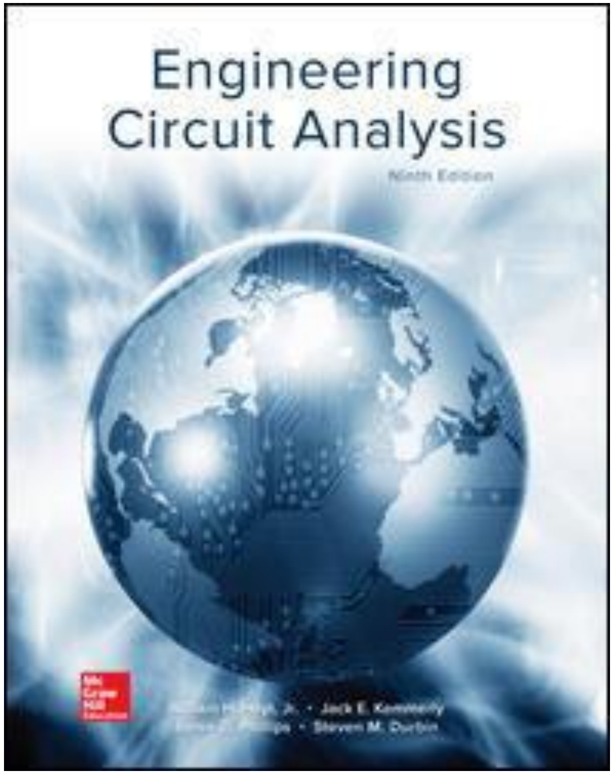
AC Circuit Power Analysis



Instantaneous Power

- This is the power at any instant in time
- It is the rate at which an element absorbs power
- Consider the generalized case where the voltage and current at the terminals of a circuit are:

$$v(t) = V_m \cos(\omega t + \theta_v) \qquad i(t) = I_m \cos(\omega t + \theta_i)$$

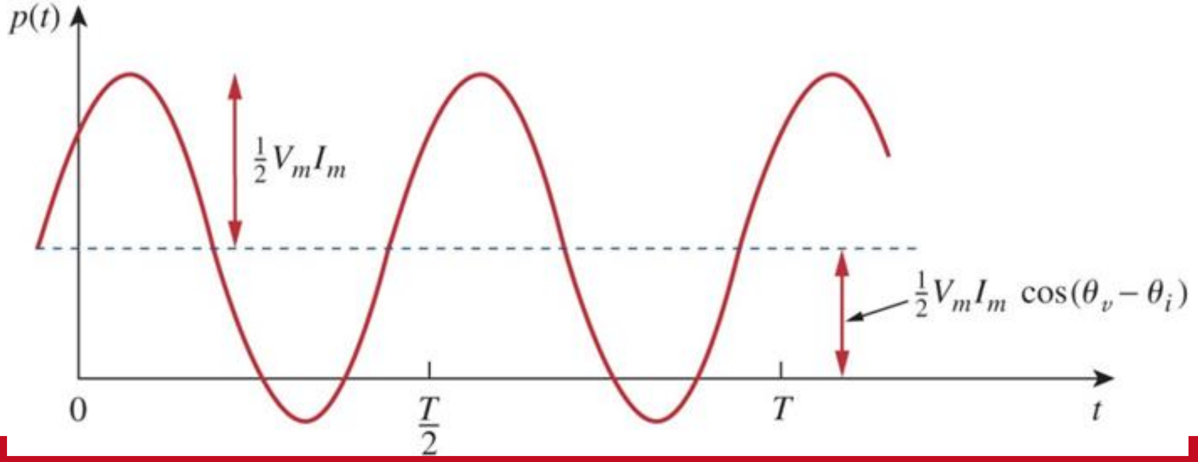
- Multiplying the two together, yields:

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Instantaneous Power

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

- One is constant, depending on the phase difference between the voltage and current
- The second is sinusoidal with a frequency twice that of the voltage and current.



Average Power

- Average power is the instantaneous power averaged over a period.
- It is given by:

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Resistive versus Reactive

- Consider the case when $\theta_v = \theta_i$ the voltage and current are in phase and the circuit is purely resistive:

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} |I|^2 R$$

- *When $\theta_v - \theta_i = \pm 90°$*, the circuit absorbs no power and is purely reactive.

$$P = \frac{1}{2} V_m I_m \cos(90°) = 0$$

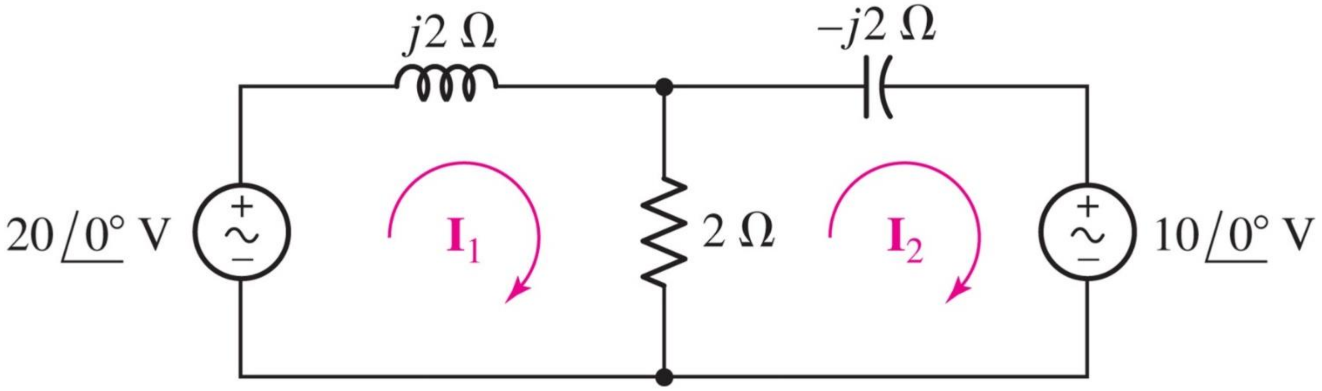
Average Power

Find the average power being delivered to an impedance $\mathbf{Z}_L = 8 - j11\ \Omega$ by a current $\mathbf{I} = 5\underline{/20^\circ}$ A.

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} \left| I \right|^2 R$$

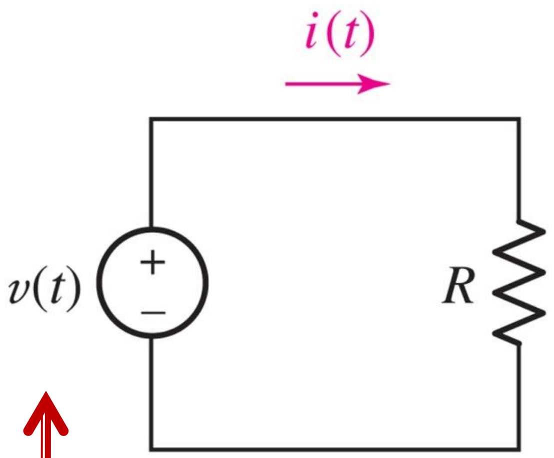
Example: Average Power

Find the average power absorbed by each element.

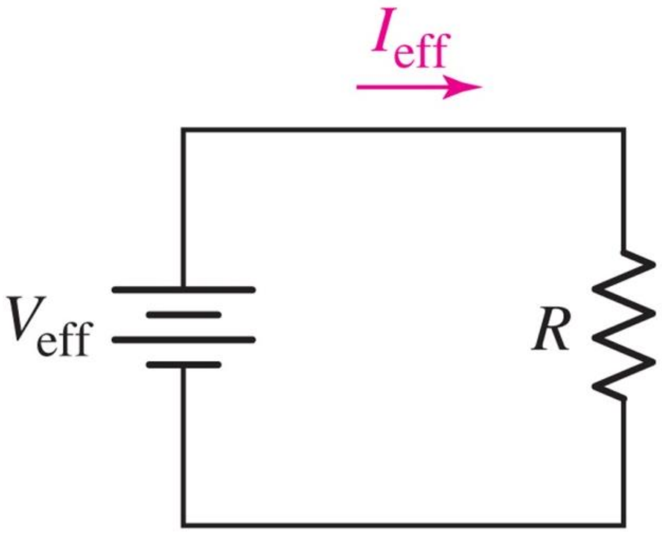


Effective Values of Current and Voltage

The same power is delivered to the resistor in the circuits shown.



Periodic, period T



$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

Effective (RMS) for Sine Wave

The effective value is often referred to as the root-mean-square or RMS value.

$$I_{\mathrm{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

For sine waves: $V_{eff} = \frac{1}{\sqrt{2}} V_m \cong 0.707 V_m$

Power is now $P = I_{eff}^2 R$

Apparent Power

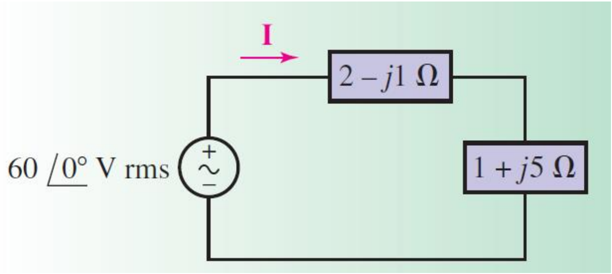
$$\text{If } v(t) = V_m \cos(\omega t + \theta) \text{ and } i(t) = I_m \cos(\omega t + \phi), \text{ then}$$

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$$

The apparent power is defined as $V_{eff}I_{eff}$ and is given the units volt-ampere V•A

Average and Apparent Power

Calculate values for the average power delivered to each of the two loads shown in Fig. 11.13, the apparent power supplied by the source, and the power factor of the combined loads.



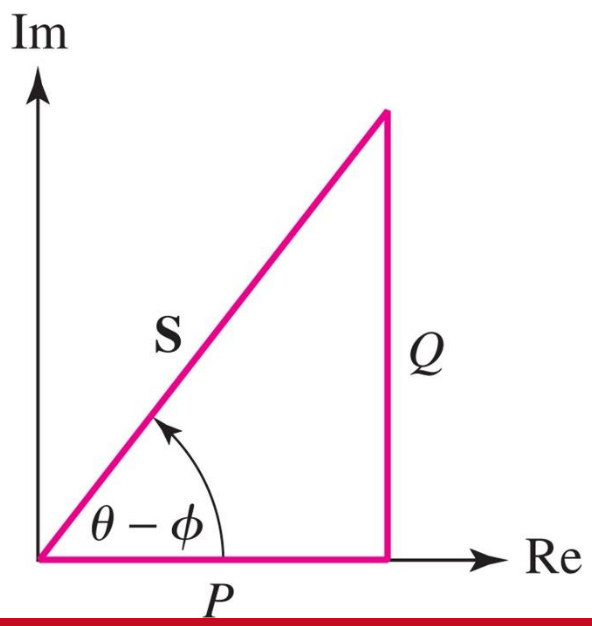
$$P = I_{eff}^{2} R$$

Apparent power = $V_{eff} I_{eff}$

Complex Power

Define the complex power **S** as

$$\mathbf{S} = \mathbf{V}_{eff}\mathbf{I}_{eff}^{*} = V_{eff}\, I_{eff}\, e^{j(\theta-\phi)} = P + jQ$$



Real part of **S** is *P*, the average power

Imaginary part of **S** is *Q*, the reactive power, which represents the flow of energy back and forth from the source (utility company) to the inductors and capacitors of the load (customer)

Summary of Complex Power Quantities

Quantity	Symbol	Formula	Units
Average power	P	$V_{\text{eff}} I_{\text{eff}} \cos(\theta - \varnothing)$	watt (W)
Reactive power	Q	$V_{\text{eff}} I_{\text{eff}} \sin(\theta - \varnothing)$	volt-ampere-reactive (VAR)
Complex power	S	$P + jQ$ $V_{\text{eff}} I_{\text{eff}} \underline{/\theta - \phi}$ $\mathbf{V}_{\text{eff}} \mathbf{I}^{*}_{\text{eff}}$	volt-ampere (VA)
Apparent power	|S|	$V_{\text{eff}} I_{\text{eff}}$	volt-ampere (VA)