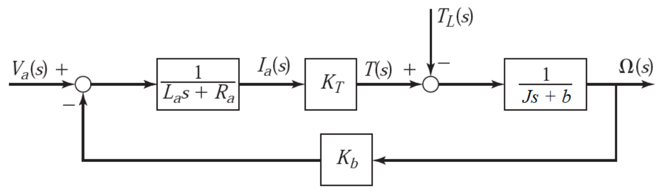
Worksheet 6 - Solution

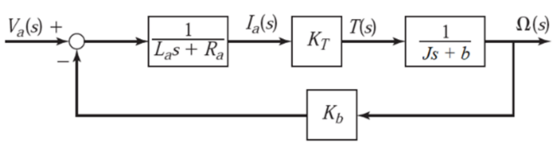
Modeling of Electromechanical Systems

1) Consider the following block diagram model of an armature-controlled DC motor. Assume the armature voltage $V_a(s)$ and the load torque $T_L(s)$ as the inputs, and the angular speed $\Omega(s)$ and the armature current $I_a(s)$ as the outputs.



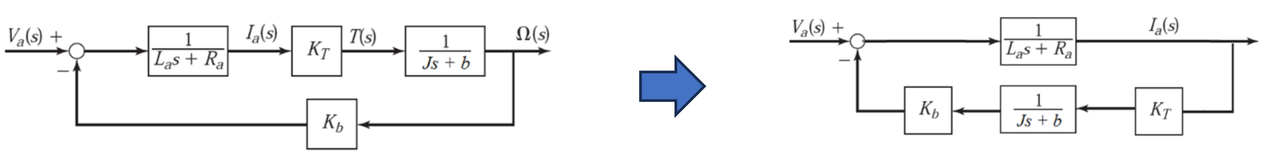
a) Obtain the following transfer functions for the DC motor by applying the superposition theorem and simplfying the block diagram. $\frac{\Omega(s)}{V_a(s)}, \frac{I_a(s)}{V_a(s)}, \frac{\Omega(s)}{T_L(s)}, \frac{I_a(s)}{T_L(s)}$.

To find the transfer function $\frac{\Omega(s)}{V_a(s)}$ set $T_L = 0$. The block diagram will be simplified as below:



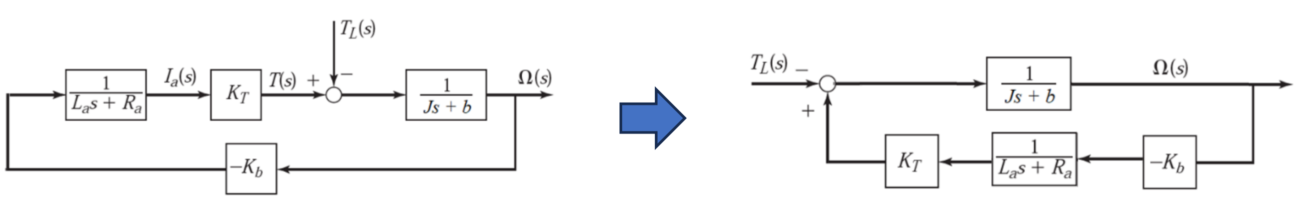
$$\frac{\Omega(s)}{V_a(s)} = \frac{\left(\frac{1}{L_a s + R_a}\right)(K_T)\left(\frac{1}{Js + b}\right)}{1 + \left(\frac{1}{L_a s + R_a}\right)(K_T)\left(\frac{1}{Js + b}\right)(K_b)} = \frac{K_T}{(L_a s + R_a)(Js + b) + K_T K_b} = \frac{K_T}{L_a J s^2 + (L_a b + J R_a)s + R_a b + K_T K_b}$$

To find the transfer function $\frac{I_a(s)}{V_a(s)}$ set $T_L = 0$. The block diagram will be simplified as below:



$$\frac{I_a(s)}{V_a(s)} = \frac{\frac{1}{L_a s + R_a}}{1 + \left(\frac{1}{L_a s + R_a}\right)(K_b)\left(\frac{1}{Js + b}\right)(K_T)} = \frac{Js + b}{(L_a s + R_a)(Js + b) + K_T K_b} = \frac{Js + b}{L_a J s^2 + (L_a b + J R_a)s + R_a b + K_T K_b}$$

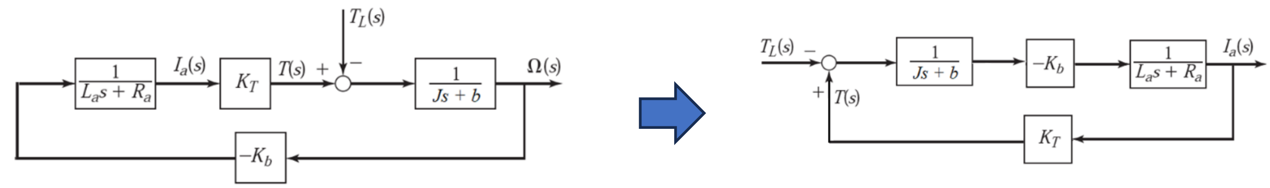
To find the transfer function $\frac{\Omega(s)}{T_L(s)}$ set $V_a = 0$. The block diagram will be simplified as below:



$$\frac{\Omega(s)}{-T_L(s)} = \frac{\dfrac{1}{Js+b}}{1-\left(\dfrac{1}{Js+b}\right)(K_T)\left(\dfrac{1}{L_a s+R_a}\right)(-K_b)}$$

$$\frac{\Omega(s)}{T_L(s)} = -\frac{L_a s + R_a}{(L_a s+R_a)(Js+b)+K_T K_b} = -\frac{L_a s + R_a}{L_a J s^2 + (L_a b + J R_a)s + R_a b + K_T K_b}$$

To find the transfer function $\frac{\mathrm{I_a}(s)}{T_L(s)}$ set $V_a = 0$. The block diagram will be simplified as below:



$$\frac{\Omega(s)}{-T_L(s)} = \frac{\left(\dfrac{1}{Js+b}\right)(-K_b)\left(\dfrac{1}{L_a s+R_a}\right)}{1-\left(\dfrac{1}{Js+b}\right)(-K_b)\left(\dfrac{1}{L_a s+R_a}\right)(K_T)}$$

$$\frac{\Omega(s)}{T_L(s)} = \frac{K_b}{(L_a s+R_a)(Js+b)+K_T K_b} = \frac{K_b}{L_a J s^2 + (L_a b + J R_a)s + R_a b + K_T K_b}$$

b) Use the ***final value theorem*** to obtain the expression for the steady-state values of the speed ω and the armature current i_a, if v_a and T_L are step functions of magnitude V_a and T_L, respectively.

The $I_a(s)$ is obtained as below:

$$I_a(s) = \frac{Js+b}{L_a J s^2 + (R_a J + b L_a)s + bR_a + K_b K_T} V_a(s) + \frac{K_b}{L_a J s^2 + (R_a J + b L_a)s + bR_a + K_b K_T} T_L(s)$$

If $V_a(s) = \frac{V_a}{s}$ and $T_L(s) = \frac{T_L}{s}$ the **steady-state value of current** is obtained as:

$$i_a(\infty) = \lim_{t\to\infty} i_a(t) = \lim_{s\to 0} s I_a(s) = \frac{bV_a}{bR_a + K_b K_T} + \frac{K_b T_L}{bR_a + K_b K_T} = \frac{bV_a + K_b T_L}{bR_a + K_b K_T}$$

The $\Omega(s)$ is obtained as below:

$$\Omega(s) = \frac{K_T}{L_a J s^2 + (R_a J + b L_a)s + bR_a + K_b K_T} V_a(s) - \frac{L_a s + R_a}{L_a J s^2 + (R_a J + b L_a)s + bR_a + K_b K_T} T_L(s)$$

If $V_a(s) = \frac{V_a}{s}$ and $T_L(s) = \frac{T_L}{s}$ the **steady-state value of speed** is obtained as:

$$\omega(\infty) = \lim_{t\to\infty} \omega(t) = \lim_{s\to 0} s\Omega(s) = \frac{K_T V_a}{bR_a + K_b K_T} - \frac{R_a T_L}{bR_a + K_b K_T} = \frac{K_T V_a - R_a T_L}{bR_a + K_b K_T}$$

c) The ***no-load speed*** and ***no-load current*** are the motor speed and the current when the load torque is zero. Determine the no-load speed and no-load current by setting $T_L = 0$ in the obtained steady-state values in part (b).

Setting $T_L = 0$ in the following equation gives the **no-load speed**, which is the highest motor speed for a given applied voltage.

$$\omega(\infty) = \frac{K_T V_a - R_a T_L}{bR_a + K_b K_T} \xrightarrow{T_L=0} \omega_{nl} = \frac{K_T V_a}{bR_a + K_b K_T}$$

Setting $T_L = 0$ in the following equation gives the **no-load current**,

$$i_a(\infty) = \frac{bV_a + K_b T_L}{bR_a + K_b K_T} \xrightarrow{T_L=0} i_{nl} = \frac{bV_a}{bR_a + K_b K_T}$$

d) The ***stall torque*** is the value of the load torque that produces zero motor speed. Determine the stall torque by setting $\omega = 0$ in the obtained steady-state value of speed in part (b).

Setting $\omega = 0$ in the following equation gives the **stall torque**,

$$\omega(\infty) = \frac{K_T V_a - R_a T_L}{bR_a + K_b K_T} \xrightarrow{\omega=0} T_{L_{stall}} = \frac{K_T V_a}{R_a}$$

f) Determine the motor parameters K_b and K_T in terms of the stall torque and the no-load speed of the motor.

From the stall torque formula:

$$T_{L_{stall}} = \frac{K_T V_a}{R_a} \quad \rightarrow \quad K_T = \frac{R_a T_{L_{stall}}}{V_a}$$

From the no-load speed formula:

$$\omega_{nl} = \frac{K_T V_a}{bR_a + K_b K_T} \quad \rightarrow \quad K_b = \frac{V_a}{\omega_{nl}} - \frac{bR_a}{K_T}$$

2) The transfer function model and parameter values for a certain armature-controlled motor are:

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_T}{L_a J s^2 + (L_a b + J R_a)s + R_a b + K_b K_T}$$

$$\frac{I_a(s)}{V_a(s)} = \frac{Js + b}{L_a J s^2 + (L_a b + J R_a)s + R_a b + K_b K_T}$$

$$\frac{\Omega(s)}{T_L(s)} = -\frac{L_a s + R_a}{L_a J s^2 + (L_a b + J R_a)s + R_a b + K_b K_T}$$

$$\frac{I_a(s)}{T_L(s)} = \frac{K_b}{L_a J s^2 + (L_a b + J R_a)s + R_a b + K_b K_T}$$

$$K_T = K_b = 0.2\ N.m/A,\ \ L_a = 4 \times 10^{-3}\ H,\ \ R_a = 0.8\ \Omega,\ \ J = 5 \times 10^{-4}\ kg.m^2,\ \ b = 5 \times 10^{-4}\ N.m.s/rad$$

a) Obtain the step response of $i_a(t)$ and $\omega(t)$ if the applied voltage is $v_a = 10\ V$.

Substituting the given parameter values into transfer functions $\frac{\Omega(s)}{V_a(s)}$ and $\frac{I_a(s)}{V_a(s)}$ gives:

$$\frac{I_a(s)}{V_a(s)} = \frac{5 \times 10^{-4}s + 5 \times 10^{-4}}{20 \times 10^{-7}s^2 + 4.02 \times 10^{-4}s + 4.04 \times 10^{-2}} = \frac{250s + 250}{s^2 + 201s + 20200}$$

$$\frac{\Omega(s)}{V_a(s)} = \frac{0.2}{20 \times 10^{-7}s^2 + 4.02 \times 10^{-4}s + 4.04 \times 10^{-2}} = \frac{100000}{s^2 + 201s + 20200}$$

If v_a is a step function of magnitude 10 V,

$$I_a(s)=\left(\frac{250s+250}{s^2+201s+20200}\right)\left(\frac{10}{s}\right)=\frac{25}{202}\left(\frac{1}{s}+\frac{-s+19999}{s^2+201s+20200}\right)$$

$$I_a(s)=\frac{25}{202}\left(\frac{1}{s}+\frac{-s+19999}{(s+100.5)^2+10099.75}\right)=\frac{25}{202}\left(\frac{1}{s}+\frac{-s-100.5}{(s+100.5)^2+10099.75}+\frac{20099.5}{(s+100.5)^2+10099.75}\right)$$

$$\boxed{i_a(t)=\frac{25}{202}\left(1-e^{-100.5t}\cos\left(\sqrt{10099.75}\,t\right)+\frac{20099.5}{\sqrt{10099.75}}\,e^{-100.5t}\sin\left(\sqrt{10099.75}\,t\right)\right)}$$

$$\Omega(s)=\left(\frac{100000}{s^2+201s+20200}\right)\left(\frac{10}{s}\right)=\frac{5000}{101}\left(\frac{1}{s}+\frac{-s-201}{s^2+201s+20200}\right)$$

$$\Omega(s)=\frac{5000}{101}\left(\frac{1}{s}+\frac{-s-201}{(s+100.5)^2+10099.75}\right)=\frac{5000}{101}\left(\frac{1}{s}+\frac{-s-100.5}{(s+100.5)^2+10099.75}+\frac{-100.5}{(s+100.5)^2+10099.75}\right)$$

$$\boxed{\omega(t)=\frac{5000}{101}\left(1-e^{-100.5t}\cos\left(\sqrt{10099.75}\,t\right)-\frac{100.5}{\sqrt{10099.75}}\,e^{-100.5t}\sin\left(\sqrt{10099.75}\,t\right)\right)}$$

b) Obtain the step response of $i_a(t)$ and $\omega(t)$ if the load torque is $T_L=0.2\ N.m.$

Substituting the given parameter values into transfer functions $\frac{\Omega(s)}{T_L(s)}$ and $\frac{I_a(s)}{T_L(s)}$ gives:

$$\frac{I_a(s)}{T_L(s)}=\frac{0.2}{20\times10^{-7}s^2+4.02\times10^{-4}s+4.04\times10^{-2}}=\frac{100000}{s^2+201s+20200}$$

$$\frac{\Omega(s)}{T_L(s)}=-\frac{4\times10^{-3}s+0.8}{20\times10^{-7}s^2+4.02\times10^{-4}s+4.04\times10^{-2}}=-\frac{2000s+400000}{s^2+201s+20200}$$

If T_L is a step function of magnitude 0.2 N.m,

$$I_a(s)=\left(\frac{100000}{s^2+201s+20200}\right)\left(\frac{0.2}{s}\right)=\frac{100}{101}\left(\frac{1}{s}+\frac{-s-201}{s^2+201s+20200}\right)$$

$$I_a(s)=\frac{100}{101}\left(\frac{1}{s}+\frac{-s-201}{(s+100.5)^2+10099.75}\right)=\frac{100}{101}\left(\frac{1}{s}+\frac{-s-100.5}{(s+100.5)^2+10099.75}+\frac{-100.5}{(s+100.5)^2+10099.75}\right)$$

$$\boxed{i_a(t)=\frac{100}{101}\left(1-e^{-100.5t}\cos\left(\sqrt{10099.75}\,t\right)-\frac{100.5}{\sqrt{10099.75}}\,e^{-100.5t}\sin\left(\sqrt{10099.75}\,t\right)\right)}$$

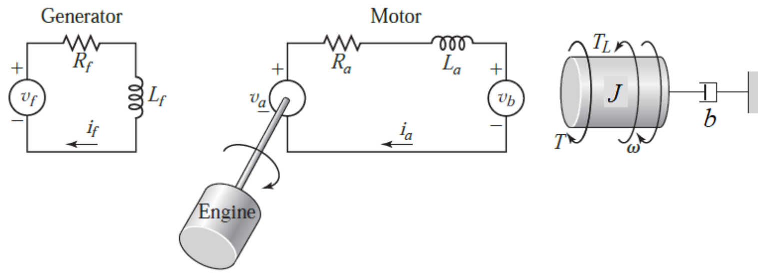
$$\Omega(s)=-\left(\frac{2000s+400000}{s^2+201s+20200}\right)\left(\frac{0.2}{s}\right)=-\frac{400}{101}\left(\frac{1}{s}+\frac{-s-100}{s^2+201s+20200}\right)$$

$$\Omega(s)=-\frac{400}{101}\left(\frac{1}{s}+\frac{-s-100}{(s+100.5)^2+10099.75}\right)=-\frac{400}{101}\left(\frac{1}{s}+\frac{-s-100.5}{(s+100.5)^2+10099.75}+\frac{0.5}{(s+100.5)^2+10099.75}\right)$$

$$\boxed{\omega(t)=-\frac{400}{101}\left(1-e^{-100.5t}\cos\left(\sqrt{10099.75}\,t\right)-\frac{0.5}{\sqrt{10099.75}}\,e^{-100.5t}\sin\left(\sqrt{10099.75}\,t\right)\right)}$$

3) The following figure is the circuit diagram of a speed-control system in which the DC motor voltage v_a is supplied by a generator driven by an engine. This system has been used on locomotives whose diesel engine operates most efficiently at one speed. The efficiency of the electric motor is not as sensitive to speed and thus can be used to drive the locomotive at various speeds. The motor voltage v_a is varied by changing the generator input voltage v_f. The voltage v_a is related to the generator field current i_f by $v_a = K_f i_f$.

Derive the system model relating the output speed ω to the voltage v_f, and obtain the transfer function $\frac{\Omega(s)}{V_f(s)}$.



There are two circuits and one inertia in this system, and we must write an equation for each.

For the generator circuit, KVL gives:

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

For the motor circuit, KVL gives:

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + v_b \quad \rightarrow \quad v_a = R_a i_a + L_a \frac{di_a}{dt} + K_b \omega$$

where $v_a = K_f i_f$. For the inertia J Newton's law gives:

$$J \frac{d\omega}{dt} = T - b\omega - T_L$$

where $T = K_T i_a$. Substitute for v_a and T, and rearrange to obtain:

$$v_f = R_f i_f + L_f \frac{di_f}{dt} \quad \rightarrow \quad V_f(s) = R_f I_f(s) + L_f s I_f(s)$$

$$K_f i_f - K_b \omega = R_a i_a + L_a \frac{di_a}{dt} \quad \rightarrow \quad K_f I_f(s) - K_b \Omega(s) = R_a I_a(s) + L_a s I_a(s)$$

$$J \frac{d\omega}{dt} = K_T i_a - b\omega - T_L \quad \rightarrow \quad J s \Omega(s) = K_T I_a(s) - b\Omega(s) - T_L(s)$$

Eliminating $I_f(s)$ and $I_a(s)$ from equations, we obtain:

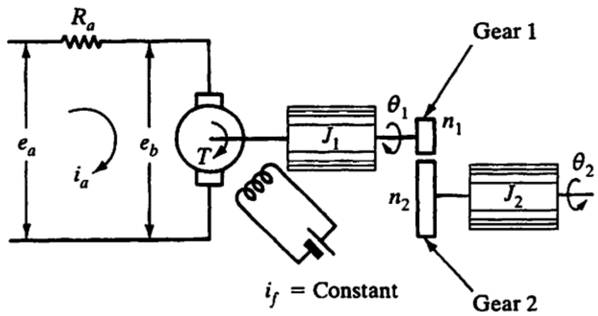
$$K_f \left(\frac{V_f}{R_f + L_f s} \right) - K_b \Omega(s) = (R_a + L_a s) \left(\frac{J s \Omega(s) + b\Omega(s) + T_L(s)}{K_T} \right)$$

Hence, the transfer function $\Omega(s)/V_f(s)$ assuming $T_L = 0$ is given by:

$$\frac{\Omega(s)}{V_f(s)} = \frac{K_f K_T}{(R_f + L_f s)\big(K_T K_b + (R_a + L_a s)(Js + b)\big)}$$

4) Consider the DC servomotor system shown below. The armature inductance is negligible and is not shown in the circuit. Obtain the transfer function between the output θ_2 and the input e_a.

R_a = Armature Resistance, Ω
L_a = Armature Inductance, H
i_a = Armature Current, A
i_f = Field Current, A
e_a = Applied Armature Voltage, V
e_b = Back-emf, V
θ_1 = Angular displacement of the motor shaft, rad
θ_2 = Angular displacement of the load element, rad
T = Torque developed by the motor, N. m
J_1 = Moment of inertia of the rotor of the motor, $kg.m^2$
J_2 = Moment of inertia of the load, $kg.m^2$
n_1 = number of teeth on gear 1
n_2 = number of teeth on gear 2



The torque T developed by the dc servomotor is:

$$T = K_T i_a$$

where K_T is the motor torque constant. The induced voltage e_b is proportional to the angular velocity ω_1, or

$$e_b = K_b\omega_1 = K_b\frac{d\theta_1}{dt}$$

where K_b is the back-emf constant.

The equation for the armature circuit if $L_a \approx 0$ is:

$$R_a i_a + e_b = e_a$$

The equivalent moment of inertia of the motor rotor plus the load inertia referred to the motor shaft is

$$J_{1eq} = J_1 + \left(\frac{n_1}{n_2}\right)^2 J_2$$

The armature current produces the torque that is applied to the equivalent moment of inertia J_{1eq}. Thus,

$$T = J_{1eq}\frac{d^2\theta_1}{dt^2}$$

Assuming that all initial conditions are zero and taking the Laplace transforms of abovementioned equations, we obtain

$$T(s) = K_T I_a(s)$$
$$E_a(s) = K_b s\theta_1(s)$$
$$R_a I_a(s) + E_b(s) = E_a(s)$$
$$T(s) = J_{1eq}s^2\theta_1(s)$$

Eliminating $T(s)$, $E_b(s)$ and $I_a(s)$ from equations, we obtain

$$\left(J_{1eq}s^2 + \frac{K_T K_b}{R_a}\right)\theta_1(s) = \frac{K_T}{R_a}E_a(s)$$

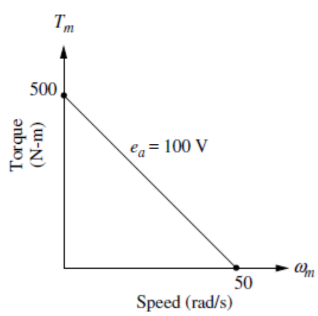
Noting that $\frac{\theta_1(s)}{\theta_2(s)} = \frac{n2}{n1}$ we can write this last equation as:

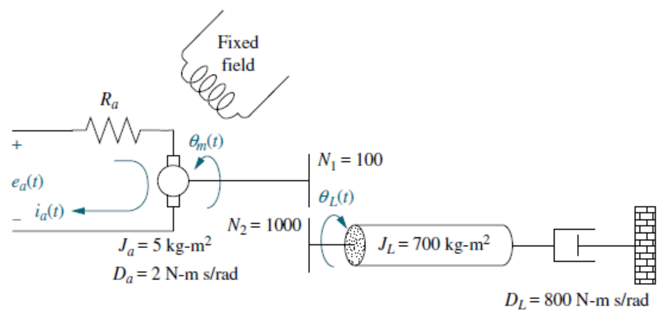
$$\left(J_{1eq}s^2 + \frac{K_T K_b}{R_a}\right)\frac{n_2}{n_1}\theta_2(s) = \frac{K_T}{R_a}E_a(s)$$

Hence, the transfer function $\theta_2(s)/E_a(s)$ is given by

$$\frac{\theta_2(s)}{E_a(s)} = \frac{\frac{n_1}{n_2}K_T}{\left(R_a\left(J_1 + \left(\frac{n_1}{n_2}\right)^2 J_2\right)s + K_T K_b\right)s}$$

5) Given the system and torque-speed curve find the transfer function $\theta_L(s)/E_a(s)$.





The total inertia J_m and damping D_m at the armature including both the armature and the reflected load to the armature:

$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 = 5 + 700\left(\frac{1}{10}\right)^2 = 12$$

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 = 2 + 800\left(\frac{1}{10}\right)^2 = 10$$

The general form of transfer function $\theta_m(s)/E_a(s)$ is:

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_T}{L_a J_m s^3 + (L_a D_m + J_m R_a)s^2 + (R_a D_m + K_b K_T)s}$$

Since $L_a = 0$, the transfer function can be simplified as:

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_T}{J_m R_a s^2 + (R_a D_m + K_b K_T)s}$$

Now, find the motor constants K_T and K_b from the given torque-speed curve:

$$K_T = \frac{R_a T_{L_{stall}}}{e_a} = \frac{500R_a}{100} = 5R_a$$

$$K_b = \frac{e_a}{\omega_{nl}} - \frac{D_a R_a}{K_T} = \frac{100}{50} - \frac{2R_a}{5R_a} = 2 - 0.4 = 1.6$$

Substitute the motor constants K_T and K_b and the J_m and D_m values:

$$\frac{\theta_m(s)}{E_a(s)} = \frac{5R_a}{12R_a s^2 + (10R_a + 1.6(5R_a))s} = \frac{5}{12s^2 + 18s}$$

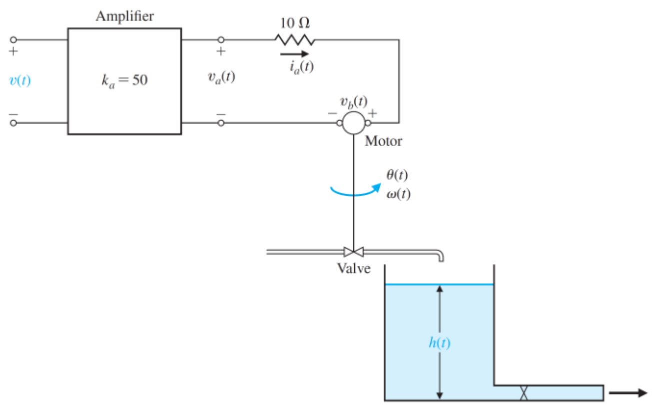
In order to find the $\theta_L(s)/E_a(s)$, we use the gear ratio $\frac{N_1}{N_2} = \frac{1}{10}$, and find

$$\frac{\theta_L(s)}{E_a(s)} = \frac{0.5}{12s^2 + 18s}$$

6) The water level $h(t)$ in a tank is controlled by an open-loop system, as shown in the following figure. A DC motor controlled by an armature current i_a turns a shaft, opening a valve. The inductance of the DC motor is negligible, that is, $L_a = 0$. Also, the rotational friction of the motor shaft and valve is negligible, that is, $b = 0$. The height of the water in the tank is

$$h(t) = \int [1.6\,\theta(t) - h(t)]dt$$

the motor constant is $K_b = K_T = 10$, and the intertia of the motor shaft and valve is $J = 6 \times 10^{-3}\ kg.m^2$.



a) Determine the differential equation for $h(t)$ and $v(t)$.

Assuming that $L_a = 0$ and $b = 0$, we have the following relationships in the DC motor electrical and mechanical dynamics:

$$J\frac{d\omega(t)}{dt} = T(t) \qquad and \qquad v_a(t) = 10i_a(t) + v_b(t)$$

$$T(t) = K_T i_a(t) \qquad and \qquad \omega(t) = K_b v_b(t)$$

Relationship for the amplifier is:

$$v_a(t) = 50v(t)$$

Combining the abovementioned equations, we can find the equation of motion for DC motor system:

$$J\frac{d\omega(t)}{dt} = T(t) \quad \rightarrow \quad J\frac{d\omega(t)}{dt} = K_T i_a(t) = K_T\left(\frac{v_a(t) - v_b(t)}{10}\right) = K_T\left(\frac{50v(t) - \frac{1}{K_b}\omega(t)}{10}\right)$$

$$J\frac{d\omega(t)}{dt} + \frac{K_T}{10K_b}\omega(t) = 5K_T v(t)$$

Using the given relationship and taking derivative we have:

$$h(t) = \int [1.6\,\theta(t) - h(t)]dt \quad \rightarrow \quad \frac{dh(t)}{dt} = 1.6\theta(t) - h(t) \quad \rightarrow \quad \frac{d^2h(t)}{dt^2} = 1.6\omega(t) - \frac{dh(t)}{dt}$$

Then find the $\omega(t)$:

$$\omega(t) = \frac{1}{1.6}\left(\frac{d^2h(t)}{dt^2} + \frac{dh(t)}{dt}\right)$$

Substitude the $\omega(t)$ in the equation of motion of the DC motor system:

$$\frac{J}{1.6}\left(\frac{d^3h(t)}{dt^3} + \frac{d^2h(t)}{dt^2}\right) + \frac{K_T}{16K_b}\left(\frac{d^2h(t)}{dt^2} + \frac{dh(t)}{dt}\right) = 5K_T v(t)$$

Simplify the equation to find the differential equation for $h(t)$ and $v(t)$:

$$\frac{d^3h(t)}{dt^3} + \left(1 + \frac{K_T}{10JK_b}\right)\frac{d^2h(t)}{dt^2} + \frac{K_T}{10JK_b}\frac{dh(t)}{dt} = \frac{8K_T}{J}v(t)$$

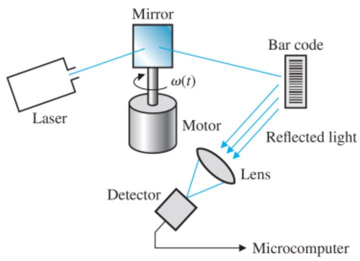
b) Determine the transfer function $H(s)/V(s)$.

Taking Laplace transform with zero initial conditions we have the transfer function:

$$s^3H(s) + \left(1 + \frac{K_T}{10JK_b}\right)s^2H(s) + \frac{K_T}{10JK_b}sH(s) = \frac{8K_T}{J}V(s)$$

$$\frac{H(s)}{V(s)} = \frac{\frac{8K_T}{J}}{s^3 + \left(1 + \frac{K_T}{10JK_b}\right)s^2 + \frac{K_T}{10JK_b}s}$$

7) In many applications, such as reading product codes in supermarkets and in printing and manufacturing, an optical scanner is utilized to read codes, as shown in the figure. As the mirror rotates, a friction force is developed that is proportional to its angular speed. The friction constant is equal to 0.06 $N.s/rad$, and the moment of inertia is equal to 0.1 $kg.m^2$. The output variable is the velocity $\omega(t)$.



a) Obtain the differential equation for the motor.

The equation of motion of the motor is:

$$J\frac{d\omega}{dt} = T_m - b\omega$$

where $J = 0.1\ kg.m^2,\ b = 0.06\ N.s/rad$ and T_m is the motor input torque.

b) Find the response of the system when the input motor torque is a unit step and the initial velocity at $t = 0$ is equal to 0.7.

Given $T_m(s) = \frac{1}{s}$ and $\omega(0) = 0.7\ rad/sec$, we take the Laplace transform of the equation of motion yielding:

$$Js\Omega(s) - \omega(0) = T_m(s) - b\Omega(s) \quad \rightarrow \quad 0.1s\Omega(s) - 0.7 = \frac{1}{s} - 0.06\Omega(s) \quad \rightarrow \quad \Omega(s) = \frac{0.7s + 10}{s(s + 0.6)}$$

Then, computing the partial fraction expansion, we find that

$$\Omega(s) = \frac{16.67}{s} - \frac{15.97}{s + 0.6}$$

The step response, determined by taking the inverse Laplace transform, is

$$\omega(t) = 16.67 - 15.97e^{-0.6t}, \qquad t \geq 0$$