CURVILINEAR MOTION: CYLINDRICAL COMPONENTS

Today's Objectives:

Students will be able to:

1. Determine velocity and acceleration components using cylindrical coordinates.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Velocity Components
- Acceleration Components
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. In a polar coordinate system, the velocity vector can be written as $\boldsymbol{v} = v_r\boldsymbol{u_r} + v_\theta\boldsymbol{u_\theta} = \dot{r}\boldsymbol{u_r} + r\dot{\theta}\boldsymbol{u_\theta}$. The term $\dot{\theta}$ is called

   A) transverse velocity. B) radial velocity.

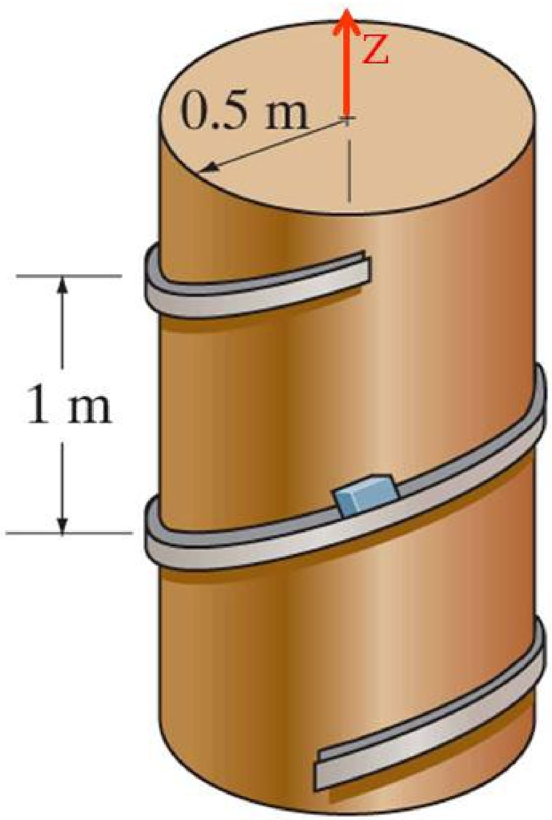
   C) angular velocity. D) angular acceleration.

2. The speed of a particle in a cylindrical coordinate system is

   A) $\dot{r}$ B) $r\dot{\theta}$

   C) $\sqrt{(r\dot{\theta})^2 + (\dot{r})^2}$ D) $\sqrt{(r\dot{\theta})^2 + (\dot{r})^2 + (\dot{z})^2}$

APPLICATIONS



A cylindrical coordinate system is used in cases where the particle moves along a 3-D curve.

In the figure shown, the box slides down the helical ramp. How would you find the box's velocity components to check to see if the package will fly off the ramp?

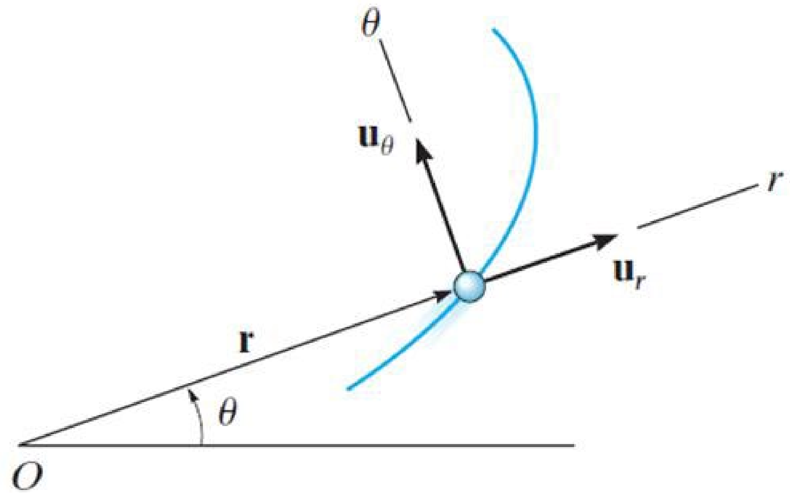
APPLICATIONS (continued)

The cylindrical coordinate system can be used to describe the motion of the girl on the slide.

Here the radial coordinate is constant, the transverse coordinate increases with time as the girl rotates about the vertical axis, and her altitude, z, decreases with time.

How can you find her acceleration components?

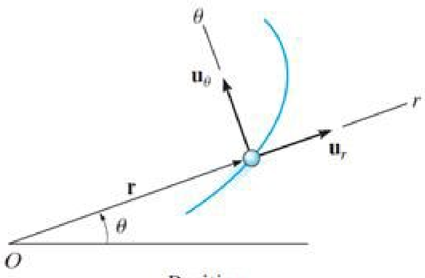
CYLINDRICAL COMPONENTS
(Section 12.8)



Position

We can express the location of P in polar coordinates as $\boldsymbol{r}$ = r $\boldsymbol{u}_r$. Note that the radial direction, r, extends outward from the fixed origin, O, and the transverse coordinate, θ, is measured counter-clockwise (CCW) from the horizontal.

VELOCITY in POLAR COORDINATES)

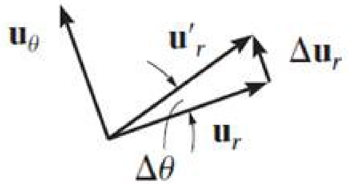


Position

The instantaneous velocity is defined as:

$$\boldsymbol{v} = d\boldsymbol{r}/dt = d(r\boldsymbol{u}_r)/dt$$

$$\boldsymbol{v} = \dot{r}\boldsymbol{u}_r + r\frac{d\boldsymbol{u}_r}{dt}$$

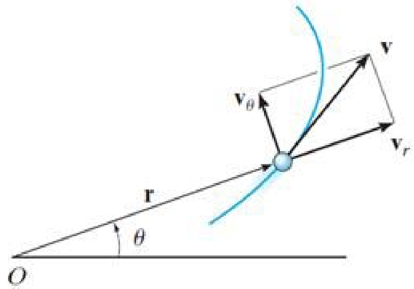


Using the chain rule:

$$d\boldsymbol{u}_r/dt = (d\boldsymbol{u}_r/d\theta)(d\theta/dt)$$

We can prove that $d\boldsymbol{u}_r/d\theta = \boldsymbol{u}_\theta$ so $d\boldsymbol{u}_r/dt = \dot{\theta}\boldsymbol{u}_\theta$

Therefore: $\boldsymbol{v} = \dot{r}\boldsymbol{u}_r + r\dot{\theta}\boldsymbol{u}_\theta$



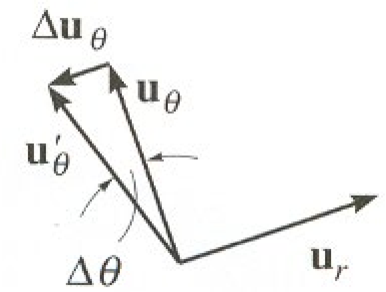
Velocity

Thus, the velocity vector has two components: $\dot{r}$, called the radial component, and $r\dot{\theta}$ called the transverse component. The speed of the particle at any given instant is the sum of the squares of both components or

$$v = \sqrt{(r\,\dot{\theta})^2 + (\,\dot{r}\,)^2}$$

ACCELERATION (POLAR COORDINATES)

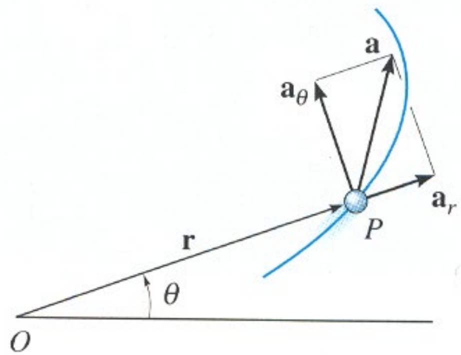
The instantaneous acceleration is defined as:



$$\mathbf{a} = d\boldsymbol{v}/dt = (d/dt)(\dot{r}\boldsymbol{u}_r + r\dot{\theta}\boldsymbol{u}_\theta)$$

After manipulation, the acceleration can be expressed as

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\boldsymbol{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\boldsymbol{u}_\theta$$



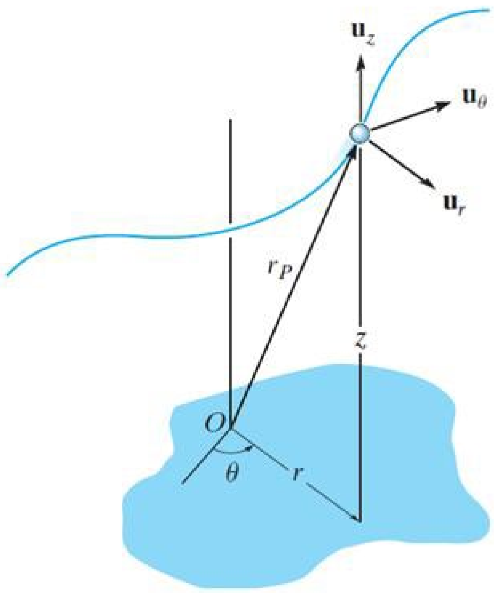
Acceleration

The term $(\ddot{r} - r\dot{\theta}^2)$ is the radial acceleration or a_r .

The term $(r\ddot{\theta} + 2\dot{r}\dot{\theta})$ is the transverse acceleration or a_θ.

The magnitude of acceleration is $a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2}$

CYLINDRICAL COORDINATES



If the particle P moves along a space curve, its position can be written as

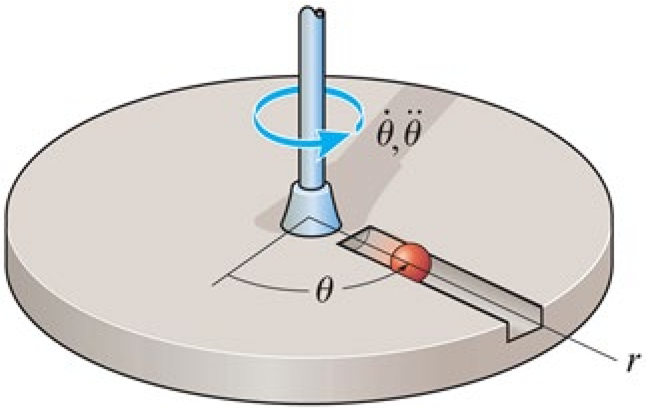
$$\boldsymbol{r_P} = \mathrm{r}\boldsymbol{u_r} + \mathrm{z}\boldsymbol{u_z}$$

Taking time derivatives and using the chain rule:

Velocity: $\boldsymbol{v_P} = \dot{\mathrm{r}}\boldsymbol{u_r} + \mathrm{r}\dot{\theta}\boldsymbol{u_\theta} + \dot{\mathrm{z}}\boldsymbol{u_z}$

Acceleration: $\mathbf{a_P} = (\ddot{\mathrm{r}} - \mathrm{r}\dot{\theta}^2)\boldsymbol{u_r} + (\mathrm{r}\ddot{\theta} + 2\dot{\mathrm{r}}\dot{\theta})\boldsymbol{u_\theta} + \ddot{\mathrm{z}}\boldsymbol{u_z}$

EXAMPLE



Given: The platform is rotating such that, at any instant, its angular position is $\theta = (4t^{3/2})$ rad, where t is in seconds.

A ball rolls outward so that its position is $r = (0.1t^3)$ m.

Find: The magnitude of velocity and acceleration of the ball when $t = 1.5$ s.

Plan: Use a polar coordinate system and related kinematic equations.

EXAMPLE (continued)

Solution:

$r = 0.1t^3, \quad \dot{r} = 0.3\,t^2, \quad \ddot{r} = 0.6\,t$

$\theta = 4\,t^{3/2}, \quad \dot{\theta} = 6\,t^{1/2}, \quad \ddot{\theta} = 3\,t^{-1/2}$

At t=1.5 s,

$r = 0.3375\text{ m}, \quad \dot{r} = 0.675\text{ m/s}, \quad \ddot{r} = 0.9\text{ m/s}^2$

$\theta = 7.348\text{ rad}, \dot{\theta} = 7.348\text{ rad/s}, \quad \ddot{\theta} = 2.449\text{ rad/s}^2$

Substitute into the equation for velocity

$$\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\dot{\theta}\,\mathbf{u}_\theta = 0.675\,\mathbf{u}_r + 0.3375\,(7.348)\,\mathbf{u}_\theta$$
$$= 0.675\,\mathbf{u}_r + 2.480\,\mathbf{u}_\theta$$

$$v = \sqrt{(0.675)^2 + (2.480)^2} = \underline{2.57\text{ m/s}}$$

PEARSON

EXAMPLE (continued)

Substitute in the equation for acceleration:

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta$$

$$\mathbf{a} = [0.9 - 0.3375(7.348)^2]\,\mathbf{u}_r + [0.3375(2.449) + 2(0.675)(7.348)]\,\mathbf{u}_\theta$$

$$\mathbf{a} = -17.33\,\mathbf{u}_r + 10.75\,\mathbf{u}_\theta \text{ m/s}^2$$

$$a = \sqrt{(-17.33)^2 + (10.75)^2} = \underline{20.4 \text{ m/s}^2}$$

CONCEPT QUIZ

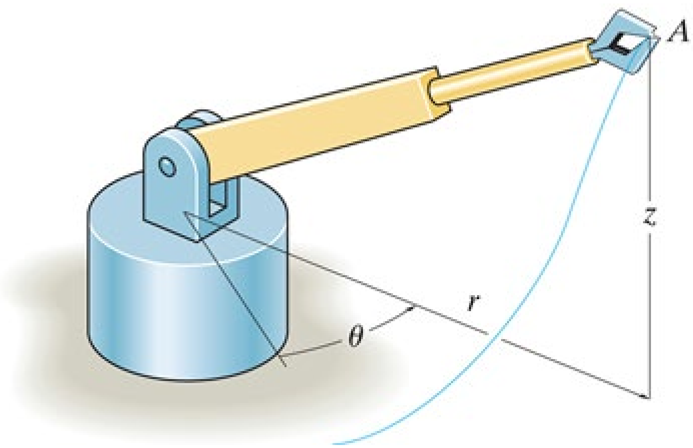
1. If $\dot{r}$ is zero for a particle, the particle is

   A) not moving.
   B) moving in a circular path.
   C) moving on a straight line.
   D) moving with constant velocity.

2. If a particle moves in a circular path with constant velocity, its radial acceleration is

   A) zero.
   B) $\ddot{r}$.
   C) $-r\dot{\theta}^2$.
   D) $2\dot{r}\dot{\theta}$.

GROUP PROBLEM SOLVING



Given: The arm of the robot is extending at a constant rate $\dot{r} = 1.5$ ft/s when r = 3 ft, z = $(4t^2)$ ft, and $\theta = (0.5\ t)$ rad, where t is in seconds.

Find: The velocity and acceleration of the grip A when t = 3 s.

Plan: Use cylindrical coordinates.

GROUP PROBLEM SOLVING (continued)

Solution:

When t = 3 s, r = 3 ft and the arm is extending at a constant rate $\dot{r} = 1.5$ ft/s. Thus $\ddot{r} = 0$ ft/s^2

$$\theta = 1.5\,t = 4.5 \text{ rad}, \quad \dot{\theta} = 1.5 \text{ rad/s}, \quad \ddot{\theta} = 0 \text{ rad/s}^2$$

$$z = 4\,t^2 = 36 \text{ ft}, \quad \dot{z} = 8\,t = 24 \text{ ft/s}, \quad \ddot{z} = 8 \text{ ft/s}^2$$

Substitute in the equation for velocity

$$\begin{aligned} \boldsymbol{v} &= \dot{r}\,\boldsymbol{u_r} + r\dot{\theta}\,\boldsymbol{u_\theta} + \dot{z}\,\boldsymbol{u_r} \\ &= 1.5\,\boldsymbol{u_r} + 3\,(1.5)\,\boldsymbol{u_\theta} + 24\,\boldsymbol{u_z} \\ &= 1.5\,\boldsymbol{u_r} + 4.5\,\boldsymbol{u_\theta} + 24\,\boldsymbol{u_z} \end{aligned}$$

Magnitude $v = \sqrt{(1.5)^2 + (4.5)^2 + (24)^2} =$ 24.5 ft/s

GROUP PROBLEM SOLVING (continued)

Acceleration equation in cylindrical coordinates

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\boldsymbol{u_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\boldsymbol{u_\theta} + \ddot{z}\boldsymbol{u_z}$$

$$= \{0 - 3\,(1.5)^2\}\boldsymbol{u_r} + \{3\,(0) + 2\,(1.5)\,1.5\,\}\,\boldsymbol{u_\theta} + 8\,\boldsymbol{u_z}$$

$$\mathbf{a} = [6.75\,\boldsymbol{u_r} + 4.5\,\boldsymbol{u_\theta} + 8\,\boldsymbol{u_z}]\ \text{ft/s}^2$$

$$a = \sqrt{(6.75)^2 + (4.5)^2 + (8)^2} = \underline{11.4\ \text{ft/s}^2}$$

ATTENTION QUIZ

1. The radial component of velocity of a particle moving in a circular path is always
   A) zero.
   B) constant.
   C) greater than its transverse component.
   D) less than its transverse component.

2. The radial component of acceleration of a particle moving in a circular path is always
   A) negative.
   B) directed toward the center of the path.
   C) perpendicular to the transverse component of acceleration.
   D) All of the above.

End of the Lecture

Let Learning Continue