HUMBER ENGINEERING

ENGI 1000 - Physics 1

WEEK 12 - MODULE 9

HUMBER

WE ARE HUMBER

Module 9

Oscillatory Motion

- Particle in Simple Harmonic Motion
  - Mass-Spring System
  - Simple Pendulum System
- Energy of the Simple Harmonic Motion
- Damped Oscillations
- Forced Oscillations

Introduction

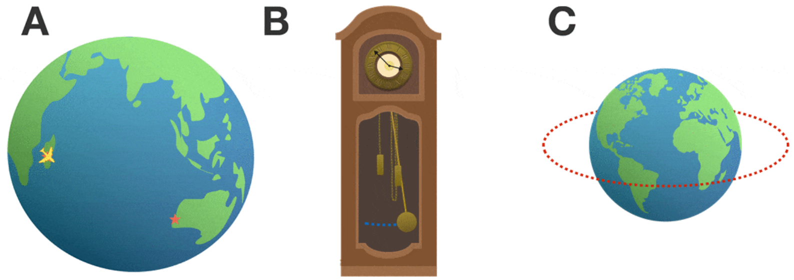
- In the previous lectures we studied the translational or linear motion, where the objects moves in straight lines at either constant velocity or constant acceleration.
- We also studied the rotational motion of the objects, where the objects moves on a circular path with constant acceleration.
- In this lecture we will study a new type of motion, which is called **oscillatory motion**, where the objects oscillate or vibrate back and forth or left and right.
- We will introduce new terms such as:
  - Period and Periodic Motion
  - Oscillation and Simple Harmonic Motion
  - Energy of the Simple Harmonic Motion

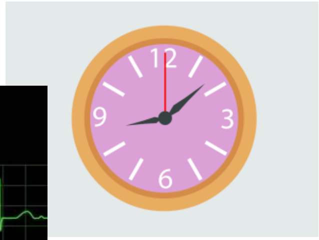
Periodic Motion

- **Periodic motion** is motion of an object that <u>regularly</u> returns to a given position after a fixed time interval, which is called **period** of the motion.

Can you bring examples of periodic motion in everyday life?

- The Earth returns to the same position in its orbit around the Sun each year.
- The electrocardiogram (ECG) signal is nearly a periodic signal, which used for the diagnosis of cardiac abnormalities.

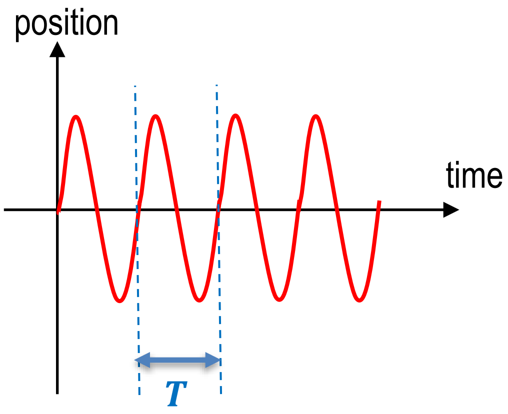


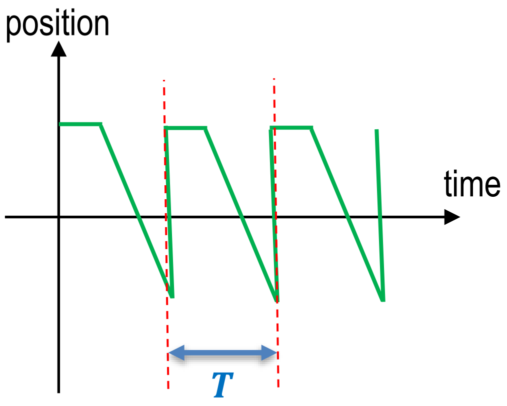


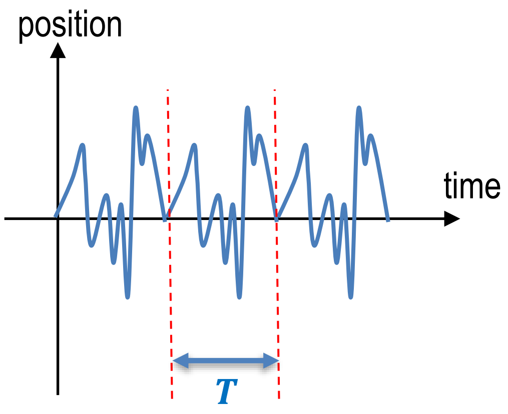
SAULT COLLEGE HUMBER Faculty of Applied Sciences & Technology

Periodic Motion

- Examples of the position-time graph for periodic motion.







Period T is the time of one complete cycle.

Motion of an object in Mass-Spring System

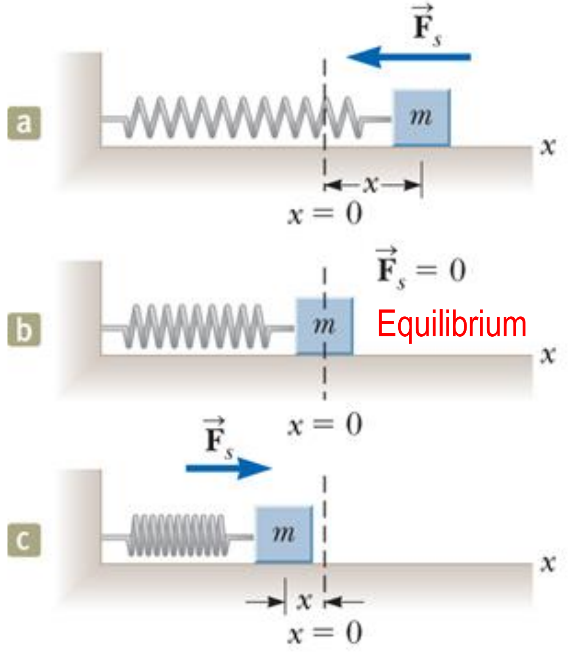
- Consider block of mass *m* attached to end of spring, with block free to move on frictionless, horizontal surface
  - When spring neither stretched nor compressed: block at rest at position called **equilibrium position** of the system $\rightarrow x = 0$
  - If the spring is disturbed from equilibrium position: system oscillates back and forth.
  - Recall the **Hook's law**: If block displaced to position x, the spring exerts a restoring force F_s on the block proportional to position

$$\boxed{F_s = -kx}$$

Restoring Force

Spring Constant

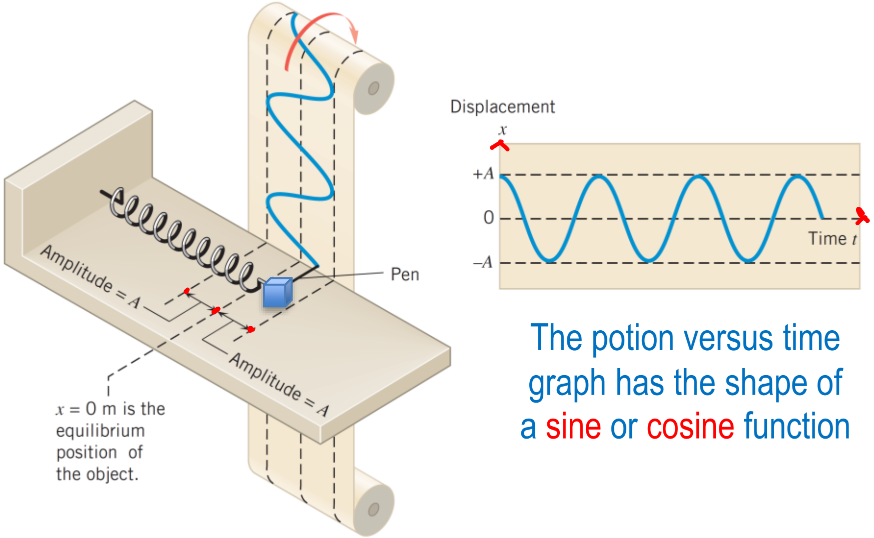
Displacement



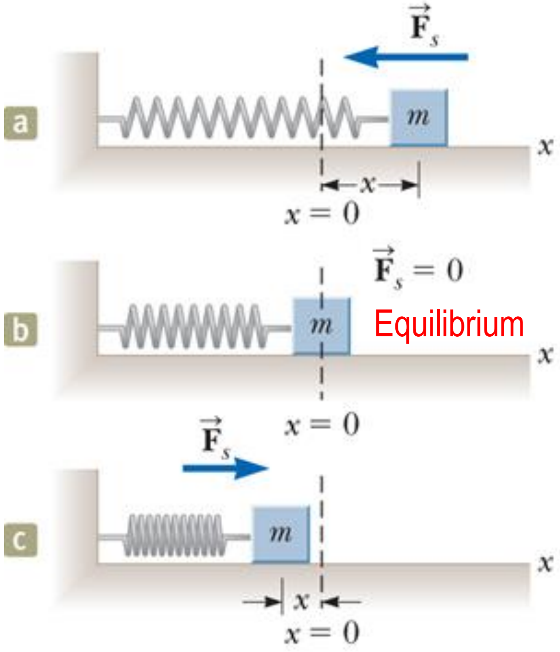
- Restoring force is always directed toward the equilibrium position and therefore opposite the displacement of the block.

Motion of an object in Mass-Spring System

- Consider block of mass *m* attached to end of spring, with block free to move on frictionless, horizontal surface
  - We can record the position of the oscillation at any time, by attaching a pen to the block and moving a strip of paper past it at a steady state rate.

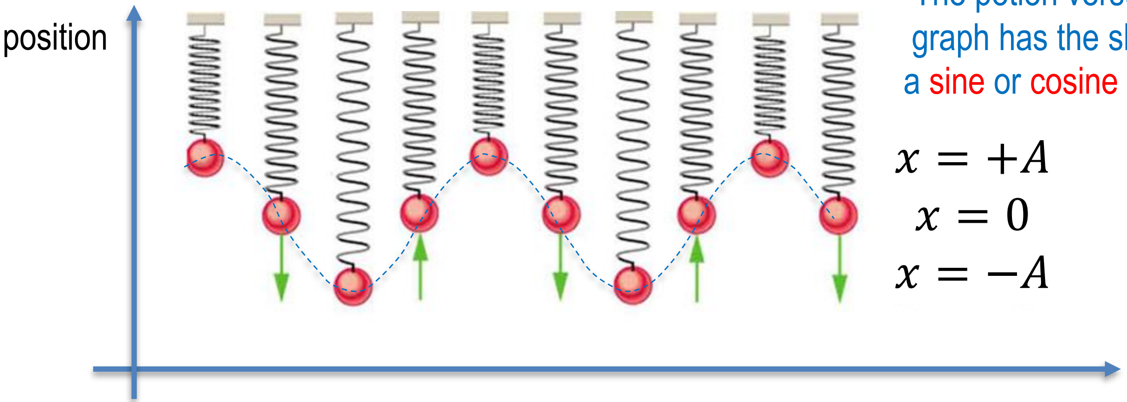


The potion versus time graph has the shape of a sine or cosine function



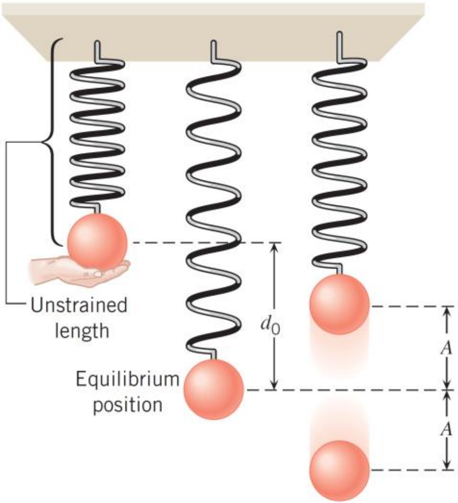
Motion of an object in Mass-Spring System

- Consider ball of mass *m* attached to end of spring in the vertical direction.
  - The restoring force also leads to sinusoidal shape oscillation when the object is attached to a vertical spring.
  - In this case, the weight of the object causes the spring to stretch, and the motion occurs with respect to the equilibrium position of the object on the stretched spring.



The potion versus time graph has the shape of a sine or cosine function

$$x = +A$$
$$x = 0$$
$$x = -A$$



SAULT COLLEGE HUMBER Faculty of Applied Sciences & Technology

Review of a Sinusoidal Function

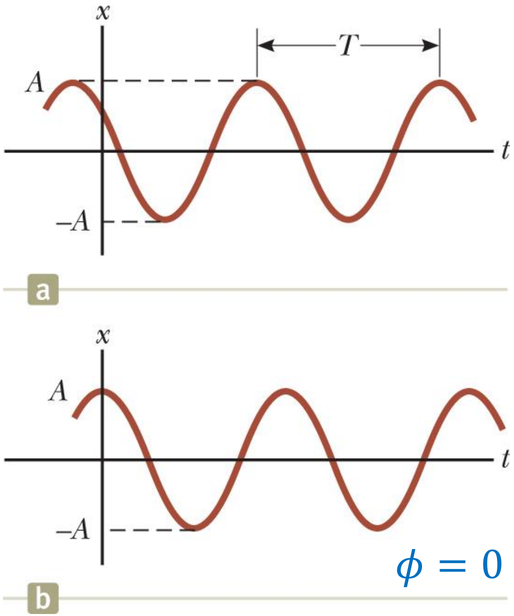
- The general form of a sinusoidal function can be represented as below:

$$\boxed{x(t) = A\cos(\omega t + \phi)}$$

Amplitude | Angular frequency | Phase constant

- **Amplitude, A**: maximum value of the function.
- **Angular frequency, ω (rad/s):** measure how rapidly oscillations occurring
- **Phase constant ϕ (rad):** initial phase angle
- **Period T (s):** time of one complete oscillation
- **Frequency f (Hz):** number of oscillations per unit time interval

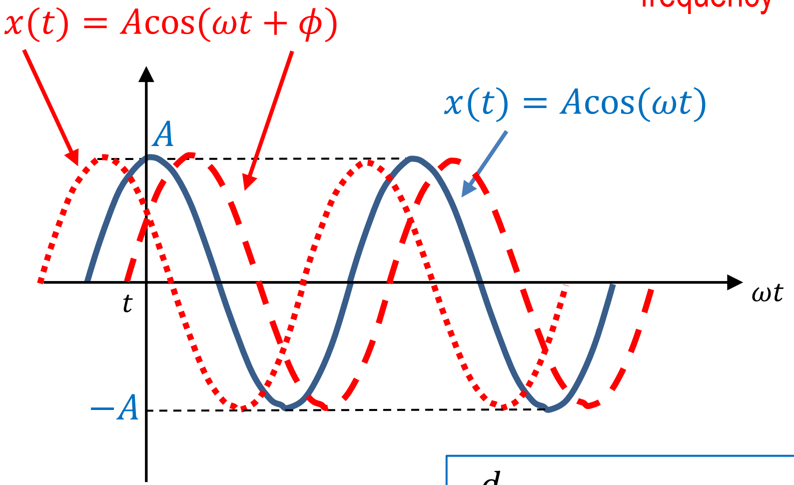
$$\boxed{f = \frac{1}{T}, \qquad \omega = 2\pi f = \frac{2\pi}{T}, \qquad T = \frac{2\pi}{\omega}}$$

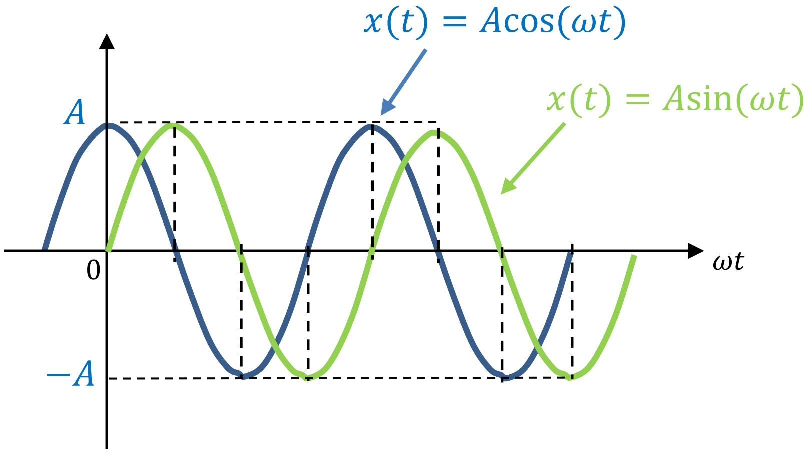


Review of a Sinusoidal Function

- The general form of a sinusoidal function can be represented as below:

$$\boxed{x(t) = \underbrace{A}_{\text{Amplitude}} \cos(\underbrace{\omega}_{\text{Angular frequency}} t + \underbrace{\phi}_{\text{Phase constant}})}$$





$$\boxed{\frac{d}{dt} A\cos(\omega t + \phi) = -A\omega \sin(\omega t + \phi)}$$

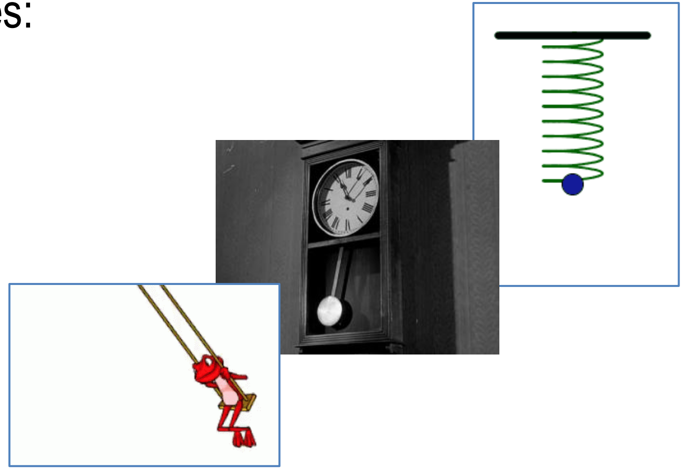
Quick Quiz 1

- A block on the end of a spring is pulled to position $x = A$ and released from rest. In one full cycle of its motion, through what total distance does it travel?

  a) $A/2$
  b) A
  c) $2A$
  d) $4A$

Simple Harmonic Motion (SHM)

- A special kind of periodic motion occurs in mechanical systems when the force acting on an object is proportional to the position of the object relative to some equilibrium position.
- If this force is always directed toward the equilibrium position, the motion is called **Simple Harmonic Motion (SHM),** which has the following properties:
  - SHM is a periodic motion.
  - The total energy of the particle exhibiting SHM is conserved.
  - SHM can be represented by a single harmonic function of sine or cosine.



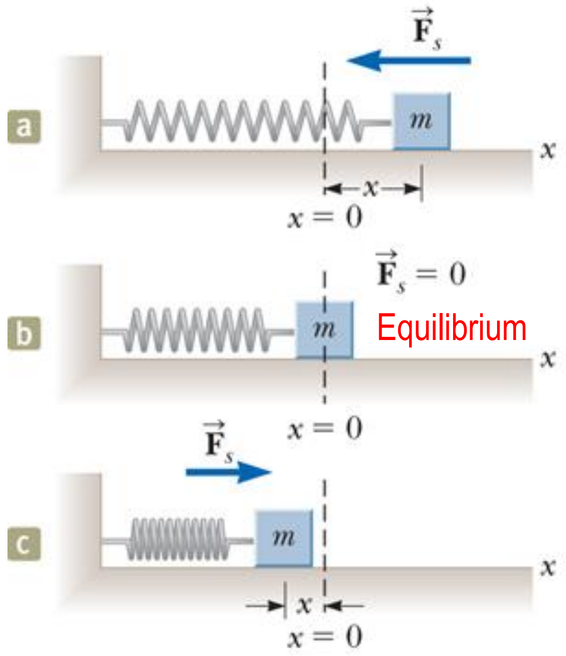
Mathematical Description of Simple Harmonic Motion

- Consider block of mass *m* attached to end of spring, with block free to move on frictionless, horizontal surface
  - When block displaced from equilibrium point and released, it can be considered as a particle under net force and consequently undergoes an acceleration.
  - Applying particle under net force model to motion of the block:

$$\sum F_x = ma_x \quad \rightarrow \quad -kx = ma_x$$

$$\boxed{a_x = -\frac{k}{m}x}$$

  - The acceleration of block is proportional to the position, but in the opposite direction of the displacement.



Mathematical Description of Simple Harmonic Motion

- Recall the relationship of acceleration and position,

$$a_x = -\frac{k}{m}x \quad \Rightarrow \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

- Find a mathematical solution for $x(t)$ that satisfies this second-order differential equation.
- We seek function whose second derivative same as the original function with negative sign and multiplied by k/m .
- Trigonometric functions sine and cosine exhibit this behavior.
- Denote the ration k/m with symbol ω^2 to simplify the equation. Then the cosine function will be the solution:

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x} \quad \Rightarrow \quad \boxed{x(t) = A\cos(\omega t + \phi)}$$

Position of the particle under SHM

Mathematical Description of Simple Harmonic Motion

- We can show that the obtained solution satisfies the second-order differential equation,

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x} \Rightarrow \boxed{x(t) = A\cos(\omega t + \phi)}$$

Position of the particle under SHM

- Find the first and second derivatives of the $x(t)$,

$$\frac{dx}{dt} = A\frac{d}{dt}\cos(\omega t + \phi) = -\omega A\sin(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -\omega A\frac{d}{dt}\sin(\omega t + \phi) = -\omega^2 A\cos(\omega t + \phi)$$

Position of a Particle under Simple Harmonic Motion

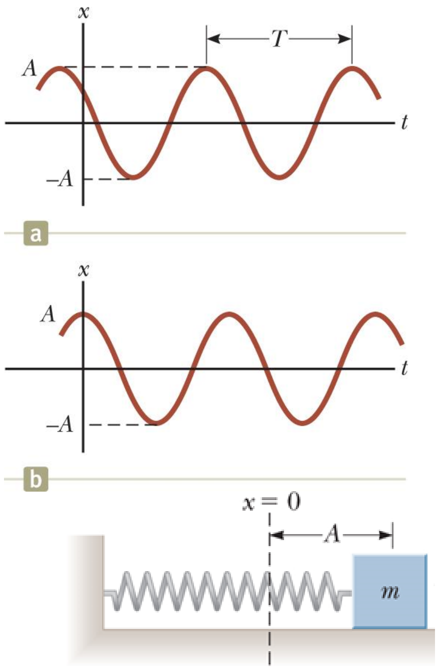
- The equation describes the **position $x(t)$** of a particle under simple harmonic motion is formulized as below,

$$\boxed{x(t) = A\cos(\omega t + \phi)}$$

Amplitude of the motion (m)

Angular frequency (rad/s)

Phase of the motion (rad)



- **Angular frequency, ω (rad/s):** measure how rapidly oscillations occurring. It is also called natural frequency.

Angular frequency

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Period

- **Amplitude, A (m)**: maximum value of the position of the particle in positive or negative x direction.
- **Phase of the motion ϕ (rad):** initial phase angle

Velocity and Acceleration of a Particle under SHM

- We can also obtain the velocity and acceleration of a particle under simple harmonic motion,

$$\boxed{x(t) = A\cos(\omega t + \phi)}$$

- Velocity of a particle under simple harmonic motion,

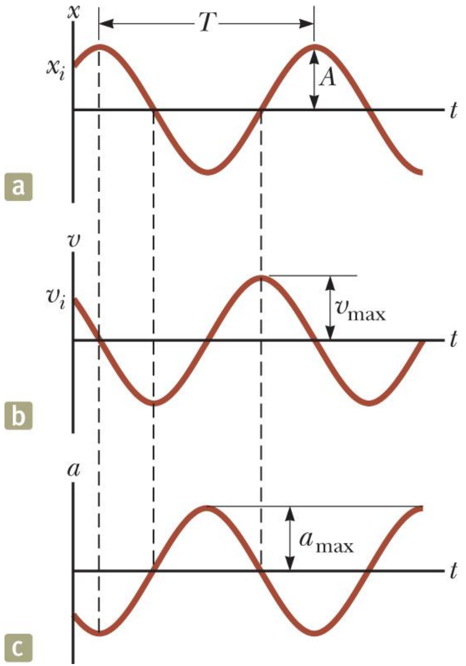
$$v = \frac{dx}{dt} = A\frac{d}{dt}\cos(\omega t + \phi) \rightarrow \boxed{v(t) = -\omega A\sin(\omega t + \phi)}$$

- Acceleration of a particle under simple harmonic motion,

$$a = \frac{d^2x}{dt^2} = -\omega A\frac{d}{dt}\sin(\omega t + \phi) \rightarrow \boxed{a(t) = -\omega^2 A\cos(\omega t + \phi)}$$

- The ***maximum*** values of the magnitudes of the velocity and acceleration are

$$\boxed{v_{max} = \omega A = \sqrt{\frac{k}{m}}A} \qquad \boxed{a_{max} = \omega^2 A = \frac{k}{m}A}$$



Initial Conditions in SHM Equations

- Consider the **position $x(t)$** and **velocity $v(t)$** of a particle under simple harmonic motion,

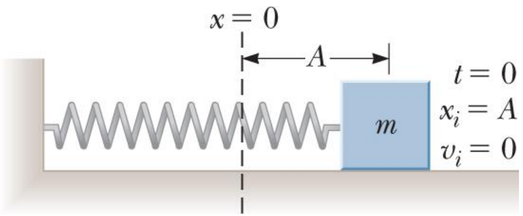
$$\boxed{x(t) = A\cos(\omega t + \phi)}$$

$$\boxed{v(t) = -\omega A\sin(\omega t + \phi)}$$

- The constants A and ϕ are evaluated from the initial conditions, that is the state of the oscillator at $t = 0$.
- Suppose a block is set into motion by pulling it from equilibrium by a distance A and realizing it from rest at $t = 0$. The solutions for $x(t)$ and $v(t)$ must obey the initial conditions at $t = 0$,

$$x_i = A \quad \rightarrow \quad x(0) = A\cos\phi = A \;\rightarrow\; \cos\phi = 1 \quad \rightarrow \quad \phi = 0$$

$$v_i = 0 \quad \rightarrow \quad v(0) = -\omega A\sin\phi = 0 \;\rightarrow\; \sin\phi = 0 \quad \rightarrow \quad \phi = 0$$



Initial Conditions in SHM Equations

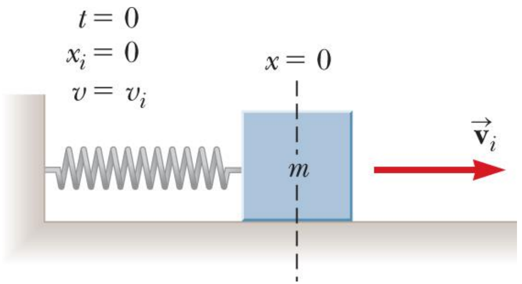
- Consider the **position $x(t)$** and **velocity $v(t)$** of a particle under simple harmonic motion,

$$\boxed{x(t) = A\cos(\omega t + \phi)} \qquad \boxed{v(t) = -\omega A\sin(\omega t + \phi)}$$

- The constants A and ϕ are evaluated from the initial conditions, that is the state of the oscillator at $t = 0$.
- Suppose system oscillating and we defined $t = 0$ as instant block passes through the equilibrium point. The solutions for $x(t)$ and $v(t)$ must obey the initial conditions at $t = 0$,

$$x_i = 0 \quad \rightarrow \quad x(0) = A\cos\phi = 0 \ \rightarrow \ \cos\phi = 0 \quad \rightarrow \quad \phi = \pm\frac{\pi}{2}$$

$$v = v_i \quad \rightarrow \quad v(0) = -\omega A\sin\phi = v_i \ \rightarrow A = -\frac{v_i}{\omega\sin\phi}$$



Since the initial velocity is positive, then amplitude must be positive, we must have

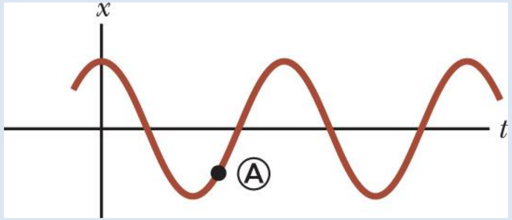
$$\phi = -\frac{\pi}{2} \ , \qquad A = \frac{v_i}{\omega}$$

Quick Quiz 2

- Consider a graphical representation of simple harmonic motion as described mathematically as

$$x(t) = A\cos(\omega t + \phi)$$

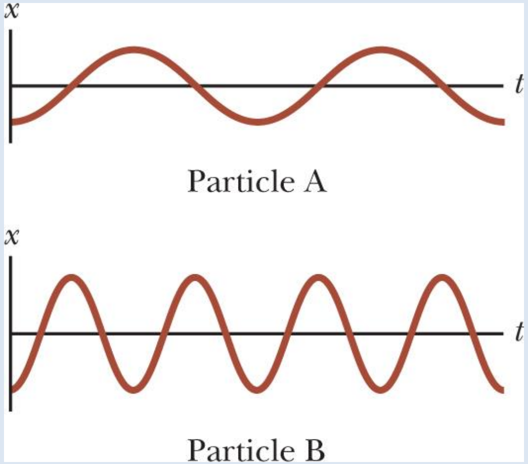
- When the particle is at point A on the graph, what can you say about its position and velocity?



  a) The position and velocity are both positive.
  b) The position and velocity are both negative.
  c) The position is positive, and the velocity is zero.
  d) The position is negative, and the velocity is zero.
  e) The position is positive, and the velocity is negative.
  f) The position is negative, and the velocity is positive.

Quick Quiz 3

- The figure shows two curves representing particles undergoing simple harmonic motion. The correct description of these two motions is that the simple harmonic motion of particle B is



a) of larger angular frequency and larger amplitude than that of particle A
b) of larger angular frequency and smaller amplitude than that of particle A
c) of smaller angular frequency and larger amplitude than that of particle A
d) of smaller angular frequency and smaller amplitude than that of particle A.

Quick Quiz 4

- An object of mass *m* is hung from a spring and set into oscillation. The period of the oscillation is measured and recorded as *T*. The object of mass *m* is removed and replaced with an object of mass 2*m*. When this object is set into oscillation, what is the period of the motion?

  $a)$ $2T$

  $b)$ $\sqrt{2}T$

  $c)$ T

  d) $\mathrm{T}/\sqrt{2}$

  $e)$ $T/2$

Simple Harmonic Motion: Mass-Spring System

Example 1 (A Block-Spring System): A 200-g block connected to a light spring for which the force constant is 5.00 N/m is free to oscillate on a frictionless, horizontal surface. The block is displaced 5.00 cm from equilibrium and released from rest as in the figure.

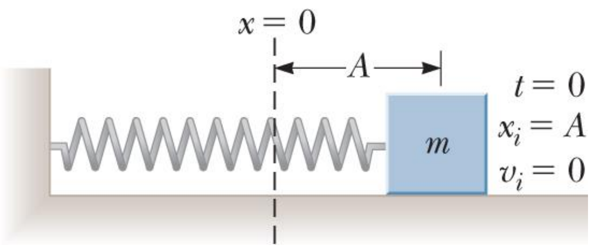
(a) Find the period of its motion.

Find the angular frequency of the block-spring system:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00\ \text{N/m}}{200 \times 10^{-3}\ \text{kg}}} = 5.00\ \text{rad/s}$$

Find the period of the system:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00\ \text{rad/s}} = \boxed{1.26\ \text{s}}$$



Simple Harmonic Motion: Mass-Spring System

Example 1 (A Block-Spring System): A 200-g block connected to a light spring for which the force constant is 5.00 N/m is free to oscillate on a frictionless, horizontal surface. The block is displaced 5.00 cm from equilibrium and released from rest as in the figure.

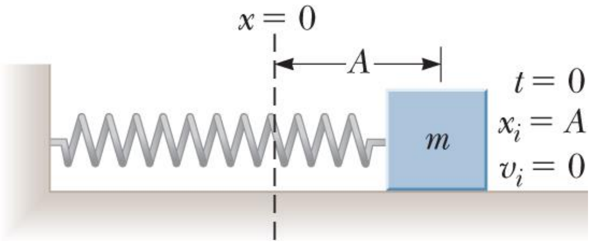
(b) Determine the maximum speed and the maximum acceleration of the block.

The maximum speed of the block:

$$v_{\text{max}} = \omega A = (5.00\ \text{rad/s})(5.00 \times 10^{-2}\ \text{m}) = \boxed{0.250\ \text{m/s}}$$

The maximum acceleration of the block:

$$a_{\text{max}} = \omega^2 A = (5.00\ \text{rad/s})^2(5.00 \times 10^{-2}\ \text{m}) = \boxed{1.25\ \text{m/s}^2}$$



Simple Harmonic Motion: Mass-Spring System

Example 1 (A Block-Spring System): A 200-g block connected to a light spring for which the force constant is 5.00 N/m is free to oscillate on a frictionless, horizontal surface. The block is displaced 5.00 cm from equilibrium and released from rest as in the figure.

(c) Express and draw the position, velocity and acceleration as functions of time in SI units.

$\boxed{x(t) = A\cos(\omega t + \phi)}$ $\boxed{v(t) = -\omega A\sin(\omega t + \phi)}$ $\boxed{a(t) = -\omega^2 A\cos(\omega t + \phi)}$

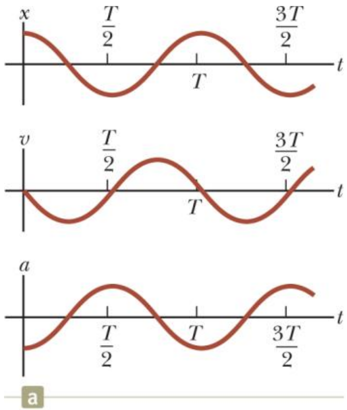
Find the phase constant ϕ from the initial condition that *x* = *A* at *t* = 0

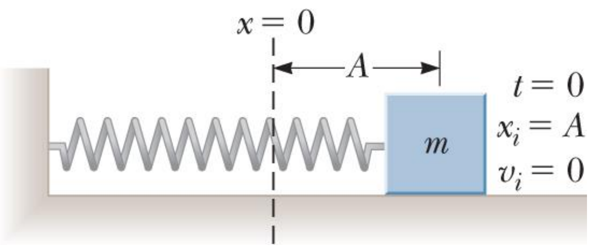
$$x(0) = A\cos\phi = A \quad \rightarrow \quad \cos\phi = 1 \quad \rightarrow \quad \phi = 0$$

$$x = A\cos(\omega t + \phi) = \boxed{0.05\cos 5.00t}$$

$$v = -\omega A\sin(\omega t + \phi) = \boxed{-0.25\sin 5.00t}$$

$$a = -\omega^2 A\cos(\omega t + \phi) = \boxed{-1.25\cos 5.00t}$$



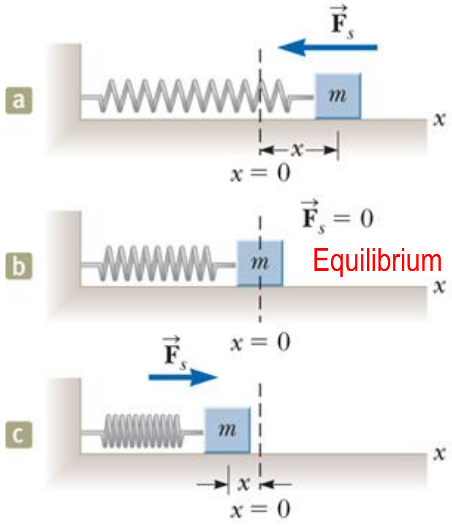


Energy of the Simple Harmonic Motion

- Mechanical energy of an isolated mass-spring system under simple harmonic motion can be determined as below
- The kinetic energy and elastic potential energy of spring are:

$$K = \frac{1}{2}mv^2 \quad \rightarrow \quad K = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi) = \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$$

$$U_s = \frac{1}{2}kx^2 \quad \rightarrow \quad U_s = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$



- The total mechanical energy:

$$E_m = K + U_s = \frac{1}{2}kA^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1} \quad \Rightarrow \quad \boxed{E_m = \frac{1}{2}kA^2}$$

Energy of the Simple Harmonic Motion

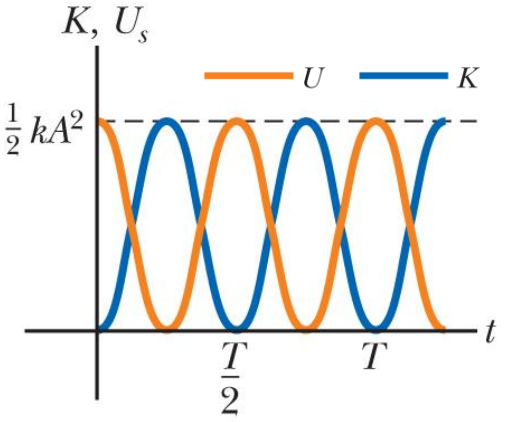
- Plots of the kinetic energy and the elastic potential energy versus time with $\phi = 0$ can be shown as:

$$K = \frac{1}{2} k A^2 \sin^2(\omega t)$$

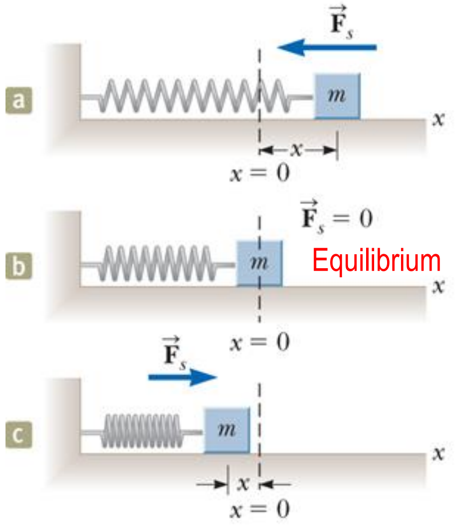
$$U_s = \frac{1}{2} k A^2 \cos^2(\omega t)$$

- At all times: The sum of kinetic and potential energies is

$$E_m = K + U_s = \frac{1}{2} k A^2$$



In isolated system, the energy continuously being transformed between potential energy stored in spring and kinetic energy of block

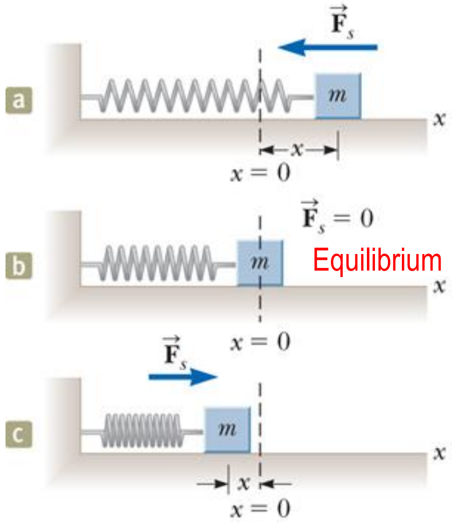


Energy of the Simple Harmonic Motion

- We can obtain the velocity of the block at an arbitrary position by expressing the total energy of the system at some arbitrary position x as

$$E_m = K + U_s = \frac{1}{2}kA^2 \quad \rightarrow \quad \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$\boxed{v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} \quad \rightarrow \quad v = \pm\omega\sqrt{A^2 - x^2}}$$

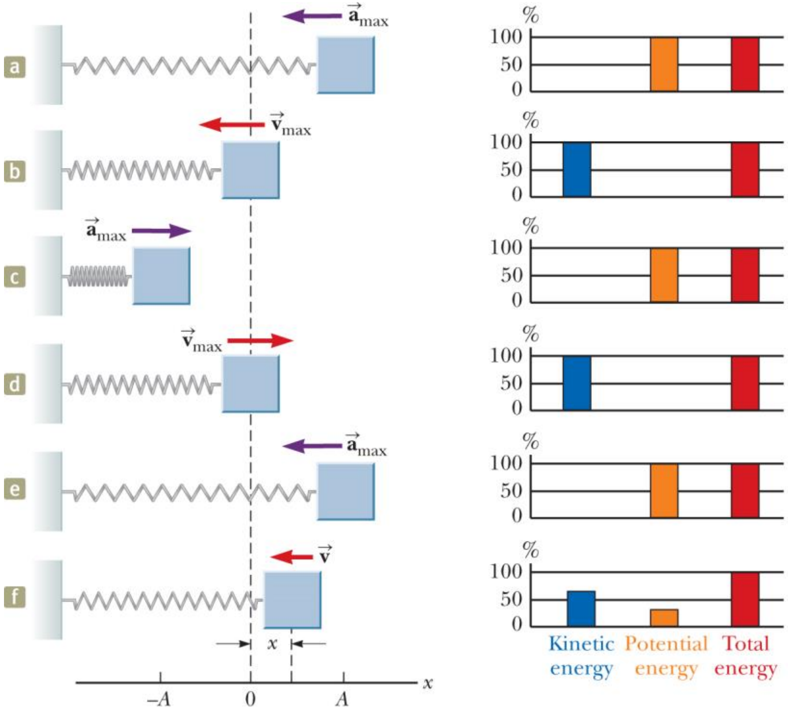


- The speed is maximum at the equilibrium point $x = 0$

- The speed is zero at the turning points $x = \pm A$

Energy of the Simple Harmonic Motion

- The following figure illustrates the position, velocity, acceleration, kinetic energy, and potential energy of the isolated mass–spring system for one full period of the motion.



t	x	v	a	K	U_s
0	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
$\frac{T}{4}$	0	$-\omega A$	0	$\frac{1}{2}kA^2$	0
$\frac{T}{2}$	$-A$	0	$\omega^2 A$	0	$\frac{1}{2}kA^2$
$\frac{3T}{4}$	0	ωA	0	$\frac{1}{2}kA^2$	0
T	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
t	x	v	$-\omega^2 x$	$\frac{1}{2}mv^2$	$\frac{1}{2}kx^2$

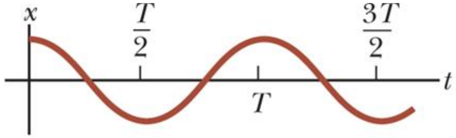
$$\boxed{v_{max} = \omega A}$$

$$\boxed{a_{max} = \omega^2 A}$$

$$\boxed{E_m = \frac{1}{2}kA^2}$$

$$\boxed{E_m = K + U_s}$$

$$\boxed{K = \frac{1}{2}mv^2} \qquad \boxed{U_s = \frac{1}{2}kx^2}$$



Energy of the Simple Harmonic Motion

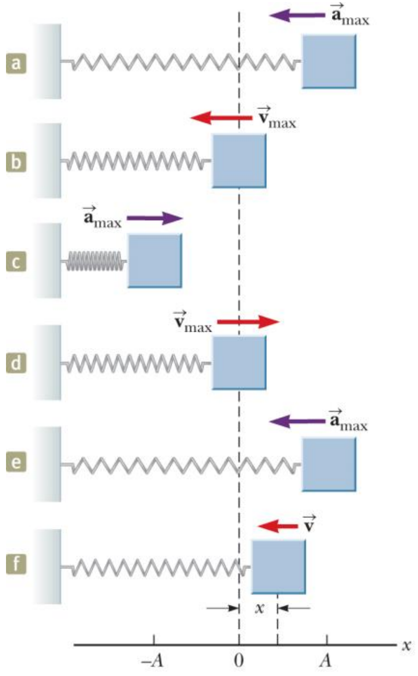
Example 2 (Oscillation on a Horizontal Surface): A 0.500-kg cart connected to a light spring for which the force constant is 20.0 N/m oscillates on a frictionless, horizontal air track. Use an energy approach to respond to the questions below.

(a) Calculate the maximum speed of the cart if the amplitude of the motion is 3.00 cm.

The cart has the maximum speed at the equilibrium position, where it has only the kinetic energy:

$$E_m = K + U_s = \frac{1}{2}kA^2 \quad \rightarrow \quad \frac{1}{2}mv_{\max}^2 + 0 = \frac{1}{2}kA^2$$

$$v_{\max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{20.0\ \mathrm{N/m}}{0.500\ \mathrm{kg}}}(0.0300\ \mathrm{m}) = \boxed{0.190\ \mathrm{m/s}}$$



Energy of the Simple Harmonic Motion

Example 2 (Oscillation on a Horizontal Surface): A 0.500-kg cart connected to a light spring for which the force constant is 20.0 N/m oscillates on a frictionless, horizontal air track. Use an energy approach to respond to the questions below.

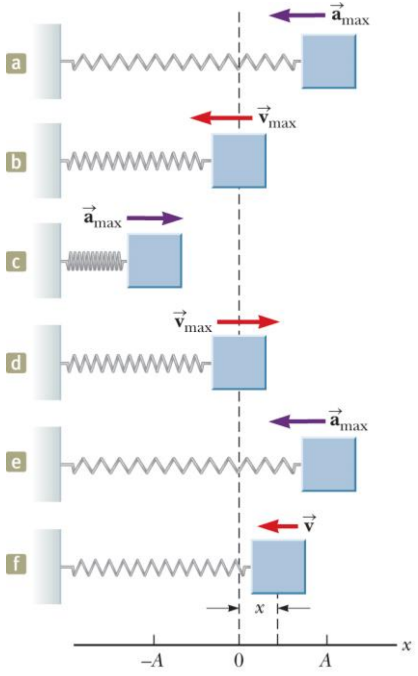
(b) What is the velocity of the cart when the position is 2.00 cm?

The velocity of the cart at any arbitrary position is obtained by the following equation:

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$$

$$= \pm\sqrt{\frac{20.0\ \mathrm{N/m}}{0.500\ \mathrm{kg}}\left((0.0300\ \mathrm{m})^2 - (0.0200\ \mathrm{m})^2\right)}$$

$$= \boxed{\pm 0.141\ \mathrm{m/s}}$$



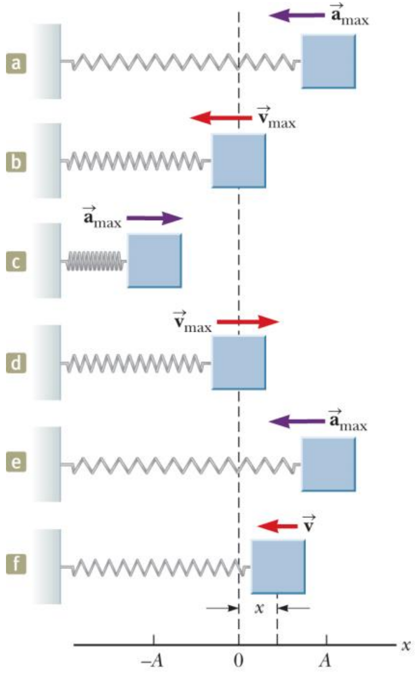
Energy of the Simple Harmonic Motion

Example 2 (Oscillation on a Horizontal Surface): A 0.500-kg cart connected to a light spring for which the force constant is 20.0 N/m oscillates on a frictionless, horizontal air track. Use an energy approach to respond to the questions below.

(c) Compute the kinetic and potential energies of the system when the position of the cart is 2.00 cm.

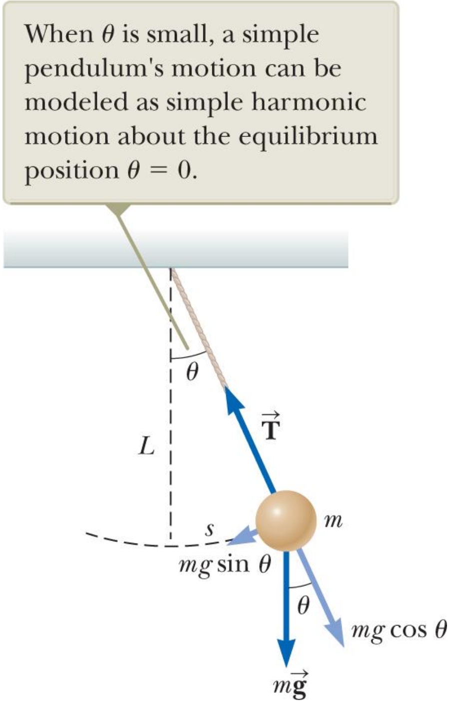
$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.500\ \text{kg})(0.141\ \text{m/s})^2 = \boxed{5.00 \times 10^{-3}\ \text{J}}$$

$$U_s = \frac{1}{2}kx^2 = \frac{1}{2}(20.0\ \text{N/m})(0.0200\ \text{m})^2 = \boxed{4.00 \times 10^{-3}\ \text{J}}$$



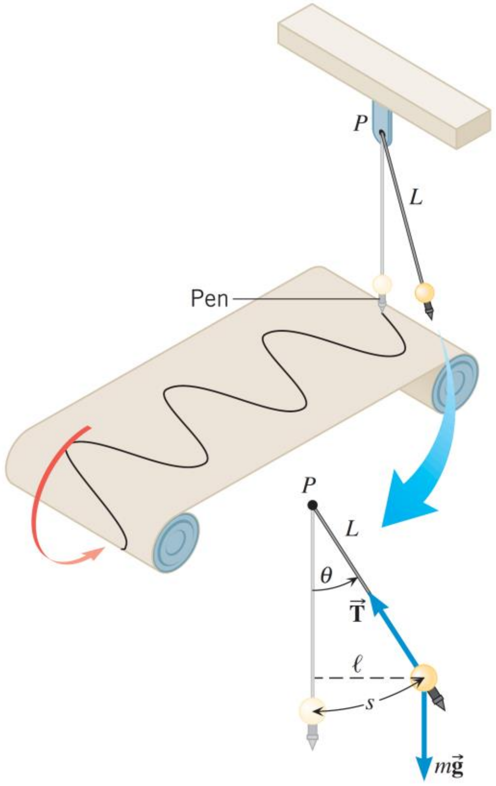
The Simple Pendulum System

- The **Simple Pendulum** system is a mechanical system that exhibits periodic motion.
- Consider a particle-like bob of mass m suspended by light string of length L fixed at upper end, and the motion occurs in vertical place.
- The forces acts on the bob are:
  - The gravitational force $F_g = mg$
  - The tension force T
- The tangential component of the gravitational force $mgsin\ \theta$ acts as the restoring force towards the equilibrium point.



The Simple Pendulum System

- We can record the position of the particle as time passes, by attaching a pen to the bottom of the swinging particle and moving a strip of paper beneath it at a steady rate.
- The graphical record reveals a pattern that is similar (but not identical) to the sinusoidal pattern for simple harmonic motion.
- For the small angles ($\theta \leq 10°$) motion close to simple harmonic motion, then we can derive the simple harmonic motion formula for the simple pendulum.



The Simple Pendulum System

- Apply Newton's second law for motion in tangential direction:

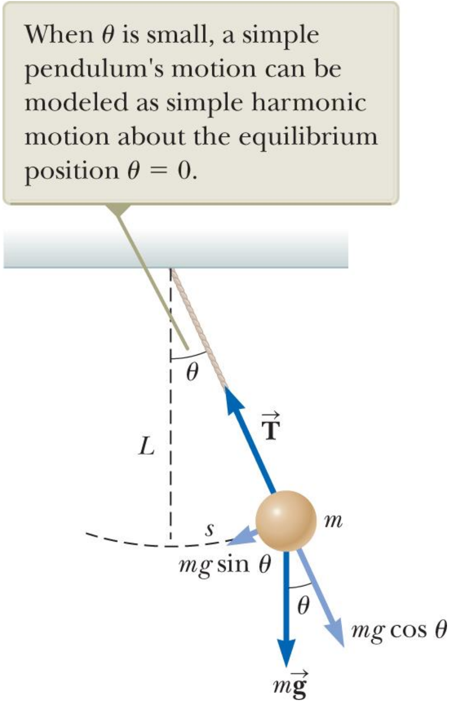
$$F_t = ma_t \quad \rightarrow \quad -mg\sin\theta = m\frac{d^2s}{dt^2}$$

Negative sign indicates tangential force acts toward equilibrium position

Bob's position measured along arc

$$s = L\theta \quad \rightarrow \quad \frac{d^2s}{dt^2} = L\frac{d^2\theta}{dt^2} \quad \rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

- Consider θ as position, compare this equation with SHM equation
- Does it have the same mathematical form?



The Simple Pendulum System

- Using the small angle approximation ($sin\theta \approx \theta \;\; for \;\; \theta \leq 10° \; or \; 0.2 \, rad$):

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta \xrightarrow{For\ the\ small\ values\ of\ \theta} \frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

TABLE 15.1 Sines and Tangents of Angles

Angle in Degrees	Angle in Radians	Sine of Angle	Percent Difference	Tangent of Angle	Percent Difference
0°	0.000 0	0.000 0	0.0%	0.000 0	0.0%
1°	0.017 5	0.017 5	0.0%	0.017 5	0.0%
2°	0.034 9	0.034 9	0.0%	0.034 9	0.0%
3°	0.052 4	0.052 3	0.0%	0.052 4	0.1%
5°	0.087 3	0.087 2	0.1%	0.087 5	0.3%
10°	0.174 5	0.173 6	0.5%	0.176 3	1.0%
15°	0.261 8	0.258 8	1.2%	0.267 9	2.3%
20°	0.349 1	0.342 0	2.1%	0.364 0	4.3%
30°	0.523 6	0.500 0	4.7%	0.577 4	10.3%

The Simple Pendulum System

- Using the small angle approximation ($sin\theta \approx \theta \;\; for \;\; \theta \leq 10° \; or \; 0.2 \, rad$):

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta \xrightarrow{For\ the\ small\ values\ of\ \theta} \frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

- The angular position of the pendulum under the simple harmonic motion is obtained as:

$$\boxed{\theta = \theta_{\max}\cos(\omega t + \phi)}$$

Angular position of the pendulum under SHM

Angular frequency

$$\boxed{\omega = \sqrt{\frac{g}{L}}}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}}$$

Period

Quick Quiz 5

- The grandfather clock in the opening storyline depends on the period of a pendulum to keep correct time. Suppose the grandfather clock is calibrated correctly and then a mischievous child slides the bob of the pendulum downward on the oscillating rod. The grandfather clock runs ……………………

  a) slow.
  b) fast.
  c) correctly.

Quick Quiz 6

- Suppose the grandfather clock is calibrated correctly at sea level and is then taken to the top of a very tall mountain.

The grandfather clock now runs ……………..

a) slow.
b) fast.
c) correctly.

Simple Harmonic Motion: Simple Pendulum

Example 3 (Connection Between Length and Time): Christian Huygens (1629–1695), the greatest clockmaker in history, suggested that an international unit of length could be defined as the length of a simple pendulum having a period of exactly 1 s.

(a) How much shorter would our length unit be if his suggestion had been followed?

Solve the period formula for the length and substitute the known values.

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \rightarrow \quad L = \frac{T^2 g}{4\pi^2} = \frac{(1.00\ \mathrm{s})^2(9.80\ \mathrm{m/s^2})}{4\pi^2} = \boxed{0.248\ \mathrm{m}}$$

The meter's length would be slightly less than one-fourth of its current length.

Simple Harmonic Motion: Simple Pendulum

Example 3 (Connection Between Length and Time): Christian Huygens (1629–1695), the greatest clockmaker in history, suggested that an international unit of length could be defined as the length of a simple pendulum having a period of exactly 1 s.

(b) What if Huygens had been born on another planet? What would the value for *g* have to be on that planet such that the meter based on Huygens's pendulum would have the same value as our meter?

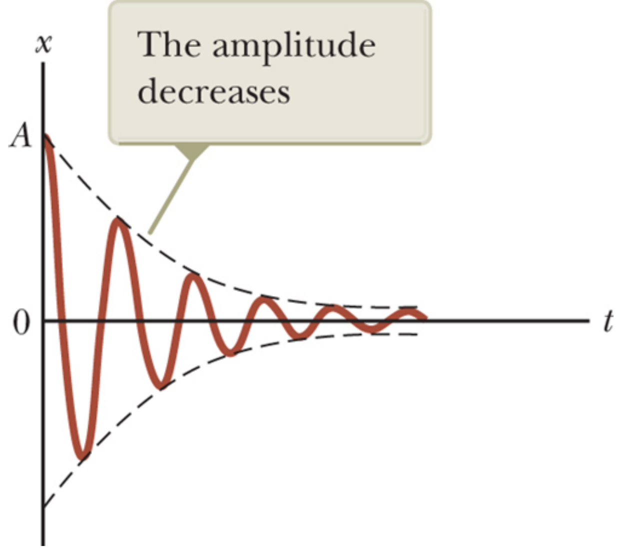
Solve the period formula for the gravitational acceleration and substitute the known values.

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \rightarrow \quad g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(1.00\ \mathrm{m})}{(1.00\ \mathrm{s})^2} = 4\pi^2 \mathrm{m/s^2} = 39.5\ \mathrm{m/s^2}$$

No planet in our solar system has an acceleration due to gravity that large.

Damped Harmonic Motion

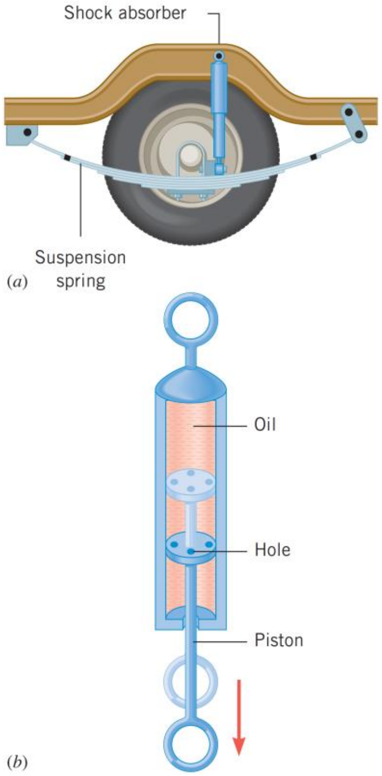
- In simple harmonic motion, we considered the isolated systems, which in the object oscillate with a constant amplitude under the action of only a linear restoring force.
- In many real systems, nonconservative forces such as friction and air resistance also act and retard the motion of the system.
- Consequently, the mechanical energy of the system diminishes in time and the oscillation amplitude decreases as time passes.
- The motion is no longer simple harmonic motion, and it is called **damped harmonic motion,** and the decrease in the amplitude is called damping.



Can you give some applications of damped harmonic motion?

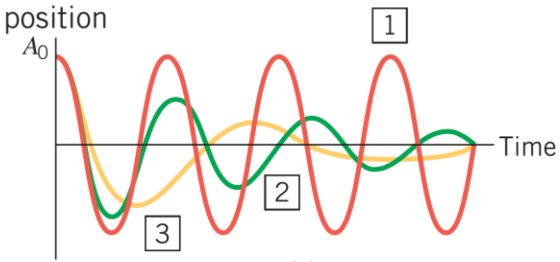
Damped Harmonic Motion

- One widely used application of damped harmonic motion is in the **suspension system** of an automobile.
  - A shock absorber been attached to a main suspension spring of a car, which designed to introduce damping forces to reduce the vibration caused by a bumpy rode.
  - It consist of a piston in a reservoir of oil. When the piston moves in response to a bump in the road, holes in the piston head permit the piston to pass through the oil. The viscous forces cause the damping.
- Different degrees of damping can exist.
  - Undamped system
  - Underdamped system
  - Critically damped system
  - Overdamped system

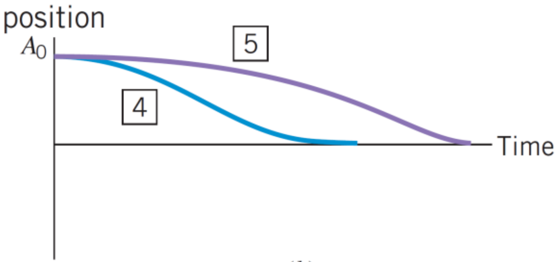


Damped Harmonic Motion

- Different degrees of damping can exist.
  - Undamped system (1): No damping. Simple harmonic motion.
  - Underdamped system (2 and 3): Amplitude of oscillations decreases rapidly. The damping is less than the critical level.
  - Critically damped system (4): No oscillation, simply returns to its equilibrium position. It is the smallest degree of damping that eliminates the oscillations
  - Overdamped system (5): No oscillation, simply returns to its equilibrium position. Takes longer time than the critical level.



(a)



(b)

Can you give an example for each damping system?

Forced Harmonic Motion

- We know that the mechanical energy of damped harmonic motion decreases in time as result of retarding forces like friction and air resistance.
- It is possible to compensate for energy decrease by applying **periodic external force** that does positive work on system, which is called driving force.
- At any instant, energy can be transferred into system by the driving force that acts in direction of motion of oscillator to keep the amplitude of the oscillations constant.
- For example: child on swing can be kept in motion by appropriately timed “pushes”

Forced Harmonic Motion

- Note that if the driving force has the same frequency as the natural frequency of the system, the amplitude of the vibration becomes larger and will increase without limit.
- The dramatic increase in amplitude near the natural frequency is called **resonance**, and the natural frequency is also called the **resonance frequency** of the system.

- A dramatic example of such resonance occurred in 1940 when the Tacoma Narrows Bridge in the state of Washington was destroyed by resonant vibrations.
- Although the winds were not particularly strong on that occasion, the "flapping" of the wind across the roadway provided a periodic driving force whose frequency matched that of the bridge.
- The resulting oscillations of the bridge caused it to ultimately collapse because the bridge design had inadequate built-in safety features.

a

b

THANK YOU

HUMBER

WE ARE HUMBER