HUMBER ENGINEERING

MENG 3510 – Control Systems
LECTURE 9

WE ARE HUMBER

HUMBER

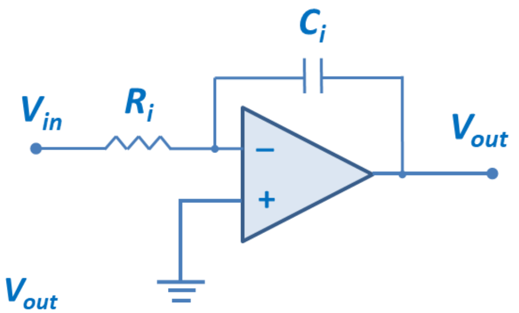
LECTURE 9
Frequency Response Analysis

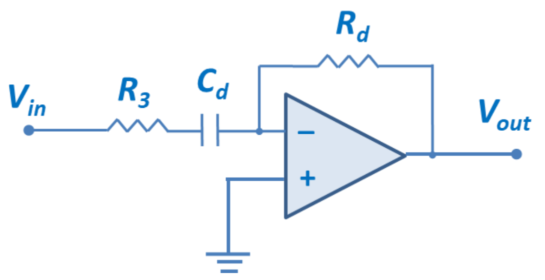
- Frequency Response Function
- Bode Diagram Plotting Techniques
- Bode Diagram of Basic Factors
  - Constant Gain
  - Integral and Derivative Factors
  - First-Order Factors
  - Second –Order Factors
- Nyquist Diagram

Frequency Response

- In the previous lectures we studied the systems in the time-domain and we analyzed the step response against time.
- Sometimes it is useful to analyze the system response in **frequency domain** by applying **sinusoidal input**.
- **Frequency response** tells us how the system responds to **sinusoidal inputs** of different frequencies.
- **Why could it be important to know how a system responds to different frequencies?**
  - **Vibration test in mechanical systems.**
    - Study of an out balanced gas turbine on an aircraft wing which is producing a sinusoidal vibration.
    - Designing motor vehicle suspension systems, if car is going along a bumpy road how it responds to oscillatory inputs.
  - **Filter circuit design in electrical and electronic systems.**
    - In designing the filter circuit, we would want to know which frequencies were attenuated by the filter.
    - Each of the controllers, PI, PD, and PID, and lead, lag compensators can be considered and studied as a filter.
    - Filtering characteristics are determined from the frequency response of the systems.
- In addition, frequency response is used for system identification and modeling, stability analysis and controller design, which will be covered later.

How do we determine the Frequency Response?

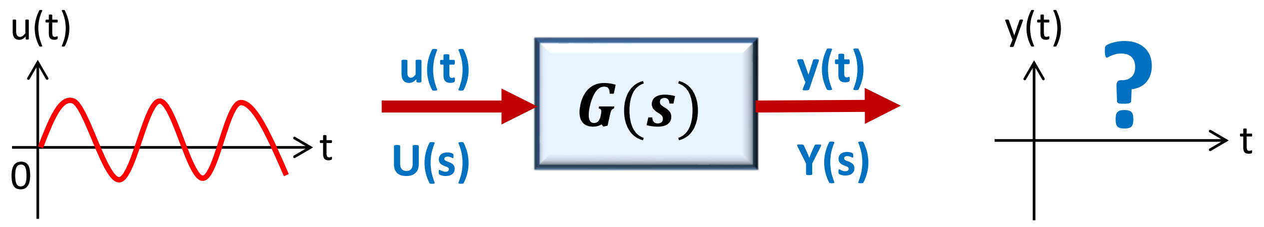




Sinusoidal Response of Linear Systems

- Consider a **linear** system with the transfer function $G(s)$ and the applied pure sinusoid input.

$$\boldsymbol{u(t) = A\sin(\omega t)}$$



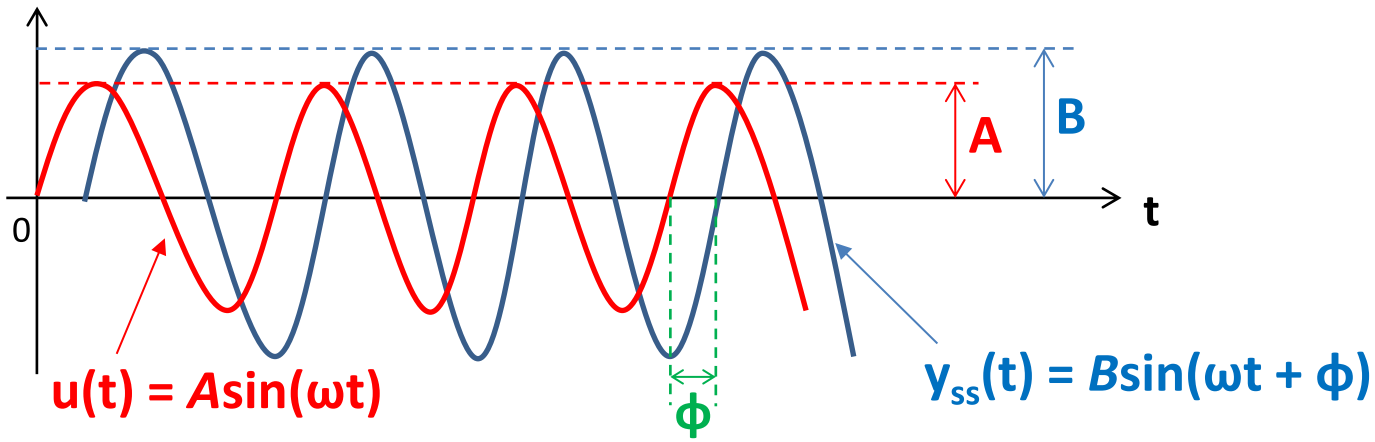
Sinusoidal Response

$$\boldsymbol{y_{ss}(t) = B\sin(\omega t + \phi)}$$

Magnitude

Phase shift

- The Steady-state Response, $y_{ss}(t)$, has the following characteristics:
  - Sinusoidal signal
  - Same frequency ω as the input signal
  - Different amplitude B and different phase Φ from the input signal, depending on the characteristic of $G(s)$

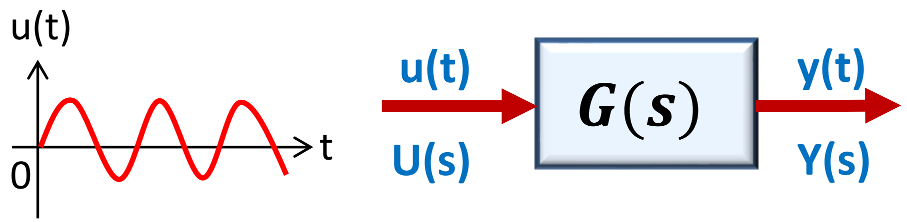


How to determine the magnitude B and the phase shift ϕ?

Sinusoidal Response of Linear Systems

- Consider a **linear** system with the transfer function $G(s)$ and the applied pure sinusoid input.

$\boldsymbol{u(t) = A\sin(\omega t)}$



$\boldsymbol{y_{ss}(t) = B\sin(\omega t + \phi)}$ **Sinusoidal Response**

- The magnitude and phase depends on the gain and phase of the $G(s)$ at frequency of ω.

- We define the **frequency response function $G(j\omega)$** as: $\boldsymbol{G(j\omega) = G(s)\big|_{s=j\omega}}$

- $\boldsymbol{G(j\omega)}$ is a complex quantity and can be represented by the magnitude and phase-angle. $\boldsymbol{G(j\omega) = |G(j\omega)|\angle G(j\omega)}$

$$\boldsymbol{Y(s) = G(s)U(s) \quad \rightarrow \quad Y(j\omega) = G(j\omega)U(j\omega)}$$

$$|Y(j\omega)| = |G(j\omega)||U(j\omega)| \quad \rightarrow \quad \boldsymbol{|G(j\omega)| = \frac{|Y(j\omega)|}{|U(j\omega)|} = \frac{|B|}{|A|}}$$ **Amplitude Ratio**

$$\angle Y(j\omega) = \angle G(j\omega) + \angle U(j\omega) \quad \rightarrow \quad \boldsymbol{\angle G(j\omega) = \angle Y(j\omega) - \angle U(j\omega) = \phi}$$ **Phase Shift**

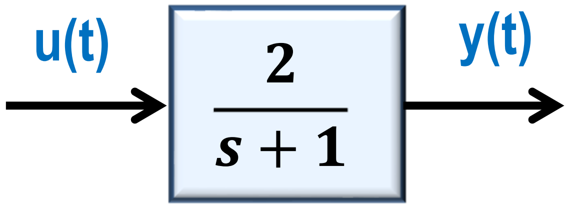
How the amplitude ratio and the phase shift changes by frequency?

Steady-state Response $\boldsymbol{y_{ss}(t) = A|G(j\omega)|\sin\big(\omega t + \angle G(j\omega)\big)}$

Frequency Response Example

Example 1

Consider the following first-order system



a) Determine the steady-state response of the system to sinusoidal input $u(t) = \sin(\omega t)$.

The steady-state response can also be determined by using the general formula

$$\boxed{y_{ss}(t) = A|G(j\omega)|\sin\big(\omega t + \angle G(j\omega)\big)}$$

First, obtain the frequency response function, $G(j\omega)$, from $G(s)$

$$G(j\omega) = G(s)\big|_{s=j\omega} \quad \rightarrow \quad G(j\omega) = \frac{2}{j\omega + 1}$$

The magnitude and phase angle of the $G(j\omega)$ are determines as follows

$$|G(j\omega)| = \left|\frac{2}{j\omega + 1}\right| = \frac{|2|}{|j\omega + 1|} = \frac{2}{\sqrt{\omega^2 + 1}}$$

$$\angle G(j\omega) = \angle\frac{2}{j\omega + 1} = \angle 2 - \angle(j\omega + 1) = 0 - \tan^{-1}\left(\frac{\omega}{1}\right) = -\tan^{-1}(\omega)$$

$$\boxed{y_{ss}(t) = \frac{2}{\sqrt{1 + \omega^2}}\sin(\omega t - \tan^{-1}(\omega))}$$

Steady-state Response

For example, if the input signal frequency is $\omega = 3\ rad/s$ the output signal is:

$$y_{ss}(t) = \frac{2}{\sqrt{10}}\sin\big(3t - \tan^{-1}(3)\big) = 0.63\sin(3t - 72°)$$

Frequency Response Example

Example 1 Consider the following first-order system

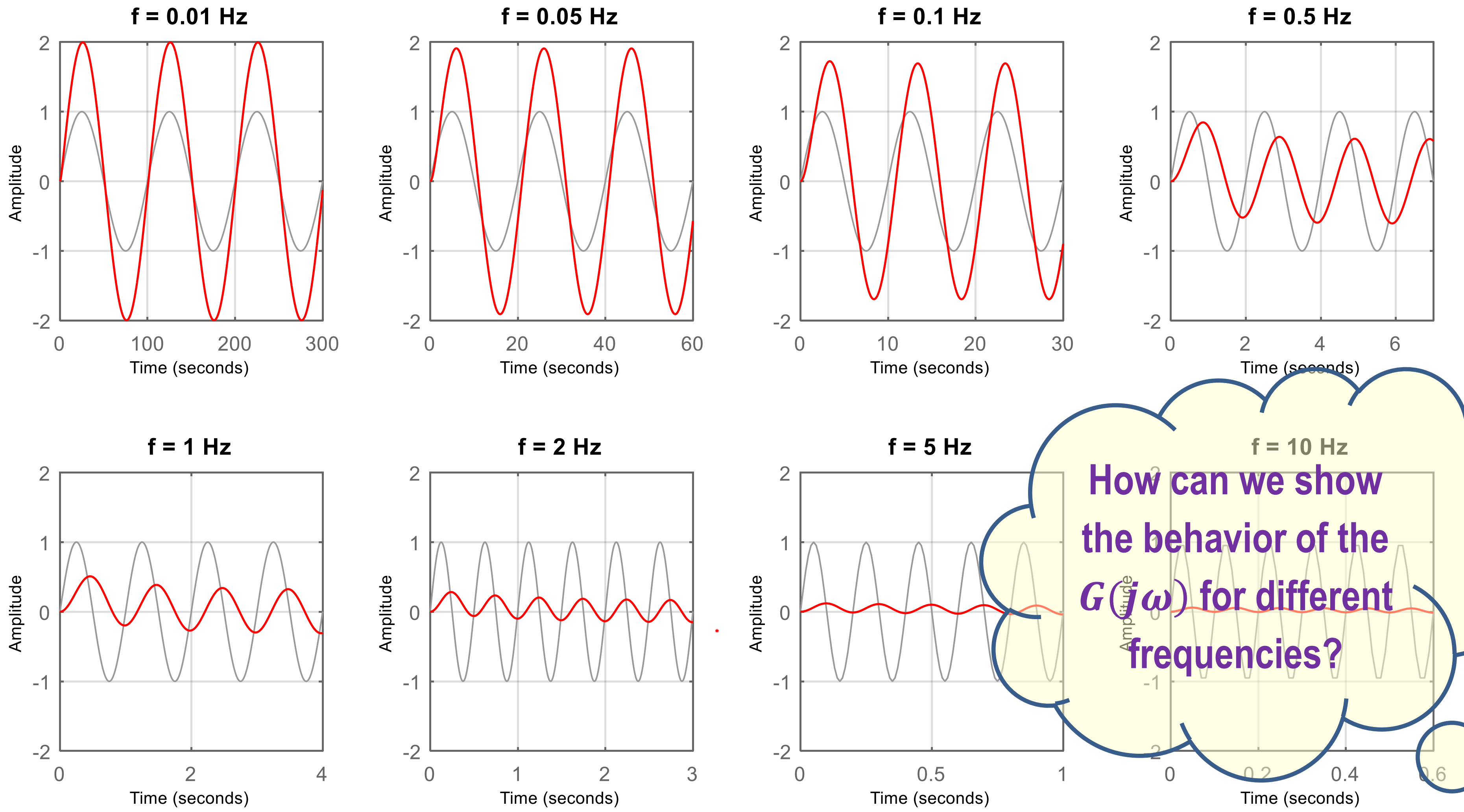
$$u(t) \longrightarrow \boxed{\frac{2}{s+1}} \longrightarrow y(t)$$

b) Plot the input and output of the system for the following range of frequency ($\omega = 2\pi f$)

$$\boxed{f = 0.01Hz, 0.05Hz, 0.1Hz, 0.5Hz, 1Hz, 2Hz, 5Hz, 10Hz}$$

$$|G(j\omega)| = \frac{|Y(j\omega)|}{|U(j\omega)|} = \frac{|B|}{|A|}$$

$$\angle G(j\omega) = \angle Y(j\omega) - \angle U(j\omega) = \phi$$



How can we show the behavior of the $G(j\omega)$ for different frequencies?

The red graph is the output

Frequency f (Hz)	Magnitude |G(jω)|	Phase Angle ∠G(jω) (deg)
0.01	1.9961	-3.59°
0.05	1.9081	-17.44°
0.1	1.6935	-32.14°
0.5	0.6066	-72.34°
1	0.3144	-80.95°
2	0.1587	-85.45°
5	0.0636	-88.17°
10	0.0318	-89.08°

Frequency Response Graphs

- **Frequency response graphs** or **Bode plots** include two separate graphs to show the variation of magnitude ratio and phase shift in terms of frequency.
- Frequency response graphs are often shown in a logarithmic plot in terms of decibels.
- Decibel is a logarithmic measurement of one variable to another.

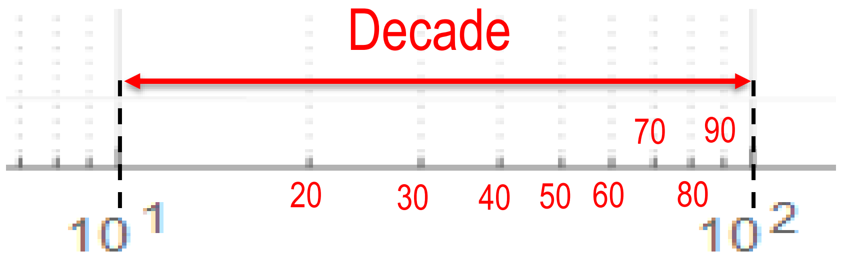
$$\boxed{\mathbf{dB} = \mathbf{20log}\left(\frac{\mathbf{Output\ magnitude}}{\mathbf{Input\ magnitude}}\right)}$$

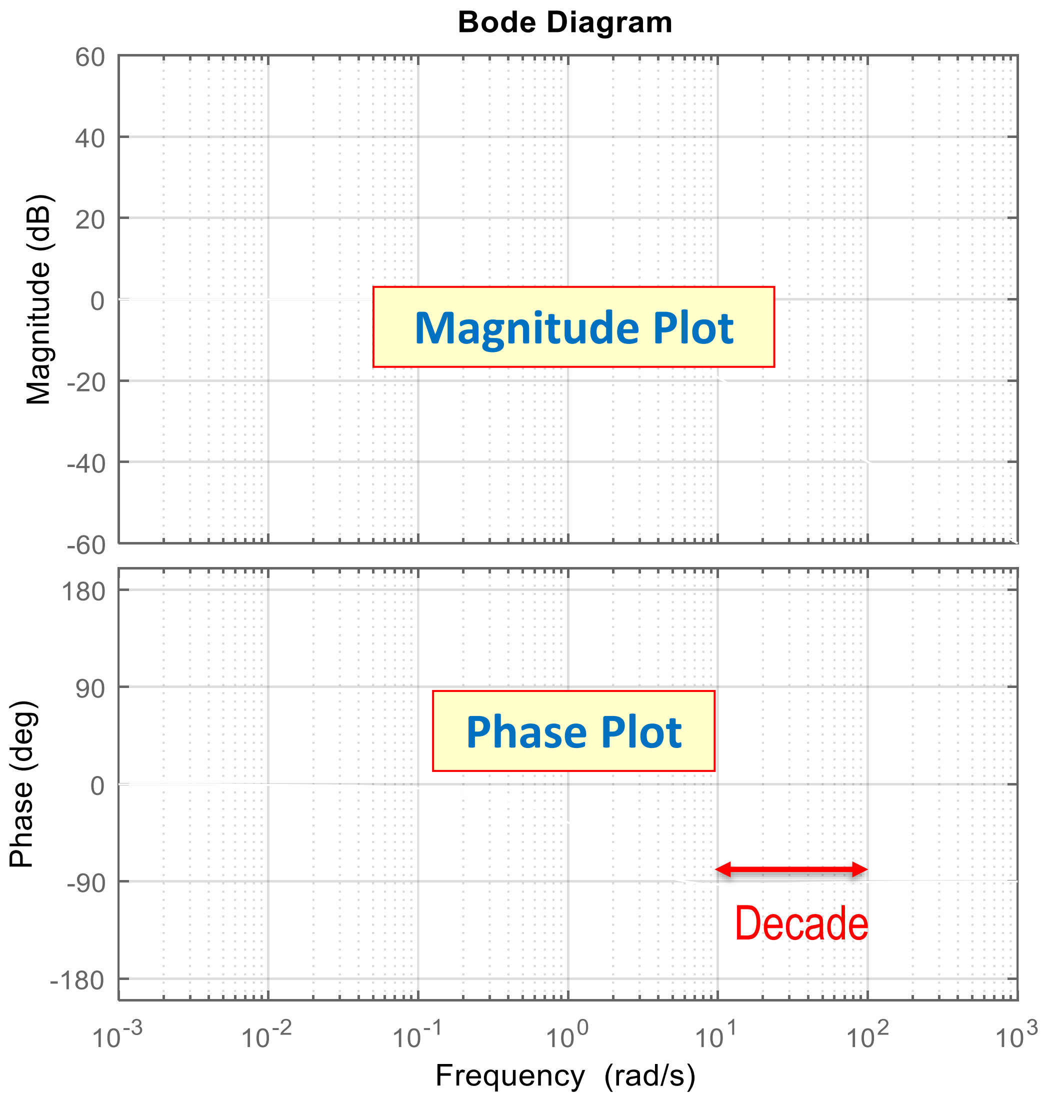
- For example, in a filter the output to input voltage ratio in decibels is obtained as:

$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{5\ V}{10\ V}\right| = 0.5$$

$$20\log\left|\frac{V_{out}}{V_{in}}\right| = 20\log(0.5) = -6.02\ dB$$

- A tenfold change in frequency is called a decade.





Frequency Response Example

Example 1 Consider the following first-order system

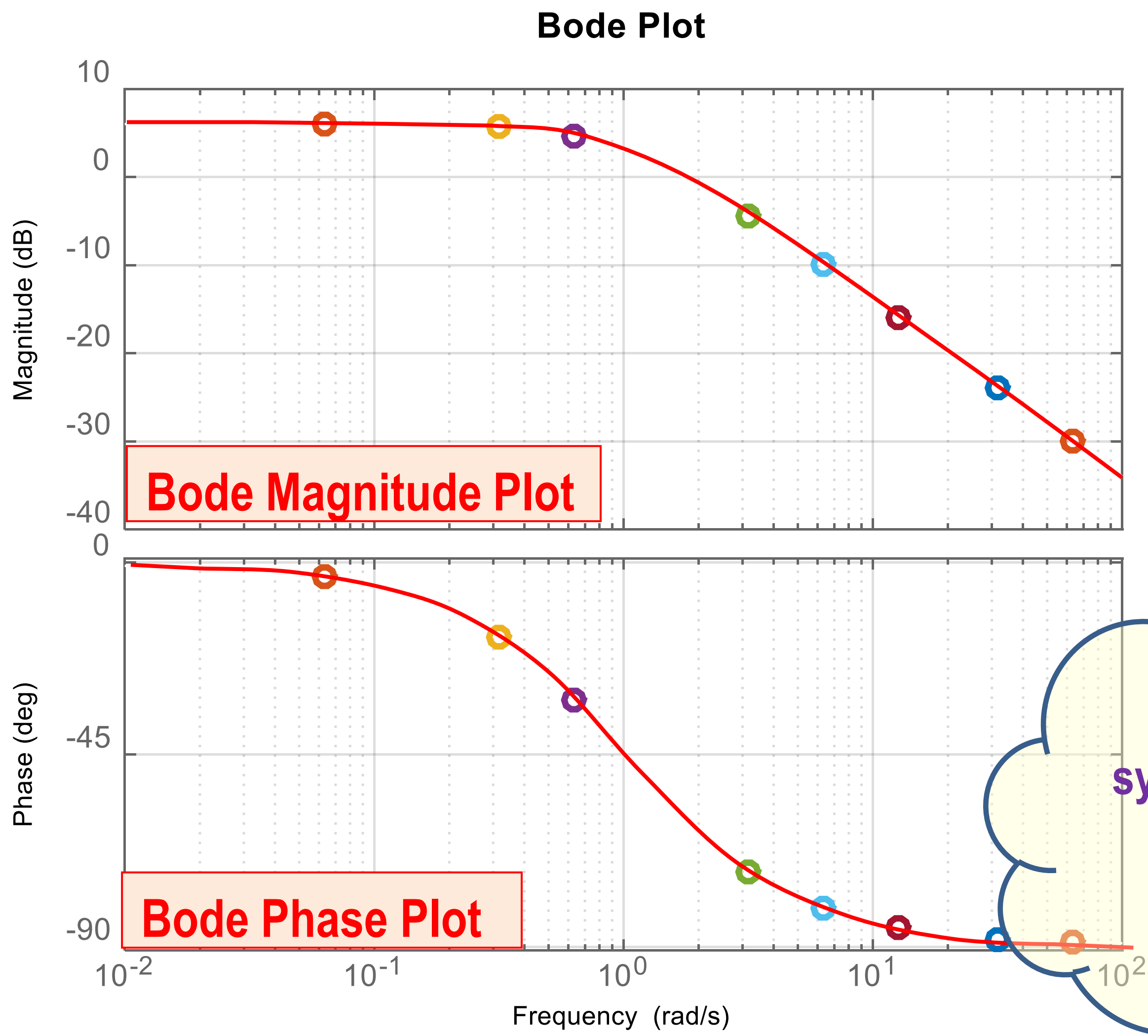
$$u(t) \rightarrow \boxed{\frac{2}{s+1}} \rightarrow y(t)$$

b) Plot the input and output of the system for the following range of frequency ($\boldsymbol{\omega = 2\pi f}$)

$$\boxed{f = 0.01Hz, 0.05Hz, 0.1Hz, 0.5Hz, 1Hz, 2Hz, 5Hz, 10Hz}$$

$$|G(j\omega)| = \frac{|Y(j\omega)|}{|U(j\omega)|} = \frac{|B|}{|A|}$$

$$\angle G(j\omega) = \angle Y(j\omega) - \angle U(j\omega) = \phi$$



Frequency f (Hz)	Magnitude |G(jω)|	Magnitude (dB) 20log|G(jω)|	Phase Angle ∠G(jω) (deg)	Frequency ω (rad/s)
0.01	1.9961	6.00 dB	-3.59°	0.063
0.05	1.9081	5.61 dB	-17.44°	0.31
0.1	1.6935	4.60 dB	-32.14°	0.63
0.5	0.6066	-4.34 dB	-72.34°	3.14
1	0.3144	-10.05 dB	-80.95°	6.28
2	0.1587	-15.99 dB	-85.45°	12.57
5	0.0636	-23.93 dB	-88.17°	31.42
10	0.0318	-29.95 dB	-89.08°	62.83

Can we find a systematic way to plot the Bode diagrams?

Bode Diagram of Basic Factors

Bode Diagram

- Consider the following general form of a transfer function

$$G(s) = \frac{K(s+z_1)(s+z_2)}{s^{\beta}(s+p_1)(s+p_2)\,(s^2+2\zeta\omega_n s+\omega_n^2)}$$

- It can be seen that the $G(s)$ is constructed from the following **basic factors**
  - Constant Gain $\rightarrow$ K
  - Integral or derivative factors $\rightarrow$ s^{β} (β is an integer number)
  - First-order factors (Single pole and zero) $\rightarrow$ $\frac{1}{s+p}$, $s+z$
  - Second-order/Quadratic factors (complex conjugated pole/zero) $\rightarrow$ $\frac{1}{s^2+2\zeta\omega_n s+\omega_n^2}$, $s^2+2\zeta\omega_n s+\omega_n^2$

- Once we become familiar with the Bode plots of these basic factors, we can easily draw the Bode plot for any general form of a transfer function $G(s)$.

- The goal is to find a systematic way to draw the Bode diagram for any system by determining some critical points of the Bode diagram.

- The Bode diagram plotting technique and the stability analysis via Bode diagram has been developed by **Hendrik Wade Bode** an American engineer.

Bode Diagram

- Note that to plot the Bode diagram we have to rewrite the transfer function $G(s)$ in the following form.

$$\boldsymbol{G(s)} = \frac{\boldsymbol{K(s+z_1)}}{\boldsymbol{s^{\beta}(s+p_1)(s^2+2\zeta\omega_n s+\omega_n^2)}} \quad \Rightarrow \quad \boldsymbol{G(s)} = \frac{\boldsymbol{K_B(s/z_1+1)}}{\boldsymbol{s^{\beta}(s/p_1+1)(s^2/\omega_n^2+2\zeta\, s/\omega_n+1)}}$$

where $\boldsymbol{K_B}$ is the total DC-gain of the transfer function $\boldsymbol{K_B = K\dfrac{z_1}{p_1\omega_n^2}}$

- The frequency response function, $G(j\omega)$ is obtained as below

$$\boldsymbol{G(j\omega)} = \frac{\boldsymbol{K_B(1+j\omega/z_1)}}{\boldsymbol{(j\omega)^{\beta}(1+j\omega/p_1)\left(1+2\zeta\left(\frac{j\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2\right)}}$$

- The log magnitude and phase of $G(j\omega)$ are obtained as below

$$|\boldsymbol{G(j\omega)}|\mathbf{dB} = 20\log(\mathrm{K_B}) + 20\log\left(\left|1+\frac{j\omega}{z_1}\right|\right) + 20\log\left(\left|\frac{1}{(j\omega)^{\beta}}\right|\right) + 20\log\left(\left|\frac{1}{1+\frac{j\omega}{p_1}}\right|\right) + 20\log\left(\left|\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+j\,\frac{2\zeta\omega}{\omega_n}}\right|\right)$$

$$\angle\boldsymbol{G(j\omega)} = \angle(\mathrm{K_B}) + \angle\left(1+\frac{j\omega}{z_1}\right) + \angle\left(\frac{1}{(j\omega)^{\beta}}\right) + \angle\left(\frac{1}{1+\frac{j\omega}{p_1}}\right) + \angle\left(\frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+j\,\frac{2\zeta\omega}{\omega_n}}\right)$$

$$\log(AB) = \log A + \log B$$

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$

$$\log A^k = k\log A$$

$$\angle(AB) = \angle A + \angle B$$

$$\angle\left(\frac{A}{B}\right) = \angle A - \angle B$$

Bode Diagram of Basic Factors

Constant Gain

$$G(s) = K \longrightarrow G(j\omega) = K$$

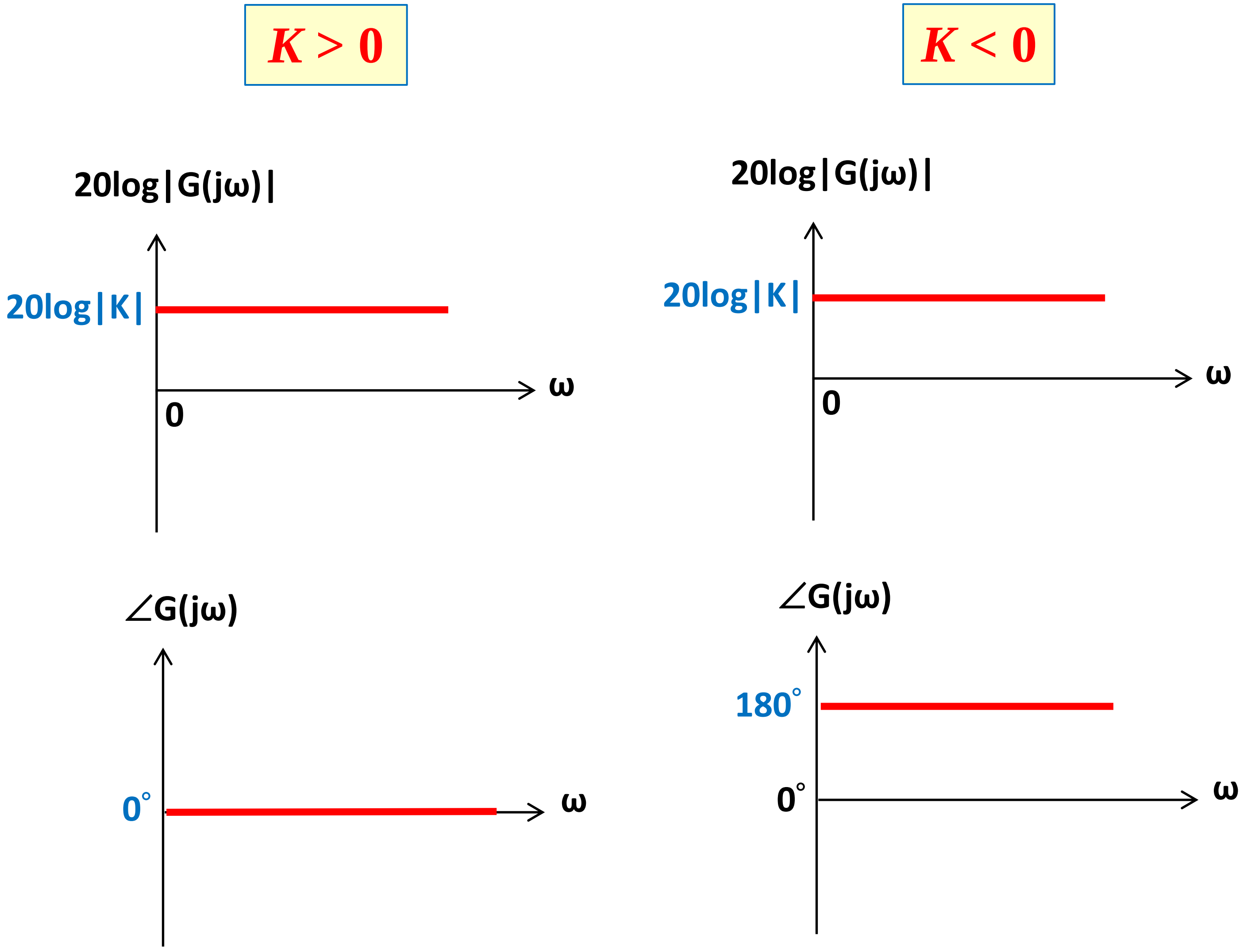
- **Bode Magnitude Plot**

$$|G(j\omega)| = |K|$$

$$20\log|G(j\omega)| = 20\log(|K|)$$

- **Bode Phase Plot**

$$\phi = \angle G(j\omega) = \begin{cases} 0^\circ & \text{if} \quad K > 0 \\ 180^\circ & \text{if} \quad K < 0 \end{cases}$$



Bode Diagram of Basic Factors

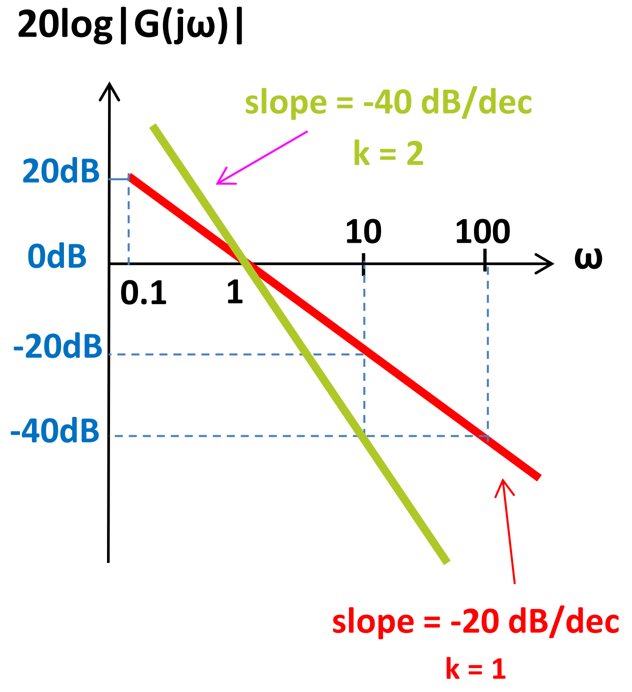
❑ Integral Factor

$$G(s) = \frac{1}{s} \longrightarrow G(j\omega) = \frac{1}{j\omega}$$

- **Bode Magnitude Plot**

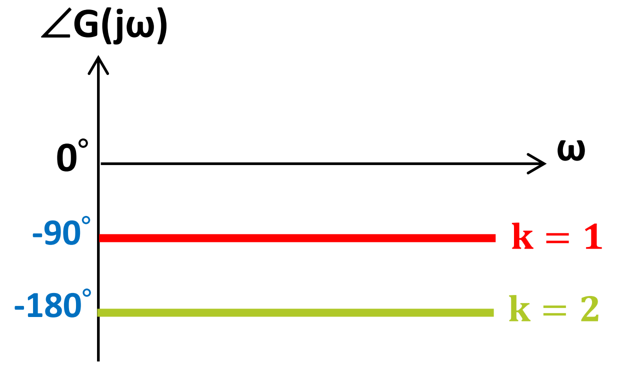
$$|G(j\omega)| = \left|\frac{1}{j\omega}\right| = \frac{1}{\omega}$$

$$20\log|G(j\omega)| = 20\log\left(\frac{1}{\omega}\right) = 20\log(1) - 20\log(\omega) = -20\log(\omega)\text{dB}$$



- **Bode Phase Plot**

$$\phi = \angle G(j\omega) = \angle\left(\frac{1}{j\omega}\right) = \angle(1) - \angle(j\omega) = -90^\circ$$



- For $G(s) = \frac{1}{s^k}$ the log magnitude and phase angle are

$$20\log|G(j\omega)| = 20\log\left|\frac{1}{(j\omega)^k}\right| = 20\log(1) - 20\log(\omega^k) = -20k\log(\omega)\text{ dB}$$

$$\phi = \angle G(j\omega) = \angle\left(\frac{1}{(j\omega)^k}\right) = -\text{k}\,90^\circ$$

Bode Diagram of Basic Factors

❑ Derivative Factor

$$\boldsymbol{G(s) = s} \longrightarrow \boldsymbol{G(j\omega) = j\omega}$$

- **Bode Magnitude Plot**

$|\boldsymbol{G(j\omega)}| = |j\omega| = \omega$

$\mathbf{20log}|\boldsymbol{G(j\omega)}| = 20\log(\omega)\mathrm{dB}$

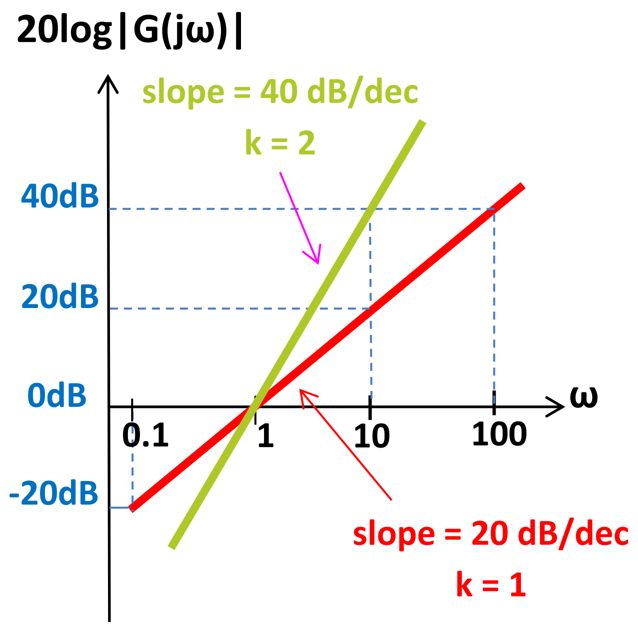
- **Bode Phase Plot**

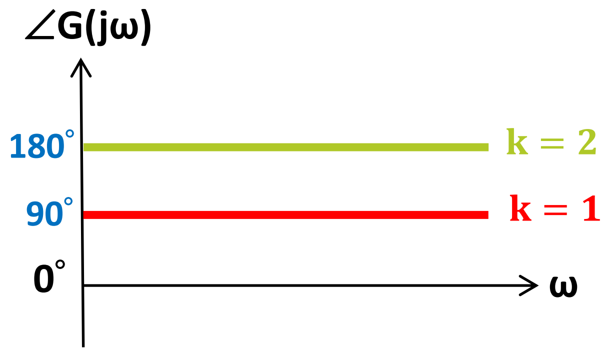
$\boldsymbol{\phi} = \angle\boldsymbol{G(j\omega)} = \angle(j\omega) = 90^\circ$

- For $\boldsymbol{G(s) = s^k}$ the log magnitude and phase angle are

$\mathbf{20log}|\boldsymbol{G(j\omega)}| = 20\log\left|(j\omega)^k\right| = 20\log(\omega^k) = 20k\log(\omega)\,dB$

$\angle\boldsymbol{G(j\omega)} = \angle\left((j\omega)^k\right) = k\,90^\circ$





Bode Diagram of Basic Factors

❑ First-Order Factor: Single Pole

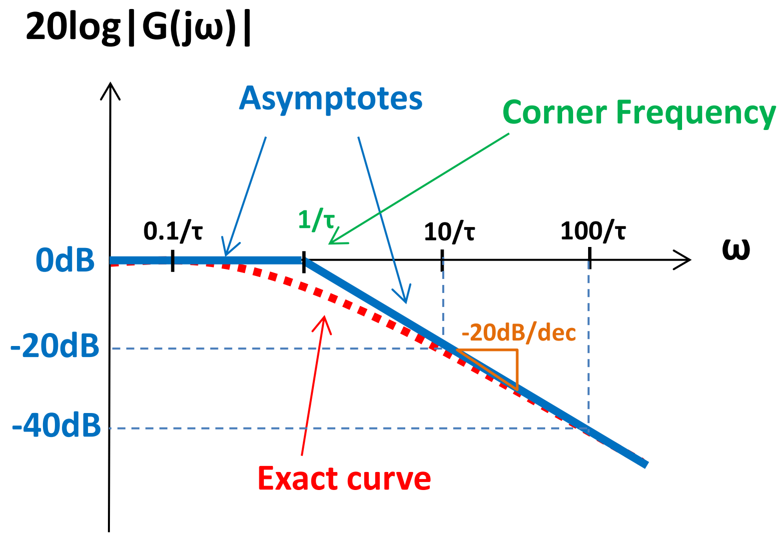
$$G(s) = \frac{1}{1+\tau s} \longrightarrow G(j\omega) = \frac{1}{1+j\omega\tau}$$

$$\omega = \frac{1}{\tau} \rightarrow \text{Corner Frequency}$$

- **Bode Magnitude Plot**

$$|G(j\omega)| = \left|\frac{1}{1+j\tau\omega}\right| = \frac{1}{\sqrt{1+(\tau\omega)^2}}$$

$$20\log|G(j\omega)| = -20\log\left(\sqrt{1+(\tau\omega)^2}\right) \rightarrow \begin{cases} \approx\ 0dB & if \quad \omega \ll \frac{1}{\tau} \\ = -3\ dB & \text{at} \quad \boldsymbol{\omega = \frac{1}{\tau}} \\ \approx -20\log(\tau\omega)\ dB & if \quad \omega \gg \frac{1}{\tau} \end{cases}$$



- The Bode magnitude curve can be estimated with two asymptote lines:
  - Low-freq. asymptote $(\omega < \frac{1}{\tau})$ $\rightarrow$ line slope = 0
  - High-freq. asymptote $(\omega > \frac{1}{\tau})$ $\rightarrow$ line slope = -20dB/dec

- The asymptotes intersect at the Corner Frequency of $\omega = \frac{1}{\tau}$

Bode Diagram of Basic Factors

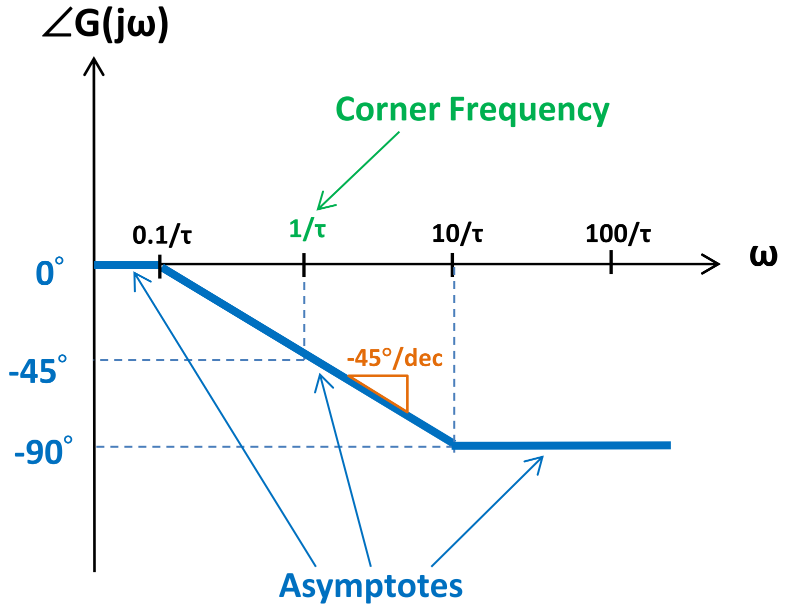
❑ First-Order Factor: Single Pole

$$G(s) = \frac{1}{1+\tau s} \longrightarrow G(j\omega) = \frac{1}{1+j\omega\tau}$$

$$\omega = \frac{1}{\tau} \rightarrow \text{Corner Frequency}$$

- **Bode Phase Plot**

$$\angle G(j\omega) = \angle\left(\frac{1}{1+j\tau\omega}\right) = -\tan^{-1}(\tau\omega) \rightarrow \begin{cases} \approx 0^\circ & if \quad \omega \ll \frac{0.1}{\tau} \\ = -45^\circ & at \quad \omega = \frac{1}{\tau} \rightarrow \text{Corner Frequency} \\ \approx -90^\circ & if \quad \omega \gg \frac{10}{\tau} \end{cases}$$



- The phase curve can be estimated with three asymptote lines:
  - High-freq. asymptote $(\omega > \frac{10}{\tau})$ $\rightarrow$ line slope = 0
  - Low-freq. asymptote $(\omega < \frac{0.1}{\tau})$ $\rightarrow$ line slope = 0
  - Middle-freq. asymptote $(\frac{0.1}{\tau} < \omega < \frac{10}{\tau})$ $\rightarrow$ line to connect the high-freq. and the low-freq. asymptote lines together

Bode Diagram of Basic Factors

❑ First-Order Factor: Single Zero

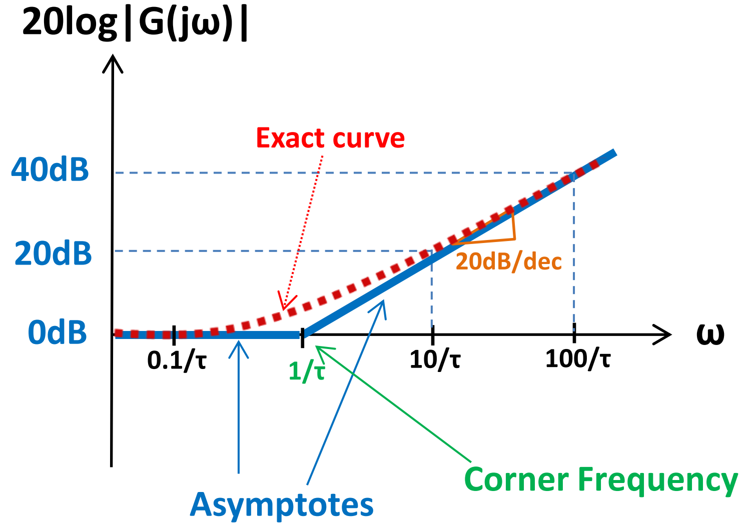
$$G(s) = 1 + \tau s \longrightarrow G(j\omega) = 1 + j\omega\tau$$

$$\omega = \frac{1}{\tau} \rightarrow \text{Corner Frequency}$$

- **Bode Magnitude Plot**

$$|G(j\omega)| = |1 + j\tau\omega| = \sqrt{1 + (\tau\omega)^2}$$

$$20\log|G(j\omega)| = 20\log\left(\sqrt{1 + (\tau\omega)^2}\right) \rightarrow \begin{cases} \approx 0dB & if \quad \omega \ll \dfrac{1}{\tau} \\ = 3dB & \text{at} \quad \boldsymbol{\omega = \dfrac{1}{\tau}} \\ \approx 20\log(\tau\omega)\, dB & if \quad \omega \gg \dfrac{1}{\tau} \end{cases}$$



- The Bode magnitude curve can be estimated with two asymptote lines:
  - Low-freq. asymptote $(\omega < \frac{1}{\tau})$ $\rightarrow$ line slope = 0
  - High-freq. asymptote $(\omega > \frac{1}{\tau})$ $\rightarrow$ line slope = +20dB/dec
- The asymptotes intersect at the **Corner Frequency** of $\boldsymbol{\omega = \frac{1}{\tau}}$

Bode Diagram of Basic Factors

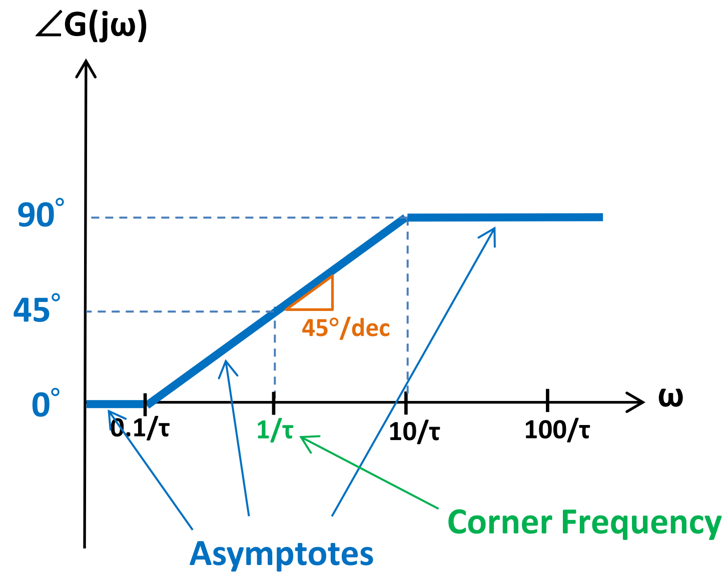
❑ First-Order Factor: Single Zero

$$G(s) = 1 + \tau s \longrightarrow G(j\omega) = 1 + j\omega\tau$$

$$\omega = \frac{1}{\tau} \rightarrow \text{Corner Frequency}$$

- **Bode Phase Plot**

$$\angle G(j\omega) = \angle(1 + j\tau\omega) = \tan^{-1}(\tau\omega) \rightarrow \begin{cases} \approx 0^\circ & if \quad \omega \ll \frac{0.1}{\tau} \\ = 45^\circ & at \quad \omega = \frac{1}{\tau} \\ \approx 90^\circ & if \quad \omega \gg \frac{10}{\tau} \end{cases}$$



- The phase curve can be estimated with three asymptote lines:
  - High-freq. asymptote ($\omega > \frac{10}{\tau}$) $\rightarrow$ line slope = 0
  - Low-freq. asymptote ($\omega < \frac{0.1}{\tau}$) $\rightarrow$ line slope = 0
  - Middle-freq. asymptote ($\frac{0.1}{\tau} < \omega < \frac{10}{\tau}$) $\rightarrow$ line to connect the high-freq. and the low-freq. asymptote lines together

Bode Diagram Example

Example 2 Draw the Bode diagram for the following system $\boldsymbol{G(s) = 50\frac{s+1}{s+5}}$

First, rewrite the transfer function in the proper form and obtain the frequency response function $G(j\omega)$ and determine the basic factors.

$$\boldsymbol{G(s) = 50\frac{s+1}{s+5}} = 50\frac{s+1}{5\left(\frac{s}{5}+1\right)} = 10\frac{s+1}{\frac{s}{5}+1} \longrightarrow \boldsymbol{G(j\omega) = 10\frac{j\omega+1}{\frac{j\omega}{5}+1}} = (10)(1+j\omega)\left(\frac{1}{1+\frac{j\omega}{5}}\right)$$

Constant Gain — (10)

Single Zero — $(1+j\omega)$

Single Pole — $\left(\frac{1}{1+\frac{j\omega}{5}}\right)$

$$|G(j\omega)|\mathrm{dB} = 20\log(|10|) + 20\log(|1+j\omega|) + 20\log\left(\left|\frac{1}{1+\frac{j\omega}{5}}\right|\right)$$

$$\angle G(j\omega) = \angle(10) + \angle(1+j\omega) + \angle\left(\frac{1}{1+\frac{j\omega}{5}}\right)$$

$$\log(AB) = \log A + \log B$$

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$

$$\angle(AB) = \angle A + \angle B$$

$$\angle\left(\frac{A}{B}\right) = \angle A - \angle B$$

Next, plot the **asymptotic Bode diagram** for each basic factor separately, and then add them together to construct the overall Bode diagram of $G(j\omega)$

Bode Diagram Example

Example 2

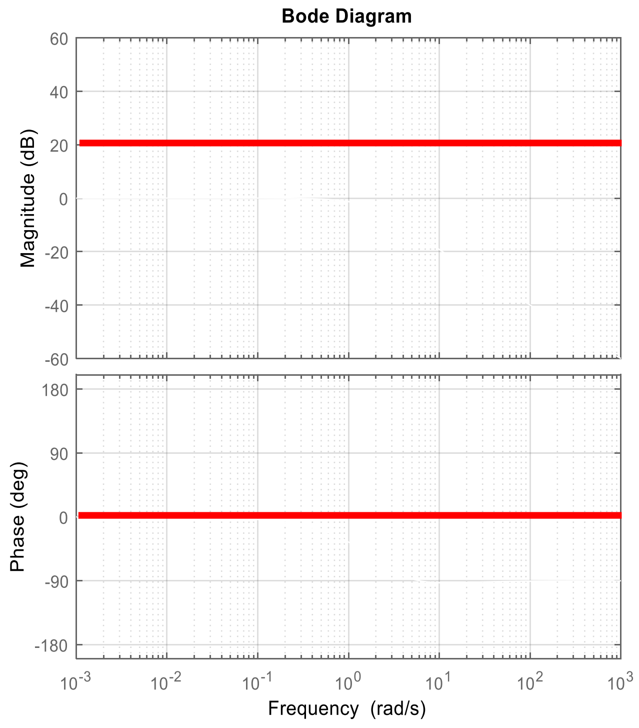
Draw the Bode diagram for the following system

$$G(s) = 50\frac{s+1}{s+5}$$

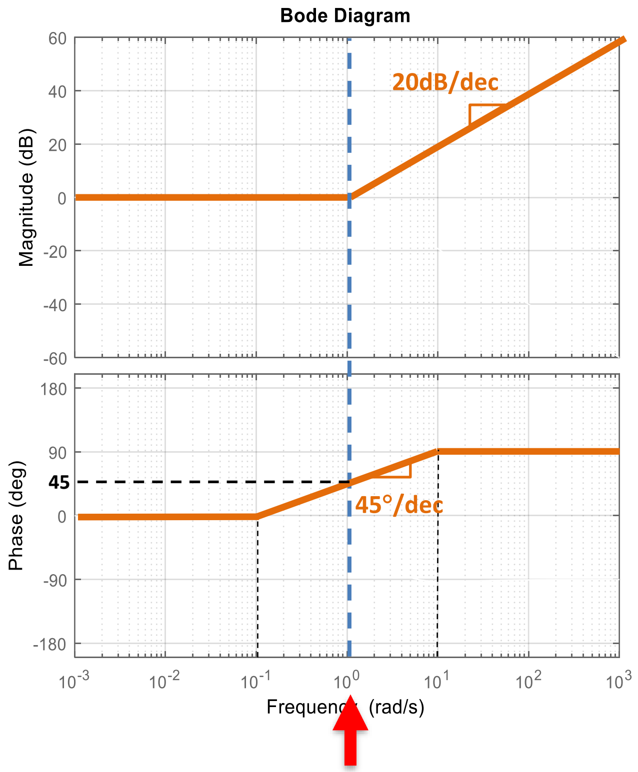
$$|\mathrm{G}(j\omega)|\mathrm{dB} = 20\log(|10|) + 20\log(|1 + j\omega|) + 20\log\left(\left|\frac{1}{1+\frac{j\omega}{5}}\right|\right)$$

$$\angle G(j\omega) = \angle(10) + \angle(1 + j\omega) + \angle\left(\frac{1}{1+\frac{j\omega}{5}}\right)$$

Constant Gain

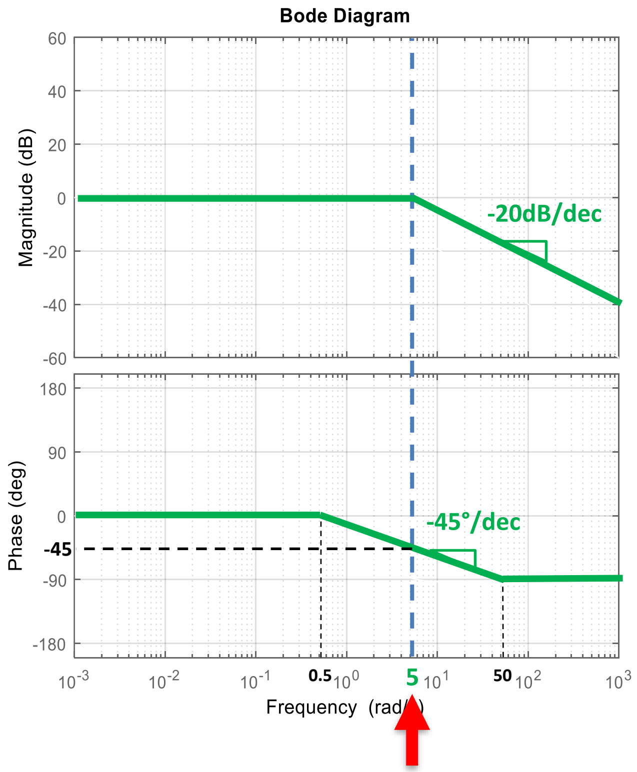


Single Zero



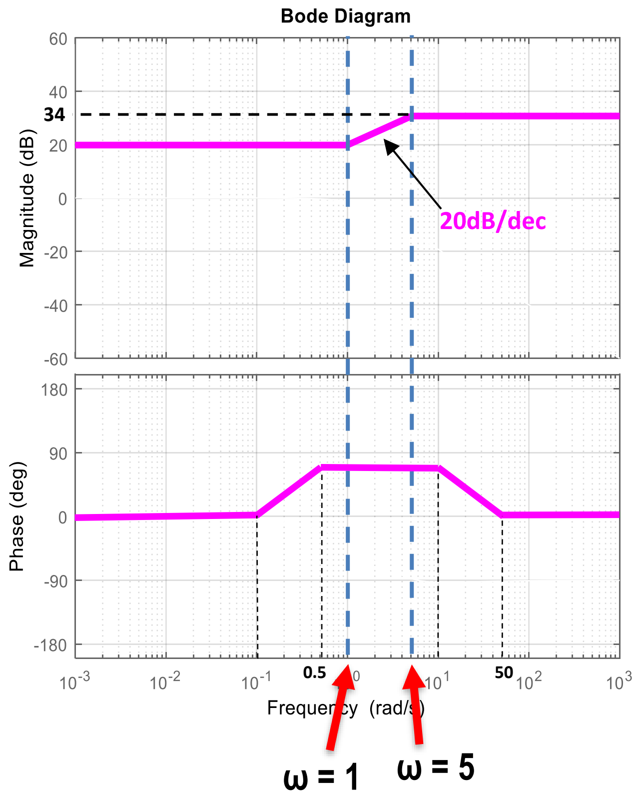
Corner frequency → ω = 1 rad/sec

Single Pole



Corner frequency → ω = 5 rad/sec

The Overall Bode Plot



Bode Diagram Example

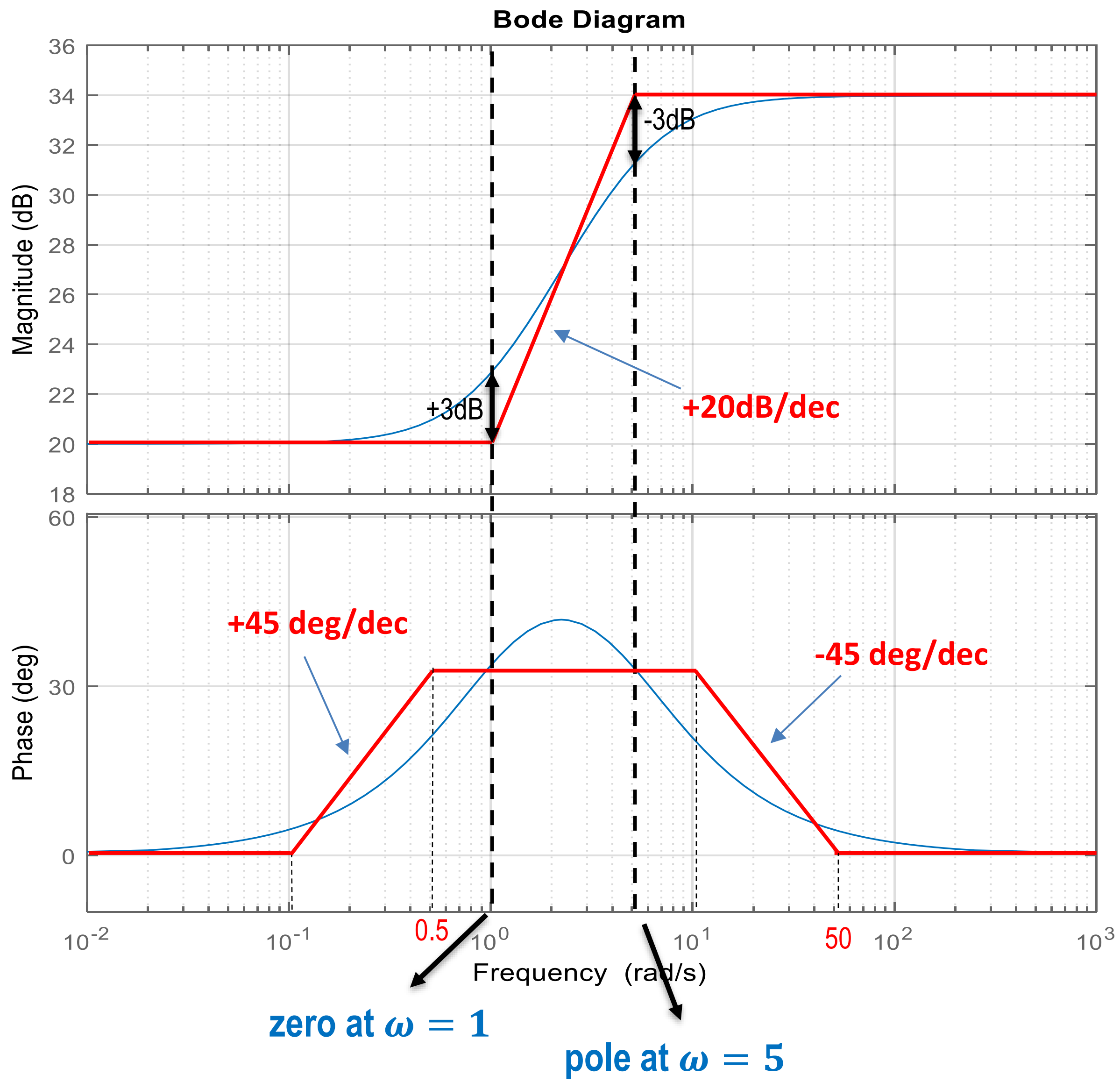
Example 2 Draw the Bode diagram for the following system

$$G(s) = 50\frac{s+1}{s+5}$$

We can plot the Bode diagram for $G(s)$ using **bode** function in MATLAB to compare.

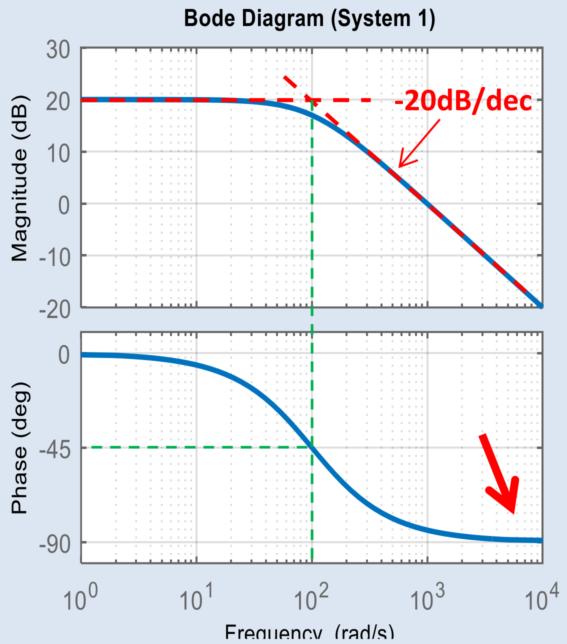
MATLAB

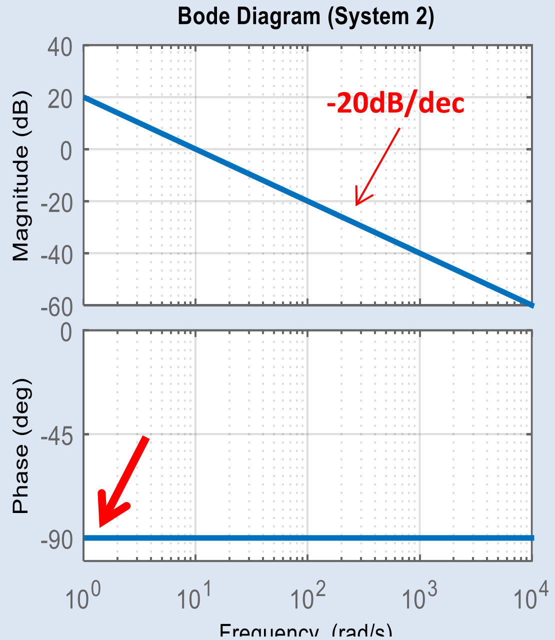
```
num = [50 50];
den = [1 5];
sys = tf(num,den);
figure; bode(sys)
```

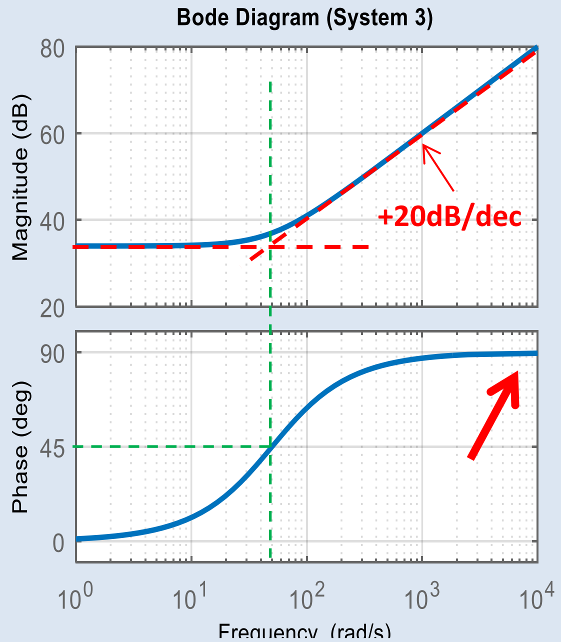


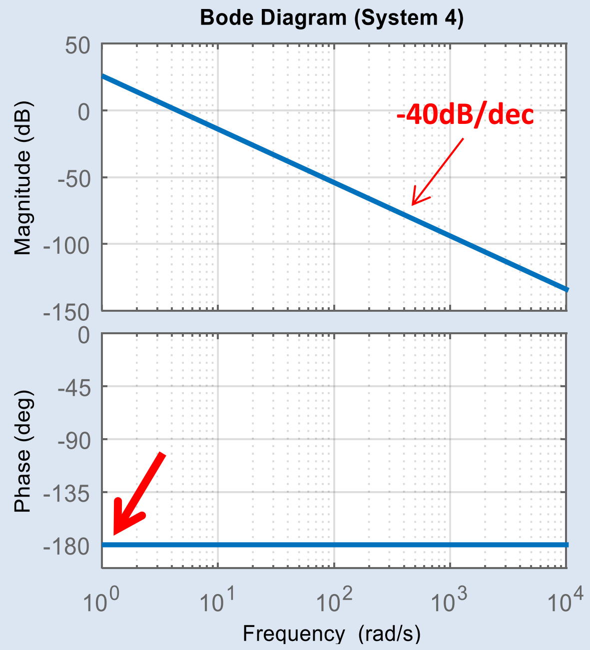
Quick Review

1. Match the Bode plots to the transfer functions.









a) $G(s) = \frac{10}{s}$

b) $G(s) = s + 50$

c) $G(s) = \frac{1000}{s+100}$

d) $G(s) = \frac{20}{s^2}$

Bode Diagram of Basic Factors

❑ Second-Order Factor: Complex Conjugate Poles ($0 < \zeta < 1$)

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\frac{\omega}{\omega_n}}$$

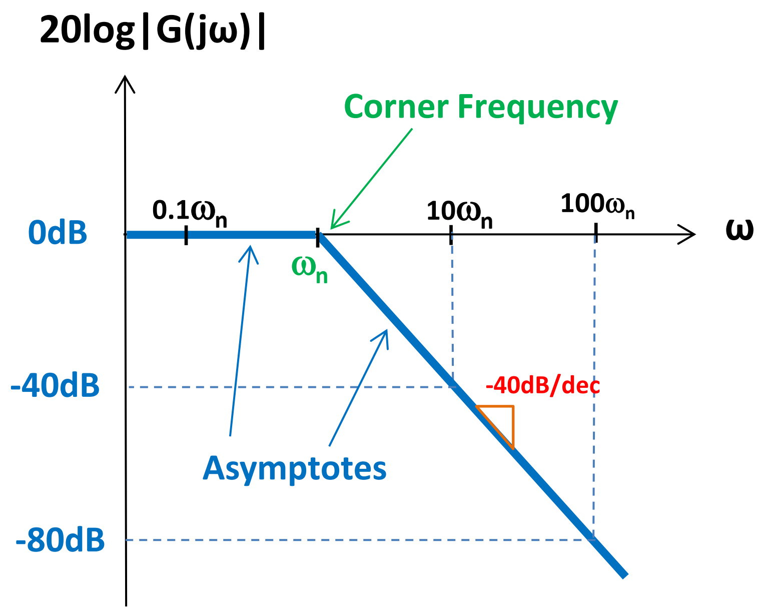
- **Bode Magnitude Plot**

$$20\log|G(j\omega)| = -20\log\left(\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}\right)$$

$$\rightarrow \begin{cases} \approx\ 0\text{dB} & \text{if } \omega \ll \omega_n \\ = -20\log(2\zeta)\ \text{dB} & \text{at } \omega = \omega_n. \\ \approx -40\log\left(\left|\frac{\omega}{\omega_n}\right|\right)\text{dB} & \text{if } \omega \gg \omega_n \end{cases}$$

The exact value of the magnitude plot at the corner frequency depends on the damping ratio ζ

$\omega = \omega_n \rightarrow$ Corner Frequency



- The Bode magnitude curve can be estimated with two asymptote lines:
  - Low-freq. asymptote ($\omega < \omega_n$) $\rightarrow$ line slope = 0
  - High-freq. asymptote ($\omega > \omega_n$) $\rightarrow$ line slope = -40dB/dec
  - The asymptotes intersect at the corner frequency $\omega = \omega_n$

Bode Diagram of Basic Factors

❑ Second-Order Factor: Complex Conjugate Poles ($0 < \zeta < 1$)

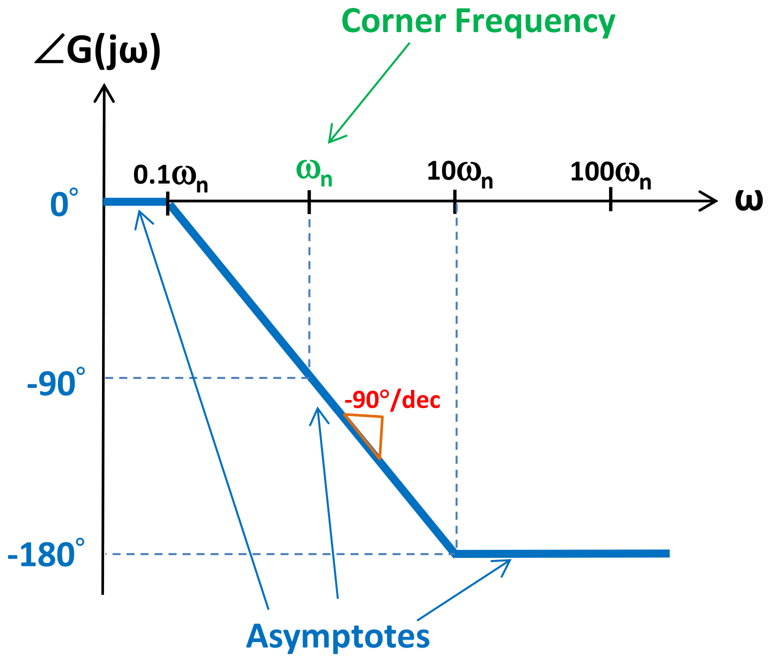
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\frac{\omega}{\omega_n}}$$

- **Bode Phase Plot**

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right) \rightarrow \begin{cases} \approx 0^\circ & if \quad \omega \ll 0.1\omega_n \\ = -90^\circ & at \quad \omega = \omega_n \rightarrow \textbf{corner freq.} \\ \approx -180^\circ & if \quad \omega \gg 10\omega_n \end{cases}$$

$\omega = \omega_n \rightarrow$ **Corner Frequency**



- The phase curve can be estimated with three asymptote lines.
  - High-freq. asymptote ($\omega > 10\omega_n$) $\rightarrow$ line slope = 0
  - Low-freq. asymptote ($\omega < 0.1\omega_n$) $\rightarrow$ line slope = 0
  - Middle-freq. asymptote ($0.1\omega_n < \omega < 10\omega_n$) $\rightarrow$ line to connect the high-freq. and the low-freq. asymptote lines together

Bode Diagram of Basic Factors

❑ Second-Order Factor: Complex Conjugate Poles $(0 < \zeta < 1)$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

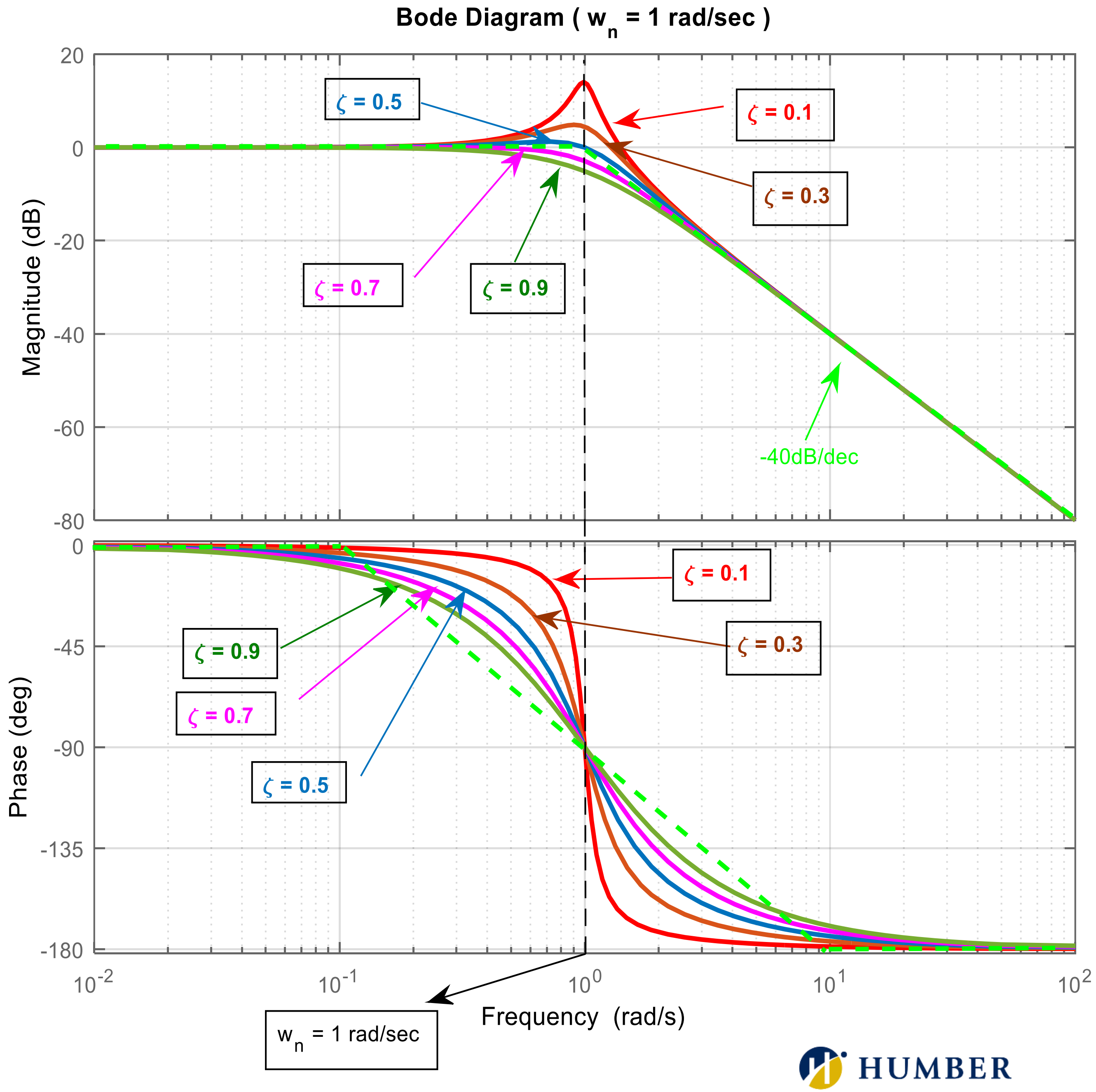
- The asymptote lines are acceptable only for damping ratio about $\zeta = 0.5$
- For $\zeta > 0.707$ there is no peak in the magnitude curve.
- For $0 < \zeta \leq 0.707$ the magnitude curve has a peak.
- Frequency and the magnitude of the peak are obtained as below:

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2}$$

Resonant Frequency

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

Resonant Peak



Bode Diagram Example

Example 3 Draw the Bode diagram for the following system

$$G(s) = \frac{36}{s^2 + 3.6s + 36}$$

First, rewrite the transfer function in the proper form and obtain the frequency response function $G(j\omega)$ and determine the basic factors.

$$G(s) = \frac{36}{s^2 + 3.6s + 36} = \frac{1}{\frac{s^2}{36} + \frac{s}{10} + 1} \longrightarrow G(j\omega) = \frac{1}{1 - \frac{\omega^2}{36} + j\frac{\omega}{10}}$$

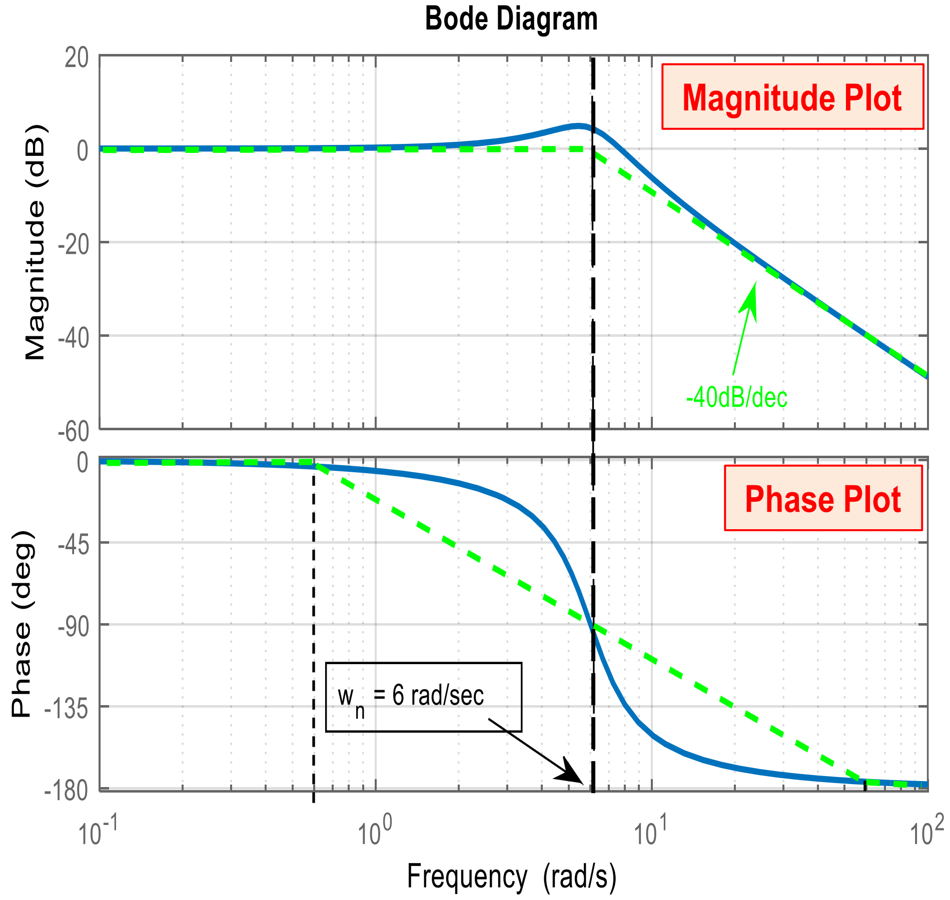
Complex Pole

$$s^2 + 3.6s + 36 = s^2 + 2\zeta\omega_n s + \omega_n^2 \longrightarrow \boxed{\zeta = 0.3} \quad \boxed{\omega_n = 6}$$

Corner frequency → ω_n = 6rad/sec

Resonant Peak → $M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1.75 = 4.86\text{dB}$

Resonant Frequency → $\omega_r = \omega_n\sqrt{1-2\zeta^2} = 5.43 \text{ rad/sec}$



Bode Diagram Example

Example 4 Draw the Bode diagram for the following system

$$G(s) = 100\frac{s+0.5}{s^2(s^2+3.6s+36)}$$

First, rewrite the transfer function in the proper form and obtain the frequency response function $G(j\omega)$ and determine the basic factors.

$$G(s) = 100\frac{s+0.5}{s^2(s^2+3.6s+36)} = 100\frac{0.5\left(\frac{s}{0.5}+1\right)}{36\,s^2\left(\frac{s^2}{36}+\frac{s}{10}+1\right)} = \frac{50}{36}\cdot\frac{\frac{s}{0.5}+1}{s^2\left(\frac{s^2}{36}+\frac{s}{10}+1\right)}$$

$$\mathrm{G}(j\omega) = \frac{50}{36}\cdot\frac{\frac{j\omega}{0.5}+1}{(j\omega)^2\left(\frac{(j\omega)^2}{36}+\frac{j\omega}{10}+1\right)} = \underbrace{\left(\frac{50}{36}\right)}_{\text{Constant Gain}}\underbrace{\left(1+\frac{j\omega}{0.5}\right)}_{\text{Single Zero}}\underbrace{\left(\frac{1}{j\omega}\right)^2}_{\text{Second-order Integrator}}\underbrace{\left(\frac{1}{1-\frac{\omega^2}{36}+j\frac{\omega}{10}}\right)}_{\text{Complex Pole}}$$

Next, plot the **asymptotic Bode diagram** for each basic factor separately, and then add them together to construct the overall Bode diagram of $G(j\omega)$

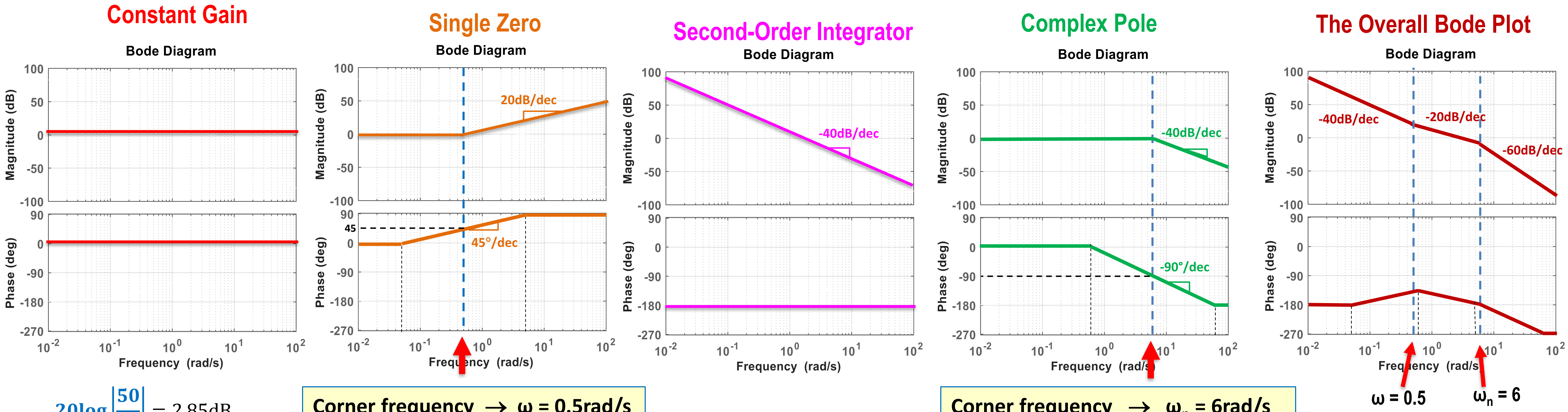
Bode Diagram Example

Example 4 Draw the Bode diagram for the following system

$$G(s) = 100\,\frac{s+0.5}{s^2(s^2+3.6s+36)}$$

$$|G(j\omega)|\text{dB} = 20\log\left(\left|\frac{50}{36}\right|\right) + 20\log\left(\left|1+\frac{j\omega}{0.5}\right|\right) + 20\log\left(\left|\frac{1}{(j\omega)^2}\right|\right) + 20\log\left(\left|\frac{1}{1-\frac{\omega^2}{36}+j\frac{\omega}{10}}\right|\right)$$

$$\angle G(j\omega) = \angle\left(\frac{50}{36}\right) + \angle\left(1+\frac{j\omega}{0.5}\right) + \angle\left(\frac{1}{(j\omega)^2}\right) + \angle\left(\frac{1}{1-\frac{\omega^2}{36}+j\frac{\omega}{10}}\right)$$



$$20\log\left|\frac{50}{36}\right| = 2.85\text{dB}$$

Corner frequency $\rightarrow$ $\omega = 0.5$rad/s

Corner frequency $\rightarrow$ $\omega_n = 6$rad/s

Bode Diagram Example

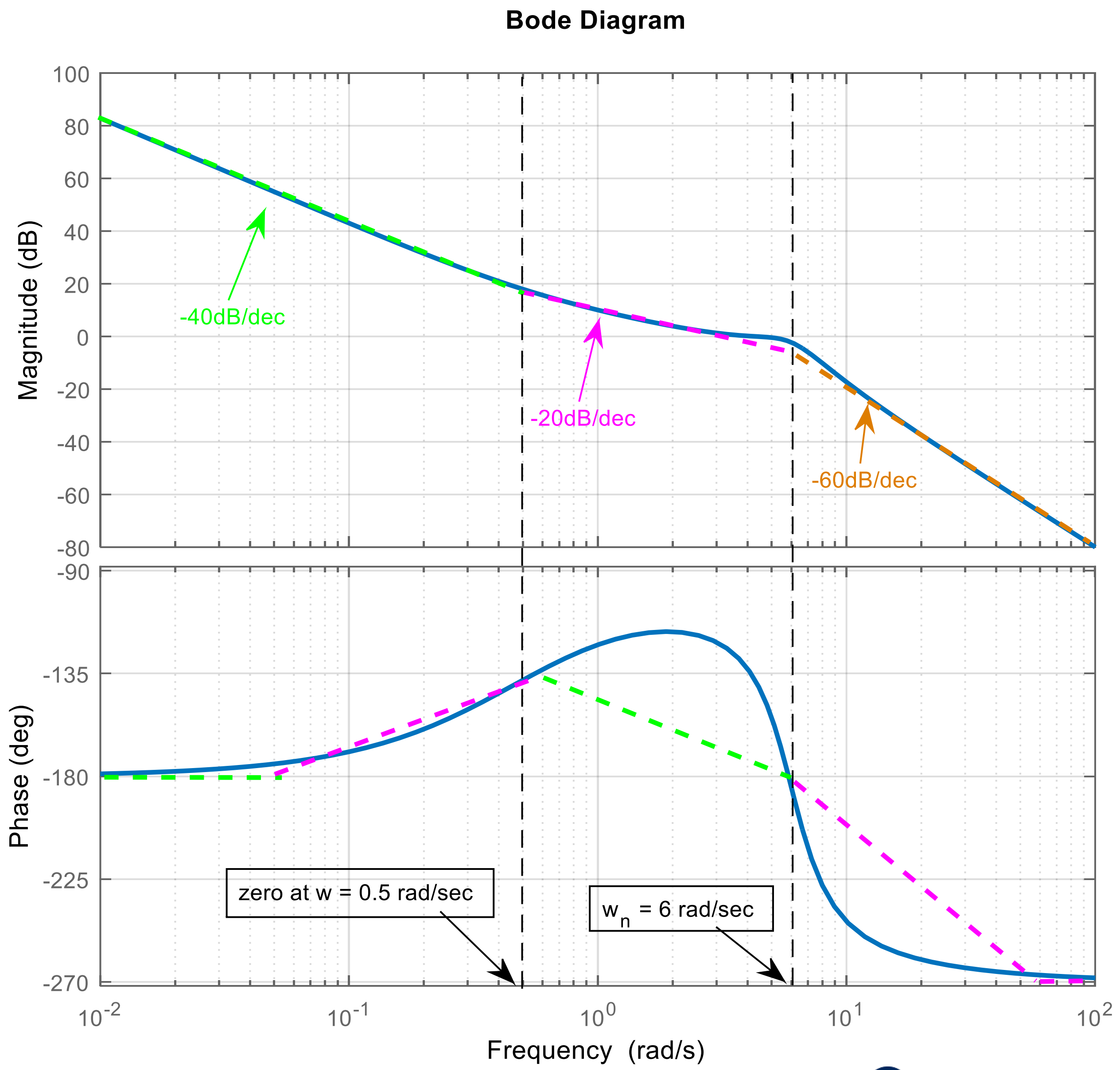
Example 4 Draw the Bode diagram for the following system

$$G(s) = 100\frac{s+0.5}{s^2(s^2+3.6s+36)}$$

We can plot the Bode diagram for $G(s)$ using **bode** function in MATLAB to compare.

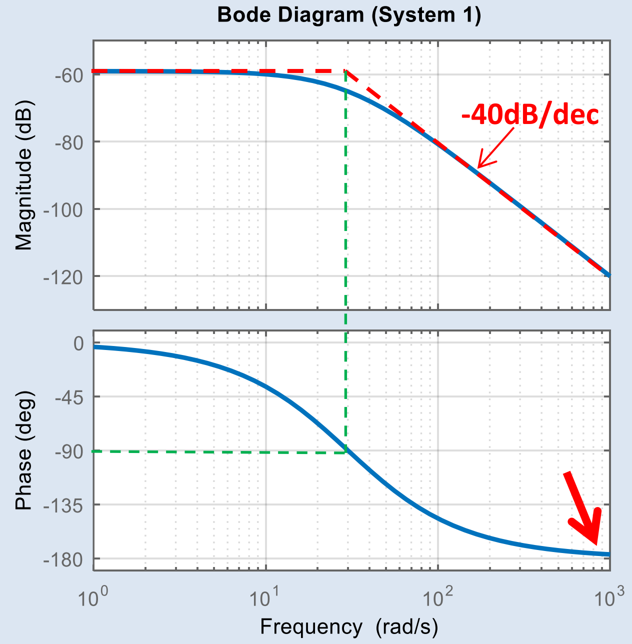
MATLAB

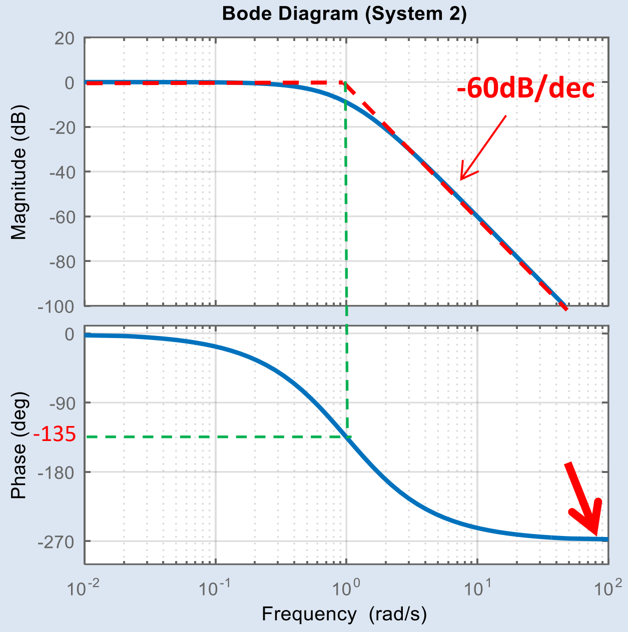
```
num = [100 50];
den = [1 3.6 36 0 0];
sys = tf(num,den);
figure; bode(sys)
```

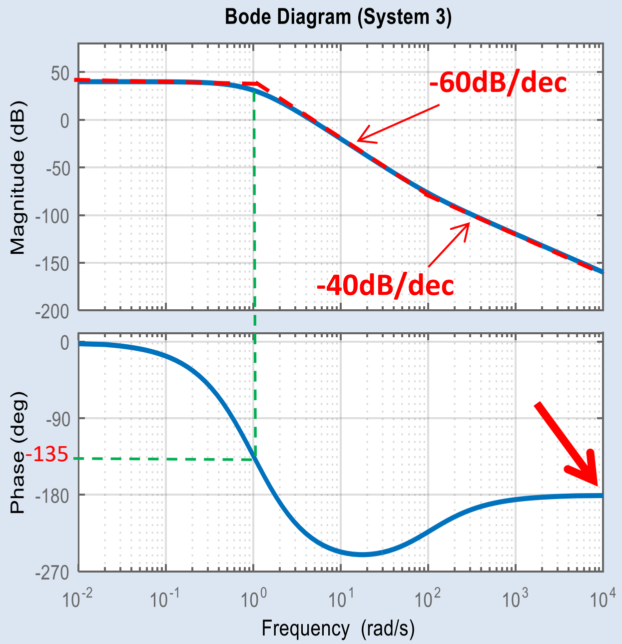


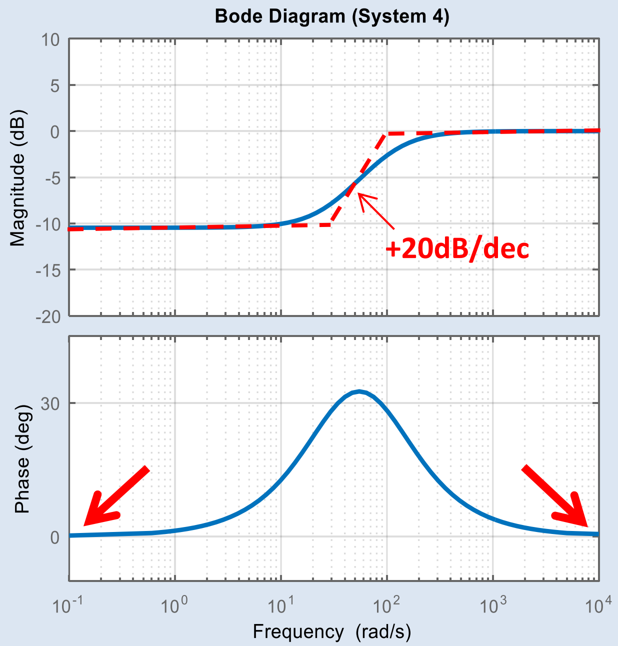
Quick Review

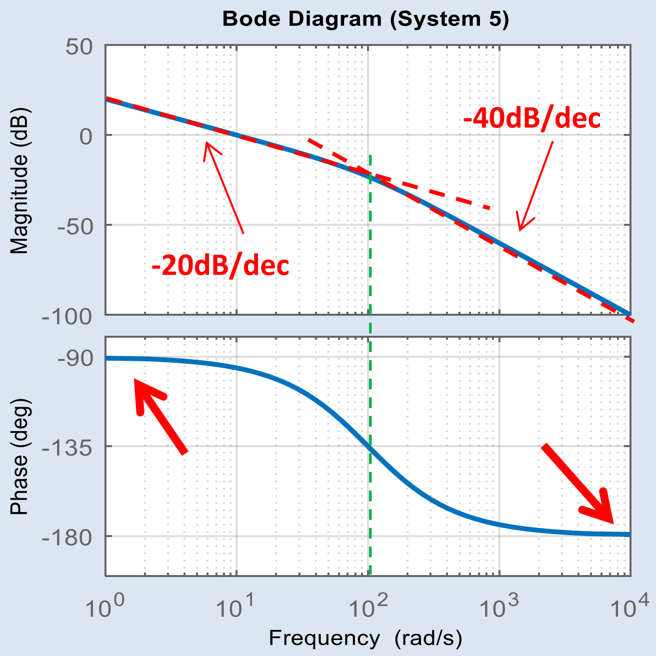
1. Match the Bode plots to the transfer functions.











a) $G(s) = \dfrac{s+100}{(s+1)^3}$

b) $G(s) = \dfrac{1}{(s+30)^2}$

c) $G(s) = \dfrac{1}{(s+1)^3}$

d) $G(s) = \dfrac{1000}{s(s+100)}$

e) $G(s) = \dfrac{s+30}{s+100}$

Bode Magnitude Diagram Plotting Guidelines

Consider the following transfer function G(s) as a product of basic factors from

$$G(s) = \frac{K(s+z_1)(s+z_2)}{s^\beta (s+p_1)(s+p_2)(s^2+2\zeta\omega_n s+\omega_n^2)}$$

Step 1: Determine all the basic factors of the $G(j\omega)$

Step 2: Determine all corner frequencies of the first-order and second-order factors.

Step 3: Find the starting point and the starting slope at low frequencies:

Starting Slope $\boxed{-20\beta \text{ dB/dec}}$ $\boxed{20\log\left|\frac{K_B}{(j\omega)^\beta}\right|}$ **Starting Point**

where K_B is the **DC-gain of** $G(s)$ when $\beta = 0$ (no derivative or integrator terms).

Step 4: Draw the asymptote lines

At each single pole's corner frequency add **-20dB/dec** to the slope.

At each single zero's corner frequency add **+20dB/dec** to the slope.

At each second-order pole's corner frequency add **-40dB/dec** to the slope.

At each second-order zero's corner frequency add **+40dB/dec** to the slope.

Step 5: The exact curve can be obtained by adding proper corrections on the graph.

Bode Magnitude Diagram – Example

Example 5 Draw the Bode magnitude diagram for the following transfer function

$$G(s) = 100\frac{s+0.5}{s^2(s^2+3.6s+36)}$$

First, rewrite the transfer function in the proper form

$$G(s) = 100\frac{s+0.5}{s^2(s^2+3.6s+36)} = 100\frac{0.5\left(\frac{s}{0.5}+1\right)}{36\,s^2\left(\frac{s^2}{36}+\frac{s}{10}+1\right)} = \frac{50}{36}\cdot\frac{\frac{s}{0.5}+1}{s^2\left(\frac{s^2}{36}+\frac{s}{10}+1\right)}$$

Step 1: Determine all the basic factors of the $G(j\omega)$

$$G(j\omega) = (1.39)\left(1+\frac{j\omega}{0.5}\right)\left(\frac{1}{j\omega}\right)^2\left(\frac{1}{1-\frac{\omega^2}{36}+j\frac{\omega}{10}}\right)$$

Constant Gain

Single Zero

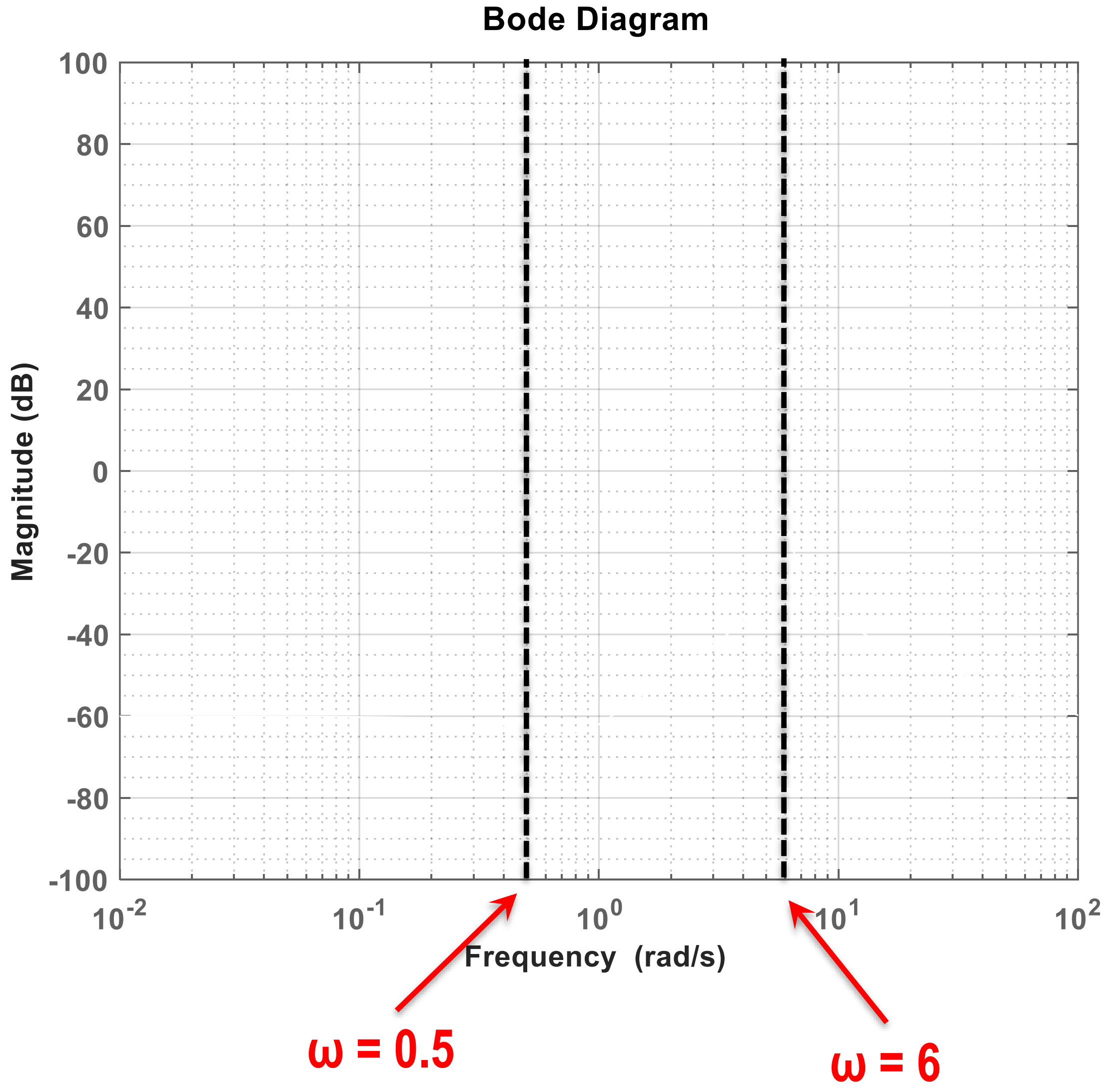
Second-order Integrator

Complex Pole

Step 2: Determine all corner frequencies of the first-order and second-order factors.

Corner frequency of the single zero $\rightarrow$ $\omega = 0.5$ rad/sec

Corner frequency of the complex pole $\rightarrow$ $\omega = \omega_n = 6$ rad/sec



Bode Magnitude Diagram – Example

Example 5 Draw the Bode magnitude diagram for the following transfer function

$$G(s) = 100\frac{s+0.5}{s^2(s^2+3.6s+36)}$$

Step 3: Find the starting point and the starting slope at low frequencies:

Starting Point $\rightarrow$ $\boxed{20\log\left|\frac{K_B}{(j\omega)^\beta}\right|}$

$$20\log\left|\frac{1.39}{(j\omega)^2}\right| = 20\log|1.39| - 20\log|(j\omega)^2| = 2.86\text{dB} - 40\log(\omega)\text{dB}$$

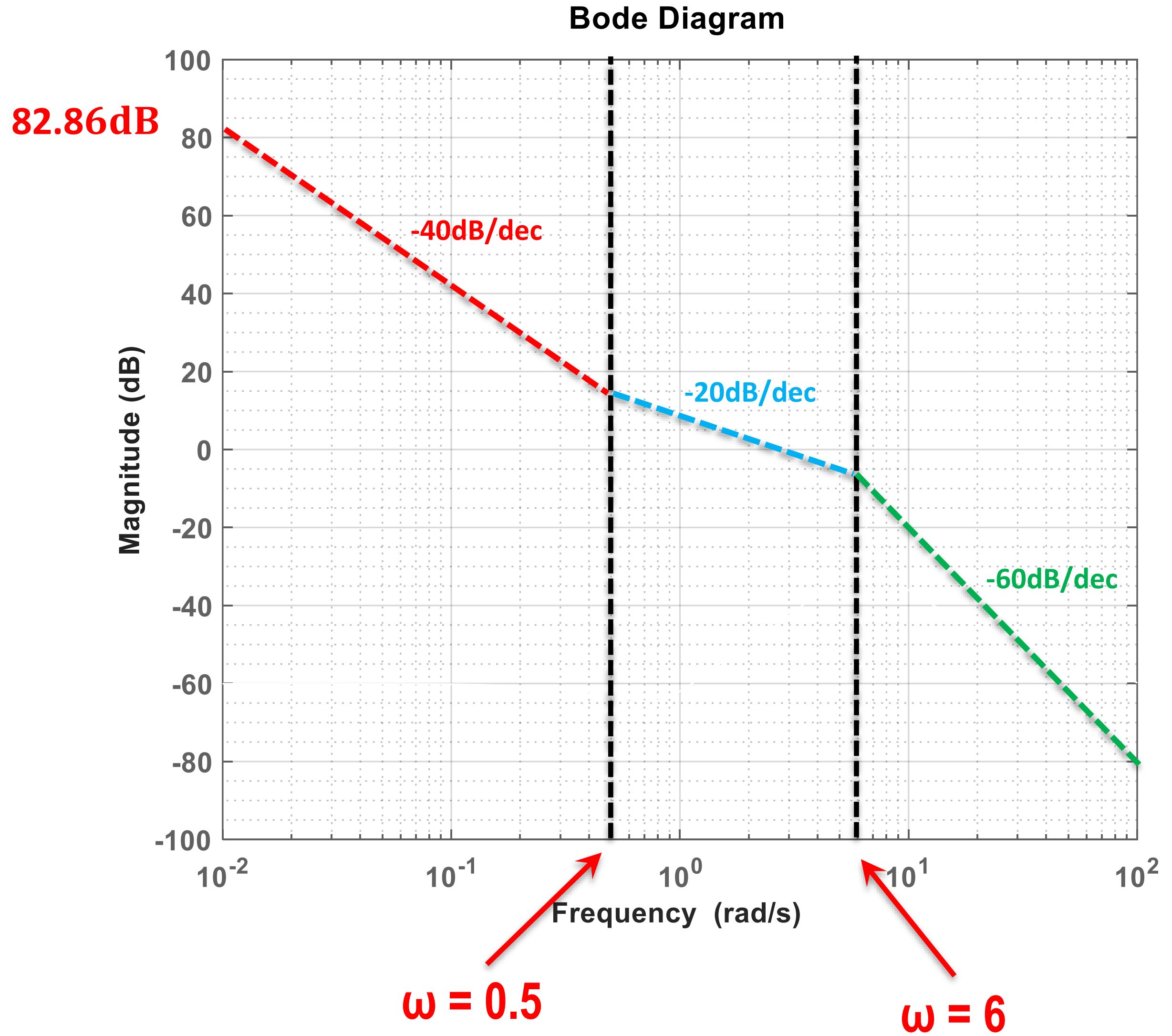
Assume that the given graph starts at frequency of $\omega = 0.01$ rad/sec

- **The Starting Point** $\rightarrow$

$$2.86\text{dB} - 40\log(0.01)\,\text{dB} = 2.86\text{dB} + 80\text{dB} = \mathbf{82.86dB}$$

Starting Slope $\rightarrow$ $\boxed{-20\beta\ \text{dB/dec}}$

- **The Starting Slope** $\rightarrow$ $-20\beta = -20 \times 2 =$ **-40dB/dec**



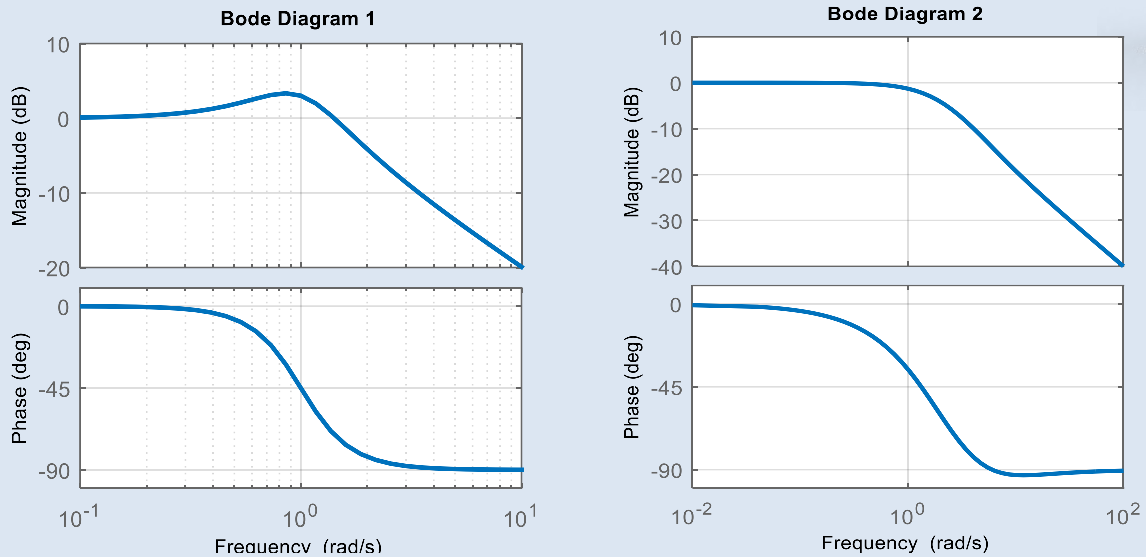
Step 4: Draw the asymptote lines

Quick Review

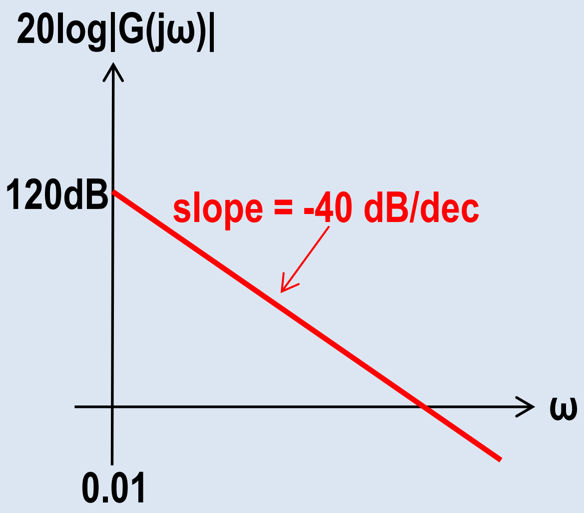
1) Match the transfer functions with the following Bode plots.

$$G_1(s) = \frac{s+1}{s^2+s+1}$$

$$G_2(s) = \frac{s+6}{s^2+5s+6}$$



2) Which system represents the following Bode magnitude plot?



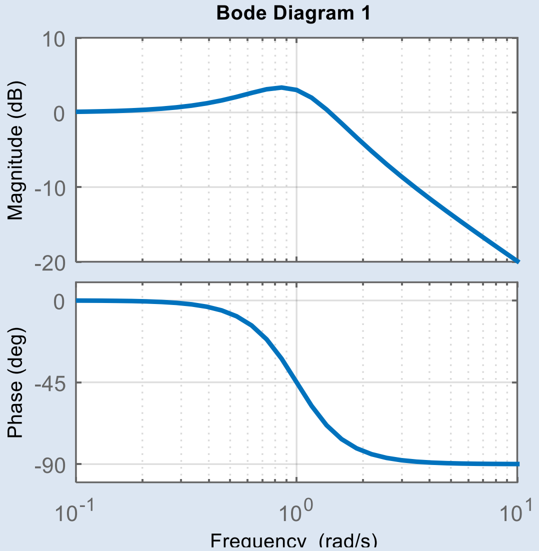
a) $G(s) = \frac{10}{s^2}$ **b)** $G(s) = \frac{100}{s(s+1)}$ **c)** $G(s) = \frac{10}{s(s+1)}$ **d)** $G(s) = \frac{100}{s^2}$

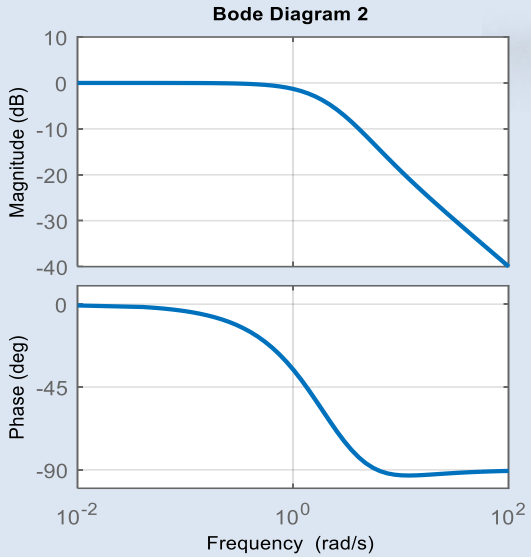
Quick Review

1) Match the transfer functions with the following Bode plots.

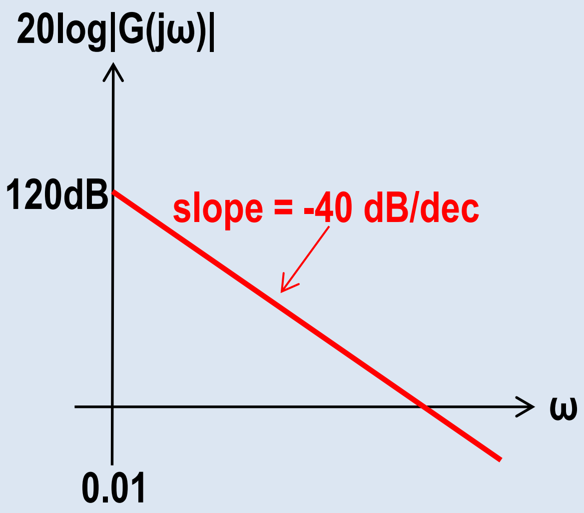
$$G_1(s) = \frac{s+1}{s^2+s+1} \qquad \boxed{1}$$

$$G_2(s) = \frac{s+6}{s^2+5s+6} \qquad \boxed{2}$$





2) Which system represents the following Bode magnitude plot?



a) $G(s) = \frac{10}{s^2}$ **b)** $G(s) = \frac{100}{s(s+1)}$ **c)** $G(s) = \frac{10}{s(s+1)}$ **d)** $G(s) = \frac{100}{s^2}$

The starting point at ω = 0.01rad/sec is 120dB

$$120\text{dB} = 20\log\left|\frac{K_B}{(j\omega)^{\beta}}\right| = 20\log\left|\frac{K_B}{(j0.01)^2}\right| = 20\log|K_B| - 20\log|0.0001|$$

$$20\log_{10}|K_B| = 120\text{dB} - 80\text{dB} = 40\text{dB} \quad \rightarrow \quad \boldsymbol{K_B = 10^{\frac{40}{20}} = 100}$$

Nyquist Diagram

Nyquist Diagram

- To study the systems in frequency-domain, we must analyze behavior of the $G(j\omega)$ in different frequencies.
- $G(j\omega)$ is a complex quantity and can be represented by
  - The magnitude and phase-angle
  - The real part and imaginary part

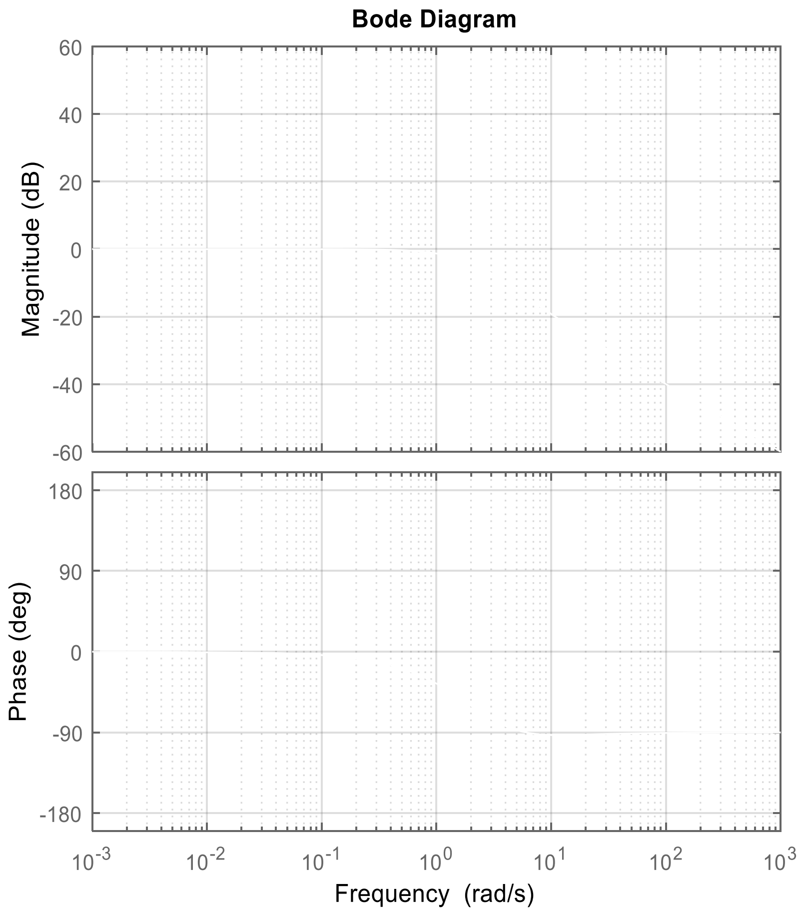
$$\boxed{G(j\omega) = |G(j\omega)|\angle G(j\omega)}$$

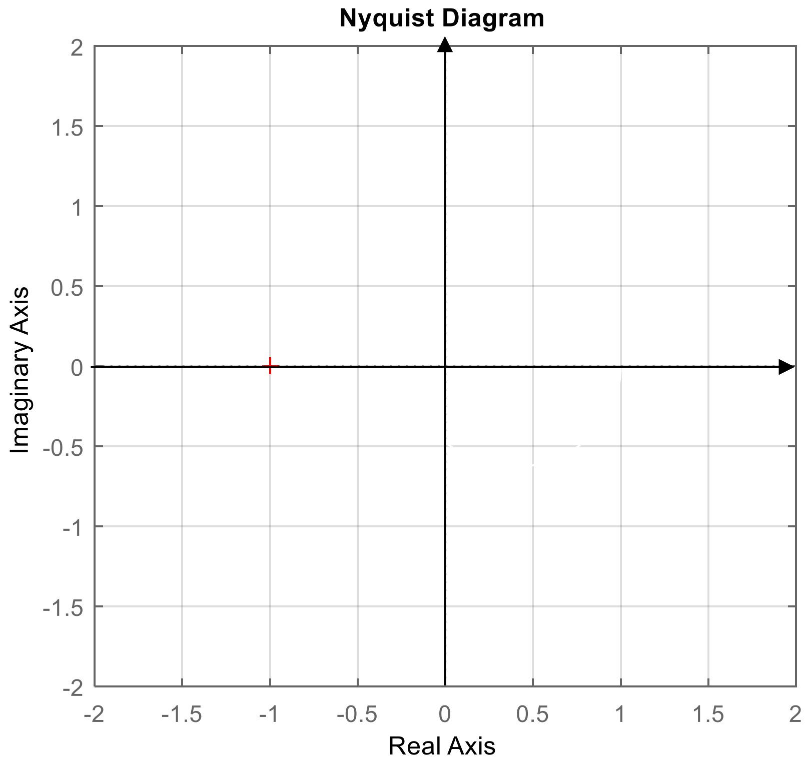
$$\boxed{G(j\omega) = \mathrm{Re}[G(j\omega)] + j\mathrm{Im}[G(j\omega)]}$$

- There are two commonly used graphical techniques to analyze the $G(j\omega)$:

Logarithmic Plot → Bode Diagram

Polar Plot → Nyquist Diagram





- Polar plots are not terribly useful as a means of visualizing and displaying a frequency response
- Useful in control system design – ***Nyquist stability criterion*** which has been developed by **Harry Nyquist** a Swedish engineer based on the Nyquist diagram..

Nyquist Diagram

Example 6 Consider the following second-order system and plot the Nyquist diagram of this system.

$$G(s) = \frac{2}{s^2 + 3s + 2}$$

First, find the Frequency response function of the system

$$\boldsymbol{G(j\omega) = G(s)\Big|_{s=j\omega}} \longrightarrow G(j\omega) = \frac{2}{(j\omega)^2 + 3(j\omega) + 2} = \frac{2}{-\omega^2 + j3\omega + 2}$$

Determine the magnitude and the phase angle of the $G(j\omega)$

$$\boxed{\boldsymbol{G(j\omega) = |G(j\omega)|\angle G(j\omega)}}$$

$$|G(j\omega)| = \frac{|2|}{|(2-\omega^2) + j(3\omega)|} = \frac{2}{\sqrt{(2-\omega^2)^2 + (3\omega)^2}} = \frac{2}{\sqrt{(\omega^2+4)(\omega^2+1)}}$$

$$\angle G(j\omega) = \angle\left(\frac{2}{(2-\omega^2) + j(3\omega)}\right) = \angle 2 - \angle(2 - \omega^2 + j3\omega) = -\tan^{-1}\left(\frac{3\omega}{2-\omega^2}\right)$$

Determine the real part and the imaginary part of the $G(j\omega)$

$$\boxed{\boldsymbol{G(j\omega) = \mathrm{Re}[G(j\omega)] + j\mathrm{Im}[G(j\omega)]}}$$

$$G(j\omega) = \frac{2}{(2-\omega^2) + j(3\omega)} \times \frac{(2-\omega^2) - j(3\omega)}{(2-\omega^2) - j(3\omega)} = \frac{2(2-\omega^2 - j3\omega)}{(2-\omega^2)^2 + (3\omega)^2}$$

$$G(j\omega) = \frac{2(2-\omega^2 - j3\omega)}{\omega^4 + 5\omega^2 + 4} = \underbrace{\frac{2(2-\omega^2)}{\omega^4 + 5\omega^2 + 4}}_{\textbf{\textit{real part}}} + j\underbrace{\frac{-2(3\omega)}{\omega^4 + 5\omega^2 + 4}}_{\textbf{\textit{imaginary part}}}$$

Nyquist Diagram

Example 6 Consider the following second-order system and plot the Nyquist diagram of this system.

$$G(s) = \frac{2}{s^2 + 3s + 2}$$

Following Table shows the values for the magnitude, phase shift, real part and imaginary part of $G(j\omega)$ for $\boldsymbol{\omega} = [\mathbf{0}, +\infty)$

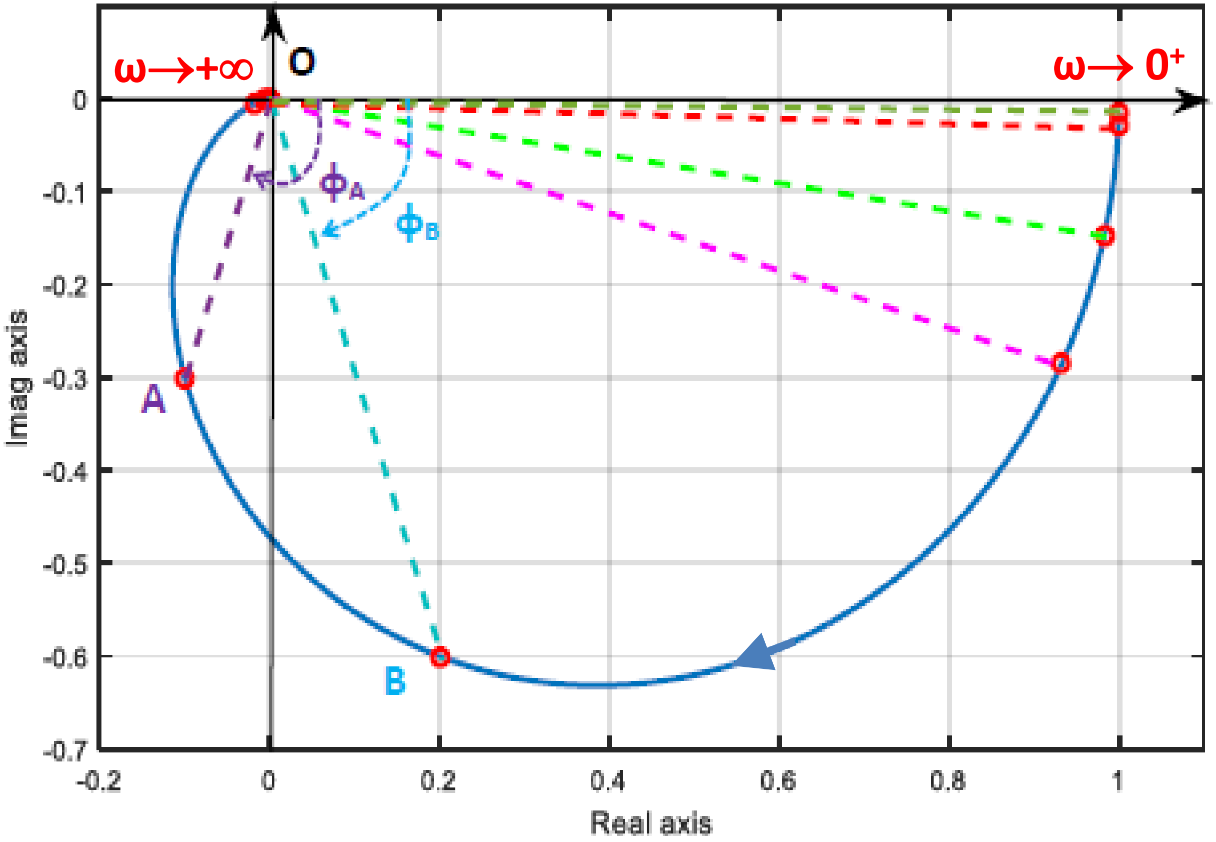
$$|G(j\omega)| = \frac{2}{\sqrt{(\omega^2+4)(\omega^2+1)}} \qquad \angle G(j\omega) = -\tan^{-1}\left(\frac{3\omega}{2-\omega^2}\right)$$

$$G(j\omega) = \underbrace{\frac{2(2-\omega^2)}{\omega^4+5\omega^2+4}}_{\textit{real part}} + j\underbrace{\frac{-2(3\omega)}{\omega^4+5\omega^2+4}}_{\textit{imaginary part}}$$

Frequency ω (rad/sec)	Real part Re[G(jω)]	Imaginary part Im[G(jω)]	Magnitude |G(jω)|	Phase shift ∠G(jω) (deg)
0.01	0.9998	-0.0150	0.9999	-0.86°
0.02	0.9993	-0.0300	0.9998	-1.72°
0.1	0.9827	-0.1481	0.9938	-8.57°
0.2	0.9330	-0.2856	0.9757	-17.02°
1	**0.2**	**-0.6000**	**0.6325**	**-71.57°**
2	**-0.1**	**-0.3000**	**0.3162**	**-108.43°**
10	-0.0187	-0.0057	0.0195	-162.97°
20	-0.0049	-0.0007	0.0050	-171.43°
100	-0.0002	-0.0000	0.0002	-178.28°
200	-0.000	-0.0000	0.0000	-179.14°

← Point B

← Point A



- Points A and B are shown as an example on the graph

Nyquist Diagram

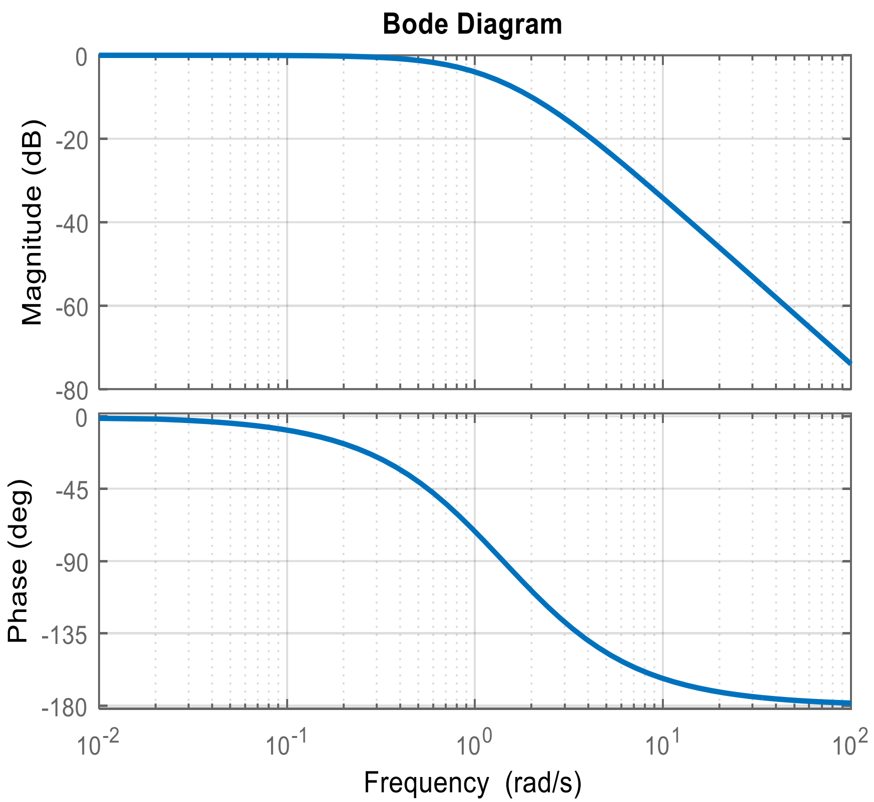
Example 6

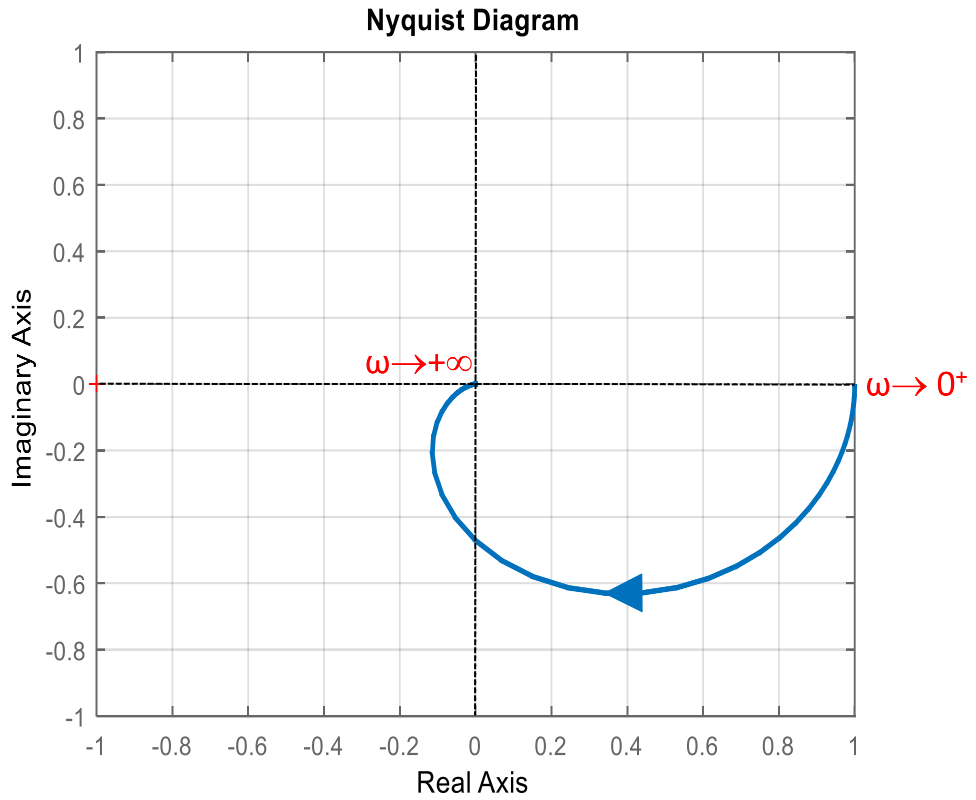
Consider the following second-order system and plot the Nyquist diagram of this system.

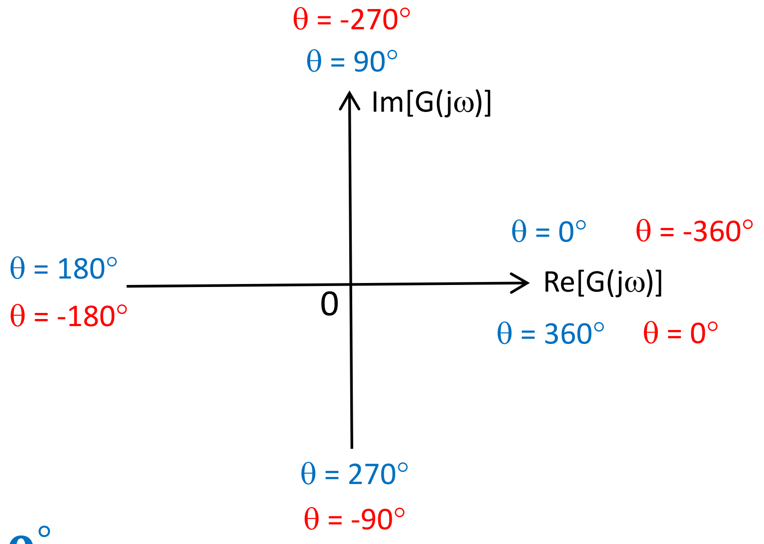
$$G(s) = \frac{2}{s^2 + 3s + 2}$$

We can plot the Bode diagram and the Nyquist diagram of G(s) using MATLAB and compare them.

```
num = [2];
den = [1 3 2];
G = tf(num,den);
figure; nyquist(G)
figure; bode(G)
```







Starting point → For $\omega \to 0^+ \quad \Rightarrow \quad \boldsymbol{G(j0) = 1 \angle 0^\circ}$

Ending point → For $\omega \to +\infty \quad \Rightarrow \quad \boldsymbol{G(j\infty) = 0 \angle -180^\circ}$

General Shape of Nyquist Diagram

- The general shape of the Nyquist diagram depends on the number of poles/zeros and the type of the system.

❑ Type 0 Systems

- Starting point ($\omega = 0^+$) is finite and located on the positive real axis.
  Polar plot starts perpendicular to the real axis.
- Ending point ($\omega \to +\infty$) is at the origin, the plot is tangent to one of the axes.

$$\text{For } \omega \to 0^+ \quad \Rightarrow \quad G(j0^+) = K_{dc} \angle 0^\circ$$

$$\text{For } \omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0 \angle \theta^\circ$$

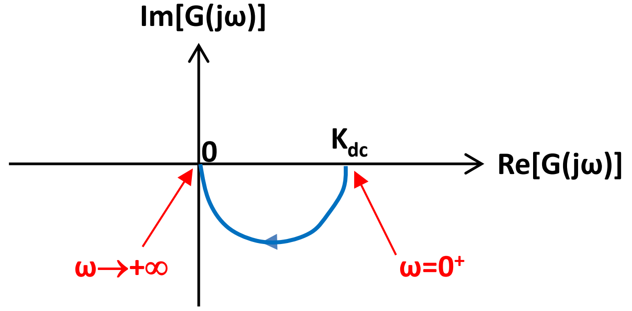
Depends on the number of poles/zeros

- Here are examples of different type 0 systems and their Nyquist diagram to compare.

$$G(j\omega) = \frac{K}{(j\omega + p_1)}$$

$$\text{For } \omega \to 0^+ \quad \Rightarrow \quad G(j0^+) = K_{dc}\angle 0^\circ$$

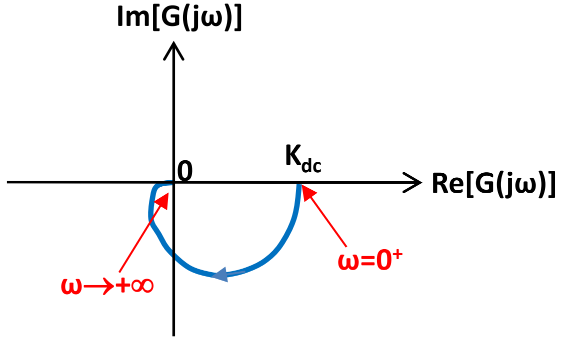
$$\text{For } \omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0\angle -90^\circ$$



$$G(j\omega) = \frac{K}{(j\omega + p_1)(j\omega + p_2)}$$

$$\text{For } \omega \to 0^+ \quad \Rightarrow \quad G(j0^+) = K_{dc}\angle 0^\circ$$

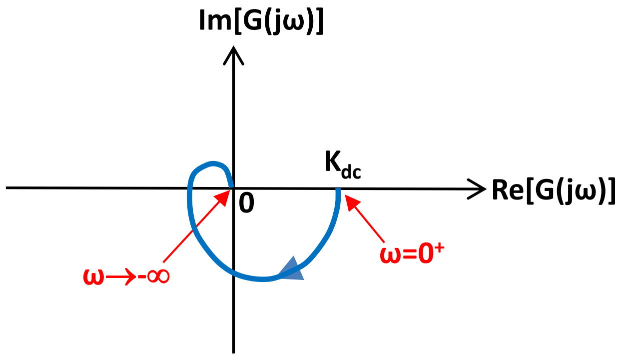
$$\text{For } \omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0\angle -180^\circ$$



$$G(j\omega) = \frac{K}{(j\omega + p_1)(j\omega + p_2)(j\omega + p_3)}$$

$$\text{For } \omega \to 0^+ \quad \Rightarrow \quad G(j0^+) = K_{dc}\angle 0^\circ$$

$$\text{For } \omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0\angle -270^\circ$$



General Shape of Nyquist Diagram

- The general shape of the Nyquist diagram depends on the number of poles/zeros and the type of the system.

❑ Type 1 Systems

- Magnitude at the starting point ($\omega = 0^+$) is infinity, and the phase angle is -90°.
- Ending point ($\omega \to +\infty$) is at the origin, the plot is tangent to one of the axes.
- At low frequencies ($\omega \to 0$) the polar plot is asymptotic to a line, which is parallel to the negative imaginary axis

$$\boxed{\textbf{For } \omega \to 0^+ \quad \Rightarrow \quad G(j0^+) = \infty\angle -90^\circ}$$

$$\boxed{\textbf{For } \omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0\angle\theta^\circ}$$

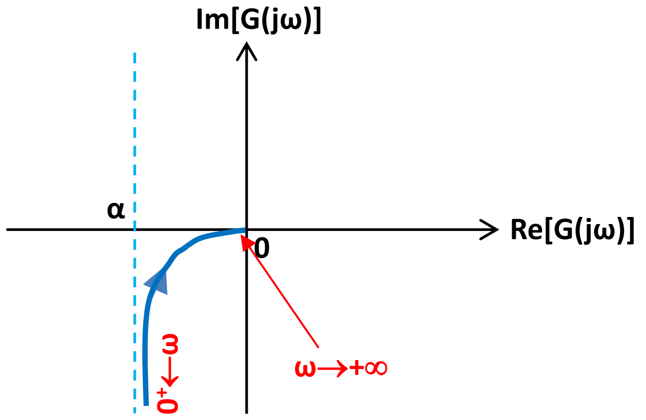
Depends on the number of poles/zeros

- Here are examples of different type 1 systems and their Nyquist diagram to compare.

$$\boxed{G(j\omega) = \frac{K}{j\omega(j\omega + p_1)}}$$

$$\text{For } \omega \to 0^+ \quad \Rightarrow \quad G(j0^+) = \infty\angle -90^\circ$$

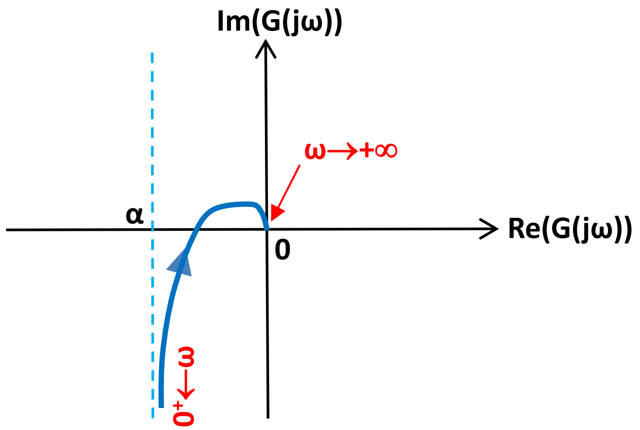
$$\text{For } \omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0\angle -180^\circ$$



$$\boxed{G(j\omega) = \frac{K}{j\omega(j\omega + p_1)(j\omega + p_2)}}$$

$$\text{For } \omega \to 0^+ \quad \Rightarrow \quad G(j0^+) = \infty\angle -90^\circ$$

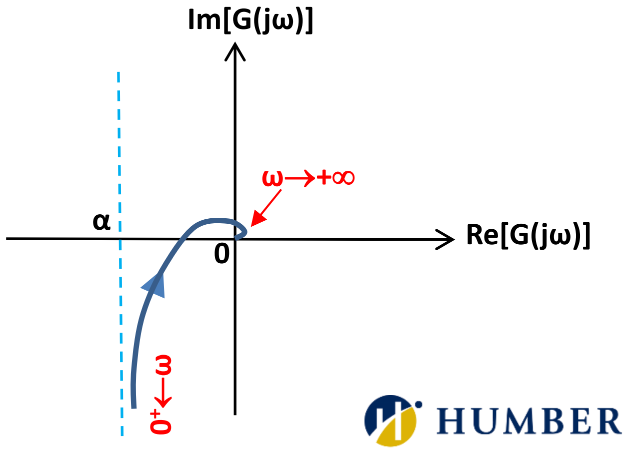
$$\text{For } \omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0\angle -270^\circ$$



$$\boxed{G(j\omega) = \frac{K}{j\omega(j\omega + p_1)(j\omega + p_2)(j\omega + p_3)}}$$

$$\text{For } \omega \to 0^+ \quad \Rightarrow \quad G(j0^+) = \infty\angle -90^\circ$$

$$\text{For } \omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0\angle -360^\circ$$



General Shape of Nyquist Diagram

Example 7 Consider the following second-order system and plot the Nyquist diagram of this system. $\boldsymbol{G(s) = \dfrac{10}{(s+1)(s+2)}}$

Find the frequency response function $G(j\omega)$

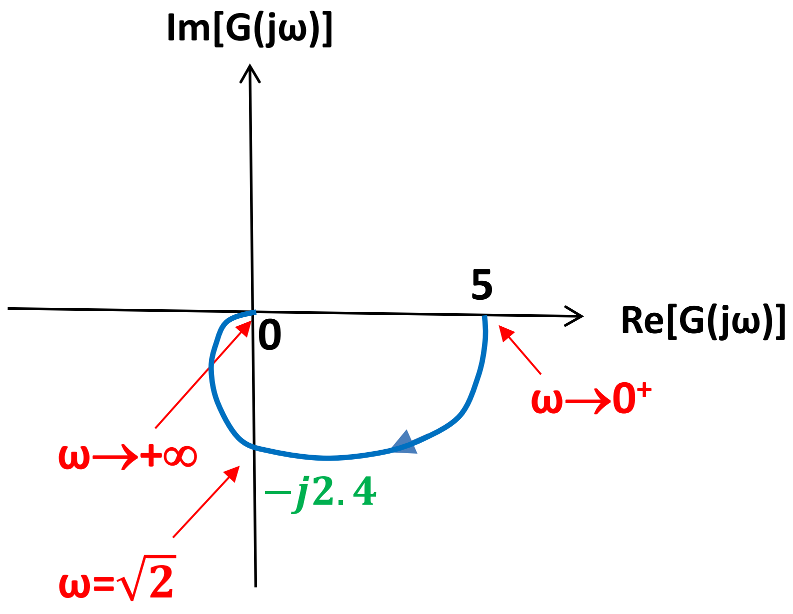
$$G(s) = \frac{10}{(s+1)(s+2)} \quad \Longrightarrow \quad G(j\omega) = \frac{10}{(j\omega+1)(j\omega+2)}$$

Determine starting point and ending point of the polar plot

Starting point $\rightarrow$ $\quad$ For $\omega \rightarrow 0^+ \quad \Rightarrow \quad G(j0) = 5\angle 0^{\circ}$

Ending point $\rightarrow$ $\quad$ For $\omega \rightarrow +\infty \quad \Rightarrow \quad G(j\infty) = 0\angle -180^{\circ}$

For $\omega \rightarrow +\infty$ the graph is tangent to the negative real axis.



Find the intersection of the Polar plot with the real and imaginary axes.

$$G(j\omega) = \frac{10}{(j\omega+1)(j\omega+2)} = \underbrace{\frac{10(2-\omega^2)}{9\omega^2+(2-\omega^2)^2}}_{real\ part} + j\underbrace{\frac{-30\omega}{9\omega^2+(2-\omega^2)^2}}_{imaginary\ part}$$

$$\mathbf{Re[G(j\omega)] = 0} \rightarrow \frac{10(2-\omega^2)}{9\omega^2+(2-\omega^2)^2} = 0 \rightarrow \omega^2 = 2 \rightarrow \boxed{\omega = \sqrt{2}} \qquad \boxed{\omega = \infty}$$

Intersection with the imaginary axis $\rightarrow$ $\boxed{G(\pm j\sqrt{2}) = -j2.4}$ $\qquad \boxed{G(\infty) = 0}$

General Shape of Nyquist Diagram

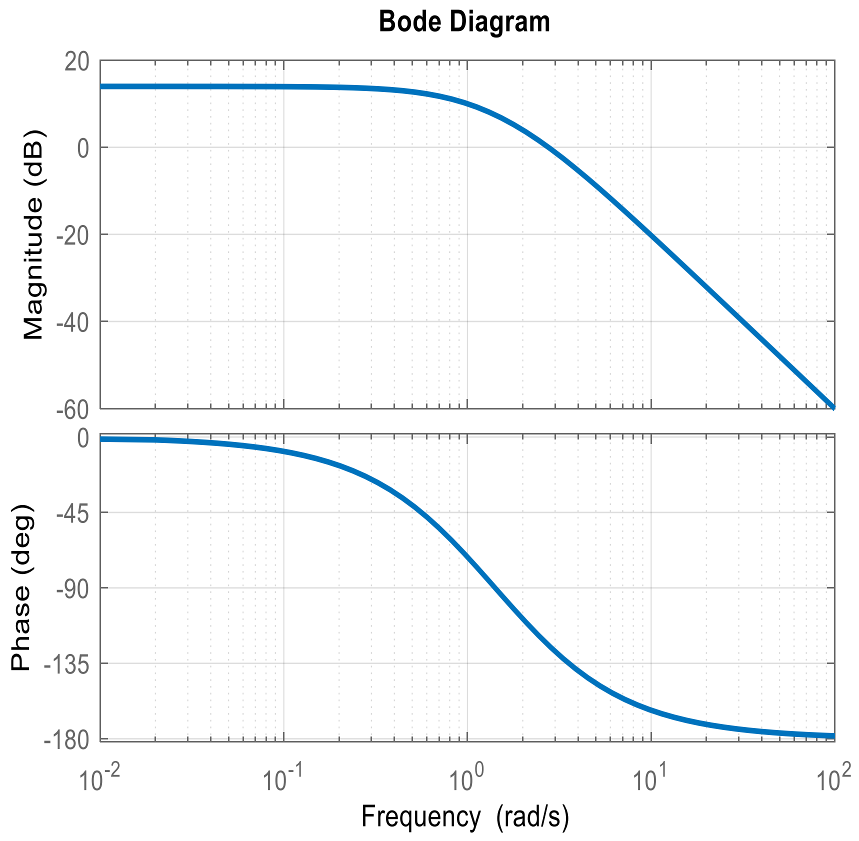
Example 7 Consider the following second-order system and plot the Nyquist diagram of this system. $G(s)=\dfrac{10}{(s+1)(s+2)}$

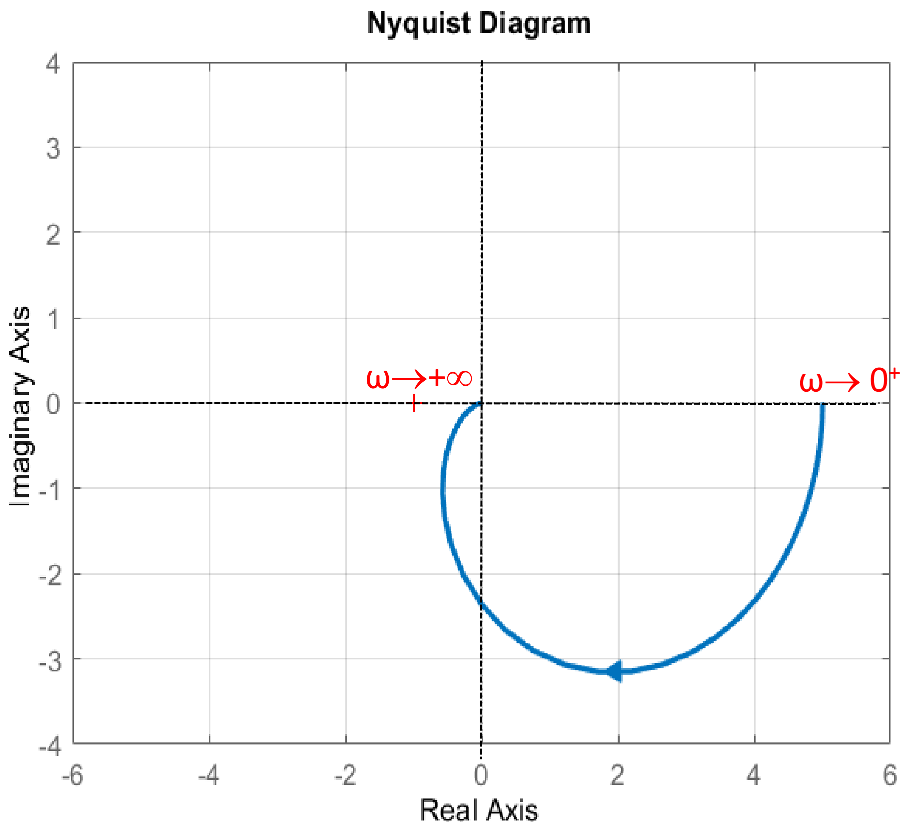
We can plot the Bode diagram and the Nyquist diagram of G(s) using MATLAB and compare them.

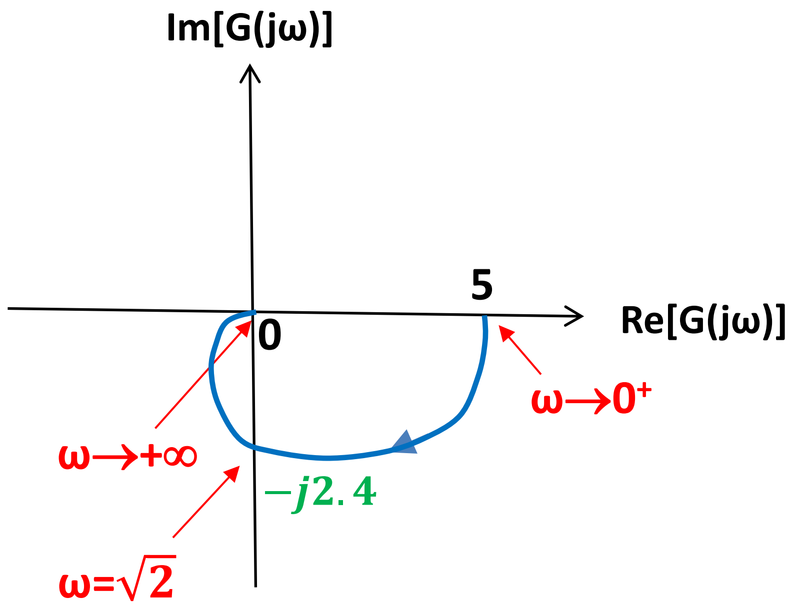
Starting point $\rightarrow$ For $\omega \rightarrow 0^{+} \Rightarrow G(j0)=5\angle 0^{\circ}$

Ending point $\rightarrow$ For $\omega \rightarrow +\infty \Rightarrow G(j\infty)=0\angle -180^{\circ}$

```
num = [10];
den = [1 3 2];
G = tf(num,den);
figure; nyquist(G)
figure; bode(G)
```







General Shape of Nyquist Diagram

Example 8 Consider the following second-order system and plot the Nyquist diagram of this system. $G(s) = \dfrac{s+1}{s(2s+1)}$

Find the frequency response function $G(j\omega)$

$$G(s) = \frac{s+1}{s(2s+1)} \quad \Rightarrow \quad G(j\omega) = \frac{j\omega+1}{j\omega(j2\omega+1)}$$

Determine starting point and ending point of the polar plot

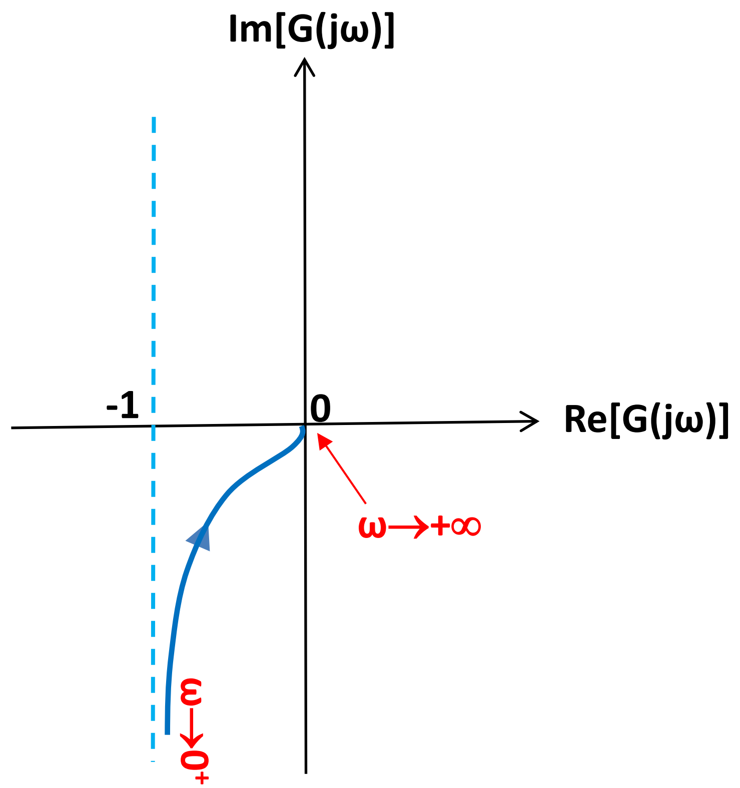
Starting point → For $\omega \to 0^+ \quad \Rightarrow \quad G(j0) = \infty \angle -90^\circ$

Ending point → For $\omega \to +\infty \quad \Rightarrow \quad G(j\infty) = 0 \angle -90^\circ$

Find the intersection of the asymptote line with the real axis

$$G(j\omega) = \frac{j\omega+1}{j\omega(j2\omega+1)} = \underbrace{\frac{-1}{(2\omega)^2+1}}_{real\ part} + j\underbrace{\frac{-(1+2\omega^2)}{\omega((2\omega)^2+1)}}_{imaginary\ part}$$

$$\alpha = \mathrm{Re}[G(j\omega)]\Big|_{\omega=0} \quad \rightarrow \quad Re[G(j0^+)] = \frac{-1}{(2\times 0)^2+1} = -1$$



General Shape of Nyquist Diagram

Example 8

Consider the following second-order system and plot the Nyquist diagram of this system.

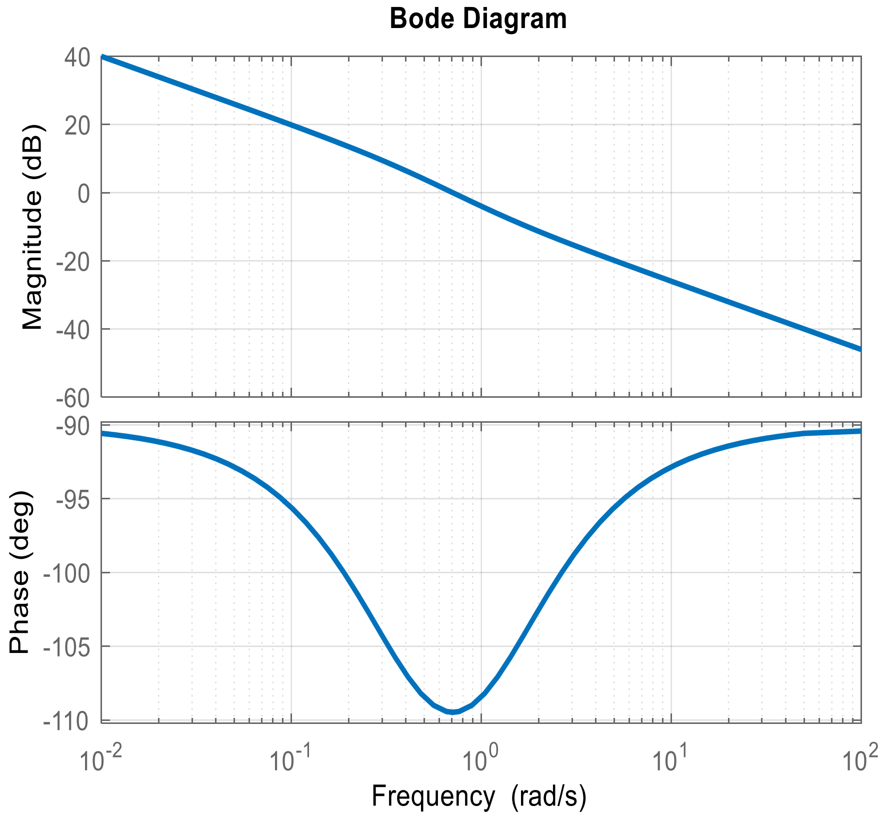
$$G(s) = \frac{s+1}{s(2s+1)}$$

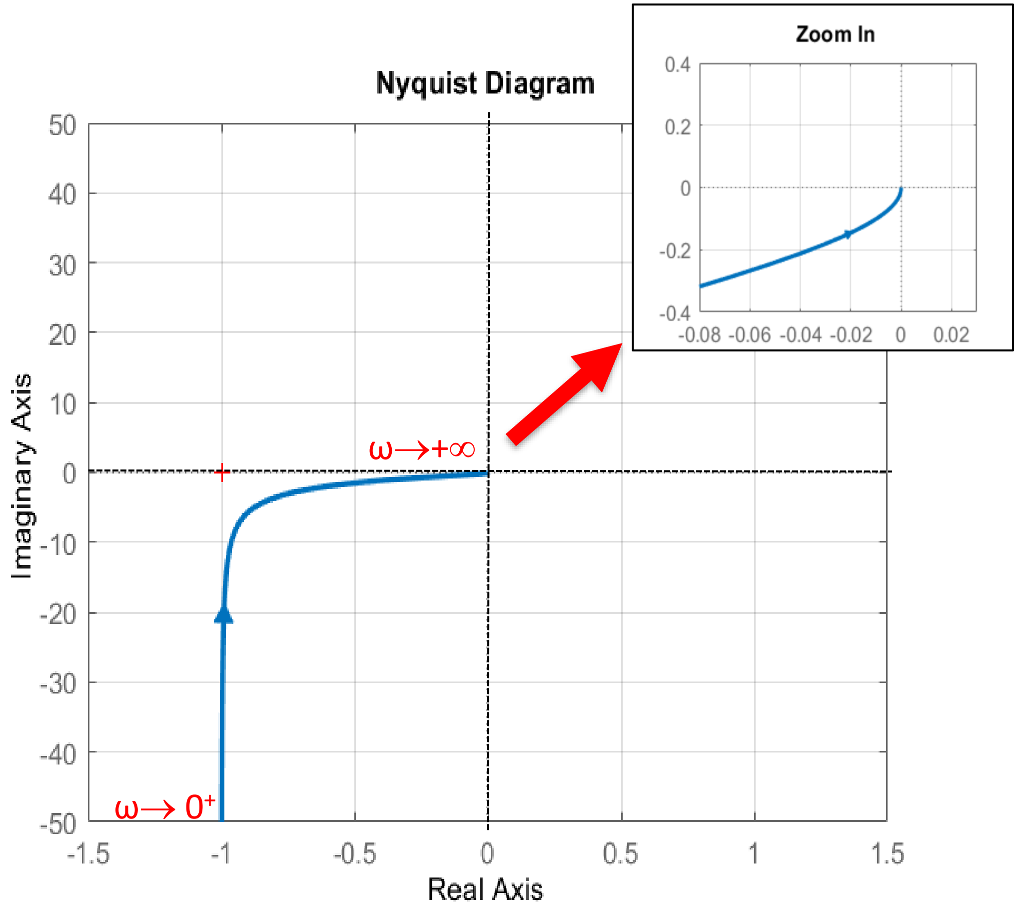
We can plot the Bode diagram and the Nyquist diagram of G(s) using MATLAB and compare them.

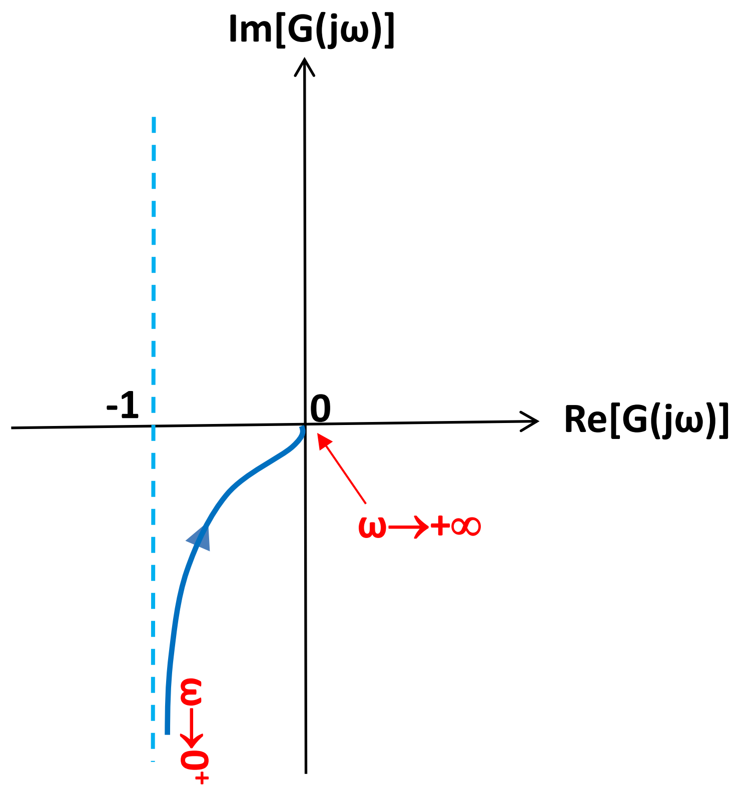
```
num = [1 1];
den = [2 1 0];
G = tf(num,den);
figure; nyquist(G)
figure; bode(G)
```

Starting point $\rightarrow$ For $\omega \rightarrow 0^+ \quad \Rightarrow \quad G(j0) = \infty \angle -90^\circ$

Ending point $\rightarrow$ For $\omega \rightarrow +\infty \quad \Rightarrow \quad G(j\infty) = 0 \angle -90^\circ$







THANK YOU

HUMBER

WE ARE HUMBER