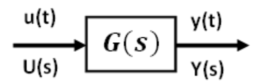
Time Response Analysis

First-order System:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1}$$



K: Steady-state Gain

τ: Time-constant

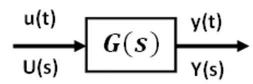
Settling-time (2%): $t_s = 4\tau$

Single real pole at $s = -\frac{1}{\tau}$

Unit-step Response: $y(t) = K - Ke^{-t/\tau}, \ t \geq 0$

Second-order System:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



K: Steady-state Gain

ω_n: Undamped natural frequency

ζ: Damping-ratio

- $\zeta > 1$: Over-damped system $\rightarrow$ Two real distinct poles at $s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$
- $\zeta = 1$: Critically damped system $\rightarrow$ Two real repeated poles at $s_1 = s_2 = -\zeta\omega_n$
- $0 < \zeta < 1$: Under-damped system $\rightarrow$ Two complex poles at $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}$
- $\zeta = 0$: Undamped system $\rightarrow$ Two complex poles at $s_{1,2} = \pm j\omega_n$

Settling-Time (2% criteria):

$$t_s \approx \frac{4}{\zeta\omega_n} \ , \ 0 < \zeta < 0.9$$

Peak Time:

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1 - \zeta^2}}$$

Rise Time:

$$t_r \cong \frac{0.8 + 2.5\zeta}{\omega_n}$$

Overshoot:

$$M_p = y(t_p) - y_{ss} = y_{ss}e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

Percent of Overshoot:

$$O.S.\% = \frac{M_p}{y_{ss}} \times 100\% = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$$

Damping Ratio from Overshoot:

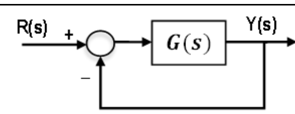
$$\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}}$$

Error constants & Steady-State Error of a Unity-feedback System:

$$k_p = \lim_{s\to 0} G(s) \quad \rightarrow \quad e_{ss} = \frac{R}{1 + k_p}$$

$$k_v = \lim_{s\to 0} sG(s) \quad \rightarrow \quad e_{ss} = \frac{R}{k_v}$$

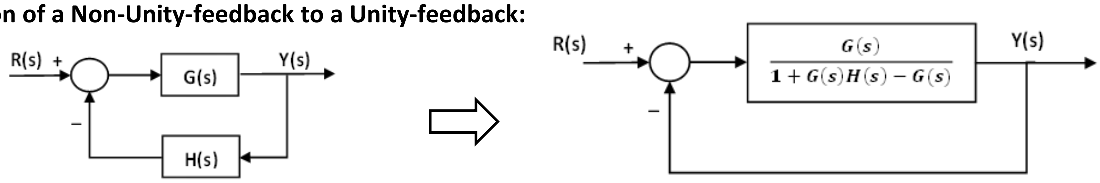
$$k_a = \lim_{s\to 0} s^2G(s) \quad \rightarrow \quad e_{ss} = \frac{R}{k_a}$$



$$E(s) = R(s) - Y(s)$$

$$e_{ss} = \lim_{s\to 0} \frac{s}{1 + G(s)} R(s)$$

Conversion of a Non-Unity-feedback to a Unity-feedback:



Steady-State Error of a Unity-feedback System with Disturbances

$$e_{ss,R} = \lim_{s\to 0} \frac{s}{1 + G_1(s)G_2(s)} R(s)$$

$$e_{ss,D} = \lim_{s\to 0} \frac{-sG_2(s)}{1 + G_1(s)G_2(s)} D(s)$$