Signal Processing (MENG3520)

Module 9

Weijing Ma, Ph. D. P. Eng.

MODULE 9

FOURIER METHODS – PART 3

Fourier Transform Family

The following four types are all part of the Fourier transform family:

- Continuous-time Fourier series (CTFS) – periodic continuous-time signals
- Continuous-time Fourier transform (CTFT) – aperiodic continuous-time signals
- Discrete-time Fourier transform (DTFT) – aperiodic discrete-time signals
- Discrete Fourier transform (DFT) – periodic discrete-time signals.

Module Outline

- 9.1 DTFS to DFT
- 9.2 Understanding DFT
- 9.3 Magnitude and Phase Spectrum of DFT

9.1

Discrete Time Fourier Series and Discrete Fourier Transform

Recall: Fourier Transform Family

The following four types are all part of the Fourier transform family:

- Continuous-time Fourier series (CTFS) – periodic continuous-time signals
- Continuous-time Fourier transform (CTFT) – aperiodic continuous-time signals
- Discrete-time Fourier transform (DTFT) – aperiodic discrete-time signals
- Discrete Fourier transform (DFT) – periodic discrete-time signals.

Implications of Different Signal Types

- CT: continuous in the time domain
- DT: discrete in the time domain

- Aperiodic: continuous in the frequency domain
- Periodic: discrete in the frequency domain

Discussion: are the following statements valid?

- Aperiodic → continuous in the frequency domain
- Periodic → discrete in the frequency domain

Characteristics of Different Fourier Transforms

	Time domain	Frequency domain
CTFS	Continuous & Periodic	Discrete
CTFT	Continuous & Aperiodic	Continuous
DTFT	Discrete & Aperiodic	Continuous
DFT	Discrete & Periodic	Discrete

Characteristics of Different Fourier Transforms

- While the first three methods, CTFS, CTFT, and DTFT are commonly used for analysis applications, DFT is the primary representation used for computational applications (the only that one can be evaluated on a computer)

- The introduction of fast computation algorithms such as fast Fourier transform (FFT) further contributes to the wide use of DFT, making it faster and less demanding on computational resource.

Recall: Continuous-Time Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

The CTFS coefficients c_k are defined as:

$$c_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$

$$x(t) \overset{CTFS}{\longleftrightarrow} c_k$$

Question: how to find a way to express a discrete-time periodic signal as a Fourier series with period N?

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$\Downarrow$$

$$x[n] = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 n}$$

Fourier Series Representation of Discrete-time Periodic Signals

Recall: a discrete-time signal $x[n]$ is periodic with period N if:

$$x[n] = x[n+N]$$

The fundamental period of this signal is:

$$\omega_0 = 2\pi/N$$

The entire set of **harmonically related** complex exponential signals that are periodic with period N:

$$\phi_k[n] = e^{jk\omega_0 n} = e^{jk\left(\frac{2\pi}{N}\right)n}, k = 0, \pm 1, \pm 2, \ldots$$

$$\phi_k[n] = e^{jk\omega_0 n} = e^{jk\left(\frac{2\pi}{N}\right)n}, \text{with } k = 0, \pm 1, \pm 2, \dots$$

Two observations on this set of signals:

1. The signal set is harmonically related with the fundamental frequency $\frac{2\pi}{N}$.
2. This signal set is periodic with period N.

Discussion 1: why are these signals harmonically related?

$$\phi_k[n] = e^{jk\omega_0 n} = e^{jk\left(\frac{2\pi}{N}\right)n}$$
$$k = 0, \pm 1, \pm 2, \ldots$$

Harmonically related: a group of complex sinusoids are harmonically related if there exists a constant ω_0 such that the **fundamental frequency** for each of these sinusoids is an integer multiple of ω_0.

Discussion 2: why are these signals periodic with period N?

$$\phi_k[n] = e^{jk\omega_0 n} = e^{jk\left(\frac{2\pi}{N}\right)n}$$
$$k = 0, \pm 1, \pm 2, \ldots$$

$$\phi_k[n+N] = e^{jk\omega_0(n+N)} = e^{jk\left(\frac{2\pi}{N}\right)(n+N)} = e^{jk\left(\frac{2\pi}{N}\right)n + 2\pi jk} = e^{jk\left(\frac{2\pi}{N}\right)n} = \phi_k[n].$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$x[n] = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 n}$$

$k = \langle N \rangle$ means k changes over the range of N consecutive integers starting from any value

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n}$$

$$\phi_k[n] = e^{jk\omega_0 n} = e^{jk\left(\frac{2\pi}{N}\right)n}, \text{with } k = 0, \pm 1, \pm 2, \dots$$

Two observations on this set of signals:

1. The signal set is harmonically related with the fundamental frequency $\frac{2\pi}{N}$.
2. This signal set is periodic with period N.

Based on these two observations, we conclude that only n signals are unique in this format:

$$\phi_k[n] = e^{jk\omega_0 n} = e^{jk\left(\frac{2\pi}{N}\right)n}, \text{with } k = 0, \pm 1, \pm 2, \dots, N-1$$

Goal of discrete-time Fourier series: to express a discrete-time periodic signal $x[n]$ as a sum of N harmonically-related **unique** complex exponentials.

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n}$$

Next question: How to compute discrete Fourier coefficients a_k?

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n}$$

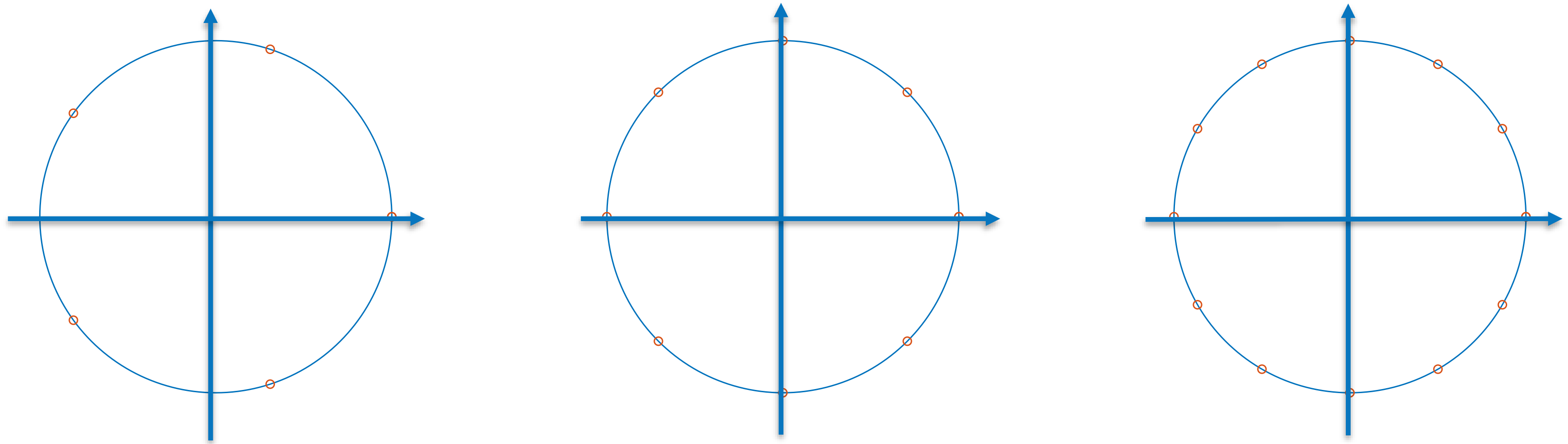
$$x[n]e^{-jr\omega_0 n} = \sum_{k=\langle N \rangle} a_k e^{j(k-r)\omega_0 n}$$

$$\sum_{n=\langle N \rangle} x[n]e^{-jr\omega_0 n} = \sum_{n=\langle N \rangle} \sum_{k=\langle N \rangle} a_k e^{j(k-r)\omega_0 n}$$

$$\sum_{n=\langle N \rangle} x[n]e^{-jr\omega_0 n} = \sum_{k=\langle N \rangle} a_k \sum_{n=\langle N \rangle} e^{j(k-r)\omega_0 n}$$

$$\sum_{n=\langle N\rangle} e^{j(k-r)\omega_0 n} = \sum_{n=\langle N\rangle} e^{j(k-r)\frac{2\pi}{N}n}$$

$$= \begin{cases} N, & when(k-r) = 0, \pm N, \pm 2N, \dots \\ 0, & otherwise. \end{cases}$$



If r is with the same range as k: $[0, N-1]$, then:

$$-N < k - r < N$$

$$\sum_{n=\langle N \rangle} e^{j(k-r)\omega_0 n}$$

$$= \begin{cases} N, \cancel{(k-r) = 0, or \pm N, \pm 2N, \ldots}\, k = r. \\ 0, \qquad\qquad otherwise. \end{cases}$$

With condition:

$$\sum_{n=\langle N\rangle} e^{j(k-r)\omega_0 n} = \begin{cases} N, if\ k = r. \\ 0, otherwise. \end{cases}$$

The original sum:

$$\sum_{n=\langle N\rangle} x[n]e^{-jr\omega_0 n} = \sum_{k=\langle N\rangle} a_k \sum_{n=\langle N\rangle} e^{j(k-r)\omega_0 n}$$

The original sum

Becomes:

$$\sum_{n=\langle N\rangle} x[n]e^{-jr\omega_0 n} = Na_r$$

Thus:

$$\boxed{a_r = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jr\omega_0 n}}$$

These are the discrete Fourier coefficients that we are looking for!

DISCRETE-TIME FOURIER SERIES PAIR

Expressing the discrete-time periodic signal into a summation of harmonically related complex exponentials:

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} a_k e^{jk\frac{2\pi}{N}n}$$

Here, Fourier series coefficients are:

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\frac{2\pi}{N}n}$$

These coefficients a_k are referred to as Fourier spectral coefficients of $x[n]$.

a_k is periodic as well with period N, i.e. $a_k = a_{k+N}$. (Why?)

Relationship of DTFS and DFT

- By analyzing a discrete-time periodic signal using DTFS, both the time domain and the Fourier domain will be discrete. DTFS satisfies the computing and storage needs of modern computers.
- For discrete-time aperiodic signals, since computationally they are of limited length, it is assumed that these signals are periodic with the period being the entire length N and then DTFS will apply: this is the generalized Discrete Fourier Transform (DFT).
- Since in Engineering, our signals and system responses are always of limited length, they can be considered periodic by duplicating the signals outside of their defined range. Therefore, DFT is widely used to analyze limited length discrete-time signals and systems.

9.2

UNDERSTANDING DFT

Understanding DFT

DFT synthesis equation (inverse DFT):

$$x[n] = \sum_{k=\langle N \rangle} X_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} X_k e^{jk\frac{2\pi}{N}n}$$

DFT analysis equation (forward DFT):

$$X[k] = X_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\frac{2\pi}{N}n}$$

Understanding DFT

DFT synthesis equation (inverse DFT):

$$x[n] = \sum_{k=\langle N \rangle} X_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} X_k e^{jk\frac{2\pi}{N}n} = \sum_{k=\langle N \rangle} X_k e^{2\pi j\frac{k}{N}n}$$

Implication: any discrete-time signal of length N can be decomposed into a series of N harmonically related complex sinusoids.

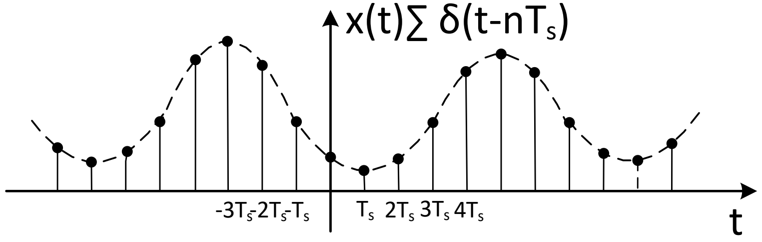
The nominal digital frequency components are: $\boxed{\mathbf{0}, \frac{\mathbf{1}}{\boldsymbol{N}}, \frac{\mathbf{2}}{\boldsymbol{N}}, \frac{\mathbf{3}}{\boldsymbol{N}}, \ldots, \frac{(\boldsymbol{N}-\mathbf{1})}{\boldsymbol{N}}.}$

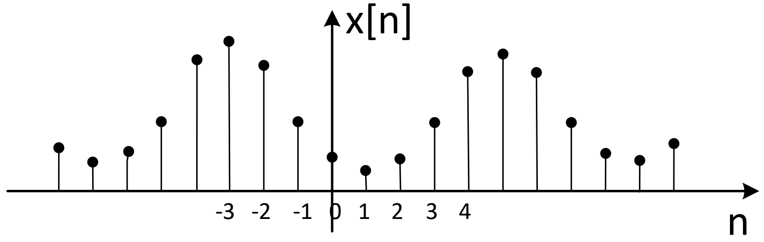
This is the normalized spectrum we used in our labs.

Understanding DFT

Frequency representation of DFT: one crucial notion is the normalized labeling of the time axis in discrete-time signal representing a continuous-time signal.

What is the difference between the following two sampled signals?





$$n = \frac{t}{T_s} = t f_s$$

Understanding DFT

$$x[n] = \sum_{k=\langle N \rangle} X_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} X_k e^{jk\frac{2\pi}{N}n}$$

Conventionally, we use $k = 0,1,..\mathrm{N} - 1$. Since, $n = tf_s$, the analog frequency components are: $0, \frac{1}{N}f_s, \frac{2}{N}f_s, \frac{3}{N}f_s, \dots, \frac{(N-1)}{N}f_s$. This means DFT discretize N frequency components equally between 0 and f_s if the DT signal is sampled from an analog signal at frequency f_s.

The frequency resolution is: $\frac{1}{N}f_s$

Understanding DFT

Impact of N: because DFT discretizes N frequency components equally between 0 and 1. increasing N will increase the frequency resolution of the DFT.

Example:

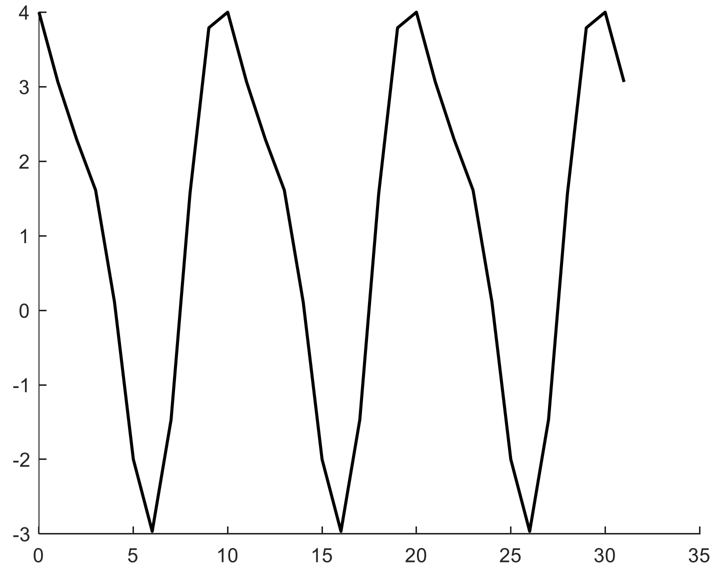
$$x[n] = 1 + \sin(\frac{2\pi}{N}n) + 3\cos(\frac{2\pi}{N}n) + \cos(\frac{4\pi}{N}n + \frac{\pi}{2})$$

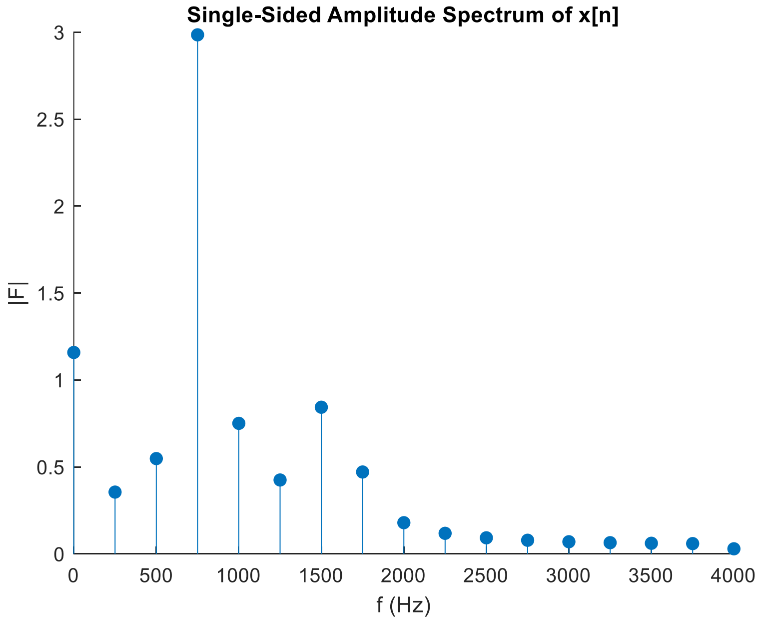
$$f_s = 8\ kHz$$

Understanding DFT

Impact of N: because DFT discretizes N frequency components equally between 0 and 1. increasing N will increase the frequency resolution of the DFT.

e.g. N=32:

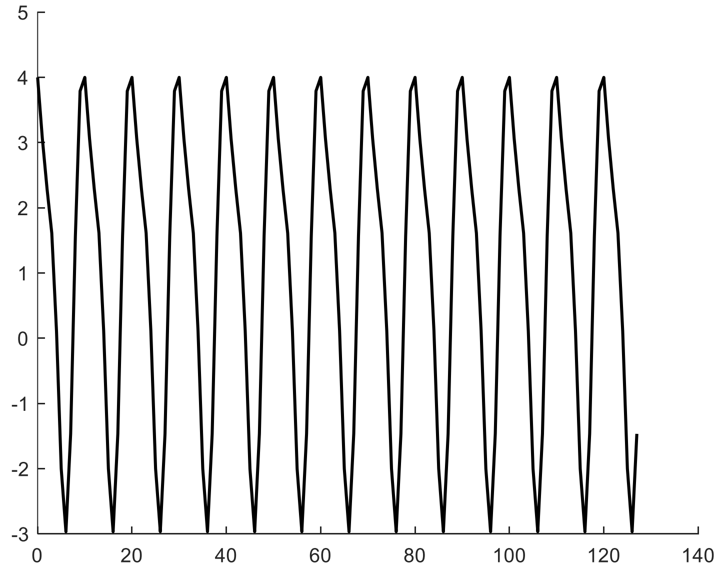


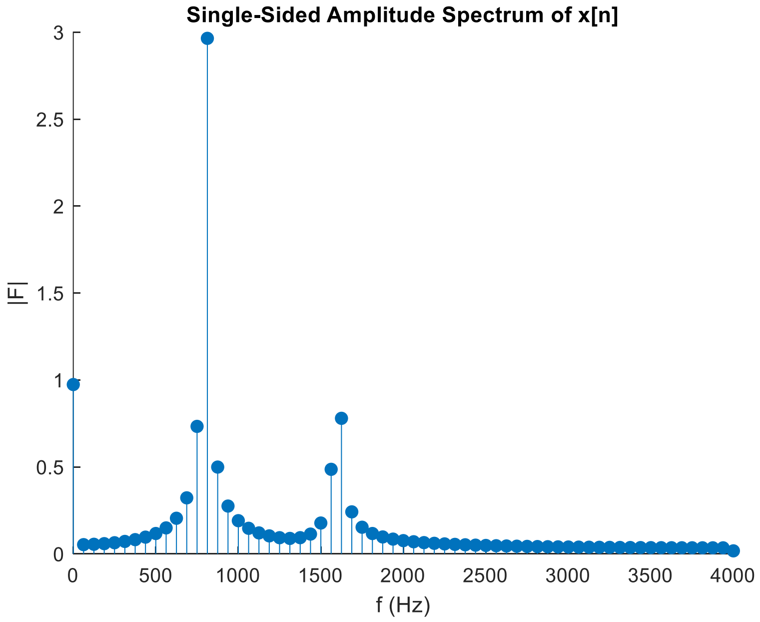


Understanding DFT

Impact of N: because DFT discretizes N frequency components equally between 0 and 1. increasing N will increase the frequency resolution of the DFT.

e.g. N=128:

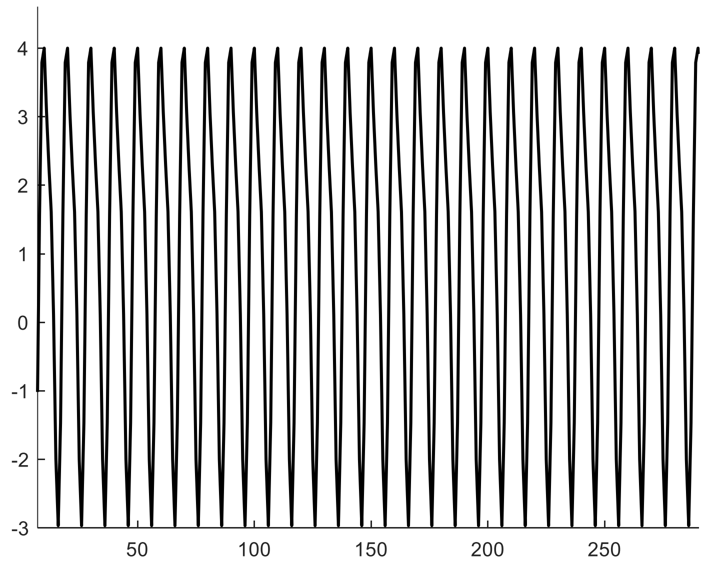


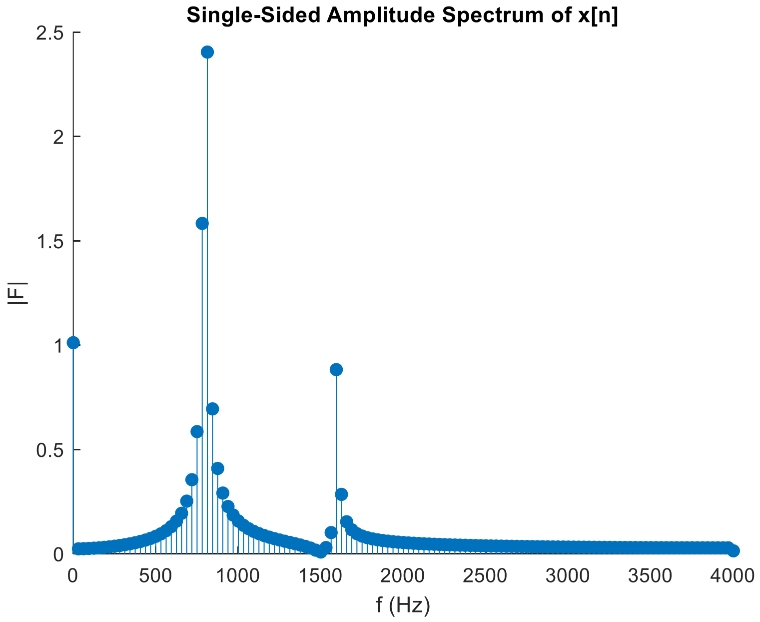


Understanding DFT

Impact of N: because DFT discretizes N frequency components equally between 0 and 1. increasing N will increase the frequency resolution of the DFT.

e.g. N=512:





Understanding DFT

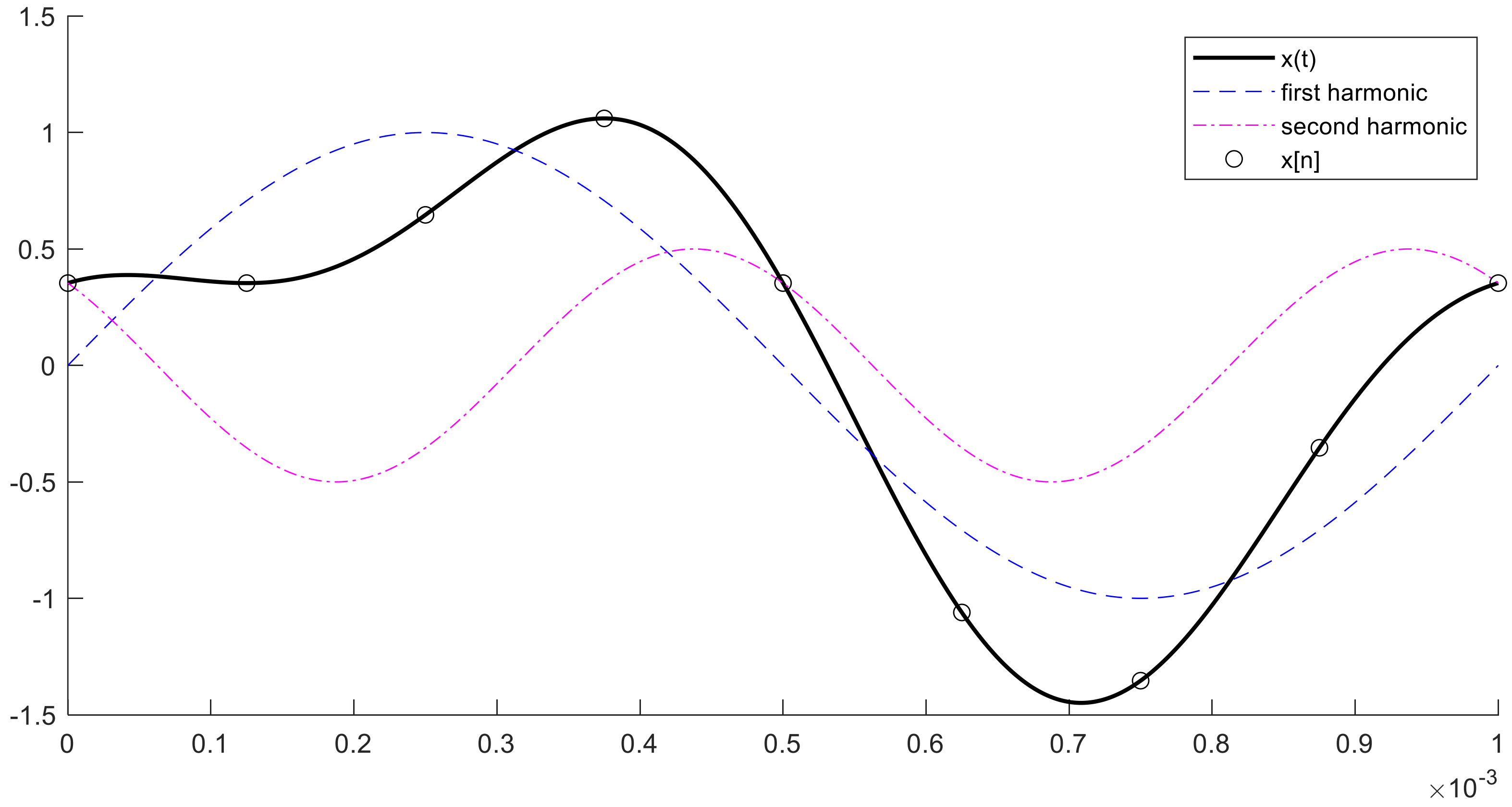
DFT analysis equation (forward DFT):

$$X[k] = X_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\omega_0 n} = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\frac{2\pi}{N}n}$$

The corresponding coefficients of each harmonically related complex sinusoid can be computed using the above equation – computationally intense.

DFT Example

$$x(t) = \sin(2\pi f_0 t) + 0.5\sin\left(4\pi f_0 t + \frac{3\pi}{4}\right)$$



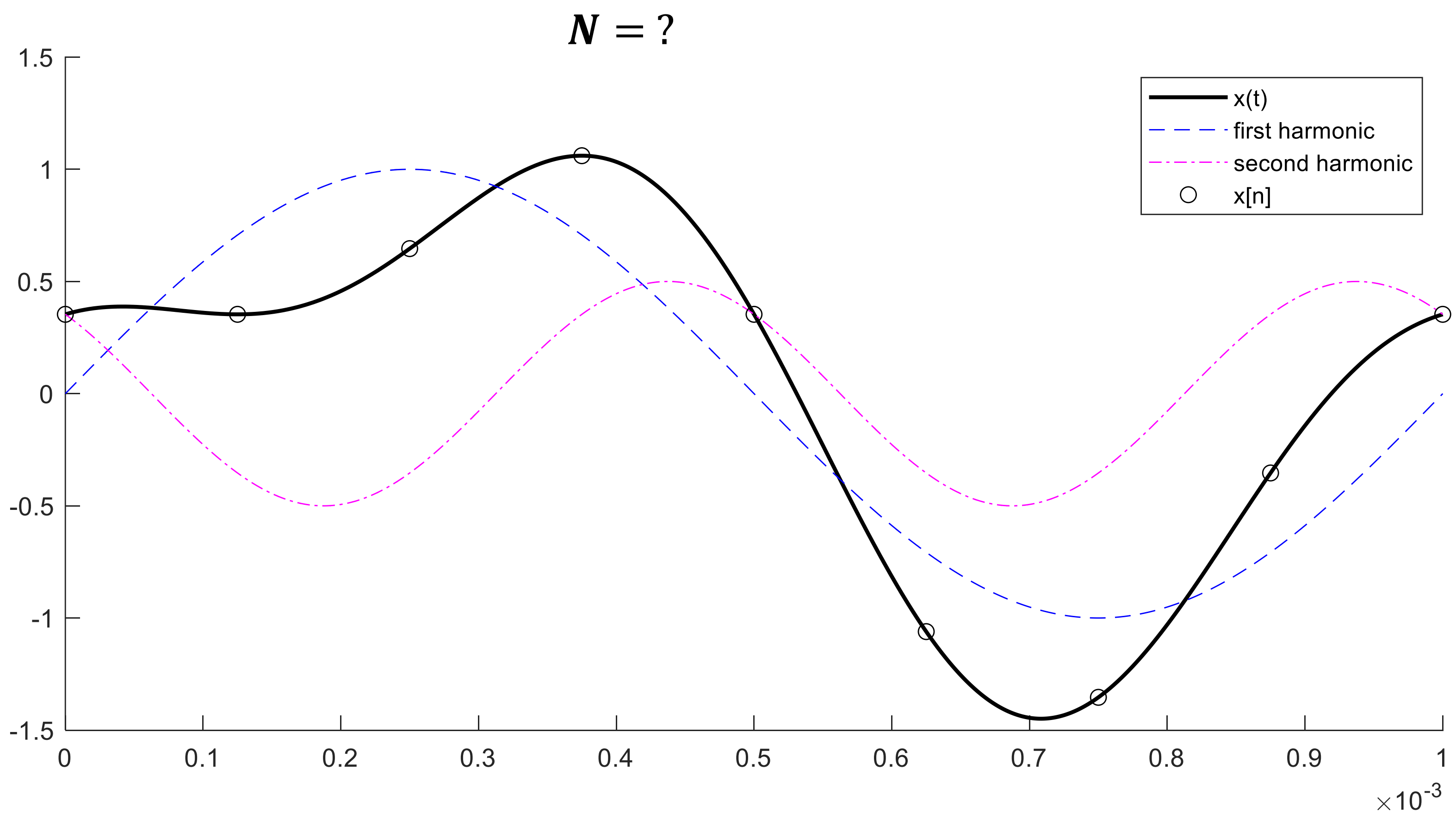
DFT Example

$$x(t) = \sin(2\pi f_0 t) + 0.5\sin\left(4\pi f_0 t + \frac{3\pi}{4}\right)$$

- Compute the following:
- Fundamental frequency of the original signal.
- Sampling frequency f_s.
- Signal length N for DFT (assuming periodic).

DFT Example

$$x(t) = \sin(2\pi f_0 t) + 0.5\sin\left(4\pi f_0 t + \frac{3\pi}{4}\right)$$



DFT EXAMPLE

$$x(t) = \sin(2\pi f_0 t) + 0.5 \sin\left(4\pi f_0 t + \frac{3\pi}{4}\right)$$

Compute the following:

- The fundamental frequency of the DFT output. (note: this is different from the fundamental frequency of the original signal).
- Harmonic frequencies of DFT outputs.
- Implement DFT on the sampled signal.

DFT Example

$$x(t) = \sin(2\pi f_0 t) + 0.5\sin\left(4\pi f_0 t + \frac{3\pi}{4}\right)$$

- Implement DFT on the sampled signal:

$$X[k] = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\left(\frac{2\pi}{N}\right)n}$$

$$X[0] = \frac{1}{8}\sum_{n=\langle N\rangle} x[n]e^{-j\times 0\times\left(\frac{2\pi}{8}\right)n}$$

$$X[1] = \frac{1}{8}\sum_{n=\langle N\rangle} x[n]e^{-j\times 1\times\left(\frac{2\pi}{8}\right)n}$$

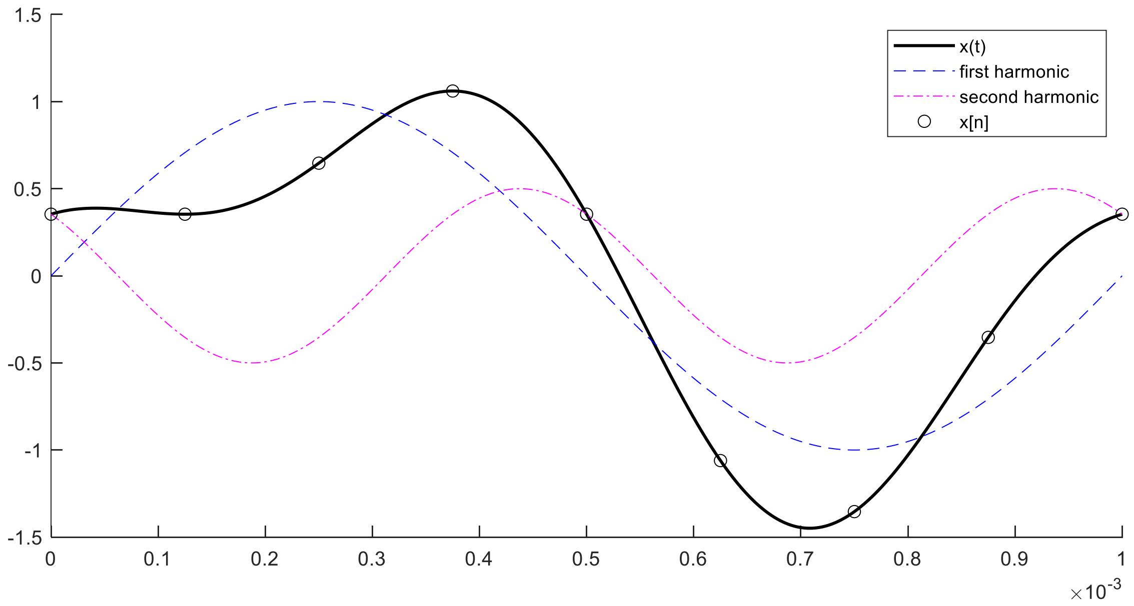
$$X[2] = \frac{1}{8}\sum_{n=\langle N\rangle} x[n]e^{-j\times 2\times\left(\frac{2\pi}{8}\right)n}$$

DFT Example

$$X[0] = \frac{1}{8} \sum_{n=\langle N \rangle} x[n] e^{-j\times 0\times\left(\frac{2\pi}{8}\right)n}$$

$$= \frac{1}{8} \sum_{n=\langle N \rangle} x[n] \times 1$$

$$= \frac{1}{8} \sum_{n=\langle N \rangle} x[n] = 0$$



DFT EXAMPLE

$$X[1] = \frac{1}{8}\sum_{n=\langle N\rangle} x[n]e^{-j\left(\frac{2\pi}{8}\right)n}$$

$$= \frac{1}{8}x[0](\cos(0) - j\sin(0))$$

$$+\frac{1}{8}x[1]\left(\cos\left(\frac{\pi}{4}\right) - j\sin\left(\frac{\pi}{4}\right)\right)$$

$$+\frac{1}{8}x[2]\left(\cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right)\right)$$

$$+\frac{1}{8}x[3]\left(\cos\left(\frac{3\pi}{4}\right) - j\sin\left(\frac{3\pi}{4}\right)\right)$$

$$+\cdots$$

$$+\frac{1}{8}x[7]\left(\cos\left(\frac{7\pi}{4}\right) - j\sin\left(\frac{7\pi}{4}\right)\right) = 0.0 - j0.5$$

DFT Example

$$X[2] = \frac{1}{8}\sum_{n=\langle N\rangle} x[n]e^{-j\left(\frac{4\pi}{8}\right)n}$$

$$= \frac{1}{8}x[0](\cos(0) - j\sin(0))$$

$$+\frac{1}{8}x[1]\left(\cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right)\right)$$

$$+\frac{1}{8}x[2](\cos(\pi) - j\sin(\pi))$$

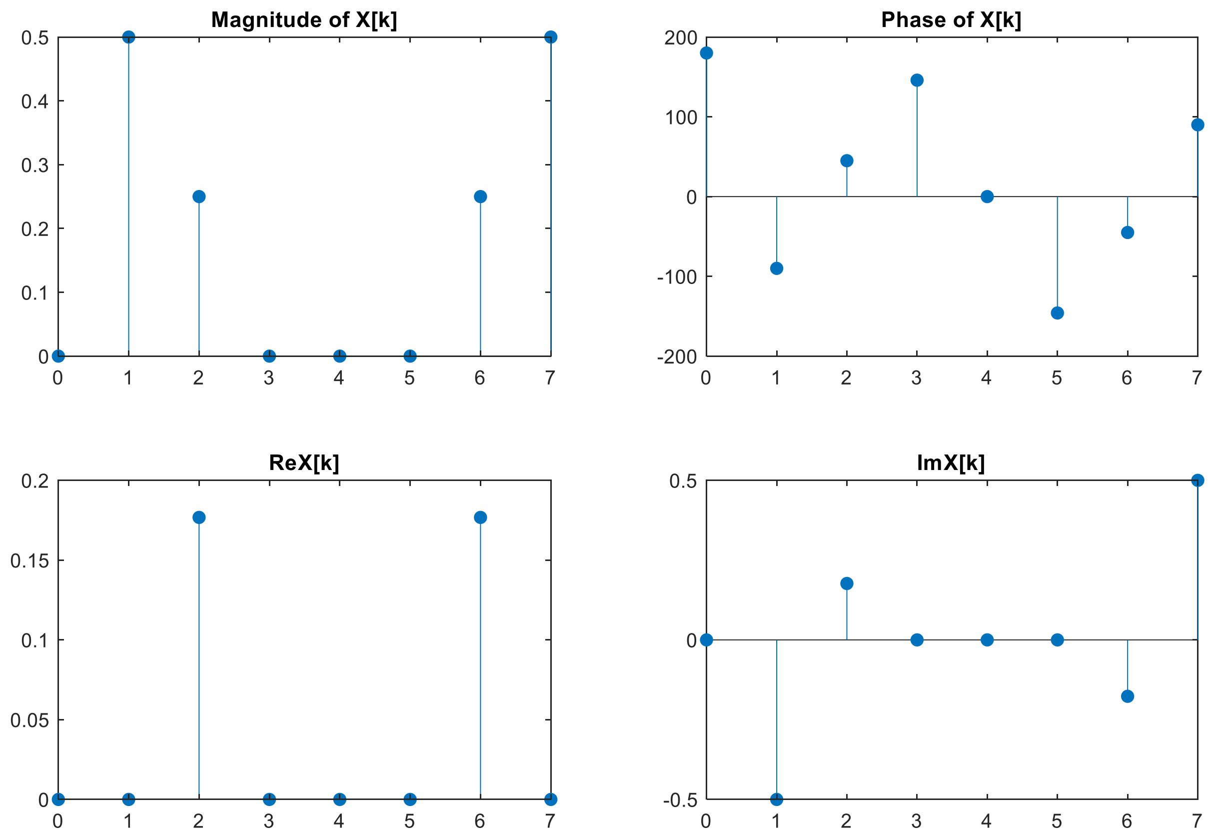
$$+\frac{1}{8}x[3]\left(\cos\left(\frac{3\pi}{2}\right) - j\sin\left(\frac{3\pi}{2}\right)\right)$$

$$+\cdots$$

$$+\frac{1}{8}x[7]\left(\cos\left(\frac{7\pi}{2}\right) - j\sin\left(\frac{7\pi}{2}\right)\right) = 0.176 + j0.176$$

DFT Example

$$x(t) = \sin(2\pi f_0 t) + 0.5\sin\left(4\pi f_0 t + \frac{3\pi}{4}\right)$$



SPECIAL CHARACTERISTICS FOR REAL SIGNALS

A signal $x[n]$ is real if and only its Fourier spectrum satisfies:

$$X[k] = X^* \, [-k] \text{ for all } k$$

i.e. $X[k]$ has conjugate symmetry.

Note that $x[n]$ being real does not necessarily imply that $X[k]$ is real.

Special Characteristics for Real Signals

Thus, the frequency spectrum satisfies:

- Real part is symmetric (even)

$$re\{X[-k]\} = re\{X[k]\}$$

- Imaginary part is antisymmetric (odd).

$$im\{X[-k]\} = -im\{X[k]\}$$

- Magnitude is symmetric (even).

$$|X[-k]| = |X[k]|$$

- Phase is antisymmetric (odd).

$$\angle X[-k] = -\angle X[k]$$

High-dimensional DFT

The high-dimensional DFT means to convert a sequence with more than one independent variables into the frequency domain. It can be generalized from the 1-dimensional case:

For high-dimensional signal $x[n_1, n_2, \dots n_d]$, you can take DFT on each independent variable respectively. Let's use two dimensional DFT on $x[n_1, n_2]$ as an example.

Recall: 1-dimensional DFT analysis equation (forward DFT) is:

$$X[k] = X_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\frac{2\pi}{N}n}$$

$$X[k_1, k_2] = \frac{1}{N_2} \sum_{n=\langle N_2 \rangle} \left(\frac{1}{N_1} \sum_{n_1=\langle N_1 \rangle} x[n_1, n_2] e^{-jk_1\omega_0 n_1} \right) e^{-jk_2\, \omega_0 n_2}$$

$$= \frac{1}{N_1\, N_2} \sum_{n=\langle N_2 \rangle} \sum_{n_1=\langle N_1 \rangle} x[n_1, n_2] e^{-j(k_1\omega_0 n_1 + k_2\, \omega_0 n_2)}$$

High-dimensional DFT

Generalized high-dimensional DFT analysis equation (forward DFT) of $x[n_1, \dots, n_d]$ is:

$$X[k_1, k_2, \dots, k_d]$$

$$= \frac{1}{\prod_{j=1}^{d} N_j} \sum_{n_1=\langle N_1\rangle} \sum_{n_2=\langle N_2\rangle} \dots \sum_{n_d=\langle N_d\rangle} x[n_1, n_2, \dots, n_d] e^{-j(k_1\omega_0 n_1 + k_2\, \omega_0 n_2 + \dots + k_d\, \omega_0 n_d)}$$

$$= \frac{1}{\prod_{j=1}^{d} N_j} \sum_{n_1=\langle N_1\rangle} \sum_{n_2=\langle N_2\rangle} \dots \sum_{n_d=\langle N_d\rangle} x[n_1, n_2, \dots, n_d] e^{-2\pi j\left(\frac{k_1 n_1}{N_1} + \frac{k_2 n_2}{N_2} + \dots + \frac{k_d n_d}{N_d}\right)}$$

Generalized high-dimensional DFT synthesis equation (inverse DFT) of $X[k_1, \dots, k_d]$is:

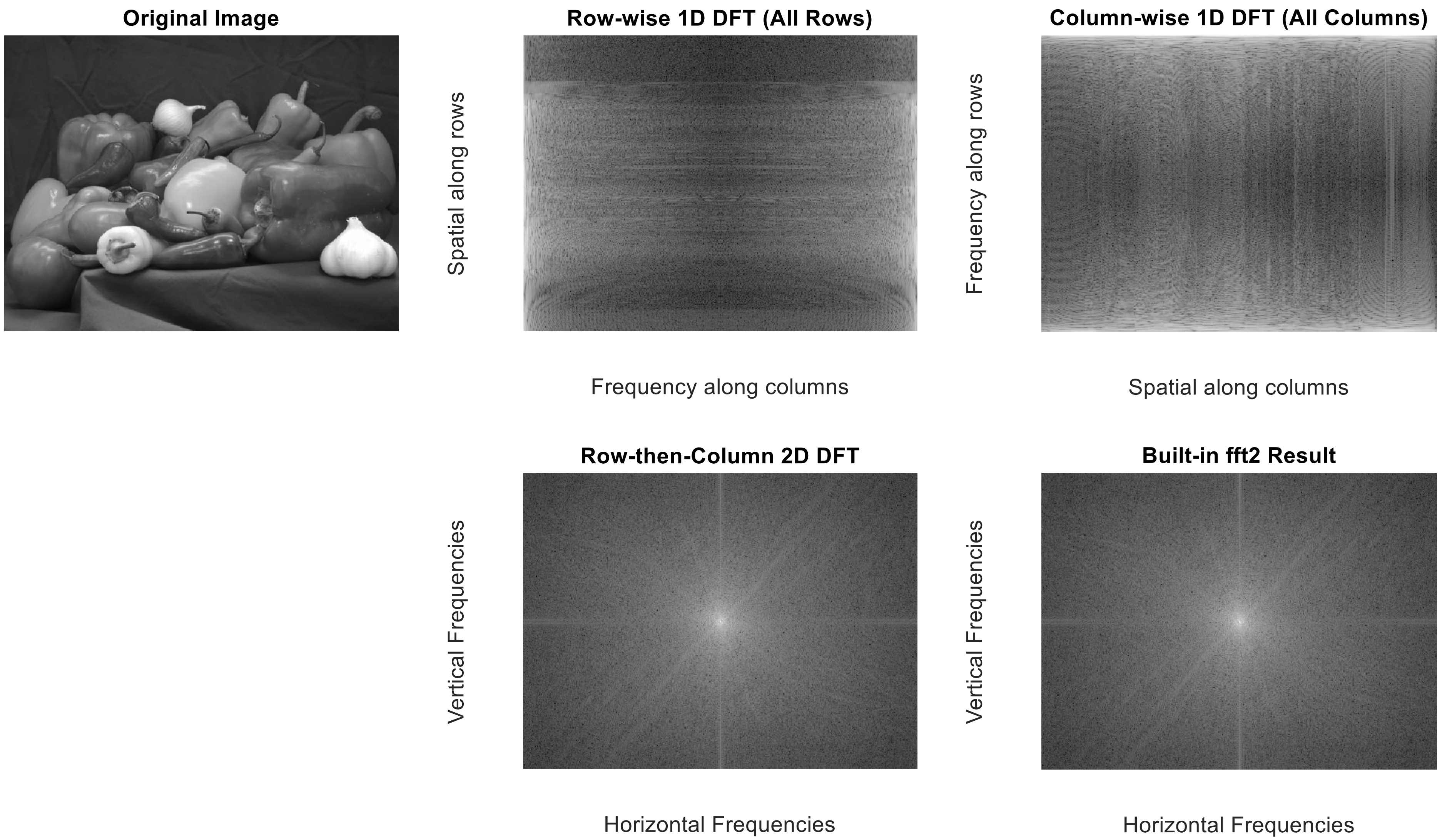
$$x[n_1, n_2, \dots, n_d]$$

$$= \sum_{k_1=\langle N_1\rangle} \sum_{k_2=\langle N_2\rangle} \dots \sum_{k_d=\langle N_d\rangle} X[k_1, k_2, \dots, k_d] e^{j(k_1\omega_0 n_1 + k_2\, \omega_0 n_2 + \dots + k_d\, \omega_0 n_d)}$$

$$= \sum_{k_1=\langle N_1\rangle} \sum_{k_2=\langle N_2\rangle} \dots \sum_{k_d=\langle N_d\rangle} X[k_1, k_2, \dots, k_d] e^{2\pi j\left(\frac{k_1 n_1}{N_1} + \frac{k_2 n_2}{N_2} + \dots + \frac{k_d n_d}{N_d}\right)}$$

High-dimensional DFT – Example

Digital images are commonly used two-dimensional signals.



9.3

MAGNITUDE AND PHASE SPECTRUM OF DFT

RECALL: PURPOSE OF FOURIER TRANSFORM

- Conversion between time domain signals and their frequency domain representations.
- Providing a new perspective of viewing a signal.
- Describing how information is distributed at different frequencies.
- The distribution of information in a signal over different frequencies is referred to as the frequency spectrum of the signal.

Recall: Frequency Spectrum

- In the context of DFT, the Fourier coefficients $X[k]$ are referred to as the **frequency spectrum** of periodic signal $x(t)$.
- The magnitude of the Fourier coefficients $|X[k]|$ is referred to as the **magnitude spectrum** of the periodic signal $x(t)$.
- The phase angle of the Fourier coefficients $\measuredangle X[k]$ is referred to as the **phase spectrum** of the periodic signal $x(t)$.
- The Fourier frequency spectrum is usually represented using two plots: the magnitude spectrum, and the phase spectrum.

Discrete Fourier Transform

DFT synthesis equation (inverse DFT):

$$x[n] = \sum_{k=\langle N \rangle} X_k e^{jk\frac{2\pi}{N}n} = \sum_{k=\langle N \rangle} |X_k| e^{j(k\frac{2\pi}{N}n+\phi_k)}$$

With X_k expressed in polar form.

Question: the term for $k = K$ is the Kth harmonic component, what is its fundamental frequency? What is its amplitude? What is its phase angle?

Question: What Type of Information is Embedded in Phase Spectrum and Magnitude Spectrum?

Let's examine the following two situations:

- Signals of the same magnitude spectrum and different phase spectrums
- Signals of different magnitude spectrums but the same phase spectrum

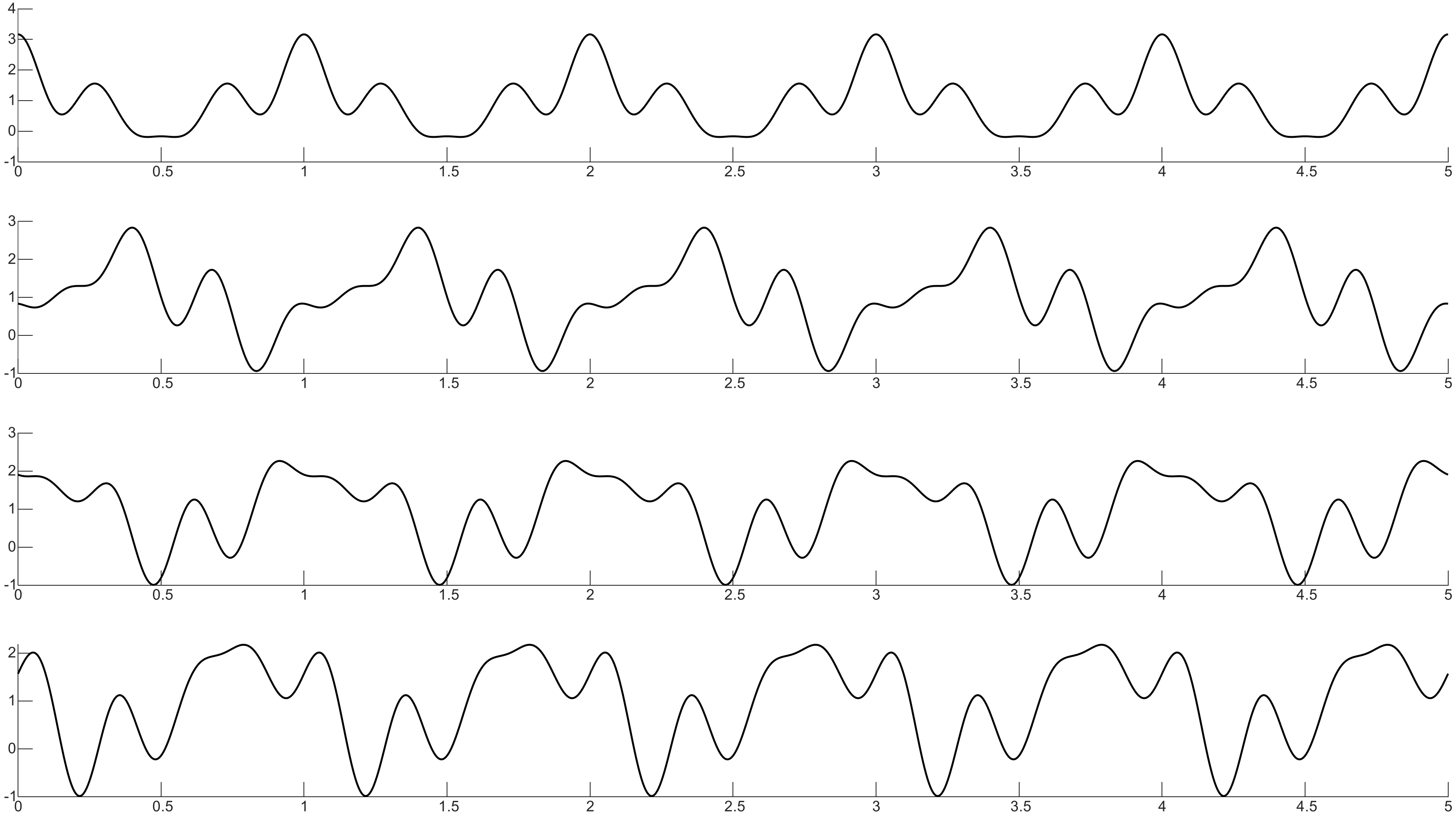
WHAT IS IN PHASE SPECTRUM AND MAGNITUDE SPECTRUM?

- Example: signals of the same magnitude spectrum and different phase spectrums

$$x(t) = A_0 + A_1 \cos(2\pi t + \phi_1)$$
$$+ A_2 \cos(6\pi t + \phi_2) + A_3 \cos(8\pi t + \phi_3)$$

Change ϕ_1, ϕ_2 and ϕ_3 and plot the resulting graphs.

SIGNALS WITH DIFFERENT PHASE ANGLES



Question: What Type of Information is Embedded in Phase Spectrum and Magnitude Spectrum?

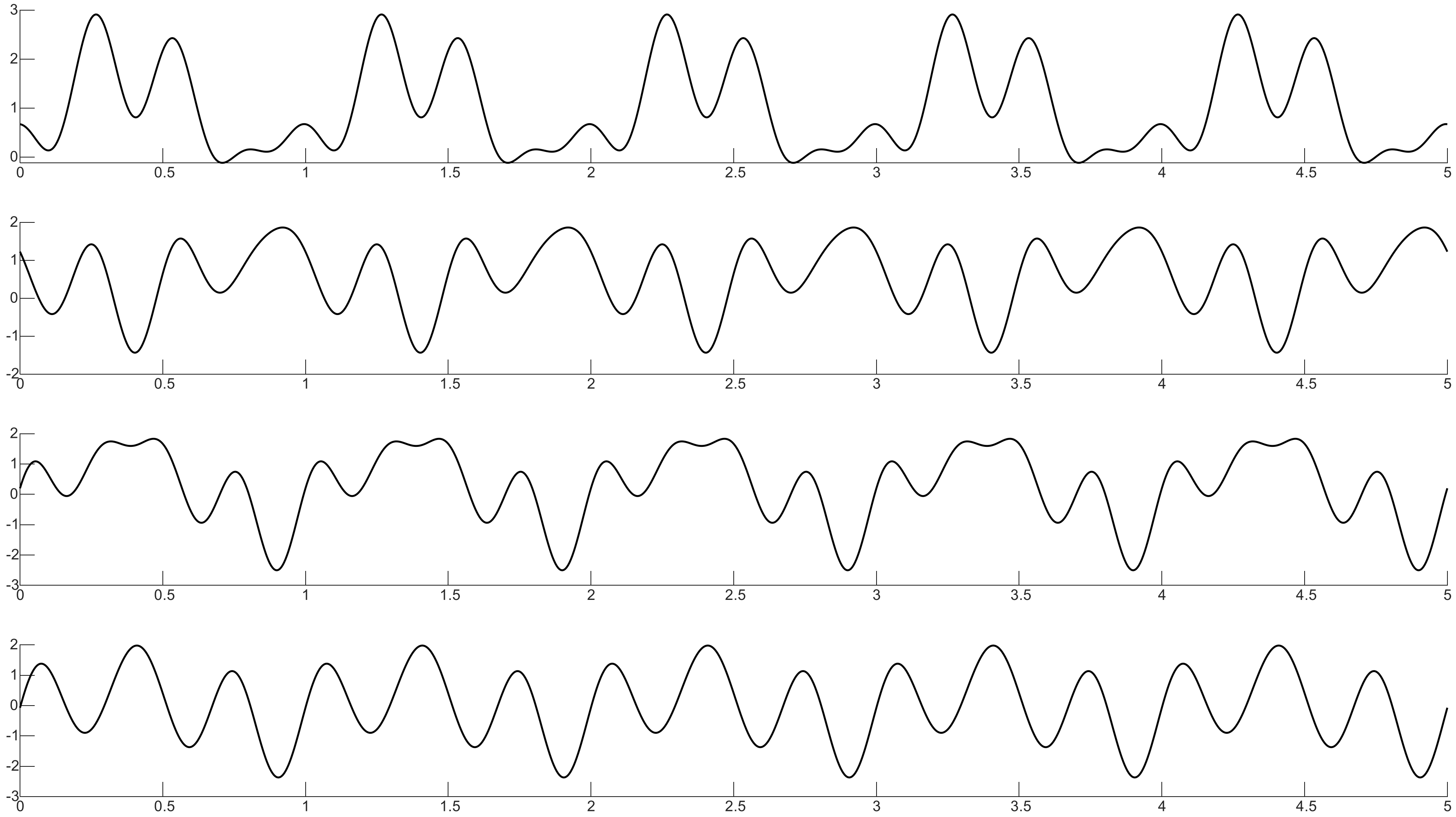
- Example: signals of different magnitude spectrum but the same phase spectrum

$$x(t) = A_0 + A_1 \cos(2\pi t + \phi_1)$$

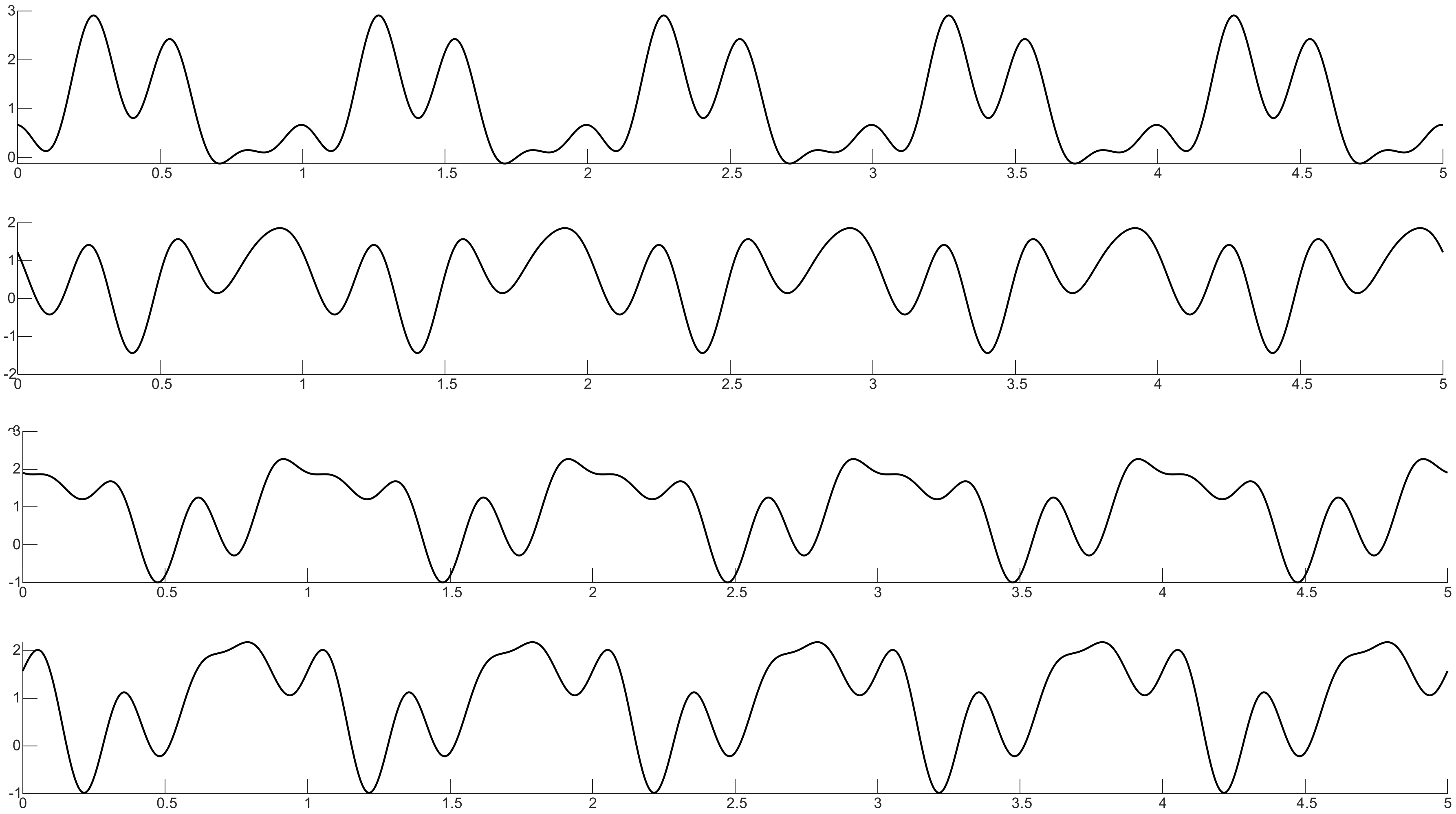
$$+ A_2 \cos(6\pi t + \phi_2) + A_3 \cos(8\pi t + \phi_3)$$

Change A_0, A_1, A_2 and A_3 and plot the resulting graphs.

SIGNALS WITH DIFFERENT AMPLITUDES



Which Pair with Different Phase Angles, and Which pair With Different Amplitude?



Conclusions

- The magnitude spectrum shows how much each frequency component contributes to the final signal.
- The phase spectrum shows the relative starting position of each frequency component (sinusoidal wave) in the time domain.

Discussion: between the phase and magnitude spectrums, which one is more important?

Discussion: between the phase and frequency spectrums, which one is more important?

- Depending on the nature of the signals.
- How signals are obtained?
- What is the receiver of such signals?
- Is the information of importance embedded mostly in the phase or in the magnitude of the frequency components?

Examples of Signals Insensitive to Phase Changes

- Audio signals are commonly more tolerant to phase distortions.
- Phase information on each individual sinusoids only affect the starting of the sinusoids not the tone of the sinusoids, thus doesn’t change the audio perception.
- Even when multiple sinusoids are blended with different phases. The resultant signals will change drastically in shape, however, will have little impact on audio perception.

Examples of Signals Insensitive to Phase Changes

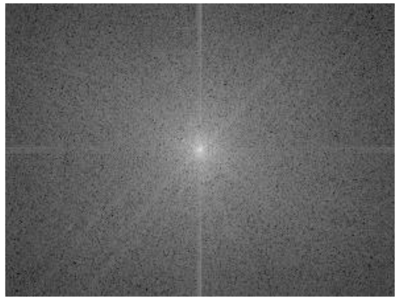
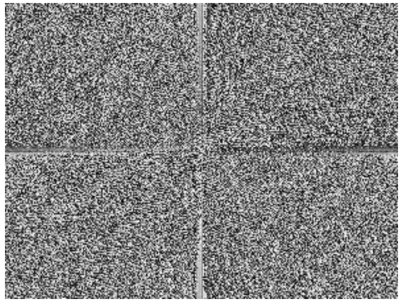
“Insensitive” does not mean immune to phase changes.

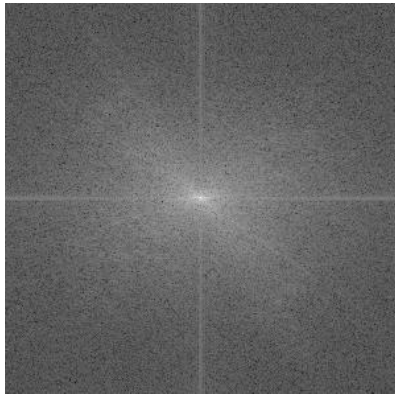
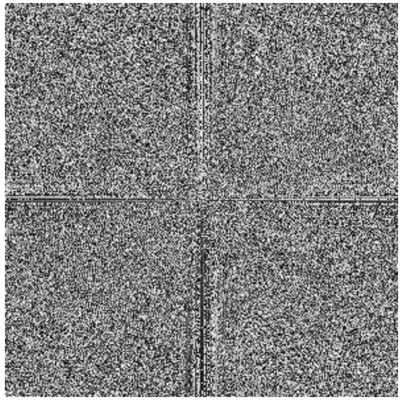
- The ears are somehow sensitive to phase differences of the sinusoids of the same frequencies: our brains process these phase differences as echoes and are very competent in building the sense of space from such information.
- Similarly, the sound perceptions produced by stereo systems of multiple speakers can be significantly affected by the relative phase of the signals produced by different speakers.

Examples of Signals Sensitive to Phase Changes

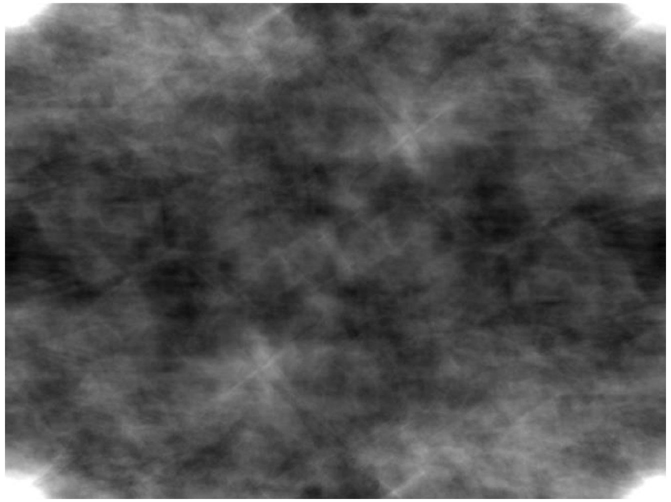
- Digital images are commonly more sensitive to phase changes.
- Phase information on each individual sinusoid reflects the relative spatial positioning of the sinusoids, which is often the edges and variations in an image, thus significantly changing the visual perception.
- On the other hand, when multiple sinusoids are blended together in an image. If the magnitude of each sinusoid got slightly distorted, the resultant signals may change drastically in shading, however, the edges and variations are still visible.

Examples of Signals Sensitive to Phase Changes

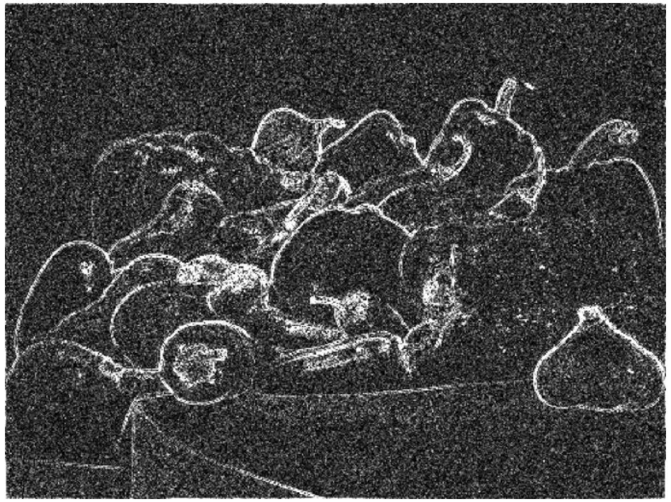




Phase = 0, magnitude unchanged



Phase unchanged, magnitude = 1



Phase unchanged, magnitude from a different image