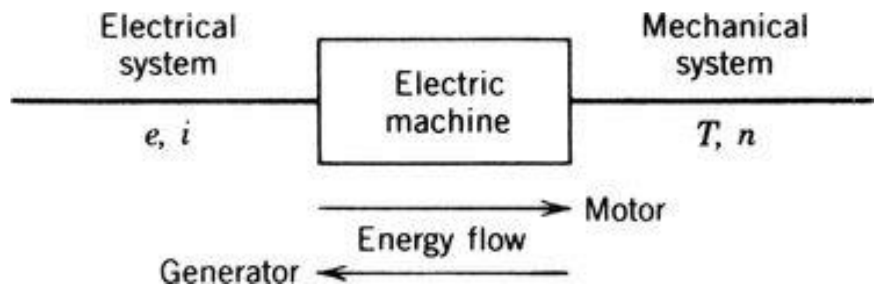
DC Machines

DC Machines



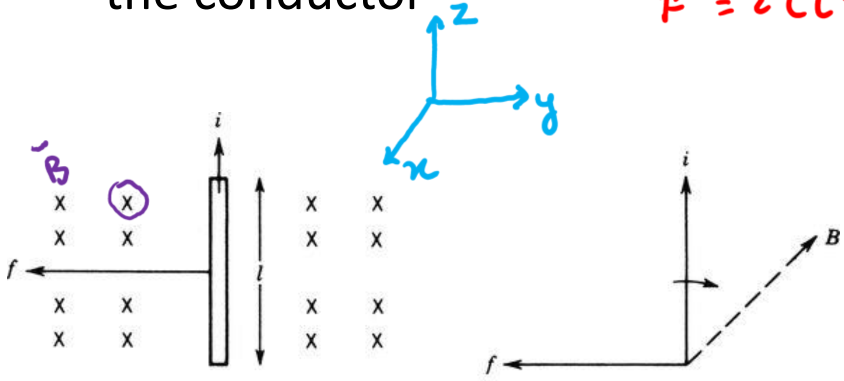
- DC machines can operate as either a Generator or a Motor
- DC machine's use as a Generator is limited because of widespread use of AC
- DC machine is extensively used as a Motor – because easy speed control is possible
- DC motors are used where high torque and variable speed is required
  - Printing press, conveyors, fans, pumps, hoists, cranes, paper mills, textile mills, rolling mills, traction (locomotives and transit cars)
  - Small dc motors are used as control devices – tachogenerators for speed sensing, servomotors for positioning and tracking

DC Machine Principle

$\hat{x}$ $\hat{y}$ $\hat{z}$

- When a current-carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force
- When a conductor moves in a magnetic field, voltage is induced in the conductor

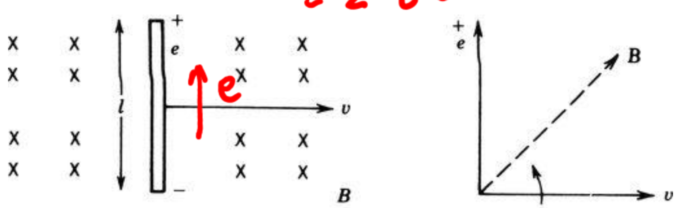
$$\bar{F} = i(\bar{l}\times\bar{B}) = i(\hat{z}\, l \times -\hat{x} B) = -\hat{y}\, i l B$$



(a) (b)

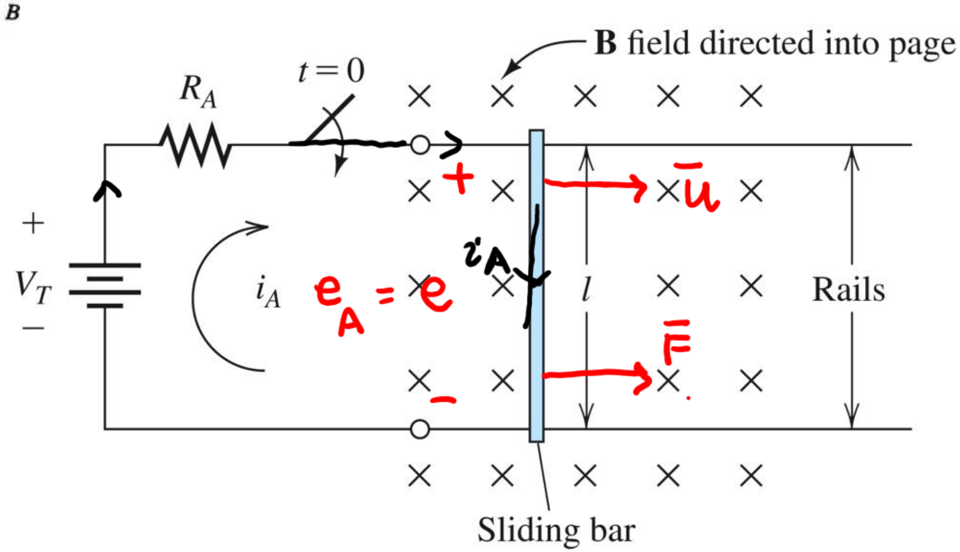
Dc Motor

$$e = l(\bar{v}\times\bar{B}) = l(\hat{y} v \times -\hat{x} B)$$
$$= \hat{z}\, l v B$$



X indicates B into the paper

(a) (b)

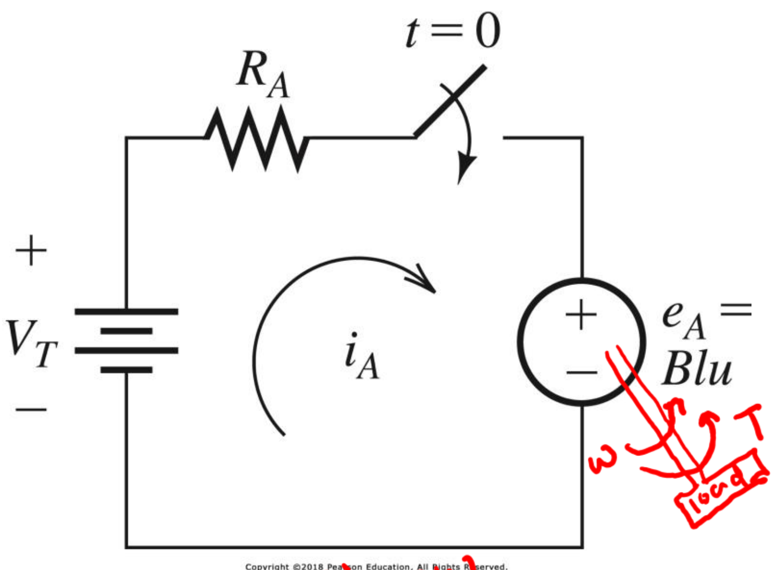


A simple dc machine consisting of a conducting bar sliding on conducting rails.

DC Machine Principle

→ Electrical quantities [voltages, i,]

- Equivalent circuit for the linear DC machine
  Motor Vs Generator



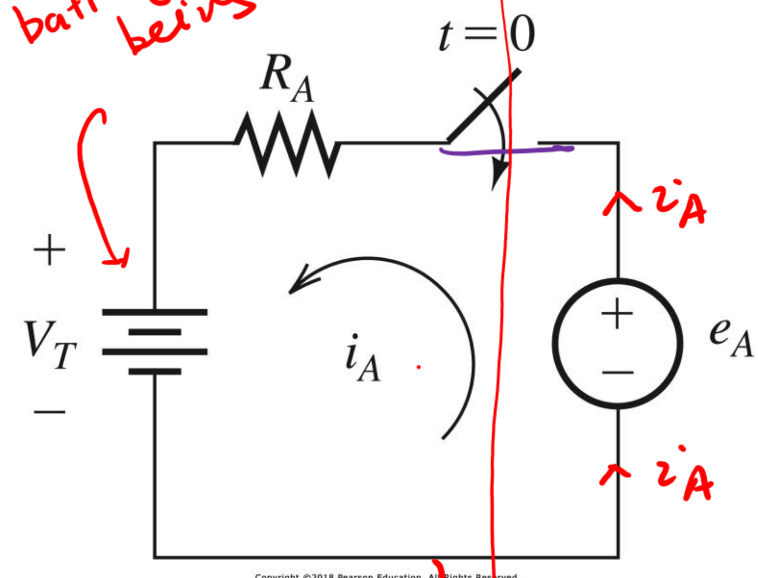
i_A : cw

operating as a Motor

$V_T > e_A$

battery which is being charged by the Gen.

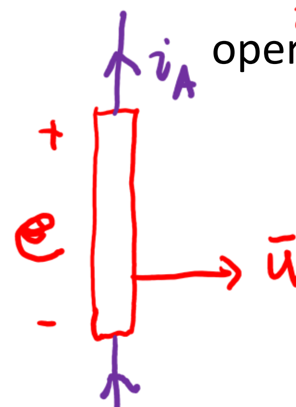
Generator



i_A : ccw

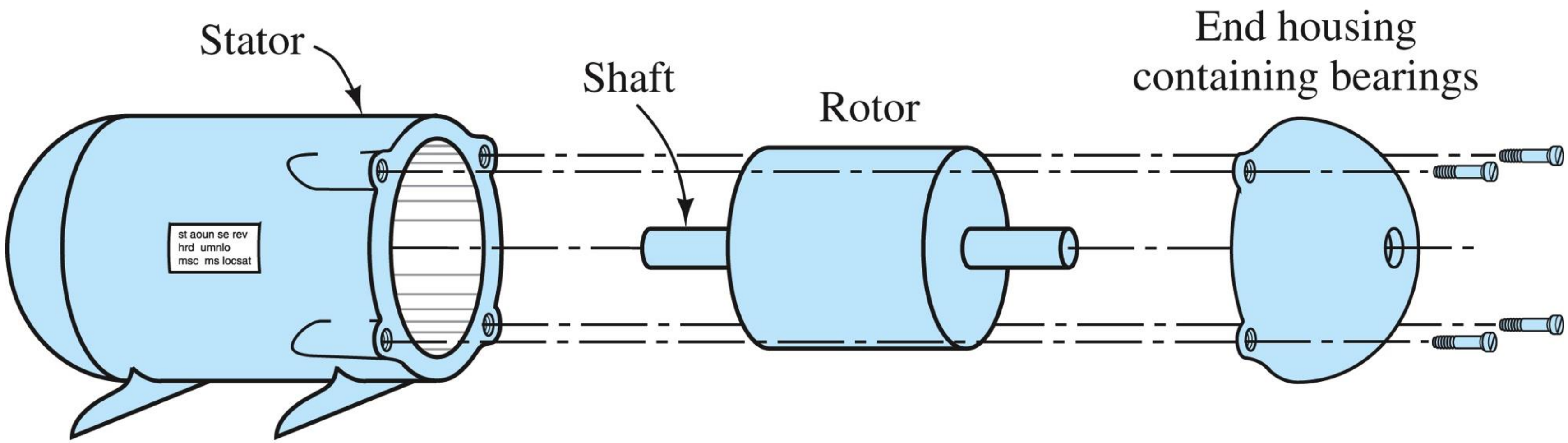
operating as a Generator

$V_T < e_A$



DC Machine Structure

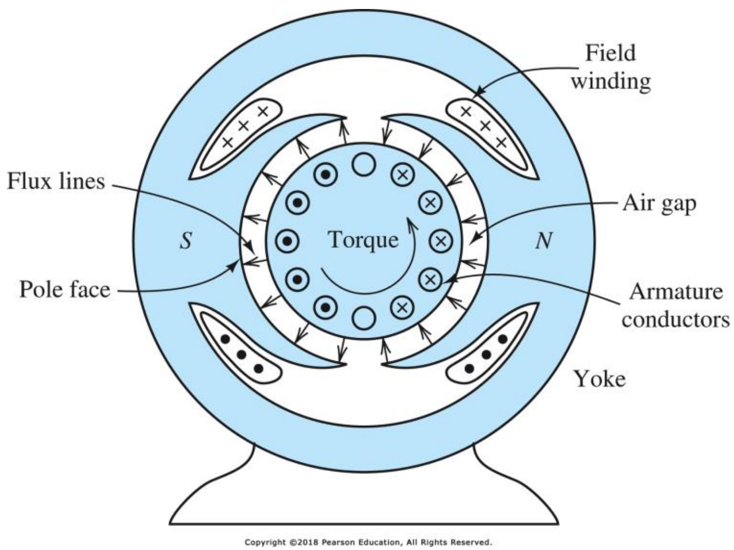
- Rotating DC Motor



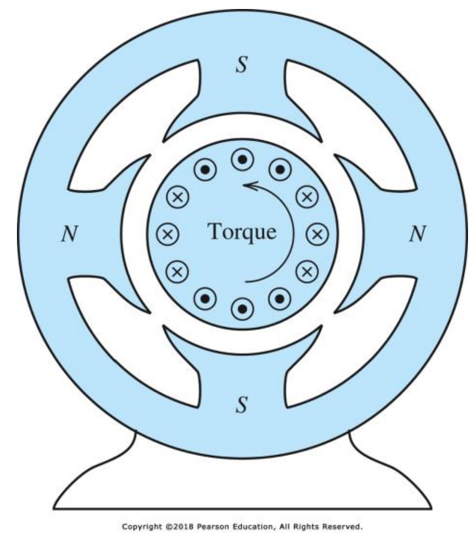
- The electrical supply makes current flow through the conductors which are placed in the magnetic field
- Each conductor placed on rotor experiences torque and rotor starts rotating

DC Machine Structure

- Rotating DC Motor - Stator



2 Pole DC Motor

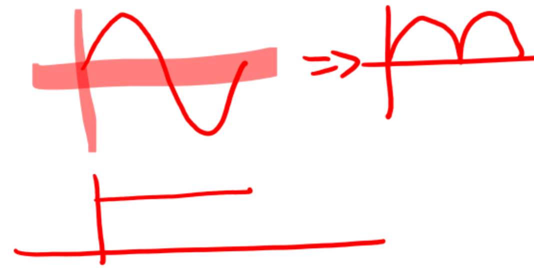


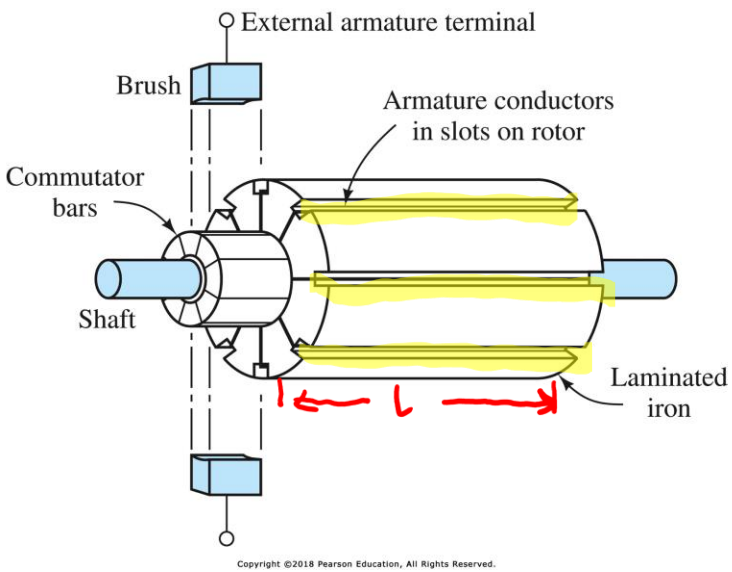
4 Pole DC Motor

- Stator : consists of Yoke and the poles
- Yoke – provides a highly permeable path for magnetic flux
- Poles – made of thin laminations stacked together and are designed to accommodate field winding → current flowing through the field winding is used to produce magnetic field

DC Machine Structure

- Rotating DC Motor – Rotor (Armature)

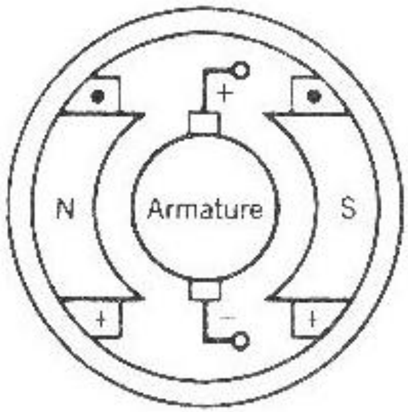


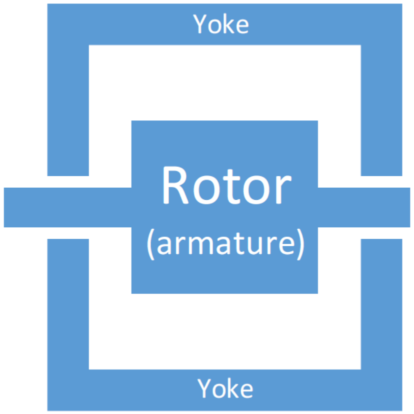


- Rotor – circular in cross section, made of thin highly permeable and electrically insulated steel laminations stacked together and mounted on the shaft
- Axial slots on the periphery to house **Armature winding**

- Commutator – made of wedge shaped copper segments; one end of armature coil is electrically connected to a copper segment of commutator
- Commutator – converts alternating voltage induced in the armature coil into a unidirectional (DC) voltage
- Brush – electrically connected to the brush holder by pigtail; through these brush holders an electrical connection can be established between external circuit and the armature coils

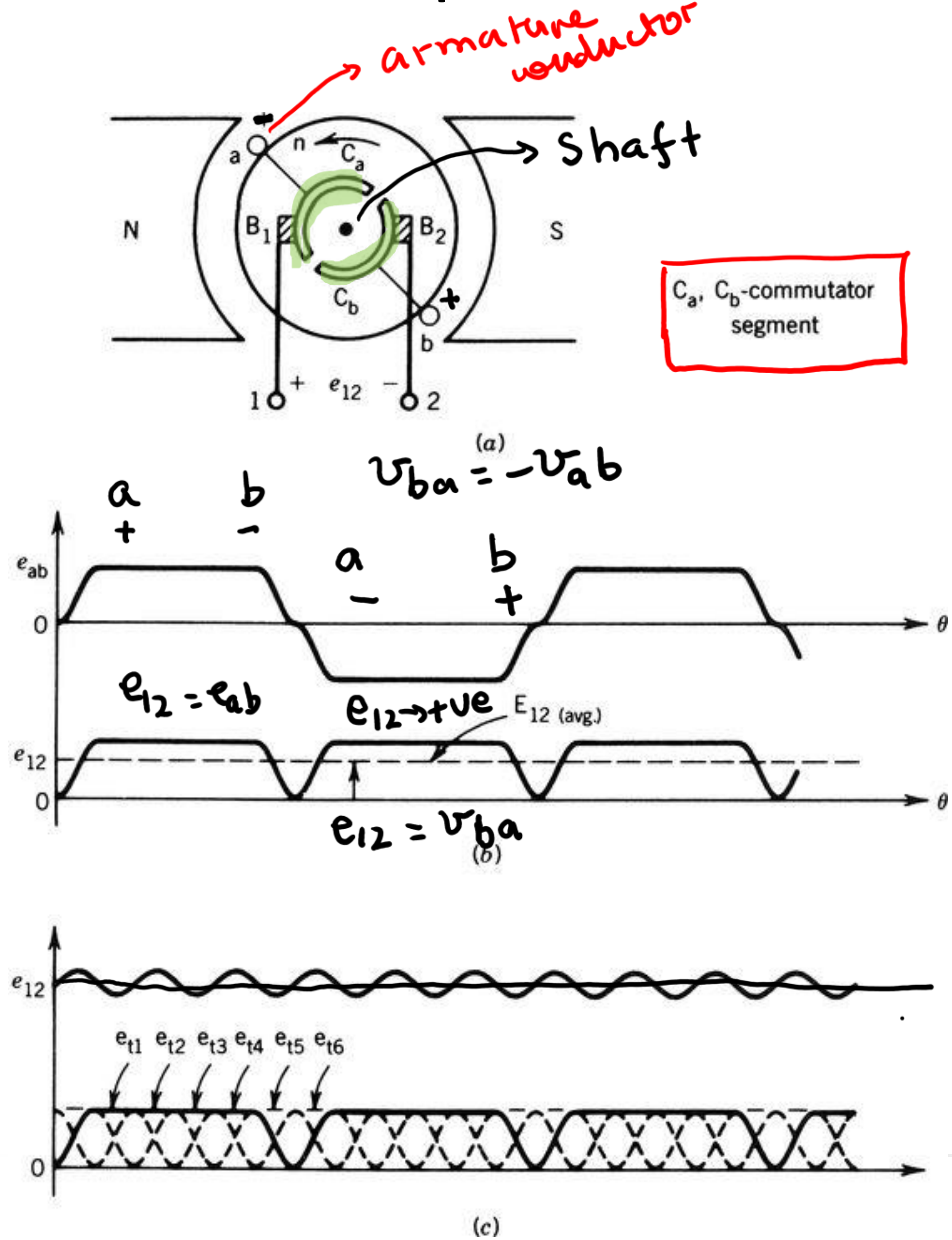
DC Machine Operation

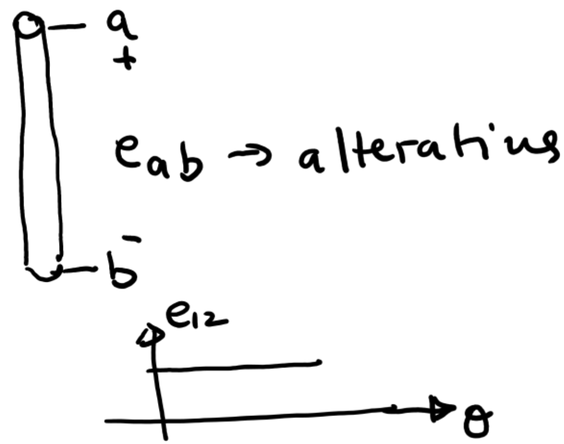




- Field winding is placed on the stator
- DC current passed through the field winding to produce magnetic field (flux)
- Armature winding is placed on the rotor. Alternating (AC) Voltage induces across the armature winding conductor as the rotating armature conductors are placed inside the magnetic field
- A mechanical commutator and brush assembly works as a rectifier and makes the armature terminal voltage unidirectional (DC)

DC Machine Operation





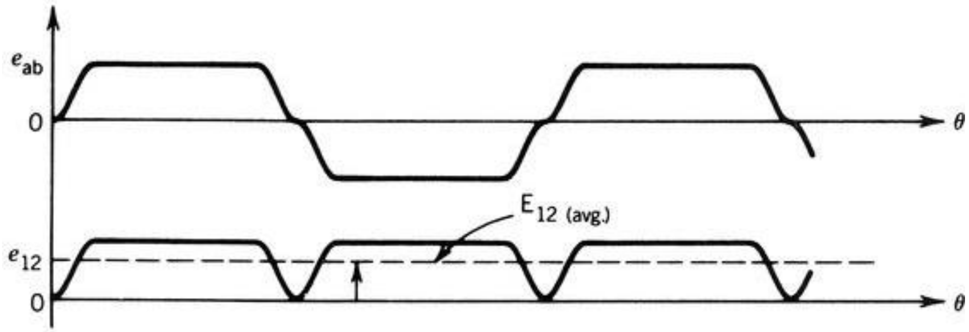
AC voltage induced in a single turn machine

DC voltage (commutation) in a single turn machine

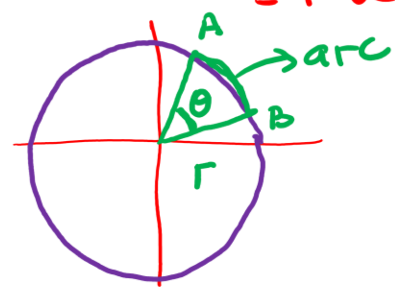
DC voltage in a multi turn machine (small ripple)

↳ AC content as a part of DC signal

DC Machine operation - Armature Voltage



$$v = \frac{s}{t} \approx \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right) = r\omega_m$$



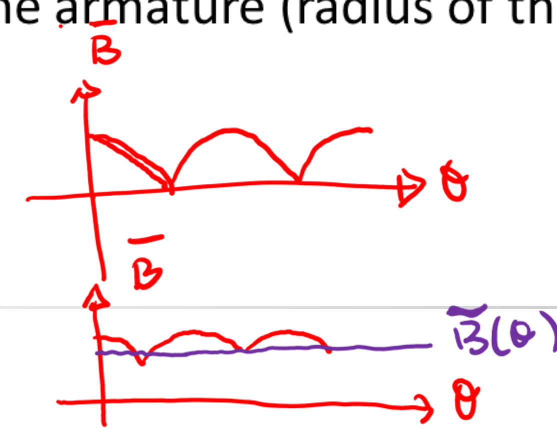
$s = r\theta$ (arc length)

$$e = Blv$$

$$e_t = 2B(\theta)l\omega_m r$$

↳ 1 turn (has 2 conductors)

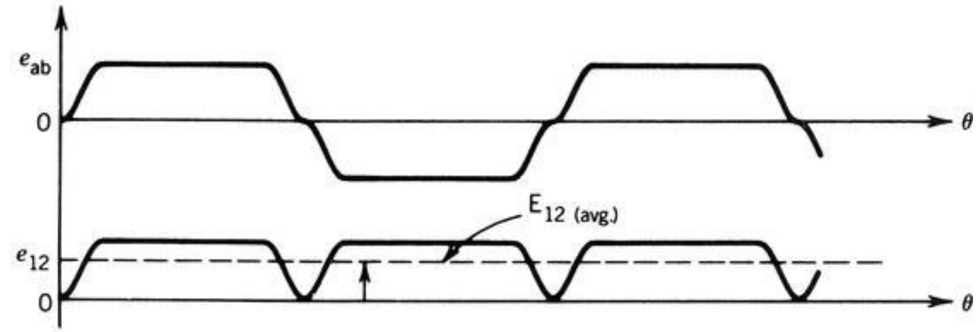
- l - length of the conductor in the armature slot
- ω_m - mechanical speed (rad/sec)
- r – distance of the conductor from the center of the armature (radius of the armature)



$$\bar{e}_t = 2\overline{B(\theta)}l\omega_m r$$

DC Machine operation - Armature Voltage

If $\bar{e}_t = 2\,\overline{B(\theta)}\, l\, \omega_m r$

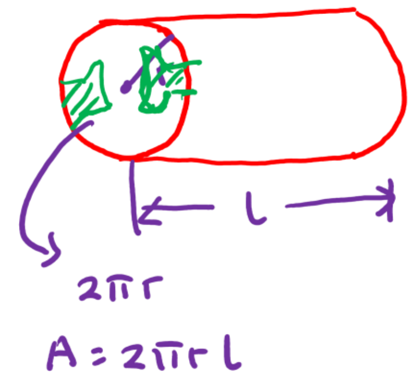


- φ - flux per pole
- A – area per pole = $\frac{2\pi r l}{p}$

Then

$$\overline{B(\theta)} = \frac{\varphi}{A} = \frac{\varphi p}{2\pi r l} = \frac{\varphi}{\left(\frac{2\pi r l}{p}\right)}$$

$\left(\frac{Wb}{m^2}\right)$



$$\bar{e}_t = \cancel{2}\frac{\varphi p}{\cancel{2}\pi r \cancel{l}}\cancel{l}\,\omega_m \cancel{r} = \frac{\varphi p}{\pi}\omega_m$$

1 turn

The average induced voltage in multi turn coil (N as # of turns)

constant

$$E_a = N\bar{e}_t = N\frac{\varphi p}{\pi}\omega_m = N\frac{p}{\pi}\varphi\omega_m = K_a\varphi\omega_m$$

constant

m/c constant

$$\boxed{E_a = K_a\varphi\omega_m}$$

Motor – back emf (counter emf)

Generator – Generated or Induced voltage

DC Machine operation - Armature Voltage

$$E_a = K_a \varphi \omega_m$$

induced armature voltage (V)

magnetic flux (Wb)

rotor (motor) speed in rad/sec

K_a - machine constant or armature constant depends on

- Type of the armature winding – lap or wave winding
- # of poles of the field
- # of coils that compose the winding
- # of turns of each coil

$\boxed{N \text{ (rpm)} \longleftrightarrow \omega \text{ rad/sec}}$

$$K_a \Longrightarrow \frac{V}{Wb \cdot \frac{rad}{sec}}$$

$$\Longrightarrow \frac{V \cdot sec}{Wb \cdot rad}$$

$$E_a = K_a \varphi N_m$$

$$K_a \Longrightarrow \frac{V}{wb \cdot rpm}$$

Armature Voltage - Example

In a particular type of winding known as the "lap winding" the constant K_a is given by $K_a = Z/2\pi$, where Z is the total number of conductors in the armature. If a DC machine has 90 lap wound coils each having 4 turns of wire, and if the air gap flux per pole is 0.04 ϕ Wb, what voltage will this machine generate if its shaft spun at 600 rpm?

$$K_a = \frac{Z}{2\pi}$$

1 rotation $= 2\pi$ rad

1 sec $= \frac{1}{60}$ min

$$E_a = K_a \varphi \omega_m$$

Z : total # of conductors $= 2 \times 4 \times 90 = 720$

$$K_a = \frac{720}{2\pi}$$

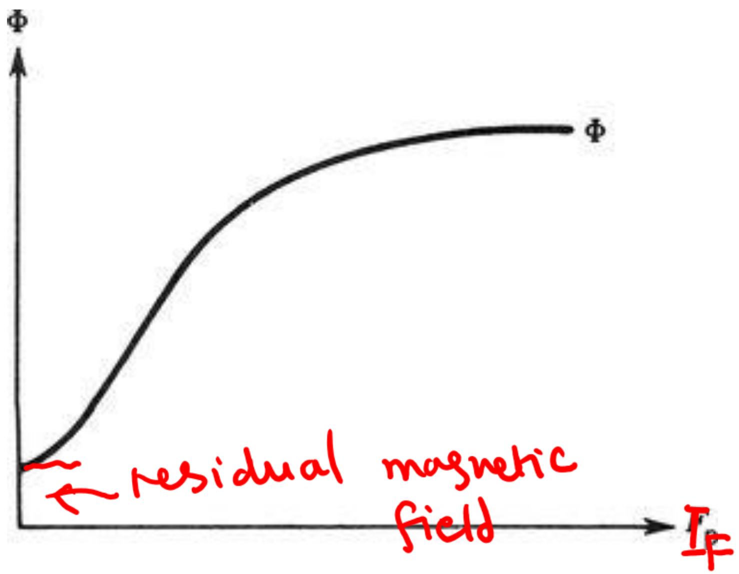
$N_m = 600$ rpm

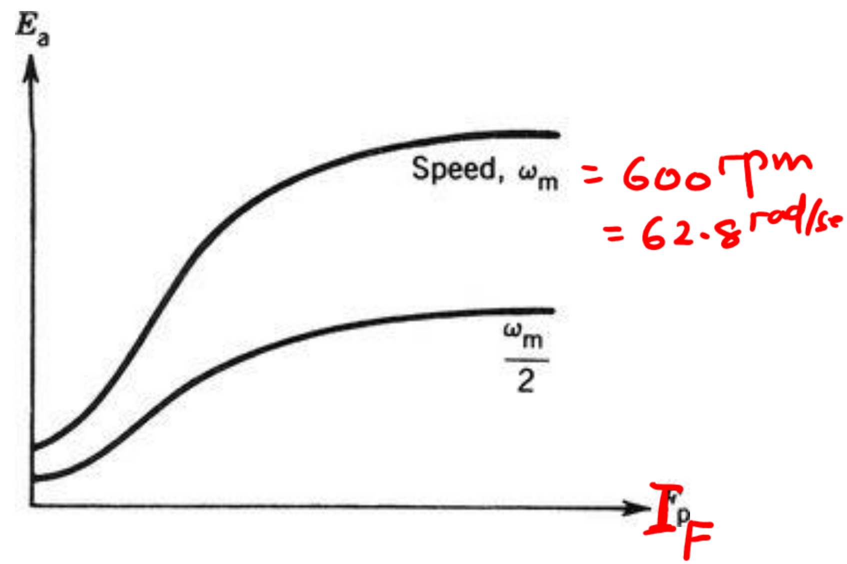
$$\omega_m = \frac{600}{\left(\frac{60}{2\pi}\right)} \Rightarrow 9.55$$

$$E_a = \frac{720}{2\pi} \times 0.04 \times 62.8 = 287.6 \text{ V}$$

$\omega_m = 62.8$ rad/sec

DC Machine operation - Armature Voltage





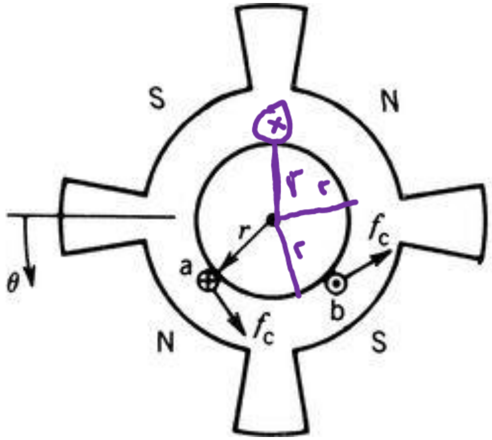
$$E_a = K_a \varphi \omega_m$$

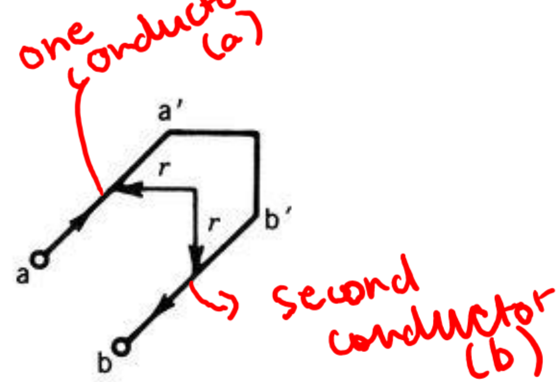
constant

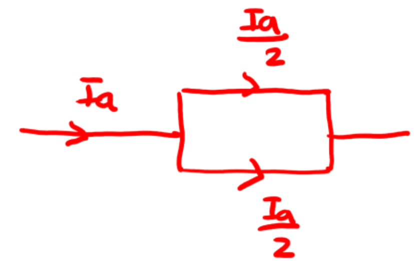
$E_a \propto \phi \Rightarrow \phi \propto I_F$

$E_a \propto I_F$

DC Machine operation – Electromagnetic Torque







slide #1

$$B(\theta) = \frac{\varphi p}{2\pi r l}$$

Consider the turn aa'b'b whose 2 conductors aa' and bb' are placed under 2 adjacent poles

$F = i l B$

The force on a conductor is

$$f_c = B(\theta) l i_c = B(\theta) l \frac{I_a}{a}$$

a → # of parallel paths

Where: i_c - current in the conductor of the armature winding

I_a - armature terminal current

$\bar{T} = \bar{F} \times \bar{d}$

↳ distance vector

Average Torque developed by a conductor:

$$T_c = f_c r = B(\theta) l \frac{I_a}{a} r = \frac{\varphi p}{2\pi \cancel{r} \cancel{l}} \cancel{l} \frac{I_a}{a} \cancel{r} = \frac{\varphi p I_a}{2\pi a}$$

The total Torque developed the armature winding:

$$T = \frac{\varphi p I_a}{2\pi a} = K_a \varphi I_a$$

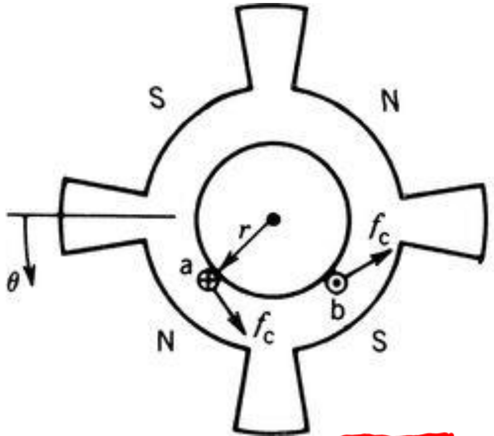
$\boxed{T = K_a \varphi I_a}$

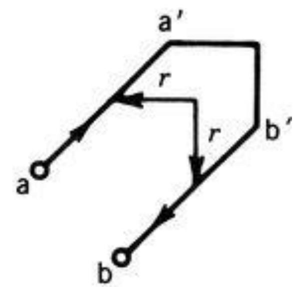
φ → Wb

I_a ↳ armature current (A)

T → N-m

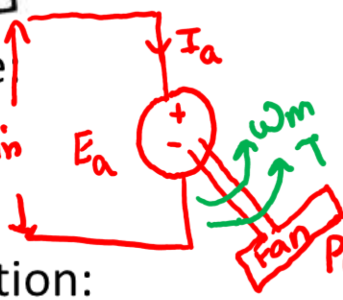
DC Machine operation – Power





The total Torque $T = K_a \varphi I_a$

$E_a I_a = P_{in}$



$P_{out} = P_{mech.}$

Motor operation:

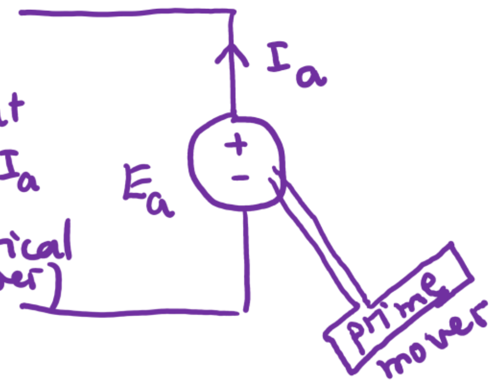
Electrical power input = $E_a I_a$

Mechanical power developed = $T\omega_m$

$P_{mech} = T\,\omega_m$

(W) (Nm) (rad/sec)

$P_{out} = E_a I_a$ (Electrical power)



$P_{in} = P_{mech}$ $(T\omega_m)$

Generator operation:

Electrical power output = $E_a I_a$

Mechanical power input = $T\omega_m$

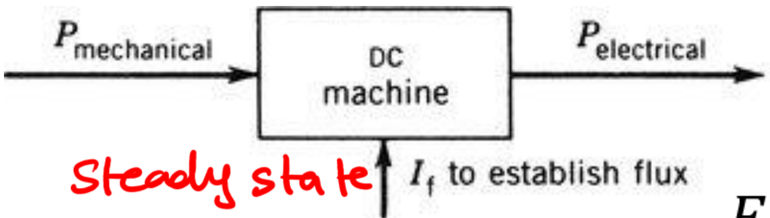
$T \rightarrow$ counter Torque

$$\textit{Electrical Power}, \quad E_a I_a = K_a \varphi \omega_m I_a = T\omega_m, \quad \textit{Mechanical power}$$

$K_a \varphi I_a \, \omega_m$

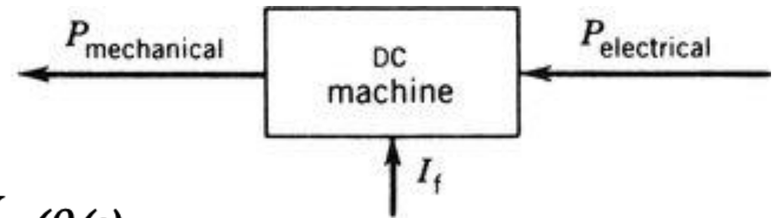
DC Machine - Modeling

Generator



Steady state

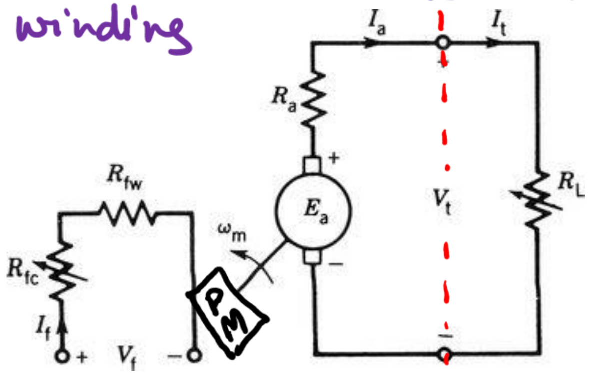
Motor



$$E_a = K_a \varphi \omega_m$$

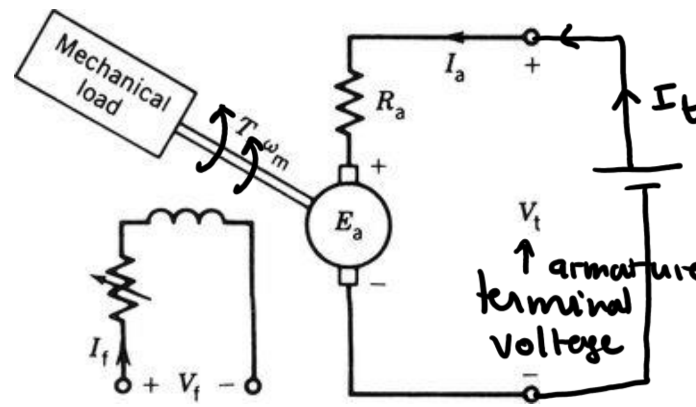
Field winding

armature winding

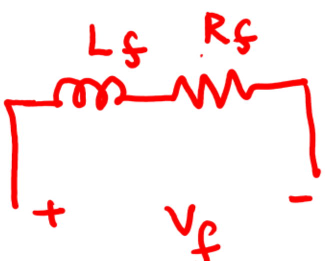


(load application)

armature terminals



armature terminal voltage



$$I_f \to \frac{V_f}{R_{fw}+R_{fc}}$$

$I_t = I_a$

line current

armature current

$R_f = R_{fc} + R_{fw}$

control resistance

winding resistance

The armature current I_a and the motor speed ω_m depend on the mechanical load connected to the motor shaft

DC Machine Modeling

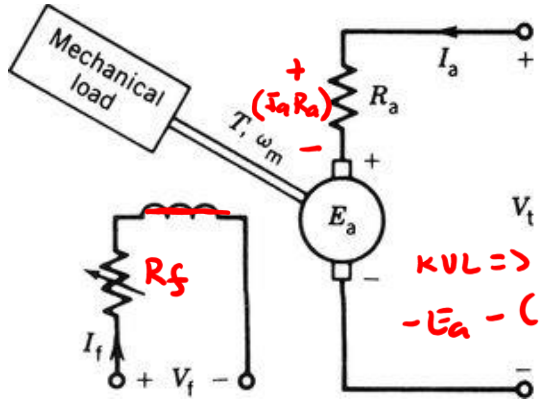
$\boxed{E_a = K_a \Phi \omega_m}$ $\boxed{T = K_a \Phi I_a}$

$R_f = R_{fc} + R_{fw}$

Motor

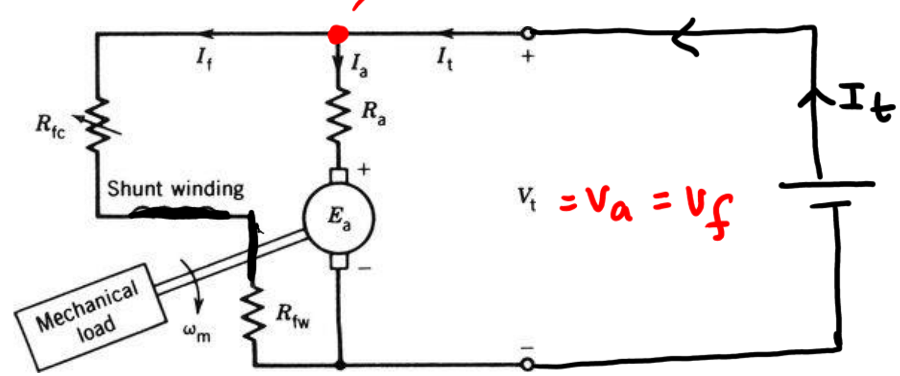
V_f and V_t are separately controlled

KCL ⇒ $I_t = I_a + I_f$



$V_t = V_a$

KVL ⇒ $-E_a - (I_a R_a) + V_t = 0$



$V_t = V_a = V_f$

① $V_f = R_f I_f$

$V_t = E_a + I_a R_a$

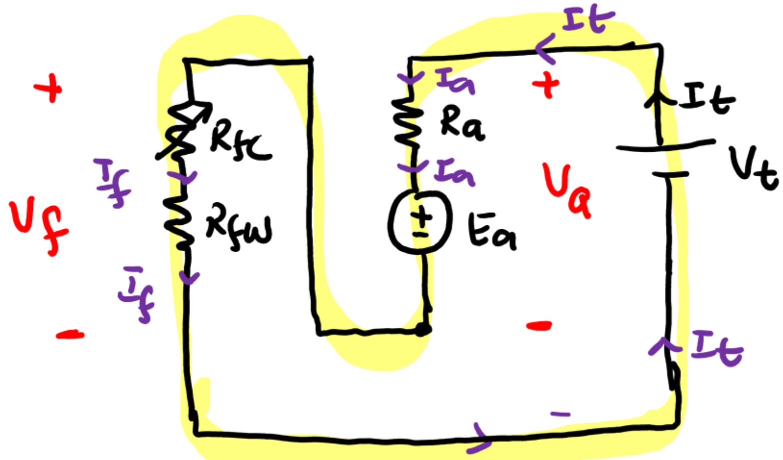
$V_a = E_a + I_a R_a$

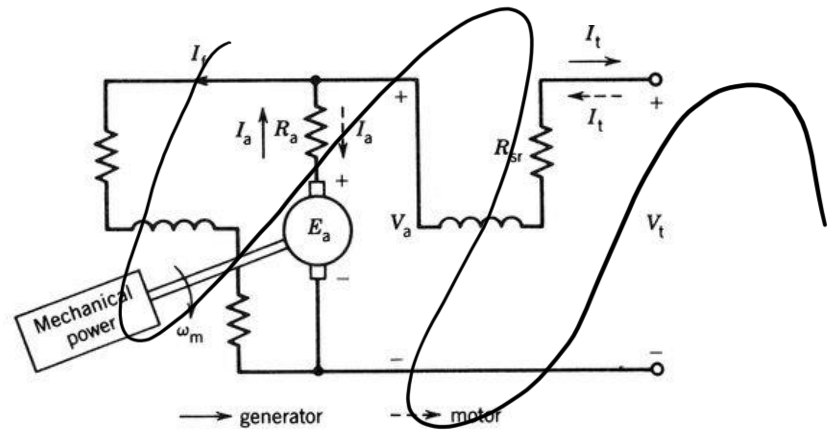
Separately Excited Motor

② Shunt Motor

$V_t = E_a + I_a R_a$

$V_t = R_f I_f$





③ Series Motor $I_t = I_a = I_f$

Compound Motor

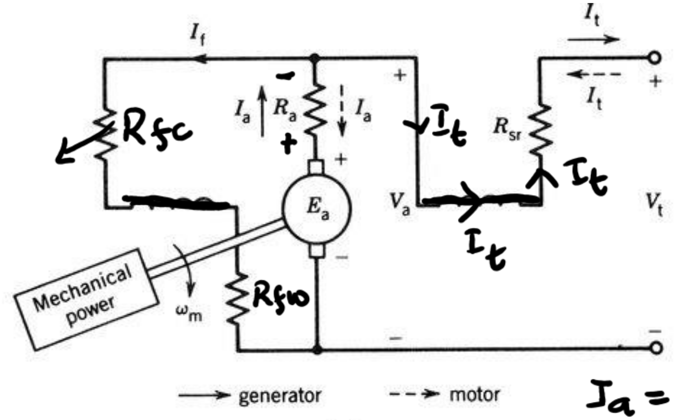
KVL ⇒ $-V_f - V_a + V_t = 0$

$\boxed{V_t = V_a + V_f}$ $\boxed{V_a = E_a + I_a R_a}$

DC Machine Modeling – Power Flow

$R_f = R_{fc} + R_{fw}$ ← shunt field

R_{sr} → series field resistance



(a)

P_{in} (Electrical) $= V_t I_t$

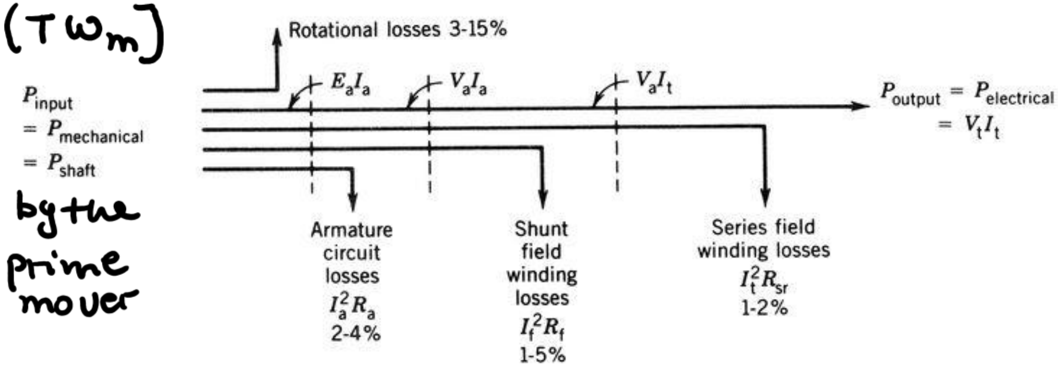
$I_a = I_f + I_t$

$E_a = V_a + I_a R_a$

$$Efficiency = \frac{P_{out}}{P_{in}}$$

$$\eta = \frac{P_{elect}}{P_{mech}} = \frac{V_t I_t}{T\omega_m} \times 100\%$$

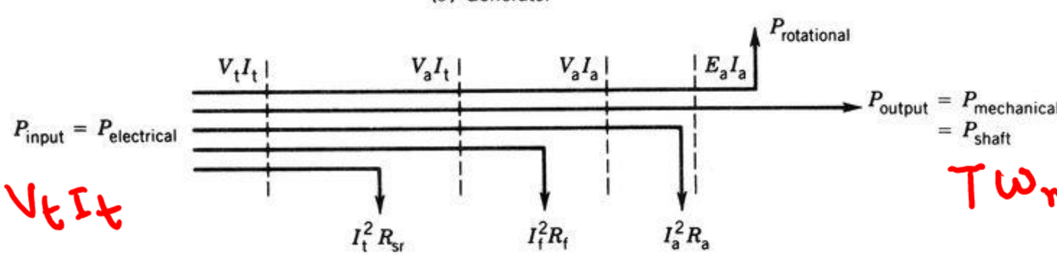
$(T\omega_m)$



by the prime mover

(b) Generator

$$\eta = \frac{P_{mech}}{P_{Elect}} = \frac{T\omega_m}{V_t I_t} \times 100\%$$



$V_t I_t$

$T\omega_m$

(c) Motor

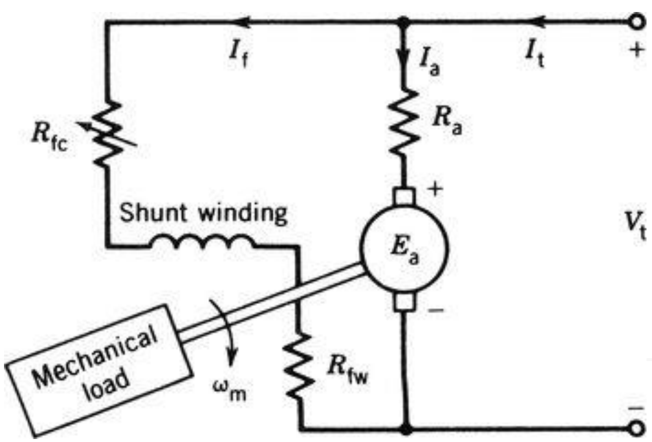
DC Motor - Example

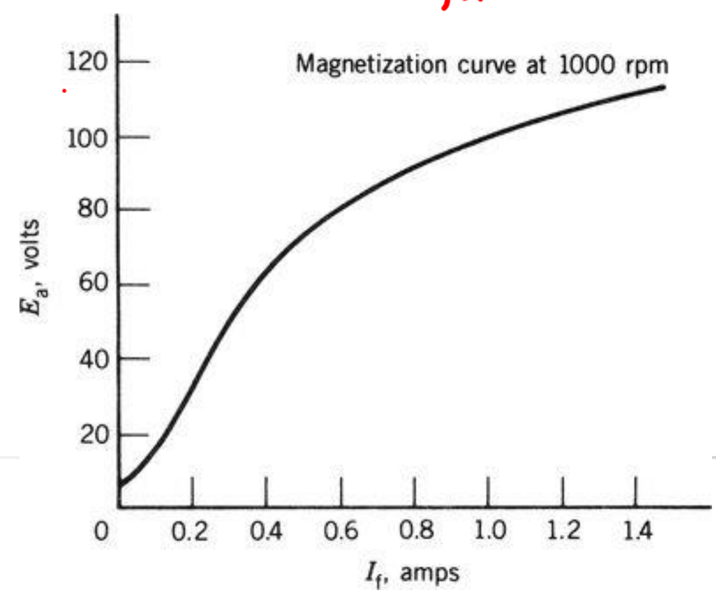
rated / full-load — V_t

A 12 kW, 100 V, 1000 rpm DC Shunt motor is connected to 100 V DC supply. The motor has armature resistance R_a = 0.1Ω and the shunt field winding resistance R_{fw} = 80 Ω. At no-load condition, the motor runs at 1000 rpm, and the armature takes 6 A. Calculate:

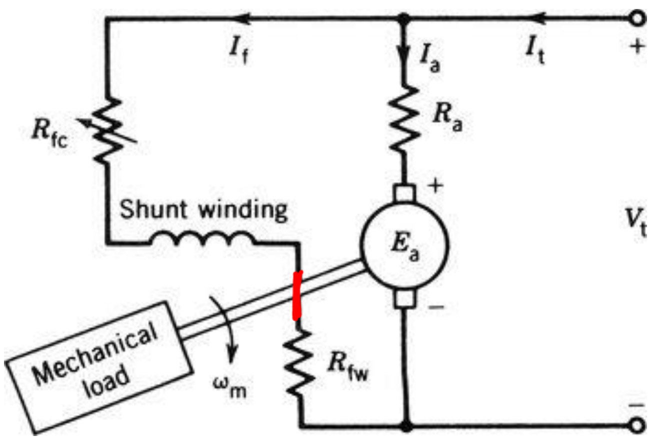
$I_a = 6A \big|_{NL}$

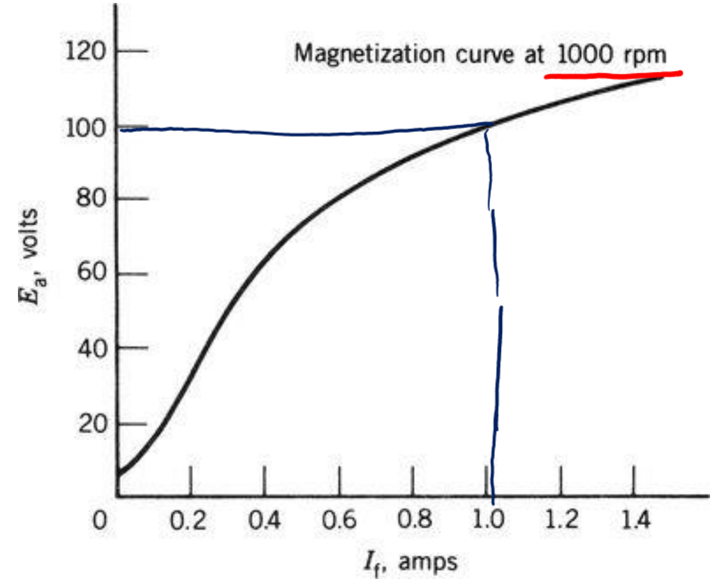
1. the value of the resistance of the shunt field rheostat (R_{fc})
2. the rotational losses at 1000 rpm
3. the speed, electromagnetic torque, and the efficiency of the motor when rated current flows in the armature (not @ no- load) $I_{a,rated}$
4. the starting torque if the starting armature current is limited to 150% of its rated value $T_{starting}$ $I_{a,st} = 1.5\ I_{rated}$



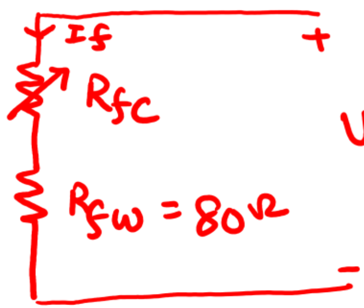


DC Motor - Example





①



$V_f = 100^V$

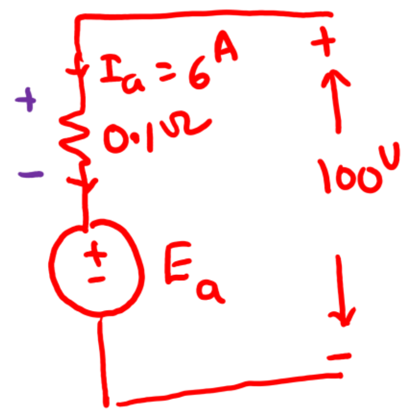
$$R_{fc} + R_{fw} = \frac{V_f}{I_f} = \frac{100^V}{0.99A}$$

$$R_{fc} = 101 - 80 = 21^{\Omega}$$

@ no-load

$N = 1000$ rpm

$I_a = 6^A$



$$E_a + I_a R_a = 100$$

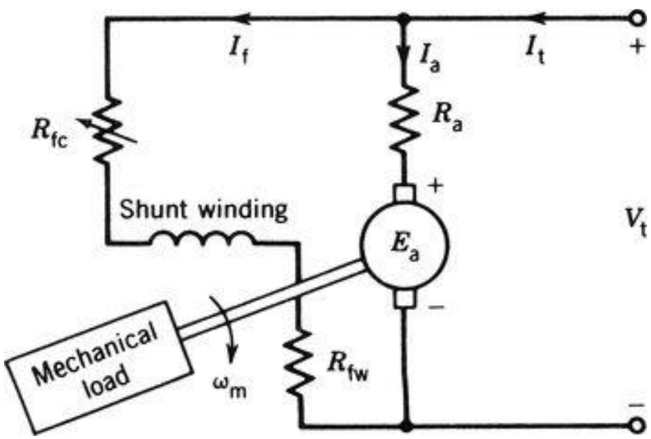
$$E_a = 100 - [0.1 \times 6]$$

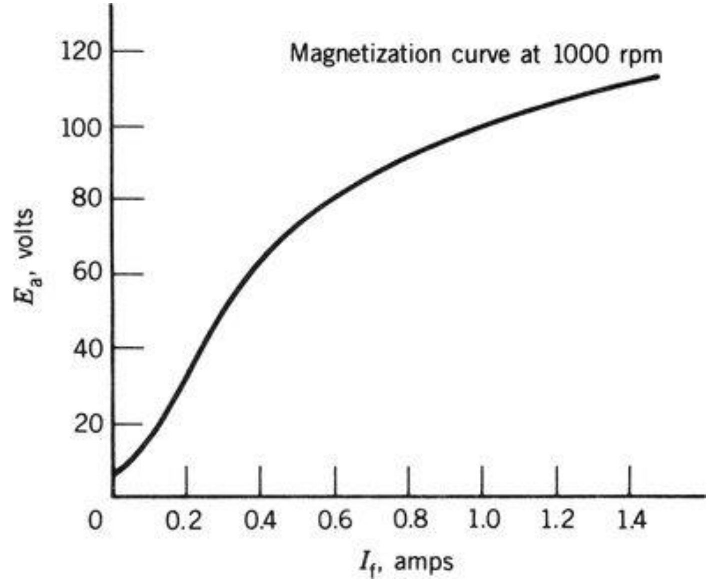
$$= 99.4^V$$

from the magnetization curve

$E_a = 99.4^V$ $\quad I_f = 0.99^A$

DC Motor - Example





(2) rotational losses, P_{rot}

@ no load $\rightarrow$ $P_{out} = P_{dev} = P_{rot} = E_a I_a = 99.4 \times 6$

$= 596.4^W$

$= 0.5964^{kW}$

(3) $I_{a,rated} = \frac{P_{rated}}{V_{rated}} = \frac{12^{kW}}{100^V} = \frac{12000^W}{100^V}$

$I_{a,rated} = 120^A$

$E_a = K_a \Phi \omega_m \Rightarrow E_a \propto \omega_m$

$E_{a1} = 99.4^V$

$\omega_{m1} = \frac{1000}{9.55}$ rad/sec

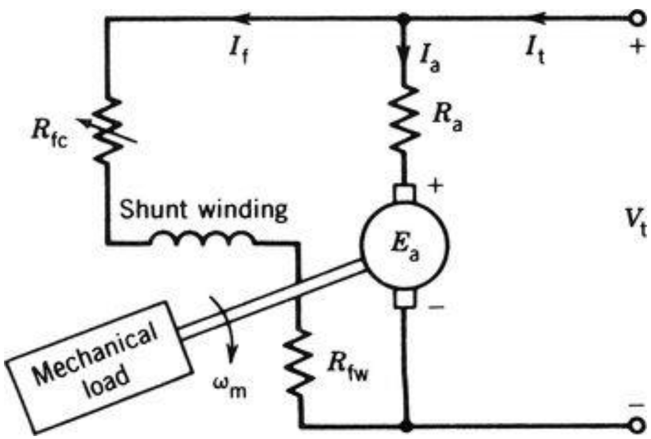
$\omega_{m1} = 1000^{rpm}$

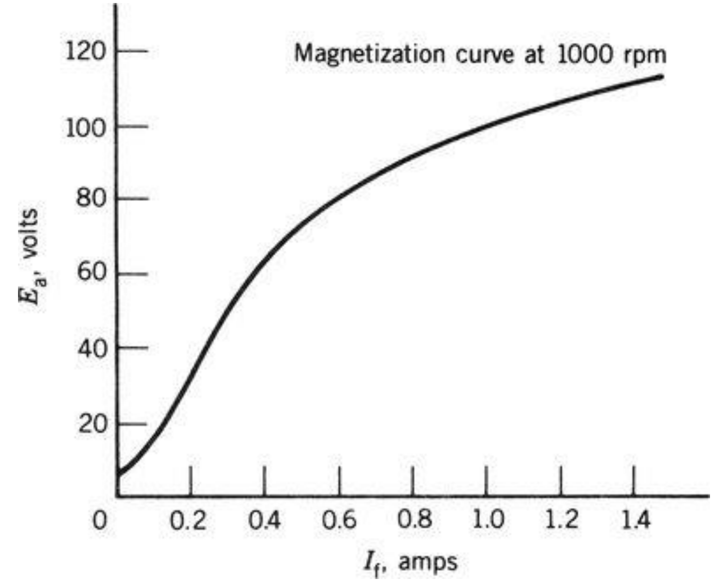
$E_{a2} = V_t - I_a R_a = 100 - [120 \times 0.1] = 88^V$

$E_{a2} = ?$

$\omega_{m2} = ?$

DC Motor - Example





$$\frac{E_{a1}}{E_{a2}} = \frac{\omega_{m1}}{\omega_{m2}} \Rightarrow \frac{99.4}{88} = \frac{1000}{\omega_{m2}} \Rightarrow \omega_{m2} = 885.31 \text{ rpm}$$

$$T = K_a \Phi I_a$$

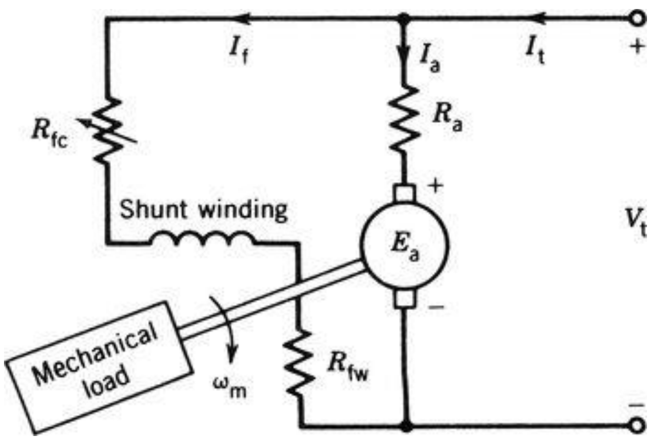
$$\boxed{K_a\Phi = \frac{E_a}{\omega_m} = \frac{99.4}{(1000/9.55)}} = \frac{88}{885.31/9.55}$$

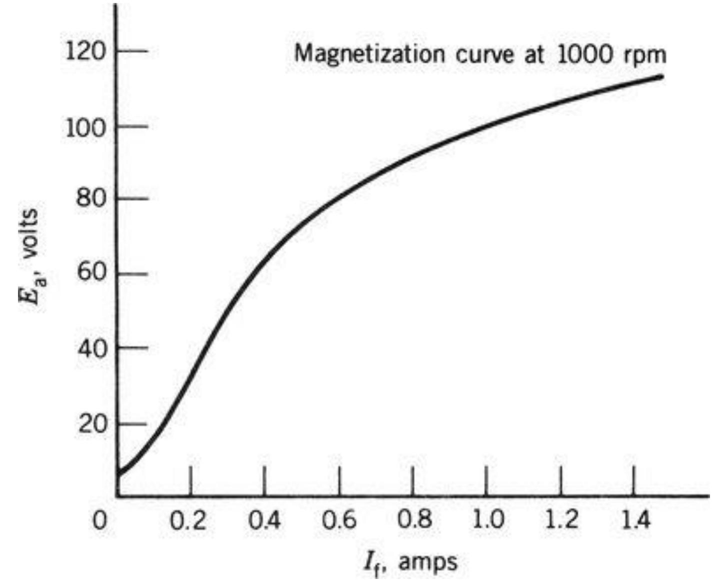
$$T = \boxed{\frac{99.4}{(1000/9.55)}} \times 120^{A} = 113.9^{\text{ N-m}}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{T\omega_m}{V_t I_t} \times 100\% = \frac{113.9 \times \left[\frac{885.31}{9.55}\right]}{100 \times [120 + 0.99]} \times 100\%$$

$$= 87.3\%$$

DC Motor - Example





(4) $T = K_a \Phi I_a$

$$T_{st} = K_a \Phi \, I_{a,st} = \underbrace{\left[\frac{99.4}{1000/9.55}\right]}_{K_a \Phi} \times \underbrace{\left[1.5 \times 120\right]}_{I_{a,st}}$$

$$T_{st} = 170.87 \text{ N-m}$$

DC Generator - Example

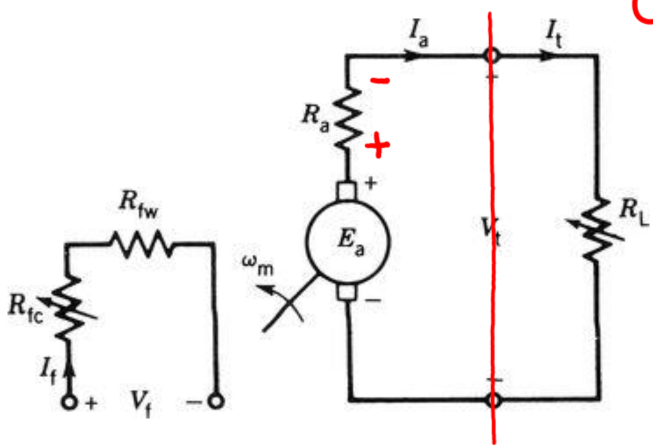
V_t : armature terminal voltage

A DC machine is used as a separately excited generator; it has an armature resistance of 1.5Ω and generates an open-circuit voltage of 200 V at 1200 rpm. Calculate:

V_t E_a $K_a \phi \omega = E_a$

1. The armature terminal voltage under a load of 5 A $I_a = I_t$
2. The torque (counter or opposing) that machine produces
3. The efficiency of the process, ignoring the energy loss of the field winding

$E_a > V_t$



① $V_t = E_a - I_a R_a$

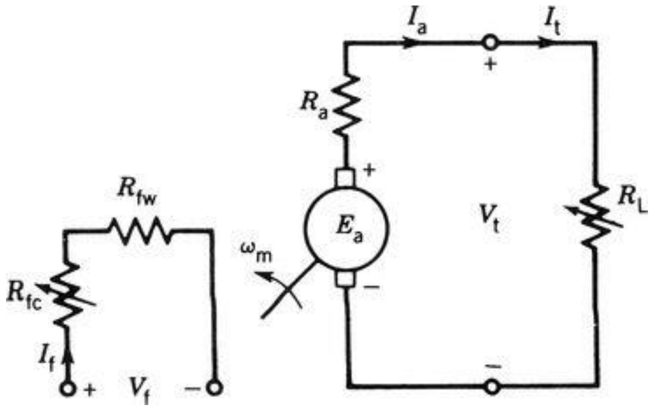
$V_t = 200 - [5 \times 1.5] = 192.5$ V

② $T = K_a \phi I_a = \left[\frac{200}{1200/9.55}\right] \times 5 = 7.96$ N-m

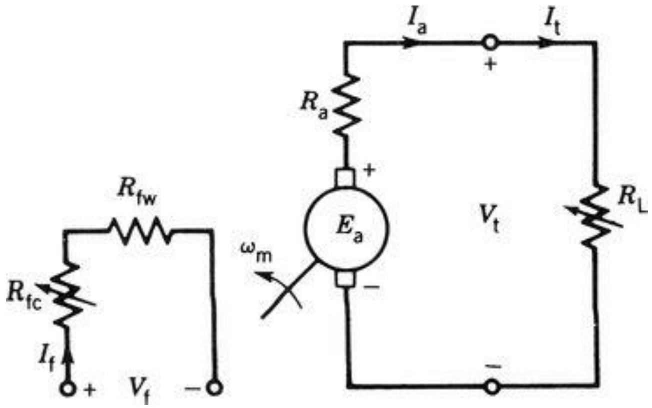
③ $$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{Elect}}{P_{mech}} \times 100\% = \frac{V_t I_t}{T \omega} \times 100\% = \frac{192.5 \times 5}{7.96 \times \left(\frac{1200}{9.55}\right)} \times 100\%$$

$= 96.23\%$

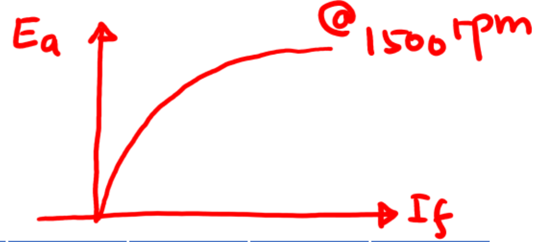
DC Generator - Example



DC Generator - Example



DC Machine - Example



The magnetization curve of a DC machine is given below:

I_f (A)	0	0.2	0.5	0.8	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
E_a (V)	14	19	31	54	72	100	124	140	156	168	180	185	187	188

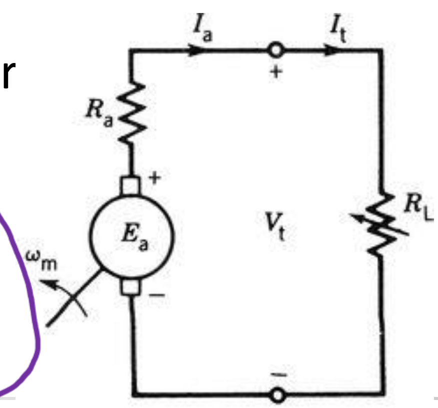
@ 1500 rpm

The shaft speed for the data above was 1500 rpm.

If $R_a = 0.15\ \Omega$, calculate:

E_a

1. The open circuit armature voltage at $I_f = 2.5$ A and speed is 1200 rpm
2. The machine torque under the conditions of part (1) and $I_a = 20$A
3. The machine's armature terminal voltage if the machine works under the conditions of part (1) and (2), as a generator
4. The machine's armature terminal voltage if the machine wor conditions of part (1) and (2), as a motor



① $E_a = K_a \phi \omega$; from the table

$E_a = 140^V$
$I_f = 2.5A$
$N = 1500$ rpm

$E_a \propto \omega \Rightarrow E_a \propto N$

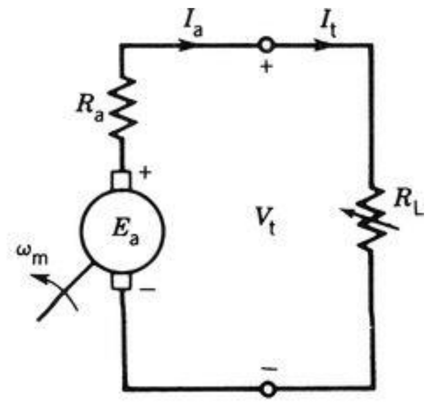
$$\frac{140}{E_{a1}} = \frac{1500}{1200} \Rightarrow E_{a1} = 112^V$$

DC Machine - Example

I_f (A)	0	0.2	0.5	0.8	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
E (V)	14	19	31	54	72	100	124	140	156	168	180	185	187	188

② $T = K_a\Phi I_a$ $\qquad I_a = 20^A$

$N = 1200^{rpm}$



$$K_a\Phi = \frac{140}{(1500/9.55)} = \frac{112}{(1200/9.55)}$$

$$T = \left[\frac{112}{(1200/9.55)}\right] \times 20 = 17.83^{N-m}$$

③ Generator $\Rightarrow E_a > V_t \Rightarrow V_t = E_a - I_a R_a$

$$V_t = 112 - [20 \times 0.15] = 109^V$$

$$E_a = 112 > V_t = 109^V$$

DC Machine - Example

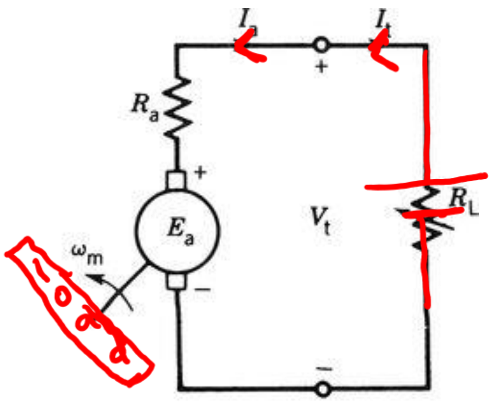
I_f (A)	0	0.2	0.5	0.8	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
E (V)	14	19	31	54	72	100	124	140	156	168	180	185	187	188

4 Motor $N = 1200\,\text{rpm}$ $I_a = 20\,A$

$V_t > E_a$: $V_t = E_a + I_a R_a$

$V_t = 112 + [20 \times 0.15] = 115\,V$

$V_t = 115\,V > E_a = 112\,V$



DC Motor – Torque Speed Characteristics

independent Var. dependent variable

$V_a \rightarrow$ armature terminal voltage

Some applications require the DC motor speed to remain constant as the mechanical load applied to the motor changes while some require that the speed be controlled over a wide range

- Separately excited DC motor

$E_a = K_a \Phi \omega$ $\quad T = K_a \Phi I_a \Rightarrow I_a = \frac{T}{K_a \Phi}$

$E_a = V_t - I_a R_a = V_a - I_a R_a$

$$\frac{K_a \Phi \omega}{K_a \Phi} = \frac{V_t}{K_a \Phi} - \left[\frac{T}{K_a \Phi}\right]\frac{R_a}{K_a \Phi}$$

$$\omega = \frac{V_t}{K_a \Phi} - \underbrace{\frac{R_a}{(K_a \Phi)^2}}_{m} T$$

$y = c - mx = -mx + c$

St. line eqn with negative or decaying slope

$$\omega_m = \frac{V_t}{K_a \varphi} - \frac{R_a}{(K_a \varphi)^2} T$$

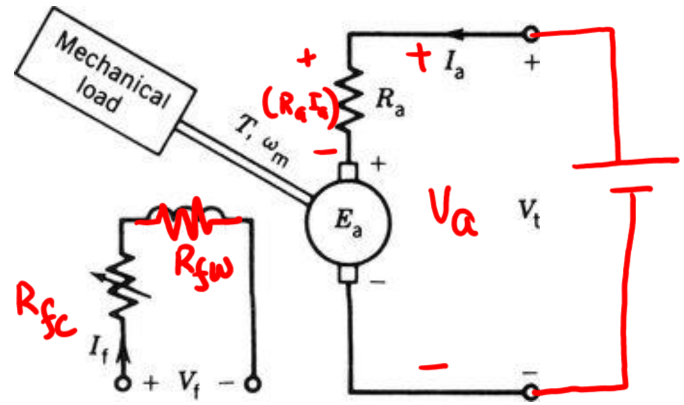
speed → (rad/sec)

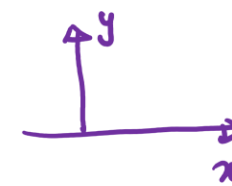
V_t → supply voltage (V_a)

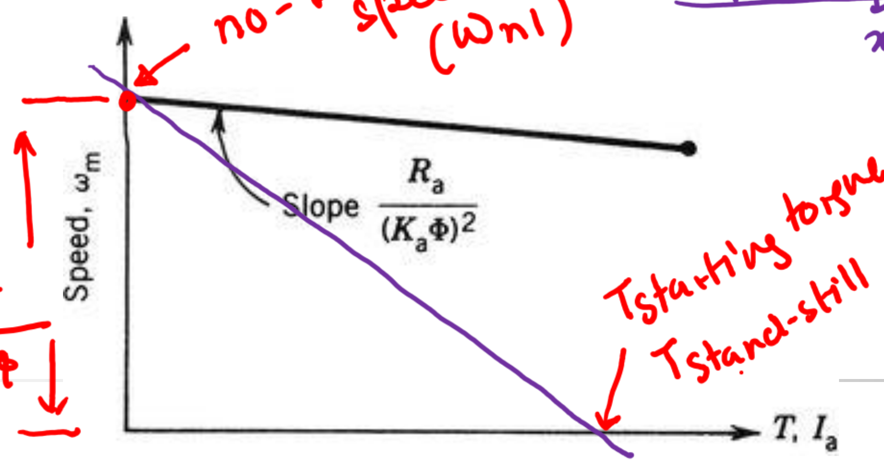
R_a → armature wdg. resistance

T → Torque (N-m)

φ → depends on $I_f < \frac{V_f}{R_f}$

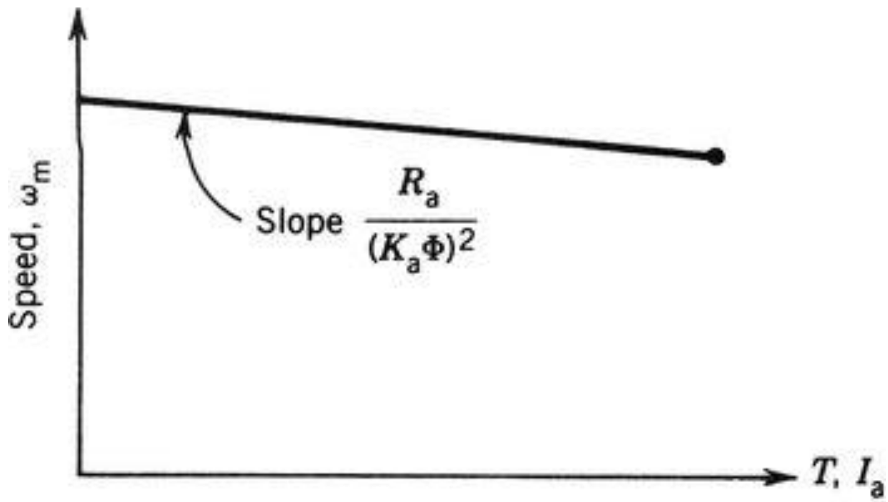






DC Motor – Torque Speed Characteristics

- Separately excited DC motor: Speed control

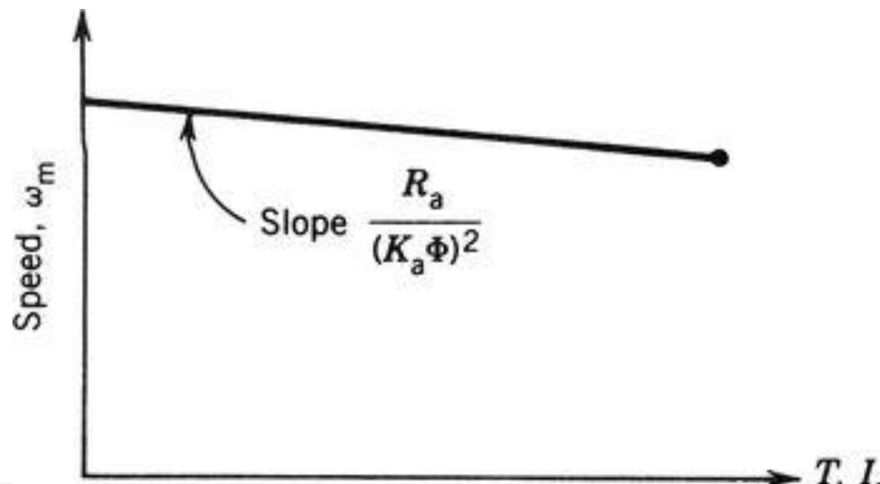


The drop in speed as the applied torque increases is small, provides a good **speed regulation**

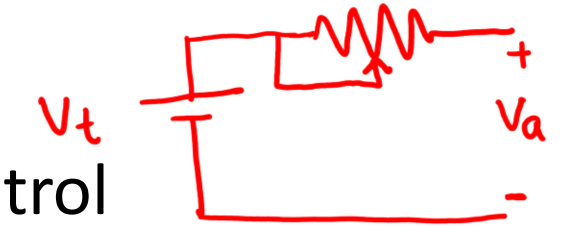
DC Motor – Speed Control

- Separately excited DC motor: Speed control
  - Armature voltage control (V_t)
    $\hookrightarrow V_a$
  - Field control (ϕ)
  - Armature resistance control (R_a)

$$\omega_m = \frac{\overset{V_a}{\cancel{V}_t}}{K_a\varphi} - \frac{R_a}{(K_a\varphi)^2}T$$



DC Motor – Speed Control



- Separately excited DC motor: Speed control
  - Armature voltage control (V_t) – the armature resistance R_a and the field current I_f are kept constant
  - For a constant load torque (elevator, hoist crane), speed will change linearly with V_t

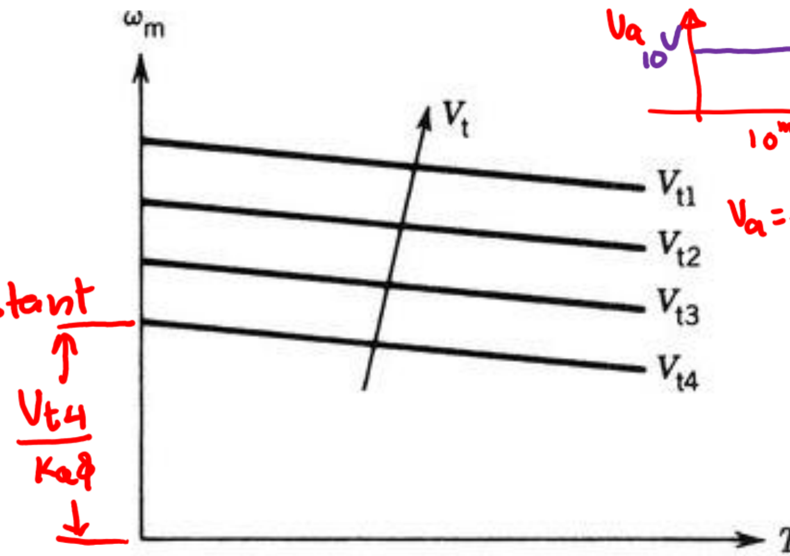
$$\omega_m = \frac{V_t}{K_a\varphi} - \frac{R_a}{(K_a\varphi)^2}T$$

constant

$y = c - mx$

$m = \frac{Ra}{(Ka\varphi)^2} \Rightarrow$ constant

$V_{t1} > V_{t2} > V_{t3} > V_{t4}$



$V_a = \frac{10 \times 10 + 0 \times 5}{15}$

DC Motor – Speed Control

- Separately excited DC motor: Speed control
  - Armature resistance control (R_a): the armature voltage V_t and the field current are I_f kept constant. The speed is controlled by changing the resistance in the armature circuit

$$\omega_m = \overbrace{\frac{V_t}{K_a\varphi}}^{\text{constant}} - \frac{R_a}{(K_a\varphi)^2}T$$

→ slope will vary

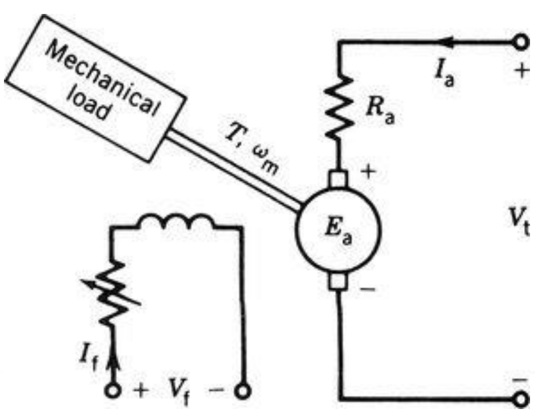
$$m = \frac{R_a}{(K_a\varphi)^2}$$

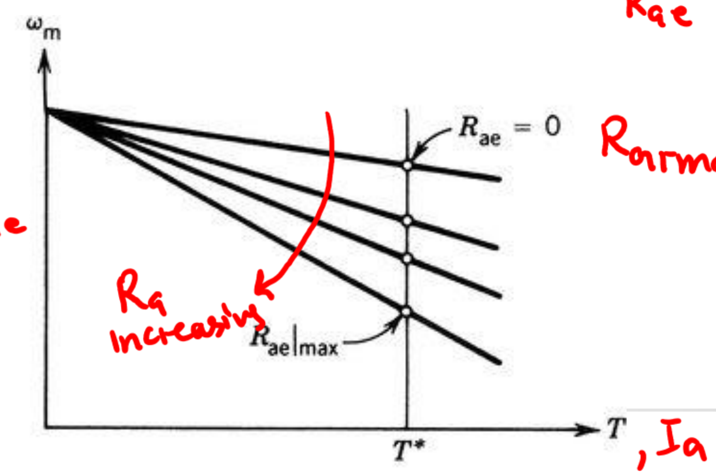
$$m \propto R_a$$

increase R_a → increase the slope

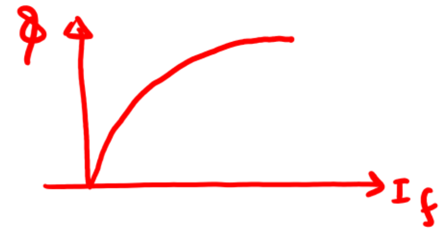
$R_{ae} \left\langle \begin{array}{l} R_{ae,max} \\ 0 \end{array}\right.$

$R_{armature} = R_a + R_{ae}$





DC Motor – Speed Control



- Separately excited DC motor: Speed control
  - Field control (ϕ): the armature resistance R_a and the armature voltage V_t are kept constant. The speed is controlled by varying the field current I_f (using a field circuit rheostat R_{fc})

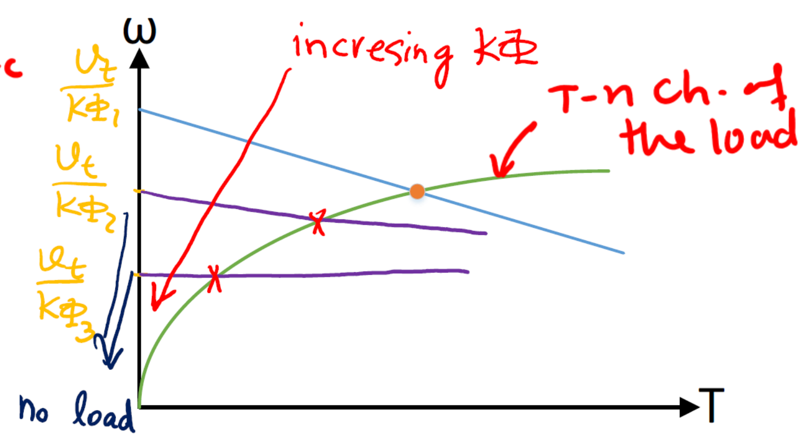
$I_f = \frac{V_f}{R_{fc}+R_{fw}}$

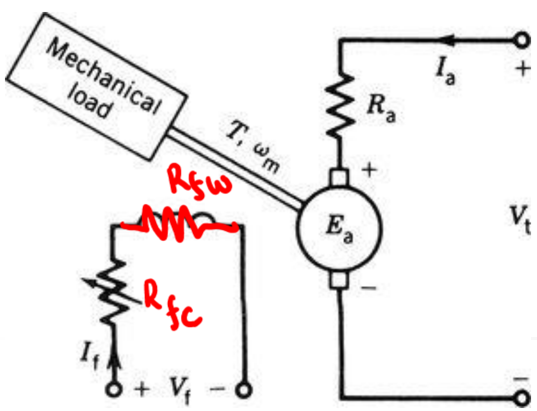
V_f

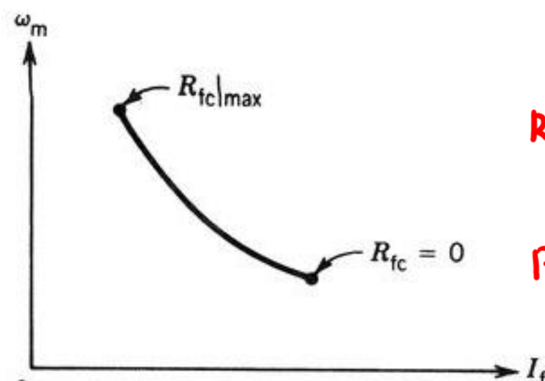
$R_{fc} \Rightarrow R_f = R_{fw} + R_{fc}$

$$\omega_m = \frac{V_t}{K_a\varphi} - \frac{R_a}{(K_a\varphi)^2}T$$

$\phi_3 > \phi_2 > \phi_1$







R_{fc} < 0 / $R_{fc,max}$

$R_{fc} = 0$; $I_f \to max$, $\phi \to max$

$\omega \to min$

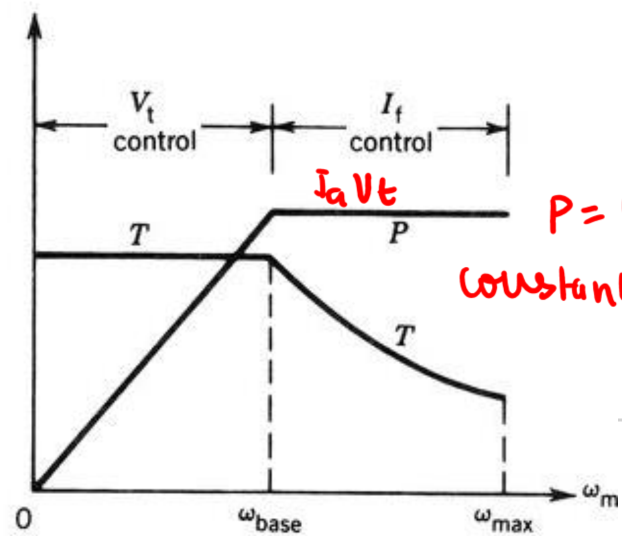
$R_{fc,max}$; $I_f \to min$,

$\phi \to min$, $\omega \to max$

DC Motor – Speed Control

rated Voltage of the motor $\uparrow \omega = \frac{V_a}{K_a \Phi} - \frac{R_a}{(K_a \Phi)^2} T$ $\downarrow$

- Separately excited DC motor: Speed control
  - Field control (ϕ): Field weakening
  - The speed control from zero to base speed is obtained by armature voltage control. The speed control beyond the base speed is obtained by decreasing the field current, called field weakening. At the base speed $V_t = V_{trated}$, the armature current is not to exceed its rated value, speed control beyond the base speed is restricted to constant power, known as constant power operation. The torque then decreases in the field weakening region



$P = \omega T$

constant = $\uparrow$ $\downarrow$

Separately excited DC Motor Speed Control - Example

A separately excited motor has the following nameplate data: 230 V, 40A, 1000rpm, Ra = 0.5 Ω .

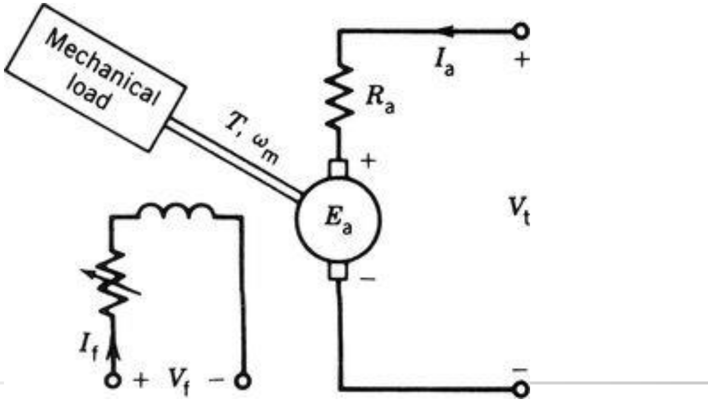
Calculate:

1. The value of $K_a\varphi$ for the operation of this motor under rated conditions
2. The resistance that would be required in series with the armature circuit of this machine to provide a torque of 62.5 Nm at a speed of 500 rpm. R_a control
3. The armature voltage that would be required to provide a torque of 62.5 Nm at 500 rpm, if we did not want to add any series resistance.
4. The efficiency of the process for the cases (2) and (3) and comment on the impact of each method on the efficiency of the machine

① @ rated conditions => $E_a = K_a\phi\,\omega$

$$E_a = V_t - I_a R_a = 230 - [40 \times 0.5] = 210^V$$

$$K_a\phi = \frac{210}{(1000/9.55)} = 2 \ \text{V/(rad/sec)}$$

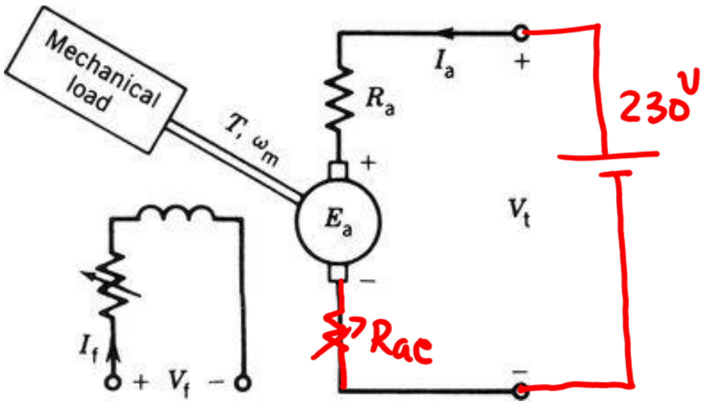


Separately excited DC Motor Speed Control - Example

② R_a control

$N_1 = 500^{rpm}$ $T_1 = 62.5^{N\text{-}m}$

$R_{armature} = R_a + R_{ae}$ find



$E_{a1} = V_t - I_{a1} R_{a1}$

$E_{a1} = K_a \Phi \omega_1 = (2)\left(\frac{500}{9.55}\right) = 105^V$

$T_1 = K_a \Phi I_{a1} \Rightarrow 62.5 = 2 I_{a1} \Rightarrow I_{a1} = 31.25^A$

$105 = 230 - [31.25 R_{a1}] \Rightarrow R_{a1} = 4^{\Omega} \Rightarrow R_{ae} = 4 - 0.5 = 3.5^{\Omega}$

④ $\eta = \frac{P_{mech}}{P_{Elect}} \times 100\% = \frac{\omega T}{V_t \times I_{a1}} \times 100\% = \frac{\left(\frac{500}{9.55}\right)(62.5)}{230 \times 31.25} \times 100\%$

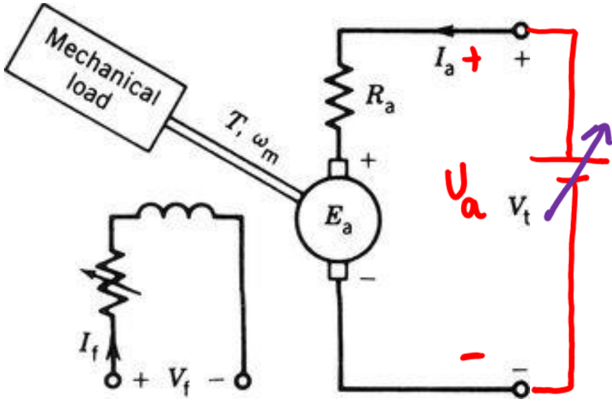
$= 45.5\%$

Separately excited DC Motor Speed Control - Example

3 armature voltage control $V_a = ?$

$T_2 = 62.5\,Nm$

$N_2 = 500\,rpm$



$$\omega = \frac{V_a}{(K_a\Phi)} - \frac{R_a}{(K_a\Phi)^2}T$$

$$\frac{500}{9.55} = \frac{V_{a2}}{(2)} - \frac{0.5}{(2^2)}\,62.5$$

$$T_2 = K_a\Phi I_{a2}$$

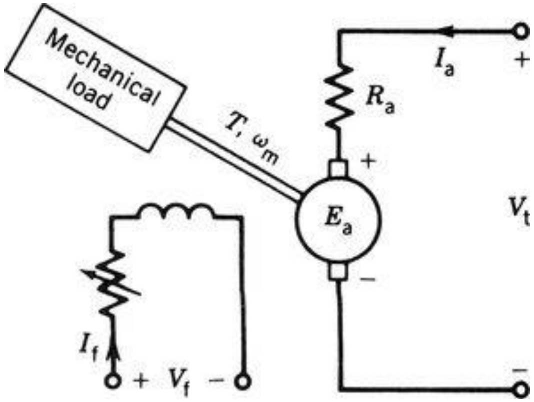
$$I_{a2} = \frac{62.5}{(2)} = 31.25\,A$$

$$V_{a2} = 120.34\,V$$

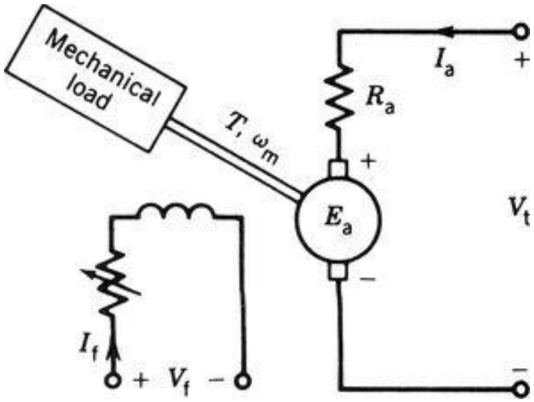
4 $$\eta = \frac{P_{mech}}{P_{Elect}} \times 100\% = \frac{\omega T}{V_{a2} I_{a2}} \times 100\% = \frac{\left(\frac{500}{9.55}\right) \times 62.5}{120.34 \times 31.25} \times 100\%$$

$$= 87\%$$

Separately excited DC Motor Speed Control - Example

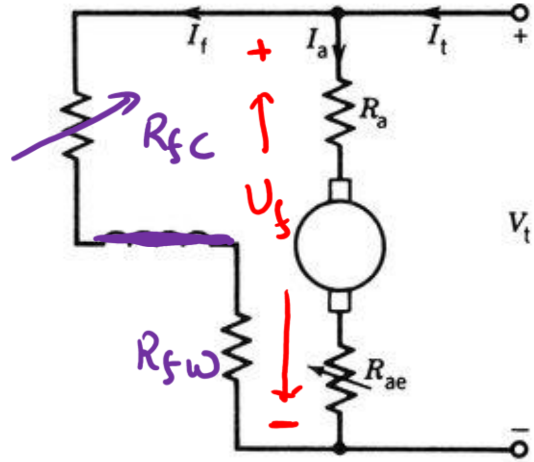


Separately excited DC Motor Speed Control - Example



DC Motor – Speed Control

- Shunt DC motor



$$\omega_m = \frac{V_t}{K_a\varphi} - \frac{R_a}{(K_a\varphi)^2}T$$

- Changing V_t changes both V_t and φ
- Speed control is through R_{fc} and R_a

$V_f = V_t \Rightarrow I_f = \frac{V_f}{R_f} \Rightarrow$ change $I_f \rightarrow$ change $\varphi \rightarrow$ change ω

Shunt DC Motor Speed Control - Example

V_t

A shunt DC motor operates off a 120 V supply. The machine has the field and the armature resistance of R_f = 80 Ω and R_a = 0.1 Ω, respectively. It draws 51.5 A from the supply when it spins at 1500 rpm. Ignoring the rotational losses and assuming linearity of the magnetic structure, calculate:

I_t

1. The load torque and efficiency
2. The load torque and efficiency, if an increase in the field resistance to 100 Ω results in an increase in the speed by 300 rpm . $N_2 = 1800$ rpm
3. The speed under the same load torque and parameters as those in Part (1) except a terminal voltage of 150 V. N_3 36.6 N-m

= 51.5 A

① $T_{load} = T_{out} = T_{shaft}$

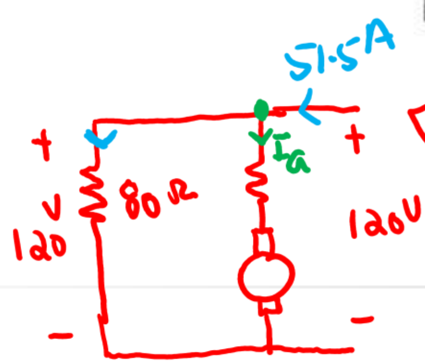
$P_{dev} = P_{shaft} + \cancel{P_{rotational\ loss}} \rightarrow 0$

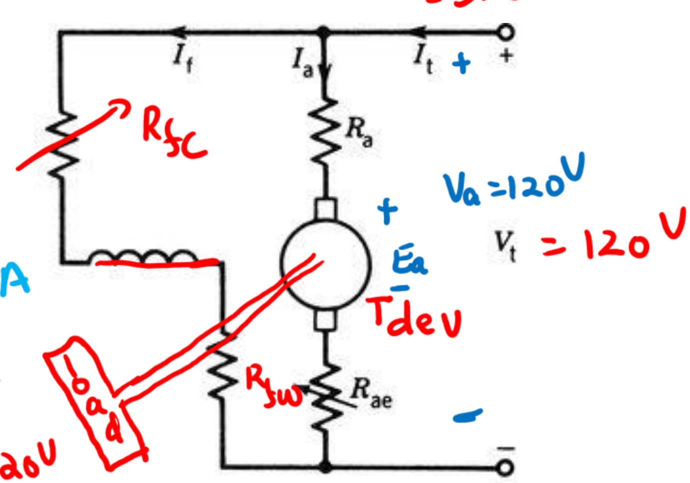
$T_{dev} = T_{load} = K_a \Phi I_a$

$I_a = 51.5 - \left[\frac{120}{80}\right] = 50$ A

$E_a = K_a \Phi \omega$

$E_a = \overset{V_t}{120} - [\overset{I_a R_a}{50 \times 0.1}] = 115$ V





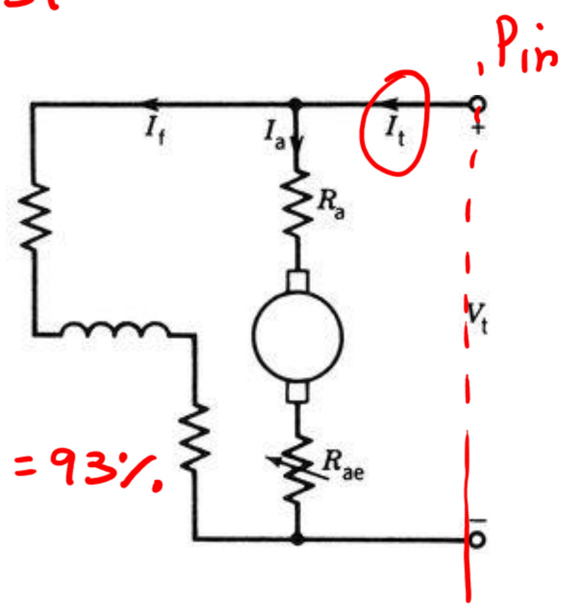
Shunt DC Motor Speed Control - Example

$$E_a = K_a \phi \omega \Rightarrow K_a \phi = \frac{115}{(1500/9.55)} = 0.731$$

$$T_{load} = 0.731 \times 50 = 36.6 \text{ N-m}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\omega T}{V_t I_t} \times 100\%$$

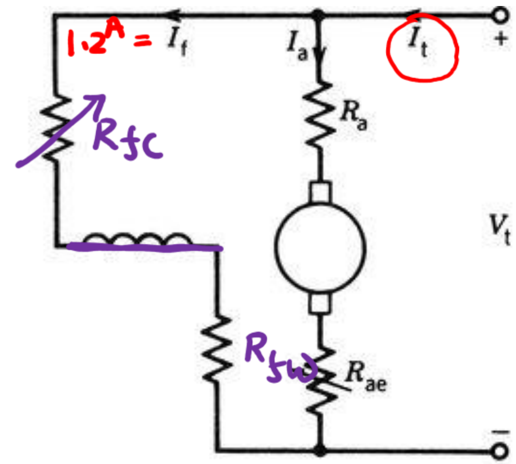
$$= \frac{\left(\frac{1500}{9.55}\right) \times 36.6}{120 \times 51.5} \times 100\% = 93\%$$



Shunt DC Motor Speed Control - Example

(2) $N_2 = 1800\,\text{rpm}$ $\quad R_f = 100\,\Omega$ $\quad T_L$

$$\omega = \frac{V_t}{K_a\phi} - \frac{R_a}{(K_a\phi)^2}\,T$$



$$\frac{1800}{9.55} = \frac{120}{K_a\phi_1} - \frac{0.1}{(K_a\phi_1)^2} \times T_1$$

$$E_a = K_a\phi\,\omega$$

$$K_a\phi \propto I_f \propto \frac{1}{R_f} \Rightarrow \frac{80}{100} = \frac{K_a\phi_1}{0.731}$$

$$K_a\phi_1 = 0.585$$

$$\frac{1800}{9.55} = \frac{120}{0.585} - \frac{0.1}{(0.585)^2}\,T_1 \Rightarrow T_1 = 56.97\ \text{N-m}$$

$T_1 = K_a\phi_1 I_{a1}$

$56.97 = 0.585\, I_{a1}$

$I_{a1} = 97.4\,\text{A}$

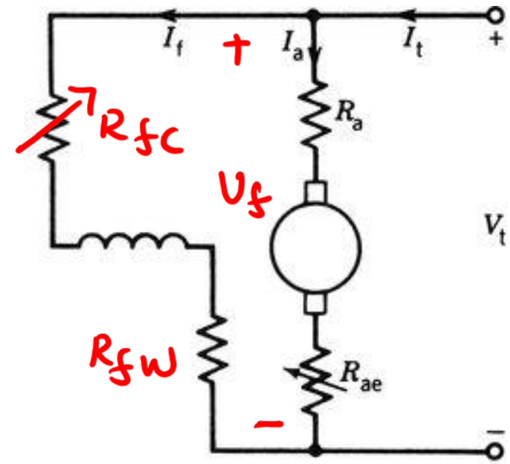
$I_t = 97.4 + 1.2 = 98.6\,\text{A}$

$$\eta = \frac{\omega_2 T_1}{V_t I_t} \times 100\% = \frac{\frac{1800}{9.55} \times 56.97}{120 \times 98.6} \times 100\%$$

$$\eta = 90.7\%$$

Shunt DC Motor Speed Control - Example

3 $N_3 = ?$ $T_L = 36.6\ \text{N-m}$ $V_t = 150\ \text{V}$



$V_t = V_f$

$V_f \uparrow \rightarrow I_f \uparrow \rightarrow \Phi \uparrow \rightarrow K_a\Phi \uparrow$

$K_a\Phi_2$

$$\omega_3 = \frac{V_t}{K_a\Phi_3} - \frac{R_a}{(K_a\Phi_3)^2} T$$

$$\omega_3 = \frac{150}{K_a\Phi_3} - \frac{0.1}{(K_a\Phi_3)^2} \times 36.6$$

$$\omega_3 = \frac{150}{0.914} - \frac{0.1}{(0.914)^2} \times 36.6$$

$\omega_3 = 159.7\ \text{rad/sec}$

$N_3 = 159.7 \times 9.55 = 1525\ \text{rpm.}$

$K_a\Phi \propto I_f \propto V_f$

$$\frac{0.731}{K_a\Phi_3} = \frac{120}{150}$$

$K_a\Phi_3 = 0.914\ \text{A}$

DC Motor – Speed Control

$\omega_m = \frac{V_t}{K_a\Phi} - \frac{R_a}{(K_a\Phi)^2} T$

- Series DC motor

$K_a\Phi \Rightarrow \Phi = f(I_f)$

$\Phi = f(I_a)$

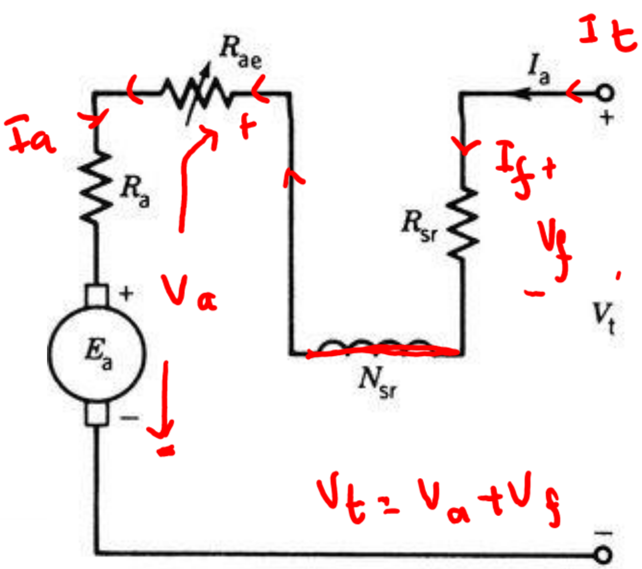
$$\omega_m = \frac{V_t}{\sqrt{K_{sr}}\sqrt{T}} - \frac{R_a + R_{sr} + R_{ae}}{K_{sr}}$$

- For a particular value of R_{ae}, the speed is inversely proportional to the square root of the torque

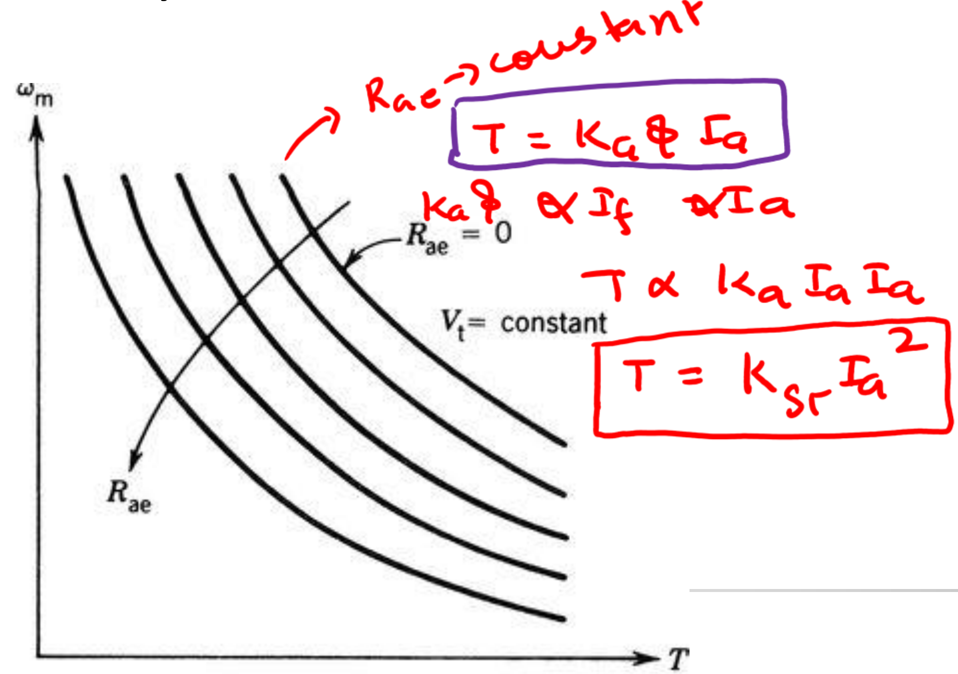
$\omega_m^2 \propto \frac{1}{T}$

- High torque at low speed (series motors are used where high starting torque is required as in subway cars, automobile starters, blenders

$I_t = I_f = I_a$



(a)



(b)

Series DC Motor Speed Control - Example

A 220 V, 7 HP series DC motor is mechanically coupled to a fan and draws 25 A and runs at 300 rpm (N) when connected to a 220 V supply with no external resistance connected to the armature circuit (i.e. $R_{ae} = 0$). The torque required by the fan is proportional to the square of the speed. Ignoring the rotational losses and assuming linearity of the magnetic structure. If $R_a = 0.6\ \Omega$ and $R_{sr} = 0.4\ \Omega$, calculate:

1. The power delivered to the fan and the torque developed by the machine
2. The value of the resistance (R_{ae}) to be inserted to the armature circuit to reduce the speed to 200 rpm. Also calculate the power delivered to the fan.

$T = K_{sr}(i_a^2)$

$155.2 = K_{sr}(25^2) \Rightarrow K_{sr} = 0.248$

$T_L \propto N^2$

(1) $P_{out} = P_{shaft} = P_{dev} - \cancel{P_{rotational}} = 0$

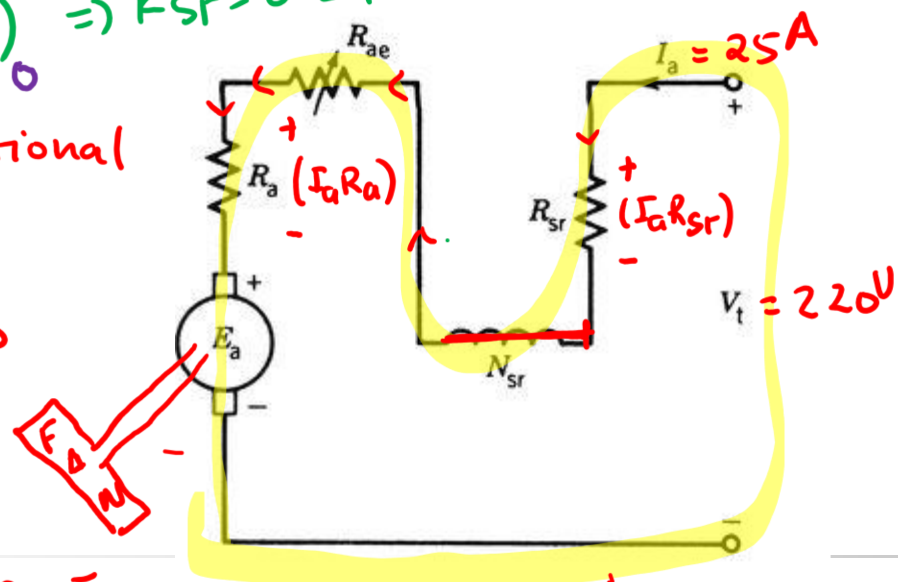
$P_{dev} = E_a I_a$

KVL $\Rightarrow -E_a - (I_a R_a) - (I_a R_{sr}) + V_t = 0$

$E_a = 220 - [25 \times 0.6] - [25 \times 0.4]$
$= 195\ V$

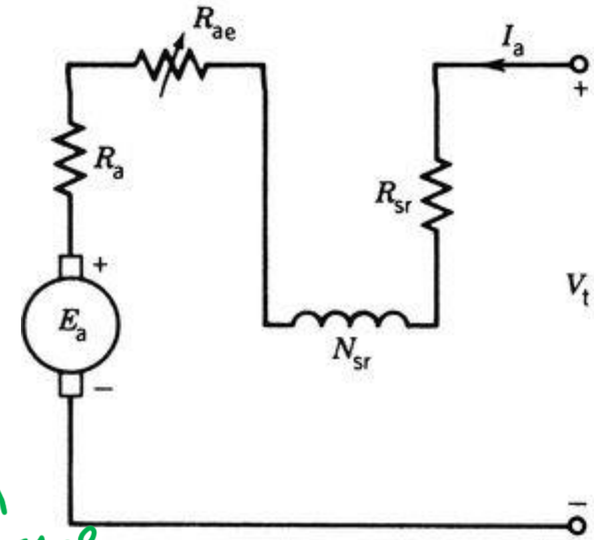
$P_{dev} = 195 \times 25 = 4875\ W$

$P_{mech} = \omega T \Rightarrow T_{dev} = \dfrac{4875}{(300/9.55)} = 155.2\ N\text{-}m$



Series DC Motor Speed Control - Example

$N_1 = 200 \text{ rpm}$ $\qquad$ $R_{ae} = ?$



$$\omega = \frac{V_t}{\sqrt{K_{sr}}\sqrt{T}} - \frac{R_a + R_{sr} + R_{ae}}{K_{sr}}$$

$$T = K_{sr}\, i_a^2$$

$$\cancel{(\omega)^2 \propto \left(\frac{1}{\sqrt{T}}\right)^2 \Rightarrow \omega^2 \propto \frac{1}{T}}$$

$$N^2 \propto T$$

Given for the fan

$$(300)^2 \propto \cancel{\frac{1}{155.2}}\ 155.2$$

$$(200)^2 \rightarrow T_1 = 155.2 \times \frac{(200)^2}{(300)^2} = 68.98 \text{ N-m}$$

$$\cancel{68.98} = K_{sr}\,\cancel{(25)^2}$$

$$\cancel{K_{sr}} = \cancel{\frac{68.98}{(25)^2}} = 0.248$$

$$\frac{200}{9.55} = \frac{220}{\sqrt{0.248}\sqrt{68.98}} - \frac{1 + R_{ae}}{0.248}$$

$$R_{ae} = 7\,\Omega$$

$$P_{dev} = E_a I_a = \omega T = \frac{200}{9.55} \times 68.98 = 1444 \text{ W}$$

Series DC Motor Speed Control - Example