HUMBER ENGINEERING

MENG 3510 – Control Systems
LECTURE 5

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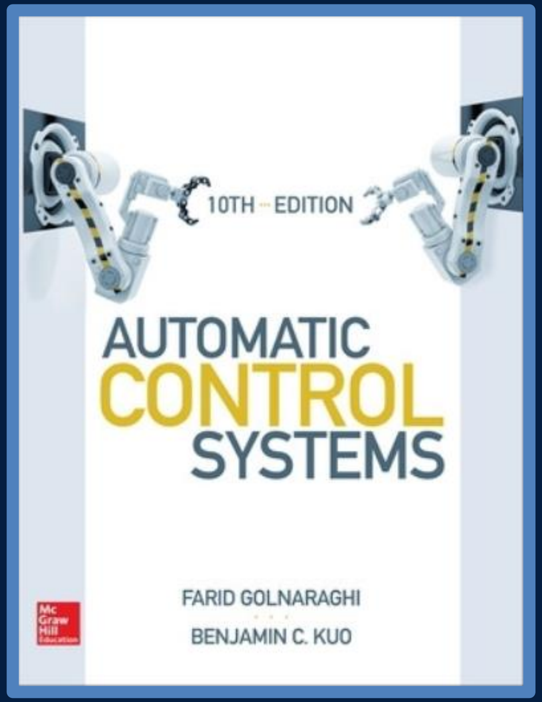
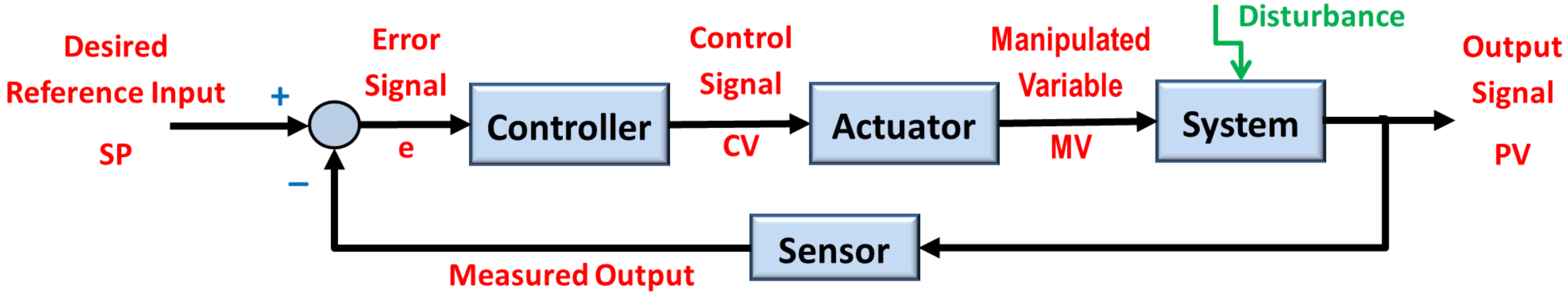
LECTURE 5
Basics of Feedback Control Design

- Introduction to Control System Design
  - Design Specifications
  - Controller Configurations
  - Fundamental Principles of Design
- Basic Control System Design
  - ON-OFF Control
  - Proportional (P) Control
  - Proportional-Integral (PI) Control
  - Proportional-Derivative (PD) Control
  - Proportional-Integral-Derivative (PID) Control



Chapter 11

Introduction to Control System Design



- **Controller:** The device that is used to control the system behavior.
  - Controller compares the actual value of the system output with the desired reference value, determines the deviation, and produces an appropriate control signal that will reduce the deviation to zero or to a small value.
  - Controllers can be implemented in analog (hardware) or digital (software) platforms.
- General objectives of control systems are stability, fast response, small tracking error, robustness, and optimization.
- Controllers can be classified according to their control actions. The most common controllers are:
  - **ON-OFF Controller**
  - **PID Controllers (P, PI, PD)**
  - **Lead Compensator**
  - **Lag Compensator**
  - **State – Feedback Controller**

Introduction to Control System Design

- Control system design involves the following steps:
  - Use design specifications to determine what the system should do and how to do.
  - Determine the controller configuration, relative to how it is connected to the controlled process.
  - Determine the parameter values of the controller to achieve the design goals.

❑ Design Specifications

- **Time-domain** performance specifications:
  - Time-constant, Rise-time, Peak-time, Settling-time, Maximum overshoot and Steady-state error
  - The controller design rely on the graphical tools, such as Root-Locus
  - Analytical solutions exist for first-order and second-order system
  - High-order systems can be approximated by low-order models to apply analytical solutions

- **Frequency-domain** performance specifications:
  - Gain-margin, Phase-margin and Resonant peak
  - The controller design rely on the graphical tools, such as Bode plot and Nyquist plot
  - High-order systems do **not** pose any particular problem

Introduction to Control System Design

- Control system design involves the following steps:
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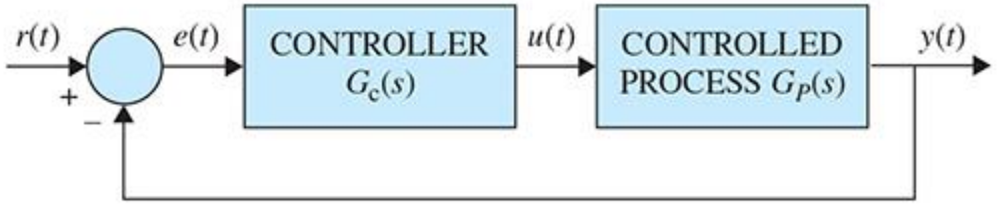
❑ Controller Configurations

- Commonly used control system configurations:

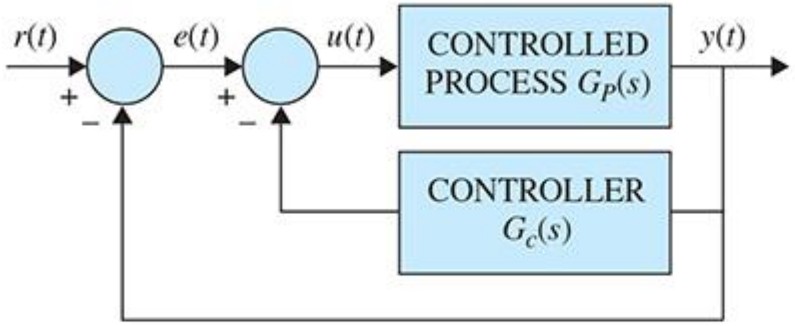
Series Compensation

- Controller placed in series with the system



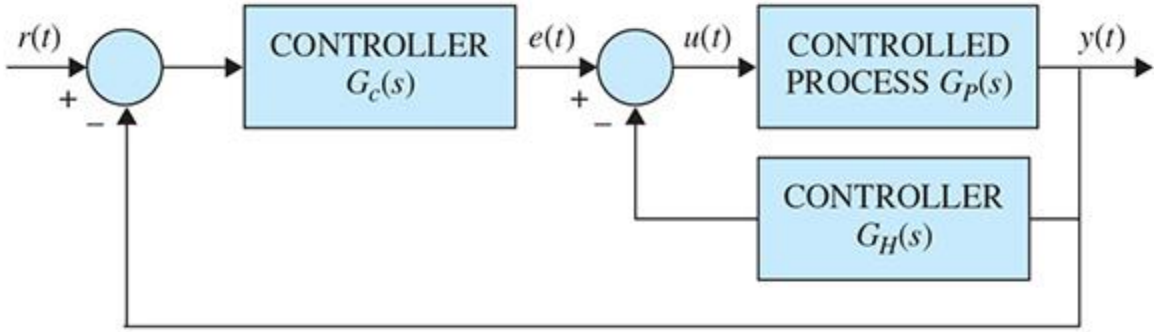
Feedback Compensation

- Controller placed in the minor feedback



Series-Feedback Compensation

- Series and feedback controllers are used



Introduction to Control System Design

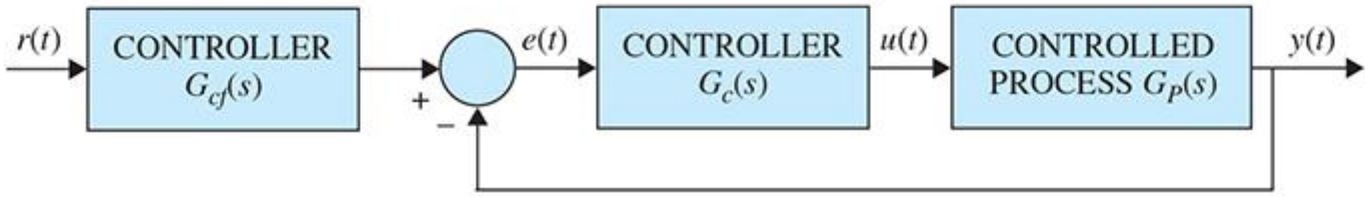
- Control system design involves the following steps:
  - Use design specifications to determine what the system should do and how to do.
  - Determine the controller configuration, relative to how it is connected to the controlled process.
  - Determine the parameter values of the controller to achieve the design goals.

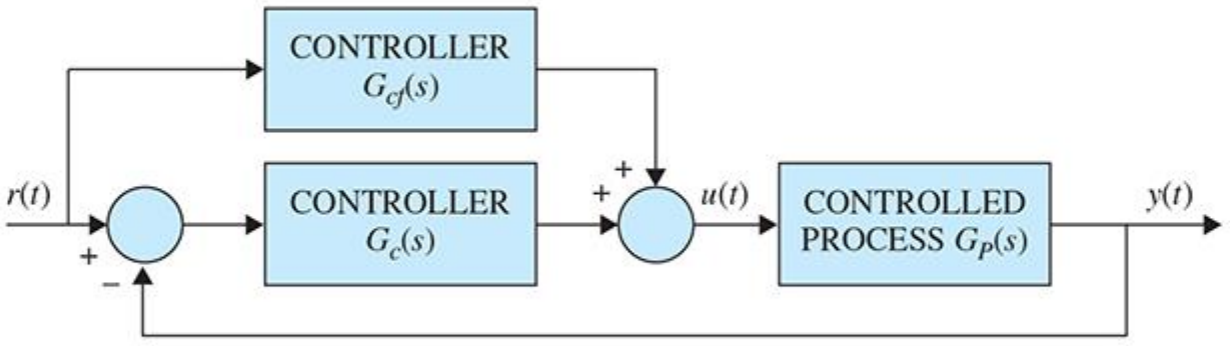
❑ Controller Configurations

- Commonly used control system configurations:

Forward Compensation

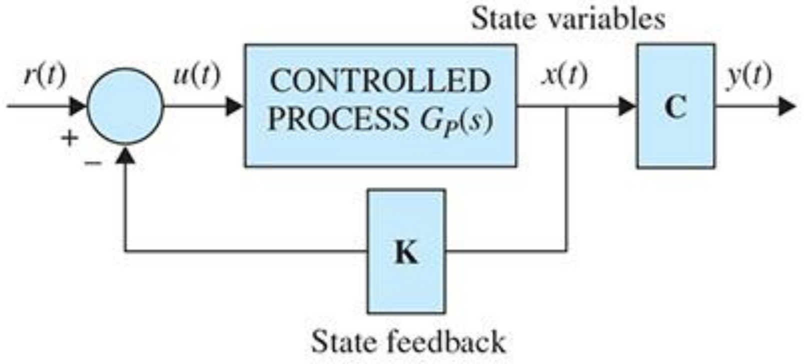
- Feedforward controller is placed in series with the closed-loop system or in parallel with the forward path.





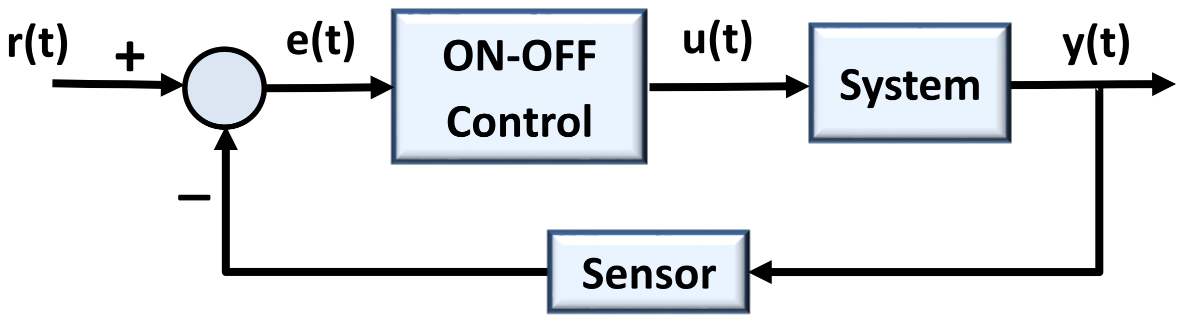
State-Feedback Compensation

- Feeding back the state variables through constant gains.



ON – OFF Control

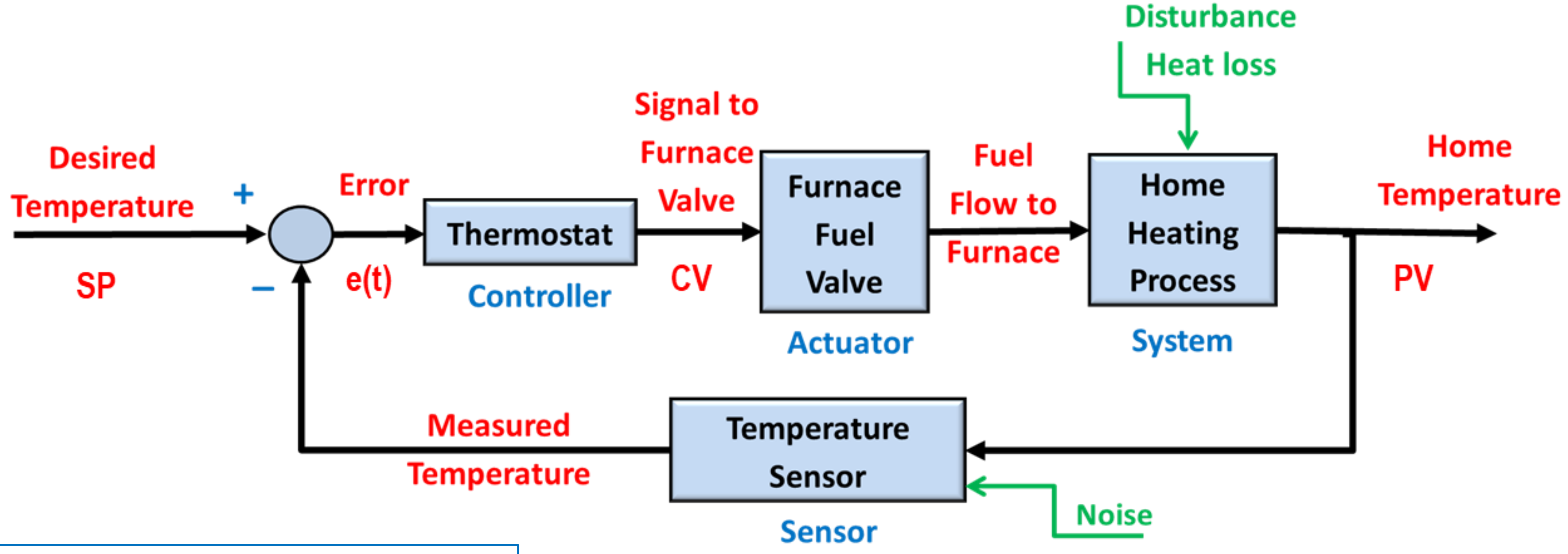
- **ON-OFF Control** is the most basic type of control.
- The controller has only two fixed positions, and the control signal $u(t)$ remains at either a maximum or minimum value, depending on the sign of the error signal $e(t)$.

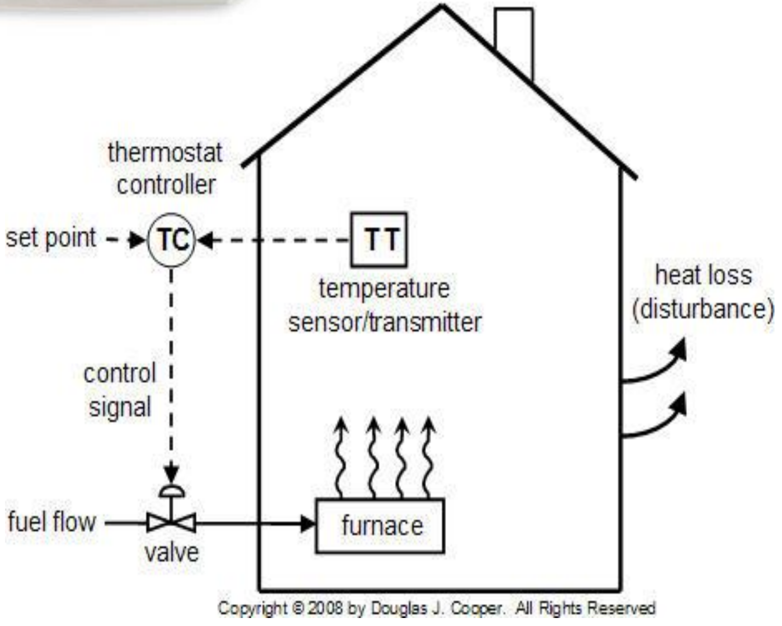


$$\boxed{e(t) = r(t) - y(t)}$$

Example

❑ Home Heating Control System





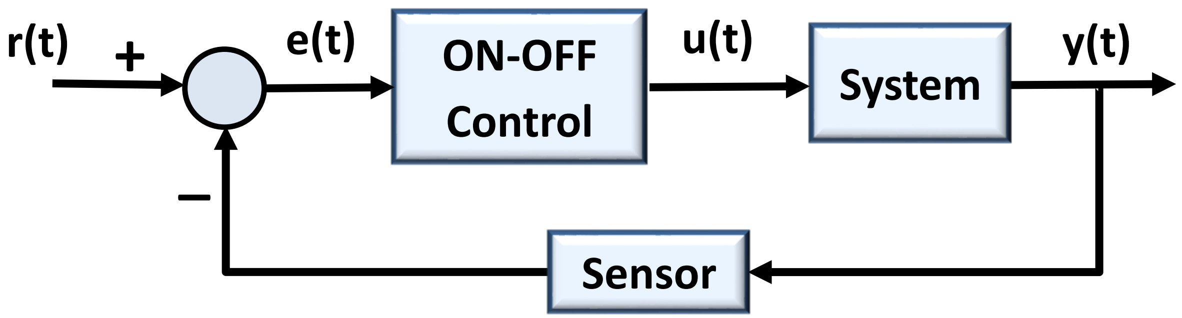
$$\text{CV} = \begin{cases} ON, & \text{for} \quad e(t) > 0 \\ OFF, & \text{for} \quad e(t) < 0 \end{cases}$$

ON – OFF Control

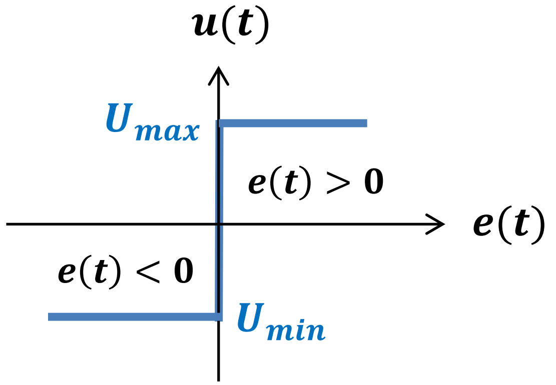
- **ON-OFF Control** is the most basic type of control.
- The controller has only two fixed positions, and the control signal $u(t)$ remains at either a maximum or minimum value, depending on the sign of the error signal $e(t)$.

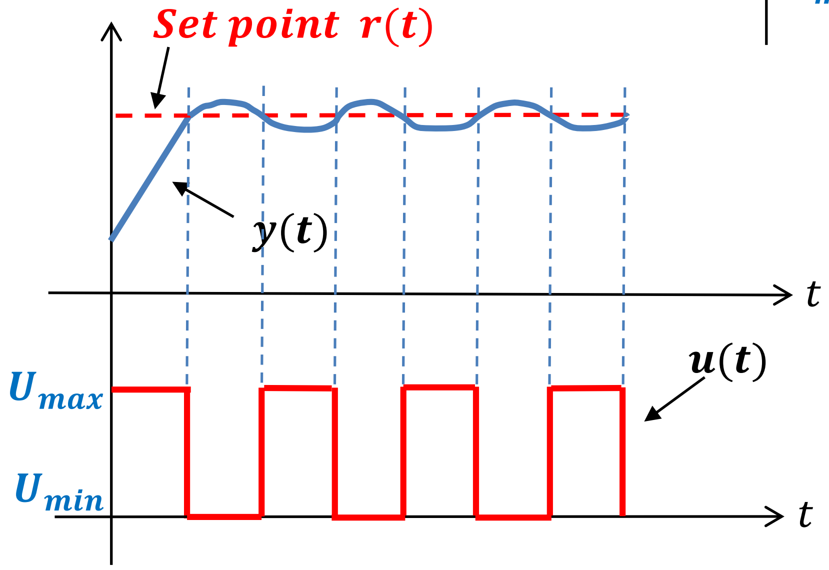
$$\boxed{e(t) = r(t) - y(t)}$$

- Relatively simple, easy to implement and inexpensive
- Effective for systems with slow dynamics and precise control is not required, such as home heating systems, water level control in swimming pools, fan control system, ….
- The drawback with this type of ON-OFF control is that
  - The control signal must switch very rapidly over the full range to maintain a certain reference.
  - A small amount of noise can make the relay switch randomly.
  - High frequency switching reduces the useful lifetime of the components.



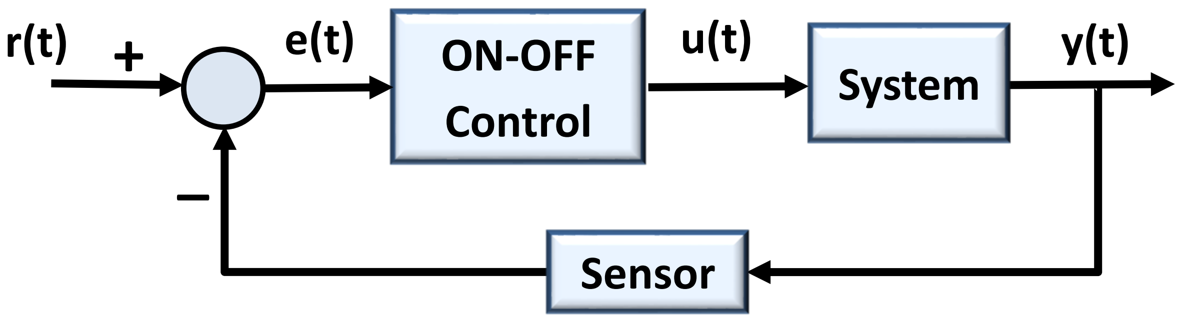
Relay Switch



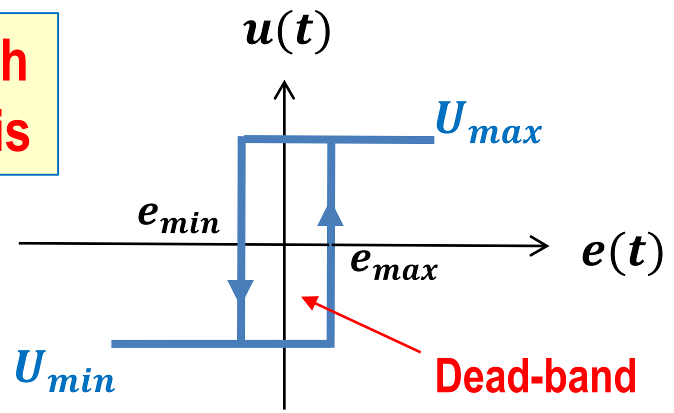


ON – OFF Control

- The ON-OFF control action can be modified by introducing **dead-band** or **hysteresis** around the set point.
- Dead-band is the range, which the error signal must move before the switching occurs.
- The dead-band causes the control signal $u(t)$ to maintain its present value until the error signal $e(t)$ has moved slightly beyond the zero value.
- This prevents too-frequent operation of the ON-OFF mechanism.
- The dead-band must be determined based on the required accuracy and the lifetime of the component.

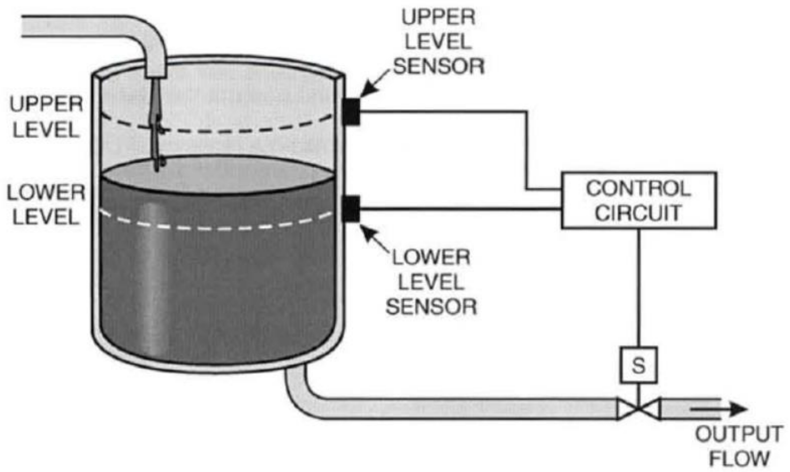


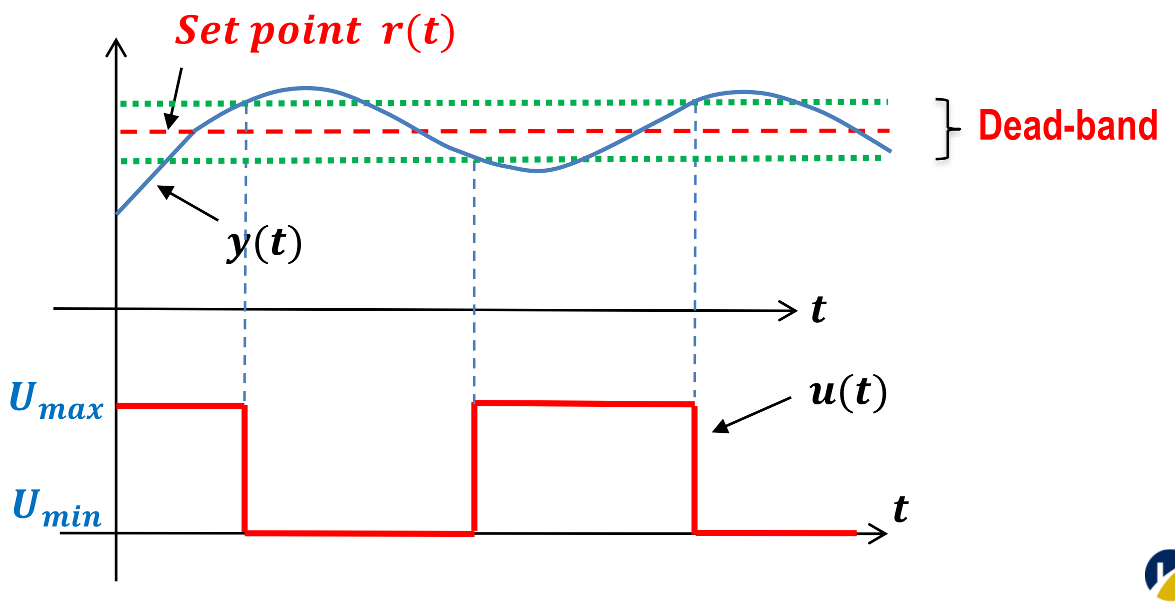
Relay with Hysteresis



Example

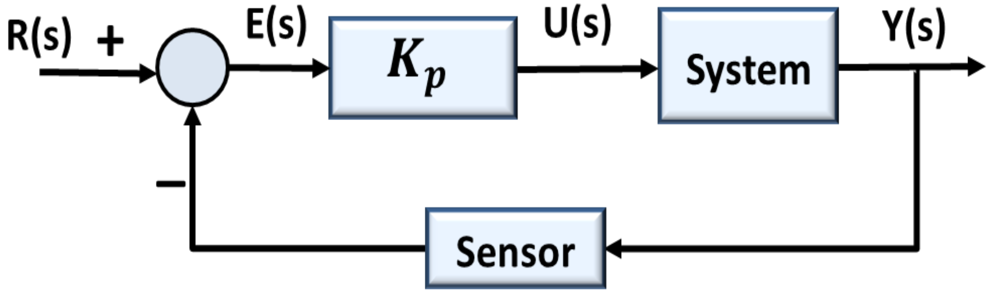
❑ Liquid Level Control System





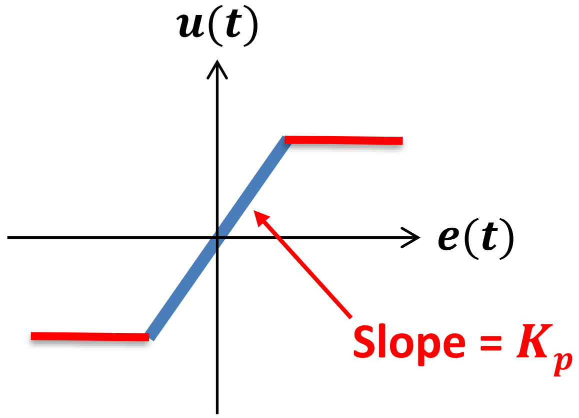
Proportional Controller

- In Proportional Control, the control signal $u(t)$ is proportional to the error signal $e(t)$ via the static adjustable gain K_p, which we can adjust as part of tuning the control system.

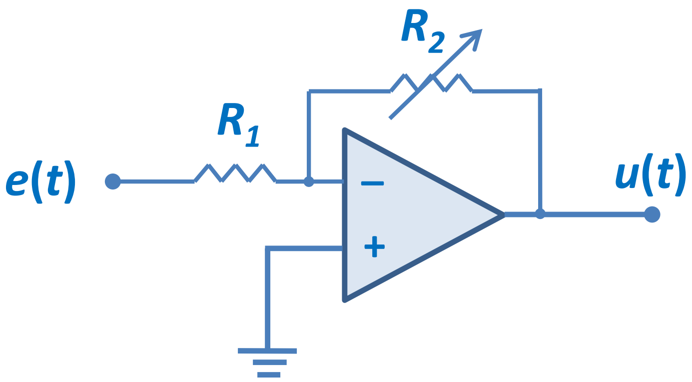


$$\boldsymbol{u(t) = K_p e(t) \quad \rightarrow \quad U(s) = K_p E(s)}$$

Proportional Gain



- Proportional Controller provides gradual changes based on the error signal $e(t)$, which can eliminate the oscillation associated with ON-OFF controllers.
- Proportional Control, is essentially an amplifier with an adjustable gain.
- Analog controller can be implemented by an op-amp circuit.

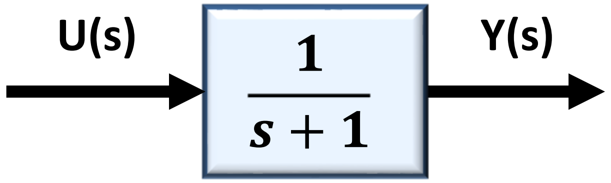


$$u(t) = -\frac{R_2}{R_1} e(t) \quad \rightarrow \quad K_p = -\frac{R_2}{R_1}$$

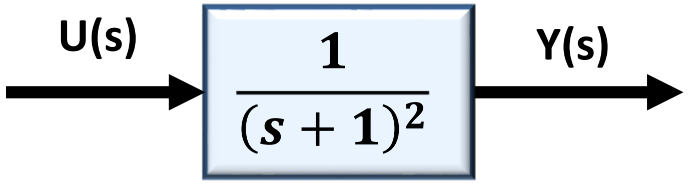
Proportional Controller

This example shows the effect of changing Proportional Gain K_p on the unit-step response of a first-order, second-order and higher-order systems.

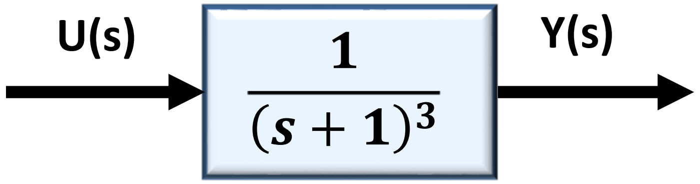
Compare the unit-step response of open-loop systems. What is the main difference between the step response of a first-order system and high-order systems?



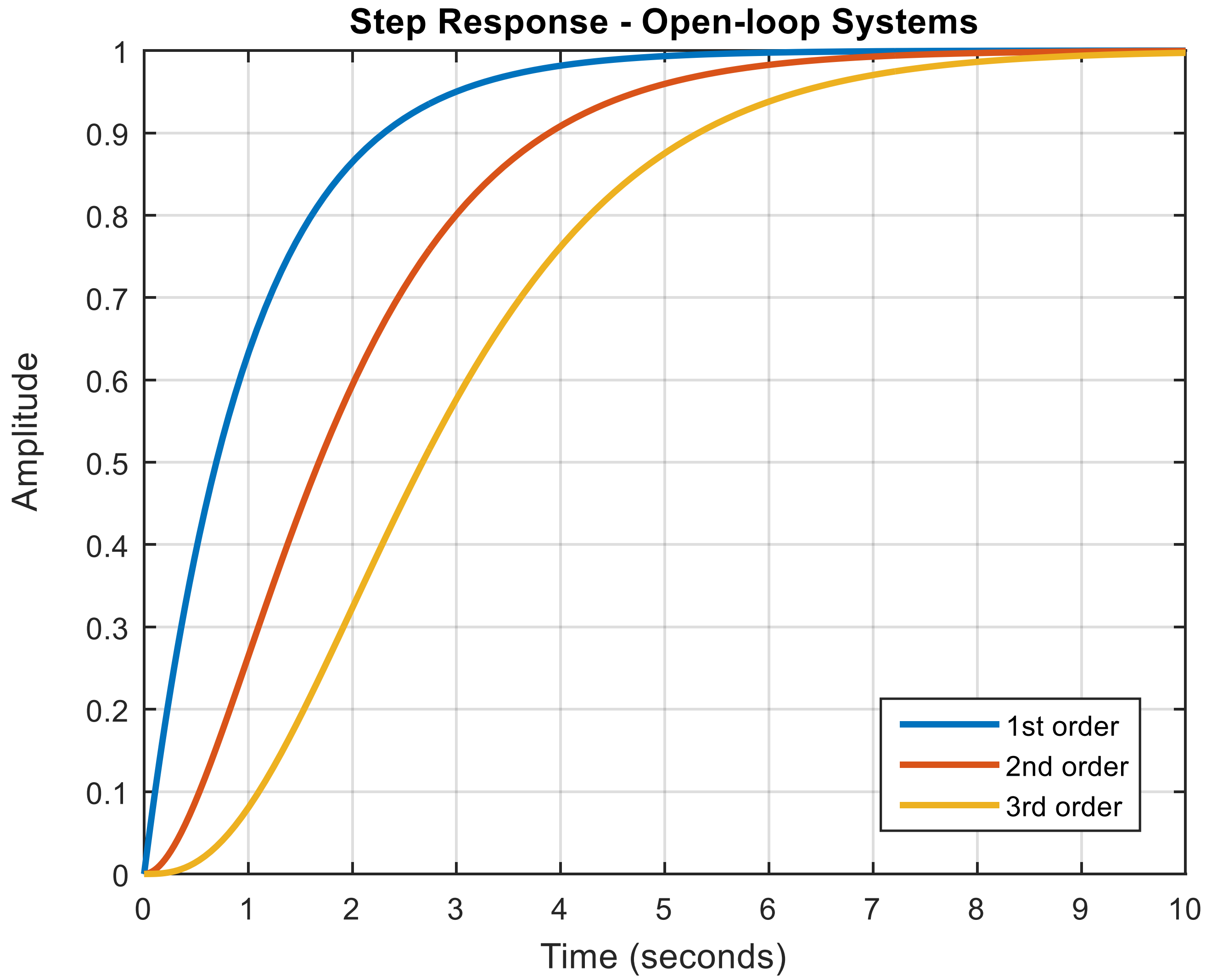
$$\frac{1}{s+1}$$



$$\frac{1}{(s+1)^2}$$

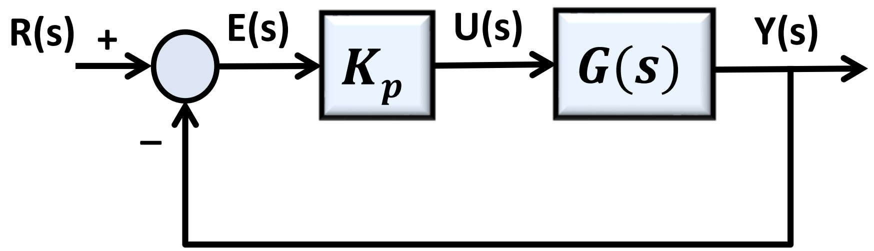


$$\frac{1}{(s+1)^3}$$



Proportional Controller

Compare the unit-step response of each close-loop system for proportional gain of $K_p = 1, 5$ and 10. Note to the effect of increasing the proportional gain on the **performance** and **stability** of the each closed-loop system,



$$G(s) = \frac{1}{s+1}$$

❑ First-Order System

- By increasing the proportional controller gain K_p :
  - Time-constant decreases $\rightarrow$ Faster response
  - Steady-state error decreases $\rightarrow$ Better tracking capability

For $K_p = 1 \rightarrow \dfrac{Y(s)}{R(s)} = \dfrac{1}{s+2}$

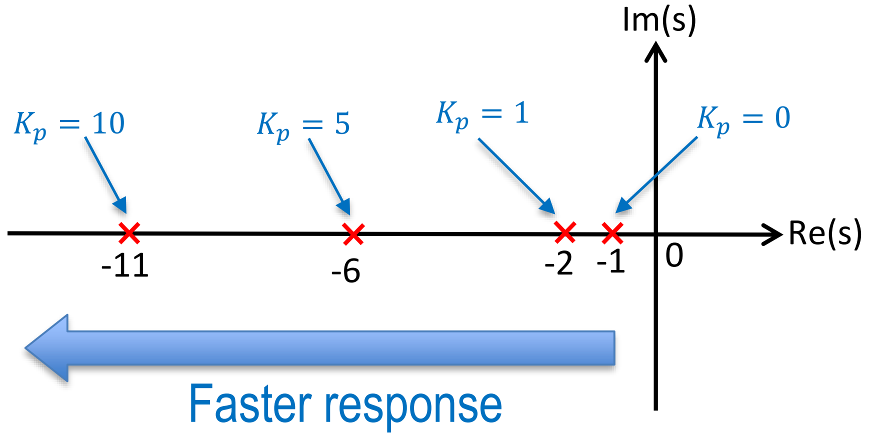
$\boxed{\text{pole} \rightarrow \quad s = -2}$

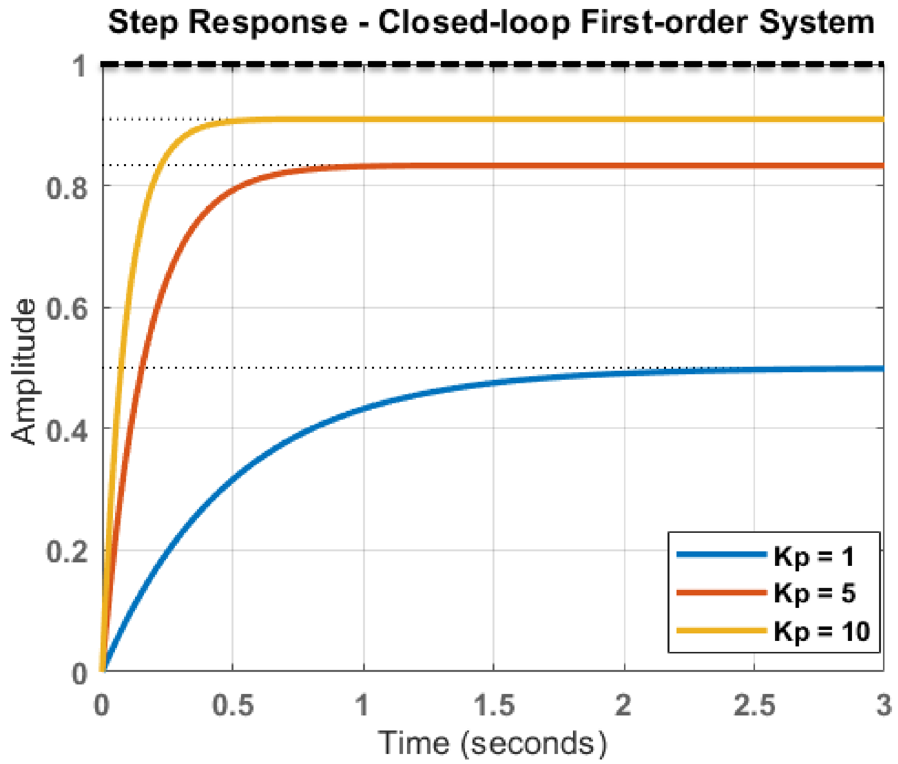
For $K_p = 5 \rightarrow \dfrac{Y(s)}{R(s)} = \dfrac{5}{s+6}$

$\boxed{\text{pole} \rightarrow \quad s = -6}$

For $K_p = 10 \rightarrow \dfrac{Y(s)}{R(s)} = \dfrac{10}{s+11}$

$\boxed{\text{pole} \rightarrow \quad s = -11}$





- Closed-loop pole moves to the left on the real axis

Proportional Controller

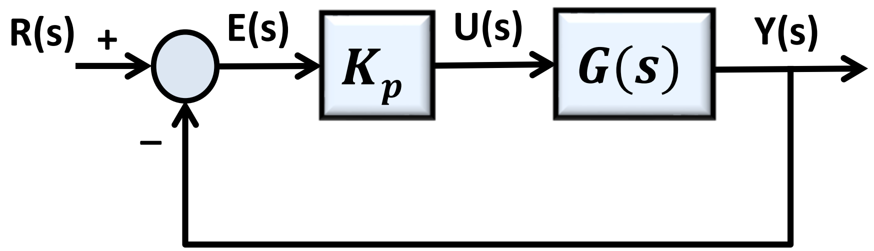
Compare the unit-step response of each close-loop system for proportional gain of $K_p = 1, 5$ and 10. Note to the effect of increasing the proportional gain on the **performance** and **stability** of the each closed-loop system,

❑ Second-Order System

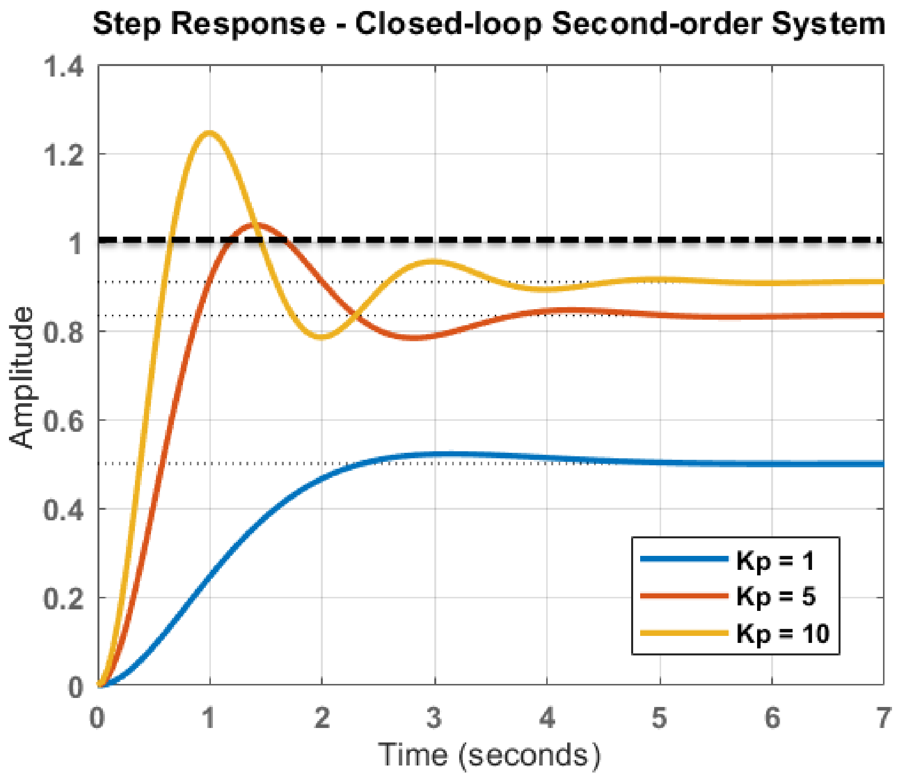
- By increasing the proportional controller gain K_p :
  - Rise time (time-constant) decreases → Faster response
  - Overshoot and oscillations increases → Reduce stability
  - Steady-state error decreases → Better tracking capability

	Rise-time	Overshoot	S.S. Error
K_p = 1	1.52 sec	4.32%	0.500
K_p = 5	0.604 sec	24.5%	0.167
K_p = 10	0.4 sec	37%	0.091

- In general, achieving to a good transient response and a good steady-state error is **not possible** by only a simple gain K_p.
- The Proportional Gain, K_p, is selected based on the desired performance criteria, such as rise-time, overshoot, ….
- Proportional controller **cannot** eliminate the steady-state error.



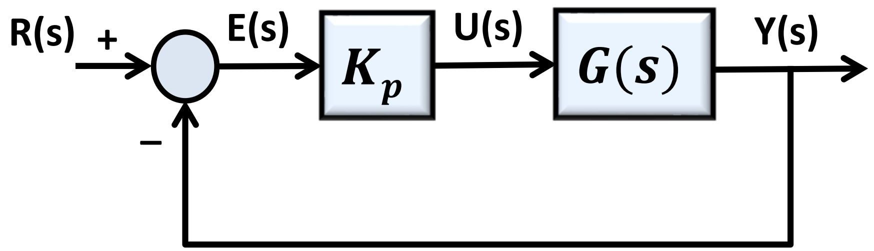
$$G(s) = \frac{1}{(s+1)^2} = \frac{1}{s^2 + 2s + 1}$$



Proportional Controller

Compare the unit-step response of each close-loop system for proportional gain of $K_p = 1, 5$ and 10. Note to the effect of increasing the proportional gain on the **performance** and **stability** of the each closed-loop system,

❑ Second-Order System



$$G(s) = \frac{1}{(s+1)^2} = \frac{1}{s^2+2s+1}$$

- By increasing the proportional controller gain K_p :
  - Closed-loop poles moves parallel to the imaginary axis

For $K_p = 1 \;\rightarrow\; \dfrac{Y(s)}{R(s)} = \dfrac{1}{s^2+2s+2}$

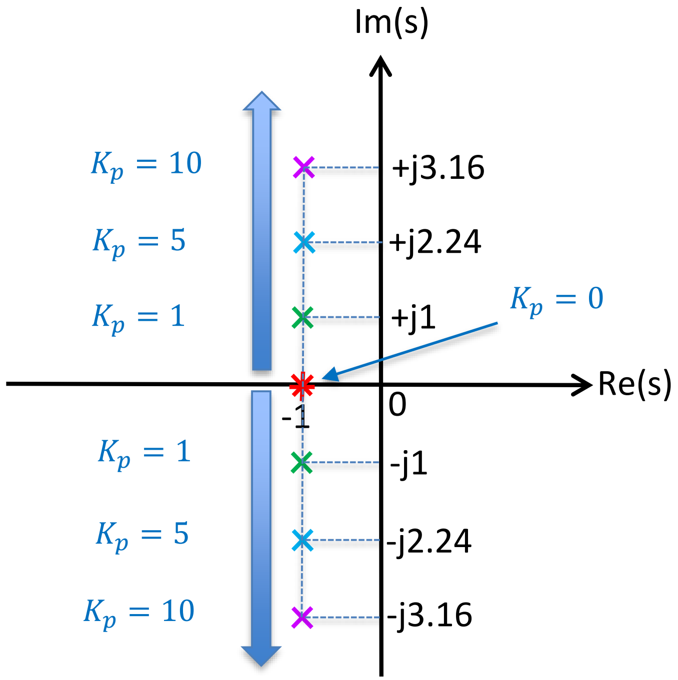
$\boxed{\text{poles} \;\rightarrow\; s = -1 \pm j1}$

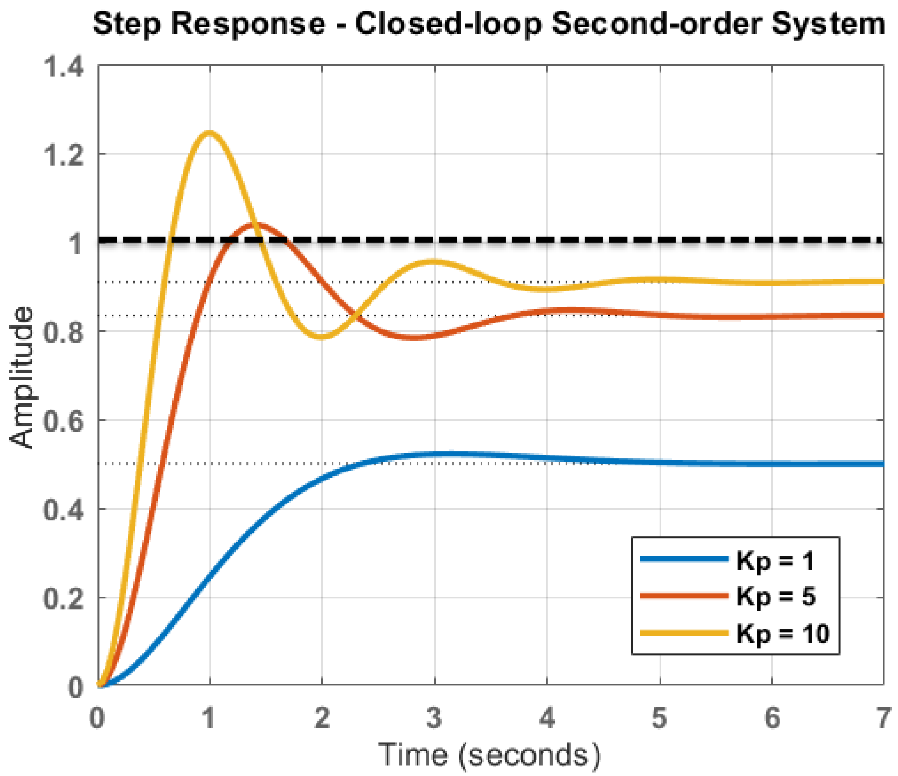
For $K_p = 5 \;\rightarrow\; \dfrac{Y(s)}{R(s)} = \dfrac{5}{s^2+2s+6}$

$\boxed{\text{poles} \;\rightarrow\; s = -1 \pm j2.24}$

For $K_p = 10 \;\rightarrow\; \dfrac{Y(s)}{R(s)} = \dfrac{10}{s^2+2s+11}$

$\boxed{\text{poles} \;\rightarrow\; s = -1 \pm j3.16}$





Proportional Controller

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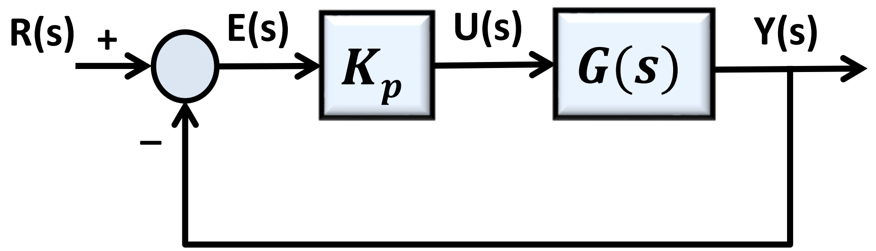


$$G(s) = \frac{1}{(s+1)^3} = \frac{1}{s^3+3s^2+3s+1}$$

❑ Third-Order System

- By increasing the proportional controller gain K_p :
  - Rise time (time-constant) decreases → Faster response
  - Overshoot and oscillations increases → Reduce stability
  - Steady-state error decreases if system is stable → Better tracking

For $K_p = 1 \rightarrow \dfrac{Y(s)}{R(s)} = \dfrac{1}{s^3+3s^2+3s+2}$

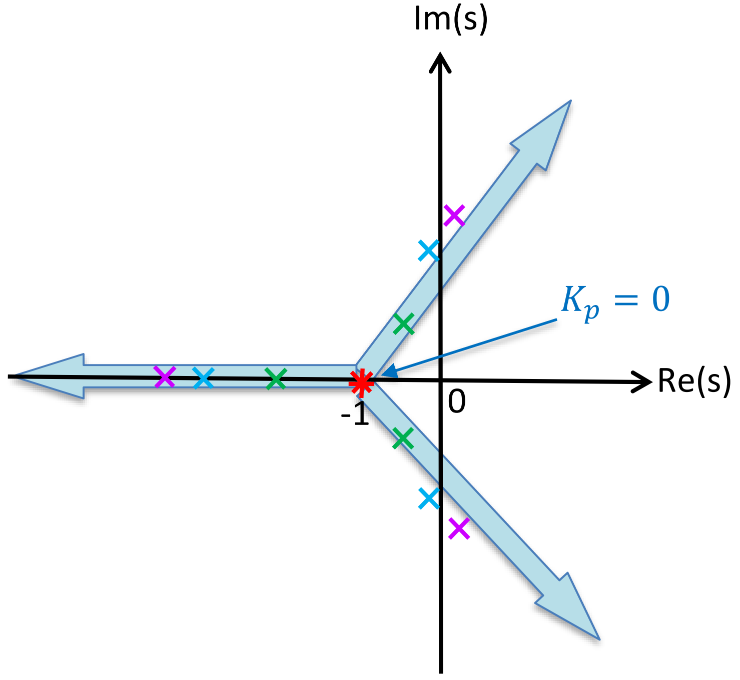
$\boxed{\text{poles} \rightarrow s = -2, s = -0.5 \pm j0.87}$

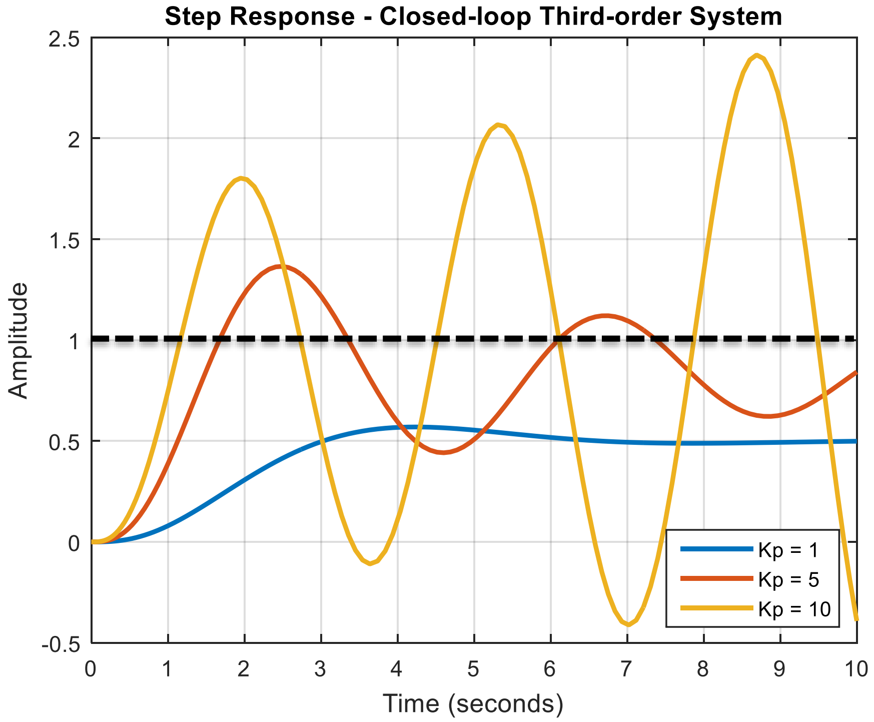
For $K_p = 5 \rightarrow \dfrac{Y(s)}{R(s)} = \dfrac{5}{s^3+3s^2+3s+6}$

$\boxed{\text{poles} \rightarrow s = -2.7, s = -0.15 \pm j1.48}$

For $K_p = 10 \rightarrow \dfrac{Y(s)}{R(s)} = \dfrac{10}{s^3+3s^2+3s+11}$

$\boxed{\text{poles} \rightarrow s = -3.15, s = +0.078 \pm j1.86}$





- Closed-loop poles may move to the right-half of s-plane (unstable system)

Proportional Controller

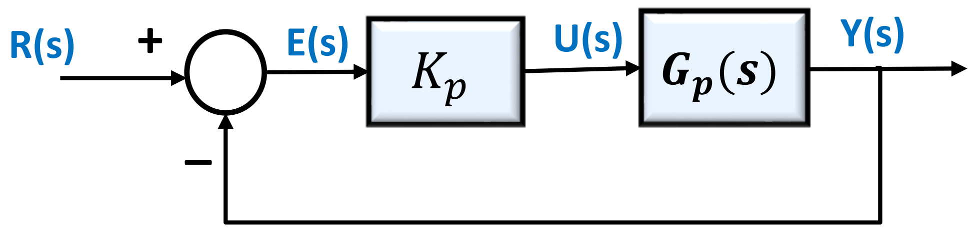
Example 1 Consider the first-order transfer function model of a cruise control system, where the input is an applied force by the engine and output is the car speed.

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{2}{10s+1}$$

a) Determine the time-constant and steady-state gain of system.

Time-constant $\rightarrow$ $\tau = 10\ sec$, Steady-state gain $\rightarrow$ $K = 2$

b) A closed-loop system with proportional control gain K_p has been developed to increase the speed of the system. Determine the required gain K_p to have a time-constant of 1 second.



First, find the closed-loop transfer function:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{K_p G_p(s)}{1 + K_p G_p(s)} = \frac{\frac{2K_p}{10s+1}}{1 + \frac{2K_p}{10s+1}} = \frac{2K_p}{10s + 1 + 2K_p}$$

Next, find the time-constant of the closed-loop transfer function and make it equal to the desired time-constant, then find the required gain K_p.

Time-constant of the closed-loop system is: $\tau_{cl} = \frac{10}{1+2K_p}$

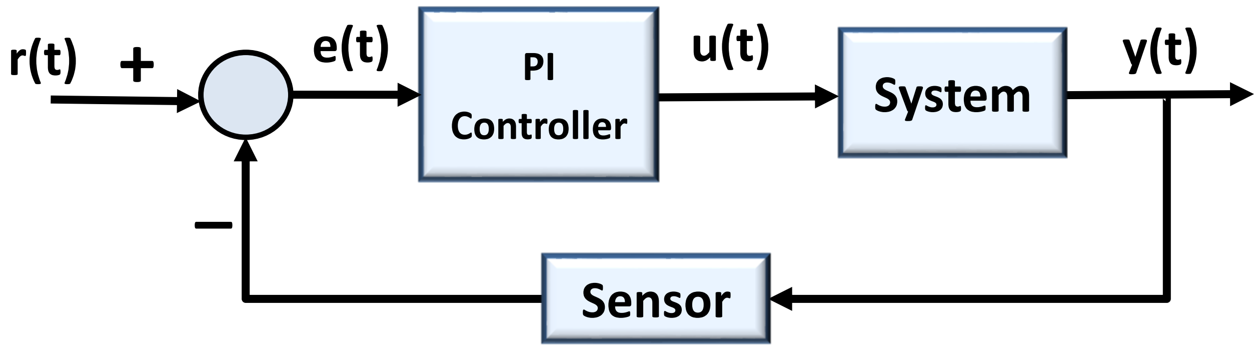
The desired time-constant is 1 sec . $\rightarrow$ $1 = \frac{10}{1+2K_p}$ $\rightarrow$ $1 + 2K_p = 10$ $\rightarrow$ $\boxed{K_p = 4.5}$

We can check the closed-loop system for the designed controller gain $K_p = 4.5$:

$$T(s) = \frac{9}{10s+10} = \frac{0.9}{s+1}$$

Proportional – Integral (PI) Controller

- **PI Control** is utilized to eliminate the steady-state error.
- It is the most widely used controller in the industry.
- In PI Control, the control signal includes two parts
  - One part is proportional to the error signal
  - The other part is proportional to the integral of the error signal



$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt \quad \rightarrow \quad U(s) = K_p E(s) + \frac{K_i}{s} E(s) = \left(K_p + \frac{K_i}{s}\right) E(s) = K_p \left(1 + \frac{1}{T_i s}\right) E(s)$$

Integral Gain

Integral Time Constant

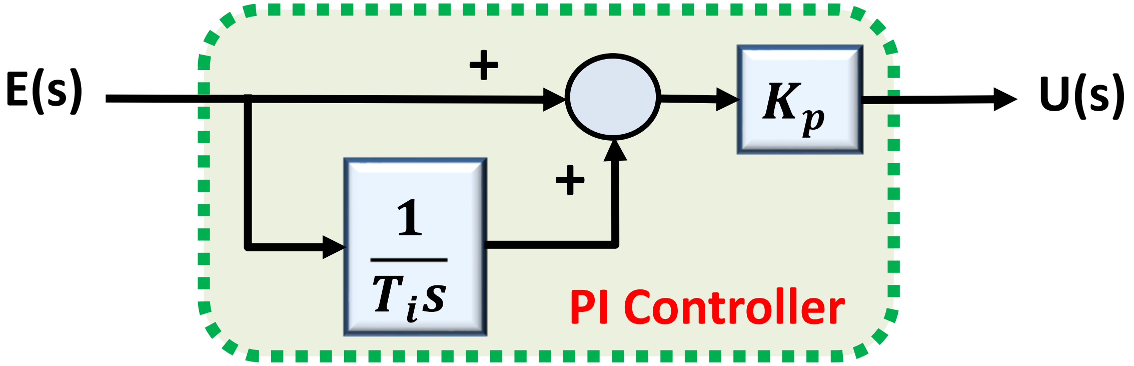
- The integral time constant T_i represent how fast the integral term reacts to eliminate the steady-state error.
- The controller parameters are related as below,

$$\boxed{T_i = \frac{K_p}{K_i}}$$

- The integrator term eliminates the steady-state error by increasing the **type** of the open-loop system.

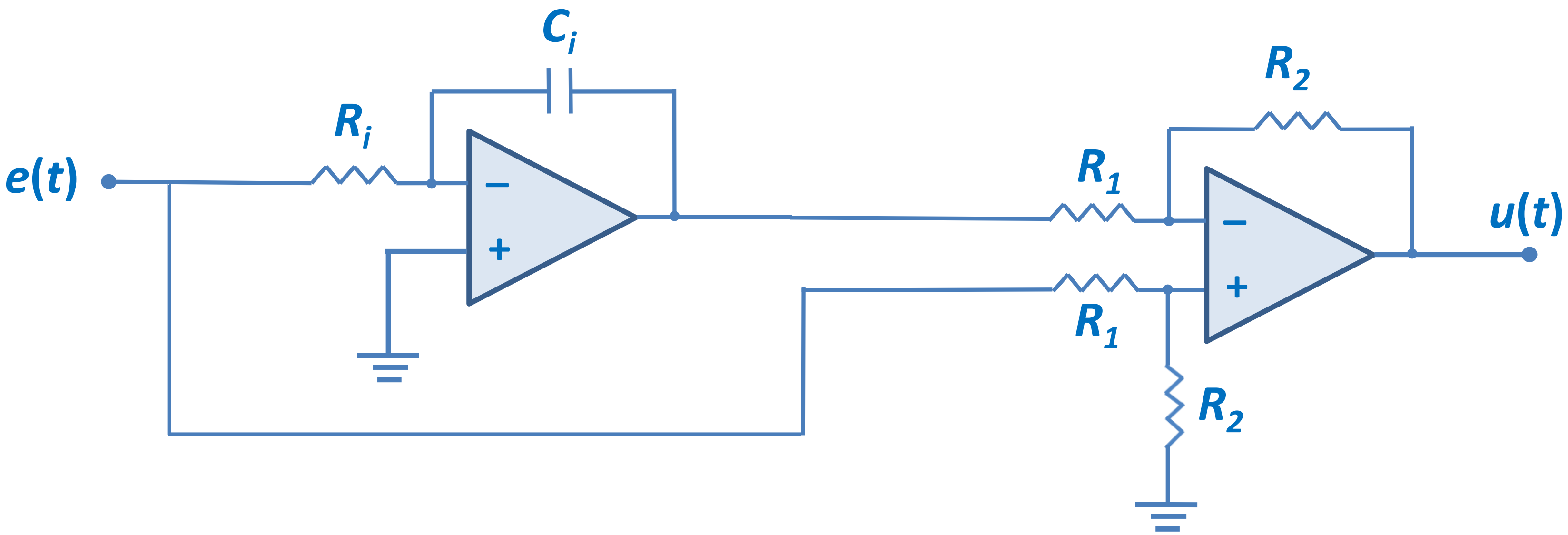
Proportional – Integral (PI) Controller

- PI Controller can be implemented as below,



$$U(s) = K_p\left(1 + \frac{1}{T_i s}\right)E(s)$$

- Analog PI controller can be realized by two OP-AMP, which provides independent adjustment of each control mode.



$$K_p = \frac{R_2}{R_1} \quad and \quad T_i = R_i C_i$$

- *For example, to implement a PI controller with $K_p = 2$ and $T_i = 1\ sec$ the components can be selected as:*

$R_1 = 500\ k\Omega,\ R_2 = 1\ M\Omega$

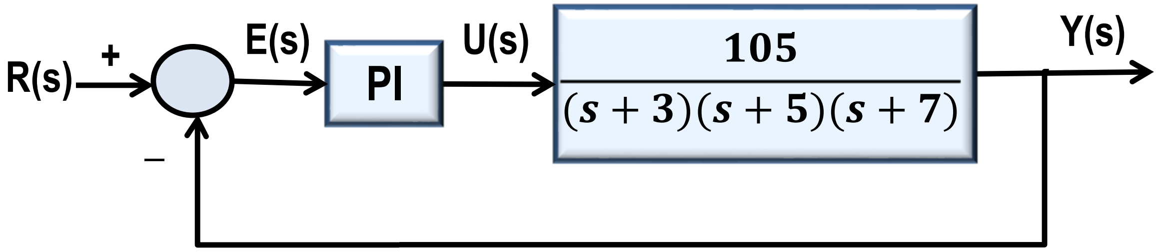
$R_i = 1\ M\Omega,\ \ C_i = 1\ \mu F$

Proportional – Integral (PI) Controller

This example shows the effect of changing Integral Time Constant T_i on the unit-step response of a high-order closed-loop system.

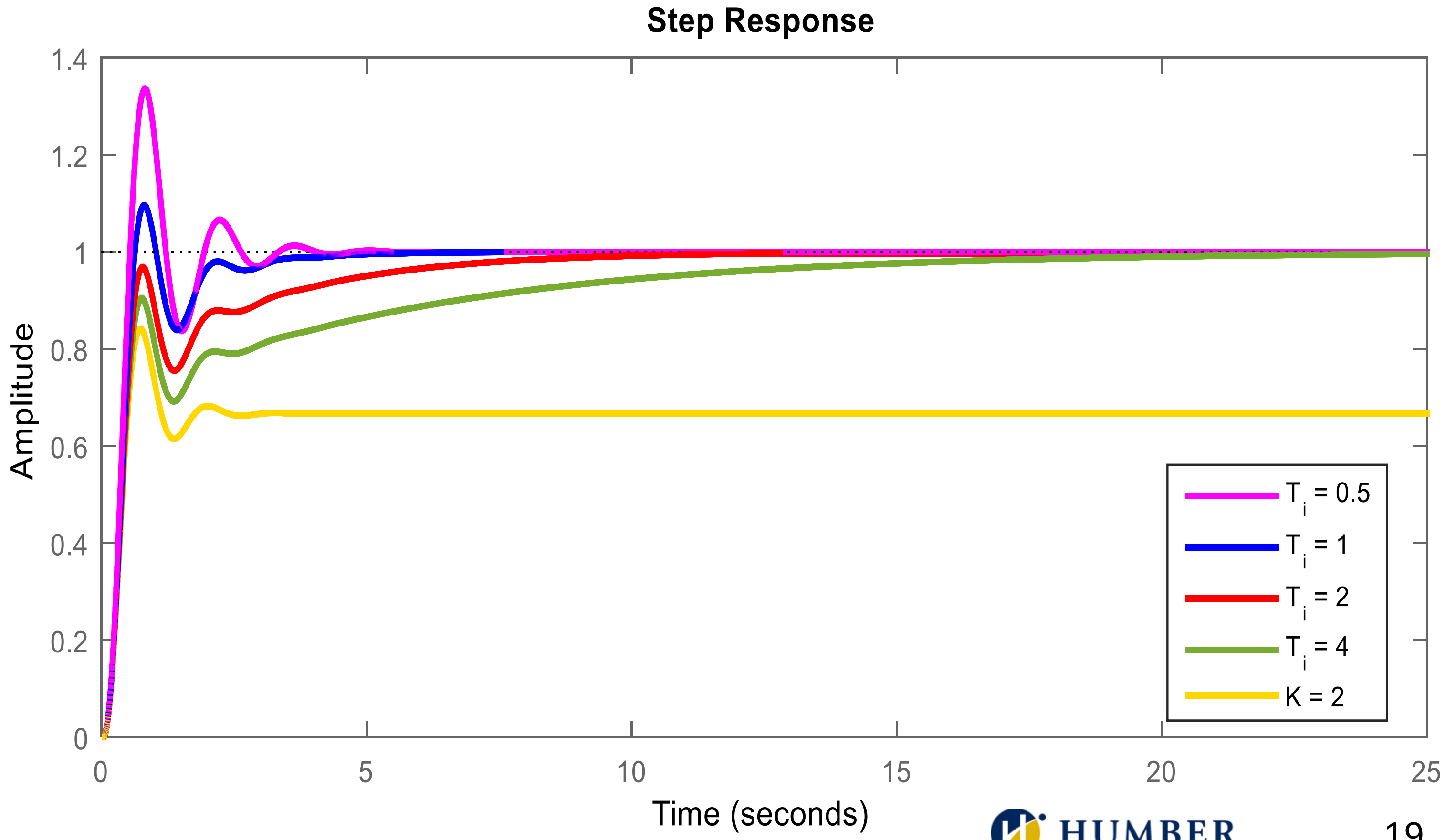
Following figure shows effect of selecting different values for $\boldsymbol{T_i}$.

$\boxed{K_p = 2}$ $\boxed{T_i = 0.5, 1, 2, 4}$



$$U(s) = K_p\left(1 + \frac{1}{T_i s}\right)E(s)$$

	Rise-time	Overshoot	Settling-time	S.S. Error
T_i = 0.5	0.324 sec	33.5%	3.12 sec	0
T_i = 1	0.383 sec	9.49%	3.22 sec	0
T_i = 2	0.447 sec	0%	7.53 sec	0
T_i = 4	0.543 sec	0%	16 sec	0
Only K_p = 2	0.301 sec	26.1%	2.11 sec	0.333



- **Large T_i** increases the settling time and results a slow response with low overshoot.
- **Small T_i** decreases the settling time but reduces relative stability of the closed-loop system and results a high overshoot.

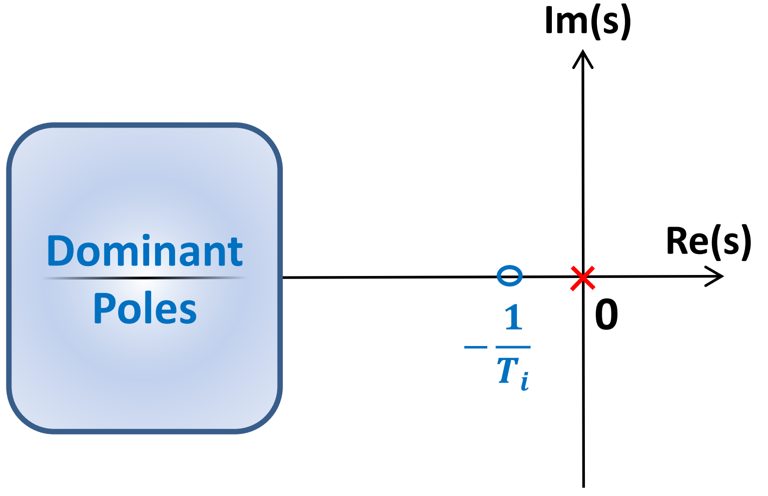
Proportional – Integral (PI) Controller

$$\frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right) = K_p\left(\frac{T_i s + 1}{T_i s}\right)$$

- Consider the PI Controller, which has one pole and one zero at:

$$s = 0 \quad \text{and} \quad s = \frac{-1}{T_i}$$

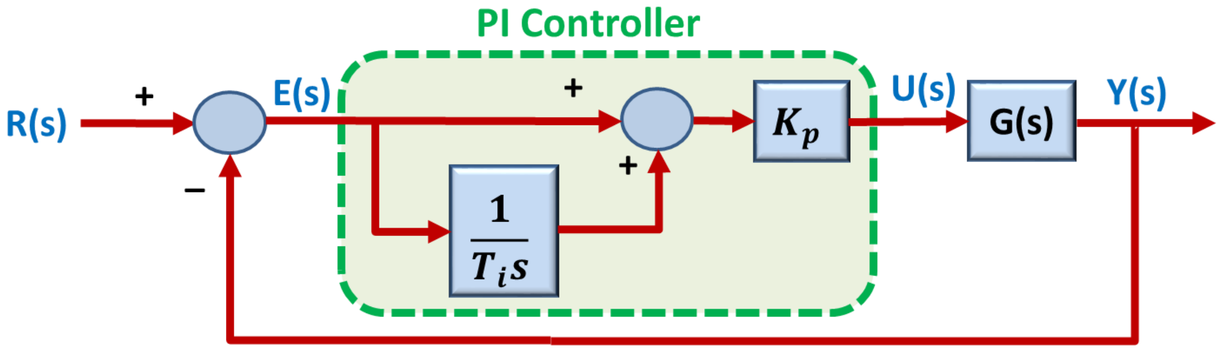
- Selecting the T_i means locating the zero of PI controller.
- The zero has to be located farther from the dominant poles of the closed-loop system with only proportional controller K_p and close to the origin.



- **Following steps show how to determine the PI controller parameters.**
  - First, find the proportional gain K_p to achieve the desired transient response specifications, such as rise-time, overshoot, or time-constant.
  - Next, determine the dominant closed-loop poles p_{cl} under the proportional control K_p.
  - Then, select the integral time constant T_i as below

$$\frac{2}{|\mathrm{Re}\{p_{cl}\}|} \leq T_i$$

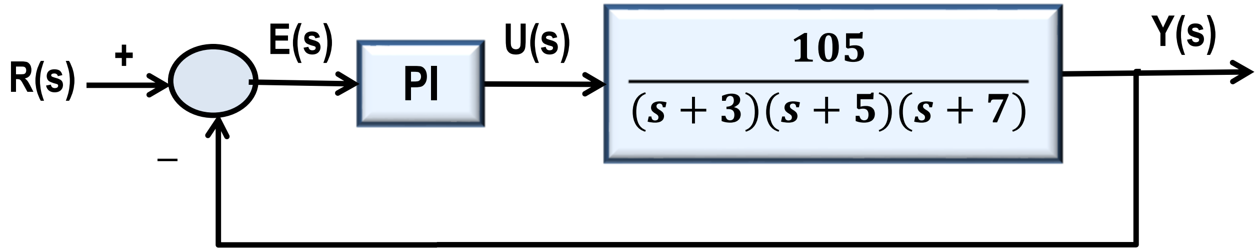
  - Fine tune the controller parameters (if required) to achieve desired transient response.



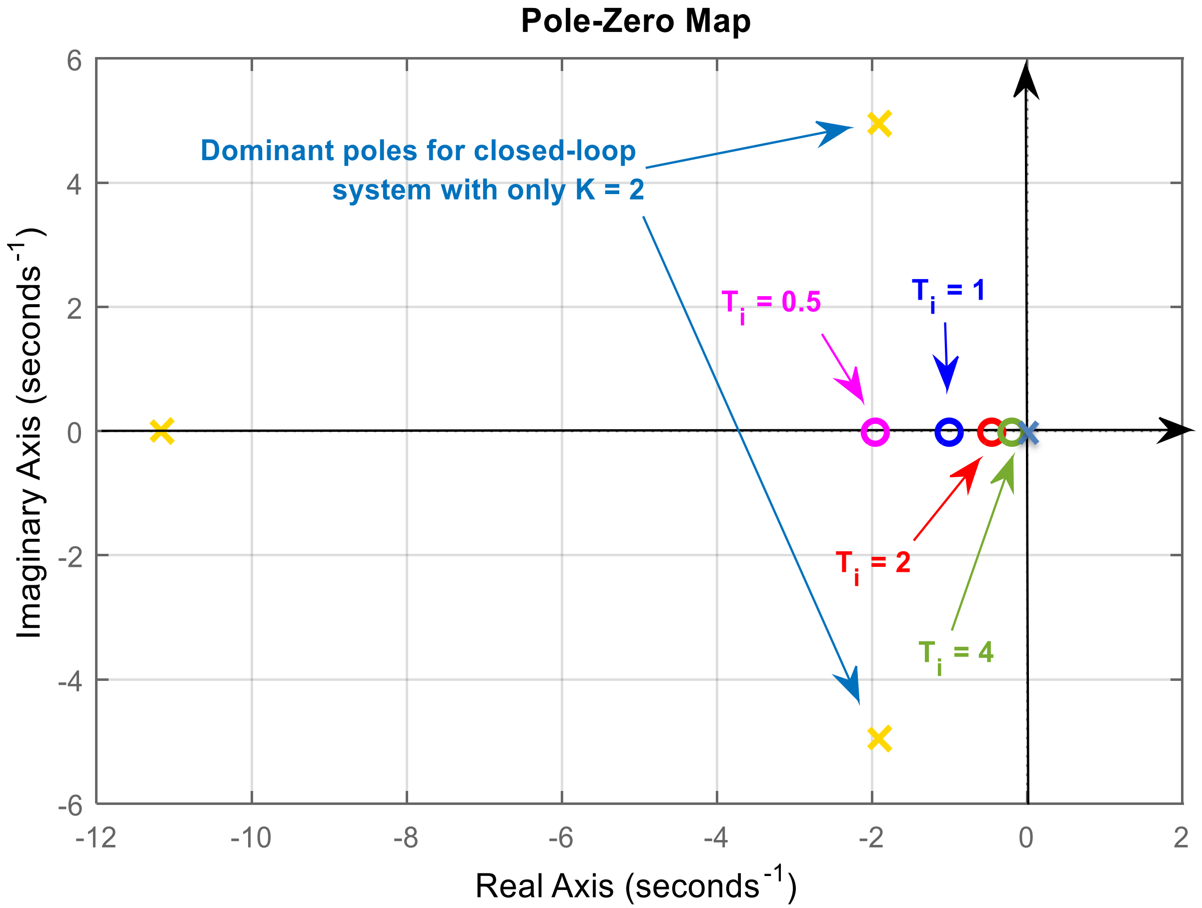
Proportional – Integral (PI) Controller

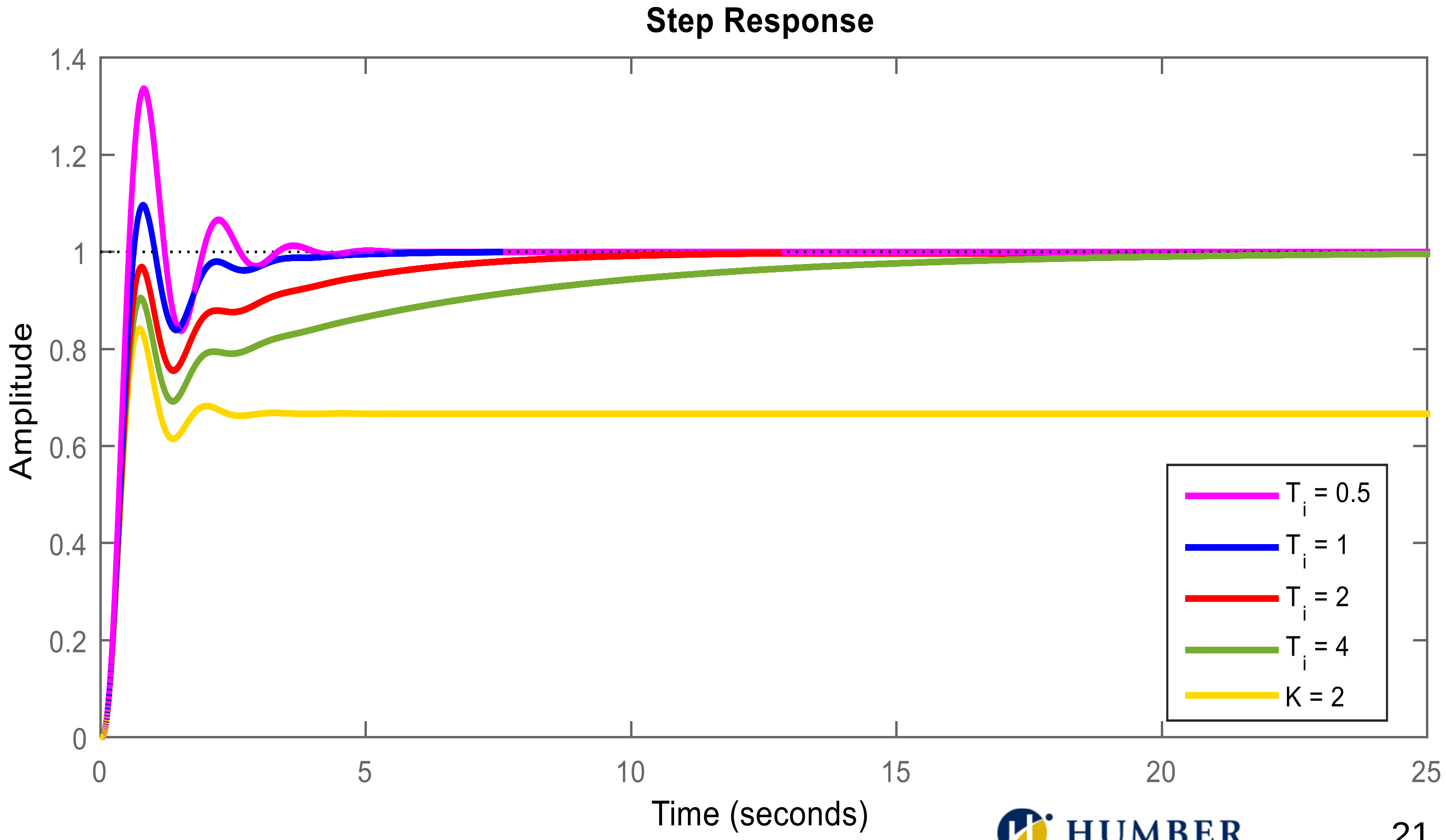
This example shows the effect of changing Integral Time Constant T_i on the unit-step response of a high-order closed-loop system.

- Following pole-zero map compares the controller zeros for different T_i values with respect to the dominant poles.



$K_p = 2 \qquad T_i = 0.5, 1, 2, 4$

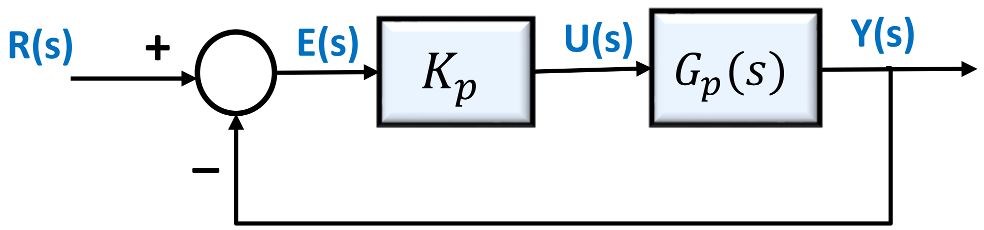




Proportional – Integral (PI) Controller

Example 2 Consider the transfer function model of a second-order dynamic system.

$$G_p(s) = \frac{1}{(s+2)(s+8)}$$



a) Determine the proportional controller gain K_p to have a 2% overshoot for unit-step response.

First find the closed-loop transfer function:

$$\boldsymbol{T(s) = \frac{Y(s)}{R(s)} = \frac{K_pG_p(s)}{1+K_pG_p(s)}} = \frac{\frac{K_p}{(s+2)(s+8)}}{1+\frac{K_p}{(s+2)(s+8)}} = \frac{K_p}{s^2+10s+16+K_p}$$

Calculate the damping ratio from the required maximum overshoot value:

$$\boldsymbol{\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2+\ln^2(O.S.)}}} \rightarrow \zeta = \frac{-\ln(0.02)}{\sqrt{\pi^2+\ln^2(0.02)}} \rightarrow \boxed{\boldsymbol{\zeta = 0.78}}$$

Desired Damping Ratio

Next, compare the characteristic equation with the <u>standard second-order system</u> to find the gain K_p.

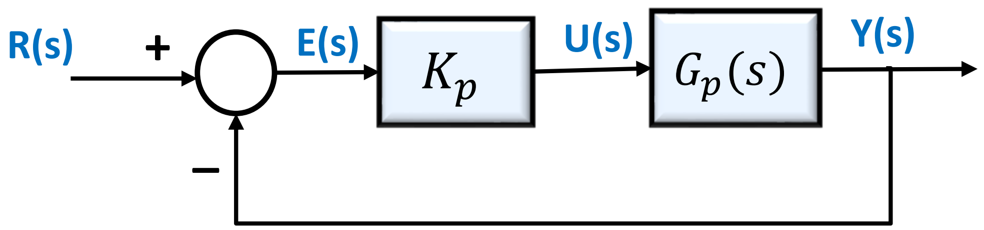
$$\boldsymbol{s^2+2\zeta\omega_n s+\omega_n^2} = s^2+10s+16+K_p \rightarrow \begin{cases} 2\zeta\omega_n = 10 & \rightarrow \quad 2(0.78)\omega_n = 10 \rightarrow \quad \omega_n = 6.41 \text{ rad/sec} \\ \omega_n^2 = 16+K_p & \rightarrow \quad (6.41)^2 = 16+K_p \rightarrow \quad \boxed{\boldsymbol{K_p = 25.1}} \end{cases}$$

Proportional – Integral (PI) Controller

Example 2

Consider the transfer function model of a second-order dynamic system.

$$G_p(s) = \frac{1}{(s+2)(s+8)}$$

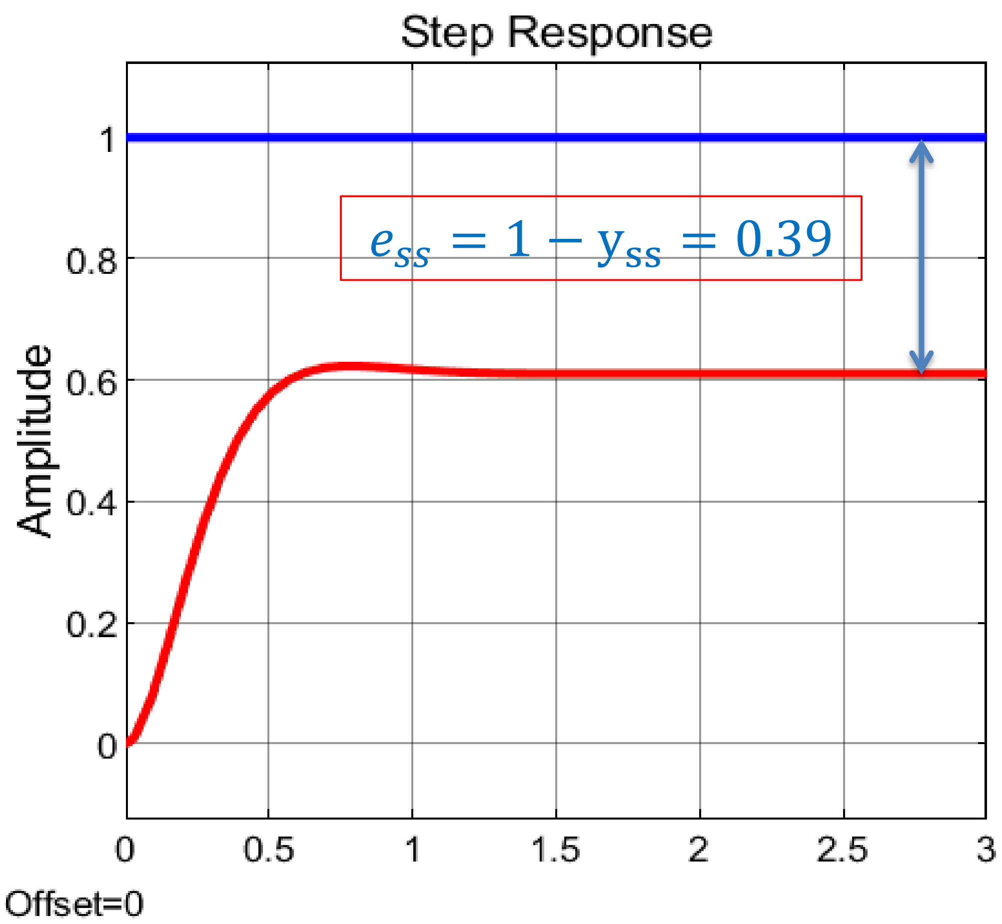


b) The tracking error is defined as $E(s) = R(s) - Y(s)$. Determine the steady-state tracking error e_{ss} due to a unit-step input, $R(s) = 1/s$ for the obtained gain K_p.

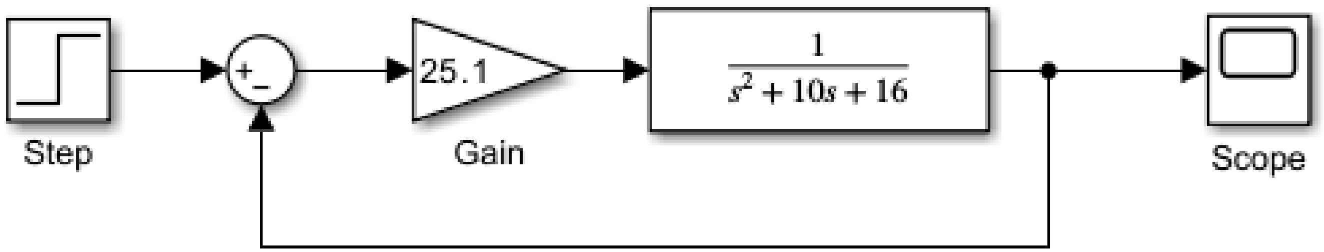
The steady-state error for a unit-step response is obtained as:

$$\boldsymbol{k_p = \lim_{s\to 0} G(s)} = \lim_{s\to 0}\left(\frac{25.1}{(s+2)(s+8)}\right) = 1.57$$

$$\boldsymbol{e_{ss} = \frac{1}{1+k_p}} = \frac{1}{1+1.57} = 0.39 \rightarrow \boxed{e_{ss} = 39\ \%}$$ **Steady-state Error**



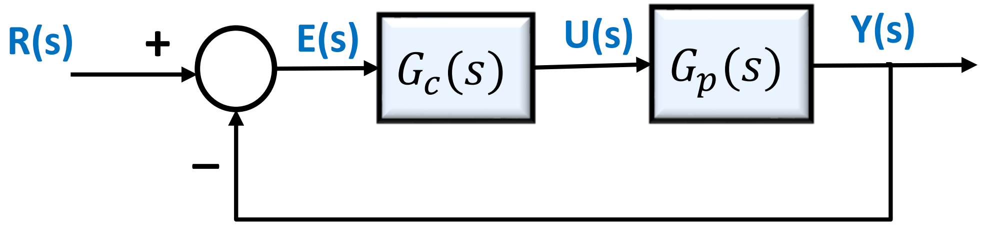
We can plot the unit-step response graph in **Simulink**.



Proportional – Integral (PI) Controller

Example 2 Consider the transfer function model of a second-order dynamic system.

$$G_p(s) = \frac{1}{(s+2)(s+8)}$$



c) Design a PI controller to achieve a zero steady-state error.

$$\boldsymbol{G_c(s) = K_p\left(1 + \frac{1}{T_i s}\right)}$$

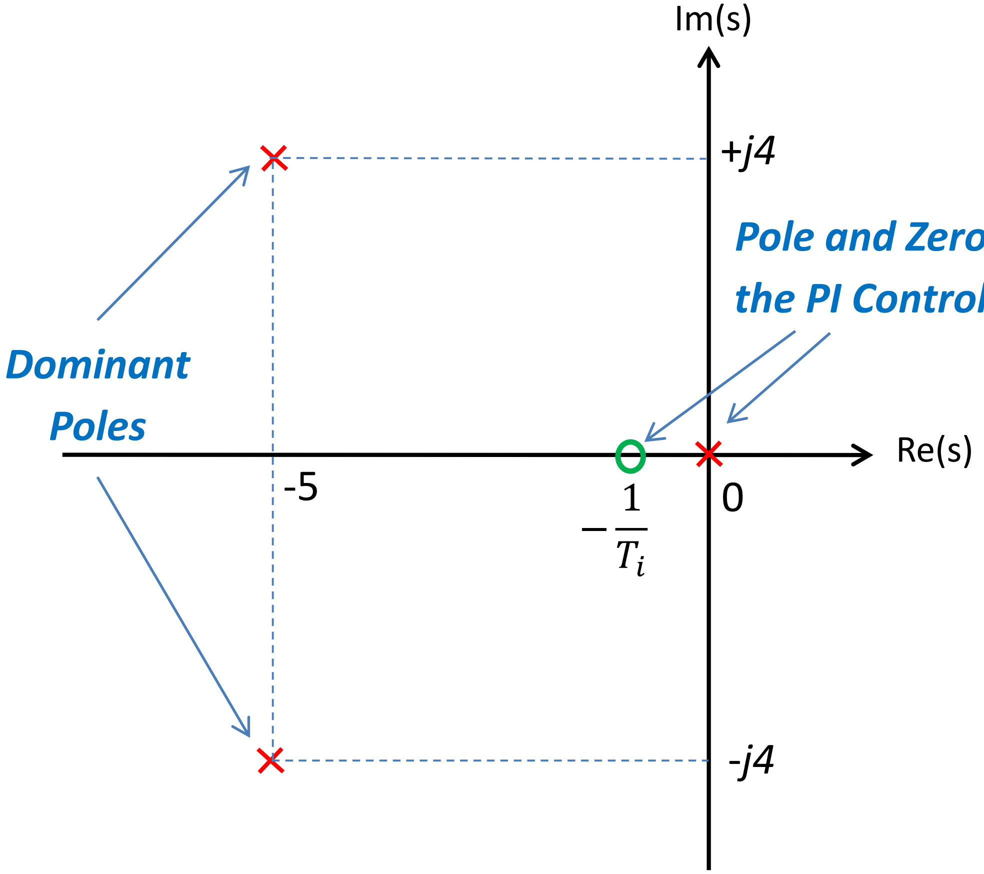
First, find the poles of the closed-loop transfer function for $K_p = 25.1$.

$$T(s) = \frac{Y(s)}{R(s)} = \frac{25.1}{s^2 + 10s + 41.1} \quad \rightarrow \quad \text{Poles:} \quad s = -5 \pm j4$$

The second-order closed-loop transfer function has one pair of complex-conjugate stable pole.

The integral time constant T_i can be selected by the following stability consideration, where p_{cl} represent the closed-loop pole under the proportional control.

$$\boldsymbol{T_i \geq \frac{2}{|Re\{p_{cl}\}|}} \quad \rightarrow \quad T_i \geq \frac{2}{5} = 0.4 \quad \rightarrow \quad T_i = 1\ sec$$

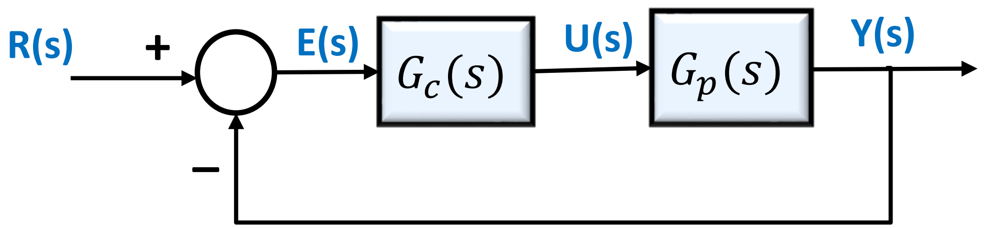


Therefore, the designed PI Controller is $\rightarrow$ $\boxed{\boldsymbol{G_c(s) = 25.1\left(1 + \frac{1}{s}\right)}}$

Proportional – Integral (PI) Controller

Example 2 Consider the transfer function model of a second-order dynamic system.

$$G_p(s) = \frac{1}{(s+2)(s+8)}$$

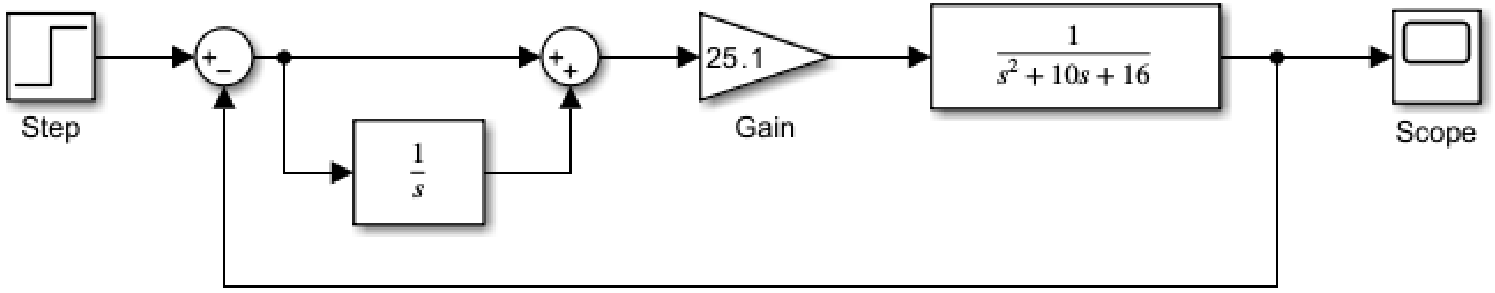


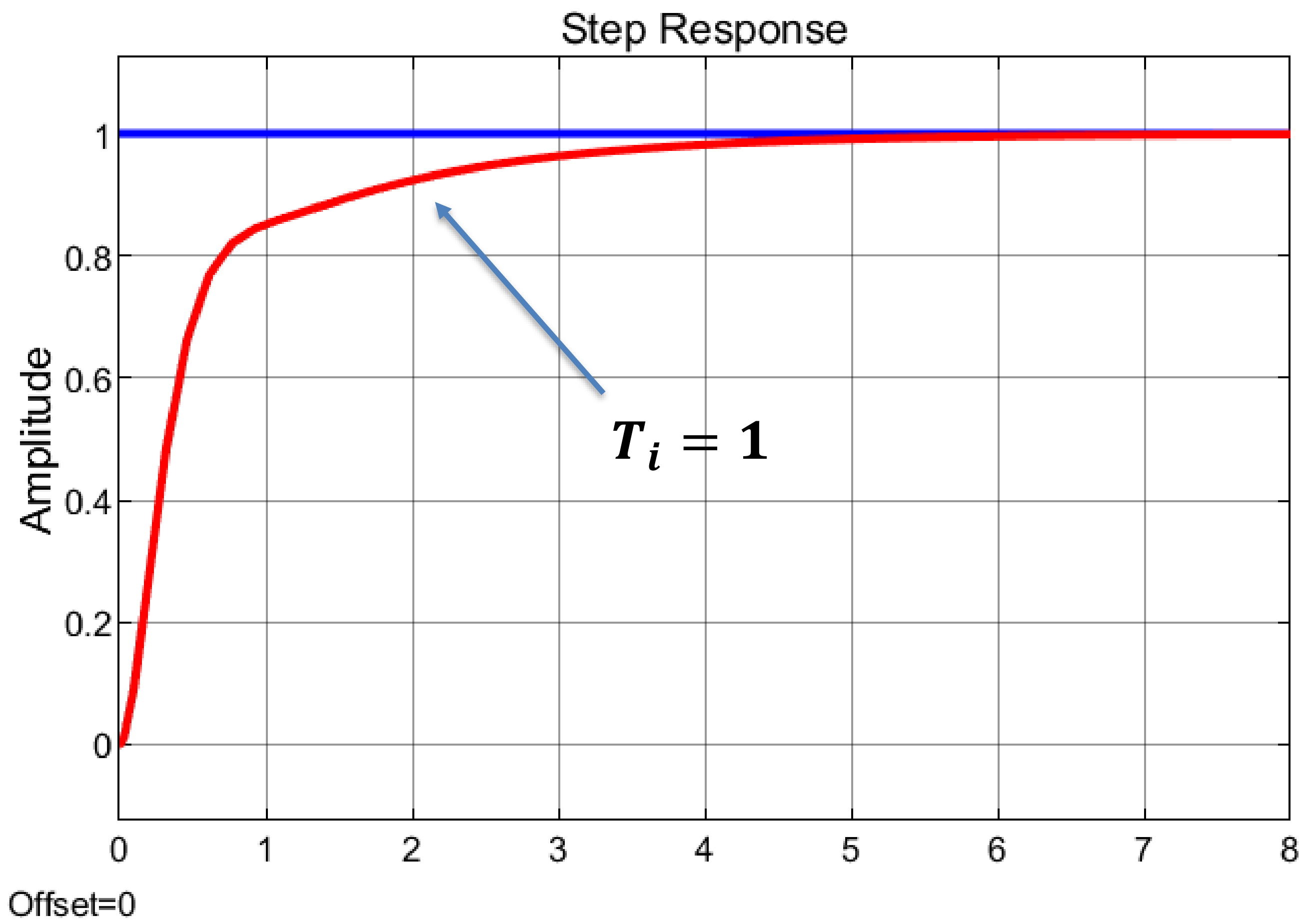
c) Design a PI controller to achieve a zero steady-state error.

$$\boldsymbol{G_c(s) = K_p\left(1 + \frac{1}{T_i s}\right)}$$

$$G_c(s) = 25.1\left(1 + \frac{1}{s}\right)$$

We can plot the unit-step response graph in **Simulink**.

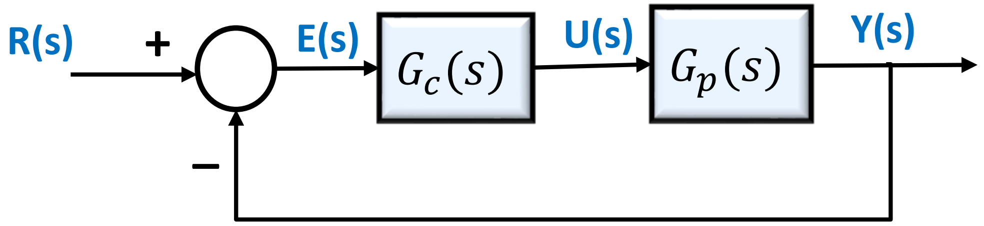




Proportional – Integral (PI) Controller

Example 2 Consider the transfer function model of a second-order dynamic system.

$$G_p(s) = \frac{1}{(s+2)(s+8)}$$

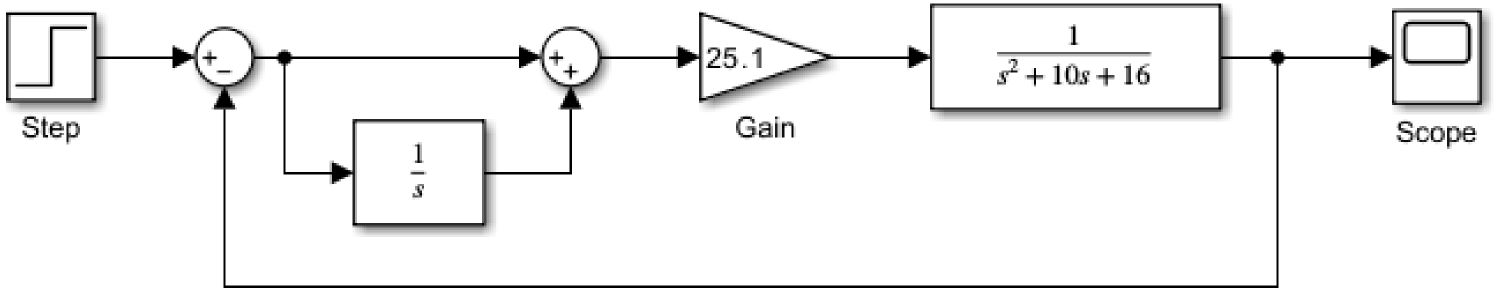


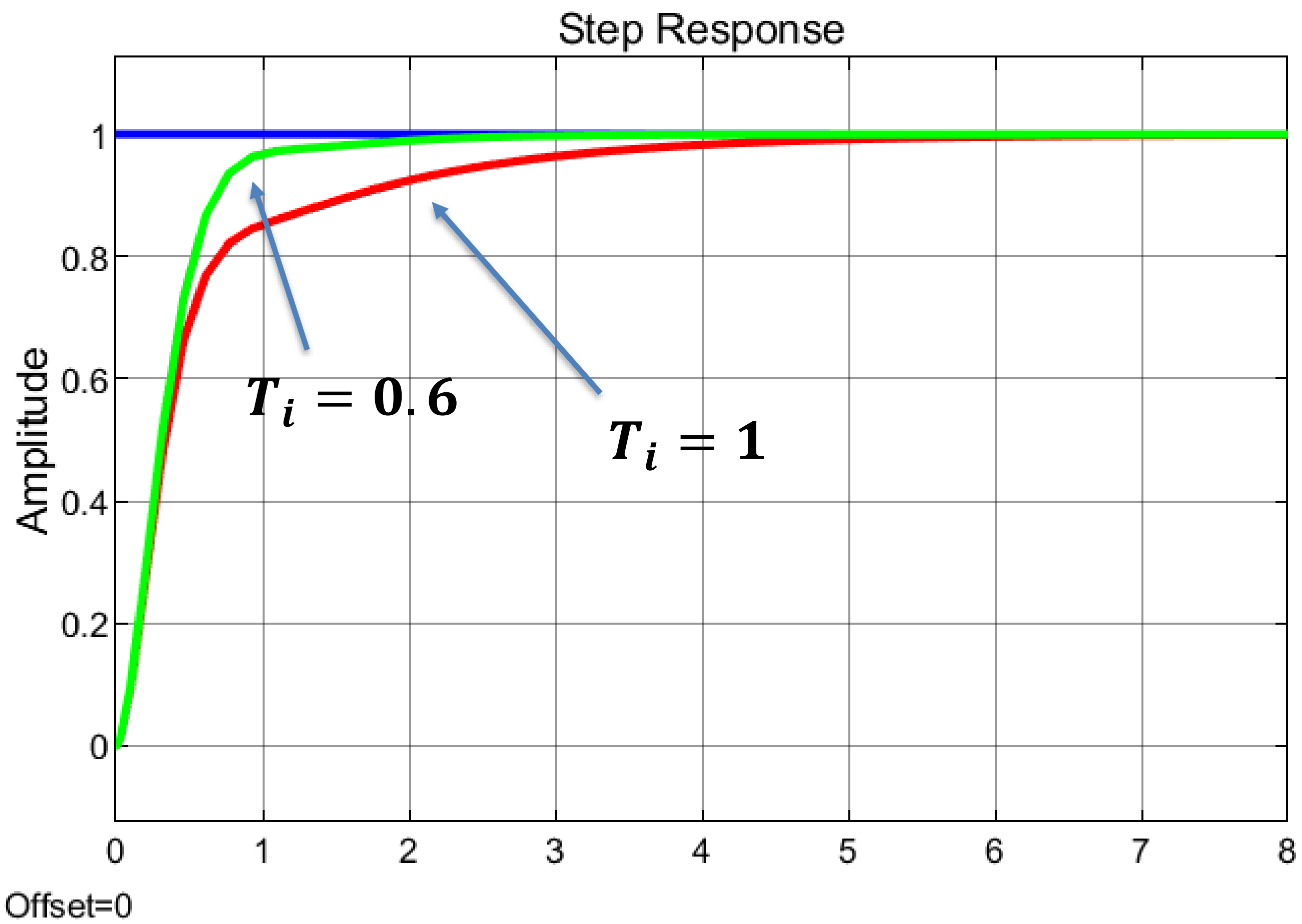
c) Design a PI controller to achieve a zero steady-state error.

$$\boldsymbol{G_c(s) = K_p\left(1 + \frac{1}{T_i s}\right)}$$

$$G_c(s) = 25.1\left(1 + \frac{1}{s}\right)$$

We can plot the unit-step response graph in **Simulink**.

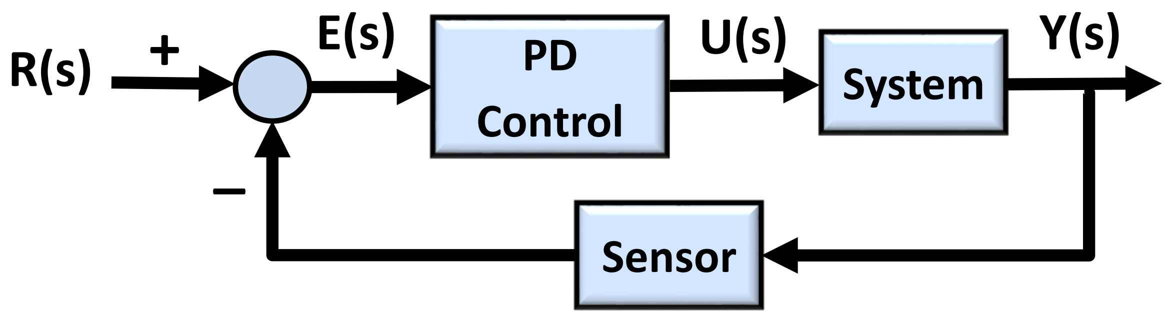




We can fine tune the T_i value to have a faster transient response.

For example, selecting $T_i = 0.6$ sec will decrease the settling time.

Proportional – Derivative (PD) Controller



- **PD Control** is utilized to increase the stability of the system and to enhance the transient response characteristics.
  - Improves stability, decreases settling time and overshoot
  - Speeds up slow systems without increasing the overshoot
  - Has no effect on the steady-state error

- In PD Control, the control signal u(t) includes two parts
  - One part is proportional to the error signal $e(t)$
  - The other part is proportional to the derivative or rate of change of the error signal

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt} \quad \rightarrow \quad U(s) = K_p E(s) + K_d s E(s) = (K_p + \underbrace{K_d}_{\text{Derivative Gain}} s) E(s) = K_p (1 + \underbrace{T_d}_{\text{Derivative Time Constant}} s) E(s)$$

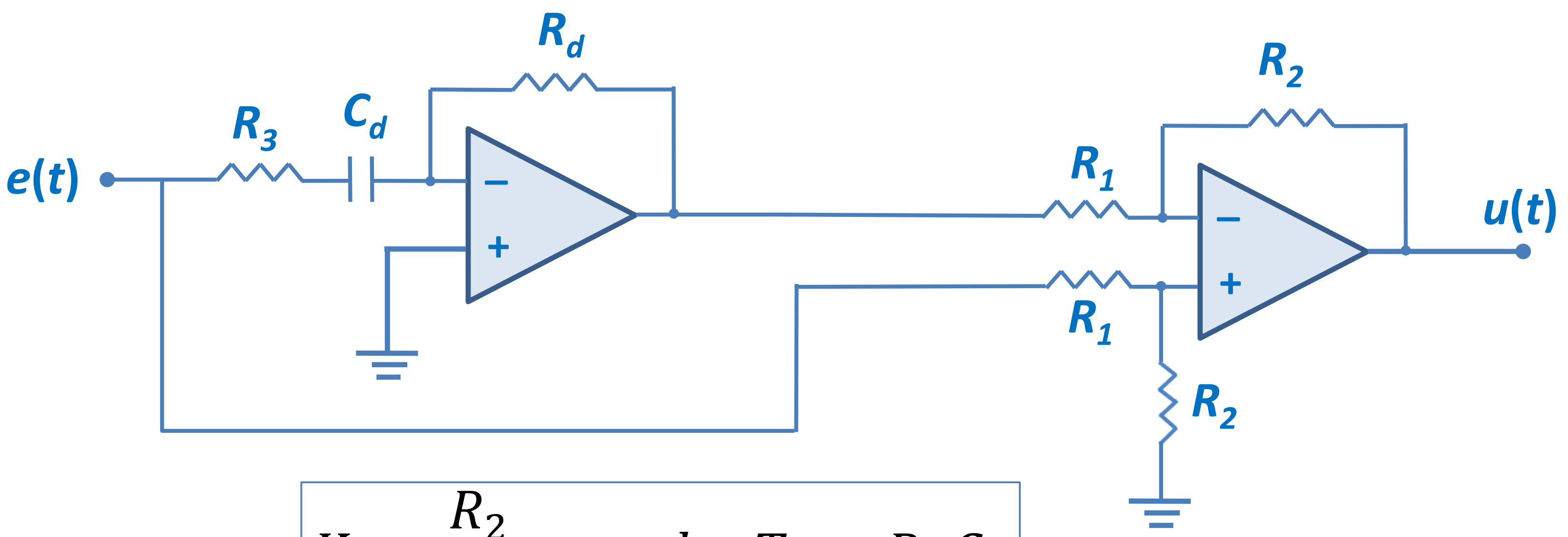
- The derivative time constant T_d represent how fast the derivative term reacts to avoid a large overshoot.
- The controller parameters are related as below,

$$\boxed{T_d = \frac{K_d}{K_p}}$$

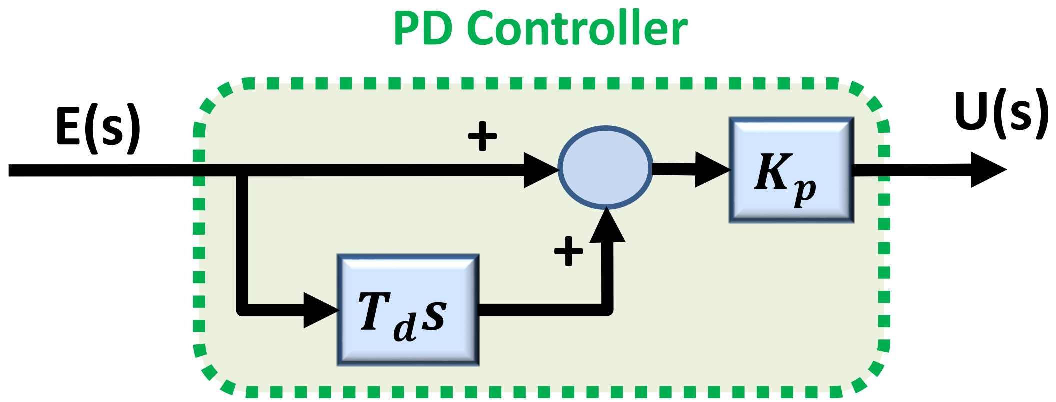
Proportional – Derivative (PD) Controller

- Analog PD controller can be realized by two OP-AMP, which provides independent adjustment of each control mode.

$$\boxed{U(s) = K_p(1 + T_d s)E(s)}$$



$$K_p = \frac{R_2}{R_1} \quad and \quad T_d = R_d C_d$$



- *For example, to implement a PD controller with $K_p = 4$ and $T_d = 0.5\ sec$ the components can be selected as:*

  $R_1 = 250\ k\Omega,\ R_2 = 1\ M\Omega$

  $R_d = 500\ k\Omega,\ \ C_d = 1\ \mu F$

  $R_3 = 0.1 R_d = 50\ k\Omega$

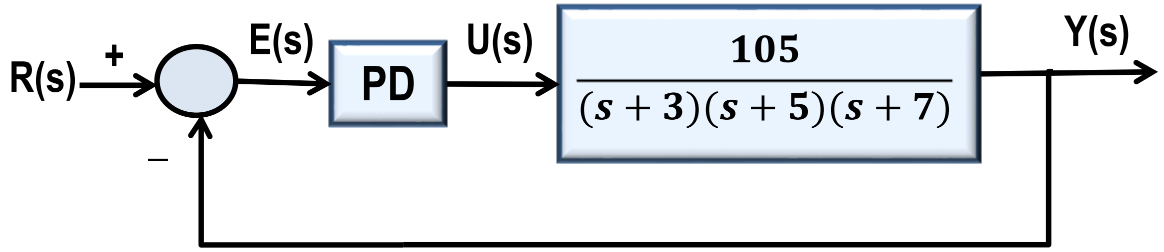
- Derivative control is unsuitable for systems that are exposed to noisy environments.
- Noisy signals contain high-frequency components that are amplified by the derivative term.
- In practical applications, a low pass filter is added to eliminate the high frequency noise.

Proportional – Derivative (PD) Controller

This example shows the effect of changing Derivative Time Constant T_d on the unit-step response of a high-order closed-loop system.

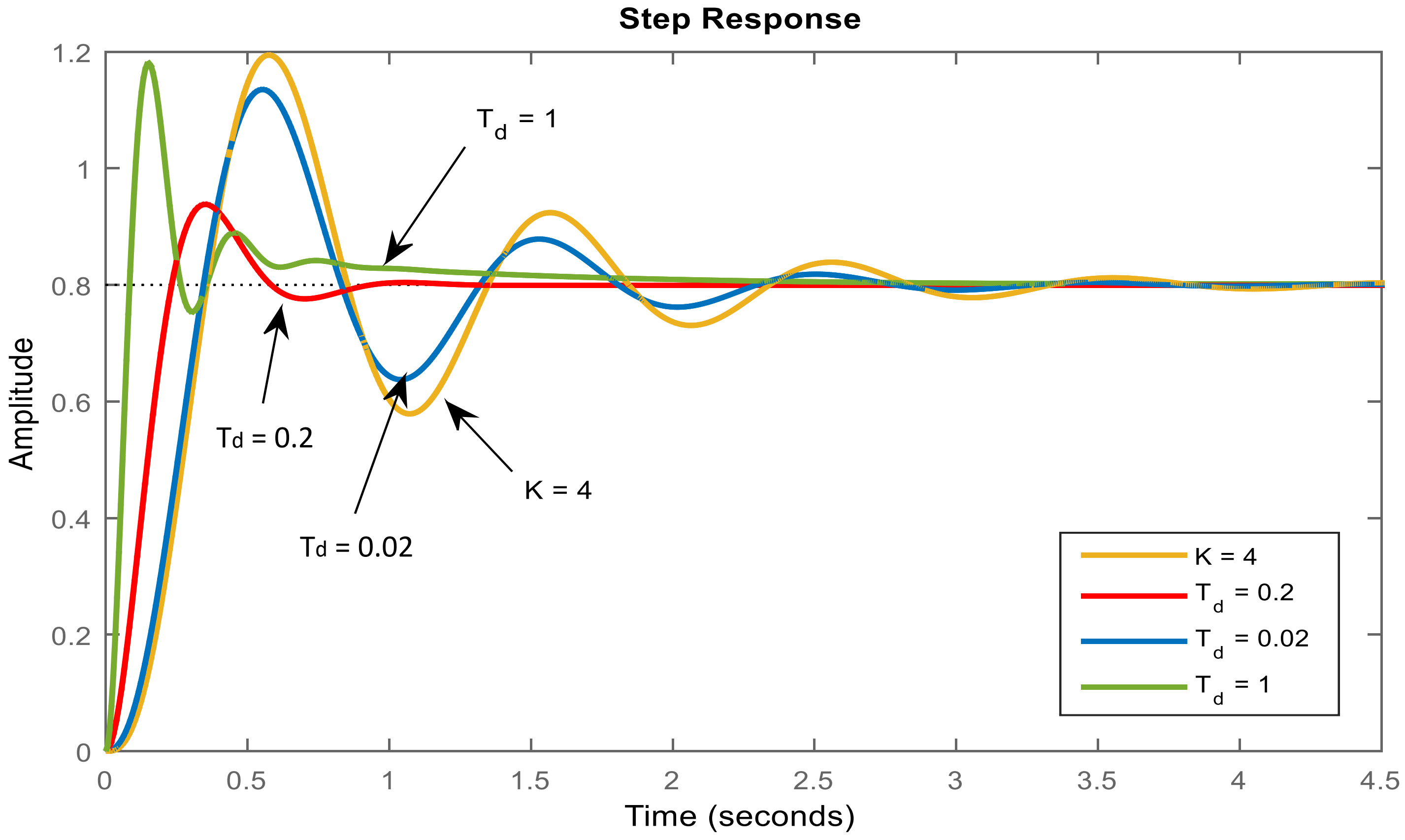
Following figure shows effect of selecting different values for T_d.

$\boxed{K_p = 4}$ $\boxed{T_d = 0.02, 0.2, 1}$



$$U(s) = K_p(1 + T_d s)E(s)$$

	Rise-time	Overshoot	Settling-time	S.S. Error
T_d = 0.02	0.216 sec	41.9%	2.59 sec	0.200
T_d = 0.2	0.158 sec	17.3%	0.798 sec	0.200
T_d = 1	0.057 sec	47.5%	1.52 sec	0.200
Only K_p = 4	0.212 sec	49.3%	3.18 sec	0.200



- Selecting an appropriate value for T_d improves stability and reduces the overshoot and settling time but has no effect on the steady-state error.

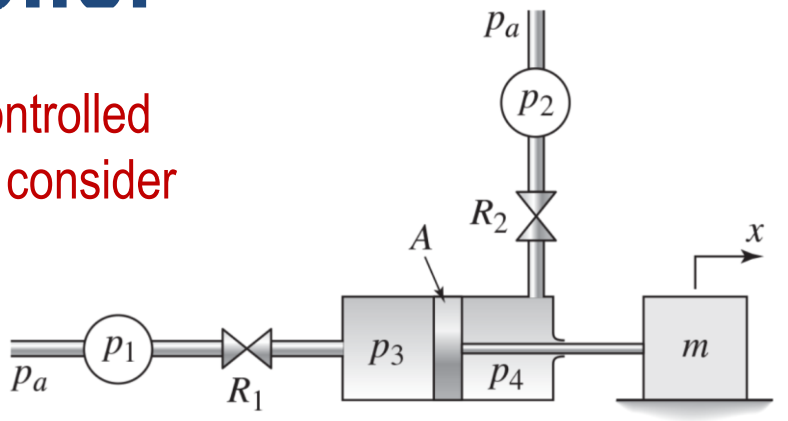
PD controller enhances the closed-loop stability

Proportional – Derivative (PD) Controller

Example 3

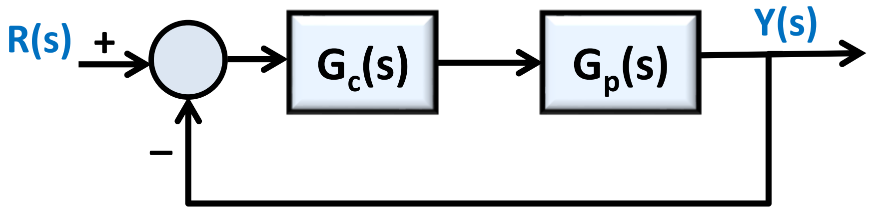
Consider the following pneumatic positioning system, where the displacement x is controlled by varying the pneumatic pressure p_1. Assume that the pressure p_2 is constant, and consider the following system model

$$G_p(s) = \frac{X(s)}{P_1(s)} = \frac{1}{s(s+2)}$$



Design a PD controller so that the unit-step response has a maximum overshoot of 5% and the peak time of $t_p = 1sec.$

$$G_c(s) = K_p(1 + T_d s)$$



Calculate the desired damping ratio from the given desired maximum overshoot of 5%

$$\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}} \rightarrow \zeta = \frac{-\ln(0.05)}{\sqrt{\pi^2 + \ln^2(0.05)}} \rightarrow \boxed{\zeta = 0.69}$$

Desired Damping Ratio

Next, calculate the undamped natural frequency from the given peak time of 1sec:

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \rightarrow 1 = \frac{\pi}{\omega_n\sqrt{1-(0.69)^2}} \rightarrow \boxed{\omega_n = 4.34 \text{ rad/sec}}$$

Desired Natural Frequency

Having the desired damping ratio and natural frequency, determine the desired characteristic equation for this closed-loop system

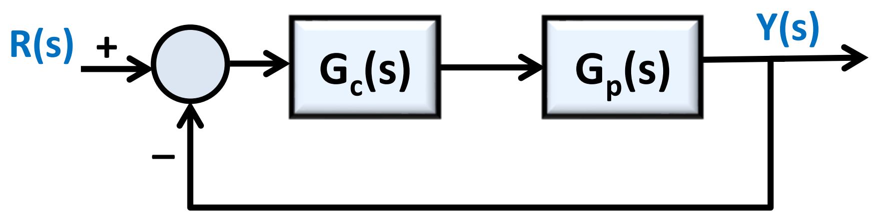
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = \boxed{s^2 + 5.99s + 18.84}$$

Desired Characteristic Equation

Proportional – Derivative (PD) Controller

Example 3

$$G_p(s) = \frac{1}{s(s+2)} \qquad G_c(s) = K_p(1 + T_d s)$$



Design a PD controller so that the unit-step response has a maximum overshoot of 5% and the peak time of $t_p = 1sec.$

Find the transfer function of the closed-loop system

$$\frac{\boldsymbol{Y(s)}}{\boldsymbol{R(s)}} = \frac{\boldsymbol{G_c(s)G_p(s)}}{\boldsymbol{1 + G_c(s)G_p(s)}} = \frac{K_p(1+T_d s)\dfrac{1}{s(s+2)}}{1 + K_p(1+T_d s)\dfrac{1}{s(s+2)}} = \frac{\dfrac{K_p(1+T_d s)}{s(s+2)}}{\dfrac{s(s+2) + K_p(1+T_d s)}{s(s+2)}} = \frac{K_p(1+T_d s)}{s^2 + (2 + K_p T_d)s + K_p}$$

Compare the desired characteristic equation with the characteristic equation of the closed-loop system

$$s^2 + 5.99s + 18.84 = s^2 + (2 + K_p T_d)s + K_p$$

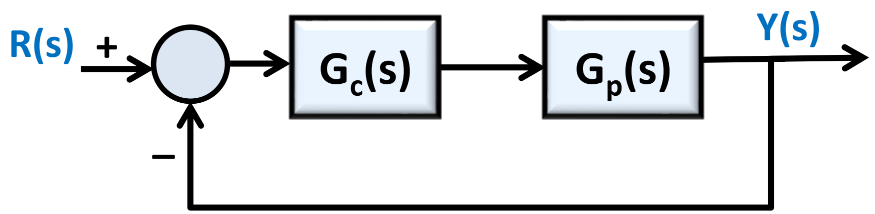
$$\begin{cases} 2 + K_p T_d = 5.99 \\ K_p = 18.84 \end{cases} \rightarrow \boxed{\boldsymbol{K_p = 18.84}}\ , \quad \boxed{\boldsymbol{T_d = 0.21}}$$

Therefore, the designed PD Controller is $\rightarrow$ $\boxed{\boldsymbol{G_c(s) = 18.84(1 + 0.21s)}}$

Proportional – Derivative (PD) Controller

Example 3

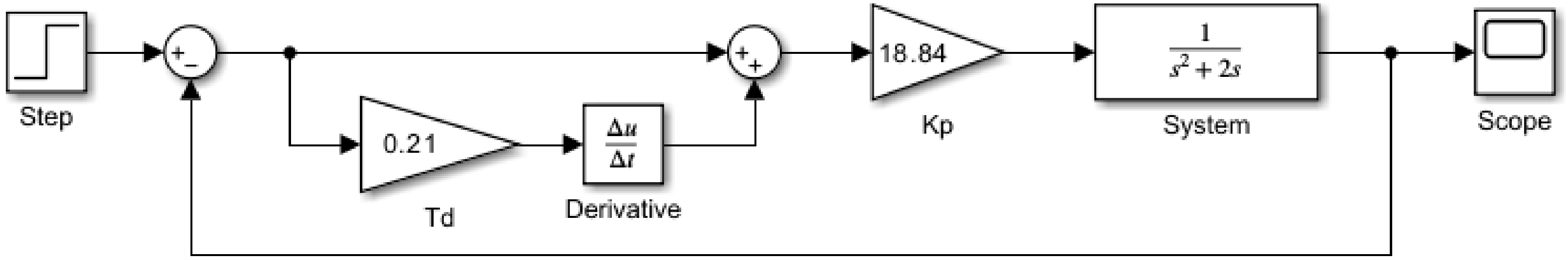
$$G_p(s) = \frac{1}{s(s+2)} \qquad G_c(s) = K_p(1 + T_d s)$$

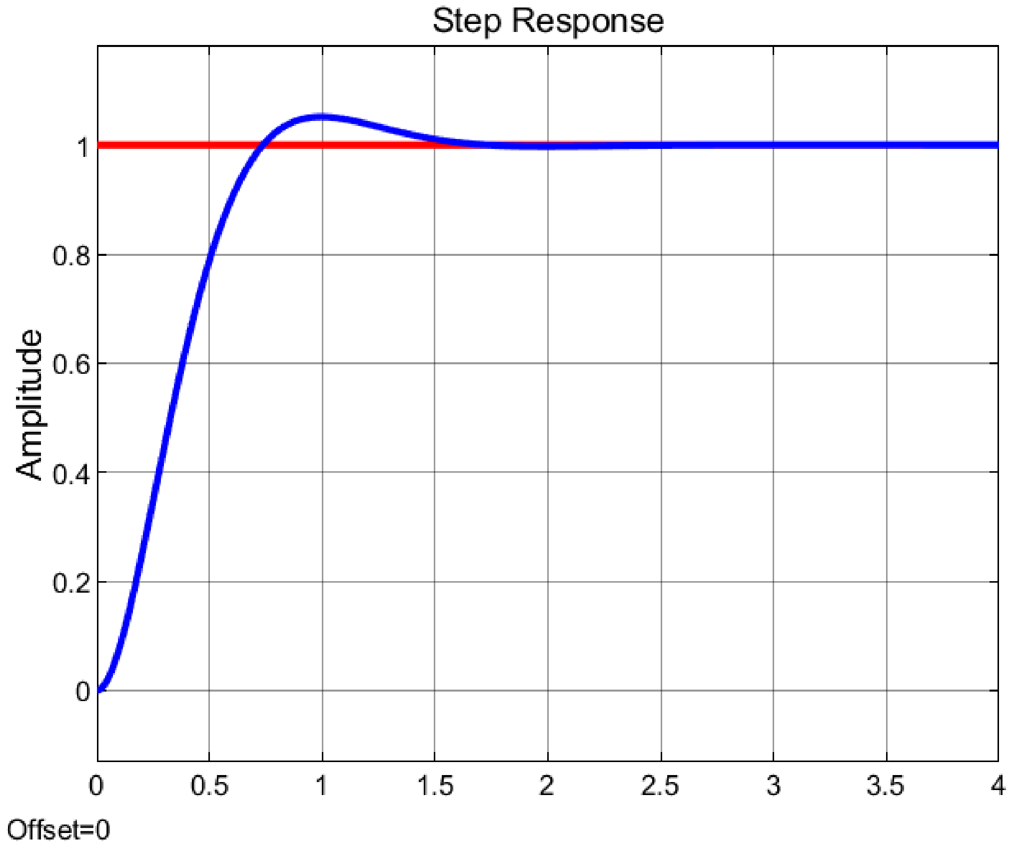


Design a PD controller so that the unit-step response has a maximum overshoot of 5% and the peak time of $t_p = 1sec$.

$$\boxed{G_c(s) = 18.84(1 + 0.21s)}$$

We can plot the unit-step response graph in **Simulink**.

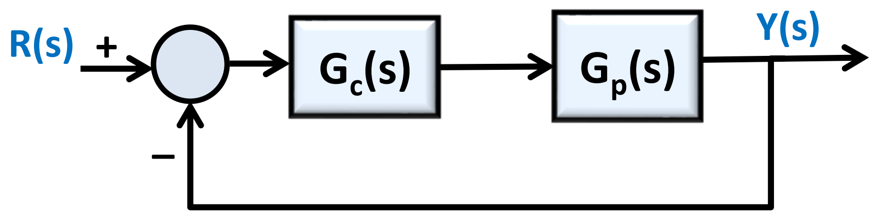




Proportional – Derivative (PD) Controller

Example 4

Consider the second-order system transfer function,



$$G_p(s) = \frac{10}{s^2 + 6s + 8}$$

a) Determine a proportional controller gain K_p to have a 2% steady-state error for unit-step input.

$$G_c(s) = K_p$$

The steady-state error for a unit-step response is obtained as:

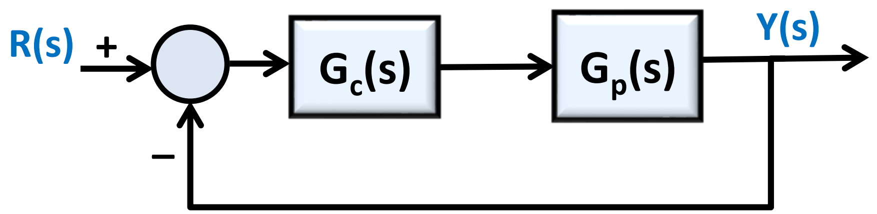
$$k_p = \lim_{s \to 0} G(s) = \lim_{s \to 0}\left(\frac{10K_p}{s^2 + 6s + 8}\right) = \frac{5K_p}{4}$$

$$e_{ss} = \frac{1}{1 + k_p} \quad \rightarrow \quad 0.02 = \frac{1}{1 + \frac{5K_p}{4}} \quad \rightarrow \quad \boxed{K_p = 39.2}$$ **Desired proportional gain**

Proportional – Derivative (PD) Controller

Example 4 Consider the second-order system transfer function,

$$G_p(s) = \frac{10}{s^2 + 6s + 8}$$



b) Calculate the percentage of overshoot for the designed proportional controller

$$G_c(s) = K_p$$

Find the transfer function of the closed-loop system

$$\frac{Y(s)}{R(s)} = \frac{K_p G_p(s)}{1 + K_p G_p(s)} = \frac{\frac{392}{s^2 + 6s + 8}}{1 + \frac{392}{s^2 + 6s + 8}} = \frac{392}{s^2 + 6s + 400}$$

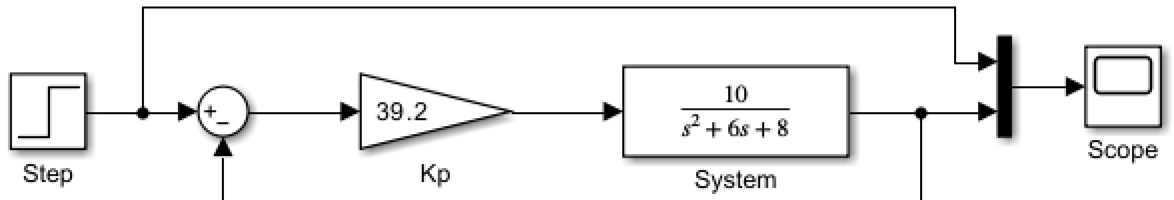
Find the damping ratio of the closed-loop system:

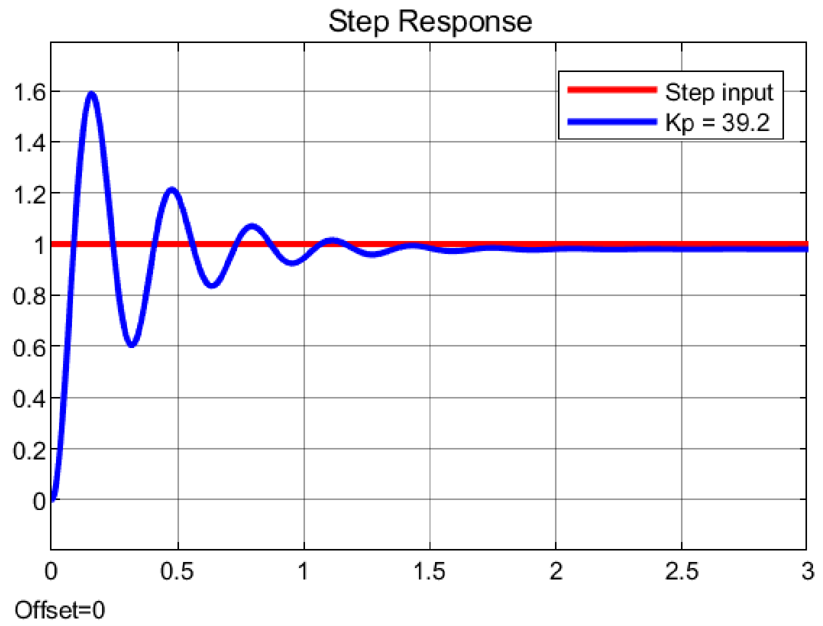
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 6s + 400 \rightarrow \begin{cases} \omega_n^2 = 400 & \rightarrow \quad \omega_n = 20 \\ 2\zeta\omega_n = 6 & \rightarrow \quad \zeta = \dfrac{6}{2\omega_n} = 0.15 \end{cases}$$

The percentage of overshoot is:

$$O.S.\% = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\% \quad \rightarrow \quad \boxed{O.S.\% = 62.1\%}$$

We can plot the unit-step response graph in **Simulink**.



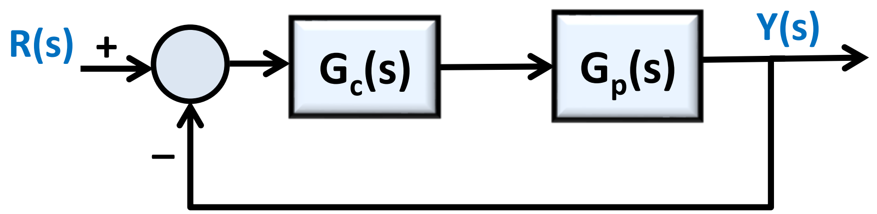


Proportional – Derivative (PD) Controller

Example 4

Consider the second-order system transfer function,

$$G_p(s) = \frac{10}{s^2 + 6s + 8}$$



c) Design a PD controller to decrease the overshoot to 5% without changing the steady-state error.

$$G_c(s) = K_p(1 + T_d s)$$

Calculate the required damping ratio to have a 5% overshoot:

$$\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}} \rightarrow \zeta = \frac{-\ln(0.05)}{\sqrt{\pi^2 + \ln^2(0.05)}} \rightarrow \boxed{\zeta = 0.69}$$

Desired Damping Ratio

Find the transfer function of the closed-loop system

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} = \frac{K_p(1 + T_d s)\frac{10}{s^2 + 6s + 8}}{1 + K_p(1 + T_d s)\frac{10}{s^2 + 6s + 8}} = \frac{10K_p(1 + T_d s)}{s^2 + (6 + 10K_pT_d)s + 8 + 10K_p}$$

Set $K_p = 39.2$ to keep the steady-state error of 2%.

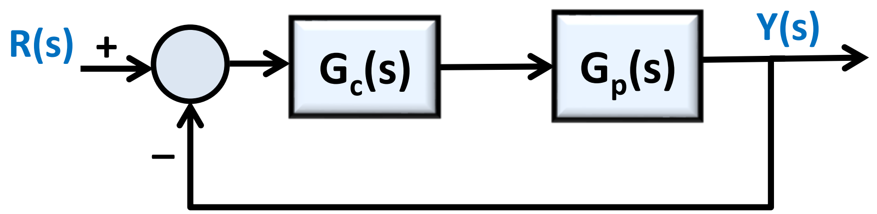
$$\frac{Y(s)}{R(s)} = \frac{392\,(1 + T_d s)}{s^2 + (6 + 392T_d)s + 400}$$

Proportional – Derivative (PD) Controller

Example 4

Consider the second-order system transfer function,

$$G_p(s) = \frac{10}{s^2 + 6s + 8}$$



c) Design a PD controller to decrease the overshoot to 5% without changing the steady-state error.

$$G_c(s) = K_p(1 + T_d s)$$

Determine the required T_d to have damping ratio of $\zeta = 0.69$

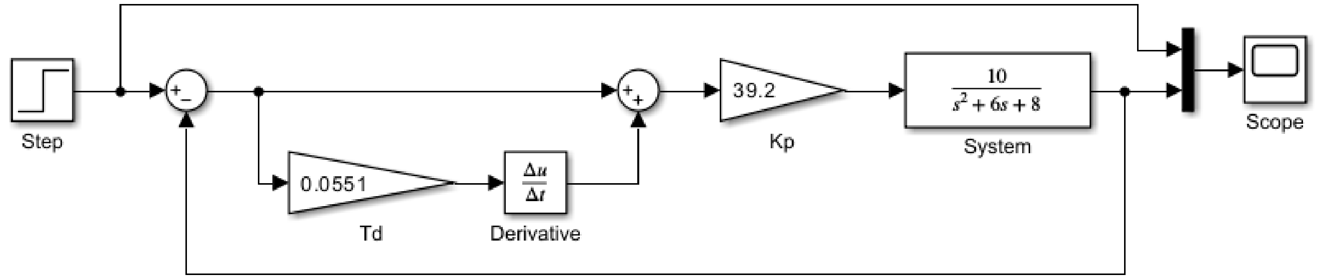
$$\frac{Y(s)}{R(s)} = \frac{392\,(1 + T_d s)}{s^2 + (6 + 392T_d)s + 400}$$

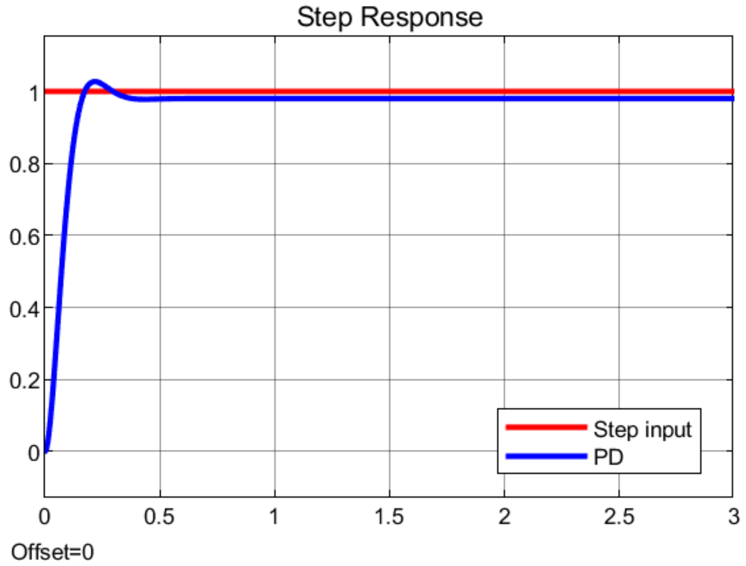
$$\begin{cases} \omega_n^2 = 400 \quad \rightarrow \quad \omega_n = 20 \\ 2\zeta\omega_n = 6 + 392T_d \quad \rightarrow \quad T_d = \dfrac{2\zeta\omega_n - 6}{392} = \dfrac{2(0.69)(20) - 6}{392} = 0.0551 \end{cases}$$

$$\boxed{T_d = 0.0551}$$

Therefore, the designed PD Controller is $\rightarrow$ $\boxed{G_c(s) = 39.2(1 + 0.0551s)}$

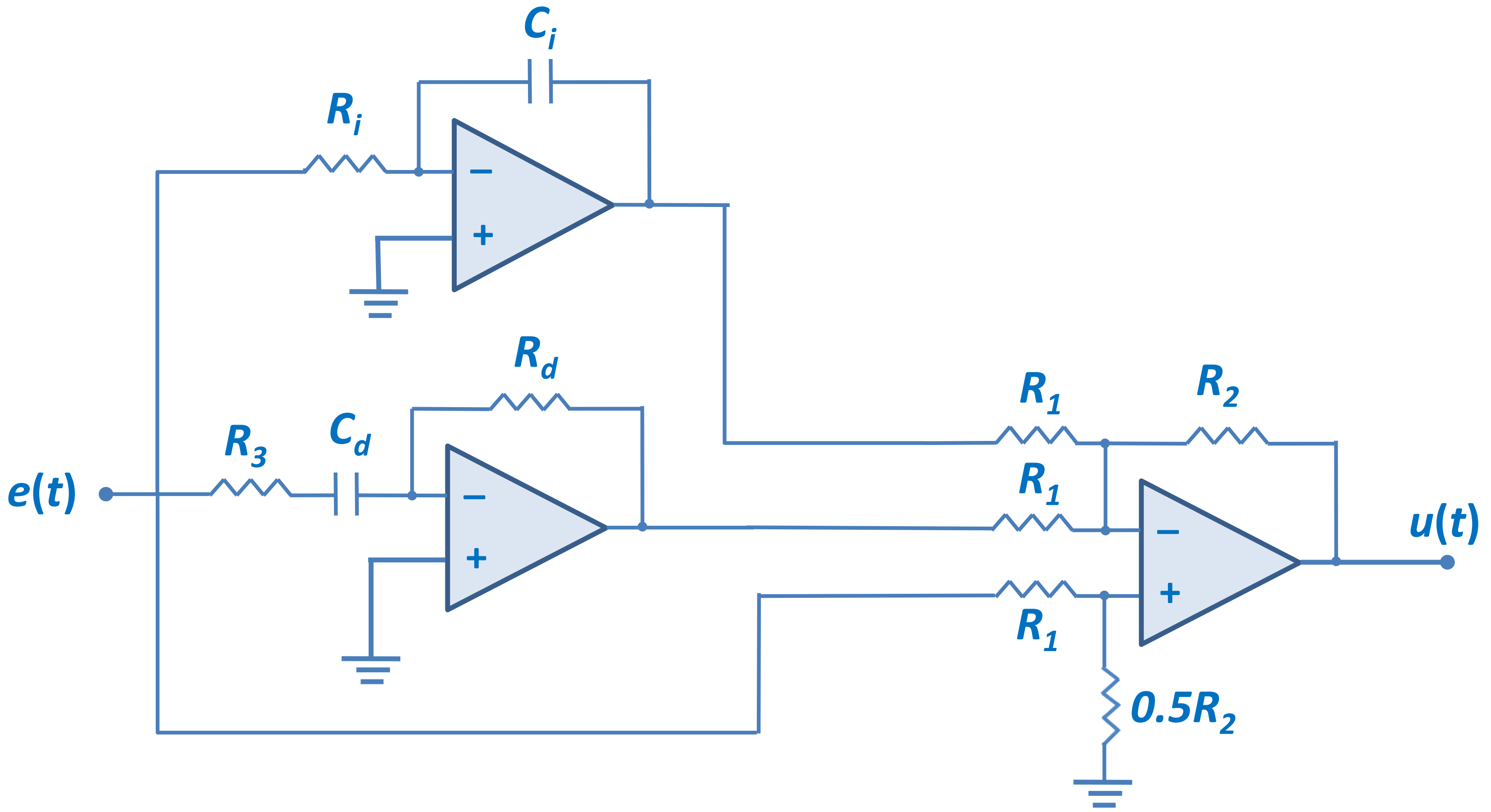
We can plot the unit-step response graph in **Simulink**.

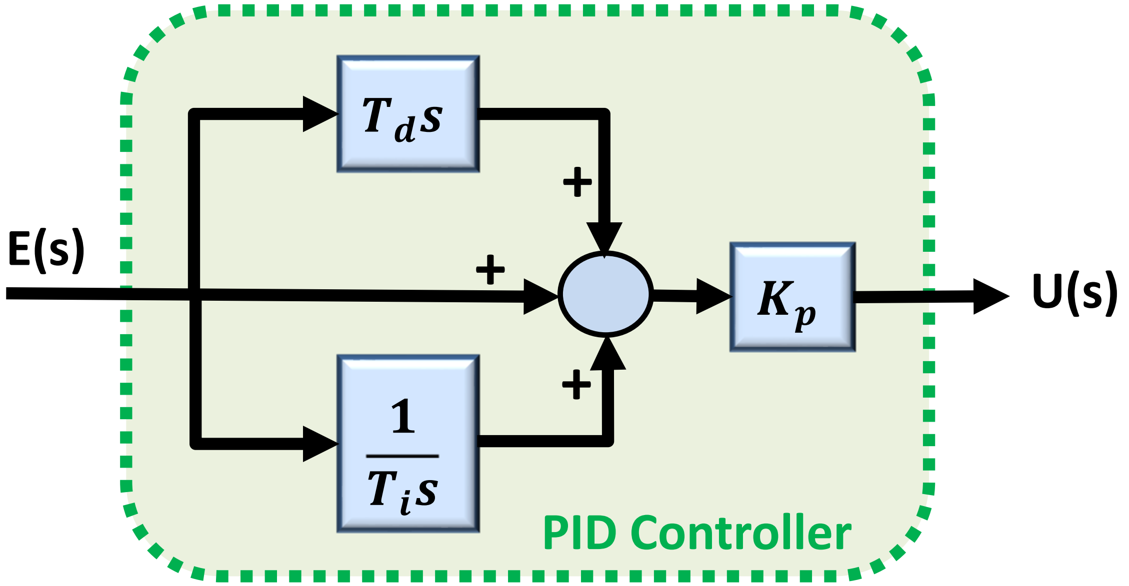




PID Controller

- **PID Controller** is obtained by combining all three modes of control (proportional, integral and derivative) that enables a controller to be produced which has no steady state error and reduces the tendency for oscillation.
- Analog PID controller can be realized by three OP-AMP, which provides independent adjustment of each control mode.





$$U(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)E(s)$$

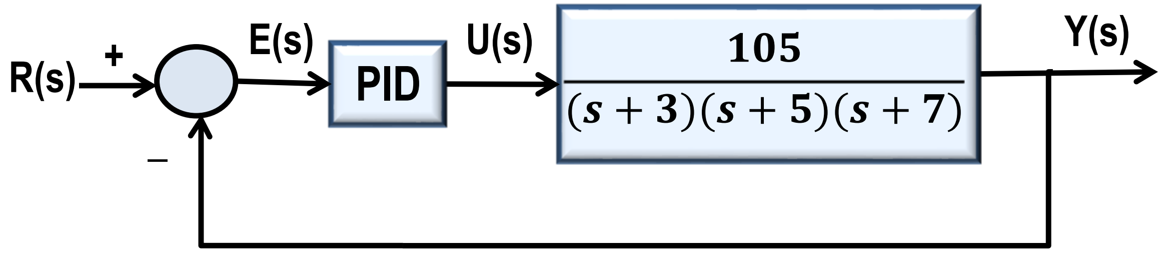
$$U(s) = \left(K_p + \frac{K_i}{s} + K_d s\right)E(s)$$

$$K_p = \frac{R_2}{R_1} \quad and \quad T_i = R_i C_i \quad and \quad T_d = R_d C_d \quad and \quad R_3 = 0.1 R_d$$

PID Controller

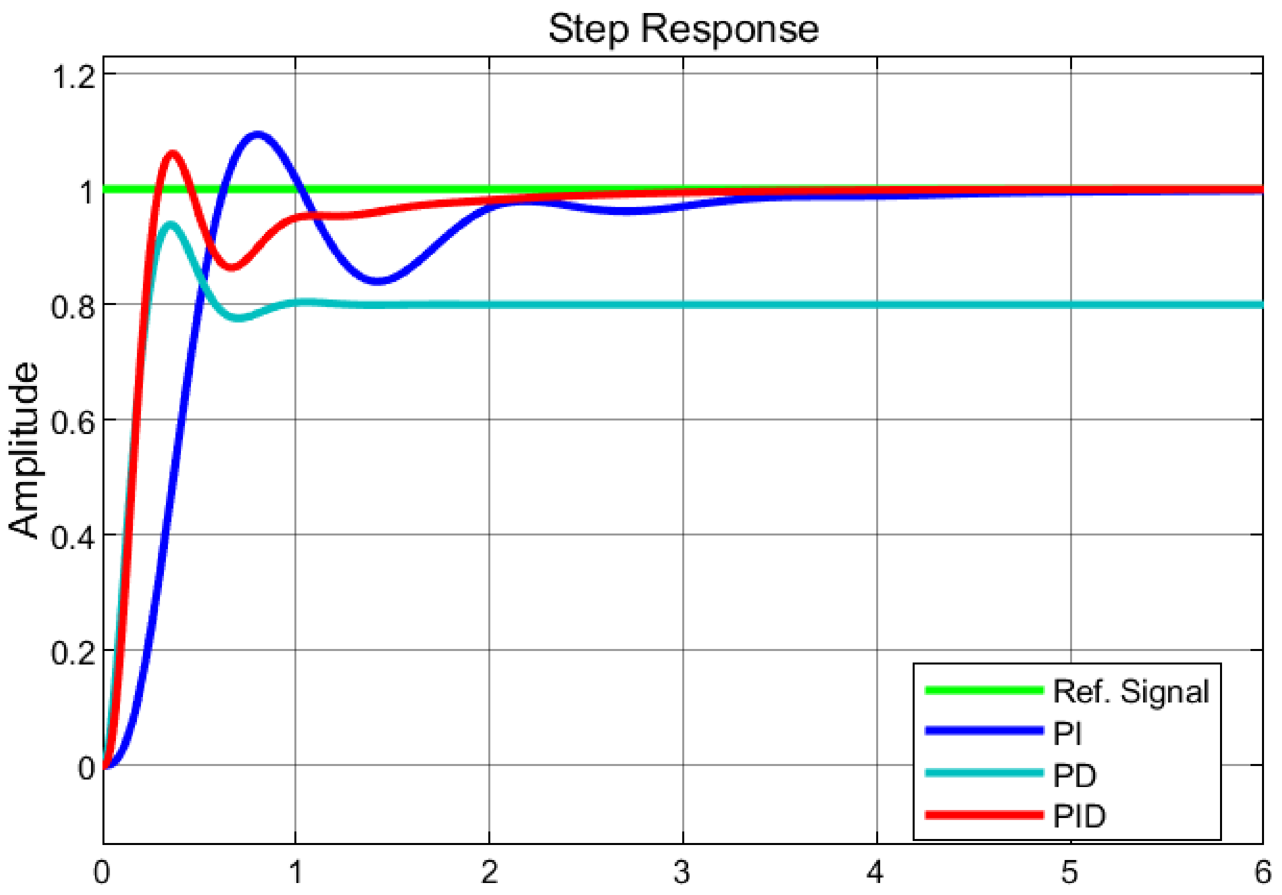
The following graph compares of the PID, PD and PI controllers.

- PID controller parameters are selected based on the previous examples of PD and PI controller.



$$U(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)E(s)$$

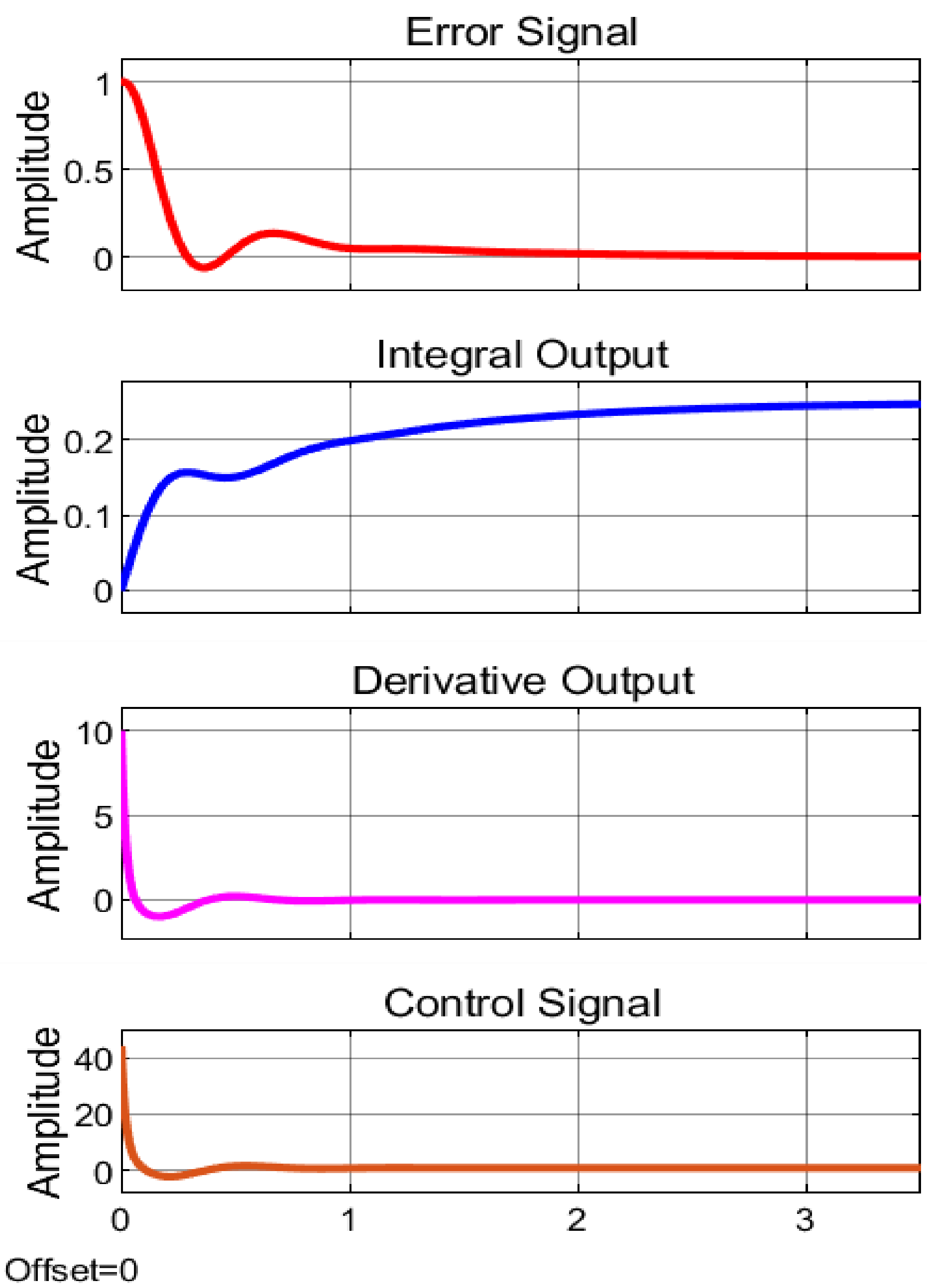
PID controller improves the transient response and eliminates the steady-state error.

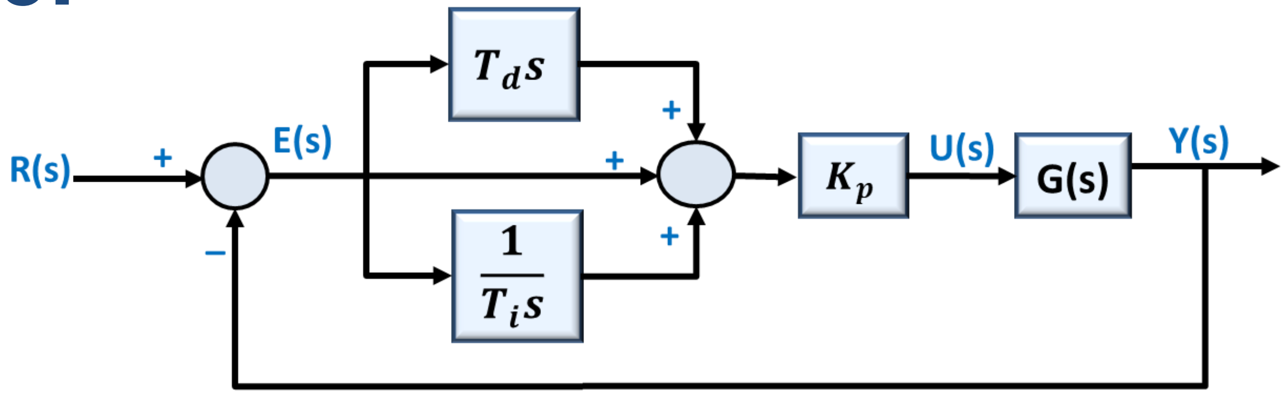


- Each of the PI and PD parts are designed separately, and then they are combined to obtain the PID controller.
- A fine tune may be required to achieve the desired performance criterion.
  - PI controller $\rightarrow$ $K = 2$ $T_i = 1$
  - PD controller $\rightarrow$ $K = 4$ $T_d = 0.2$
  - PID controller $\rightarrow$ $K = 4$ $T_i = 1$ $T_d = 0.2$

PID Controller

Following graph shows the Control signal, Error signal and output of the integral and derivative terms.



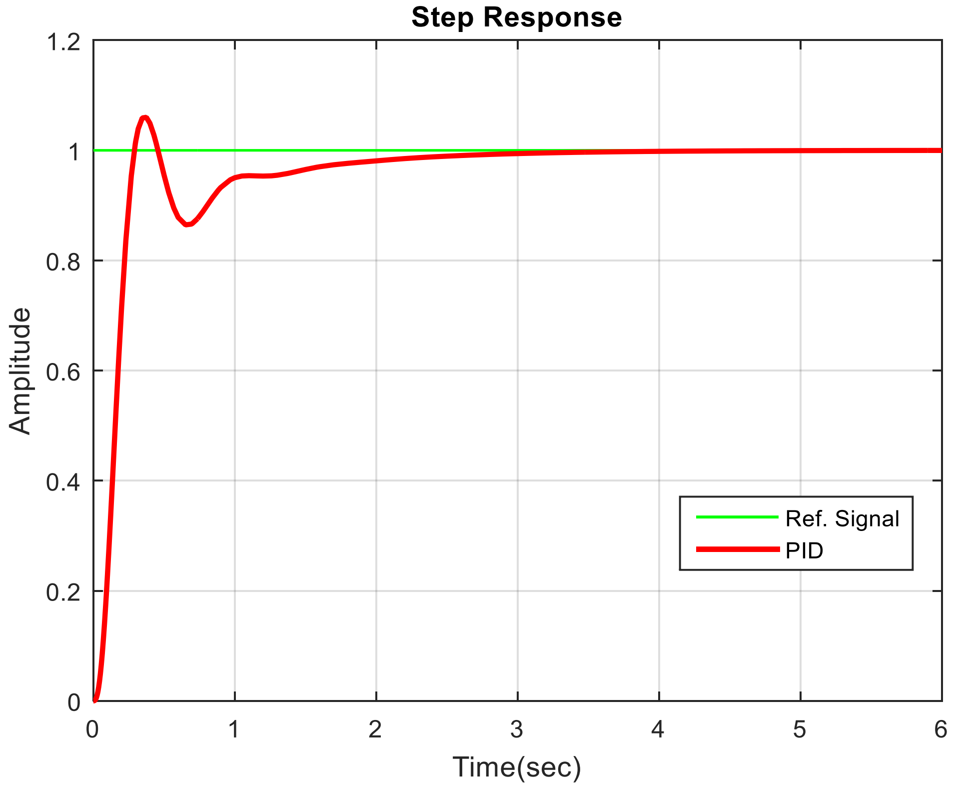


$$U(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)E(s)$$

- Error signal is simply $e(t) = r(t) - y(t)$. It will be zero whenever the output signal y(t) is equal to the reference signal r(t).

- The integral output at any time instant is the area under the error signal $e(t)$ curve up to that instant. It can have a nonzero value when the error signal $e(t)$ is zero.

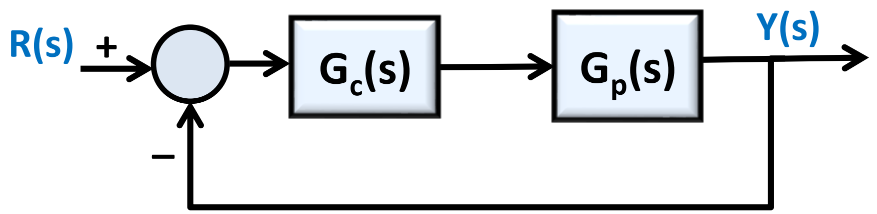
- The derivative output is proportional to the rate of change of the error signal $e(t)$ to initiates an early correction to avoid a large overshoot. It is zero when the error signal e(t) has no change.



PID Controller

Example 5 Consider the second-order system transfer function,

$$G_p(s) = \frac{1}{8s^2 + 7}$$



a) Determine range of the PD controller parameters to have a stable closed-loop system.

$$G_c(s) = K_p(1 + T_d s)$$

b) Design a PD controller so that the unit-step response is critically-damped, which is the fastest response with no overshoot and no oscillation.

$$G_c(s) = K_p(1 + T_d s)$$

c) Design an integral control action to eliminate the steady-state error.

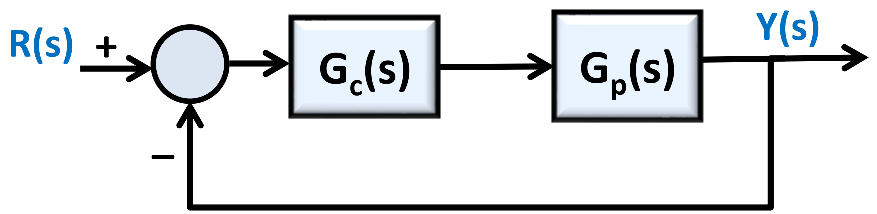
$$G_c(s) = K_p\left(1 + T_d s + \frac{1}{T_i s}\right)$$

PID Controller

Example 5

Consider the second-order system transfer function,

$$G_p(s) = \frac{1}{8s^2 + 7}$$



a) Determine range of the PD controller parameters to have a stable closed-loop system.

$$G_c(s) = K_p(1 + T_d s)$$

First, find the transfer function of the closed-loop system

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} = \frac{K_p(1+T_d s)\frac{1}{8s^2+7}}{1 + K_p(1+T_d s)\frac{1}{8s^2+7}} = \frac{\frac{K_p(1+T_d s)}{8s^2+7}}{\frac{8s^2+7+K_p(1+T_d s)}{8s^2+7}} = \frac{K_p(1+T_d s)}{8s^2 + K_p T_d s + K_p + 7}$$

Create the Routh-Hurwitz table for the characteristic equation.

s^2	8	$K_p + 7$
s^1	$K_p T_d$	0
s^0	$K_p + 7$	0

For stability, all terms in the first column must be positive:

$$K_p T_d > 0 \quad \rightarrow \quad T_d > 0, \qquad K_p > 0$$

$$K_p + 7 > 0 \quad \rightarrow \quad K_p > -7$$

$$\boxed{K_P > 0, \qquad T_d > 0}$$

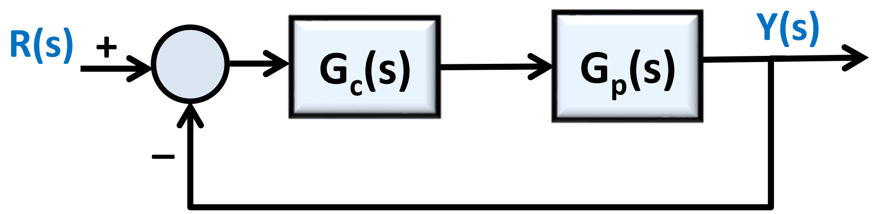
Stability Condition

PID Controller

Example 5

Consider the second-order system transfer function,

$$G_p(s) = \frac{1}{8s^2 + 7}$$



b) Design a PD controller so that the unit-step response is critically-damped, which is the fastest response with no overshoot and no oscillation.

$$G_c(s) = K_p(1 + T_d s)$$

To have a critically-damped the damping ratio must be $\boldsymbol{\zeta = 1}$.

Match the characteristic equation of the closed-loop system with the standard form:

The characteristic equation of the closed-loop system is $\rightarrow\ 8s^2 + K_pT_ds + K_p + 7 = 0$

$$\boldsymbol{s^2 + 2\zeta\omega_n s + \omega_n^2} = s^2 + \frac{K_pT_d}{8}s + \frac{K_p + 7}{8} \rightarrow \begin{cases} 2\zeta\omega_n = \dfrac{K_pT_d}{8} \rightarrow \omega_n = \dfrac{K_pT_d}{16} & Eqn.(1) \\ \omega_n^2 = \dfrac{K_p + 7}{8} \rightarrow \omega_n = \sqrt{\dfrac{K_p + 7}{8}} & Eqn.(2) \end{cases}$$

Substitute the ω_n from Eqn. (2) into Eqn. (1) to find a relationship between the controller parameters:

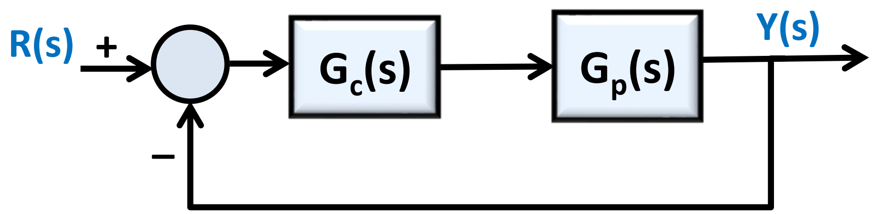
$$\omega_n = \frac{K_pT_d}{16} \rightarrow \sqrt{\frac{K_p + 7}{8}} = \frac{K_pT_d}{16} \rightarrow \frac{\sqrt{K_p + 7}}{2\sqrt{2}} = \frac{K_pT_d}{16} \rightarrow \boxed{\boldsymbol{T_d = \frac{8\sqrt{K_p + 7}}{K_p\sqrt{2}}}}$$

Any combination of positive non-zero values for K_p and T_d that satisfied the above relationship is acceptable.

PID Controller

Example 5 Consider the second-order system transfer function,

$$G_p(s) = \frac{1}{8s^2 + 7}$$



b) Design a PD controller so that the unit-step response is critically-damped, which is the fastest response with no overshoot and no oscillation.

$$G_c(s) = K_p(1 + T_d s)$$

For example, we can select the controller parameters as below:

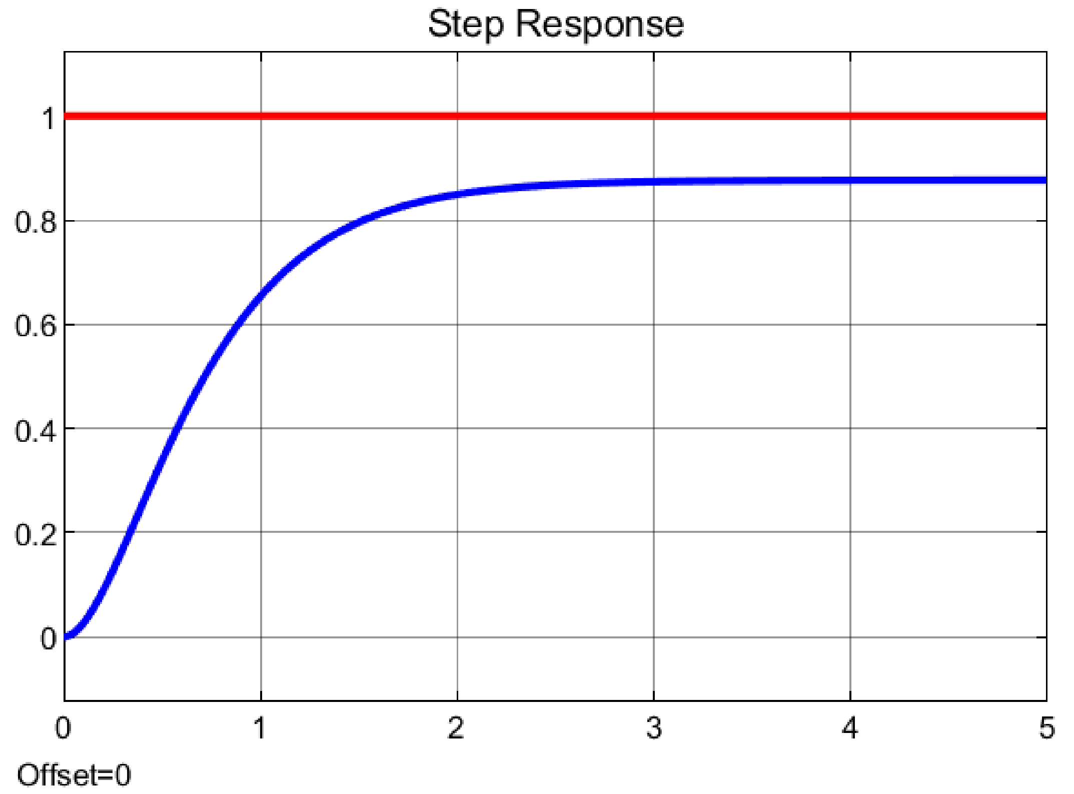
$$\boxed{K_p = 50} \longrightarrow T_d = \frac{8\sqrt{K_p + 7}}{K_p\sqrt{2}} = \frac{8\sqrt{57}}{50\sqrt{2}} \longrightarrow \boxed{T_d = 0.8542}$$

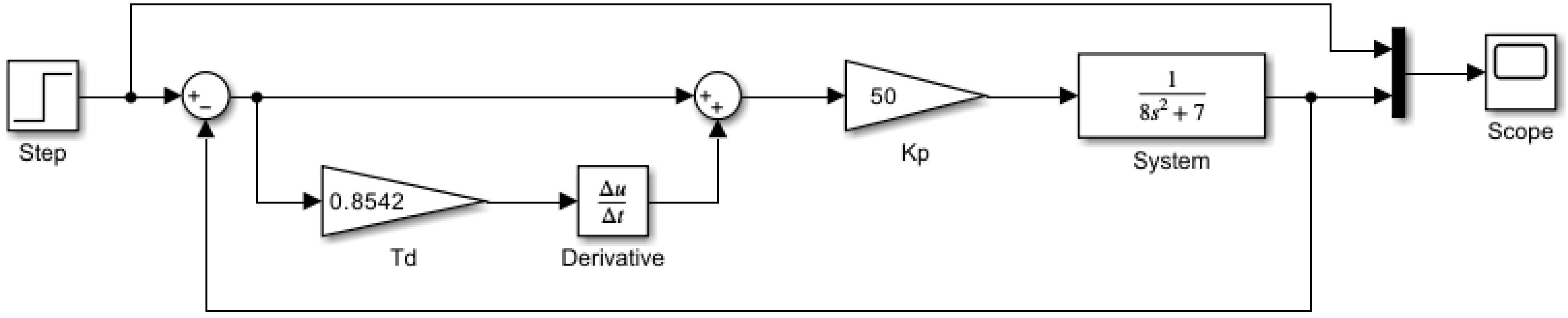
Therefore, the designed PD Controller is $\rightarrow$ $\boxed{G_c(s) = 50(1 + 0.8542s)}$

We can plot the unit-step response graph in **Simulink**.

The graph shows a critically-damped response. No overshoot.

However, the step response has a steady-state error.



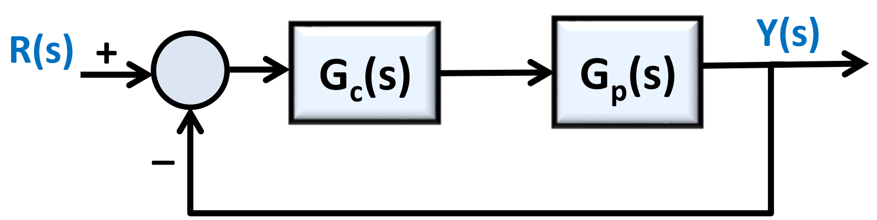


PID Controller

Example 5

Consider the second-order system transfer function,

$$G_p(s) = \frac{1}{8s^2 + 7}$$



c) Design an integral control action to eliminate the steady-state error.

$$G_c(s) = K_p\left(1 + T_d s + \frac{1}{T_i s}\right)$$

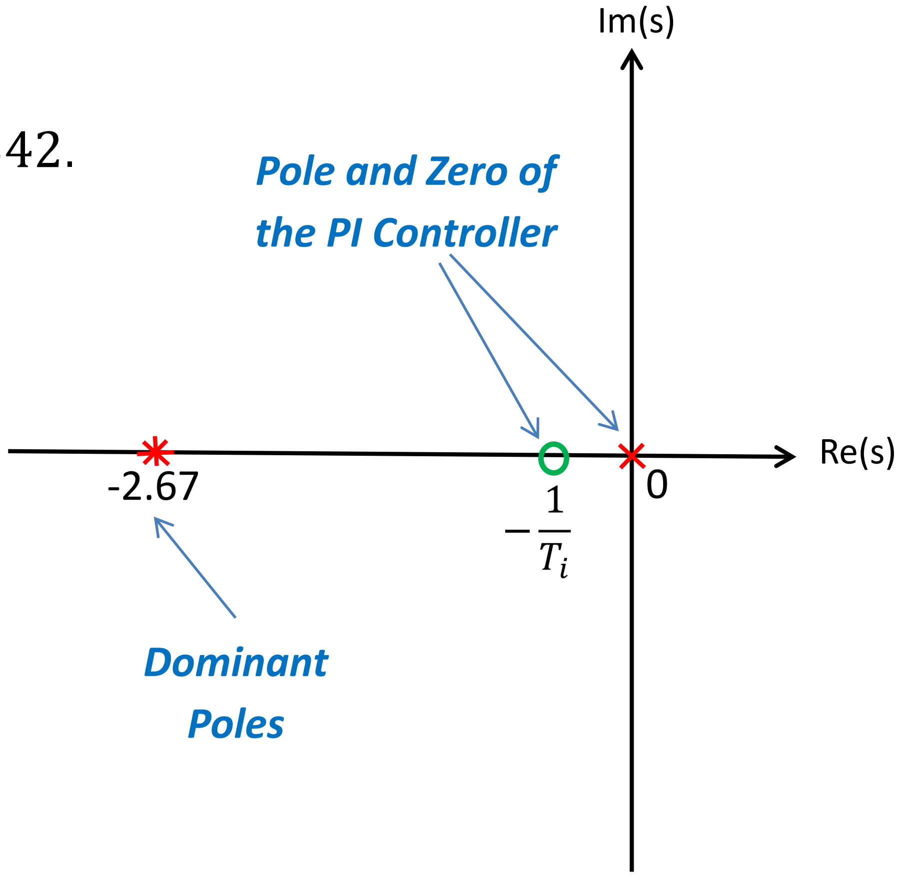
First, find the dominant poles of the closed-loop system with PD controller for $K_p = 50$ and $T_d = 0.8542$.

$$\frac{Y(s)}{R(s)} = \frac{K_p(1 + T_d s)}{8s^2 + K_p T_d s + K_p + 7} = \frac{50(1 + 0.8542s)}{8s^2 + 42.7083s + 57} \rightarrow \text{Poles:} \quad s_1 = s_2 = -2.6693$$

The second-order closed-loop transfer function has two repetitive real stable poles.

The integral time constant T_i can be selected by the following stability consideration, where p_{cl} represent the closed-loop pole under the PD control.

$$\boldsymbol{T_i \geq \frac{2}{|Re\{p_{cl}\}|}} \quad \rightarrow \quad T_i \geq \frac{2}{2.6693} = 0.7493\ sec$$



PID Controller

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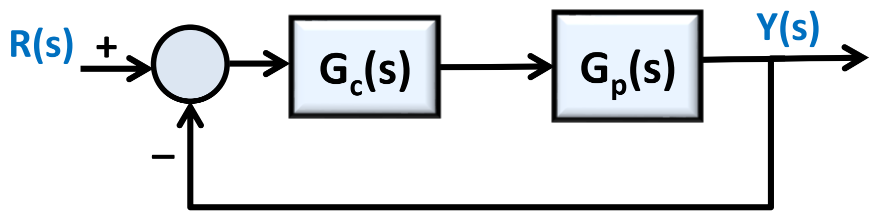
$$G_p(s) = \frac{1}{8s^2 + 7}$$



c) Design an integral control action to eliminate the steady-state error.

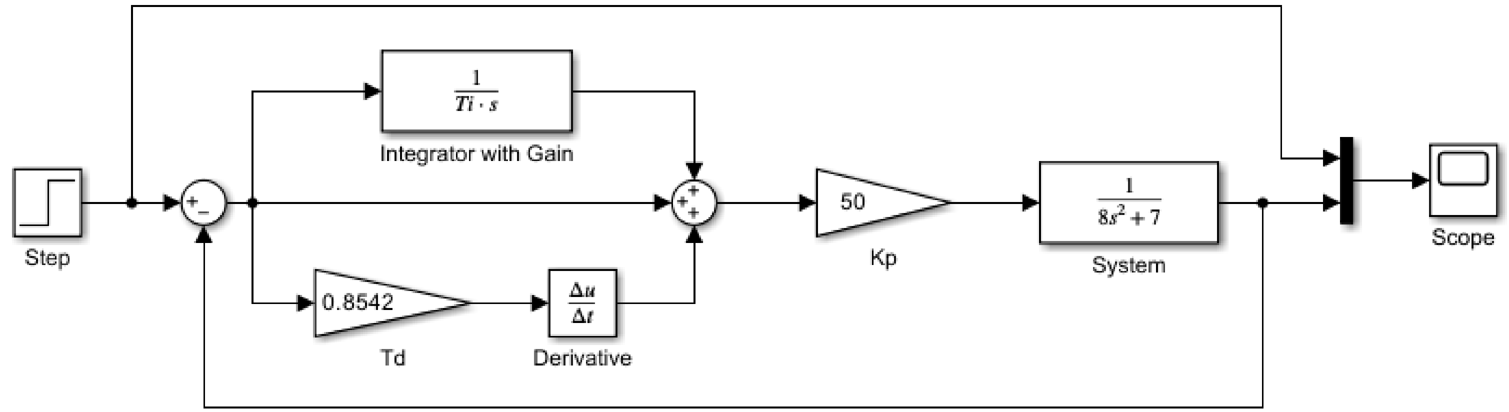
$$G_c(s) = K_p\left(1 + T_d s + \frac{1}{T_i s}\right)$$

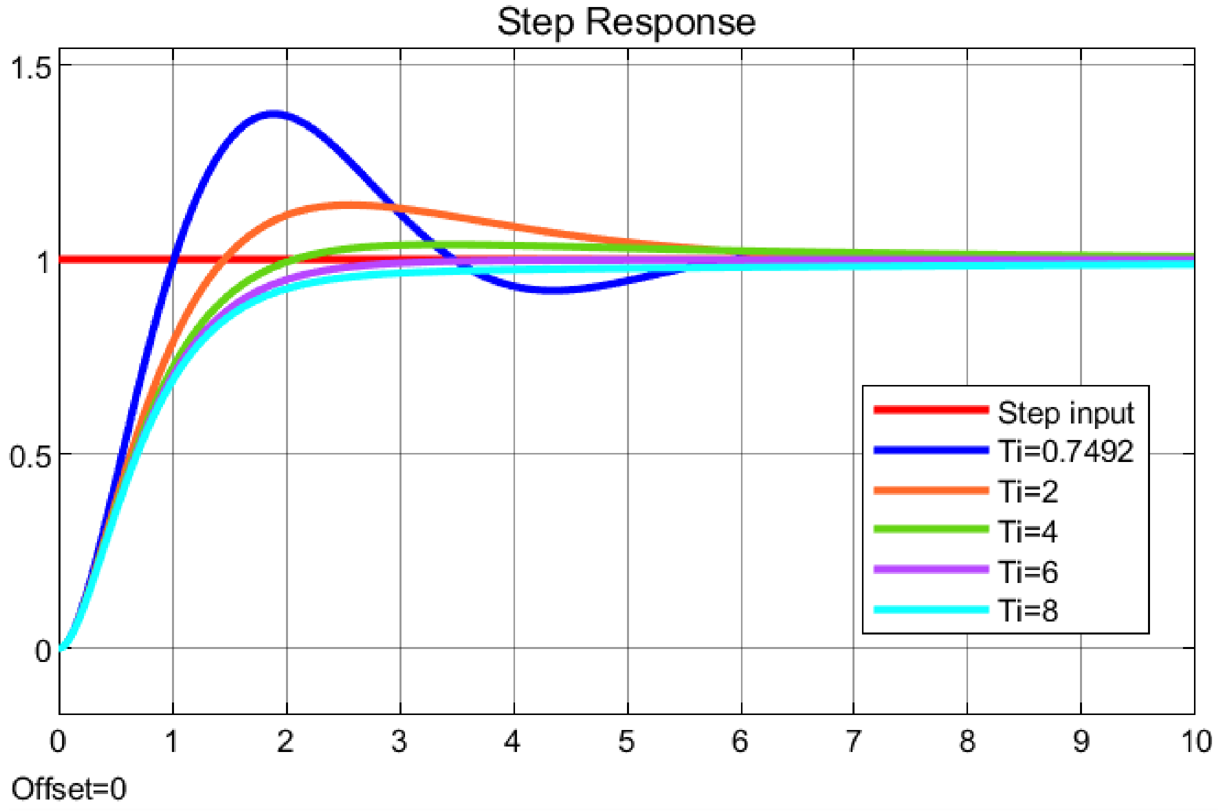
We can plot the unit-step response graph for different values of $T_i \geq 0.7493$ to select the desired transient response.

For example, results are shown in the figure to compare the effect on the transient response and the stability of the system.

The selected PID controller is:

$$\boxed{G_c(s) = 50\left(1 + 0.8542s + \frac{1}{6s}\right)}$$





THANK YOU

HUMBER

WE ARE HUMBER