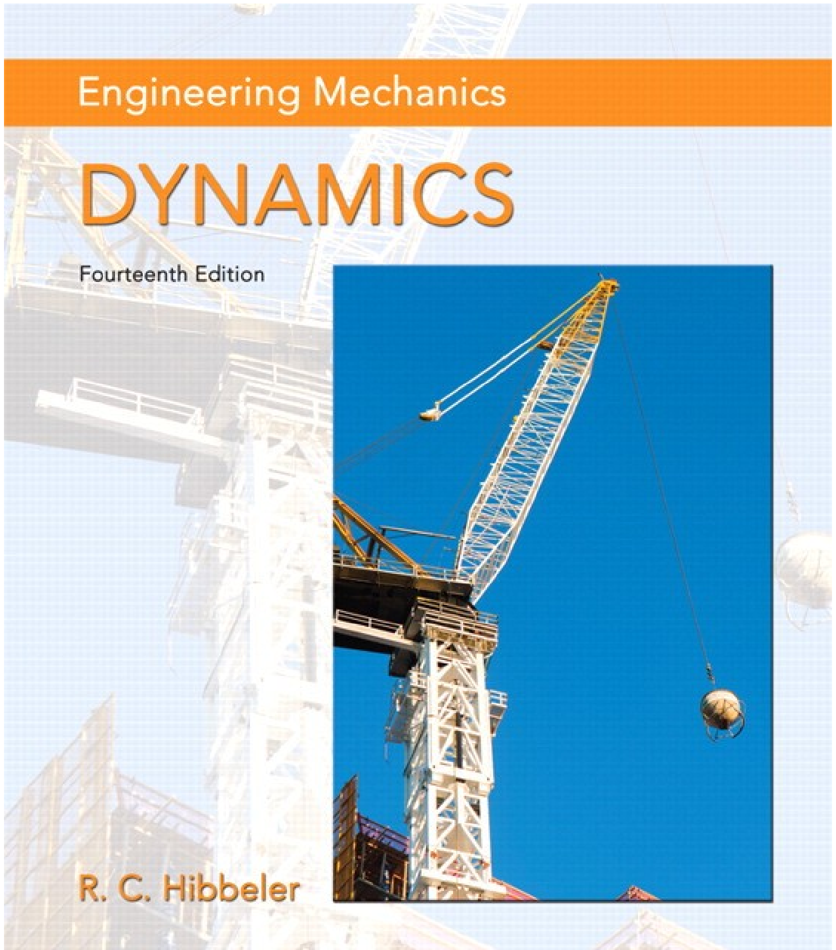
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 15

Kinetics of a Particle: Impulse and Momentum

Principle of Linear Impulse and Momentum and Conservation of Linear Momentum for Systems of Particles

Today's Objectives:

Students will be able to:

1. Apply the principle of linear impulse and momentum to a system of particles.
2. Understand the conditions for conservation of momentum.

Principle of Linear Impulse and Momentum and Conservation of Linear Momentum for Systems of Particles

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- **Linear Impulse and Momentum for a System of Particles**
- **Conservation of Linear Momentum**
- Concept Quiz
- Group Problem Solving
- Attention Qui

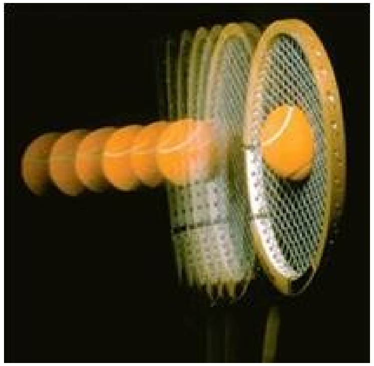
Reading Quiz

1) The internal impulses acting on a system of particles always ____

   A) equal the external impulses.
   B) sum to zero.
   C) equal the impulse of weight.
   D) None of the above.

2) If an impulse-momentum analysis is considered during **the very short time of interaction,** as shown in the picture, weight is a/an ___________

   A) impulsive force.
   B) explosive force.
   C) non-impulsive force.
   D) internal force.

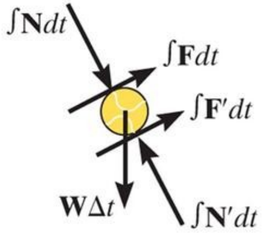


Applications (1 of 2)

As the wheels of this pitching machine rotate, they apply frictional impulses to the ball, thereby giving it linear momentum in the direction of $F\,dt$ and $F'dt$.

The weight impulse, $W\Delta t$ is very small since the time the ball is in contact with the wheels is very small.

Does the release velocity of the ball depend on the mass of the ball?



Applications (2 of 2)

This large crane-mounted hammer is used to drive piles into the ground.

Conservation of momentum can be used to find the velocity of the pile just after impact, assuming the hammer does not rebound off the pile

If the hammer rebounds, does the pile velocity change from the case when the hammer doesn't rebound? Why?

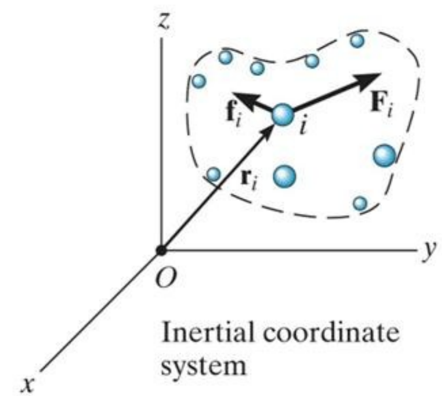
In the impulse-momentum analysis, do we have to consider the impulses of the weights of the hammer and pile and the resistance force? Why or why not?

Section 15.2

Principle of linear impulse and momentum for a system of particles

Principle of linear impulse and momentum for a system of particles

For the system of particles shown, the internal forces f_i **between** particles always occur in pairs with equal magnitude and opposite directions. Thus the internal **impulses sum to zero.**



The linear impulse and momentum equation for this system only includes the impulse of **external** forces.

$$\sum m_i (v_i)_1 + \sum \int_{t_1}^{t_2} F_i \, dt = \sum m_i (v_i)_2$$

Motion of the Center of Mass

For a system of particles, we can define a "fictitious" center of mass of an aggregate particle of mass m_{tot}, where m_{tot} is the sum $\left(\sum m_i\right)$ of all the particles. This system of particles then has an aggregate velocity of $v_G = \left(\sum m_i v_i\right) / m_{tot}$

The motion of this fictitious mass is based on motion of the center of mass for the system.

The position vector $\boldsymbol{r_G} = \left(\sum \boldsymbol{m_i r_i}\right) / \boldsymbol{m_{tot}}$ describes the **motion of the center of mass.**

Section 15.3

Conservation Of Linear Momentum For A System Of Particles

Conservation of linear momentum for a system of particles

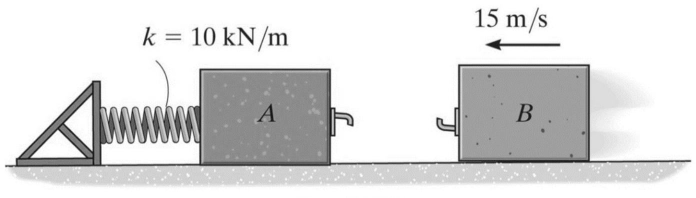
When the **sum of external impulses** acting on a system of objects is **zero**, the linear impulse-momentum equation simplifies to

$$\sum m_i (v_i)_1 = \sum m_i (v_i)_2$$

This equation is referred to as the **conservation of linear momentum**. Conservation of linear momentum is often applied when particles collide or interact. When particles impact, only **impulsive forces cause** a change of linear momentum.

The sledgehammer applies an impulsive force to the stake. The weight of the stake is considered negligible, or non-impulsive, as compared to the force of the sledgehammer. Also, provided the stake is driven into soft ground with little resistance, the impulse of the ground acting on the stake is considered non-impulsive.

Example 1 (1 of 2)



Given: Spring constant $k = 10\ kN/m$

$m_A = 15\ kg,\ v_A = 0\ m/s,\ \ m_B = 10\ kg,\ v_B = 15\ m/s$

The blocks couple together at impact.

Find: The maximum compression of the spring.

Plan: 1) We can consider both blocks as a single system and apply **the conservation of linear momentum** to find the velocity after impact, but before the spring compresses.

2) Then use the **energy conservation** to find the compression of the spring.

Example 1 (2 of 2)

Solution:

1) **Conservation of linear momentum**

$$+\rightarrow \sum m_i (v_i)_0 = \sum m_i (v_i)_1$$

$$10(-15i) = (15+10)(vi)$$

$$v = -6 m/s$$

$$= 6\ m/s \leftarrow$$

2) **Energy conservation equation**

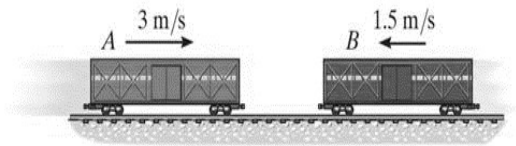
$$T_1 + V_1 = T_2 + V_2$$

$$0.5(15+10)(-6)^2 + 0 = 0 + 0.5(10000)(s_{max})^2$$

So the maximum compression of the spring is $s_{max} = 0.3m$.

Example 2 (1 of 2)

Given: Two rail cars with masses of $m_A = 20Mg$ and $m_B = 15Mg$ and velocities as shown.



Find: The speed of the car A after collision if the cars collide and rebound such that B moves to the right with a speed of $2\,\mathrm{m/s}$. Also find the average impulsive force between the cars if the collision place in $0.5\ s$.

Plan: Use **conservation of linear momentum** to find the velocity of the car A after collision (all internal impulses cancel). Then use the **principle of impulse and momentum** to find the impulsive force by looking at only one car.

Example 2 (2 of 2)

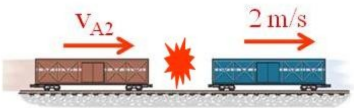
Solution:

Conservation of linear momentum (x-dir):

$$m_A(v_{A1})+m_B(v_{B1})=m_A(v_{A2})+m_B(v_{B2})$$

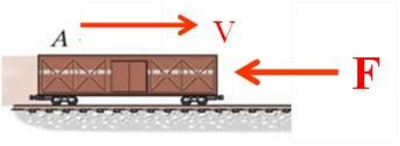
$$20{,}000(3)+15{,}000(-1.5)$$

$$=(20{,}000)v_{A2}+15{,}000(2)$$



$$v_{A2}=0.375 m/s$$

Impulse and momentum on car A (x-dir):



$$m_A(v_{A1})+\int F dt=m_A(v_{A2})$$

$$20{,}000(3)-\int F dt=20{,}000(0.375)$$

$$\int F dt=52{,}000 N.s$$

The average force is

$$\int F dt=52{,}500 N.s=F_{avg}(0.5\sec);=105 kN$$

Concept Quiz

1. Over the short time span of a tennis ball hitting the racket during a player's serve, the ball's weight can be considered ________________

   A) nonimpulsive. B) impulsive.

   C) not subject to Newton's second law. D) Both A and C.

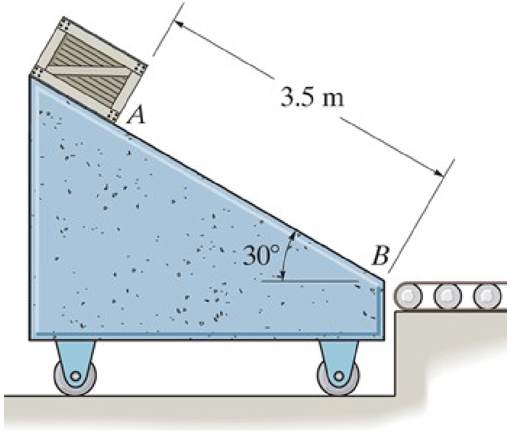
2. A drill rod is used with a air hammer for making holes in hard rock so explosives can be placed in them. How many impulsive forces act on the drill rod during the drilling?

   A) None B) One

   C) Two D) Three

Group Problem Solving 1 (1 of 3)

Given: The free-rolling ramp has a mass of 40 kilogram. The 10 kilogram crate slides from rest at A, 3.5 meters down the ramp to B. Assume that the ramp is smooth, and neglect the mass of the wheels.



Find: The ramp's speed when the crate reaches B.

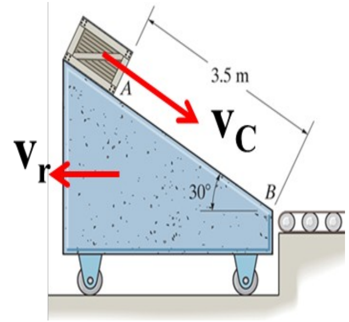
Plan: Use **the energy conservation equation** as well as **conservation of linear momentum** and the relative velocity equation (you really thought you could safely forget it?) to find the velocity of the ramp

Group Problem Solving 1 (2 of 3)

Solution:

To find the relations between v_c **and** v_r

use **conservation of linear momentum:**



$$\overset{+}{\rightarrow}\ 0 = (40)(-v_r) + (10)v_{cx}$$

$$\Rightarrow v_{cx} = 4v_r \qquad (1)$$

Since $v_C = v_r + v_{C/r}$

$$v_{Cx}i - v_{Cy}j = -v_r i + v_{C/r}(\cos 30 i - \sin 30 j)$$

$$v_{Cx} = -v_r + v_{C/r}\cos 30 \qquad (2)$$

$$v_{Cy} = v_{C/r}\sin 30$$

Eliminating $v_{C/r}$ from Eqs. (2) and (3), and substituting

Eq. (1) results in $v_C = 8.660\, v_r$

Group Problem Solving 1 (3 of 3)

Then, **energy conservation equation** can be written ;

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 10(9.81)(3.5 \sin 30) = 0.5(10)(v_C)^2 + 0.5(40)(v_r)^2$$

$$\Rightarrow \quad 0 + 10(9.81)(3.5 \sin 30)$$

$$= 0.5(10)\left[(v_{Cx})^2 + (v_{Cy})^2 + 0.5(40)(v_r)^2\right]$$

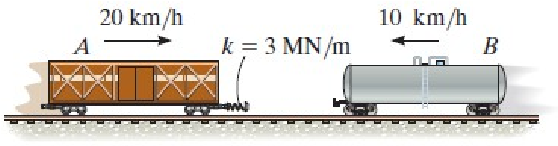
$$= 0.5(10)\left[(4.0 v_r)^2 + (8.660 v_r)^2 + 0.5(40)(v_r)^2\right]$$

$$\Rightarrow \quad 171.7 = 475.0(v_r)^2$$

$$v_r = 0.601\, m/s$$

Group Problem Solving 2 (1 of 3)

Given: Two rail cars with masses of $m_A = 30 Mg$ and $m_B = 15 Mg$ and velocities as shown.



Find: The maximum compression of the spring mounted on car *A*.

Plan: Use **conservation of linear momentum** to find the velocity of the cars after collision. Then use **the energy conservation equation** to find the compression of the spring

Solution

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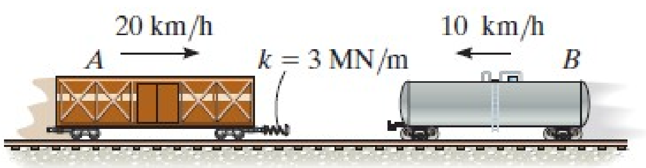
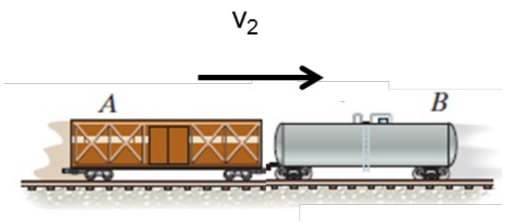




Conservation of linear momentum along x-axis:

$$m_A(v_{A1}) + m_B(v_{B1}) = (m_A + m_B)v_2$$

where $v_{A1} = 20 km/h = 5.556 m/s,$

$$v_{B1} = -10 km/h = -2.778 m/s$$

$$30{,}000\,(5.556) + 15{,}000\,(-2.778) = (30{,}000 + 15000)\,v_2$$

$$v_2 = 2.778\ m/s \rightarrow$$

Group Problem Solving 2 (3 of 3)

Then, **energy conservation equation** to find the maximum compression of the spring ;

$$T_1 + V_1 = T_2 + V_2$$

$$\left[0.5\,(30000)\,(5.556)^2 + 0.5\,(15000)\,(-2.778)^2\right] + 0$$
$$= 0.5\,(45000)\,(2.778)^2 + 0.5\,(3 \times 10^6)\,(s\max)^2$$

The maximum compression of the spring

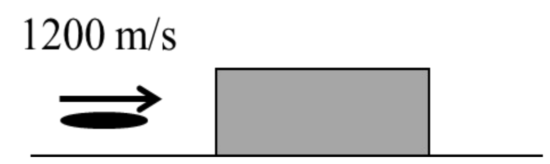
$$s_{\max} = 0.4811\ m = 481\ mm$$

Attention Quiz

1. The 20 g bullet is fired horizontally at $1200\ m/s$ into the 300 g block resting on a smooth surface. If the bullet becomes embedded in the block, what is the velocity of the block immediately after impact

   A) 1125 m/s B) 80 m/s

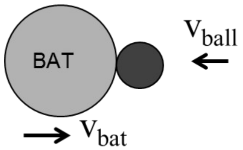
   C) 1200 m/s D) 75 m/s



2. The 200-g baseball has a horizontal velocity of $30\ m/s$ when it is struck by the bat, B, weighing 900-g, moving $47 m/s$. During the impact with the bat, how many impulses of importance are used to find the final velocity of the ball?

   A) Zero B) One

   C) Two D) Three



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