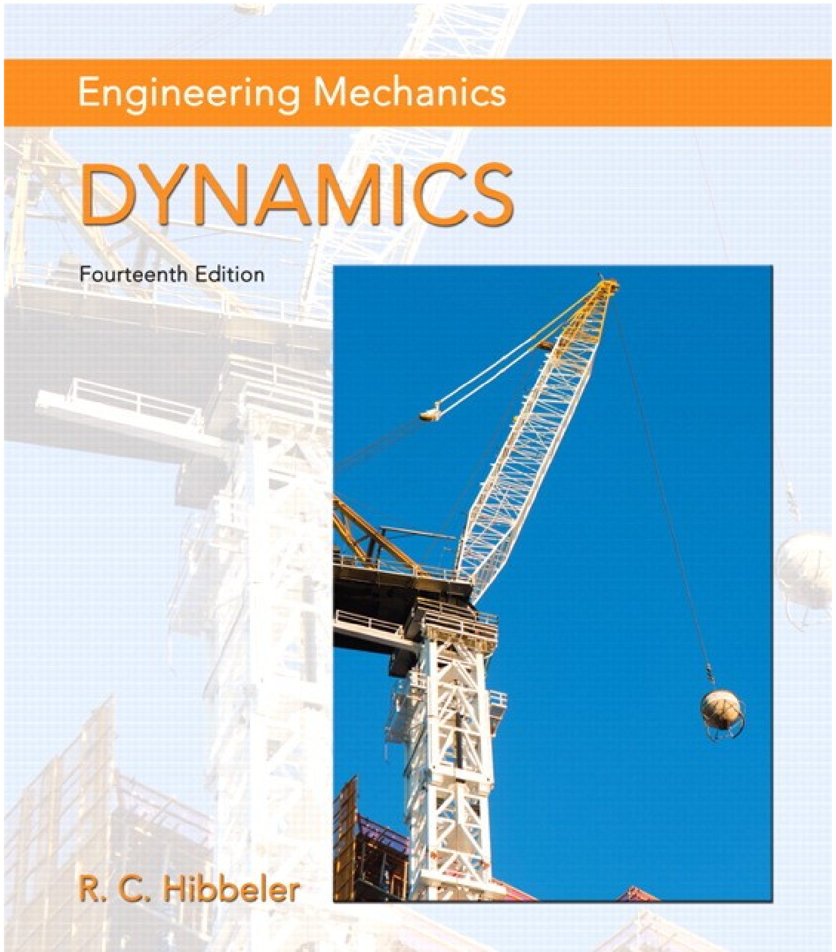
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 15

Kinetics of a Particle:
Impulse and
Momentum
Chapter Objectives

The Work of A Force, The Principle of Work And Energy & Systems of Particles (1 of 2)

Today's Objectives:

Students will be able to:

1. Calculate the work of a force.
2. Apply the principle of work and energy to a particle or system of particles.

The Work of A Force, The Principle of Work And Energy & Systems of Particles (2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- **Linear Momentum and Impulse**
- **Principle of Linear Impulse and Momentum**
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1) The linear impulse and momentum equation is obtained by integrating the _______ with respect to time.

   A) friction force
   B) equation of motion
   C) kinetic energy
   D) potential energy

2) Which parameter is not involved in the linear impulse and momentum equation?

   A) Velocity
   B) Displacement
   C) Time
   D) Force

Applications (1 of 3)

A dent in an trailer fender can be removed using an impulse tool, which delivers a force over a very short time interval. To do so, the weight is gripped and jerked upwards, striking the stop ring.

How can we determine the magnitude of the linear impulse applied to the fender?

Could you analyze a carpenter's hammer striking a nail in the same fashion?

Sure!

Applications (2 of 3)

A good example of impulse is the action of hitting a ball with a bat.

The impulse is the average force exerted by the bat multiplied by the time the bat and ball are in contact.

Is the impulse a vector? Is the impulse pointing in the same direction as the force being applied?

Given the situation of hitting a ball, how can we predict the resultant motion of the ball?

Applications (3 of 3)

When a stake is struck by a sledgehammer, a large impulse force is delivered to the stake and drives it into the ground.

If we know the initial speed of the sledgehammer and the duration of impact, how can we determine the magnitude of the impulsive force delivered to the stake?

Section 15.1

Principle Of Linear Impulse And Momentum

Principle of Linear Impulse And Momentum (1 of 4)

The next method we will consider for solving particle kinetics problems is obtained by **integrating the equation of motion with respect to time.**

The result is referred to as the **principle of impulse and momentum**. It can be applied to problems involving both linear and angular motion.

This principle is useful for solving problems that involve **force**, **velocity**, and **time**. It can also be used to analyze the mechanics of **impact** (taken up in a later section).

Principle of Linear Impulse And Momentum (2 of 4)

The **principle of linear impulse and momentum** is obtained by integrating the equation of motion with respect to time. The equation of motion can be written

$$\sum F = m\,a = m(dv/dt)$$

Separating variables and integrating between the limits

$v = v_1$ at $t = t_1$ and $v = v_2$ at $t = t_2$ results in

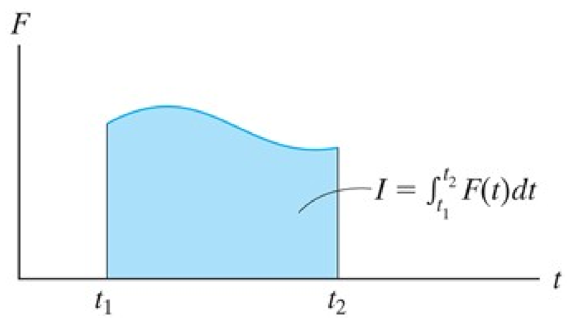
$$\sum \int_{t_1}^{t_2} F\,dt = m\int_{v_1}^{v_2} dv = mv_2 - mv_1$$

This equation represents the principle of linear impulse and momentum. It relates the particle's final velocity (v_2) and initial velocity (v_1) and the forces acting on the particle as a function of time.

Principle of Linear Impulse And Momentum (3 of 4)

Linear momentum: The vector mv is called the linear momentum, denoted as *L*. This **vector** has the **same direction** as v. The linear momentum vector has units of $(kg \cdot m)/s$ or $(slug \cdot ft)/s$.

Linear impulse: The integral $\int F\, dt$ is the linear impulse, denoted I. It is a **vector quantity** measuring the effect of a force during its time interval of action. I acts in the **same direction** as F and has units of $N \cdot s$ or $lb \cdot s$.



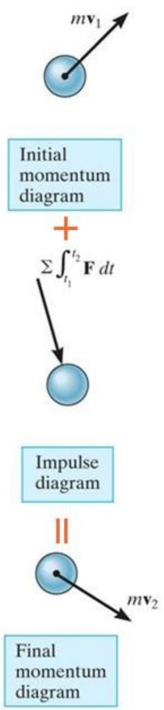
The impulse may be determined by **direct integration**. Graphically, it can be represented by the **area under the force versus time curve**. If *F* is constant, then

$$I = F(t_2 - t_1)$$

Principle of Linear Impulse And Momentum (4 of 4)

The principle of linear impulse and momentum in **vector** form is written as

$$mv_1 + \sum \int_{t_1}^{t_2} F \, dt = mv_2$$



The particle's initial momentum plus the sum of all the impulses applied from t_1 *to* t_2 is equal to the particle's final momentum.

The two **momentum diagrams** indicate direction and magnitude of the particle's initial and final momentum, mv_1 *and* mv_2. The **impulse diagram** is similar to a free body diagram, but includes the time duration of the forces acting on the particle.

Impulse And Momentum: Scalar Equations

Since the principle of linear impulse and momentum is a vector equation, it can be resolved into its x, y, z component **scalar equations**:

$$m(v_x) + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

$$m(v_z)_1 + \sum \int_{t_1}^{t_2} F_z \, dt = m(v_z)_2$$

The scalar equations provide a convenient means for applying the principle of linear impulse and momentum once the velocity and force vectors have been resolved into x, y, z components.

Problem Solving

Establish the x, y, z **coordinate system.**

Draw the particle's **free body diagram** and establish the **direction** of the particle's initial and final **velocities**, drawing the **impulse and momentum diagrams** for the particle. Show the linear momenta and force impulse vectors.

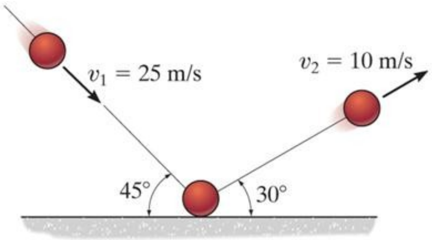
Resolve the force and velocity (or impulse and momentum) vectors into their **x, y, z components**, and apply the **principle of linear impulse and momentum** using its scalar form.

Forces as **functions of time** must be **integrated** to obtain impulses. If a force is constant, its impulse is the product of the force's magnitude and time interval over which it acts.

Example I (1 of 3)

Given: A 0.5 kilogram ball strikes the rough ground and rebounds with the velocities shown. Neglect the ball's weight during the time it impacts the ground.

Find: The magnitude of impulsive force exerted on the ball.



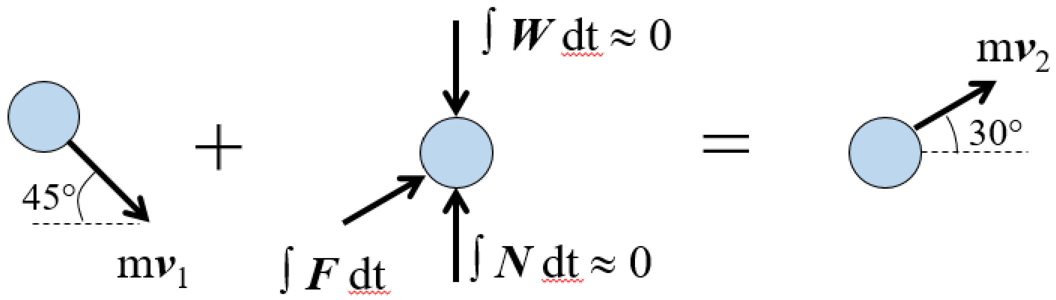
Plan:

1) Draw **momentum and impulse diagrams** of the ball as it hits the surface.
2) Apply the principle of impulse and momentum to determine the impulsive force.

Example I (2 of 3)

Solution:

1) **Impulse and momentum diagrams** can be drawn as:



The impulse caused by the ball's weight and the normal force ***N*** can be neglected because their magnitudes are very small as compared to the impulse from the ground.

Example I (3 of 3)

2) **The principle of impulse and momentum** can be applied along the direction of motion:

$$mv_1 + \sum \int_{t_1}^{t_2} F\,dt = mv_2$$

$$\Rightarrow 0.5(25\cos 45^\circ i - 25\sin 45^\circ j) + \int_{t_1}^{t_2} \sum F\,dt$$

$$= 0.5(10\cos 30^\circ i + 10\sin 30^\circ j)$$

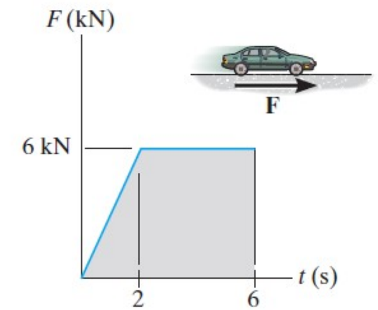
The impulsive force vector is

$$I = \int_{t_1}^{t_2} \sum F dt = (4.509i + 11.34j) N.s$$

Magnitude: $I = \sqrt{4.509^2 + 11.34^2} = 12.2 N.s$

Example II (1 of 3)

Given: The wheels of the 1500 kilogram car generate a traction force **F** described by the graph.



Find: The speed of the car when t = 6 s if the car starts from rest.

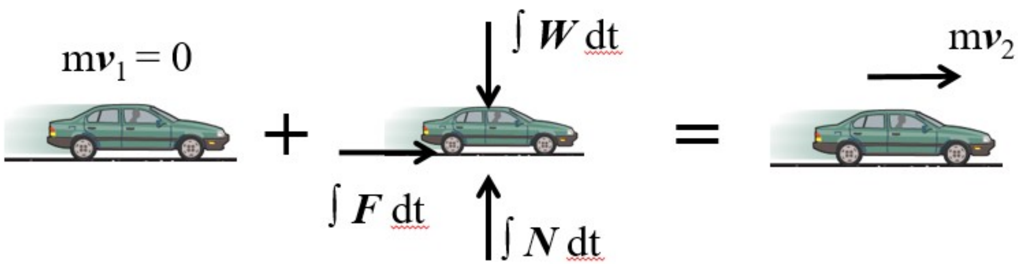
Plan:

1) Draw the **momentum and impulse diagrams** of the car.
2) Apply the principle of impulse and momentum to determine the speed.

Example II (2 of 3)

Solution:

1) **The impulse and momentum diagrams** can be drawn as:



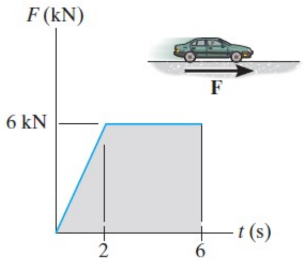
Why is the traction force ***F*** drawn acting to the right?

The impulse caused by the weight ***W*** and the normal force ***N*** can be canceled. Why?

Example II (3 of 3)

2) The principle of impulse and momentum can be applied along the direction of motion:

$$\xrightarrow{+} \quad mv_1 + \sum \int_{t_1}^{t_2} F\,dt = mv_2$$



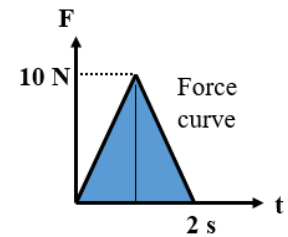
$$(1500\,kg)(0\,\text{m/s}) + 0.5(6000\,\text{N})(2\,\text{s}) + (6000\,\text{N})(4\,\text{s})$$
$$= (1500\,\text{kg})(v_2\, m/s)$$

$$v_2 = 20\, m/s$$

Pearson

Concept Quiz

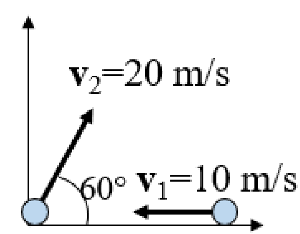
1) Calculate the impulse due to the force.



A) $20\ kg{\cdot}m/s$ B) $10\ kg{\cdot}m/s$

C) $5\ N{\cdot}s$ D) $15\ N{\cdot}s$

2) A constant force **F** is applied for 2 s to change the particle's velocity from v_1 to v_2. Determine the force **F** if the particle's mass is $2\ kg$.



A) $(17.3j)\,N$ B) $(-10i+17.3j)\ N$

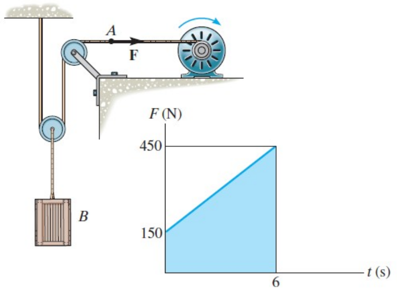
C) $(20i+17.3j)\,N$ D) $(10i+17.3j)\,N$

Group Problem Solving (1 of 3)

Given: The 40 kilogram crate is moving downward at $10\ m/s$.

The motor M pulls on the cable with a force of **F,** which has a magnitude that varies as shown on the graph.

Find: The speed of the crate when $t = 6s$.



Plan:

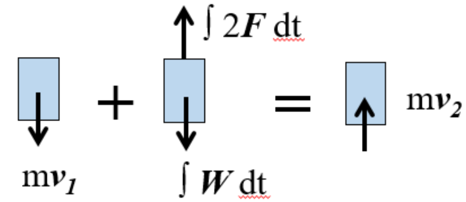
1) Draw the **momentum and impulse diagrams** of the crate.

2) Apply the principle of impulse and momentum to determine the speed.

Group Problem Solving (2 of 3)

Solution

The **momentum and impulse diagrams are**

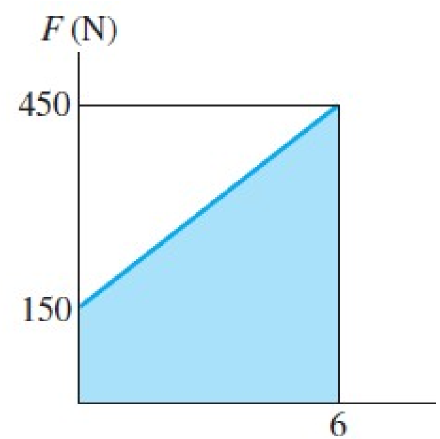


The impulse due to cable force is:

$$+\rightarrow \int_0^6 2F\,dt = 2[0.5(150+450)6] = 3600\,N.s$$

The impulse due to weight is:

$$+\rightarrow \int_0^6 (-W)\,dt = -40(9.81)(6) = -2354\,N.s$$



Group Problem Solving (3 of 3)

2) Apply the principle of impulse and momentum to determine the velocity.

$$mv_1 + \sum \int_{t_1}^{t_2} F\,dt = mv_2$$

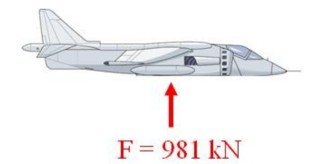
$$+\uparrow \quad mv_1 - \int_0^6 W\,dt + \int_0^6 2F\,dt = mv_2$$

$$40(-10) - 2354 + 3600 = 40v_2$$

$$\Rightarrow \; v_2 = 21.1\, m/s$$

Attention Quiz

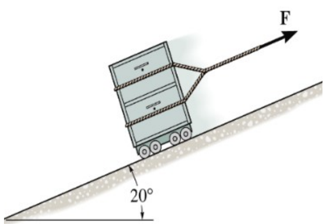
1) Jet engines on the $100\ Mg$ VTO L aircraft exert a constant vertical force of $981 kN$ as it hovers. Determine the net impulse on the aircraft over $t = 10\ s$.



A) $-981\ kN{\cdot}s$ B) $0\ kN{\cdot}s$

C) $981\ kN{\cdot}s$ D) $9810\ kN{\cdot}s$

2) A $100\ lb$ cabinet is placed on a smooth surface. If a force of $100\ lb$ is applied for $2\ s$, determine the net impulse on the cabinet during this time interval.



A) $0\ lb{\cdot}s$ B) $100\ lb{\cdot}s$

C) $200\ lb{\cdot}s$ D) $300\ lb{\cdot}s$

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