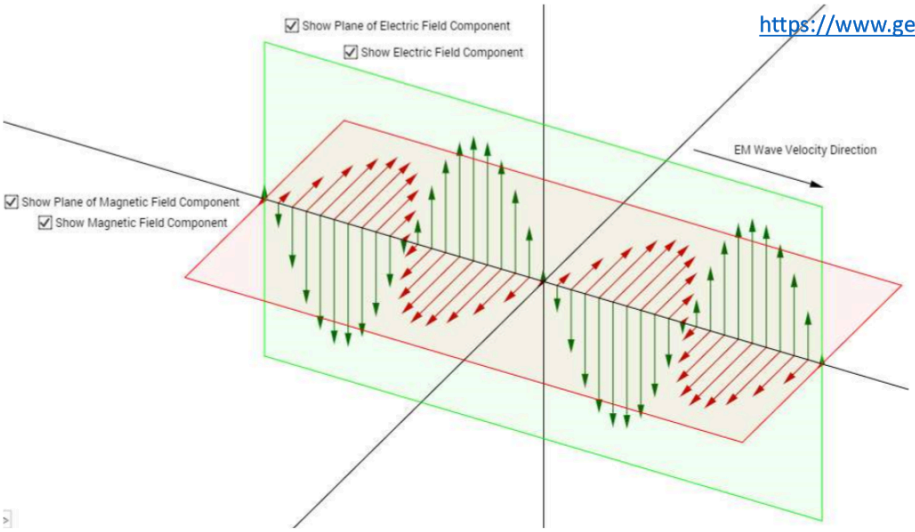
Class Note 1.1

September 8, 2022 9:48 PM



https://www.geogebra.org/m/xhYwXSsH

mathematical model
3D coordinate system
vector

sine function(period, amplitude)
equations

Fundamental Principles of Calculus

- Discovered independently by Isaac Newton (English) and Gottfried Leibniz (German) in late 17th century;
- Motivated by 4 major classes of scientific problems of the time:
  - Find the tangent line to a general curve at a given point
  - Find the area of a general region, the length of a general curve, and the volume of a general solid.
  - Find the maximum or minimum value of a quantity (optimization).

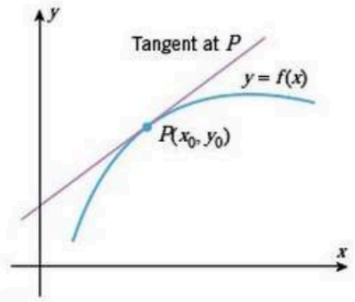
- https://www.maa.org/press/periodicals/convergence/historical-activities-for-calculus-module-3-optimization-galileo-and-the-brachistochrone-problem
- Distance traveled by a body – velocity – acceleration of the body at any instant.

- Calculus provides **efficient computational methods**

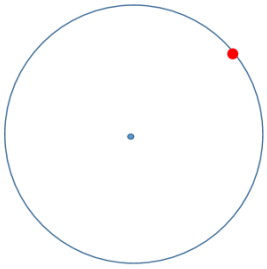
CALCULUS: Differential and Integral

The TANGENT LINE problem

Given a function f and a point $P(x_0, y_0)$ on its graph, find an equation of the line that is tangent to the graph at point P



Tangent to a Circle



Differential

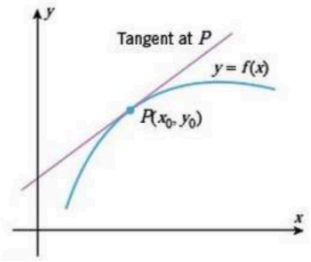
The TANGENT LINE problem

Integral

The AREA problem

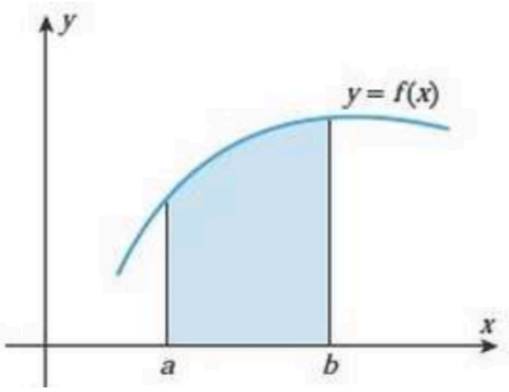
Given a function f, find the area of a

Given a function f and a point $P(x_0, y_0)$ on its graph, find an equation of the line that is tangent to the graph at point P



RATE OF CHANGE

Given a function f, find the area of a shaded region bounded by the graph of f and an interval $[a, b]$ on the x-axis



Fundamental Building Blocks

Function

G.Leibniz, 1673; L.Euler

- Definition, Notation
- Dependent and Independent Variables
- Domain and Range
- Function is represented
  - Numerically by data tables
  - Algebraically by formulas/equations
  - Geometrically by graphs
  - Verbally
- Linear, Polynomial, Rational, Power, Exponential, Logarithmic, Trigonometric , Composite.

- Limits
- Derivatives
- Integrals
- Applications

Functions

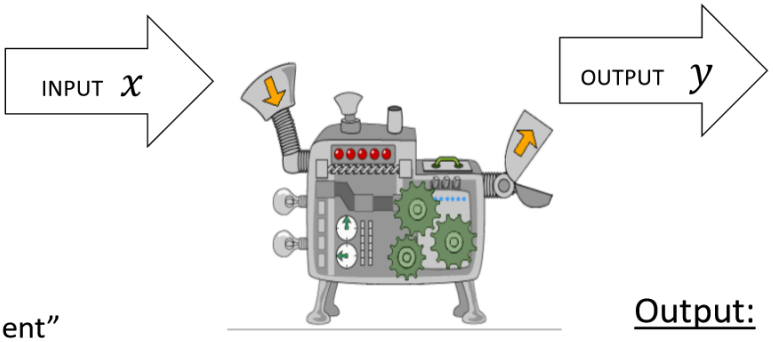
- Term “function” is used to indicate the dependence of one quantity on another.
- If a variable y depends on a variable x in such way that each value of x determines exactly one value of y, then we say that y is a function of x.

$$y = f(x)$$

- Another way: a function is a rule that associates

a unique **output**(y) with each **input** (x).

Function “machine” or “computer program”



Input:
- aka “argument”
- Independent variable
- Displayed on x-axis
- Associated with DOMAIN

Output:
- Returned value
- Dependent variable
- Displayed on y-axis
- Associated with RANGE

https://www.geogebra.org/m/rmqjtbxS

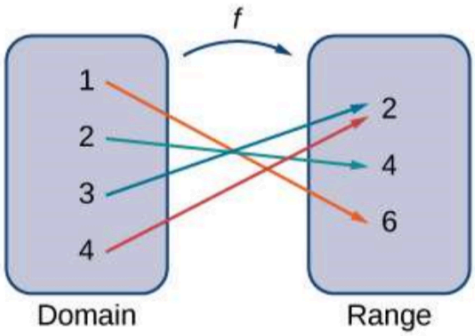
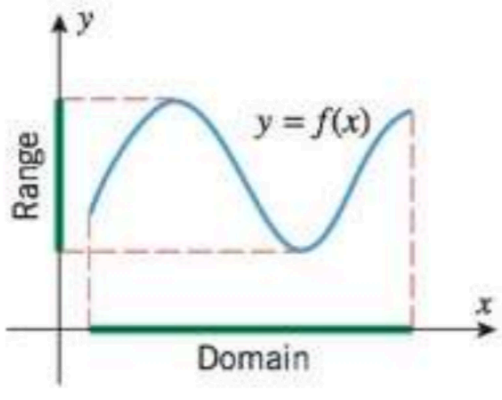


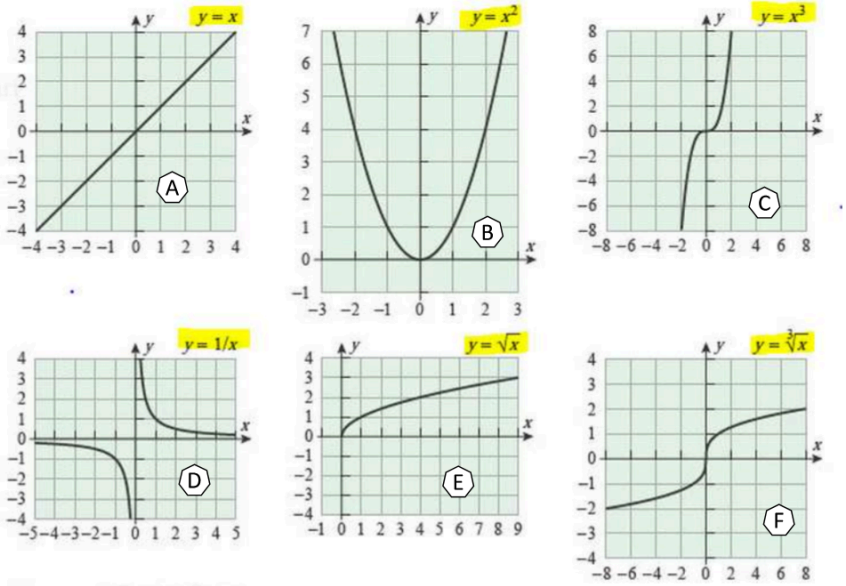
Figure 1.3 A function maps every element in the domain to exactly one element in the range. Although each input can be sent to only one output, two different inputs can be sent to the same output.

https://openstax.org/books/calculus-volume-1/pages/1-1-review-of-functions

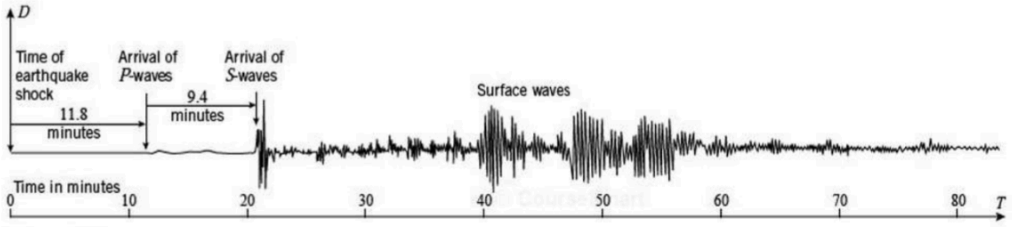


▲ **Figure 0.1.12** The projection of $y = f(x)$ on the x-axis is the set of allowable x-values for f, and the projection on the y-axis is the set of corresponding y-values.

Graphs of Functions
(visualizing the algebraic equation)

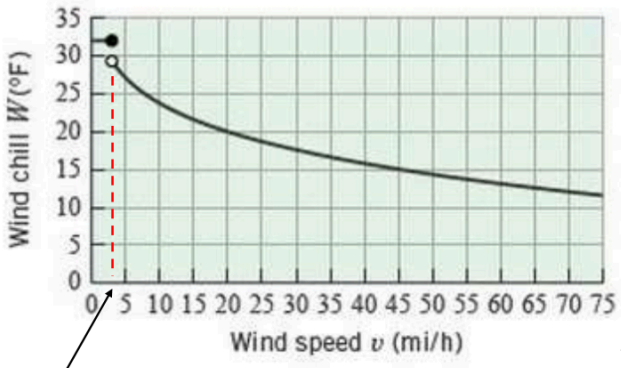


Graphs of Functions
(empirical, based on experimental data)



Deflection of the seismograph needle as a function of time elapsed since the wave left the earthquake's epicenter.

Graphs of Functions
(empirical, based on experimental data)



$v = 3(mi/h)$ is a BREAKPOINT

$$W = \begin{cases} 32, & 0 \le v \le 3 \\ 55.628 - 22.07v^{0.16}, & 3 < v \end{cases}$$

W(0) =

W(3) =

W(20) =

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The wind chill index measures the sensation of coldness that we feel from the combined effect of temperature and wind speed.

Limits

- The concept of a “limit” is the fundamental building block on which all calculus concepts are based
- Intuitive approach. “Terminal velocity” example.

Air resistance, “drag force”, prevents the velocity of a skydiver from increasing indefinitely. The velocity approaches a limit, called the “terminal velocity”.

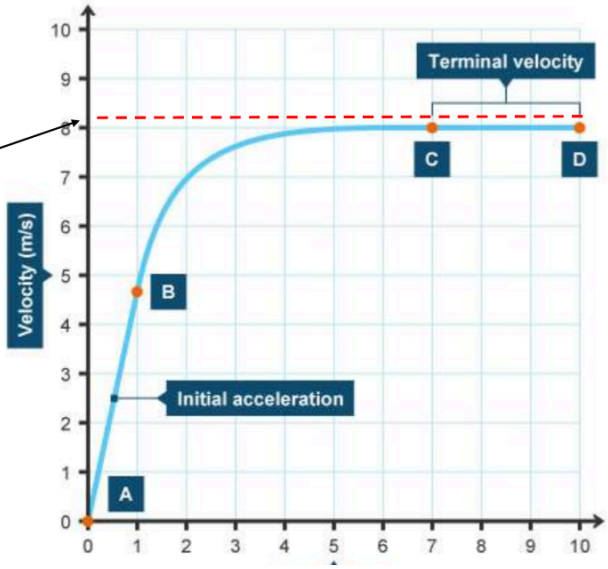
“Terminal velocity” is a limiting value of the velocity of a falling object.

Horizontal asymptote

$$V_t = \sqrt{\frac{2mg}{\rho A C_d}}$$

where

- V_t is terminal velocity,
- m is the mass of the falling object,
- g is the acceleration due to gravity,
- C_d is the drag coefficient,
- ρ is the density of the fluid through which the object is falling,

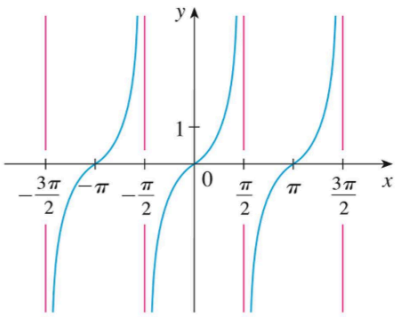


• A is the projected area of the object.

Time (s)

Domain and Vertical Asymptotes

$f(x) = \tan x$; the lines
$x = (2k + 1)\pi/2$ are vertical asymptotes for any integer number *k*.



$f(x) = \ln x$: the line x = 0 is a vertical asymptote