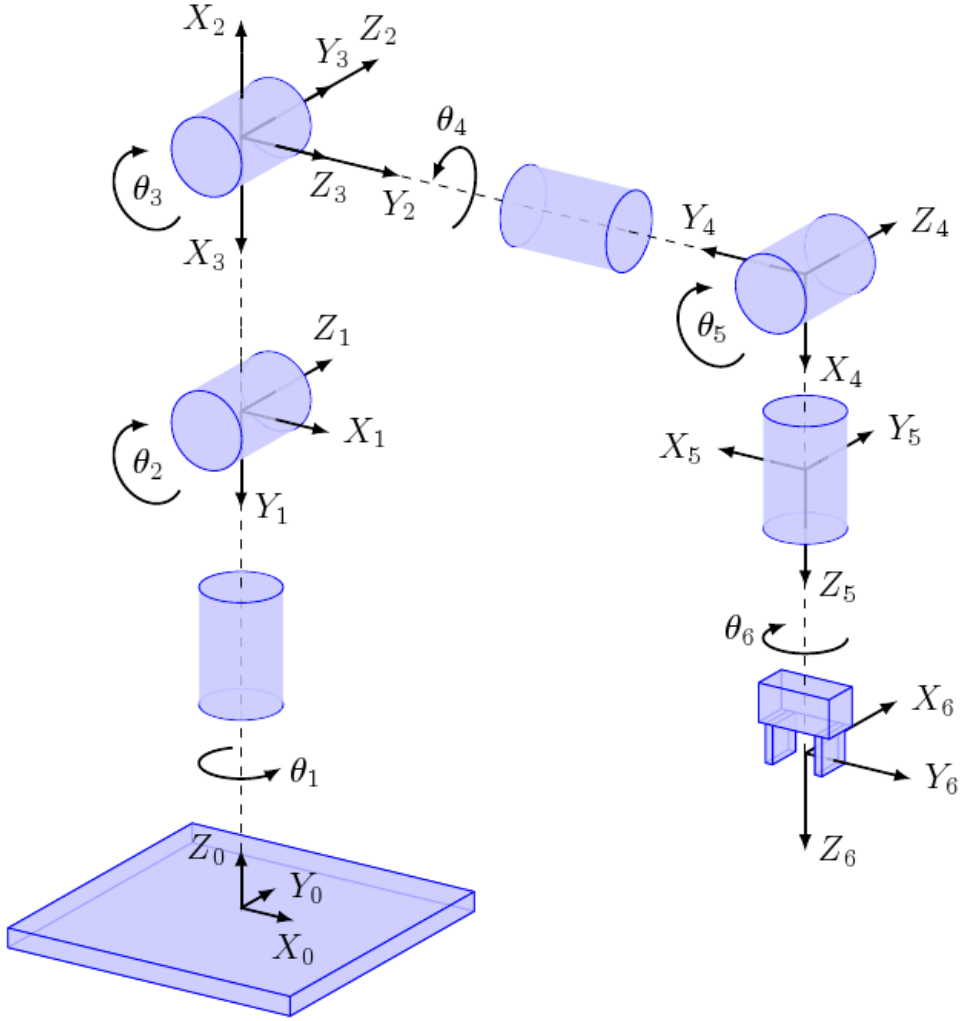
Kinematics and Dynamics of Robots

Module 9

Inverse Kinematics 1



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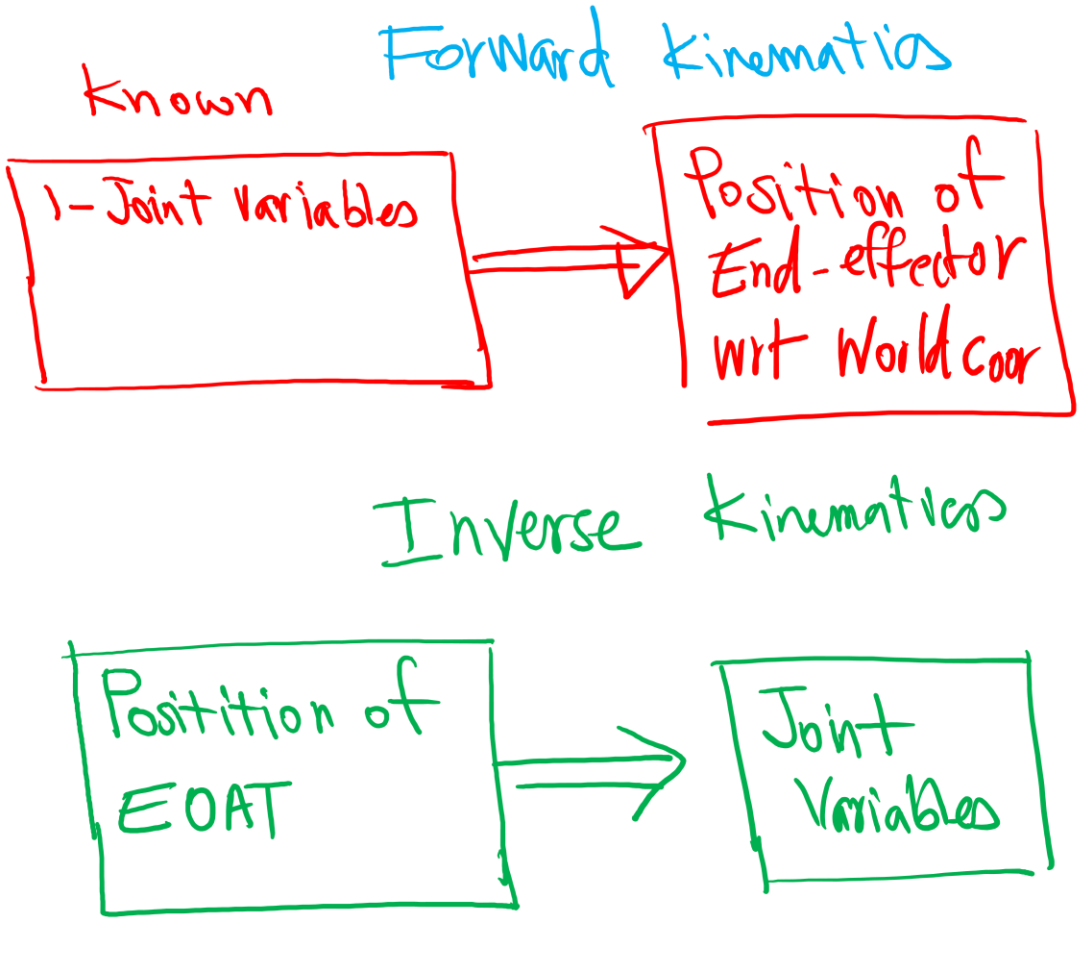
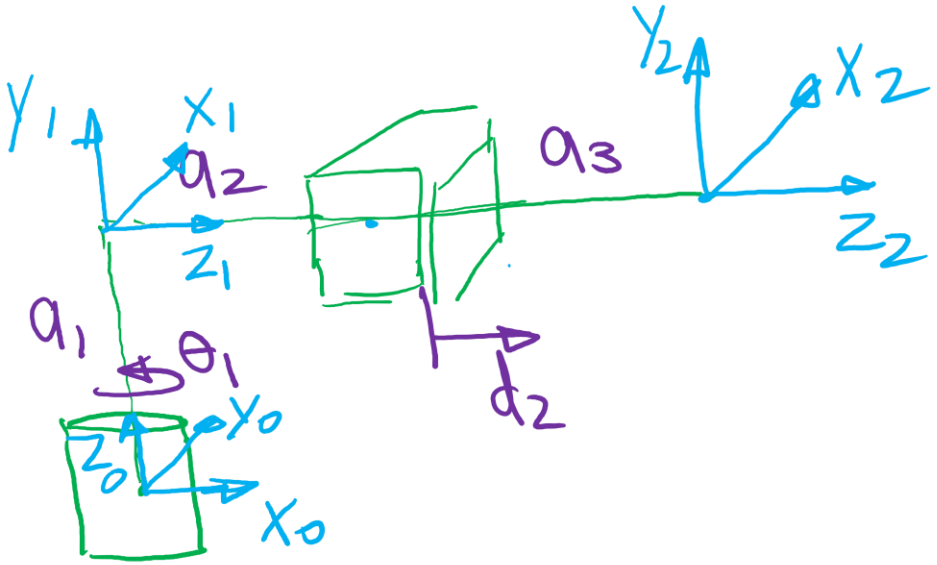
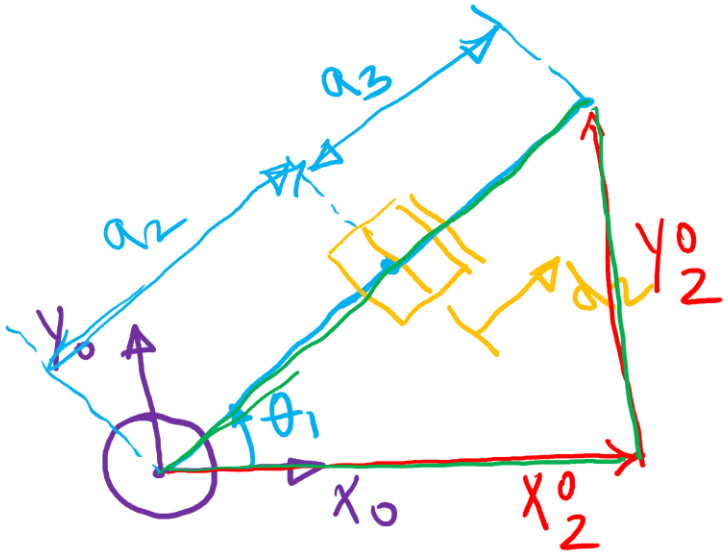
Inverse Kinematics



$$P^0 = H_n^0 P^n$$

Inverse Kinematics

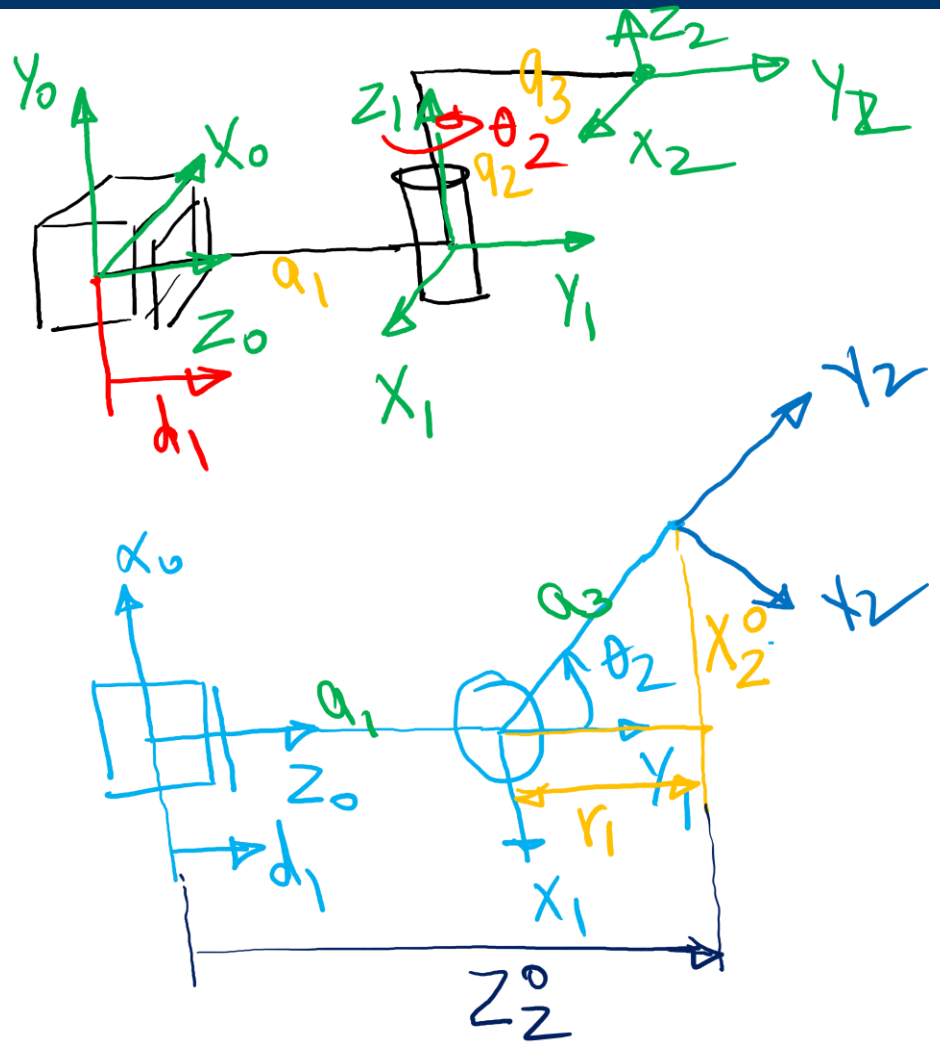




Inputs: X_2^0, Y_2^0, Z_2^0 Unknown: θ_1, d_2

$$a_2 + d_2 + a_3 = \sqrt{X_2^{0^2} + Y_2^{0^2}} \Rightarrow d_2 = \sqrt{X_2^{0^2} + Y_2^{0^2}} - a_2 - a_3$$

$$\theta_1 = \tan^{-1} \frac{Y_2^0}{X_2^0}$$



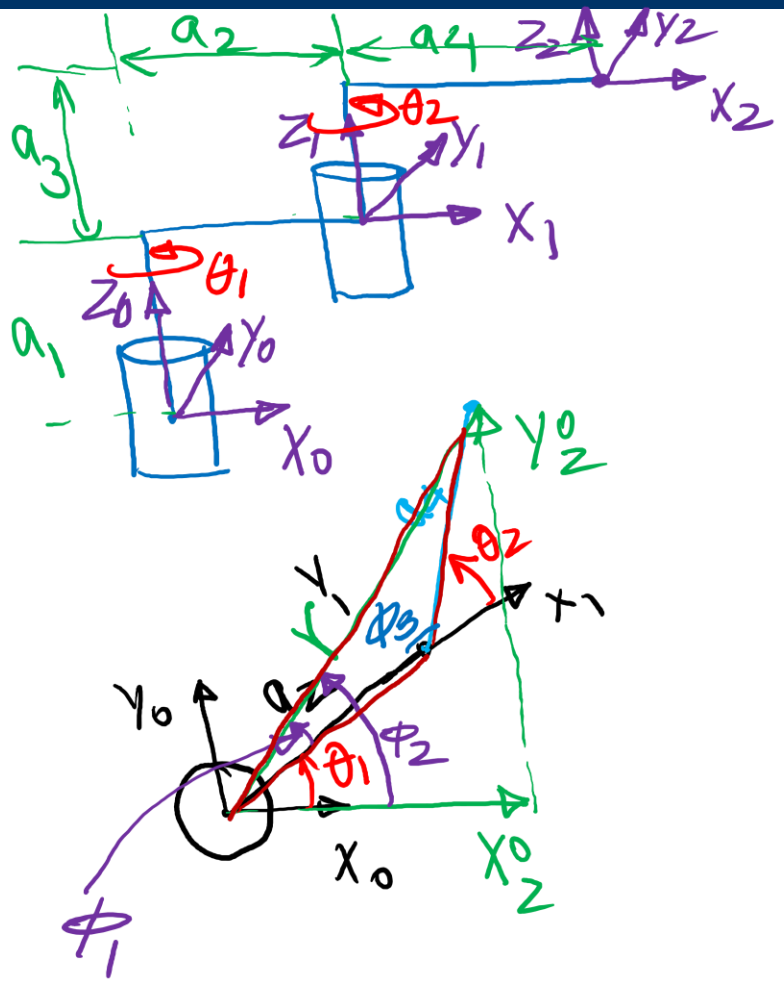
Known: (X_2^0, Y_2^0, Z_2^0) unknown (d_1, θ_2)

$$r_1 = \sqrt{q_3^2 - X_2^{02}}$$

$$\theta_2 = \tan^{-1}\left(\frac{X_2^0}{r_1}\right)$$

$$Z_2^0 = q_1 + d_1 + r_1 \Rightarrow d_1 = Z_2^0 - q_1 - r_1 = Z_2^0 - q_1 - \sqrt{q_3^2 - X_2^{02}}$$

Inverse Kinematics



Known (X_2^0, Y_2^0, Z_2^0) Unknown (θ_1, θ_2)

Coslaw: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

$$r = \sqrt{X_2^{0\,2} + Y_2^{0\,2}}$$

$$\theta_1 = \Phi_2 - \Phi_1$$

$$\Phi_2 = \tan^{-1}\frac{Y_2^0}{X_2^0}$$

Write Coslaw: $\Phi_1 = \cos^{-1}\left(\frac{a_4^2 - a_2^2 - r^2}{-2ra_2}\right)$

Plug Φ_1, Φ_2 in $\theta_1 = \Phi_2 - \Phi_1$ ✓

$$\theta_2 = 180 - \Phi_3$$

$$\Phi_3 = \cos^{-1}\left(\frac{r^2 - a_2^2 - a_4^2}{-2a_2a_4}\right)$$