Practice Questions

Please note that this is NOT a comprehensive coverage of all Final materials. The exam scope has been communicated during the class.

This exam will be covering everything after the midterm (labs and lectures). It will be in the same format as the midterm, (10 multiple choice questions 3’ each; 5 short answer questions, 5’ each; 3-4 computational questions, 45’ in total).

Section I. Multiple choice questions.

MC-1. If a discrete-time system is represented by the system function $H(z)$, which statement about this system below is correct?

(a) With input signal being $x[n]$, its output will be in the form of $Y[z] = H(z)\sum_{k=-\infty}^{\infty} x[k]z^{-k}$ in the z-domain.

(b) The system is fully characterized by $H(z)$.

(c) If when the input is $x[n]$, the output is $y[n]$, then if the input is $x[n-10]$, then the output is $y[n-10]$.

(d) $H(z)$ is discrete.

(e) $h[n] = \mathcal{Z}^{-1}\{H(z)\}$ is the unit step response of the system.

Answer: (b)

MC-2. How to compute the stability of a DT system using system analysis tools?

(a) Compute its Fourier transform and see if it converges.

(b) Compute its z-transform to check its pole locations.

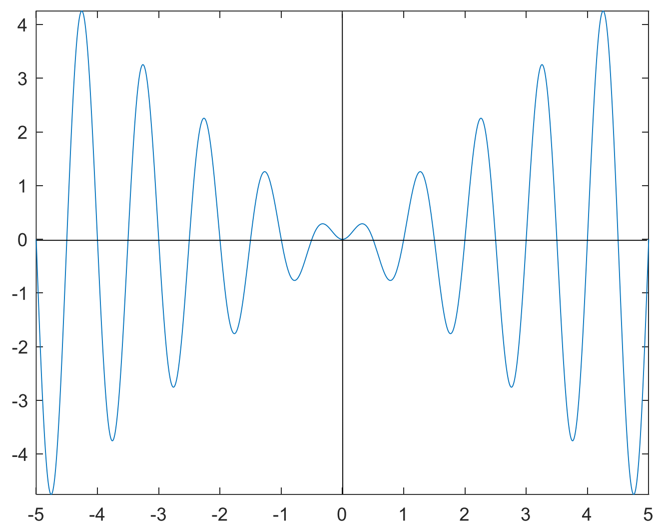
(c) Compute its Fourier series expansion to see if it is periodic.

(d) Compute its Laplace transform to find its zero locations.

(e) none of the above.

Answer: (b)

MC-3. A signal s(t) is shown in the Figure below, outside of this window is all zero. Which one of the following statements is correct?



(a) The fundamental frequency of this signal is 1Hz.

(b) The Fourier spectrum of this signal is purely imaginary and even.

(c) This signal doesn't have a Fourier transform.

(d) This signal has a continuous Fourier spectrum.

(e) None is correct.

Answer: (d)

MC-4. What does Fourier convergence imply?

(a) A Fourier representation of a signal is not necessarily the same at every point as the original signal.

(b) Signals with discontinuities won't converge pointwise at the discontinuities.

(c) The ripples due to Gibbs phenomenon is part of the limitations of the Fourier transform.

(d) The Gibbs ripples will remain even if enough harmonics are included.

(e) All of the above.

Answer: (e)

Section II: Short answer questions

SA-1. A transfer function is defined as below:

$$X(z) = \frac{1}{1 - \frac{2}{3}z^{-1} + \frac{1}{9}z^{-2}}$$

Please determine the difference equation of this system. Is it possible for this system to be causal and stable? If yes, what is the ROC of this system? Please provide rationale.

Answer: possible. ROC|z|>1/3.

SA-2. A periodic waveform is represented as: $x(t) = \sin(2\pi t) + 0.2\sin(4\pi t + \pi/2) + \sin(5\pi t) + e^{7j\pi t}$, what is the fundamental frequency of this signal and please write out its corresponding exponential Fourier coefficients.

Fundamental frequency is $f = 2\pi/\omega_0 = 2$, where:

$$\omega_0 = \pi$$

$$x(t) = \frac{1}{2j}(e^{2j\omega_0 t} - e^{-2j\omega_0 t}) + 0.2\cos(4\omega_0 t) + \frac{1}{2j}(e^{5j\omega_0 t} - e^{-5j\omega_0 t}) + e^{7j\omega_0 t}$$

$$= \frac{1}{2j}(e^{2j\omega_0 t} - e^{-2j\omega_0 t}) + 0.2\frac{1}{2}(e^{4j\omega_0 t} + e^{-4j\omega_0 t}) + \frac{1}{2j}(e^{5j\omega_0 t} - e^{-5j\omega_0 t}) + e^{7j\omega_0 t}$$

$a_2 = \frac{1}{2j}, a_{-2} = -\frac{1}{2j}$

$a_4 = 0.1, a_{-4} = 0.1$

$a_5 = \frac{1}{2j}, a_{-5} = -\frac{1}{2j}$

$a_5 = 1.$

Section III: Computational questions

CQ-1. A periodic discrete time signal of period $N = 6$ is defined as f[n]:

$$f[n] = \begin{cases} n, 0 \le n < 3 \\ 6 - n, 3 \le n < 6 \end{cases}$$

After going through a system, the output signal is

$$g[n] = f[n] - f[n-3]$$

Show that $g[n]$ also has a period of $N = 6$, and condition for this system to be causal and stable.

Answer:

(a)

$$g[n+6] = f[n+6] - f[n+6-3] = f[n] - f[n-3] = g[n]$$

So $N = 6$ is the period of g[n].

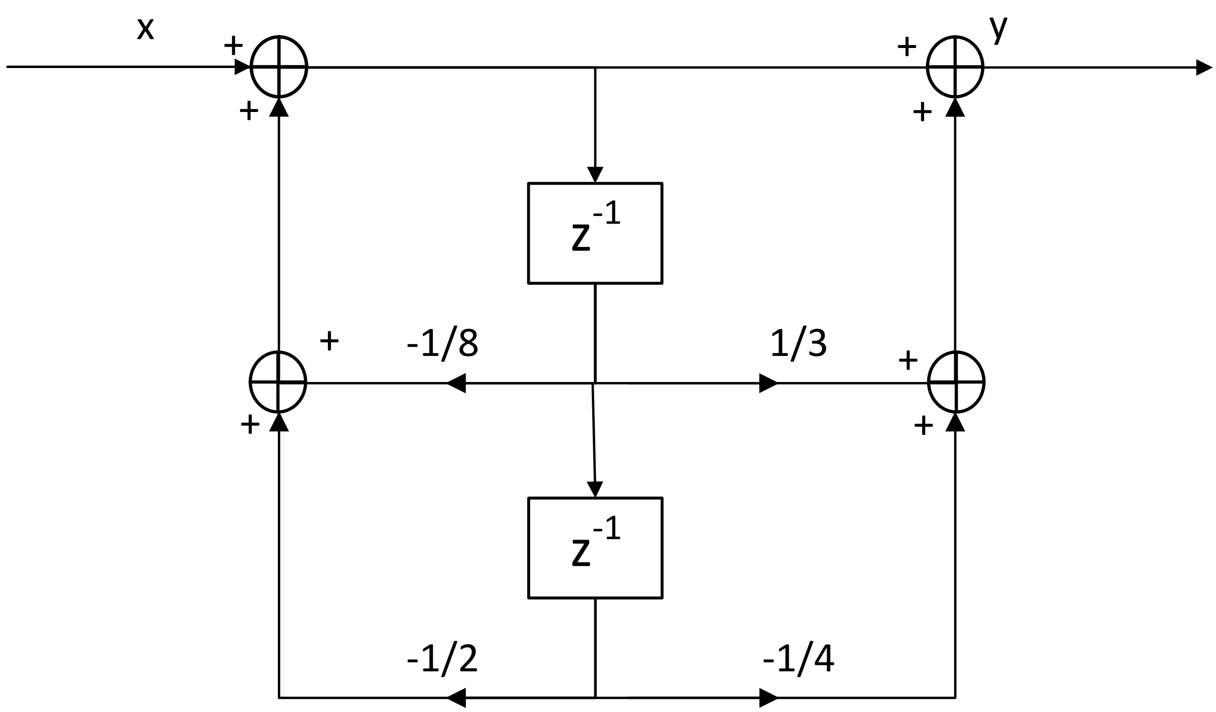
(b) Since this system is DT LTI, and the system function is:

$$H(z) = \frac{z^3 - 1}{z^3}$$

The numerator and denominator both is of order of three, so this system will be causal if and only if the system will be causal if and only the ROC is the exterior of a circle.

(c) This system is stable since it's causal and its poles are within the unit circle.

CQ-2. If a block diagram of a system is shown below. Please find H(z). What type of system is this (cascade, parallel, feedback)?



Answer: set the intermediate signal to be v[n]

$$v[n] + \frac{1}{8}v[n-1] + \frac{1}{2}v[n-2] = x[n]$$

$$H1(z) = \frac{1}{1 + \frac{1}{8}z^{-1} + \frac{1}{2}z^{-2}}$$

$$v[n] + \frac{1}{3}v[n-1] - \frac{1}{4}v[n-2] = y[n]$$

$$H2(z) = 1 + \frac{1}{3}z^{-1} - \frac{1}{4}z^{-2}$$

$$H(z) = H1(z) \cdot H2(z) = \frac{\left(1 + \frac{1}{3}z^{-1} - \frac{1}{4}z^{-2}\right)}{\left(1 + \frac{1}{8}z^{-1} + \frac{1}{2}z^{-2}\right)}$$

This is a cascade system.