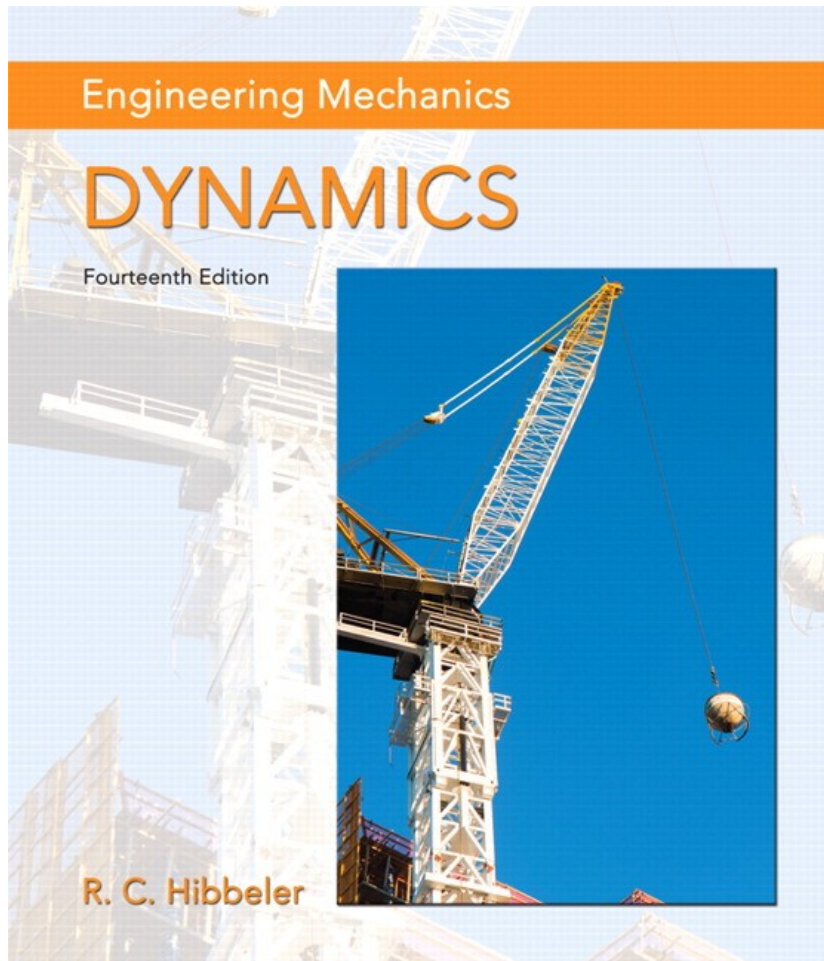


Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 14

Kinetics of a Particle:
Work and Energy

Conservative Forces, Potential Energy and Conservation of Energy (1 of 2)

Today's Objectives:

Students will be able to:

1. Use the concept of conservative forces and determine the potential energy of such forces.
2. Apply the principle of conservation of energy.



Conservative Forces, Potential Energy and Conservation of Energy (2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- **Conservative Force**
- **Potential Energy**
- **Conservation of Energy**
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1) The potential energy of a spring is _____

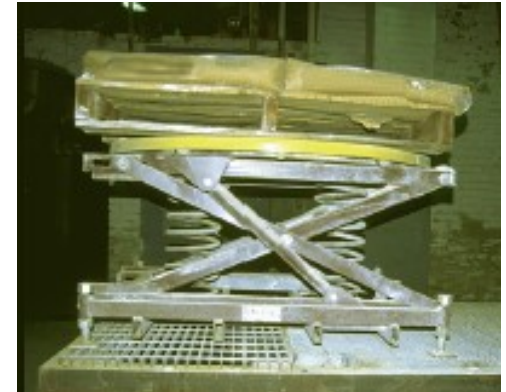
- A) always negative.
- B) always positive.
- C) positive or negative.
- D) equal to kx .

2) When the potential energy of a conservative system increases, the kinetic energy _____

- A) always decreases.
- B) always increases.
- C) could decrease or
- D) does not change. increase.

Applications (1 of 2)

1. The weight of the sacks resting on this platform causes potential energy to be stored in the supporting springs.
2. As each sack is removed, the platform will **rise** slightly since some of the potential energy within the springs will be transformed into an increase in gravitational potential energy of the remaining sacks.



If the sacks weigh 100 lb and the equivalent spring constant is $k = 500 \text{ lb} / \text{ft}$, what is the energy stored in the springs?

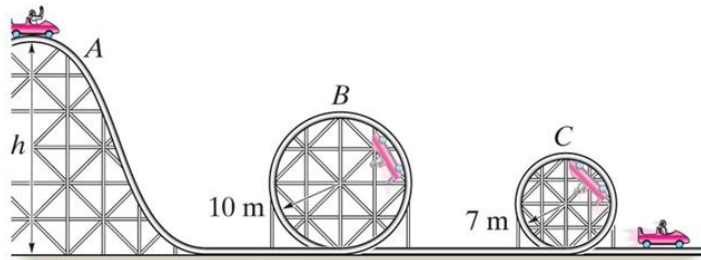
Applications (2 of 2)

The young woman pulls the water balloon launcher back, stretching each of the four elastic cords.

If we know the unstretched length and stiffness of each cord, can we estimate the maximum height and the maximum range of the water balloon when it is released from the current position? Would we need to know any other information?



Applications (3 of 3)



The roller coaster is released from rest at the top of the hill A. As the coaster moves down the hill, potential energy is transformed into kinetic energy.

What is the velocity of the coaster when it is at B and C?

Also, how can we determine the minimum height of hill A so that the car travels around both inside loops without leaving the track?

Section 14.5

Conservative Force

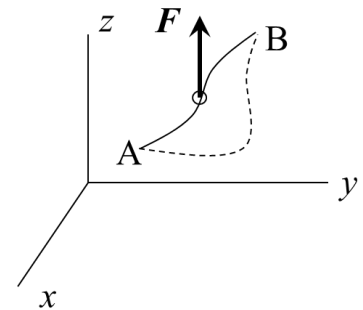
Conservative Force (1 of 2)

A force F is said to be conservative if the work done is **independent of the path** followed by the force acting on a particle as it moves from A to B. This also means that the work done by the force F in a closed path (i.e., from A to B and then back to A) is zero.

Thus, we say the work is **conserved**.

$$\oint F \cdot dr = 0$$

The work done by a conservative force depends **only** on the positions of the particle, and is **independent** of its velocity or acceleration.



Conservative Force (2 of 2)

A more rigorous definition of a conservative force makes use of a potential function (V) and partial differential calculus, as explained in the text. However, even without the use of these more complex mathematical relationships, much can be understood and accomplished.

The “conservative” potential energy of a particle/system is typically written using the potential function V . There are two major components to V commonly encountered in mechanical systems, the potential energy from gravity and the potential energy from springs or other elastic elements.

$$V_{total} = V_{gravity} + V_{springs}$$

Potential Energy (1 of 2)

Potential energy is a measure of the amount of work a **conservative force** will do when a body changes position.

In general, for any conservative force system, we can define the potential function (V) as a function of position. The work done by conservative forces as the particle moves equals the change in the value of the potential function (e.g., the sum of $V_{gravity}$ *and* $V_{springs}$).

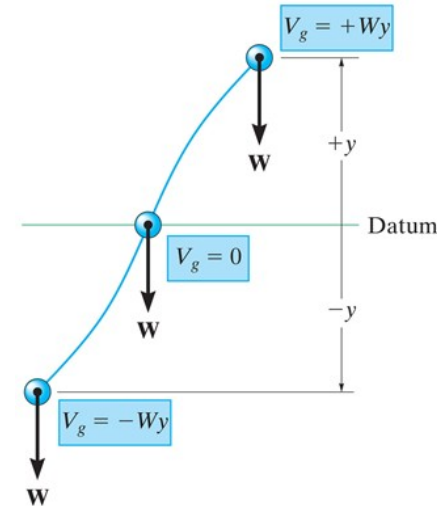
It is important to become familiar with the two types of potential energy and how to calculate their magnitudes.

Potential Energy due to Gravity

The potential function (formula) for a gravitational force, e.g., weight ($W = mg$), is the force multiplied by its elevation from a datum. The datum can be defined at any convenient location.

$$V_g = \pm W y$$

V_g is **positive** if y is above the datum and **negative** if y is below the datum. Remember, **you** get to set the datum.



Elastic Potential Energy

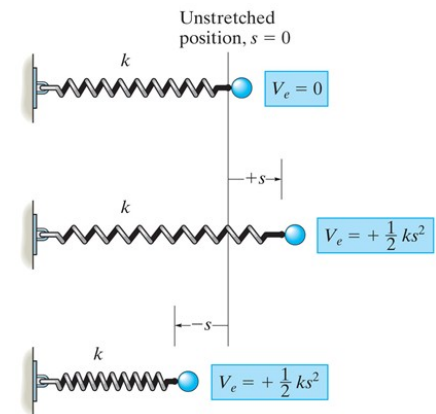
Recall that the **force** of an elastic spring is $F = ks$.

It is important to realize that the **potential energy** of a spring, while it looks similar, is a **different** formula.

V_e (where 'e' denotes an elastic spring) has the distance "s" raised to a power (the result of an integration) or

$$V_e = \frac{1}{2} ks^2$$

Notice that the potential function V_e always yields positive energy.



Section 14.6

Conservation of Energy

Conservation of Energy

When a particle is acted upon by a system of conservative forces, the work done by these forces is conserved and the **sum of kinetic energy and potential energy remains constant**. In other words, as the particle moves, kinetic energy is converted to potential energy and vice versa. This principle is called the principle of conservation of energy and is expressed as

$$T_1 + V_1 = T_2 + V_2 = \text{Constant}$$

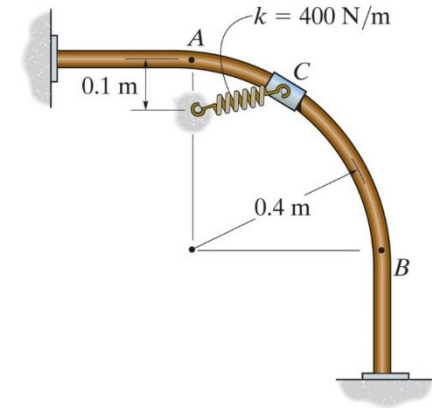
T_1 stands for the kinetic energy at state 1 and V_1 is the potential energy function for state 1. T_1 and V_2 represent these energy states at state 2. Recall, the kinetic energy is defined as $T = \frac{1}{2} mv^2$.

Example (1 of 2)

Given: The 4 kilogram collar, C, has a velocity of 2 m/s at A.

The spring constant is 400 N/m .

The unstretched length of the spring is 0.2 m .



Find: The velocity of the collar at B.

Plan: Apply the conservation of energy equation between A and B. Set the gravitational potential energy datum at point A or point B (in this example, choose point A-why?).

Example (2 of 2)

Solution

Note that the potential energy at B has two parts.

$$V_B = (V_B)_e + (V_B)_g$$

$$V_B = 0.5(400)(0.5 - 0.2)^2 - 4(9.81)(0.4)$$

The kinetic energy at B is $T_B = 0.5(4)v_B^2$

Similarly, the potential and kinetic energies at A will be

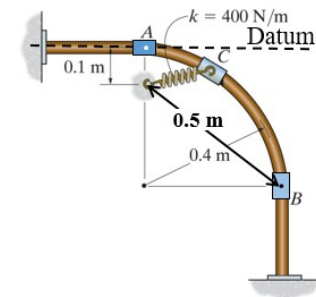
$$V_A = 0.5(400)(0.1 - 0.2)^2, \quad T_A = 0.5(4)2^2$$

The energy conservation equation becomes $T_A + V_A = T_B + V_B$.

$$[0.5(400)(0.5 - 0.2)^2 - 4(9.81)0.4] + 0.5(4)v_B^2$$

$$= [0.5(400)(0.1 - 0.2)^2] + 0.5(4)2^2$$

$$\Rightarrow v_B = 1.96 \text{ m/s}$$



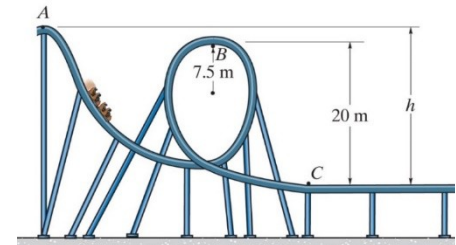
Concept Quiz

- 1) If the work done by a conservative force on a particle as it moves between two positions is $-10\text{ ft}\cdot\text{lb}$, the change in its potential energy is _____
- A) $0\text{ ft}\cdot\text{lb}$.
B) $10\text{ ft}\cdot\text{lb}$
C) $+10\text{ ft}\cdot\text{lb}$.
D) None of the above.
- 2) Recall that the work of a spring is $U_{1-2} = -\frac{1}{2} k(s_2^2 - s_1^2)$ and can be either positive or negative. The potential energy of a spring is $V = \frac{1}{2} ks^2$. Its value is _____
- A) always negative.
B) either positive or negative.
C) always positive.
D) an imaginary number!

Group Problem Solving I (1 of 3)

Given: The 800 kilogram roller coaster car is released from rest at A.

Find: The minimum height, h , of Point A so that the car travels around inside loop at B without leaving the track. Also find the velocity of the car at C for this height, h , of A.



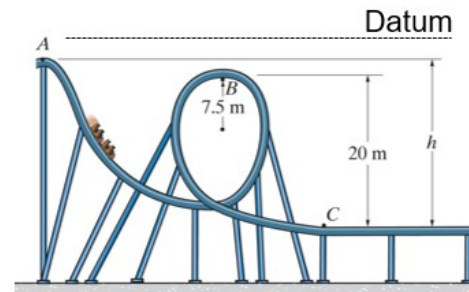
Plan: Note that only kinetic energy and potential energy due to gravity are involved. Determine the velocity at B using the equation of motion and then apply the conservation of energy equation to find minimum height h .

Group Problem Solving I (2 of 3)

Solution:

1) Placing the datum at A:

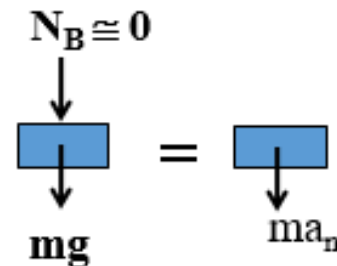
$$\begin{aligned}T_A + V_A &= T_B + V_B \\ \Rightarrow 0.5(800)0^2 + 0 \\ &= 0.5(800)(v_B)^2 - 800(9.81)(h - 20)\end{aligned}$$



2) Find the required velocity of the coaster at B so it doesn't leave the track.

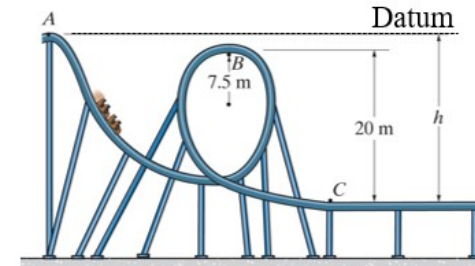
Equation of motion applied at B:

$$\begin{aligned}\sum F_n &= ma_n = m \frac{v^2}{\rho} & 800(9.81) &= 800 \frac{(v_B)^2}{7.5} \\ & & \Rightarrow v_B &= 8.578 \text{ m/s}\end{aligned}$$



Group Problem Solving I (3 of 3)

Now using the energy conservation, eq. (1), the minimum h can be determined



$$0.5(800)0^2 + 0 = 0.5(800)(8.578)^2 - 800(9.81)(h - 20)$$

$$\Rightarrow h = 23.75 \text{ m}$$

3) Find the velocity at C applying the energy conservation.

$$T_A + V_A = T_C + V_C$$

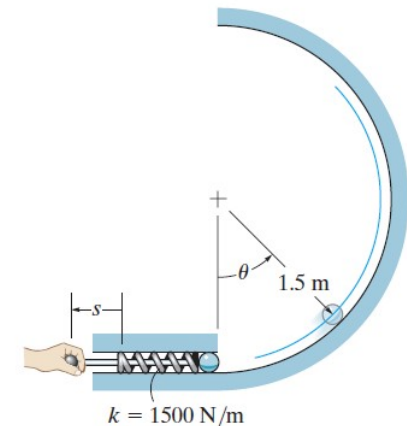
$$\Rightarrow 0.5(800)0^2 + 0 = 0.5(800)(v_C)^2 - 800(9.81)(23.75)$$

$$\Rightarrow V_C = 21.6 \text{ m/s}$$

Group Problem Solving II (1 of 3)

Given: The arm is pulled back such that $s = 100 \text{ mm}$ and released.

When $s = 0$, the spring is unstretched. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



Find: The speed of the 0.3-kilogram ball and the normal reaction of the circular track on the ball when $\theta = 60^\circ$.

Plan: Determine the velocity at $\theta = 60^\circ$ using the conservation of energy equation and then apply the equation of motion to find the normal reaction on the ball

Group Problem Solving II (2 of 3)

1) Placing the datum at A:

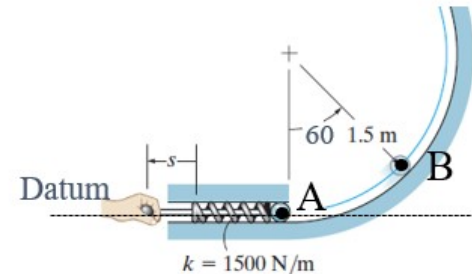
$$T_A + V_A = T_B + V_B$$

where

$$T_A = 0.5(0.3)v_A^2$$
$$V_A = 0 + 0.5(1500)(0.1)^2$$

$$T_B = 0.5(0.3)v_B^2$$

$$V_B = 0.3(9.81)(1.5)(1 - \cos 60^\circ)$$



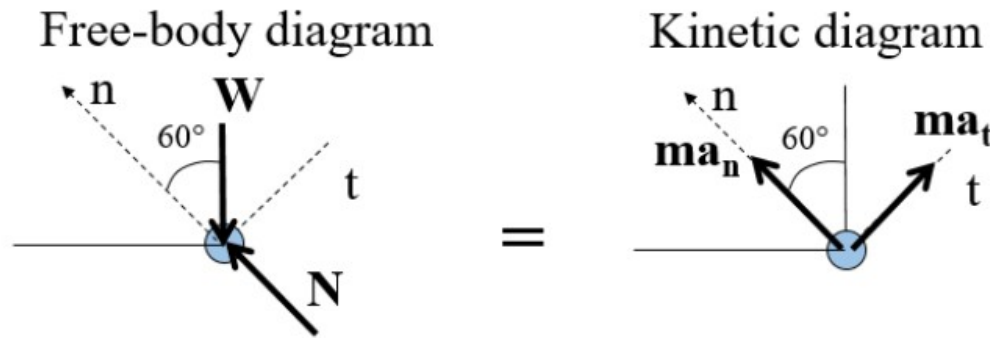
The conservation of energy equation is

$$0 + 0.5(1500)(0.1)^2 = 0.5(0.3)(v_B)^2 + 0.3(9.81)(1.5)(1 - \cos 60^\circ)$$

$$v_B = 5.94 \text{ m/s}$$

Group Problem Solving II (3 of 3)

2) Find the normal reaction on the ball when $\theta = 60^\circ$.



Equation of motion applied at $\theta = 60^\circ$:

$$\sum F_n = ma_n = m \frac{v_B^2}{\rho}$$

$$N - 0.31(9.81) \cos 60^\circ = 0.3 \frac{5.94^2}{1.5}$$

$$N = 8.53 \text{ N}$$

Attention Quiz

1) The principle of conservation of energy is usually _____ to apply than the principle of work & energy.

A) harder

B) easier

C) the same amount of work

D) It is a mystery!

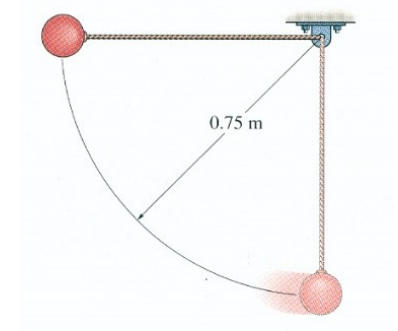
2) If the pendulum is released from the horizontal position, the velocity of its bob in the vertical position is _____

A) 3.8 m/s

B) 6.9 m/s .

C) 14.7 m/s .

D) 21 m/s



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