Instrumentation & Measurement

Digitization

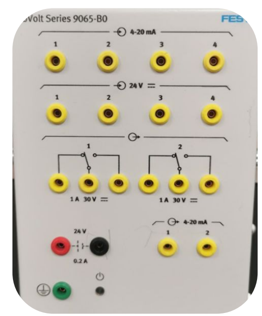
Winter 2024

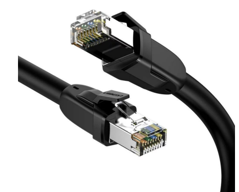
Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Sensor Components

Actual value to measure → Input → **Measurement Element (Sensor)** → Output → Measured Value

Sensing Element → Signal Conditioning → Digital processing → Output or Display

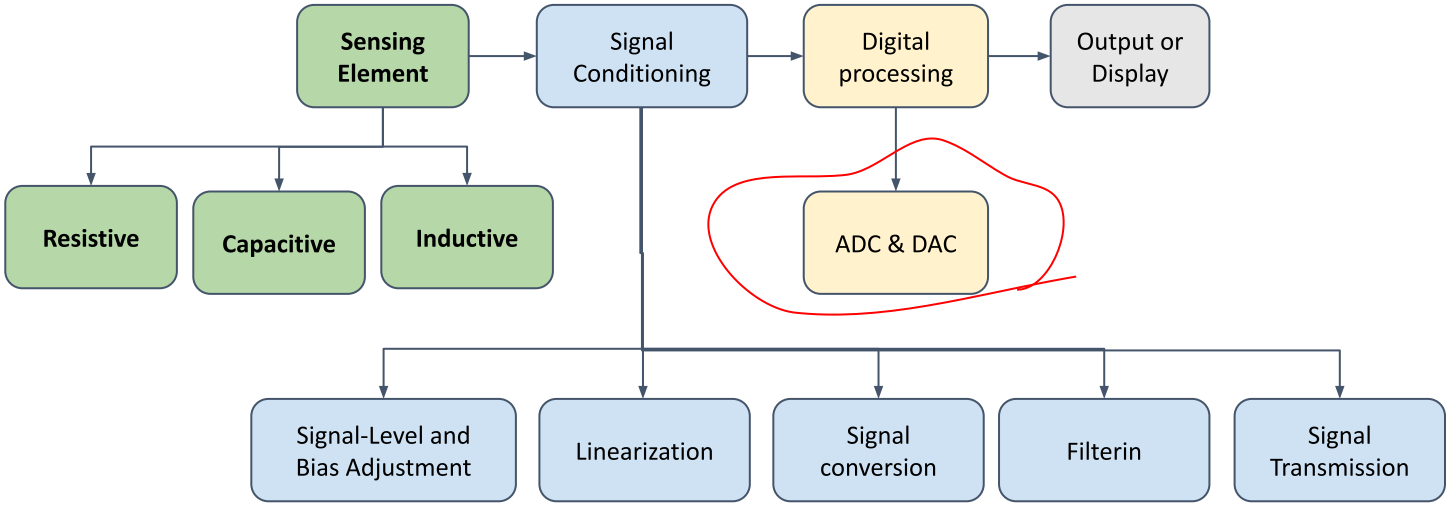




A sensor might or might not include signal conditioning, digital processing or display.

Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Sensor Components

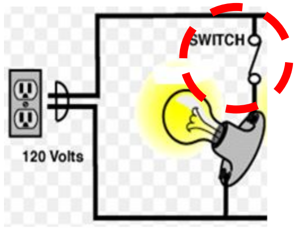


Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Digital Processing

- Digitalization Concept
- Analog to Digital Converter
- Data Processing and Filtering
- Digital to Analog Converter

Digital Concept - Information Coding with one switch



	Light	power	Switch
States	Off	No	open
	On	Yes	Close

Association

→ 0

→ 1

	Light	power	Switch	State
States	Off	No	open	0
	On	Yes	Close	1

With one switch two states can be made 0 and 1

This is called Binary state

Two states information such as

Black and white

High and not High

Hot and not hot

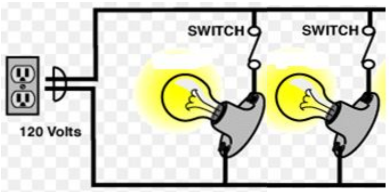
…

can be coded

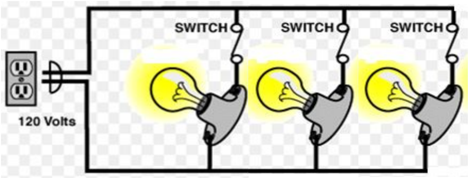
Sensor Switches like level switch, temperature switch are used to do such information coding.

Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Digital Concept - Information Coding with switches



Codes
00
01
10
11



Codes
000
001
010
011
100
101
110
111

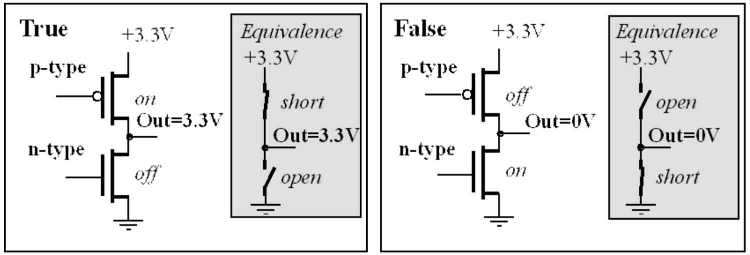
Number of Switches	Number of states	
1	2	
2	4	
3	8	
4	16	
5	32	
N		?

N is number of switches

$$2^N$$

Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

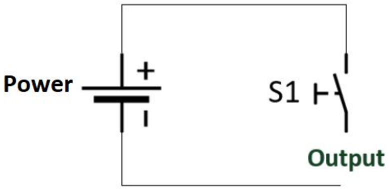
Bit & Byte



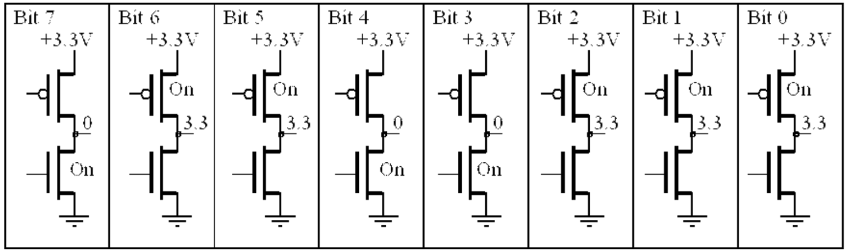
[Source: Digital Logic (utexas.edu)]

Bit is the smallest and basic unit of digital information.

A bit can be modeled as one switch



8 Bits = Byte



Byte

Bit 7	**Bit 6**	**Bit 5**	**Bit 4**	**Bit 3**	**Bit 2**	**Bit 1**	**Bit 0**

Text Coding with one byte

ASCII Code: Character to Binary

0	0011 0000	O	0100 1111	m	0110 1101
1	0011 0001	P	0101 0000	n	0110 1110
2	0011 0010	Q	0101 0001	o	0110 1111
3	0011 0011	R	0101 0010	p	0111 0000
4	0011 0100	S	0101 0011	q	0111 0001
5	0011 0101	T	0101 0100	r	0111 0010
6	0011 0110	U	0101 0101	s	0111 0011
7	0011 0111	V	0101 0110	t	0111 0100
8	0011 1000	W	0101 0111	u	0111 0101
9	0011 1001	X	0101 1000	v	0111 0110
A	0100 0001	Y	0101 1001	w	0111 0111
B	0100 0010	Z	0101 1010	x	0111 1000
C	0100 0011	a	0110 0001	y	0111 1001
D	0100 0100	b	0110 0010	z	0111 1010
E	0100 0101	c	0110 0011	.	0010 1110
F	0100 0110	d	0110 0100	,	0010 0111
G	0100 0111	e	0110 0101	:	0011 1010
H	0100 1000	f	0110 0110	;	0011 1011
I	0100 1001	g	0110 0111	?	0011 1111
J	0100 1010	h	0110 1000	!	0010 0001
K	0100 1011	i	0110 1001	'	0010 1100
L	0100 1100	j	0110 1010	"	0010 0010
M	0100 1101	k	0110 1011	(	0010 1000
N	0100 1110	l	0110 1100	)	0010 1001
				space	0010 0000

$$2^8 = 256$$

0000 0000
0000 0001
0000 0010
0000 0011
…
0001 0000
0001 0001
…
1111 1101
1111 1110
1111 1111

Amir Jalali, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Integer Number Coding

Codes	Number
0	--> 0
1	--> 1

Codes		Number
0	0	--> 0
0	1	--> 1
1	0	--> 2
1	1	--> 3

0	0	0	--> 0
0	0	1	--> 1
0	1	0	--> 2
0	1	1	--> 3
1	0	0	--> 4
1	0	1	--> 5
1	1	0	--> 6
1	1	1	--> 7

Codes				number
0	0	0	0	--> 0
0	0	0	1	--> 1
0	0	1	0	--> 2
0	0	1	1	--> 3
0	1	0	0	--> 4
0	1	0	1	--> 5
0	1	1	0	--> 6
0	1	1	1	--> 7
1	0	0	0	--> 8
1	0	0	1	--> 9
1	0	1	0	--> 10
1	0	1	1	--> 11
1	1	0	0	--> 12
1	1	0	1	--> 13
1	1	1	0	--> 14
1	1	1	1	--> 15

For coding larger number than 15 for example 20 we need more bits, minimum 5 bits

Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Integer Number Coding

With on byte

2^8 = 256

Numbers from 0 to 255 can be coded

$$
\begin{array}{l}
0000\ 0000 \rightarrow\ 0 \\
0000\ 0001 \rightarrow 1 \\
0000\ 0010 \rightarrow 2 \\
0000\ 0011 \\
\ldots \\
0001\ 0000 \\
0001\ 0001 \\
\ldots \\
1111\ 1101 \\
1111\ 1110 \rightarrow 254 \\
1111\ 1111 \rightarrow \\
255
\end{array}
$$

One word is made of two bytes

2^16 = 65536

Numbers from 0 to 65535 can be coded

One double word (DB) is made of two words (4 Bytes)

2^32 = 4 294 967 296

Numbers from 0 to 4 294 967 295 can be coded

Analog Value Coding

In general a value is changing from 0 to 10 as below. If we have three bits how can we code these values?

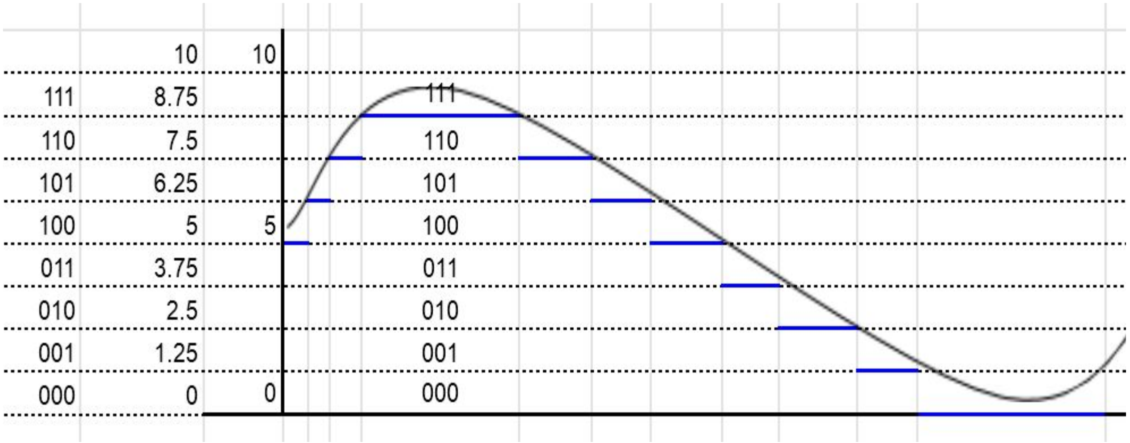
With 3 bits
Eight codes

idea??

Analog Value Coding

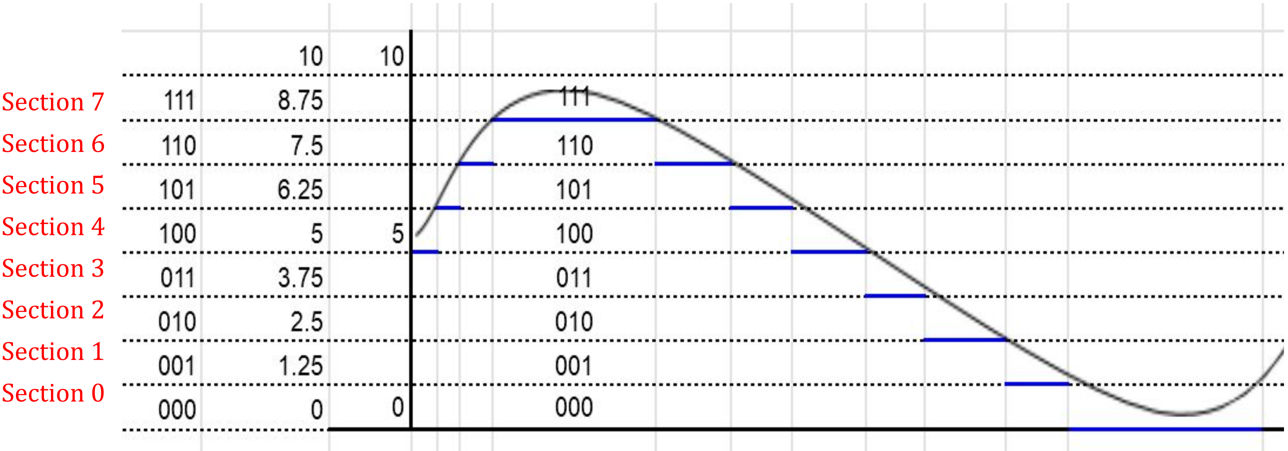
Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Analog Value Coding



What is the code for 2.45?

Analog Value Coding



What is the code for 2.45?

Solution 1: By looking at above picture we can say 2.45 is in the section of (1.25 - 2.5). All numbers in this section including 2.45 are represented by 001 or 1.

Solution 2: each section length is 1.25 , then 2.45/1.25=1.96 , then 2.45 is in the section 1. Convert 1 to its binary code → 001

Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Definition of Resolution and Quantization Error

There is N bit available for coding these values.

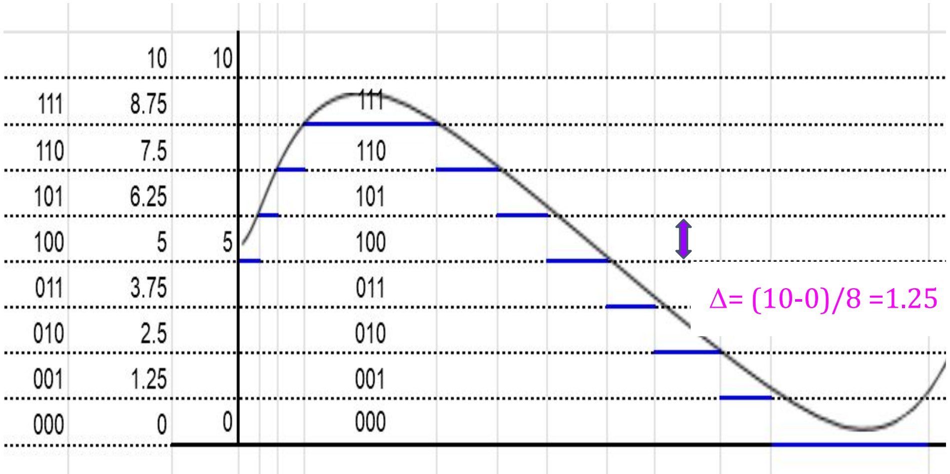
Q is number of the available integer number for coding.

Q= 2^N

Resolution is defined as quantization interval or each section length.

If an integer number is produced by quantization it will represent an actual value.

Quantization error is defined as difference between analog (True) value and the represented value after the coding.



Q= 2^3=8

Δ= (10-0)/8 =1.25

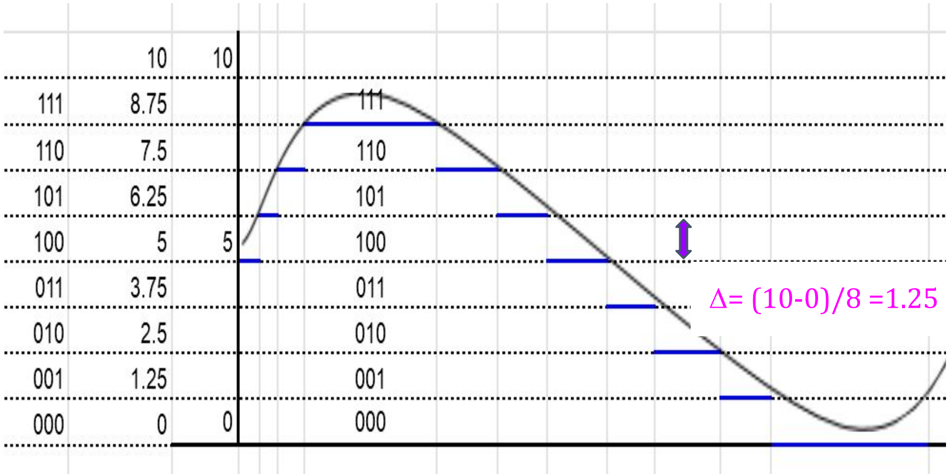
V= integer of (2.45/1.25)= integer of (1.96)=1

e = 2.45-1= 1.45

Reducing Quantization Error

In front example the error is big value and hardly represents the actual value.

how can we reduce the quantization error?



Q= 2^3=8

Δ= (10-0)/8 =1.25

V= integer of (2.45/1.25)= integer of (1.96)=1

e = 2.45-1.25= 1.2

Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

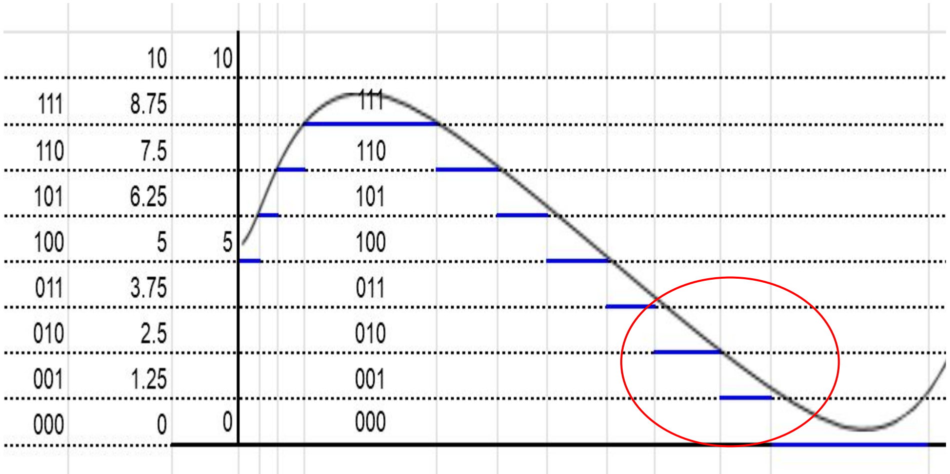
Reducing Quantization Error

In front example the error is big value and hardly represents the actual value.

how can we reduce the quantization error?

1- One way is to change the coding procedure .

For example instead of rounding down to find integer value, the numbers are coded to middle value.



2- The other way is to increase number of the codes and make the resolution (quantization interval) smaller.

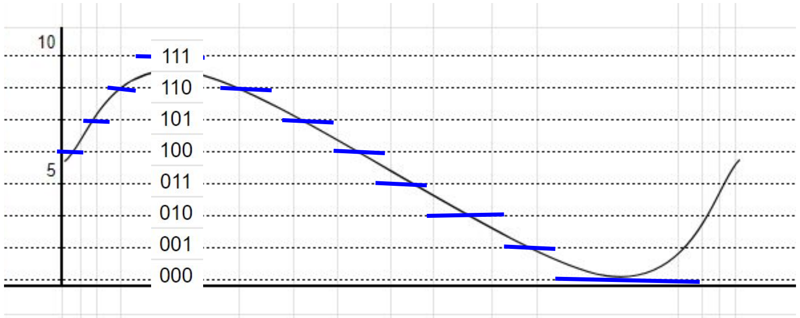
Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Calculation of Resolution and Quantization Error

Let's assume analog value changes from Y_{min} to Y_{max}

There is N bit available for coding these values.

To round to middle value, the Ymin will be presented by 0 and everything between Ymin and Ymin + Δ will be presented by 1. Therefore it will be Q-1 quantization interval.



Number of bits N

Number of codes $Q=2^N$

$$\text{Resolution(Quantization Step Size) } \Delta = \frac{y_{\max} - y_{min}}{Q-1}$$

$$\text{Quantized Value } \ V = Round \text{ of } \left(\frac{y}{y_{\max} - y_{min}}\right) \times (Q-1)$$

$$e = y - V \times \Delta$$

$$\max(e) = \pm\frac{\Delta}{2}$$

Maximum of quantization error would be half of resolution

Q= 2^3=8

Δ= (10-0)/7 =1.4285

V= round of (2.45/1.4285)= round of (1.715)=2

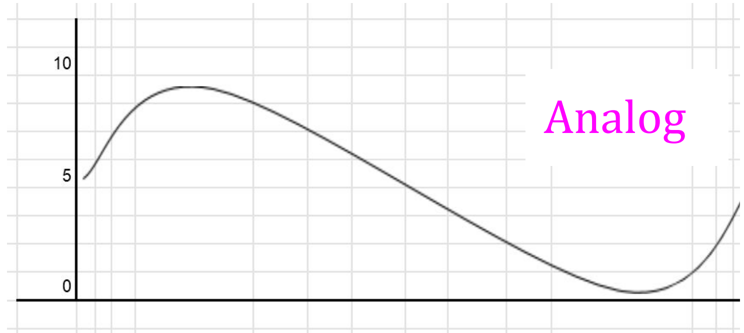
e = 2.45-2.875= - .407

Max (e) =± Δ/2= 0.7143

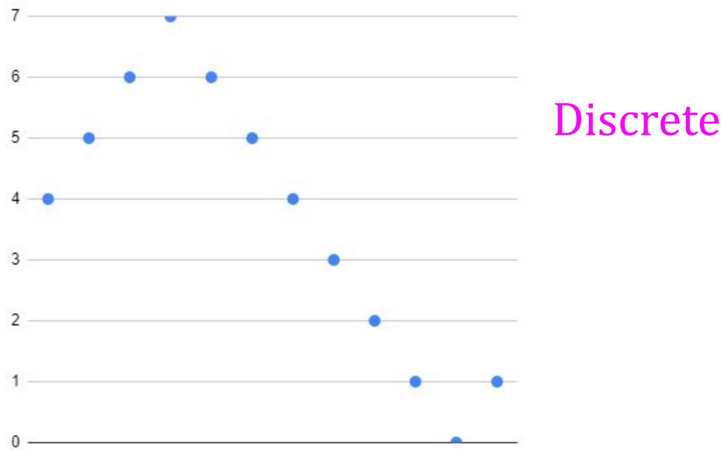
Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Analog & Digital

Analog and Discrete Values (Ramp and Stairs)



Continuous numbers represent analog values like room temperature. The numbers change continuously, it means there is always one number between any two chosen numbers. Example: {from 0 to 10}. In nature the values are analog value



Digital numbers represent discrete values, the numbers change stepwise like integer number.

Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Example 1

A temperature sensor in the picture measures temperature from 0-100 degree celsius.

If we have 10 bits to code the measured temperature, what would be the binary code for 25.6 degrees?

How many Codes will we get with 10 bits?

Example1

A temperature sensor in the picture measures temperature from 0-100 degree celsius.

If we have 10 bits to code the measured temperature.

What would be the binary code for 25.6 degrees?

How many Codes will we get with 10 bits?

2^10 = 1024

100/1024 = 0.09765 quantization interval

25.6 / 0.09765625 = 262.144 → 262 → 01 0000 0110

Example 2

There is a sensor with 4-20 mA output connected to a controller. The controller input has a 10 bit ADC which converts this signal to integer numbers.

1- What would be the number of codes used for coding? what would be the largest integer number you can read inside the controller?

2- What would be the resolution?

3- if the sensor is sending out 13 mA, what will be integer value inside the controller?

4- What would be the quantization error value? in mA and in percentage of full scale?

5- What would be the maximum quantization error value in mA and in percentage of full scale?

6- if inside the controller you see 376 as reading, how much in mA the sensor is sending?

Example 2

There is a sensor with 4-20 mA output connected to a controller. The controller input has a 10 bit ADC which converts this signal to integer numbers.

1- What would be the number of codes used for coding? what would be the largest integer number you can read inside the controller?

2- What would be the resolution?

3- if the sensor is sending out 13 mA, what will be integer value inside the controller?

4- What would be the quantization error value? in mA and in percentage of full scale?

5- What would be the maximum quantization error value in mA and in percentage of full scale?

6- if inside the controller you see 376 as reading, how much in mA the sensor is sending?

1- 2^10= 1024

largest integer number = 1023

2) 20-4 mA=16 mA   16 mA/ 1023= 0.0156 mA per step

Resolution = 0.0156 mA per step

3) 13-4 = 9 mA   9/16*1023= 575.4375 → 575

4) 9- (575* 16/1023)= 9 - 8.9931= 0.0068 mA

0.0068/16 mA *100 = 0.042 % FS

5) ±Δ/2= ±(20-4)/(1023x2)=± 0.0078 mA

±(16/(1023x2))/16 x 100=± 100/(1023x2)= ±0.0488 % FS

6) 376/1023* 16 mA +4 = 9.88 mA

Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Example 3

There is a laser sensor measuring distance of 0-5 meter. There is a 12 bit ADC to convert the measurement to digital value.

1- What would be the minimum change of distance which can change the the reading on the display?

2- What should be the display resolution to cover the device measuring resolution, one, two or three decimal points?

3- The distance is 2.65 meter, what would be the quantization value? What would quantization error?

4- What would be the maximum quantization error value for this device?

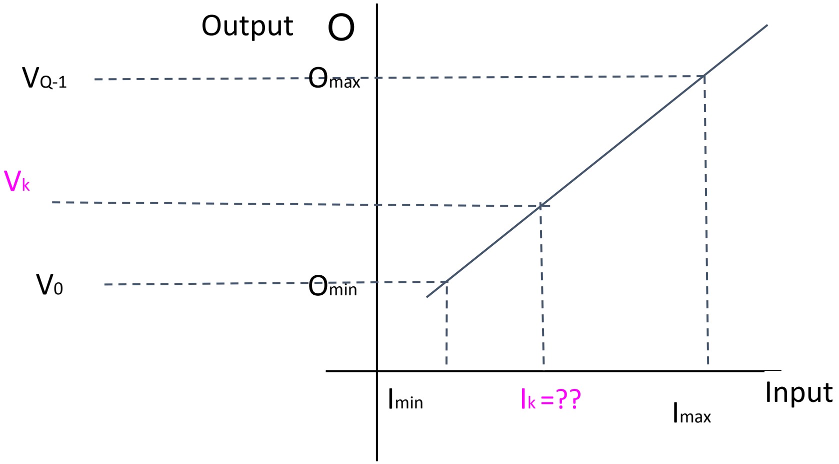
Example 4

A sensor measuring range is I_{min} to I_{max}. An the output range is O_{min} to O_{max} .

The sensor output is given to a controller with N bit ADC.

The ADC produces Q integer number V_i based on the input value.

Make an equation for programming so that for every given integer number k it provides the a decimal number of measured input value.



Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

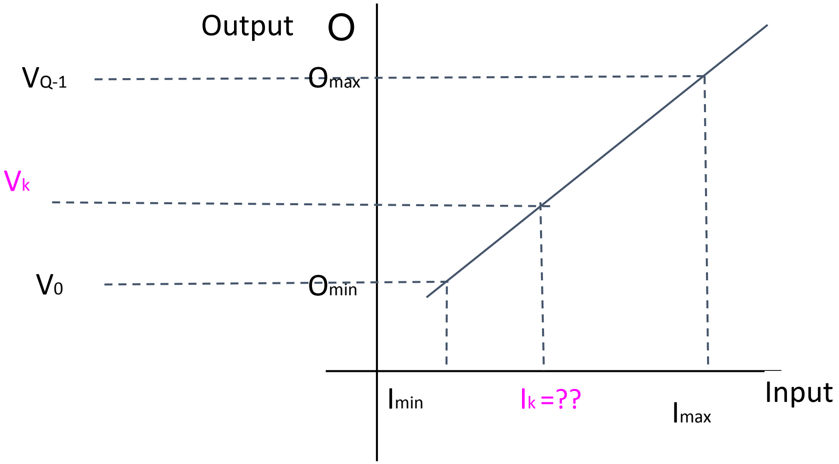
Example 4

A sensor measuring range is I_{min} to I_{max}. An the output range is O_{min} to O_{max} .

The sensor output is given to a controller with N bit ADC.

The ADC produces Q integer number V_i based on the input value.

Make an equation for programming so that for every given integer number k it provides the a decimal number of measured input value.



$Q = 2^N$

$V_{q-1} = Q - 1$

$\Delta = \dfrac{I_{\max} - I_{min}}{V_{q-1}}$

V_k is given

presented *value* for k $\Rightarrow I_k = V_k \times \Delta = \dfrac{V_k}{V_{q-1}} \times (I_{\max} - I_{min}) + I_{min}$

Example 1 and Example 2-6 are numerical example of example 4

Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Sampling

Next question is that how often should be the analog value should be read and converted to digital value? For example every 1 ms , 100 ms, 1 sec or 10 second.

This time interval for conversion is referred to as sampling period, T_s.

Inverse of sampling period is referred to as sampling frequency $f_s=1/T_s$.

At first glance having small sampling time (Higher sampling frequency) will make digital signal closer to analog signal but this desire is limited by some factors therefore the possible minimum value for sampling frequency should be found then a compromise should be made between the high and low sampling frequency limits.

The high sampling frequency (low sample time) limits:

1- Conversion from analog to digital by electronic circuit takes time. Then sampling time can not be smaller than conversion time.

2- A memory is needed to save the digital numbers. For example to save digital numbers produced by sample time of 100 ms compare to 1 sec, 10 times more memory is needed.

3- If the converted value is supposed to be transmitted over the network, the transmission line and speed can be considered as limiting factor also

Sampling Time

The high sampling frequency (low sample time) limits:

1- Conversion from analog to digital by electronic circuit takes time. Then sampling time can not be smaller than conversion time.

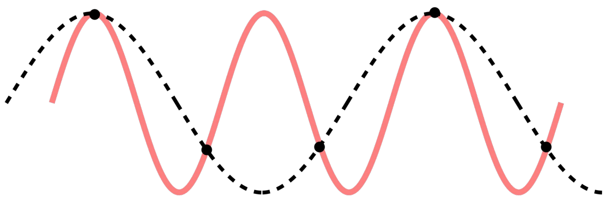
2- A memory is needed to save the digital numbers. For example to save digital numbers produced by sample time of 100 ms compare to 1 sec, 10 times more memory is needed.

3- If the converted value is supposed to be transmitted over the network, the transmission line and speed can be considered as limiting factor also

The low sampling frequency (high sample time) problems:

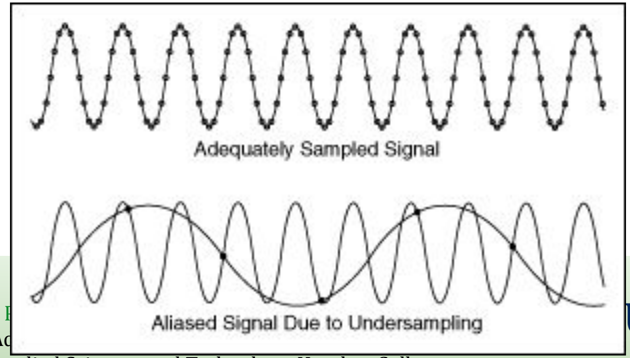
1- If the number of the sample are too low the sampled value hardly present actual value and rise and fall of the actual values will be missed.

Intuitively if a rate (change over time) is high, then higher sampling rate is better. If something changes slowly the high sampling rate will not be needed.



Nyquist–Shannon sampling theorem & Optimal sampling Rate

- The minimum of sampling frequency should be double of maximum frequency of a analog value.
- If this condition is not applied some part original analog value will be lost and the original analog value can not be constructed from digital codes. In other word the correlation between analog and digital value will be lost.



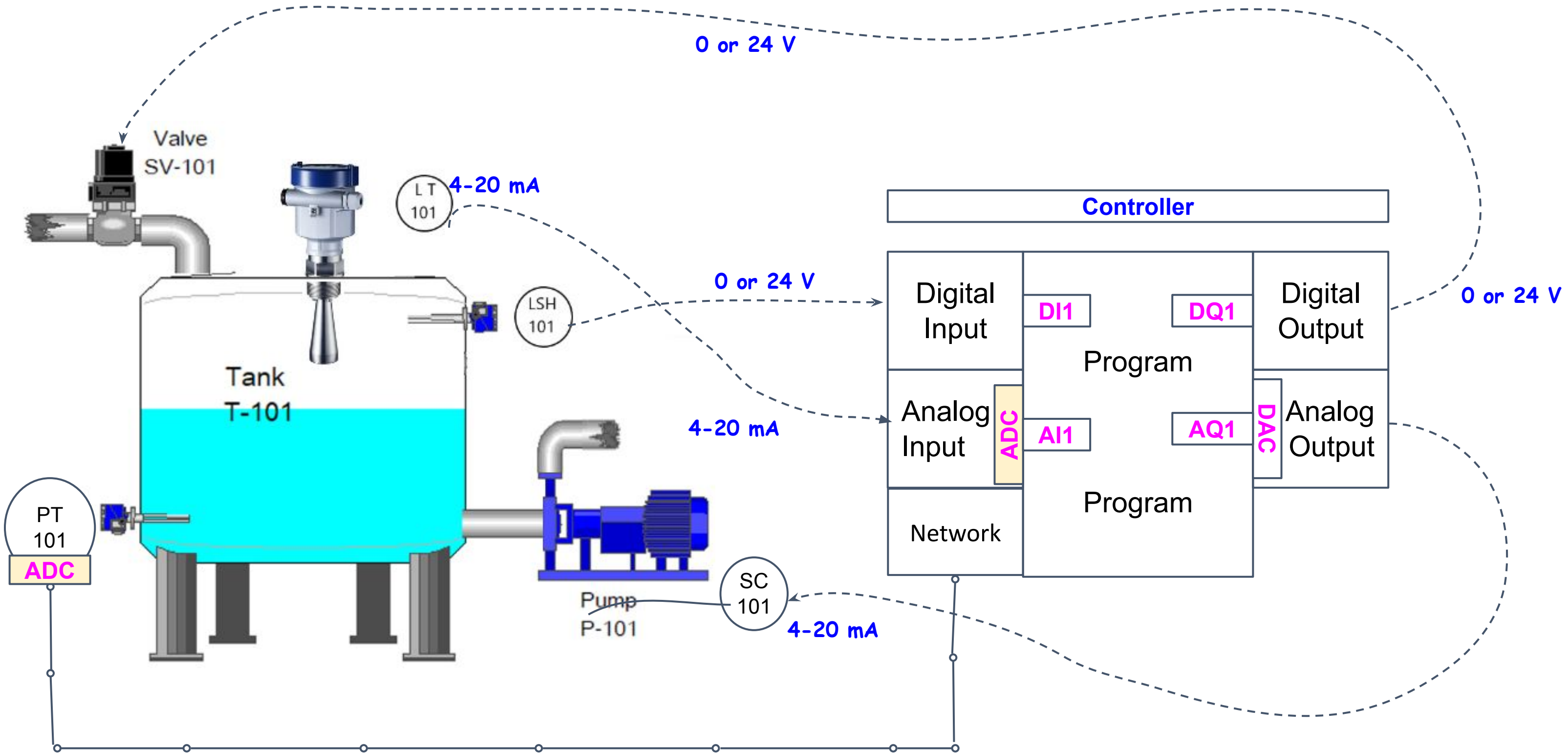
- Nyquist Theorem defines theoretical and mathematical condition for minimum of sampling frequency of one signal. In reality conditions of whole control system such as frequency response of the process have role to find minimum and optimal sample frequency
- the sample frequency can be selected 10 times of maximum frequency of the analog signal.

Sensor Response Time

- Low sample increases the sensor response time and make it slower.
- Slow response device in fast changing environment make the measurement and the control system unreal. The measurement will lose some values and the system lags to follow actual changes.
- a low sample rate might make the system unstable also.

- In case that the ADC conversion time is high and make the system slow:
    - And ADC with different method of conversion might be found that which has smaller conversion time.
    - If ADC is done inside the controller, A sensor with ADC in its own sensor head might be better solution or another way around.

Practical Wiring Connection Example



Amir Jafari, Ph.D., P.Eng.
Professor, Advanced Manufacturing
Faculty of Applied Sciences and Technology, Humber College

Example 5

The line voltage frequency is 60 Hz.

To convert this voltage reading to digital value what should minimum sampling interval?

what would be sampling frequency if we want to have 10 samples per one period?