# Measurements and Vectors

Tuesday, September 6, 2022 2:03 PM

# HUMBER ENGINEERING

ENGI 1000 - Physics 1

WEEK 1 - MODULE 1

HUMBER

WE ARE HUMBER

# Module 1
# Measurements and Vectors

- Physics and Measurements
  - Standards of Length, Mass, and Time
  - Conversion of Units
  - Dimensional Analysis
- Vectors and Scalars
  - Coordinate Systems
  - Vectors and Scalar Quantities
  - Some Properties of Vectors
  - Components of a Vector and Unit Vectors



## Physics and Measurement

- If you measure a quantity, how do you know it is accurate?
- If someone tells you a quantity measured by someone else is accurate, how can you verify this?
- In order for us to be able to know whether our measurements are accurate and to verify that others' measurements are accurate, we need agreed-upon standard units of measurement, plus a means to convert between these and other units

other units.



## Standard Units

How tall are you?

- Write down your height.



## Standard Units

What units did you refer to?

- Meters?
- Centimeters?
- Inches?
- Feet and Inches?



## Standard Units

- We are only able to compare one another's heights to each other because we have agreed-upon units of length.
- While we may not agree whether it is better to use feet and inches vs. meters, these are well-established units, and a conversion factor is known which enables us to express these measurements in a common unit.
- Consider the following:
  - Student #1 has a height equal to 8.7 times the length of their own hand.
  - Student #2 has a height equal to 9.1 times the length of their own hand.

**…who is taller?**



## Standard Units

# Standard Units

- Consider the following:
  - Student #1 has a height equal to 8.7 times the length of their own hand.
  - Student #2 has a height equal to 9.1 times the length of their own hand.

**…who is taller?**

- It is impossible to know without first knowing the length of the respective students' hands, according to a universal standard. We need to know:
  - Student #1's hand length in standard units
  - Student #2's hand length in standard units

- And then we can compute their respective heights in standard length units in order to compare their heights objectively.



# Standard Units

- Imperial length units were determined scientifically, right? Nope!
  - The Inch used to be the width of a person's thumb
  - The foot used to be the length of a person's booted foot

- In the 1200s, King Henry of England, recognizing the need for a standard unit of length, decided that the yard would be the distance from the King's nose to the end of his outstretched hand. (or so the legend goes)



# Standard Units


**The INTERNATIONAL SYSTEM OF UNITS (SI units)**
- Developed in 1960
- The only system with an official status in most countries worldwide
- SI unit system is also called metric system.

  - LENGTH (meter)
  - MASS (kilogram)
  - TIME (second)
  - ELECTRIC CURRENT (ampere)
  - TEMPERATURE (kelvin)
  - AMOUNT OF SUBSTANCE (mole)
  - BRIGHTNESS (candela)

**These are the seven base units**



# Standard Units of Length


- **Length:** The distance between two points in space

- SI fundamental unit of length is meter (m).

- **Meter (m):** The distance traveled by light in vacuum during a time interval of 1/299,792,458 second
  - Redefined in 1983
  - Light-speed in vacuum is precisely 299,792,458 meter per second
  - Light is the same everywhere in the Universe



# Standard Units of Mass


- **Mass:** A measurement of how much matter is in an object.
- SI fundamental unit of mass is kilogram (kg).
- **Kilogram (kg):** The mass of a specific platinum-iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sevres, France.
  - Established in 1887
  - The duplicate is kept at the National Institute of Standards and Technology (NIST) in Maryland

An accurate copy is housed under a double bell jar in a vault at NIST



# Standard Units of Time


- **Time:** The progression of events from past to the present into the future
- SI fundamental unit of time is second (s).
- **Second (s):** 9,192,631,770 times the period of vibration of radiation from the cesium-133 atom.
  - Redefined in 1967
  - Based on high precision of atomic clock
  - The clock neither gain nor lose a second in 20 million years

A cesium fountain atomic clock



# Conversion of Units

- **Conversion factor** is a number that tells us how many of one unit are equivalent to one of another unit measuring the same quantity.

| | |
|---|---|
| 1 mile = 1.609 km | 1 kg = 2.205 pounds (lbs) |
| 1 mile = 1609 meters | 1 kg = 35.274 ounce (oz) |
| 1 mile = 5280 feet | 1 ton = 1000 kg |
| 1 km = 1/1.609 miles = 0.622 miles | 1 ton = 2205 pounds (lbs) |
| 1 km = (5280 feet)/1.609 = 3282 feet | 1 US ton = 907.185 kg |
| 1 m = 1.094 yards | 1 US ton = 2000 pounds (lbs) |

- 1 hour = 60 minutes
- 1 hour = 3600 seconds



# Conversion of Units

**Example 1:** Convert 15.0 in. (inch) to cm (centimeter)

Since 1 in. is defined as exactly 2.54 cm, we find that:

- 1 in. = 2.54 cm ➡ 15.0 in. = 15.0 × 2.54 cm = 38.1 cm

$$15.0\text{ in.} = (15.0\,\cancel{\text{in.}})\left(\frac{2.54\text{ cm}}{1\,\cancel{\text{in.}}}\right) = 38.1\text{ cm}$$

15.0 in. = 38.1 cm



# Conversion of Units

**Example 2:** Convert 55 mph (mile per hour) to m/s (meter per second)

Since speed is length divided by time, we have to convert miles to meters, and hours to seconds:

- 1 mile = 1.609 km
- 1 km = 1000 m

➡ 55 miles = 55 × 1.609 km = 88 km = 88 × 1000 m = 88000 m

- 1 hour = 3600 seconds ➡ 55 mph = 88000m/1h = 88000m/3600s ≈ 24 m/s

$$55\text{ mph} = \left(\frac{55\,\cancel{\text{mile}}}{1\,\cancel{\text{h}}}\right)\left(\frac{1.609\,\cancel{\text{km}}}{1\,\cancel{\text{mile}}}\right)\left(\frac{1000\text{ m}}{1\,\cancel{\text{km}}}\right)\left(\frac{1\,\cancel{\text{h}}}{3600\text{ s}}\right) \approx 24\text{ m/s}$$

55 mph ≈ 24 m/s



# Conversion of Units


- An important feature of the SI unit system is the use of prefixes to express larger and smaller values of a quantity.
- The prefixes are defined based on power of ten.

Prefixes for the various powers of ten and their abbreviations

| Power | Prefix | Abbreviation |
|---|---|---|
| $10^{-24}$ | yocto | y |
| $10^{-21}$ | zepto | z |
| $10^{-18}$ | atto | a |
| $10^{-15}$ | femto | f |
| $10^{-12}$ | picto | p |
| $10^{-9}$ | nano | n |
| $10^{-6}$ | micro | μ |
| $10^{-3}$ | milli | m |
| $10^{-2}$ | centi | c |
| $10^{-1}$ | deci | d |

| Power | Prefix | Abbreviation |
|---|---|---|
| $10^{3}$ | kilo | k |
| $10^{6}$ | mega | M |
| $10^{9}$ | giga | G |
| $10^{12}$ | tera | T |
| $10^{15}$ | peta | P |
| $10^{18}$ | exa | E |
| $10^{21}$ | zetta | Z |
| $10^{24}$ | yotta | Y |



# Conversion of Units

## Conversion of Units

- An important feature of the SI unit system is the use of prefixes to express larger and smaller values of a quantity.
- The prefixes are defined based on power of ten.

- 1 km = $10^3$ m
- 1 m = 10 dm = $10^2$ cm = $10^3$ mm
- 1 kg = $10^3$ g = $10^6$ mg
- 1 µg = $10^{-3}$ g = $10^{-6}$ kg
- 1 s = $10^3$ ms = $10^6$ µs
- 1 A = $10^3$ mA = $10^6$ µA



## Dimensional Analysis

- Remember the seven SI base units?
- You can think of the SI base units as the "primary colors" of dimensional analysis
- Other quantities can be derived by combining these primary colors, similar to how new colors can be derived by mixing the primary colors



## Dimensional Analysis

**Example 3:** Density describes how much mass is contained within a given volume of substance.

$$\boxed{Density = \frac{mass}{Volume} \quad \rightarrow \quad \rho = \frac{m}{V}}$$

- Mass: We have a base unit for that (kg)
- Volume:
  - $V$ = length × width × height
  - $V$ = length × length × length
  - Units of volume, derived from base units is $km^3$ ...or, we can use $m^3$
- Density = mass/volume, so unit of density is $kg/m^3$



## Dimensional Analysis

**Example 4:** Force (unit is the Newton (N))

$$\boxed{Force = mass \times acceleration \quad \rightarrow \quad F = ma}$$

- Mass is a base unit (kg)

- Acceleration is the rate of change of velocity over time
    - $a = \frac{v}{t}$
    - $v$ is measured in meters per second (m/s)
    - $t$ is measured in second (s)
    - Unit of acceleration is m/s$^2$ (these are base units)
- So, if $F = ma$, then the Newton expressed in SI base units is (kg x m)/s$^2$

$$\boxed{1\ N = 1\,\frac{kg\ m}{s^2}}$$



# Coordinate Systems

- Coordinate systems are used to describe object's location and motion in space.
- There are two popular coordinate systems:
    - **Cartesian Coordinate System**
    - **Polar Coordinate System**

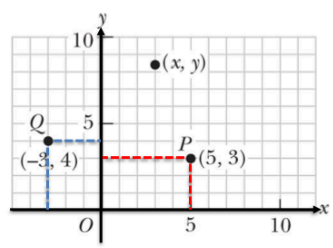


# Cartesian Coordinate Systems

- It is also called rectangular coordinates
- Axes are perpendicular
- Intersection of axes defined as origin
- In two-dimension there are two axes
    - $x$-axis and $y$-axis
- It is also called $xy$ plane
- Every point is labeled with coordinates $(x, y)$

$P(5,3)$ and $Q(-3,4)$

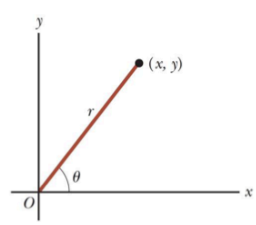
Two-dimensional cartesian coordinate system





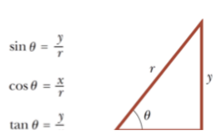

# Polar Coordinate Systems

- Every point is represented as $(r, \theta)$
    - $r$ is the distance from origin
    - $\theta$ is the angle that measured counterclockwise from positive $x$-axis
- Relation between cartesian coordinate and polar coordinate of a point:




$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

$$\boxed{\begin{aligned} \tan\theta &= \frac{y}{x} \\ r &= \sqrt{x^2 + y^2} \end{aligned}}$$



# Polar Coordinate Systems

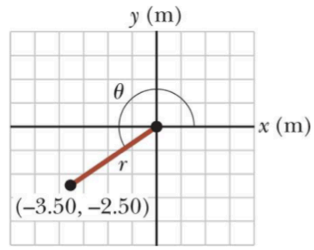
**Example 5:** The Cartesian coordinates of a point in the $xy$ plane are $(x, y) = (-3.50, -2.50)$ m as shown in the figure. Find the polar coordinates of this point.

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2} = \boxed{4.30\text{ m}}$$

$$\tan\theta = \frac{y}{x}$$

$$= \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$

$$\boxed{\theta = 216°}$$





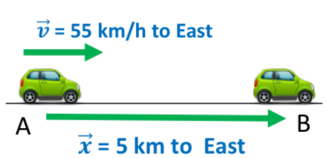

# Vector and Scalar Quantities

- A **scalar quantity** is completely specified by a single value with an appropriate unit (also called magnitude) and has no direction.
- A **vector quantity** is completely specified by a number with an appropriate unit (the magnitude of the vector) plus a direction.



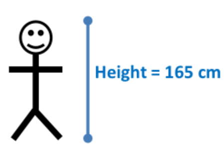

# Vector and Scalar Quantities

- Some examples of scalar and vector quantities



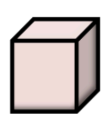

Displacement
Velocity


Force




Mass = 65 kg
Age = 16 years

Temperature = 25°C



Volume = 125 m³



# Quick Quiz 1

- Think, pair and share:

Which of the followings are vector quantities and which are scalar quantities?

a) your age
b) acceleration
c) velocity
d) speed
e) mass
f) time
g) temperature



# Representing Vectors

- A vector can be represented by an arrow.
  - The magnitude of the vector is represented by the length of the arrow.
  - The direction of the vector is represented by the direction of the arrow.

Tail  Head

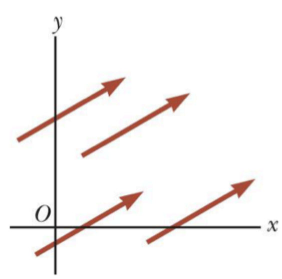


# Equality of Two Vectors

- Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ defined as equal if have same magnitude $(A = B)$ and point in same direction along parallel lines

$$\boxed{\vec{\mathbf{A}} = \vec{\mathbf{B}}}$$

- In this figure all vectors are equal even though have different starting points
- We can move vector to position parallel to itself in diagram without affecting the vector






# Adding and Subtracting Vectors

- Recall that a vector is a quantity that has both magnitude and direction.


$\boxed{\vec{\mathbf{A}} + \vec{\mathbf{B}} = ?}$ $\boxed{\vec{\mathbf{A}} - \vec{\mathbf{B}} = ?}$

- Therefore, the process of addition and subtraction must consider both the magnitude and the direction of the vectors.
- There are two methods of adding and subtracting the vectors:
  - **Head-to-Tail Method:** A graphical method using ruler and protractor.
  - **Components Method:** An analytical method using simple geometry and trigonometry, which is more efficient and more accurate than graphical method.



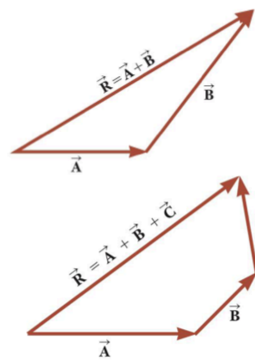
# Adding Vectors: Graphic Method

- Steps of vector addition via the graphical Head-to-Tail Method

$$\boxed{\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{R}}}$$

1) Draw the first vector $\vec{\mathbf{A}}$.
2) Draw the second vector $\vec{\mathbf{B}}$ with its tail at the head of the first vector.
3) The resultant vector $\vec{\boldsymbol{R}}$ is drawn from the tail of the first vector $\vec{\mathbf{A}}$ to the head of the second vector $\vec{\mathbf{B}}$.

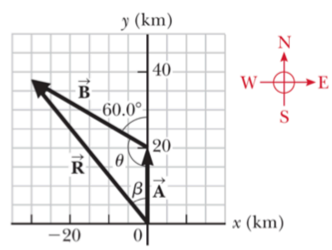
- For more than two vectors, the resultant vector $\vec{\boldsymbol{R}}$ is drawn from the tail of the first vector to the head of the last vector.






# Adding Vectors: Graphic Method

**Example 6 (A Vacation Trip): (a)** A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north as shown in the figure. Find the magnitude and direction of the car's resultant displacement.






# Adding Vectors: Graphic Method

**Example 6 (A Vacation Trip): (a)** A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north as shown in the figure. Find the magnitude and direction of the car's resultant displacement.

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$R = \sqrt{(20.2\text{ km})^2 + (35.0\text{ km})^2 - 2(20.2\text{ km})(35.0\text{ km})\cos 120°}$$

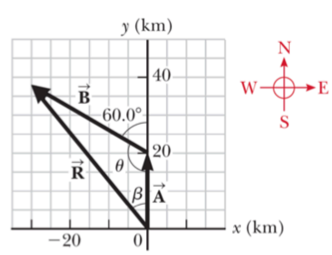
$$= \boxed{48.2\text{ km}}$$

magnitude

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta$$

35.0 km



$$= \frac{\ldots}{48.2\text{ km}} \sin 120° = 0.629$$

$\beta = \boxed{38.9°}$ direction

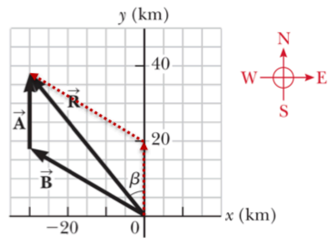


# Adding Vectors: Graphic Method

**Example 6 (A Vacation Trip): (b)** Suppose the trip were taken with the two vectors in reverse order: 35.0 km at 60.0° west of north first and then 20.0 km due north. How would the magnitude and the direction of the resultant vector change?

They would not change.
The commutative law for vector addition tells us that the order of vectors in an addition is irrelevant.

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$$





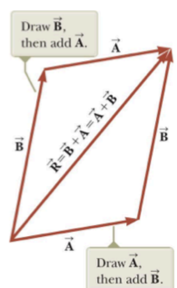

# Properties of Vectors

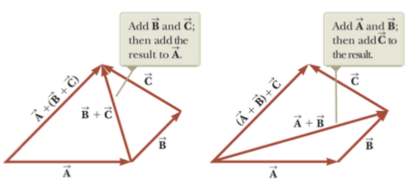
- **Cumulative Law of Addition:** The sum of two vectors is independent of the order of the addition.

$$\boxed{\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}}$$

- **Associative Law of Addition:** The sum of three or more vectors is independent of the way in which individual vectors grouped together.

$$\boxed{\vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}}}$$



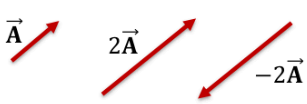



# Properties of Vectors

- **Negative of a Vector:** The negative of vector $\vec{\mathbf{A}}$ is defined as the vector that when added to $\vec{\mathbf{A}}$ gives zero for the vector sum.
- The vectors $\vec{\mathbf{A}}$ and $-\vec{\mathbf{A}}$ have the same magnitude but opposite directions.

$$\boxed{\vec{\mathbf{A}} + (-\vec{\mathbf{A}}) = \mathbf{0}}$$


- **Multiplying a Vector by a Scalar:** If vector $\vec{\mathbf{A}}$ is multiplied by a positive scalar quantity $m$, the product $m\vec{\mathbf{A}}$ is a vector that has the same direction as $\vec{\mathbf{A}}$ and magnitude $mA$.
- If $m$ is a negative scalar, then product $m\vec{\mathbf{A}}$ is directed opposite $\vec{\mathbf{A}}$.



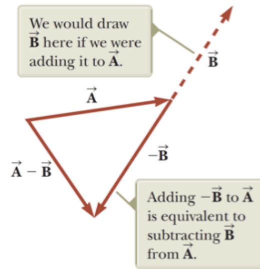



# Subtracting Vectors: Graphic Method

- Steps of vector subtraction via the graphical Tail-to-Head Method

$$\boxed{\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}})}$$

1) Draw the first vector $\vec{\mathbf{A}}$.
2) Draw the negative of second vector $-\vec{\mathbf{B}}$ with its tail at the head of the first vector.
3) The resultant vector $\vec{\boldsymbol{R}}$ is drawn from the tail of the first vector $\vec{\mathbf{A}}$ to the head of the vector $-\vec{\mathbf{B}}$.






# Quick Quiz 2

- The magnitude of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are $A = 12$ units and $B = 8$ units. Which pair of numbers represents the largest and smallest possible values for the magnitude of the resultant vector? $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$

Same Direction $\vec{A}$ $\vec{B}$

a) 14.4 units, 4 units

b) 12 units, 8 units

c) 20 units, 4 units

d) none of these answers



# Quick Quiz 3

- If vector $\vec{\mathbf{B}}$ is added to vector $\vec{\mathbf{A}}$ which two of the following choices must be true for the resultant vector to be equal to zero? $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \mathbf{0}$

*a)* $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are parallel and in the same direction

*b)* $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are parallel and in the opposite directions

*c)* $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitude

*d)* $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular



# Adding and Subtracting Vectors

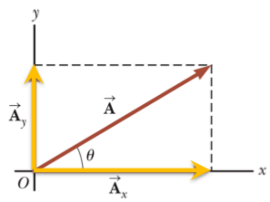
- There are two methods of adding and subtracting the vectors:
  - **Head-to-Tail Method:** A graphical method using ruler and protractor.
    - Not highly accurate
    - Not recommended for three-dimensional problems

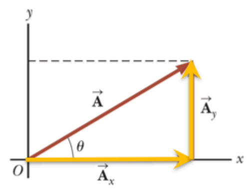
- **Components Method:** An analytical method using simple <u>geometry</u> and <u>trigonometry</u>.
  - Uses projections of vectors along coordinate axes
  - The projections are called components of the vector or rectangular components
  - Describes the vector by its components
  - More efficient and more accurate than graphical method



# Components of a Vector

- Consider vector $\vec{\mathbf{A}}$ lying in the $xy$ plane and making arbitrary angle $\theta$ with positive $x$-axis.
- We can express vector $\vec{\mathbf{A}}$ as sum of two component vectors:
  - $\vec{\mathbf{A}}_x$ parallel to $x$-axis, which is called projection of $\vec{\mathbf{A}}$ along $x$-axis
  - $\vec{\mathbf{A}}_y$ parallel to $y$-axis, which is called projection of $\vec{\mathbf{A}}$ along $y$-axis




$$\boxed{\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y}$$



# Components of a Vector

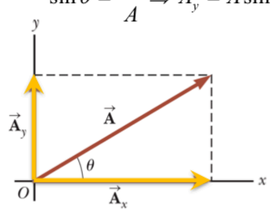
- Magnitude of the component vectors are the lengths of the two sides of a right triangle with the hypotenuse of length $A$.

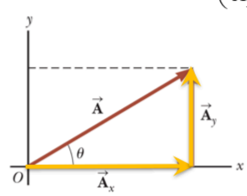
$$\cos\theta = \frac{A_x}{A} \Rightarrow A_x = A\cos\theta$$

$$\sin\theta = \frac{A_y}{A} \Rightarrow A_y = A\sin\theta$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



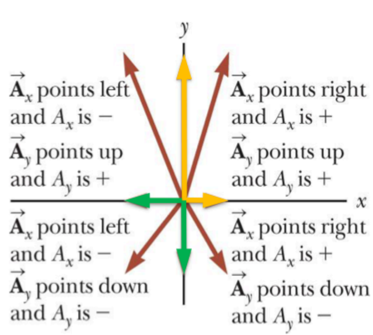




$$\boxed{\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y}$$



# Components of a Vector

- These components can be positive or negative.
  - The component $A_x$ is positive if vector $\vec{\mathbf{A}}_x$ points in the <u>positive $x$ direction</u>.
  - The component $A_x$ is negative if vector $\vec{\mathbf{A}}_x$ points in the <u>negative $x$ direction</u>.
  - A similar statement is made for the component $A_y$.



# Quick Quiz 4

- Choose the correct response to make the sentence true:

A component of a vector is ………………. larger than the magnitude of the vector.

a) always

b) never

c) sometimes

Pythagorean theorem

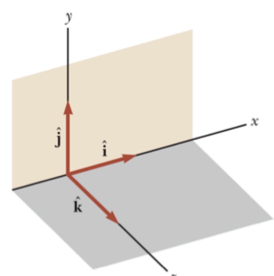


# Unit Vectors

- Vector quantities often expressed in terms of unit vectors.

**A unit vector is a dimensionless vector having a magnitude exactly 1.**

- Unit vectors are used to specify the direction in the coordinate systems.
- Symbols $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ represent unit vectors pointing in positive $x$, $y$, and $z$ directions, respectively.
- They form a set of mutually perpendicular vectors in right-handed coordinate system.






# Unit Vectors and Components of a Vector

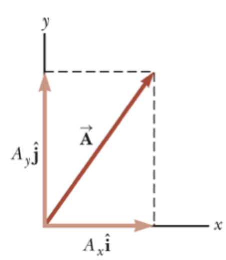
- Consider vector $\vec{\mathbf{A}}$ lying in $xy$ plane
- The component vectors $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ can be shown in terms of the unit vectors:

$$\vec{\mathbf{A}}_x = A_x\hat{\mathbf{i}}$$

$$\vec{\mathbf{A}}_y = A_y\hat{\mathbf{j}}$$

- The unit-vector notation for the vector $\vec{\mathbf{A}}$ is:

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y \quad \rightarrow \quad \boxed{\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}}$$



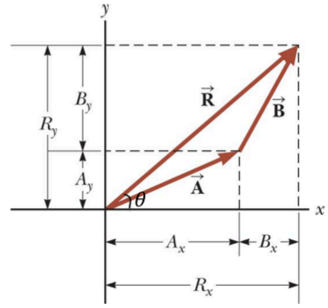



# Vector Addition using Components of a Vector

- Consider vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ lying in $xy$ plane
- The component of two vectors are shown in the figure.
- Sum of the vectors are determined as below

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{R}} = \underbrace{\left(A_x + B_x\right)}_{R_x}\hat{\mathbf{i}} + \underbrace{\left(A_y + B_y\right)}_{R_y}\hat{\mathbf{j}} \Rightarrow \boxed{\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}}$$




Magnitude → $R = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{\left(A_x + B_x\right)^2 + \left(A_y + B_y\right)^2}$

Angle → $\tan\theta = \dfrac{R_y}{R_x} = \dfrac{A_y + B_y}{A_x + B_x}$



# Vector Addition in Three-Dimensional Space

- Consider vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ located in 3D $xyz$ space with the vector components of $A_x, A_y, A_z$ and $B_x, B_y, B_z$.

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

- Sum of the vectors in 3D coordinate system are determined as below

$$\vec{\mathbf{R}} = \underbrace{\left(A_x + B_x\right)}_{R_x}\hat{\mathbf{i}} + \underbrace{\left(A_y + B_y\right)}_{R_y}\hat{\mathbf{j}} + \underbrace{\left(A_z + B_z\right)}_{R_z}\hat{\mathbf{k}} \Rightarrow \vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} + R_z\hat{\mathbf{k}}$$

Magnitude → $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

Angles → $\cos\theta_x = \dfrac{R_x}{R}$ $\quad\cos\theta_y = \dfrac{R_y}{R}$ $\quad\cos\theta_z = \dfrac{R_z}{R}$



# Quick Quiz 5

- For which of the following vectors is the magnitude of the vector equal to one of the components of the vector?

*a)* $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$

*b)* $\vec{\mathbf{B}} = -3\hat{\mathbf{j}}$

*c)* $\vec{\mathbf{C}} = 5\hat{\mathbf{k}}$



# Vector Addition using Components

**Example 7 (The Sum of Two Vectors):** Find the sum of two vector $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ lying in the $xy$ plane and given by

$$\vec{\mathbf{A}} = (2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}) \quad \text{and} \quad \vec{\mathbf{B}} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}})$$

$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}} \quad \rightarrow \quad A_x = 2.0, \quad A_y = 2.0, \quad A_z = 0.0$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} \quad \rightarrow \quad B_x = 2.0, \quad B_y = -4.0, \quad B_z = 0.0$$

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} = (2.0 + 2.0)\hat{\mathbf{i}} + (2.0 - 4.0)\hat{\mathbf{j}} = 4.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{j}}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0)^2 + (2.0)^2} = \sqrt{20} = \boxed{4.5}$$ Magnitude

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0}{4.0} = -0.50 \rightarrow \boxed{\theta = 333°}$$ Direction



# Vector Addition using Components

**Example 8 (The Resultant Displacement):** A particle undergoes three consecutive displacements. Find unit-vector notation for the resultant displacement and magnitude.

$\Delta\vec{\mathbf{r}}_1 = (15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$cm, $\Delta\vec{\mathbf{r}}_2 = (23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}})$cm and $\Delta\vec{\mathbf{r}}_3 = (-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}})$cm

$$\begin{aligned}\Delta\vec{\mathbf{r}} &= \Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_3 \\ &= (15+23-13)\hat{\mathbf{i}}\text{ cm} + (30-14+15)\hat{\mathbf{j}}\text{ cm} \\ &\quad + (12-5.0+0)\hat{\mathbf{k}}\text{ cm} \\ &= (25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}})\text{cm}\end{aligned}$$

Resultant displacement

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(25\text{ cm})^2 + (31\text{ cm})^2 + (7.0\text{ cm})^2} = \boxed{40\text{ cm}}$$

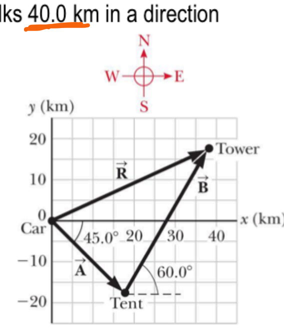
Magnitude



# Vector Addition using Components

**Example 9 (Taking a Hike):** A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

**(a)** Determine the components of the hiker's displacement for each day.




$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = \boxed{17.7\text{ km}}$$

$$A_y = A\sin(-45.0°) = (25.0\text{ km})(-0.707) = \boxed{-17.7\text{ km}}$$

$$B_x = B\cos(60.0°) = (40.0\text{ km})(0.500) = \boxed{20.0\text{ km}}$$

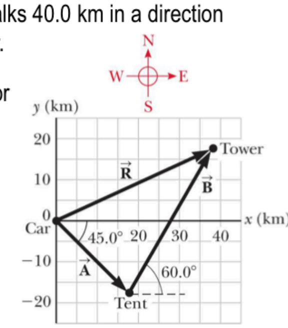
$$B_y = B\sin(60.0°) = (40.0\text{ km})(0.866) = \boxed{34.6\text{ km}}$$



# Vector Addition using Components

**Example 9 (Taking a Hike):** A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

**(b)** Determine the components of the hiker's resultant displacement $\vec{\mathbf{R}}$ for the trip. Find an expression for $\vec{\mathbf{R}}$ in terms of unit vectors.




$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = \boxed{37.3\text{ km}}$$

$$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = \boxed{17.0\text{ km}}$$

$$\vec{\mathbf{R}} = (37.7\hat{\mathbf{i}} + 17.0\hat{\mathbf{j}})\text{ km}$$

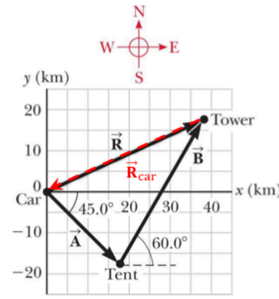
# Vector Addition using Components

**Example 9 (Taking a Hike):** A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

**(c)** After reaching the tower, the hiker wishes to return to her car along a single straight line. What are the components of the vector representing this hike? What should the direction of the hike be?

$$\vec{\mathbf{R}}_{car} = -\vec{\mathbf{R}} = \left(-37.7\hat{\mathbf{i}} - 17.0\hat{\mathbf{j}}\right) \text{ km}$$

$$\tan\theta = \frac{R_{car,y}}{R_{car,x}} = \frac{-17.0 \text{ km}}{-37.7 \text{ km}} = 0.450 \Rightarrow \theta = 204.2°, \text{ or } 24.2° \text{ south of west}$$






THANK YOU

HUMBER

WE ARE HUMBER