Signal Processing (MENG3520)

Module 1

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MODULE 1

INTRODUCTION TO SIGNALS AND SYSTEMS

Module Outline

1.0 Overview of signals and systems

1.1 Models of signals

1.2 Properties of signals

1.3 Continuous-time signals

- 1.3.1 Commonly used continuous-time functions
- 1.3.2 Independent- and dependent- variable transformation of CT signals

1.4 Discrete-time signals

- 1.4.1 Sampling and discrete-time signals
- 1.4.2 Common discrete-time sequences
- 1.4.3 Independent- and dependent- variable transformation of DT signals
- 1.4.4 Differencing and accumulation of DT signals

1.4

DISCRETE-TIME SIGNALS

1.4.1

Sampling and Discrete-Time Signals

- Discrete-time (DT) signals and systems applications have increased over continuous-time (CT) ones.
- Operations that once done by CT are replaced with DT signals and systems.
- It is possible that a system is inherently DT, but most of the DT systems on DT signals are created by sampling CT signals.
- Most of the functions and methods in CT signals have similar counterparts. However, some operations are fundamentally different.

Sampling: obtaining the values of a signal at discrete points in time.

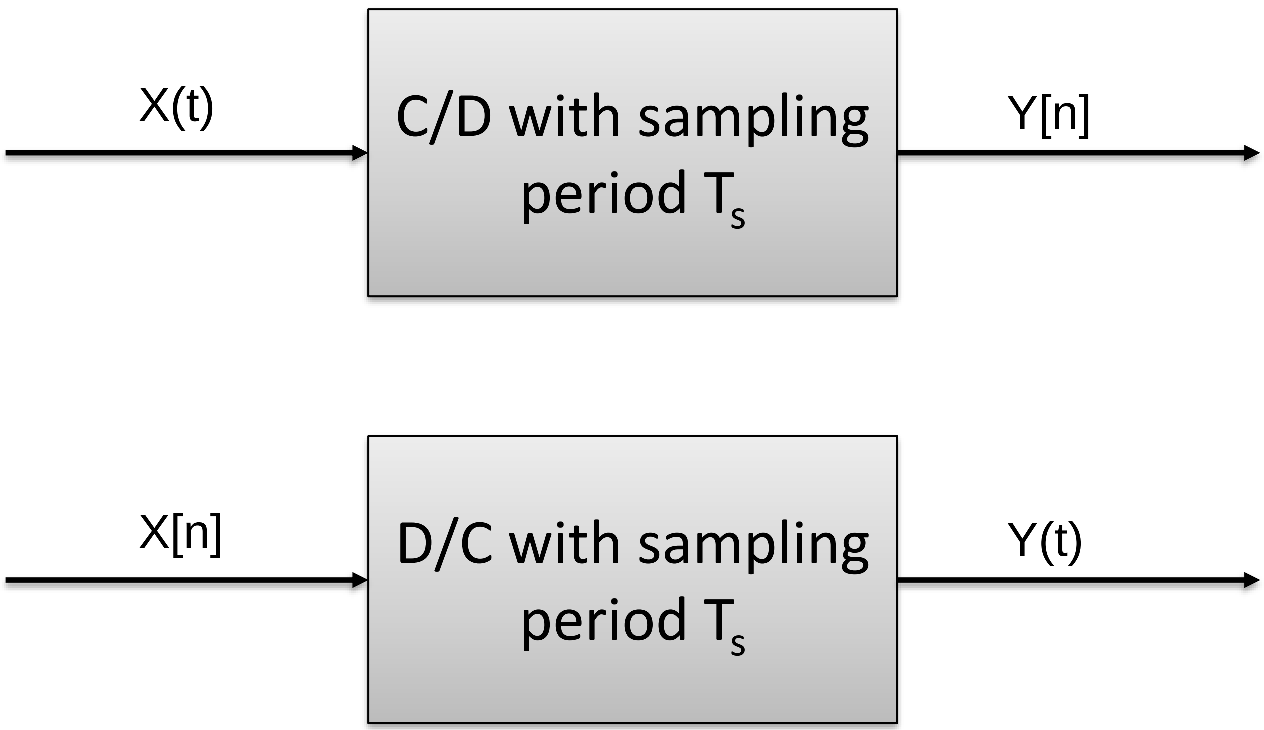
Sampling is performed by an ideal continuous-time to discrete-time (C/D) converter.

Interpolation: the opposite procedure to sampling, reconstructing the values of a signal in the continuous-time domain.

Interpolation is performed by an ideal discrete-time to continuous-time (D/C) converter.

Unless special conditions are met, the sampling process from CT to DT is a **lossy** process, i.e. loses information.

How to ensure the sampling process retains all the information? We will discuss this further in future lectures.


X(t)
C/D with sampling period T_s
Y[n]
X[n]
D/C with sampling period T_s
Y(t)

Sampling can be considered as the multiplication of a CT signal ($x(t)$): with a sampling function.

The ideal sampling function is a periodic sequence of impulses of period T_s ($\delta_{T_s}(t)$):

$$\delta_{T_s(t)} = \sum_{n} \delta(t - nT_s)$$

Recall: features of the unit impulse function (Dirac function) $\delta(t)$:

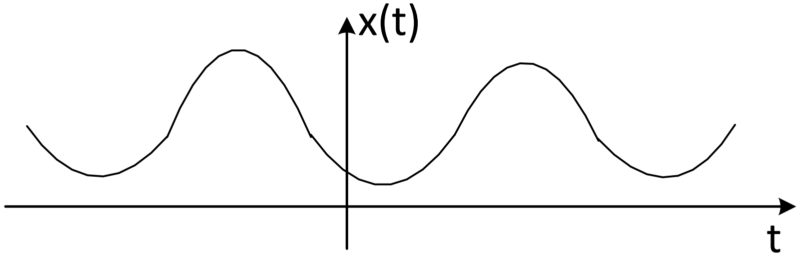
$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$

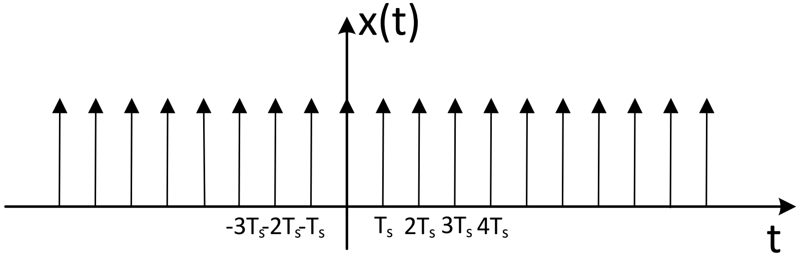
The sampling process becomes:

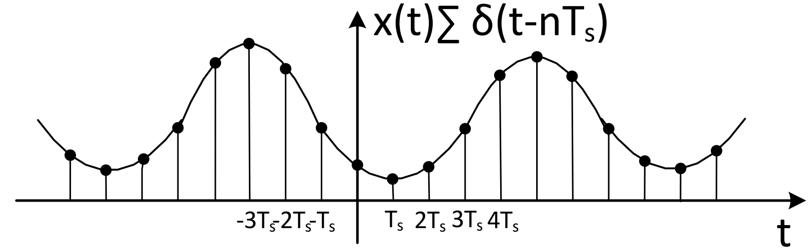
$$x(t)\delta_{T_s(t)} = \sum_n x(t)\delta(t - nT_s)$$

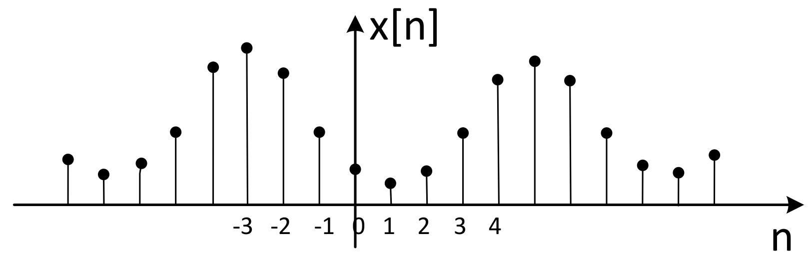
$$= \sum_n x(nT_s)\delta(t - nT_s) \stackrel{\text{def}}{=} x[n]$$

Activity: identify the relations between these four figures.



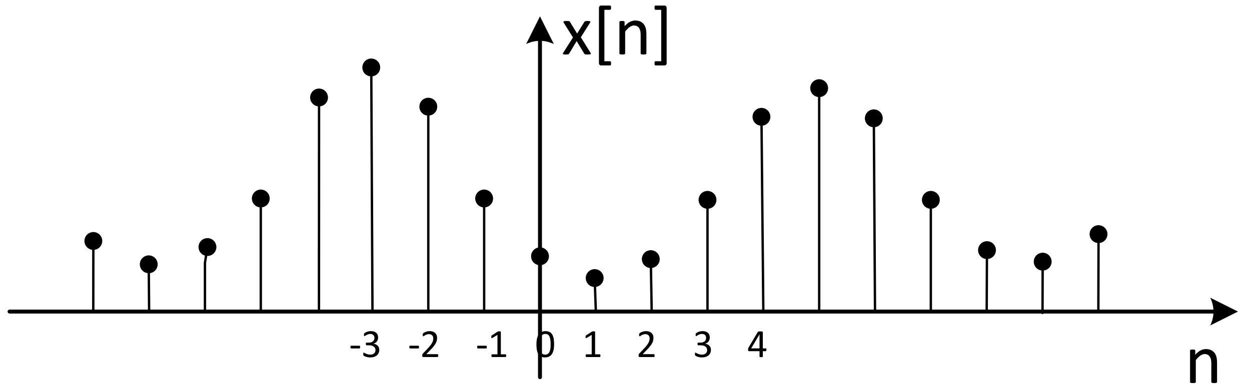






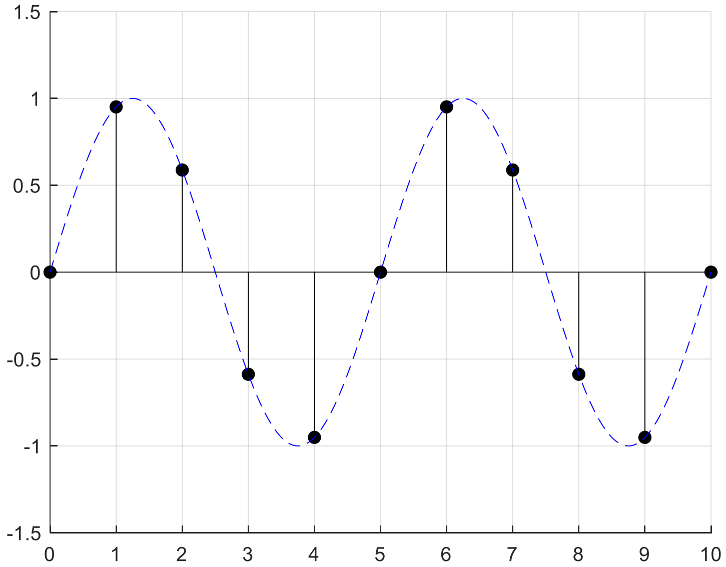
IMPORTANCE OF SAMPLING PERIOD T_S

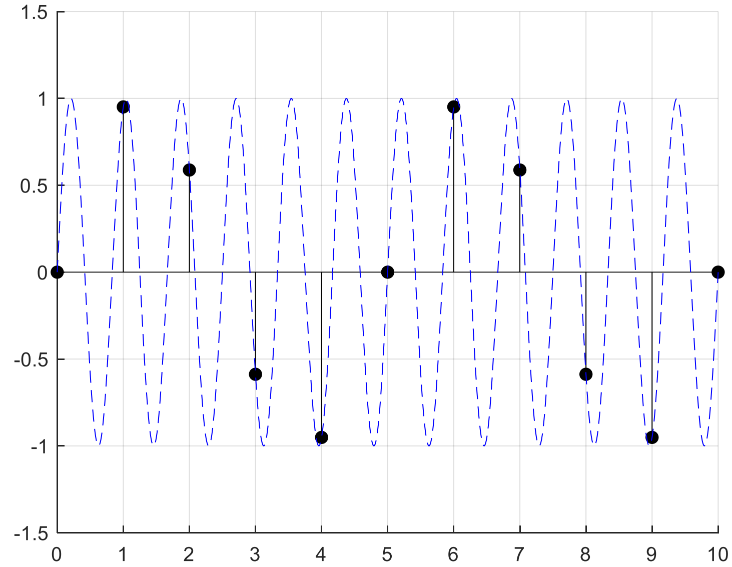
- How is T_s represented in a DT sequence x[n]?
- Sampling frequency f_s
- Sampling ω_s



Importance of Sampling Period T_S

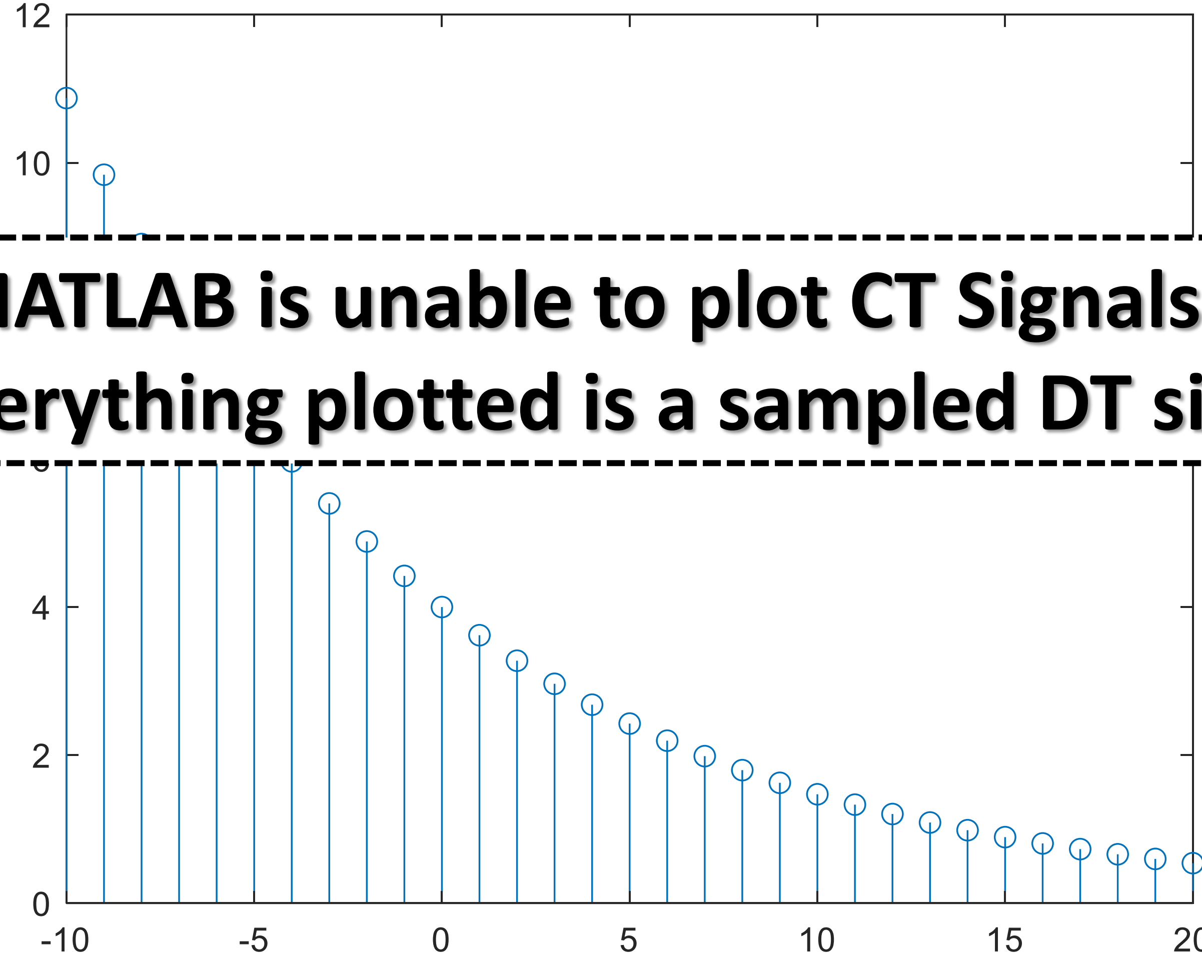
- Sometimes, sampling on different CT signals may result in identical DT signals.





SOME TYPICAL SAMPLING APPLICATIONS

SOME TYPICAL SAMPLING APPLICATIONS



MATLAB is unable to plot CT Signals, so everything plotted is a sampled DT signal

1.4.2

Common Discrete-time Sequences

Most commonly used DT sequences are similar to CT functions.

- Exponentials and sinusoids
- Unit step sequence
- Unit impulse sequence

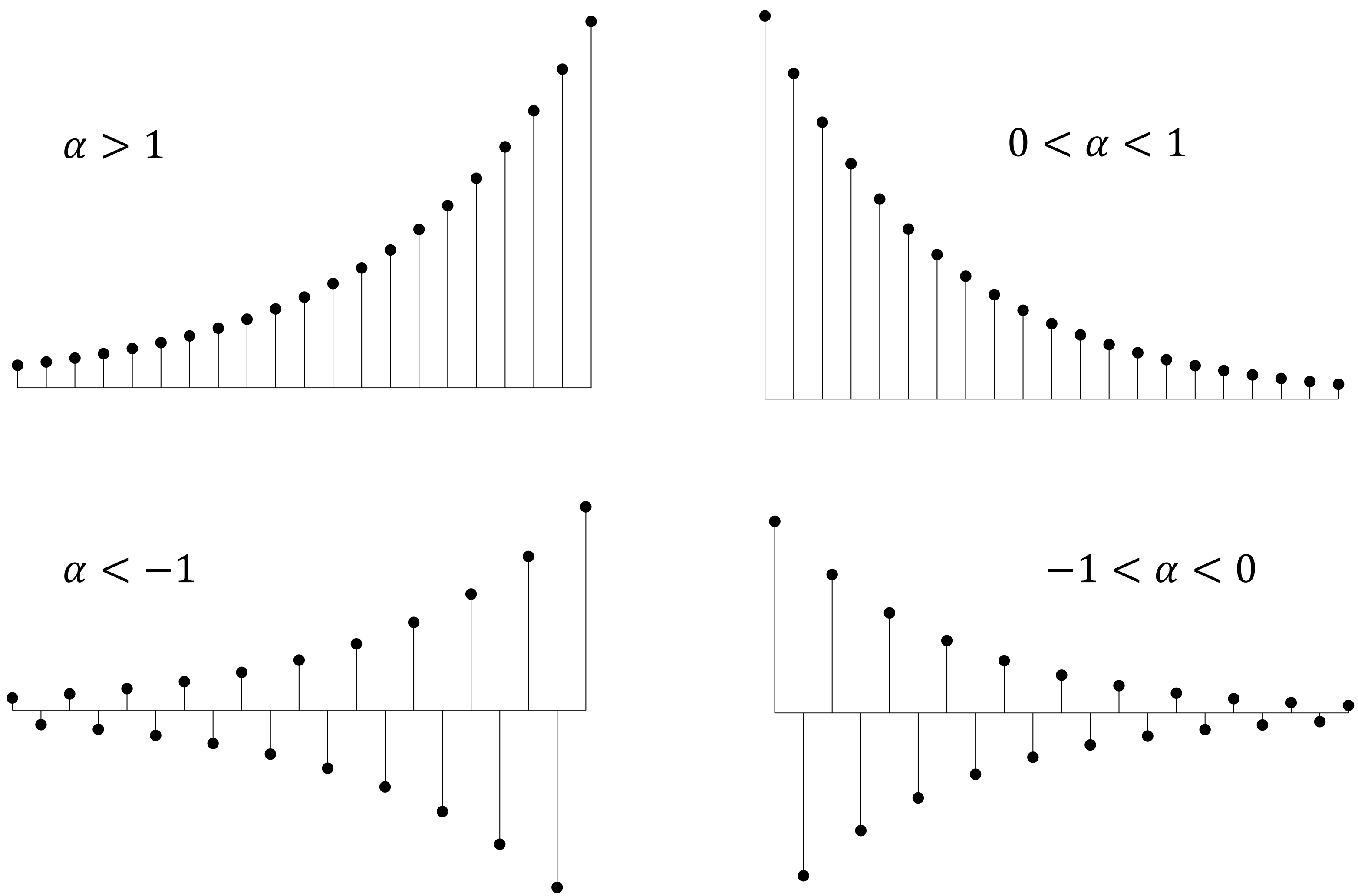
DT Complex Exponentials

- General format: $x[n] = Ce^{\beta n}$, but more commonly written as $x[n] = C\alpha^n$, where $\alpha = e^{\beta}$.
- Different cases:
  - Case 1: C real, α real
  - Case 2: C real, β purely imaginary
  - Case 3: C complex, α complex

DT COMPLEX EXPONENTIALS

- Case 1: $x[n] = C\alpha^n$, where C real, α real.
- Depending on the value of α, the discrete exponential can look very different.

DT COMPLEX EXPONENTIALS: $x[n] = C\alpha^n$



DT Complex Exponentials

- Case 2: $x[n] = Ce^{\beta n}$, where C is real, and β is purely imaginary, which means $|\alpha| = 1$.
- For simplicity, let $C = 1$, this signal becomes:

$$x[n] = e^{j\omega_0 n} = \cos(\omega_0 n) + j\sin(\omega_0 n)$$

Question: is $x[n]$ periodic?

DT Complex Exponentials

- Recall: $x[n] = e^{j\omega_0 n} = \cos(\omega_0 n) + j\sin(\omega_0 n)$
- For $x[n]$ to be periodic, there must exist a smallest integer N, s.t. $x[n] = x[n+N]$.
- $\begin{cases} \cos(\omega_0 n) = \cos(\omega_0(n+N)) \\ \sin(\omega_0 n) \ = \sin(\omega_0(n+N)) \end{cases}$
- Thus: $N = \frac{2\pi\, m}{\omega_0}$. (m is an integer.)

DT Complex Exponentials: $x[n] = e^{j\omega_0 n}$

- Only certain ω_0 and f_0 will make $N = \frac{2m\pi}{\omega_0} = \frac{m}{f_0}$ to be an integer. Others will result in an aperiodic signal $x[n]$.

- Question 1: what type of f_0 will make $N = \frac{2m\pi}{\omega_0} = \frac{m}{f_0}$ to be an integer?
- Answer: $f_0 = \frac{m}{N}$ has to be a rational number.

DT Complex Exponentials

- Only certain ω_0 and f_0 will make $N = \frac{2m\pi}{\omega_0} = \frac{m}{f_0}$ an integer. Others will result in an aperiodic signal $x[n]$.

- Question 2: what is the fundamental period and frequency of this DT signal?

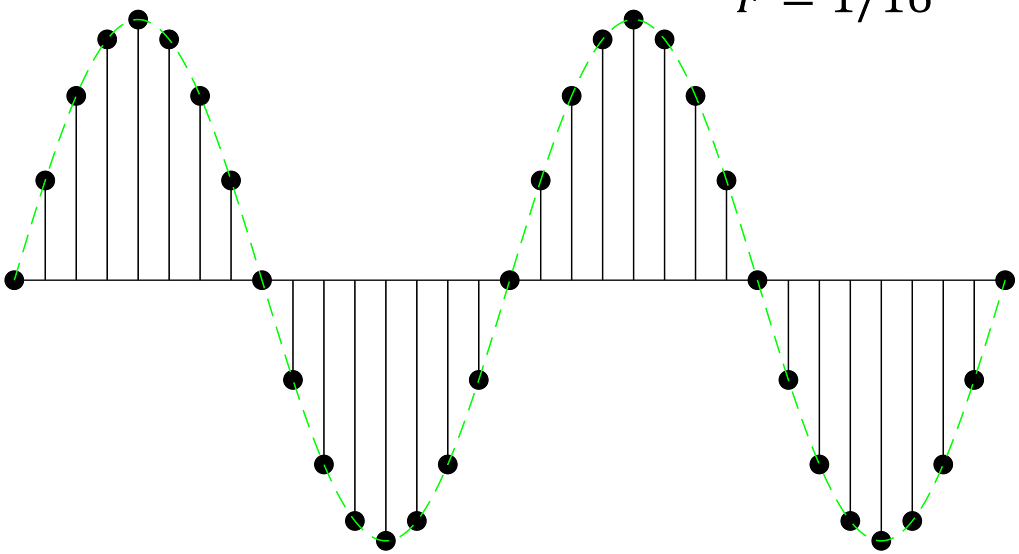
- Answer: fundamental period is $N = \frac{2m\pi}{\omega_0}$; fundamental frequency is $f = \frac{2\pi}{N} = \frac{\omega_0}{m}$.

SOME DT SINUSOIDAL SIGNAL EXAMPLES

$$Fundametal\ period\ N = 16$$

$$Fundametal\ frequency\ f = \frac{1}{16}$$

$$\mathrm{x[n]} = \sin(2\pi Fn)\,,$$
$$F = 1/16$$

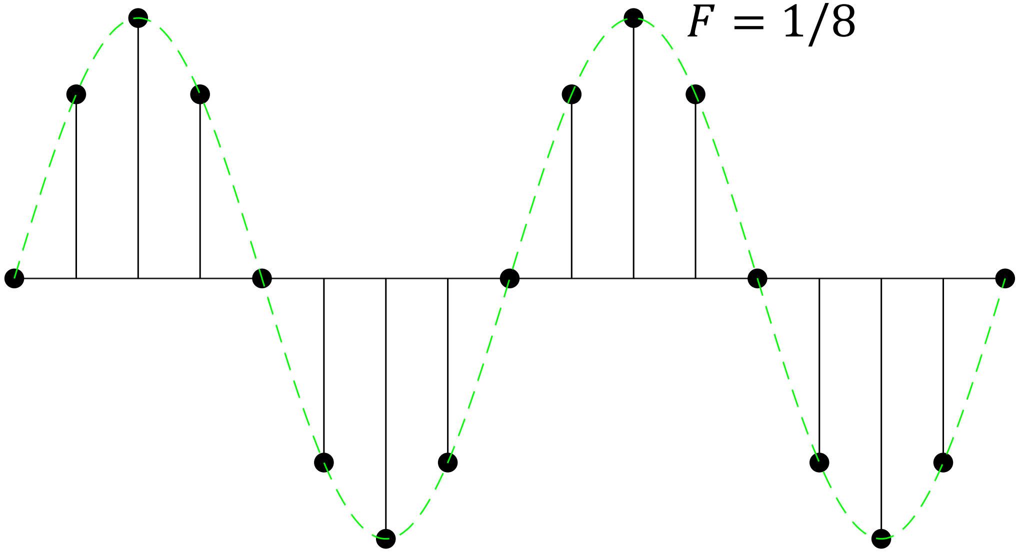


SOME DT SINUSOIDAL SIGNAL EXAMPLES

$$Fundametal\ period\ N = 8$$

$$Fundametal\ frequency\ f = \frac{1}{8}$$

$$\mathrm{x[n]} = \sin(2\pi Fn)\,,$$
$$F = 1/8$$



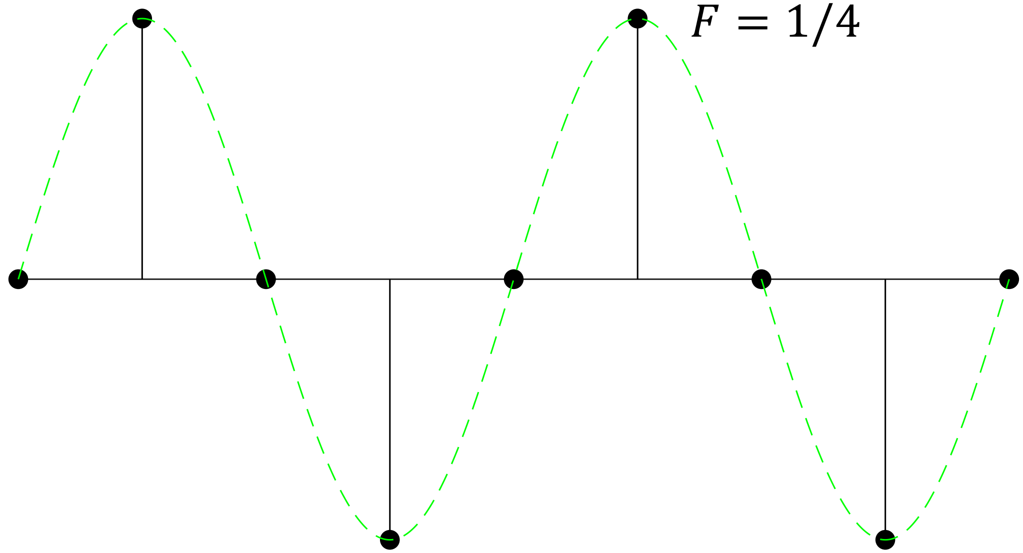
Some DT Sinusoidal Signal Examples

$$Fundametal\ period\ N = 4$$

$$Fundametal\ frequency\ f = \frac{1}{4}$$

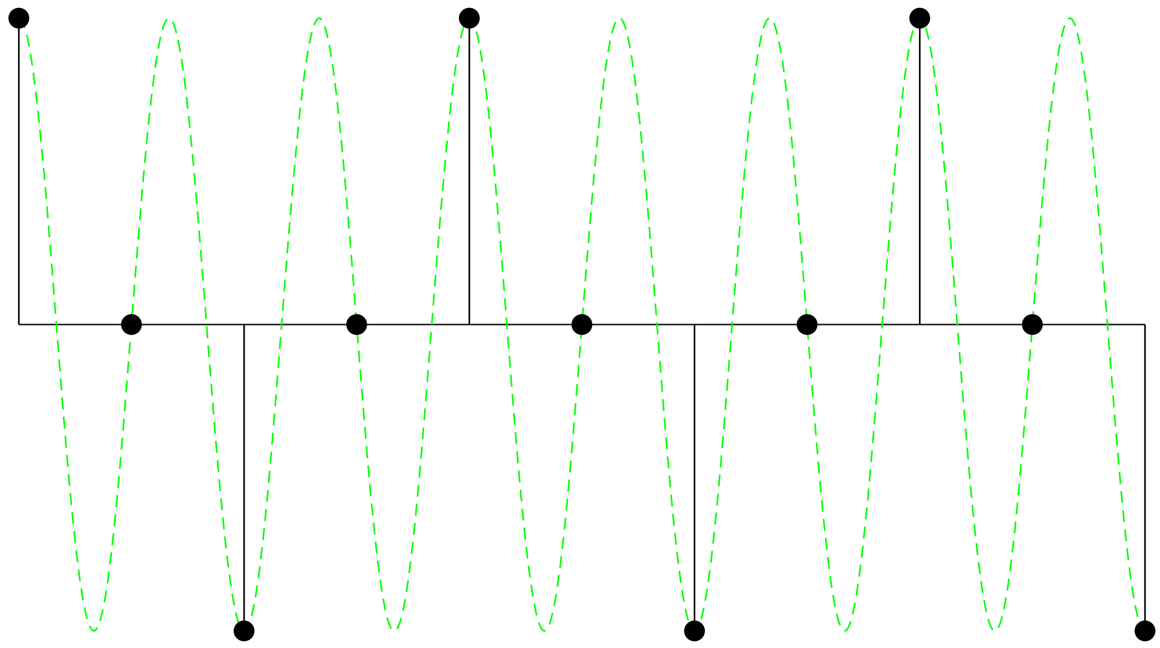
$$\mathrm{x[n]} = \sin(2\pi F n)\,,$$

$$F = 1/4$$



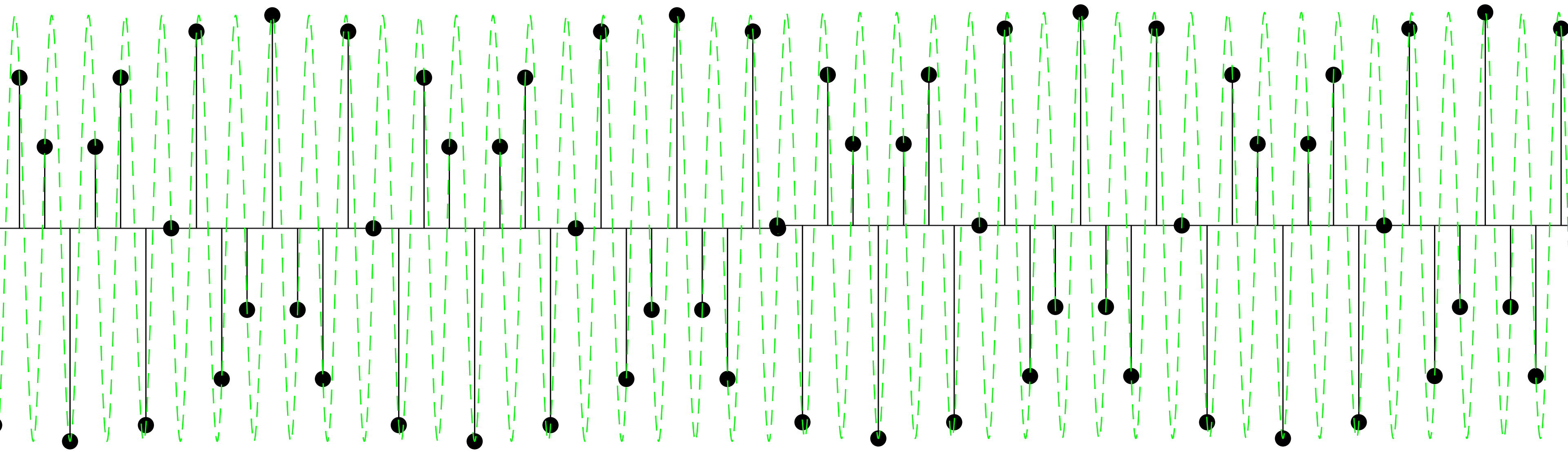
Some DT Sinusoidal Signal Examples

$$Fundametal\ period\ N = 4$$
$$Fundametal\ frequency\ f = \frac{1}{4}$$
$$x[n] = sin(2\pi Fn)\,,$$
$$F = 3/4$$



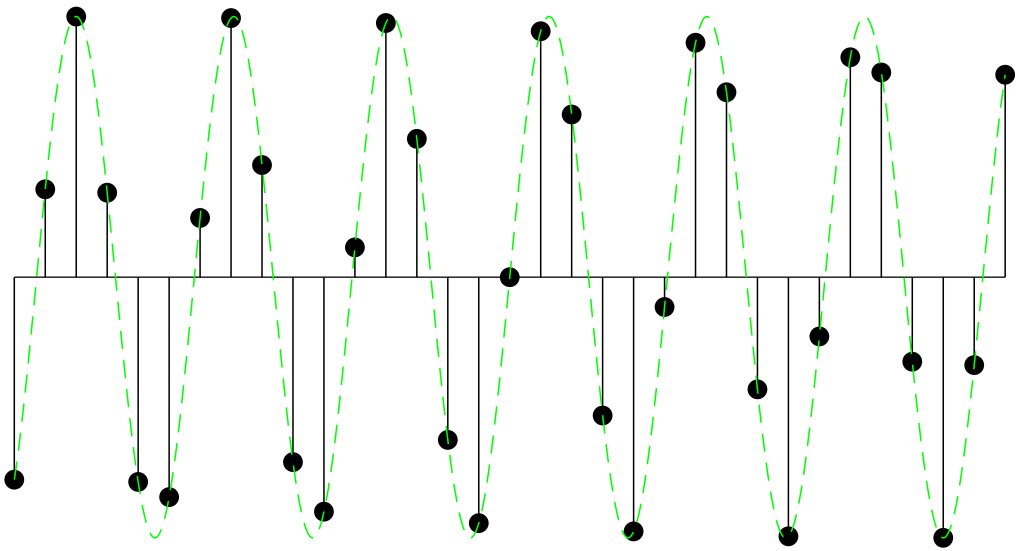
Some DT Sinusoidal Signal Examples

$$Fundametal\ period\ N = 16$$
$$Fundamental\ frequency\ = \ 1/16,$$
$$x[n] = sin(2\pi Fn)\ ,$$
$$F = 11/16$$



SOME DT SINUSOIDAL SIGNAL EXAMPLES

$$\begin{gathered} Aperiodic \\ no\ fundamental\ frequency \\ x[n] = \sin(2\pi F n)\,, \\ F = \pi/16 \end{gathered}$$



DT Complex Exponentials

- Question 3: What is the fundamental period of signal $x[n] = \sin(7\pi n/4)$?

- $\sin\left(\frac{7\pi n}{4}\right) = \sin(\frac{7\pi}{4}(n+N))$
- $N = 8m/7$ has to be an integer.
- $N = 8, m = 7$
- Fundamental frequency $\omega = \frac{\omega_0}{N} = \frac{\pi}{4}$

DT COMPLEX EXPONENTIALS

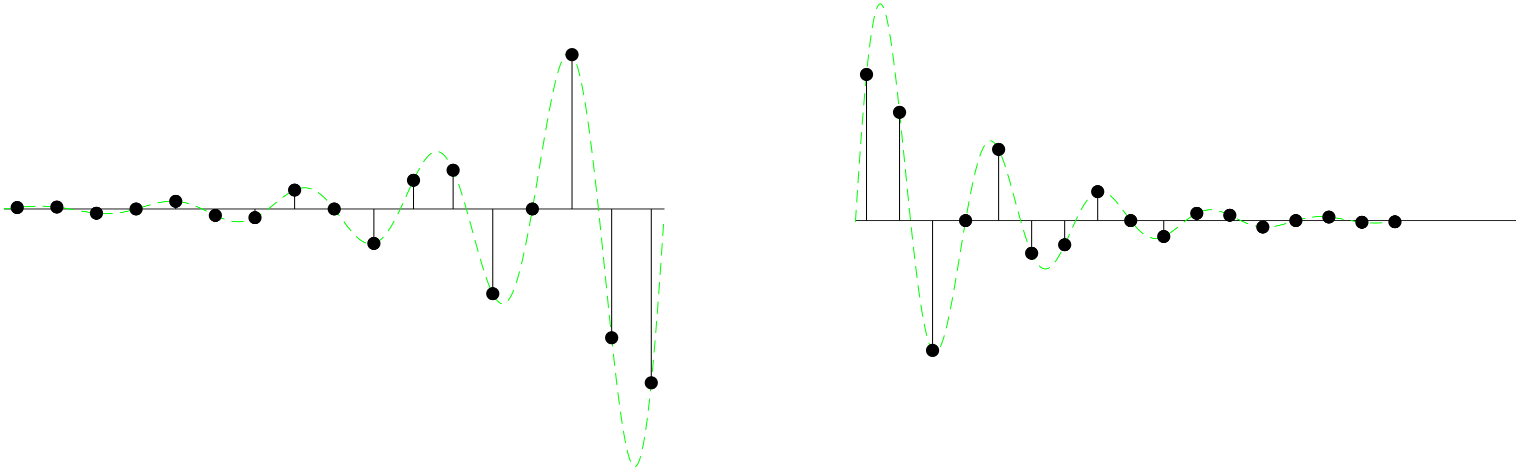
- Question 4: determine the fundamental period of the DT signal below:
- $x[n] = e^{j(2\pi/3)n} + e^{j(3\pi/4)n}$
- Note: for the entire signal to repeat, each term need to go through an integer number of its own fundamental period.

DT Complex Exponentials

- Case 3: $x[n] = C\alpha^n$, where C and α are both complex.
- Express C in polar format and α in polar format: $C = |C|e^{j\theta}$ and $\alpha = |\alpha|e^{j\omega_0}$
- Thus:
- $x(t) = C\alpha^n = |C||\alpha|^n e^{jn\omega_0} e^{j\theta} = |C||\alpha|^n e^{j(\omega_0 n+\theta)}$
  $|C||\alpha|^n$: case 1, decaying or growing exponential
  $e^{j(\omega_0 n+\theta)}$: case 2, sampled DT sinusoids in real and imaginary planes.

DT Complex Exponentials

- Case 3: $x(t) = C\alpha^n$, where C and α are both complex.

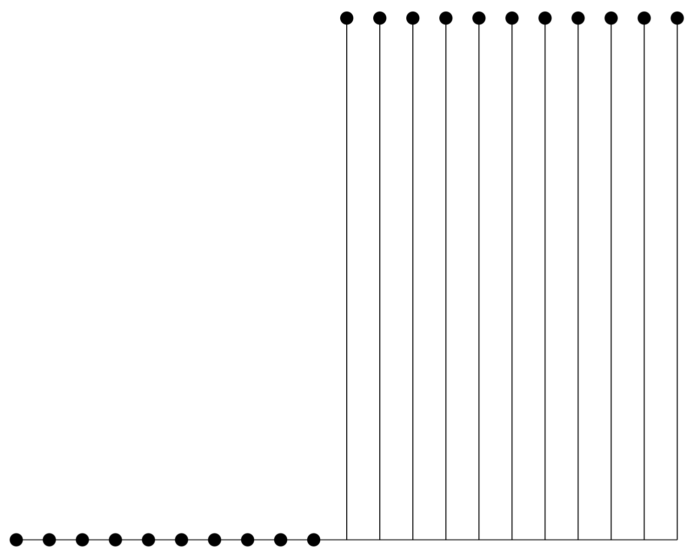


Most commonly used DT sequences are similar to CT functions.

- Exponentials and sinusoids
- Unit step sequence
- Unit impulse sequence

Discrete-time unit step sequence

- $u[n] = \begin{cases} 0, n < 0 \\ 1, n \geq 0 \end{cases}$

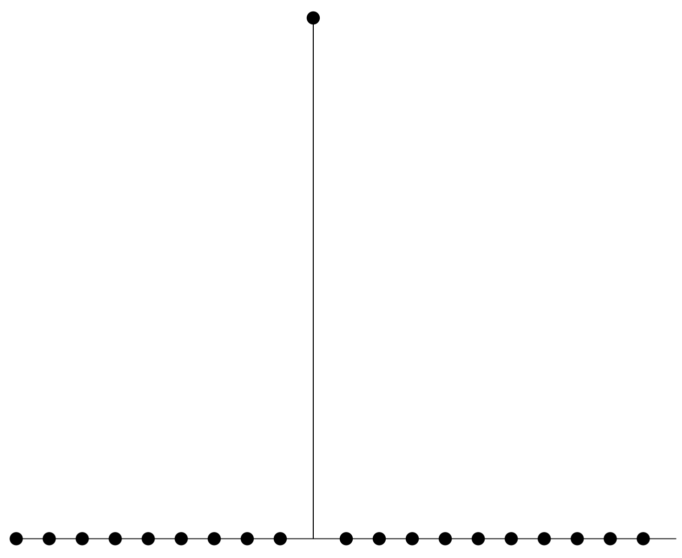


Most commonly used DT sequences are similar to CT functions.

- Exponentials and sinusoids
- Unit step sequence
- Unit impulse sequence

Unit impulse sequence

$$\delta[n] = \begin{cases} 0, n \neq 0 \\ 1, n = 0 \end{cases}$$



Relationship between unit impulse sequence and unit step sequence:

$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

1.4.3

Independent- and Dependent-Variable Transformation of DT Signals

Basic Signal Transformation

$$f = y[n]$$

Dependent variable transformation:

- Shifting: amplitude translation
- Scaling: amplitude scaling

Independent variable transformation:

- Shifting: time translation/time shifting
- Scaling: time scaling.

BASIC SIGNAL TRANSFORMATION

Dependent variable transformation:

- Shifting: DT amplitude translation is exactly the same as CT
- $y[n] \longrightarrow y[n] + D$

Basic Signal Transformation

Dependent variable transformation:

- Scaling: DT amplitude scaling is exactly the same as CT
- $y[n] \longrightarrow Ay[n]$

BASIC SIGNAL TRANSFORMATION

Independent variable transformation:

- Scaling: time scaling
- $y[n] \longrightarrow y[an]$
- $|a| > 1$: Time compression
- $|a| < 1$: Time expansion
- If an is not an integer, i.e. $y[an]$ is not defined, then interpolation is needed to figure out the value $y[an]$.

BASIC SIGNAL TRANSFORMATION

Independent variable transformation:

- Shifting: time translation
- $y[n] \longrightarrow y[n-N]$
- N has to be an integer to ensure $y[n-N]$ is defined.

1.4.4

DIFFERENCING AND ACCUMULATION OF DT SIGNALS

Differencing and Accumulation

- Often times, processing DT signals requires operations such as differencing and accumulation.
- Because of the discrete nature of DT signals. Differencing and accumulation is significantly simpler in DT format.

DIFFERENCING

- First derivative of a CT signal g(t) is defined numerically as one of the following:

$$\frac{d}{dt} g(t) = \lim_{\Delta t \to 0} \frac{g(t + \Delta t) - g(t)}{\Delta t}$$

$$\frac{d}{dt} g(t) = \lim_{\Delta t \to 0} \frac{g(t) - g(t - \Delta t)}{\Delta t}$$

$$\frac{d}{dt} g(t) = \lim_{\Delta t \to 0} \frac{g(t + \Delta t) - g(t - \Delta t)}{2\Delta t}$$

DIFFERENCING

- Differencing of a DT signal g[n]:
- Forward difference:

$$diff(g[n]) = g[n+1] - g[n]$$

- Backward difference:

$$diff(g[n]) = g[n] - g[n-1]$$

- Central difference:

$$diff(g[n]) = \frac{1}{2}(g[n+1] - g[n-1])$$

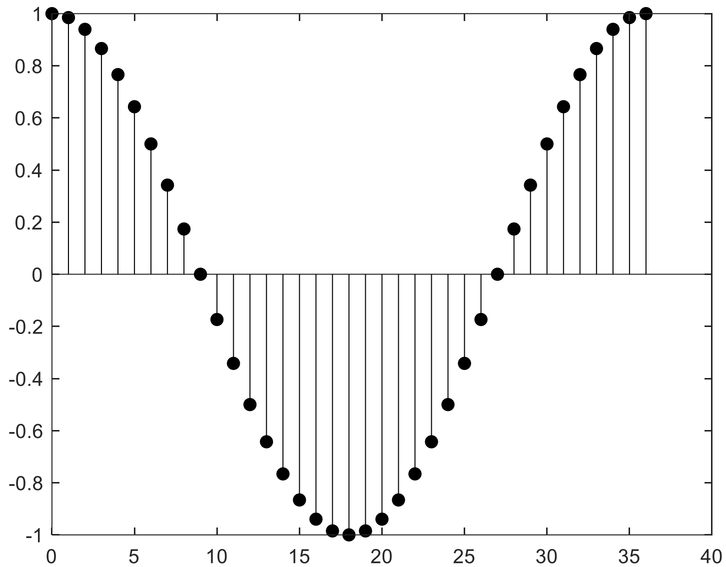
ACCUMULATION

- The equivalent of Integration of a CT signal g(t) in DT signals is defined as accumulation:

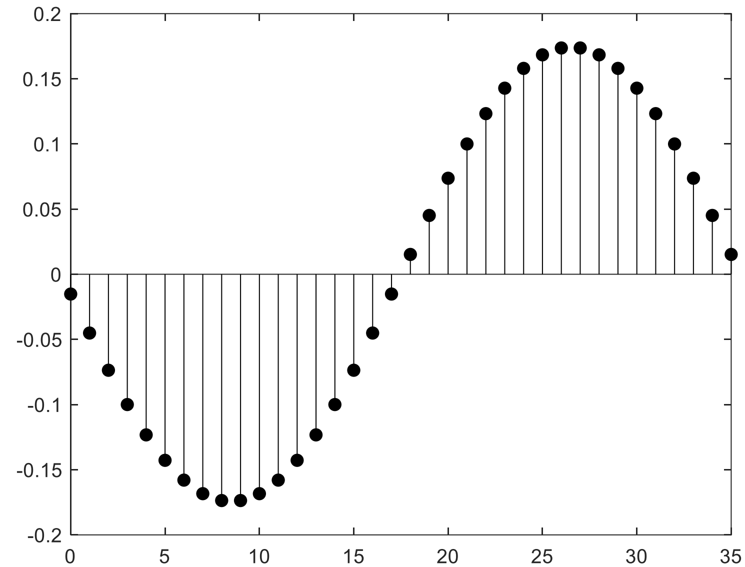
$$\sum_{m=-\infty}^{n} g[m]$$

- Just like integration, accumulation is not unique: multiple functions can have the same difference.
- As a result, if you take difference of a DT sequence and then compute its accumulation, the result may not be the original sequence.

DIFFERENCING AND ACCUMULATION

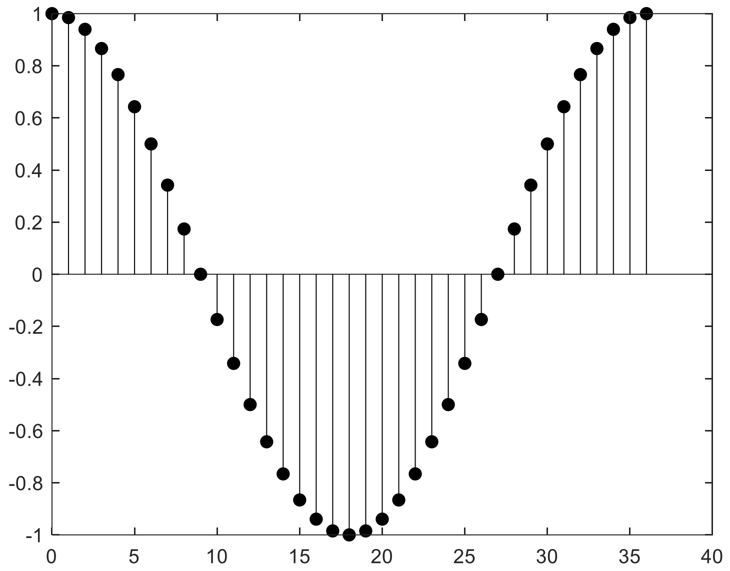


g[n]

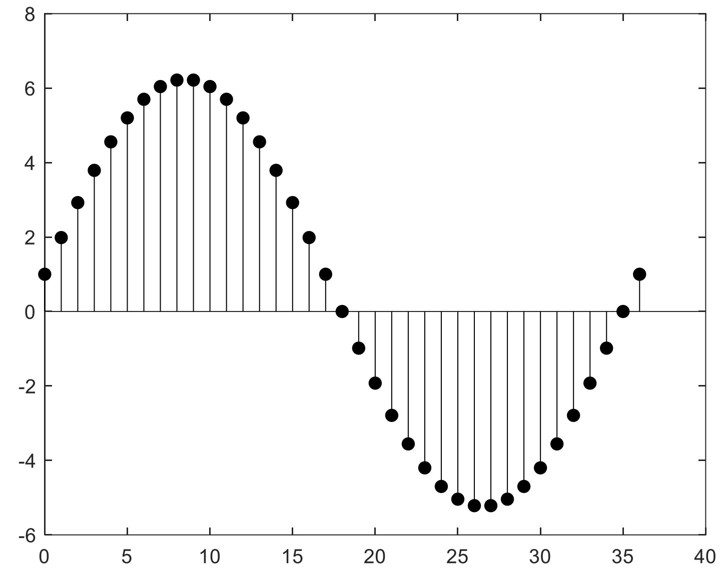


Forward Diff g[n]

DIFFERENCING AND ACCUMULATION



g[n]



Accumulation g[n]