

CURVILINEAR MOTION: GENERAL & RECTANGULAR COMPONENTS

Today's Objectives:

Students will be able to:

1. Describe the motion of a particle traveling along a curved path.
2. Relate kinematic quantities in terms of the rectangular components of the vectors.



In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- General Curvilinear Motion
- Rectangular Components of Kinematic Vectors
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. In curvilinear motion, the direction of the instantaneous velocity is always
 - A) tangent to the hodograph.
 - B) perpendicular to the hodograph.
 - C) tangent to the path.
 - D) perpendicular to the path.
2. In curvilinear motion, the direction of the instantaneous acceleration is always
 - A) tangent to the hodograph.
 - B) perpendicular to the hodograph.
 - C) tangent to the path.
 - D) perpendicular to the path.

APPLICATIONS



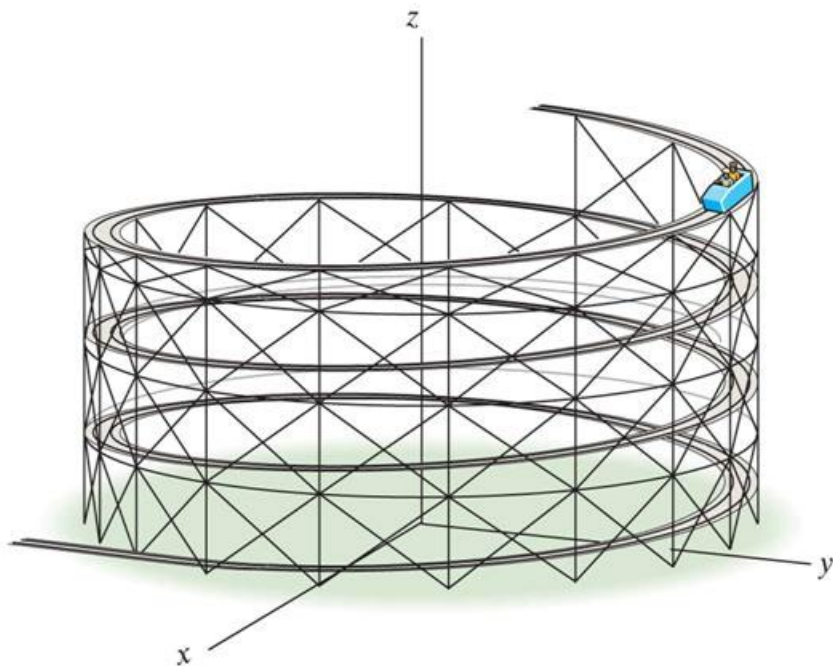
The path of motion of a plane can be tracked with radar and its x , y , and z -coordinates (relative to a point on earth) recorded as a function of time.

How can we determine the velocity or acceleration of the plane at any instant?

APPLICATIONS (continued)

A roller coaster car travels down a fixed, helical path at a constant speed.

How can we determine its position or acceleration at any instant?



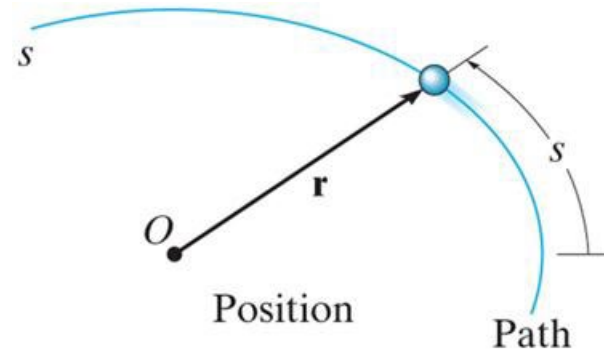
If you are designing the track, why is it important to be able to predict the acceleration of the car?

GENERAL CURVILINEAR MOTION

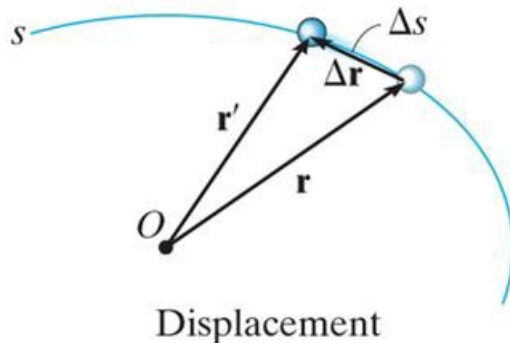
(Section 12.4)

A particle moving along a curved path undergoes **curvilinear motion**. Since the motion is often three-dimensional, **vectors** are usually used to describe the motion.

A particle moves along a curve defined by the path function, s .



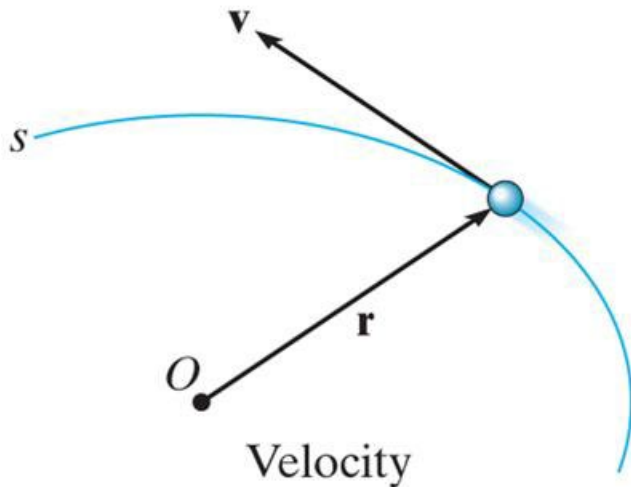
The **position** of the particle at any instant is designated by the vector $\mathbf{r} = \mathbf{r}(t)$. Both the **magnitude** and **direction** of \mathbf{r} may vary with time.



If the particle moves a distance Δs along the curve during time interval Δt , the **displacement** is determined by **vector subtraction**: $\Delta \mathbf{r} = \mathbf{r}' - \mathbf{r}$

VELOCITY

Velocity represents the rate of change in the position of a particle.



The **average velocity** of the particle during the time increment Δt is

$$\mathbf{v}_{avg} = \Delta \mathbf{r} / \Delta t .$$

The **instantaneous velocity** is the time-derivative of position

$$\mathbf{v} = d\mathbf{r} / dt .$$

The **velocity vector**, \mathbf{v} , is **always** tangent to the path of motion.

The magnitude of \mathbf{v} is called the **speed**. Since the arc length Δs approaches the magnitude of $\Delta \mathbf{r}$ as $t \rightarrow 0$, the speed can be obtained by differentiating the path function ($v = ds/dt$). Note that this is not a vector!

ACCELERATION

Acceleration represents the rate of change in the velocity of a particle.

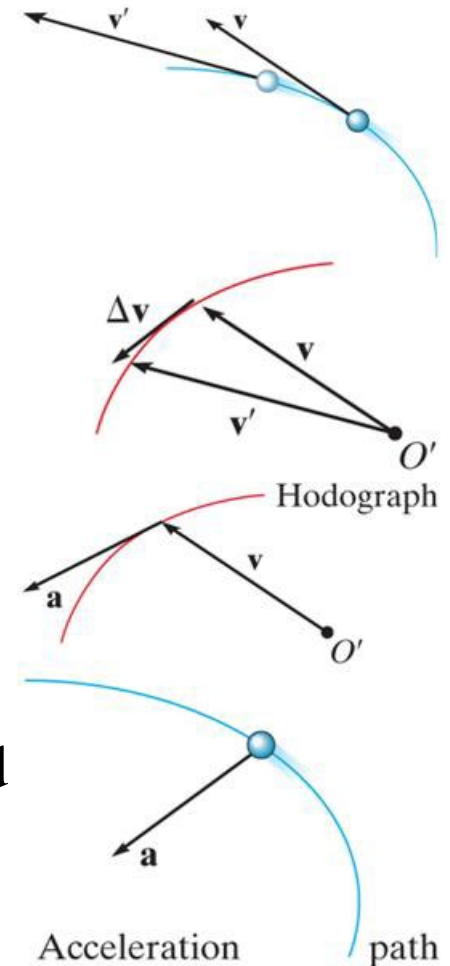
If a particle's velocity changes from \mathbf{v} to \mathbf{v}' over a time increment Δt , the **average acceleration** during that increment is:

$$\mathbf{a}_{avg} = \Delta \mathbf{v} / \Delta t = (\mathbf{v} - \mathbf{v}') / \Delta t$$

The **instantaneous acceleration** is the time-derivative of velocity:

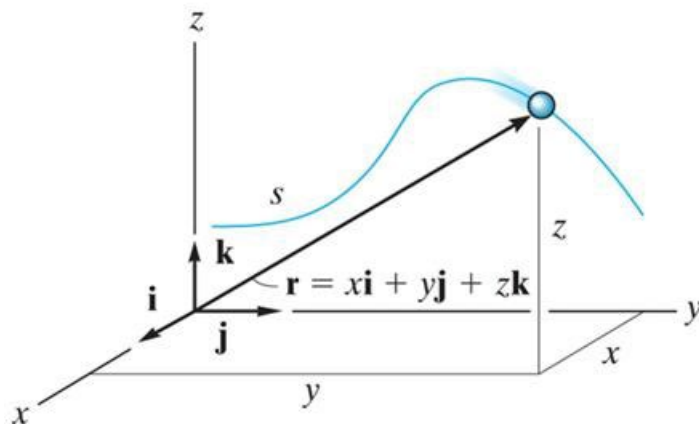
$$\mathbf{a} = d\mathbf{v}/dt = d^2\mathbf{r}/dt^2$$

A plot of the locus of points defined by the arrowhead of the velocity vector is called a **hodograph**. The acceleration vector is tangent to the hodograph, but not, in general, tangent to the path function.



CURVILINEAR MOTION: RECTANGULAR COMPONENTS (Section 12.5)

It is often convenient to describe the motion of a particle in terms of its x , y , z or **rectangular components**, relative to a **fixed frame of reference**.



Position

The position of the particle can be defined at any instant by the **position vector**

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} .$$

The x , y , z -components may all be **functions of time**, i.e.,

$$x = x(t), y = y(t), \text{ and } z = z(t) .$$

The **magnitude** of the position vector is: $r = (x^2 + y^2 + z^2)^{0.5}$

The **direction** of \mathbf{r} is defined by the unit vector: $\mathbf{u}_r = (1/r)\mathbf{r}$

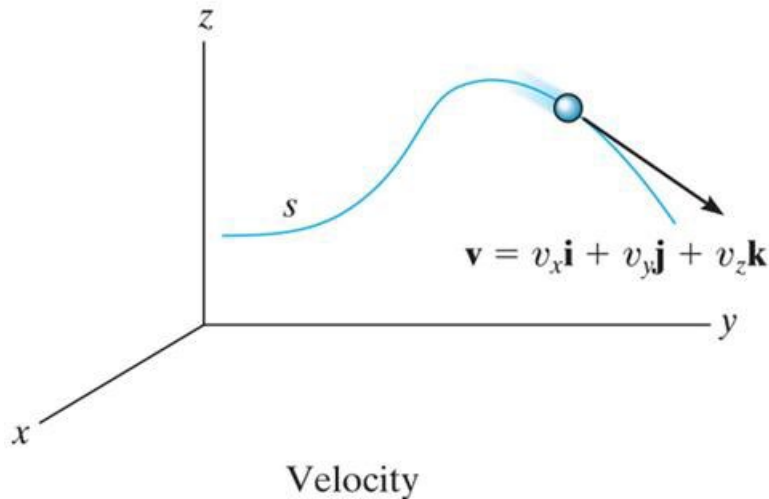
RECTANGULAR COMPONENTS: VELOCITY

The **velocity vector** is the time derivative of the position vector:

$$\mathbf{v} = d\mathbf{r}/dt = d(x \mathbf{i})/dt + d(y \mathbf{j})/dt + d(z \mathbf{k})/dt$$

Since the **unit vectors** \mathbf{i} , \mathbf{j} , \mathbf{k} are **constant** in **magnitude** and **direction**, this equation reduces to $\mathbf{v} = v_x \mathbf{i} + v_y \mathbf{j} + v_z \mathbf{k}$

where $v_x = \dot{x} = dx/dt$, $v_y = \dot{y} = dy/dt$, $v_z = \dot{z} = dz/dt$



The **magnitude** of the velocity vector is

$$v = [(v_x)^2 + (v_y)^2 + (v_z)^2]^{0.5}$$

The **direction** of \mathbf{v} is **tangent** to the path of motion.

RECTANGULAR COMPONENTS: ACCELERATION

The **acceleration vector** is the time derivative of the velocity vector (second derivative of the position vector).

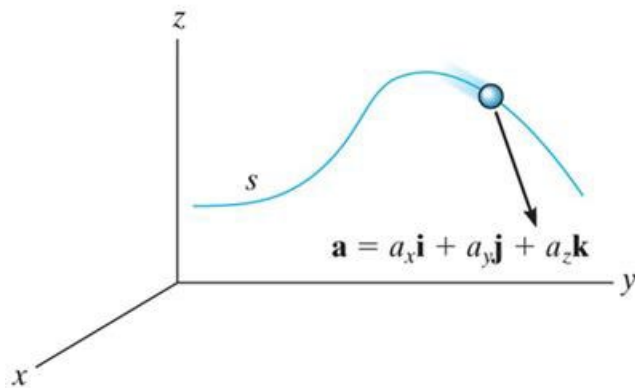
$$\mathbf{a} = d\mathbf{v}/dt = d^2\mathbf{r}/dt^2 = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

where $a_x = \dot{v}_x = \ddot{x} = dv_x/dt$, $a_y = \dot{v}_y = \ddot{y} = dv_y/dt$,

$$a_z = \dot{v}_z = \ddot{z} = dv_z/dt$$

The **magnitude** of the acceleration vector is

$$a = \sqrt{(a_x)^2 + (a_y)^2 + (a_z)^2}$$



Acceleration

The **direction** of \mathbf{a} is **usually not tangent** to the path of the particle.

EXAMPLE

Given: The box slides down the slope described by the equation $y = (0.05x^2)$ m, where x is in meters.
 $v_x = -3$ m/s, $a_x = -1.5$ m/s² at $x = 5$ m.

Find: The y components of the velocity and the acceleration of the box at $x = 5$ m.

Plan: Note that the particle's velocity can be found by taking the first time derivative of the path's equation. And the acceleration can be found by taking the second time derivative of the path's equation.

Take a derivative of the position to find the component of the velocity and the acceleration.

EXAMPLE (continued)

Solution:

Find the y-component of velocity by taking a time derivative of the position $y = (0.05x^2)$

$$\Rightarrow \dot{y} = 2 (0.05) x \dot{x} = 0.1 x \dot{x}$$

Find the acceleration component by taking a time derivative of the velocity \dot{y}

$$\Rightarrow \ddot{y} = 0.1 \dot{x} \dot{x} + 0.1 x \ddot{x}$$

Substituting the x-component of the acceleration, velocity at $x=5$ into \dot{y} and \ddot{y} .

EXAMPLE (continued)

Since $\dot{x} = v_x = -3 \text{ m/s}$, $\ddot{x} = a_x = -1.5 \text{ m/s}^2$ at $x = 5 \text{ m}$

$$\Rightarrow \dot{y} = 0.1 x \dot{x} = 0.1 (5) (-3) = -1.5 \text{ m/s}$$

$$\begin{aligned}\Rightarrow \ddot{y} &= 0.1 \dot{x} \dot{x} + 0.1 x \ddot{x} \\ &= 0.1 (-3)^2 + 0.1 (5) (-1.5) \\ &= 0.9 - 0.75 \\ &= 0.15 \text{ m/s}^2\end{aligned}$$

At $x = 5 \text{ m}$

$$v_y = -1.5 \text{ m/s} = \underline{1.5 \text{ m/s}} \downarrow$$

$$a_y = \underline{0.15 \text{ m/s}^2} \uparrow$$

CONCEPT QUIZ

1. If the position of a particle is defined by $\mathbf{r} = [(1.5t^2 + 1) \mathbf{i} + (4t - 1) \mathbf{j}]$ (m), its speed at $t = 1$ s is _____.
A) 2 m/s
B) 3 m/s
C) 5 m/s
D) 7 m/s
2. The path of a particle is defined by $y = 0.5x^2$. If the component of its velocity along the x-axis at $x = 2$ m is $v_x = 1$ m/s, its velocity component along the y-axis at this position is _____.
A) 0.25 m/s
B) 0.5 m/s
C) 1 m/s
D) 2 m/s

GROUP PROBLEM SOLVING

Given: The particle travels along the path $y = 0.5 x^2$.
When $t = 0$, $x = y = z = 0$.

Find: The particle's distance and the magnitude of its acceleration when $t = 1$ s, if $v_x = (5 t)$ ft/s, where t is in seconds.

Plan:

- 1) Determine x and a_x by integrating and differentiating v_x , respectively, using the initial conditions.
- 2) Find the y -component of velocity & acceleration by taking a time derivative of the path.
- 3) Determine the magnitude of the acceleration & position.

GROUP PROBLEM SOLVING (continued)

Solution:

1) x-components:

Velocity known as: $v_x = \dot{x} = (5t) \text{ ft/s} \Rightarrow \underline{5 \text{ ft/s at } t=1\text{s}}$

Position: $\int v_x dt = \int_0^t (5t) dt \Rightarrow x = 2.5 t^2 \Rightarrow \underline{2.5 \text{ ft at } t=1\text{s}}$

Acceleration: $a_x = \ddot{x} = d/dt (5t) \Rightarrow \underline{5 \text{ ft/s}^2 \text{ at } t=1\text{s}}$

2) y-components:

Position known as : $y = 0.5 x^2 \Rightarrow \underline{3.125 \text{ ft at } t=1\text{s}}$

Velocity: $\dot{y} = 0.5 (2) x \dot{x} = x \dot{x} \Rightarrow \underline{12.5 \text{ ft/s at } t=1\text{s}}$

Acceleration: $a_y = \ddot{y} = \dot{x} \dot{x} + x \ddot{x} \Rightarrow \underline{37.5 \text{ ft/s}^2 \text{ at } t=1\text{s}}$

GROUP PROBLEM SOLVING (continued)

3) The position vector and the acceleration vector are

Position vector: $\mathbf{r} = [x \mathbf{i} + y \mathbf{j}]$ ft

where $x = 2.5$ ft, $y = 3.125$ ft

Magnitude: $r = \sqrt{2.5^2 + 3.125^2} = \underline{4.00 \text{ ft}}$

Acceleration vector: $\mathbf{a} = [a_x \mathbf{i} + a_y \mathbf{j}]$ ft/s²

where $a_x = 5$ ft/s², $a_y = 37.5$ ft/s²

Magnitude: $a = \sqrt{5^2 + 37.5^2} = \underline{37.8 \text{ ft/s}^2}$

ATTENTION QUIZ

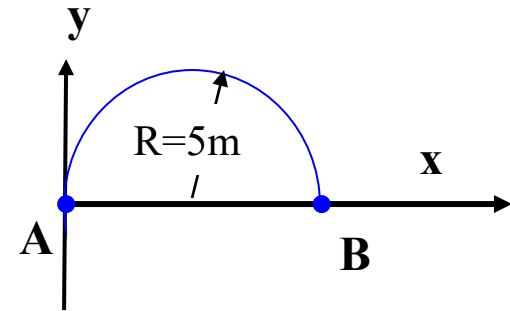
1. If a particle has moved from A to B along the circular path in 4s, what is the average velocity of the particle?

A) $2.5 \mathbf{i}$ m/s

B) $2.5 \mathbf{i} + 1.25 \mathbf{j}$ m/s

C) $1.25 \pi \mathbf{i}$ m/s

D) $1.25 \pi \mathbf{j}$ m/s



2. The position of a particle is given as $\mathbf{r} = (4t^2 \mathbf{i} - 2x \mathbf{j})$ m. Determine the particle's acceleration.

A) $(4 \mathbf{i} + 8 \mathbf{j})$ m/s²

B) $(8 \mathbf{i} - 16 \mathbf{j})$ m/s²

C) $(8 \mathbf{i})$ m/s²

D) $(8 \mathbf{j})$ m/s²

End of the Lecture

Let Learning Continue