LAB 3: DC MOTOR PARAMETER ESTIMATION & MODELING

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LAB 3 Grading Sheet

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Part 1: Transfer Function Model from Nominal Values	**/15**
Part 2: DC Motor Parameter Estimation	**/30**
General Formatting: **Clarity, Writing style, Grammar, Spelling, Layout of the report**	**/5**
Total Mark	**/50**

LAB 3: DC MOTOR PARAMETER ESTIMATION & MODELING

OBJECTIVES

- To obtain equation of motion of the DC motor in QUBE-Servo 3 system
- To determine transfer function model of the DC motor using equation of motion
- To estimate the DC motor parameters and update the transfer function model

DISCUSSIONS OF FUNDAMENTALS

FIRST-PRINCIPLES SYSTEM MODELING

The **QUBE-Servo 3** employs a brushed DC motor connected to a disc payload. The system schematic is shown in Figure 1 and the electrical and mechanical parameters are given in **Table 1**. The rotating shaft of the DC motor with mass moment of inertia J_m is connected to the ***load hub***. This hub is a metal disk that has an inertia of J_h and contains permanent magnets used to attach the disc or rotary pendulum payload. The disc has a moment of inertia of J_d. The total moment of inertia, J_{eq}, of the rotating parts can be expressed as,

$$J_{eq} = J_m + J_h + J_d \tag{1}$$

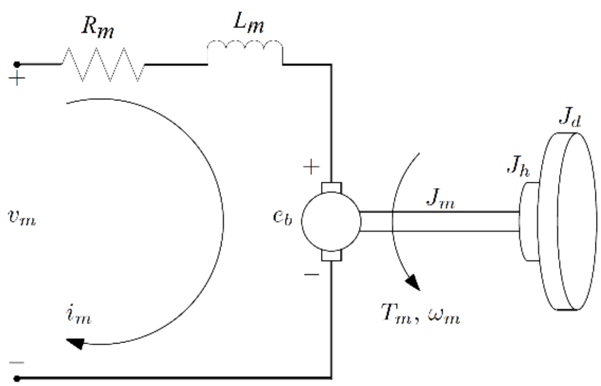


Figure 1. QUBE-Servo 3 DC motor and load

Table 1: QUBE-Servo 3 System Parameters

Symbol	Description	Value
R_m	Rotor resistance	$7.5\ \Omega$
L_m	Rotor inductance	$1.15\ \times 10^{-3}\ H$
J_m	Rotor inertia	$1.4 \times 10^{-6}\ kg.m^2$
k_m	Motor back-emf constant	$0.0422\ V/(rad/s)$
k_t	Motor torque constant	$0.0422\ N.m/A$
J_h	Load hub inertia	$0.6 \times 10^{-6}\ kg.m^2$
m_d	Load disc mass	$0.053\ kg$
r_d	Load disc radius	$0.0248\ m$

The back-emf (electromotive) voltage $e_b(t)$ depends on the speed of the motor shaft, $\omega_m(t)$, and the back-emf voltage constant of the motor k_m. The back-emf voltage opposes the current flow (the applied voltage $v_m(t)$),

$$e_b(t) = k_m \omega_m(t) \tag{2}$$

Applying Kirchoff's Voltage Law to the armature circuit, we can obtain the ***electrical subsystem equation***:

$$v_m(t) - R_m i_m(t) - L_m \frac{di_m(t)}{dt} - k_m \omega_m(t) = 0 \tag{3}$$

Since the motor inductance L_m is much less than its resistance, it can be ignored. Then, the ***electrical subsystem equation*** becomes:

$$v_m(t) - R_m i_m(t) - k_m \omega_m(t) = 0 \tag{4}$$

Solving for $i_m(t)$, the motor current can be found as:

$$i_m(t) = \frac{v_m(t) - k_m \omega_m(t)}{R_m} \tag{5}$$

Applying Newton's second law to the motor shaft yields the ***mechanical subsystem equation***:

$$J_{eq} \frac{d\omega_m(t)}{dt} = T_m(t) \tag{6}$$

where T_m is the applied torque from the DC motor, which is a linear function of armature current,

$$T_m(t) = k_t i_m(t) \tag{7}$$

TRANSFER FUNCTION MODEL

The electrical equation and the mechanical equation are brought together to get an expression that represents the motor shaft speed $\omega_m(t)$ in terms of the applied motor voltage $v_m(t)$.

$$J_{eq} \frac{d\omega_m(t)}{dt} = \tau_m(t) \quad \rightarrow \quad J_{eq} \frac{d\omega_m(t)}{dt} = k_t i_m(t) \quad \rightarrow \quad J_{eq} \frac{d\omega_m(t)}{dt} = k_t \left(\frac{v_m(t) - k_m \omega_m(t)}{R_m} \right)$$

After collecting the terms, the ***equation of motion*** becomes,

$$J_{eq} \frac{d\omega_m(t)}{dt} + \frac{k_t k_m}{R_m} \omega_m(t) = \frac{k_t}{R_m} v_m(t) \tag{8}$$

Taking Laplace transform assuming the zero initial condition to find the **voltage-to-speed** transfer function of the servo system,

$$J_{eq} s \Omega_m(s) + \frac{k_t k_m}{R_m} \Omega_m(s) = \frac{k_t}{R_m} V_m(s) \tag{9}$$

The transfer function model is obtained as follows,

$$G(s) = \frac{\Omega_m(s)}{\mathrm{V}_m(s)} = \frac{k_t}{J_{eq} R_m s + k_t k_m} \tag{10}$$

It can be reform as the standard first-order transfer function model,

$$G(s) = \frac{\Omega_m(s)}{\mathrm{V}_m(s)} = \frac{\frac{1}{k_m}}{\frac{J_{eq} R_m}{k_t k_m} s + 1} \coloneqq \frac{K}{\tau s + 1}$$

where K is the ***steady-state gain*** or the ***DC gain*** of the model, and τ is the ***time-constant*** of the model.

$$K = \frac{1}{k_m}, \qquad \tau = \frac{J_{eq} R_m}{k_t k_m} \tag{11}$$

PART 1: Transfer Function Model from Nominal Values

1. The moment of inertia of a disk J_d with mass m_d and radius r_d about its pivot is obtained as,

$$J_d = \frac{1}{2} m_d r_d^2$$

Determine the mass moment of inertia of the load inertia disk, J_d, based on the given disc radius, r_d, and disk mass, m_d, from **Table 1**. Then calculate the equivalent mass moment of inertia J_{eq} from Eqn. (1) based on the given values from **Table 1**. Show your calculations below.

$$J_d = \frac{1}{2} m_d r_d^2 = \frac{1}{2}(0.053)(0.0248)^2 = 1.63 * 10^{-5}\ kg.m^2$$

$$J_{eq} = J_m + J_h + J_d = (1.4 * 10^{-6}) + (0.6 * 10^{-6}) + (16 * 10^{-6}) = 1.83 * 10^{-5}\ kg.m^2$$

2. Given the parameters in **Table 1** and using Eqn. (11) and (10) calculate the **DC gain** and the **time constant** of the first-order transfer function model and derive **transfer function**. Show your calculations below and provide the results in **Table 2**.

$$K = \frac{1}{K_m} = \frac{1}{0.0422} = 23.7 \frac{\frac{rads}{s}}{V}$$

$$Time\ constant = \tau = \frac{J_{eq} R_m}{k_t K_m} = \frac{(1.8 * 10^{-5})(7.5)}{(0.0422)(0.0422)} = 0.076\ seconds$$

$$Transfer\ function = G(s) = \frac{K}{\tau s + 1} = \frac{23.7}{0.076s + 1}$$

Table 2 – Transfer Function Model from Nominal Values

DC-Gain (rad/s/V)	Time Constant (sec)	Transfer Function Model
23.7	0.076	$\frac{23.7}{0.076s + 1}$

PART 2: DC Motor Parameters Estimation

Stall is the condition in which the motor shaft velocity is **zero**, $\omega_m = 0$. In this case, assuming the armature inductance L_m is negligible, it is possible to determine the ***armature resistance*** R_m of the motor. From Eqn. (4) we can derive an expression to determine the ***armature resistance*** having the voltage and current:

$$v_m(t) - R_m i_m(t) - k_m \omega_m(t) = 0 \quad \xrightarrow{\omega_m = 0} \quad \boldsymbol{R_m = \frac{v_m(t)}{i_m(t)}} \tag{12}$$

Assuming the armature resistance R_m is known, we can identify the ***motor back-emf constant*** k_m experimentally. This is done by allowing the motor to rotate freely and allowing it to reach a steady-state speed for a given voltage $v_m(t)$ and measuring the corresponding current $i_m(t)$. Then we can determine the ***motor back-emf constant*** from Eqn. (4),

$$v_m(t) - R_m i_m(t) - k_m \omega_m(t) = 0 \quad \rightarrow \quad \boldsymbol{k_m = \frac{v_m(t) - R_m i_m(t)}{\omega_m(t)}} \tag{13}$$

3. Similar to the method in **Lab 2**, create the following system in **Simulink** to apply constant voltage to the motor and read the servo velocity and the current. You can find the required blocks below:

   QUARC Targets > Data Acquisition > Generic > Immediate I/O > HIL Write Analog

   QUARC Targets > Data Acquisition > Generic > Immediate I/O > HIL Read

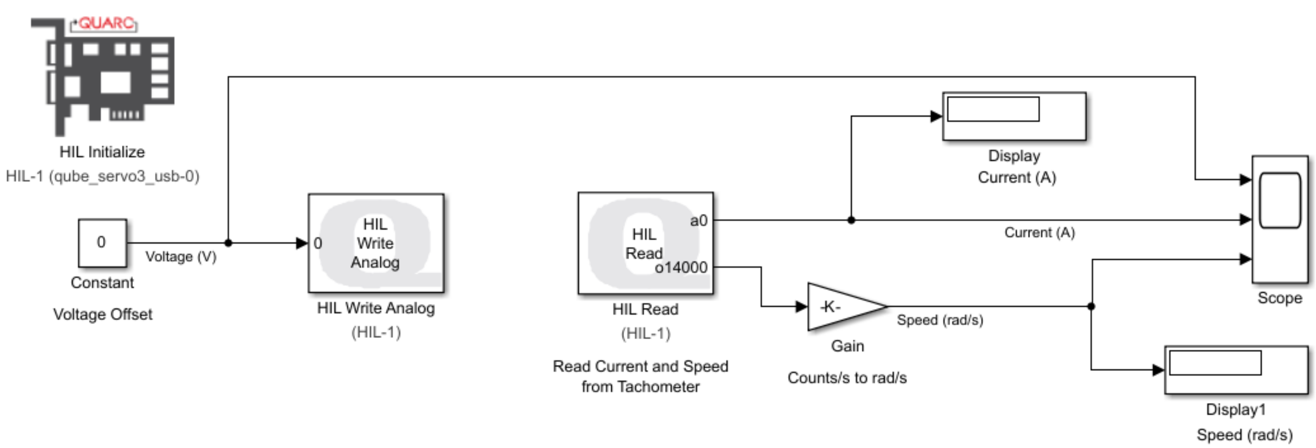
   Simulink > Sources > Constant

   Simulink > Sinks > Scope

   Simulink > Sink > Display

   Simulink > Math Operations > Gain

   Remark*: We are using a conversion gain of $2\pi/2048$ to convert the tachometer output to rad/s.*



4. Open the **HIL Initialize** block and set the **Board type** to **qube_servo3_usb**.

5. Click on the **Model Settings** icon in the **MODELING** tab to open the **Configuration Parameters** window. Click on the **Solver** drop down menu and select the **Type** of **Fixed step** and set the **Solver** to **ode1** solver. Then click **OK**.

6. The **HIL Read** block can read the servo velocity using the ***tachometer output on channel 14000***. It can also read from the current sensor on ***analog input channel #0***. Set the **HIL Read** block parameters as below:

   Analog channels = 0, Encoder channels = [], Digital channels = [], Other channels = [14000]

7. Set the **Constant** block to **zero**.
8. Set the **Display** block **Decimation** value to **50**.
9. **Save** the Simulink file as **Lab3.slx**. Set the **Stop Time** to **inf**. Click on **Monitor & Tune** to run your code.

ESTIMATE the MOTOR RESISTANCE

10. To experimentally estimate the **motor resistance** R_m, apply a set of voltages to the DC motor by setting the **Constant** block value according to the given values in **Table 3**. For each measurement, hold the motor shaft stationery by grasping the inertial disc load to *stall the motor* and *record the current measurement* displayed in the **Current (A) Display**. Fill the following table with the measured current for different voltages and calculate the corresponding resistance from **Eqn. (12)**.

NOTE: Do NOT hold the motor in stall mode for more than 10 seconds.

Table 3 – Motor Resistance Experimental Results

Applied Voltage (V) $v_m(t)$	Measured Current (A) $i_m(t)$	Motor Resistance (Ω) R_m
-5.0 V	-0.69	7.24
-4.0 V	-0.55	7.27
-3.0 V	-0.42	7.14
-2.0 V	-0.29	6.89
-1.0 V	-0.15	6.67
+1.0 V	0.15	6.67
+2.0 V	0.29	6.89
+3.0 V	0.42	7.14
+4.0 V	0.55	7.27
+5.0 V	0.69	7.24

11. Take the average of all the measured resistance values in **Table 3** and compare this with the motor resistance R_m nominal value from **Table 1.** Calculate the percentage of difference.

The average of the measured resistance values is 7.042 Ohms.
When compared to the motor resistance, the average resistance is lower.
Percentage difference $= \frac{7.5-7.042}{7.5} * 100\% = 6.11\%$

ESTIMATE the MOTOR BACK-EMF CONSTANT

12. To experimentally estimate the **motor back-emf constant** k_m, repeat the same procedure by applying different voltage to the DC motor with the *motor free to spin* (i.e. **do not stall the motor**) and record the *measured speed* and *current* in **Table 4**. Fill the following table with the measured speed and current for different voltages and calculate the corresponding back-emf constant from **Eqn. (13).** Use the calculated average motor resistance R_m from **Step 11**.

NOTE: Consider the average speed and average current values if there are fluctuations on the waveforms.

Table 4 – Back-emf Experimental Results

Applied Voltage (V)	Measured Speed (rad/s)	Measured Current (A)	Motor back-emf constant k_m
-5.0 V	-129	-0.007	0.0384
-4.0 V	-106	-0.007	0.0373
-3.0 V	-83	-0.006	0.0356
-2.0 V	-59	-0.005	0.0333
-1.0 V	-32	-0.005	0.0302
+1.0 V	32	0.005	-0.0302
+2.0 V	59	0.006	0.0332
+3.0 V	83	0.006	0.0356
+4.0 V	106	0.007	0.0373
+5.0 V	129	0.007	0.0384

13. Take the average of all the measured back-emf constant values in **Table 4** and compare this with the given nominal k_m value in **Table 1.** Calculate the percentage of difference.

The average of the measured back-emf constant values is 0.03494.
When compared to the km, the average back-emf constat is much lower.
Percentage difference $= \frac{0.0422-0.03494}{0.0422} * 100\% = 17.2\%$

14. Based on the estimated R_m and k_m and using Eqn. (11) and (10) recalculate the **DC gain** and the **time constant** of the first-order transfer function model and derive **transfer function**. Show your calculations below and provide the results in **Table 5**.

$$K = \frac{1}{K_m} = \frac{1}{0.03494} = 28.6\frac{\frac{rads}{s}}{V}$$
$$Time\ constant = \tau = \frac{J_{eq}R_m}{k_t K_m} = \frac{(1.83 * 10^{-5})(7.042)}{(0.0422)(0.03494)} = 0.087\ seconds$$
$$Transfer\ function = G(s) = \frac{K}{\tau s+1} = \frac{28.6}{0.087s+1}$$

Table 5 – Transfer Function Model from Estimated Parameters

DC-Gain (rad/s/V)	Time Constant (sec)	Transfer Function Model
28.6	0.084	$\frac{28.6}{0.087s+1}$

15. Compare the result in **Table 5** with the transfer function obtained from the nominal values in **Table 2**.

When comparing the result in Table 5 with the transfer function obtained in Table 2, we observed that the result from Table 2 has a longer response time that that from table 1. The steady state value for the nominal values lower than the DC-gain for the estimated parameters. Hence, we can conclude that these results are different and not very similar in nature. This can be due to more environmental factors such as friction from the motor and human error when handling the QUBE-Servo 3.

16. **Stop** the model.
17. **Power OFF the QUBE-Servo 3 system.**