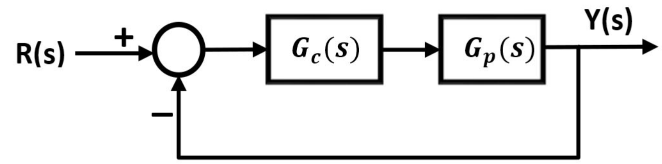
MENG 3510 – Quiz 1 Solution – Winter 2025

Question 1. [10 marks] Consider the following closed-loop control system,

$$G_c(s) = K_p$$

$$G_p(s) = \frac{1}{s(s+1)}$$



a) Find the overall closed-loop transfer function $T(s) = \frac{Y(s)}{R(s)}$ in terms of parameter K_p. Show your work.

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)H(s)} = \frac{(K_p)\left(\frac{1}{s(s+1)}\right)}{1 + (K_p)\left(\frac{1}{s(s+1)}\right)(1)} = \frac{\frac{K_p}{s(s+1)}}{1 + \frac{K_p}{s(s+1)}} = \frac{K_p}{s^2 + s + K_p}$$

b) Determine the controller gain K_p so that the unit-step response has a maximum overshoot of 5%. Show your work.

Find the required damping ratio for the desired overshoot:

$$\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}} = \frac{-\ln(0.05)}{\sqrt{\pi^2 + \ln^2(0.05)}} = 0.6901$$

Find the desired characteristic equation based on the desired damping ratio:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 2(0.6901)\omega_n s + \omega_n^2 = s^2 + 1.3802\omega_n s + \omega_n^2$$

Match the desired characteristic equation with the characteristic equation of the closed-loop system to find the required controller gain K_p.

$$s^2 + 1.3802\omega_n s + \omega_n^2 = s^2 + s + K_p$$

$$1.3802\omega_n = 1 \quad \rightarrow \quad \omega_n = \frac{1}{1.3802} = 0.72 \ rad/s$$

$$\omega_n^2 = K_p \quad \rightarrow \quad K_p = (0.72)^2 \quad \rightarrow \quad K_p = 0.5184$$

c) Assuming that $K_p = 0.52$, determine the steady-state error of the closed-loop system for the unit-step and the unit-ramp inputs. Show your work.

This is a unity-feedback system with the overall forward path transfer function of:

$$G(s) = \frac{0.52}{s(s+1)}$$

Steady-state error for **unit-step** input:

$$k_p = \lim_{s\to 0} G(s) = \lim_{s\to 0}\left(\frac{0.52}{s(s+1)}\right) = \infty$$

$$e_{ss} = \frac{1}{1+k_p} = \frac{1}{\infty} = 0$$

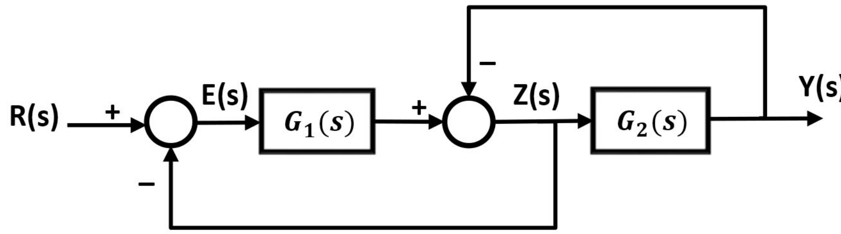
Since the feedback is unity and the forward path transfer function is Type 1, the unit-step error is zero.

Steady-state error for **unit-ramp** input:

$$k_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} s\left(\frac{0.52}{s(s+1)}\right) = 0.52$$

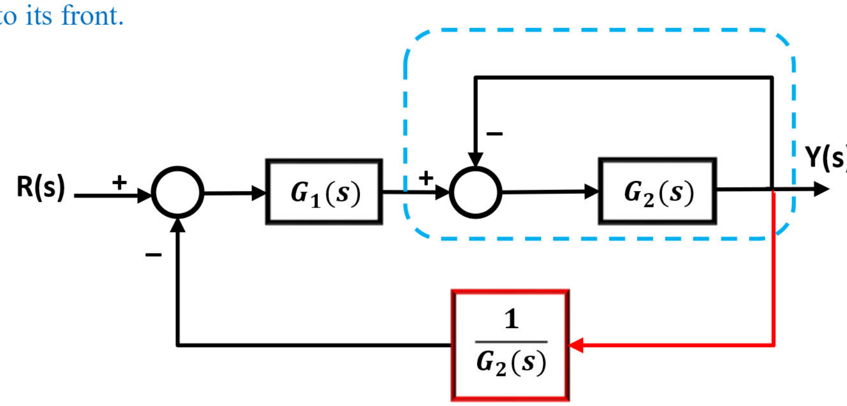
$$e_{ss} = \frac{1}{k_v} = \frac{1}{0.52} = 1.92$$

Question 2. [10 marks] Consider the following block diagram of a control system.

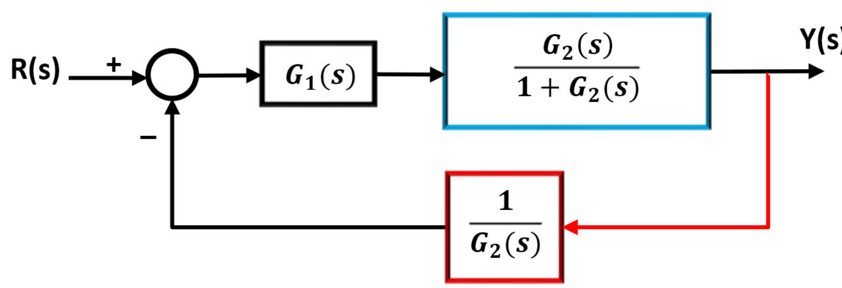


a) Simplify the block diagram and find the overall transfer function $\frac{Y(s)}{R(s)}$. Show your work.

1) Move the branch from behind of block G_2 to its front.



2) Simplify the internal feedback loop



3) Simplify the overall feedback loop

$$T(s)=\frac{Y(s)}{R(s)}=\frac{\dfrac{G_1(s)G_2(s)}{1+G_2(s)}}{1+\left(\dfrac{G_1(s)G_2(s)}{1+G_2(s)}\right)\left(\dfrac{1}{G_2(s)}\right)}=\frac{\dfrac{G_1(s)G_2(s)}{1+G_2(s)}}{\dfrac{\big(1+G_2(s)\big)G_2(s)+G_1(s)G_2(s)}{\big(1+G_2(s)\big)G_2(s)}}=\frac{G_1(s)G_2(s)}{1+G_2(s)+G_1(s)}$$

b) Draw the signal-flow graph of the block diagram. Simplify the graph if required. (Place the required nodes and branches. Label all signals and gains on the graph. Show the direction of branches.)

First place all nodes and then draw the branches.