Topic 7

April 3, 2023 2:50 PM

I. CENTER OF GRAVITY, CENTER OF MASS AND CENTROID OF A BODY
II. COMPOSITE BODIES

ENGI 1510 - ENGINEERING DESIGN
Winter 2023

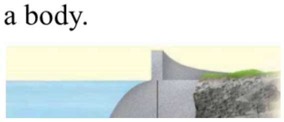
CENTER OF GRAVITY, CENTER OF MASS AND CENTROID OF A BODY

Today's Objective :

Students will:

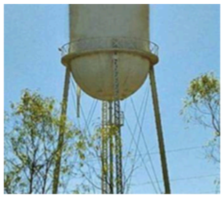
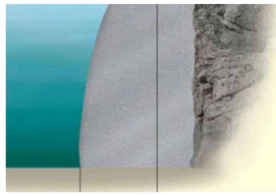
a) Understand the concepts of center of gravity, center of mass, and centroid.

b) Be able to determine the location of these points for a body.

In-Class Activities:

- Check Homework, if any
- Reading Quiz
- Applications
- Center of Gravity

- Determine CG Location
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. The __________ is the point defining the geometric center of an object.

   A) Center of gravity B) Center of mass

   C) Centroid D) None of the above

2. To study problems concerned with the motion of matter under the influence of forces, i.e., dynamics, it is necessary to locate a point called ________.

   A) Center of gravity B) Center of mass

   C) Centroid D) None of the above

APPLICATIONS

To design the structure for supporting a water tank, we will need to know the weight of the tank and water as well as the locations where the resultant forces representing these distributed loads act.

How can we determine these resultant weights and their lines of action?

APPLICATIONS (continued)

One concern about a sport utility vehicle (SUV) is that it might tip over when taking a sharp turn.

One of the important factors in determining its stability is the SUV's center of mass.

Should it be higher or lower to make a SUV more stable?

How do you determine the location of the SUV's center of mass?

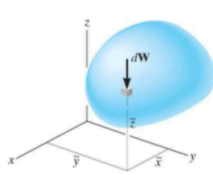
APPLICATIONS (continued)

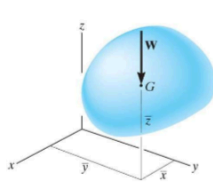
To design the ground support structure for a goal post, it is critical to find total weight of the structure and the center of gravity's location.

Integration must be used to determine total weight of the goal post due to the curvature of the supporting member.

How do you determine the location of overall center of gravity?

CONCEPT OF CENTER OF GRAVITY (CG)





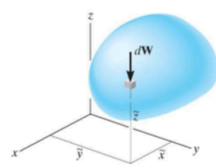
A body is composed of an infinite number of particles, and so if the body is located within a gravitational field, then each of these particles will have a weight $d\boldsymbol{W}$.

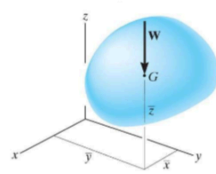
The center of gravity (CG) is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.

From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G.

Also, note that the sum of moments due to the individual particle's weights about point G is equal to zero.

CONCEPT OF CG (continued)





The location of the center of gravity, measured from the y axis, is determined by equating the moment of ***W*** about the y-axis to the sum of the moments of the weights of the particles about this same axis.

If $d\boldsymbol{W}$ is located at point $(\tilde{x}, \tilde{y}, \tilde{z})$, then

$$\bar{x}\,\boldsymbol{W} = \int \tilde{x}\, d\boldsymbol{W}$$

Similarly, $\bar{y}\,\boldsymbol{W} = \int \tilde{y}\, d\boldsymbol{W}$ $\quad \bar{z}\,\boldsymbol{W} = \int \tilde{z}\, d\boldsymbol{W}$

Therefore, the location of the center of gravity G with respect to the x, y, z-axes becomes

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

CM & CENTROID OF A BODY

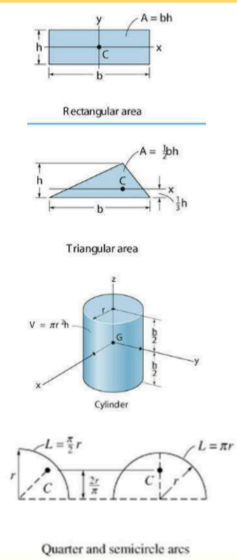
$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

By replacing the ***W*** with a m in these equations, the coordinates of the center of mass can be found.

$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm} \qquad \bar{y} = \frac{\int \tilde{y}\, dm}{\int dm} \qquad \bar{z} = \frac{\int \tilde{z}\, dm}{\int dm}$$

Similarly, the coordinates of the centroid of volume, area, or length can be obtained by replacing ***W*** by ***V***, ***A***, or ***L***, respectively.

CONCEPT OF CENTROID



The centroid, C, is a point defining the geometric center of an object.

The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogenous (density or specific weight is constant throughout the body).

If an object has an axis of symmetry, then the centroid of object lies on that axis.

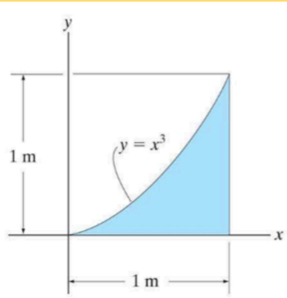
In some cases, the centroid may not be located on the object.

STEPS TO DETERME THE CENTROID OF AN AREA

1. Choose an appropriate differential element dA at a general point (x,y).
   Hint: Generally, if y is easily expressed in terms of x (e.g., $y = x^2 + 1$), use a vertical rectangular element. If the converse is true, then use a horizontal rectangular element.
2. Express dA in terms of the differentiating element dx (or dy).
3. Determine coordinates $(\tilde{x}, \tilde{y})$ of the centroid of the rectangular element in terms of the general point (x, y).
4. Express all the variables and integral limits in the formula using either x or y depending on whether the differential element is in terms of dx or dy, respectively, and integrate.

Note: Similar steps are used for determining the CG or CM. These steps will become clearer by doing a few examples.

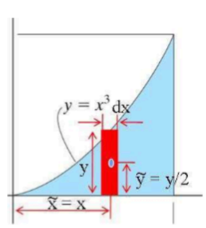
EXAMPLE I



Given: The area as shown.

Find: The centroid location $(\bar{x}, \bar{y})$

Plan: Follow the steps.



Solution:

1. Since y is given in terms of x, choose dA as a vertical rectangular strip.

2. $dA = y\,dx = x^3\,dx$

3. $\tilde{x} = x$ and $\tilde{y} = y / 2 = x^3 / 2$

EXAMPLE I (continued)

4. $\bar{x} = (\int_A \tilde{x} \ dA) / (\int_A dA)$

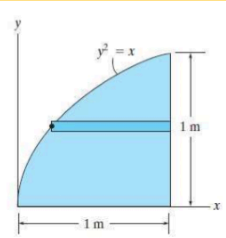
$$= \frac{{}_0\int^1 x\,(x^3)\,d\,x}{{}_0\int^1 (x^3)\,d\,x} = \frac{1/5\,[\,x^5\,]_0^1}{1/4\,[\,x^4\,]_0^1}$$

$= (1/5) / (1/4) = 0.8 \text{ m}$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{{}_0\int^1 (x^3/2)\,(x^3)\,dx}{{}_0\int^1 x^3\,dx} = \frac{1/14[x^7]_0^1}{1/4}$$

$= (1/14) / (1/4) = 0.2857 \text{ m}$

EXAMPLE II

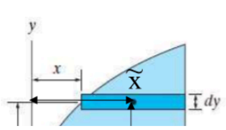


Given: The shape and associated horizontal rectangular strip shown.

Find: dA and $(\tilde{x}, \tilde{y})$

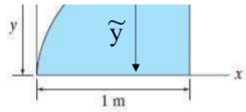
Plan: Follow the steps.

Solution:



1. $dA = x\,dy = y^2\,dy$

2. $\tilde{x} = x + (1 - x) / 2 = (1 + x) / 2 = (1 + y^2)/2$

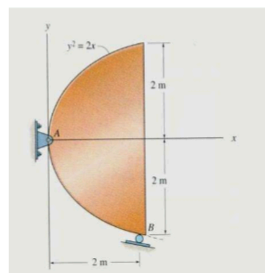


2. $\tilde{x} = x + (1-x) / 2 = (1 + x) /2 = (1 + y^2)/2$

3. $\tilde{y} = \overline{y}$

CONCEPT QUIZ

1. The steel plate, with known weight and non-uniform thickness and density, is supported as shown. Of the three parameters CG, CM, and centroid, which one is needed for determining the support reactions? Are all three parameters located at the same point?

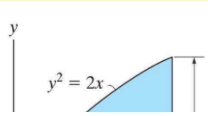


   A) (center of gravity, yes)
   B) (center of gravity, no)
   C) (centroid, yes)
   D) (centroid, no)

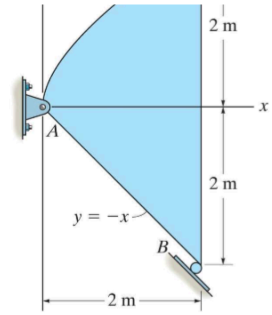
2. When determining the centroid of the area above, which type of differential area element requires the least computational work?

   A) Vertical
   B) Horizontal
   C) Polar
   D) Any one of the above.

GROUP PROBLEM SOLVING



Given: The steel plate is 0.3 m thick and has a density of 7850 kg/m^3.



Find: The location of its center of mass. Also compute the reactions at A and B.

Plan: Follow the solution steps to find the CM by integration. Then use 2-dimensional equations of equilibrium to solve for the external reactions.

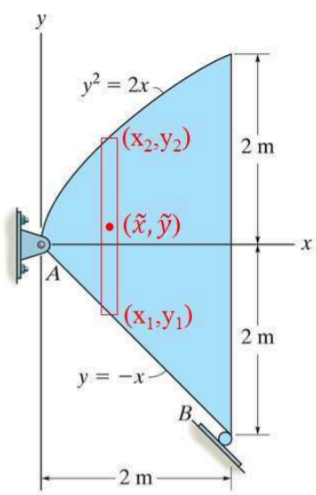
GROUP PROBLEM SOLVING (continued)

Solution:

1. Choose dA as a vertical rectangular strip.

2. $dA = (y_2 - y_1)\ dx$
   $= (\sqrt{2x} + x)\ dx$

3. $\tilde{x} = x$
   $\tilde{y} = (y_1 + y_2) / 2$
   $= (\sqrt{2x} - x) / 2$



GROUP PROBLEM SOLVING (continued)

4.

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^2 x(\sqrt{2x}+x)\,dx}{\int_0^2 (\sqrt{2x}+x)\,dx} = \frac{\left[\left(\frac{2\sqrt{2}}{5}\right)x^{\frac{5}{2}}+\frac{1}{3}x^3\right]_0^2}{\left[\left(\frac{2\sqrt{2}}{3}\right)x^{\frac{3}{2}}+\frac{1}{2}x^2\right]_0^2}$$

$$= \frac{5.867}{4.667} = 1.257 \text{ m}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^2 \{(\sqrt{2x}-x)/2\}(\sqrt{2x}+\ )}{\int_0^2 (\sqrt{2x}+x)\,dx} = \frac{\left[\frac{x^2}{2}-\frac{1}{6}x^3\right]_0^2}{\left[\left(\frac{2\sqrt{2}}{3}\right)x^{\frac{3}{2}}+\frac{1}{2}x^2\right]_0^2}$$

$$= \frac{0.6667}{4.667} = 0.143 \text{ m}$$

$\bar{x} = 1.26$ m and $\bar{y} = 0.143$ m

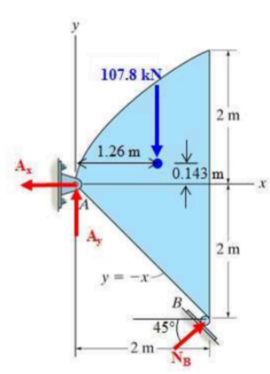
GROUP PROBLEM SOLVING (continued)

Place the weight of the plate at the centroid G.

Area, $A = 4.667\ m^2$

Weight, $W = (7850)\ (9.81)\ (4.667)\ 0.3 = 107.8$ kN

Here is FBD to find the reactions at A and B.



Applying Equations of Equilibrium:

$+ \sum M_A = N_B\,(2\sqrt{2}) - 107.8\,(1.26) = 0$

$N_B = 47.92 = 47.9$ kN

$+\rightarrow\ \sum F_X = -A_x + 47.92 \sin 45° = 0$

$A_X = 33.9$ kN

$+\uparrow\ \sum F_Y = A_y + 47.92 \cos 45° - 107.8 = 0$

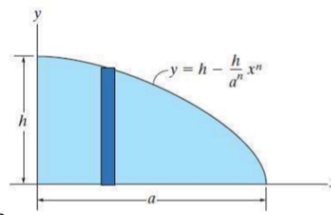
$A_Y = 73.9$ kN

ATTENTION QUIZ

1. If a vertical rectangular strip is chosen as the differential element, then all the variables, including the integral limit, should be in terms of _____ .

   A) x B) y

   C) z D) Any of the above.



2. If a vertical rectangular strip is chosen, then what are the values of $\tilde{x}$ and $\tilde{y}$?

   A) (x , y) B) (x / 2 , y / 2)

   C) (x , 0) D) (x , y / 2)

COMPOSITE BODIES

<u>Today's Objective</u>:

Students will be able to determine:

a) The location of the center of gravity (CG),

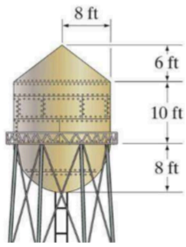
b) The location of the center of mass,

c) And, the location of the centroid using the method of composite

<u>In-Class Activities:</u>

- Check homework, if any
- Reading Quiz

bodies.



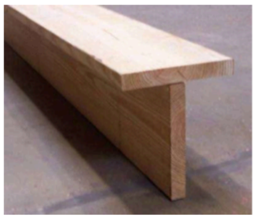
- Applications
- Method of Composite Bodies
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. A composite body in this section refers to a body made of ____.
   A) Carbon fibers and an epoxy matrix in a car fender
   B) Steel and concrete forming a structure
   C) A collection of “simple” shaped parts or holes
   D) A collection of “complex” shaped parts or holes

2. The composite method for determining the location of the center of gravity of a composite body requires _______.
   A) Simple arithmetic
   B) Integration
   C) Differentiation
   D) All of the above.

APPLICATIONS

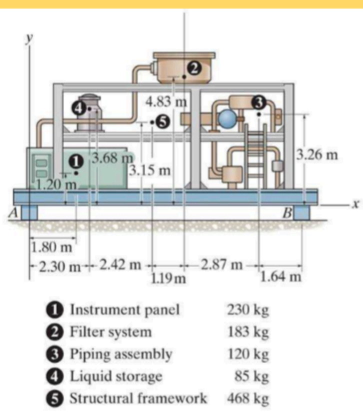


The I-beam (top) or T-beam (bottom) shown are commonly used in building various types of structures.

When doing a stress or deflection analysis for a beam, the location of its centroid is very important.

How can we easily determine the location of the centroid for different beam shapes?

APPLICATIONS (continued)

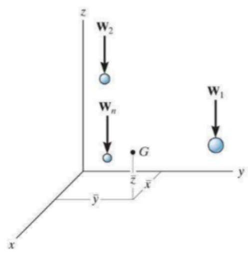


The compressor is assembled with many individual components.

In order to design the ground support structures, the reactions at blocks A and B have to be found. To do this easily, it is important to determine the location of the compressor's center of gravity (CG).

If we know the weight and CG of individual components, we need a simple way to determine the location of the CG of the assembled unit.

CG/CM OF A COMPOSITE BODY



Consider a composite body which consists of a series of particles (or bodies) as shown in the figure. The net or resultant weight is given as $W_R = \sum W$.

Summing the moments about the y-axis, we get

$$\bar{x}\, W_R = \tilde{x}_1 W_1 + \tilde{x}_2 W_2 + \ldots\ldots\ldots. + \tilde{x}_n W_n$$

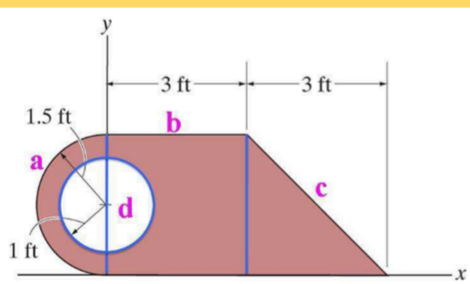
where $\tilde{x}_1$ represents x coordinate of W_1, etc..

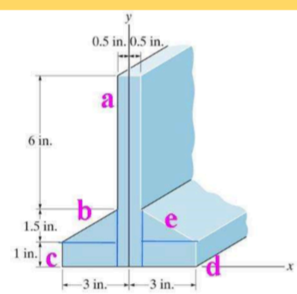
Similarly, we can sum moments about the x- and z-axes to find the coordinates of the CG.

$$\bar{x} = \frac{\sum \tilde{x} W}{\sum W} \qquad \bar{y} = \frac{\sum \tilde{y} W}{\sum W} \qquad \bar{z} = \frac{\sum \tilde{z} W}{\sum W}$$

By replacing the W with a M in these equations, the coordinates of the center of mass can be found.

CONCEPT OF A COMPOSITE BODY

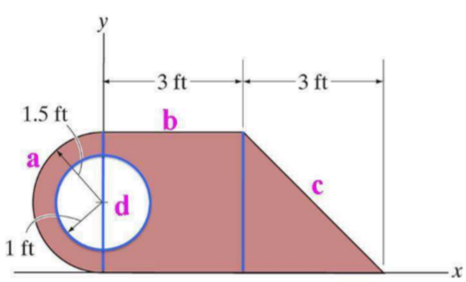


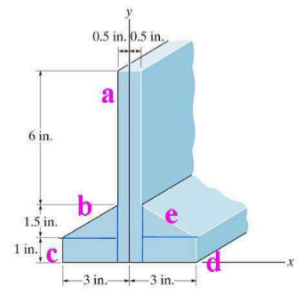


Many industrial objects can be considered as composite bodies made up of a series of connected "simple-shaped" parts, like a rectangle, triangle, and semicircle, or holes.

Knowing the location of the centroid, C, or center of gravity, CG, of the simple-shaped parts, we can easily determine the location of the C or CG for the more complex composite body.

CONCEPT OF A COMPOSITE BODY (continued)





This can be done by considering each part as a "particle" and following the procedure as described in Section 9.1.

This is a simple, effective, and practical method of determining the location of the centroid or center of gravity of a complex part, structure or machine.

STEPS FOR ANALYSIS

1. Divide the body into pieces that are known shapes.
   Holes are considered as pieces with negative weight or size.

2. Make a table with the first column for segment number, the second

column for weight, mass, or size (depending on the problem), the next set of columns for the moment arms, and, finally, several columns for recording results of simple intermediate calculations.

3. Fix the coordinate axes, determine the coordinates of the center of gravity of centroid of each piece, and then fill in the table.

4. Sum the columns to get $\bar{x}$, $\bar{y}$, and $\bar{z}$. Use formulas like

   $\bar{x} = (\Sigma\, \tilde{x}_i A_i) / (\Sigma A_i)$ or $\bar{x} = (\Sigma\, \tilde{x}_i W_i) / (\Sigma W_i)$

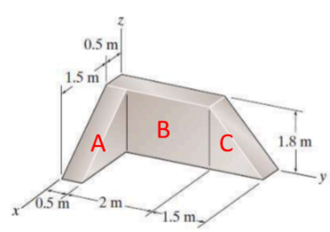
   This approach will become straightforward after doing examples!

EXAMPLE

Given: Three blocks are assembled as shown.

Find: The center of volume of this assembly.

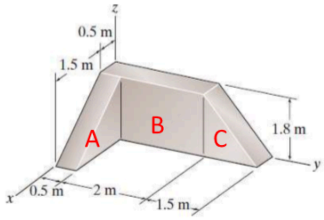
Plan: Follow the steps for analysis.



Solution:

1. In this problem, the blocks A, B and C can be considered as three pieces (or segments).

EXAMPLE (continued)



Volumes of each shape:

$V_A = (0.5)\,(1.5)\,(1.8)\,(0.5) = 0.675\ m^3$

$V_B = (2.5)\,(1.8)\,(0.5) = 2.25\ m^3$

$V_C = (0.5)\,(1.5)\,(1.8)\,(0.5) = 0.675\ m^3$

Segment	V (m^3)	$\tilde{x}$ (m)	$\tilde{y}$ (m)	$\tilde{z}$ (m)	$x\tilde{V}$ (m^4)	$\tilde{y}V$ (m^4)	$z\tilde{V}$ (m^4)
A	0.675	1.0	0.25	0.6	0.675	0.1688	0.405
B	2.25	0.25	1.25	0.9	0.5625	2.813	2.025
C	0.675	0.25	3.0	0.6	0.1688	2.025	0.405
Σ	**3.6**				**1.406**	**5.007**	**2.835**

EXAMPLE (continued)

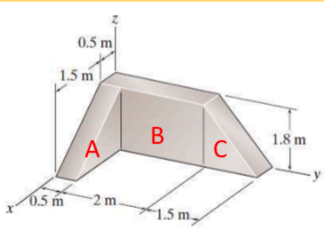


Table Summary

V (m^3)	$\tilde{x}$ V (m^4)	$\tilde{y}$ V (m^4)	$\tilde{z}$ V (m^4)
3.6	**1.406**	**5.007**	**2.835**

Substituting into the Center of Volume equations:

$\bar{x} = (\Sigma\,\tilde{x}\ V) / (\Sigma V) = 1.406 / 3.6 = 0.391\ m$

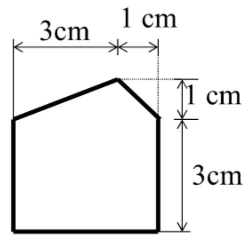
$\bar{y} = (\Sigma\,\tilde{y}\ V) / (\Sigma V) = 5.007 / 3.6 = 1.39\ m$

$\bar{z} = (\Sigma\,\tilde{z}\ V) / (\Sigma V) = 2.835 / 3.6 = 0.788\ m$

CONCEPT QUIZ

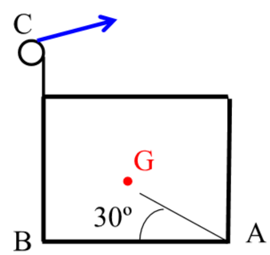
1. Based on typical available centroid information, what are the minimum number of pieces to consider for determining the centroid of the area shown at the right?

   A) 4 B) 3 C) 2 D) 1

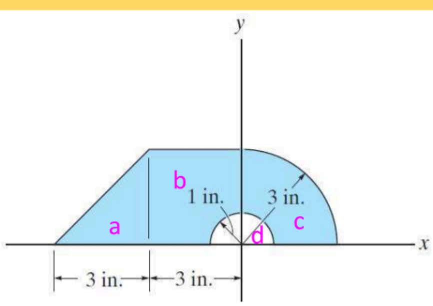


2. A storage box is tilted up to clean the rug underneath the box. It is tilted up by pulling the handle C, with edge A remaining on the ground. What is the maximum angle of tilt possible (measured between bottom AB and the ground) before the box tips over?

   A) 30° B) 45 ° C) 60 ° D) 90 °



GROUP PROBLEM SOLVING



Given: The part shown.

Find: The centroid of the part.

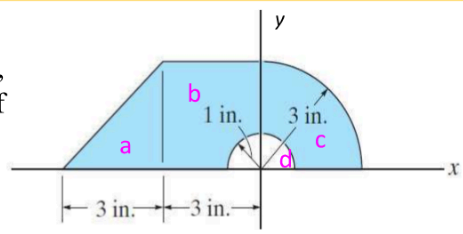
Plan: Follow the steps for analysis.

Solution:

1. This body can be divided into the following pieces:
   triangle (a) + rectangle (b) + quarter circular (c)
   – semicircular area (d).
   Note that a negative sign should be used for the hole!

GROUP PROBLEM SOLVING (continued)

Steps 2 & 3: Create and complete the table using parts a, b, c, and d. Note the location of the axis system.



Segment	Area A (in^2)	$\tilde{x}$ (in)	$\tilde{y}$ (in)	$\tilde{x}$ A (in^3)	$\tilde{y}$ A (in^3)
Triangle a	4.5	– 4	1	– 18	4.5
Rectangle b	9.0	– 1.5	1.5	– 13.5	13.5
Qtr. Circle c	$9\pi / 4$	$4(3) / (3\pi)$	$4(3) / (3\pi)$	9	9
Semi-Circle d	$-\pi / 2$	0	$4(1) / (3\pi)$	0	– 0.67
Σ	**19.00**			**– 22.5**	**26.33**

GROUP PROBLEM SOLVING (continued)

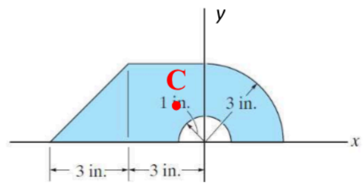
4. Now use the table data results and the formulas to find the

coordinates of the centroid.

Area A	$\tilde{x}A$	$\tilde{y}A$
19.00	– 22.5	26.33

$\bar{x}$ = $(\Sigma\, \tilde{x}\, A) / (\Sigma\, A)$ = – 22.5 in^3/ 19.0 in^2 = – 1.18 in

$\bar{y}$ = $(\Sigma\, \tilde{y}\, A) / (\Sigma\, A)$ = 26.33 in^3 / 19.0 in^2 = 1.39 in

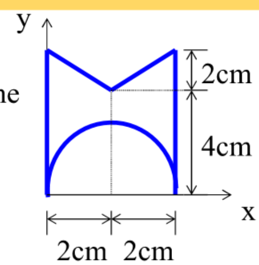


ATTENTION QUIZ

1. A rectangular area has semicircular and triangular cuts as shown. For determining the centroid, what is the minimum number of pieces that you can use?

   A) Two B) Three

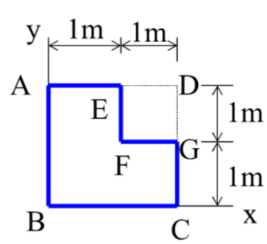
   C) Four D) Five



2. For determining the centroid of the area, two square segments are considered; square ABCD and square DEFG. What are the coordinates $(\tilde{x}, \tilde{y})$ of the centroid of square DEFG?

   A) (1, 1) m B) (1.25, 1.25) m

   C) (0.5, 0.5) m D) (1.5, 1.5) m



End of the Lecture

Let Learning Continue