Instrumentation & Measurement

Winter 2024

Force and Mass

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Introduction to Process Control

- Source Book:

Process Control Instrumentation Technology
Curtis D. Johnson
Eighth Edition
Pearson Education Limited 2014
ISBN 10: 1-292-02601-4
ISBN 13: 978-1-292-02601-5

Chapter 5 section 3

- Source Book:

Instrumentation and Process Control
Franklyn Kirk, Thomas Weedon, Philip Kirk
American Tech Publishers 7th Edition (May 2019)
ISBN-10: 0826934463
ISBN-13: 978-0826934468

- Source Book:

Measurement and Instrumentation: Theory and Application, Alan S. Morris, Reza Langari, Third edition Academic Press (Oct 2020)
ISBN-13: 978-0128171417
ISBN-10: 0128171413

Mass Balanced Method

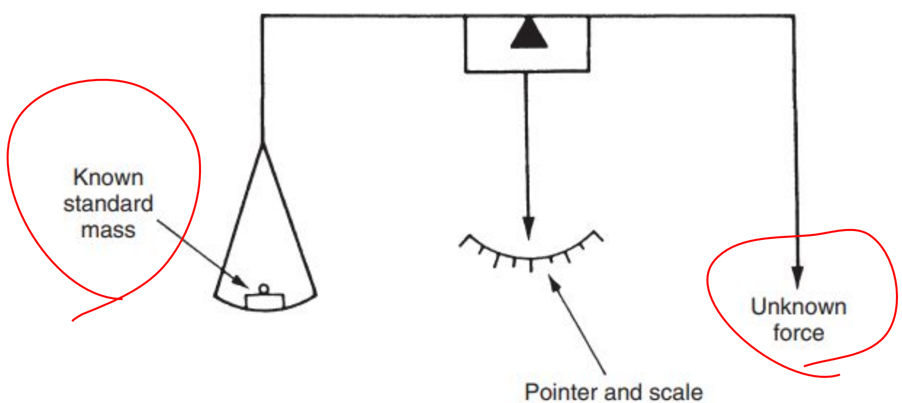

Figure 18.5
Beam balance (Equal arm balance).

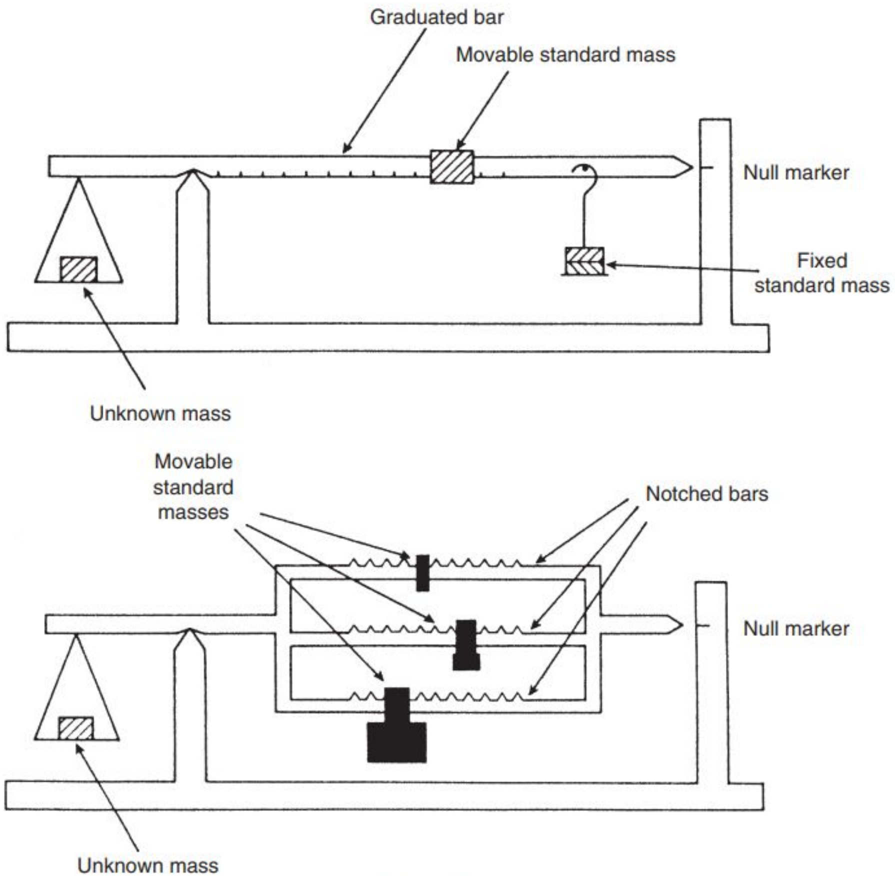

Figure 18.6
Two alternative forms of weigh beam.

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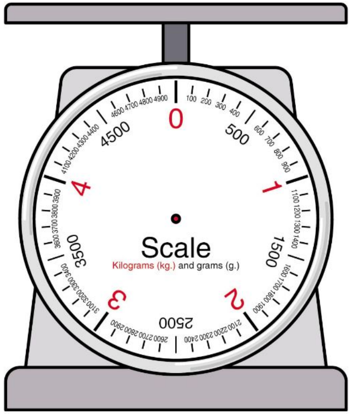
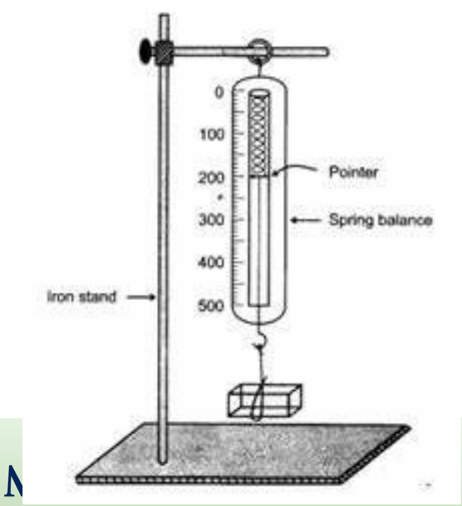
Spring Balanced

$$F = K \,.\, \Delta x \qquad \Rightarrow \; m = \frac{K}{g}\Delta x \; \Rightarrow \; m \,\alpha\, \Delta x \;\; \text{Linearly}$$

$$F = mg$$



To make it electrical the displacement should be converted to electrical signal.



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Strain Gauge

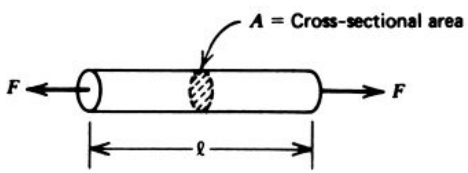
Stress & Strain Definition

- The force applied over unit of area is referred to as stress.
- The deformation resulted from applied force referred to as strain.
- Stress is defined as force per unit area.
- Strain is defined as deformation of the material per unit size of the material
- More specifically in a bar, the change of length over unit length is defined as strain

$$\text{stress} = \frac{F}{A}$$

where F = applied force in N
A = cross-sectional area of the sample in m^2

We see that the units of stress are N/m^2 in SI units (or $lb/in.^2$ in English units)



$$\text{strain} = \frac{\Delta l}{l}$$

where Δl = change in length in m (in.)
l = original length in m (in.)

Strain is thus a unitless quantity.

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Stress & Strain Types

Tensile stress:

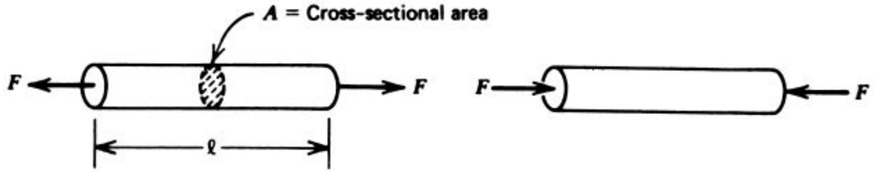
- The applied forces are aligned along a single axis. The applied forces cause the material to be pulled apart or stretched along that axis

Compressive stress:

- The applied forces are aligned on a single axes and cause the material compression.

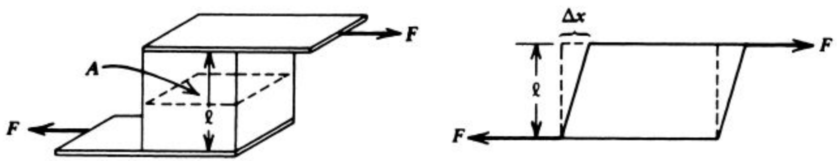
Shear stress:

- The applied force are not aligned on one axes and lead to two dimensional deformation e.g length and width



a) Tensile stress applied to a rod

b) Compressional stress applied to a rod



a) Shear stress results from a force couple

b) Shear stress tends to deform an object as shown

$$\text{shear stress} = \frac{F}{A} \qquad \text{shear strain} = \frac{\Delta x}{l}$$

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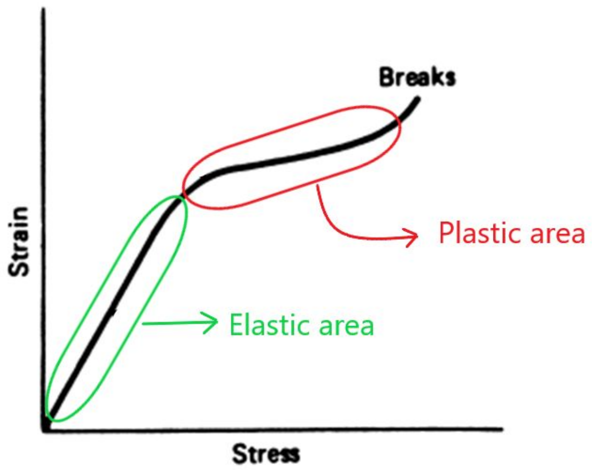
Deformation: Elastic and Plastic area

- **Elastic area:** deformation is not permanent.
- **Plastic area:** deformation will be permanent.
- **Neck:** If the stress increases material will break finally



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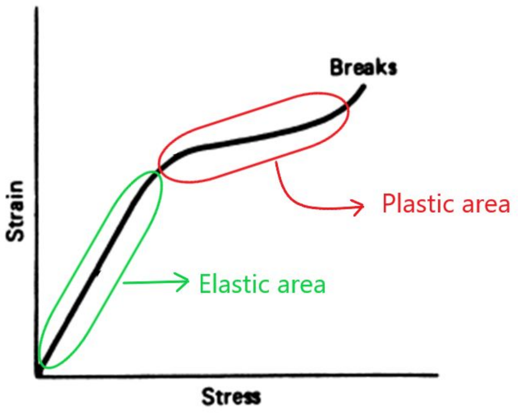
Elasticity Modulus

- For some materials the slope of line depends on the type of material not dimension.
- The inverse of the line slope in elastic area for compressional and tensile strain is referred to as elasticity modulus or Young's modulus.



$$E = \frac{\text{stress}}{\text{strain}} = \frac{1}{\textit{slope in elasticity area}}$$

TABLE 1
Modulus of elasticity

Material	Modulus (N/m^2)
Aluminum	6.89×10^{10}
Copper	11.73×10^{10}
Steel	20.70×10^{10}
Polyethylene (plastic)	3.45×10^{8}

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Example 1

Find the strain that results from a tensile force of 1000 N applied to a 10-m aluminum beam having a 4 x 10^4 m2 cross-sectional area.

Solution:

The modulus of elasticity of aluminum is found from Table 1 to be E = 6.89 x 10^10 N/m2.

$$E = \frac{Stress}{strain}$$

$$Stress = \frac{F}{A} = \frac{1000\ N}{4\times10^{-4}} = 250 \times 10^{4}\ \frac{N}{m^{2}}$$

$$Strain = \frac{Stress}{E} = \frac{250\times 10^{4}}{6.89\times10^{10}}$$

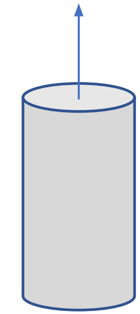
$$= 3.63 \times 10^{-5}\ or\ \ 36.3\ \mu m/m$$

$$\Delta l = strain \times l = 36.3 \times 10 = 363\ \mu m$$

Example 2

There is aluminum bar as front. A person stands on it.
What kind of stress is applied on this bar ?
If the length of the bar changes 2 µm, what is the weight of the person?
Assume g=10 N/Kg

A= 7.25 cm^2



h= 20 cm

A board to stand on

Aluminum Bar

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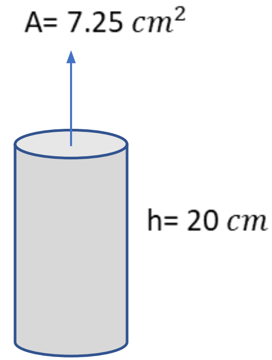
Example 2



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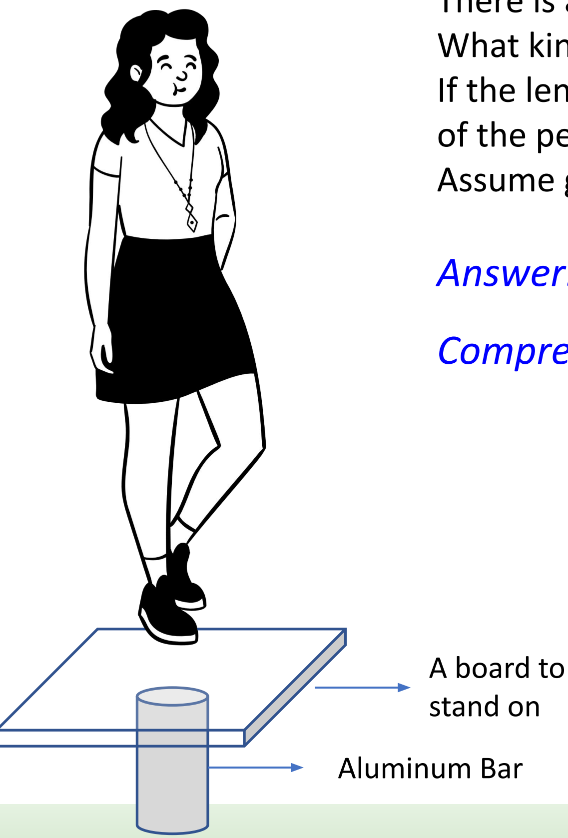


Answer:

Compressional

$$\text{Strain} = \frac{\Delta l}{l} = \frac{2 \times 10^{-6}}{20 \times 10^{-2}} = 1 \times 10^{-5}$$

$$\text{Alumininum elasticity modulus E} = 6.89 \times 10^{10}$$

$$E = \frac{stress}{strain}$$

$$Stress = E \times \text{strain} = 6.89 \times 10^{10} \times 10^{-5} = 6.89 \times 10^{5}$$

$$\text{Sterss} = \frac{F}{A} \implies F = stress \times A = 6.89 \times 10^{5} \times 7.25 \times 10^{-4} = 499.525\,N$$

$$F = mg \implies m = \frac{499.525\,N}{10} = 49.95\,kg \approx 50\,kg$$

Stress and Wire Resistance

- Let's assume a tensile stress is applied to a wire.
- The wire length will increase and the wire cross-sectional will decrease therefore resistance increases.
- In case of compressional stress the resistance decreases

$$R = \rho \frac{L}{A}$$

Tensile Stress:

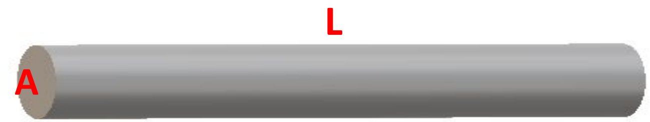
$L \uparrow \;\Rightarrow\; R \uparrow$

$A \downarrow \;\Rightarrow\; R \uparrow$

Compressional Stress:

$L \downarrow \;\Rightarrow\; R \downarrow$

$A \uparrow \;\Rightarrow\; R \downarrow$



$$R_0 = \rho \frac{l_0}{A_0}$$

After stress is applied:

$$R_0 + \Delta R = \rho \frac{l_0 + \Delta l}{A_0 - \Delta A}$$

$$\text{volume stays constant} \implies l_0 \times A_0 = (l_0 + \Delta l) \times (A_0 - \Delta A)$$

$$\Rightarrow A_0 - \Delta A = \frac{l_0 \times A_0}{l_0 + \Delta l}$$

$$\Rightarrow R_0 + \Delta R = \rho \frac{l_0 + \Delta l}{\frac{l_0 \times A_0}{l_0 + \Delta l}} = \rho \frac{l_0}{A_0} \times \left(1 + \frac{\Delta l}{l_0}\right)^2$$

$$\frac{\Delta l}{l_0} \leqslant 1 \implies R_0 + \Delta R = \rho \frac{l_0}{A_0} \times \left(1 + 2\frac{\Delta l}{l_0}\right)$$

$$\implies \boxed{\Delta R = R_0 \times 2\frac{\Delta l}{l_0}}$$

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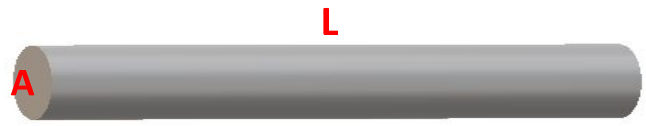
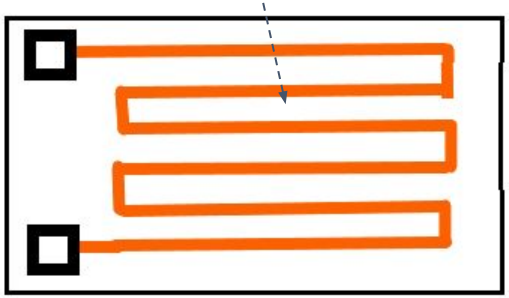
Metal Strain Gauge Construct

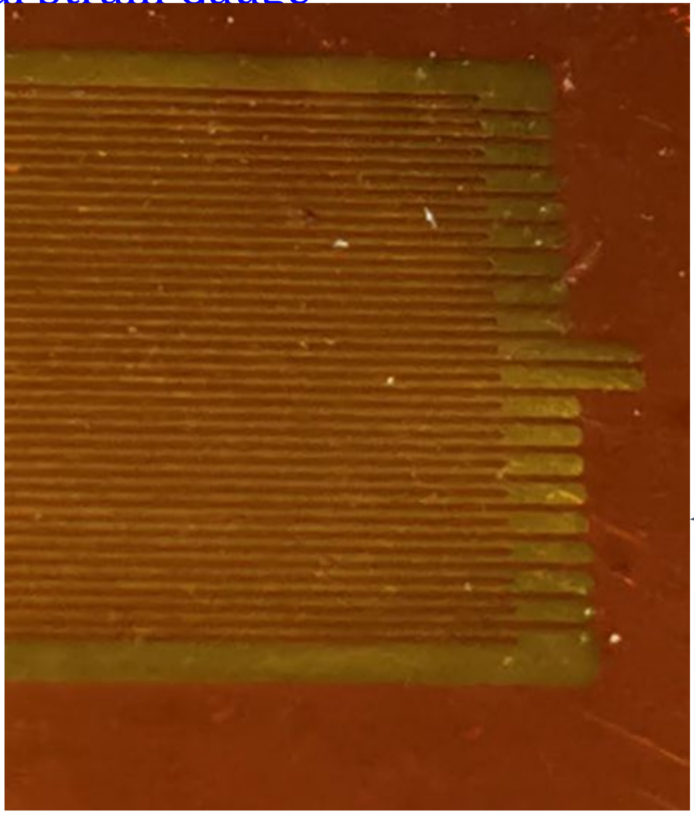
- Applying stress results in change of of wire resistance.
- To make resistance change greater, The wire placed on a pad as the front figure then glued to the pad.
- Now change of resistance will be multiplied.
- This pad is referred to as metal strain gauge

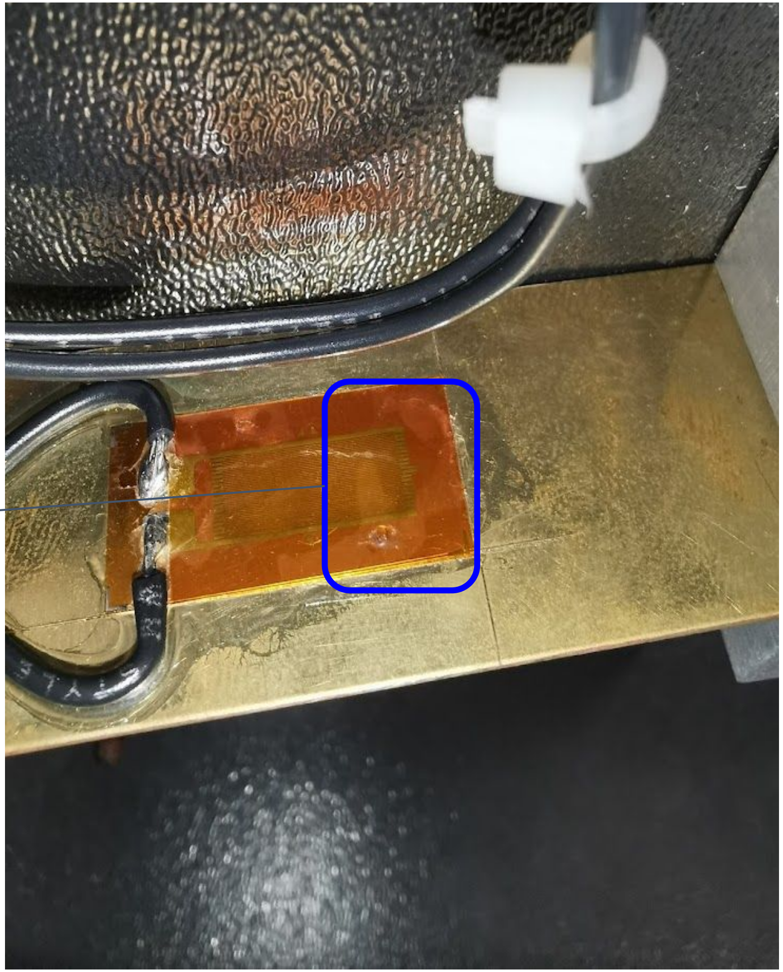


$$\Delta R = R_0 \times 2\frac{\Delta l}{l_0}$$



Metal Strain Gauge





Gauge Factor

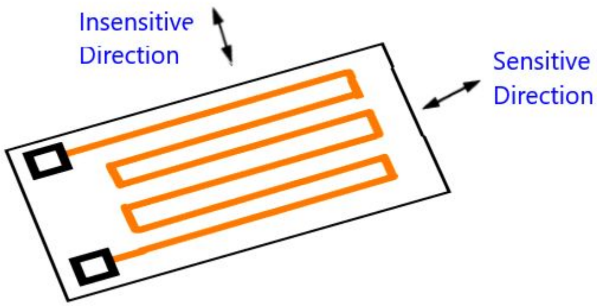
The strain gauges come with predefined factor called gauge factor.

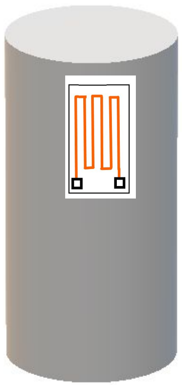
The gauge factor is defined as resistance change per resistance unit over the strain

$$\mathrm{GF} = \frac{\Delta R/R}{\text{strain}}$$

where $\Delta R/R$ = fractional change in gauge resistance because of strain
$\text{strain} = \Delta l/l$ = fractional change in length

The pad shows that the sensitivity of the strain gauge is directional





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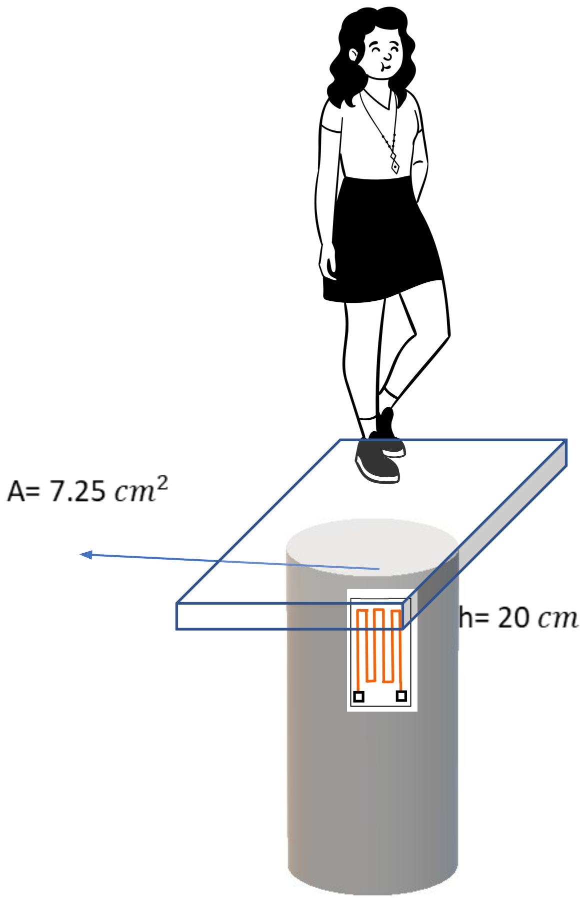
Example:

A strain gauge pad is attached to a bar. When somebody stands on it, the change of length is 2 µm.

The gauge factor for this pad is 2 and its resistance is 500 ohm. How much would be the change of resistance?

The resistance increase or decrease?



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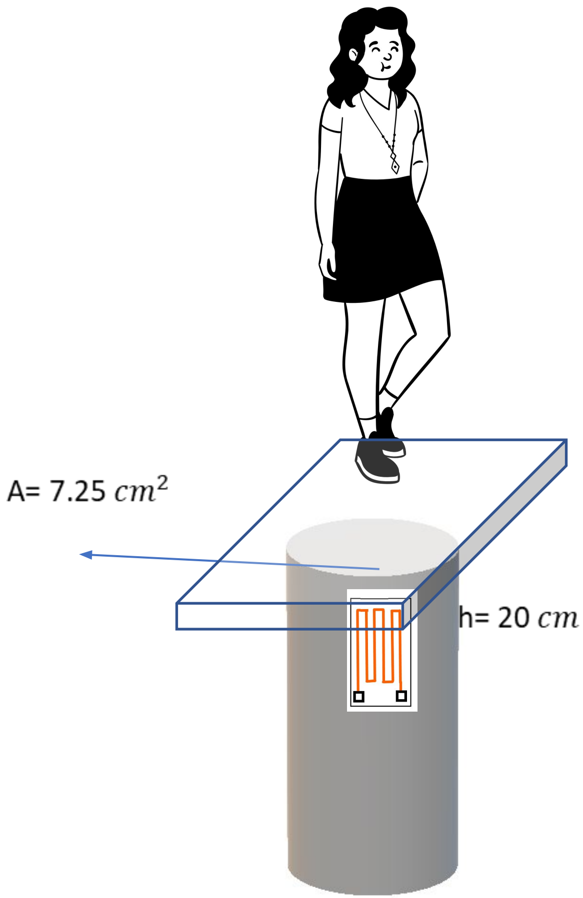

$$Stain = \frac{\Delta l}{l} = 10 \ \frac{\mu m}{m}$$

$$\mathrm{GF} = \frac{\frac{\Delta R}{R}}{Strain}$$

$$2 = \frac{\frac{\Delta R}{500}}{10 \times 10^{-6}} \xrightarrow{\text{yields}} \Delta R = 10 \ m\Omega$$

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Temperature Effect Cancelation

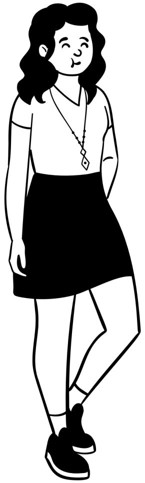
The resistance of the pad does not just depends on the stress , but also depends on the environmental temperature. In case of measuring the force it is considered to be disturbance.

To cancel the temperature effect, one more gauge is added on the insensitive direction. This gauge is referred to as dummy gauge. then the resistance of the dummy gauge is subtracted from the active gauge.

In metal wires

$$T\uparrow \;\Rightarrow\; R\uparrow$$

$$T\downarrow \;\Rightarrow\; R\downarrow$$

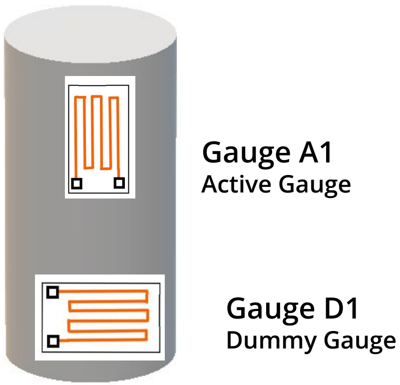


$$\Delta R_{D1} = \Delta R^{Temperature}$$

$$\Delta R_{A1} = \Delta R^{Stress} + \Delta R^{Temperature}$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

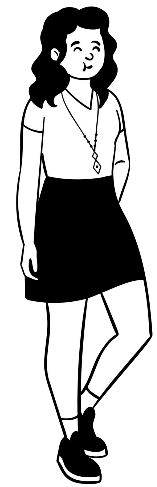
$$\Delta R_{A1} - \Delta R_{D1} = \Delta R^{Stress}$$



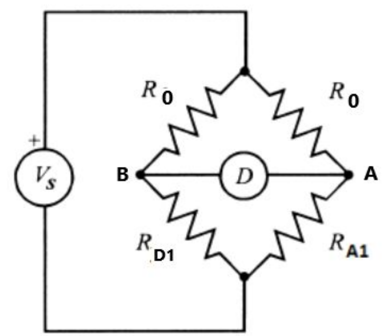
Conversion to Voltage & Subtraction

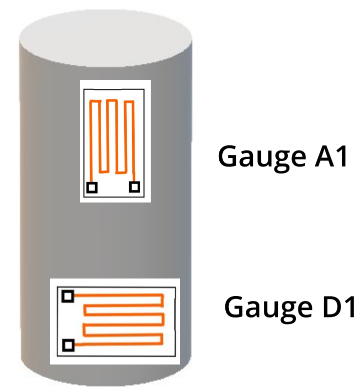
The resistance of the pad does not just depends on the stress , but also depends on the environmental temperature. In case of measuring the force it is considered to be disturbance.

To cancel the temperature effect, one more gauge is added to insensitive direction then their resistance are subtracted.



$$V_D = V_A - V_B$$





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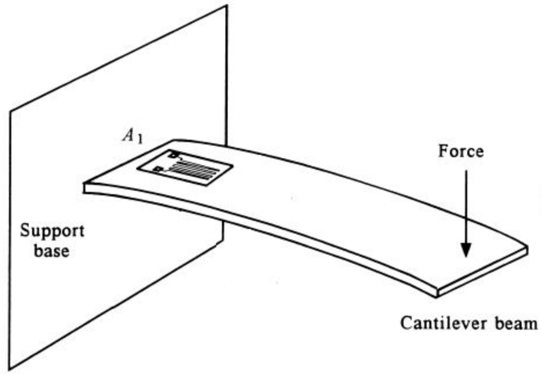
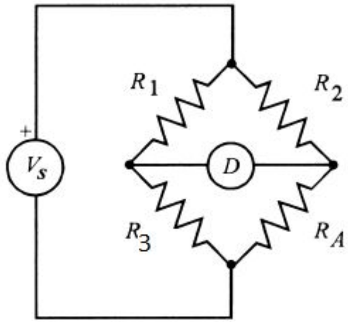
Converting Resistance Change to Voltage

- In signal conditioning section we saw the change of resistance can be converted to change of voltage by using a bridge circuit.
- The resistance of the pad does not just depends on the stress , but also depends on the environmental temperature. In case of measuring the force it is considered to be disturbance.





$$V_D = \frac{V_s}{4 \times R_{0A}} \times \Delta R_A = \frac{V_s}{4} \times GF \times strain$$

$$R(T) = R(T_0) \times (1 + \alpha \times \Delta T)$$

$$\Delta R = \Delta R^{Stress} + \Delta R^{\text{Temperature}}$$

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Temperature Effect Cancelation

$$V_D = V_s \times \left[\frac{R_{A1}}{R_{A1}+R_2} - \frac{R_{A3}}{R_{A3}+R_1}\right] \text{ and taking } R_{A1} = R_{A3} = R_2 = R_1 = R_0$$

After stress applied

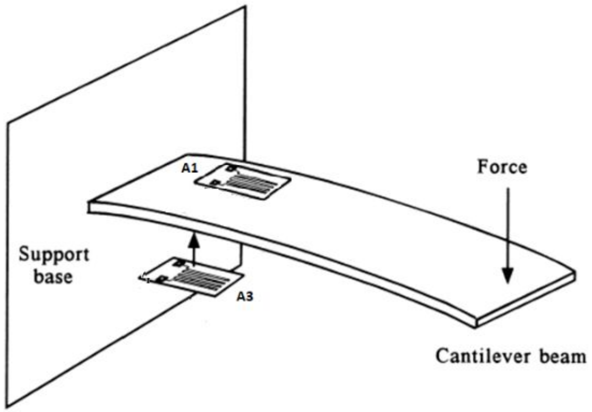
$$V_D = V_s \times \left[\frac{R_0 + \Delta R_0^{stress} + \Delta R_0^{Temp}}{R_0 + R_0 + \Delta R_0^{stress} + \Delta R_0^{Temp}} - \frac{R_0 - \Delta R_0^{stress} + \Delta R_0^{Temp}}{R_0 + R_0 - \Delta R_0^{stress} + \Delta R_0^{Temp}}\right]$$

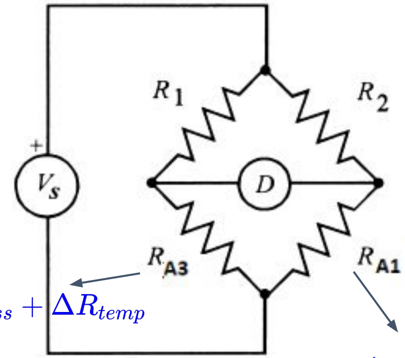
$$V_D = \frac{V_s}{2} \times \frac{\Delta R_0^{stress}}{R_0}$$

These two cancel each other

The A1 resistance increases and A3 is under compressional stress and resistance decreases.

Compare to when there was one pad the sensitivity has doubled also.





$R_0 - \Delta R_{stress} + \Delta R_{temp}$

$R_0 + \Delta R_{stress} + \Delta R_{temp}$

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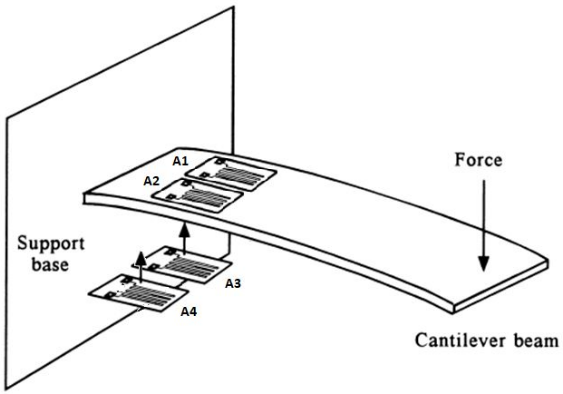
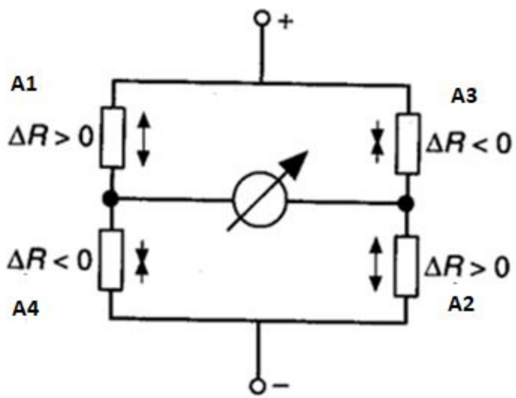
Full Bridge of Strain Gauges





$$V_D = V_s \times \left[\frac{R_{A2}}{R_{A1} + R_{A2}} - \frac{R_{A4}}{R_{A3} + R_{A1}} \right]$$

and taking $R_{A1} = R_{A3} = R_2 = R_1 = R_0$

After stress applied

$$V_D = V_s \times \left[\frac{R_0 + \Delta R_0^{stress} + \Delta R_0^{Temp}}{R_0 + R_0 + 2 \times \Delta R_0^{Temp}} - \frac{R_0 - \Delta R_0^{stress} + \Delta R_0^{Temp}}{R_0 + R_0 + 2 \times \Delta R_0^{Temp}} \right]$$

$$V_D \approx V_s \times \frac{\Delta R_0^{stress}}{R_0}$$

1- With this configuration the sensitivity is increased four folds compare to when there was just one gauge.

2- The temperature effect is reduced sonsiedeably

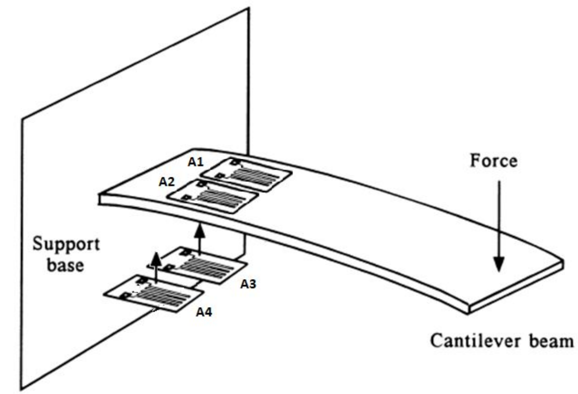
Example 4

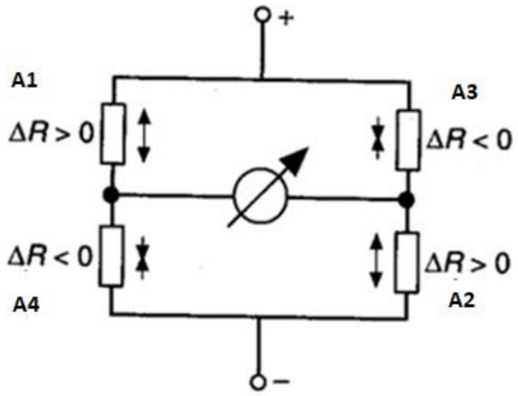
For the strain gauges GF=2.03

R=350 and V_s=10

If the strain is 1450 μm/m

V_D?





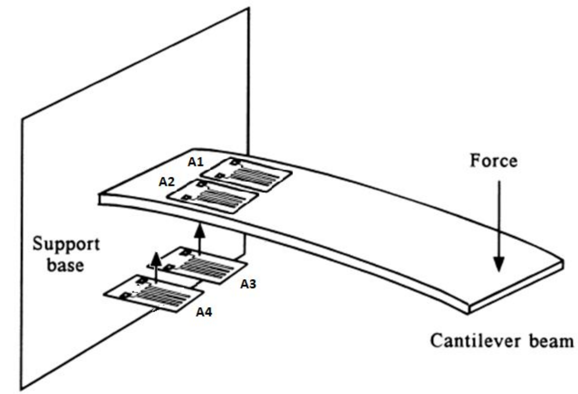
Example 4

For the strain gauges GF=2.03

R=350 and V_s=10

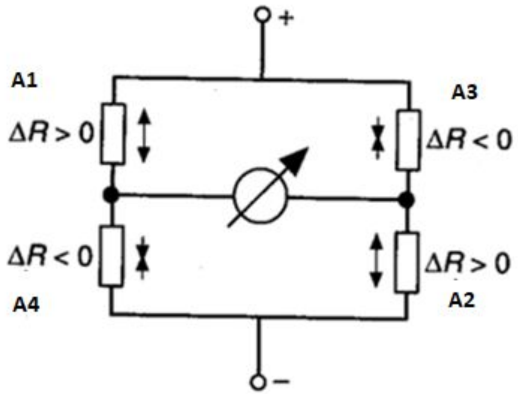
If the strain is 1450 µm/m

V_D?



V_D= Vs x GF x Strain =

10x2.03x1450x 10^-6= 0.03 volt



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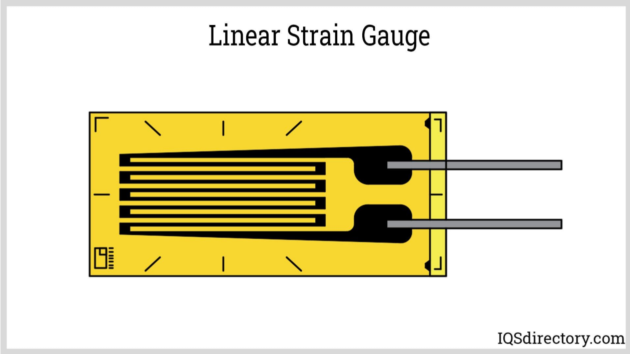
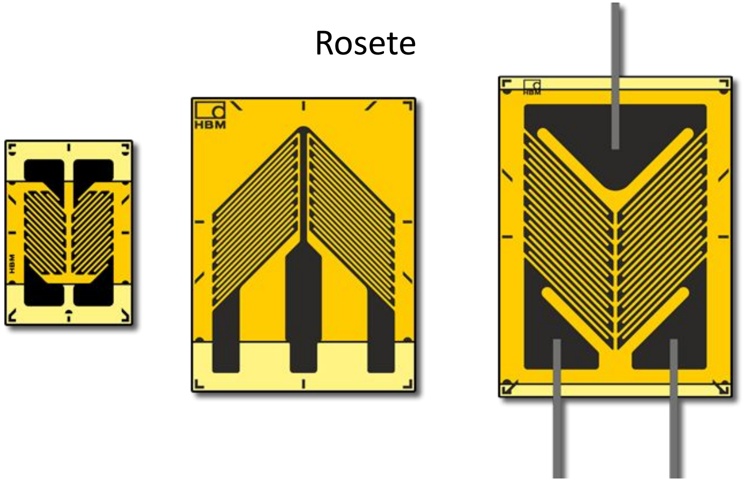
Linear & Rosette Strain gauges

Rosete can cover two dimensions



Rosete



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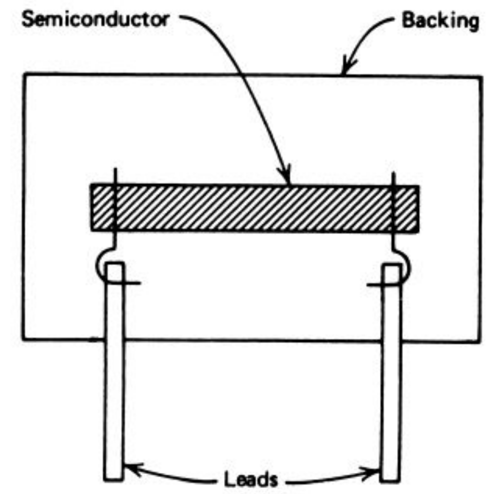
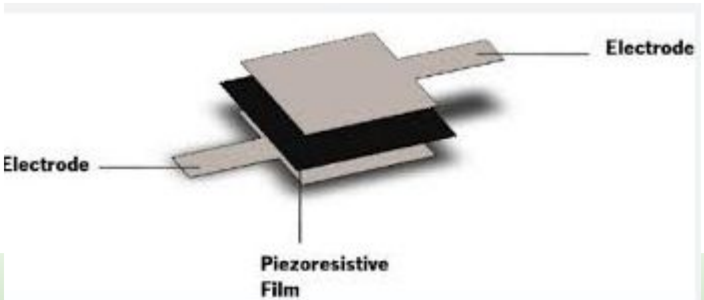
Semiconductor Strain Gauge - Piezoresistive Effect

- The physical dimension of semiconductor material changes with stress. This will lead to change of resistance. This effect is also known as piezoresistive effect
- GF (Gauge factor) is negative. It means for compressive stress the resistance increases.
- GF can be much larger than metal strain gauge, as much as -200.
- Device Highly nonlinear, For example for small strain the GF could be about -150 and for 5000 µm/m, the GF could be -50





Example 5

There are two strain gauges, one is metal gauge with GF=2.13 and another is semiconductor with GF -151. The nominal resistance of both is 120 Ω. The strain is 150 µm/m.

what would be the resistance change in these gauges?

$$
\begin{aligned}
GF &= \frac{\Delta R/R}{\text{strain}} \\
\text{Metal Gauge} \Rightarrow \Delta R &= GF \times \text{strain} \times R \\
&= 120 \times 2.13 \times 150 \times 10^{-6} = 0.038\,\Omega \\
Semiconductor\ \text{Gauge} \Rightarrow \Delta R &= GF \times \text{strain} \times R \\
&= 120 \times 151 \times 150 \times 10^{-6} = 2.72\,\Omega
\end{aligned}
$$

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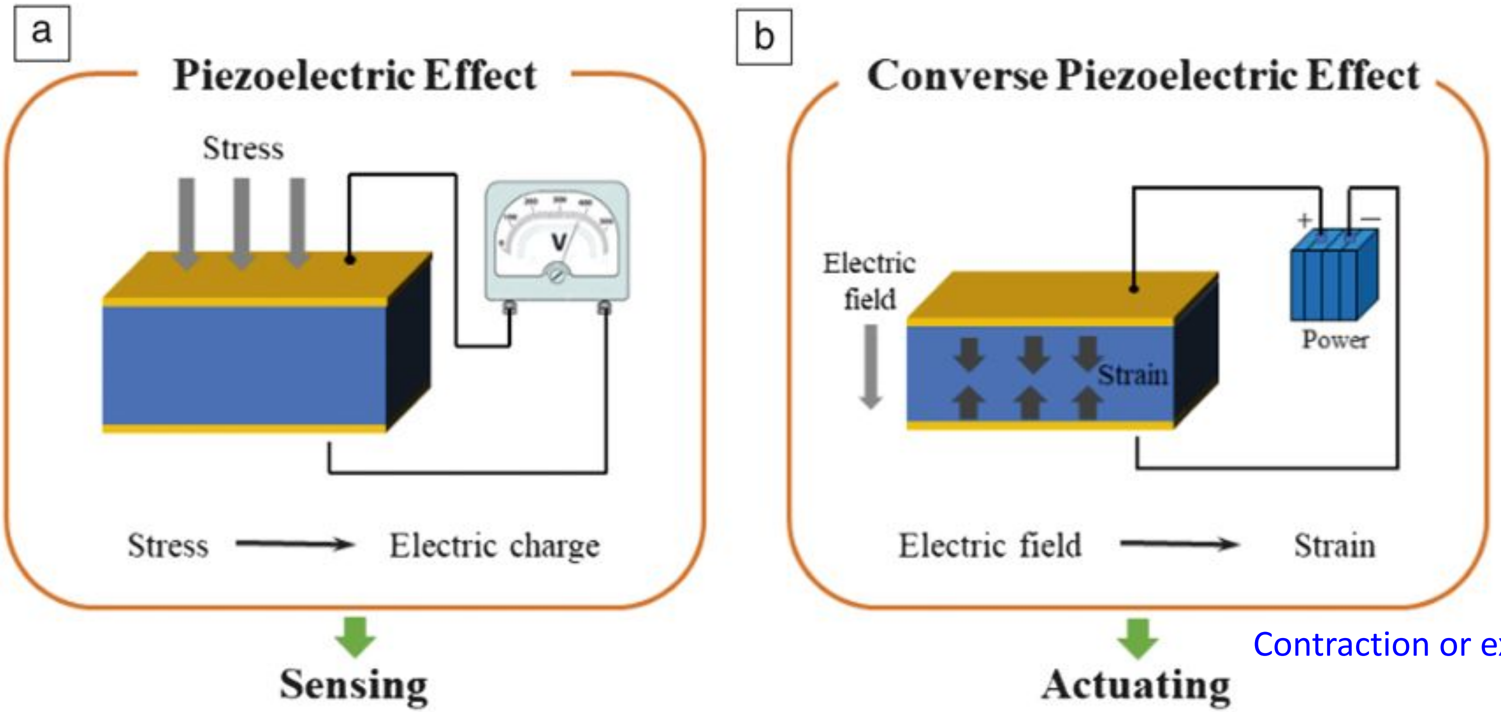
Piezoelectric

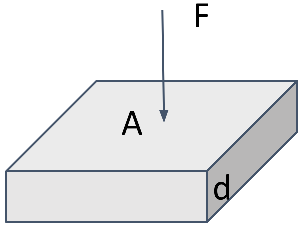
Piezoelectric Effect (Sensing - Actuating)



Contraction or expansion

[Source: Lead-free piezoceramics: Status and perspectives | Cambridge Core]

Piezoelectric Sensors



- If a deformation (force) is applied to the piezoelectric material they generate electricity and in reverse if electricity applied to them they deform (generate force) and this behaviour is known as piezoelectric property.
- Natural piezoelectric material: Quartz
- Synthetic piezoelectric material: lithium sulfate, and ferroelectric ceramics such as barium titanate.
- Application in
  - ultrasonic sender and receiver
  - vibrator
  - Accelerometer and load cells
  - Microphone

$$V = \frac{k \times d}{A} \times F = k \times d \times \text{stress}$$

$F \rightarrow Applied\,Force$ in g

$A \rightarrow Area\,of\,material$ in mm

$d \rightarrow Thickness$

$k \rightarrow$ Piezoelectric constant

Quartz $\rightarrow k = 2.3$

Barium titanate $\rightarrow k = 140$

Example : $force$ 1 g applied on $A = 100\ \text{mm}^2$ & d$=$ 1 mm

quartz produce 23 μV

Barium titanate produce 1.4 mV

Piezoelectric Property

Piezoelectric material react to dynamic force not static. Therefore they are more suitable to detect dynamic forces such as acceleration, vibration or dynamic pressure measurement.

Strain Gauge can be used to measure both dynamic and static.

If the piezoelectric sensor remains under the load for long time, its property might change and output value drifts. Then the repeatability of the sensor depends on working condition.

Strain gauge is more linear than piezoelectric.

Load Cells

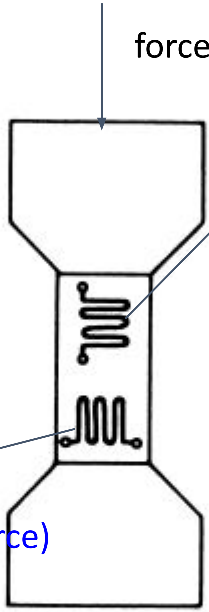
- Strain Gauge
- Hydraulic
- Pneumatic
- Capacitive

Weighing- Load cells with strain gauges

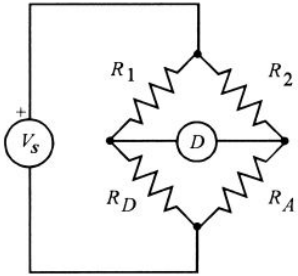
The most common configurations for load cells are compression and tension.

force

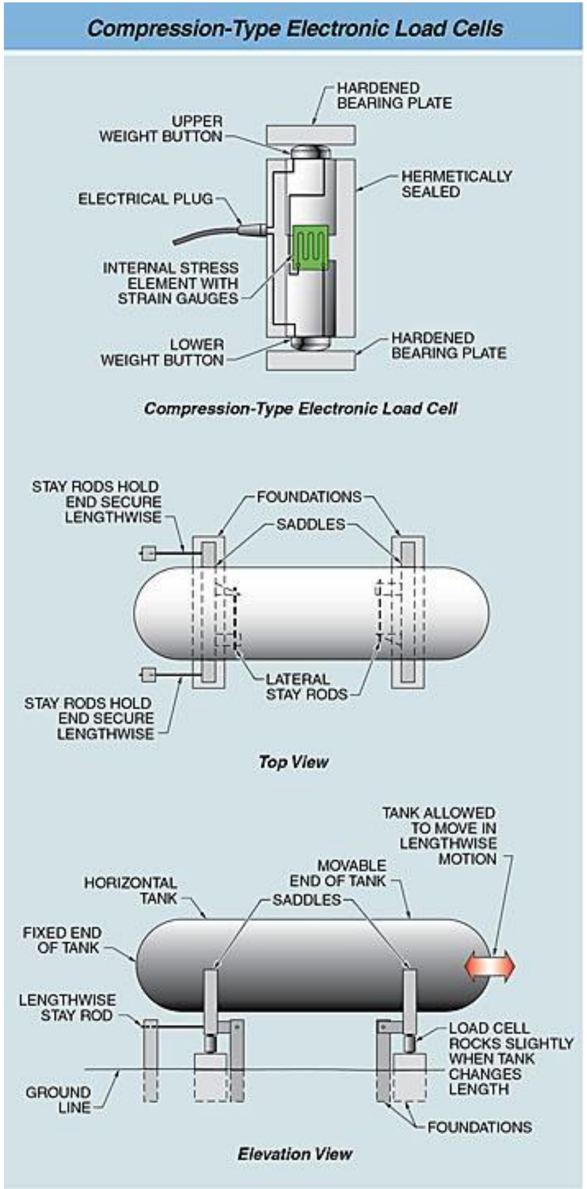
Dummy gauge (Insensitive to force) is added to cancel the temperature effect



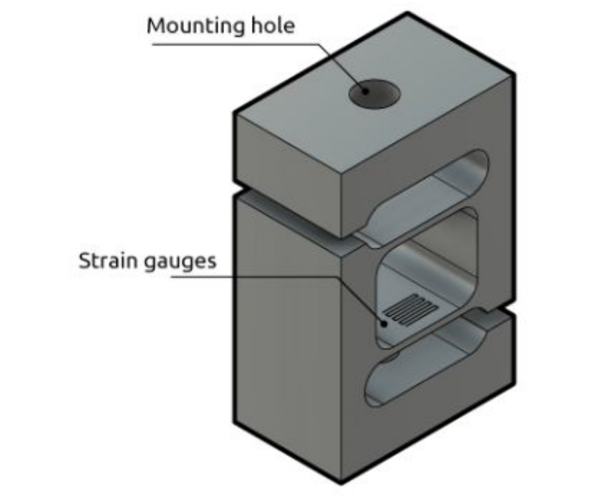
Active gauge (sensitive to force)

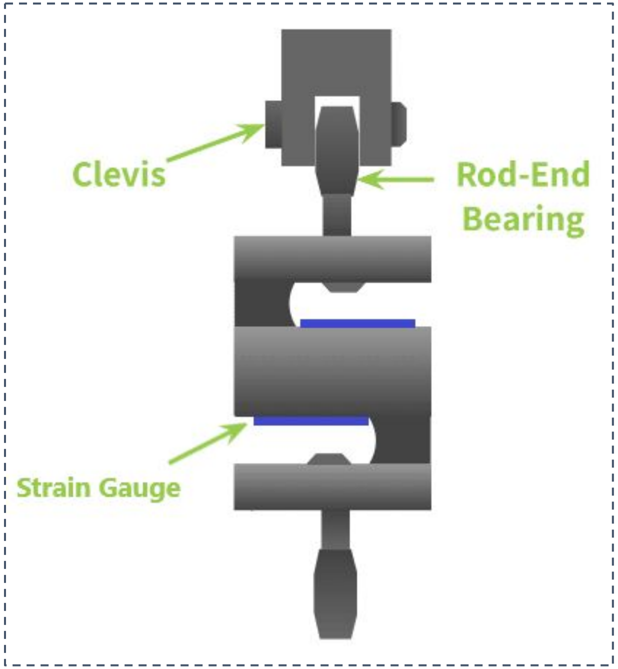


A compression-type electronic load cell measures the applied stress to a compressive strain gauge to determine weight.

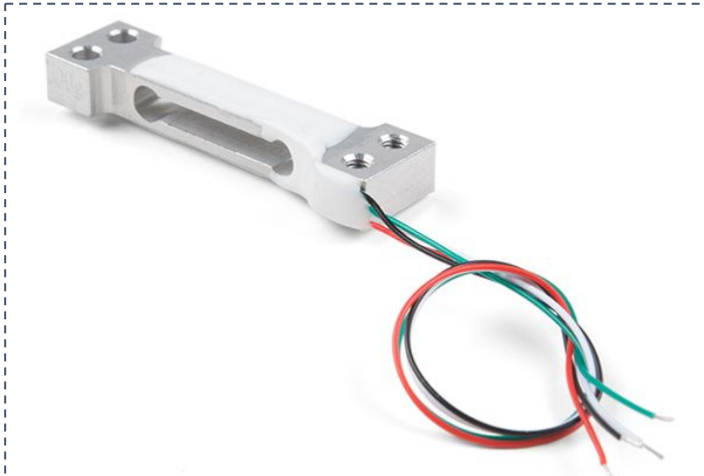


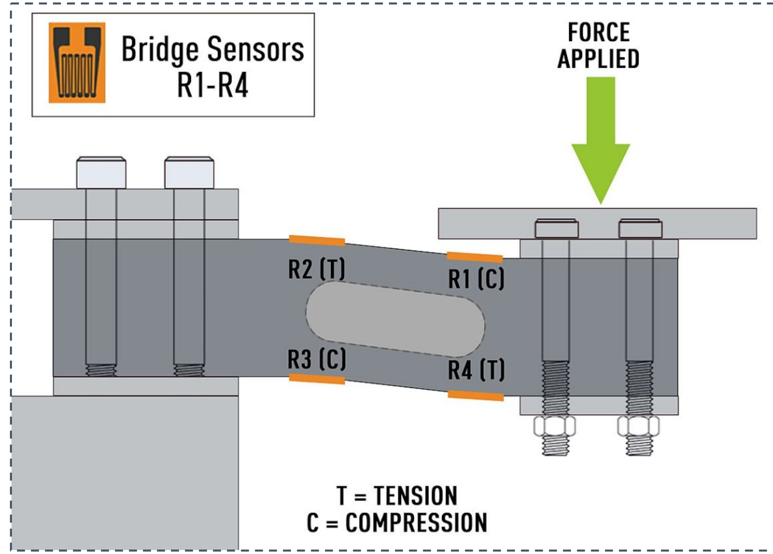
S-Beam load cell

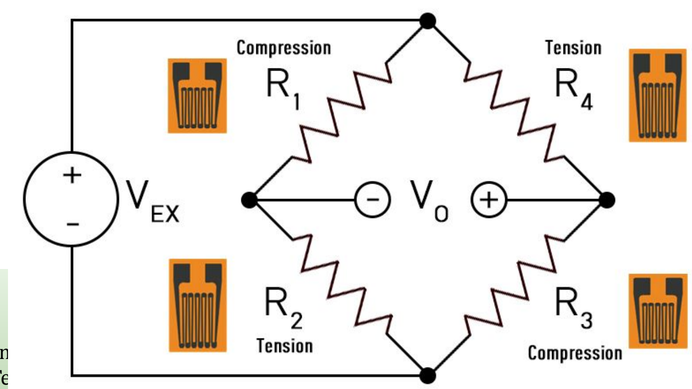




Bending Beam Load Cell





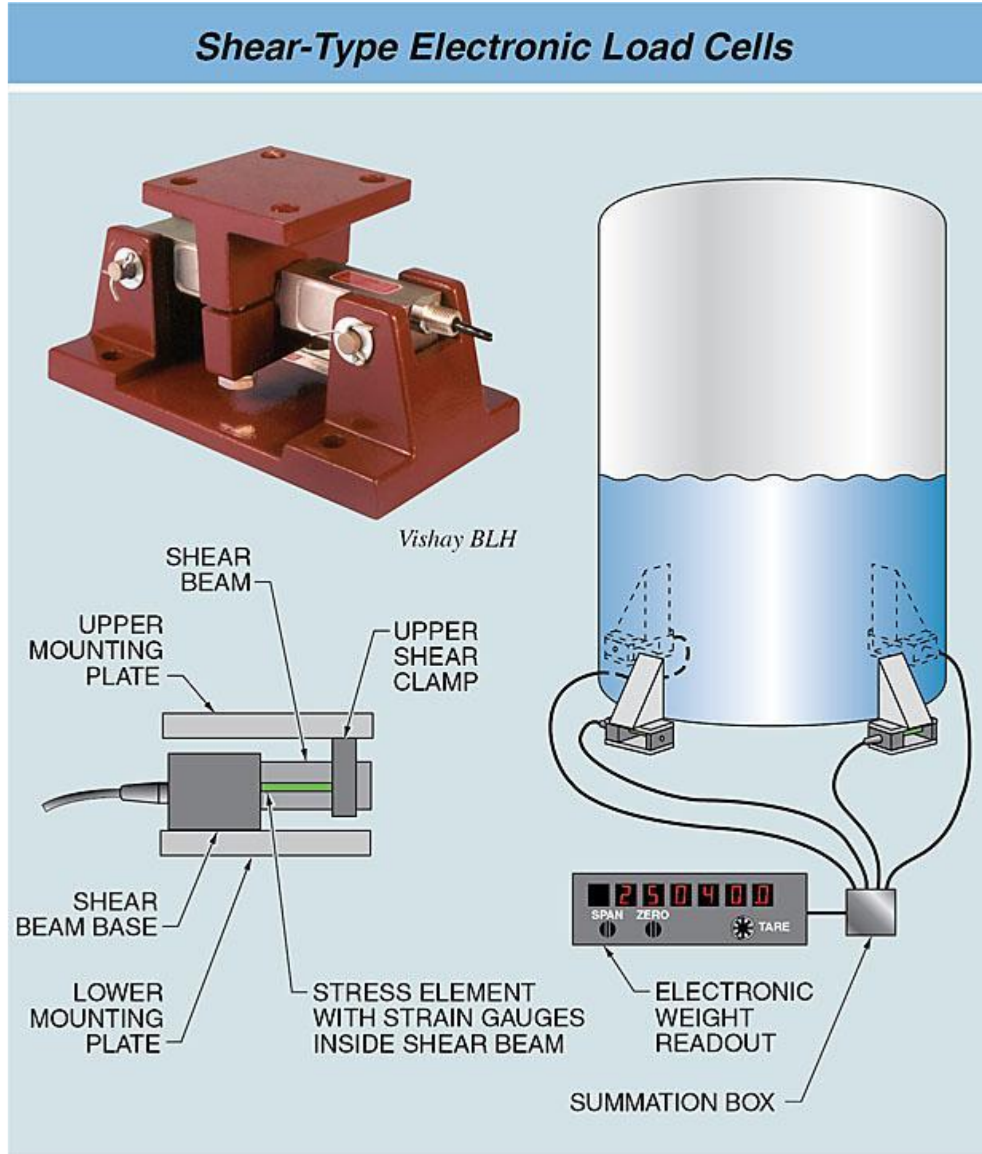


[Photos source](#)
[wikipedia](#)

Level measurement with load cell

Shear-type electronic load cells are placed under the feet of a tank to measure the weight of the contents in the tank.

By knowing the density of the liquid, the level can be calculated



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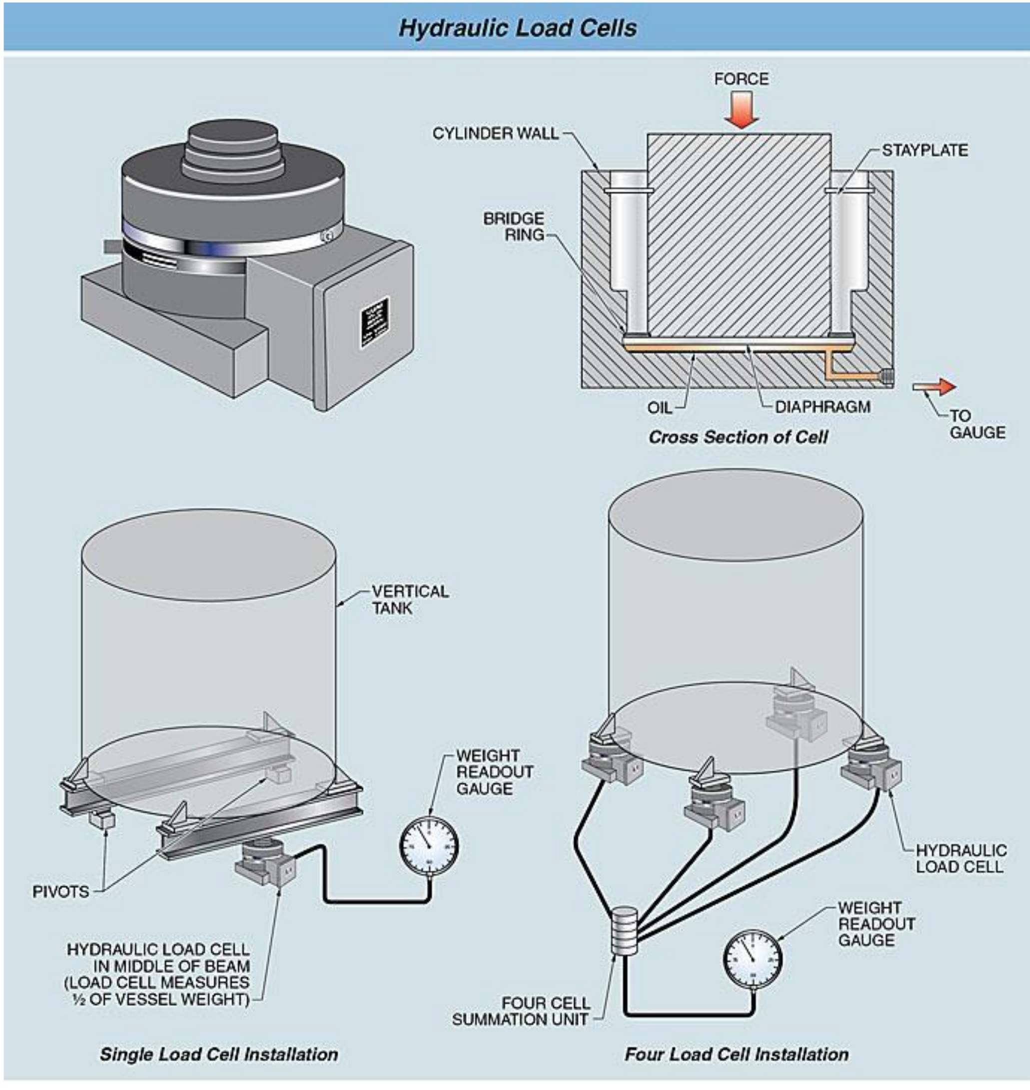
Hydraulic Load Cell

Single or multiple hydraulic load cells can be used to weigh the contents of a tank.

In hydraulic load cell the force is converted to oil pressure and then oil pressure is measured



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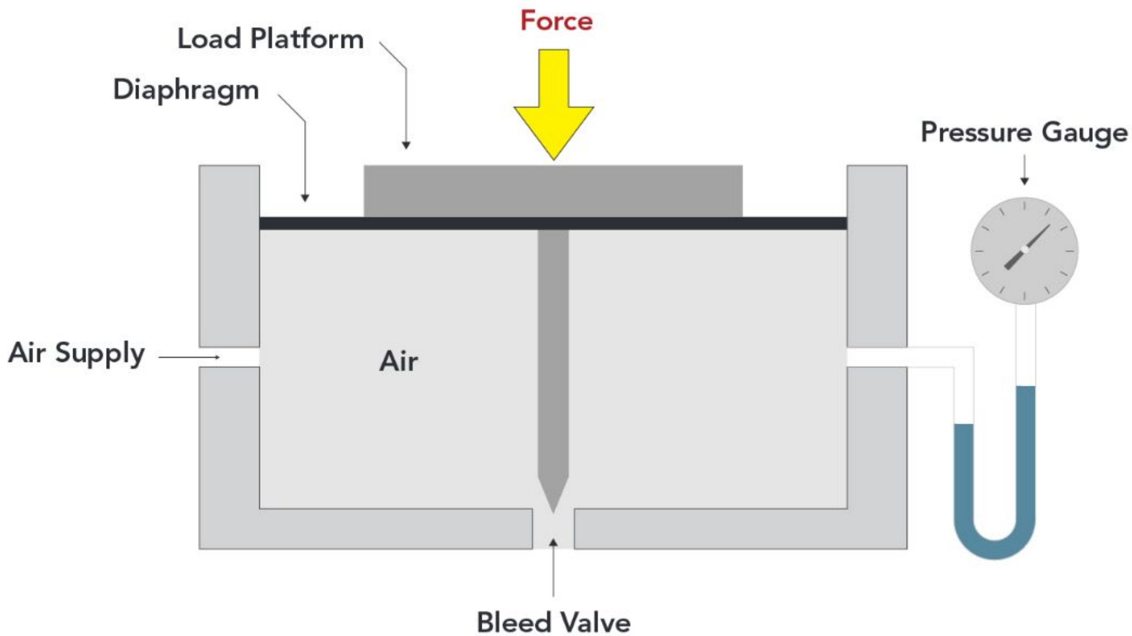
Pneumatic Cell



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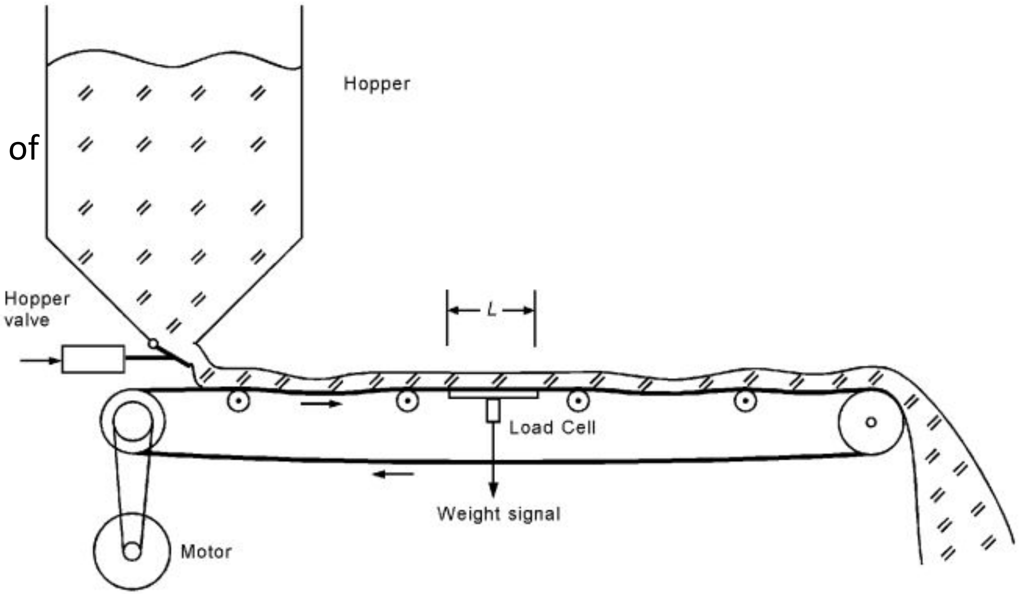
Solid Mass Flowmeter

Mass flow rate is mass per time unit

Encoder measure the the speed of conveyor

A load cell is under the weighing platform, the length of platform is known.

mass flow rate can be calculated as below



$$\text{Mass Flow rate } Q = \frac{\text{Measured weight}}{\text{time}} = \frac{W}{t}$$

$$t = \frac{\text{Length of weighing platform}}{\text{speed of conveyor}} = \frac{L}{\nu}$$

$$\Rightarrow Q = \frac{W \times \nu}{L}$$

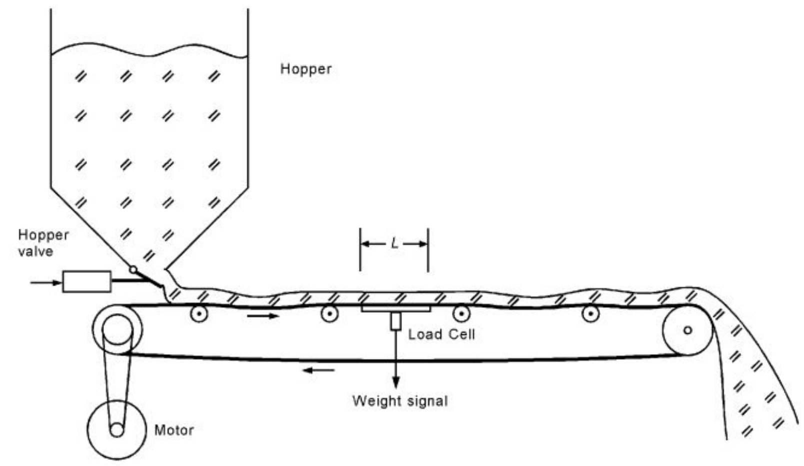
Example 6

There is an incremental encoder on the shaft of the conveyor. The encoder resolution is 100 ppr and sending 33 pps. The radius of conveyor roller is 25 cm. The length of the weighing platform is 1 meter. The load cell shows 40 kg on average.

A) How much of grain will be delivered in one minute?

B) If we want to know how much grain is conveyed in one hour, how would you calculate? discuss

C) What factors mentioned in above have impact on accuracy on this calculation and how the accuracy can be improved?



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Example 6

Answer:

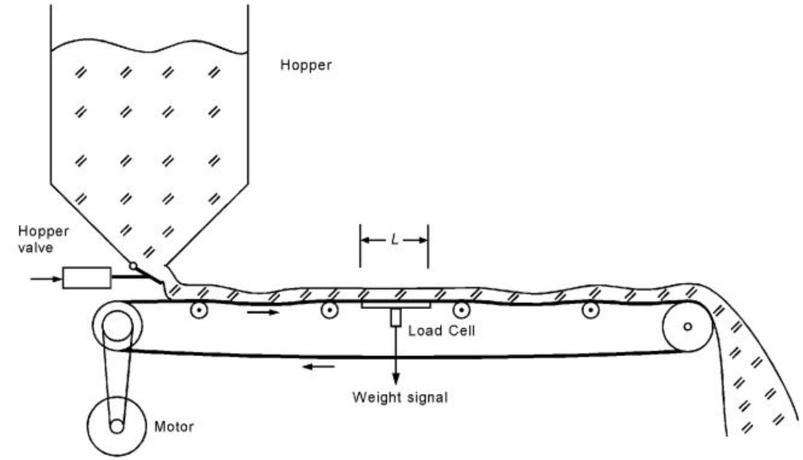
B) One way in theory is to to multiply the mass rate by 3600. In this way, a small error in mass rate calculation can become 3600 times bigger. The other practical point would be that the mass rate will not stay constant during one hour, then the average should be calculated.

The other way , to read encoder and load cell value every 0.1, 1 or 10 seconds to calculate the mass during that time and then add up all calculations for one hour. This way the error due to encoder resolution and weight platform will be more random and by adding the positive and negative error exist in each calculation they will cancel each other therefore the accuracy will improve compare to first approach.

C) Resolution of encoder, by higher resolution the rps will be more accurate and consequently the calculated time will be more accurate.

The length of the platform, provide the weight of a limited piece of conveyor. By making the longer platform the measured weight will be more precise (closer to average of weight on the conveyor on that moment)

Improving the load cell accuracy



$$rps = \frac{33}{100} = 0.33$$

$$\text{conveyor speed} = \text{v} = 2\pi r \times rps = 0.518\ \frac{m}{s}$$

$$L = v \times t \Rightarrow t = \frac{L}{v} = \frac{1}{0.518} = 1.93\, s$$

$$Mass\, rate = \frac{W}{t} = \frac{40}{1.93} = 20.73\, \frac{kg}{s}$$

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Mass rate =W/t= W/L x V

To improve the accuracy

Load cell accuracy can be increased

L can be selected larger to make error of w and V smaller

to reduce encoder error the resolution should increase

End

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