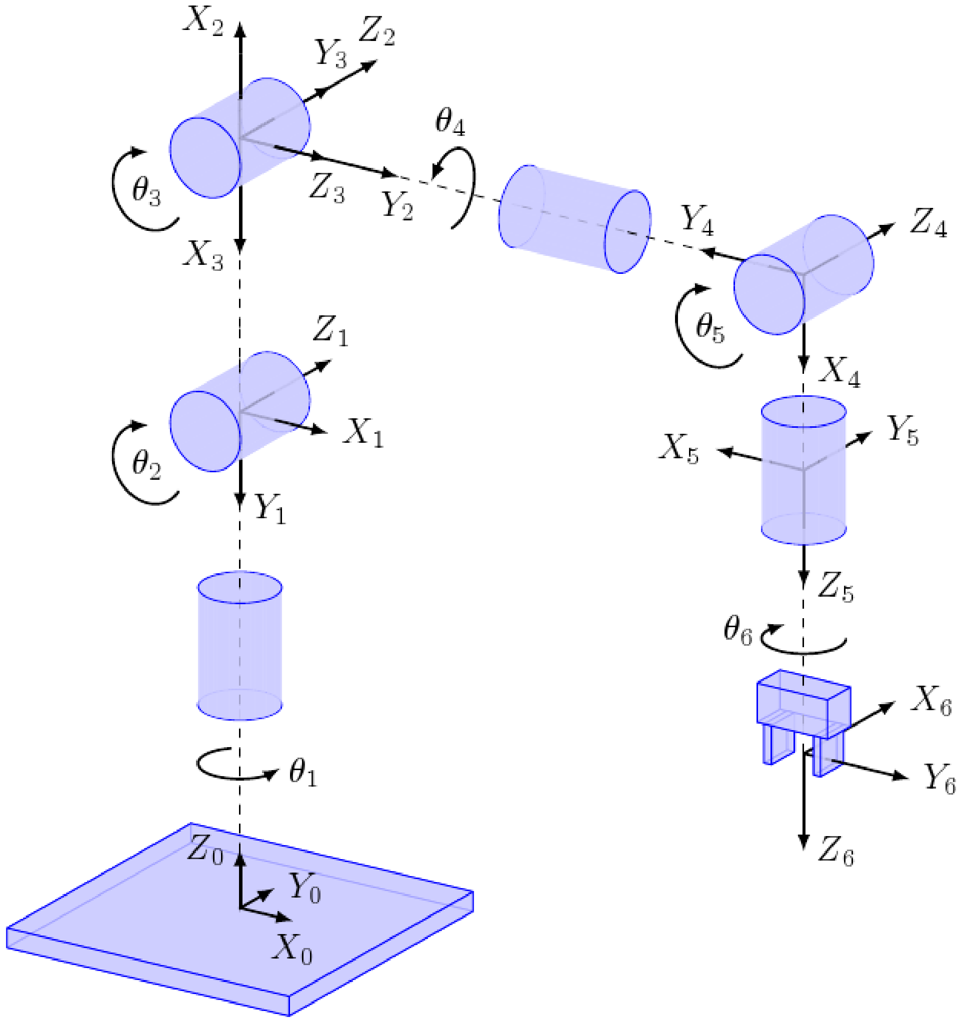
Kinematics and Dynamics of Robots

Module 8

Forward Kinematics Using D-H Parameters



Forward Kinematics Using D-H Parameters

Step 1: Locate and label the joint axes $z_0, \ldots, z_{n-1}$.

Step 2: Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axes are chosen conveniently to form a right-handed frame.

- **For** $i = 1, \ldots, n - 1$ perform Steps 3 to 5.

Step 3: Locate the origin o_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate o_i at this intersection. If z_i and z_{i-1} are parallel, locate o_i in any convenient position along z_i.

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through o_i, or in the direction normal to the $z_{i-1} - z_i$ plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right-handed frame.

Step 6: Establish the end-effector frame $o_n x_n y_n z_n$. Assuming the n^{th} joint is revolute, set $z_n = a$ parallel to z_{n-1}. Establish the origin o_n conveniently along z_n, preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set $y_n = s$ in the direction of the gripper closure and set $x_n = n$ as $s \times a$. If the tool is not a simple gripper set x_n and y_n conveniently to form a right-handed frame.

Step 7: Create a table of DH parameters a_i, d_i, α_i, θ_i.

a_i	=	distance along x_i from the intersection of the x_i and z_{i-1} axes to o_i.
d_i	=	distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. If joint i is prismatic, d_i is variable.
α_i	=	the angle from z_{i-1} to z_i measured about x_i.
θ_i	=	the angle from x_{i-1} to x_i measured about z_{i-1}. If joint i is revolute, θ_i is variable.

Step 8: Form the homogeneous transformation matrices A_i by substituting the above parameters into Equation 6.1.

Step 9: Form $T_n^0 = A_1 \ldots A_n$. This then gives the position and orientation of the tool frame expressed in base coordinates.

Forward Kinematics Using D-H Parameters

$$H_n^{n-1} = R_{z,\theta_n} T_{z,d_n} T_{x,r_n} R_{x,\alpha_n}$$

		θ	α	r	d
$0 \to 1$	1				
$1 \to 2$	2				
$2 \to 3$	3				

$$A_i = \mathrm{Rot}_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}\mathrm{Rot}_{x,\alpha_i}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

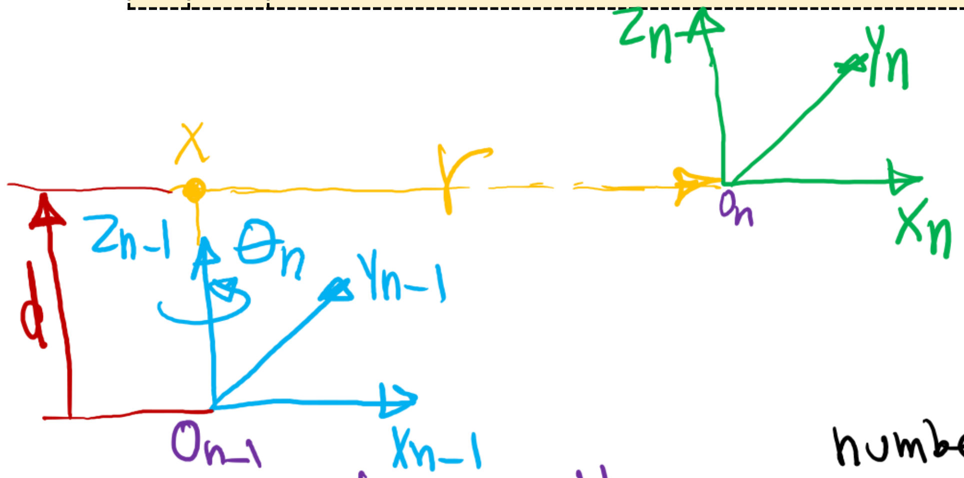
$$\times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \leftarrow \begin{bmatrix} c\theta_n & -s\theta_n c\alpha_n & s\theta_n s\alpha_n & r_n c\theta_n \\ s\theta_n & c\theta_n c\alpha_n & -c\theta_n s\alpha_n & r_n s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_n^0 = H_1^0 H_2^1 H_3^2 H_4^3 H_5^4 H_6^5$$

r_n

a_i	=	distance along x_i from the intersection of the x_i and z_{i-1} axes to o_i.
d_i	=	distance along z_{i-1} from o_{i-1} to the intersection of the x_i and z_{i-1} axes. If joint i is prismatic, d_i is variable.
α_i	=	the angle from z_{i-1} to z_i measured about x_i.
θ_i	=	the angle from x_{i-1} to x_i measured about z_{i-1}. If joint i is revolute, θ_i is variable.



$0 \rightarrow 1$

number of rows = #frames − 1

$1 \rightarrow 2$

θ	α	r	d
θ_n	ϕ	r	d

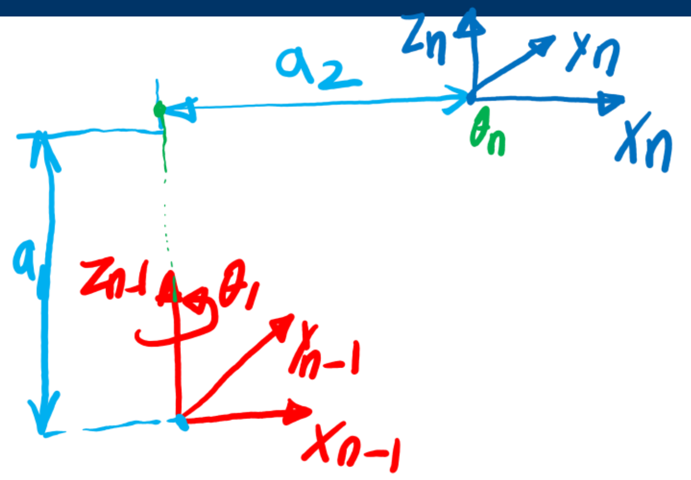
θ: rotation about Z_{n-1} (of frame n−1) that is required to match X_{n-1} to X_n including rotation of θ_n

α: rotation of frame(n−1) about X_n that is required to match Z_{n-1} to Z_n

r: the distance from the center of (n−1) frame (the intersection of (X_n, Z_{n-1})) to frame n O_n in X_n direction

d: the distance from the center of (n−1) frame (O_{n-1}) to frame n (intersection of (X_n, Z_{n-1})) in Z_{n-1} direction including d translation

Forward Kinematics Using D-H Parameters



θ: rotation about Z_{n-1} that is required to match X_{n-1} to X_n

α: rotation of n-1 frame about X_n that is required to match Z_{n-1} to Z_n

r: The distance between the center of (n-1) frame to n frame in X_n direction (intersection of X_n and Z_{n-1} to O_n in X_n direction)

d: The distance between the center of (n-1) frame (O_{n-1}) to n frame (O_n) in Z_{n-1} direction (O_{n-1} to intersection of X_n and Z_{n-1} in the direction of Z_{n-1})

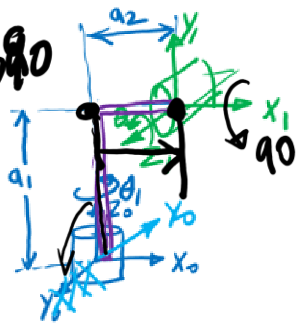
# of rows = Number of frames − 1

H		θ	α	r	d
H_1^0	$0 \rightarrow 1$	θ_1	0	a_2	a_1
H_2^1	$1 \rightarrow 2$				
⋮	⋮				

$$H_1^0 = R_{z,\theta} T_{z,d} T_{x,r} R_{x,\alpha} = R_{z,\theta_1} T_{z,a_1} T_{x,a_2} R_{x,0}$$

	θ	α	r	d
$0\to1$	θ_1	90	a_2	a_1

$$H_1^0 = R_{z,\theta_1} T_{z,a_1} T_{x,a_2} R_{x,90}$$



$$H_1^0 = \begin{bmatrix} R_1^0 & d_1^0 \\ 0\,0\,0 & 1 \end{bmatrix}$$

$d_1^0 = \{$ 1- frame movement $= d^0_{1_{fm}} = \begin{bmatrix} a_2 \\ 0 \\ a_1 \end{bmatrix}$

$$d_0^1 = R_{\theta_1, Z_0} \begin{bmatrix} a_2 \\ 0 \\ a_1 \end{bmatrix} = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & r_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & r_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1- θ: rotation of n-1 around Z_{n-1} to match $X_{n-1} \to X_n$

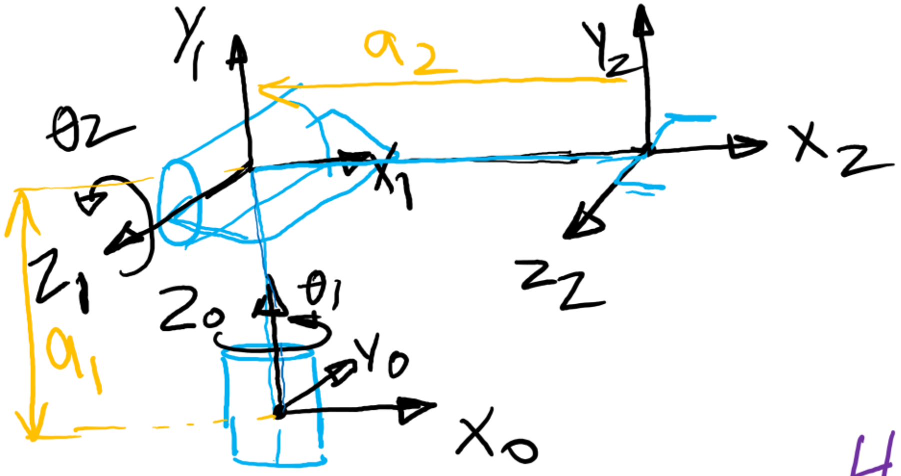
2- α: " " " X_n to match $Z_{n-1} \to Z_n$

r: distance of (X_1, Z_0) to O_1 in X_n

d: distance of O_0 to (X_1, Z_0) in Z_0

$$H_1^0 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & a_2 c\theta_1 \\ s\theta_1 & 0 & -c\theta_1 & a_2 s\theta_1 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c90 & -s90 & 0 \\ 0 & s90 & c90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & a_2 c\theta_1 \\ s\theta_1 & 0 & -c\theta_1 & a_2 s\theta_1 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



		θ	α	r	d
$0 \to 1$	1	θ_1	90	0	a_1
$1 \to 2$	2	θ_2	0	a_2	0

Number of rows $= 3 - 1 = 2$

$\uparrow$ # frames

$$H_2^0 = H_1^0 H_2^1$$

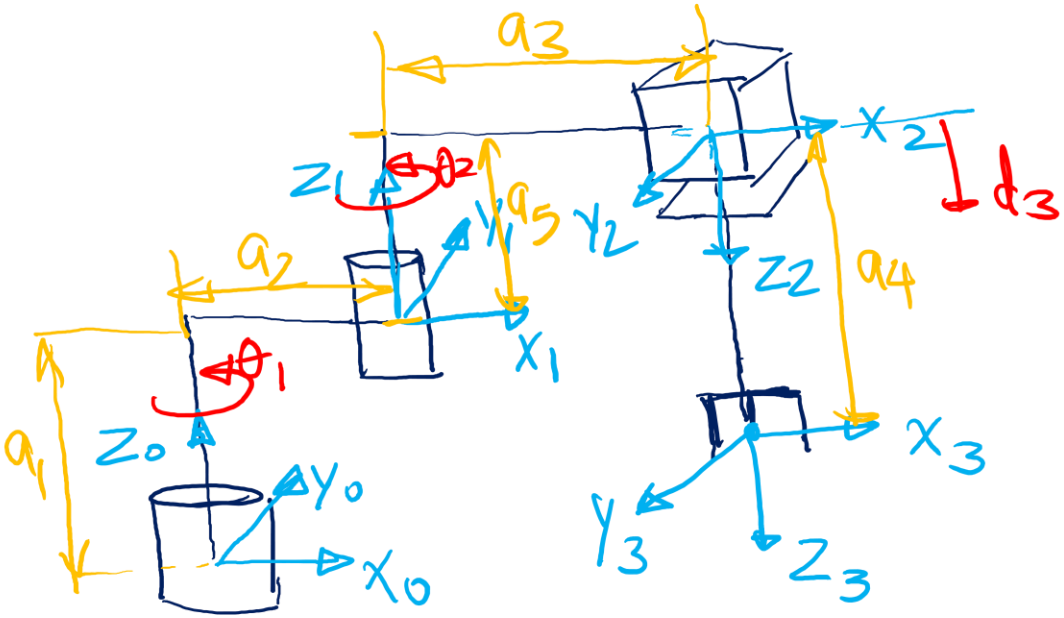
$$H_1^0 = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Plug in $\theta_1 = \theta_1$ $\alpha_1 = 90$ $r = 0$ $d = a_1$

$$H_2^1 = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

plug in $\theta_2 = \theta_2$ $\alpha_2 = 0$ $r_2 = a_2$ $d_2 = 0$

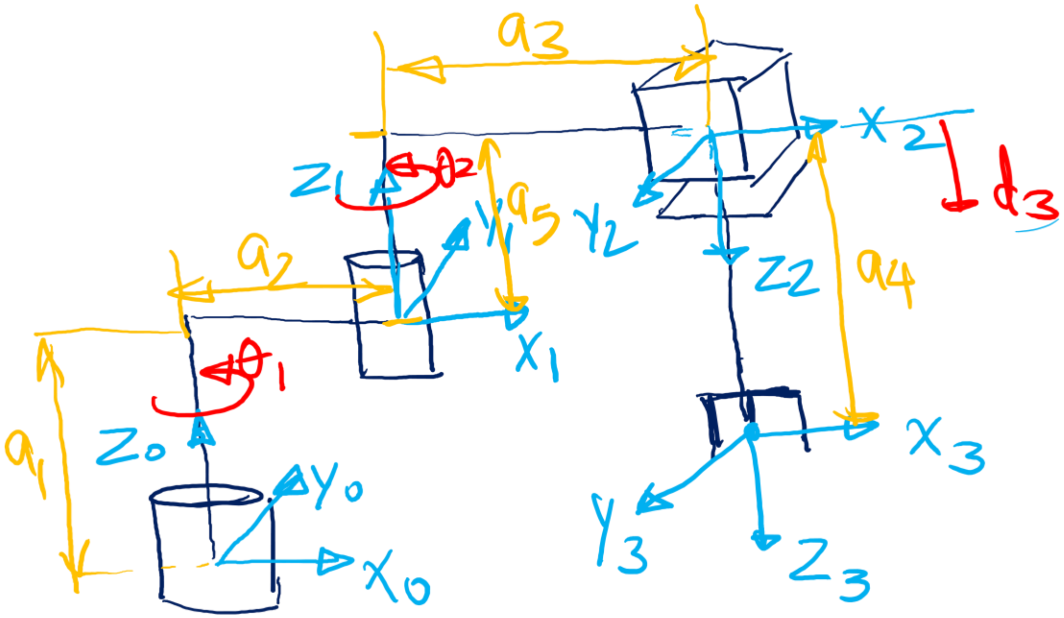
Forward Kinematics Using D-H Parameters



		θ	α	r	d
$0 \to 1$	1	θ_1	0	a_2	a_1
$1 \to 2$	2	θ_2	180	a_3	a_5
$2 \to 3$	3	0	0	0	$a_4 + d_3$

Forward Kinematics Using D-H Parameters

HUMBER



		θ	α	r	d	
$0 \rightarrow 1$	1	θ_1	0	a_2	a_1	H_1^0
$1 \rightarrow 2$	2	θ_2	180	a_3	a_5	H_2^1
$2 \rightarrow 3$	3	0	0	0	a_4+d_3	H_3^2

$$H_3^0 = H_1^0 H_2^1 H_3^2$$

Forward Kinematics Using D-H Parameters

HUMBER

$$H_1^0 = \begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \\ x_0 \\ y_0 \\ z_0 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & y_1 & z_1 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_1^0 = \underbrace{\begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R_{z,\theta_1}} \begin{bmatrix} a_2 \\ 0 \\ a_1 \\ 1 \end{bmatrix} =$$