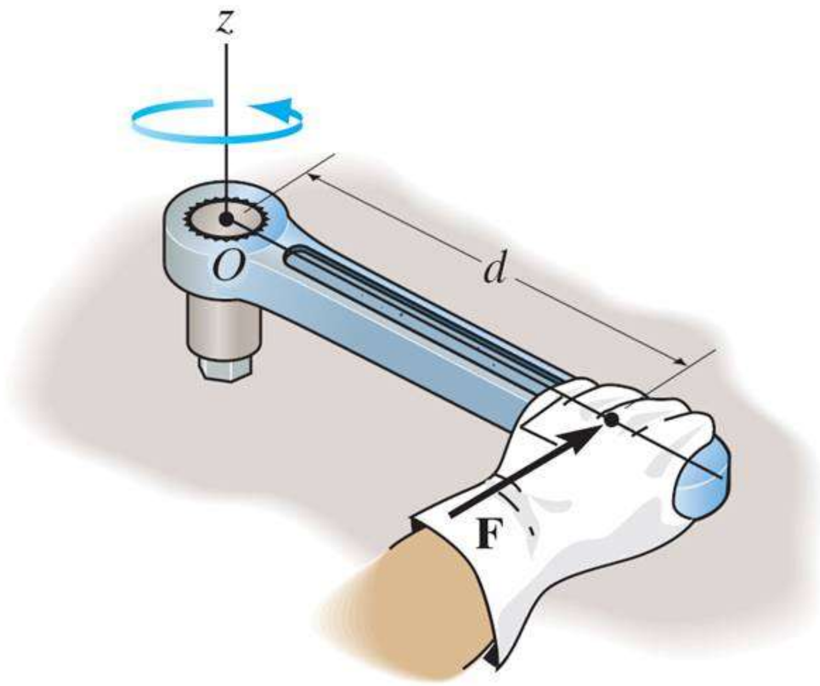
What do you think is happening at points A and B?

APPLICATIONS (continued)



Carpenters often use a hammer in this way to pull a stubborn nail. Through what sort of action does the force F_H at the handle pull the nail? How can you mathematically model the effect of force F_H at point O?

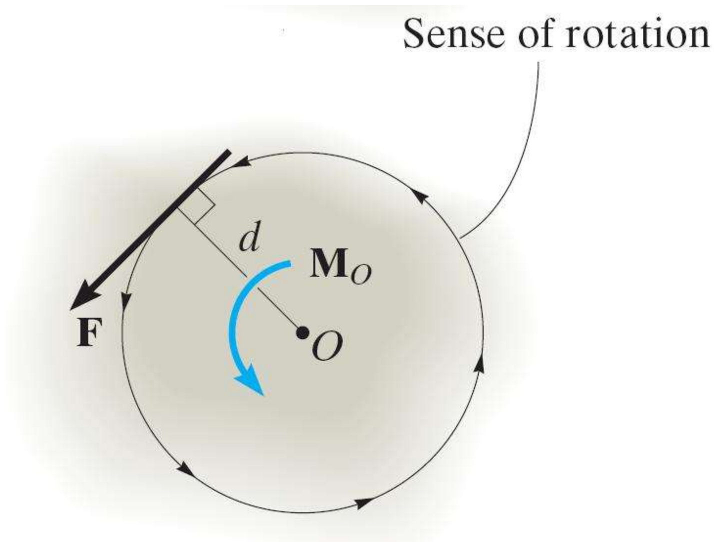
MOMENT OF A FORCE - SCALAR FORMULATION (Section 4.1)



The moment of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

MOMENT OF A FORCE - SCALAR FORMULATION (continued)

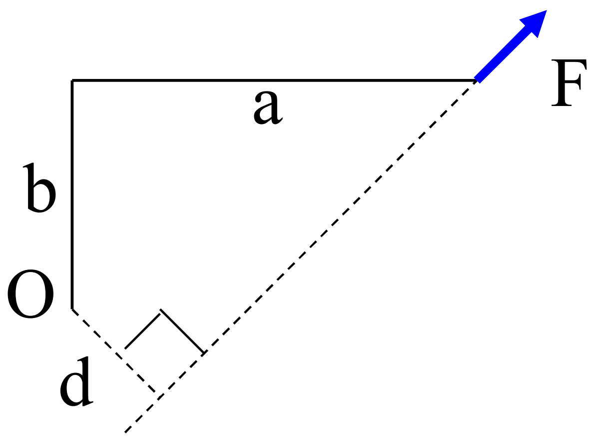
In a 2-D case, the magnitude of the moment is $M_o = F\,d$



As shown, d is the perpendicular distance from point O to the line of action of the force.

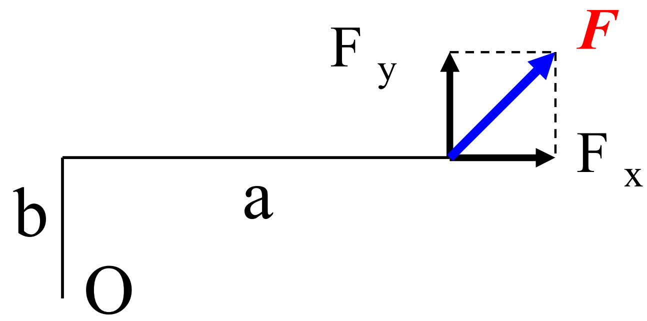
In 2-D, the direction of M_O is either clockwise (CW) or counter-clockwise (CCW), depending on the tendency for rotation.

MOMENT OF A FORCE - SCALAR FORMULATION (continued)



For example, $M_O = F\ d$ and the direction is counter-clockwise.

Often it is easier to determine M_O by using the components of ***F*** as shown.



Then $M_O = (F_y\ a) - (F_x\ b)$. Note the different signs on the terms! The typical sign convention for a moment in 2-D is that counter-clockwise is considered positive. We can determine the direction of rotation by imagining the body pinned at O and deciding which way the body would rotate because of the force.

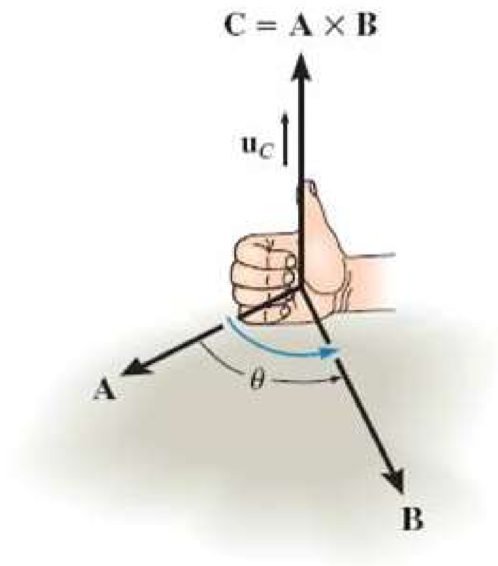
VECTOR CROSS PRODUCT (Section 4.2)

While finding the moment of a force in 2-D is straightforward when you know the perpendicular distance d, finding the perpendicular distances can be hard—especially when you are working with forces in three dimensions.

So a more general approach to finding the moment of a force exists. This more general approach is usually used when dealing with three dimensional forces but can be used in the two dimensional case as well.

This more general method of finding the moment of a force uses a vector operation called the cross product of two vectors.

CROSS PRODUCT (Section 4.2)



In general, the cross product of two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ results in another vector, $\boldsymbol{C}$, i.e., $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$. The magnitude and direction of the resulting vector can be written as

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = \mathrm{A\ B} \sin \theta\ \boldsymbol{u_C}$$

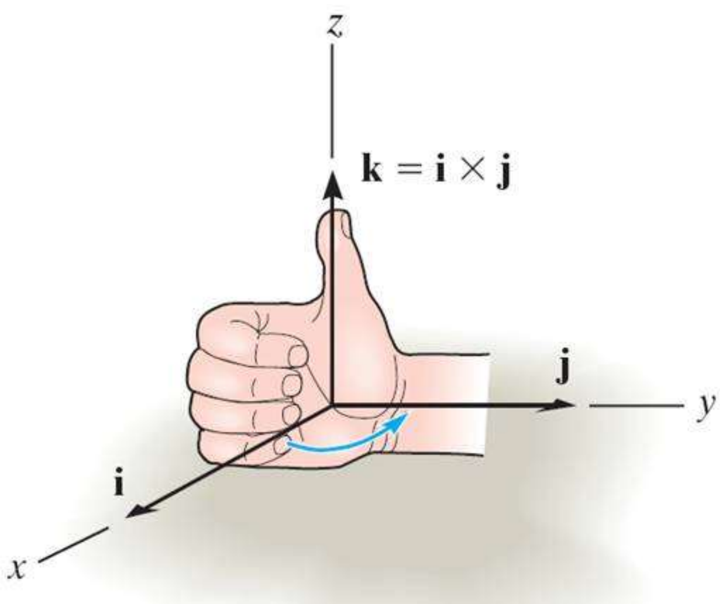
As shown, $\boldsymbol{u_C}$ is the unit vector perpendicular to both A and B vectors (or to the plane containing the A and B vectors).

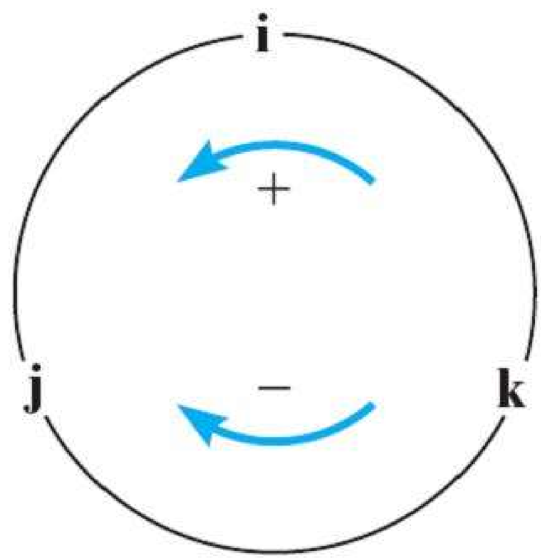
CROSS PRODUCT (continued)

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product.

For example: $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}$

Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{0}$





CROSS PRODUCT (continued)

Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2 × 2 determinants.

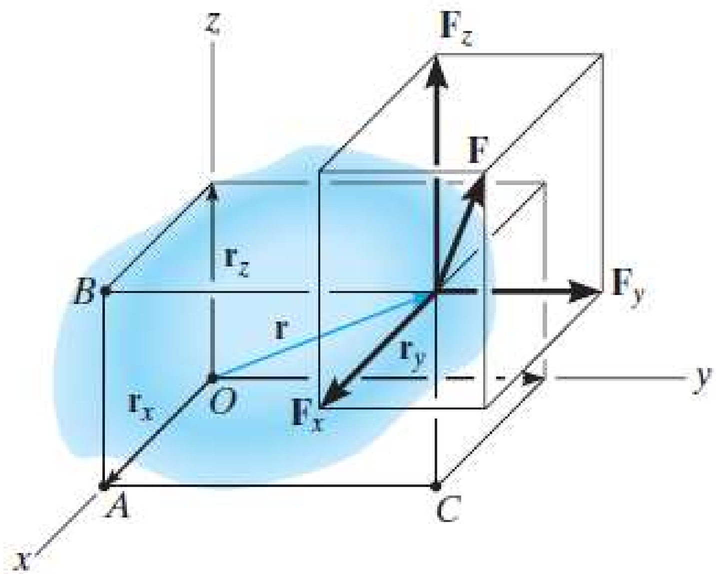
For element **i**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$

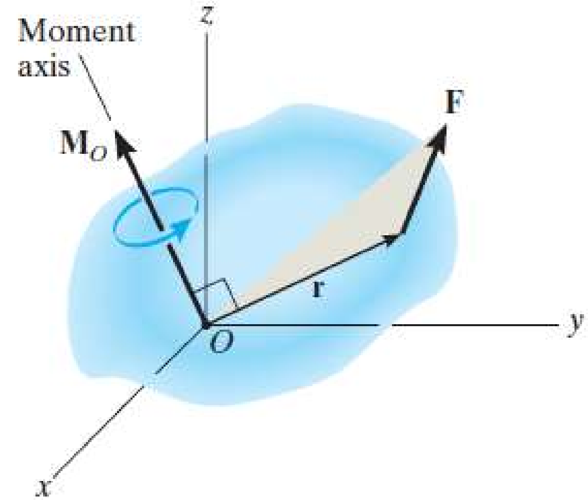
Remember the negative sign

For element **j**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$

For element **k**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{k}(A_xB_y - A_yB_x)$

MOMENT OF A FORCE – VECTOR FORMULATION (Section 4.3)





Moments in 3-D can be calculated using scalar (2-D) approach, but it can be difficult and time consuming. Thus, it is often easier to use a mathematical approach called the vector cross product.

Using the vector cross product, $\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$.

Here $\boldsymbol{r}$ is the position vector from point O to any point on the line of action of $\boldsymbol{F}$.

MOMENT OF A FORCE – VECTOR FORMULATION (continued)

So, using the cross product, a moment can be expressed as

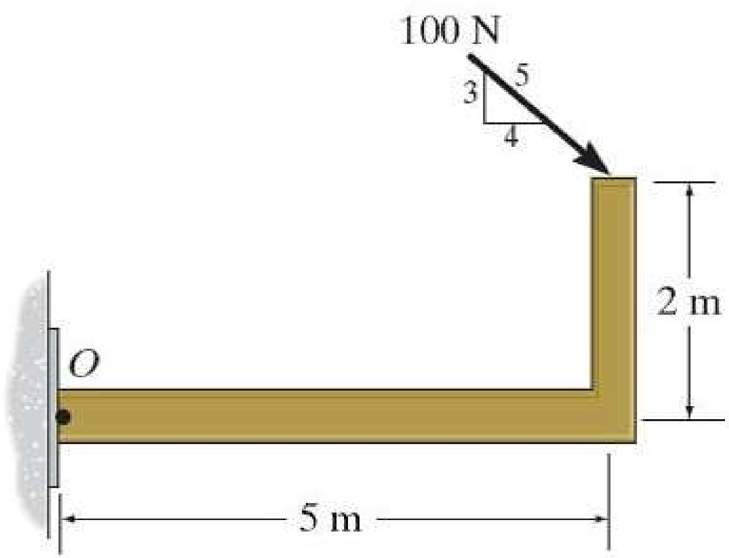
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

By expanding the above equation using 2 × 2 determinants (see Section 4.2), we get (sample units are N - m or lb - ft)

$$\boldsymbol{M_O} = (r_y\ F_z\ - r_z\ F_y)\ \boldsymbol{i}\ -\ (r_x\ F_z\ -\ r_z\ F_x\ )\ \boldsymbol{j}\ +\ (r_x\ F_y\ - r_y\ F_x\ )\ \boldsymbol{k}$$

The physical meaning of the above equation becomes evident by considering the force components separately and using a 2-D formulation.

EXAMPLE I



Given: A 100 N force is applied to the frame.

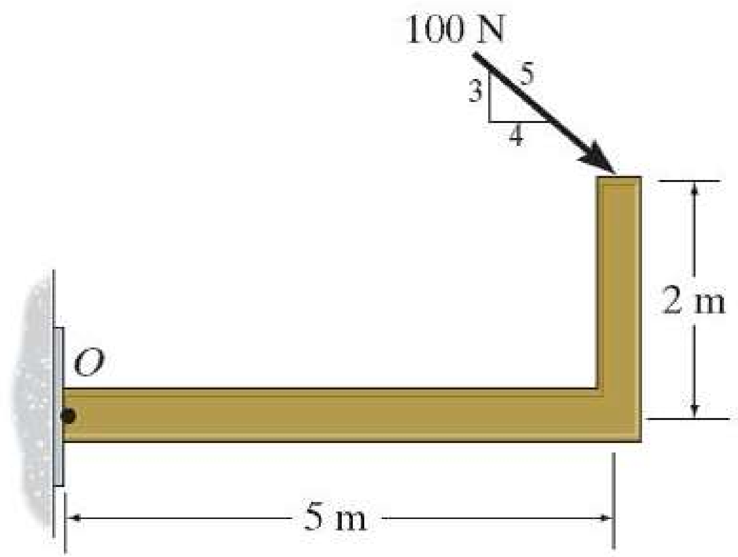
Find: The moment of the force at point O.

Plan:

1) Resolve the 100 N force along x and y-axes.

2) Determine M_O using a scalar analysis for the two force components and then add those two moments together.

EXAMPLE I (continued)



Solution:

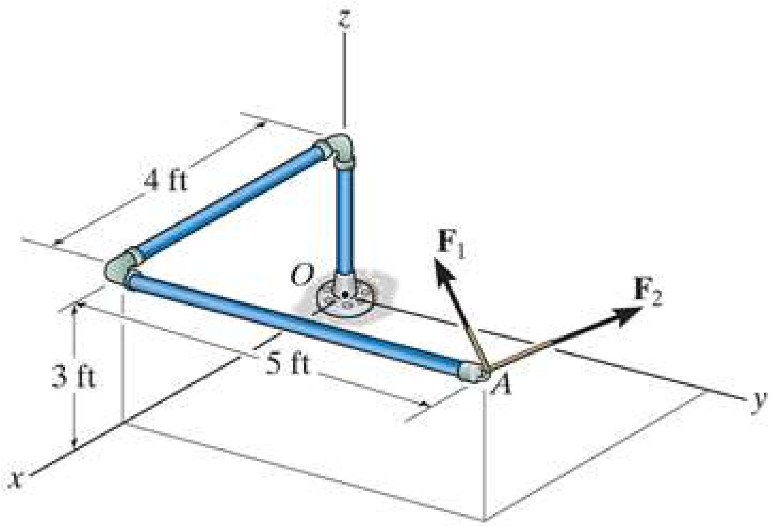
$+\uparrow F_y = \underline{-\ 100\ (3/5)\ N}$

$+\rightarrow F_x = \underline{100\ (4/5)\ N}$

$+\circlearrowleft M_O = \{-\ 100\ (3/5)N\ (5\ m) - (100)(4/5)N\ (2\ m)\}\ N\cdot m$

$= \underline{-\ 460\ N\cdot m}\circlearrowleft$ or $\underline{460\ N\cdot m\ CW}$

EXAMPLE II



Given: $\boldsymbol{F_1}$={100 $\boldsymbol{i}$ - 120 $\boldsymbol{j}$ + 75 $\boldsymbol{k}$}lb

$\boldsymbol{F_2}$={-200 $\boldsymbol{i}$ +250 $\boldsymbol{j}$ + 100 $\boldsymbol{k}$}lb

Find: Resultant moment by the forces about point O.

Plan:

1) Find $\boldsymbol{F} = \boldsymbol{F_1} + \boldsymbol{F_2}$ and $\boldsymbol{r_{OA}}$.

2) Determine $\boldsymbol{M_O} = \boldsymbol{r_{OA}} \times \boldsymbol{F}$.

EXAMPLE II (continued)

Solution:

First, find the resultant force vector $\boldsymbol{F}$

$$\boldsymbol{F} = \boldsymbol{F_1} + \boldsymbol{F_2}$$

$$= \{ (100 - 200)\, \boldsymbol{i} + (-120 + 250)\, \boldsymbol{j} + (75 + 100)\, \boldsymbol{k}\} \text{ lb}$$

$$= \{-100\, \boldsymbol{i} + 130\, \boldsymbol{j} + 175\, \boldsymbol{k}\} \text{ lb}$$

Find the position vector $\boldsymbol{r_{OA}}$

$$\boldsymbol{r_{OA}} = \{4\, \boldsymbol{i} + 5\, \boldsymbol{j} + 3\, \boldsymbol{k}\} \text{ ft}$$

Then find the moment by using the vector cross product.

$$\boldsymbol{M_O} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 4 & 5 & 3 \\ -100 & 130 & 175 \end{vmatrix} = [\{5(175) - 3(130)\}\, \boldsymbol{i} - \{4(175) - 3(-100)\}\, \boldsymbol{j} + \{4(130) - 5(-100)\}\, \boldsymbol{k}] \text{ ft·lb}$$

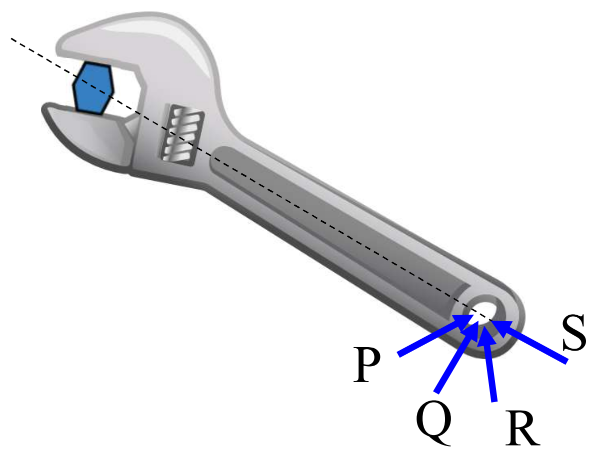
$$= \{\underline{485}\, \boldsymbol{i}\ \underline{-\ 1000}\, \boldsymbol{j} + \underline{1020}\, \boldsymbol{k}\}\ \underline{\text{ft·lb}}$$

CONCEPT QUIZ

1. If a force of magnitude F can be applied in four different 2-D configurations (P,Q,R, & S), select the cases resulting in the maximum and minimum torque values on the nut. (Max, Min).

A) (Q, P) B) (R, S)

C) (P, R) D) (Q, S)

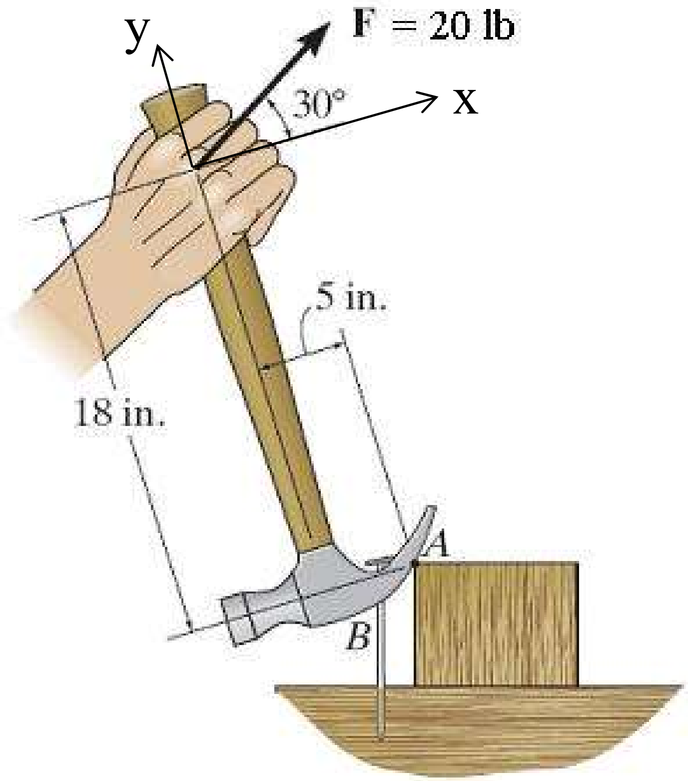


2. If $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$, then what will be the value of $\boldsymbol{M} \bullet \boldsymbol{r}$?

A) 0 B) 1

C) $r^2 F$ D) None of the above.

GROUP PROBLEM SOLVING I



Given: A 20 lb force is applied to the hammer.

Find: The moment of the force at A.

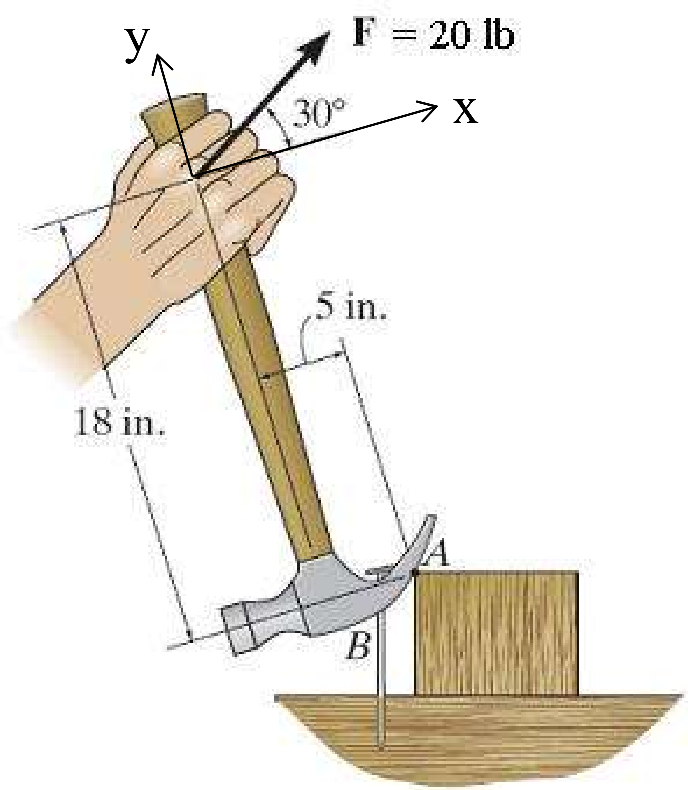
Plan:

Since this is a 2-D problem:

1) Resolve the 20 lb force along the handle's x and y axes.

2) Determine M_A using a scalar analysis.

GROUP PROBLEM SOLVING I (continued)



Solution:

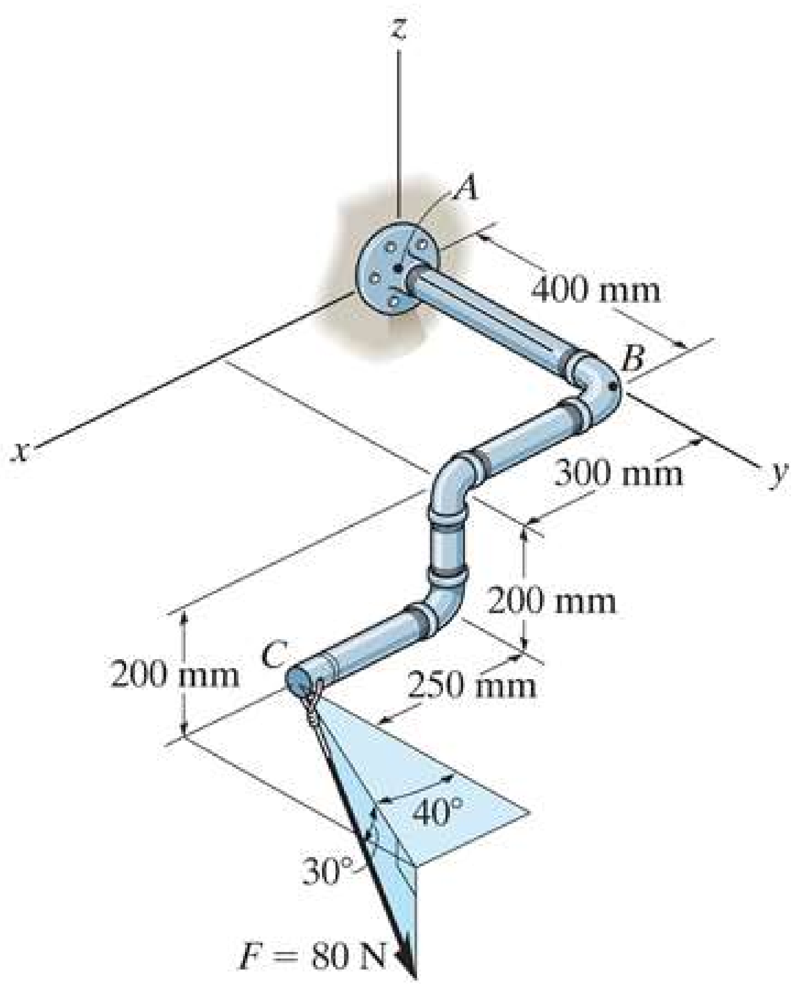
$+\uparrow F_y = 20 \sin 30° \text{ lb}$

$+\rightarrow F_x = 20 \cos 30° \text{ lb}$

$+\circlearrowleft M_A = \{-(20 \cos 30°)\text{lb } (18 \text{ in}) - (20 \sin 30°)\text{lb } (5 \text{ in})\}$

$= -361.77 \text{ lb}\cdot\text{in} =$ <u>362 lb·in (clockwise or CW)</u>

GROUP PROBLEM SOLVING II



Given: The force and geometry shown.

Find: Moment of F about point A

Plan:

1) Find $\boldsymbol{F}$ and $\boldsymbol{r}_{AC}$.

2) Determine $\boldsymbol{M}_A = \boldsymbol{r}_{AC} \times \boldsymbol{F}$

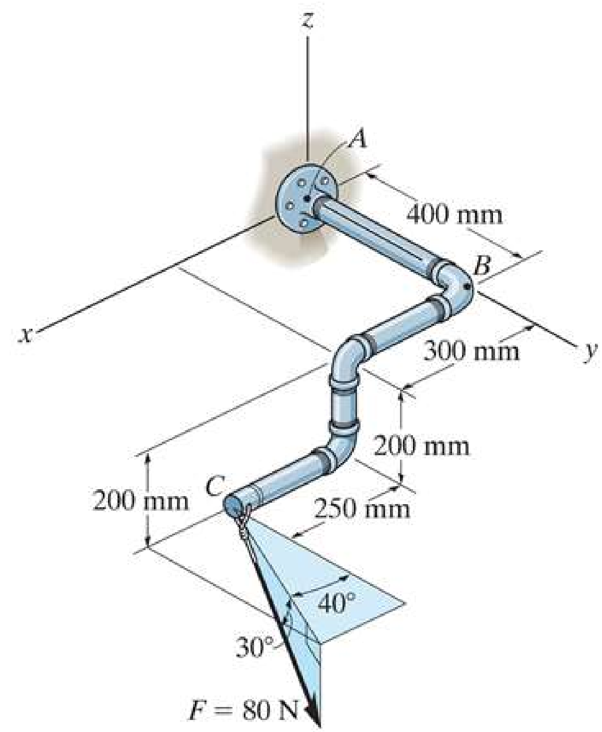
GROUP PROBLEM SOLVING II (continued)

Solution:

$\boldsymbol{F} = \{\, (80 \cos 30) \sin 40 \; \boldsymbol{i}$

$\quad + (80 \cos 30) \cos 40 \; \boldsymbol{j} - 80 \sin 30 \; \boldsymbol{k} \} \text{ N}$

$= \{44.53 \; \boldsymbol{i} + 53.07 \; \boldsymbol{j} - 40 \; \boldsymbol{k} \} \text{ N}$



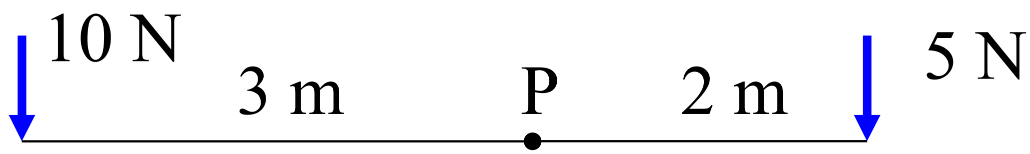
$\boldsymbol{r}_{AC} = \{0.55 \; \boldsymbol{i} + 0.4 \; \boldsymbol{j} - 0.2 \; \boldsymbol{k} \} \text{ m}$

Find the moment by using the cross product.

$$\boldsymbol{M}_A = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40 \end{vmatrix}$$

$= \{ -5.39 \; \boldsymbol{i} + 13.1 \; \boldsymbol{j} + 11.4 \; \boldsymbol{k} \} \text{ N·m}$

ATTENTION QUIZ



1. Using the CCW direction as positive, the net moment of the two forces about point P is

A) 10 N • m B) 20 N • m C) - 20 N • m

D) 40 N • m E) - 40 N • m

2. If $\boldsymbol{r} = \{ 5\, \boldsymbol{j} \}$ m and $\boldsymbol{F} = \{ 10\, \boldsymbol{k} \}$ N, the moment $\boldsymbol{r} \times \boldsymbol{F}$ equals { ________ } N·m.

A) $50\, \boldsymbol{i}$ B) $50\, \boldsymbol{j}$ C) $-50\, \boldsymbol{i}$

D) $-50\, \boldsymbol{j}$ E) 0

MOMENT ABOUT AN AXIS

Today's Objectives:

Students will be able to determine the moment of a force about an axis using

a) scalar analysis, and,

b) vector analysis.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Scalar Analysis
- Vector Analysis
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. When determining the moment of a force about a specified axis, the axis must be along ______________.

   A) the x axis   B) the y axis   C) the z axis

   D) any line in 3-D space   E) any line in the x-y plane

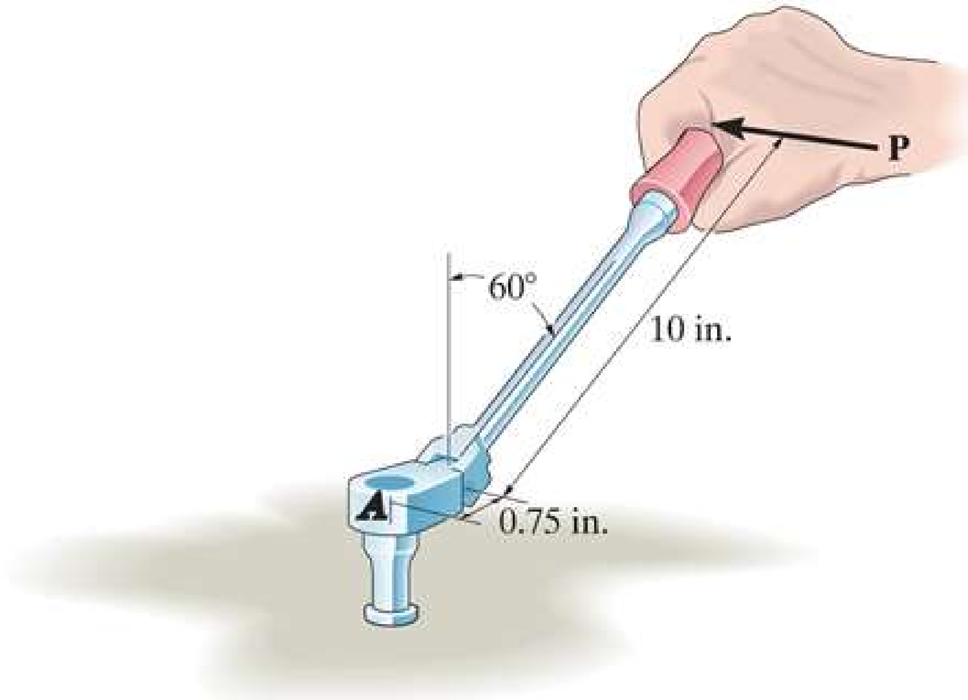
2. The triple scalar product $\boldsymbol{u} \bullet (\boldsymbol{r} \times \boldsymbol{F})$ results in

   A) a scalar quantity (+ or -).   B) a vector quantity.

   C) zero.   D) a unit vector.

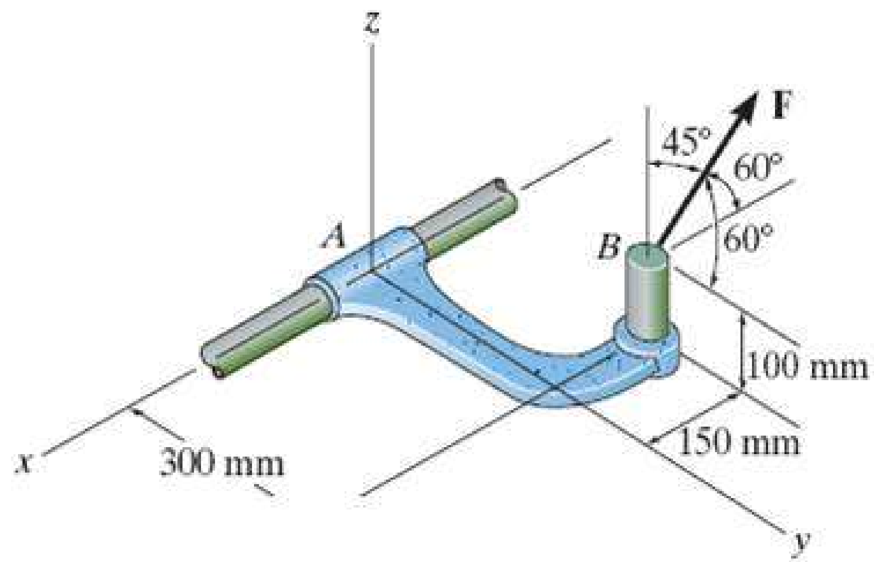
   E) an imaginary number.

APPLICATIONS



With the force ***P***, a person is creating a moment $\boldsymbol{M_A}$ using this flex-handle socket wrench. Does all of $\boldsymbol{M_A}$ act to turn the socket? How would you calculate an answer to this question?

APPLICATIONS (continued)



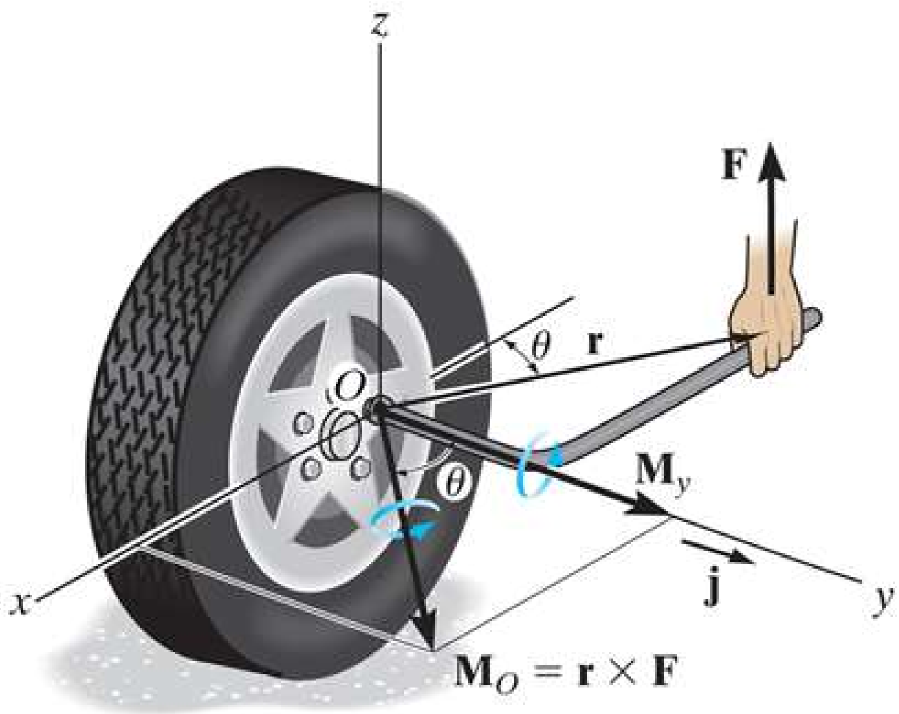
Sleeve A of this bracket can provide a maximum resisting moment of 125 N·m about the x-axis. How would you determine the maximum magnitude of ***F*** before turning about the x-axis occurs?

SCALAR ANALYSIS

Recall that the moment of a scalar force about any point O is $M_O = F\ d_O$ where d_O is the perpendicular (or shortest) distance from the point to the force's line of action. This concept can be extended to find the moment of a force about an axis.

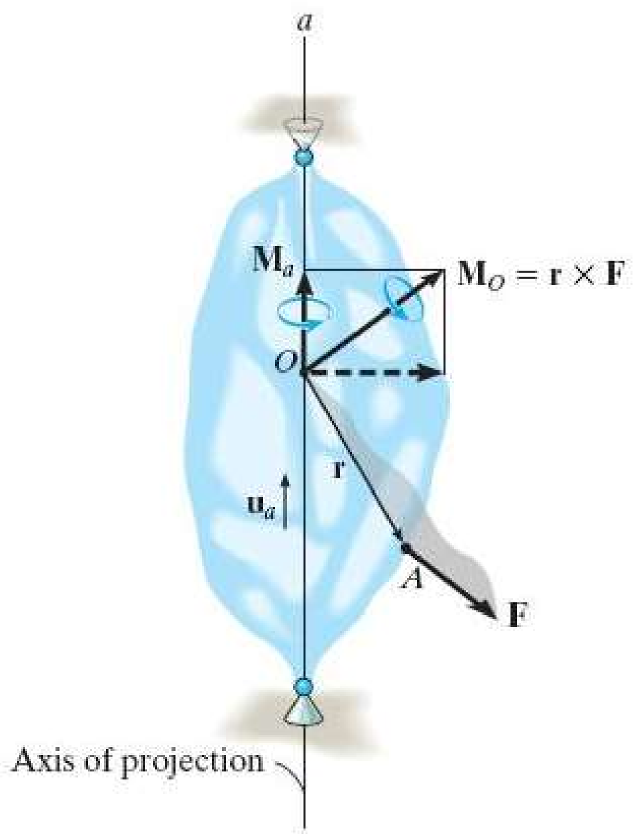
Finding the moment of a force about an axis can help answer the types of questions we just considered.

SCALAR ANALYSIS (continued)



In the figure above, the moment about the y-axis would be $M_y = F_z\,(d_x) = F\,(r \cos \theta)$. However, unless the force can easily be broken into components and the “d_x” found quickly, such calculations are not always trivial and vector analysis may be much easier (and less likely to produce errors).

VECTOR ANALYSIS



Our goal is to find the moment of ***F*** (the tendency to rotate the body) about the *a*-axis.

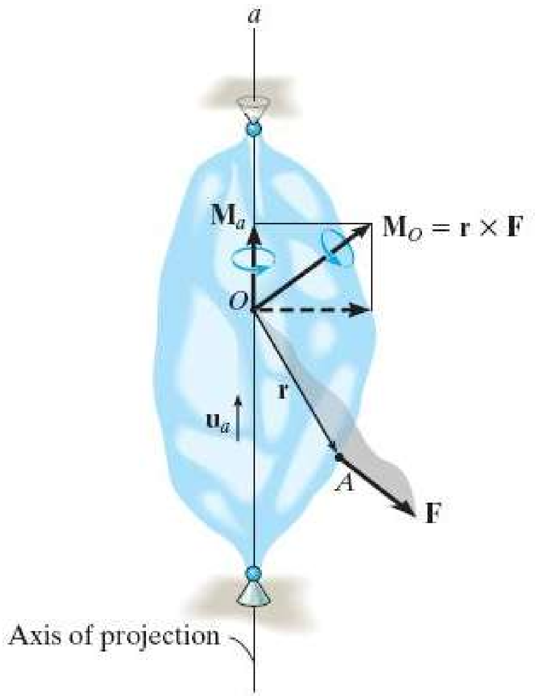
First compute the moment of ***F*** about any arbitrary point O that lies on the *a*-axis using the cross product.

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

Now, find the component of $\boldsymbol{M_O}$ along the *a*-axis using the dot product.

$$M_a = \boldsymbol{u_a} \bullet \boldsymbol{M_O}$$

VECTOR ANALYSIS (continued)



M_a can also be obtained as

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The above equation is also called the triple scalar product.

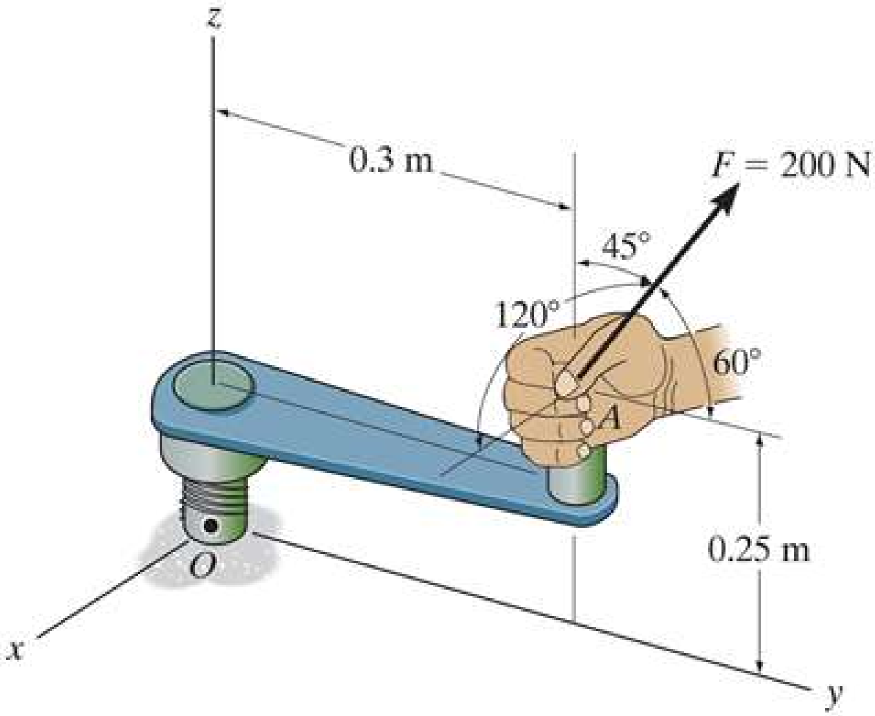
In the this equation,

$\boldsymbol{u_a}$ represents the unit vector along the *a*-axis,

$\boldsymbol{r}$ is the position vector from any point on the *a*-axis to any point A on the line of action of the force, and

$\boldsymbol{F}$ is the force vector.

EXAMPLE



Given: A force is applied to the tool as shown.

Find: The magnitude of the moment of this force about the x axis of the value.

Plan:

1) Use $M_z = \boldsymbol{u} \bullet (\boldsymbol{r} \times \boldsymbol{F})$.

2) First, find ***F*** in Cartesian vector form.

3) Note that ***u*** $= 1$ ***i*** in this case.

4) The vector ***r*** is the position vector from O to A.

EXAMPLE (continued)

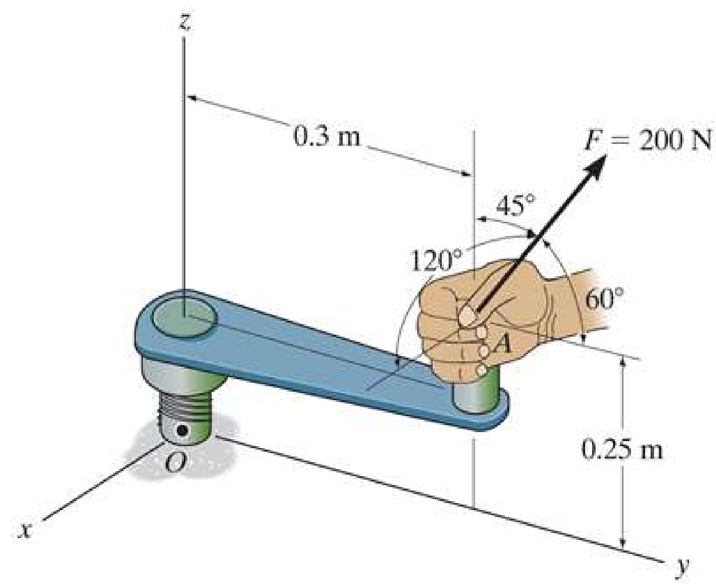
Solution:

$\boldsymbol{u} = 1\ \boldsymbol{i}$

$\boldsymbol{r}_{OA} = \{0\ \boldsymbol{i} + 0.3\ \boldsymbol{j} + 0.25\ \boldsymbol{k}\}$ m

$\boldsymbol{F} = 200\ (\cos 120\ \boldsymbol{i} + \cos 60\ \boldsymbol{j} + \cos 45\ \boldsymbol{k})$ N

$= \{-100\ \boldsymbol{i} + 100\ \boldsymbol{j} + 141.4\ \boldsymbol{k}\}$ N



Now find $M_z = \boldsymbol{u} \bullet (\boldsymbol{r}_{OA} \times \boldsymbol{F})$

$$M_z = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.25 \\ -100 & 100 & 141.4 \end{vmatrix} = 1\{0.3\ (141.4) - 0.25\ (100)\}\ \text{N}\cdot\text{m}$$

$M_z = 17.4$ N·m CCW

CONCEPT QUIZ

1. The vector operation $(\boldsymbol{P} \times \boldsymbol{Q}) \bullet \boldsymbol{R}$ equals

A) $\boldsymbol{P} \times (\boldsymbol{Q} \bullet \boldsymbol{R})$.

B) $\boldsymbol{R} \bullet (\boldsymbol{P} \times \boldsymbol{Q})$.

C) $(\boldsymbol{P} \bullet \boldsymbol{R}) \times (\boldsymbol{Q} \bullet \boldsymbol{R})$.

D) $(\boldsymbol{P} \times \boldsymbol{R}) \bullet (\boldsymbol{Q} \times \boldsymbol{R})$.

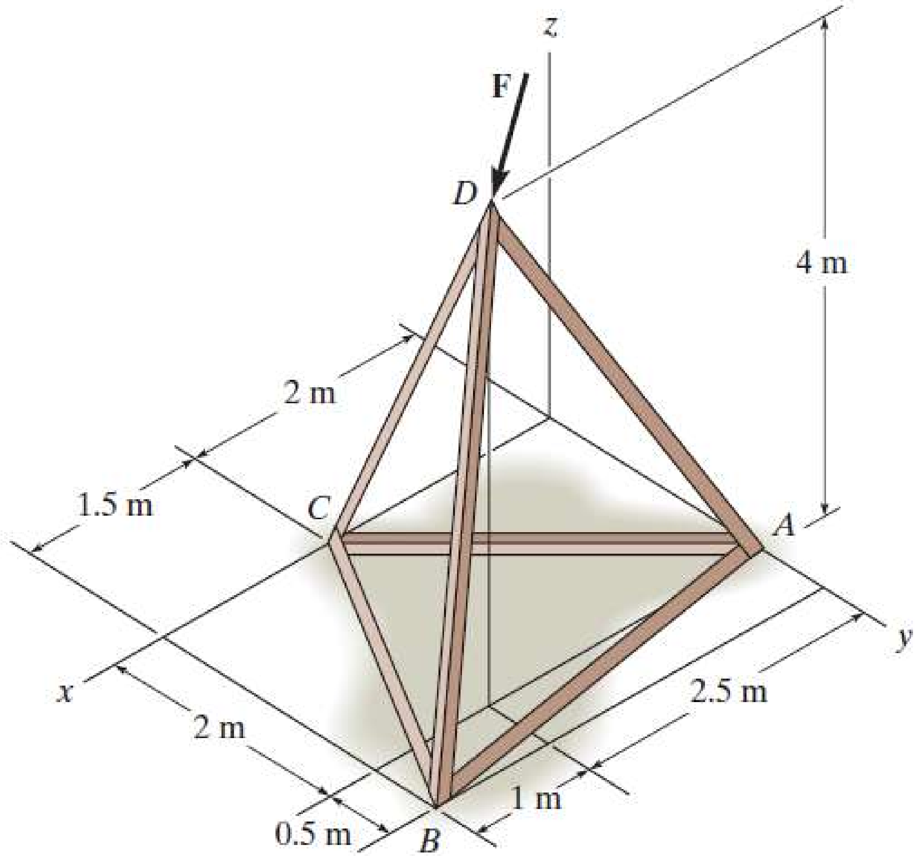
CONCEPT QUIZ (continued)

2. The force ***F*** is acting along DC. Using the triple scalar product to determine the moment of ***F*** about the bar BA, you could use any of the following position vectors except ______.

   A) $\boldsymbol{r}_{BC}$ B) $\boldsymbol{r}_{AD}$

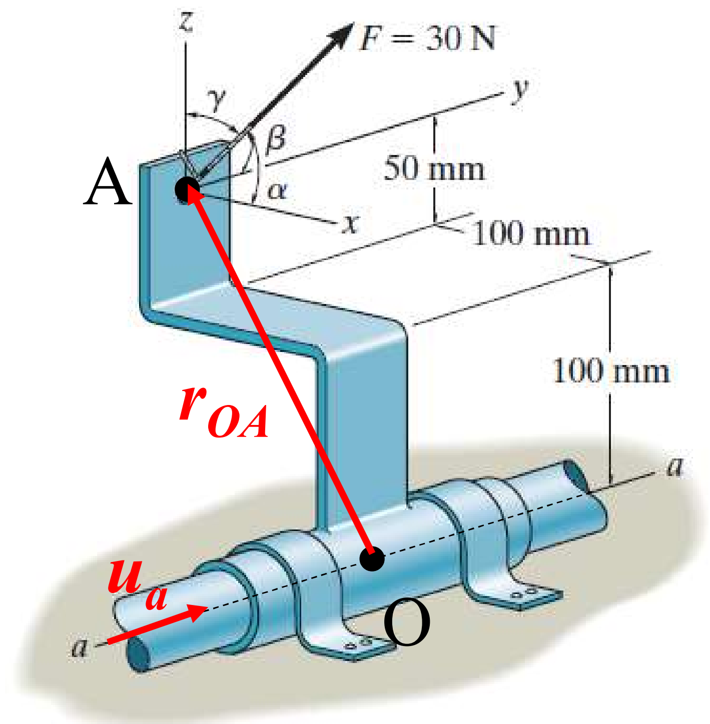
   C) $\boldsymbol{r}_{AC}$ D) $\boldsymbol{r}_{DB}$

   E) $\boldsymbol{r}_{BD}$



GROUP PROBLEM SOLVING

Given: The force of F = 30 N acts on the bracket. $\alpha = 60°$, $\beta = 60°$, $\gamma = 45°$.



Find: The moment of ***F*** about the a-a axis.

Plan:

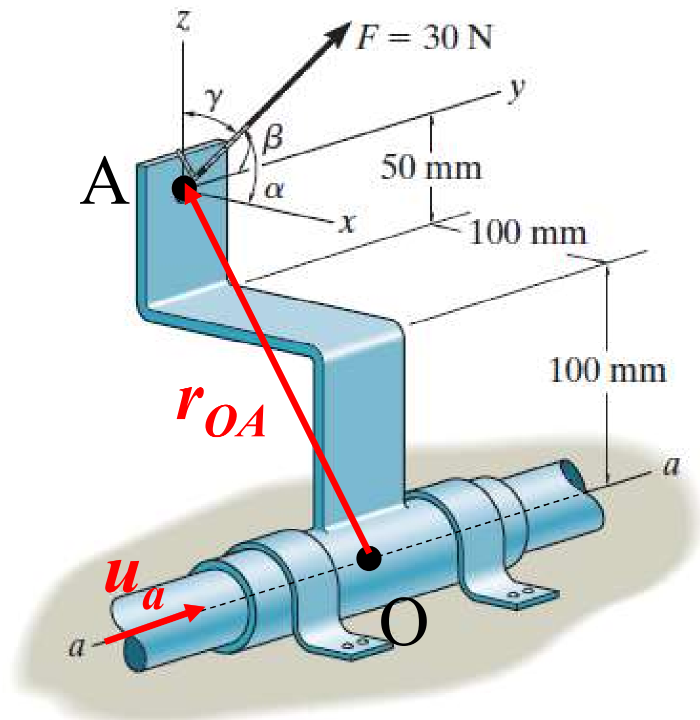
1) Find $\boldsymbol{u_a}$ and $\boldsymbol{r_{OA}}$
2) Find ***F*** in Cartesian vector form.
3) Use $M_a = \boldsymbol{u_a} \cdot (\boldsymbol{r_{OA}} \times \boldsymbol{F})$

GROUP PROBLEM SOLVING (continued)

Solution:

$\boldsymbol{u_a} = \boldsymbol{j}$

$\boldsymbol{r_{OA}} = \{-\,0.1\ \boldsymbol{i} + 0.15\ \boldsymbol{k}\}\ \text{m}$



$$\boldsymbol{F} = 30\ \{\cos 60°\ \boldsymbol{i} + \cos 60°\ \boldsymbol{j} + \cos 45°\ \boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{F} = \{\ 15\ \boldsymbol{i} + 15\ \boldsymbol{j} + 21.21\ \boldsymbol{k}\}\ \text{N}$$

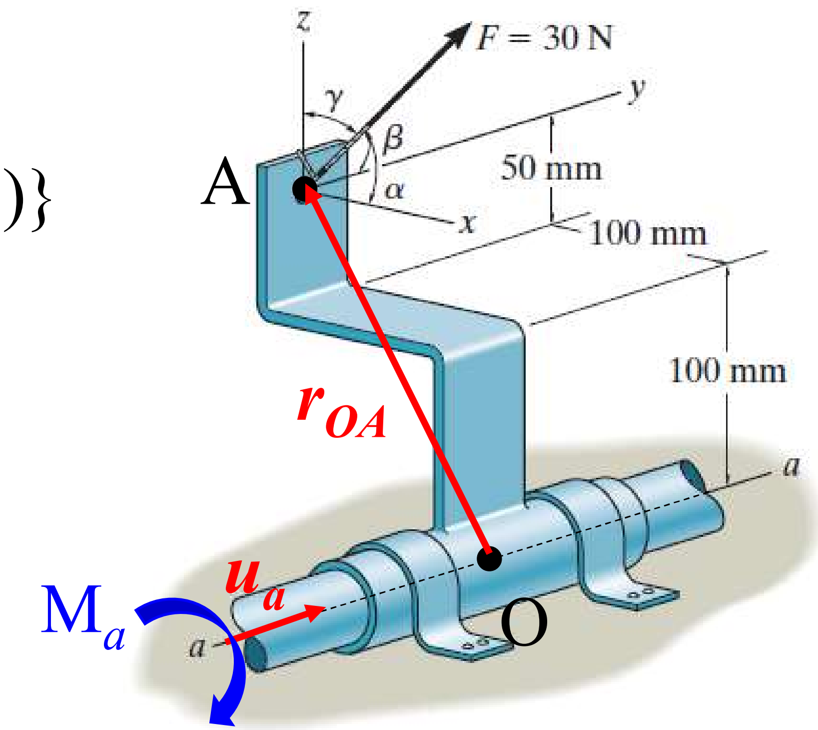
GROUP PROBLEM SOLVING (continued)

Now find the triple product, $M_a = \boldsymbol{u_a} \bullet (\boldsymbol{r_{OA}} \times \boldsymbol{F})$

$$M_a = \begin{vmatrix} 0 & 1 & 0 \\ -0.1 & 0 & 0.15 \\ 15 & 15 & 21.21 \end{vmatrix} \text{ N}\cdot\text{m}$$

$$M_a = -1 \, \{-0.1\,(21.21) - 0.15\,(15)\}$$

$$= \underline{4.37 \text{ N}\cdot\text{m}}$$

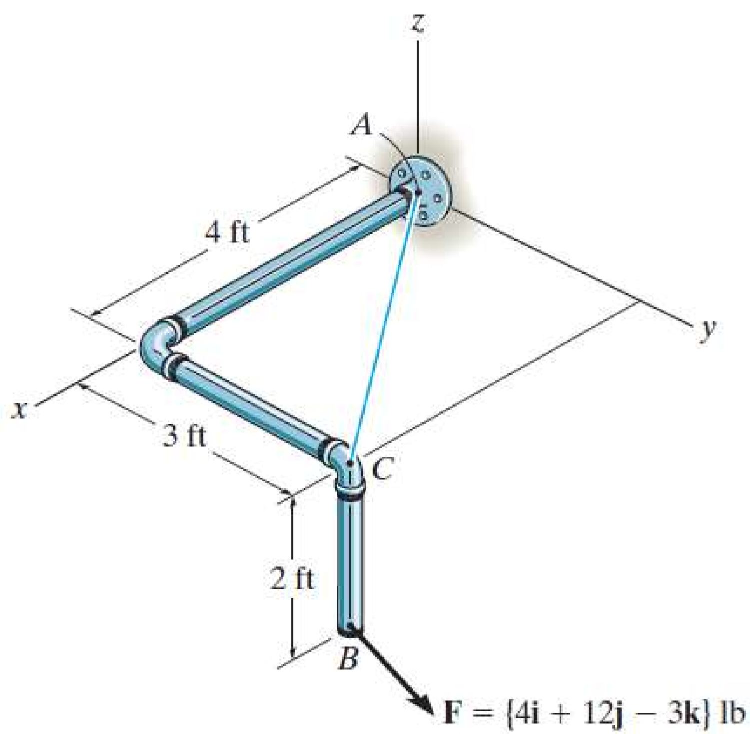


ATTENTION QUIZ

1. For finding the moment of the force ***F*** about the x-axis, the position vector in the triple scalar product should be ___ .

A) $\boldsymbol{r}_{AC}$ B) $\boldsymbol{r}_{BA}$

C) $\boldsymbol{r}_{AB}$ D) $\boldsymbol{r}_{BC}$



2. If $\boldsymbol{r} = \{1\ \boldsymbol{i} + 2\ \boldsymbol{j}\}$ m and $\boldsymbol{F} = \{10\ \boldsymbol{i} + 20\ \boldsymbol{j} + 30\ \boldsymbol{k}\}$ N, then the moment of ***F*** about the y-axis is ____ N·m.

A) 10   B) -30

C) -40   D) None of the above.

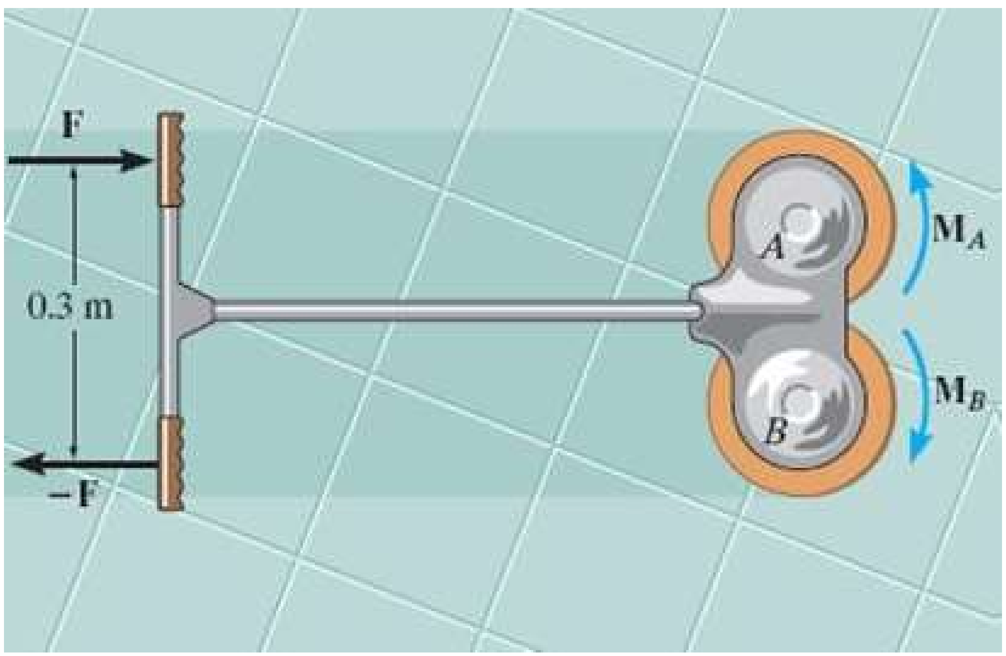
MOMENT OF A COUPLE

Today's Objectives:

Students will be able to

a) define a couple, and,

b) determine the moment of a couple.



In-Class activities:

- Check Homework
- Reading Quiz
- Applications
- Moment of a Couple
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. In statics, a couple is defined as __________ separated by a perpendicular distance.

   A) two forces in the same direction

   B) two forces of equal magnitude

   C) two forces of equal magnitude acting in the same direction

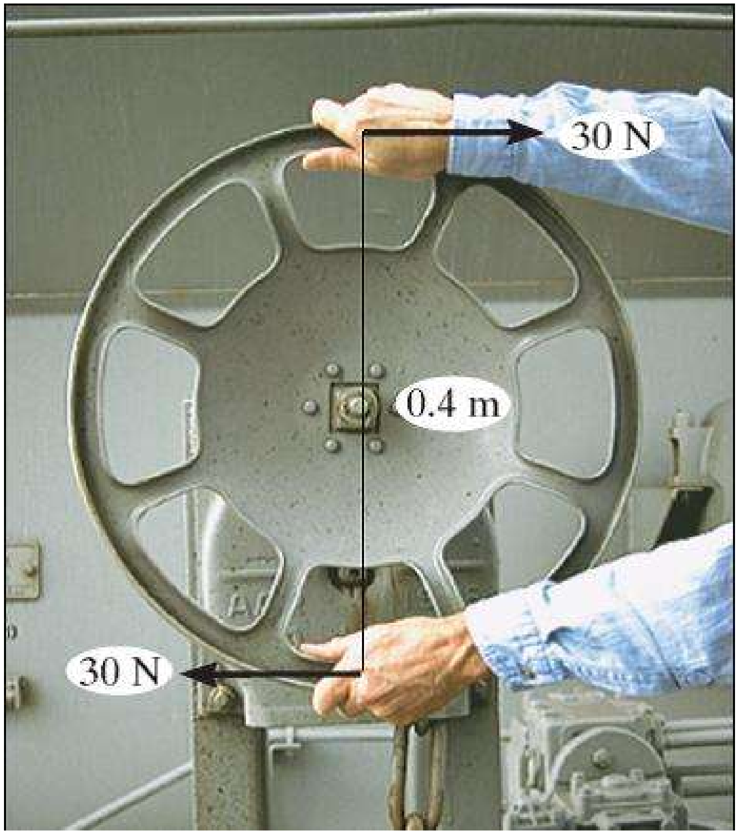
   D) two forces of equal magnitude acting in opposite directions

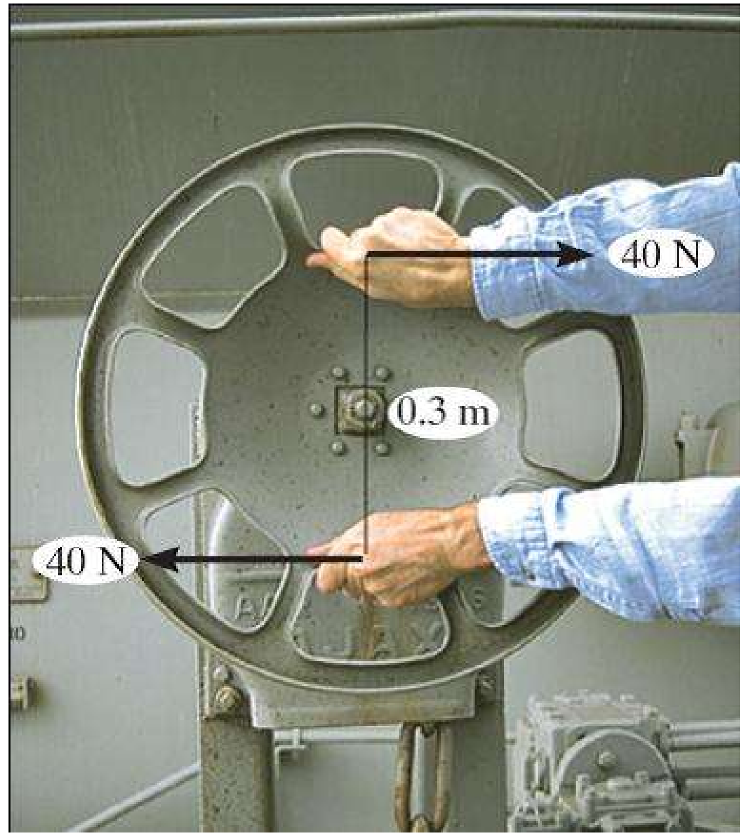
2. The moment of a couple is called a _________ vector.

   A) Free B) Spinning

   C) Fixed D) Sliding

APPLICATIONS





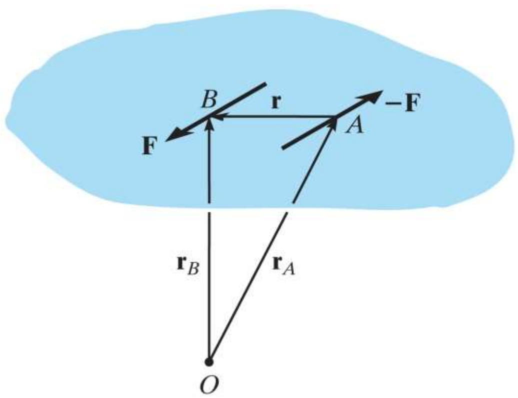
A torque or moment of 12 N·m is required to rotate the wheel. Why does one of the two grips of the wheel above require less force to rotate the wheel?

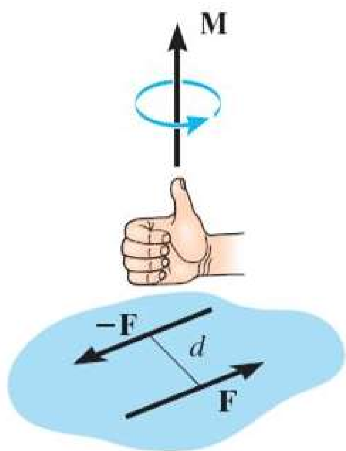
APPLICATIONS (continued)

When you grip a vehicle's steering wheel with both hands and turn, a couple moment is applied to the wheel.

Would older vehicles without power steering have needed larger or smaller steering wheels?

MOMENT OF A COUPLE





A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance "d."

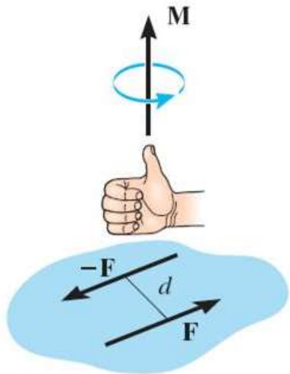
The moment of a couple is defined as

$M_O = F\ d$ (using a scalar analysis) or as

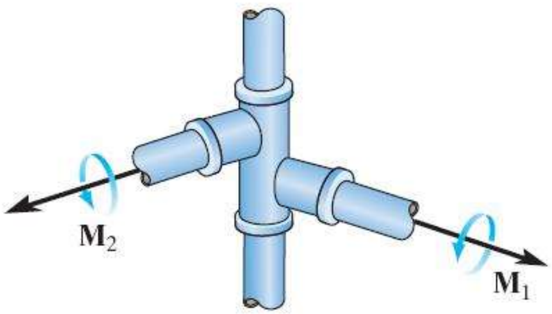
$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$ (using a vector analysis).

Here $\boldsymbol{r}$ is any position vector from the line of action of $\boldsymbol{F}$ to the line of action of $\boldsymbol{F}$.

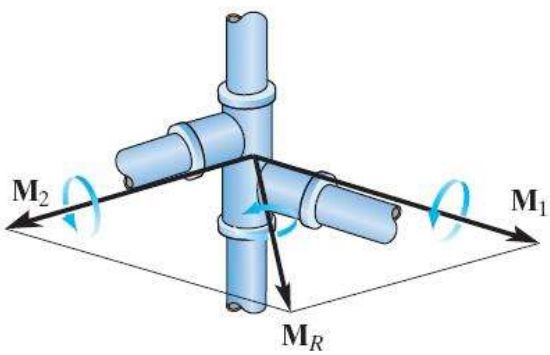
MOMENT OF A COUPLE (continued)



The net external effect of a couple is that the net force equals zero and the magnitude of the net moment equals F *d.

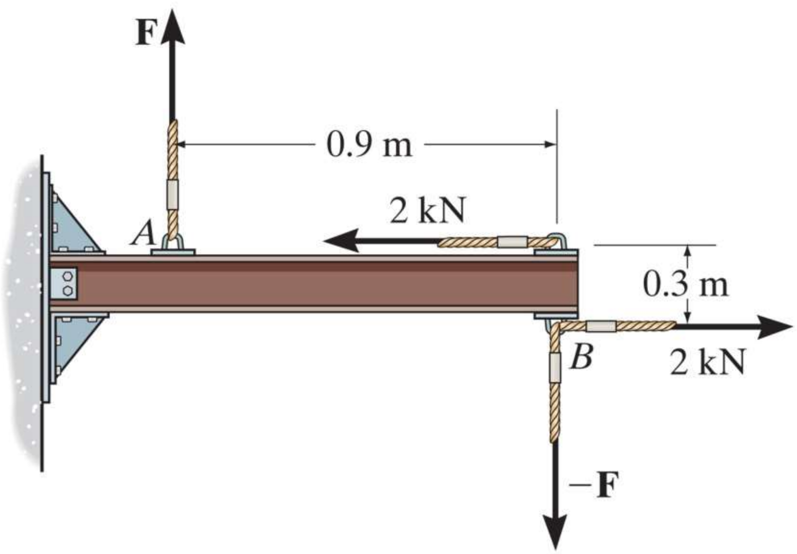


Since the moment of a couple depends only on the distance between the forces, the moment of a couple is a free vector. It can be moved anywhere on the body and have the same external effect on the body.



Moments due to couples can be added together using the same rules as adding any vectors.

EXAMPLE I : SCALAR APPROACH



Given: Two couples act on the beam with the geometry shown.

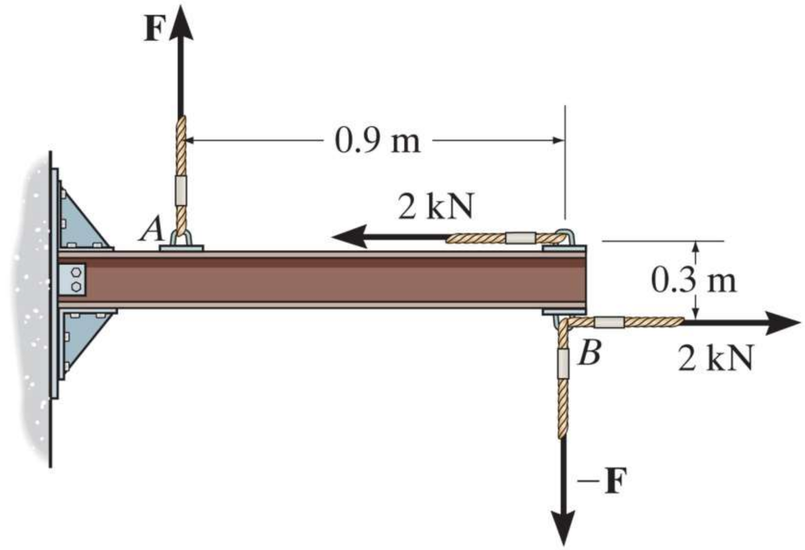
Find: The magnitude of F so that the resultant couple moment is 1.5 kN·m clockwise.

Plan:

1) Add the two couples to find the resultant couple.
2) Equate the net moment to 1.5 kN·m clockwise to find F.

EXAMPLE I : SCALAR APPROACH (continued)

Solution:



The net moment is equal to:

$$+ \Sigma M = - F\,(0.9) + (2)\,(0.3)$$

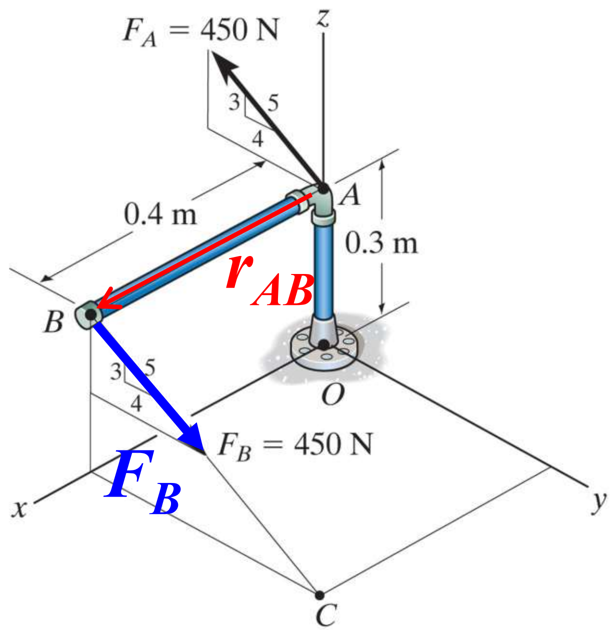
$$= - 0.9\,F + 0.6$$

$$- 1.5\ \text{kN·m} \quad = - 0.9\,F + 0.6$$

Solving for the unknown force F, we get

F = 2.33 kN

EXAMPLE II : VECTOR APPROACH



Given: A 450 N force couple acting on the pipe assembly.

Find: The couple moment in Cartesian vector notation.

Plan:

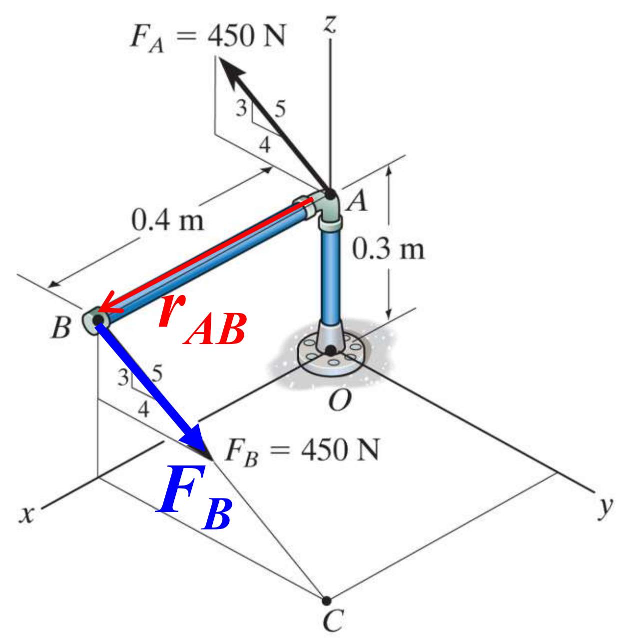
1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \boldsymbol{F}_B$.

3) Calculate the cross product to find $\boldsymbol{M}$.

EXAMPLE II: VECTOR APPROACH (continued)

Solution:



$$r_{AB} = \{ 0.4\ \boldsymbol{i} \}\ \text{m}$$

$$F_B = \{0\ \boldsymbol{i} + 450(4/5)\ \boldsymbol{j} - 450(3/5)\ \boldsymbol{k}\}\ \text{N}$$

$$= \{0\ \boldsymbol{i} + 360\ \boldsymbol{j} - 270\ \boldsymbol{k}\}\ \text{N}$$

$$M = r_{AB} \times F_B$$

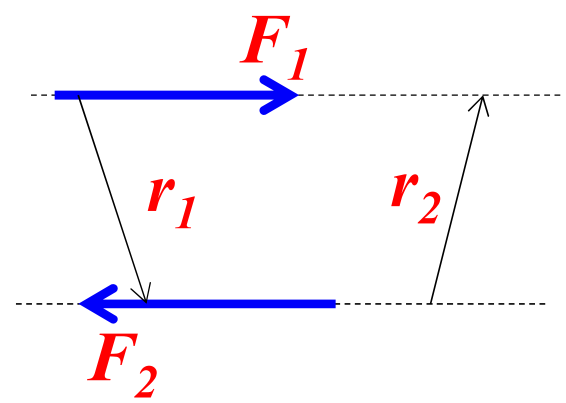
$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.4 & 0 & 0 \\ 0 & 360 & -270 \end{vmatrix}\ \text{N}\cdot\text{m}$$

$$= [\{0(-270) - 0(360)\}\ \boldsymbol{i} - \{4(-270) - 0(0)\}\ \boldsymbol{j} + \{0.4(360) - 0(0)\}\ \boldsymbol{k}]\ \text{N}\cdot\text{m}$$

$$= \{\underline{0\ \boldsymbol{i}} + \underline{108\ \boldsymbol{j}} + \underline{144\ \boldsymbol{k}}\}\ \underline{\text{N}\cdot\text{m}}$$

CONCEPT QUIZ

1. F_1 and F_2 form a couple. The moment of the couple is given by ____ .



A) $r_1 \times F_1$ B) $r_2 \times F_1$

C) $F_2 \times r_1$ D) $r_2 \times F_2$

2. If three couples act on a body, the overall result is that

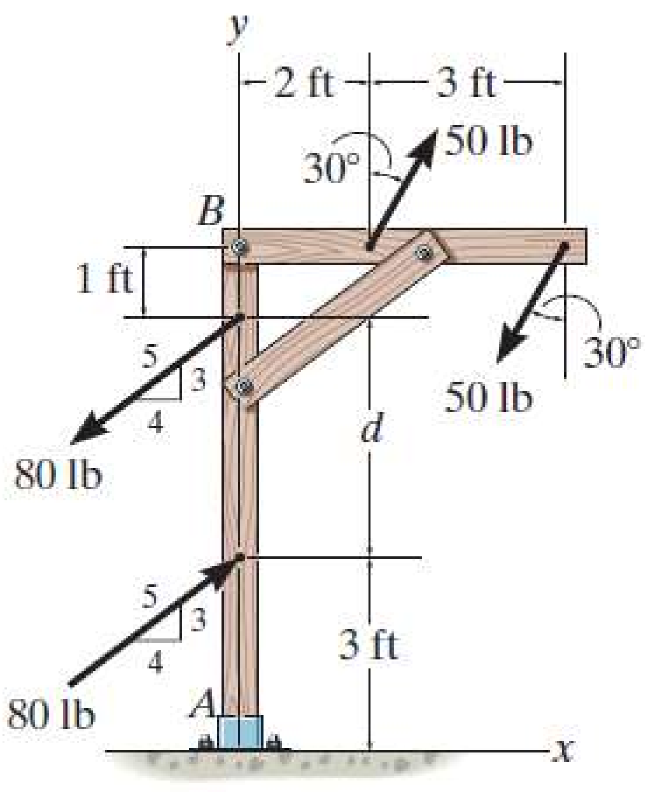
A) The net force is not equal to 0.

B) The net force and net moment are equal to 0.

C) The net moment equals 0 but the net force is not necessarily equal to 0.

D) The net force equals 0 but the net moment is not necessarily equal to 0 .

GROUP PROBLEM SOLVING I



Given: Two couples act on the beam with the geometry shown and d = 4 ft.

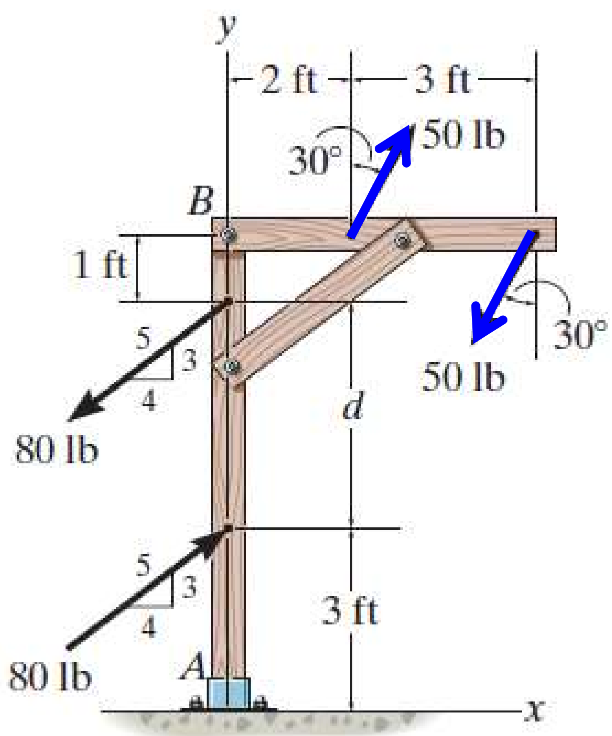
Find: The resultant couple

Plan:

1) Resolve the forces in x and y-directions so they can be treated as couples.

2) Add these two couples to find the resultant couple.

GROUP PROBLEM SOLVING I (continued)



The x and y components of the upper-left 50 lb force are:

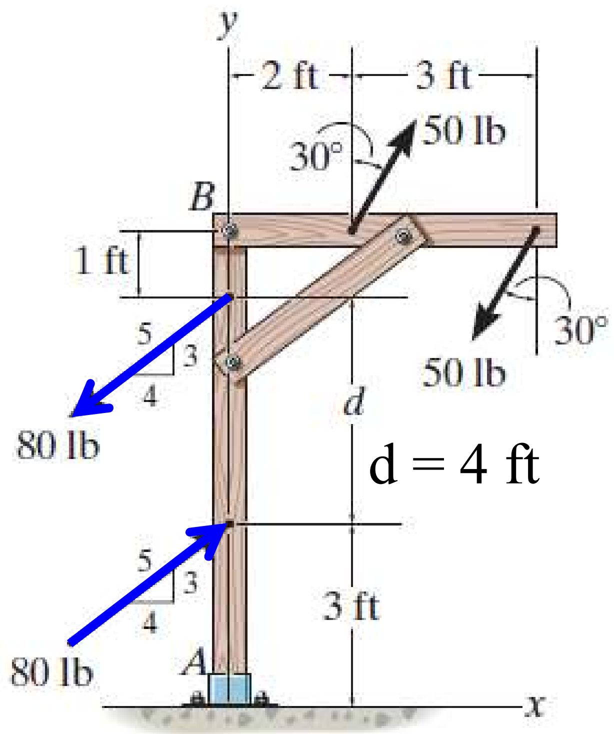
50 lb (cos 30°) = 43.30 lb vertically up

50 lb (sin 30°) = 25 lb to the right

Do both of these components form couples with their matching components of the other 50 force?

No! Only the 43.30 lb components create a couple. Why?

GROUP PROBLEM SOLVING I (continued)



Now resolve the lower 80 lb force:

(80 lb) (3/5), acting up

(80 lb) (4/5), acting to the right

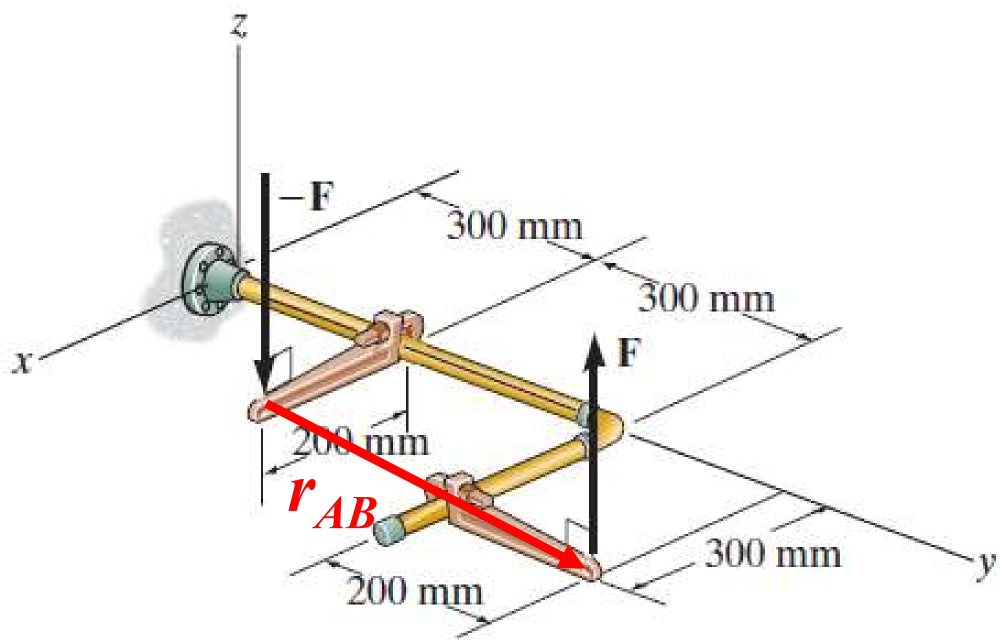
Do both of these components create a couple with components of the other 80 lb force?

The net moment is equal to:

$$+\circlearrowleft \Sigma M = -(43.3 \text{ lb})(3 \text{ ft}) + (64 \text{ lb})(4 \text{ ft})$$

$$= -129.9 + 256 = \underline{126 \text{ ft}\cdot\text{lb CCW}}$$

GROUP PROBLEM SOLVING II



Given: $\boldsymbol{F} = \{80\ \boldsymbol{k}\}$ N and

$-\boldsymbol{F} = \{-80\ \boldsymbol{k}\}$ N

Find: The couple moment acting on the pipe assembly using Cartesian vector notation.

Plan:

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \{80\ \boldsymbol{k}\}$ N.

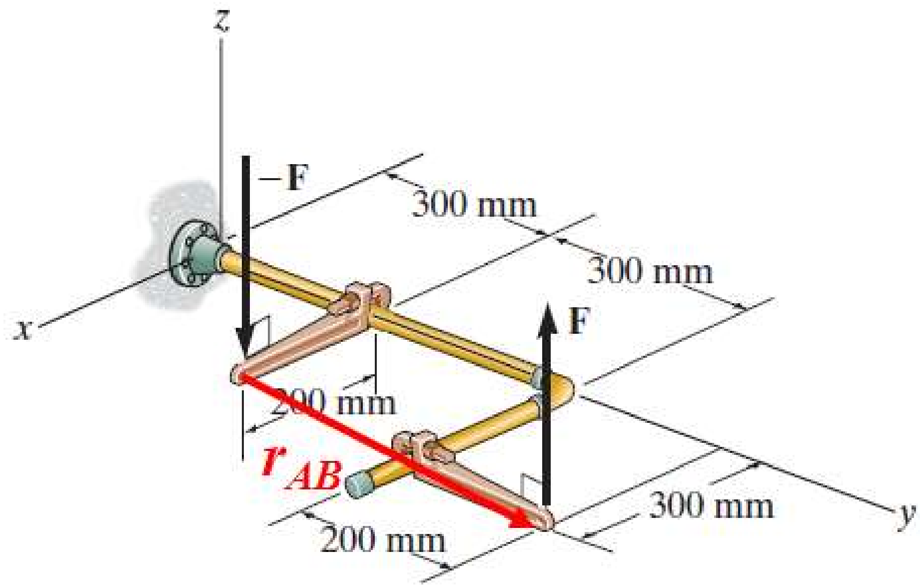
3) Calculate the cross product to find $\boldsymbol{M}$.

GROUP PROBLEM SOLVING II (continued)

$$\boldsymbol{r}_{AB} = \{\, (0.3 - 0.2\,)\,\boldsymbol{i} + (0.8 - 0.3)\,\boldsymbol{j} \;+ (0 - 0)\,\boldsymbol{k} \,\}\ \text{m}$$

$$= \{\, 0.1\,\boldsymbol{i} \;+\; 0.5\,\boldsymbol{j} \,\}\ \text{m}$$

$$\boldsymbol{F} \;= \{80\,\boldsymbol{k}\}\ \text{N}$$



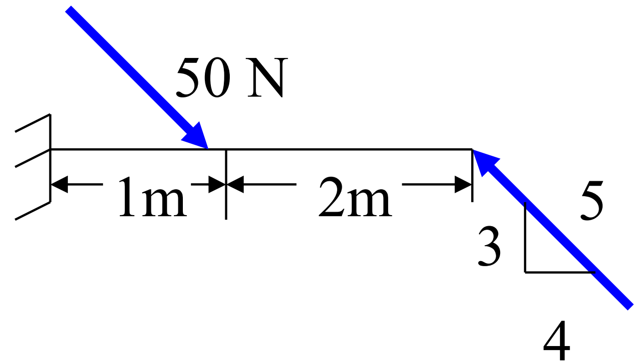
$$\boldsymbol{M} \;=\; \boldsymbol{r}_{AB} \times \boldsymbol{F} \;=\; \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & 80 \end{vmatrix}\ \text{N}\cdot\text{m}$$

$$= \{(40 - 0)\,\boldsymbol{i} - (8 - \;0)\,\boldsymbol{j} + (0)\,\boldsymbol{k}\}\ \text{N}\cdot\text{m}$$

$$= \{\, \underline{40}\,\boldsymbol{i}\ \underline{-\,8}\,\boldsymbol{j} \;\}\ \underline{\text{N}\cdot\text{m}}$$

ATTENTION QUIZ

1. A couple is applied to the beam as shown. Its moment equals _____ N·m.

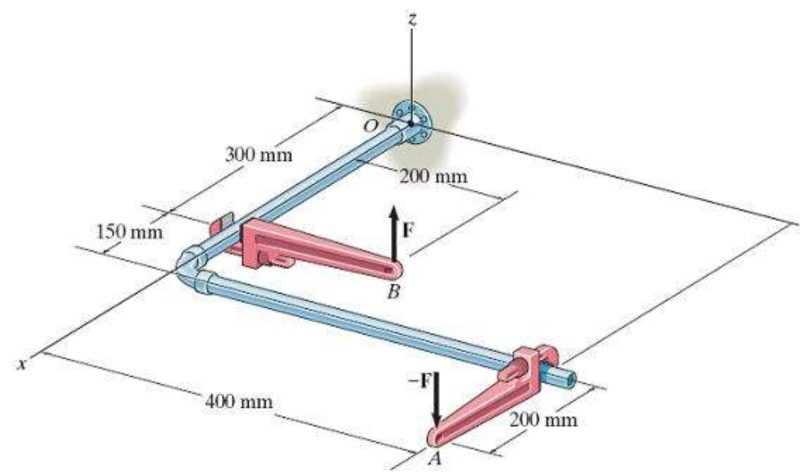


A) 50 B) 60

C) 80 D) 100

2. You can determine the couple moment as $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$

If $\boldsymbol{F} = \{ -20\ \boldsymbol{k}\}$ lb, then $\boldsymbol{r}$ is



A) $\boldsymbol{r}_{BC}$ B) $\boldsymbol{r}_{AB}$

C) $\boldsymbol{r}_{CB}$ D) $\boldsymbol{r}_{BA}$

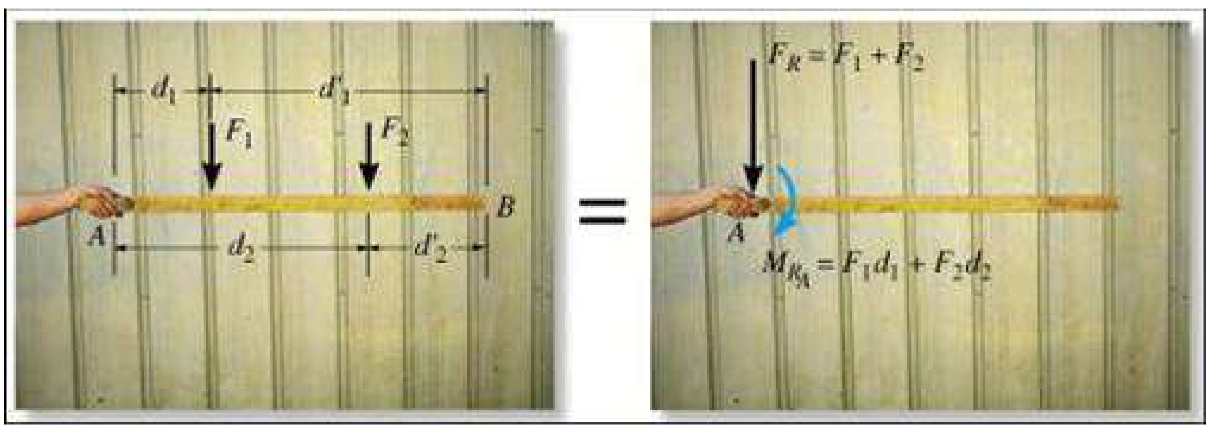
SIMPLIFICATION OF FORCE AND COUPLE SYSTEMS & THEIR FURTHER SIMPLIFICATION

Today's Objectives:

Students will be able to:

a) Determine the effect of moving a force.

b) Find an equivalent force-couple system for a system of forces and couples.



In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Equivalent Systems
- System Reduction
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. A general system of forces and couple moments acting on a rigid body can be reduced to a ___ .

   A) single force

   B) single moment

   C) single force and two moments

   D) single force and a single moment

2. The original force and couple system and an equivalent force-couple system have the same _____ effect on a body.

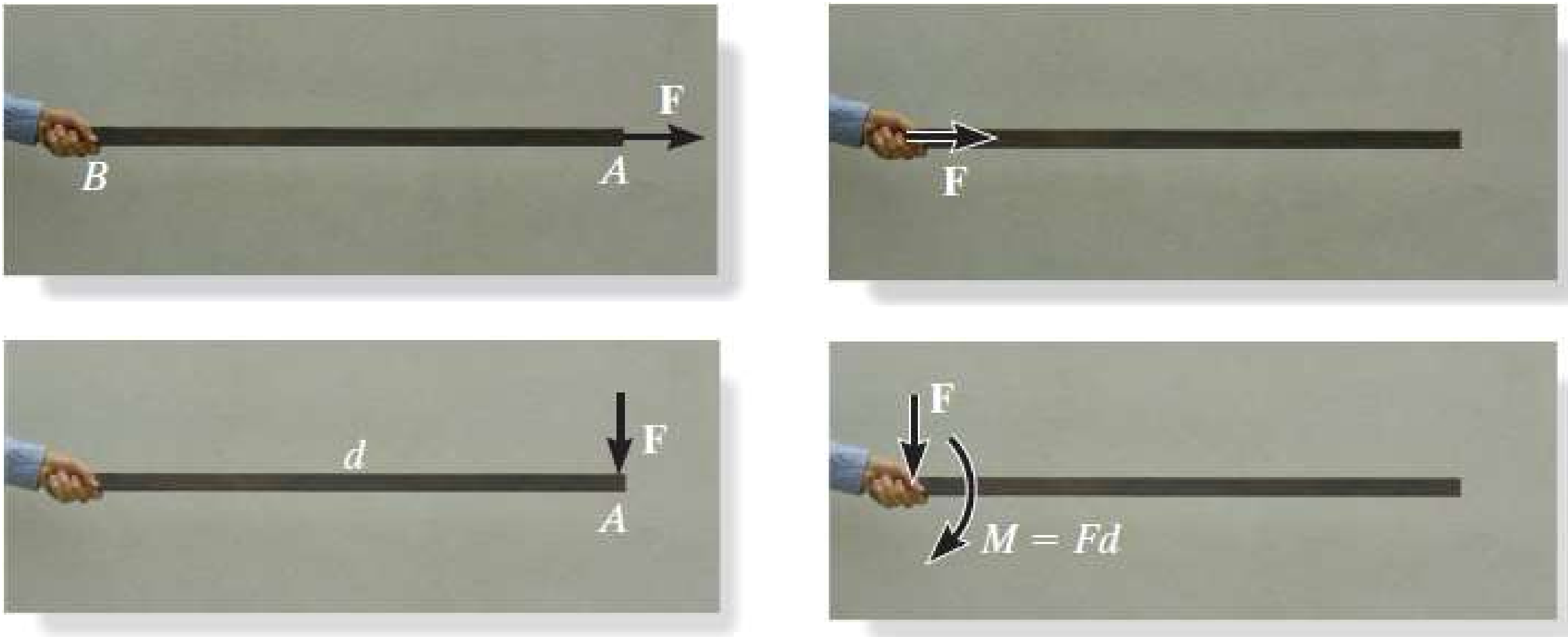
   A) internal

   B) external

   C) internal and external

   D) microscopic

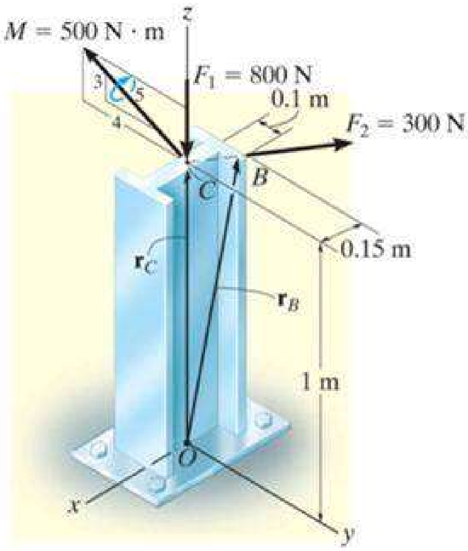
APPLICATIONS



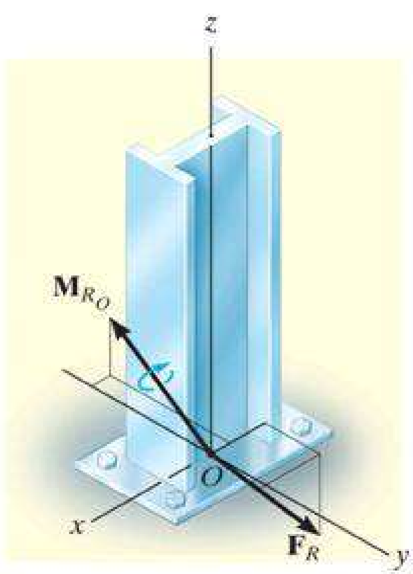
What are the resultant effects on the person's hand when the force is applied in these four different ways?

Why is understanding these differences important when designing various load-bearing structures?

APPLICATIONS (continued)



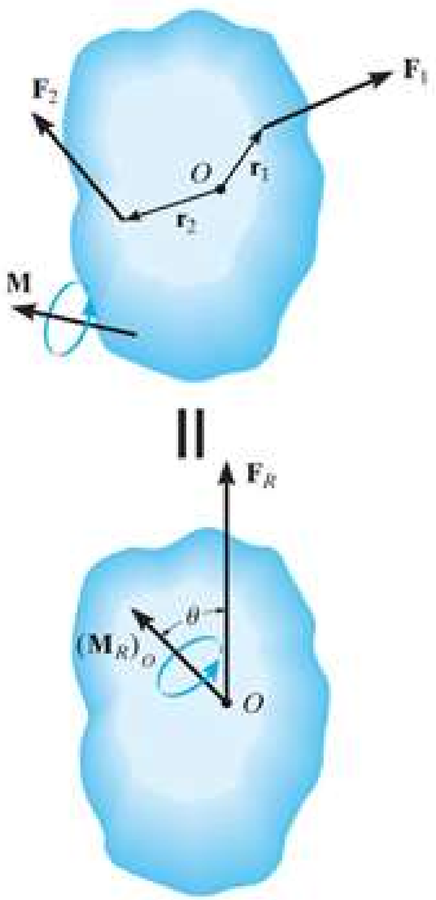
|| ??



Several forces and a couple moment are acting on this vertical section of an I-beam.

For the process of designing the I-beam, it would be very helpful if you could replace the various forces and moment just one force and one couple moment at point O with the same external effect? How will you do that?

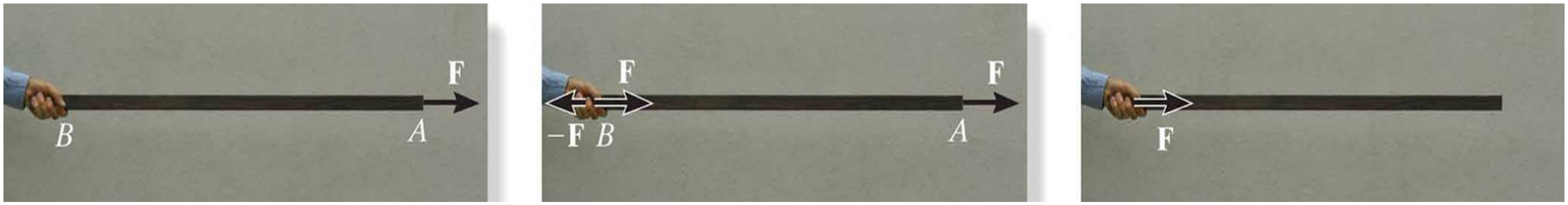
SIMPLIFICATION OF FORCE AND COUPLE SYSTEM (Section 4.7)



When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect.

The two force and couple systems are called equivalent systems since they have the same external effect on the body.

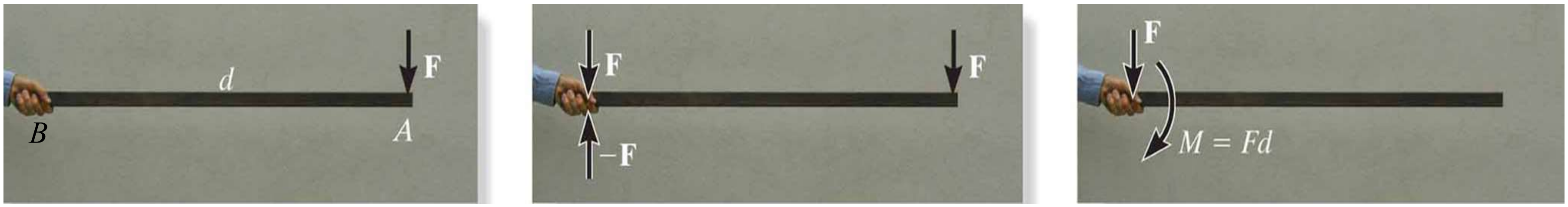
MOVING A FORCE ON ITS LINE OF ACTION



Moving a force from A to B, when both points are on the vector's line of action, does not change the external effect.

Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).

MOVING A FORCE OFF OF ITS LINE OF ACTION

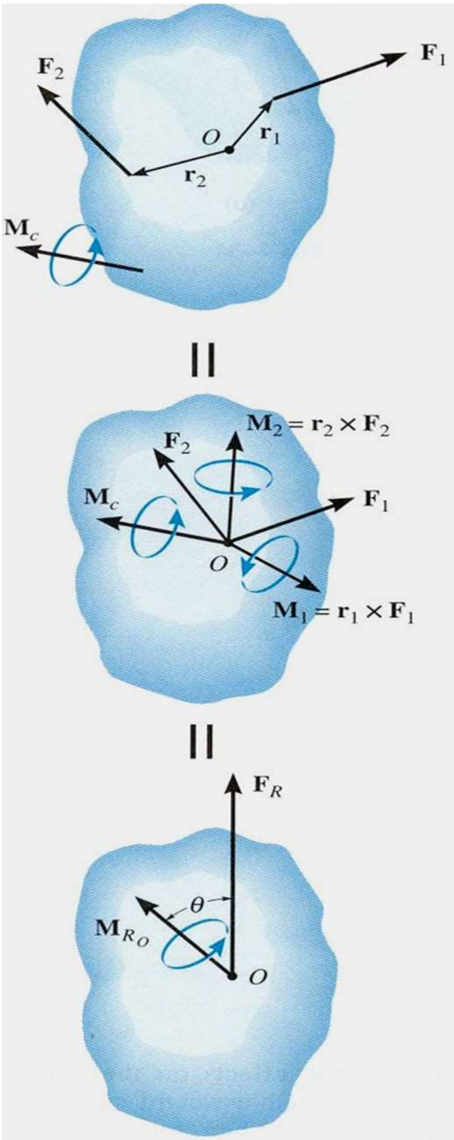


When a force is moved, but not along its line of action, there is a change in its external effect!

Essentially, moving a force from point A to B (as shown above) requires creating an additional couple moment. So moving a force means you have to “add” a new couple.

Since this new couple moment is a “free” vector, it can be applied at any point on the body.

SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM



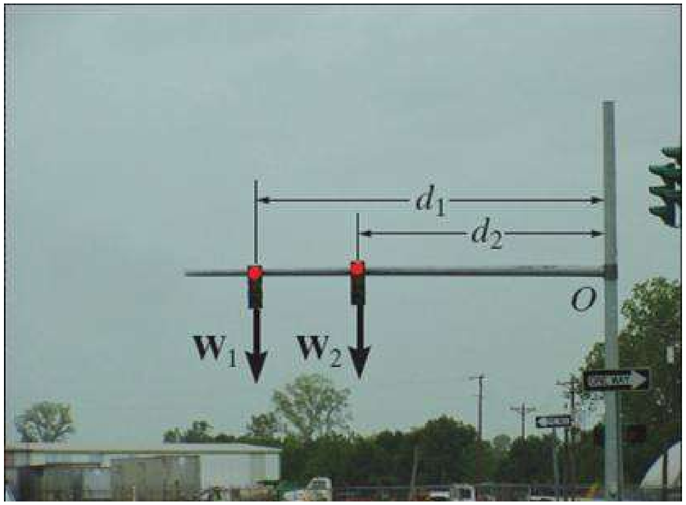
When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point O.

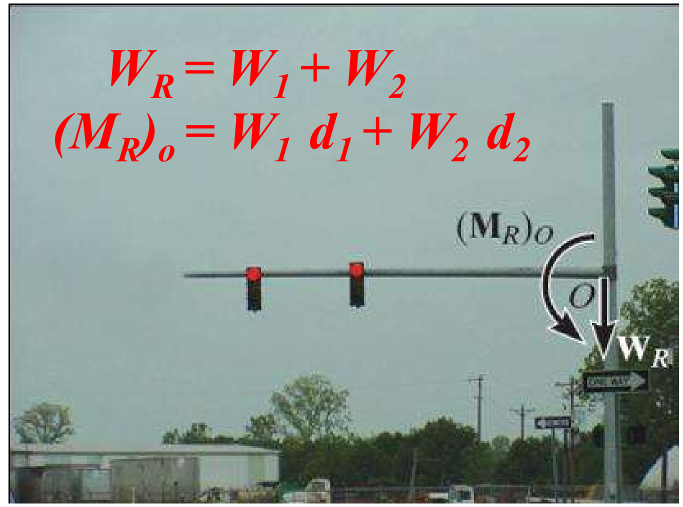
Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$

$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM (continued)

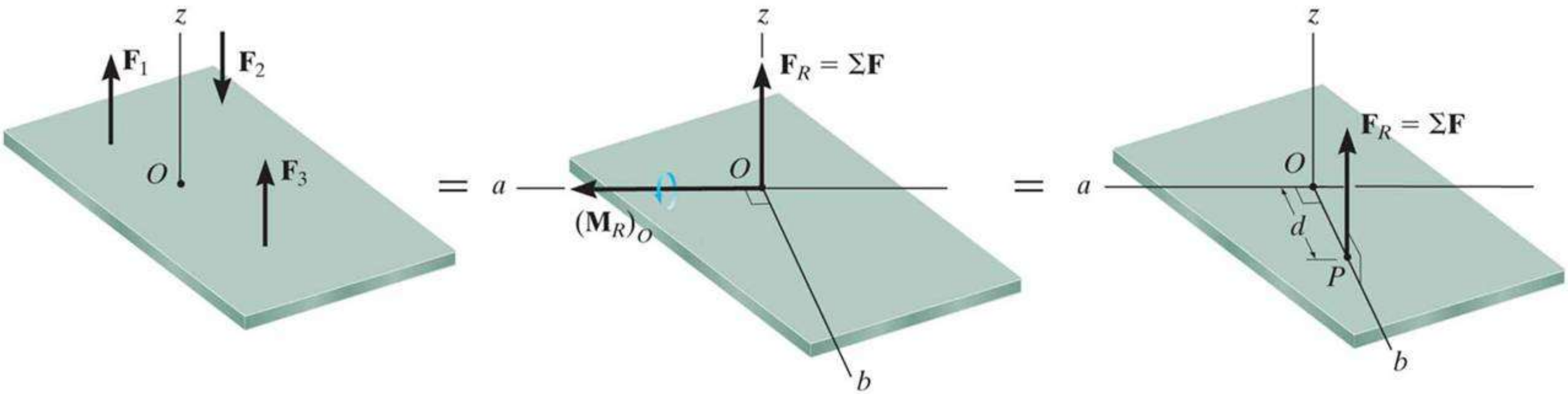




If the force system lies in the x-y plane (a 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$F_{R_x} = \Sigma F_x$$
$$F_{R_y} = \Sigma F_y$$
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

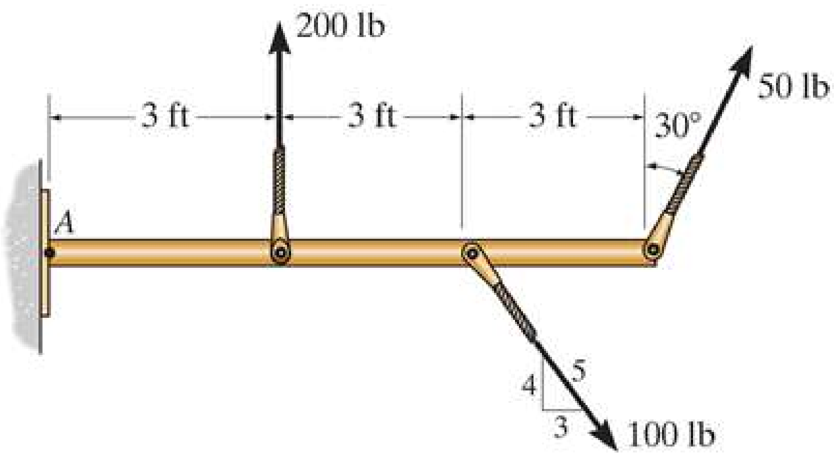
FURTHER SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM (Section 4.8)



If F_R and M_{RO} are perpendicular to each other, then the system can be further reduced to a single force, F_R , by simply moving F_R from O to P.

In three special cases, concurrent, coplanar, and parallel systems of forces, the system can always be reduced to a single force.

EXAMPLE I



Given: A 2-D force system with geometry as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location measured from A.

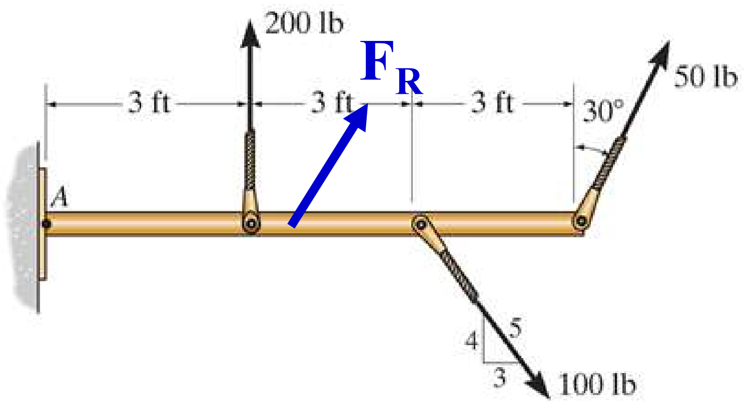
Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force component to A.

3) Shift F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

EXAMPLE I (continued)



$+\rightarrow \Sigma F_{Rx} = 50(\sin 30) + 100(3/5)$
$= 85 \text{ lb}$

$+\uparrow \Sigma F_{Ry} = 200 + 50(\cos 30) - 100(4/5)$
$= 163.3 \text{ lb}$

$+\circlearrowleft M_{RA} = 200\ (3) + 50\ (\cos 30)\ (9)$
$- 100\ (4/5)\ 6 =$ 509.7 lb·ft CCW

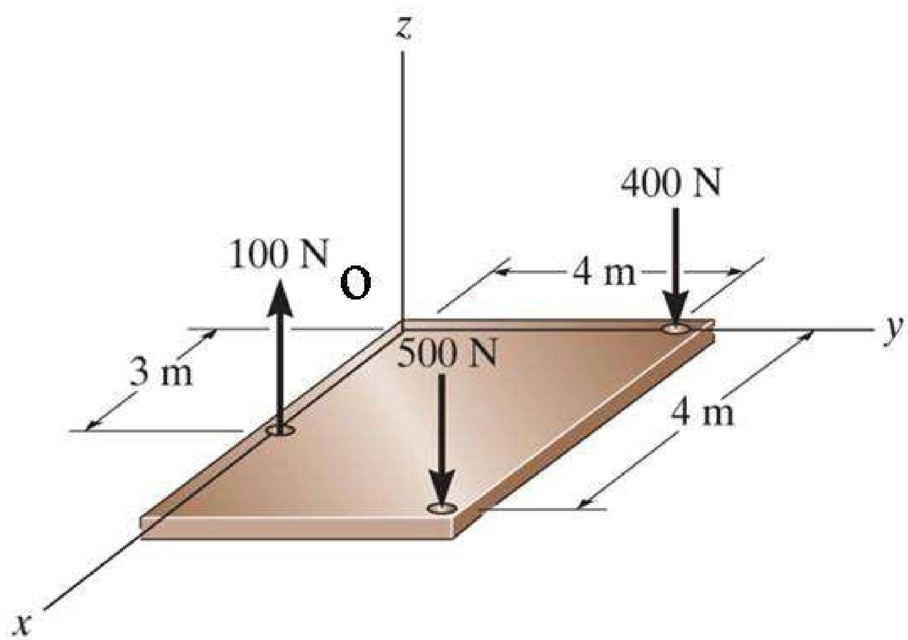
$$F_R = (\ 85^2 + 163.3^2\ )^{1/2} = \underline{184 \text{ lb}}$$

$$\angle\ \theta = \tan^{-1}(\ 163.3/85) = \underline{62.5°}$$

The equivalent single force F_R can be located at a distance d measured from A.

$$d = M_{RA}/F_{Ry} = 509.7\ /\ 163.3 = \underline{3.12 \text{ ft}}$$

EXAMPLE II



Given: The slab is subjected to three parallel forces.

Find: The equivalent resultant force and couple moment at the origin O. Also find the location (x, y) of the single equivalent resultant force.

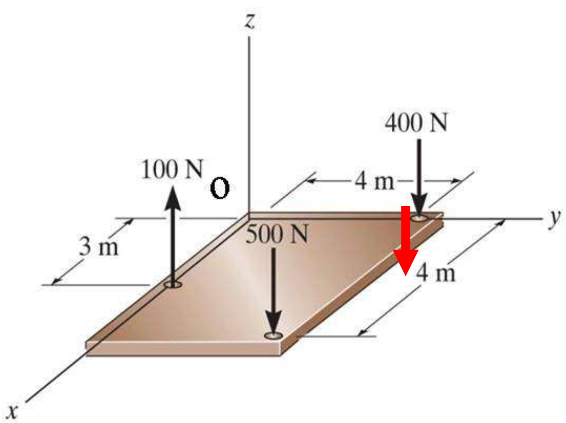
Plan:

1) Find $\boldsymbol{F_{RO}} = \sum \boldsymbol{F_i} = F_{Rzo}\,\boldsymbol{k}$

2) Find $\boldsymbol{M_{RO}} = \sum (\boldsymbol{r_i} \times \boldsymbol{F_i}) = M_{RxO}\,\boldsymbol{i} + M_{RyO}\,\boldsymbol{j}$

3) The location of the single equivalent resultant force is given as $x = -M_{RyO} / F_{RzO}$ and $y = M_{RxO} / F_{RzO}$

EXAMPLE II (continued)



$\boldsymbol{F_{RO}} = \{100\ \boldsymbol{k} - 500\ \boldsymbol{k} - 400\ \boldsymbol{k}\} = -\ 800\ \boldsymbol{k}$ N

$\boldsymbol{M_{RO}} = (3\ \boldsymbol{i}) \times (100\ \boldsymbol{k}) + (4\ \boldsymbol{i} + 4\ \boldsymbol{j}) \times (-500\ \boldsymbol{k})$

$+ (4\ \boldsymbol{j}) \times (-400\ \boldsymbol{k})$

$= \{-300\ \boldsymbol{j} + 2000\ \boldsymbol{j} - 2000\ \boldsymbol{i} - 1600\ \boldsymbol{i}\}$

$= \{\ \underline{-\ 3600\ \boldsymbol{i}} + \underline{1700\ \boldsymbol{j}}\ \}$ N·m

The location of the single equivalent resultant force is given as,

$x = -\ M_{Ryo} / F_{Rzo} = (-1700) / (-800) = \underline{2.13\text{ m}}$

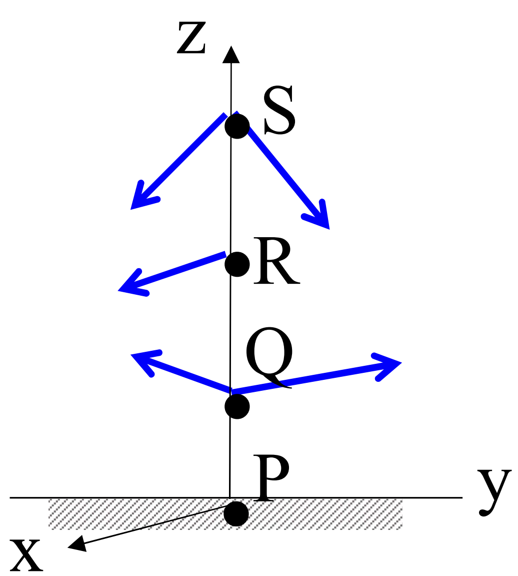
$y = M_{Rxo} / F_{Rzo} = (-3600) / (-800) = \underline{4.5\text{ m}}$

CONCEPT QUIZ

1. The forces on the pole can be reduced to a single force and a single moment at point ____ .

   A) P B) Q C) R

   D) S E) Any of these points.



2. Consider two couples acting on a body. The simplest possible equivalent system at any arbitrary point on the body will have

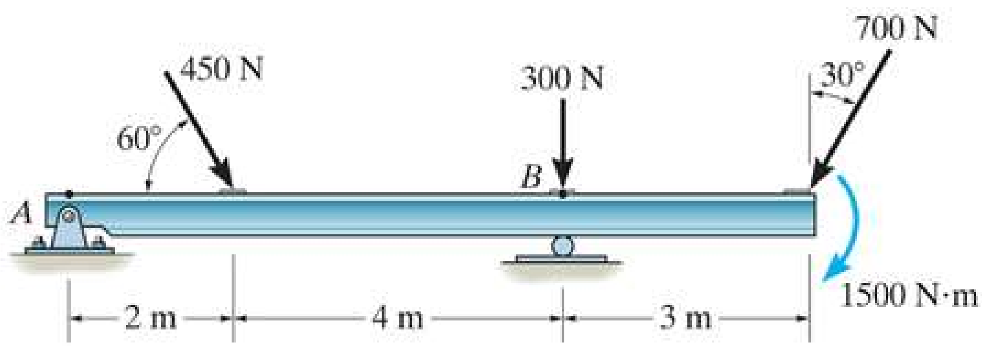
   A) One force and one couple moment.

   B) One force.

   C) One couple moment.

   D) Two couple moments.

GROUP PROBLEM SOLVING I



Given: A 2-D force and couple system as shown.

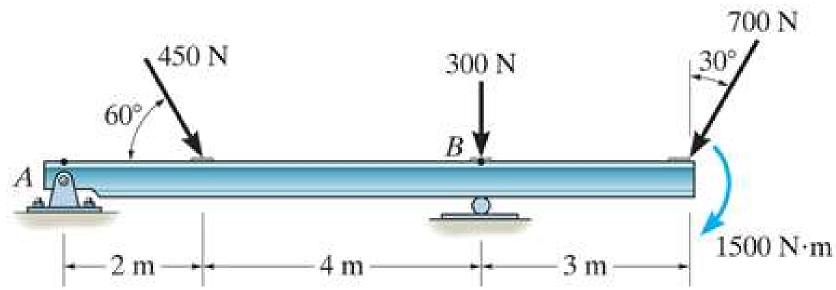
Find: The equivalent resultant force and couple moment acting at A.

Plan:

1) Sum all the x and y components of the two forces to find F_{RA}.
2) Find and sum all the moments resulting from moving each force to A and add them to the 1500 N·m free moment to find the resultant M_{RA} .

GROUP PROBLEM SOLVING I (continued)

Summing the force components:



$+\rightarrow \Sigma F_x = 450 (\cos 60) - 700 (\sin 30)$

$= -125 \text{ N}$

$+ \uparrow \Sigma F_y = -450 (\sin 60) - 300 - 700 (\cos 30)$

$= -1296 \text{ N}$

Now find the magnitude and direction of the resultant.

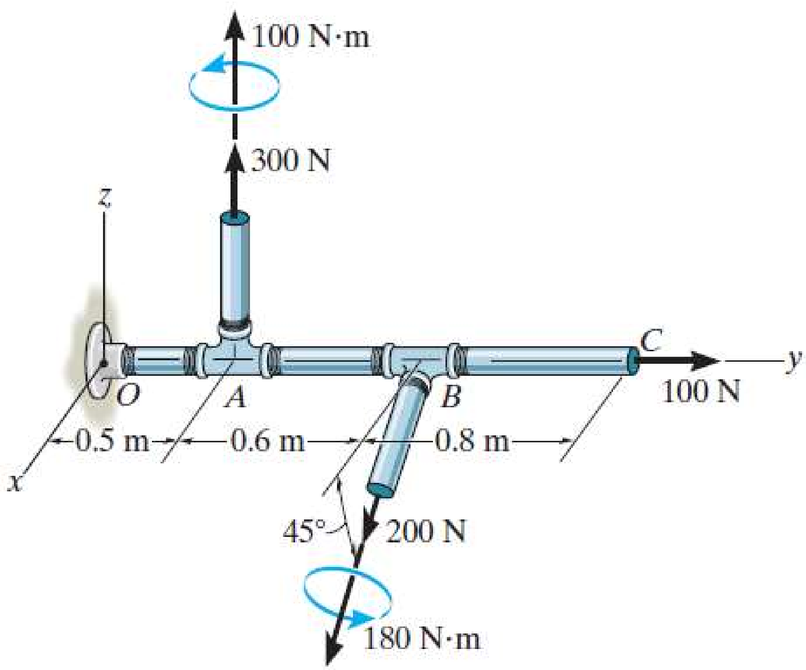
$F_{RA} = (125^2 + 1296^2)^{1/2} = \underline{1302 \text{ N}}$ and $\theta = \tan^{-1} (1296 / 125)$

$= \underline{84.5°}$

$+\circlearrowleft M_{RA} = 450 (\sin 60) (2) + 300 (6) + 700 (\cos 30) (9) + 1500$

$= \underline{9535 \text{ N·m}} \circlearrowleft$

GROUP PROBLEM SOLVING II



Given: Forces and couple moments are applied to the pipe.

Find: An equivalent resultant force and couple moment at point O.

Plan:

a) Find $\boldsymbol{F_{RO}} = \Sigma\, \boldsymbol{F_i} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$

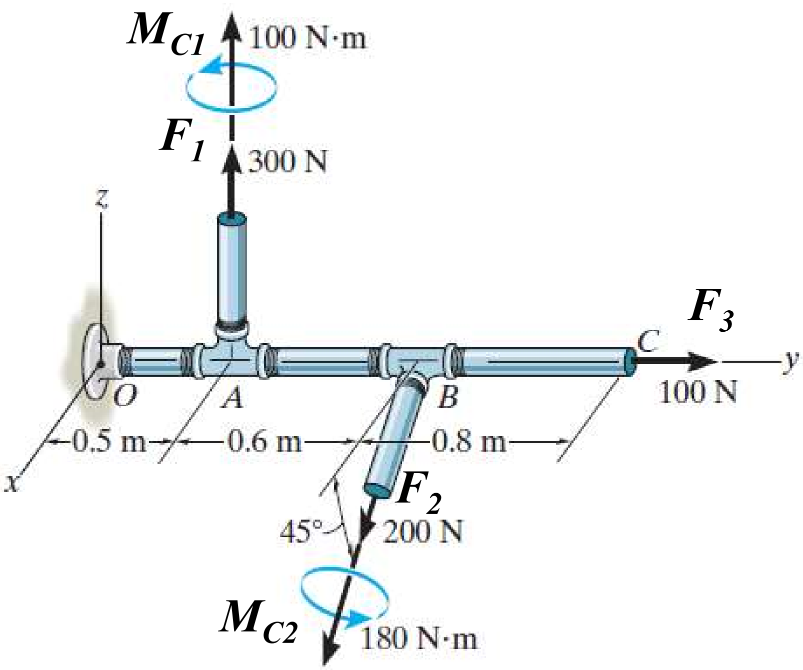
b) Find $\boldsymbol{M_{RO}} = \Sigma\, \boldsymbol{M_C} + \Sigma\, (\, \boldsymbol{r_i} \times \boldsymbol{F_i}\,)$

where,

$\boldsymbol{M_C}$ are any free couple moments.

$\boldsymbol{r_i}$ are the position vectors from the point O to any point on the line of action of $\boldsymbol{F_i}$.

GROUP PROBLEM SOLVING II (continued)



$$\boldsymbol{F_1} = \{300\ \boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{F_2} = 200\{\cos 45°\ \boldsymbol{i} - \sin 45°\ \boldsymbol{k}\}\ \text{N}$$

$$= \{141.4\ \boldsymbol{i} - 141.4\ \boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{F_3} = \{100\ \boldsymbol{j}\}\ \text{N}$$

$\boldsymbol{r_1} = \{0.5\ \boldsymbol{i}\}$ m, $\boldsymbol{r_2} = \{1.1\ \boldsymbol{i}\}$ m,

$\boldsymbol{r_3} = \{1.9\ \boldsymbol{i}\}$ m

Free couple moments are:

$$\boldsymbol{M_{C1}} = \{100\ \boldsymbol{k}\}\ \text{N·m}$$

$$\boldsymbol{M_{C2}} = 180\{\cos 45°\ \boldsymbol{i} - \sin 45°\boldsymbol{k}\}\text{N·m}$$

$$= \{127.3\ \boldsymbol{i} - 127.3\boldsymbol{k}\}\text{N·m}$$

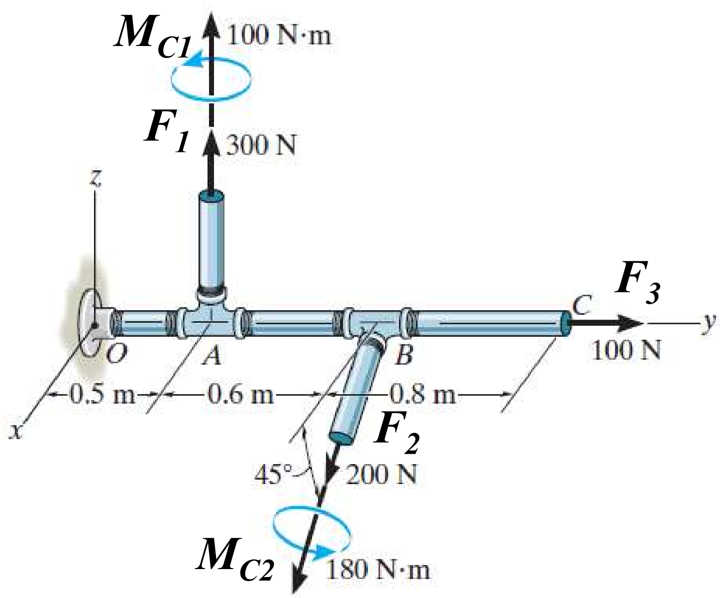
GROUP PROBLEM SOLVING II (continued)

Resultant force and couple moment at point O:

$$\boldsymbol{F}_{RO} = \Sigma\, \boldsymbol{F}_i = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$$

$$= \{300\,\boldsymbol{k}\} + \{141.4\,\boldsymbol{i} - 141.4\,\boldsymbol{k}\} + \{100\,\boldsymbol{j}\}$$

$$\boldsymbol{F}_{RO} = \{\underline{141}\,\boldsymbol{i} + \underline{100}\,\boldsymbol{j} + \underline{159}\,\boldsymbol{k}\}\ \underline{\text{N}}$$



$$\boldsymbol{M}_{RO} = \Sigma\, \boldsymbol{M}_C + \Sigma\,(\, \boldsymbol{r}_i \times \boldsymbol{F}_i\,)$$

$$\boldsymbol{M}_{RO} = \{100\,\boldsymbol{k}\} + \{127.3\,\boldsymbol{i} - 127.3\boldsymbol{k}\}$$

$$+ \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 0.5 & 0 \\ 0 & 0 & 300 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1.1 & 0 \\ 141.4 & 0 & -141.4 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1.9 & 0 \\ 0 & 100 & 0 \end{vmatrix}$$

$$\boldsymbol{M}_{RO} = \{\underline{122}\,\boldsymbol{i}\ \underline{-\,183}\,\boldsymbol{k}\}\ \underline{\text{N·m}}$$

ATTENTION QUIZ

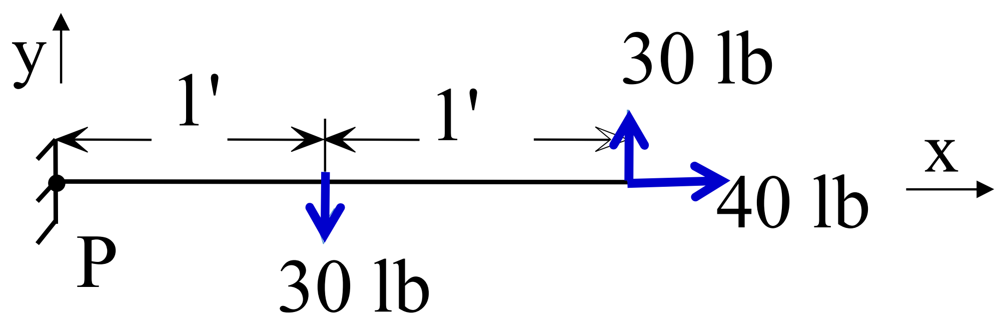
1. For this force system, the equivalent system at P is _____________ .

   A) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +60 ft ·lb

   B) F_{RP} = 0 lb and M_{RP} = +30 ft · lb

   C) F_{RP} = 30 lb (along +y-dir.) and M_{RP} = -30 ft ·lb

   D) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +30 ft ·lb



ATTENTION QUIZ

2. Consider three couples acting on a body. Equivalent systems will be ________ at different points on the body.

   A) Different when located

   B) The same even when located

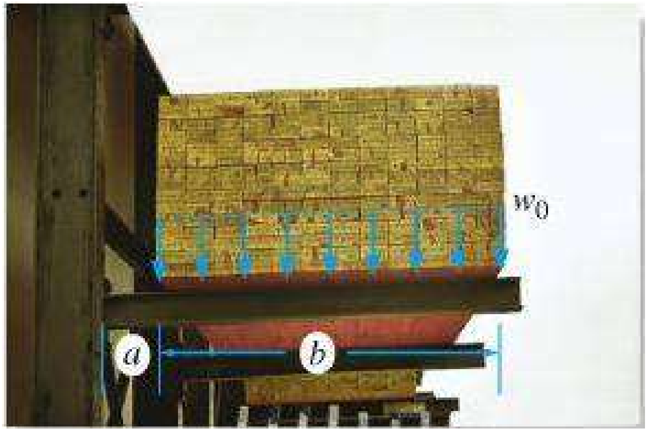
   C) Zero when located

   D) None of the above.

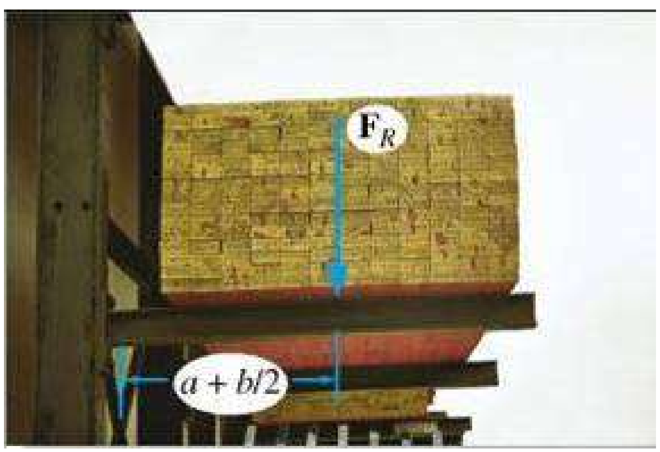
REDUCTION OF A SIMPLE DISTRIBUTED LOADING

<u>Today's Objectives</u>:

Students will be able to determine an equivalent force for a distributed load.



=

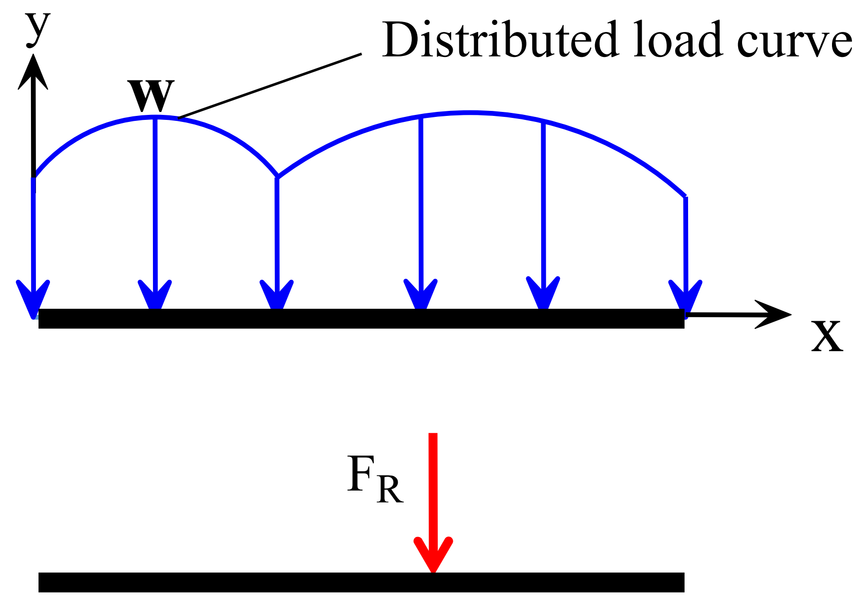


<u>In-Class Activities</u>:

- Check Homework
- Reading Quiz
- Applications
- Equivalent Force
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. The resultant force (F_R) due to a distributed load is equivalent to the ______ under the distributed loading curve, w = w(x).



A) Centroid B) Arc length

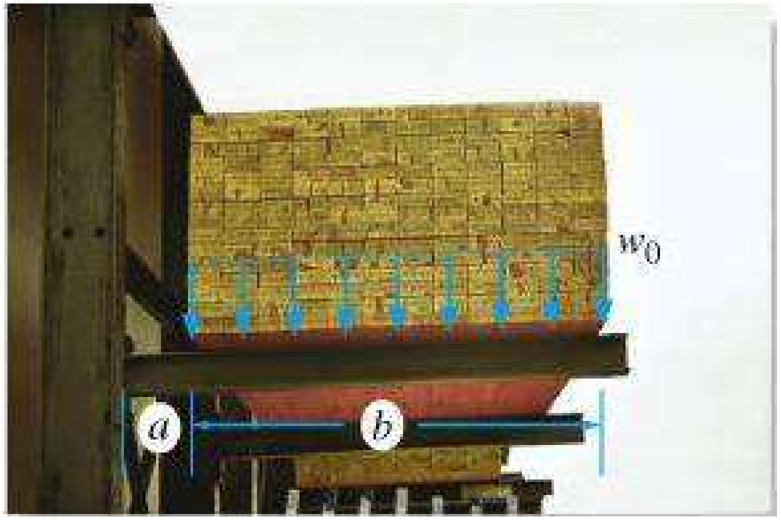
C) Area D) Volume

2. The line of action of the distributed load's equivalent force passes through the ______ of the distributed load.

A) Centroid B) Mid-point

C) Left edge D) Right edge

APPLICATIONS

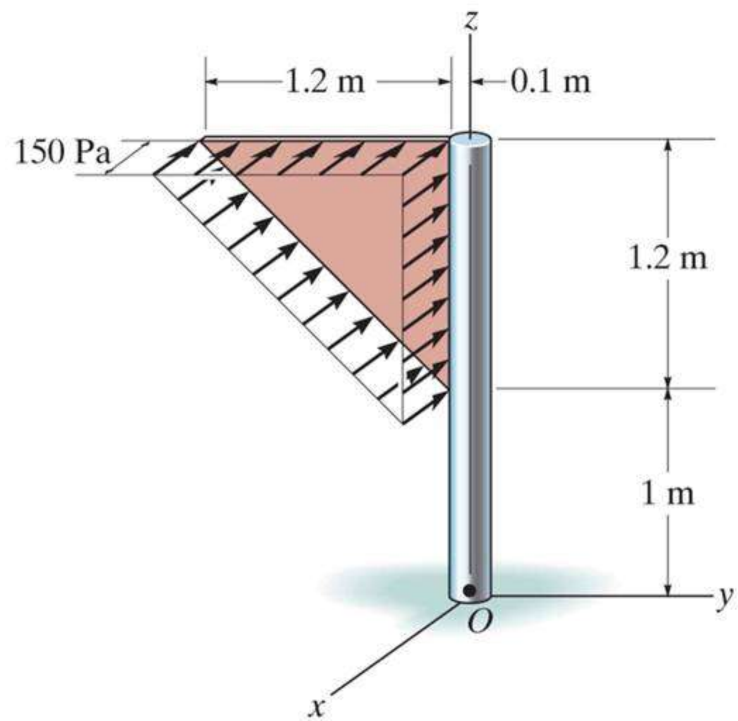


There is a bundle (called a bunk) of 2" x 4" boards stored on a storage rack. This lumber places a distributed load (due to the weight of the wood) on the beams holding the bunk.

To analyze the load's effect on the steel beams, it is often helpful to reduce this distributed load to a single force. How would you do this?

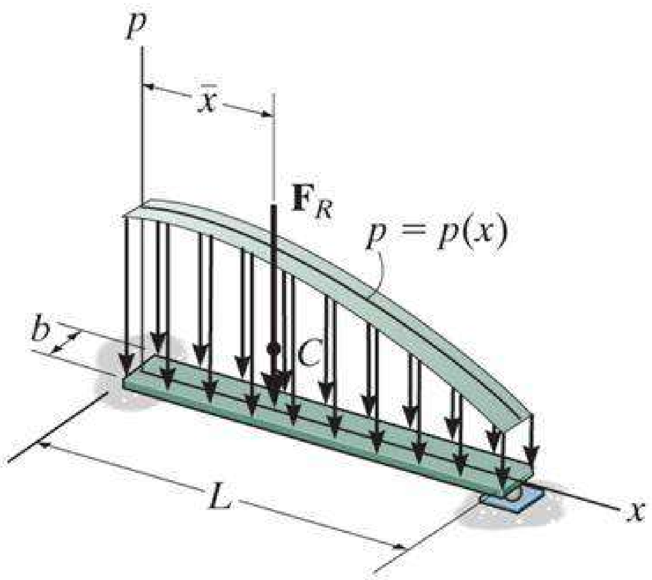
APPLICATIONS (continued)

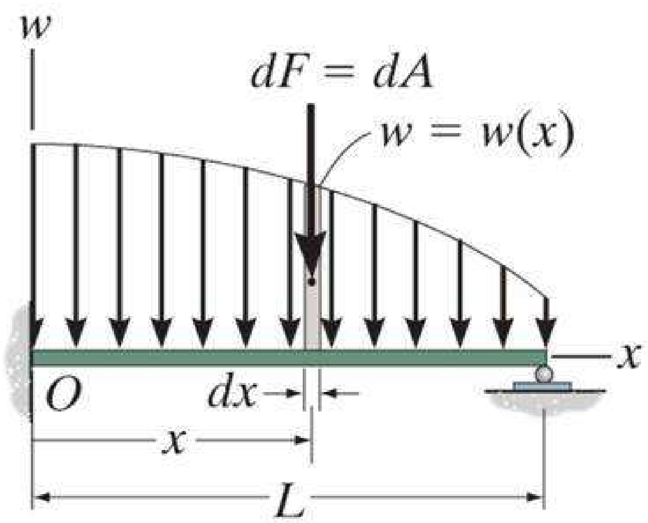
The uniform wind pressure is acting on a triangular sign (shown in light brown).



To be able to design the joint between the sign and the sign post, we need to determine a single equivalent resultant force and its location.

DISTRIBUTED LOADING



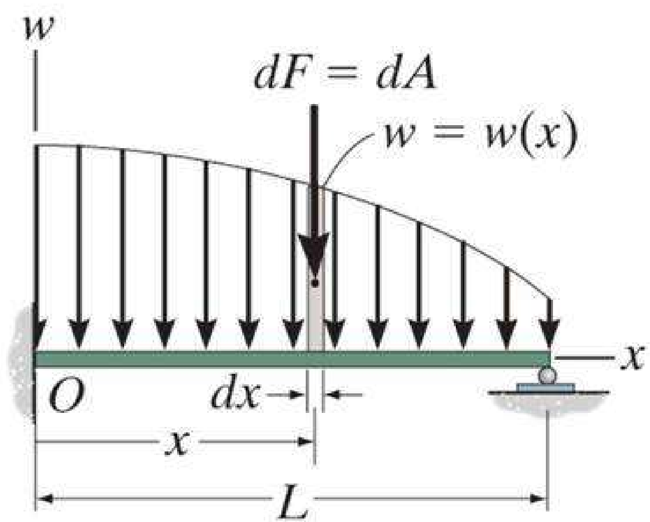


In many situations, a surface area of a body is subjected to a distributed load. Such forces are caused by winds, fluids, or the weight of items on the body's surface.

We will analyze the most common case of a distributed pressure loading. This is a uniform load along one axis of a flat rectangular body.

In such cases, w is a function of x and has units of force per length.

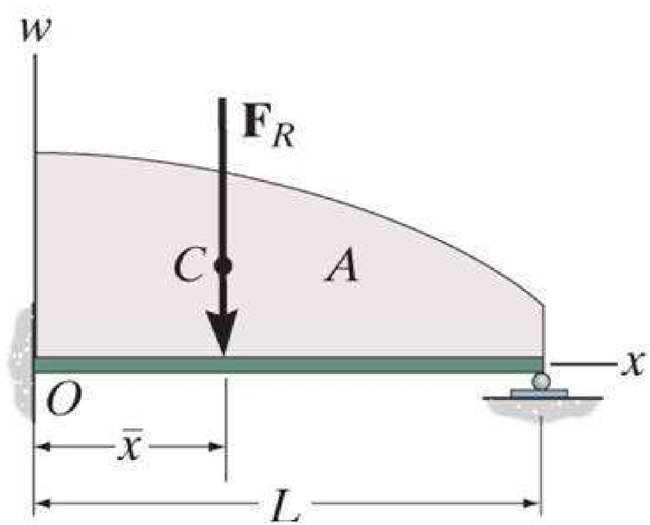
MAGNITUDE OF RESULTANT FORCE



Consider an element of length dx.

The force magnitude dF acting on it is given as

$$dF = w(x)\,dx$$

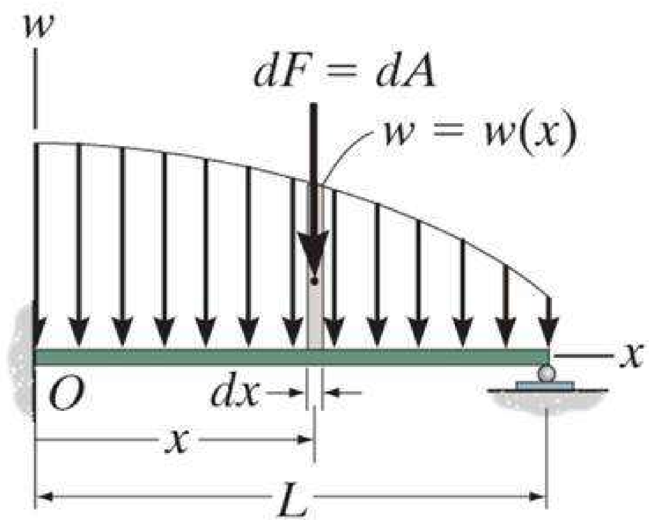


The net force on the beam is given by

$$+\downarrow\ F_R = \int_L dF = \int_L w(x)\,dx = A$$

Here A is the area under the loading curve w(x).

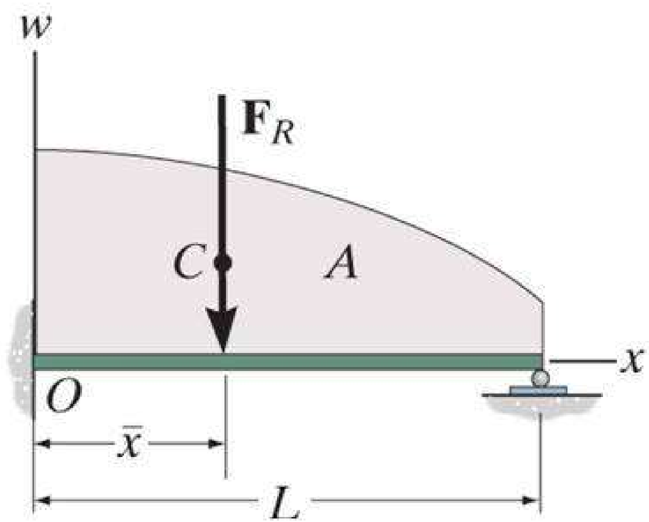
LOCATION OF THE RESULTANT FORCE



The force dF will produce a moment of (x)(dF) about point O.

The total moment about point O is given as

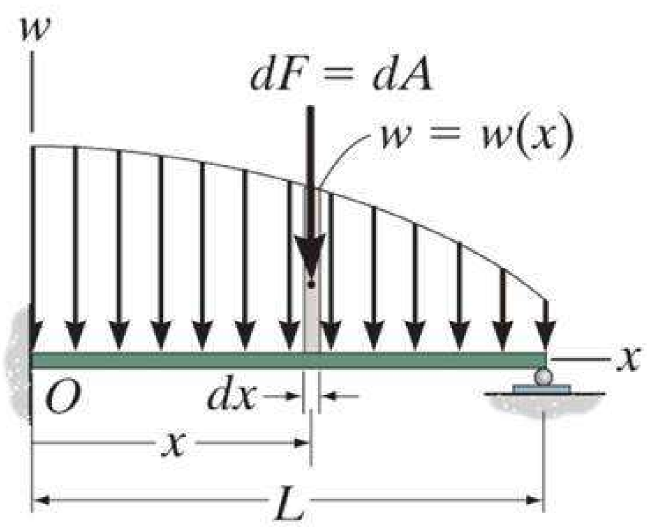
$$\curvearrowleft + \; M_{RO} = \int_L x \, dF = \int_L x \, w(x) \, dx$$



Assuming that F_R acts at $\bar{x}$, it will produce the moment about point O as

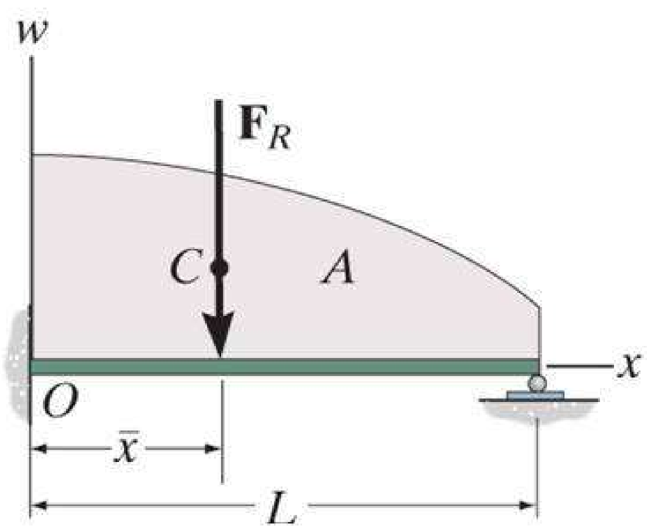
$$\curvearrowleft + \; M_{RO} = (\bar{x})\,(F_R) = \bar{x} \int_L w(x) \, dx$$

LOCATION OF THE RESULTANT FORCE (continued)



Comparing the last two equations, we get

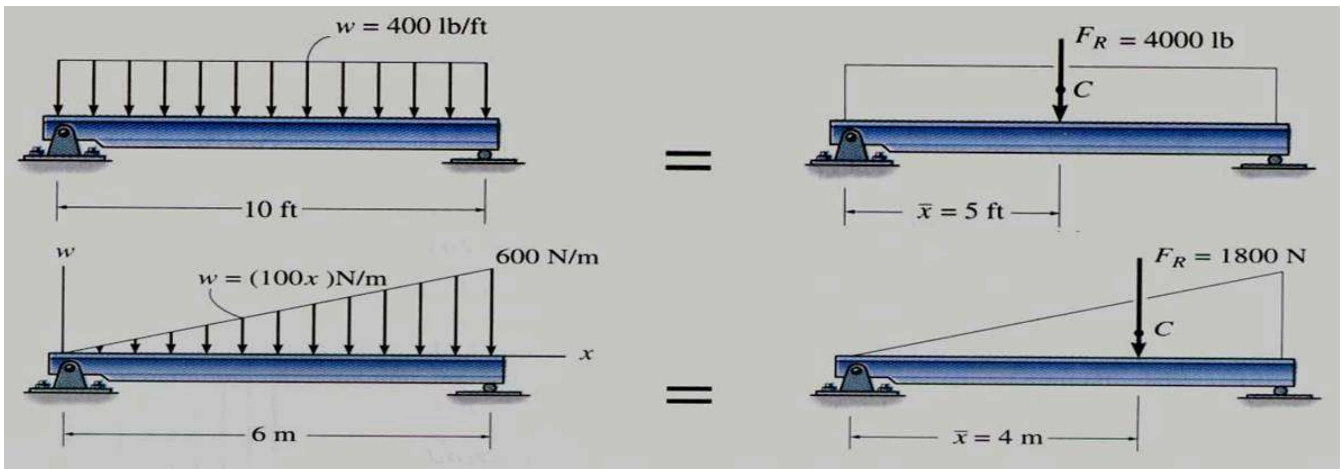
$$\bar{x} = \frac{\int_L x w(x)\, dx}{\int_L w(x)\, dx} = \frac{\int_A x\, dA}{\int_A dA}$$



You will learn more detail later, but F_R acts through a point "C," which is called the geometric center or centroid of the area under the loading curve w(x).

EXAMPLE I

Until you learn more about centroids, we will consider only rectangular and triangular loading diagrams whose centroids are well defined and shown on the inside back cover of your textbook.

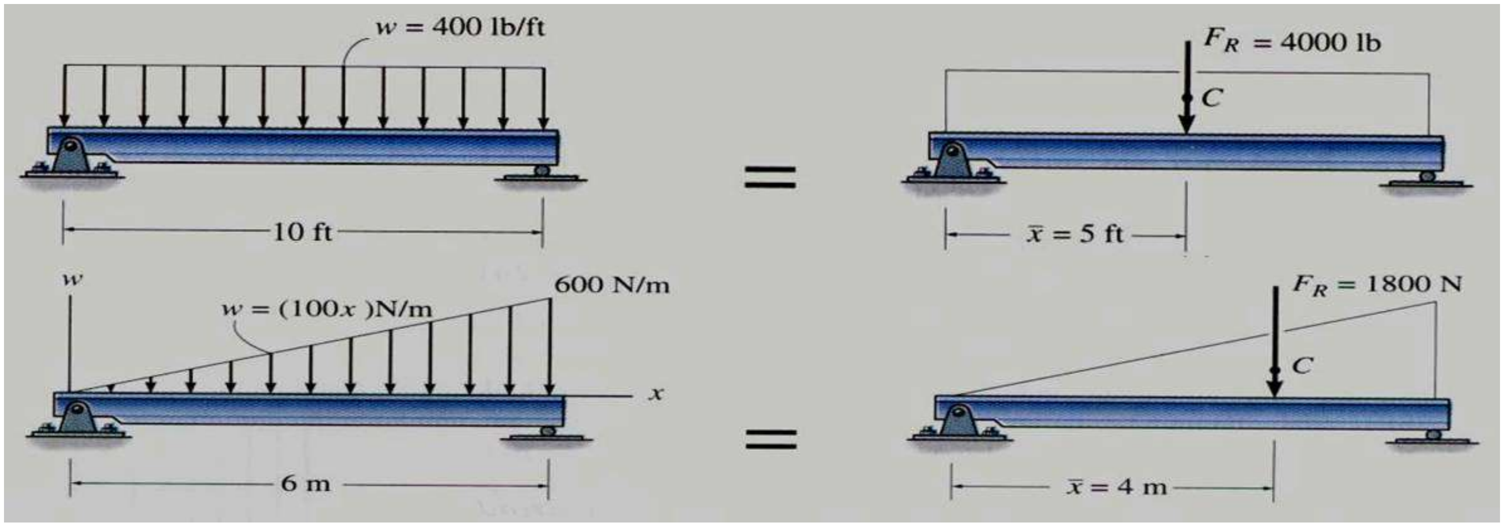


Look at the inside back cover of your textbook. You should find the rectangle and triangle cases. Finding the area of a rectangle and its centroid is easy!

Note that triangle presents a bit of a challenge but still is pretty straightforward.

EXAMPLE I (continued)

Now let's complete the calculations to find the concentrated loads (which is a common name for the resultant of the distributed load).



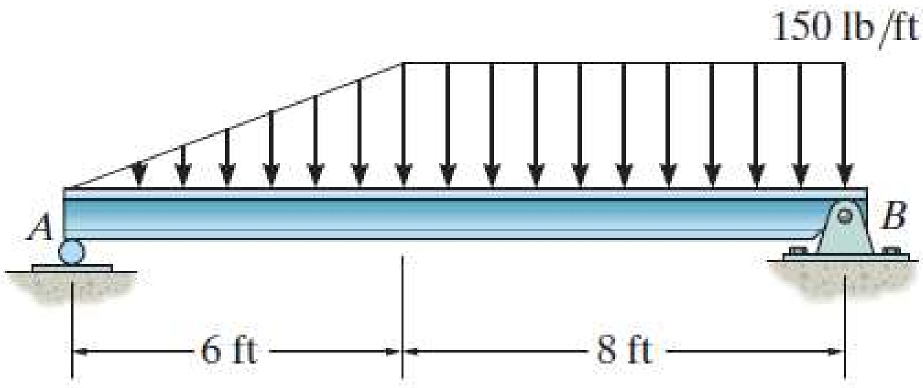
The rectangular load: $F_R = 400 \times 10 =$ 4,000 lb and $\bar{x}$ = 5 ft.

The triangular loading:

$F_R = (0.5)(600)(6) =$ 1,800 N and $\bar{x} = 6 - (1/3)\,6 =$ 4 m.

Please note that the centroid of a right triangle is at a distance one third the width of the triangle as measured from its base.

EXAMPLE II



Given: The loading on the beam as shown.

Find: The equivalent force and its location from point A.

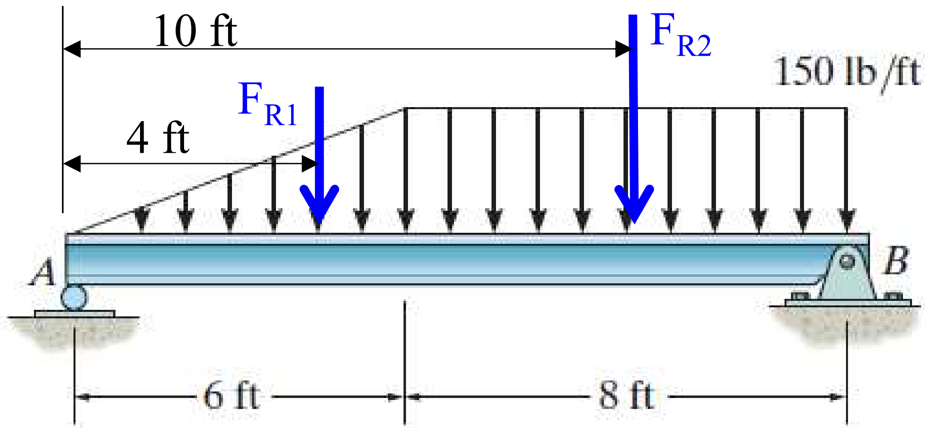
Plan:

1) The distributed loading can be divided into two parts. (one rectangular loading and one triangular loading).

2) Find F_R and its location for each of the distributed loads.

3) Determine the overall F_R of the point loadings and its location.

EXAMPLE II (continued)



For the triangular loading of height 150 lb/ft and width 6 ft,

F_{R1} = (0.5) (150) (6) = 450 lb

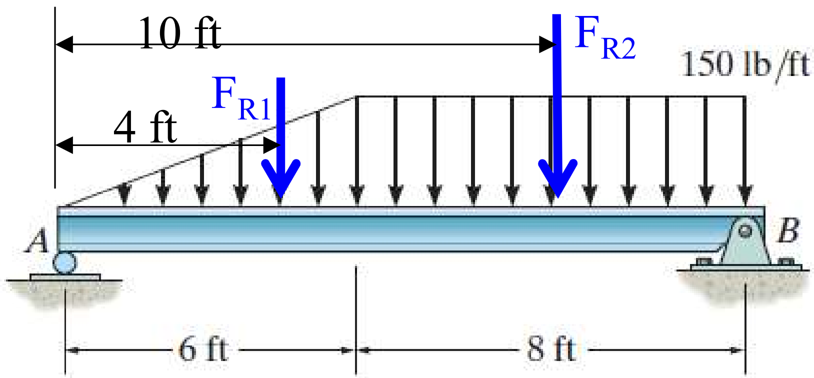
and its line of action is at $\bar{x}_1$ = (2/3)(6) = 4 ft from A

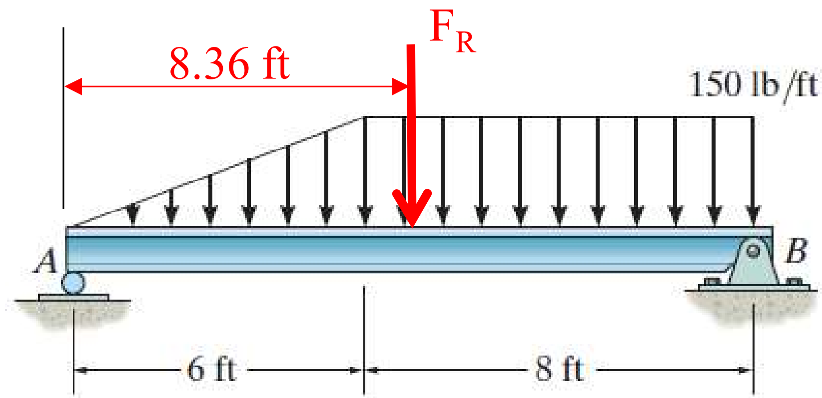
For the rectangular loading of height 150 lb/ft and width 8 ft,

F_{R2} = (150) (8) = 1200 lb

and its line of action is at $\bar{x}_2$ = 6 + (1/2)(8) = 10 ft from A

EXAMPLE II (continued)





The equivalent force and couple moment at A will be

$$F_R = 450 + 1200 = \underline{1650 \text{ lb}}$$

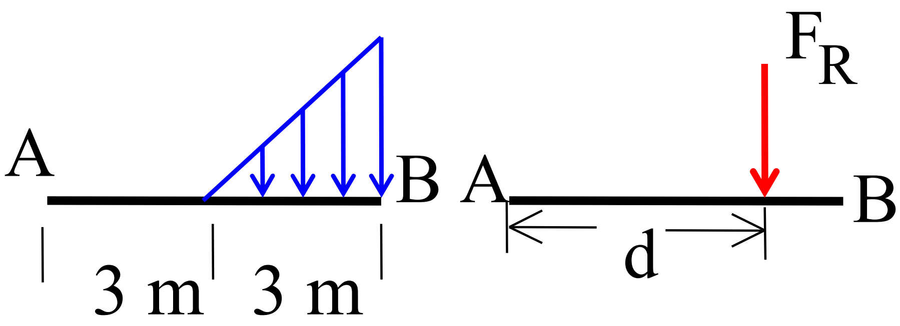
$$+\circlearrowleft M_{RA} = 4\,(450) + 10(1200) = \underline{13800 \text{ lb}\cdot\text{ft}}$$

Since ($F_R\,\bar{x}$) has to equal M_{RA} : $1650\,\bar{x} = 13800$

Solve for $\bar{x}$ to find the equivalent force's location.

$\bar{x}$ = <u>8.36 ft from A</u>.

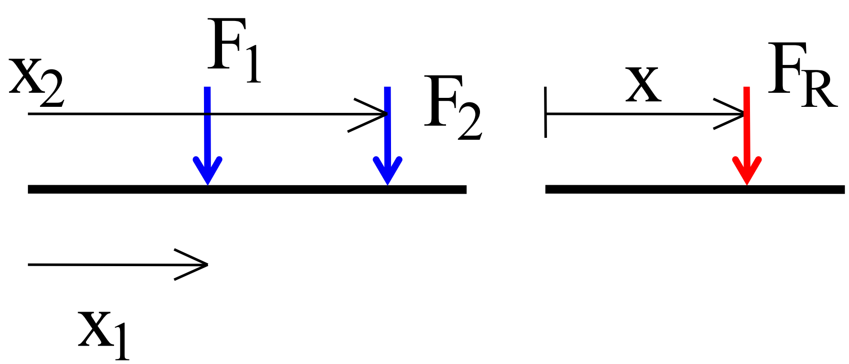
CONCEPT QUIZ



1. What is the location of F_R, i.e., the distance d?

A) 2 m B) 3 m C) 4 m

D) 5 m E) 6 m

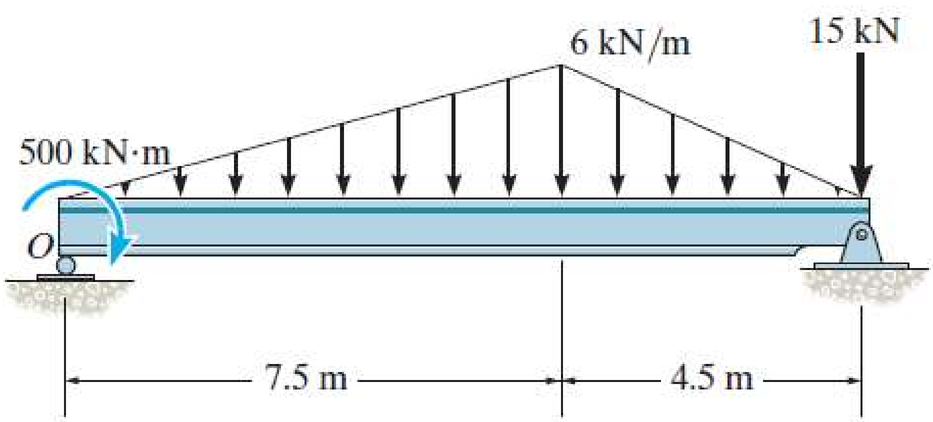


2. If $F_1 = 1$ N, $x_1 = 1$ m, $F_2 = 2$ N and $x_2 = 2$ m, what is the location of F_R, i.e., the distance x.

A) 1 m B) 1.33 m C) 1.5 m

D) 1.67 m E) 2 m

GROUP PROBLEM SOLVING



Given: The distributed loading on the beam as shown.

Find: The equivalent force and couple moment acting at point O.

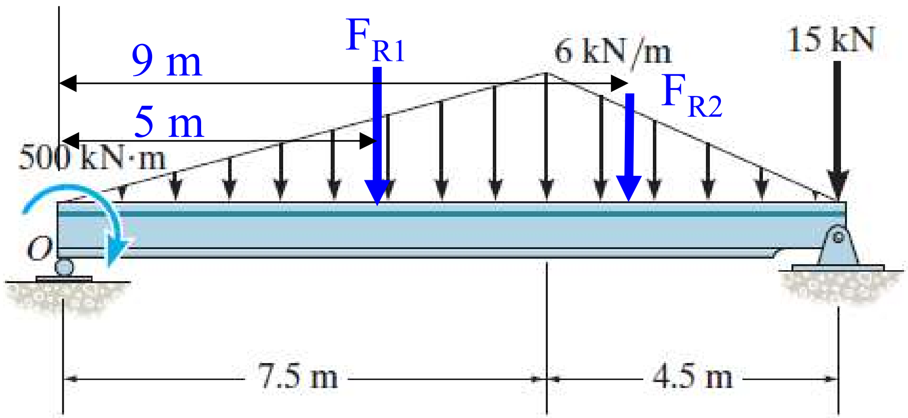
Plan:

1) The distributed loading can be divided into two parts--two triangular loads.

2) Find F_R and its location for each of these distributed loads.

3) Determine the overall F_R of the point loadings and couple moment at point O.

GROUP PROBLEM SOLVING (continued)



For the left triangular loading of height 6 kN/m and width 7.5 m,

F_{R1} = (0.5) (6) (7.5) = 22.5 kN

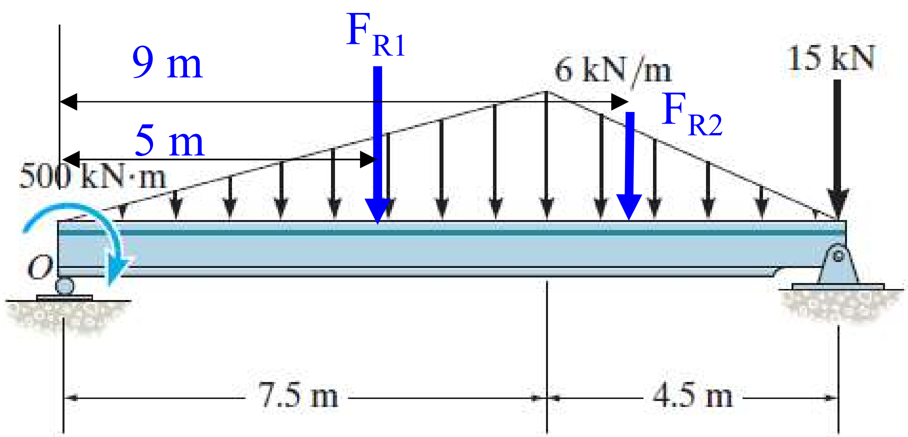
and its line of action is at $\bar{x}_1 = (2/3)(7.5) = 5$ m from O

For the right triangular loading of height 6 kN/m and width 4.5 m,

F_{R2} = (0.5) (6) (4.5) = 13.5 kN

and its line of action is at $\bar{x}_2 = 7.5 + (1/3)(4.5) = 9$ m from O

GROUP PROBLEM SOLVING (continued)



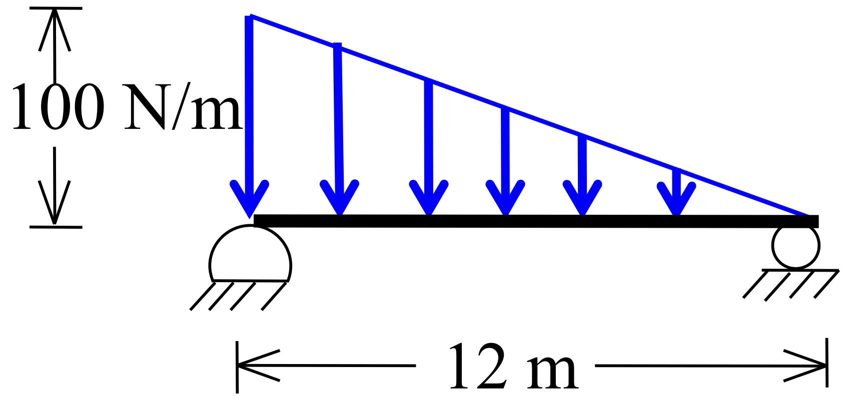
For the combined loading of the three forces, add them.

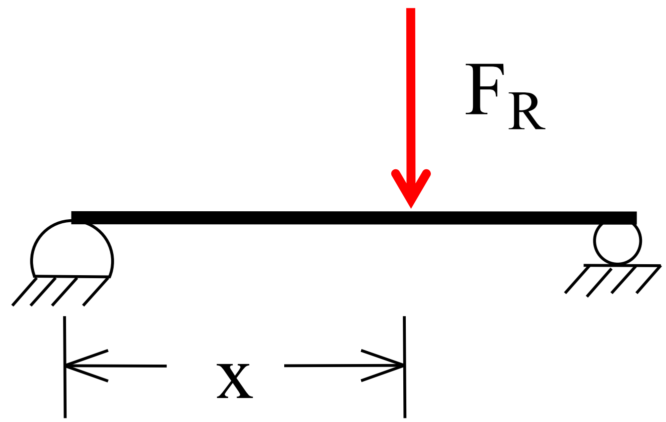
$$F_R = 22.5 + 13.5 + 15 = \underline{51 \text{ kN}}$$

The couple moment at point O will be

$$+\circlearrowleft M_{RO} = 500 + 5\,(22.5) + 9\,(13.5) + 12\,(15) = \underline{914 \text{ kN}\cdot\text{m}}$$

ATTENTION QUIZ





1. F_R = ________________

A) 12 N B) 100 N

C) 600 N D) 1200 N

2. x = ______________.

A) 3 m B) 4 m

C) 6 m D) 8 m

End of the Lecture

Let Learning Continue