Class Note 2.2

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Module 2.2 Class Notes Basic Rules of Differentiation

Assume that all given functions are functions of a single variable and differentiate with respect to this variable.

1. Constant Rule	**EXAMPLE 1**
The derivative of a constant is 0. For any constant C: $\frac{d}{dx}[C] = 0$	a. $y = 0.0071$, then $y' = \frac{dy}{dx} = 0$ b. $s(t) = Ke^{-0.025}$, where K is constant, then $s' = \frac{ds}{dt} = 0,$

EXAMPLE 2. (Self-check)

a) $y = 15$, then $y' =$

b) $y = -\sqrt{12}$, then $y' =$

c) $\theta(t) = \frac{\pi}{2}$; then $\theta'(t) = \frac{d\theta}{dt}$=

d) $u(x) = 10^{-5}, \quad u' = \frac{du}{dx} =$

e) $s(t) = V \sin\frac{\pi}{6}\omega$, Assume that V, ω are constants.
$s' = \frac{ds}{dt} =$

2. Power Rule	**EXAMPLE 3**
For any real number n, $\frac{d}{dx}[x^n] = nx^{n-1}$ **Verbally**: To differentiate the power function with respect to the *input*, place the constant exponent in front of the expression and multiply by the *input* raised to the exponent reduced by 1. Those steps(operations) produce the derivative. **Note**: this rule is valid for the general power function: x^r, where $r \in \mathbb{R}$	a. $y = x^2$, then $y' = 2x^{2-1} = 2x^1 = 2x$ b. $y = x^{-\frac{1}{2}}, \quad y' = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}}$ $= -\frac{1}{2x^{\frac{3}{2}}}$ c. $\frac{d}{dt}\left[t^{\sqrt{2}}\right] = \sqrt{2}t^{\sqrt{2}-1}$

3. Constant times a function rule	**EXAMPLE 4**
The derivative of a constant times a function is the constant times the derivative of the function. If C is a constant, then for any function $u(x)$ $$\frac{d}{dx}[Cu(x)] = C\frac{d}{dx}[u(x)]$$	a. $y = 4x^3$, then $y' = 4\frac{d}{dx}[x^3] = 4(3x^2) = 12x^2$
	b. $s(t) = -\frac{3}{4}t^{-4}$, then $s' = \frac{ds}{dt}$ $= -\frac{3}{4}\frac{d}{dt}[t^{-4}] = -\frac{3}{4}(-4t^{-5}) = 3t^{-5} = \frac{3}{t^5}$

EXAMPLE 5. (Self-check)

a) $y = 6x^{\frac{1}{3}}$, then $y' =$

b) $y = 5\sqrt{x}, \quad y' =$

c) $y = \frac{1}{2x^2}$, then $y' =$

d) $y = \frac{1}{7\sqrt[3]{x}}$; then y'=

e) $y = 0.025t^{-0.4}$, then $y' = \frac{dy}{dt} =$

f) $u = 5x$, then $u' =$

4. Sum and difference rules	**EXAMPLE 6**
If u & v are functions of x, then $$\frac{d}{dx}[u \pm v] = \frac{d}{dx}[u] \pm \frac{d}{dx}[v]$$ or $[u \pm v]' = u' \pm v$ **Verbally:** the derivative of the sum(difference) of the functions is the sum(difference) of the derivatives of the addends.	Differentiate the polynomial function: $y = 5x^3 - 4x^2 + 12x - 8.$ The rule can be extended to any number of functions combined by addition and subtraction. $y' = 15x^2 - 8x + 12$

EXAMPLE 7. (Self-check) Combine the rules to find the derivatives.

a) Find $\frac{d}{dx}[x^{-2} + 6x^3 + 7] =$

b) Find $\frac{d}{dx}\left[2x^5 - 3x^{-7} + x + 2\sqrt{x}\right] =$

EXAMPLE 8. Calculate derivatives.

a) $y = \frac{x^3+1}{x}$ b) $f(x) = \frac{x^4+3\sqrt{x}}{x}$

Hint. It is often useful to manipulate an expression algebraically prior to differentiation. In this case divide the numerator through by the denominator and reduce the resulting fractions.

a) $y = \frac{x^3}{x} + \frac{1}{x} = x^2 + x^{-1} \quad \to \quad y' = 2x - x^{-2}$

b)

$$f(x) = \frac{x^4 + 3\sqrt{x}}{x} = \frac{x^4}{x} + \frac{3\sqrt{x}}{x} = x^3 + 3x^{-\frac{1}{2}}$$

Then, $f'(x) = 3x^2 - \frac{3}{2}x^{-\frac{3}{2}}$

EXAMPLE 9. Evaluating derivatives at the given values: If $f(x) = 7 - 4x^2$, find $f'(1)$ and $f'(-3)$

Find the derivative first: $f'(x) = -8x$. Treat the derivative as a function of x to be evaluated for the given values

For $x = 1$ $f'(1) = -8(1) = -8$; and for $x = -3$ $f'(-3) = -8(-3) = 24$

Note,

- The derivative of a function is a function. Differentiation results in a function.
- Alternative notations may be used in applications: for any $x = x_0$: $f'(x_0) = \left.\frac{df}{dx}\right|_{x=x_0}$

EXAMPLE 10. Find an equation to the tangent line to the curve $y = \frac{2}{x}$ at the point (2,1) on this curve.

Hint. Equation of the tangent line T to the curve at the point $\boldsymbol{P}\big(x_0, f(x_0)\big)$ in the *point –slope form*:

$$y = f(x_0) + f'(x_0)(x - x_0)$$

or in the slope-intercept form: $y = mx + b$

Answers to Self-Check Examples

2. all 0; **5.** a. $2x^{-\frac{2}{3}}$; b. $2.5x^{-\frac{1}{2}}$; c. $-x^{-3}$; d. $-\frac{1}{21}x^{-\frac{4}{3}}$; e. $0.01t^{-1.4}$; f. 5;

7. a. $-2x^{-3} + 18x^2$; b. $10x^4 + 21x^{-8} + 1 + x^{-\frac{1}{2}}$;

10. $y = -\frac{1}{2}x + 2$