Worksheet 8 – Solution

1) Find the frequency response function $G(j\omega)$ and all the corner frequencies of poles and zeros, then plot the Bode diagram of the following system transfer functions in a semi-log graph.

a) $G(s) = 5$

Find the frequency response function, basic factors, and corner frequencies.

$G(s) = 5 \quad \rightarrow \quad G(j\omega) = 5$

The basic factor is:

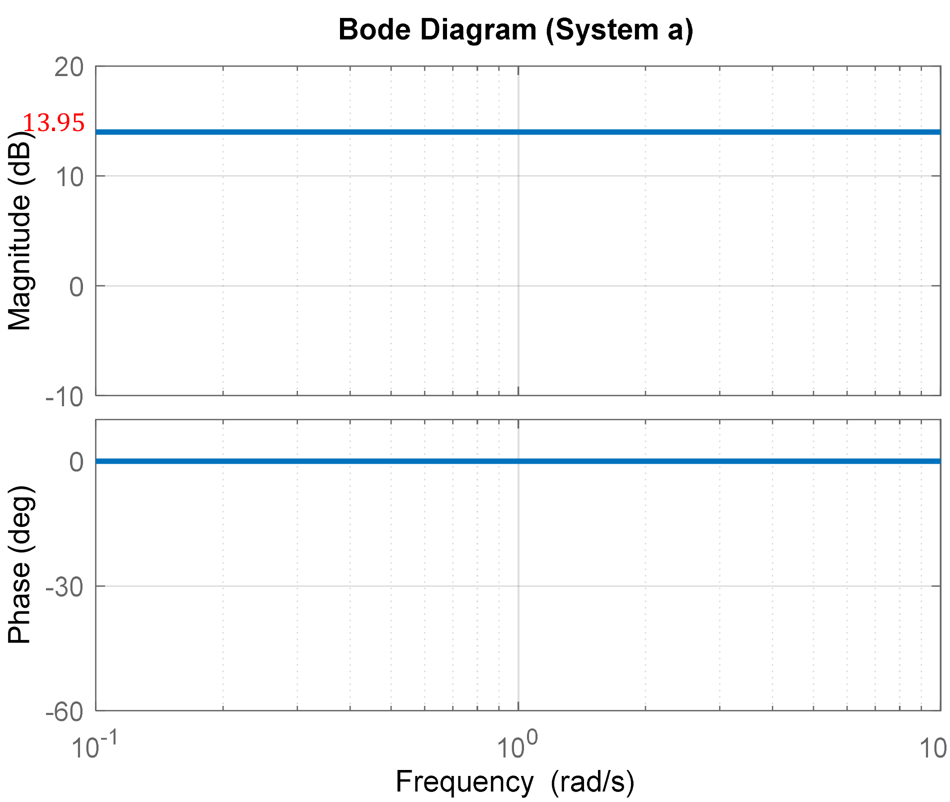
constant gain of 5

Magnitude:

$20\log|G(j\omega)| = 20\log|5| = 13.95\text{dB}$

Phase Angle:

$\angle G(j\omega) = \tan^{-1}(\frac{0}{5}) = 0$ deg



b) $\boldsymbol{G(s) = \frac{1}{4}}$

Find the frequency response function, basic factors, and corner frequencies.

$$G(s) = \frac{1}{4} \quad \rightarrow \quad G(j\omega) = \frac{1}{4}$$

The basic factor is:

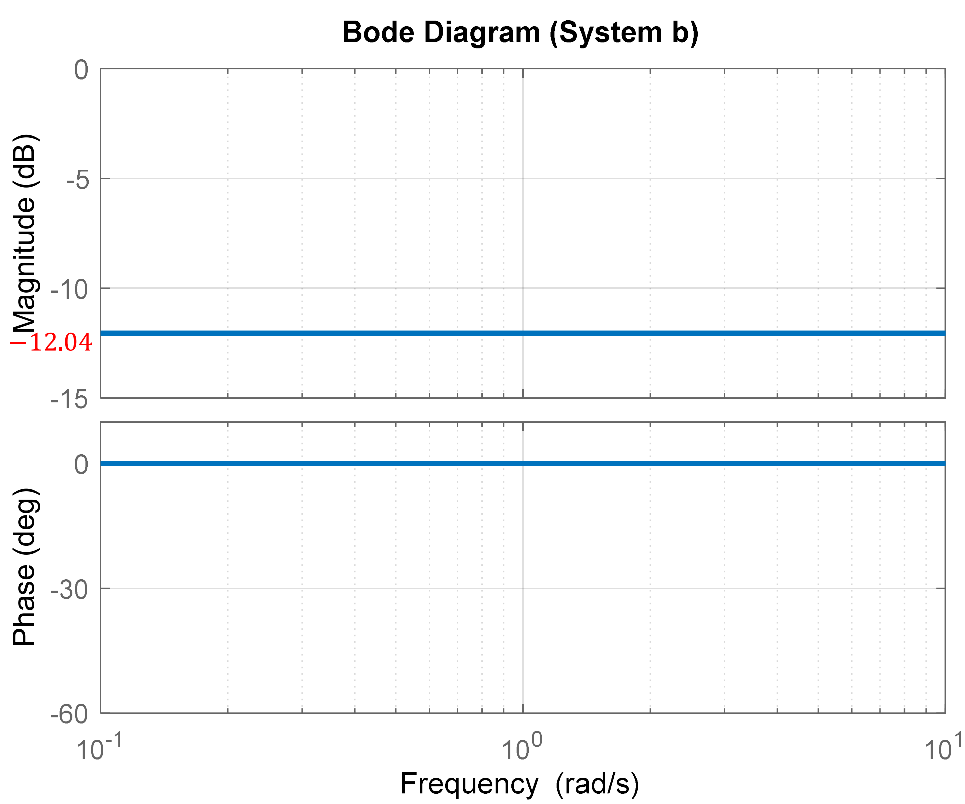
constant gain of 1/4

Magnitude:

$20\log|G(j\omega)| = 20\log|1/4| = -12.04\text{dB}$

Phase Angle:

$\angle G(j\omega) = \tan^{-1}(\frac{0}{1/4}) = 0$ deg



c) $\boldsymbol{G(s) = \frac{1}{4s+1}}$

Find the frequency response function, basic factors, and corner frequencies.

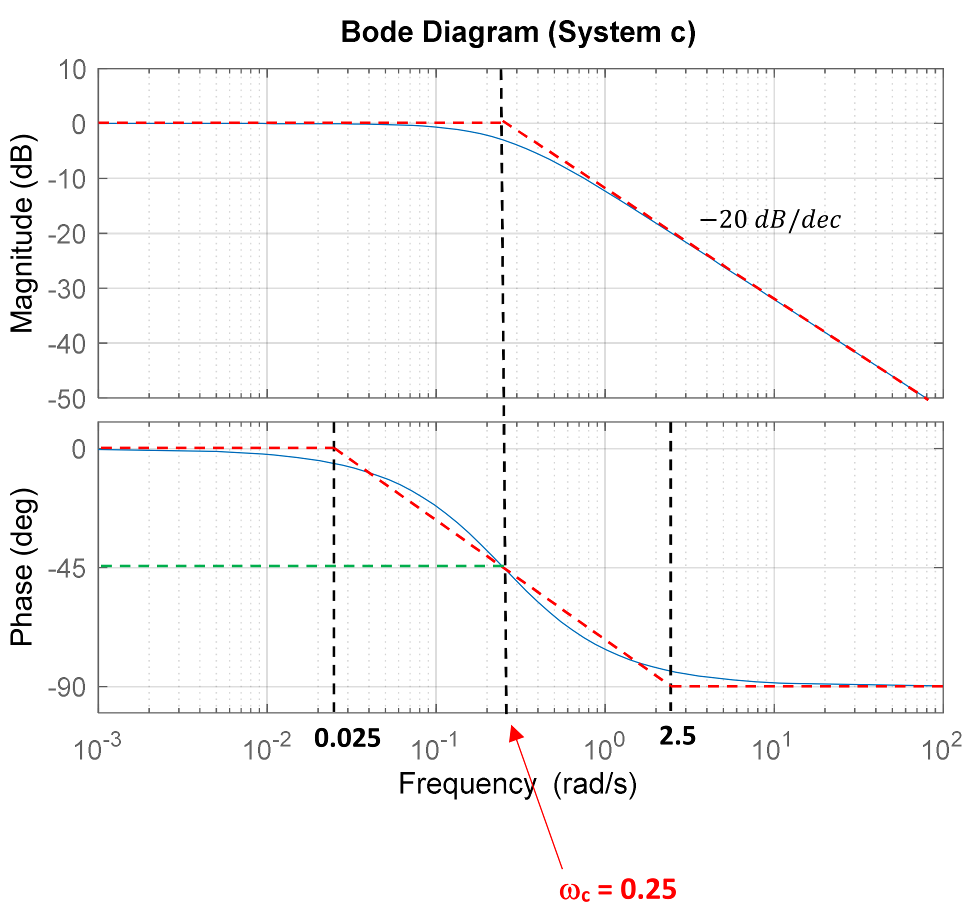
$$G(s) = \frac{1}{4s+1} \quad \rightarrow \quad G(j\omega) = \frac{1}{j4\omega + 1}$$

The basic factor is:

first-order pole with corner frequency of $\omega_c = \frac{1}{4}$

Starting Slope: $-20\beta \frac{dB}{dec} = -20(0)\ \frac{dB}{dec} = 0\ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{1}{(j0.001)^0}\right| = 20\log(1) - 20\log(1) = 0\ dB$



d) $G(s) = \frac{5}{4s+1}$

Find the frequency response function, basic factors, and corner frequencies.

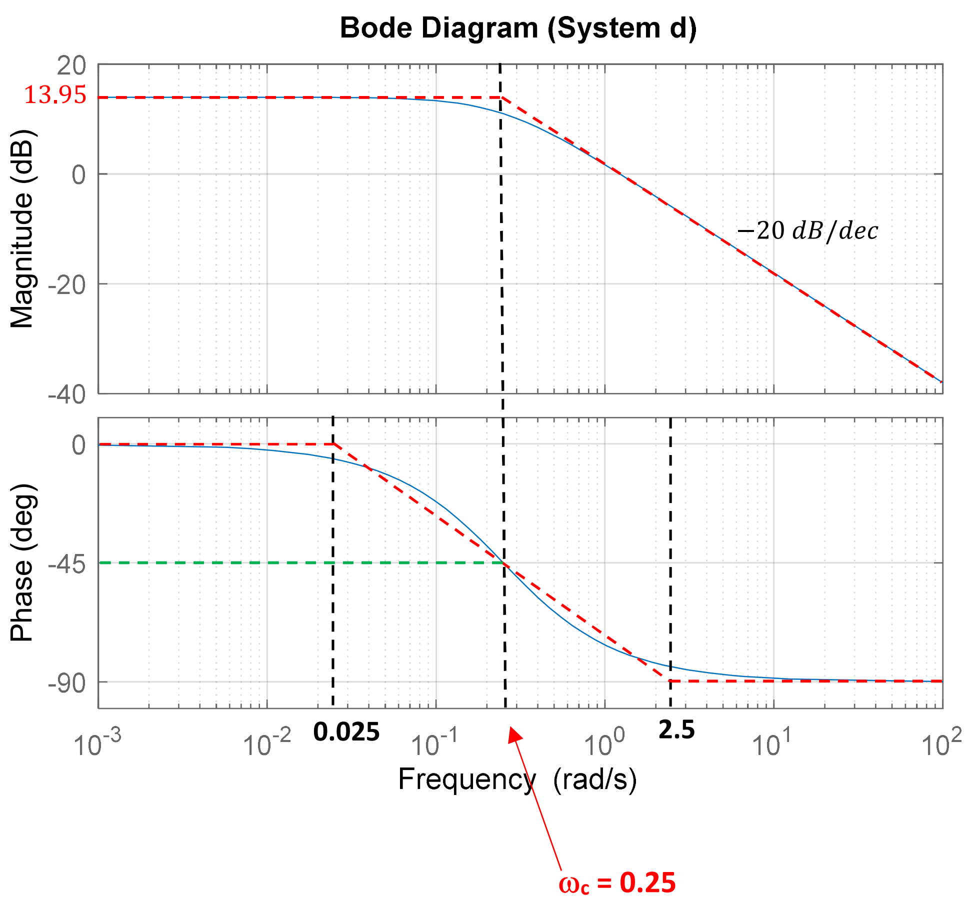
$$G(s) = \frac{5}{4s+1} = (5)\left(\frac{1}{4s+1}\right) \quad \rightarrow \quad G(j\omega) = (5)\left(\frac{1}{j4\omega+1}\right)$$

The basic factors are:

1) constant gain of 5 $\rightarrow$ 20log(5) = 13.95dB
2) first-order pole with corner frequency of $\omega_c = \frac{1}{4}$

Starting Slope: $-20\beta \frac{dB}{dec} = -20(0)\ \frac{dB}{dec} = 0\ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{5}{(j0.001)^0}\right| = 20\log(5) - 20\log(1) = 13.95\ dB$



e) $G(s) = \frac{3}{s+2}$

Find the frequency response function, basic factors, and corner frequencies.

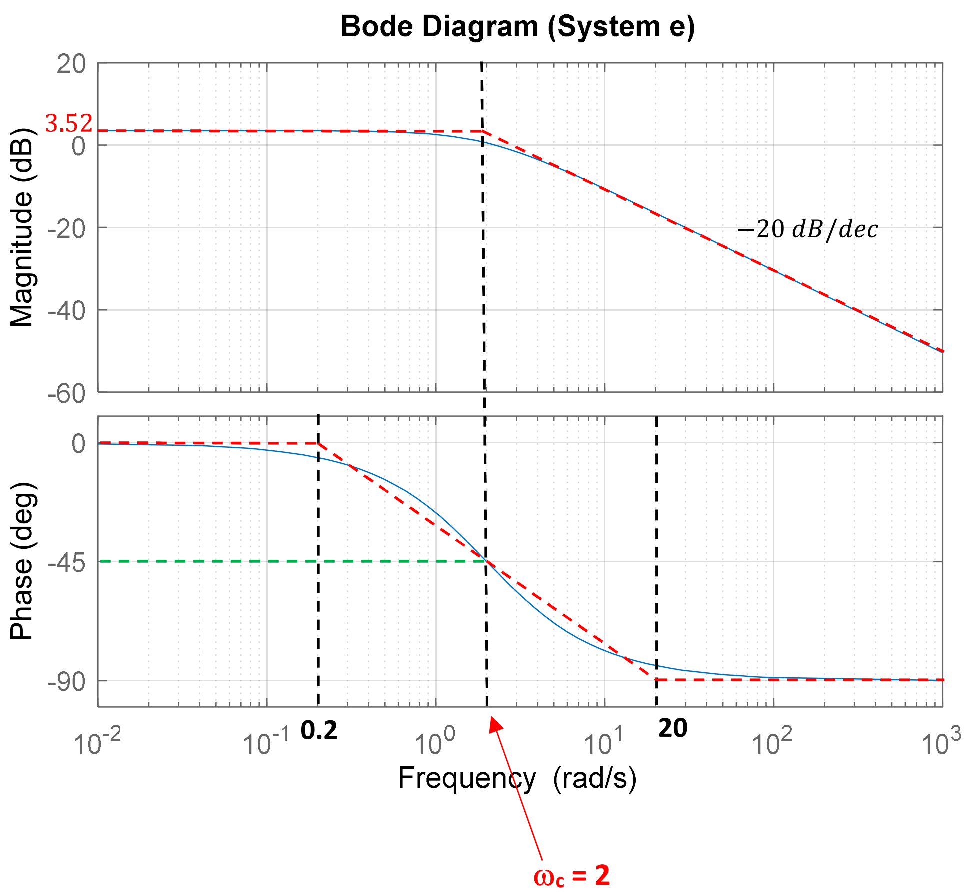
$$G(s) = \frac{3}{s+2} = \left(\frac{3}{2}\right)\left(\frac{1}{\frac{1}{2}s+1}\right) \quad \rightarrow \quad G(j\omega) = \frac{3}{j\omega+2} = (1.5)\left(\frac{1}{j0.5\omega+1}\right)$$

The basic factors are:

1) constant gain of 1.5 $\rightarrow$ 20log(1.5) = 3.52dB
2) first-order pole with corner frequency of $\omega_c = 2$

Starting Slope: $-20\beta \frac{dB}{dec} = -20(0)\ \frac{dB}{dec} = 0\ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{1.5}{(j0.01)^0}\right| = 20\log(1.5) - 20\log(1) = 3.52\ dB$



f) $\boldsymbol{G(s) = 7s + 1}$

Find the frequency response function, basic factors, and corner frequencies.

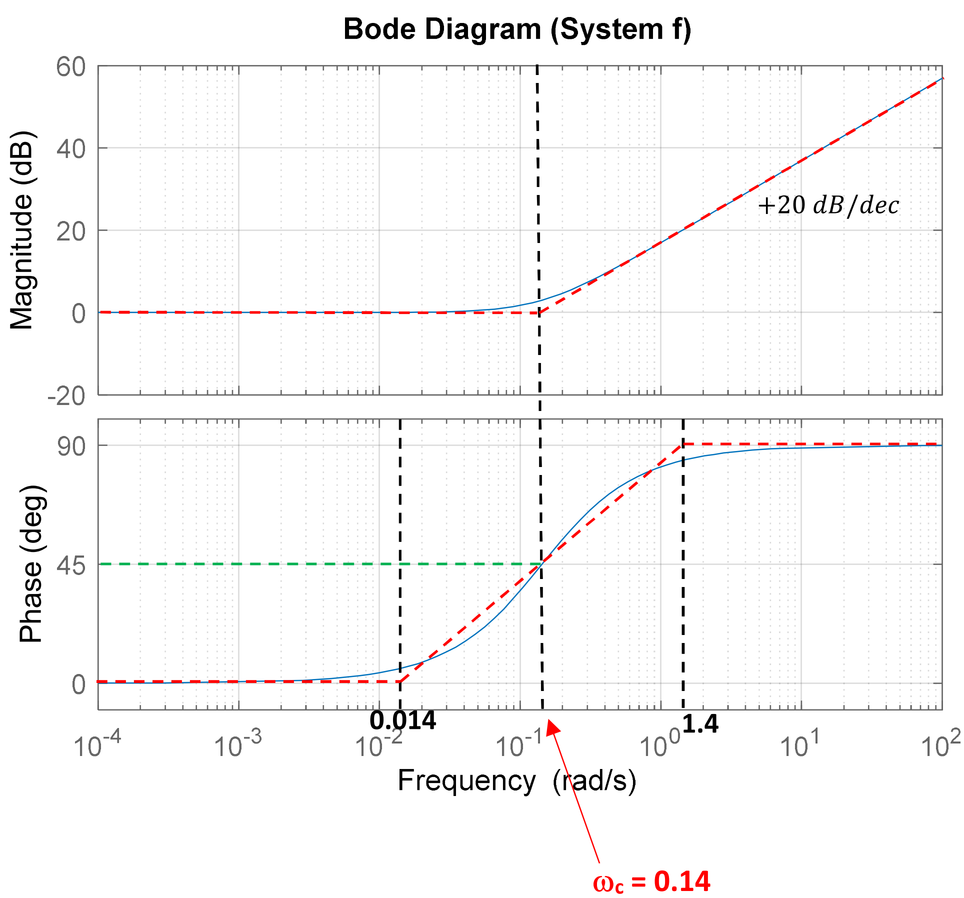
$G(s) = 7s + 1 \quad \rightarrow \quad G(j\omega) = j7\omega + 1$

The basic factor is:

first-order zero with corner frequency of $\omega_c = \frac{1}{7} = 0.14$

Starting Slope: $-20\beta \frac{dB}{dec} = -20(0)\ \frac{dB}{dec} = 0\ \ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{1}{(j0.0001)^0}\right| = 20\log(1) - 20\log(1) = 0\ dB$



g) $\boldsymbol{G(s) = s + 8}$

Find the frequency response function, basic factors, and corner frequencies.

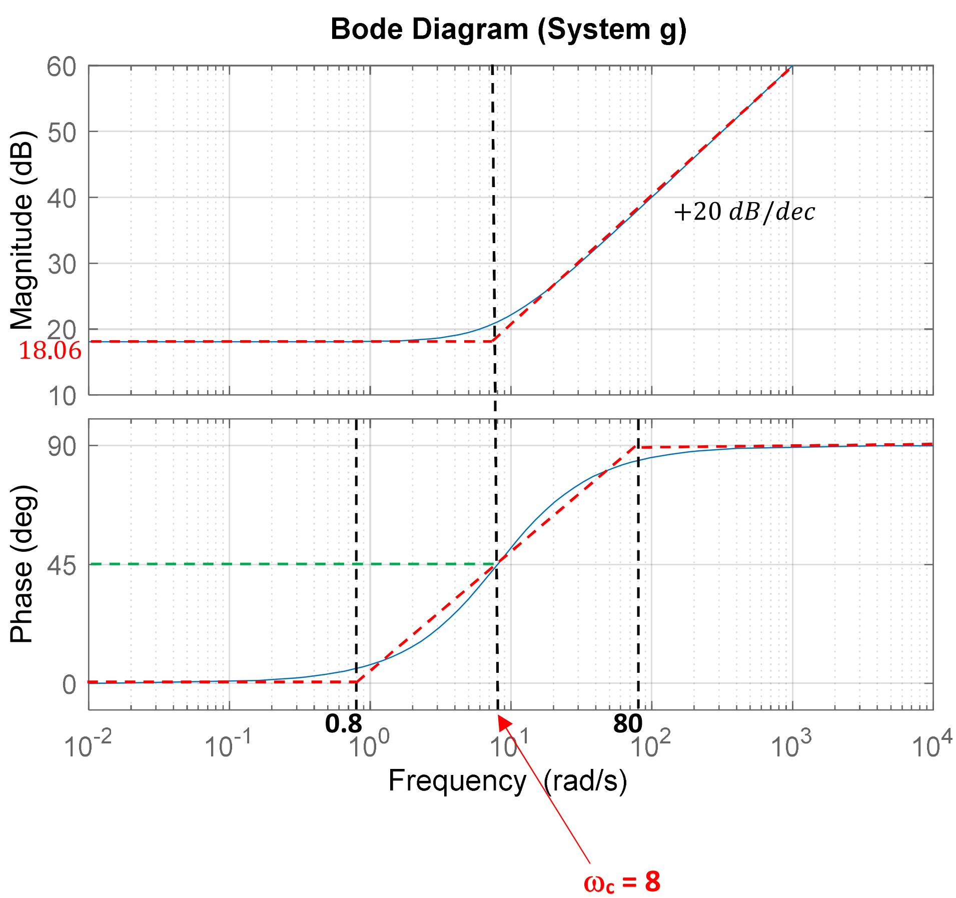
$$G(s) = s + 8 = (8)\left(\frac{1}{8}s + 1\right) \qquad G(j\omega) = (8)\left(\frac{j\omega}{8} + 1\right)$$

The basic factors are:

1) constant gain of 8 $\rightarrow$ 20log(8) = 18.06dB
2) first-order zero with corner frequency of $\omega_c = 8$

Starting Slope: $-20\beta \frac{dB}{dec} = -20(0) \ \frac{dB}{dec} = 0 \ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{8}{(j0.001)^0}\right| = 20\log(8) - 20\log(1) = 18.06\ dB$



h) $\boldsymbol{G(s) = \frac{1}{s}}$

Find the frequency response function, basic factors, and corner frequencies.

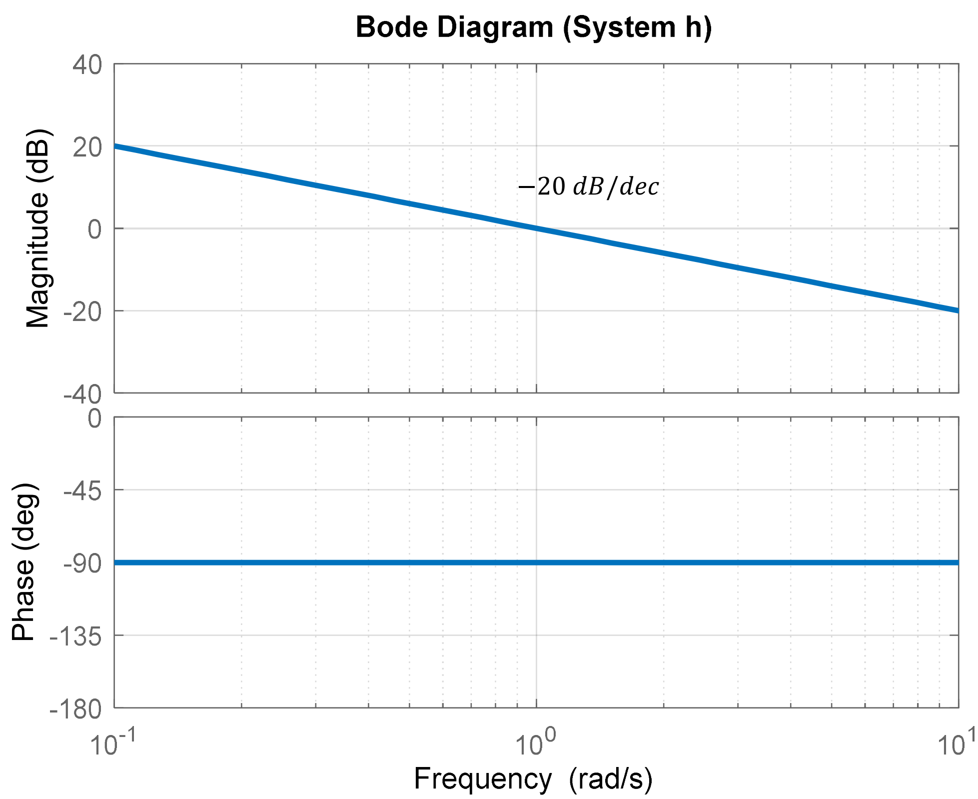
$$G(s) = \frac{1}{s} \quad \rightarrow \quad G(j\omega) = \frac{1}{j\omega}$$

The basic factor is:

first-order integrator

Starting Slope: $-20\beta \, \frac{dB}{dec} = -20(1) \, \frac{dB}{dec} = -20 \, \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{1}{(j0.1)^1}\right| = 20\log(1) - 20\log(0.1) = 20\ dB$



i) $\boldsymbol{G(s) = \frac{1}{5s}}$

Find the frequency response function, basic factors, and corner frequencies.

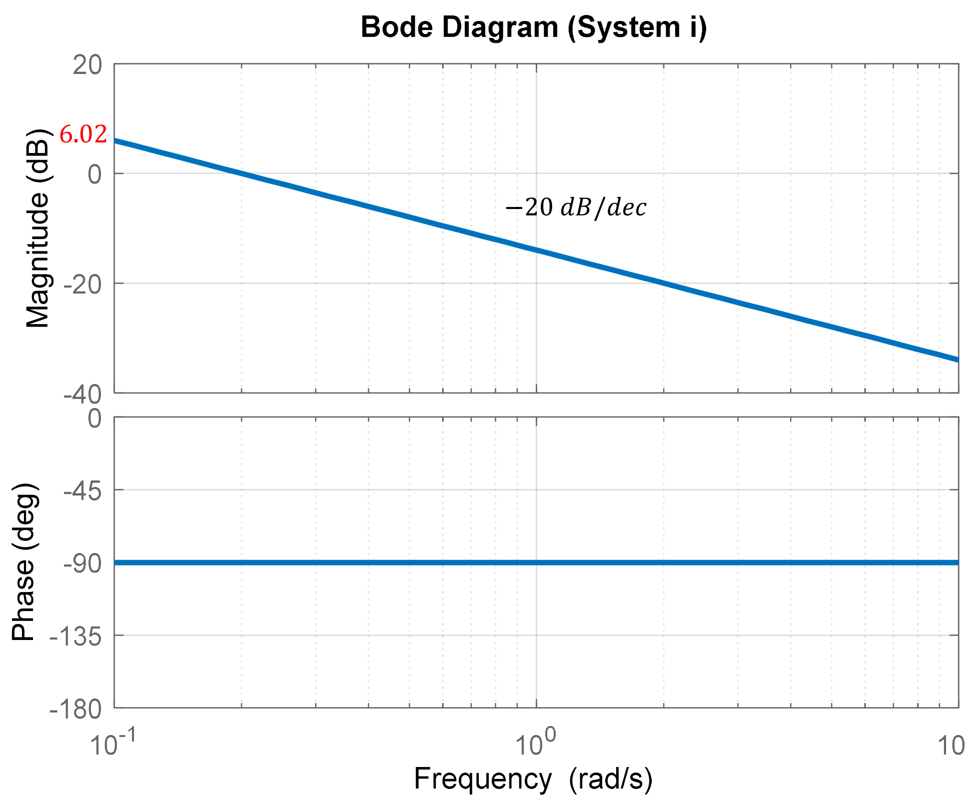
$$G(s) = \frac{1}{5s} = \left(\frac{1}{5}\right)\left(\frac{1}{s}\right) \quad \rightarrow \quad G(j\omega) = \left(\frac{1}{5}\right)\left(\frac{1}{j\omega}\right)$$

The basic factor is:

1) constant gain of 1/5 $\rightarrow$ 20log(0.2) = -13.98dB
2) first-order integrator

Starting Slope: $-20\beta \frac{dB}{dec} = -20(1)\ \frac{dB}{dec} = -20\ \ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{1/5}{(j0.1)^1}\right| = 20\log(1/5) - 20\log(0.1) = 6.02\ dB$



j) $\boldsymbol{G(s) = \frac{1}{s^2}}$

Find the frequency response function, basic factors, and corner frequencies.

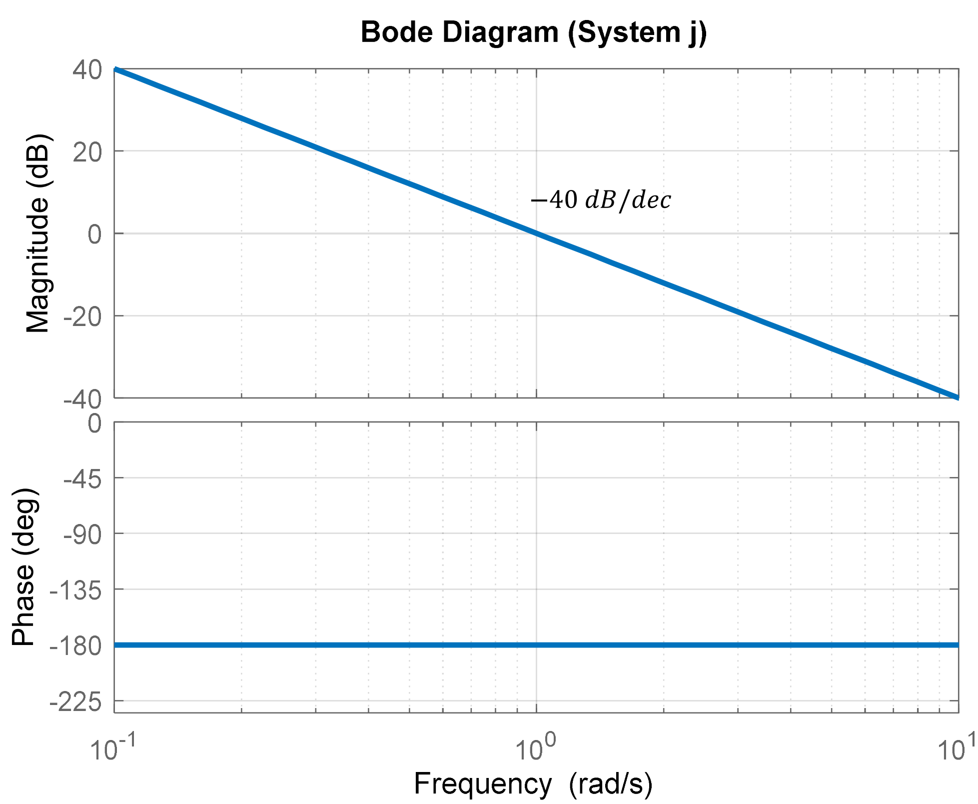
$$G(s) = \frac{1}{s^2} \quad \rightarrow \quad G(j\omega) = \frac{1}{(j\omega)^2}$$

The basic factor is:

second-order integrator

Starting Slope: $-20\beta \frac{dB}{dec} = -20(2)\ \frac{dB}{dec} = -40\ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{1}{(j0.1)^2}\right| = 20\log(1) - 20\log(0.01) = 40\ dB$



k) $\boldsymbol{G(s) = s}$

Find the frequency response function, basic factors, and corner frequencies.

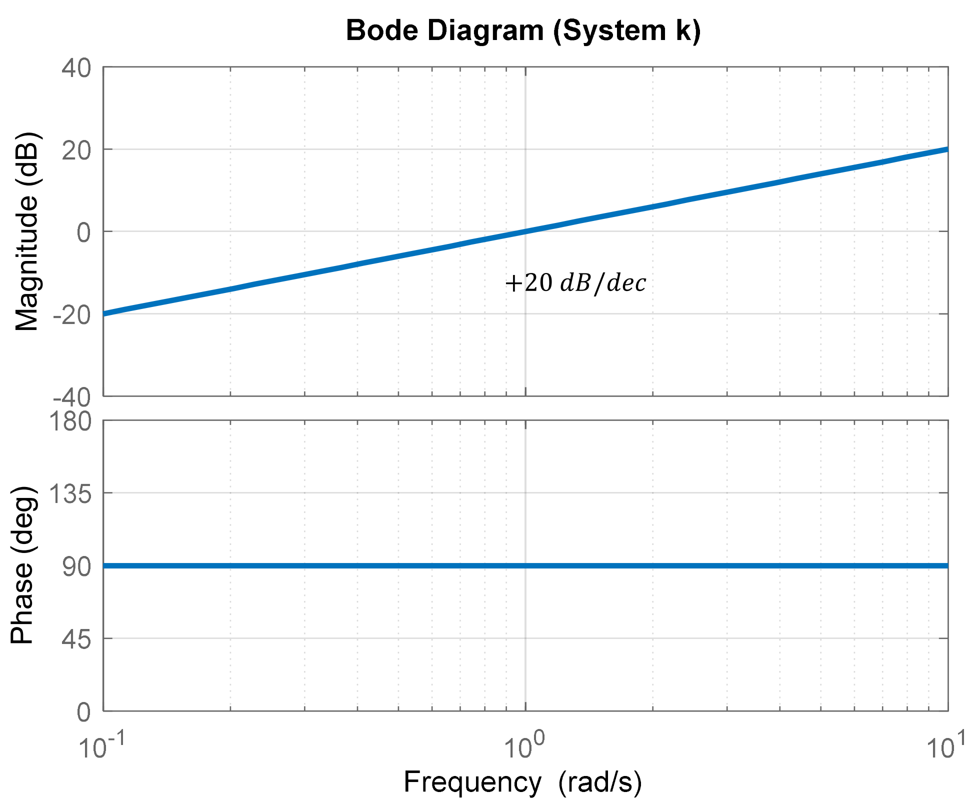
$$G(s) = s \quad \rightarrow \quad G(j\omega) = j\omega$$

The basic factor is:

first-order derivative

Starting Slope: $-20\beta \frac{dB}{dec} = -20(-1)\ \frac{dB}{dec} = 20\ \ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{1}{(j0.1)^{-1}}\right| = 20\log(1) - 20\log(10) = -20\ dB$



I) $G(s) = 5s$

Find the frequency response function, basic factors, and corner frequencies.

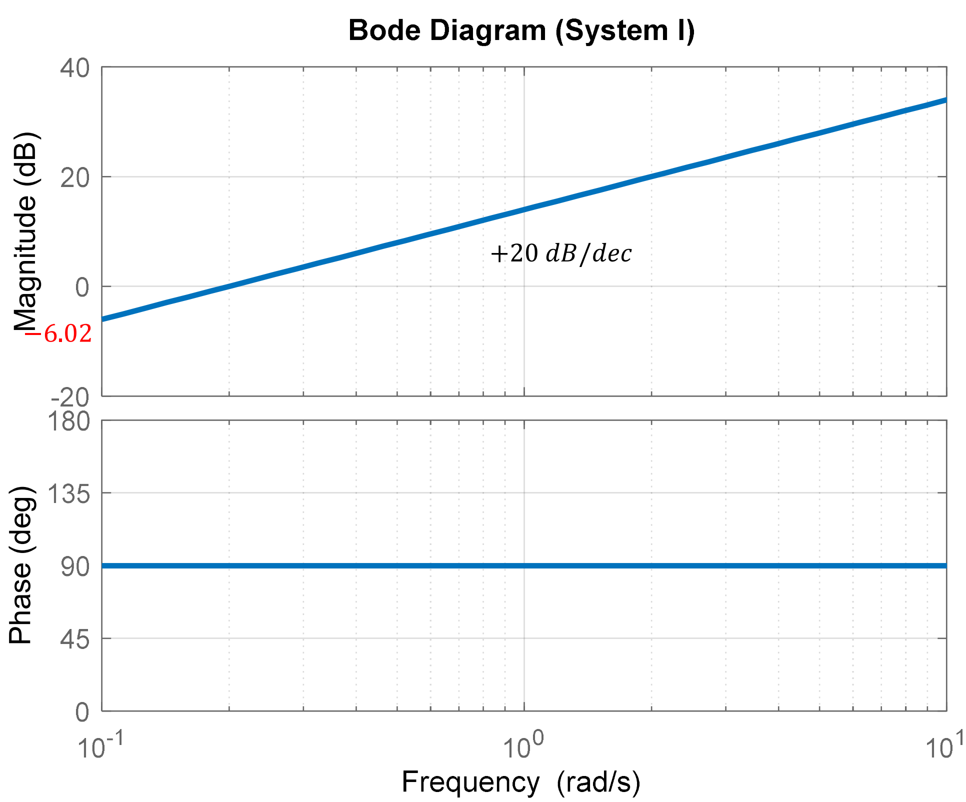
$$G(s) = 5s \quad \rightarrow \quad G(j\omega) = j5\omega$$

The basic factor is:

1) constant gain of 5 $\rightarrow$ 20log(5) = 13.98dB
2) first-order derivative

Starting Slope: $-20\beta \frac{dB}{dec} = -20(-1)\ \frac{dB}{dec} = 20\ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^{\beta}}\right| = 20\log\left|\frac{5}{(j0.1)^{-1}}\right| = 20\log(5) - 20\log(10) = -6.02\ dB$



m) $G(s) = \frac{s+2}{s+20}$

Find the frequency response function, basic factors, and corner frequencies.

$$G(s) = \frac{s+2}{s+20} = \frac{2(0.5s+1)}{20(0.05s+1)} = (0.1)(0.5s+1)\left(\frac{1}{0.05s+1}\right)$$

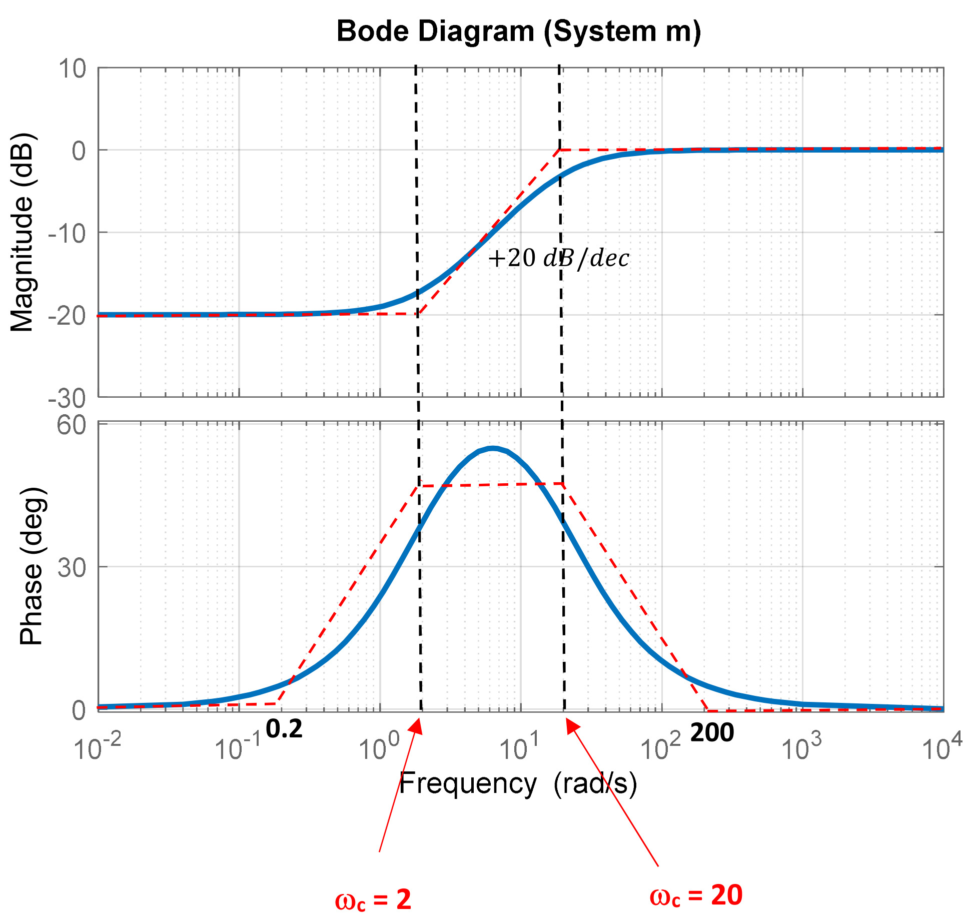
$$G(j\omega) = (0.1)(j0.5\omega+1)\left(\frac{1}{j0.05\omega+1}\right)$$

The basic factors are:

1) constant gain of 0.1 $\rightarrow$ 20log(0.1) = -20dB
2) first-order zero with corner frequency of $\omega_c = 2$
3) first-order pole with corner frequency of $\omega_c = 20$

Starting Slope: $-20\beta \frac{dB}{dec} = -20(0)\ \frac{dB}{dec} = 0\ \ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{0.1}{(j0.01)^0}\right| = 20\log(0.1) - 20\log(1) = -20\ dB$



n) $G(s) = \frac{5(s+0.1)}{s}$

Find the frequency response function, basic factors, and corner frequencies.

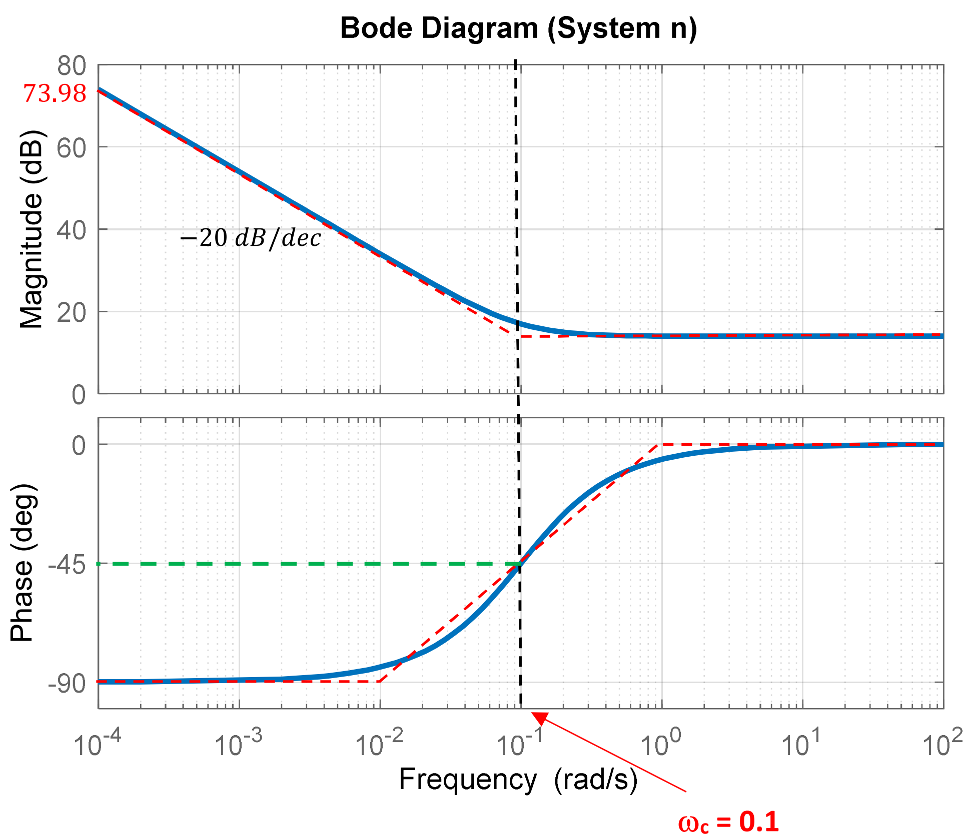
$$G(s) = \frac{5(s+0.1)}{s} = \frac{0.5(10s+1)}{s} \qquad G(j\omega) = (0.5)(j10\omega + 1)\left(\frac{1}{j\omega}\right)$$

The basic factors are:

1) constant gain of 0.5 $\rightarrow$ 20log(0.5) = -6.02dB
2) first-order zero with corner frequency of $\omega_c = 0.1$
3) first-order integrator

Starting Slope: $-20\beta \frac{dB}{dec} = -20(1)\frac{dB}{dec} = -20 \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{0.5}{(j0.0001)^1}\right| = 20\log(0.5) - 20\log(0.0001) = 73.98\ dB$



o) $G(s) = \frac{2}{s(s+1)}$

Find the frequency response function, basic factors, and corner frequencies.

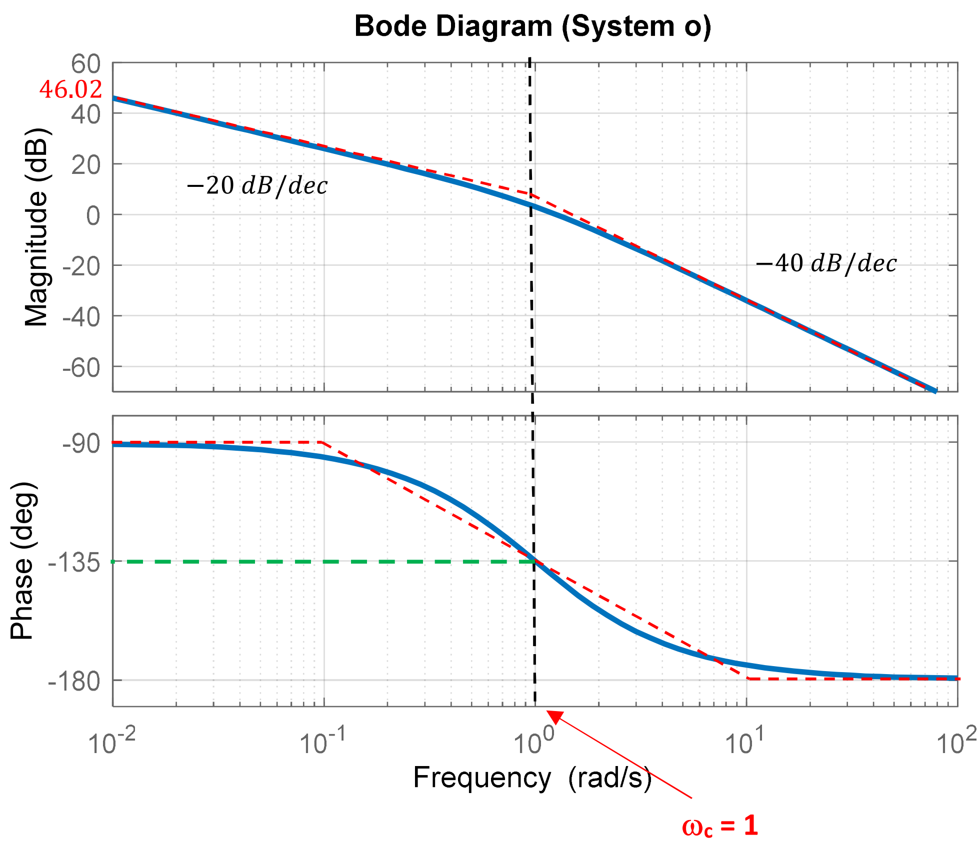
$$G(s) = \frac{2}{s(s+1)} \qquad G(j\omega) = (2)\left(\frac{1}{j\omega}\right)\left(\frac{1}{j\omega+1}\right)$$

The basic factors are:

1) constant gain of 2 $\rightarrow$ 20log(2) = 6.02dB
2) first-order pole with corner frequency of $\omega_c = 1$
3) first-order integrator

Starting Slope: $-20\beta \frac{dB}{dec} = -20(1)\ \frac{dB}{dec} = -20\ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{2}{(j0.01)^1}\right| = 20\log(2) - 20\log(0.01) = 46.02\ dB$



p) $G(s) = \frac{3(s+4)}{s+10}$

Find the frequency response function, basic factors, and corner frequencies.

$$G(s) = \frac{3(s+4)}{s+10} = \frac{12(0.25s+1)}{10(0.1s+1)} = (1.2)(0.25s+1)\left(\frac{1}{0.1s+1}\right)$$

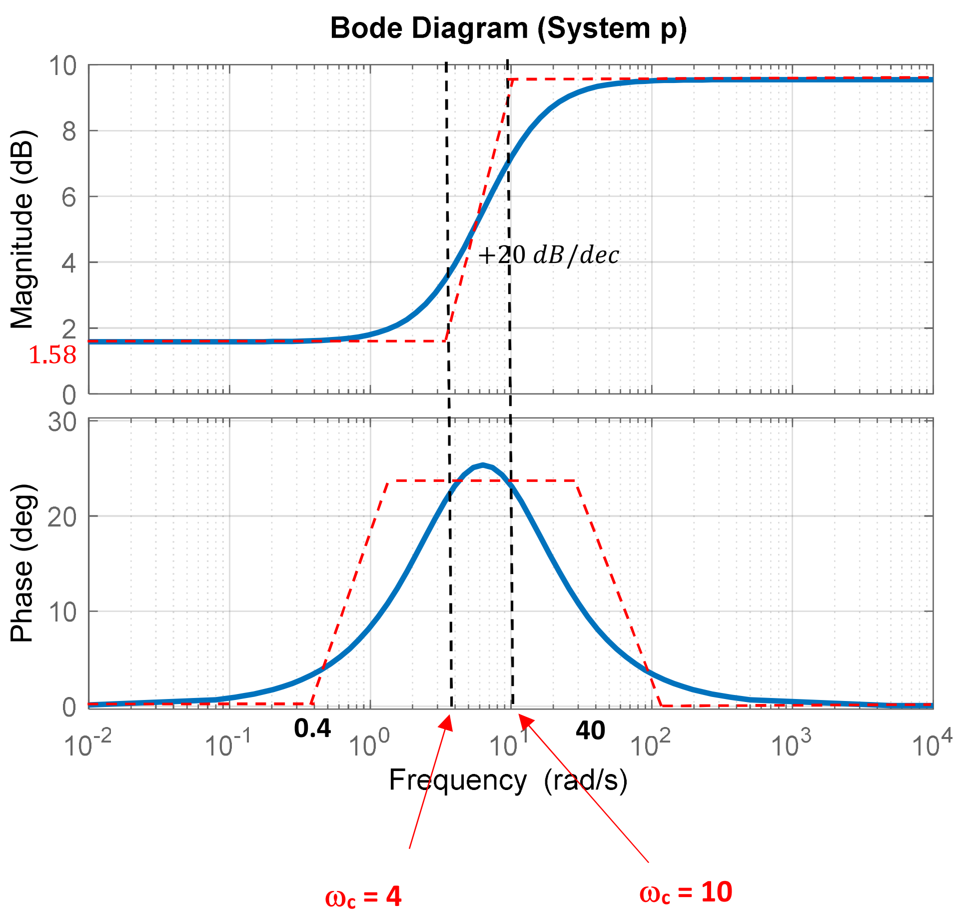
$$G(j\omega) = (1.2)(j0.25\omega+1)\left(\frac{1}{j0.1\omega+1}\right)$$

The basic factors are:

1) constant gain of 1.2 $\rightarrow$ 20log(1.2) = 1.58dB
2) first-order zero with corner frequency of $\omega_c = 4$
3) first-order pole with corner frequency of $\omega_c = 10$

Starting Slope: $-20\beta \frac{dB}{dec} = -20(0)\ \frac{dB}{dec} = 0\ \ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{1.2}{(j0.01)^0}\right| = 20\log(1.2) - 20\log(1) = 1.58\ dB$



q) $\boldsymbol{G(s) = \frac{2(s+10)}{s+0.1}}$

Find the frequency response function, basic factors, and corner frequencies.

$$G(s) = \frac{2(s+10)}{s+0.1} = \frac{20(0.1s+1)}{0.1(10s+1)} = (200)(0.1s+1)\left(\frac{1}{10s+1}\right)$$

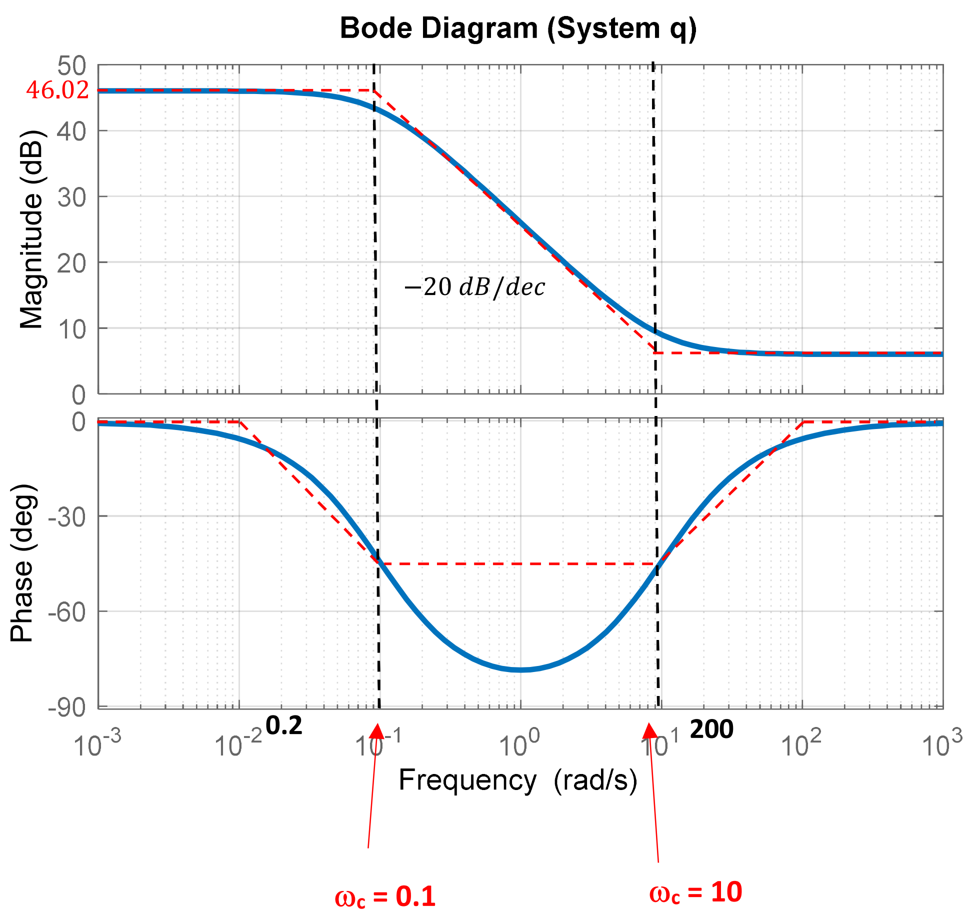
$$G(j\omega) = (200)(j0.1\omega+1)\left(\frac{1}{j10\omega+1}\right)$$

The basic factors are:

1) constant gain of 200 $\rightarrow$ 20log(200) = 46.02dB
2) first-order zero with corner frequency of $\omega_c = 10$
3) first-order pole with corner frequency of $\omega_c = 0.1$

Starting Slope: $-20\beta \frac{dB}{dec} = -20(0) \frac{dB}{dec} = 0 \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{200}{(j0.001)^0}\right| = 20\log(200) - 20\log(1) = 46.02\ dB$



2) Find the frequency response function $G(j\omega)$ and all the corner frequencies of poles and zeros and indicate them on the given graph. Then draw the asymptote lines of the Bode diagram and indicate the slopes on the graph.

a) $G(s) = \frac{6}{s^2(s+2)}$

Find the frequency response function, basic factors, and corner frequencies

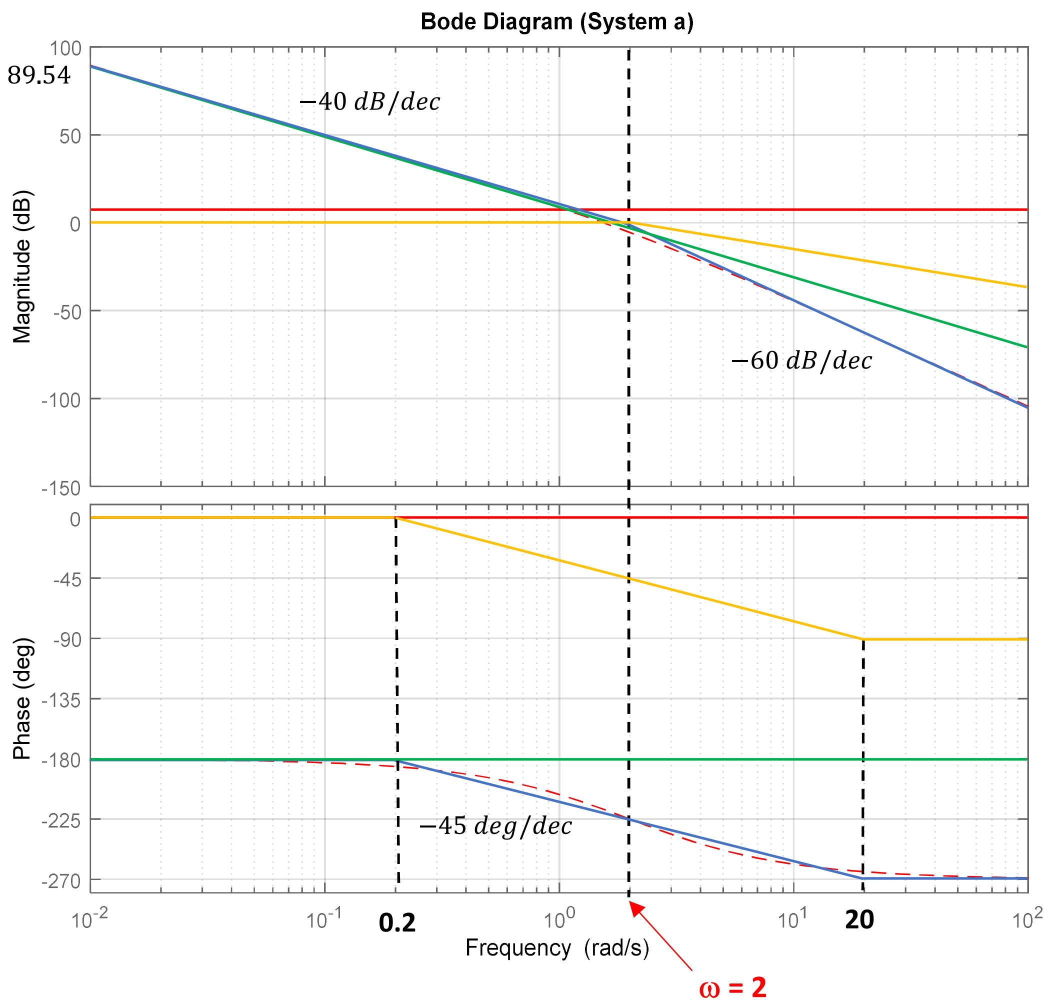
$$G(s) = \left(\frac{6}{2}\right)\left(\frac{1}{s^2}\right)\left(\frac{1}{0.5s+1}\right) \quad \rightarrow \quad G(j\omega) = 3\frac{1}{(j\omega)^2}\left(\frac{1}{0.5j\omega+1}\right)$$

The basic factors are:

1) constant gain of 3 $\rightarrow$ 20log(3) = 9.54dB (phase is all zero)
2) second-order integrator (green graph)
3) first-order pole with corner frequency of $\omega_c = 2$ (yellow graph)

Starting Slope: $-20\beta \frac{dB}{dec} = -20(2)\frac{dB}{dec} = -40\frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{3}{(j0.01)^2}\right| = 89.54\ dB$



b) $G(s) = \frac{2}{s(s+3)}$

Find the frequency response function, basic factors, and corner frequencies

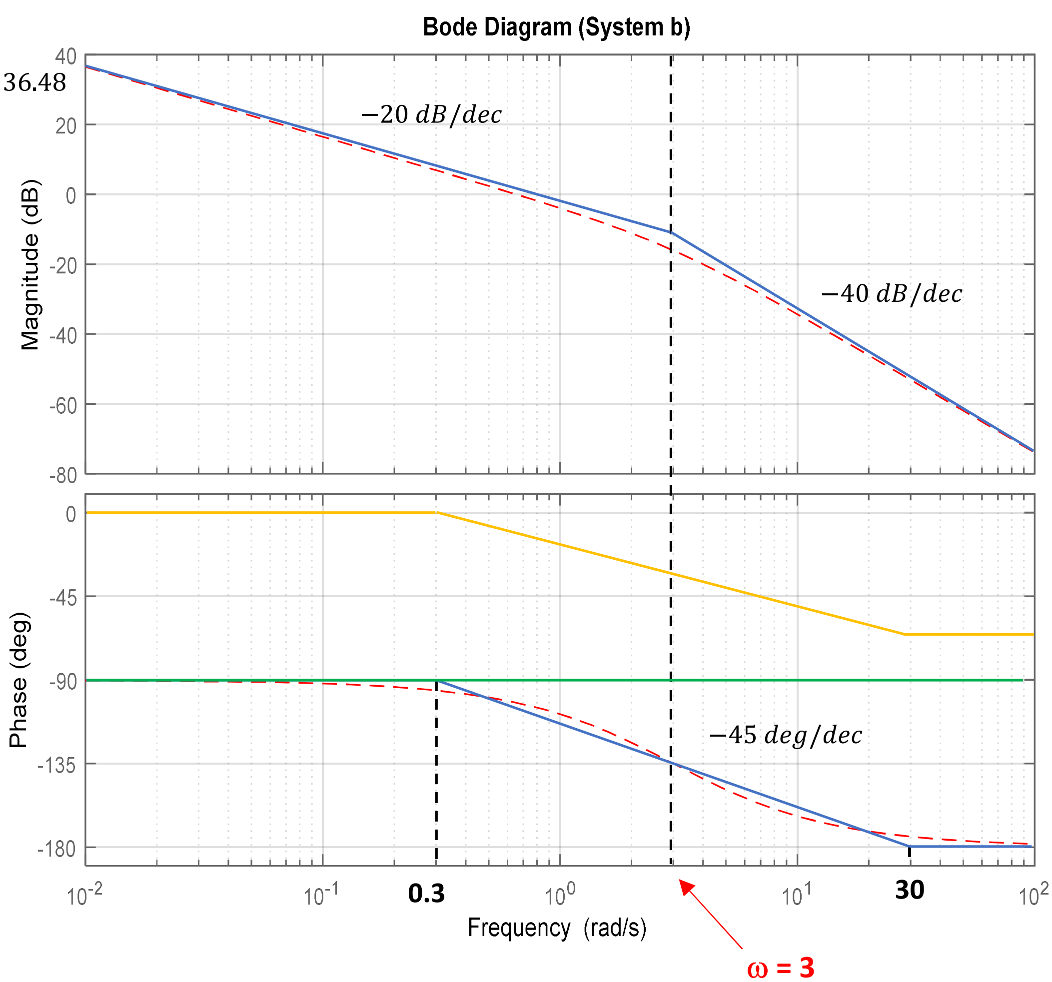
$$G(s) = \left(\frac{2}{3}\right)\left(\frac{1}{s}\right)\left(\frac{1}{\frac{1}{3}s+1}\right) \quad \rightarrow \quad G(j\omega) = \left(\frac{2}{3}\right)\left(\frac{1}{j\omega}\right)\left(\frac{1}{\frac{1}{3}j\omega+1}\right)$$

The basic factors are:

1) constant gain of 2/3 → 20log(2/3) = -3.52dB (phase is all zero)
2) first-order integrator (yellow graph)
3) first-order pole with corner frequency of $\omega_c = 3$ (green graph)

Starting Slope: $-20\beta \frac{dB}{dec} = -20(1)\ \frac{dB}{dec} = -20\ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{2/3}{(j0.01)^1}\right| = 36.48\ dB$



c) $G(s) = \frac{10(s+5)}{s(s+2)}$

Find the frequency response function, basic factors, and corner frequencies

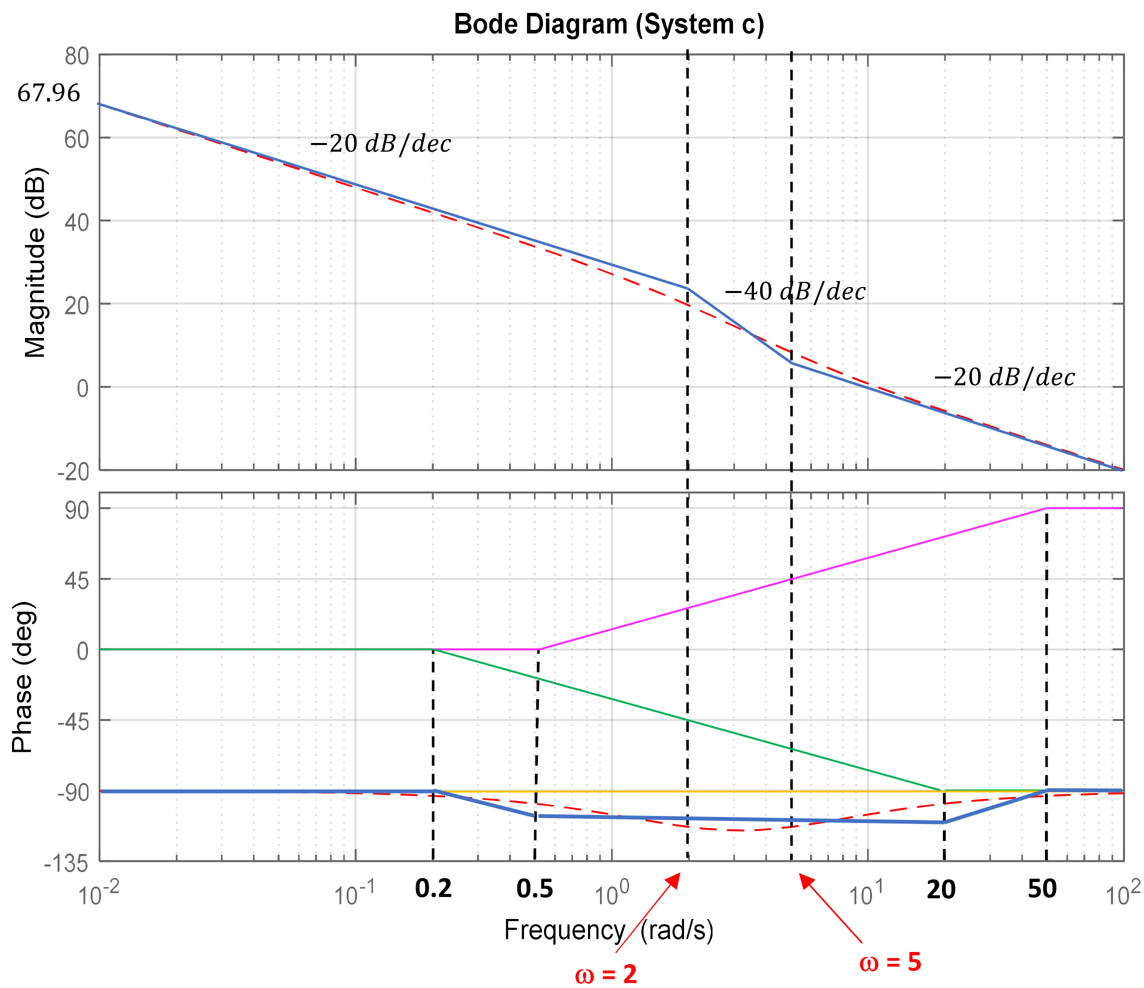
$$G(s) = \left(\frac{10\times 5}{2}\right)(0.2s+1)\left(\frac{1}{s}\right)\left(\frac{1}{0.5s+1}\right) \quad \rightarrow \quad G(j\omega) = (25)(0.2j\omega+1)\left(\frac{1}{j\omega}\right)\left(\frac{1}{0.5j\omega+1}\right)$$

The basic factors are:

1) constant gain of 25 → 20log(25) = 27.96dB (phase is all zero)
2) first-order zero with corner frequency of $\omega_c = 5$ (purple graph)
3) first-order integrator (yellow graph)
4) first-order pole with corner frequency of $\omega_c = 2$ (green graph)

Starting Slope: $-20\beta\,\frac{dB}{dec} = -20(1)\,\frac{dB}{dec} = -20\,\frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{25}{(j0.01)^1}\right| = 67.96\ dB$



d) $G(s) = \frac{10}{s^2+0.4s+1}$

Find the frequency response function, basic factors, and corner frequencies

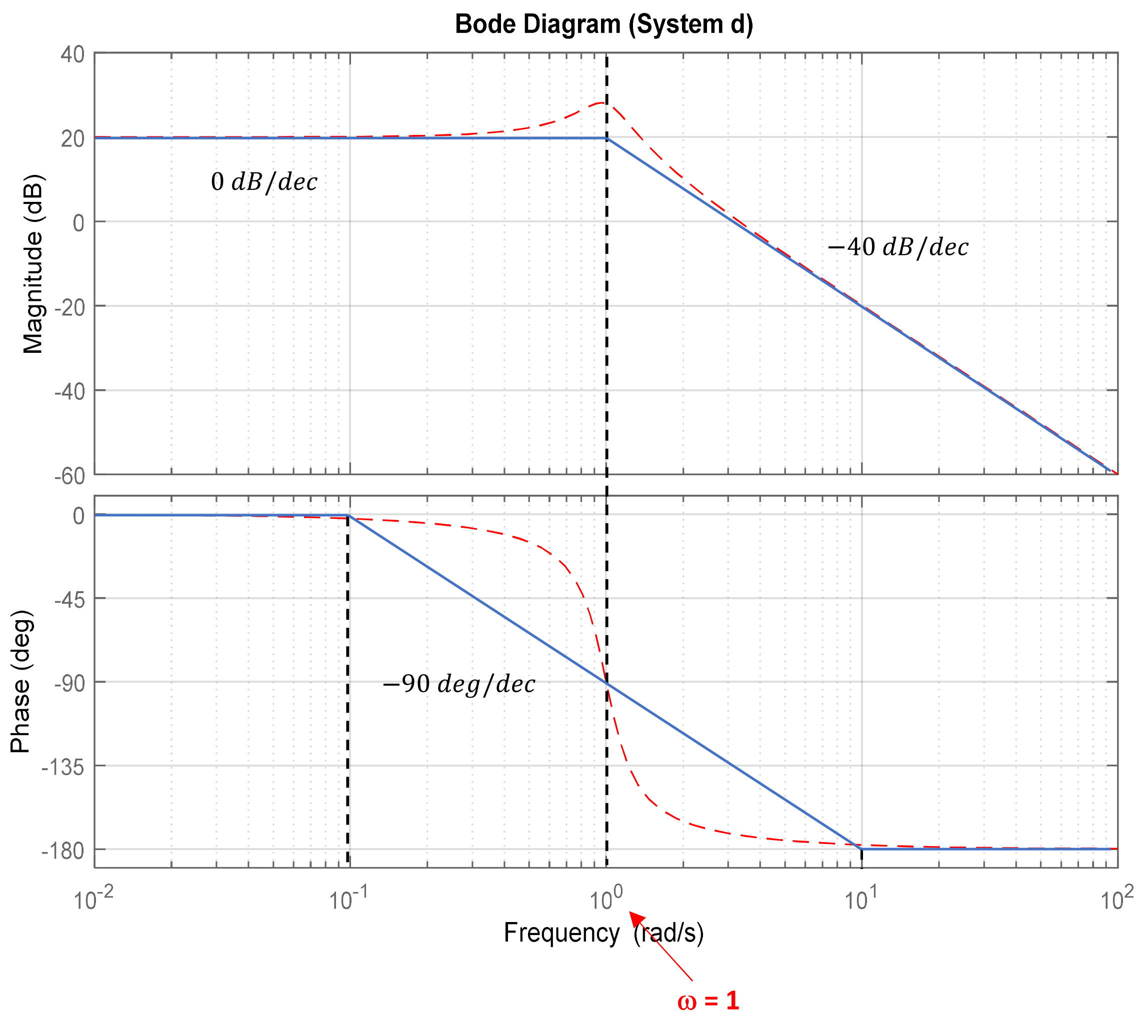
$$G(s) = \frac{10}{s^2 + 0.4s + 1} \quad \rightarrow \quad G(j\omega) = \frac{10}{(j\omega)^2 + 0.4j\omega + 1}$$

The basic factors are:

1) constant gain of 10 $\rightarrow$ 20log(10) = 20dB (phase is all zero)
2) second-order complex poles with corner frequency of $\omega_c = 1$ and damping ratio of $\zeta = 0.2$

Starting Slope: $-20\beta \frac{dB}{dec} = -20(0)\ \frac{dB}{dec} = 0\ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{10}{(j0.01)^0}\right| = 20\ dB$



e) $G(s) = \frac{20}{s(s+10)(s^2+s+0.5)}$

Find the frequency response function, basic factors, and corner frequencies

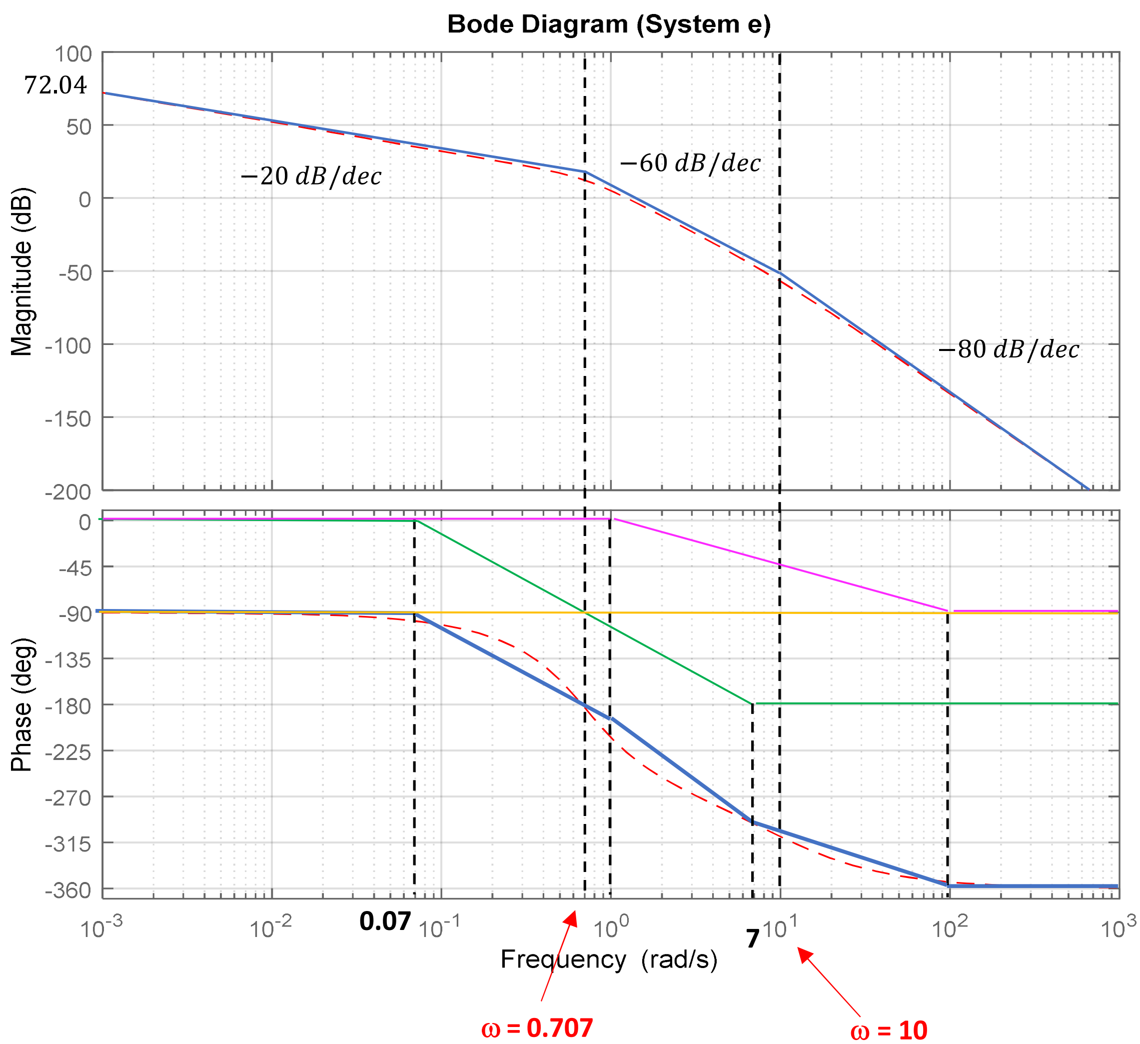
$$G(s) = \left(\frac{20}{10\times 0.5}\right)\left(\frac{1}{s}\right)\left(\frac{1}{0.1s+1}\right)\left(\frac{1}{\frac{s^2}{0.5}+\frac{s}{0.5}+1}\right) \rightarrow G(j\omega) = (4)\left(\frac{1}{j\omega}\right)\left(\frac{1}{0.1j\omega+1}\right)\left(\frac{1}{\frac{(j\omega)^2}{0.5}+\frac{j\omega}{0.5}+1}\right)$$

The basic factors are:

1) constant gain of 4 $\rightarrow$ 20log(4) = 12.04dB (phase is all zero)
2) first-order integrator (yellow graph)
3) first-order pole with corner frequency of $\omega_c = 10$ (purple graph)
4) second-order complex poles with corner frequency of $\omega_c = 0.707$ and damping ratio of $\zeta = 0.707$ (green graph)

Starting Slope: $-20\beta \frac{dB}{dec} = -20(1) \frac{dB}{dec} = -20 \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{4}{(j0.001)^1}\right| = 72.04\ dB$



f) $G(s) = \frac{100(s+0.5)}{s^2(s^2+3.6s+36)}$

Find the frequency response function, basic factors, and corner frequencies

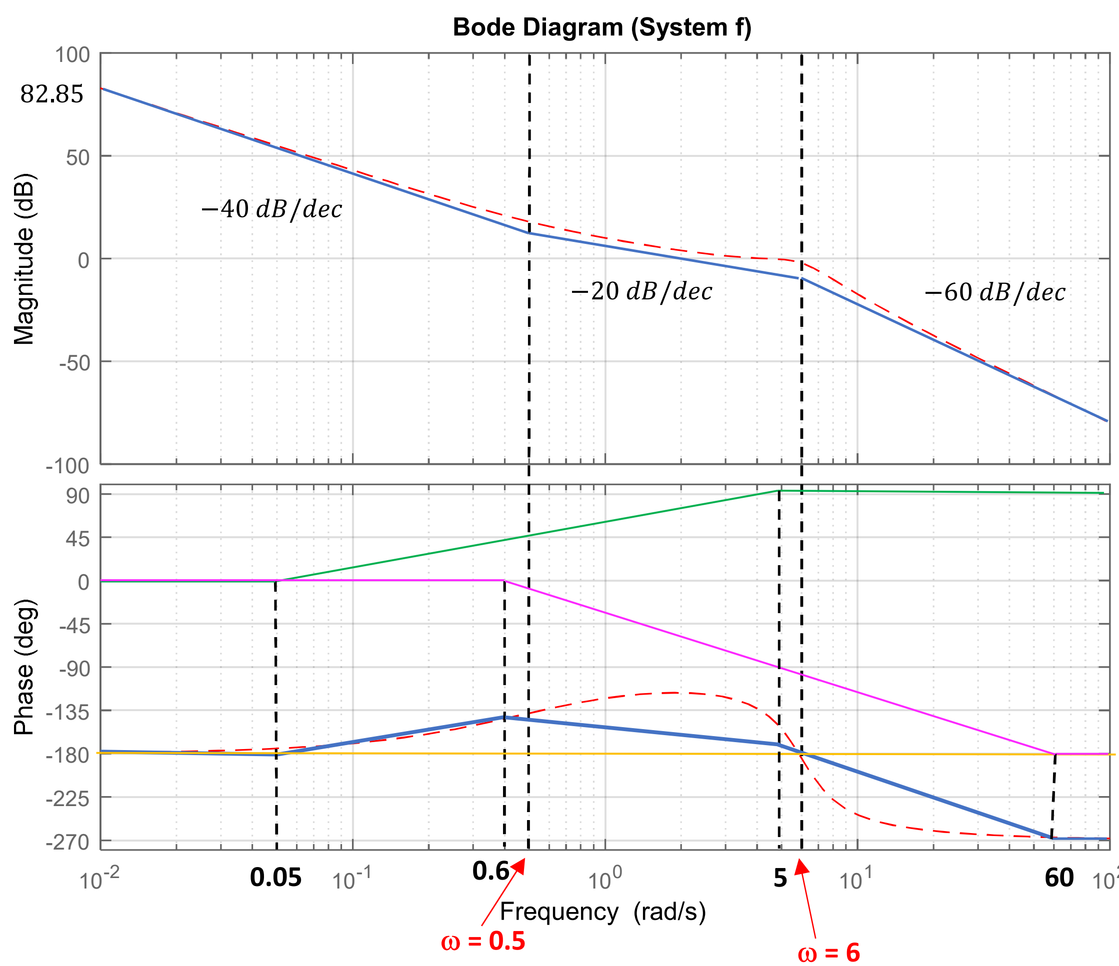
$$G(s) = \left(\frac{100 \times 0.5}{36}\right)(2s+1)\left(\frac{1}{s^2}\right)\left(\frac{1}{\frac{s^2}{36}+\frac{s}{10}+1}\right) \quad \rightarrow \quad G(j\omega) = \left(\frac{25}{18}\right)(2j\omega+1)\frac{1}{(j\omega)^2}\left(\frac{1}{\frac{(j\omega)^2}{36}+\frac{j\omega}{10}+1}\right)$$

The basic factors are:

1) constant gain of 25/18 → 20log(25/18) = 2.85dB (phase is all zero)
2) second-order integrator (yellow graph)
3) first-order zero with corner frequency of $\omega_c = 0.5$ (green graph)
4) second-order complex poles with corner frequency of $\omega_c = 6$ and damping ratio of $\zeta = 0.3$ (purple graph)

Starting Slope: $-20\beta \frac{dB}{dec} = -20(2)\frac{dB}{dec} = -40 \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{25/18}{(j0.01)^2}\right| = 82.85\ dB$



g) $G(s) = \frac{2000(s+0.5)}{s(s+10)(s+50)}$

Find the frequency response function, basic factors, and corner frequencies

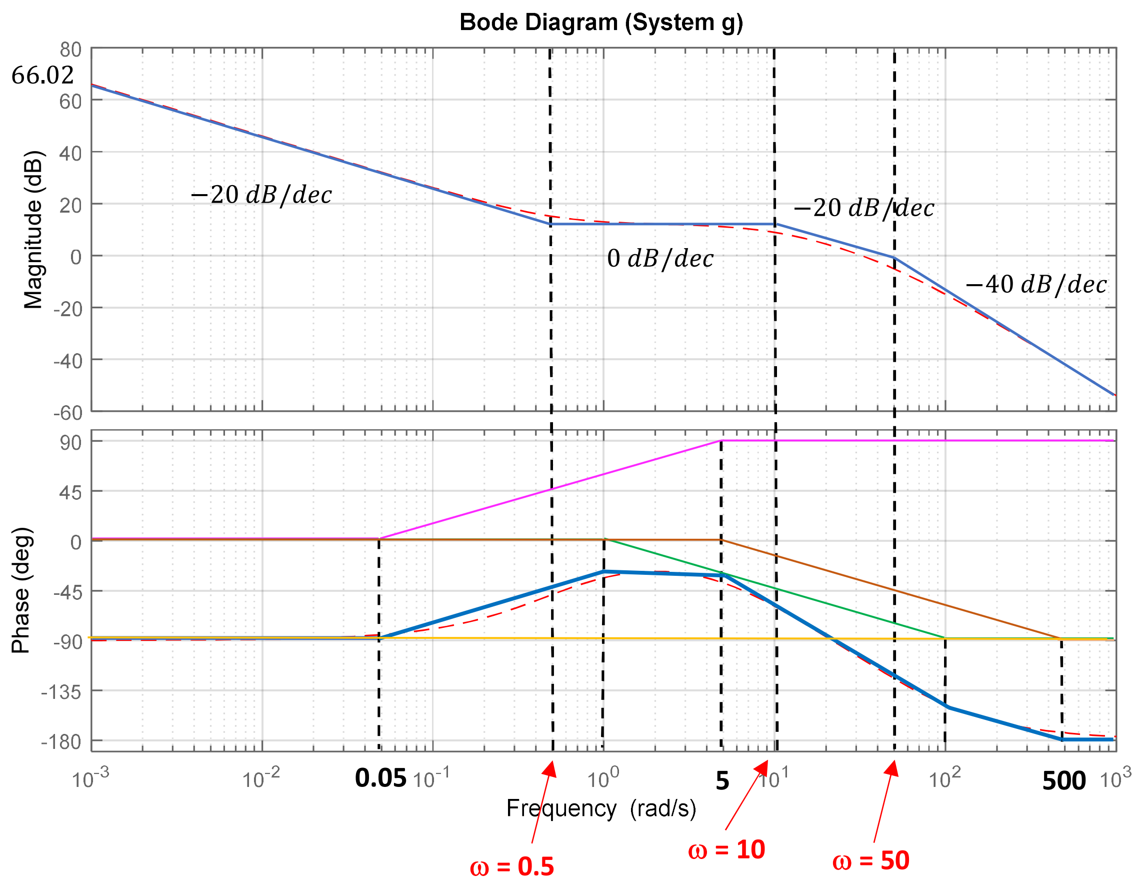
$$G(s) = \left(\frac{2000 \times 0.5}{10 \times 50}\right)(2s+1)\left(\frac{1}{s}\right)\left(\frac{1}{0.1s+1}\right)\left(\frac{1}{0.02s+1}\right) \quad \rightarrow \quad G(j\omega) = (2)(2j\omega+1)\frac{1}{j\omega}\left(\frac{1}{0.1j\omega+1}\right)\left(\frac{1}{0.02j\omega+1}\right)$$

The basic factors are:

1) constant gain of 2 $\rightarrow$ 20log(2) = 6.02dB (phase is all zero)
2) first-order integrator (yellow graph)
3) first-order zero with corner frequency of $\omega_c = 0.5$ (purple graph)
4) first-order pole with corner frequency of $\omega_c = 10$ (green graph)
5) first-order pole with corner frequency of $\omega_c = 50$ (brown graph)

Starting Slope: $-20\beta \frac{dB}{dec} = -20(1) \frac{dB}{dec} = -20 \frac{dB}{dec}$

Starting Point: $20 \log \left|\frac{K_B}{(j\omega)^\beta}\right| = 20 \log \left|\frac{2}{(j0.001)^1}\right| = 66.02\ dB$



h) $G(s) = \frac{20(s+1)}{s(s+10)(s^2+s+0.5)}$

Find the frequency response function, basic factors, and corner frequencies

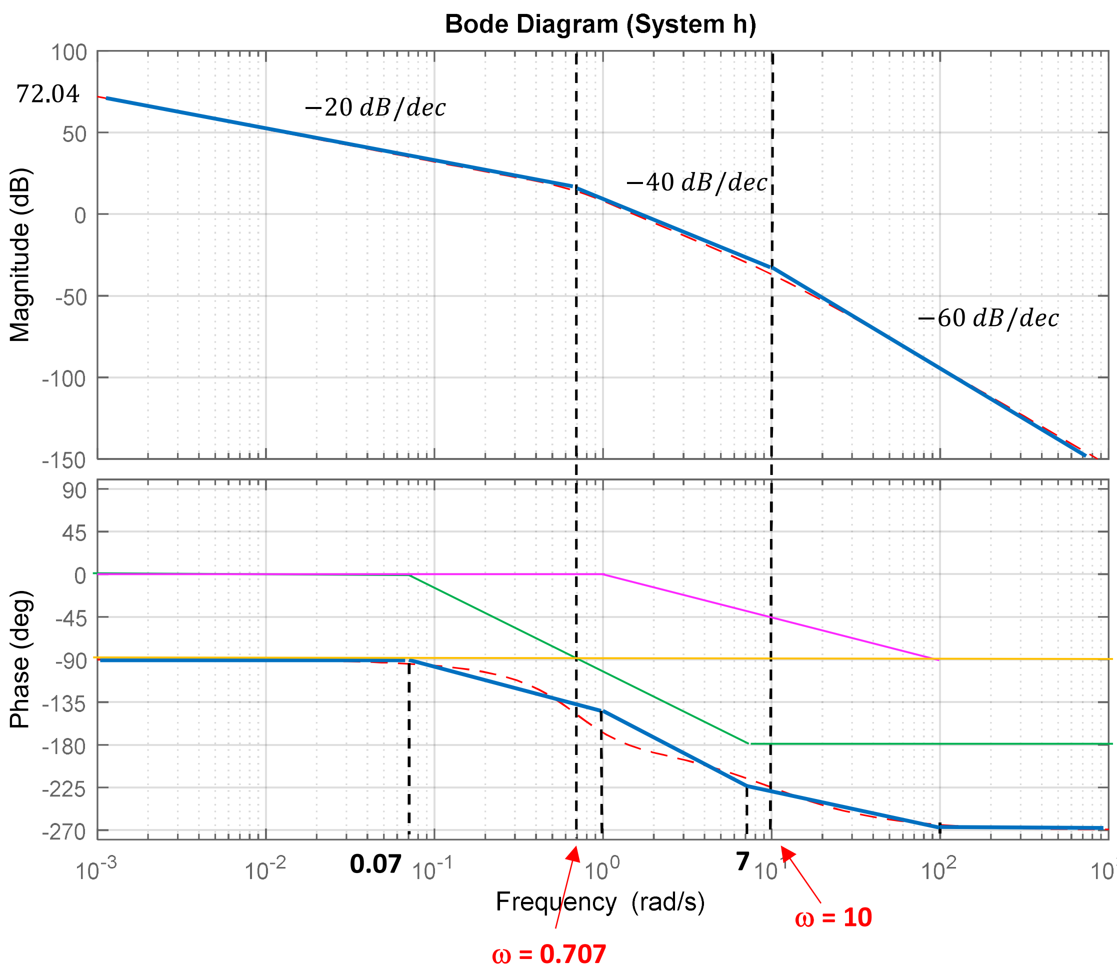
$$G(s) = \left(\frac{20}{10 \times 0.5}\right)(s+1)\left(\frac{1}{s}\right)\left(\frac{1}{0.1s+1}\right)\left(\frac{1}{\frac{s^2}{0.5}+\frac{s}{0.5}+1}\right) \quad \rightarrow \quad G(j\omega) = (4)(j\omega+1)\frac{1}{j\omega}\left(\frac{1}{0.1j\omega+1}\right)\left(\frac{1}{\frac{(j\omega)^2}{0.5}+\frac{j\omega}{0.5}+1}\right)$$

The basic factors are:

1) constant gain of 4 $\rightarrow$ 20log(4) = 12.04dB (phase is all zero)
2) first-order integrator (yellow graph)
3) first-order pole with corner frequency of $\omega_c = 10$ (purple graph)
4) second-order complex poles with corner frequency of $\omega_c = 0.707$ and damping ratio of $\zeta = 0.707$ (green graph)

Starting Slope: $-20\beta \frac{dB}{dec} = -20(1)\ \frac{dB}{dec} = -20\ \ \frac{dB}{dec}$

Starting Point: $20\log\left|\frac{K_B}{(j\omega)^\beta}\right| = 20\log\left|\frac{4}{(j0.001)^1}\right| = 72.04\ dB$



3) Sketch the polar plot (Nyquist diagram) of the following transfer functions.

a) $\boldsymbol{G(s) = \frac{1}{s(\tau s+1)}}$

The frequency response function is obtained as below

$$G(j\omega) = \frac{1}{j\omega(j\omega\tau + 1)} \quad \rightarrow \quad G(j\omega) = \frac{-\tau}{(\omega\tau)^2 + 1} + j\frac{-1}{\omega((\omega\tau)^2 + 1)}$$

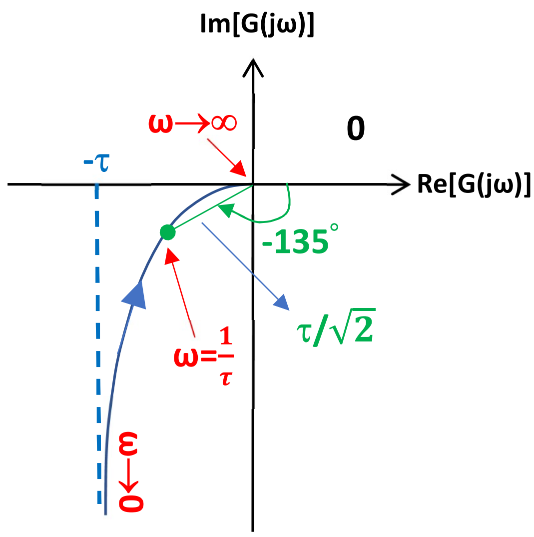
The magnitude and phase angle are obtained as below

$$\begin{cases} |G(j\omega)| = \left|\frac{1}{j\omega(j\omega\tau + 1)}\right| = \frac{1}{|j\omega||j\omega\tau + 1|} \\ \angle G(j\omega) = \angle\left(\frac{1}{j\omega(j\omega\tau + 1)}\right) = -90^\circ - \tan^{-1}(\tau\omega) \end{cases}$$

Starting point: For $\omega \rightarrow 0 \quad \Rightarrow \quad G(j0) = \infty\angle{-90^\circ}$

For $\omega = \frac{1}{\tau} \quad \Rightarrow G\left(j\frac{1}{\tau}\right) = \frac{\tau}{\sqrt{2}}\angle - 135^\circ$

Ending point: For $\omega \rightarrow \infty \quad \Rightarrow \quad G(j\infty) = 0\angle - 180^\circ$



For $\omega \rightarrow \infty$ the graph is tangent to the negative real axis.

b) $G(s) = \frac{10}{s(s+1)(s+2)}$

The frequency response function is

$$G(j\omega) = \frac{5}{j\omega(j\omega+1)(j\frac{\omega}{2}+1)}$$

The real part and the imaginary part of the $G(j\omega)$ are obtained as below

$$G(j\omega) = \frac{5}{j\omega(j\omega+1)(j\frac{\omega}{2}+1)} \times \frac{(-j\omega)(-j\omega+1)(-j\frac{\omega}{2}+1)}{(-j\omega)(-j\omega+1)(-j\frac{\omega}{2}+1)}$$

$$G(j\omega) = \underbrace{\frac{-30}{9\omega^2+(2-\omega^2)^2}}_{real\ part} + j\underbrace{\frac{-10(2-\omega^2)}{9\omega^3+\omega(2-\omega^2)^2}}_{imaginary\ part}$$

The magnitude and phase angle are obtained as below

$$\begin{cases} |G(j\omega)| = \left|\frac{5}{j\omega(j\omega+1)(j\frac{\omega}{2}+1)}\right| = \frac{5}{|j\omega||j\omega+1||j\frac{\omega}{2}+1|} \\ \angle G(j\omega) = \angle\left(\frac{5}{j\omega(j\omega+1)(j\frac{\omega}{2}+1)}\right) = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) \end{cases}$$

Starting point: For $\omega \to 0^+ \quad \Rightarrow \quad G(j0) = \infty\angle -90^\circ$

Ending point: For $\omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0\angle -270^\circ$

We can also find the intersection of the Polar plot with the real and imaginary axis.

$$\mathrm{Re}[G(j\omega)] = 0 \quad \rightarrow \quad \frac{-30}{9\omega^2+(2-\omega^2)^2} = 0 \quad \rightarrow \quad \omega = \infty$$

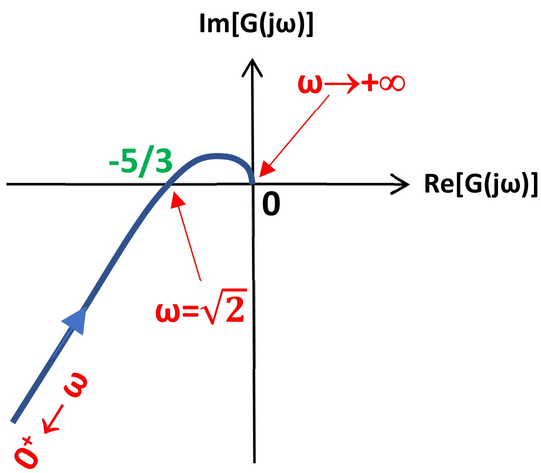
Therefore, the Polar plot intersects the imaginary axis only at the origin.

$$\mathrm{Im}[G(j\omega)] = 0 \quad \rightarrow \quad \frac{-10(2-\omega^2)}{9\omega^3+\omega(2-\omega^2)^2} = 0 \quad \rightarrow \quad \omega = \pm\sqrt{2}$$

We can find the value of $G(j\omega)$ at these frequencies

$$G(\pm j\sqrt{2}) = -\frac{5}{3}$$

Note that $\omega = -\sqrt{2}$ represent the point on the Polar plot for $-\infty < \omega < 0$. We know that the Polar plot for $-\infty < \omega < 0$ is mirror image of the Polar plot of $0 < \omega < +\infty$ with respect to the real axis.

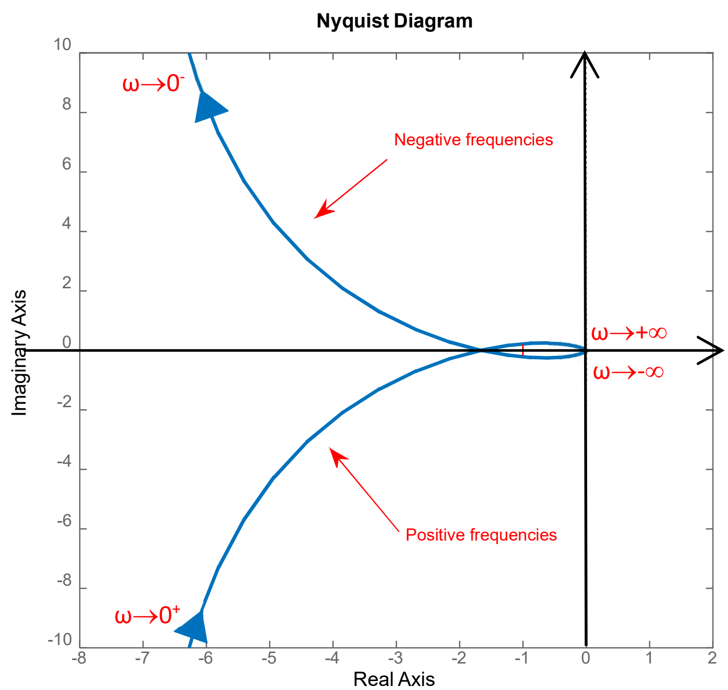


For $\omega \to \infty$ the graph is tangent to the positive imaginary axis.

We can also plot the Polar plot (Nyquist diagram) of $G(s)$ by MATLAB as below.

$$G(s) = \frac{10}{s(s+1)(s+2)}$$

```
num = [10];
den = [1 3 2 0];
sys = tf(num,den);
nyquist(sys)
```



c) $G(s) = \frac{\mathbf{10}}{(s+\mathbf{1})(s+\mathbf{2})}$

The frequency response function is

$$G(j\omega) = \frac{5}{(j\omega + 1)(j\frac{\omega}{2} + 1)}$$

The real part and the imaginary part of the $G(j\omega)$ are obtained as below

$$G(j\omega) = \frac{5}{(j\omega + 1)(j\frac{\omega}{2} + 1)} \times \frac{(-j\omega + 1)(-j\frac{\omega}{2} + 1)}{(-j\omega + 1)(-j\frac{\omega}{2} + 1)}$$

$$G(j\omega) = \underbrace{\frac{10(2-\omega^2)}{9\omega^2 + (2-\omega^2)^2}}_{real\ part} + j\underbrace{\frac{-30\omega}{9\omega^2 + (2-\omega^2)^2}}_{imaginary\ part}$$

The magnitude and phase angle are obtained as below

$$\begin{cases} |G(j\omega)| = \left|\frac{5}{(j\omega + 1)(j\frac{\omega}{2} + 1)}\right| = \frac{5}{|j\omega + 1||j\frac{\omega}{2} + 1|} \\ \angle G(j\omega) = \angle\left(\frac{5}{(j\omega + 1)(j\frac{\omega}{2} + 1)}\right) = -\tan^{-1}(\omega) - \tan^{-1}(\frac{\omega}{2}) \end{cases}$$

Starting point: For $\omega \to 0^+ \quad \Rightarrow \quad G(j0) = 5\angle 0^\circ$

Ending point: For $\omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0\angle -180^\circ$

We can also find the intersection of the Polar plot with the real and imaginary axes.

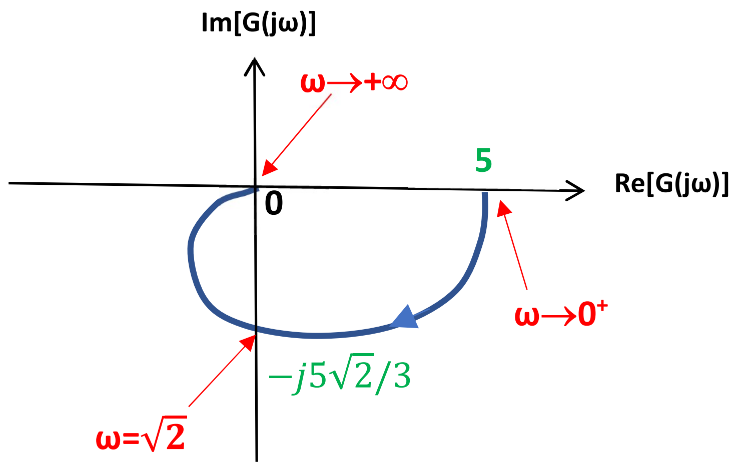
$$\mathrm{Re}[G(j\omega)] = 0 \quad \rightarrow \quad \frac{10(2-\omega^2)}{9\omega^2 + (2-\omega^2)^2} = 0 \quad \rightarrow \quad \omega = \pm\sqrt{2}$$

$$\mathrm{Im}[G(j\omega)] = 0 \quad \rightarrow \quad \frac{-30\omega}{9\omega^2 + (2-\omega^2)^2} = 0 \quad \rightarrow \quad \omega = 0$$

We can find the value of $G(j\omega)$ at these frequencies

$$G(\pm j\sqrt{2}) = \mp j\frac{5\sqrt{2}}{3} \qquad \text{and} \qquad G(j0) = 5$$

Note that $\omega = -\sqrt{2}$ represent the point on the Polar plot for $-\infty < \omega < 0$. We know that the Polar plot for $-\infty < \omega < 0$ is mirror image of the Polar plot of $0 < \omega < +\infty$ with respect to the real axis.

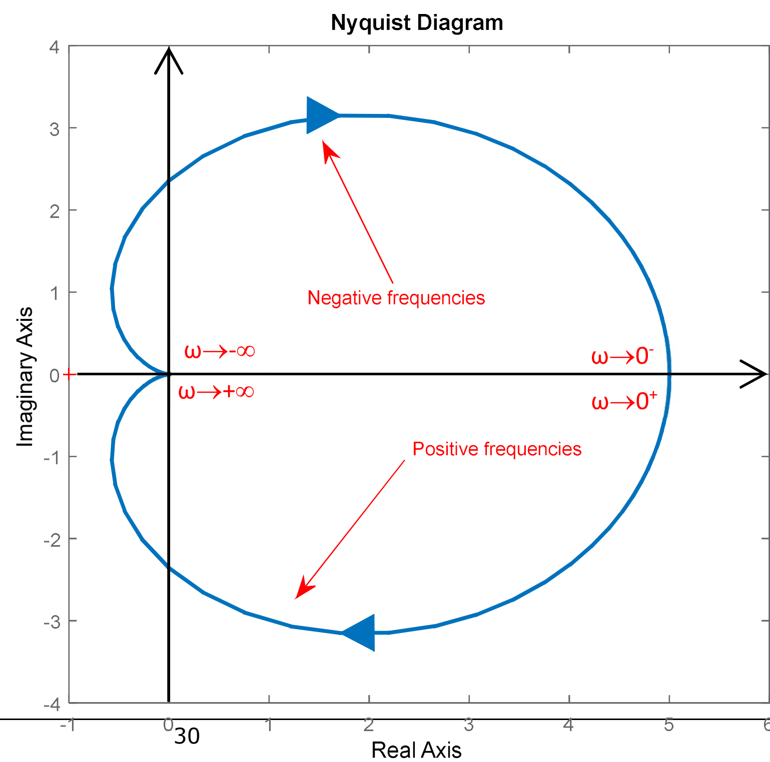


For $\omega \to \infty$ the graph is tangent to the negative real axis.

We can also plot the Polar plot (Nyquist diagram) of $G(s)$ by MATLAB as below.

$$G(s) = \frac{10}{(s+1)(s+2)}$$

```
num = [10];
den = [1 3 2];
sys = tf(num,den);
nyquist(sys)
```



d) $G(s) = \frac{s+5}{s+10}$

The frequency response function is

$$G(j\omega) = 0.5\frac{(j\frac{\omega}{5}+1)}{(j\frac{\omega}{10}+1)}$$

The real part and the imaginary part of the $G(j\omega)$ are obtained as below

$$G(j\omega) = 0.5\frac{(j\frac{\omega}{5}+1)}{(j\frac{\omega}{10}+1)} \times \frac{(-j\frac{\omega}{10}+1)}{(-j\frac{\omega}{10}+1)}$$

$$G(j\omega) = \underbrace{\frac{50+\omega^2}{\omega^2+100}}_{real\ part} + j\underbrace{\frac{5\omega}{\omega^2+100}}_{imaginary\ part}$$

The magnitude and phase angle are obtained as below

$$\begin{cases} |G(j\omega)| = \left|\dfrac{0.5(j\frac{\omega}{5}+1)}{(j\frac{\omega}{10}+1)}\right| = \dfrac{0.5\left|j\frac{\omega}{5}+1\right|}{|j\frac{\omega}{10}+1|} \\ \angle G(j\omega) = \angle\left(\dfrac{0.5(j\frac{\omega}{5}+1)}{(j\frac{\omega}{10}+1)}\right) = \tan^{-1}\left(\dfrac{\omega}{5}\right) - \tan^{-1}(\dfrac{\omega}{10}) \end{cases}$$

Starting point: For $\omega \to 0^+ \Rightarrow G(j0) = 0.5\angle 0^\circ$

For $\omega = 5 \Rightarrow G(j5) = \sqrt{\frac{2}{5}}\angle 18.5^\circ$

For $\omega = 10 \Rightarrow G(j5) = \sqrt{\frac{5}{8}}\angle 18.5^\circ$

Ending point: For $\omega \to +\infty \Rightarrow G(j\infty) = 1\angle 18.5^\circ$

We can also find the intersection of the Polar plot with the real and imaginary axes.

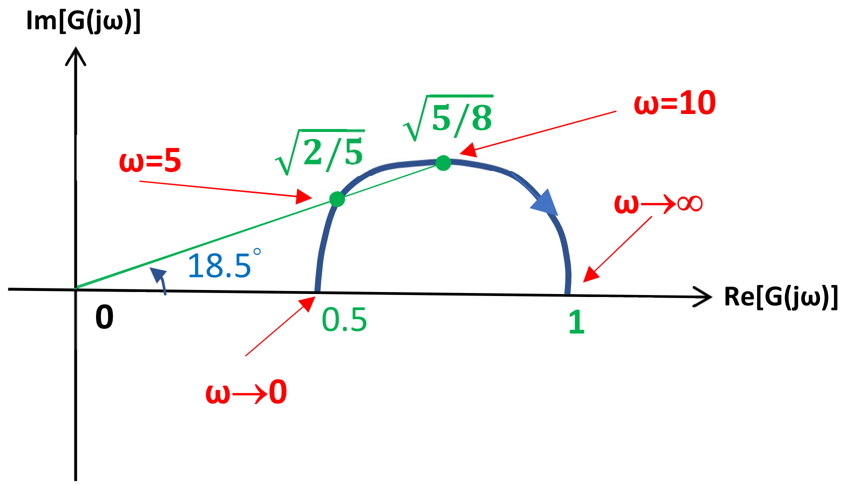
$$\mathrm{Re}[G(j\omega)] = 0 \quad \to \quad \frac{50+\omega^2}{\omega^2+100} = 0 \quad \to \quad \omega^2 = -50$$

It can be seen that $G(j\omega)$ does not intersect the imaginary axis

$$\mathrm{Im}[G(j\omega)] = 0 \quad \to \quad \frac{5\omega}{\omega^2+100} = 0 \quad \to \quad \omega = 0, \qquad \omega = \infty$$

We can find the value of $G(j\omega)$ at these frequencies

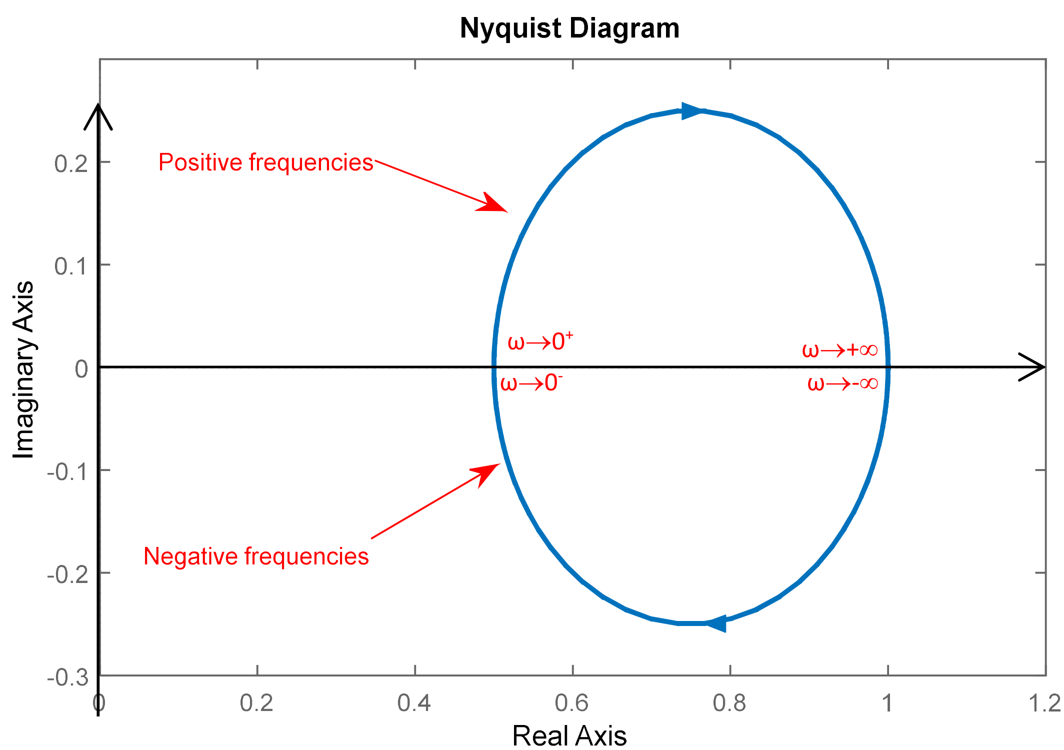
$$G(j0) = 0.5 \qquad \text{and} \qquad G(j\infty) = 1$$



We can also plot the Polar plot (Nyquist diagram) of $G(s)$ by MATLAB as below.

$$G(s) = \frac{s+5}{s+10}$$

```
num = [1 5];
den = [1 10];
sys = tf(num,den);
nyquist(sys)
```



e) $G(s) = \frac{6}{(s+1)(s+2)(s+3)}$

The frequency response function is

$$G(j\omega) = \frac{1}{(j\omega + 1)(j\frac{\omega}{2} + 1)(j\frac{\omega}{3} + 1)}$$

The real part and the imaginary part of the $G(j\omega)$ are obtained as below

$$G(j\omega) = \frac{1}{(j\omega + 1)(j\frac{\omega}{2} + 1)(j\frac{\omega}{3} + 1)} \times \frac{(-j\omega + 1)(-j\frac{\omega}{2} + 1)(-j\frac{\omega}{3} + 1)}{(-j\omega + 1)(-j\frac{\omega}{2} + 1)(-j\frac{\omega}{3} + 1)}$$

$$G(j\omega) = \underbrace{\frac{1 - \omega^2}{(\omega^2 + 1)(\frac{\omega^2}{4} + 1)(\frac{\omega^2}{9} + 1)}}_{real\ part} + j\underbrace{\frac{\omega(\omega^2 - 11)}{6(\omega^2 + 1)(\frac{\omega^2}{4} + 1)(\frac{\omega^2}{9} + 1)}}_{imaginary\ part}$$

The magnitude and phase angle are obtained as below

$$\begin{cases} |G(j\omega)| = \dfrac{1}{|j\omega + 1|\left|j\frac{\omega}{2} + 1\right||j\frac{\omega}{3} + 1|} \\ \angle G(j\omega) = \angle\left(\dfrac{1}{(j\omega + 1)(j\frac{\omega}{2} + 1)(j\frac{\omega}{3} + 1)}\right) = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}(\frac{\omega}{3}) \end{cases}$$

Starting point: For $\omega \to 0^+ \quad \Rightarrow \quad G(j0) = 1\angle 0^{\circ}$

Ending point: For $\omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0\angle{-270^{\circ}}$

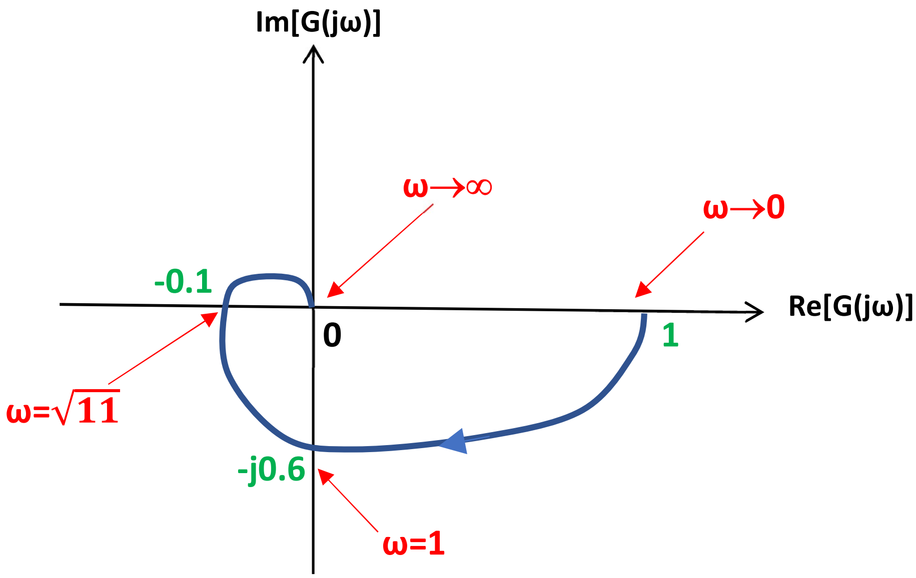
We can also find the intersection of the Polar plot with the real and imaginary axes.

$$\mathrm{Re}[G(j\omega)] = 0 \quad \to \quad \frac{1 - \omega^2}{(\omega^2 + 1)(\frac{\omega^2}{4} + 1)(\frac{\omega^2}{9} + 1)} = 0 \quad \to \quad \omega = \pm 1$$

$$\mathrm{Im}[G(j\omega)] = 0 \quad \to \quad \frac{\omega(\omega^2 - 11)}{6(\omega^2 + 1)(\frac{\omega^2}{4} + 1)(\frac{\omega^2}{9} + 1)} = 0 \quad \to \quad \omega = 0, \omega = \pm\sqrt{11}$$

We can find the value of $G(j\omega)$ at these frequencies

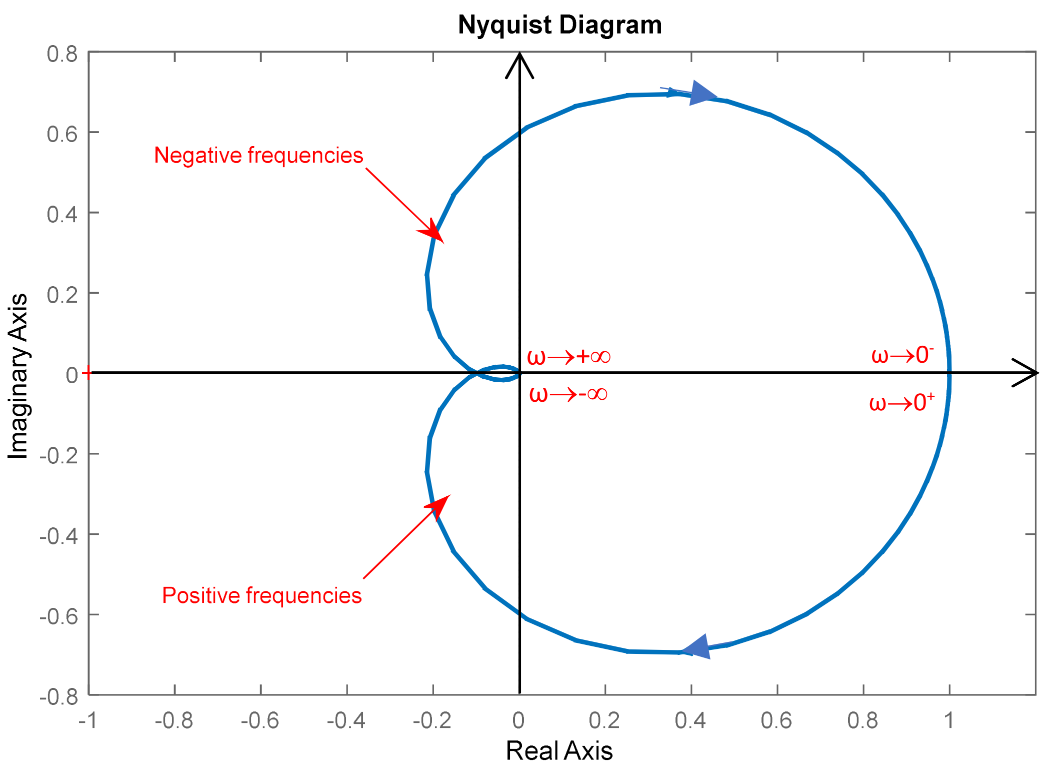
$$G(\pm j1) = \mp j0.6 \quad \text{and} \quad G(j0) = 1 \quad \text{and} \quad G\left(\pm j\sqrt{11}\right) = -0.1$$



We can also plot the Polar plot (Nyquist diagram) of $G(s)$ by MATLAB as below.

$$G(s) = \frac{6}{(s+1)(s+2)(s+3)}$$

```
num = [6];
den = [1 6 11 6];
sys = tf(num,den);
nyquist(sys)
```



f) $G(s) = \frac{1}{s(2s+1)}$

The frequency response function is obtained as below

$$G(j\omega) = \frac{1}{j\omega(j2\omega+1)}$$

The magnitude and phase angle are obtained as below

$$\begin{cases} |G(j\omega)| = \left|\frac{1}{j\omega(j2\omega+1)}\right| = \frac{1}{|j\omega||j2\omega+1|} \\ \angle G(j\omega) = \angle\left(\frac{1}{j\omega(j2\omega+1)}\right) = -90^\circ - \tan^{-1}(2\omega) \end{cases}$$

The real part and the imaginary part are obtained as below

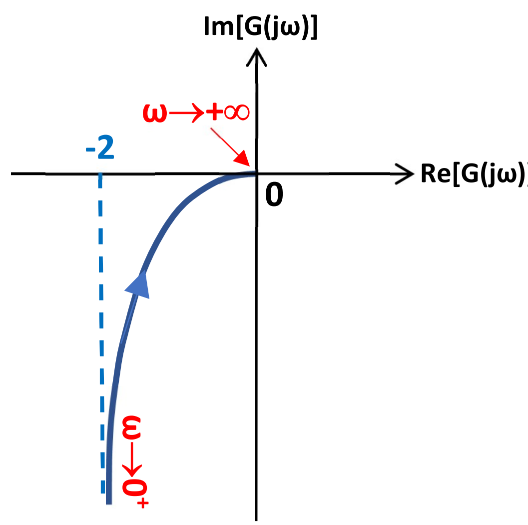
$$\mathrm{Re}[G(j\omega)] + j\mathrm{Im}[G(j\omega)] = \frac{-2}{(2\omega)^2+1} - j\frac{1}{\omega((2\omega)^2+1)}$$

Starting point: For $\omega \to 0^+ \Rightarrow G(j0^+) = \infty\angle -90^\circ$

Ending point: For $\omega \to +\infty \Rightarrow G(j\infty) = 0\angle -180^\circ$

$$Re[G(j\omega)] = \frac{-2}{(2\omega)^2+1}$$

$$Re[G(j0^+)] = \frac{-2}{(2\times 0)^2+1} = -2$$



For $\omega \to 0^+$ the graph is tangent to the line of $Re[G(j0^+)] = -2$.

For $\omega \to +\infty$ the graph is tangent to the negative real axis.

g) $G(s) = \frac{s+1}{s(2s+1)}$

The frequency response function is obtained as below

$$G(j\omega) = \frac{j\omega + 1}{j\omega(j2\omega + 1)}$$

The magnitude and phase angle are obtained as below

$$\begin{cases} |G(j\omega)| = \left|\frac{j\omega + 1}{j\omega(j2\omega + 1)}\right| = \frac{|j\omega + 1|}{|j\omega||j2\omega + 1|} \\ \angle G(j\omega) = \angle\left(\frac{j\omega + 1}{j\omega(j2\omega + 1)}\right) = \tan^{-1}(\omega) - 90^{\circ} - \tan^{-1}(2\omega) \end{cases}$$

The real part and the imaginary part are obtained as below

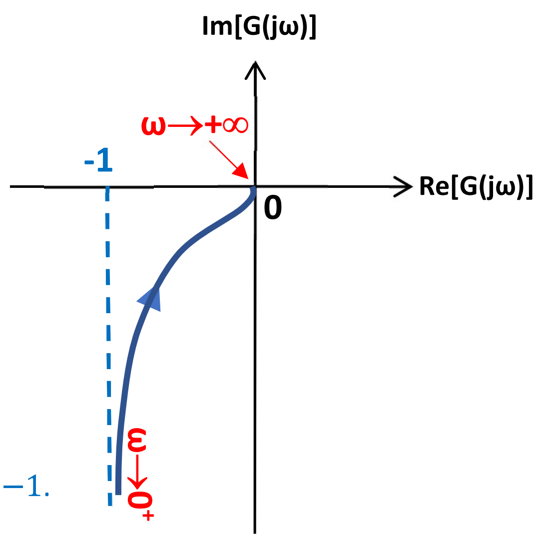
$$\mathrm{Re}[G(j\omega)] + j\mathrm{Im}[G(j\omega)] = \frac{-1}{(2\omega)^2 + 1} - j\frac{1 + 2\omega^2}{\omega((2\omega)^2 + 1)}$$

Starting point: For $\omega \to 0^+ \quad \Rightarrow \; G(j0^+) = \infty\angle{-90^{\circ}}$

Ending point: For $\omega \to +\infty \quad \Rightarrow \; G(j\infty) = 0\angle - 90^{\circ}$

$$Re[G(j\omega)] = \frac{-1}{(2\omega)^2 + 1}$$

$$Re[G(j0^+)] = \frac{-1}{(2 \times 0)^2 + 1} = -1$$



For $\omega \to 0^+$ the graph is tangent to the line of $Re[G(j0^+)] = -1$.

For $\omega \to +\infty$ the graph is tangent to the negative imaginary axis.

h) $G(s) = \frac{10(2s+1)}{s^2(0.5s+1)}$

The frequency response function is obtained as below

$$G(j\omega) = \frac{10(j2\omega + 1)}{(j\omega)^2(j0.5\omega + 1)}$$

The magnitude and phase angle are obtained as below

$$\begin{cases} |G(j\omega)| = \left|\frac{10(j2\omega + 1)}{(j\omega)^2(j0.5\omega + 1)}\right| = \frac{10|j2\omega + 1|}{|j\omega||j\omega||j0.5\omega + 1|} \\ \angle G(j\omega) = \angle\left(\frac{10(j2\omega + 1)}{(j\omega)^2(j0.5\omega + 1)}\right) = \tan^{-1}(2\omega) - 180^\circ - \tan^{-1}(0.5\omega) \end{cases}$$

The real part and the imaginary part are obtained as below

$$\mathrm{Re}[G(j\omega)] + j\mathrm{Im}[G(j\omega)] = \frac{-10(\omega^2 + 1)}{\omega^2((0.5\omega)^2 + 1)} - j\frac{15}{\omega((0.5\omega)^2 + 1)}$$

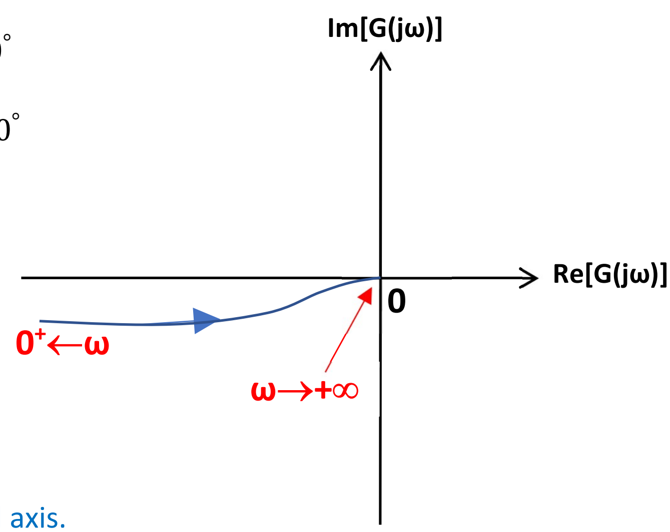
Strating point: For $\omega \to 0^+ \Rightarrow G(j0) = \infty\angle{-180^\circ}$

Ending point: For $\omega \to +\infty \Rightarrow G(j\infty) = 0\angle - 180^\circ$

Here, for $\omega \to 0^+$ we have:

$\mathrm{Re}[G(j0^+)] < 0$

$\mathrm{Im}[G(j0^+)] < 0$



For $\omega \to 0^+$ the graph is tangent to the negative real axis.

For $\omega \to +\infty$ the graph is tangent to the negative real axis.