Coding with MATLAB – Part I

Example 1. The sine function can be evaluated by the following infinite series:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

Create an M-file to implement this formula so that it computes and displays the values of $\sin x$ as each term in the series is added. In other words, compute and display in sequence the values for

$$\sin x = x$$
$$\sin x = x - \frac{x^3}{3!}$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$
$$\vdots$$

up to the order term of your choosing. For each of the preceding, compute and display the percent relative error as

$$\%\ \text{of error} = \frac{\text{true} - \text{series approximation}}{\text{true}} \times 100\%$$

As a test case, employ the program to compute sin(0.9) for up to and including eight terms - that is, up to the term $\frac{x^{15}}{15!}$.

Solution. Let's call the function as `SineSeries(x,n)` that receives x and then number of terms n and returns an approximation for $\sin x$ based on the first n terms.

```
function SineSeries (x,n)
```

```
% add a description
i = 1;
tru = sin(x);
ser = 0;
fprintf('\n');
fprintf('order  true value  approximation  error\n');
while (1)
    if i > n
        break
    end
    ser = ser + (-1)^(i - 1) * x^(2*i-1) / factorial(2*i-1);
    er = (tru - ser) / tru * 100;
    fprintf('%3d %14.10f %14.10f %12.8f\n',i,tru,ser,er);
    i = i + 1;
end
```

Modify the code so that the procedure stops when the relative error is less than a given tolerance ε_s. Note that in this case, ε_s will be replaced by n in the input arguments. (**Practice at home**)

Example 2. An amount of money ***P*** is invested in an account where interest is compounded at the end of the period. The future worth ***F*** yielded at an interest rate i after n periods may be determined from the following formula:

$$F = P(1 + i)^n$$

Write an M-file that will calculate the future worth of an investment for each year from 1 through n. The input to the function should include the initial investment P, the interest rate i (as a decimal), and the number of years n for which the future worth is to be calculated. The output should consist of a table with headings and columns for n and F. Run the code for $P = \$100{,}000, i = 0.05$, and $n = 10$ years.

Solution.

```
function TotalValue(P, i, n)
% add a description
nn = 0:n;
F = P*(1+i).^nn;
y = [nn;F];
fprintf('\n year    future worth\n');
fprintf('%5d %14.2f\n',y);
end
```

or it can be coded by for loop as follows.

```
function TotalValue(P, i, n)
% add a description
fprintf('\n year    future worth\n');
for j=0:n
    F = P*(1+i)^j;
    y = [j; F];
    fprintf('%5d %14.2f\n',y);
end
```