Feb. 7, 2024

Solutions.

**Humber College** 

Student Name:

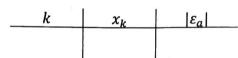
(1)

Student Number:

1. Consider the following equation:

$$x - \cos(x) = 0.$$

- a) [2 marks] Show this equation has a unique solution in the interval [0, 1].
- **b)** [2 marks] Show that the solution of this equation is a fixed point of the function  $g(x) = 1 \frac{\sin^2 x}{x+1}$ .
- c) [5 marks] Use the *first 8 iterations* of the fixed-point algorithm with the function g(x) given in **b**) to estimate the root of equation (1) starting from  $x_0 = 1$ . Create a table with three columns as below:



Round your answers to 5-decimal digits.

(1) 
$$f_{(x)} = x - G_{5x}$$
  
)  $f_{(0)} = 0 - G_{(0)} = -1$   
 $f_{(1)} = 1 - G_{(1)} = 6.45970$ 

fex) is a continuous function and feo) fei) Ko. So, by IVT, it has at least one Solution on [0,1].

To show that the solution is ornique, we consider the derivative of few.

f(x)=1+Siix >0, for all x. So, fex) is an increasing fraction on [0,1]. This shows that the root is unique.

b) We need to show that oczag(x) will lead to fox =0.

$$\chi_{2}g(x) \Rightarrow \chi_{2}l - \frac{S_{11}^{2}x}{\chi_{+1}} \Rightarrow \chi_{2}\frac{\eta_{+1}-S_{11}^{2}x}{\chi_{+1}}$$

→ x(x+1)=x+1-Six => x2+x=1-Six+x

>> x2= Cosx =0 x- Cix=0 =0 (x-65x)(x+ Cix)=0

K	XK	18a		
٥				
1	0.64596	0.35404		
2_	0.77985	0.20726		
3	0.72220	0.07392		
4	0.74627	0.03334		1
5	0.73606	0.01369	ŧ	
6	0.74037	0.00585		
7	0.73854	0.00246		
8	0.73931	0.00104		
	,			
÷ 4				

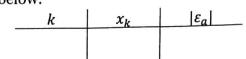
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2. [5 marks] The fourth-degree polynomial

 $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$ 

has a zero in [-1, 0]. Use the modified secant method with  $\delta = 10^{-2}$  and the midpoint of the interval as the initial approximation to estimate the root. Stop the iterative scheme if either  $|\varepsilon_a| < 10^{-4}$  or the number of iterations exceeds 6. Provide a table as below:



Round your answers to 5-decimal digits.

K	$\chi_{\mathbf{k}}$	[ Eal
0 1 2 3 4	-0.5 -0.15218 -0.04190 -0.04066 -0.04066	0.69564 0.72470 0.02951 0.00000208 STOP 18al < 10 <sup>4</sup>

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Feb. 7, 2024

- 3. a) [4 marks] Apply 5 iterations of the bisection method on the interval [2.5, 3.5] to find an approximation to  $\sqrt[3]{25}$ .
  - b) [2 marks] Find a bound for the number of iterations needed by the bisection method to achieve an approximation to  $\sqrt[3]{25}$  in the above interval with accuracy

a)	$\int_{(x)=x}^{3} -25$	
$[a_0,b_0]$	= [a,b] = [2.5, 3.5]	
	16)= -9.375	
fa	00) = 17.875	

10 <sup>-3</sup> . Round your answers to 4-decir	mal digits	s.			Sign
n a	1, 1	akl	bx 1	*XK	f(xx)
$f(x) = x^{2} - 25$	K 0	2.5	3.5	3.0000	+
[a,b]=[2.5,3.5]		2.5	3.0	2.75	-
$(\alpha_{ib})=(2.37.53)$	1	2 7		2.875	_
= -9.375	2_	2.17		2.9375	+
= 17.875	3	12.873	3.0	- 2 9063	
	4	2.875	2.931	12 92-19	_
	5	2.9063	2.9375	2.9375 -2.9063 2.9219	
			(	ι	1

b) Upper bound for the error = 
$$\frac{b-a}{2^n} = \frac{1}{2^n}$$

When want to have  $\frac{1}{2^n} \leq 10^{-3}$ 
 $\frac{1}{10^{-3}} = 1000 \text{ km} > \frac{1}{3} = 1000 \text{ k$