HUMBER

ROBOTICS II

Trajectory Planning

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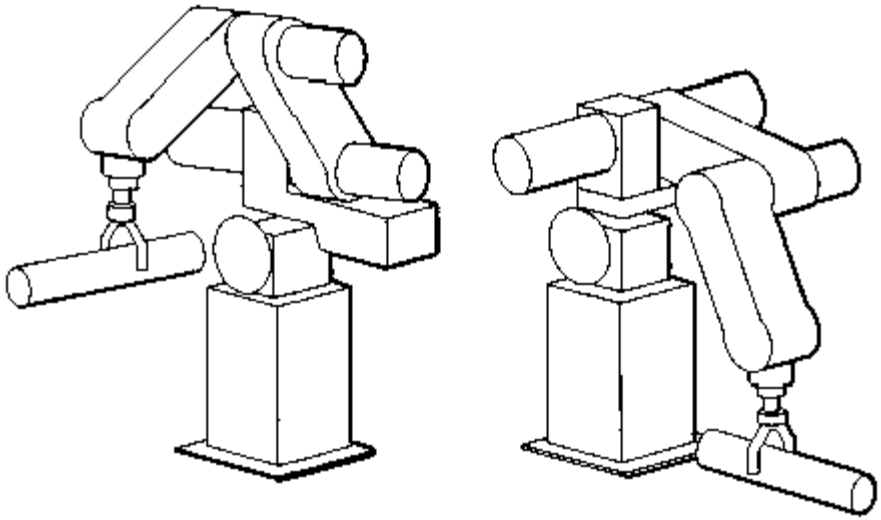
Trajectory



Joint Trajectory

- Point to Point
- Point to Point with Via Point

Point to Point

- Cubical Polynomial Approach for a trajectory

Cubical Polynomial

- In making a single smooth motion at least four constraints are required.
- Two constraints on the function value selection on the basis of initial and final position

$$\theta(0) = \theta_0,$$

$$\theta(t_f) = \theta_f.$$

Cubical Polynomial

- The Other two constraints are continuous velocity.

$$\dot{\theta}(0) = 0,$$

$$\dot{\theta}(t_f) = 0.$$

- These four constrained can be satisfied by the polynomial of at least third degree.

Cubical Polynomial

- Cubical equation is for joint position

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3,$$

- For Velocity and acceleration

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2,$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t.$$

Cubical Polynomial

- Solving the equations for constraints with the given information, we obtain

$$a_0 = \theta_0,$$

$$a_1 = 0,$$

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0),$$

$$a_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0).$$

Example

A single link robot with a rotary joint is motionless at $\Theta = 15$ degree. It is desired to move the joint in a smooth manner to $\Theta = 75$ degree in 3 seconds. Find the coefficients of a cubic which accomplishes this motion and brings the manipulator to rest at the goal. Plot the position, velocity, and acceleration of the joint as a function of time.

Cubical Polynomial

- We can determine the constrained by using the above equation.

$$a_0 = 15.0,$$

$$a_1 = 0.0,$$

$$a_2 = 20.0,$$

$$a_3 = -4.44.$$

Cubical Polynomial

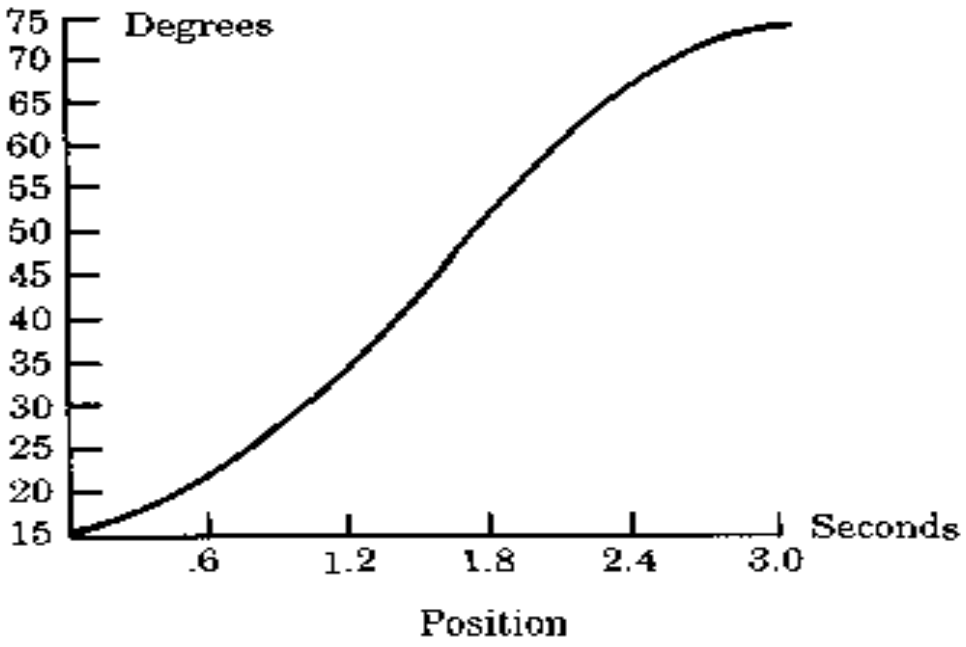
- Equation of trajectory are

$$\theta(t) = 15.0 + 20.0t^2 - 4.44t^3,$$

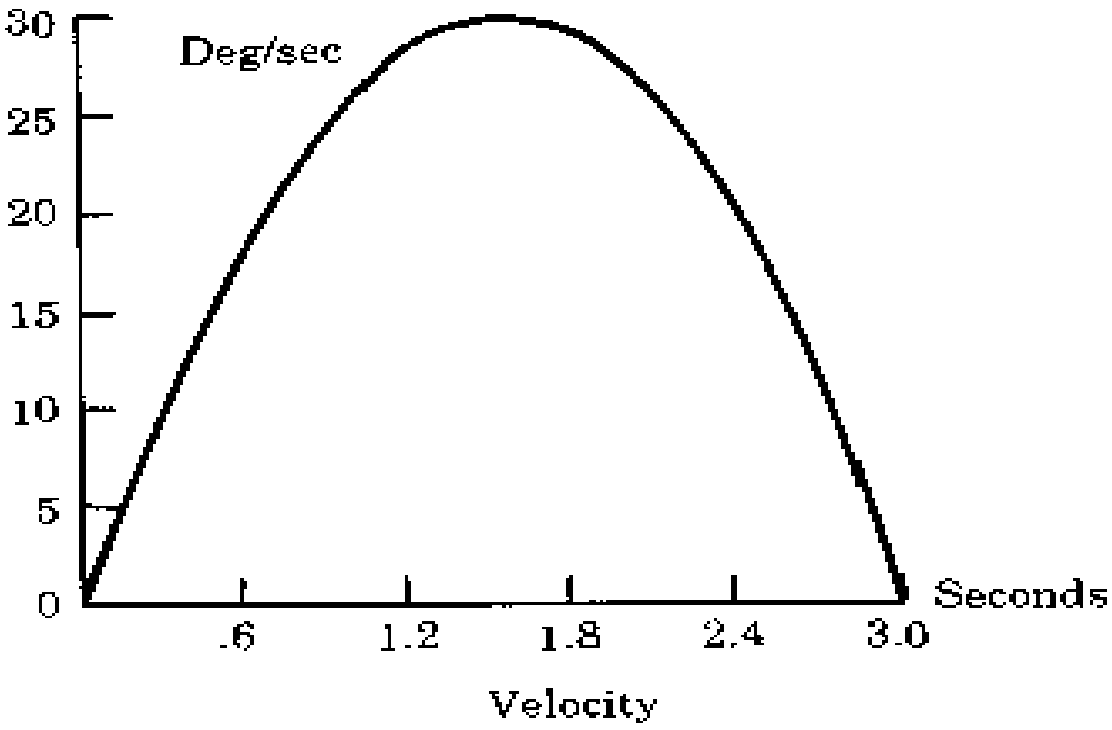
$$\dot{\theta}(t) = 40.0t - 13.33t^2,$$

$$\ddot{\theta}(t) = 40.0 - 26.66t.$$

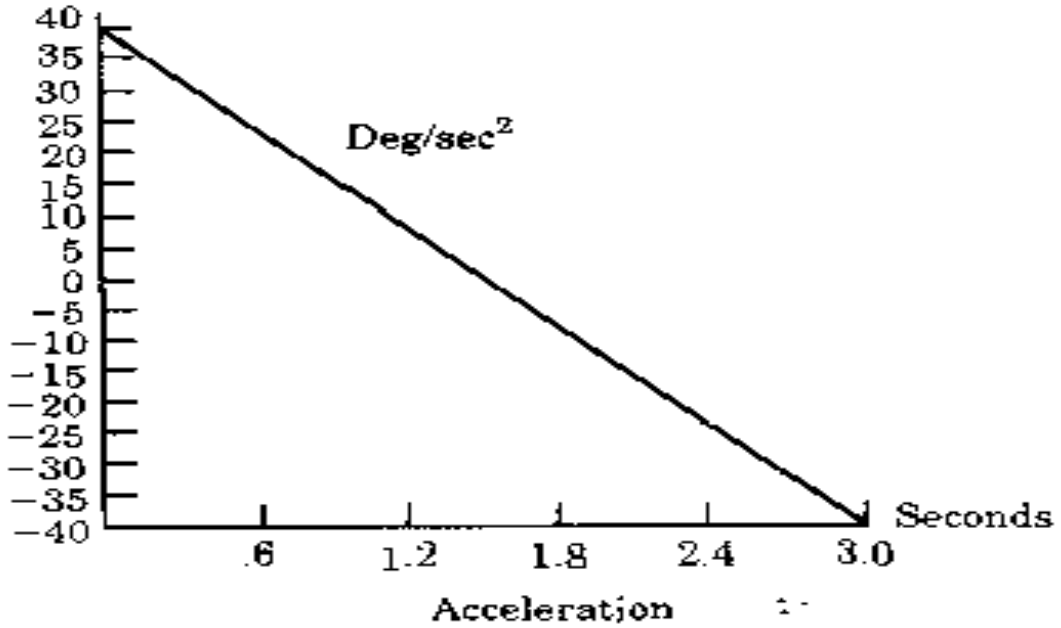
Position


75
70
65
60
55
50
45
40
35
30
25
20
15
Degrees
.6
1.2
1.8
2.4
3.0
Seconds
Position

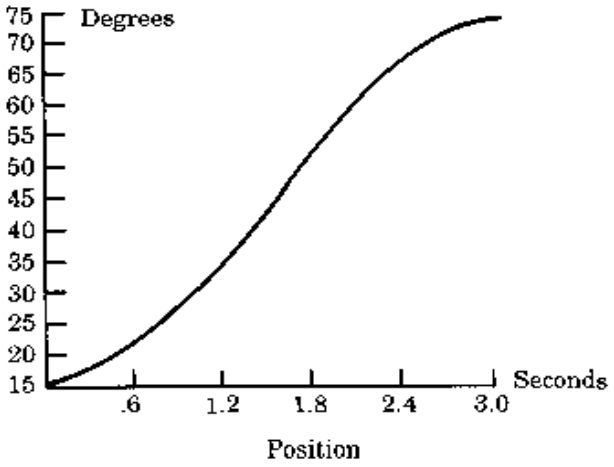
Velocity Diagram

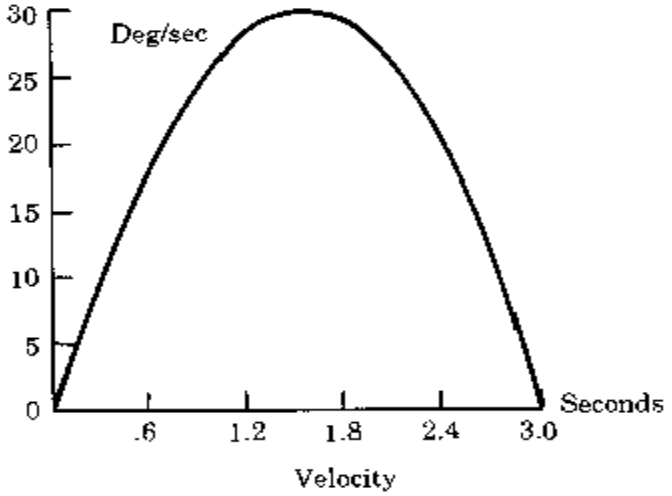


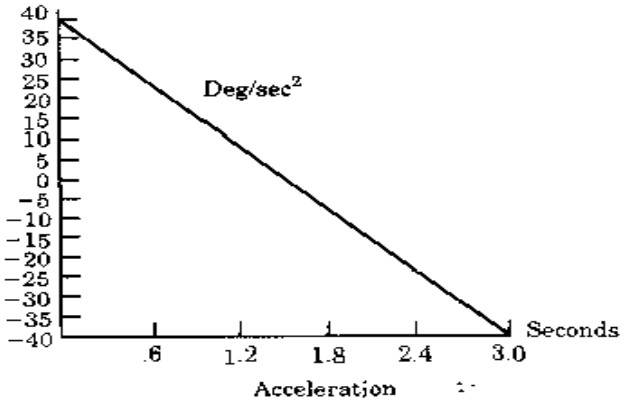
Acceleration Diagram



Position, Velocity and Acceleration diagrams







Cubical Polynomial Via point