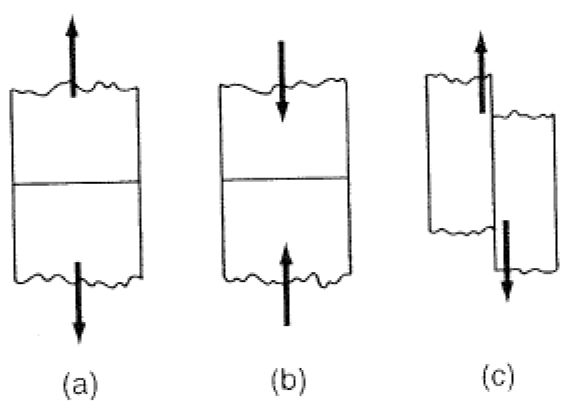
MENG 3050 - Power Transmission Components

Lesson 02-A-Stress and Deformation (review)

Objectives

- Review the types of stresses caused from axial, bending, shear, and torsion loading..
- Review the relationship between stresses in the part and the strength or stress-carrying ability of the part and begin to appreciate the relationship between the two.
- Distinguish between the ability of a material to carry loads in shear versus axial loading, and the relationship between these types of stresses.
- Review the principles of deformation and whether those levels of deformation are acceptable to the design being analyzed.
- Review beam deflection formulas and their use in design problems.
- Learn how to analyze columns using the Euler and Johnson methods.
- Review the meaning of factor of safety and its use in determining safe stress levels and/or allowable load limits and their relationship to the unknowns in the design process.

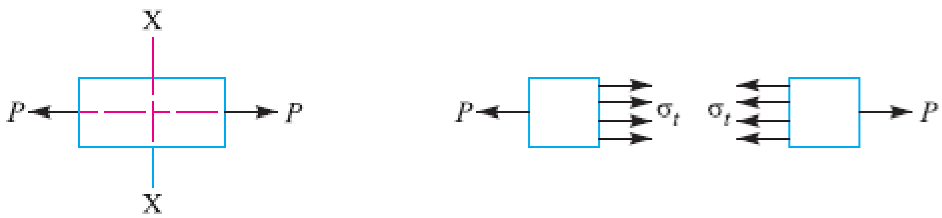
Tensile and Compressive Axial Loads

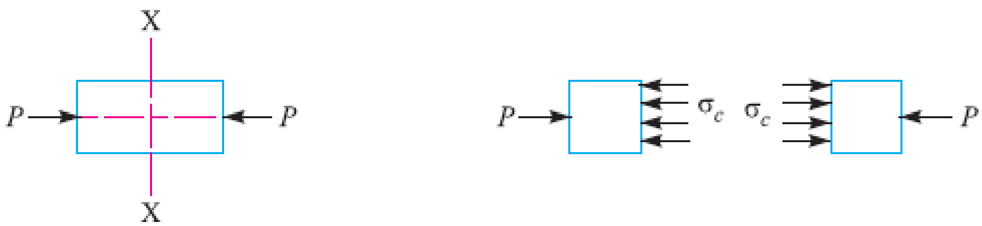


a) Tensile force: causes normal stresses
b) Compression force: causes normal stresses
c) Axial load: causes Sheer stresses

Tensile/ compression Stress and Strain:

When a body is subjected to two equal and opposite axial pulls/ push P (also called tensile/ compression load), then the stress induced at any section of the body is known as tensile/ compression stress,





Stress, $\sigma = P/A$

P = Force or load acting on a body, and

A = Cross-sectional area of the body.

Strain:

When a system of forces or loads act on a body, it undergoes some deformation. This deformation per unit length is known as unit strain or simply a strain. It is denoted by a Greek letter epsilon (ε). Mathematically

Strain, $\varepsilon = \delta l \,/\, l$ or $\delta l = \varepsilon . l$

δl = Change in length of the body, and

l = Original length of the body.

Young's Modulus or Modulus of Elasticity:

Hooke's law; states that when a material is loaded within elastic limit, the stress is directly proportional to strain

$$\sigma \propto \varepsilon \qquad \text{or} \qquad \sigma = E.\varepsilon$$

$$E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$$

$$\delta = \frac{FL}{AE}$$

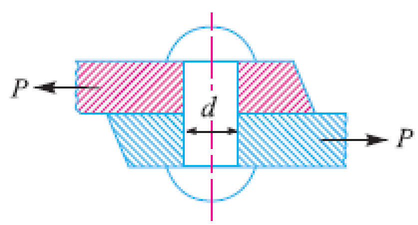
where δ = deflection

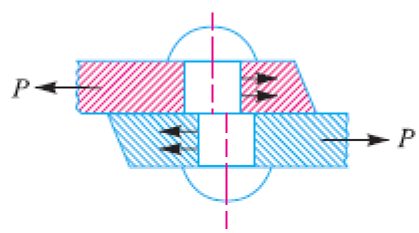
L = length

E = modulus of elasticity

Shear Stress and Strain

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called shear stress





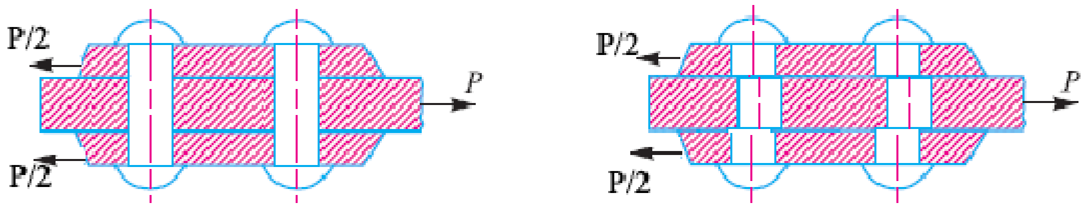
$$\text{Shear stress, } \tau = \frac{\text{Tangential force}}{\text{Resisting area}}$$

Consider a body consisting of two plates connected by a rivet as shown above: the shear stress on the rivet cross-section,

$$\tau = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{4P}{\pi d^2}$$

when the shearing takes place at two cross-sections of the rivet, then the rivets are said to be in double shear. In such a case, the area resisting the shear off the rivet,

$$\tau = \frac{P}{A} = \frac{P}{4 \times \frac{\pi}{4} \times d^2} = \frac{P}{\pi d^2}$$



Shear Modulus or Modulus of Rigidity:

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$$\tau \propto \phi \quad \text{or} \quad \tau = C \,.\, \phi \quad \text{or} \quad \tau / \phi = C$$

τ = Shear stress,

ϕ = Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by N or G

Torsion:

Any moment vector that is collinear with an axis of a mechanical part is called a torque vector, because the moment causes the part to be twisted about that axis. A bar subjected to such a moment is said to be in torsion.
As shown in Fig. 3–21, the torque T applied to a bar can be designated by drawing arrows on the surface of the bar to indicate direction or by drawing torque-vector arrows along the axes of twist of the bar. Torque vectors are the hollow arrows shown on the x axis in Fig. 3–21. Note that they conform to the right-hand rule for vectors.
The angle of twist, in radians, for a solid round bar is

$$\theta = \frac{Tl}{GJ}$$

where T = torque

l = length

G = modulus of rigidity

J = polar second moment of area

Shear stress develops throughout the cross section. For a round bar in torsion,
These stresses are proportional to the radius ρ and are given by

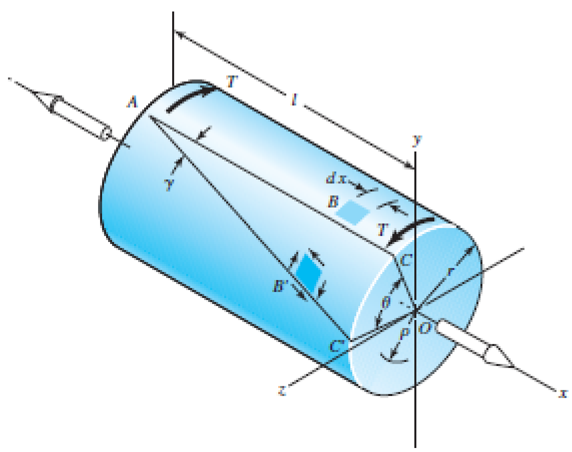
$$\tau = \frac{T\rho}{J}$$

Designating ρ as the radius to the outer surface, we have

$$\tau_{max} = \frac{Tr}{J}$$

The assumptions used in the analysis are:

- The bar is acted upon by a pure torque, and the sections under consideration are remote from the point of application of the load and from a change in diameter.
- The material obeys Hooke's law.
- Adjacent cross sections originally plane and parallel remain plane and parallel after twisting, and any radial line remains straight.



The last assumption depends upon the axisymmetric of the member, so it does not hold true for noncircular cross sections. apply *only* to circular sections. For a solid round section,

$$J = \frac{\pi d^4}{32}$$

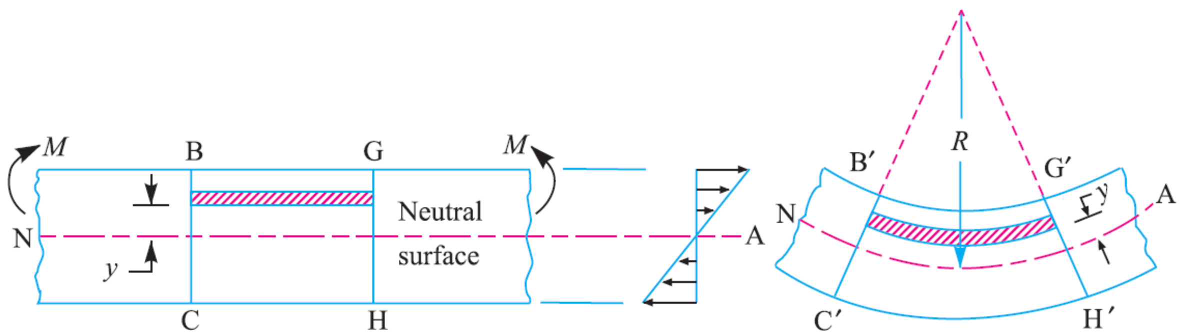
where *d* is the diameter of the bar. For a hollow round section,

$$J = \frac{\pi}{32}(d_o^4 - d_i^4)$$

Bending Stress in Straight Beams

Consider a straight beam subjected to a bending moment M. The following assumptions are usually made while deriving the bending formula.

1. The material of the beam is perfectly homogeneous
2. The material of the beam obeys Hooke's law.
3. The transverse sections (i.e. BC or GH) which were plane before bending, remain plane after bending also.
4. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.
5. The Young's modulus (E) is the same in tension and compression.
6. The loads are applied in the plane of bending.



The bending equation is given by:

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

M = Bending moment acting at the given section,

σ = Bending stress,

Where:

I= Moment of inertia of the cross-section about the neutral axis,

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

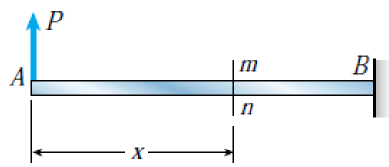
Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$$

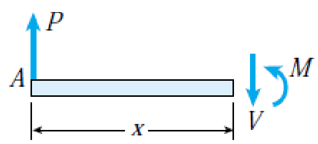
The ratio I/y is known as section modulus and is denoted by Z.

SHEAR FORCES AND BENDING MOMENTS:

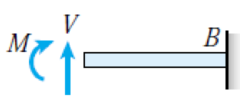
Method of section applied to a beam:
Shear force V and bending moment M in a beam:



(a)

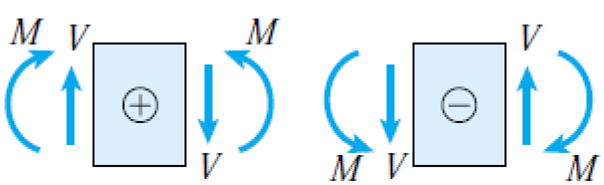


(b)

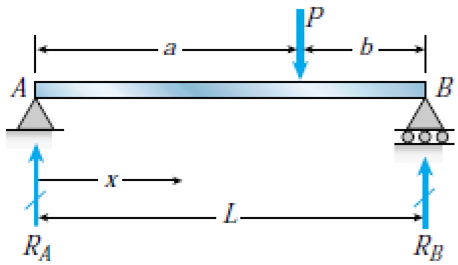


(c)

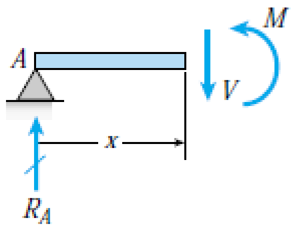
Sign convention for the Shear Force and Bending Moment Diagrams:



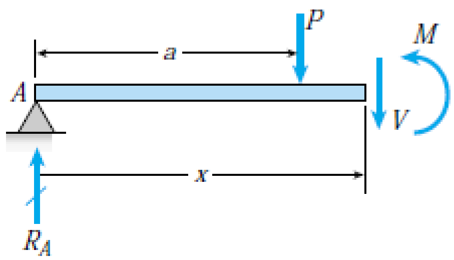
shear force (SF) is defined as the algebraic sum of all the vertical forces, either to the left or to the right-hand side of the section
Shear Force Diagram: is graph connecting Shear Forces at various locations on the beam


P
a
b
A
B
x
L
R_A
R_B

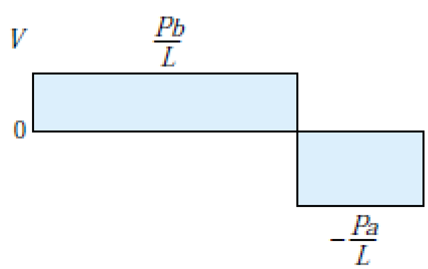
(a)


M
A
V
x
R_A

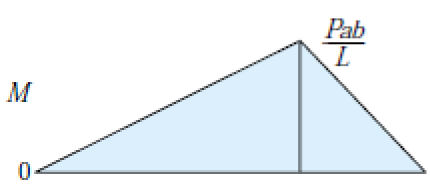
(b)


P
a
M
A
V
x
R_A

(c)

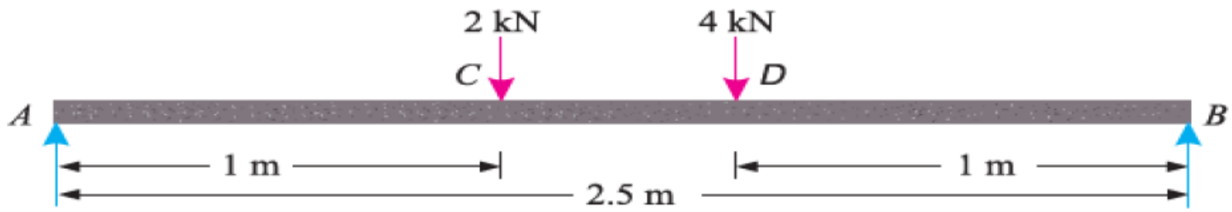

V
$\frac{Pb}{L}$
0
$-\frac{Pa}{L}$

(d)

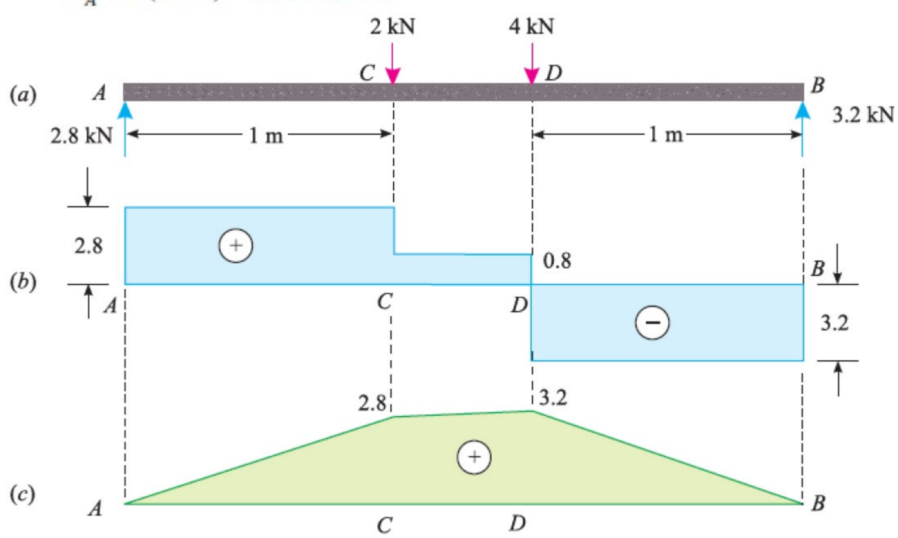

$\frac{Pab}{L}$
M
0

(e)

Example:

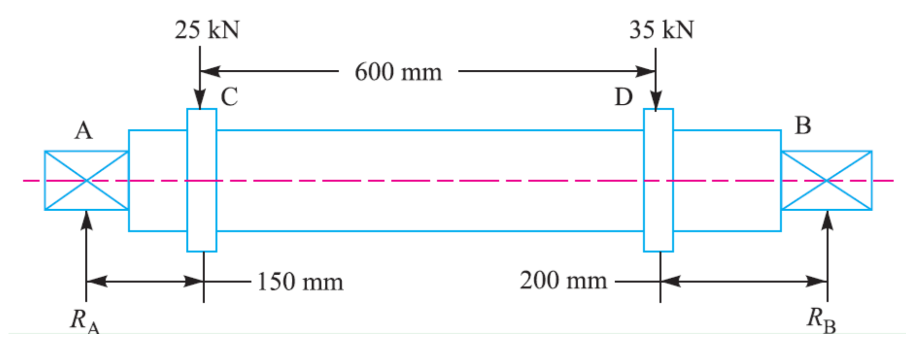


$$R_B \times 2.5 = (2 \times 1) + (4 \times 1.5) = 8$$
$$R_B = 8/2.5 = 3.2 \text{ kN}$$
$$R_A = (2 + 4) - 3.2 = 2.8 \text{ kN}$$



Example

A pump lever rocking shaft is shown. The pump lever exerts forces of 25 kN and 35 kN concentrated at 150 mm and 200 mm from the left and right hand bearing respectively Find the diameter of the central portion of the shaft if the stress is not to exceed 100 MPa.



Solution

$\sigma^b = 100$ MPa = 100 N/mm^2

Let RA and RB = Reactions at A and B respectively.

Taking moments about A, we have

RB × 950 = 35 × 750 + 25 × 150 = 30 000

∴ RB = 30 000 / 950 = 31.58 kN = 31.58 × 10^3 N

and

RA = (25 + 35) – 31.58 = 28.42 kN = 28.42 × 10^3 N

∴ Bending moment at C

= RA × 150 = 28.42 × 10^3 × 150 = 4.263 × 10^6 N-mm

and bending moment at D = RB × 200 = 31.58 × 10^3 × 200 = 6.316 × 10^6 N-mm

We see that the maximum bending moment is at D, therefore maximum bending moment,

M= 6.316 × 10^6 N-mm.

Let d = Diameter of the shaft.

∴ Section modulus

$$Z = \frac{\pi}{32} \times d^3$$
$$= 0.0982\, d^3$$

We know that bending stress (σ_b),

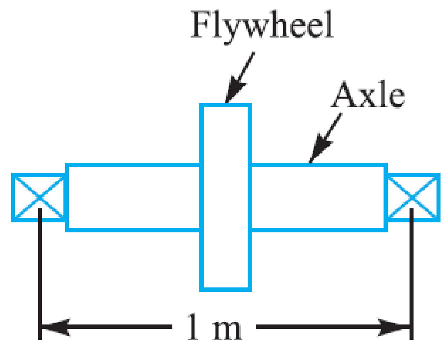
$$100 = \frac{M}{Z}$$

$$= \frac{6.316 \times 10^6}{0.0982\, d^3} = \frac{64.32 \times 10^6}{d^3}$$

$d^3 = 64.32 \times 10^6/100 = 643.2 \times 10^3$ or $d = 86.3$ say 90 mm

Example

An axle 1 m long supported in bearings at its ends carries a fly wheel weighing 30 kN at the center. If the stress (bending) is not to exceed 60 MPa, find the diameter of the axle,



Solution:

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$$

Maximum bending moment at the centre of the axle,

$$M = \frac{W.L}{4} = \frac{30 \times 10^3 \times 1000}{4} = 7.5 \times 10^6 \text{ N-mm}$$

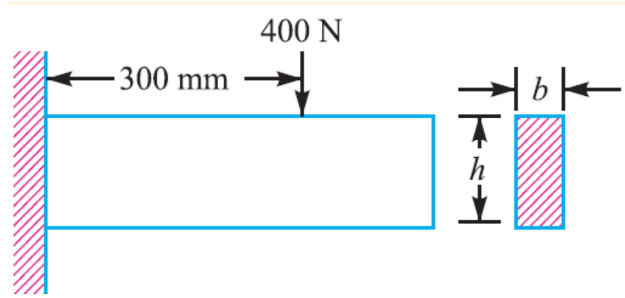
We know that bending stress (σ_b),

$$60 = \frac{M}{Z} = \frac{7.5 \times 10^6}{0.0982\, d^3} = \frac{76.4 \times 10^6}{d^3}$$

$d^3 = 76.4 \times 106/60 = 1.27 \times 10^6$ or $d = 108.3$ say 110 mm

Example

A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width



Solution:

Section modulus:

$$Z = \frac{b\,.\,h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3} \text{ mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3 \text{ N-mm}$$

We know that bending stress (σ_b),

$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2\,b^3} = \frac{180 \times 10^3}{b^3}$$

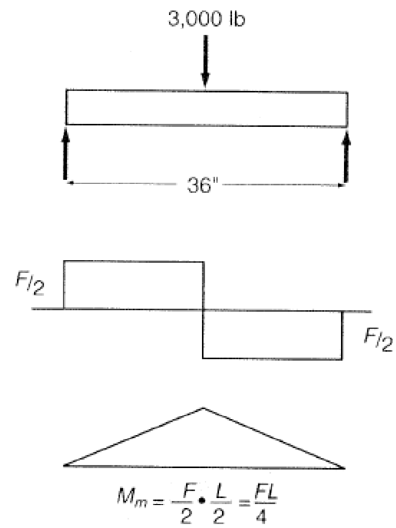
$\therefore$ $b^3 = 180 \times 10^3/40 = 4.5 \times 10^3$ or $b = 16.5$ mm

and $h = 2b = 2 \times 16.5 = 33$ mm **Ans.**

Example:

For the 2"x 2" simply supported beam made from the same 1020, assume a safety factor of 2 based on the ultimate stress allowable, ultimate stresses = 55ksi

What is the maximum stress in the beam?



Solution:

a-

$$I = \frac{bh^3}{12} = \frac{2\,in\,(2\,in)^3}{12} = 1.33\,in^4$$

$$M_m = \frac{FL}{4}$$

$$S = \frac{Mc}{I} = \frac{(FL)\,c}{(4)\,I}$$

$$S = 3{,}000\,lb\ \frac{36\,in\,1\,in}{4\,(1.33\,in)^4} = 20{,}300\ \ lb/in^2$$

b-

$$\delta = \frac{F\,L^3}{-48\ EI}$$

$$\delta = \frac{3000lb\,(36\ in)^3}{48\ (30x10^6)\ lb/in^2 1.33in^4} = -.073\ in$$

c- Yes, the stress of 20,300 lb/in^2 is less than the yield of S_y= 30,000 lb/in^2.

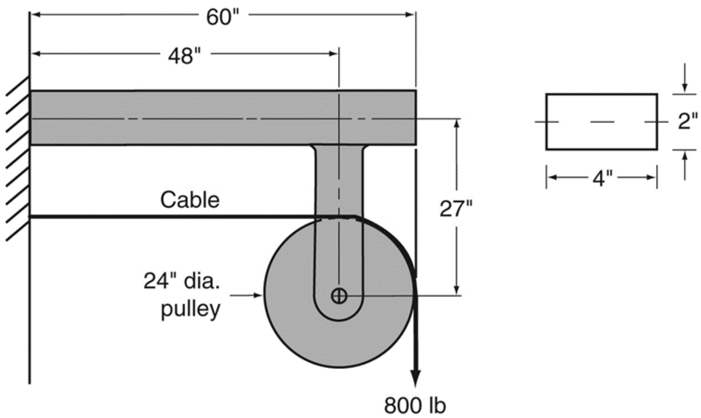
d-

$$\frac{S_u}{N} = \frac{55{,}000\,lb/in^2}{2} = 27{,}500\ lb/in^2$$

It is acceptable as this is less than the actual stress of 20,300 lb/in^2

Example:

- Using the figure below, in the cantilevered beam assembly that supports a pulley and cable, determine the maximum combined stress.
- Select the appropriate theorem based on the fact that this assembly is made from a low-strength ductile steel material.



The maximum stress is at the wall caused by a moment, couple, and axial load.

– The moment and couple can be added:

$$M_m = Fd + Fd$$

$$M_m = 800 \text{ lb } (48 \text{ in}) + 800 \text{ lb } (27 \text{ in})$$

$$M_m = 60{,}000 \text{ in-lb}$$

– Stress from bending:

$$S = \frac{M}{Z} \qquad Z = \frac{b\,h^2}{6}$$

$$S = \frac{60{,}000 \text{ in-lb}}{2.67 \text{ in}^3} \qquad Z = \frac{4 \text{ in } (2 \text{ in})^2}{6}$$

$$S = 22{,}500 \text{ lb/in}^2 \qquad Z = 2.67 \text{ in}^3$$

– Axial stress:

$$S = \frac{F}{A} = \frac{800 \text{ lb}}{2 \text{ in} \cdot 4 \text{ in}}$$

$$S = 100 \text{ lb/in}^2$$

$$S = S_B + S_A$$

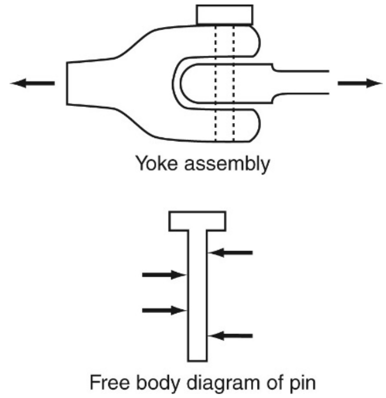
$$S = 22{,}500 + 100$$

$$S = 22{,}600 \text{ lb/in}^2$$

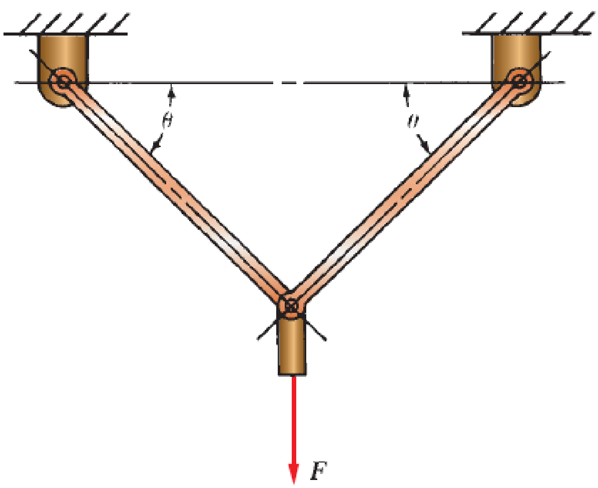
As the bending stress was so much larger than the axial stress, the axial stress could have been ignored.

Example:

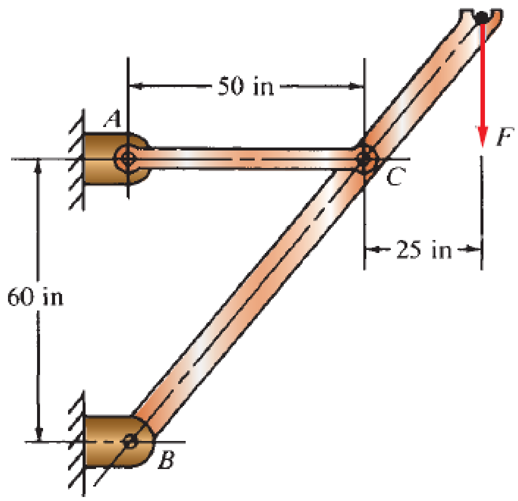
The yoke shown has a ½-inch diameter pin made from AISI 1040 cold drawn steel. (Conceder shear force)
For a load of 20,000 pounds, will this fail? Would it exceed the ultimate stresses?, assume ultimate stress = 55 ksi, yield stress = 27 ksi., ultimate shear force = 42 ksi



ompute the forces in the two angled rods for an applied force, F = 1500 lb, if the angle u is 45°. If the rods from are circular, determine their required diameter if the load is static and the allowable stress is 18 000 psi.



Compute the stress in the middle portion of rod AC if the vertical force on the boom is 2500 lb. The rod is rectangular, 1.50 in by 3.50 in.
Each of the pins at A, B, and C has a diameter of 0.50 in and is loaded in double shear. Compute the shear stress in each pin.
Compute the maximum tensile and compressive stresses in the member B-C if the load F is 1800 lb. The cross section of B-C is a HSS 6x4x1/4 rectangular tube.


50 in
A
F
C
25 in
60 in
B