LAB 6: Time Response Analysis of Dynamic Systems

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Student Name	Signature*	Total Mark
Michael McCorkell	Michael M	48 / 50

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LAB 6 Grading Sheet

Student First Name: Michael	**Student Last Name:** McCorkell

Part A: Analysis of First-Order Transfer Function Models	9.5 **/10**
PART B: Transfer Function Model of Nonlinear Systems from Step Response	**/15** 10/10
Part C: Analysis of Second-Order Transfer Function Models	**/10** 13.5/15
Post Lab Assignment	10 **/10**
General Formatting: **Clarity, Writing style, Grammar, Spelling, Layout of the report**	5 **/5**
Total Mark	48 **/50**

LAB 6: Time Response Analysis of Dynamic Systems

OBJECTIVES

- To understand some of the basic concepts behind the dynamic systems analysis, such as stability, time response characteristics, nonlinearity, and linear models.
- To explore step response of first-order and second-order systems and derive first-order and second-order transfer function model of a dynamic system from step response.

INTRODUCTION

POLES and ZEROES

Recall that the ***poles*** of a transfer function model are those values of the Laplace transform variable, s, for which the denominator of the transfer function is zero, including any roots shared with the numerator, i.e., roots of the denominator which `cancel' out with the numerator. Similarly, the zeros of a transfer function are those values of s that cause the transfer function to become zero, including roots shared with the denominator.

STABILITY of LTI SYSTEMS

The time response of a linear time invariant (LTI) dynamic system $y(t) = y_{forced}(t) + y_{free}(t)$, where $y_{forced}(t)$ is the ***forced response*** (driven by the input applied to the system) and $y_{free}(t)$ is the ***free response*** (driven by the system's initial states).
A linear, time-invariant system is called:

1. ***stable*** if the free response approaches zero as time approaches infinity.
2. ***unstable*** if the free response grows without bound as time approaches infinity.
3. ***marginally stable*** if the free response neither decays nor grows but remains constant or oscillates as time approaches infinity.

For LTI dynamic systems one can discuss stability easily in terms of the locations of the poles of the system's transfer function. A system is ***stable*** if all poles lie in the ***left-half*** of the complex s-plane (LHP). A system is ***unstable*** if its transfer function has at least one pole in the ***right-half*** of the complex plane (RHP), or a pole of multiplicity greater than one on the imaginary axis. A system is ***marginally stable*** if it has no poles in the RHP and only poles of multiplicity one on the ***imaginary axis***.

FIRST-ORDER and SECOND-ORDER TRANSFER FUNCTION MODELS

First-order transfer function models has the following general form,

$$G(s) = \frac{K}{\tau s + 1}$$

where, K: is the steady-state gain,
τ: is the time-constant,

The **steady-state gain** affects the final value of the step response, and the **time-constant** represents how fast the system response reaches the final value.

Second-order transfer function models has the following general form,

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where, K: is the steady-state gain,
ζ: is the damping ratio,
ω_n: is the natural undamped frequency,

The **steady-state gain** affects the final value of the step response, and the **damping ratio** and the **natural undamped frequency** determined the speed and stability of the response.

1. Under-damped Systems: $0 < \zeta < 1$
2. Critically-damped Systems: $\zeta = 1$
3. Over-damped Systems: $\zeta > 1$
4. Undamped Systems: $\zeta = 0$

Part A: Analysis of First-Order Transfer Function Models

Consider the following transfer function models.

$$G_1(s) = \frac{0.5}{s+0.5}, \qquad G_2(s) = \frac{s+1}{0.1s+1}, \qquad G_3(s) = \frac{s+3}{s+0.3}, \qquad G_4(s) = \frac{1}{s-1}$$

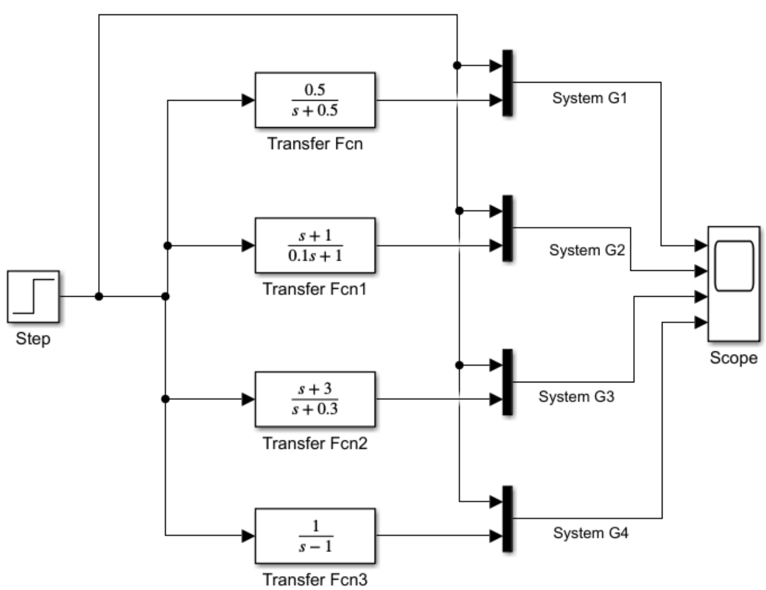
Complete the required information in **Table 1** for each transfer function:

Table 1

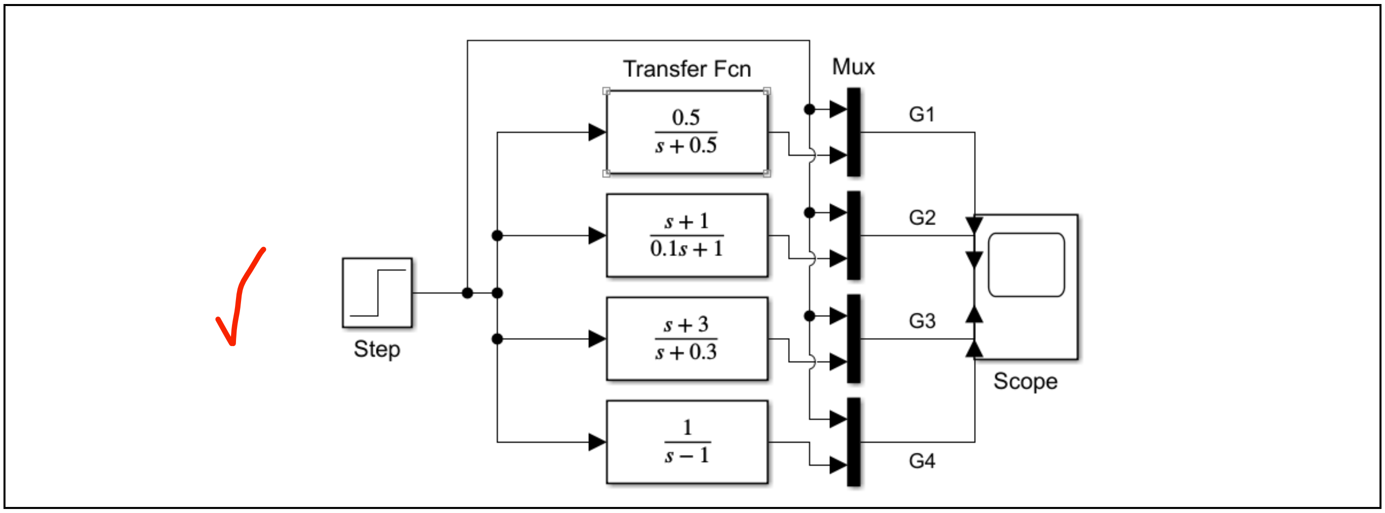
Transfer Function	Poles	Zeros	Initial Value of Step Response	Steady-State Value of Step Response	Stability (YES/NO)	Time Constant
$G_1(s)$	-0.5	None	0	1	Yes	2
$G_2(s)$	-10	-1	10	1	Yes	0.1
$G_3(s)$	-0.3	-3	1	10	Yes	3.33
$G_4(s)$	1	None	Undefined	Undefined	Yes	1

[-0.25] [-0.25]

1. Start **Simulink** and open a new window.
2. Create the following block diagram in **Simulink**:



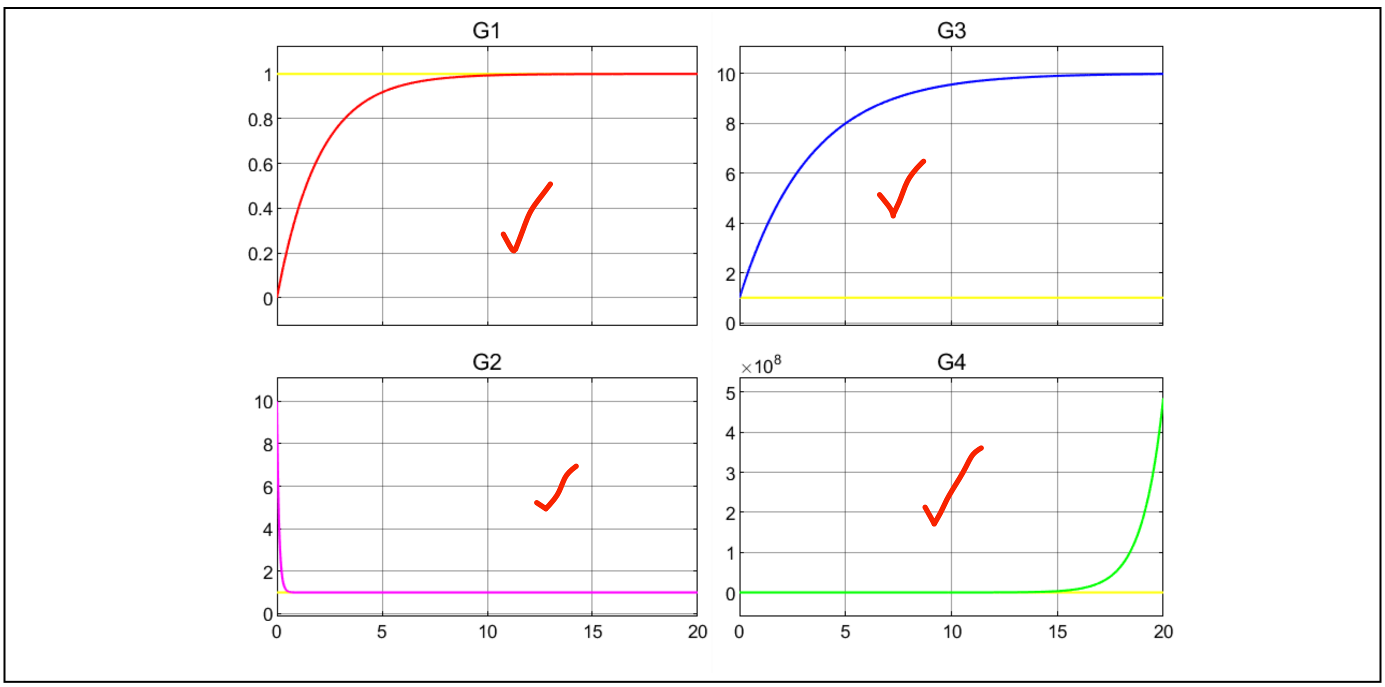
3. Provide your **Simulink** model diagram below:



4. Click on the **Model Settings** icon in the **MODELING** tab to open the **Configuration Parameters** window, click on the **Solver details**. Set the **Relative tolerance** value to **1e-12** to increase the resolution of the simulation. Then click **OK**.

5. **Save** the model file as **Lab6_PartA.slx**.

6. Set the **Step Time** of the **Step** block to **0**. Set the simulation **Stop Time 20** sec and **Run** the simulation. Open the **Scope** block. You will see the **input-output** curves for each transfer function. Provide the plot below:



7. Compare the step response graphs with the results of **Table 1**. Are the results consistent with each other?

G1(s) and G3(s) are consistent between the table and the graphs.
G2(s) has a mismatch in the initial value.
G4(s) shows an inconsistency between the stability designation in the table and the graph's behavior.

? G4(s) is not stable.

PART B: Transfer Function Model of Nonlinear Systems from Step Response

Dynamic models are essential for understanding the system behavior and design control systems. These models can either be derived mathematically from the first principles of physics of process or ***empirically*** from the **step response** of the process. The purpose of this part is to obtain a transfer function model of the following Single-Tank system experimentally from the input- output data.

Consider the following **Single-Tank** System as shown below:

The **Single-Tank** system consists of a pump with a water basin and a tank. The pump drives the water vertically from the bottom water basin up to the top of the system. The flow from the tank flows through an outlet orifice located at the bottom of the tank into the main water reservoir.

Liquid level control is common in many industries, such as pulp and paper mills, petrochemical plants and water treatment facilities.

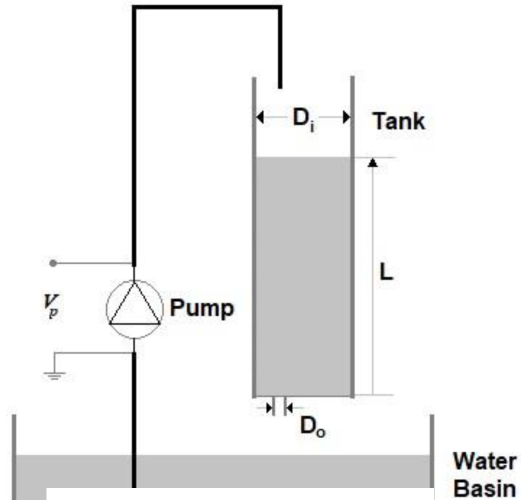


Figure 1: Single-Tank System

Parameters of the Single-Tank system are defined as follows:

- V_p: Voltage to the pump ($0V \leq V_p \leq 200V$)
- D_i: Tank inside diameter ($30\ cm$)
- D_o: Out flow orifice diameter ($1\ cm$)
- L: The water level in the tank

The rate of flow can be changed by changing the voltage of the pump (V_p) and/or by using outflow orifices with different diameters. Here, the *voltage to the pump* (V_p) is considered as the ***input*** to the process and the *water level in the tank* (L) is the ***output*** of the system.

Applying the mass balance principle to the water level in the Tank, the ***equation of motion*** of the Tank Is derived as:

$$A\frac{dL(t)}{dt} = F_i - F_o$$

where, A is the ***area of the Tank***,

$$A = \frac{\pi D_i^2}{4}$$

F_i and F_o are the ***inflow rates*** and ***outflow rates*** of the Tank, respectively. Assume that the volumetric ***inflow rate*** into the Tank is directly proportional to the applied pump voltage:

$$F_i = K_p V_p$$

where, K_p is the ***pump flow constant***. Applying the Bernoulli's equation for small orifices, the ***outflow velocity*** from Tank can be expresses by the following relationship,

$$v_o = \sqrt{2gL}$$

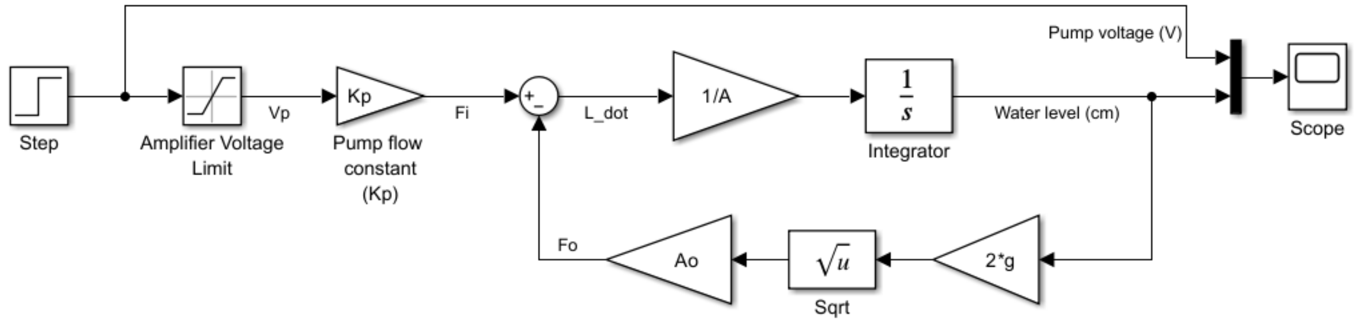
where, g is the ***gravitational acceleration***. Lastly, the outflow rate from Tank can be expressed as below:

$$F_o = A_o v_o$$

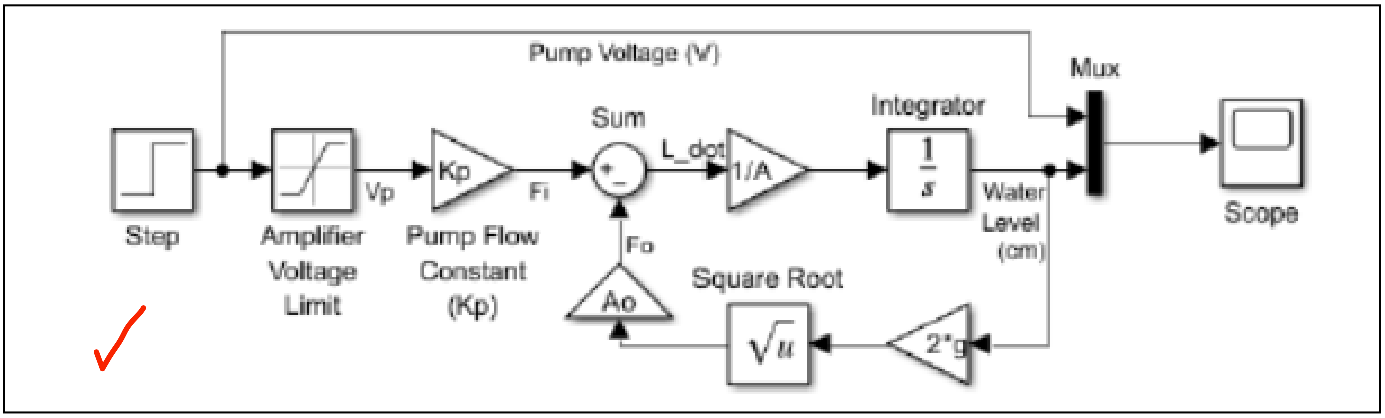
where, A_o is the ***outlet area of Tank.***

$$A_o = \frac{\pi D_o^2}{4}$$

8. Open a new **Simulink** model window.
9. Create the following nonlinear model of the Single-Tank system in **Simulink**.



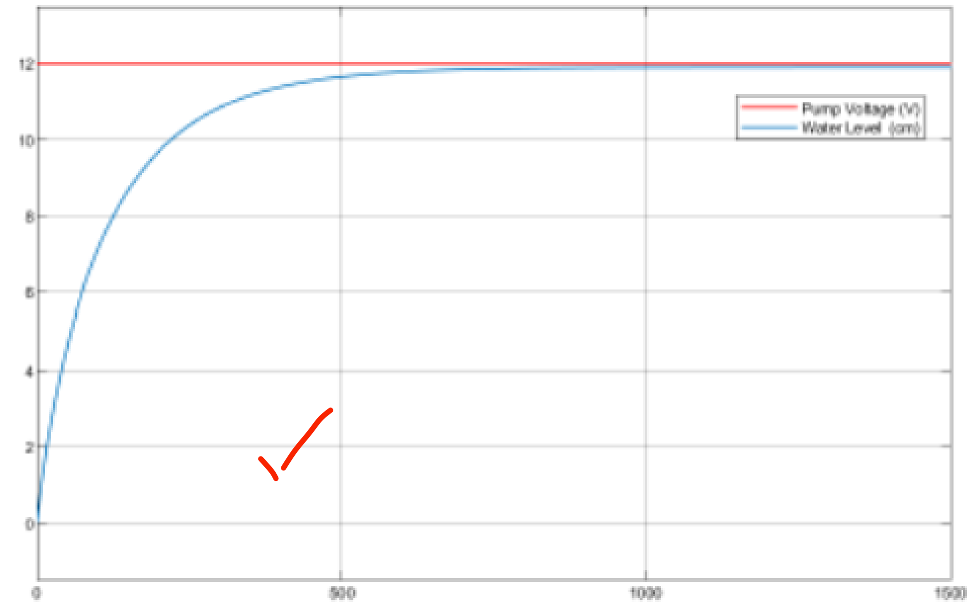
10. Provide your **Simulink** model diagram below:



11. Click on the **Model Settings** icon in the **MODELING** tab to open the **Configuration Parameters** window, click on the **Solver details**. Set the **Relative tolerance** value to **1e-12** to increase the resolution of the simulation. Then click **OK**.
12. **Save** the model file as **Lab6_PartB.slx**.
13. In the **Saturation** block, set the pump voltage limits as $0V \leq V_p \leq 50V$.
14. In the **Step** block, set **Step Time** to **0** and apply the **Final value** of **12V** as the pump voltage.
15. ***Right-click*** in the **Simulink** model window, select **Model Properties** from drop down menu. This will bring up the **Model Properties** dialog box.
16. Select the **Callbacks** tab.
17. Select **InitFcn** from the list of **Model callbacks**.
18. Type the following **MATLAB** commands into the panel under **Model initialization function**. Click **Ok**. These commands will be executed at the start of the model simulation.

```
Di = 30;            % Tank inside diameter [cm]
Do = 1;             % Out flow orifice diameter [cm]
A = pi*Di^2/4;      % Area of the Tank [cm^2]
Ao = pi*Do^2/4;     % Outlet area of Tank [cm^2]
g = 981;            % Gravitational acceleration [cm/s^2]
Kp = 10;            % Pump flow constant [cm^3/V.s]
```

19. Set the simulation **Stop Time 1500** sec and, **Run** the simulation. Open the **Scope** block. You will see the **pump voltage** and the **water level** curves. Provide the plot below:



20. Activate the **Cursor Measurements** of **Scope** and find the **DC-gain** and the **time constant** from the step response plot. Insert the required values in the first row of **Table 2.**

21. Determine a ***first-order*** transfer function model for the system. Provide the transfer function model in the standard form as:

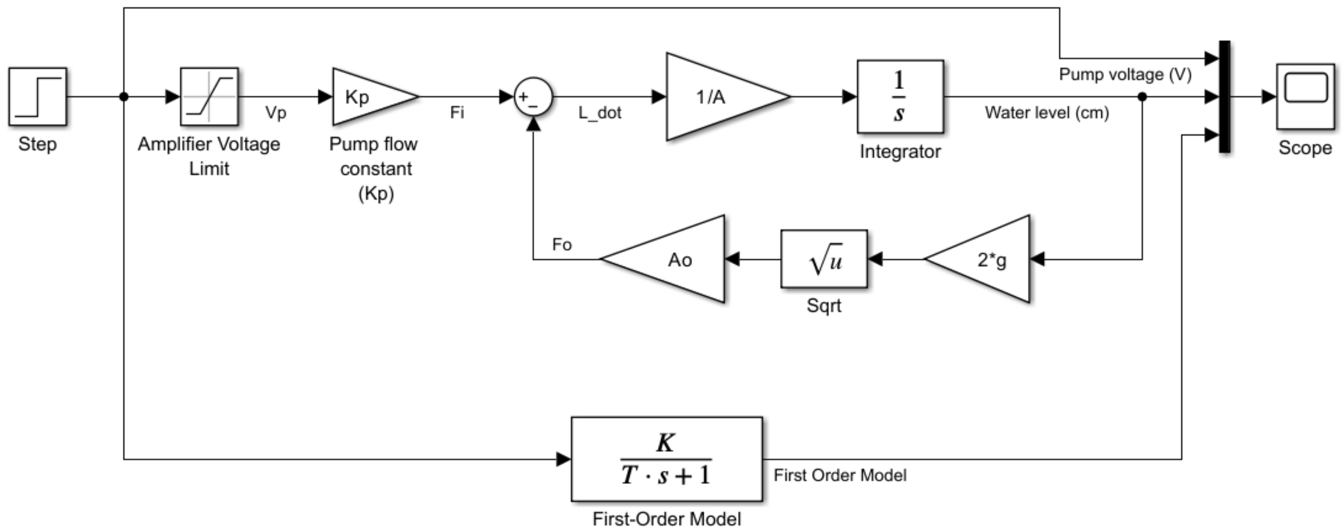
$$G(s) = \frac{K}{\tau s + 1}$$

Complete the first row of **Table 2** based on your calculations.

Table 2

Applied Voltage (V)	Water Level (cm)	Time Constant (sec)	DC-gain (cm/V)	Transfer Function Model G(s)
12 V	11.9 cm	110 sec	0.99167	$G(s) = \frac{0.99167}{110s + 1}$
15 V	18.59 cm	137.5 sec	1.2393	$G(s) = \frac{1.2393}{137.5s + 1}$
18 V	26.75 cm	164.7 sec	1.4861	$G(s) = \frac{1.4861}{164.7s + 1}$

22. Create the first-order transfer function model in **Step 21** and add it to your **Simulink** model as shown below:

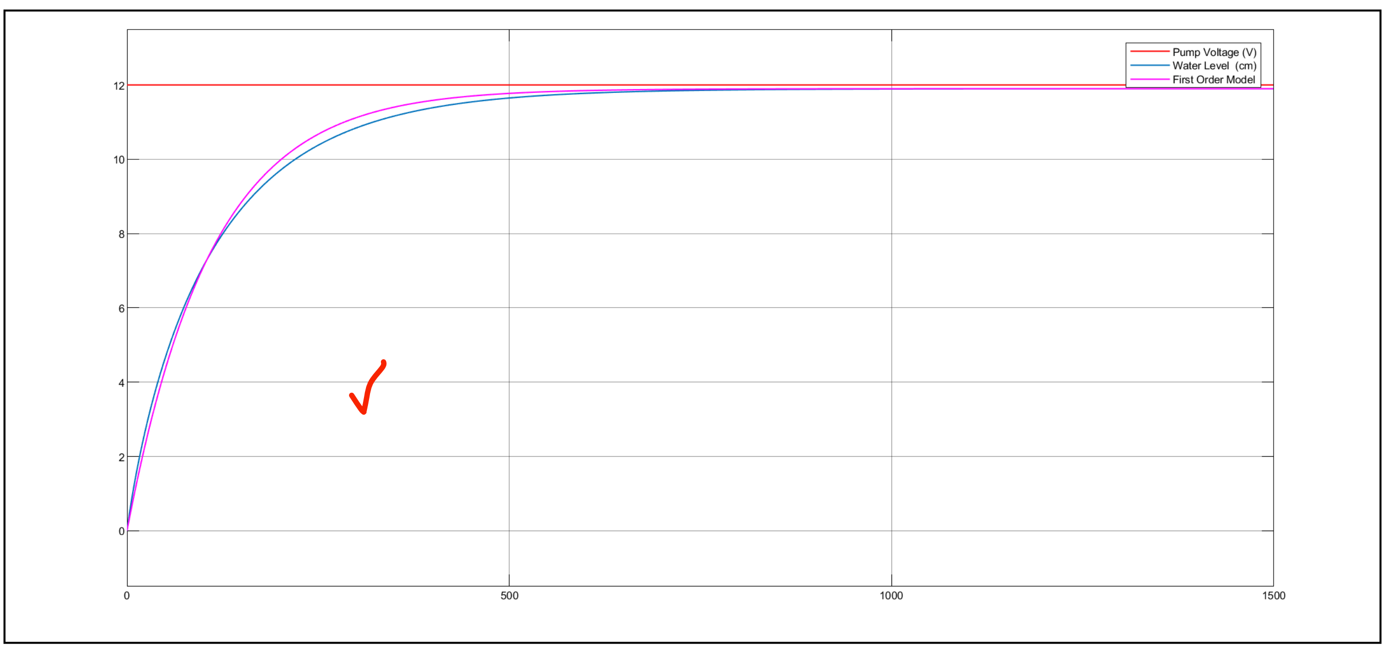


23. Run the simulation. Open the **Scope** block. Compare the step response of the nonlinear system and the linear first-order transfer function model. Is the linear model acceptable?

The linear first-order model appears to be a reasonable approximation, particularly for steady-state analysis and cases where an approximate transient reaction is adequate. However, because of its inability to accurately represent nonlinear dynamics, the linear model may introduce modest flaws for very precise transient response analysis. Overall, the linear model may be deemed sufficiently accurate for a variety of practical applications.

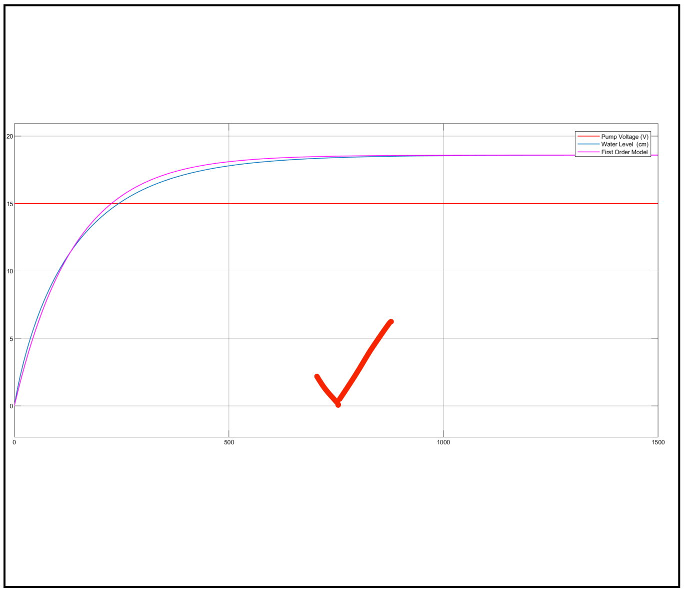
Provide the plot below:

Pump Voltage = 12V & Water Level (cm)

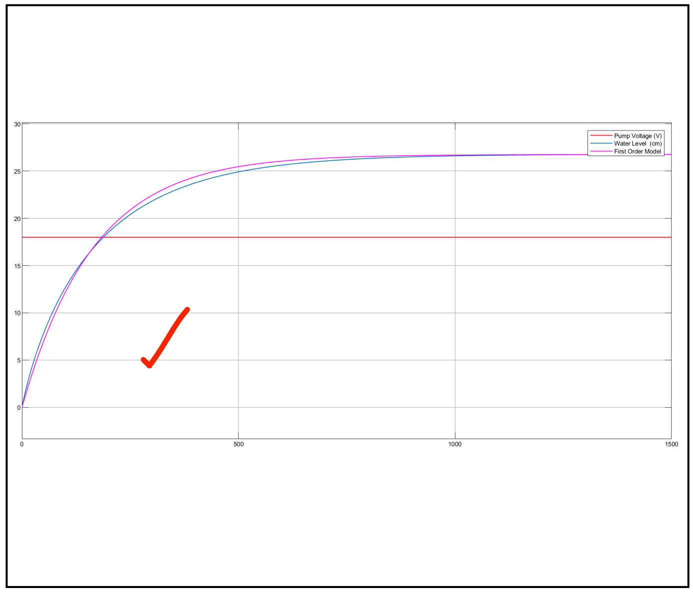


24. Change the ***applied voltage*** according to the provided values in **Table 2**. Plot the graphs and provide them below.

Pump Voltage = 15V & Water Level (cm)



Pump Voltage = 18V & Water Level (cm)



25. Find the **DC-gain**, the **time constant** and the **transfer function model** for each system and complete the **Table 2**.
26. Compare the obtained linear transfer function models in **Table 2** for each operation point. Explain effect of changing the operating point of the nonlinear Single-Tank system on the linear model. Can we use a single model for all operation points?

The change in both time constant and DC gain shows that the system's dynamics vary with the operating point. As the water level increases, the system:
• Responds more slowly (higher time constant), likely due to the increased resistance to further increases in water level as the tank fills.
• Becomes more sensitive to input changes (higher DC gain), meaning that a small change in voltage has a larger impact on the water level at higher points.

This variability in the model parameters is characteristic of nonlinear systems, where the system behavior depends on the operating point.

In conclusion, a single linear model is insufficient for accurately representing the system’s behavior across all operating points. Instead, multiple linear models or a nonlinear model would better capture the system’s dynamics across different voltage levels and water heights.

Part C: Analysis of Second-Order Transfer Function Models

Consider the series RLC circuit as shown in Figure 2,

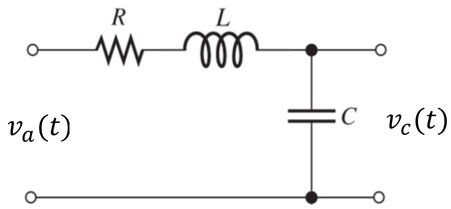


Figure 2: Series RLC Circuit

The **dynamic model** of this system is described by the following ordinary differential equation,

$$\boldsymbol{v_a(t) = LC\frac{d^2v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t)}$$

where R: Resistance (Ω),
C: Capacitance (F),
L: Inductance (H),
$v_a(t)$: Applied voltage (V),
$v_c(t)$: Voltage across the capacitor (V),

Considering the $v_a(t)$ as the input and $v_c(t)$ as the output, the **transfer function model** is obtained as below,

$$G(s) = \frac{V_c(s)}{V_a(s)} = \frac{1}{LCs^2 + RCs + 1}$$

27. Find the transfer function model in the ***standard*** second-order transfer function form and determine the **DC-gain** K, **damping ratio** ζ and the **undamped natural frequency** ω_n in terms of the R, C and L values and complete **Table 3**. Show your work below.

$$\textbf{Standard Form} \quad \rightarrow \quad \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(s) = \frac{1}{LCs^2 + RCs + 1} \rightarrow LCs^2 + RCs + 1 = s^2 + \frac{R}{L}s + \frac{1}{LC}$$

Transfer Function in standard second-order form is

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \rightarrow \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

DC-gain K=1

Damping Ratio ζ: $\frac{R}{2}\sqrt{\frac{C}{L}}$

Undamped Natural Frequency ω_n: $\sqrt{\frac{1}{LC}}$

Table 3

Transfer Function Model	DC – gain K	Damping Ratio ζ	Undamped Natural Frequency ω_n
$G(s) = \dfrac{1}{LCs^2 + RCs + 1}$	1	$\dfrac{R}{2}\sqrt{\dfrac{C}{L}}$	$\sqrt{\dfrac{1}{LC}}$

28. Assume that $R = 100\ \Omega,\ C = 0.05\ F,\ \ L = 5H$. Open a new **script** file and run the following code in **MATLAB** to create the transfer function model of the system.

```
R = 100;                       % Resistance (Ohms)
C = 0.05;                      % Capacitance (F)
L = 5;                         % Inductance (H)

num = [1/(L*C)];               % Numerator coefficients
den = [1 R/L 1/(L*C)];         % Denominator coefficients
G = tf(num,den)                % Create transfer function

z = R*sqrt(C/L)/2              % Damping Ratio
wn = 1/sqrt(L*C)               % Undamped Natural Frequency (rad/sec)

P = pole(G)                    % Poles

figure; pzmap(G)               % Pole-Zero Map
figure; step(G)                % Step response plot
xlim([0 30])                   % Time axis limit of step response plot

stepinfo(G)                    % Step response specifications
```

?

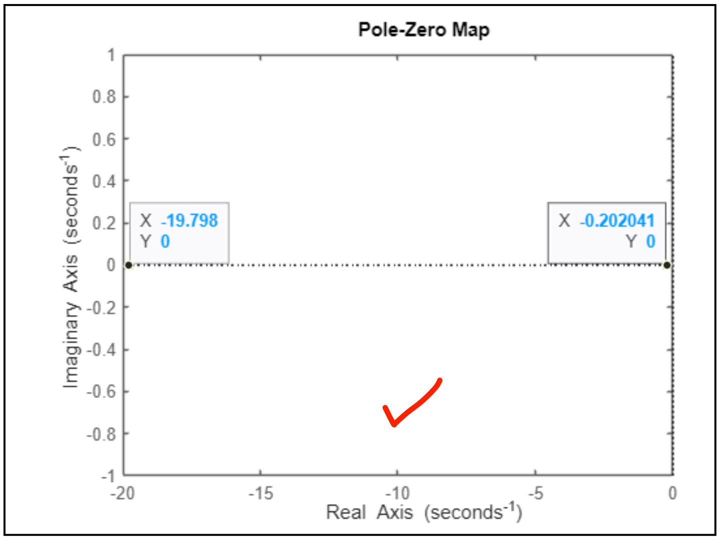
Provide the results from **MATLAB Command** window and complete the first row of **Table 4**:

Rise Time: 10.8747
Transient Time: 19.4140
Settling Time: 19.4140
Settling Min: 0.9035
Settling Max: 0.9993
Overshoot: 0
Undershoot: 0
Peak: 0.9993
Peak Time: 36.2413

Where are the transfer function, damping ratio, natural frequency and the poles?

[-1.5]

Pole-Zero Map (R = 100Ω)



Step Response Plot (R = 100Ω)

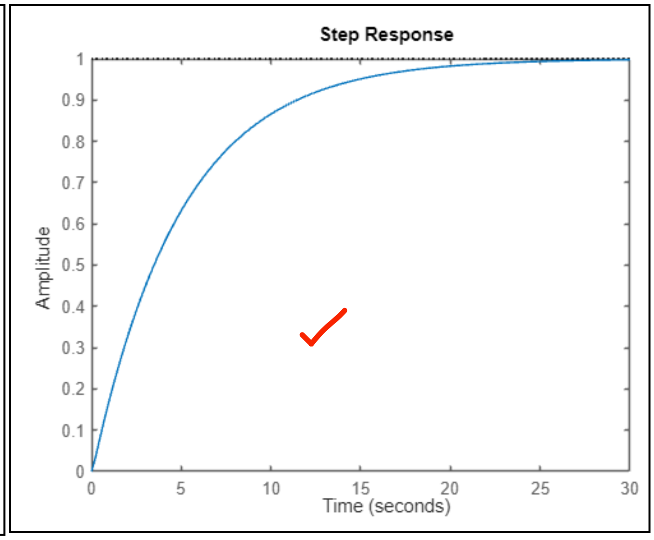


Table 4

Resistance (Ohms)	Poles	Damping Ratio ζ	Undamped Natural Frequency ω_n	System damping type	Rise Time	Percent of Overshoot	Settling Time
100 Ω	-19.8 -0.202	5	2	Overdamping	10.8747	0	19.4140
50 Ω	-9.5826 -0.4174	2.5	2	Overdamping	5.2678	0	9.4787
20 Ω	-2 -2	1	2	Overdamping	1.6790	0	2.9170
10 Ω	-1+1.7321i -1-1.7321i	0.5	2	Underdamped	0.8195	16.2929	4.0379
5 Ω	-0.5+1.9365i -0.5-1.9365i	0.25	2	Underdamped	0.6343	44.3235	7.0579
0 Ω	0+2i 0-2i	0	2	Undamped	NaN	NaN	NaN

29. Change the ***Resistance*** according to the given values in **Table 4.** Provide the required graphs, calculations, and data from the simulation and complete **Table 4**.

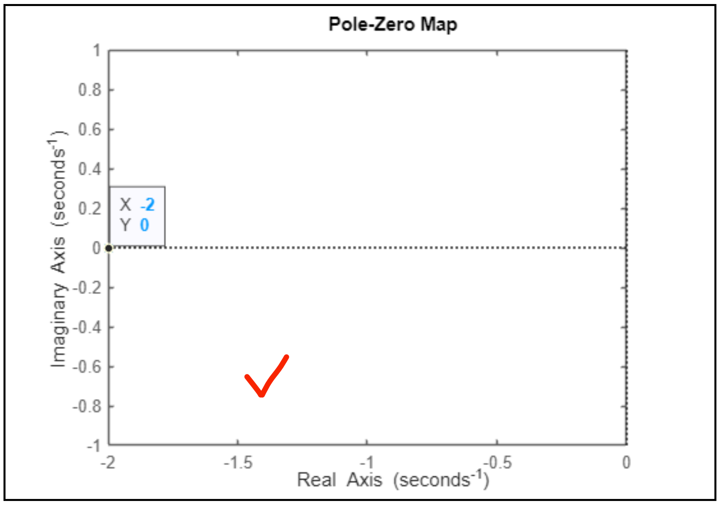
30. Explain the effect of changing the **Resistance** on the <u>damping ratio</u> and the <u>natural frequency</u> of the system.

Increasing resistance results in higher damping ratios, making the system more overdamped and reducing oscillations.

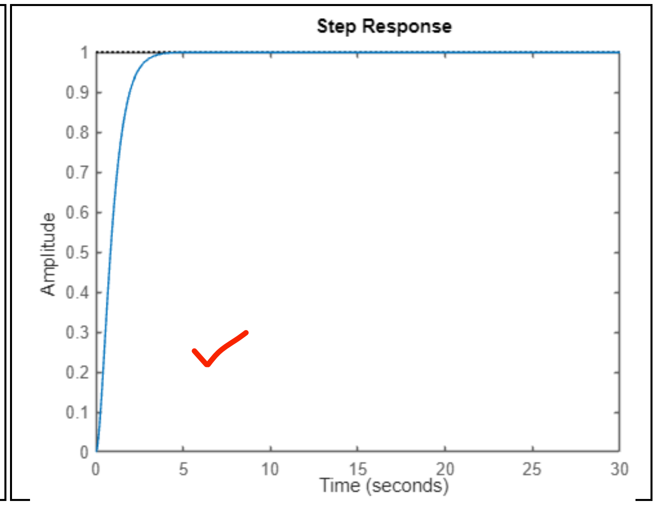
Decreasing resistance lowers the damping ratio, moving the system toward underdamped behavior with more oscillations and higher overshoot.

Natural frequency remains unchanged at ωn =2 for all resistance values, indicating that it is independent of the resistance in this setup.

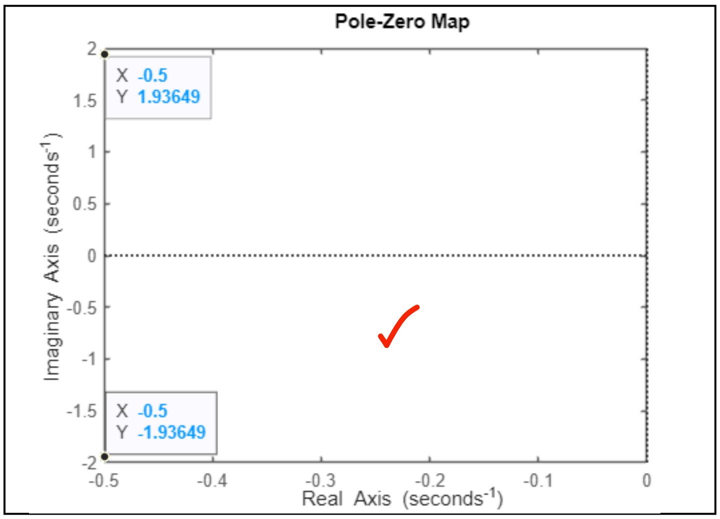
Pole-Zero Map (R = 20Ω)



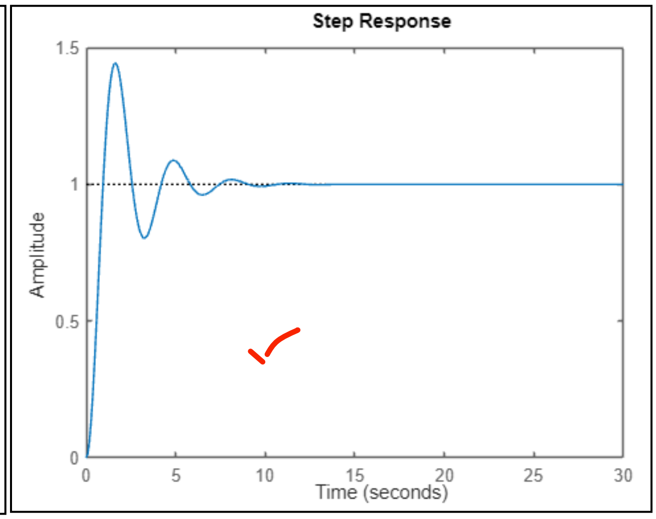
Step Response Plot (R = 20Ω)



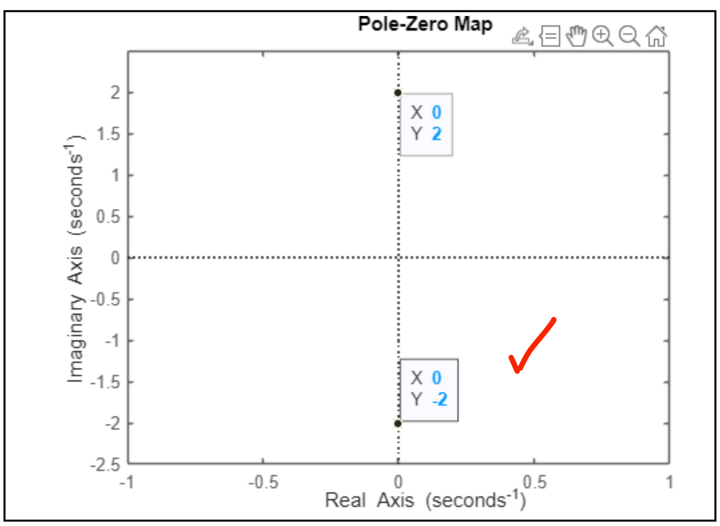
Pole-Zero Map (R = 5Ω)



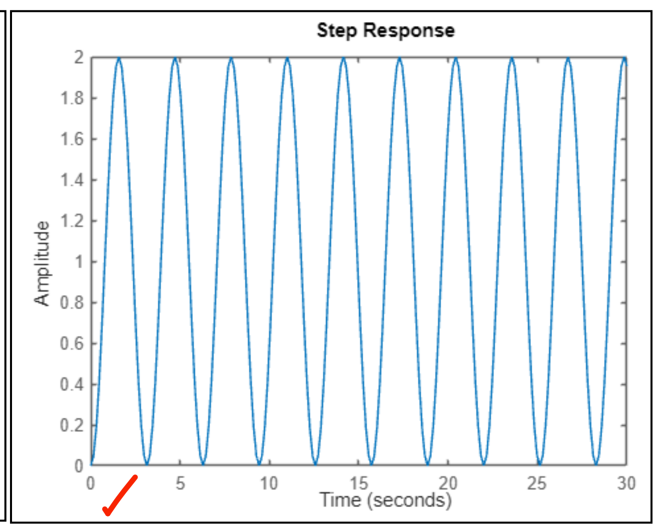
Step Response Plot (R = 5Ω)



Pole-Zero Map (R = 0Ω)



Step Response Plot (R = 0Ω)



Post Lab Assignment

1. Consider the following transfer function models:

$$G_1(s) = \frac{2.5}{s^2 + 3s + 2}, \quad G_2(s) = \frac{2.5}{s^2 + 2s + 1}, \quad G_3(s) = \frac{5}{s^2 + 4s + 13}, \quad G_4(s) = \frac{10}{s^2 + 10}$$

   a) Create a **MATLAB** code to obtain the transfer function and pole locations. Provide your code and the results.

   b) Plot the step response and the pole-zero map of each model. Provide the graphs.

   c) Determine the damping ratio and the undamped natural frequency of the models. Show your calculations.

   d) Determine the damping type of each system. Over-damped, under-damped, critically-damped or undamped? Justify your answer.

2. For a system output given in the following figure and the following information, determine a second-order transfer function model. Show your calculations.

$$\boldsymbol{G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

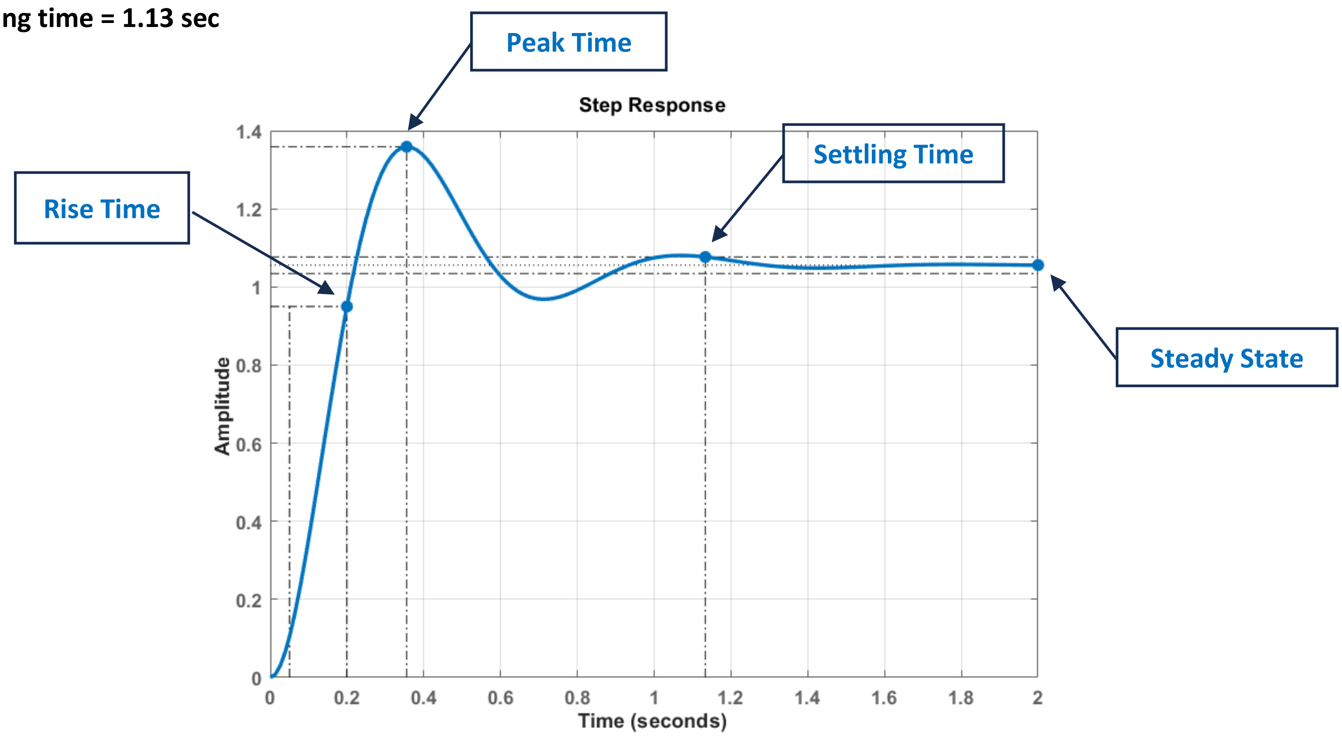
Peak time = 0.355 sec

Overshoot = 28.7%

Steady state value = 1.06

Rise time = 0.15 sec

Settling time = 1.13 sec

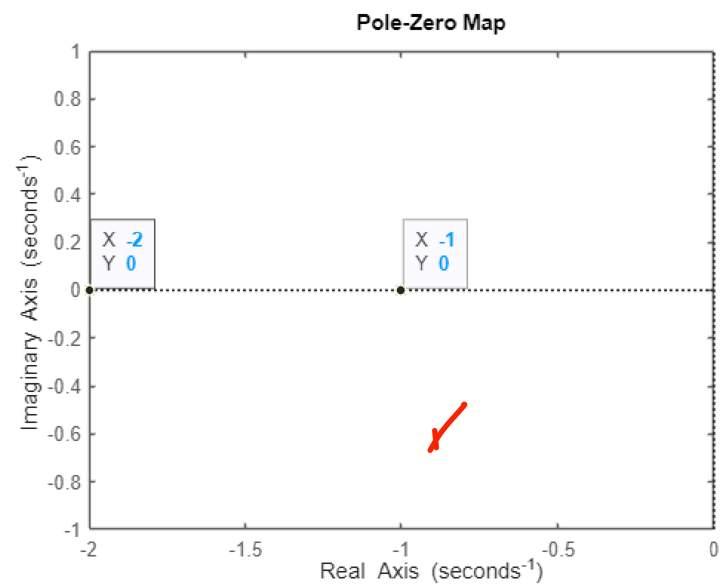


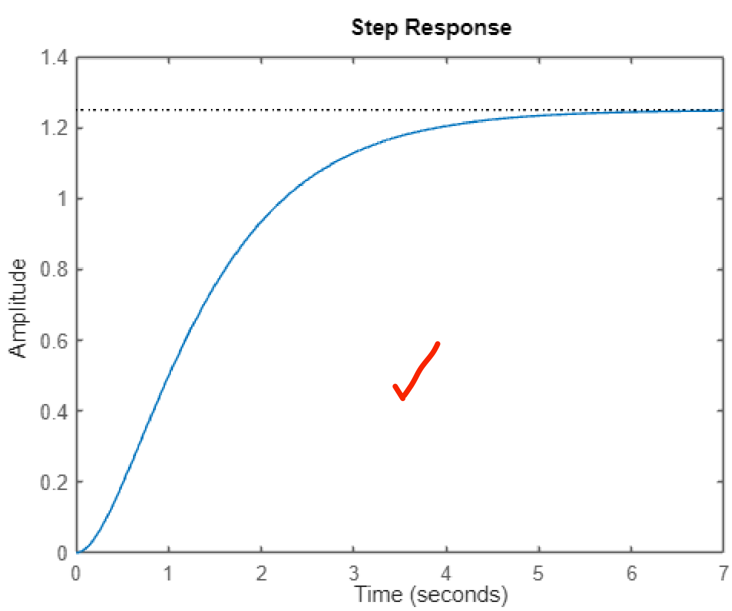
Plot the step response of the model and show the peak time, overshoot, rise-time, settling-time and steady-state value on the graph. Provide your code and the graph.

a)

```
num1 = 2.5;                    % Numerator coefficients
den1 = [1 3 2];                % Denominator coefficients
G1 = tf(num1,den1)             % Create transfer function

num2 = 2.5;                    % Numerator coefficients
den2 = [1 2 1];                % Denominator coefficients
G2 = tf(num2,den2)             % Create transfer function

num3 = 5;                      % Numerator coefficients
den3 = [1 4 13];               % Denominator coefficients
G3 = tf(num3,den3)             % Create transfer function

num4 = 10;                     % Numerator coefficients
den4 = [1 0 10];               % Denominator coefficients
G4 = tf(num4,den4)             % Create transfer function

P1 = pole(G1)                  % Poles
P2 = pole(G2)                  % Poles
P3 = pole(G3)                  % Poles
P4 = pole(G4)                  % Poles
```

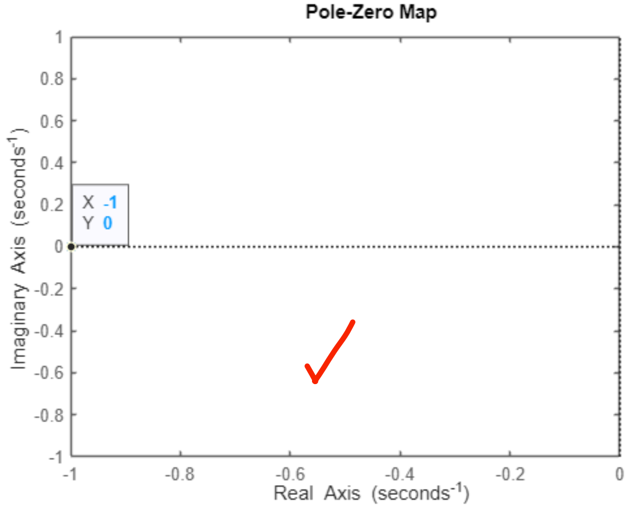
G1

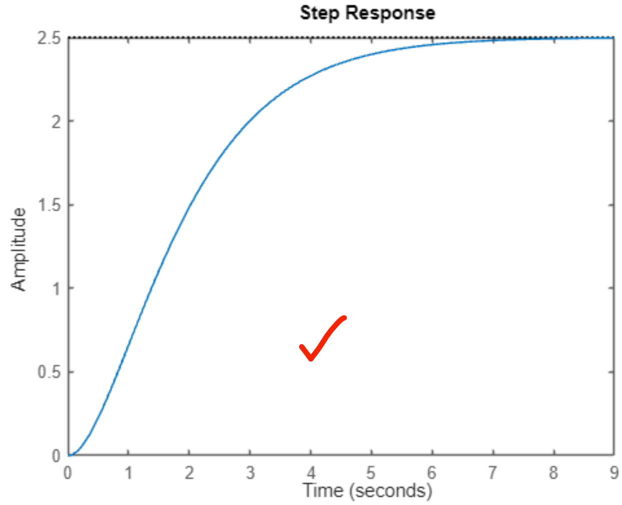




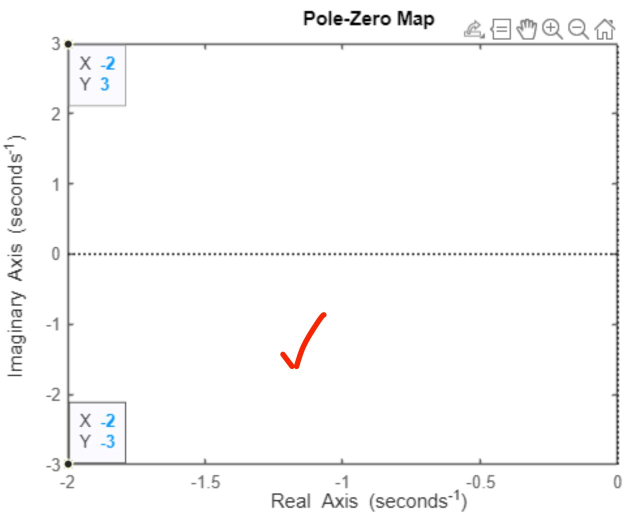

```
G1 =

      2.5
  -------------
  s^2 + 3 s + 2

Continuous-time transfer function.
Model Properties
G2 =

      2.5
  -------------
  s^2 + 2 s + 1

Continuous-time transfer function.
Model Properties
G3 =

       5
  --------------
  s^2 + 4 s + 13

Continuous-time transfer function.
Model Properties
G4 =

     10
  --------
  s^2 + 10

Continuous-time transfer function.
Model Properties
P1 = 2×1
    -2
    -1

P2 = 2×1
    -1
    -1

P3 = 2×1 complex
  -2.0000 + 3.0000i
  -2.0000 - 3.0000i

P4 = 2×1 complex
   0.0000 + 3.1623i
   0.0000 - 3.1623i
```

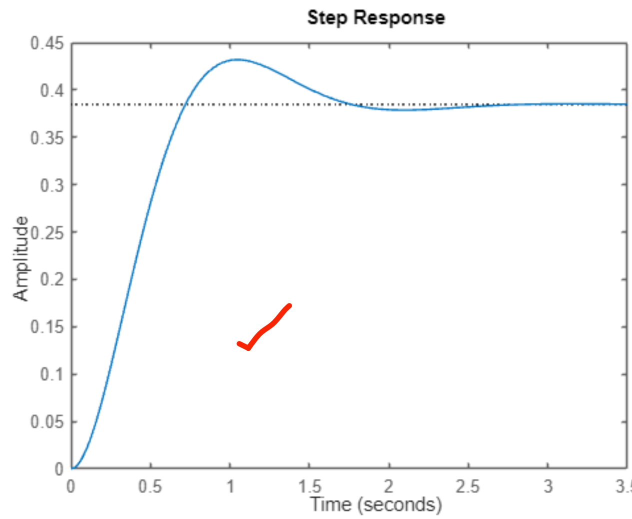
G2


Pole-Zero Map
X -1
Y 0
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})

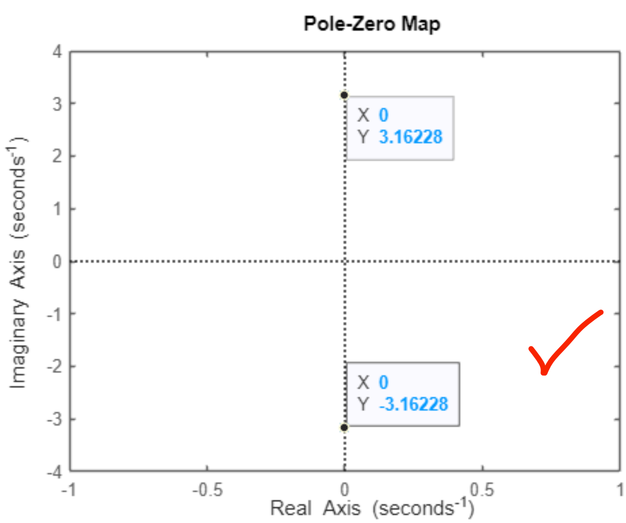

Step Response
Amplitude
Time (seconds)

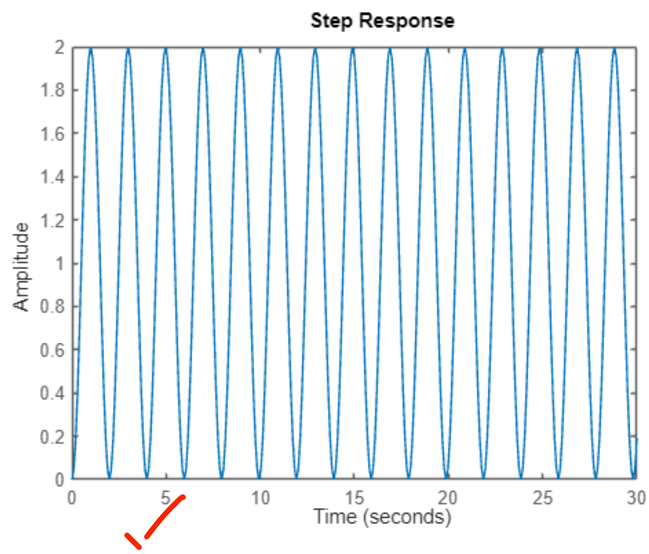
G3


Pole-Zero Map
X -2
Y 3
X -2
Y -3
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})


Step Response
Amplitude
Time (seconds)

G4


Pole-Zero Map
X 0
Y 3.16228
X 0
Y -3.16228
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})


Step Response
Amplitude
Time (seconds)

G1

$$w_n = \sqrt{2} \approx 1.414$$

$$2\zeta w_n \to 3 \Rightarrow \zeta = \frac{3}{2w_n} = \frac{3}{2 * 1.414} = 1.061$$

1.061 -> Overdamped

The damping ratio is slightly greater than 1, meaning the system is overdamped, and it will return to steady-state without oscillating but more slowly than a critically damped system.

G2

$$w_n = \sqrt{1} = 1$$

$$2\zeta w_n \to 2 \Rightarrow \zeta = \frac{2}{2w_n} = \frac{2}{2 * 1} = 1$$

1 -> Critically Damped

With a damping ratio of exactly 1, the system is critically damped. This is the boundary condition between oscillation and non-oscillation, where the system returns to steady-state as quickly as possible without oscillating.

G3

$$w_n = \sqrt{13} \approx 3.606$$

$$2\zeta w_n \to 4 \Rightarrow \zeta = \frac{4}{2w_n} = \frac{4}{2 * 3.606} \approx 0.554$$

0.554 -> Underdamped

Since 0<ζ<1, the system is underdamped, meaning it will exhibit oscillations as it approaches steady-state.

G4

$$w_n = \sqrt{10} \approx 3.162$$

$$\zeta = 0$$

0 -> Undamped

With a damping ratio of 0, there is no damping in the system, and it will oscillate indefinitely without settling to a steady-state value.

$$\%OS = 100 * e^{\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)}$$

$$\ln(0.287) = -\frac{\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$-1.251 = -\frac{\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$1.251 = \frac{\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$\frac{1.251}{\pi} = \frac{\zeta}{\sqrt{1-\zeta^2}}$$

$$0.398^2 = \frac{\zeta^2}{1-\zeta^2}$$

$$0.1586(1-\zeta^2) = \zeta^2$$

$$1.1586\zeta^2 = 0.1586$$

$$\zeta^2 = \frac{0.1586}{1.1586} = 0.13689$$

$$\sqrt{\zeta^2} = \sqrt{0.13689}$$

$$\zeta = 0.369$$

$$w_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}}$$

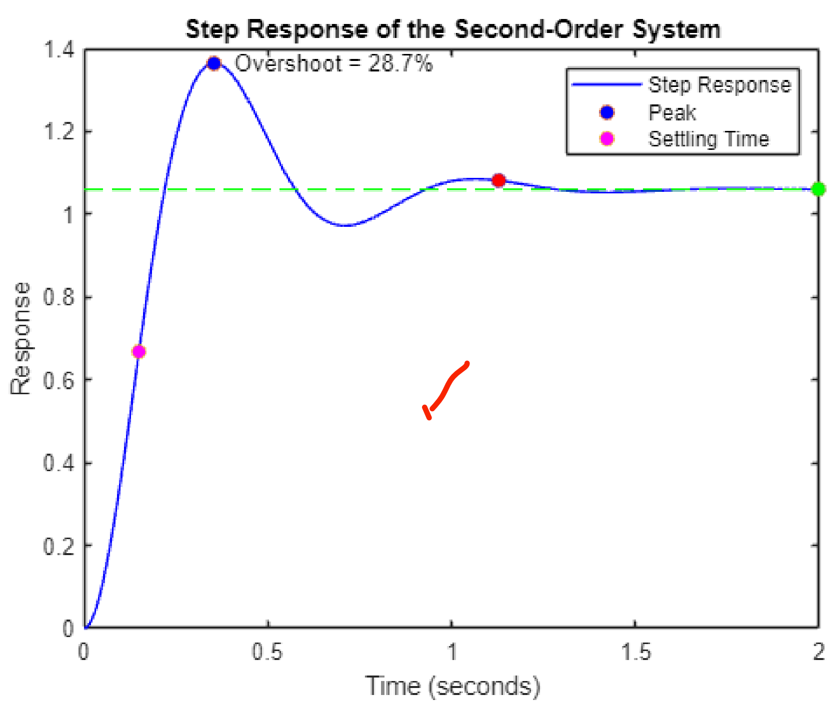
$$w_n = \frac{\pi}{0.355\sqrt{1-(0.369)^2}}$$

$$w_n = 9.5215$$

$$G(s) = \frac{K * w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

$$G(s) = \frac{1.06 * (9.5215)^2}{s^2 + 2(0.369)(9.5215)s + (9.5215)^2}$$

$$G(s) = \frac{96.0985}{s^2 + 7.027s + 90.659}$$



```
% Define the transfer function
numerator = 96.0985;
denominator = [1, 7.027, 90.659];
system = tf(numerator, denominator);

% Mark steady-state value, peak time, rise time, and settling time
steady_state_value = 1.06; peak_time = 0.355; rise_time = 0.15; settling_time = 1.13;

% Plot the step response
t = 0:0.001:2;
[y, t] = step(system, t); figure;
plot(t, y, 'b-', 'MarkerSize', 1); hold on; % Plot response with points

% Plot individual markers for the key points
plot(t(find(t >= peak_time, 1)), y(find(t >= peak_time, 1)), 'o', 'MarkerFaceColor', 'blue', 'DisplayName', 'Peak Time');
plot(t(find(t >= rise_time, 1)), y(find(t >= rise_time, 1)), 'o', 'MarkerFaceColor', 'magenta', 'DisplayName', 'Rise Time');
plot(t(find(t >= settling_time, 1)), y(find(t >= settling_time, 1)), 'o', 'MarkerFaceColor', 'red', 'DisplayName', 'Settling Time');
plot(t(end), steady_state_value, 'o', 'MarkerFaceColor', 'green', 'DisplayName', 'Steady-State Value');
plot([0, max(t)], [y_final, y_final], 'g--', 'DisplayName', ['Steady-State Value = ' num2str(y_final)]);
text(T_p, y(idx_peak), ['   Overshoot = ' num2str(OS) '%']);

% Customize the plot
title('Step Response of the Second-Order System');
xlabel('Time (seconds)'); ylabel('Response');
legend('Step Response', 'Peak', 'Settling Time');
```