ENGI-1500
Physics -2

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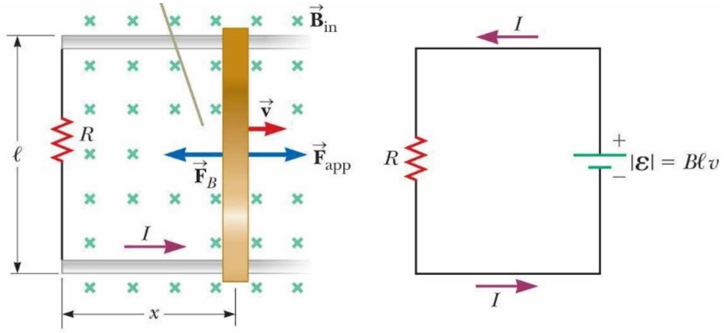
Winter 2023

HUMBER

Reminder of the previous week

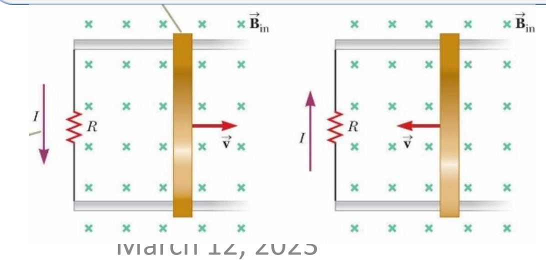
Motional EMF

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt} \Rightarrow \varepsilon = -B\ell v$$



Lenz's Law

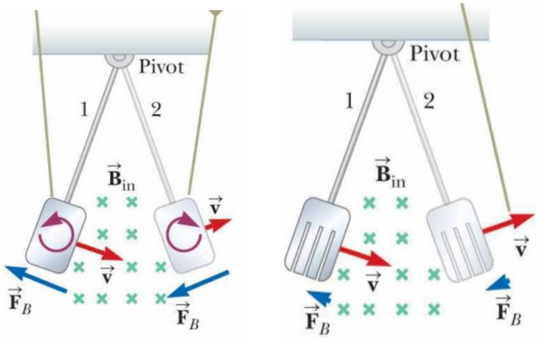
The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.



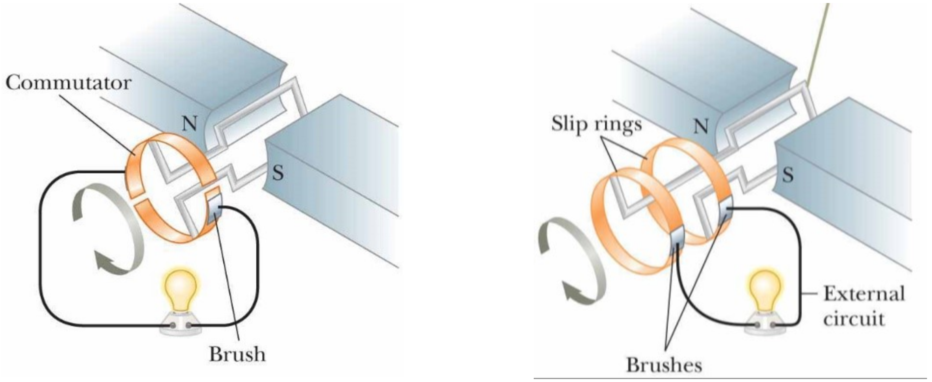
Faraday's Law - Generalized

$$\Delta V = \oint \vec{E}\cdot d\vec{s} \Rightarrow \boxed{\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$

Eddy Currents



DC and AC Generators



Motors

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Week 10 / Class 8

Inductance (Ch. 31)
Alternating Circuits (Ch. 32)

Outline of Week 10 / Class 8

- Reminder of the previous week
- Midterm results
- Inductance (Ch. 31)
    - Self Induction and Inductance
    - RL Circuits
    - Energy in a Magnetic Field
    - Mutual Inductance
    - Oscillations in an LC Circuit
    - The RLC Circuit [Reading from Textbook]
- Alternating Current Circuits (Ch. 32)
    - AC Sources
    - Resistors in an AC Circuit
    - Inductors in an AC Circuit
    - Capacitors in an AC Circuit
    - The RLC Series Circuit [Reading from Textbook]
    - Power in an AC Circuit [Next Week]
    - Resonance in a Series RLC Circuit [Reading from Textbook]
    - The Transformer and Power Transmission [Next Week]
- Examples
- Next week's topic

Inductance (Ch. 31)

Self Induction and Inductance

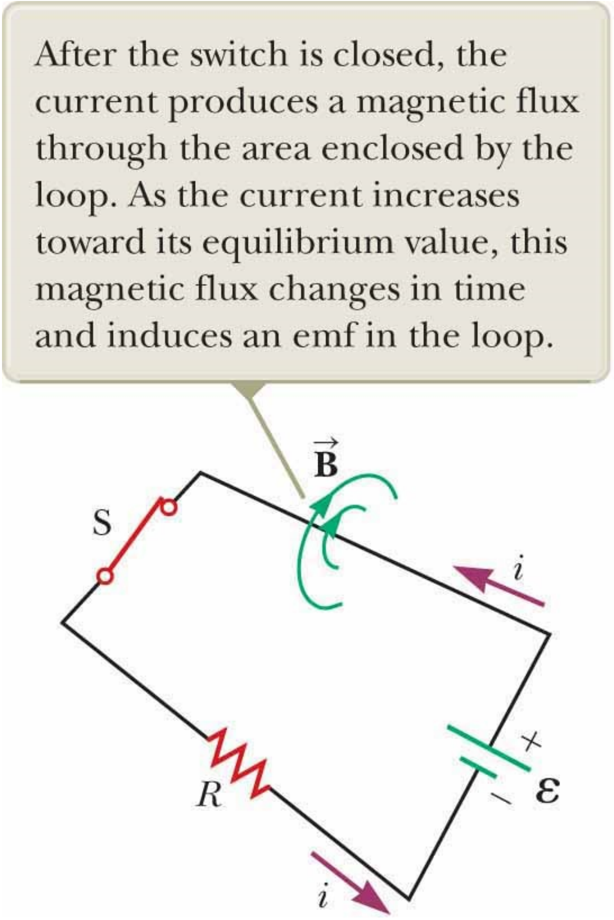
Self induction and inductance

- ***Induced***: emfs and currents caused by changing magnetic field

- When switch ***S*** closed:
  - Current does not immediately jump from zero to maximum value **ε/R**

- Circuit is a current loop → source of magnetic field
  - As current increases → increasing magnetic flux through loop of circuit
    - Increasing flux creates induced emf in circuit: ***back emf*** with opposite direction of emf of battery
    - Causing induced current in loop → establishing magnetic field opposing change in original magnetic field
  - Gradual rather than instantaneous increase in current to its final equilibrium value

- ***Self-induction***: Changing flux through circuit and resultant induced emf arising from circuit itself



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Self induction and inductance

- Self-induced emf is always proportional to the time rate of change of the current. For any loop of wire, we can write this proportionality as (L: inductance of the loop):

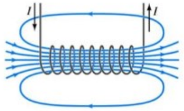
$$\boxed{\mathcal{E}_L = -L\frac{di}{dt}}$$

- From Faraday's law:

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}} \qquad \mathcal{E} = -N\frac{d\Phi_B}{dt}$$

$$\boxed{L = \frac{N\Phi_B}{i}} \qquad \boxed{L = -\frac{\mathcal{E}_L}{di/dt}}$$

$$B = \mu_o nI$$



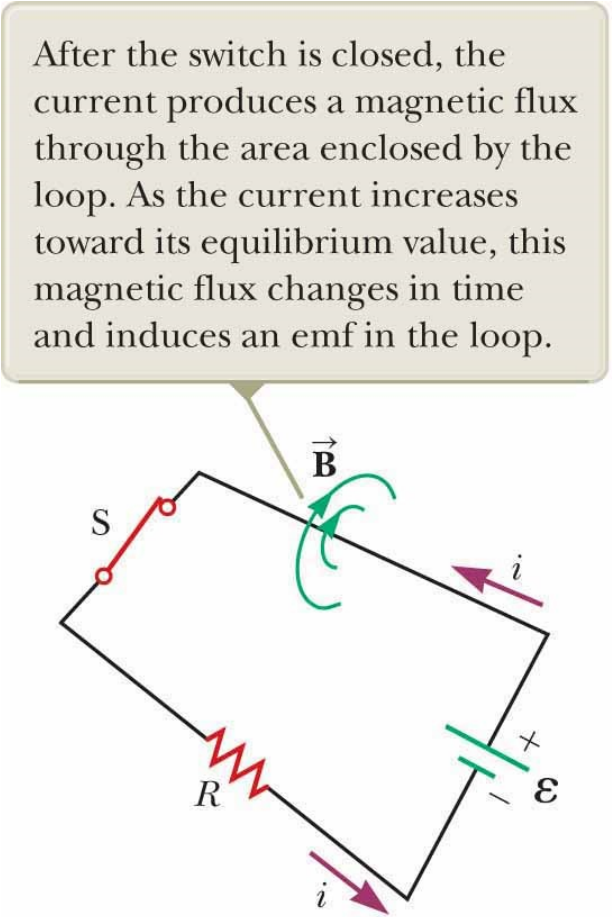
$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{d(\mu_o nIA)}{dt} = -N\mu_o nA\frac{dI}{dt} = -\frac{NBA}{I}\frac{dI}{dt}$$

$$\varepsilon = -\frac{N\Phi_B}{I}\frac{dI}{dt} = -L\frac{dI}{dt}$$

Define: Self Inductance $\qquad L = \dfrac{N\Phi_B}{I}$

- (SI Unit: Henry (H), 1 H = 1 V · s/A)

- Compare inductance with resistance
  - Resistance: measurement of opposition to current
  - Inductance: measurement of opposition to change in current



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Inductance (Ch. 31)

RL Circuits

RL Circuits

Charging

- Suppose S_2 set to *a* and S_1 open for $t < 0$, then thrown closed at $t = 0$

$$\varepsilon - iR - L\frac{di}{dt} = 0$$

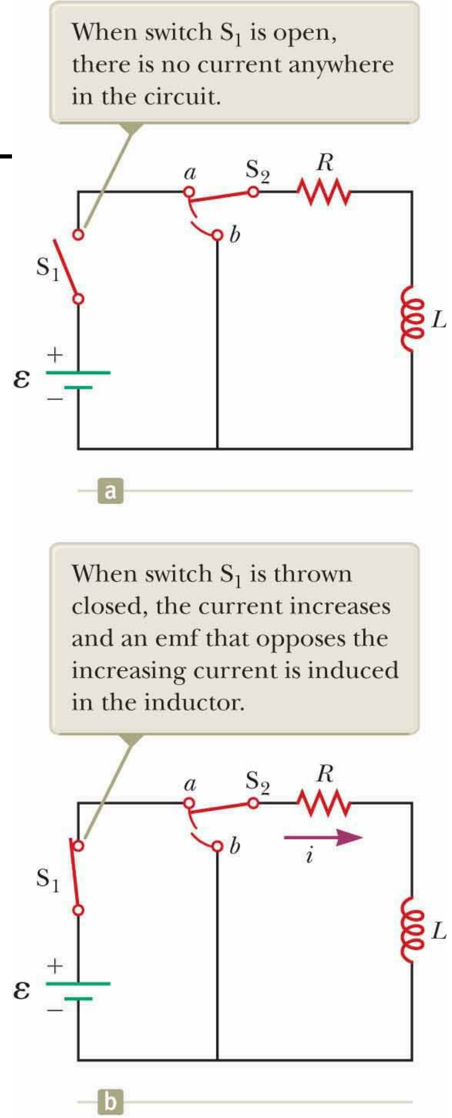
$$\text{let } x = (\varepsilon/R) - i \Rightarrow dx = -di \quad \Rightarrow \quad x + \frac{L}{R}\frac{dx}{dt} = 0$$

$$\Rightarrow \int_{x_0}^{x} \frac{dx}{x} = -\frac{R}{L}\int_0^t dt \Rightarrow \ln\frac{x}{x_0} = -\frac{R}{L}t \quad \Rightarrow \quad x = x_0 e^{-Rt/L}$$

(i=0 at t=0; $x_0 = \varepsilon/R$)

$$\Rightarrow \frac{\varepsilon}{R} - i = \frac{\varepsilon}{R}e^{-Rt/L} \Rightarrow \boxed{i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)}$$

- Current does not increase instantly to its final equilibrium value
- Without an inductor, current increases instantly to its final value



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RL Circuits

Charging

- Rewrite expression *i*:

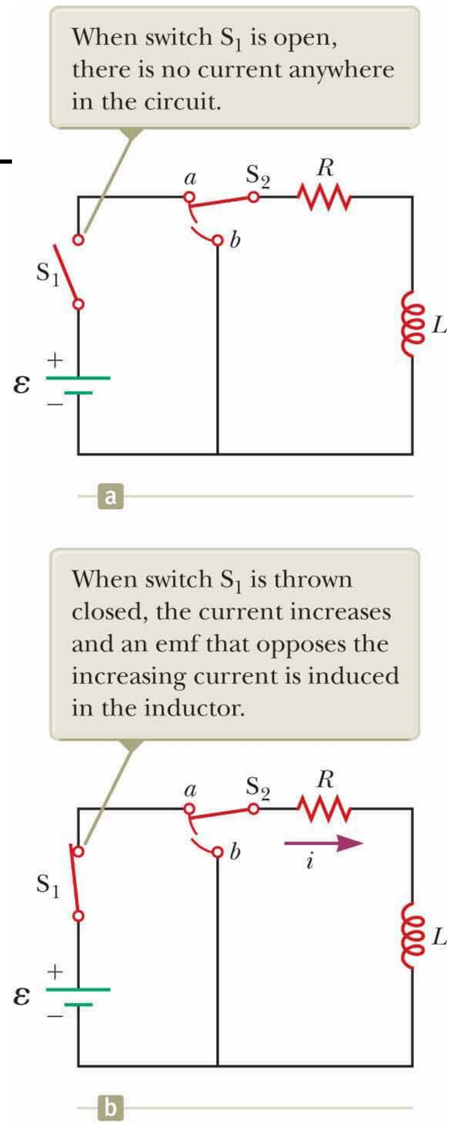
$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right) \quad \boxed{\Rightarrow i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)}$$

τ = **time constant** of the *RL* circuit: $\boxed{\tau = \frac{L}{R}}$

- Physically, τ = time interval required for current in circuit to reach:

$$\left(1 - e^{-1}\right) = 0.632 = 63.2\%$$

- Time constant: useful parameter for comparing time responses of various circuits



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RL Circuits

Charging

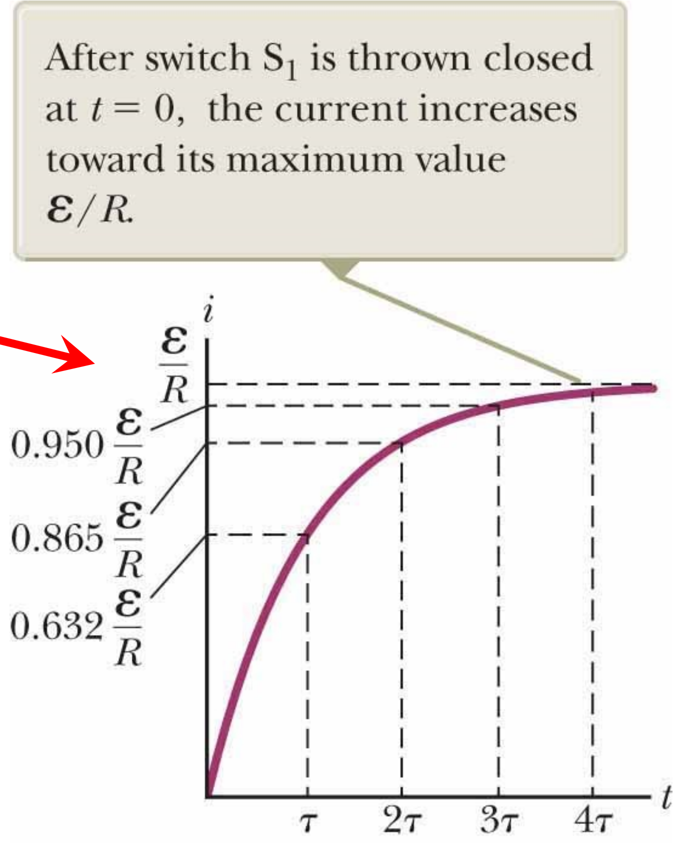
$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$$

as $t \to \infty$, $\quad \frac{di}{dt} \to 0$

$\Rightarrow \varepsilon - iR = 0$

$\Rightarrow i = \varepsilon/R$

Similar to charge Q on a capacitor



- Current initially increases rapidly from 0
- Then gradually approaches equilibrium value ***ε/R*** as $t \to \infty$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

RL Circuits

Discharging

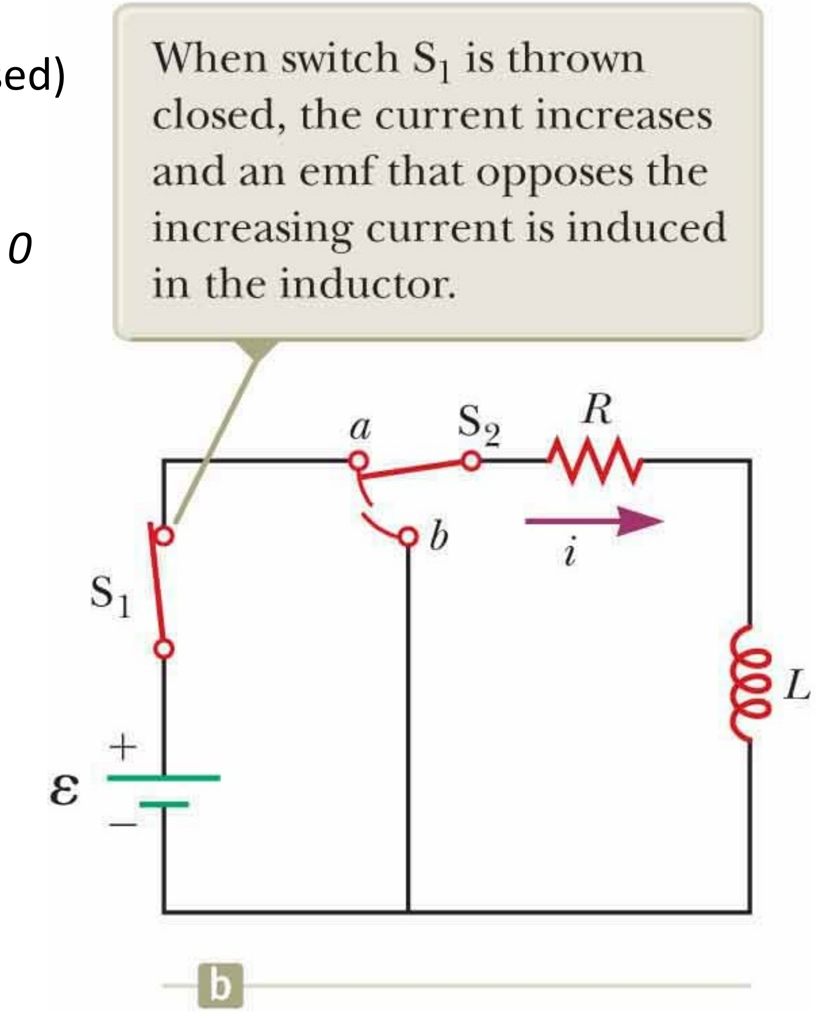
- Suppose switch S_2 set at position *a* long enough (and switch S_1 remains closed) to allow current to reach equilibrium value $I = \varepsilon/R$

- If S_2 is thrown from *a* to *b* $\rightarrow$ circuit described by only right-side loop $\rightarrow$ $\varepsilon = 0$

$$iR + L\frac{di}{dt} = 0$$

Solving the differential equation yields:

$$\boxed{i = \frac{\varepsilon}{R}e^{-t/\tau} = I_i e^{-t/\tau}} \qquad (\tau = \frac{L}{R})$$

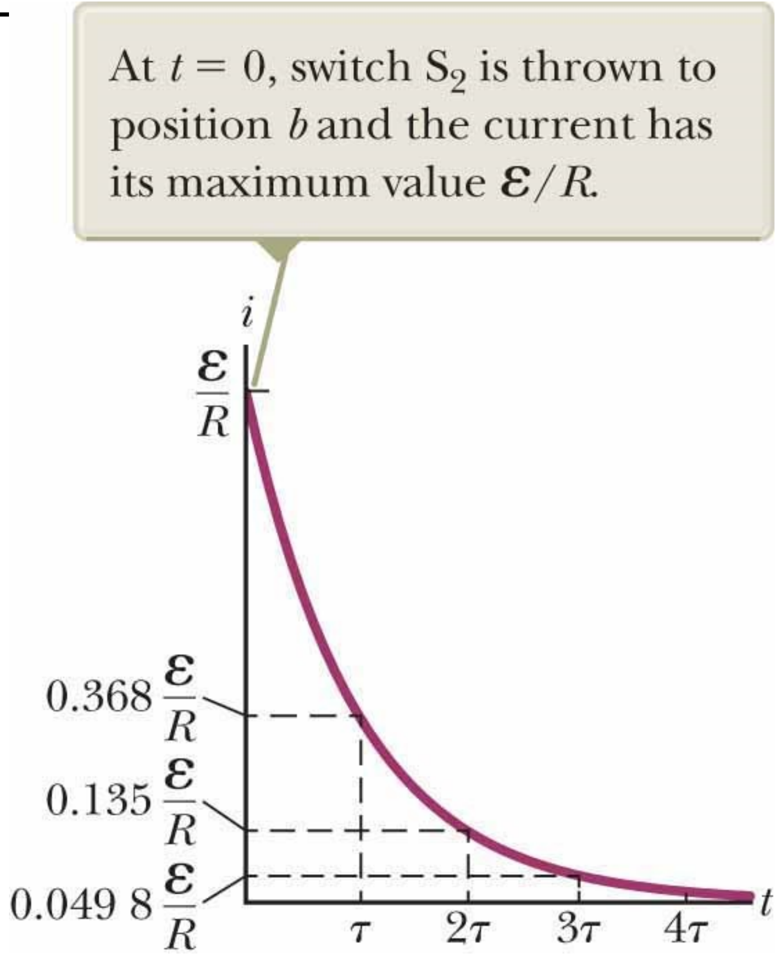


RL Circuits

Discharging

$$i = \frac{\mathcal{E}}{R} e^{-t/\tau} = I_i e^{-t/\tau} \quad (\tau = \frac{L}{R})$$

- Current falls off exponentially to zero as $t \rightarrow \infty$
- If circuit did not contain an inductor, current would immediately decrease to zero when battery was removed



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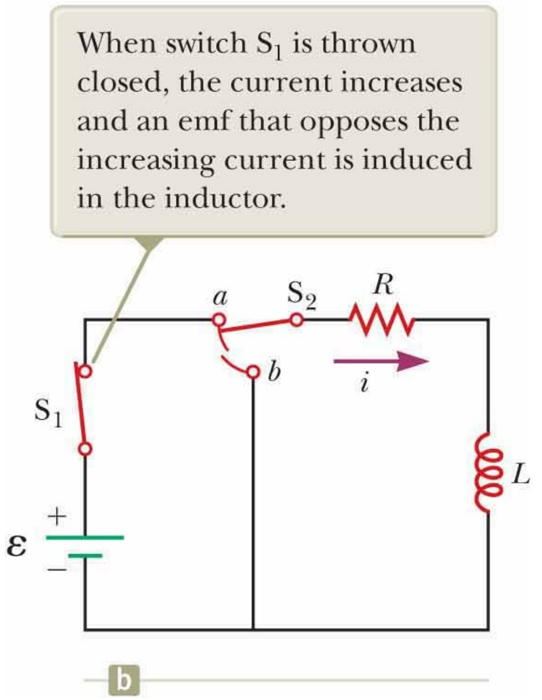
Time Constant of an RL Circuit

Example 31.2

Considering the circuit in the figure, suppose ***Ɛ=12V, R=6.00Ω*** and ***L=30.0mH***.

A) Find the time constant of the circuit

B) Switch S_2 is at position ***a***, and switch S_1 is thrown closed at ***t = 0***. Calculate the current in the circuit at ***t=2.00ms***.



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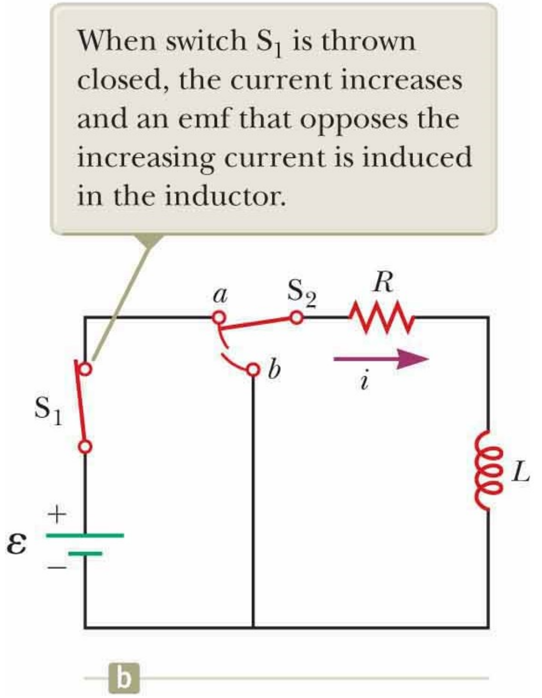

Solution

A) Find the time constant of the circuit

$$\tau = \frac{L}{R}$$

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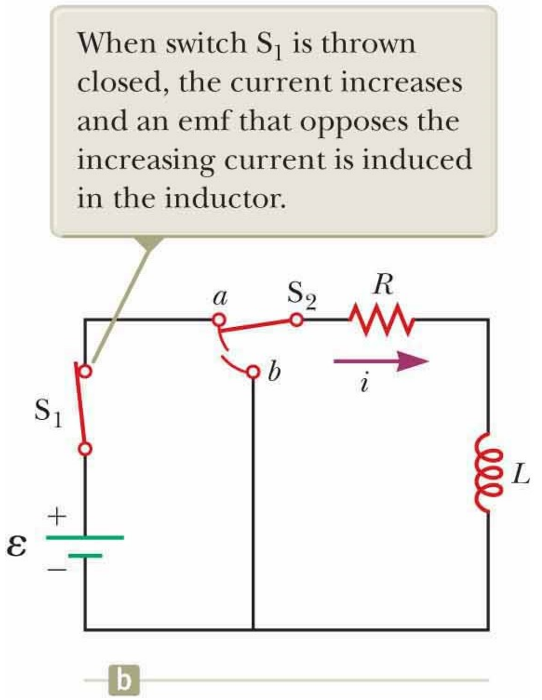

Solution

A) Find the time constant of the circuit

$$\tau = \frac{L}{R} = \frac{30.0 \times 10^{-3}\ \mathrm{H}}{6.00\ \Omega} = \boxed{5.00\ \mathrm{ms}}$$

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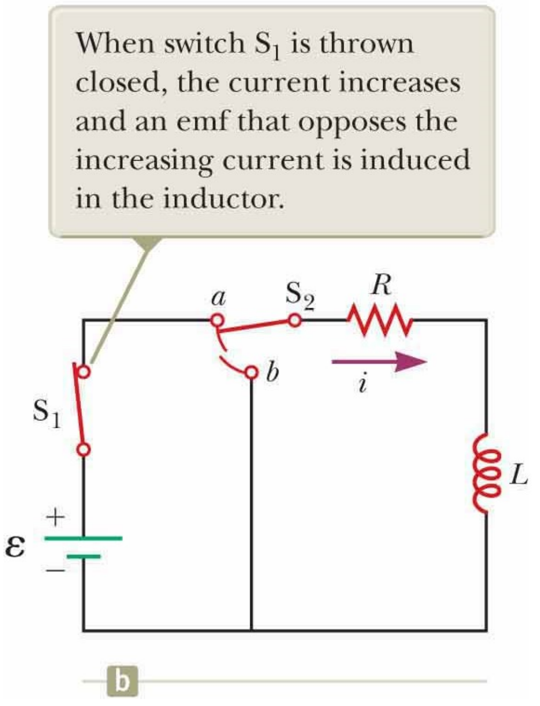

Solution

A) Find the time constant of the circuit

$$\tau = \frac{L}{R} = \frac{30.0 \times 10^{-3}\ \mathrm{H}}{6.00\ \Omega} = 5.00\ \mathrm{ms}$$

B) Switch S_2 is at position ***a***, and switch S_1 is thrown closed at ***t = 0***. Calculate the current in the circuit at ***t=2.00ms***.

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

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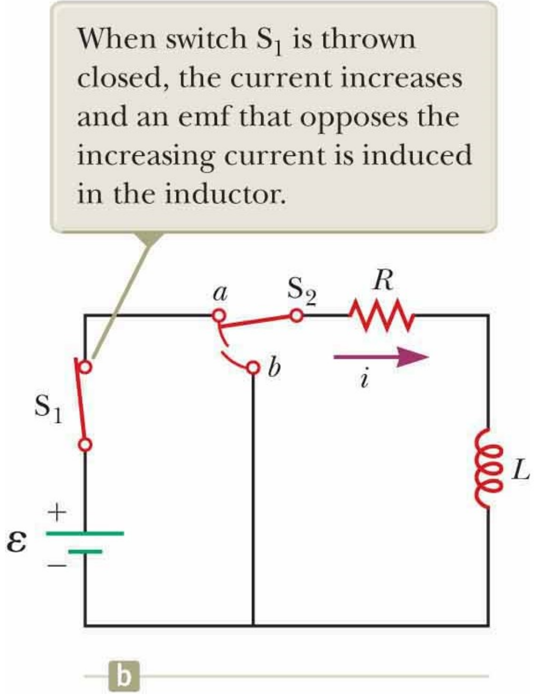

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Solution

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B) Switch S_2 is at position ***a***, and switch S_1 is thrown closed at ***t = 0***. Calculate the current in the circuit at ***t=2.00ms***.

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \frac{12.0\ \mathrm{V}}{6.00\ \Omega}\left(1 - e^{-2.00\ \mathrm{ms}/5.00\ \mathrm{ms}}\right) = 2.00\ \mathrm{A}\left(1 - e^{-0.400}\right)$$

$$= \boxed{0.659\ \mathrm{A}}$$

Inductance (Ch. 31)

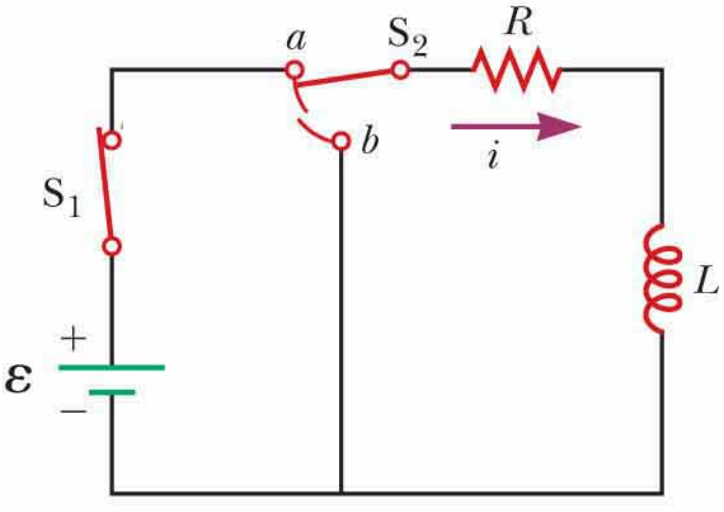
Energy in a Magnetic Field

Energy in a Magnetic Field

- When switch S_1 is closed, the energy supplied by battery
  - partly appears as internal energy in the resistance in the circuit (dissipated)
  - with remaining energy stored in the magnetic field of the inductor

$$i\varepsilon = i^2 R + Li\frac{di}{dt}$$

- $i\varepsilon$ = rate at which energy supplied by battery
- i^2R = rate at which energy delivered to resistor
- $Li(di/dt)$ = rate at which energy stored in inductor



- If U_B = energy stored in inductor at any time:

$$\frac{dU_B}{dt} = Li\frac{di}{dt}$$

- Total energy stored in inductor at any instant:

$$U_B = \int dU_B = \int_0^i Li\,di = L\int_0^i i\,di$$

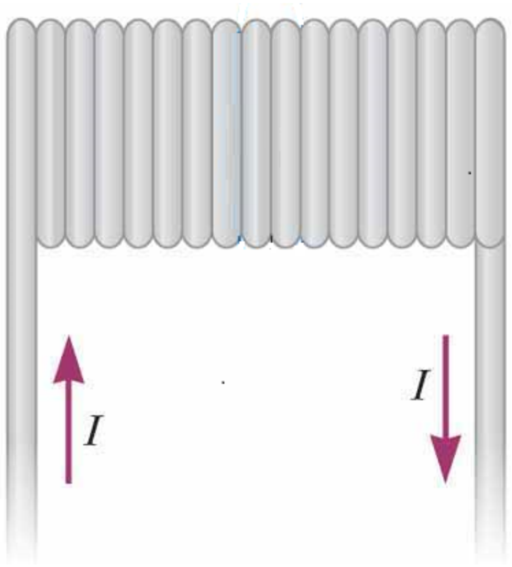
$$\boxed{U_B = \frac{1}{2}Li^2}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Energy in a Magnetic Field

- Determine energy density of magnetic field in a solenoid inductor with given inductance

Inductance: $L = \mu_0 n^2 V$ Magnetic Field: $B = \mu_0 n i$



- Total magnetic energy in the solenoid:

$$U_B = \frac{1}{2} L i^2 = \frac{1}{2} \mu_0 n^2 V \left(\frac{B}{\mu_0 n} \right)^2 = \frac{B^2}{2\mu_0} V$$

- Magnetic energy density (energy stored per unit volume in magnetic field of the solenoid):

$$u_B = \frac{U_B}{V} \Rightarrow \boxed{u_B = \frac{B^2}{2\mu_0}}$$

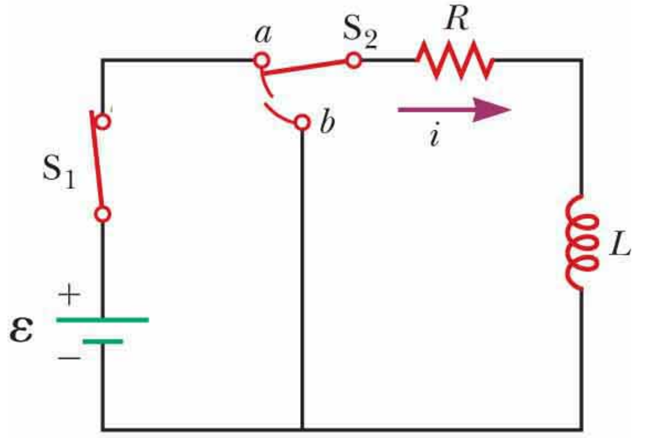
- Expression valid for any region of space in which a magnetic field exists
- Energy density $\propto$ square of field magnitude

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

What Happens to the Energy in the Inductor?

Example 31.3

Consider once again the RL circuit shown in the figure, with switch S2 at position a and the current having reached its steady-state value. When S2 is thrown to position b, the current in the right-side loop decays exponentially with time according to the expression $\mathbf{i = \varepsilon/R\ e^{-t/\tau} = I_i\ e^{-t/\tau}}$. Show that all the energy initially stored in the magnetic field of the inductor appears as internal energy in the resistor (dissipated) as the current decays to zero.



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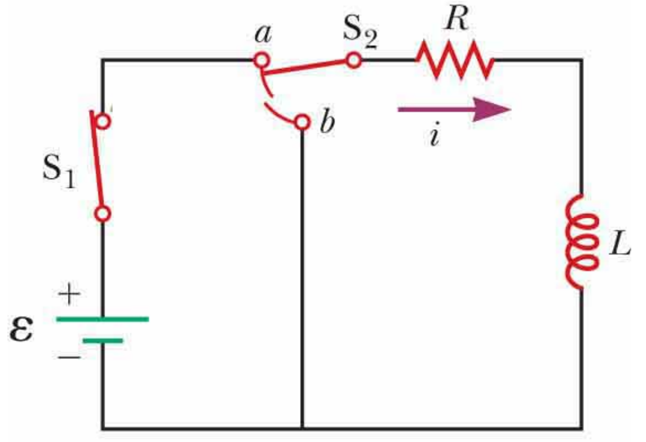

Solution

Power delivered to the resistor:

$$\frac{dE_{\text{int}}}{dt} = P = i^2R$$

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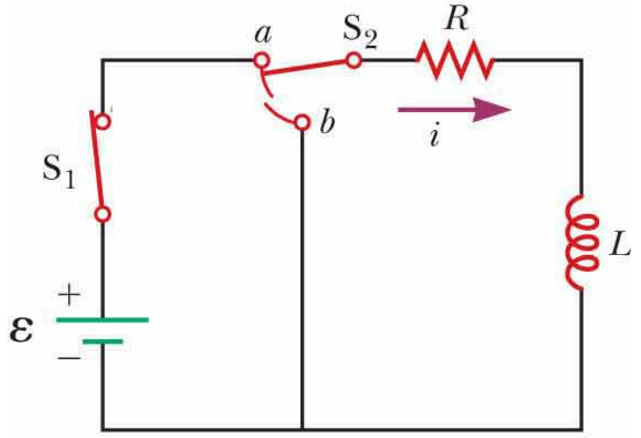

Solution

Power delivered to the resistor:

$$\frac{dE_{\text{int}}}{dt} = P = i^2R$$

Substitute the inductor current given in the question:

$$\frac{dE_{\text{int}}}{dt} = i^2R = (I_i e^{-Rt/L})^2 R = I_i^{\,2} R e^{-2Rt/L}$$

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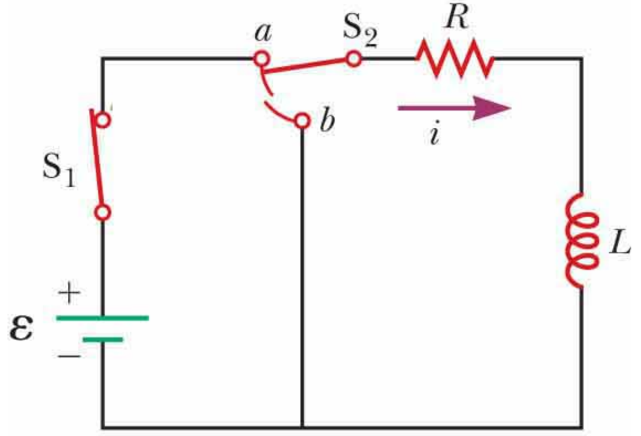

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Solve for E_{int} by integrating

$$E_{\text{int}} = \int_0^\infty I_i^{\,2} R e^{-2Rt/L}\, dt = I_i^{\,2} R \int_0^\infty e^{-2Rt/L}\, dt$$

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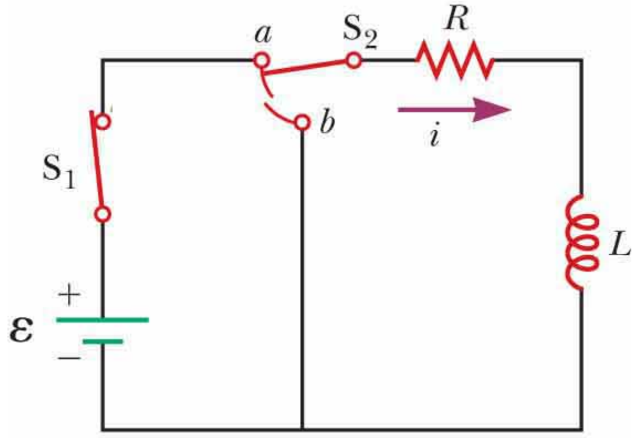

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Substitute the inductor current given in the question:

$$\frac{dE_{\text{int}}}{dt} = i^2 R = (I_i e^{-Rt/L})^2 R = I_i^{\,2} R e^{-2Rt/L}$$

Solve for E_{int} by integrating

$$E_{\text{int}} = \int_0^\infty I_i^{\,2} R e^{-2Rt/L}\, dt = I_i^{\,2} R \int_0^\infty e^{-2Rt/L}\, dt$$

Taking the integral yields:

$$E_{\text{int}} = I_i^{\,2} R \left(\frac{L}{2R}\right) = \frac{1}{2} L I_i^2$$

The initial energy stored in the magnetic field of the inductor

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Inductance (Ch. 31)

Mutual Inductance

Mutual Inductance

- Enclosed circuits with time-varying currents induces emf in nearby circuits through ***mutual induction***

Mutual inductance M_{12} of coil 2 with respect to coil 1:

$$\boxed{M_{12} = \frac{N_2 \Phi_{12}}{i_1}}$$

- If i_1 exists and varies with time, emf induced by coil 1 in coil 2 is:

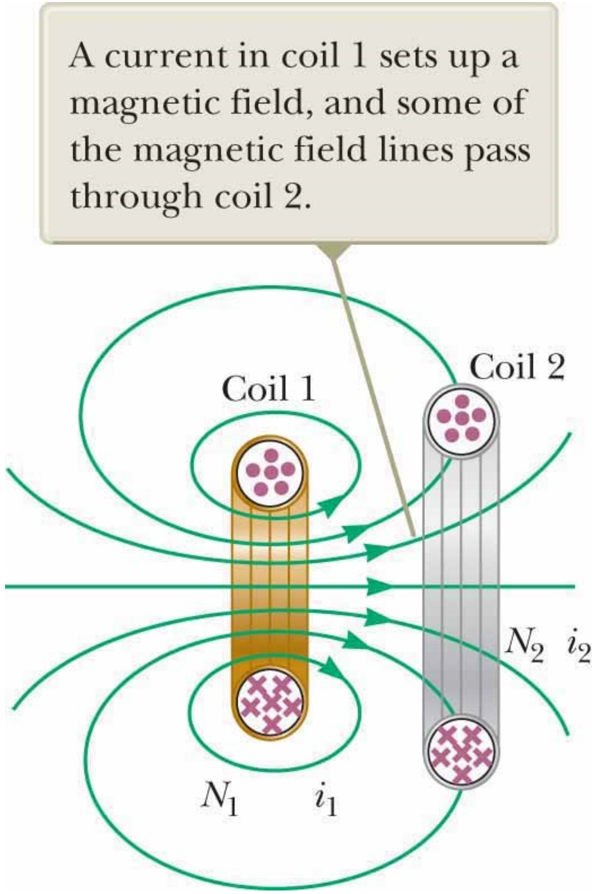
$$\varepsilon_2 = -N_2 \frac{d\Phi_{12}}{dt} = -N_2 \frac{d}{dt}\left(\frac{M_{12} i_1}{N_2}\right) = -M_{12} \frac{di_1}{dt}$$

- If i_2 exists and varies with time, emf induced by coil 2 in coil 1 is:

$$\varepsilon_1 = -M_{21} \frac{di_2}{dt}$$

Although the proportionality constants M12 and M21 have been treated separately, it can be shown that they are equal.

- $M_{12} = M_{21} = M$ ⇒ $\varepsilon_2 = -M \frac{di_1}{dt}$ and $\varepsilon_1 = -M \frac{di_2}{dt}$ (M unit: Henry)



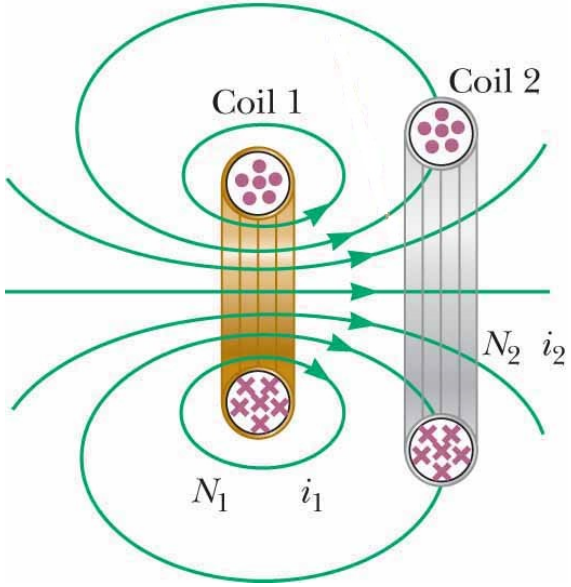
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Mutual Inductance

Quick Quiz

In the figure, coil 1 is moved closer to coil 2, with the orientation of both coils remaining fixed. Because of this movement, what happens to the mutual inductance of the two coils?

(a) increases,
(b) decreases, or
(c) is unaffected.



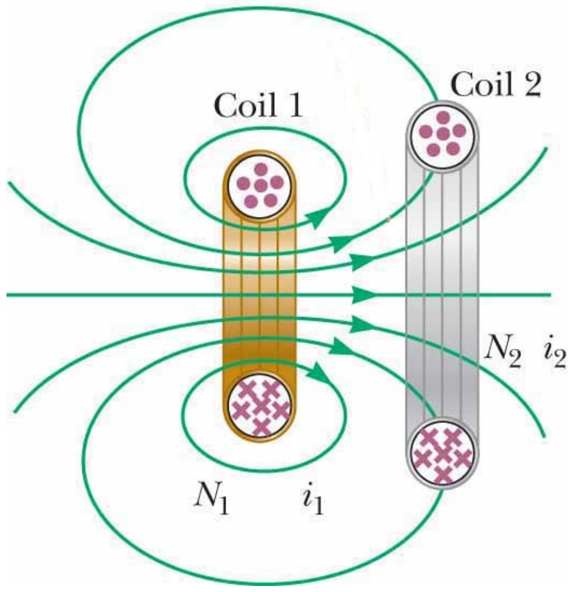
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Mutual Inductance

Quick Quiz

In the figure, coil 1 is moved closer to coil 2, with the orientation of both coils remaining fixed. Because of this movement, what happens to the mutual inductance of the two coils?

(a) increases,
(b) decreases, or
(c) is unaffected.



$$M_{12} = \frac{N_2 \Phi_{12}}{i_1}$$

Mutual inductance increases because the magnetic flux through coil2 increases.

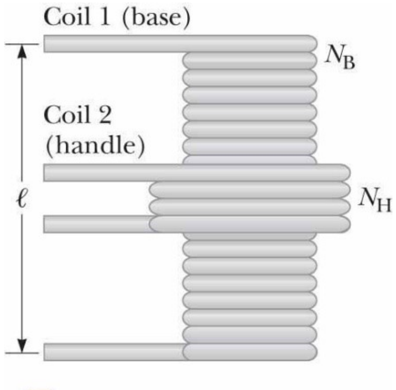
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Wireless Battery Charger

Example 31.5

An electric toothbrush has a base designed to hold the toothbrush handle when not in use. As shown in the photo, the handle has a cylindrical hole that fits loosely over a matching cylinder on the base. When the handle is placed on the base, a changing current in a solenoid inside the base cylinder induces a current in a coil inside the handle. This induced current charges the battery in the handle. We can model the base as a solenoid of length $\boldsymbol{l}$ with $\boldsymbol{N_B}$ turns, carrying a current $\boldsymbol{i}$, and having a cross-sectional area $\boldsymbol{A}$. The handle coil contains $\boldsymbol{N_H}$ turns and completely surrounds the base coil. Find the mutual inductance of the system.

a



b

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

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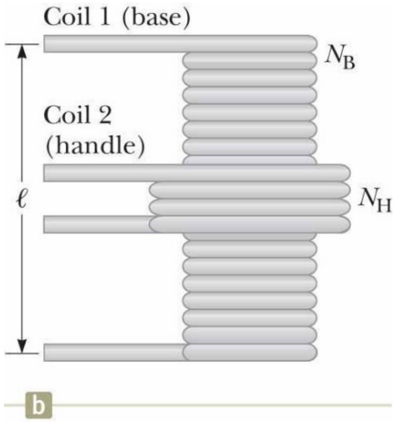

Solution

Let's express the magnetic field in the interior of the base solenoid:

$$B = \mu_0 \frac{N_B}{\ell} i$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Wireless Battery Charger

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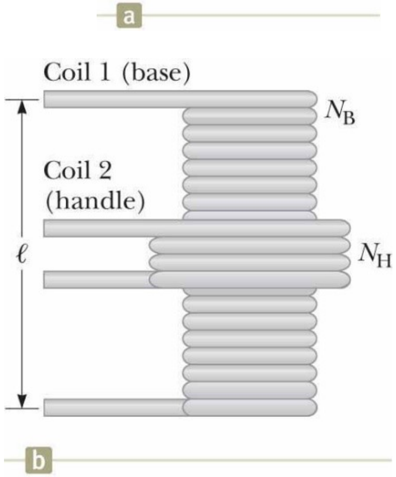

Solution

Let's express the magnetic field in the interior of the base solenoid:

$$B = \mu_0 \frac{N_B}{\ell} i$$

Then we can find the mutual inductance, noting that the magnetic flux Φ_{BH} through the handle's coil caused by the magnetic field of the base coil is BA:

$$M = \frac{N_H \Phi_{BH}}{i} = \frac{N_H BA}{i} = \boxed{\mu_0 \frac{N_B N_H}{\ell} A}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

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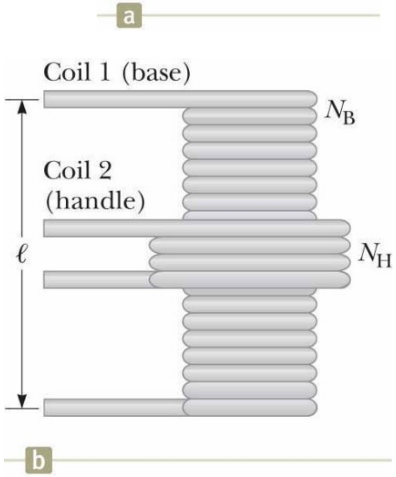

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$$M = \frac{N_H \Phi_{BH}}{i} = \frac{N_H BA}{i} = \boxed{\mu_0 \frac{N_B N_H}{\ell} A}$$

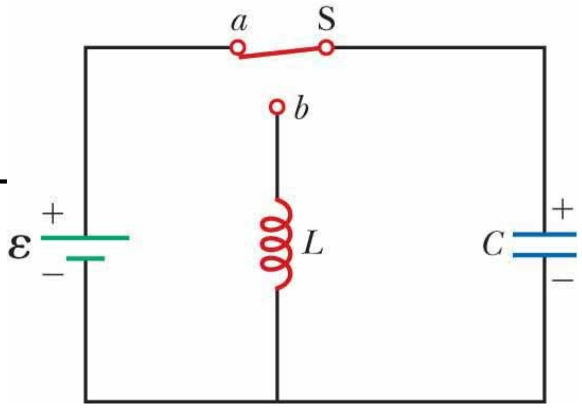
Other 'cordless' devices use similar technique to avoid metal to metal contact

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Inductance (Ch. 31)

Oscillations in an LC Circuit

Oscillations in an LC Circuit



When the capacitor is fully charged, the energy ***U*** in the circuit is stored in the capacitor's electric field and is equal to:

$$U_E = \frac{Q_{\max}^2}{2C}$$

The current in the circuit is zero; therefore, no energy is stored in the inductor. After the switch is closed (position-b), the capacitor begins to discharge. The capacitor's discharge represents a current in the circuit, and some energy is now stored in the magnetic field of the inductor. Therefore, energy is transferred from the electric field of the capacitor to the magnetic field of the inductor.

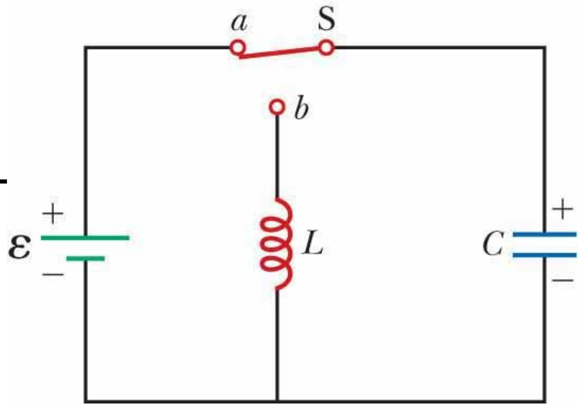
When the capacitor is fully discharged, it stores no energy. At this time, the current reaches its maximum value and all the energy in the circuit is stored in the inductor.

The current continues in the same direction, decreasing in magnitude, with the capacitor eventually becoming fully charged again but with the polarity of its plates now opposite the initial polarity. This process is followed by another discharge until the circuit returns to its original state of maximum charge.

The energy continues to oscillate between inductor and capacitor.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Oscillations in an LC Circuit



At an arbitrary time ***t*** after the switch is closed so that the capacitor has a charge $q < Q_{max}$ and the current is $i < I_{max}$. At this time, both circuit elements store energy. But the sum of the two energies must equal the total initial energy ***U*** stored in the fully charged capacitor at t=0 (assuming no losses):

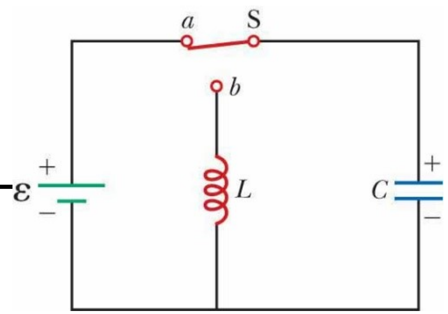
$$U = U_E + U_B = \frac{q^2}{2C} + \tfrac{1}{2}Li^2 = \frac{Q_{\max}^2}{2C}$$

q and ***i*** varies with time, so we can take the derivative with respect to time

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{q^2}{2C} + \tfrac{1}{2}Li^2\right) = \frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = 0 \xrightarrow{i = \frac{dq}{dt} \to \frac{di}{dt} = \frac{d^2q}{dt^2}} \left\{ \begin{array}{l} \dfrac{q}{C} + L\dfrac{d^2q}{dt^2} = 0 \\ \dfrac{d^2q}{dt^2} = -\dfrac{1}{LC}q \end{array} \right.$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Oscillations in an LC Circuit



$$\frac{q}{C} + L\frac{d^2q}{dt^2} = 0$$

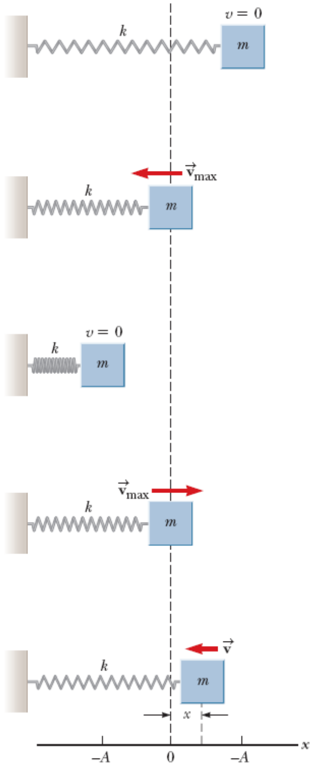
$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q$$

Solving for q noting that this expression is of the same form as particle in simple harmonic motion

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

$$x = A\cos(\omega t + \phi)$$

k: spring constant
m: mass of the block
w = √k/m



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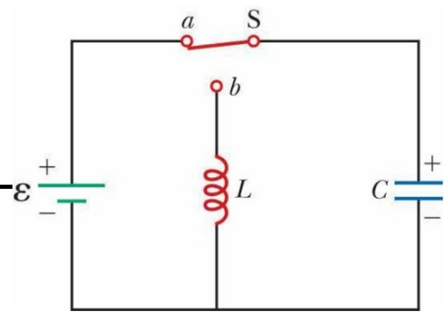
Oscillations in an LC Circuit



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k: spring constant
m: mass of the block
w = √k/m

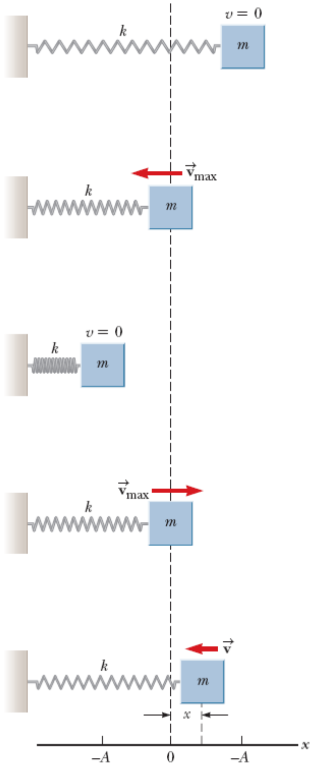
$$x = A\cos(\omega t + \phi)$$

$$q = Q_{\max}\cos(\omega t + \phi)$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

ω: natural frequency of oscillation of the LC circuit

$$i = \frac{dq}{dt} = -\omega Q_{\max}\sin(\omega t + \phi)$$



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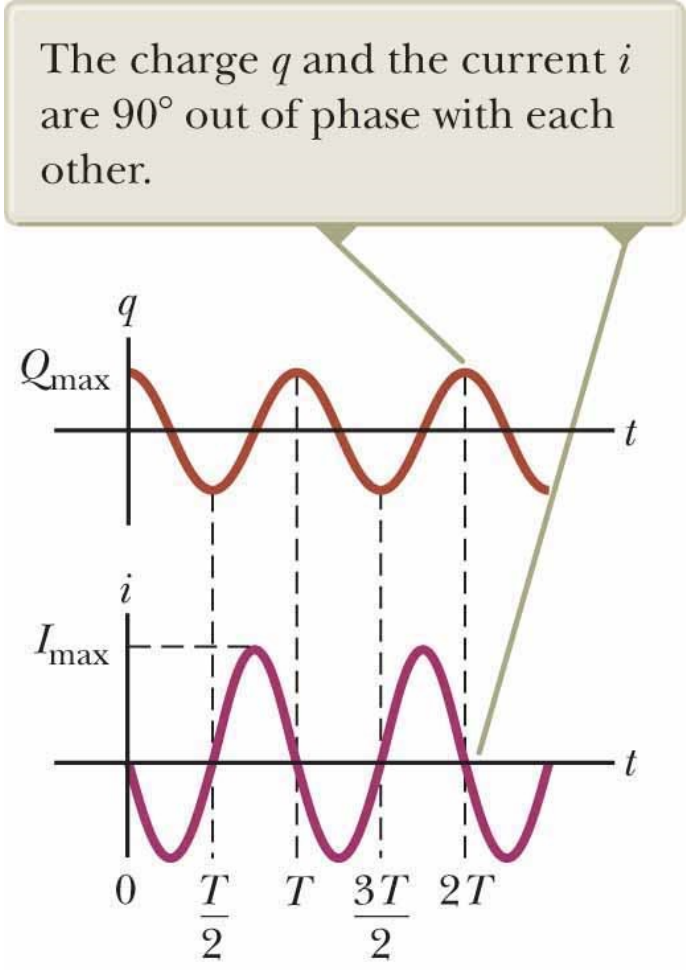
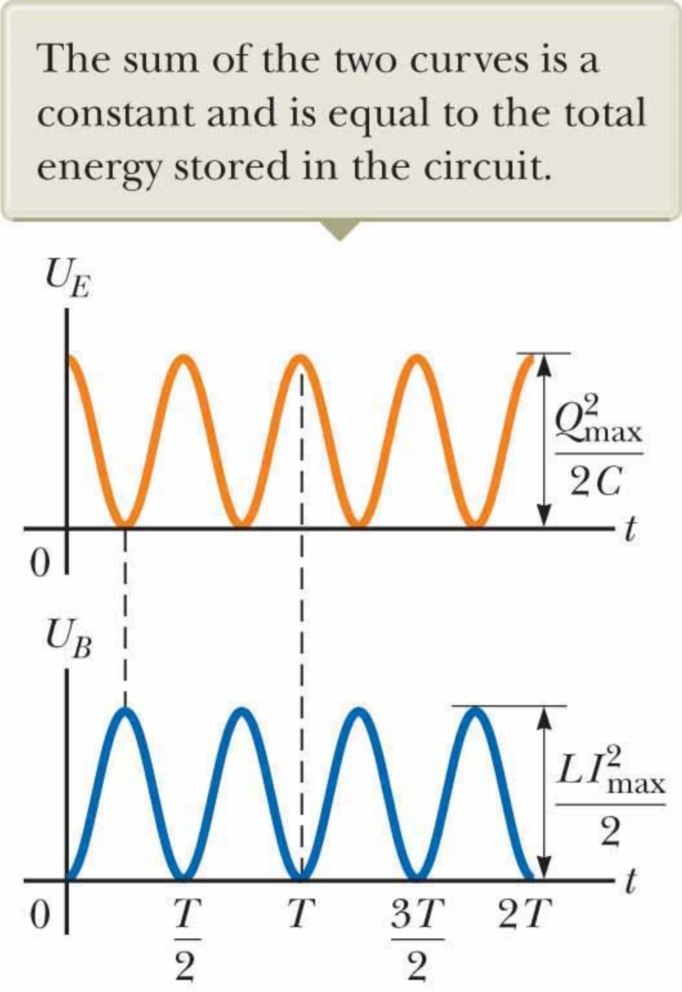




$$\frac{Q_{max}^2}{2C} = \frac{LI_{max}^2}{2}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

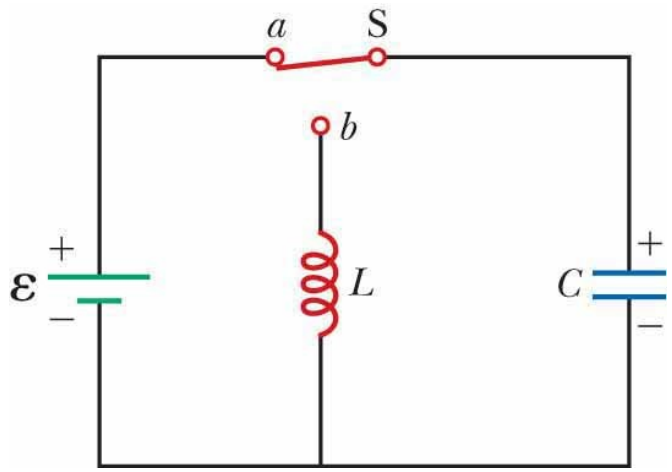
Oscillations in an LC Circuit

Example 31.6

In the figure, the battery has an emf of ***12.0 V***, the inductance is ***2.81 mH***, and the capacitance is ***9.00 pF***. The switch has been set to position ***a*** for a long time so that the capacitor is charged. The switch is then thrown to position ***b***, removing the battery from the circuit and connecting the capacitor directly across the inductor.

A) Find the frequency of oscillation of the circuit.

B) What are the maximum values of charge on the capacitor and current in the circuit?



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A) Find the frequency of oscillation of the circuit.

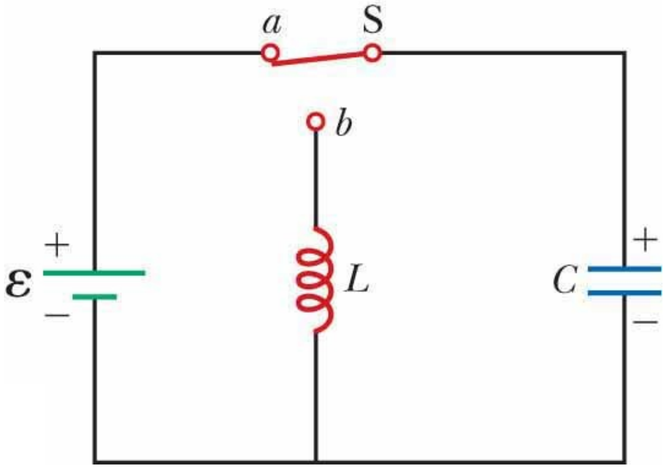
B) What are the maximum values of charge on the capacitor and current in the circuit?

Solution

Using the formula and substituting the numerical values:

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi[(2.81 \times 10^{-3}\ \mathrm{H})(9.00 \times 10^{-12}\ \mathrm{F})]^{1/2}} = 1.00 \times 10^{6}\ \mathrm{Hz}$$



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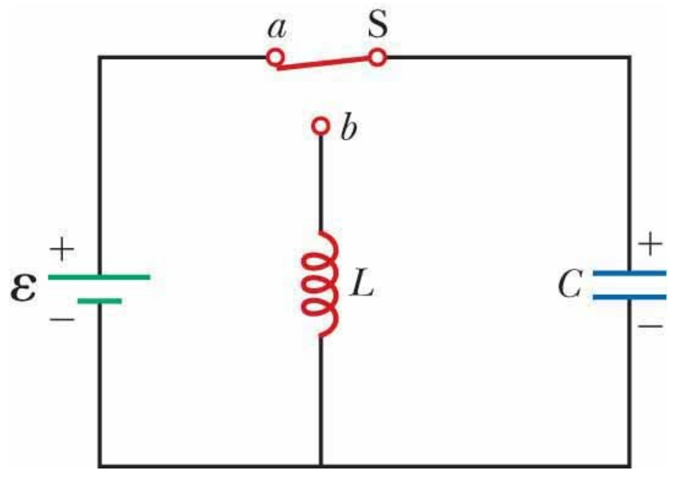
Then we can find the initial charge on the capacitor, which equals the maximum charge:

$$Q_{\max} = C\Delta V = (9.00 \times 10^{-12}\text{ F})(12.0\text{ V}) = \boxed{1.08 \times 10^{-10}\text{ C}}$$



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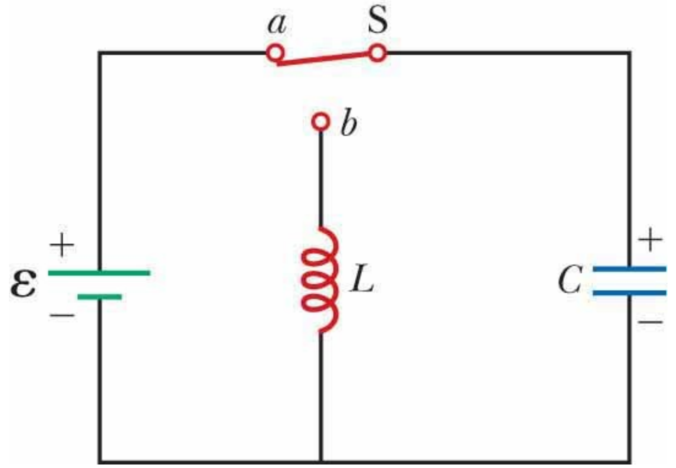
$$Q_{\max} = C\Delta V = (9.00 \times 10^{-12}\text{ F})(12.0\text{ V}) = 1.08 \times 10^{-10}\text{ C}$$

We can find the maximum current from the maximum charge:

$$I_{\max} = \omega Q_{\max} = 2\pi f Q_{\max} = (2\pi \times 10^{6}\text{ s}^{-1})(1.08 \times 10^{-10}\text{ C})$$
$$= 6.79 \times 10^{-4}\text{ A}$$



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Inductance (Ch. 31)

The RLC Circuit [Reading from Textbook]

The RLC Circuit

Suppose the switch is at position ***a*** so that the capacitor has an initial charge Q_{max}. The switch is now thrown to position ***b***. At this instant, the total energy stored in the capacitor and inductor is $Q^2_{max}/2C$. This total energy, however, is no longer constant as it was in the ***LC*** circuit because the resistor causes transformation to internal energy.

$$\frac{dU}{dt} = -i^2R \qquad U = U_E + U_B$$

$$\frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = -i^2R$$

$i = dq/dt$

$$Li\frac{d^2q}{dt^2} + i^2R + \frac{q}{C}i = 0$$

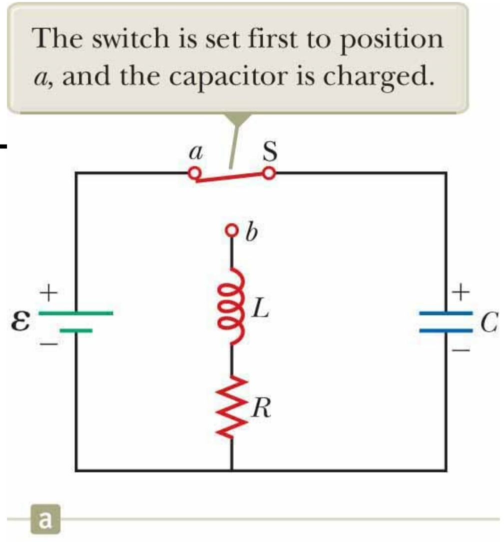
divide by *i*

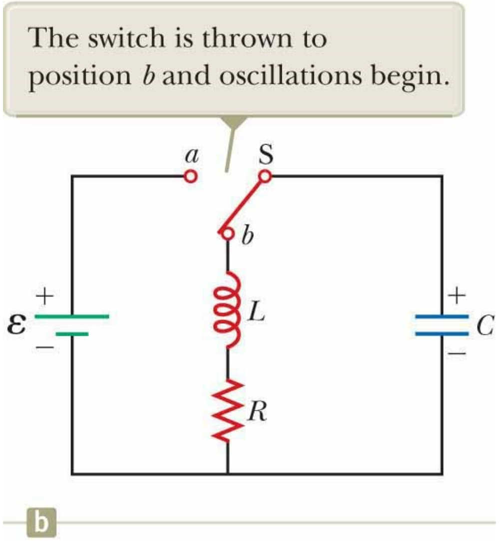
$$L\frac{d^2q}{dt^2} + iR + \frac{q}{C} = 0$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

The RLC circuit is analogous to the damped harmonic oscillator

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$





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$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

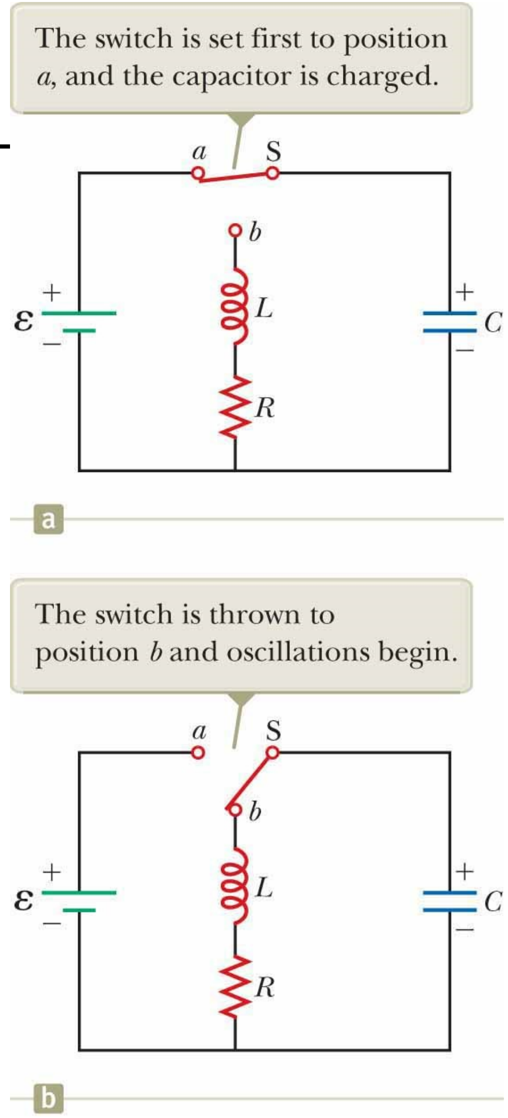
$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$q = Q_{\max}e^{-Rt/2L}\cos\omega_d t$$

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$

The RLC circuit is analogous to the damped harmonic oscillator:

- q → position x of particle at any instant
- L → mass m of particle
- R → damping coefficient b
- C → 1/k
    - k = force constant of spring

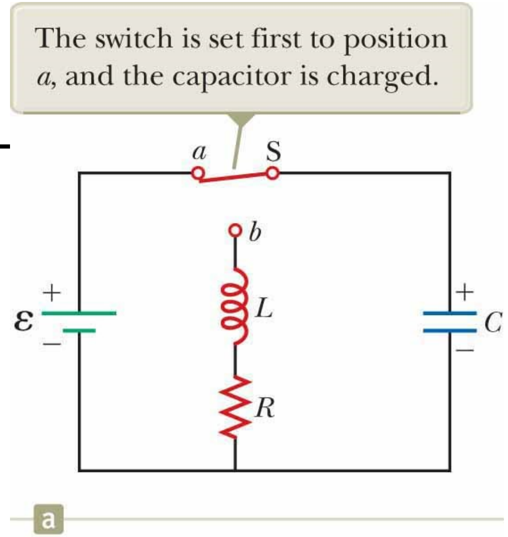


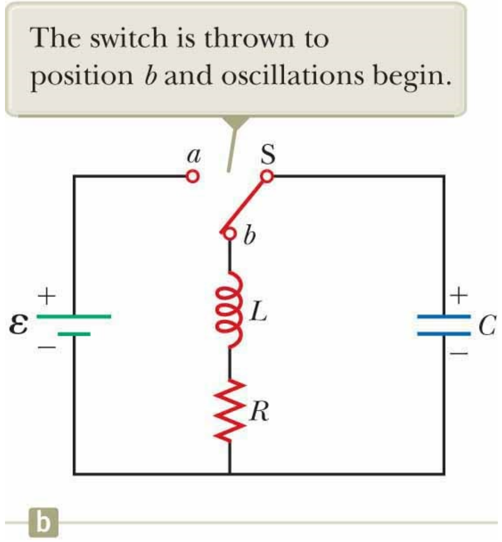
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The RLC Circuit

TABLE 31.1 Analogies Between the *RLC* Circuit and the Particle in Damped Harmonic Motion

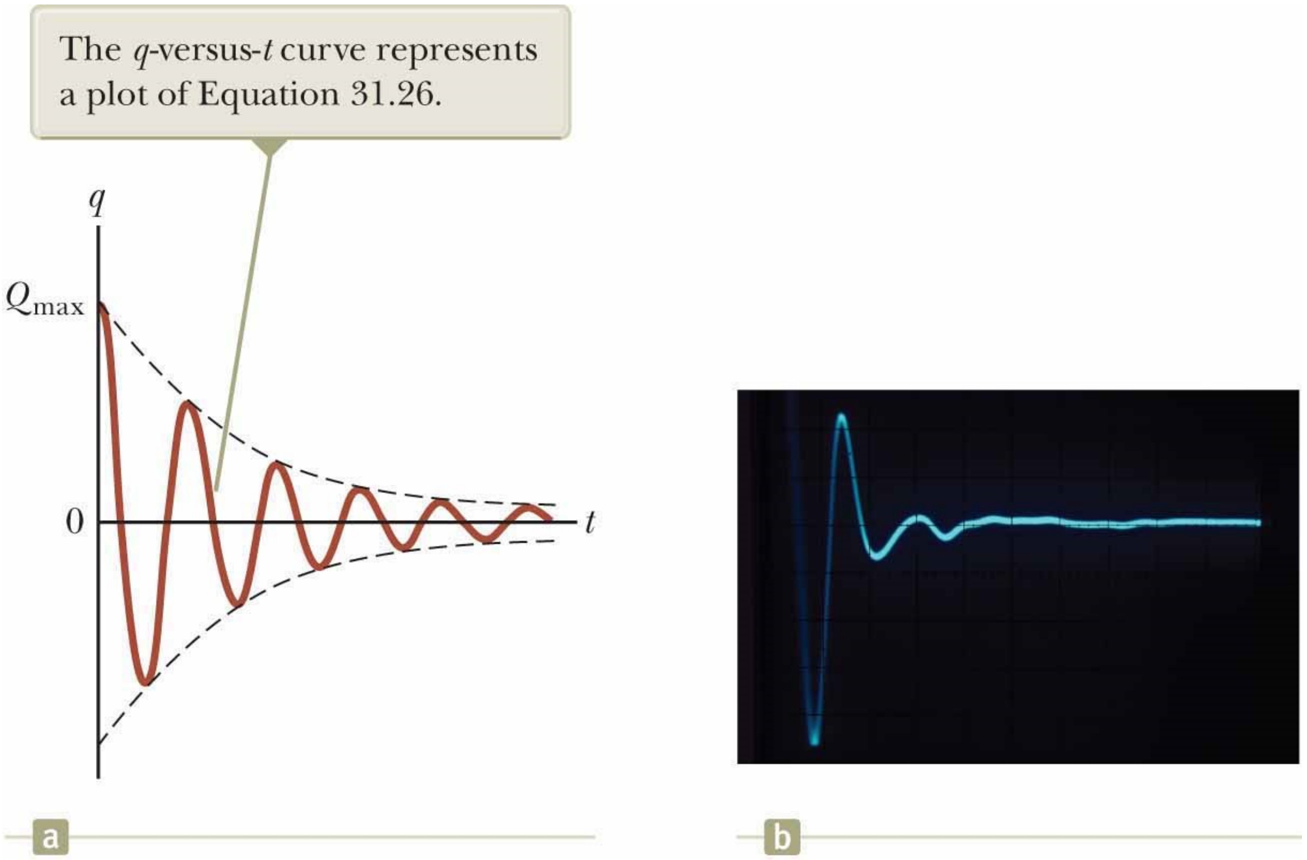
RLC Circuit		One-Dimensional Particle in Damped Harmonic Motion
Charge	$q \leftrightarrow x$	Position
Current	$i \leftrightarrow v_x$	Velocity
Potential difference	$\Delta V \leftrightarrow F_x$	Force
Resistance	$R \leftrightarrow b$	Viscous damping coefficient
Capacitance	$C \leftrightarrow 1/k$	(k = spring constant)
Inductance	$L \leftrightarrow m$	Mass
Current = time derivative of charge	$i = \dfrac{dq}{dt} \leftrightarrow v_x = \dfrac{dx}{dt}$	Velocity = time derivative of position
Rate of change of current = second time derivative of charge	$\dfrac{di}{dt} = \dfrac{d^2q}{dt^2} \leftrightarrow a_x = \dfrac{dv_x}{dt} = \dfrac{d^2x}{dt^2}$	Acceleration = second time derivative of position
Energy in inductor	$U_B = \frac{1}{2}Li^2 \leftrightarrow K = \frac{1}{2}mv^2$	Kinetic energy of moving particle
Energy in capacitor	$U_E = \frac{1}{2}\dfrac{q^2}{C} \leftrightarrow U = \frac{1}{2}kx^2$	Potential energy stored in a spring
Rate of energy loss due to resistance	$i^2R \leftrightarrow bv^2$	Rate of energy loss due to friction
RLC circuit	$L\dfrac{d^2q}{dt^2} + R\dfrac{dq}{dt} + \dfrac{q}{C} = 0 \leftrightarrow m\dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} + kx = 0$	Particle in damped harmonic motion

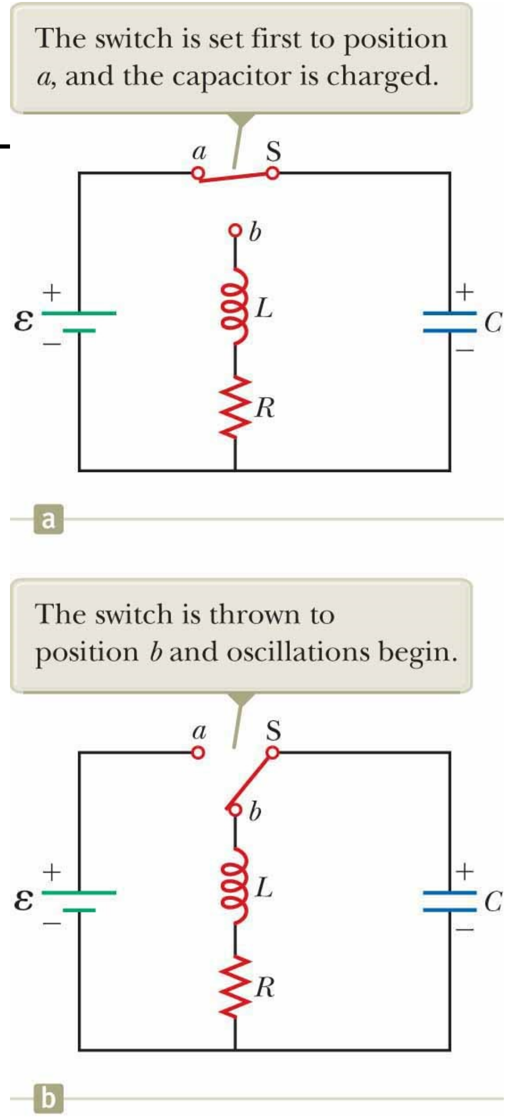




Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The RLC Circuit





$$R_c = \sqrt{4L/C}$$

Critical resistance value

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

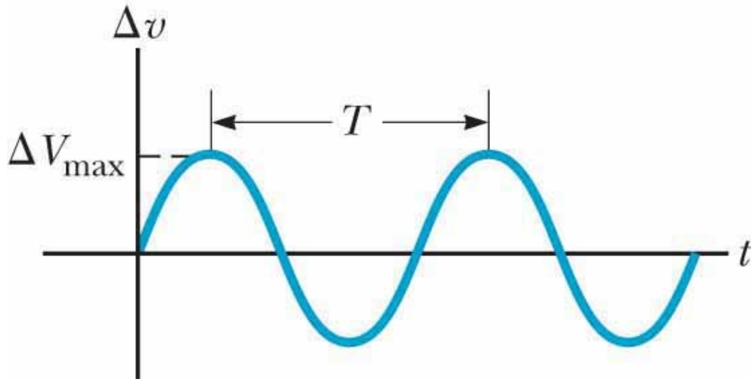
Alternating Current Circuits (Ch. 32)

AC Sources

AC Sources

An ***AC*** circuit consists of circuit elements and a power source that provides an alternating voltage.
This *time-varying* voltage from the source is described by:

$$\Delta v = \Delta V_{\max} \sin \omega t$$



ΔV_{max}: Maximum output voltage of the source (voltage amplitude)
ω: angular frequency of the source

$$\omega = 2\pi f = \frac{2\pi}{T}$$

f: frequency of the source
T: period

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

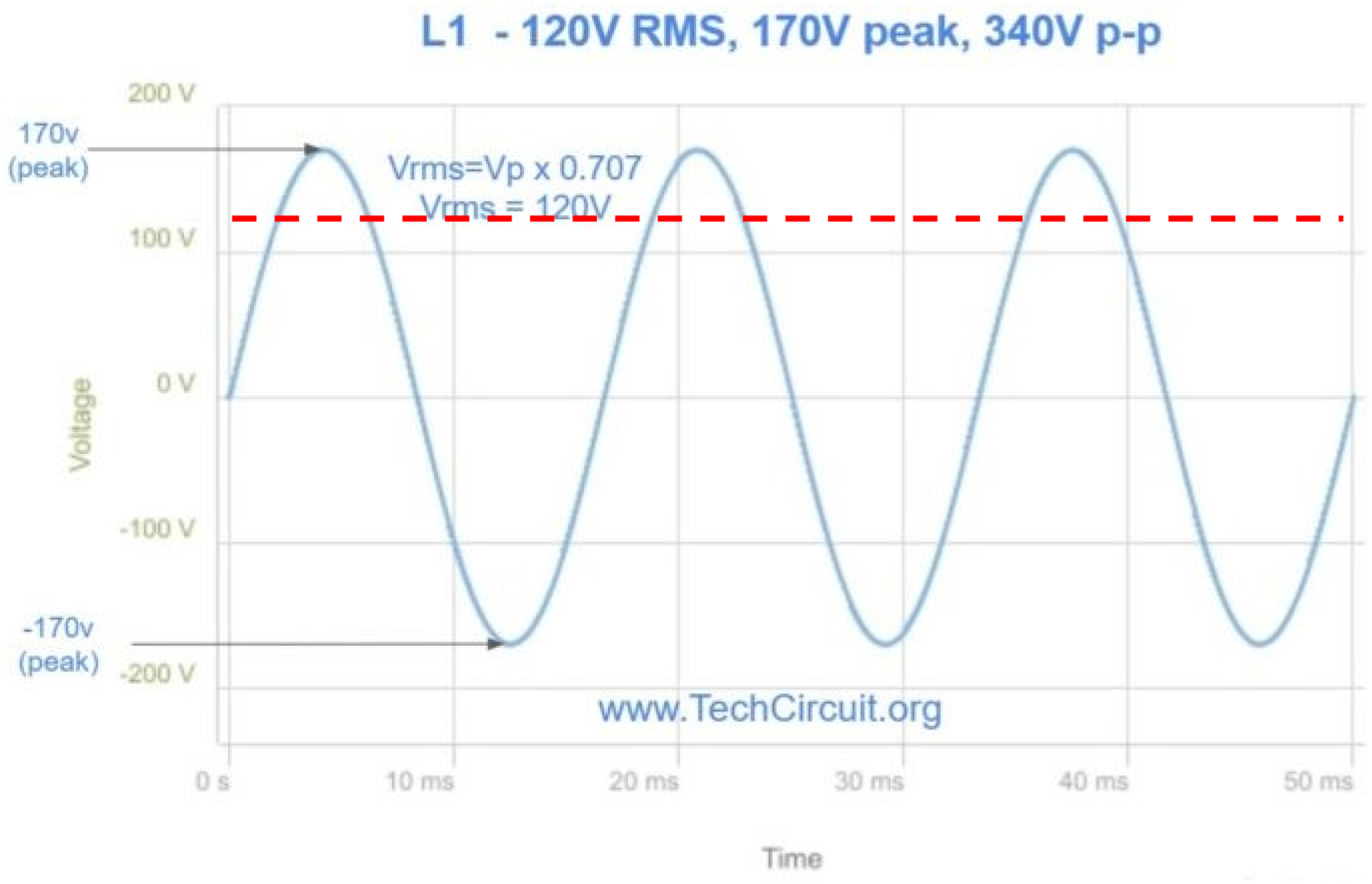
AC Sources

In a home, each electrical outlet serves as an AC source.

Because the output voltage of an AC source varies sinusoidal with time, the voltage is positive during one half of the cycle and negative during the other half.

In Canada;

- V_{max}=~170V
- V_{rms}=120V
- f=60Hz



Source: https://techcircuit.org/2020/02/08/understanding-single-split-phase-residential-line-voltage/

Alternating Current Circuits (Ch. 32)

Resistors in an AC Circuit

Resistors in an AC Circuit

Consider a simple AC circuit consisting of a resistor and an AC source.
At any instant, the algebraic sum of the voltages around a closed loop in a circuit must be zero (Kirchhoff's loop rule):

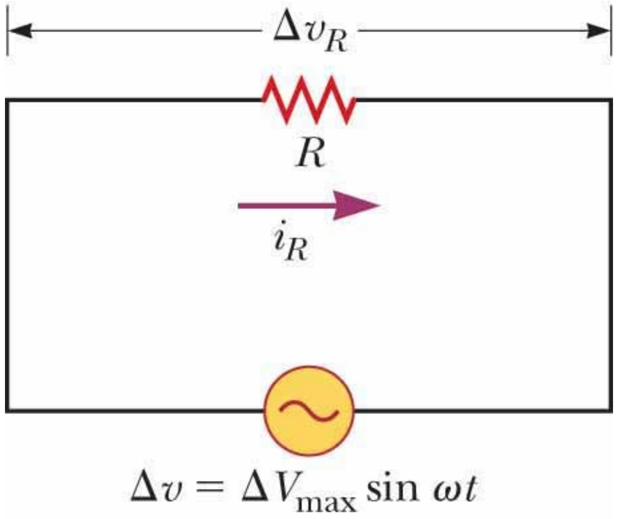
$$\Delta v - i_R R = 0$$

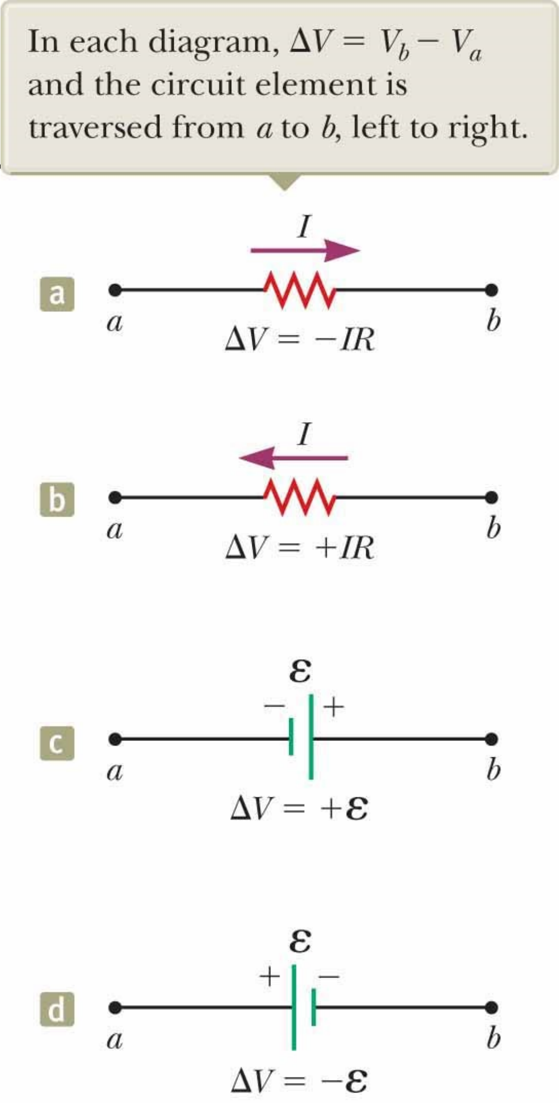
$$i_R = \frac{\Delta v}{R} = \frac{\Delta V_{\max}}{R} \sin \omega t = I_{\max} \sin \omega t$$

$$I_{\max} = \frac{\Delta V_{\max}}{R}$$

Instantaneous voltage across resistor is:

$$\Delta v_R = i_R R = I_{\max} R \sin \omega t$$





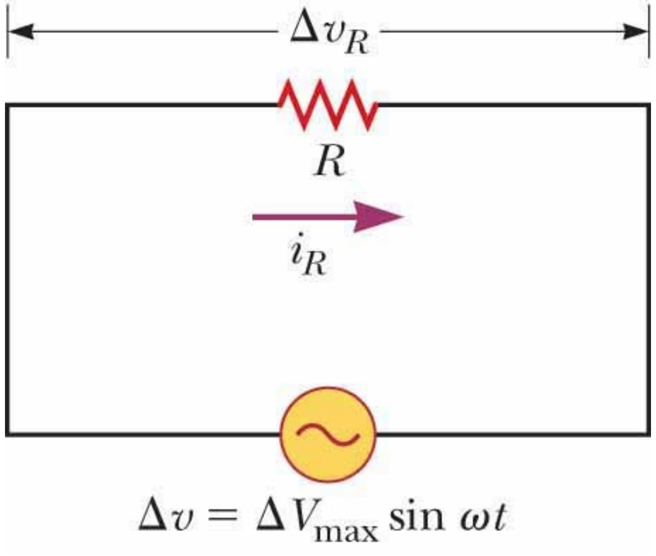
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

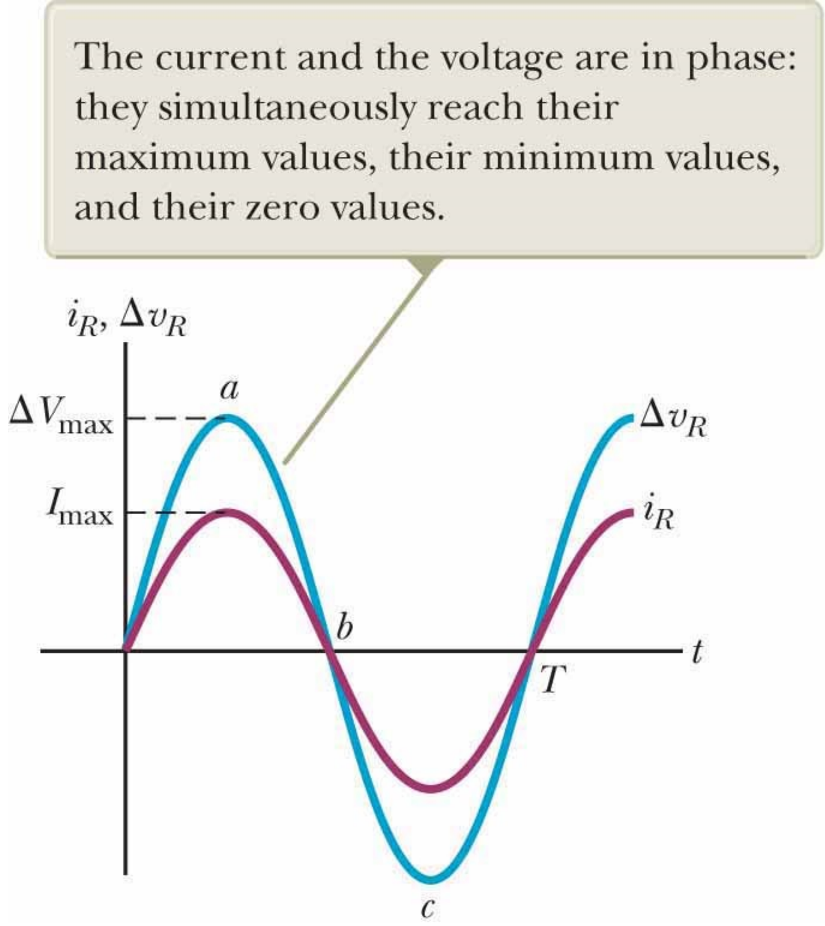
Resistors in an AC Circuit

The current and voltage are in step with each other because they vary identically with time.

Because i_R and Δv_R both vary as ***sin(ωt)*** and reach their maximum values at the same time, they are said to be ***in phase.***

Resistors behave essentially the same way in both DC and AC circuits. That, however, is not the case for capacitors and inductors.





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Phasor Diagrams

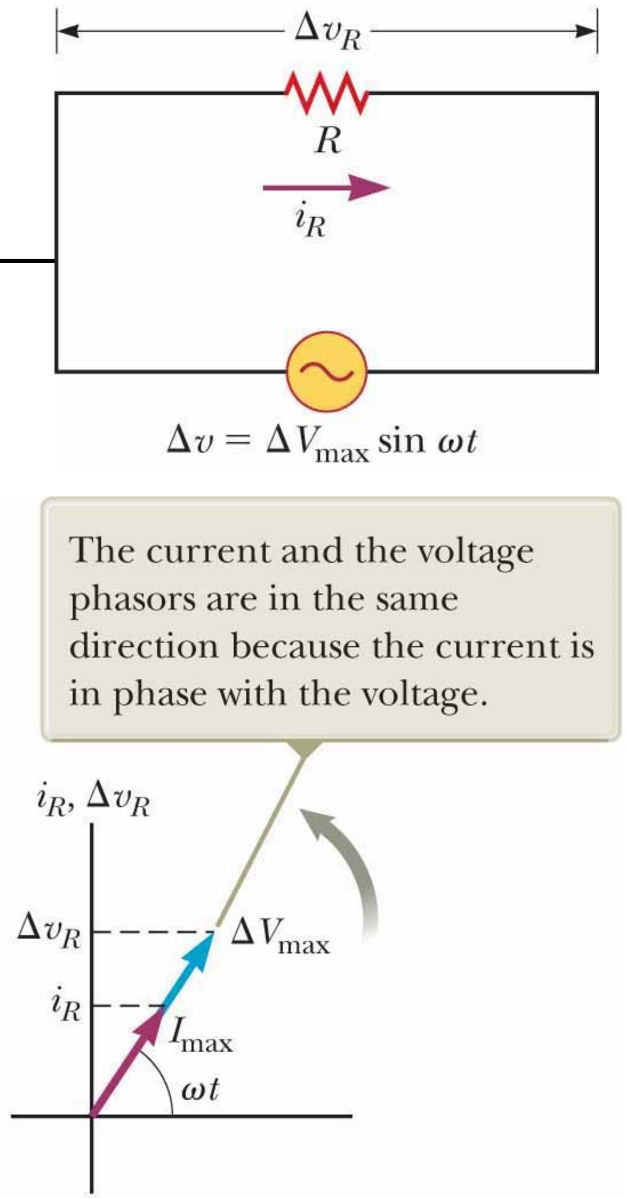
To simplify our analysis of circuits containing two or more elements, we use a graphical representation called a ***phasor diagram***.

A phasor is a vector whose length is proportional to the maximum value of the variable it represents (ΔV_{max} for voltage and I_{max} for current in this discussion).

The phasor rotates counter-clockwise at an angular speed equal to the angular frequency associated with the variable.

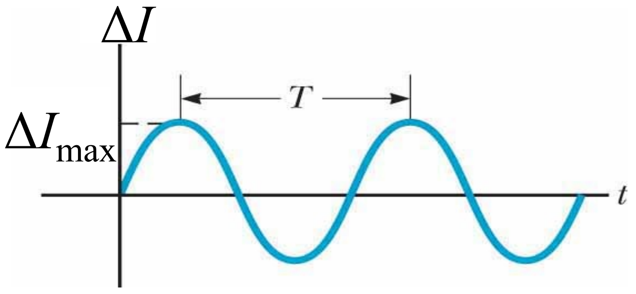
The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.

For single-loop resistive circuit:
Current and voltage phasors point to the same direction in phasor diagram because $\boldsymbol{i_R}$ and $\boldsymbol{\Delta v_R}$ are in phase.



<u>Source:</u> Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Resistors in an AC Circuit

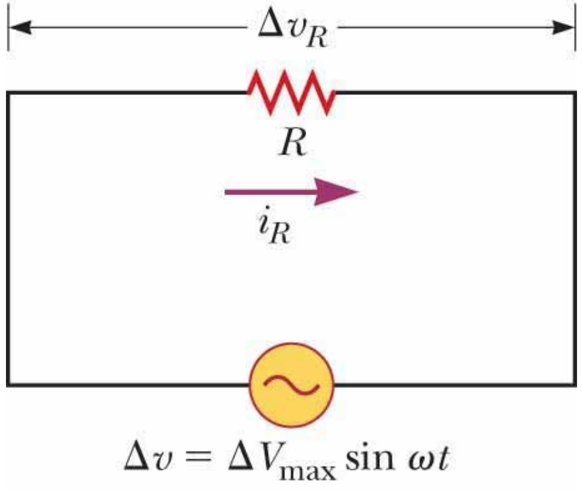


Average value of current over one cycle = 0

- Current maintained in the positive direction for the same amount of time and at same magnitude as maintained in the negative direction.

Direction of current:

- No effect on behavior of resistor
- Collisions between electrons and fixed atoms of resistor result in increase in resistor's temperature (internal energy) at all times
- Doesn't matter which way electrons are going



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

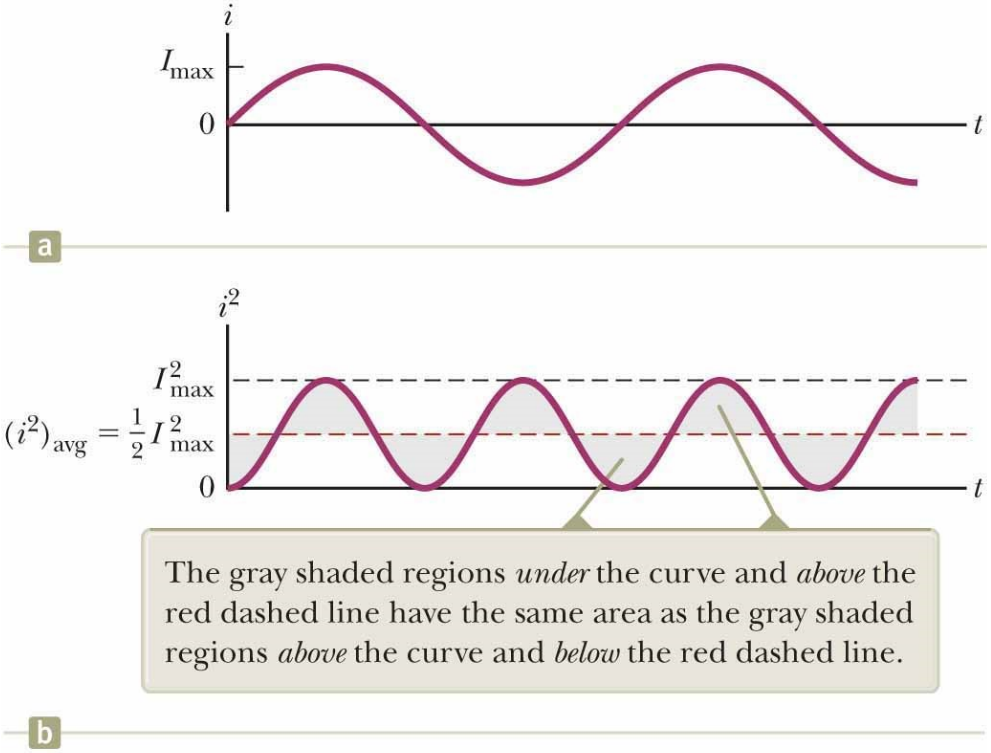
Resistors in an AC Circuit

rms Current and Voltage

- Rate of energy delivered to the resistor is power: $P = i^2R$
  - i = instantaneous current in resistor
  - No difference whether current is direct or alternating
- Average value important in AC circuit (**rms current**):

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max} \longrightarrow P_{avg} = I_{rms}^2 R$$

- Why rms?
  - AC ammeters and voltmeters designed to read rms values
  - Many equations with the same form as their direct-current counterparts

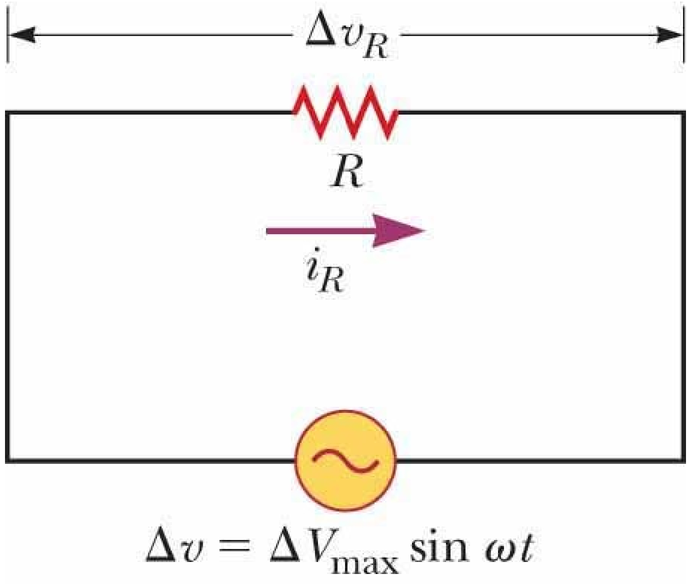


Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

What's the rms Current

Example 32.1

The voltage output of an AC source is given by the expression ***$\Delta v = 200 \sin(\omega t)$*****,** where Δv is in volts. Find the rms current in the circuit when this source is connected to a ***47.0Ω*** resistor.



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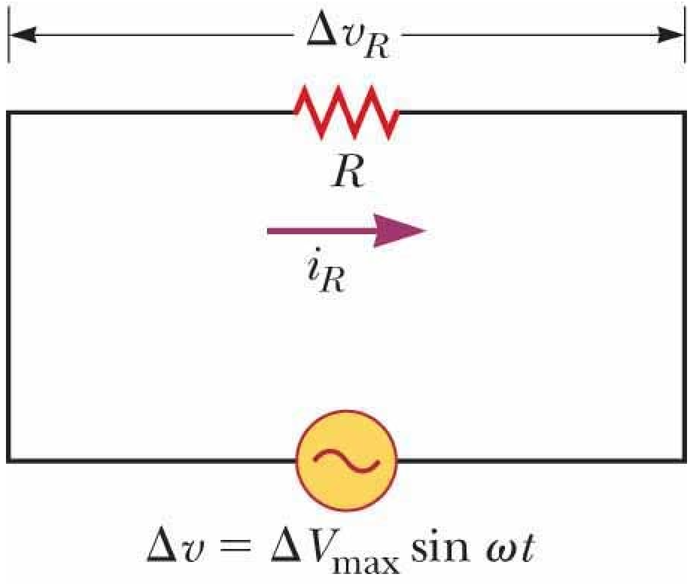


Solution

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = \frac{\Delta V_{\max}}{\sqrt{2}R}$$

$$I_{\text{rms}} = \frac{200 \text{ V}}{\sqrt{2}(47.0 \ \Omega)} = \boxed{3.01 \text{ A}}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Alternating Current Circuits (Ch. 32)

Inductors in an AC Circuit

Inductors in an AC Circuit

Now let's consider an AC circuit consisting only of an inductor connected to the terminals of an AC source. Using Kirchhoff's loop rule:

$$\Delta v - L\frac{di_L}{dt} = 0$$

$$\Delta v = L\frac{di_L}{dt} = \Delta V_{\max} \sin \omega t$$

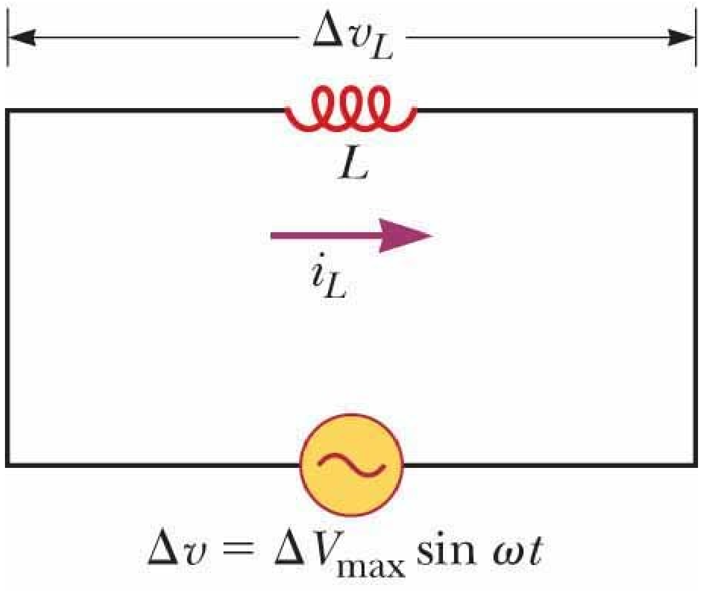
$$di_L = \frac{\Delta V_{\max}}{L} \sin \omega t \, dt$$

$$i_L = \frac{\Delta V_{\max}}{L}\int \sin \omega t \, dt = -\frac{\Delta V_{\max}}{\omega L} \cos \omega t$$

$$\cos \omega t = -\sin(\omega t - \pi/2)$$

$$i_L = \frac{\Delta V_{\max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

The instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are out of phase by $\pi/2$ rad = 90°.



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Inductors in an AC Circuit

$$\Delta v = \Delta V_{\max} \sin \omega t$$

$$i_L = \frac{\Delta V_{\max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

When voltage Δv_L across inductor maximum (point *a*) $\rightarrow$

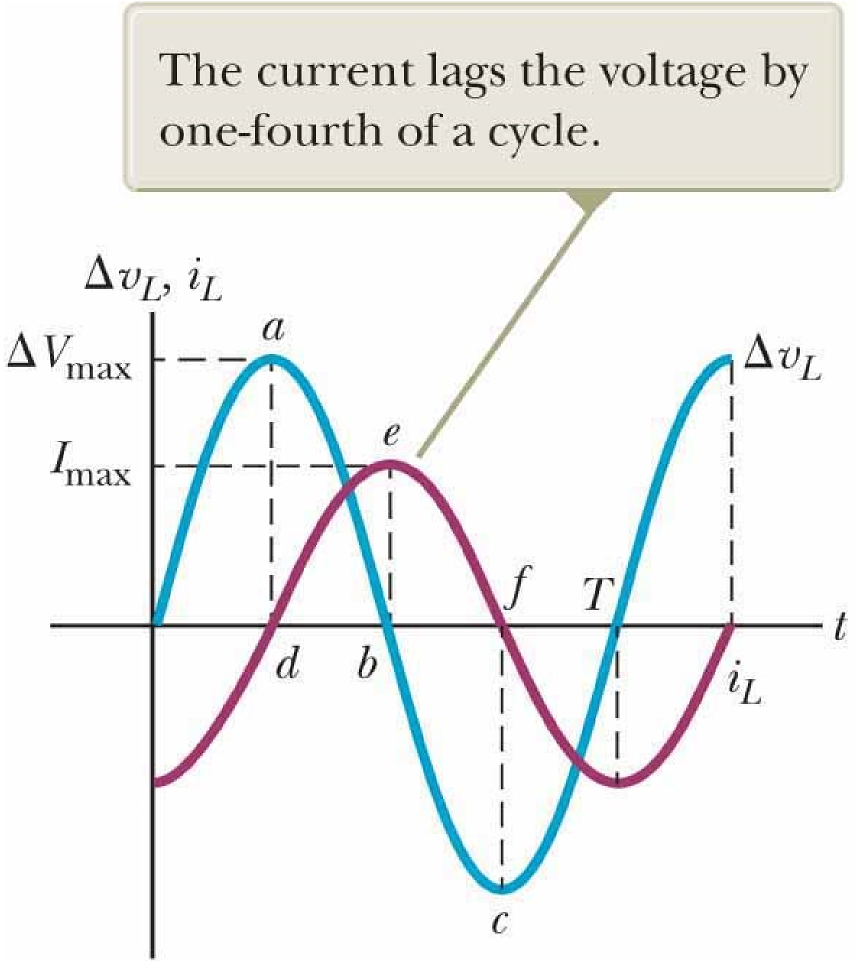
- Current in inductor = 0 (point *d*)
  - Changing at its highest rate

When voltage = 0 (point *b*) $\rightarrow$

- Current has maximum value (point *e*)

Voltage reaches maximum value one-quarter of period before current reaches maximum value.

- For sinusoidal applied voltage: current in inductor always *lags* behind voltage across inductor by 90°
  - One-quarter cycle in time



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Inductors in an AC Circuit

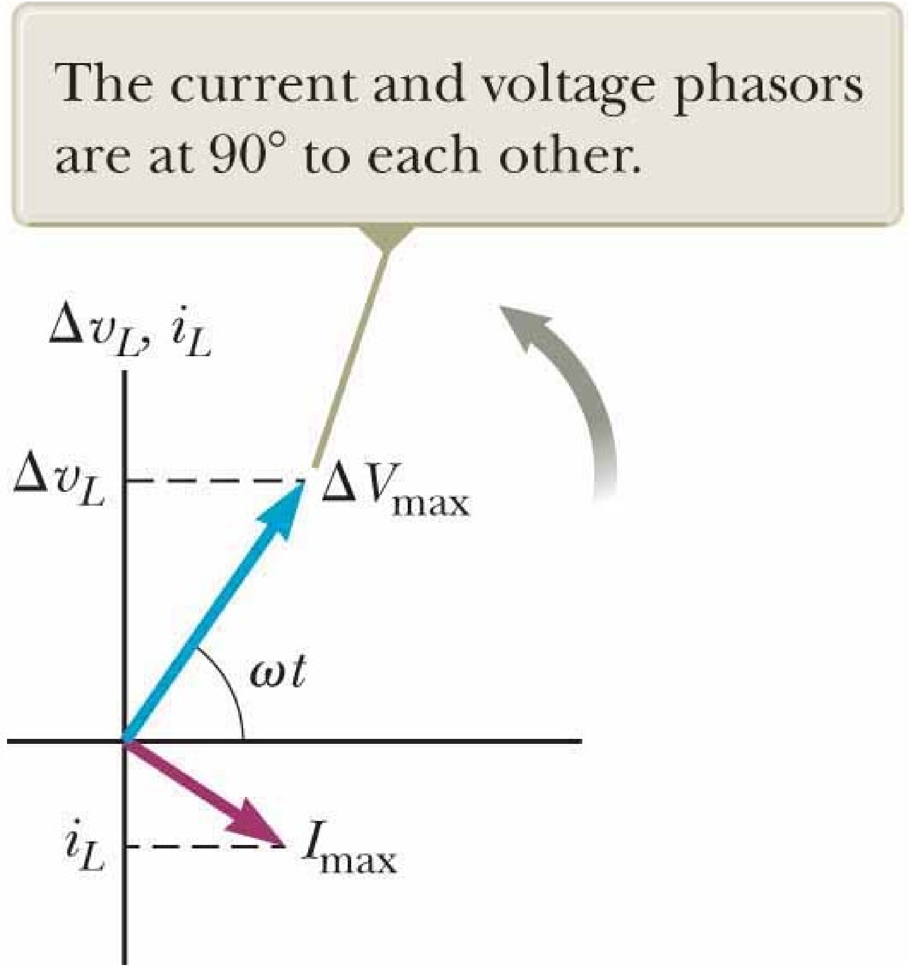
$$\Delta v = \Delta V_{\max} \sin \omega t$$

$$i_L = \frac{\Delta V_{\max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$I_{\max} = \frac{\Delta V_{\max}}{\omega L} \longleftrightarrow I = \Delta V / R$$

We define ***ωL*** as inductive reactance ***X_L:***

$$\boxed{X_L \equiv \omega L} \rightarrow \boxed{I_{\max} = \frac{\Delta V_{\max}}{X_L}}$$



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

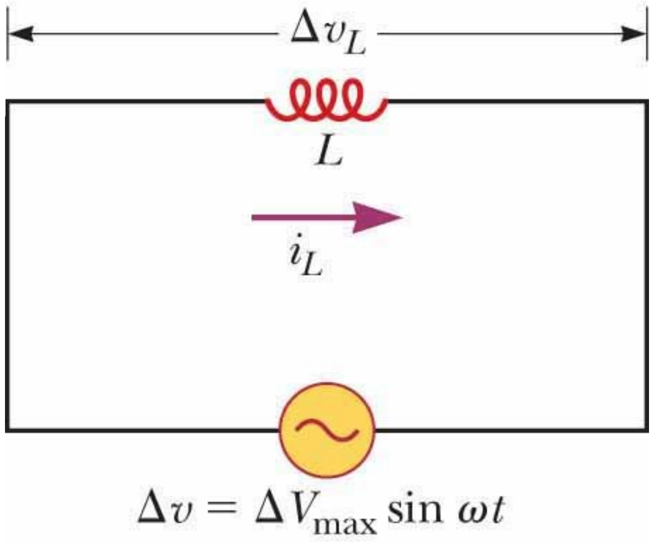
A Purely Inductive AC Circuit

Example 32.2

In a purely inductive AC circuit, ***L = 25.0 mH*** and the rms voltage is ***150 V***. Calculate the inductive reactance (***X_L***) and rms current in the circuit if the frequency is ***60.0 Hz***.

$$X_L \equiv \omega L$$

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{X_L}$$



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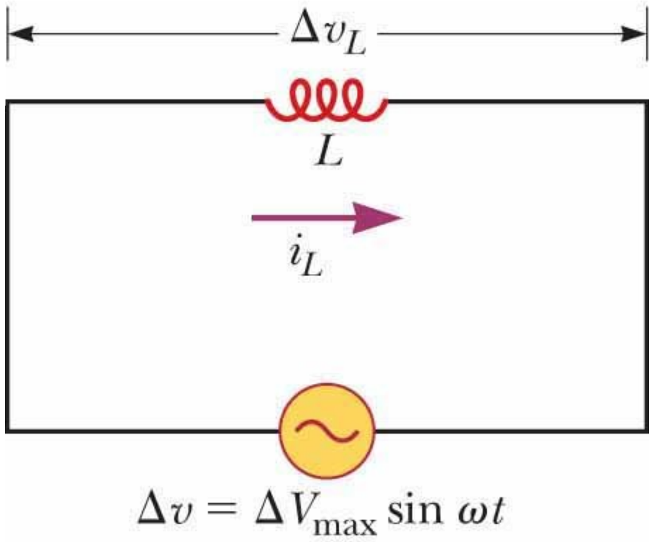

Solution

$$X_L = \omega L = 2\pi f L = 2\pi(60.0\ \text{Hz})(25.0 \times 10^{-3}\ \text{H})$$

$$= \boxed{9.42\ \Omega}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

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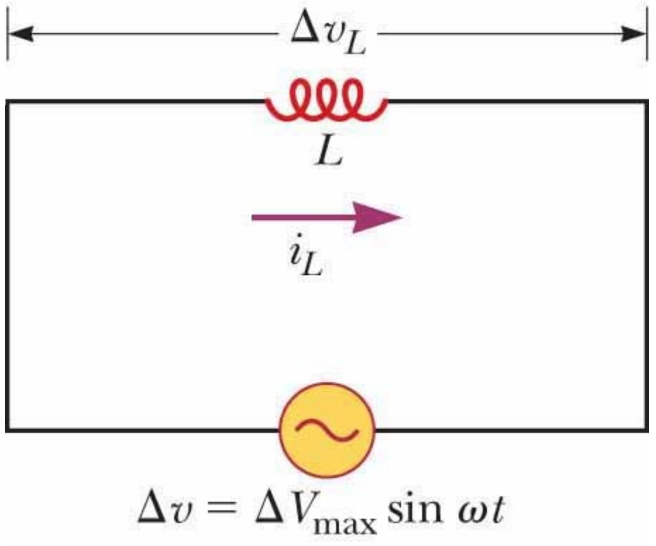


Solution

$$X_L = \omega L = 2\pi f L = 2\pi (60.0 \text{ Hz})(25.0 \times 10^{-3} \text{ H})$$
$$= \boxed{9.42\ \Omega}$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{150 \text{ V}}{9.42\ \Omega} = \boxed{15.9 \text{ A}}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Alternating Current Circuits (Ch. 32)

Capacitors in an AC Circuit

Capacitors in an AC Circuit

Now let's consider an AC circuit consisting only of a capacitor connected to the terminals of an AC source. Using Kirchhoff's loop rule:

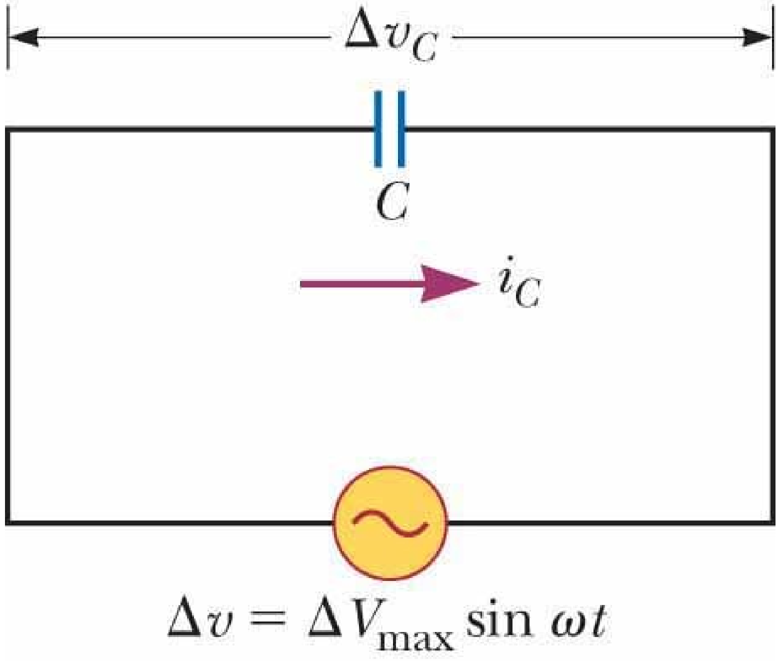
$$\Delta v - \frac{q}{C} = 0$$

$$q = C\,\Delta V_{\max} \sin \omega t$$

$$i_C = \frac{dq}{dt} = \omega C\,\Delta V_{\max} \cos \omega t$$

$$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i_C = \omega C\,\Delta V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

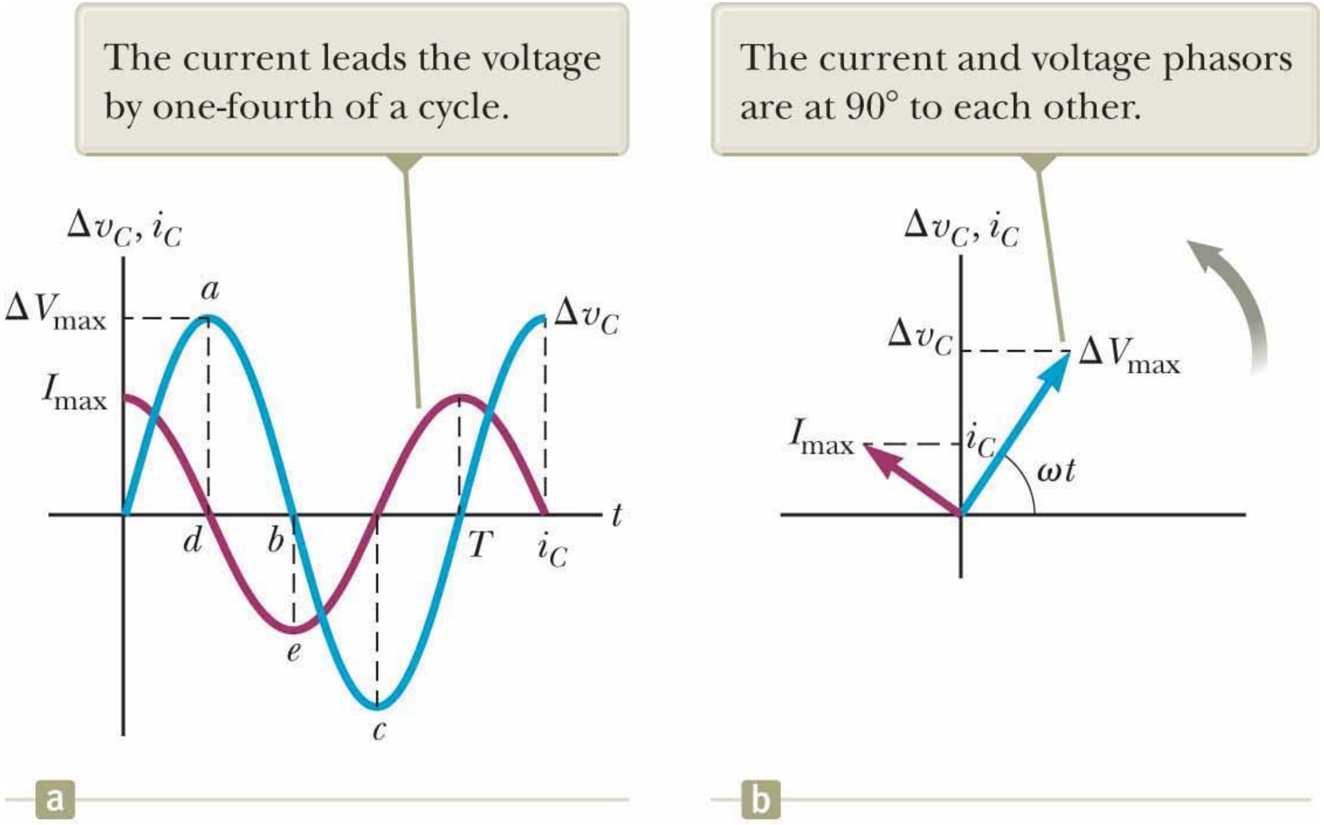


The instantaneous current i_C of the capacitor and the instantaneous voltage Δv_L across the capacitor are out of phase by π/2 rad = 90°.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Capacitors in an AC Circuit

Phasor diagram for current and voltage:
→ for sinusoidally applied voltage, current ***leads*** voltage across capacitor by **90°**



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Capacitors in an AC Circuit

$$i_C = \omega C \Delta V_{\max} \cos \omega t$$

Current in circuit reaches maximum value when $\cos\omega t = \pm 1$:

$$I_{\max} = \omega C \Delta V_{\max} = \frac{\Delta V_{\max}}{(1/\omega C)}$$

The denominator plays the role of resistance, with units of ohms.

We define ***1/ωC*** as capacitive reactance $\boldsymbol{X_C}$:

$$X_C \equiv \frac{1}{\omega C} \rightarrow I_{\max} = \frac{\Delta V_{\max}}{X_C}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

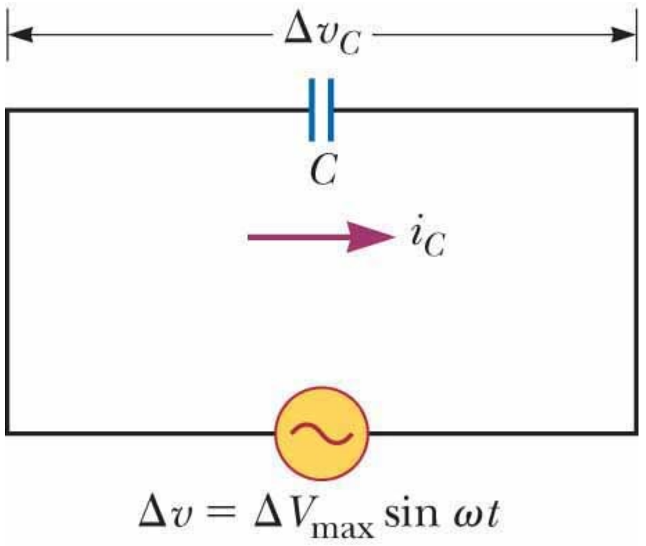
A Purely Capacitive AC Circuit

Example 32.3

An ***8.00- μF*** capacitor is connected to the terminals of a ***60.0-Hz*** AC source whose rms voltage is ***150 V***. Find the capacitive reactance (X_C) and the rms current in the circuit.

$$X_C \equiv \frac{1}{\omega C}$$

$$I_{max} = \frac{\Delta V_{max}}{X_C}$$



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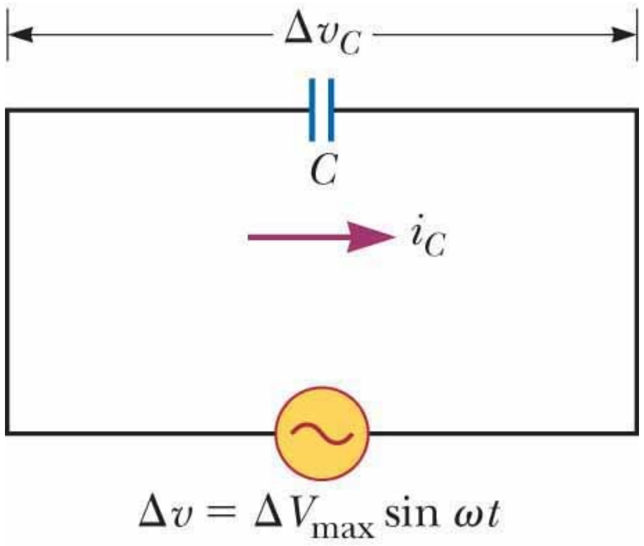

Solution

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\ \mathrm{Hz})(8.00 \times 10^{-6}\ \mathrm{F})} = \boxed{332\ \Omega}$$

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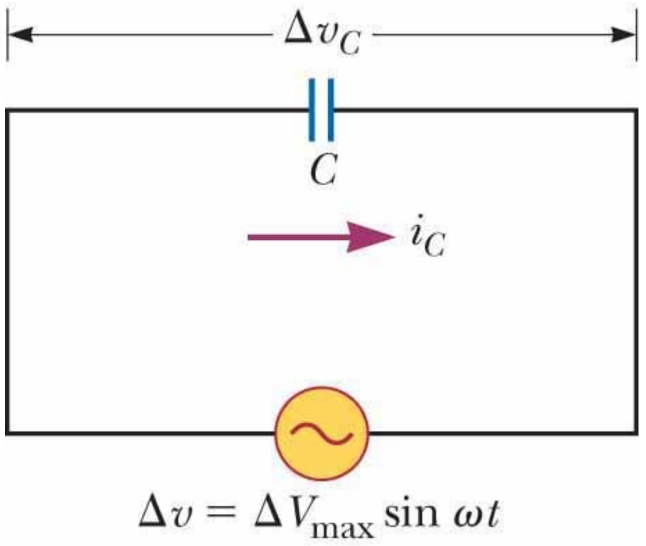

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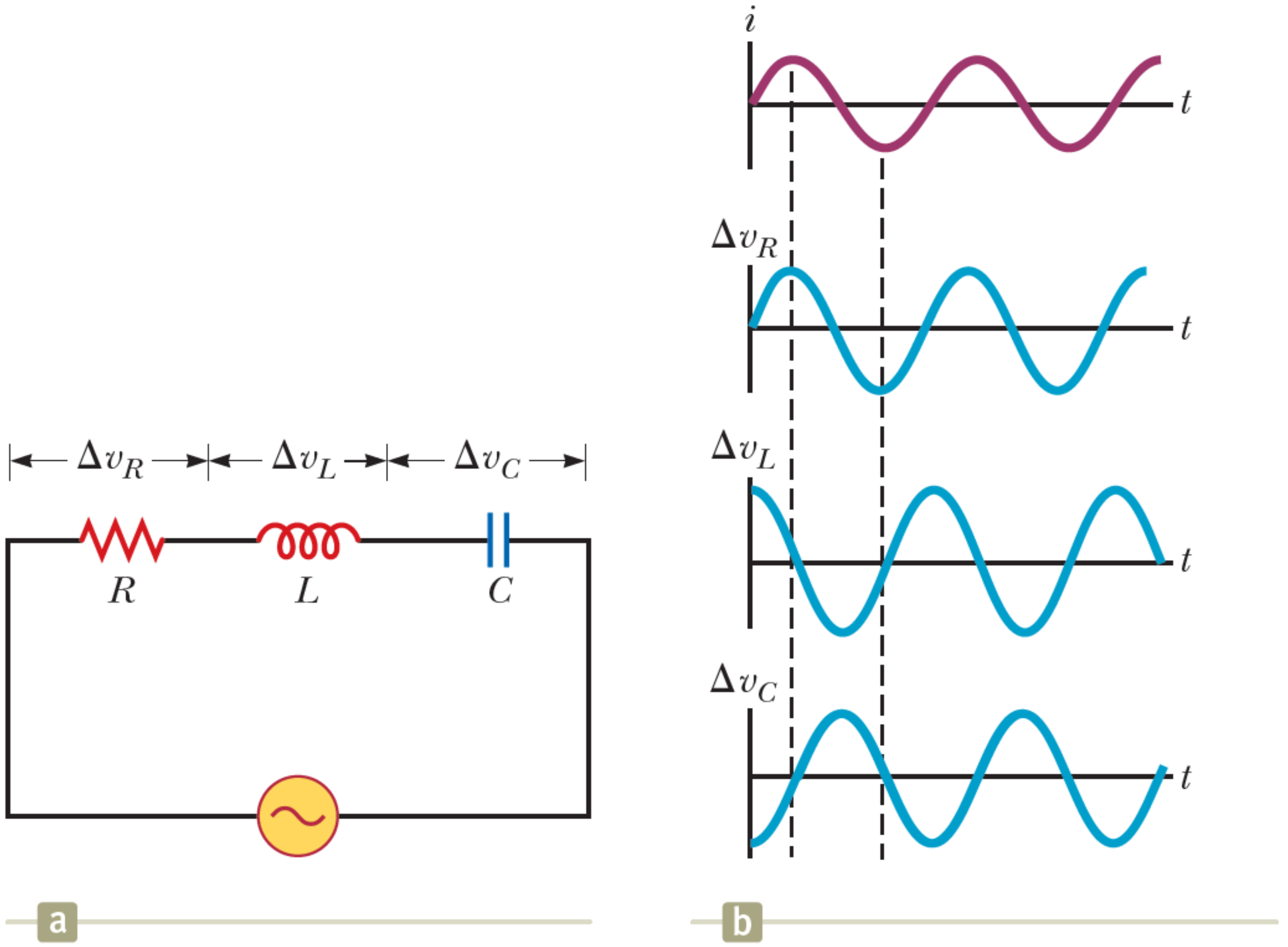
$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{150\text{ V}}{332\ \Omega} = \boxed{0.452\text{ A}}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Alternating Current Circuits (Ch. 32)

The RLC Series Circuit [Reading from Textbook]

The RLC Series Circuit



$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

$$I_{\max} = \frac{\Delta V_{\max}}{Z}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Alternating Current Circuits (Ch. 32)

Power in AC Circuits

Power in AC Circuits

- In DC circuits, the power delivered by a battery to an external DC circuit is equal to the product of the current and the terminal voltage of the battery. Likewise, the instantaneous power delivered by an AC source to a circuit is the product of the current and the applied voltage.

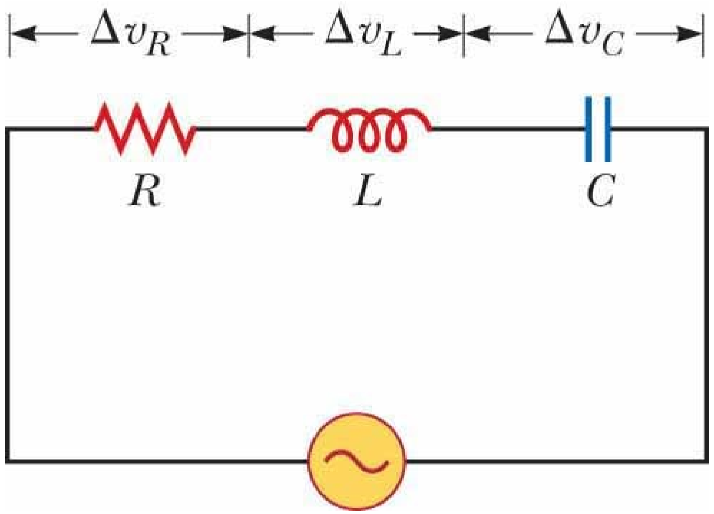
- For the RLC circuit in the figure, we can express the instantaneous power ***P*** as:

$$P = i\,\Delta v = I_{\max} \sin(\omega t - \phi)\, \Delta V_{\max} \sin \omega t$$

$$P = I_{\max}\, \Delta V_{\max} \sin \omega t \sin(\omega t - \phi)$$

- This result is a complicated function of time and is therefore not very useful from a practical viewpoint.

- What is generally of interest is the ***average power*** over one or more cycles.



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Power in AC Circuits

- Average power can be computed by first using the trigonometric identity:

$$\sin(\omega t - \phi) = \sin \omega t \cos \phi - \cos \omega t \sin \phi$$

- Substituting this identity gives us:

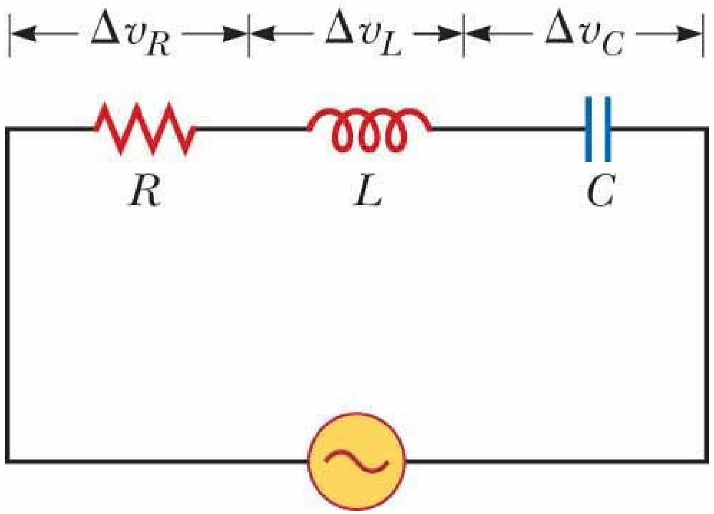
$$P = I_{\max} \Delta V_{\max} \underbrace{\sin^2 \omega t}_{½} \cos \phi - I_{\max} \Delta V_{\max} \underbrace{\sin \omega t \cos \omega t \sin \phi}_{0}$$

Taking the time average

$$P_{\text{avg}} = \frac{1}{2} I_{\max} \Delta V_{\max} \cos \phi$$

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = 0.707 I_{\max} \qquad \Delta V_{\text{rms}} = \frac{\Delta V_{\max}}{\sqrt{2}} = 0.707\, \Delta V_{\max}$$

$$\boxed{P_{\text{avg}} = I_{\text{rms}} \Delta V_{\text{rms}} \underbrace{\cos \phi}_{\text{Power factor}}}$$



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

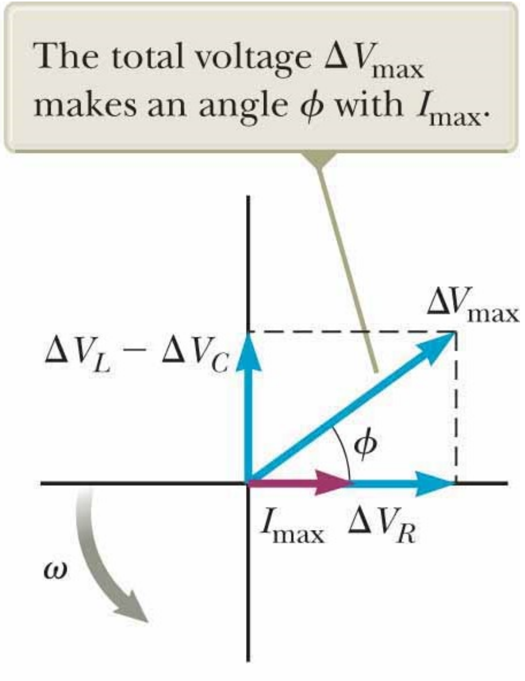
Power in AC Circuits

$$P_{avg} = I_{rms}\,\Delta V_{rms}\,\underbrace{\cos\phi}_{\text{Power factor}}$$

- When the load is purely resistive, ϕ=0, cosϕ=1;

$$P_{avg} = I_{rms}^2 R$$

$$P_{avg} = I_{rms}\,\Delta V_{rms}$$



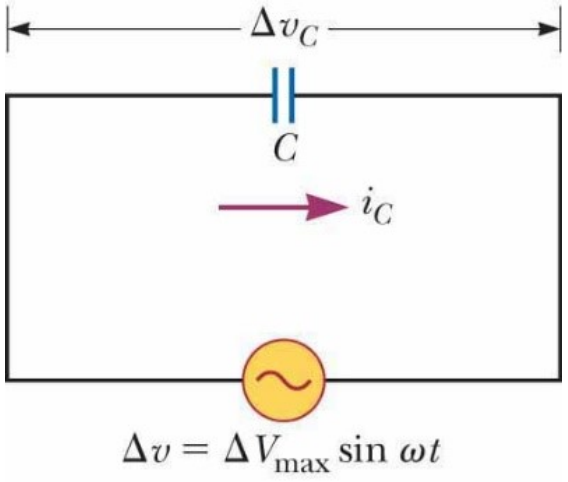
- The average power delivered by the source is converted to internal energy in the resistor (dissipated), just as in the case of a DC circuit.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Power in AC Circuits

- Note that no power losses are associated with ***pure capacitors*** and ***pure inductors*** in an AC circuit.

Capacitor

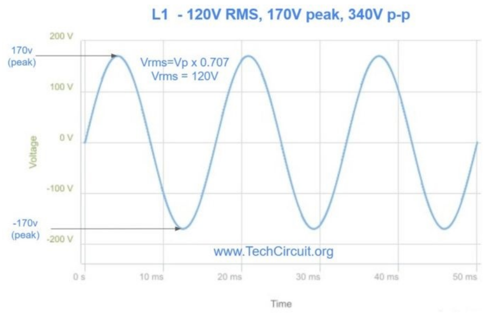


- When the current begins to increase in one direction in an AC circuit, charge begins to accumulate on the capacitor and a voltage appears across it.

- When this voltage reaches its maximum value, the energy stored in the capacitor as electric potential energy is **½ $C(\Delta V_{max})^2$**

- This energy storage, however, is only momentary. The capacitor is charged and discharged twice during each cycle

- Therefore, the average power supplied by the source is zero. In other words, no power losses occur in a capacitor in an AC circuit.



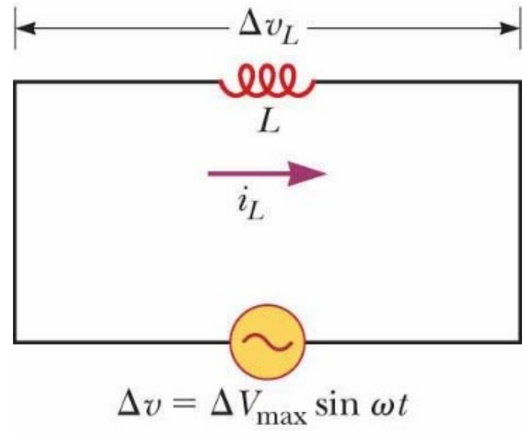
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

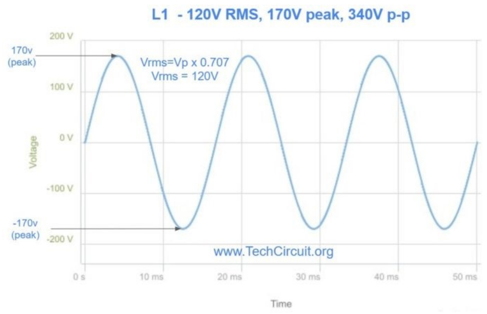
Power in AC Circuits

- Note that no power losses are associated with ***pure capacitors*** and ***pure inductors*** in an AC circuit.

Inductor

- When the current in an inductor reaches its maximum value, the energy stored in the inductor is a maximum and is given by **$\frac{1}{2} L(I_{max})^2$**
- When the current begins to decrease in the circuit, this stored energy in the inductor returns to the source as the inductor attempts to maintain the current in the circuit.



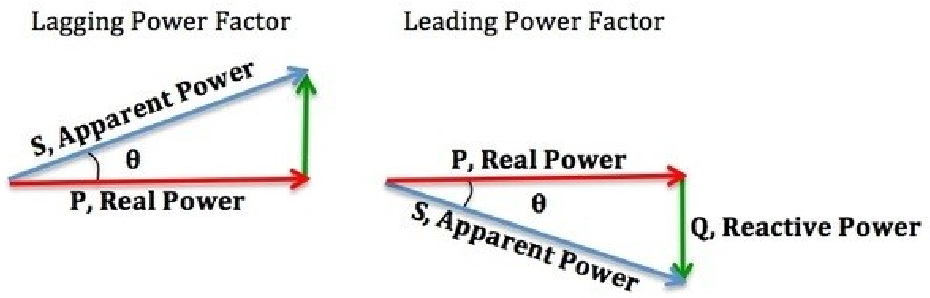


Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Power in AC Circuits

Power factor correction

https://www.youtube.com/watch?v=NF4VRKa7LSM [2:26]



Source:
https://electricalenergyworld.blogspot.com/2019/02/what-is-power-factor-power-factor-pf-is.html

Alternating Current Circuits (Ch. 32)

Resonance in a Series RLC Circuit [Reading from Textbook]

Alternating Current Circuits (Ch. 32)

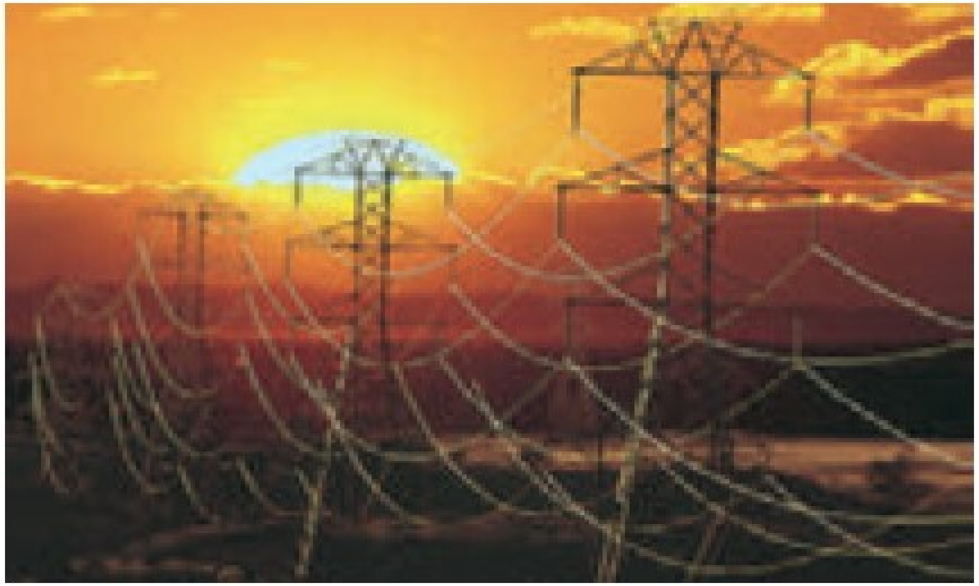
The Transformer and Power Transmission

The Transformer and Power Transmission

- The power lines transfer energy from the electric production facilities to homes and businesses. The energy is transferred at a very high voltage, possibly hundreds of thousands of volts in some cases. Even though it makes power lines very dangerous, the high voltage results in less loss of energy due to resistance in the wires.

- It is economical to use a high voltage and a low current to minimize the $\boldsymbol{I^2R}$ loss in transmission lines when electric power is transmitted over great distances.

- Consequently, ***350-kV*** lines are common, and in many areas, even higher-voltage (***765-kV***) lines are used.



Lester Lefkowitz/Taxi/Getty Images

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Transformer and Power Transmission

- At the receiving end of such lines, the consumer requires power at a low voltage (for safety and for efficiency in design).

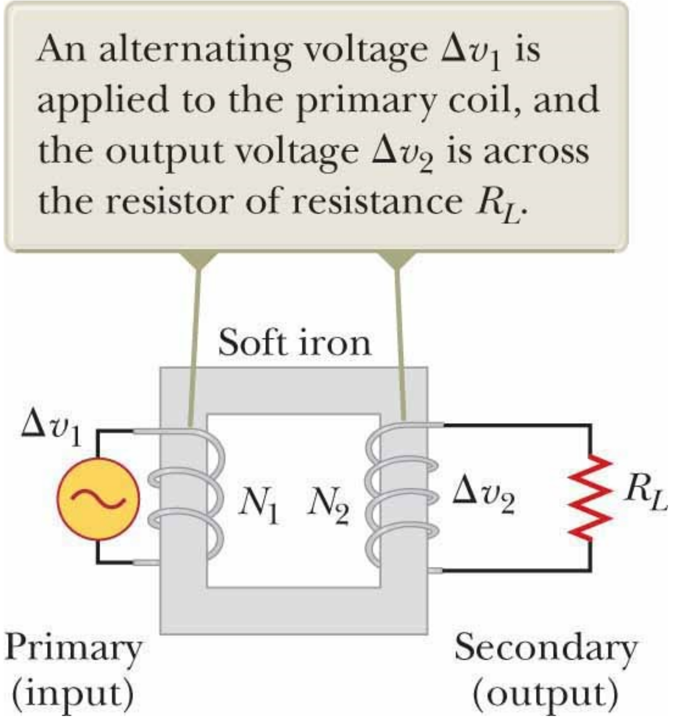
- In practice, the voltage is decreased to approximately ***20,000V*** at a distribution substation, then to ***4,000 V*** for delivery to residential areas, and finally to ***120V*** and ***240V*** at the customer's site.

- Therefore, a device is needed that can change the alternating voltage and current without causing appreciable changes in the power delivered. The ***AC transformer*** is that device.

Source:
https://energyeducation.ca/encyclopedia/Electrical_substation
https://www.pplelectric.com/utility/about-us/connect-newsletter/residential/2020/september-2020-residential-connect.aspx
Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Transformer and Power Transmission

- In its simplest form, the AC transformer consists of two coils of wire wound around a core of iron.
- The coil on the left, which is connected to the input alternating-voltage source and has N_1 turns, is called the ***primary winding*** (or the ***primary***). The coil on the right, consisting of N_2 turns and connected to a load resistor R_L, is called the ***secondary winding*** (or the ***secondary***).
- The purposes of the iron core are to increase the magnetic flux through the coil and to provide a medium in which nearly all the magnetic field lines through one coil pass through the other coil.
- Eddy-current losses are reduced by using a laminated core.
- Typical transformers have ***power efficiencies from 90% to 99%.***



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Transformer and Power Transmission

Assuming an ***ideal transformer***, Faraday's law states:

$$\Delta v_1 = -N_1 \frac{d\Phi_B}{dt}$$

where Φ_B is the magnetic flux through each turn. Since we assumed all magnetic field lines remain within the iron core, the flux through each turn of the primary equals the flux through each turn of the secondary. Hence, the voltage across the secondary is:

$$\Delta v_2 = -N_2 \frac{d\Phi_B}{dt}$$

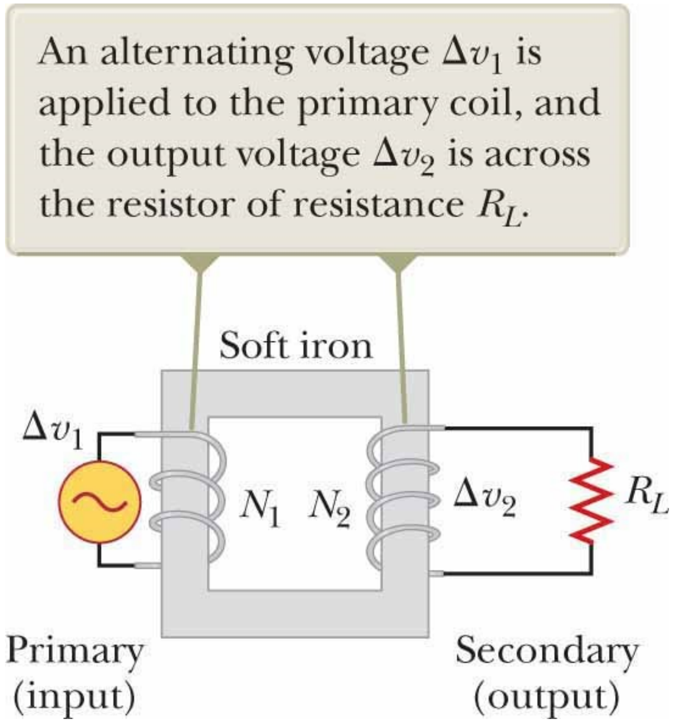
$$\Delta v_2 = \frac{N_2}{N_1} \Delta v_1$$

When N2 > N1 →

Output voltage V_2 exceeds input voltage V_1:
Step-up transformer

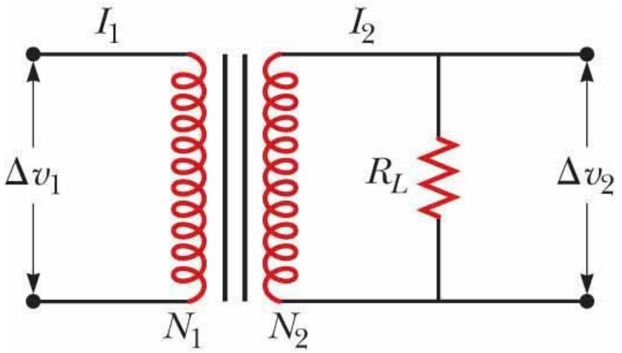
When N2 < N1 →

Output voltage less than input voltage:
Step-down transformer



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

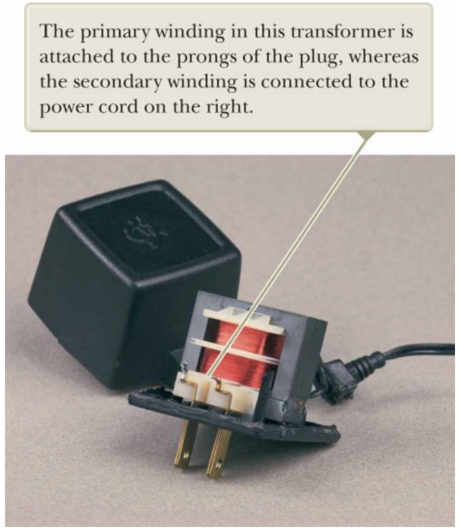
The Transformer and Power Transmission



When a current I_1 exists in the primary circuit, a current I_2 is induced in the Secondary (uppercase I and ΔV refer to rms values).

In an ideal transformer where there are no losses, the power supplied by the source is equal to the power in the secondary circuit:

$$I_1 \Delta V_1 = I_2 \Delta V_2$$



To operate properly, many common household electronic devices require low voltages. A small transformer that plugs directly into the wall like the one illustrated in the figure can provide the proper voltage. This particular transformer converts the ***120-V*** AC in the wall socket to ***12.5***-V AC.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Economics of AC Power

Example 32.7

An electricity-generating station needs to deliver energy at a rate of ***20MW*** to a city ***1.0km*** away. A common voltage for commercial power generators is ***22kV***, but a step-up transformer is used to boost the voltage to ***230kV*** before transmission.

A) If the resistance of the wires is ***2.0Ω*** and the energy costs are about ***11¢/kWh***, estimate the cost of the energy converted to internal energy in the wires (dissipated on the resistance) during one day.

B) Repeat the calculation for the hypothetical situation in which the power plant delivers the energy at its original voltage of 22 kV.

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Solution

Solution for part A

Let's first calculate the I_{rms} in the wires:

$$I_{\text{rms}} = \frac{P_{\text{avg}}}{\Delta V_{\text{rms}}} = \frac{20 \times 10^6 \text{ W}}{230 \times 10^3 \text{ V}} = 87 \text{ A}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

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Now we can calculate the power dissipated / lost on wires:

$$P_{\text{wires}} = I_{\text{rms}}^2 R = (87 \text{ A})^2 (2.0 \ \Omega) = 15 \text{ kW}$$

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Let's calculate the total energy lost on wires in one day (24 hrs):

$$T_{\text{ET}} = P_{\text{wires}} \Delta t = (15 \text{ kW})(24 \text{ h}) = \boxed{363 \text{ kWh}}$$

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We can now calculate the cost of the lost energy:

$$\text{Cost} = (363 \text{ kWh})(\$0.11/\text{kWh}) = \$40$$

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B) Repeat the calculation for the hypothetical situation in which the power plant delivers the energy at its original voltage of 22 kV.

Solution

Solution for part B

Let's first calculate the I_{rms} in the wires:

$$I_{\text{rms}} = \frac{P_{\text{avg}}}{\Delta V_{\text{rms}}} = \frac{20 \times 10^6 \text{ W}}{22 \times 10^3 \text{ V}} = 909 \text{ A}$$

Now we can calculate the power dissipated / lost on wires:

$$P_{\text{wires}} = I_{\text{rms}}^2 R = (909 \text{ A})^2(2.0 \ \Omega) = 1.7 \times 10^3 \text{ kW}$$

Let's calculate the total energy lost on wires in one day (24 hrs):

$$T_{\text{ET}} = P_{\text{wires}} \Delta t = (1.7 \times 10^3 \text{ kW})(24 \text{ h}) = 4.0 \times 10^4 \text{ kWh}$$

We can now calculate the cost of the lost energy:

$$\text{Cost} = (4.0 \times 10^4 \text{ kWh})(\$0.11/\text{kWh}) = \boxed{\$4.4 \times 10^3}$$

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Solution

Solution for part A

$$\text{Cost} = (363\ \text{kWh})(\$0.11/\text{kWh}) = \boxed{\$40}$$

Solution for part B

$$\text{Cost} = (4.0 \times 10^4\ \text{kWh})(\$0.11/\text{kWh}) = \boxed{\$4.4 \times 10^3}$$

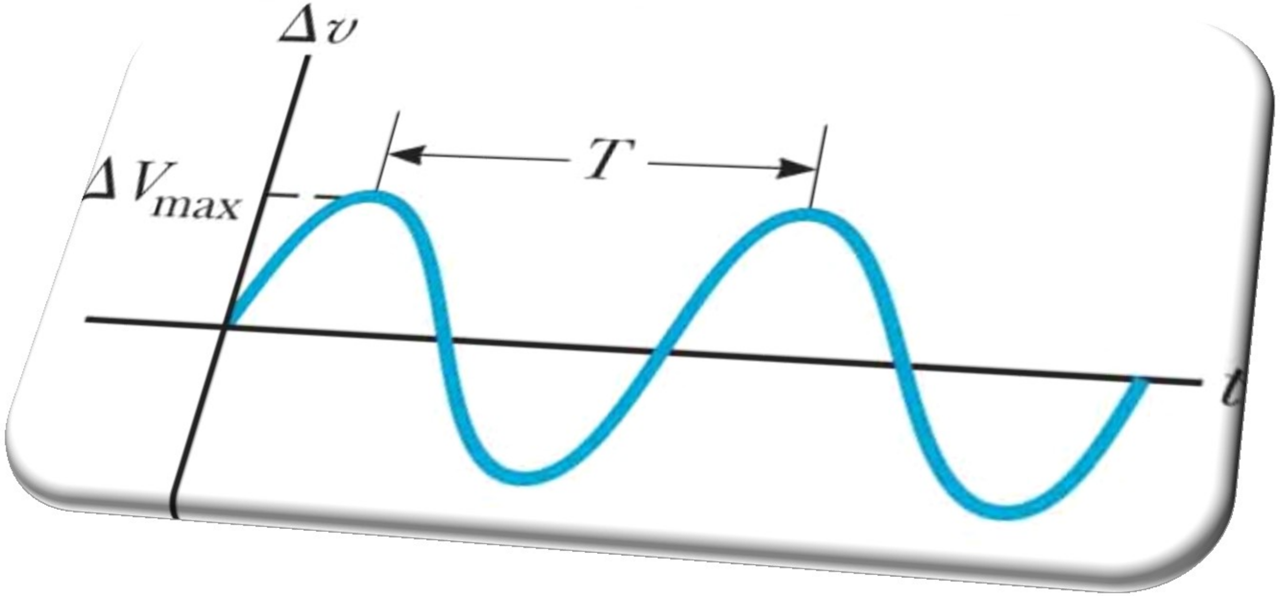
Note the tremendous savings that are possible through the use of transformers and high-voltage transmission lines. Such savings in combination with the efficiency of using alternating current to operate motors led to the universal adoption of alternating current instead of direct current for commercial power grids.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Summary of Week 10, Class 8

- Inductance (Ch. 31)
  - Self Induction and Inductance
  - RL Circuits
  - Energy in a Magnetic Field
  - Mutual Inductance
  - Oscillations in an LC Circuit
  - The RLC Circuit [Reading from Textbook]

- Alternating Current Circuits (Ch. 32)
  - AC Sources
  - Resistors in an AC Circuit
  - Inductors in an AC Circuit
  - Capacitors in an AC Circuit
  - The RLC Series Circuit [Reading from Textbook]
  - Power in an AC Circuit [Next Week]
  - Resonance in a Series RLC Circuit [Reading from Textbook]
  - The Transformer and Power Transmission [Next Week]



Reading / Preparation for Next Week

Topics for next week:

- Electromagnetic Waves (Ch. 33)