Tutorial 2

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Tutorial 2: Motion in One Dimension

Part A: Distance and Displacement

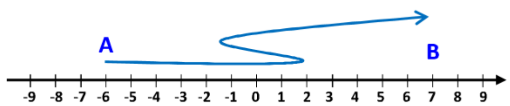
Kinematics is studying of ......**motion**...... without thinking about its ......**causes**......

......**Displacement**...... is the change in position of an object that is a vector quantity.

Distance is the ......**length**...... of the path traveled by the object.

Displacement: $\Delta x = x_f - x_i$

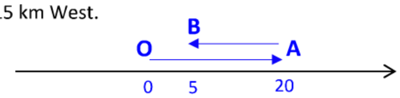
1) What is the displacement of the path in the diagram?



$\Delta x = x_f - x_i = 7 - (-6) = 13\ units$

2) You drive 20 km East, then turn around and drive 15 km West.

a) What is your displacement?



$\Delta x_{total} = \Delta x_{O\to A} + \Delta x_{A\to B}$

$= (x_A - x_O) + (x_B - x_A)$

$= (20\ km - 0\ km) + (5\ km - 20\ km)$

$= 20\ km + (-15\ km) = 5\ km$

b) What was your distance traveled?

$d = 20\ km + 15\ km = 35\ km$

Part B: Velocity and Speed

......**Velocity**...... is the rate at which the object changes its position.

Velocity is a ......**vector**...... quantity, and speed is a ......**scalar**...... quantity, SI units of both are ......**m/s**......

Instantaneous velocity is the ......**slope**...... of the ......**displacement**...... versus time graph.

Average Velocity: $v_{x,avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$ **Instantaneous Velocity:** $v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

Average Speed: $v_{avg} = \frac{d}{\Delta t}$ **Instantaneous Speed = Magnitude of instantaneous velocity**

1) A position-time graph for a particle moving along the x-axis is shown in the following figure.

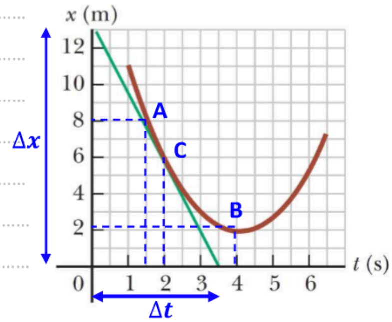
a) Find the average velocity in the time interval $t = 1.50\ s$ to $t = 4.00\ s$.

$t_A = 1.50\ s, \quad x_A = 8.00\ m$

$t_B = 4.00\ s, \quad x_B = 2.00 m$

$$v_{x,avg} = \frac{x_B - x_A}{t_B - t_A} = \frac{2.00\ m - 8.00\ m}{4.00\ s - 1.50\ s}$$

$$v_{x,avg} = -\frac{6.00\ m}{2.5\ s} = -2.4\ m/s$$



b) Determine the instantaneous velocity at $t = 2.00\ s$ by measuring the slope of the tangent line shown in the graph.

v_x at point C is the slope of the line at point C

$$slope = \frac{0\ m - 13\ m}{3.5s - 0s} = \frac{-13\ m}{3.5\ s} = -3.7\ m/s$$

The negative sign shows that the direction of v_x is along the negative x direction.

c) At what value of t is the velocity zero?

The velocity, v_x, will be zero when the slope of the tangent line is zero.

This occurs for the point B on the graph at $t = 4.00\ s$, where x has its minimum value.

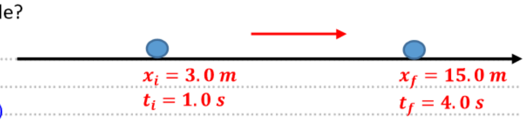
Part C: Particle under Constant Velocity

Particle moves in a straight line with a constant velocity of v_x

Constant Velocity: $v_x = \frac{\Delta x}{\Delta t}$

Position: $x_f = x_i + v_x t$

1) A particle is moving at constant velocity. Its position at t = 1.0 s is 3.0 m and its position at t = 4.0 s is 15.0 m. What is the velocity of the particle?

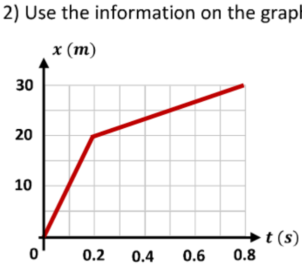


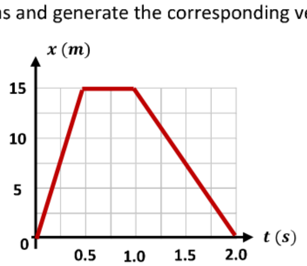
$x_f = x_i + v_x t$

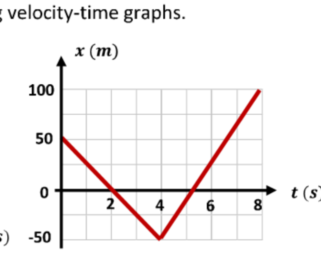
$15.0\ m = 3.0\ m + v_x(4.0\ s - 1.0\ s)$

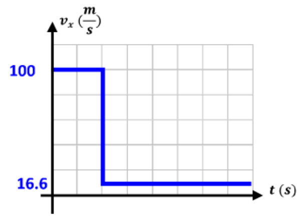
$$v_x = \frac{12\ m}{3\ s} = 4\ m/s$$

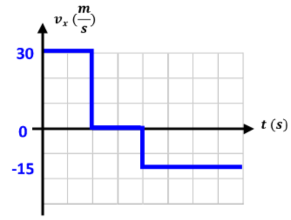
2) Use the information on the graphs and generate the corresponding velocity-time graphs.

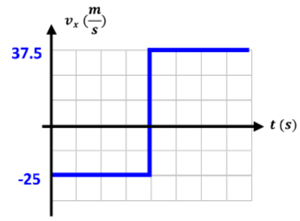












$0 \le t \le 0.2 \;\rightarrow\; v_x = \frac{20\,m - 0\,m}{0.2\,s - 0\,s} = 100\frac{m}{s}$ *and* $0.2 \le t \le 0.8 \;\rightarrow\; v_x = \frac{30\,m - 20\,m}{0.8\,s - 0.2\,s} = 16.7\frac{m}{s}$

$0 \le t \le 0.5 \;\rightarrow\; v_x = \frac{15\,m - 0\,m}{0.5\,s - 0\,s} = 30\frac{m}{s}$ *and* $1.0 \le t \le 2.0 \;\rightarrow\; v_x = \frac{0\,m - 15\,m}{2.0\,s - 1.0\,s} = -15\frac{m}{s}$

$0 \le t \le 4 \;\rightarrow\; v_x = \frac{-50\,m - 50\,m}{4\,s - 0\,s} = -25\frac{m}{s}$ *and* $4 \le t \le 8 \;\rightarrow\; v_x = \frac{100\,m - (-50\,m)}{8\,s - 4\,s} = 37.5\frac{m}{s}$

Part D: Acceleration

Acceleration is how quickly....**velocity**........ is changing.

Acceleration is a ...**vector**..... quantity and its SI unit is ....**m/s²**.....

Acceleration at any time is the ....**slope**........ of the ....**velocity**...... versus time graph at that time.

Average Acceleration: $a_{x,avg} = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$

Instantaneous Acceleration: $a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$

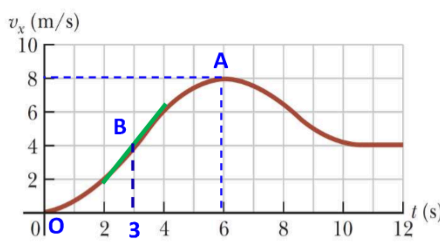
1) Following figure shows a graph of v_x versus t for the motion of a motorcyclist as he starts from rest and moves along the road in a straight line.

a) Find the average acceleration for the time interval $t = 0$ to $t = 6.00\ s$.

$t_A = 6.00\ s\,, \quad v_A = 8.00\ m$

$a_{x,avg} = \frac{v_A - v_O}{t_A - t_O} = \frac{8.00\ m/s - 0\ m/s}{6.00\ s - 0\ s}$

$a_{x,avg} = \frac{8.00\ m/s}{6.00\ s} = 1.3\ m/s^2$



b) Estimate the time at which the acceleration has its greatest positive value and the value of the acceleration at that instant.

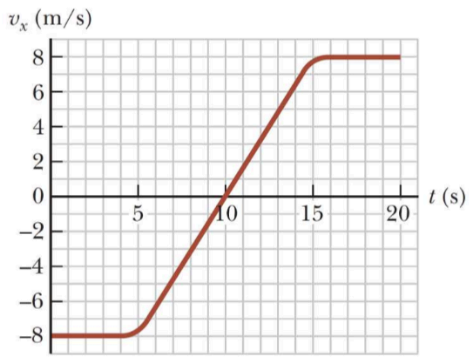
Maximum positive acceleration occurs when the slope of the velocity-time curve is greatest, at point B at $t = 3\ s$, and is equal to the slope of the graph at point B:

$Slope = \frac{6\frac{m}{s} - 2\frac{m}{s}}{4\ s - 2\ s} = 2\ m/s^2$

c) When is the acceleration zero?

The acceleration is zero, when the slope of the tangent line to the velocity-time graph is zero, which occurs at point O at $t = 0\ s$, at point A at $t = 6\ s$ and also for $t > 10\ s$.

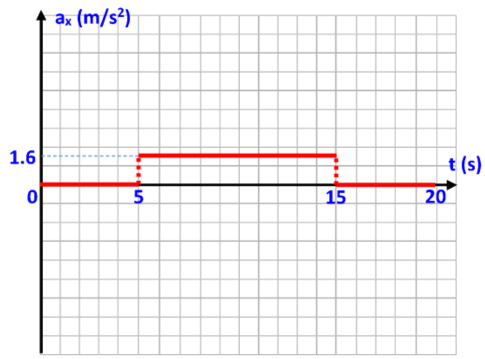
2) A velocity-time graph for an object moving along x-axis is shown as below.



a) Plot a graph of the acceleration versus time.

From $t = 0$ to $t = 5\ s$, the velocity is constant, $v_x = -8\ m/s$. So, the acceleration is zero.

From $t = 15\ s$ to $t = 20\ s$, the velocity is constant, $v_x = 8\ m/s$. So, the acceleration is zero.



From $t = 5\ s$ to $t = 15\ s$, the velocity changes linearly. The acceleration is the slope of the line from $t = 5\ s$ to $t = 15\ s$.

$$Slope = \frac{8\frac{m}{s} - (-8\frac{m}{s})}{15\ s - 5\ s}$$

$$= \frac{16\ m/s}{10\ s} = 1.6\ m/s^2$$

b) Determine the average acceleration of the object in the time interval $t = 5.00\ s$ to $t = 15.0\ s$.

The average acceleration from $t = 5\ s$ to $t = 15\ s$ is the slope of the line from $t = 5\ s$ to $t = 15\ s$.

$$Slope = \frac{8\frac{m}{s} - (-8\frac{m}{s})}{15\ s - 5\ s} = \frac{16\ m/s}{10\ s} = 1.6\ m/s^2$$

Part E: Motion Diagrams and Graphs

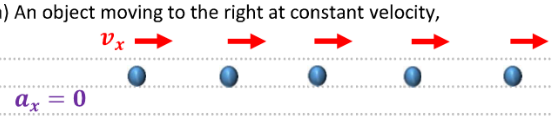
If acceleration is zero, then the object moves with ....**constant**.... velocity.

If the velocity and acceleration are in the same direction, then the object is ....**speeding up**....

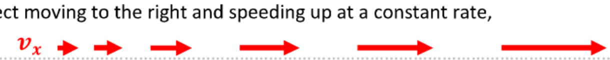
If the velocity and acceleration are in opposite direction, then the object is ....**slowing down**....

1) Draw motion diagrams for each of the following scenarios:

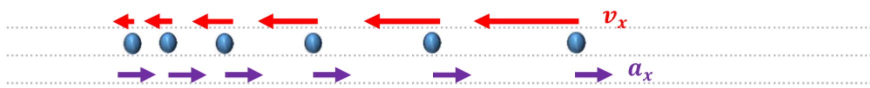
a) An object moving to the right at constant velocity,



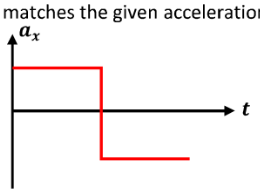
b) An object moving to the right and speeding up at a constant rate,

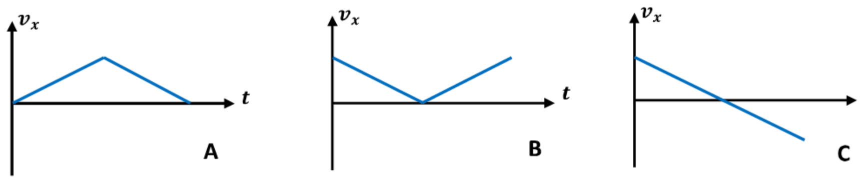


c) An object moving to the left and slowing down at a constant rate.



2) Which of the velocity-time graphs matches the given acceleration-time graph.





The given graph represents a motion with a positive acceleration, which means increase in velocity, and thereafter a constant negative acceleration, which means a decrease in velocity. So, the correct answer is A.

Part F: Particle under Constant Acceleration

Particle moves in a straight line with a constant acceleration of a_x

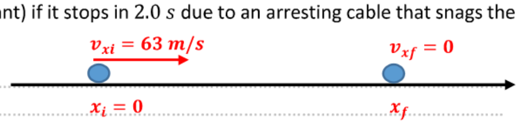
Average velocity: $v_{x,avg} = \frac{v_{xi}+v_{xf}}{2}$

Velocity: $v_{xf} = v_{xi} + a_x t$, $\quad v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

Position: $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$, $\quad x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$

1) A jet lands on an aircraft carrier at a speed of 140 mi/h ($\approx 63\ m/s$).

a) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the jet and brings it to a stop?



$$v_{xf} = v_{xi} + a_x t$$

$$0\frac{m}{s} = 63\ m/s + a_x(2.0\ s)$$

$$a_x = \frac{-63\ m/s}{2.0\ s} = -31.5\ m/s^2$$

b) If the jet touches down at position $x_i = 0$ m, what is its final position?

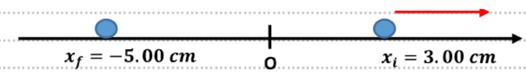
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$x_f = 0\ m + \left(63\frac{m}{s}\right)(2.0\ s) + \frac{1}{2}\left(-31.5\frac{m}{s^2}\right)(2.00)^2 = 63\ m$$

2) An object moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is $-5.00\ cm$, what is its acceleration?

$v_{xi} = 12.0\ cm/s$

$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$



$-5.00\ cm = 3.00\ cm + (12\ cm/s)(2.00\ s) + \frac{1}{2}a_x(2.00)^2$

$a_x = \frac{-32\ cm}{2.00\ s^2} = -16\ cm/s^2$

Part G: Freely Falling Objects

Freely falling is when an object is moving freely only under the influence of ……**gravity**……………….

The magnitude of gravitational acceleration near the surface of Earth is ….**9.8**…. m/s² and its direction is …**downward**….

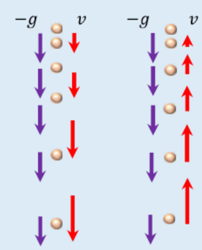
Ignoring the air resistance, the free fall motion can be modeled as a constant acceleration motion in vertical direction with the acceleration of ……**- 9.8**…… m/s².

Particle moves vertically with a constant acceleration of $a_y = -g = -9.8\ m/s^2$

Velocity: $v_{yf} = v_{yi} + a_y t$, $\quad v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$

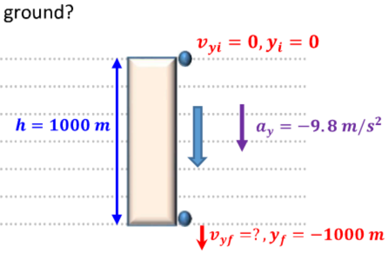
Position: $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$, $\quad y_f = y_i + \frac{1}{2}(v_{yi} + v_{yf})t$

Average velocity: $v_{y,avg} = \frac{v_{yi}+v_{yf}}{2}$



1) You drop a coin from top of a hundred story (1000 m) building.

a) If you ignore air resistance, what is its velocity before it hits the ground?



$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$

$v_{yf}^2 = 0 + 2(-9.8\ m/s^2)(-1000\ m - 0)$

$v_{yf}^2 = 19600 \quad \rightarrow \quad v_{yf} = -140\ m/s$

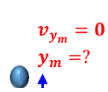
b) How long does it take to hit the ground?

$v_{yf} = v_{yi} + a_y t$

$-140\frac{m}{s} = 0\frac{m}{s} + \left(-9.8\frac{m}{s^2}\right)t \quad \rightarrow \quad t = \frac{-140\frac{m}{s}}{-9.8\frac{m}{s^2}} = 14.3\ s$

2) A baseball is hit straight up into the air. If the initial velocity was 20 m/s,

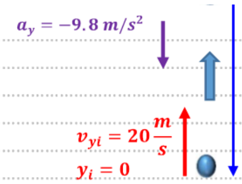
a) How high will the ball go?



$$v_{y_m}^2 = v_{yi}^2 + 2a_y(y_m - y_i)$$

$$0 = (20\ m/s)^2 + 2(-9.8\ m/s^2)(y_m - 0)$$

$$y_m = \frac{-\left(20\frac{m}{s}\right)^2}{2\left(-9.8\frac{m}{s^2}\right)} = 20.4\ m$$



b) How long will it be until the catcher catches the ball at the same height it was hit?

First determine the time for travelling up:

$$v_{y_m} = v_{yi} + a_y t$$

$$0 = 20\frac{m}{s} + \left(-9.8\frac{m}{s^2}\right)t \quad \rightarrow \quad t = \frac{-20\frac{m}{s}}{-9.8\frac{m}{s^2}} = 2.0\ s$$

Since the ball spends half of the time travelling up and half for travelling down, the total travel time is: $t_{total} = 2 \times 2.0\ s = 4.0\ s$

c) How fast is it going when catcher catches it?

Due to the symmetry in free-fall motion, velocity of the ball when catcher catches it is same as the initial velocity, but in the opposite direction:

$$v_{yf} = -20\ m/s$$