NUMM 2500 **Humber College** **Quiz 4 (5%)** **Quiz Period: 50 min.** **April 03, 2024**

Student Name:

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Q 1. [**4 marks**] A <u>natural cubic</u> spline S on $[0, 2]$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & 0 \le x < 1 \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & 1 \le x \le 2 \end{cases}$$

Find b, c, and d.

$S_0'(x) = 2 - 3x^2$, $S_1'(x) = b + 2c(x-1) + 3d(x-1)^2$

$S_0''(x) = -6x$ $S_1''(x) = 2c + 6d(x-1)$

1. $S_0(1) = S_1(1) \mapsto 2 = 2$ ✓

2. $S_0'(1) = S_1'(1) \mapsto \boxed{-1 = b}$

3. $S_0''(1) = S_1''(1) \Rightarrow -6 = 2c \mapsto \boxed{c = -3}$

4. natural condition: $S''(2) = 0 \Rightarrow 2c + 6d = 0 \rightarrow \boxed{d = 1}$

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Q 2. [**10 marks**] Use the upper bound for the error in Simpson's 1/3 rule to find the values of n and therefore h required to approximate

$$\int_0^2 \frac{1}{x+4}\,dx$$

to within 10^{-5} and compute the approximation using composite Simpson's 1/3 rule.

Let $f(x) = \frac{1}{x+4}$. We find the fourth derivative of $f(x)$ first.

$$f'(x) = \frac{-1}{(x+4)^2} \mapsto f''(x) = \frac{2}{(x+4)^3} \mapsto f'''(x) = \frac{-6}{(x+4)^4} \mapsto f^{(4)}(x) = \frac{24}{(x+4)^5}$$

$$M = \max_{0 \le x \le 2} |f^{(4)}(x)| = \max_{0 \le x \le 2} \frac{24}{|x+4|^5} = \frac{24}{|0+4|^5} = 0.02344$$

$$|E_a| \le \frac{b-a}{180} h^4 M \le 10^{-5} \Rightarrow \frac{2}{180} h^4 (0.02344) \le 10^{-5}$$

$$\Rightarrow h^4 \le 0.0384 \Rightarrow h \le 0.44266$$

$$\Rightarrow \frac{2-0}{n} \le 0.44266 \Rightarrow n \ge 4.52$$

We choose $\boxed{n=6}$ and therefore $h = \frac{b-a}{n} = 0.333 = \frac{1}{3}$

$$\int_0^2 \frac{1}{x+4}\,dx = \frac{h}{3}\left[f(0) + 4f(\tfrac{1}{3}) + 2f(\tfrac{2}{3}) + 4f(1) + 2f(\tfrac{4}{3}) + 4f(\tfrac{5}{3}) + f(2)\right]$$

$$\cong 0.4055$$

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Q 3. **[6 marks]** Determine constants $a, b, c,$ and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

that has degree of precision 3.

$f(x)=1 \longmapsto \quad 2 = \int_{-1}^{1} 1\, dx = a+b$ ~~[illegible]~~

$f(x)=x \longmapsto \quad 0 = \int_{-1}^{1} x\, dx = -a+b+c+d$

$f(x)=x^2 \longmapsto \quad \frac{2}{3} = \int_{-1}^{1} x^2 dx = a+b-2c+2d$

$f(x)=x^3 \longmapsto \quad 0 = \int_{-1}^{1} x^3 dx = -a+b+3c+3d$

$$\begin{cases} a+b=2 \\ -a+b+c+d=0 \\ a+b-2c+2d=\frac{2}{3} \\ -a+b+3c+3d=0 \end{cases} \longmapsto \begin{cases} a=1 \\ b=1 \\ c=1/3 \\ d=-1/3 \end{cases}$$

So,

$$\int_{-1}^{1} f(x)dx = f(-1)+f(1)+\frac{1}{3}f'(-1)-\frac{1}{3}f'(1)$$