NUMM 2500 **Quiz 1 (5%)** **Quiz Period: 50 min.**

Humber College - Jan. 24, 2024

Student Name:

Student Number:

Solutions

1. The first four nonzero terms of the Maclaurin series for the arctangent function are:

$$P(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7.$$

Compute the absolute error and relative error in the following approximations of π using the polynomial $P(x)$ in place of the arctangent:

$$4\left[\arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{3}\right)\right]$$

Use **6 digit rounding** arithmetic for this question.

$$4\left[\arctan\left(\tfrac{1}{2}\right) + \arctan\left(\tfrac{1}{3}\right)\right] \simeq 4\left[P\left(\tfrac{1}{2}\right) + P\left(\tfrac{1}{3}\right)\right]$$

$$= 4\left[0.463467 + 0.321745\right]$$

$$= 3.140851 = \hat{\pi}$$

$$\text{abs. error} = |\pi - \hat{\pi}| = 0.000742$$

$$\text{rel. error} = \frac{|\pi - \hat{\pi}|}{\pi} = \frac{0.000742}{\pi} = 0.000236$$

2. Let

$$f(x) = \frac{x\sin x + \cos x - 1}{1 - x^2 - \cos x}.$$

a. Replace each trigonometric function with its second Maclaurin polynomial,

$$\sin x \cong x - \frac{1}{6}x^3$$
$$\cos x \cong 1 - \frac{1}{2}x^2$$

and use **six-digit rounding** arithmetic to evaluate $f(0.01)$.

b. The actual value is $f(0.01) = -0.9999666667$. Find the relative error for the value obtained in part **(a).**

a)

$$f(x) = \frac{x\sin x + \cos x - 1}{1 - x^2 - \cos x} \simeq \frac{x\left(x - \frac{1}{6}x^3\right) + \left(1 - \frac{1}{2}x^2\right) - 1}{1 - x^2 - \left(1 - \frac{1}{2}x^2\right)}$$

$$f(0.01) \simeq -0.999967 = \hat{f}$$

b)

$$\text{rel. error} = \frac{|f(0.01) - \hat{f}|}{|f(0.01)|} = 0.00000033$$

3. Use **five-digit rounding** arithmetic and the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

to find the approximations to the roots of the following quadratic equation.

$$x^2 - 5000.002x + 10 = 0$$

Compute the relative error for each root considering the fact that the actual roots accurate to 13-digits are

$$x_1 = 5000, \qquad x_2 = 0.001999999999953$$

Do you see any issue using the quadratic formula to compute the roots? If so, explain the reason(s). Do you have any idea on how to fix this issue.

$a=1$, $b=-5000.002$, $c=10$

$$x_1 = \frac{5000.002 + \sqrt{5000.002^2 - 40}}{2} = 5000.$$

rel. error $= 0$

$$x_2 = \frac{5000.002 - \sqrt{5000.002^2 - 40}}{2} = 0.002$$

rel. error $= 2.36 \times 10^{-11}$

In computing x_2, we encounter the subtractive cancellation as two close numbers 5000.002 and $\sqrt{5000.002^2 - 40}$ are being subtracted.

A suggestion: To avoid subtractive cancellation, we use the following formula to compute x_2.

$$x_2 = \frac{40}{2\left[5000.002 + \sqrt{5000.002^2 - 40}\right]}$$