# HUMBER ENGINEERING

## MENG-3020
## SYSTEMS MODELING & SIMULATION

## LECTURE 9

HUMBER

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# LECTURE 9
# Introduction to Data-Driven Modeling

- High-Order Systems Approximation
- System Modeling via Transient Response Analysis
  - First-Order & Second-Order Systems Modeling via Step Response
  - Effect of Extra Stable Pole and Stable Zero on Step Response
- System Identification Procedure
  - Experiment Design & Data Examination
  - Model Structure Selection
  - Model Estimation
  - Model Validation

# What We Already Know?

- **Time Response of First-Order Systems**

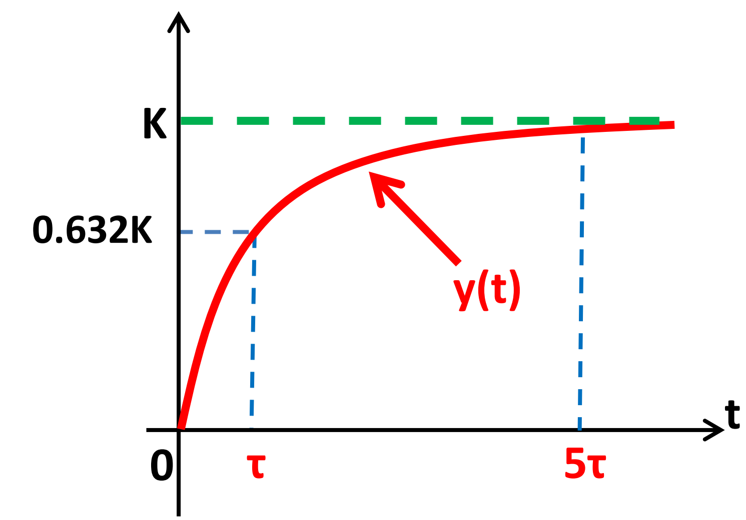
$$G(s) = \frac{K}{\tau s + 1}$$

$K \rightarrow$ Steady-state Gain

$\tau \rightarrow$ Time Constant

Pole $\rightarrow$ $s = -\frac{1}{\tau}$

$5\tau \rightarrow$ Settling-time




$$y(t) = K\left(1 - e^{-1/\tau}\right), \qquad t \geq 0$$

- **Time Response of Second-Order Systems**

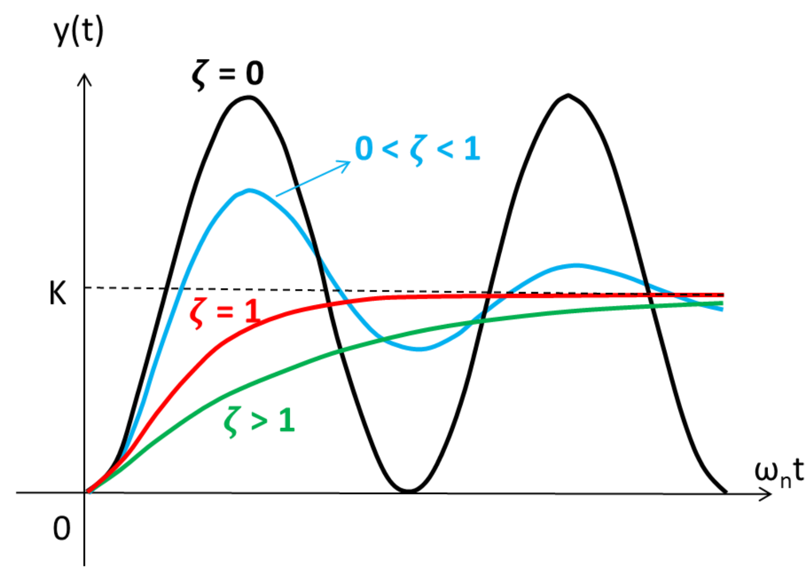
$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$K \rightarrow$ Steady-state Gain

$\zeta \rightarrow$ Damping ratio

$\omega_n \rightarrow$ Undamped Natural Frequency

Poles $\rightarrow$ $s_{1,2} =- -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$




- **Stable Systems $\zeta > 0$**
  - Over-damped Systems $\zeta > 1$
  - Critically-damped Systems $\zeta = 1$
  - Under-damped systems $0 < \zeta < 1$
- **Marginally Stable Systems $\zeta = 0$**
  - Undamped Systems
- **Unstable Systems $\zeta < 0$**
  - Negatively-damped systems

# What We Already Know?

- **Time Response Specifications of Second-Order Systems**

$$G(s) = \frac{K\,\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

**Rise time ( $t_r$ ):**

$$t_r \cong \frac{0.8 + 2.5\zeta}{\omega_n}$$

**Peak time ($t_p$):**

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

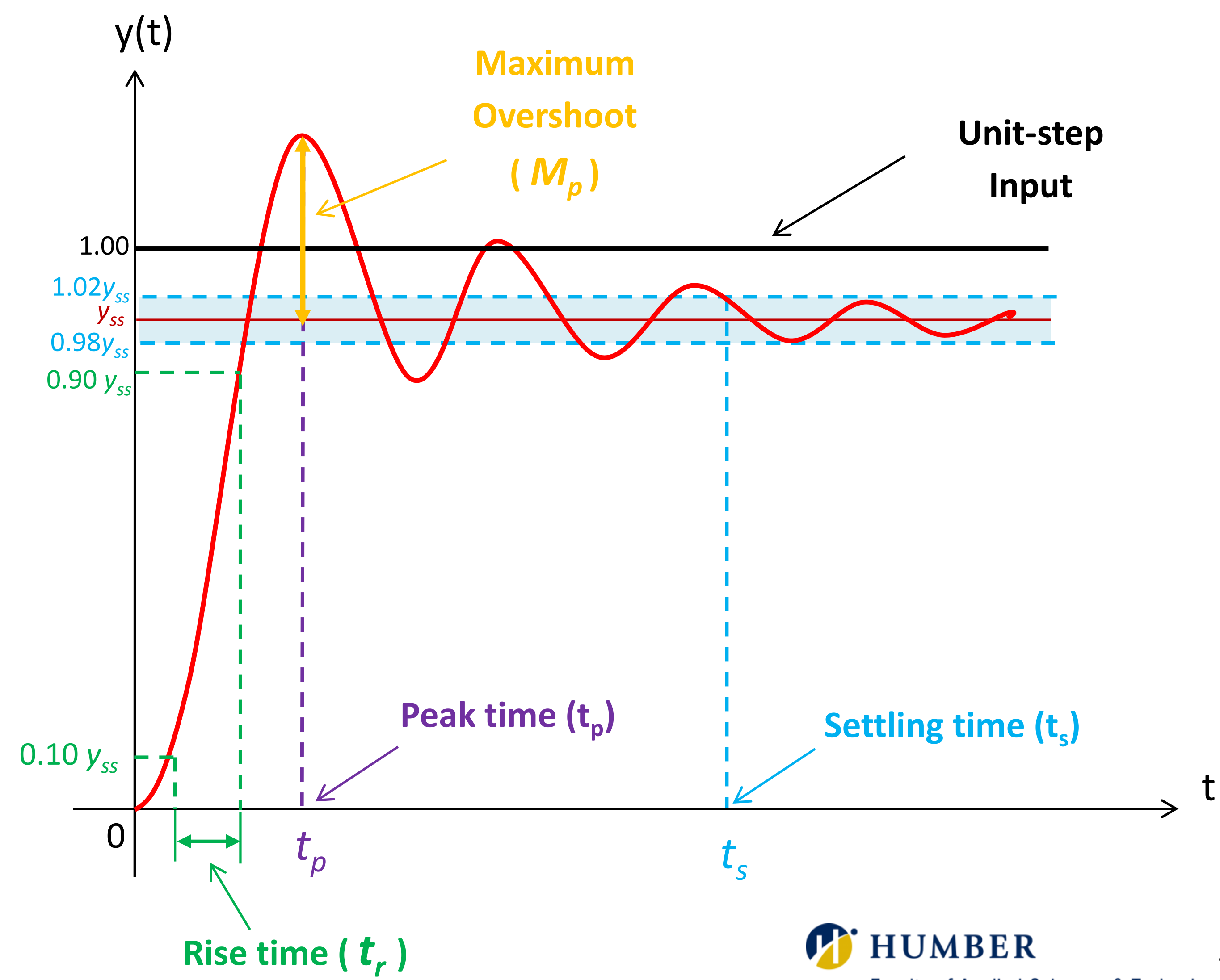
**Maximum overshoot ( $M_p$ ):**

$$M_p = y(t_p) - y_{ss} = y_{ss}e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$$\%\text{O.S.} = \frac{M_p}{y_{ss}} \times 100\%$$

**Settling time ( $t_s$ ):**

$$2\% \ \text{criteria} \rightarrow \quad t_s \approx \frac{4}{\zeta\omega_n}\ , \qquad 0 < \zeta < 0.9$$



# High-Order Systems Approximation

## ❑ Reduced-Order Models


- Consider transfer function of a high-order LTI system as follows:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

- By factorization of the denominator polynomial G(s) can be written in pole-zero form:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_r)\cdots(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{A_1}{s+p_1} + \cdots + \frac{A_r}{s+p_r} + \cdots + \frac{Bs+C}{s^2+2\zeta\omega_n s+\omega_n^2}$$

- The LTI system can be modeled as a combination of several first-order and second-order systems in series or parallel form.

- The pole-zero form of a high-order LTI system can be approximated by a **lower-order** one by eliminating some insignificant poles and zeroes.

# High-Order Systems Approximation

## ❑ Order Approximation Rules

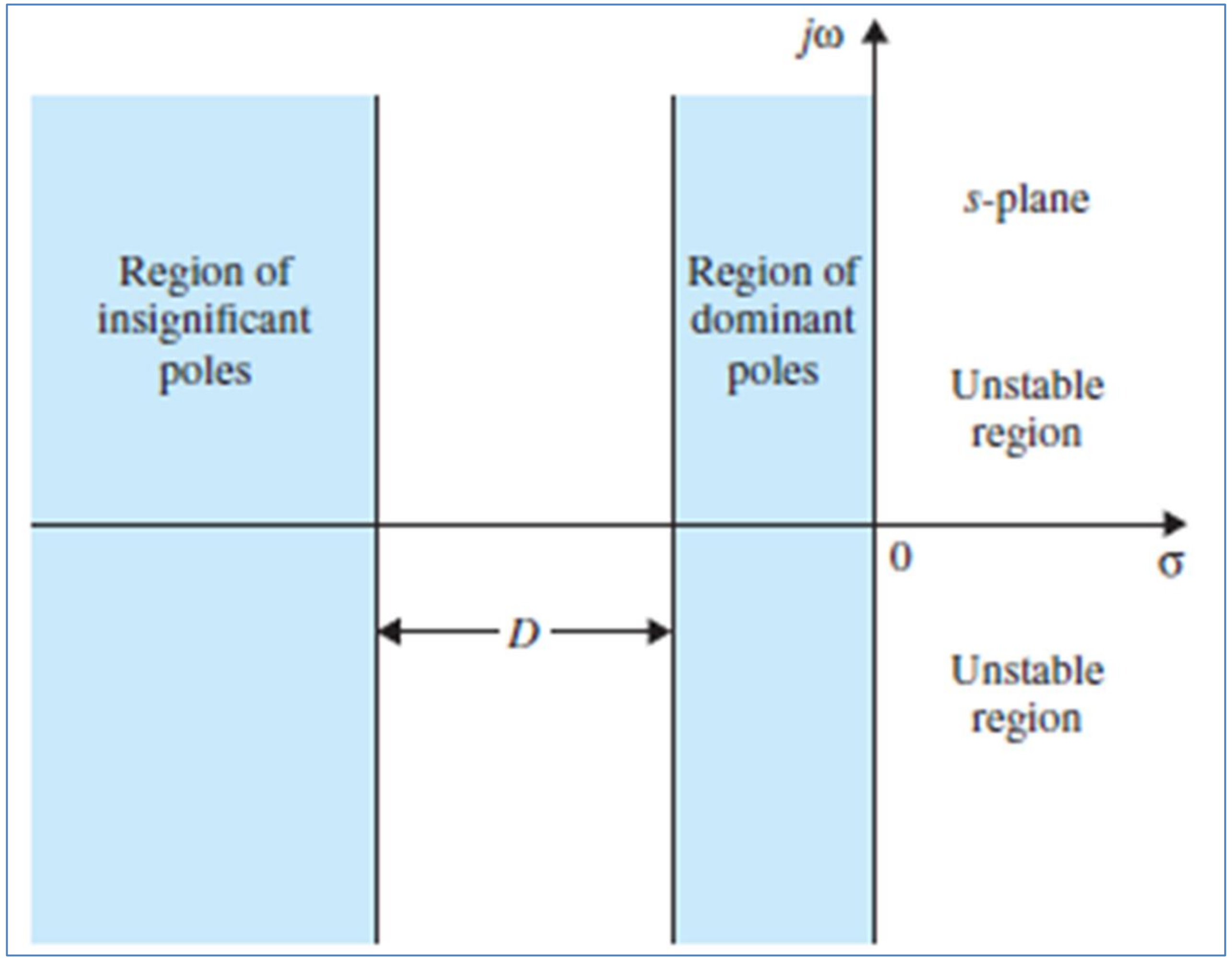

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_r)\cdots(s^2+2\zeta\omega_n s+\omega_n^2)}$$

$$G(s) = \frac{A_1}{s+p_1} + \cdots + \frac{A_r}{s+p_r} + \cdots + \frac{Bs+C}{s^2+2\zeta\omega_n s+\omega_n^2}$$

- The poles close to the imaginary axis are considered as the Dominant Poles of the system. Because, they have larger time constant and have more effect on the transient response.

- The stable poles are located very far from the origin (10 times farther from dominant poles) have low effect on the transient response and may be neglected.

- A pair of closely located stable poles and stable zeros can effectively cancel each other, if the residue of the pole is **much smaller** than the other poles.



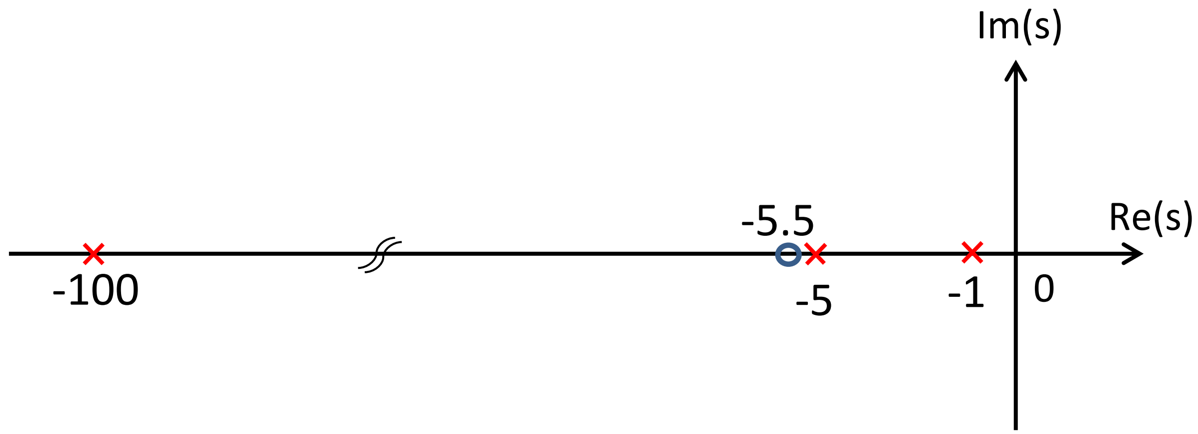
# High-Order Systems Approximation

**Example 1** Determine a low-order approximation for the following high-order system.

$$G(s) = \frac{50s + 275}{s^3 + 106s^2 + 605s + 500}$$

First, we have to find the steady-state gain and pole-zero locations for this system.

$$G(s) = \frac{50s + 275}{s^3 + 106s^2 + 605s + 500} = \frac{50(s + 5.5)}{(s+1)(s+5)(s+100)}$$




Steady-state gain $\rightarrow$ $G(0) = \frac{50 \times 5.5}{1 \times 5 \times 100} = 0.55$

Poles $\rightarrow$ $p_1 = -1, \;\; p_2 = -5, \;\; p_3 = -100$

Zeros $\rightarrow$ $z_1 = -5.5$

- The pole-zero pair of $p_2 = -5$ , $z_1 = -5.5$ are both stable and close to each other, can be canceled if the residue is small enough.
- The pole $p_3 = -100$ is very far from the origin, so it can be neglected.
- The pole $p_1 = -1$ is the dominant pole.

How to compensate the steady-state gain?

# High-Order Systems Approximation

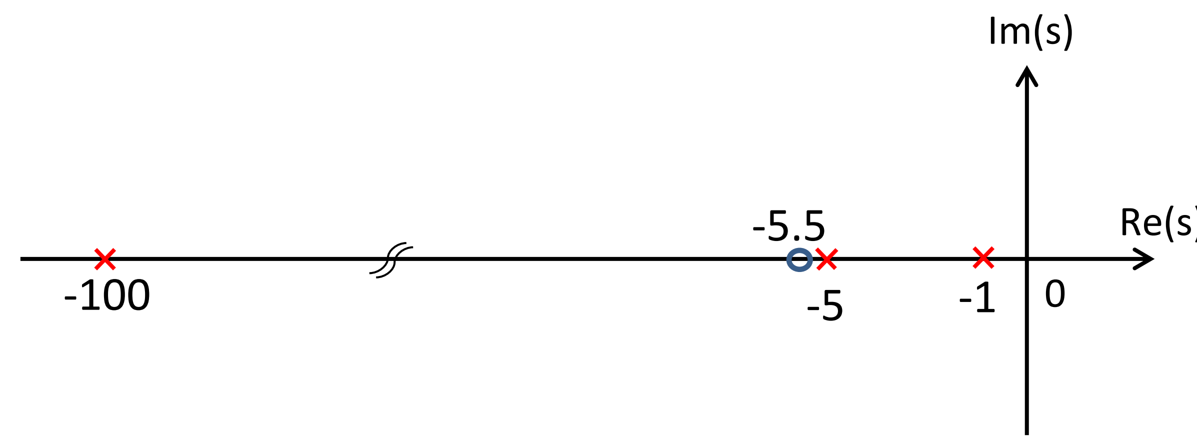
**Example 1**

Determine a low-order approximation for the following high-order system.

$$\boldsymbol{G(s) = \frac{50s + 275}{s^3 + 106s^2 + 605s + 500}}$$

First, we have to find the steady-state gain and pole-zero locations for this system.

$$G(s) = \frac{50s + 275}{s^3 + 106s^2 + 605s + 500} = \frac{50(s + 5.5)}{(s+1)(s+5)(s+100)}$$




Steady-state gain $\rightarrow$ $G(0) = \frac{50\times5.5}{1\times5\times100} = 0.55$

Poles $\rightarrow$ $p_1 = -1,\ \ p_2 = -5,\ \ p_3 = -100$

Zeros $\rightarrow$ $z_1 = -5.5$

To keep the same steady-state gain we have to find the partial fraction expansion of G(s)

$$G(s) = \frac{50(s + 5.5)}{(s+1)(s+5)(s+100)} = \frac{0.568}{s+1} + \frac{-0.066}{s+5} + \frac{-0.502}{s+100}$$

$$\boxed{\boldsymbol{G(s) \cong \frac{0.568}{s+1}}}$$

Since the residue of the pole at $-5$ is much smaller than the other poles, the pole-zero cancellation is **valid**.

The dominant pole is $p_1 = -1$

# High-Order Systems Approximation

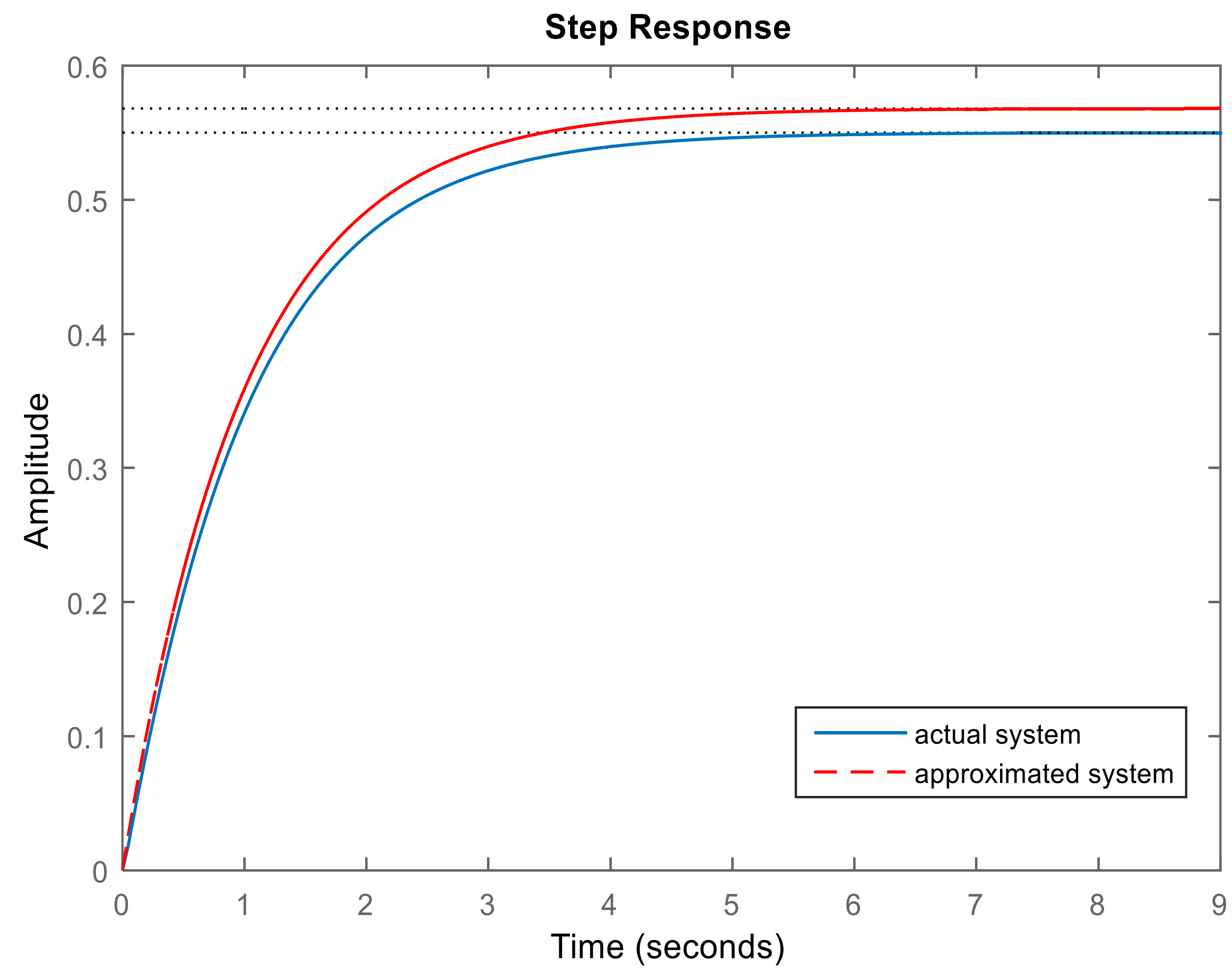
**Example 1** Determine a low-order approximation for the following high-order system.

$$G(s) = \frac{50s + 275}{s^3 + 106s^2 + 605s + 500}$$

**Model Verification:** We can plot unit-step responses of the original system and its approximated version by MATLAB to compare them.

$$\boxed{G(s) \cong \frac{0.568}{s+1}}$$

```
num1 = [50  275];
den1 = [1  106  605  500];
sys1 = tf(num1,den1);

num2 = [0.568];
den2 = [1  1];
sys2 = tf(num2,den2);

step(sys1)
hold on
step(sys2)
```



# Quick Review

1. Which transfer function can be the best low-order approximation of $G(s)$?

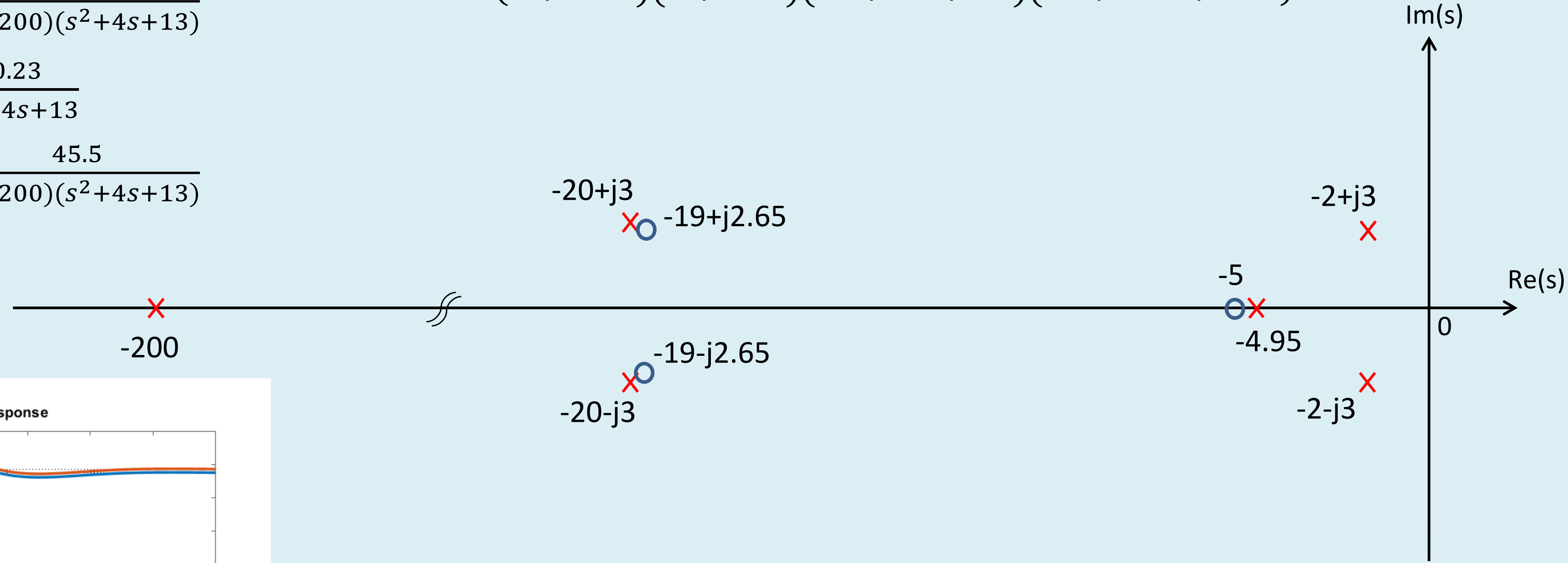
   a) $G(s) = \frac{50}{s^2+4s+13}$

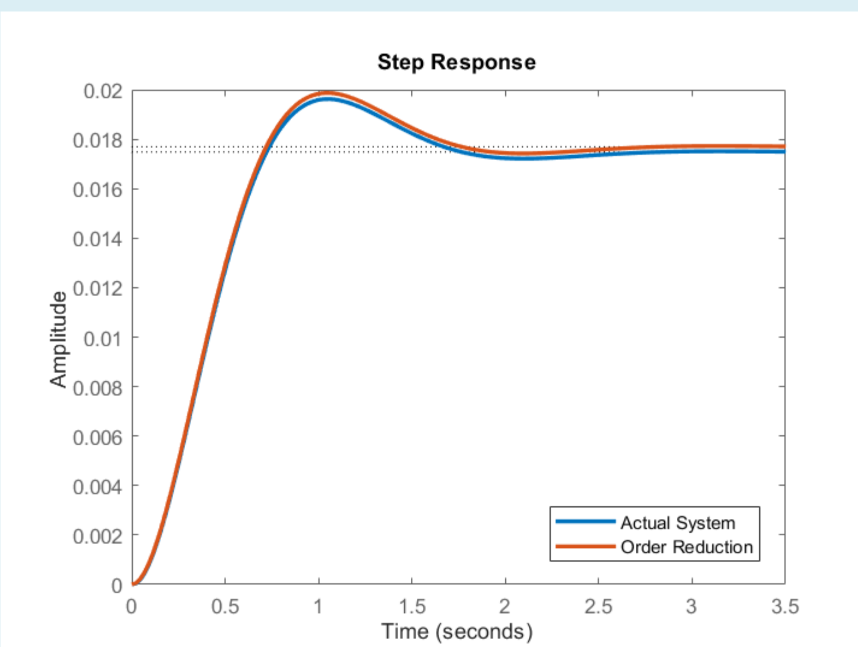
   b) $G(s) = \frac{1}{(s+200)(s^2+4s+13)}$

   c) $G(s) = \frac{0.23}{s^2+4s+13}$

   d) $G(s) = \frac{45.5}{(s+200)(s^2+4s+13)}$

$$G(s) = \frac{50(s+5)(s^2+38s+368)}{(s+4.95)(s+200)(s^2+4s+13)(s^2+40s+409)}$$

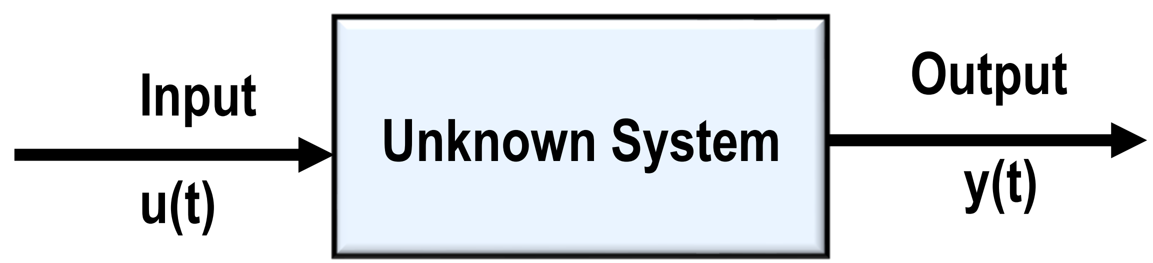



$$G(s) = \frac{0.00063}{s+4.95} + \frac{0.013}{s+200} + \frac{-0.00029s+0.23}{s^2+4s+13} - \frac{0.0016s+0.0319}{s^2+40s+409}$$

# Introduction to System Identification

- System identification refers to obtaining the transfer function $G(s)$ of a system by only considering its ***output response*** to a given particular ***input signal***.
- This is useful when we have not much information about a system at hand.
- For example, it is a *black box* for us or *too complicated* to be modeled, and we need to find out its transfer function for simulation purposes, for instance.
- Although we assume that no information is provided by the system, some hypothesis must be considered, like for instance the **order** of the system.




- In this course we will focus on system identification and transfer function modeling via
  - Transient response analysis of first-order and second-order systems.
  - MATLAB System Identification toolbox for Black-box system modeling.

# System Modeling via Step Response

- In Transient Response Analysis approach, we model the systems based on the step response of the system.

## ❑ First-Order Model

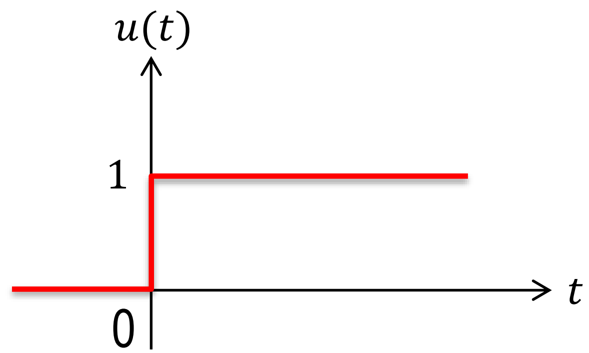
$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1}$$

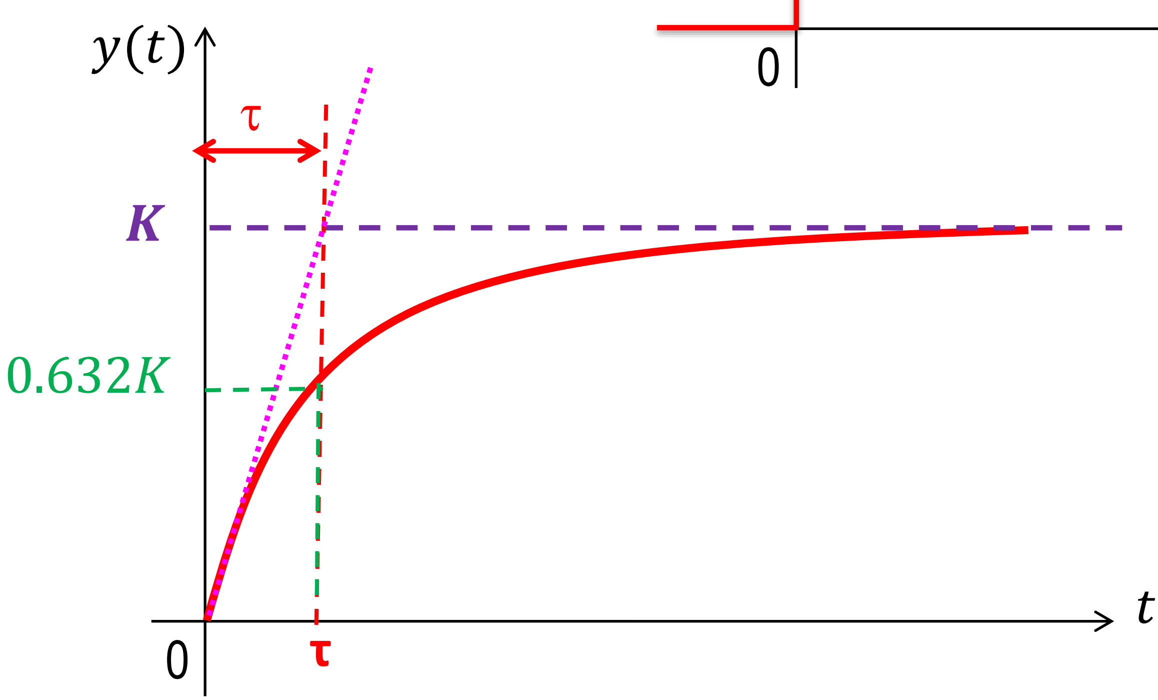
$K$ : DC-gain

$\tau$ : Time constant

- **Unit-step Response**

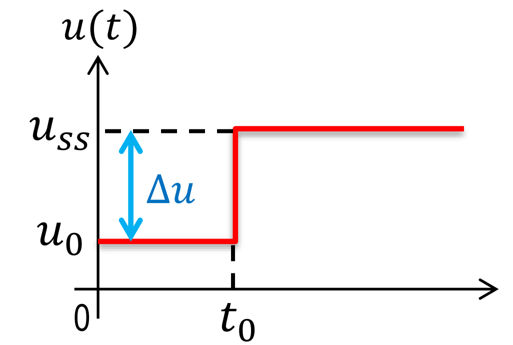
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

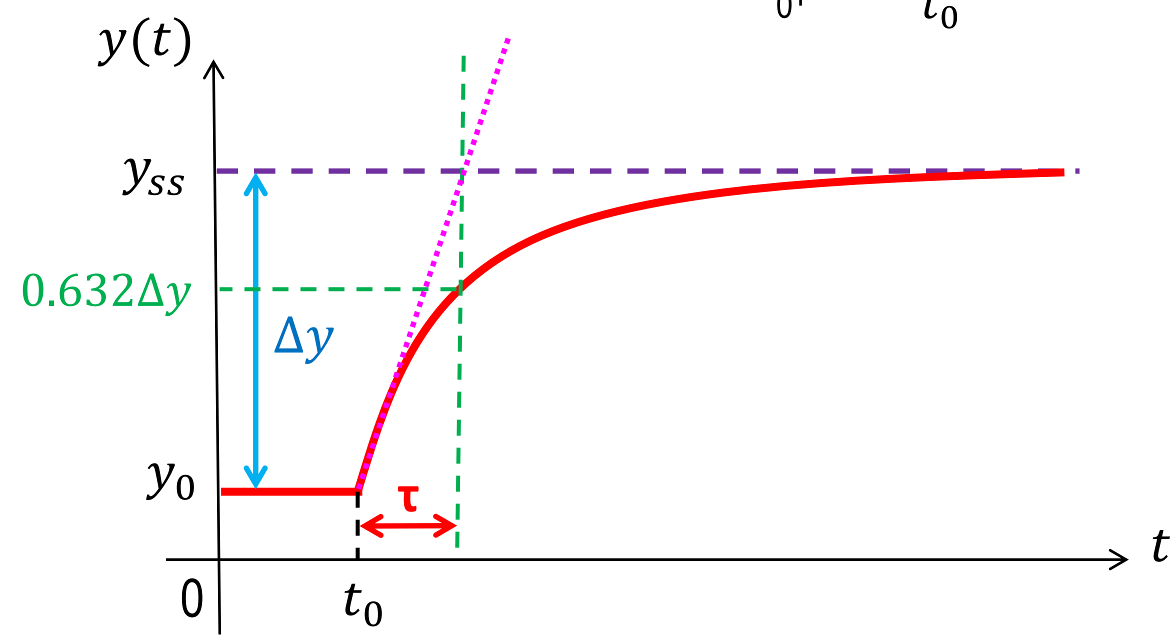



- **General Step Response**

$$u(t) = \begin{cases} u_{ss} & t \geq t_0 \\ u_0 & 0 \leq t < t_0 \\ 0 & t < 0 \end{cases}$$




$$K = \frac{\Delta y}{\Delta u} = \frac{y_{ss} - y_0}{u_{ss} - u_0}$$

# System Modeling via Step Response

- In Transient Response Analysis approach, we model the systems based on the step response of the system.

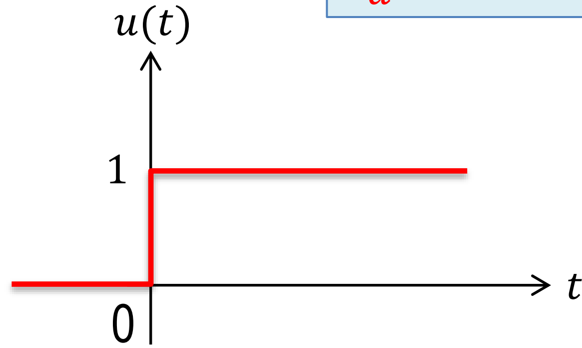
## ❑ First-Order Model with Transportational Delay

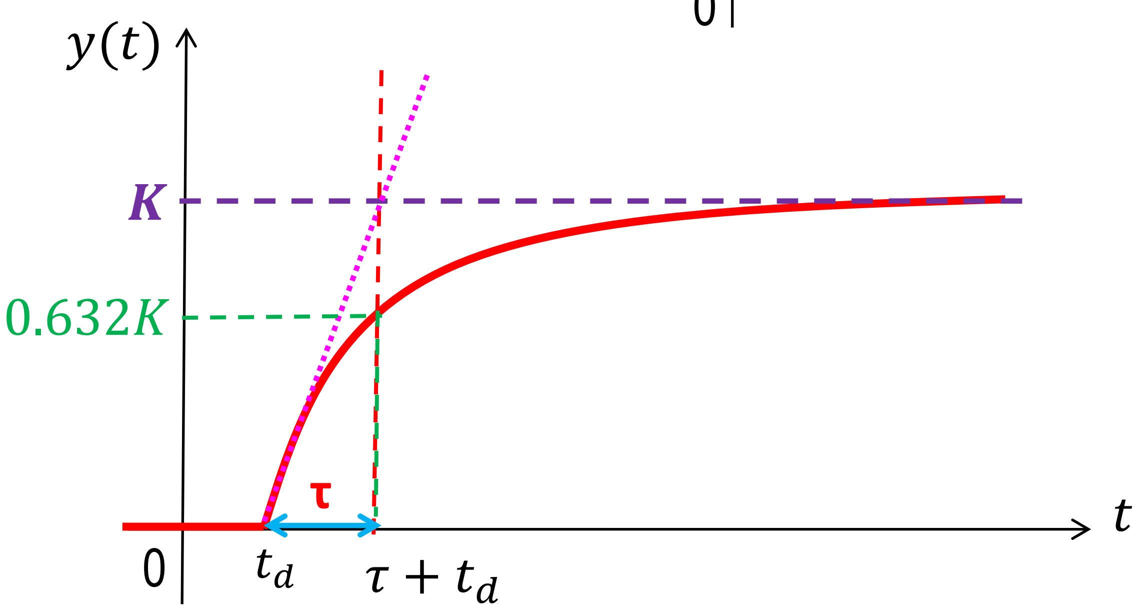
$$\boldsymbol{G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} e^{-t_d s}}$$

| |
|---|
| $K$ : DC-gain |
| $\tau$ : Time constant |
| $t_d$ : Time-delay |

- **Unit-step Response**

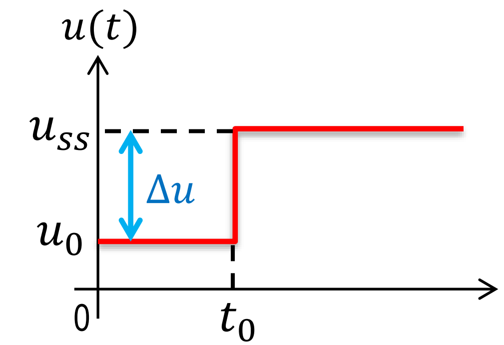
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

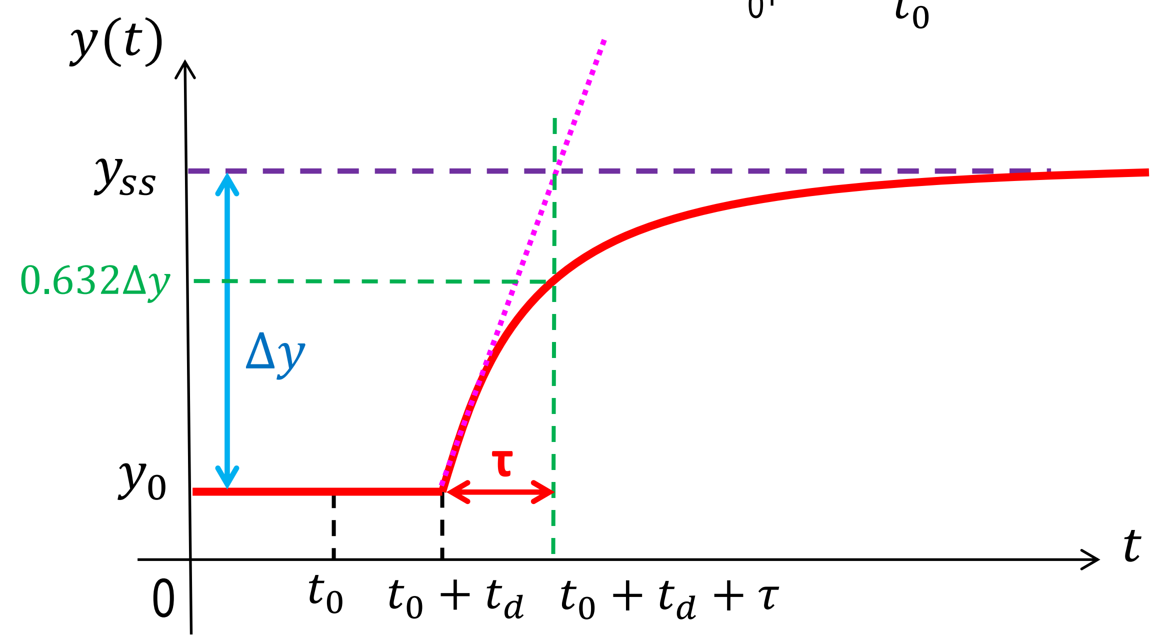



- **General Step Response**

$$u(t) = \begin{cases} u_{ss} & t \geq t_0 \\ u_0 & 0 \leq t < t_0 \\ 0 & t < 0 \end{cases}$$




$$\boxed{K = \frac{\Delta y}{\Delta u} = \frac{y_{ss} - y_0}{u_{ss} - u_0}}$$

# System Modeling via Step Response

- In Transient Response Analysis approach, we model the systems based on the step response of the system.

## ❑ First-Order Model with Delay-Time

- We can approximate high-order overdamped systems with a **First-Order Plus Delay-Time (FOPDT)** model.
- The method is called Ziegler-Nichols Approach based the name of the persons who introduced the method.
- The method consists of applying a tangent line to the curve at the inflection point, to determine the DC-gain, delay-time and time constant.

$$G(s) = \frac{K}{\tau s + 1} e^{-t_d s}$$

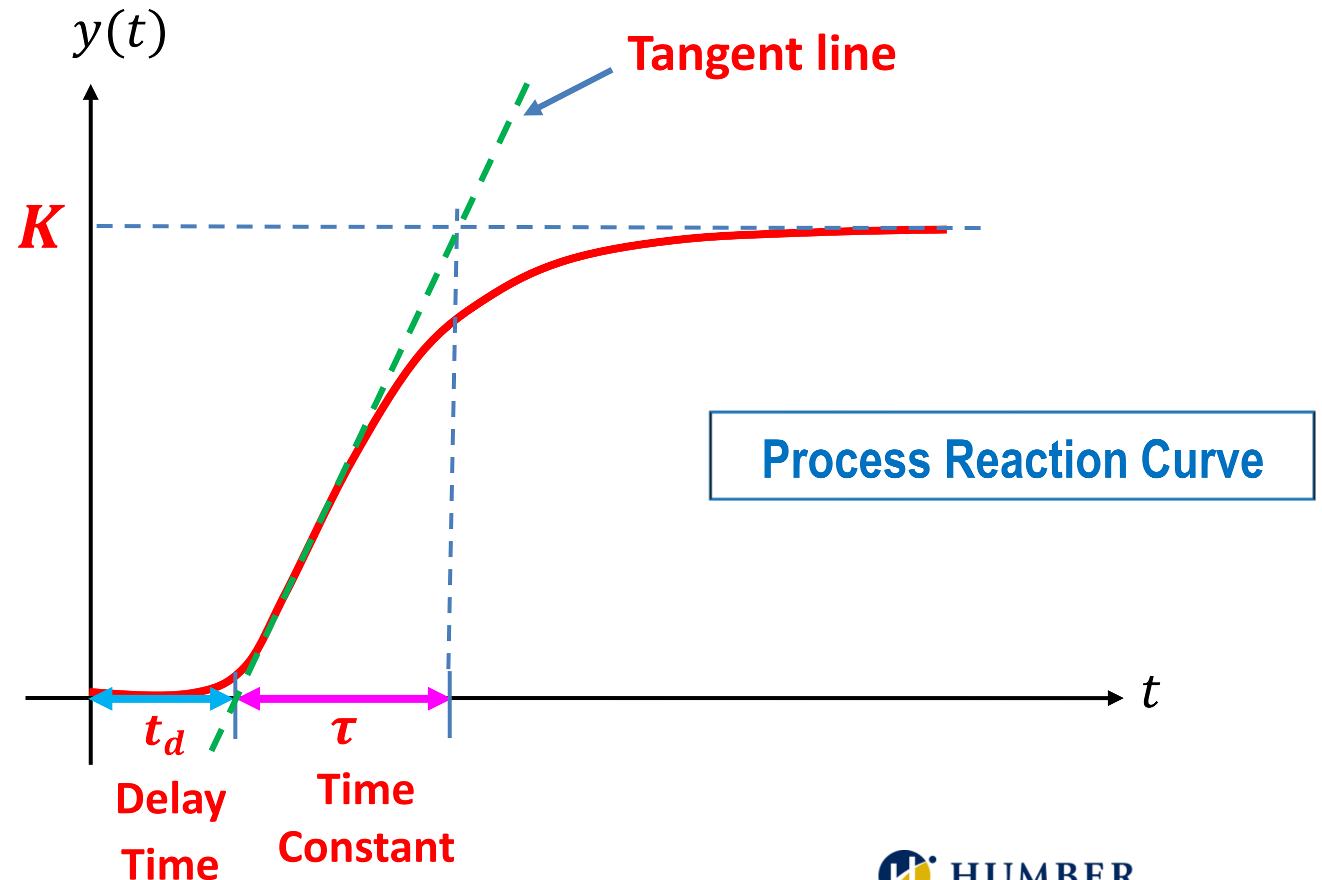
- **Unit-step response**

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$K$ : DC-gain

$\tau$ : Time constant

$t_d$ : Time-delay



# System Modeling via Step Response

**Example 2**

Determine a FOPDT model for a third-order system based on the given unit-step response (process reaction curve).

$$G(s) = \frac{105}{(s+3)(s+5)(s+7)}$$

$$G(s) = \frac{K}{\tau s + 1} e^{-t_d s}$$

From the unit-step response graph we can determine the DC-gain, time constant and the delay-time of the FOPDT model :

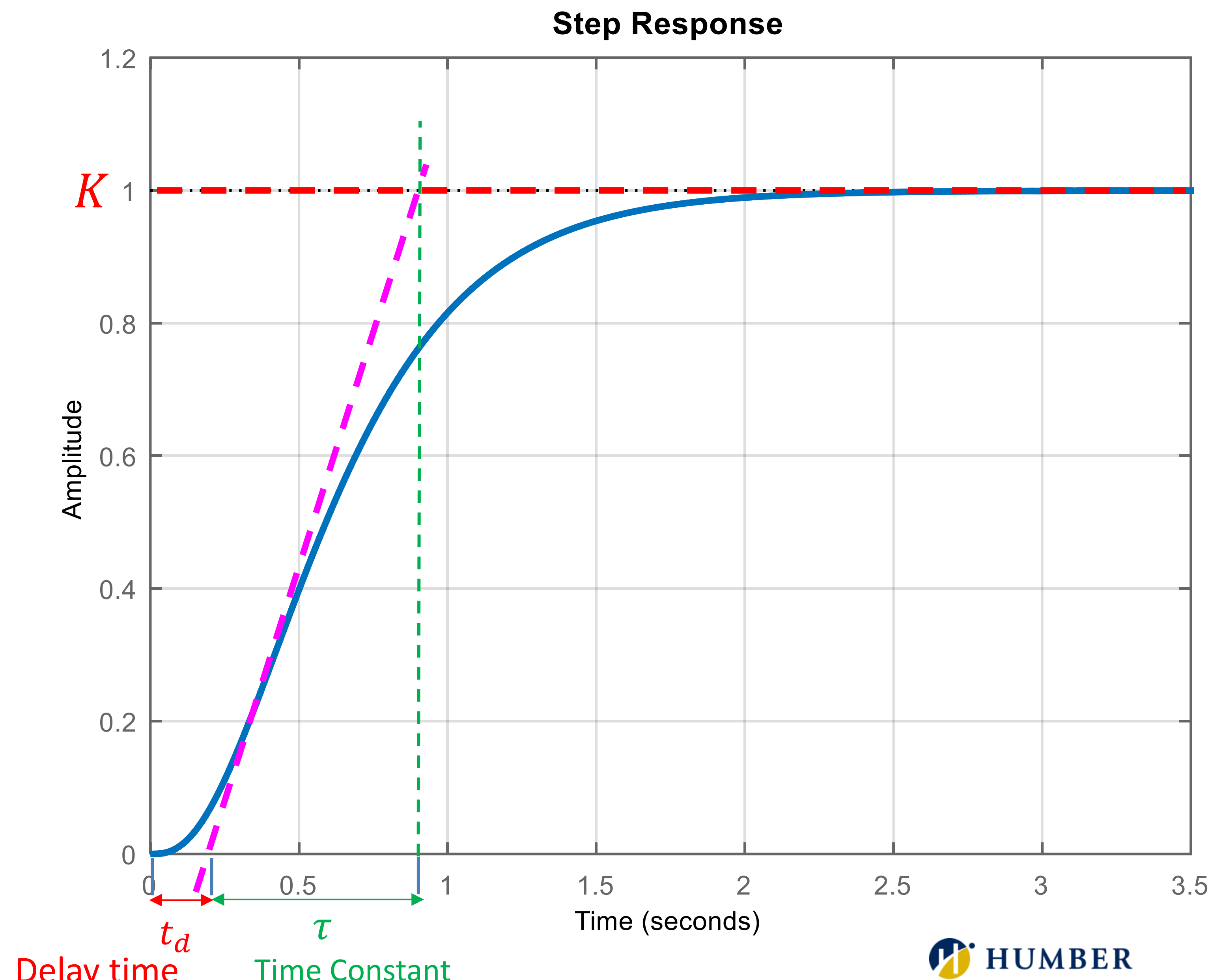
$$\boxed{K = 1}$$

$$\boxed{t_d = 0.21\ sec}$$

$$\boxed{\tau = 0.59\ sec}$$

$$\boxed{G(s) = \frac{1}{0.59s + 1} e^{-0.21s}}$$

The difficulty in applying this method is that it first becomes necessary to find the inflection point of the curve, where the curve changes direction and the second derivative is equal to zero.



# System Modeling via Step Response

**Example 2**

Determine a FOPDT model for a third-order system based on the given unit-step response (process reaction curve).

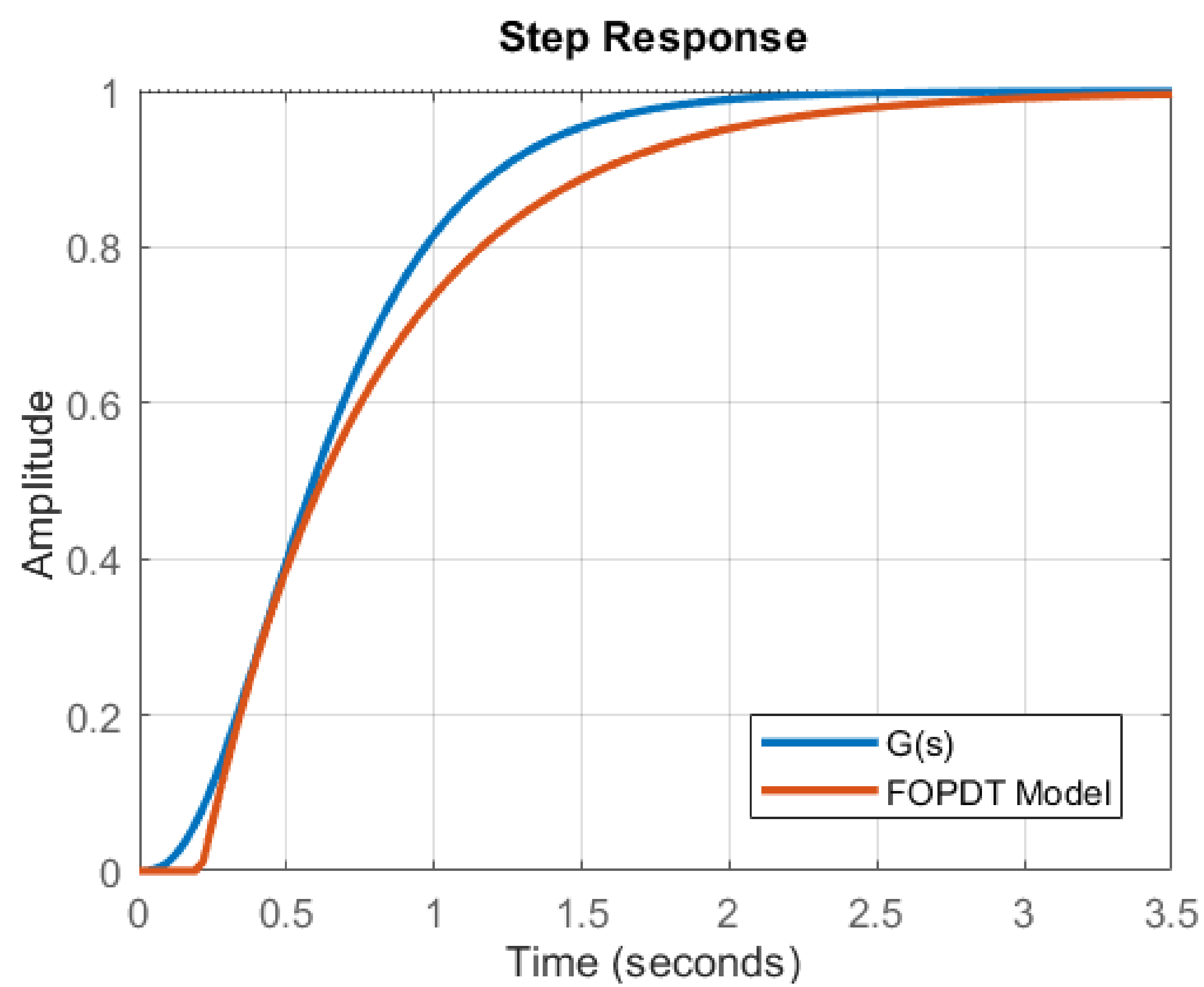
$$G(s) = \frac{105}{(s+3)(s+5)(s+7)}$$

$$G(s) = \frac{K}{\tau s + 1} e^{-t_d s}$$

**Model Verification:** We can plot unit-step responses of G(s) an approximated FOPDT model by MATLAB to compare them.

$$\boxed{G(s) = \frac{1}{0.59s + 1} e^{-0.21s}}$$

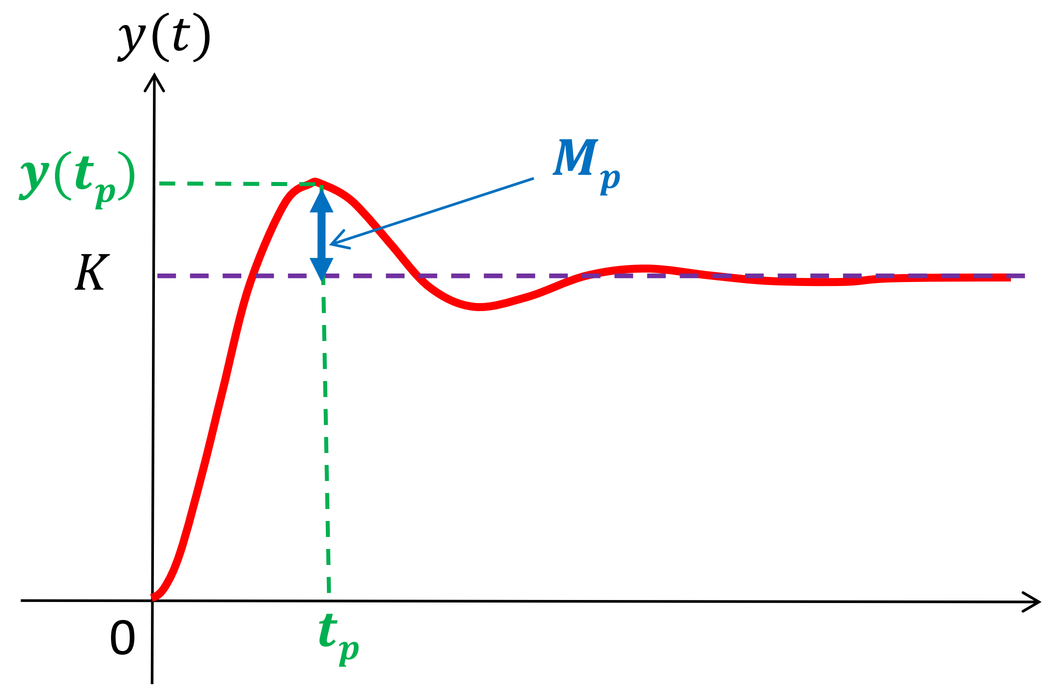
```
num1 = [105];
den1 = poly([-3,-5,-7]);
sys1 = tf(num1,den1);

num2 = [1];
den2 = [0.59 1];
sys2 = tf(num2,den2,'OutputDelay', 0.21);

figure; step(sys1)
hold on
step(sys2)
```



# System Modeling via Step Response

## ❑ Second-Order Model (Under-damped Systems)

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$




$K$ : DC-gain

$\zeta$ : Damping ratio

$\omega_n$ : Undamped natural frequency

1) Measure $y_{ss}$ and $y(t_p)$ and compute the damping ratio $\zeta$

$$M_p = y(t_p) - y_{ss} \quad \rightarrow \quad O.S. = \frac{M_p}{y_{ss}} \quad \rightarrow \quad \boxed{\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}}}$$

2) Measure peak time $t_p$ and compute the natural frequency $\omega_n$

$$\boxed{t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}}$$

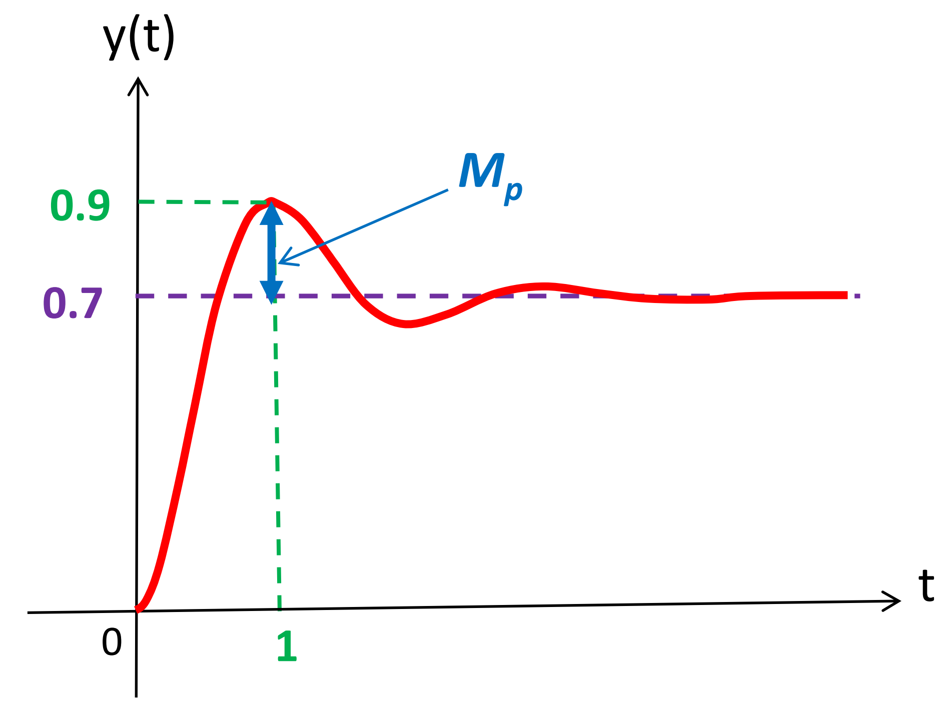
3) The DC-gain $K$ is obtained with the previously explained approach

# System Modeling via Step Response

**Example 3**

Determine TF model of the system based on the given unit-step response.

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$




From the unit-step response graph we have:

$\boxed{K = 0.7}$ $\boxed{t_p = 1sec, \quad M_p = 0.9 - 0.7 = 0.2}$

The damping ratio is determined as below

$$O.S. = \frac{M_p}{y_{ss}} = \frac{0.2}{0.7} = 0.29$$

$$\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}} = \frac{-\ln(0.29)}{\sqrt{\pi^2 + \ln^2(0.29)}} \rightarrow \boxed{\zeta = 0.367}$$

The undamped natural frequency is calculated as below

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \rightarrow 1 = \frac{\pi}{\omega_n\sqrt{1-(0.367)^2}} \rightarrow \boxed{\omega_n = 3.38 \text{ rad/s}}$$

The second-order model is obtained as:

$$\boxed{G(s) = 0.7\frac{11.43}{s^2 + 2.48s + 11.43}}$$

# System Modeling via Step Response

**Example 3**

Determine TF model of the system based on the given unit-step response.

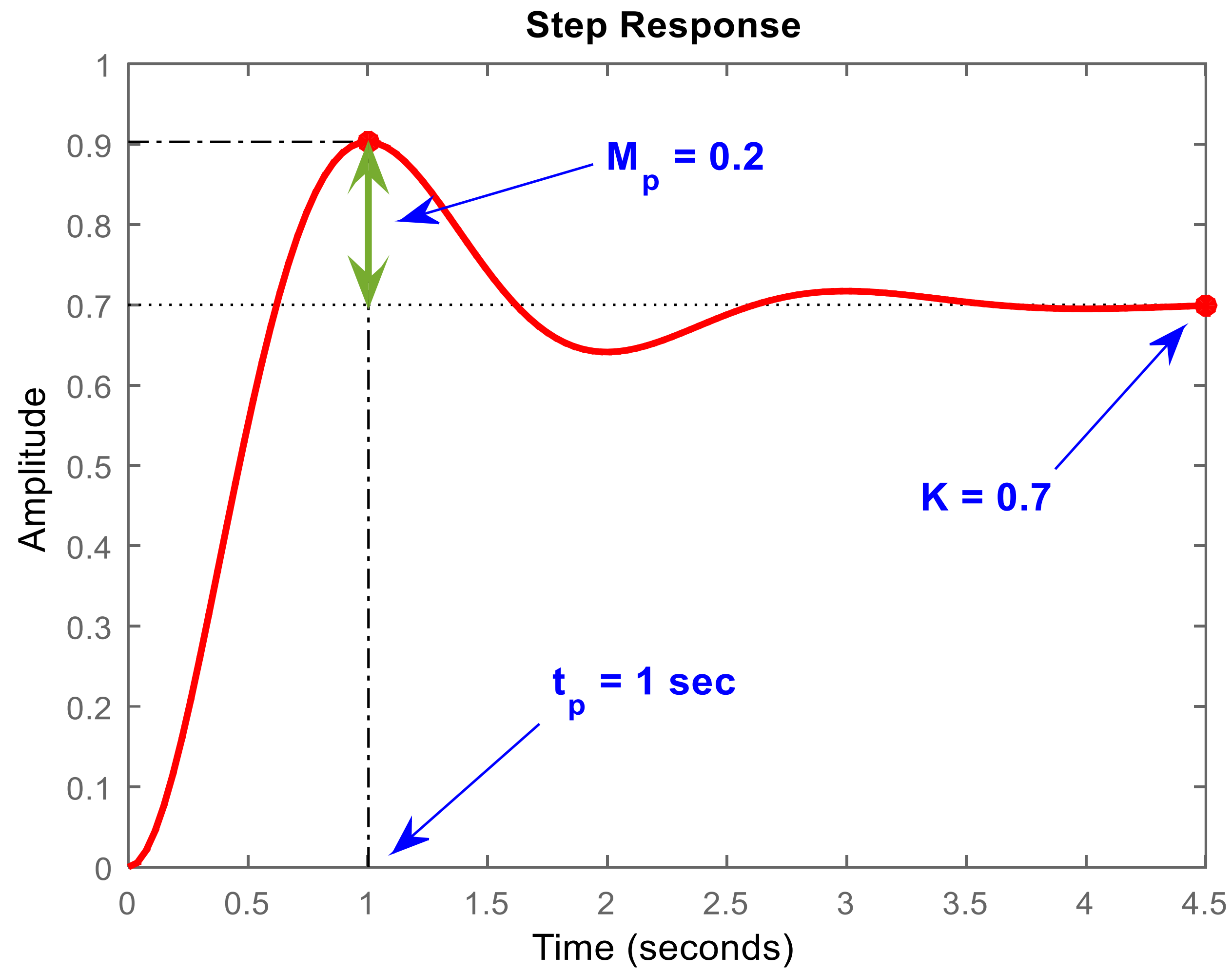
$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

**Model Verification:** We can plot unit-step responses of the second-order model by MATLAB to compare the time response specifications them.

$$K = 0.7, \qquad t_p = 1sec, \qquad M_p = 0.9 - 0.7 = 0.2$$

$$G(s) = 0.7\frac{11.43}{s^2 + 2.48s + 11.43}$$

```
num = 0.7*[11.43];
den = [1 2.48 11.43];
sys = tf(num,den);

stepplot(sys)
```



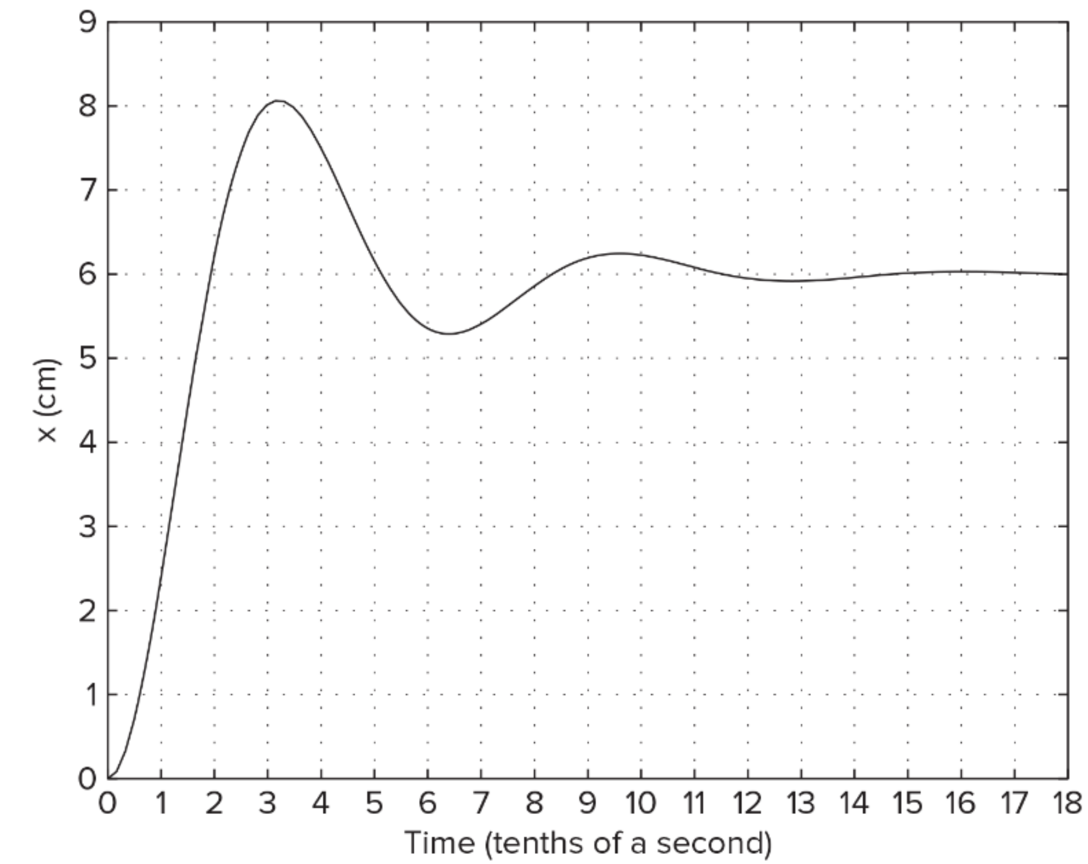
# System Modeling via Step Response

**Example 4**

Figure shows the response of a mass-spring-damper system to a step input of magnitude $6 \times 10^3\ N$.
The equation of motion is:

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = f(t)$$

Estimate the values of $m, b,$ and $k$.




From the equation of motion, the transfer function model of the system is obtained as:

$$ms^2X(s) + bsX(s) + kX(s) = F(s)$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

Since the model is a second-order transfer function, we can find the model parameters in terms of the DC-gain, damping ratio and undamped natural frequency and determine those values from the step response and estimate the system parameters.

$$G(s) = \frac{\frac{1}{m}}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$

$$\boldsymbol{G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

$$\left\{ \begin{aligned} \omega_n^2 &= \frac{k}{m} \\ K\omega_n^2 &= \frac{1}{m} \\ 2\zeta\omega_n &= \frac{b}{m} \end{aligned} \right.$$

$$\boxed{k = \frac{1}{K}, \qquad m = \frac{1}{K\omega_n^2}, \qquad b = \frac{2\zeta}{K\omega_n}}$$

# System Modeling via Step Response

## Example 4

Figure shows the response of a mass-spring-damper system to a step input of magnitude $6 \times 10^3\ N$.
The equation of motion is:

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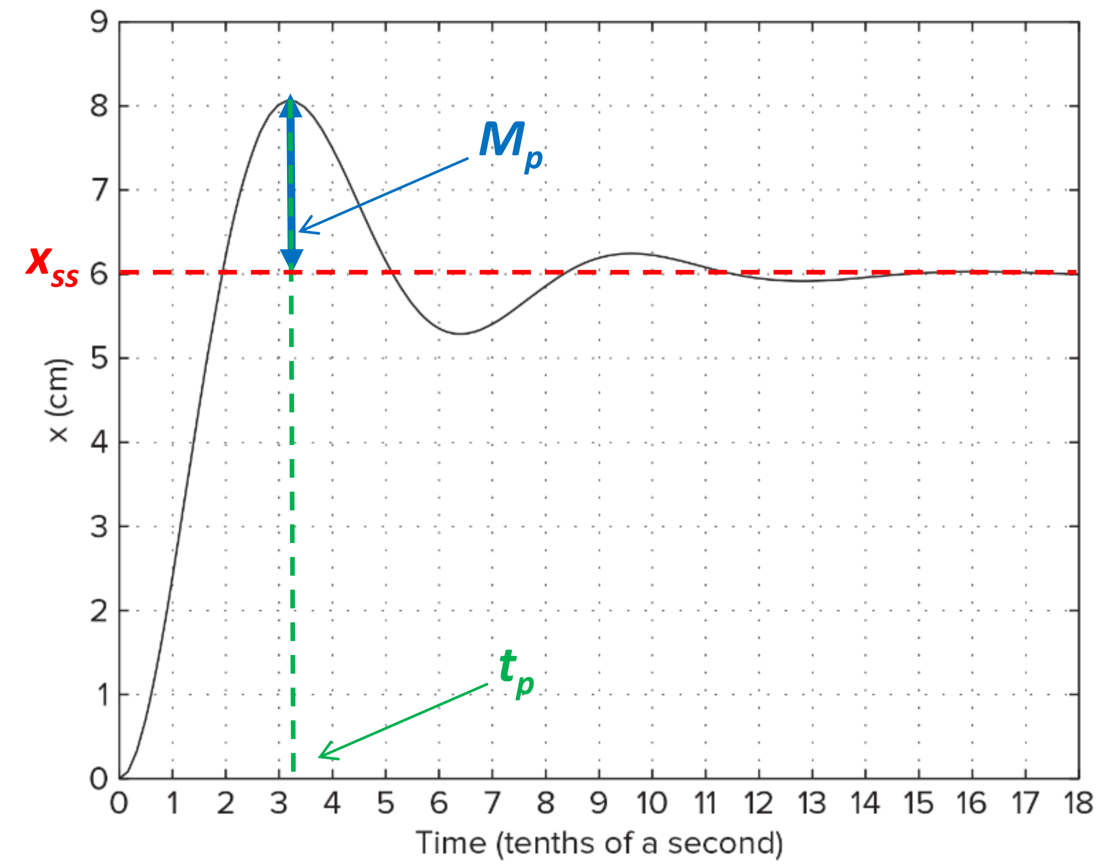
Estimate the values of $m, b$, and $k$.




From the step response we have steady-state value, peak-time and maximum deviation from the steady-state value:

$$x_{ss} = 0.06 \text{ m}, \qquad t_p = 0.32sec, \qquad M_p = 0.081 - 0.06 = 0.021 \text{ m}$$

The DC-gain of system is obtained as:

$$K = \frac{\Delta x}{\Delta f} = \frac{x_{ss} - x_0}{f_{ss} - f_0} = \frac{0.06 - 0}{6000 - 0} = 10^{-5} m/N \quad \rightarrow \quad \boxed{K = 10^{-5}\ m/N}$$

The damping ratio is determined as below

$$O.S. = \frac{M_p}{x_{ss}} = \frac{0.021}{0.06} = 0.35$$

$$\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}} = \frac{-\ln(0.35)}{\sqrt{\pi^2 + \ln^2(0.35)}} \quad \rightarrow \quad \boxed{\zeta = 0.32}$$

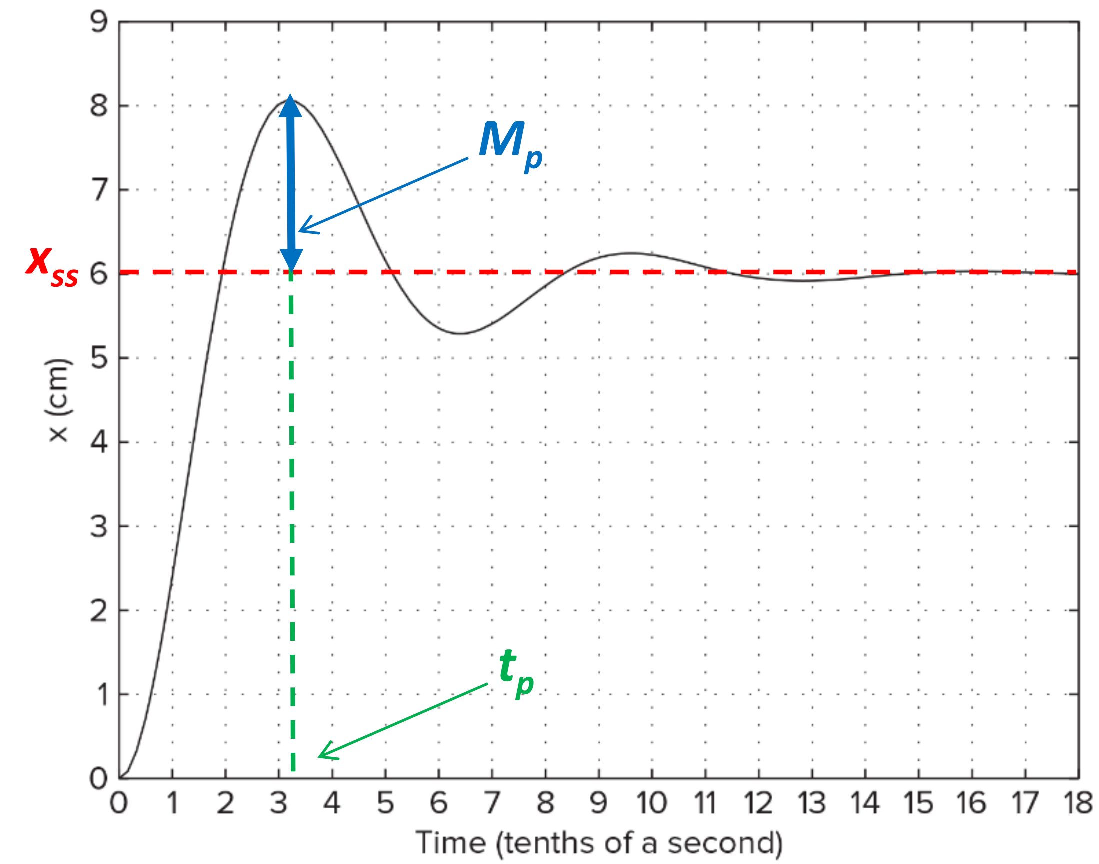
# System Modeling via Step Response

**Example 4**

Figure shows the response of a mass-spring-damper system to a step input of magnitude $6 \times 10^3\ N$.
The equation of motion is:

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = f(t)$$

Estimate the values of $m$, $b$, and $k$.




The undamped natural frequency is calculated as below

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \rightarrow 0.32 = \frac{\pi}{\omega_n\sqrt{1-(0.32)^2}} \rightarrow \boxed{\omega_n = 10.36\ \text{rad/s}}$$

The system parameters are estimated as:

$$k = \frac{1}{K} \rightarrow k = \frac{1}{10^{-5}} = 10^5\ \ N/m$$

$$m = \frac{1}{K\omega_n^2} \rightarrow m = \frac{1}{10^{-5}(10.36)^2} = 932\ kg$$

$$b = \frac{2\zeta}{K\omega_n} \rightarrow b = \frac{2(0.32)}{10^{-5}(10.36)} = 6178\ N.s/m$$

$$\boxed{G(s) = \frac{X(s)}{F(s)} = \frac{1}{932s^2 + 6178s + 10^5}}$$

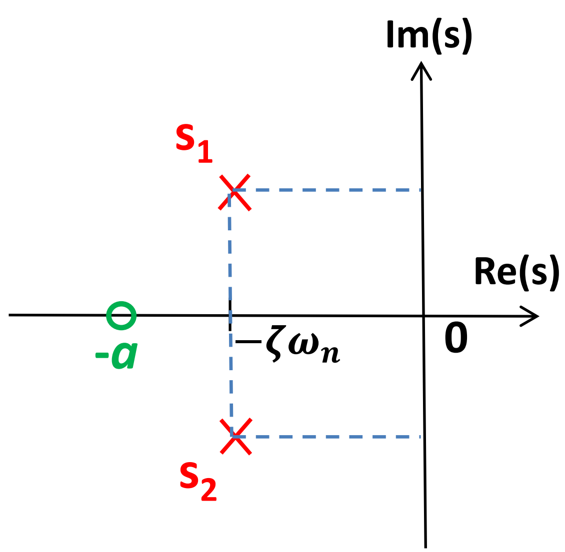
# System Modeling via Step Response

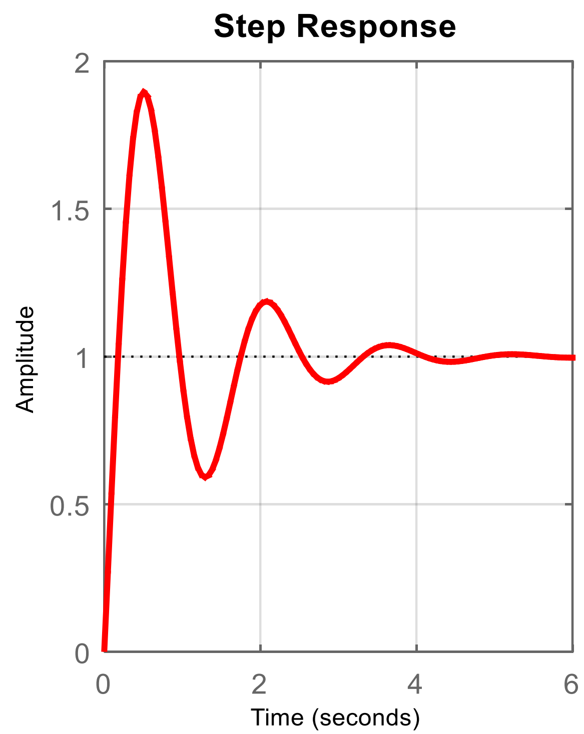
## ❑ Second-Order Model (Undamped System) with a Stable Zero

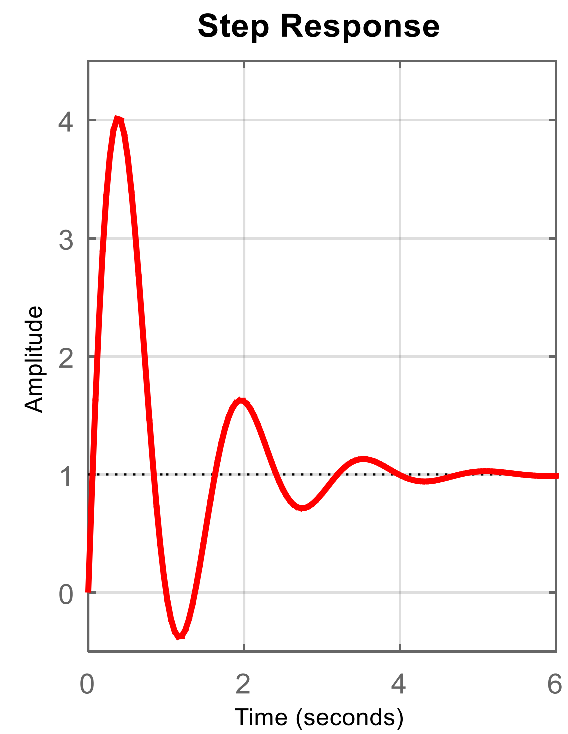
$$G(s) = K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \times \frac{s+a}{a} \quad , \qquad a > 0$$

$K$ : DC-gain
$\zeta$ : Damping ratio
$\omega_n$ : Un-damped natural frequency
$-a$ : Zero location

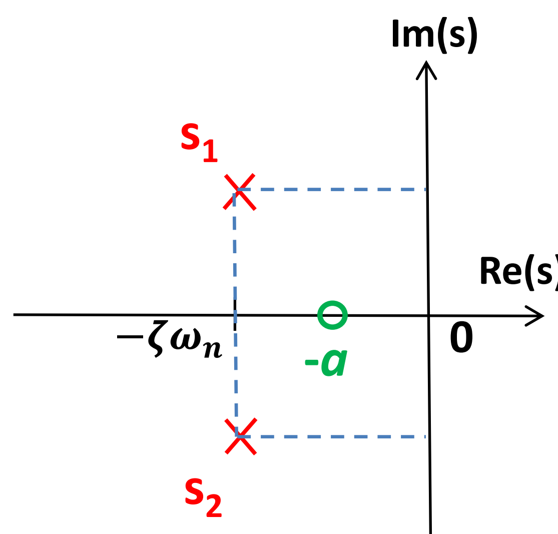
$4\zeta\omega_n > a > \zeta\omega_n$




$0 < a < \zeta\omega_n$



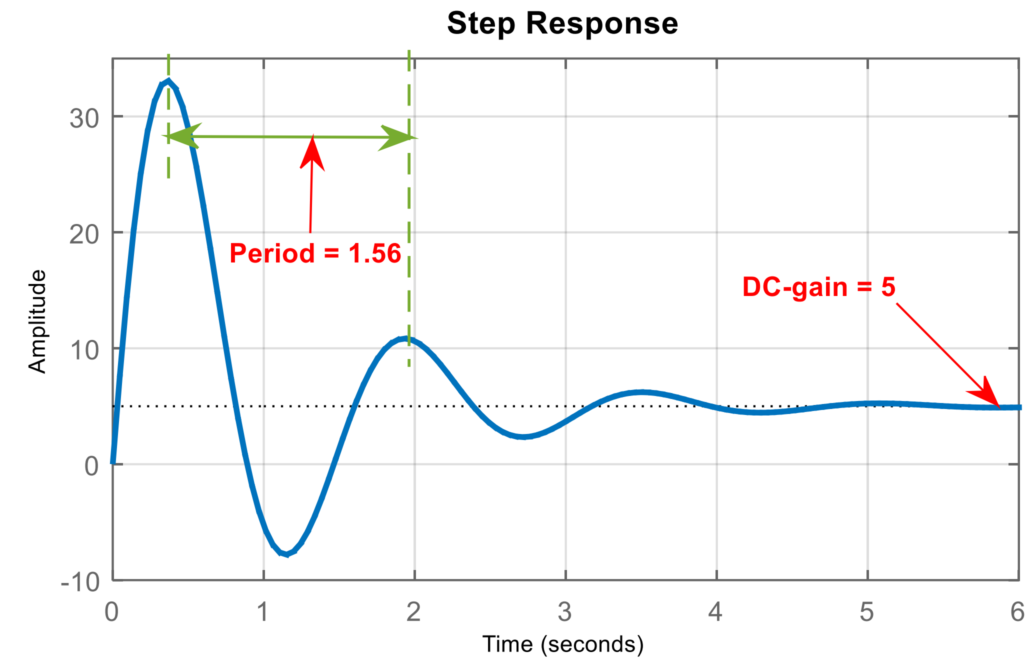

- Effects of the stable real zero at $s = -a$ on the unit-step response is:
  - Increasing the maximum overshoot
  - Decreasing the rise time

# System Modeling via Step Response

**Example 5** Determine TF model of the system based on the given step response.

$$G(s) = K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{s+a}{a}$$

$$\boxed{0 < a < \zeta\omega_n}$$




From the unit-step response graph we have the DC-gain:

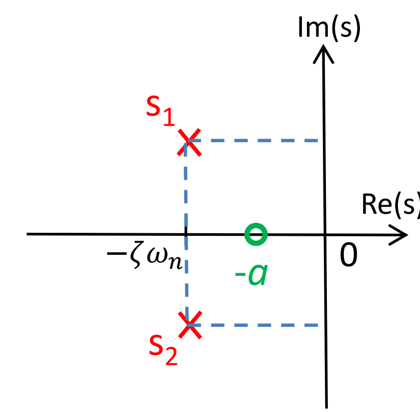
$$\boxed{K = 5}$$

The damped natural frequency ($\omega_d$) can be obtained from the period of the oscillations in step response

$$\omega_d = \frac{2\pi}{\text{period}} = \frac{2\pi}{1.56} = 4.03 \text{ rad/sec}$$

The relationship between the $\omega_d$, $\omega_n$ and $\zeta$ $\rightarrow$ $\boxed{\omega_d = \omega_n\sqrt{1-\zeta^2}}$

Since the step response is oscillatory, we can estimate the damping ratio as $\zeta = 0.2$ and determine the undamped natural frequency as:




$\boxed{\zeta = 0.2}$ ➡ $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{4.03}{\sqrt{1-0.2^2}} \rightarrow \boxed{\omega_n = 4.11 \text{ rad/sec}}$$

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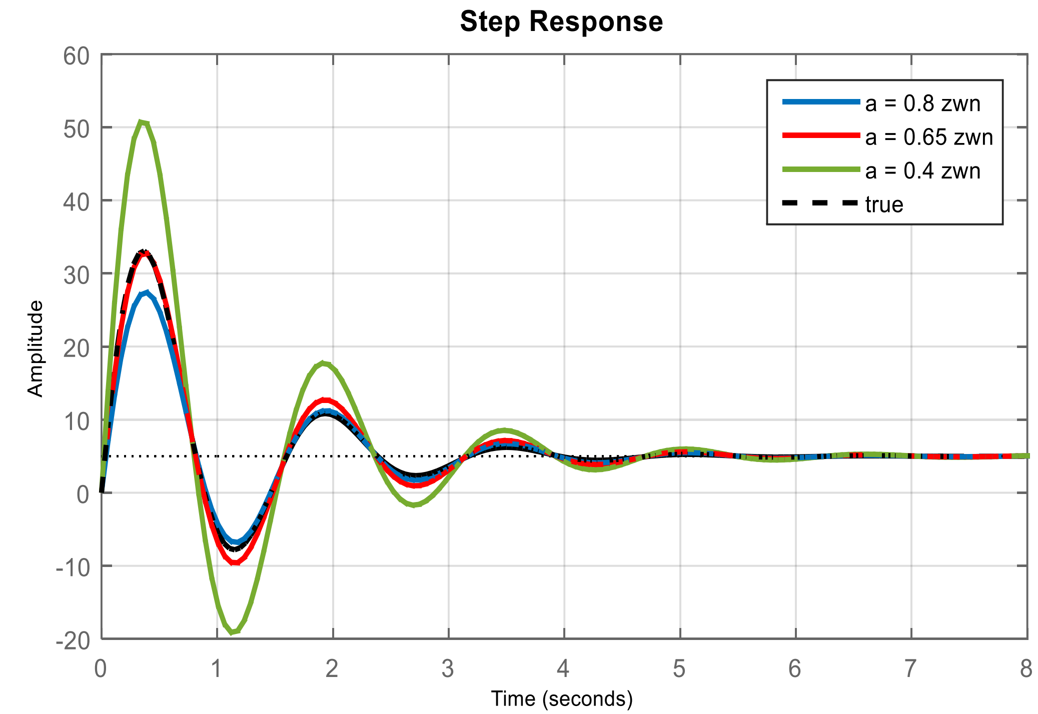
# System Modeling via Step Response

**Example 5** Determine TF model of the system based on the given step response.

$$G(s) = K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\cdot\frac{s+a}{a}$$

$\boxed{0 < a < \zeta\omega_n}$

$\boxed{K = 5}$ $\boxed{\zeta = 0.2}$ $\boxed{\omega_n = 4.11}$




Therefore, having the $\zeta$ and the $\omega_n$ the complex-conjugate poles location can be determined

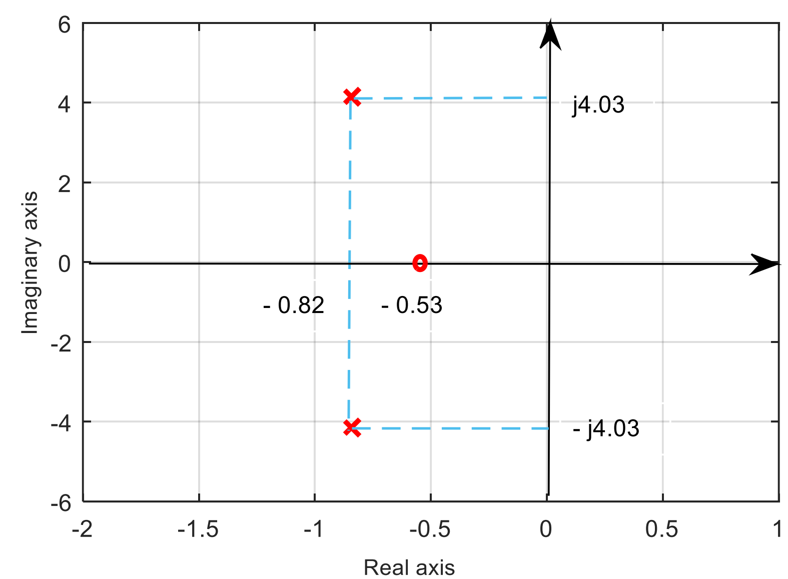
**Poles** $\rightarrow$ $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -0.82 \pm j4.03$

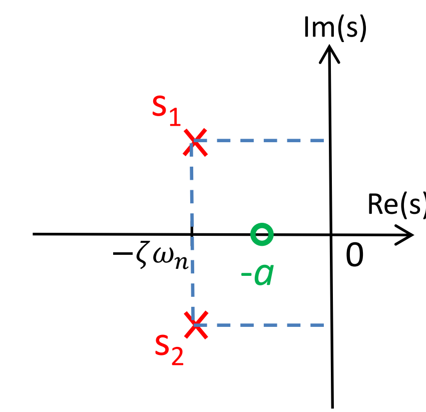
Location of the additional real zero $(s+a)$ must be selected between the complex conjugate poles and imaginary axis.

$\boxed{0 < a < \zeta\omega_n \quad \rightarrow \quad 0 < a < 0.82}$

$$a = 0.65\zeta\omega_n = 0.65 \times 0.82$$

$\boxed{a = 0.53}$ **The zero location**




$$G(s) = 5\frac{4.11^2}{s^2 + 2\times 0.82s + 4.11^2}\cdot\frac{s+0.53}{0.53} \rightarrow \boxed{G(s) = \frac{84.46s + 44.76}{0.53s^2 + 0.8692s + 8.953}}$$

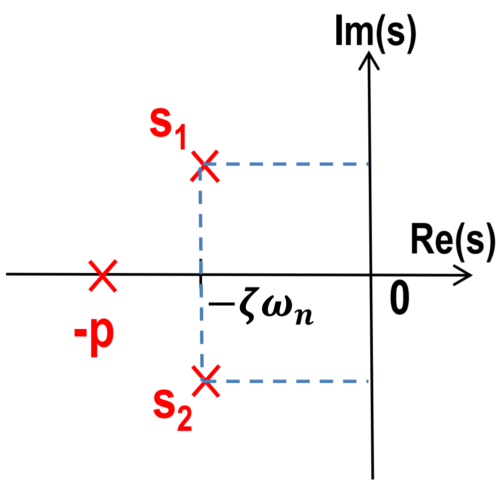
# System Modeling via Step Response

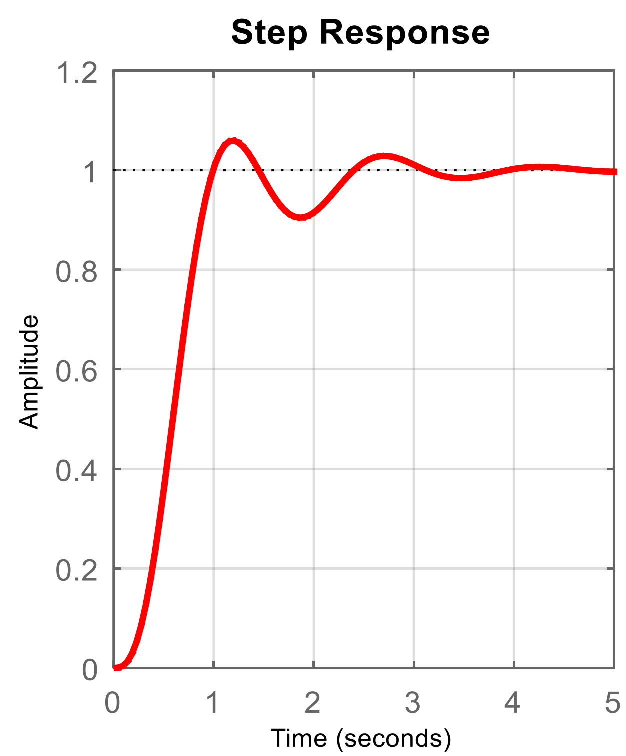
## ❑ Second-Order Model (Undamped System) with a Stable Pole

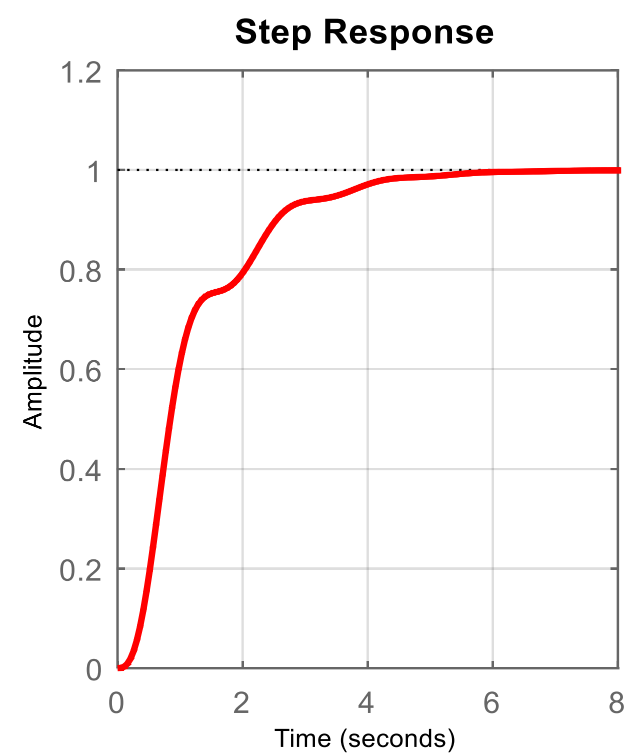
$$G(s) = K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \times \frac{p}{s+p}, \qquad p > 0$$

$K$ : DC-gain
$\zeta$ : Damping ratio
$\omega_n$ : Un-damped natural frequency
$-a$ : Zero location

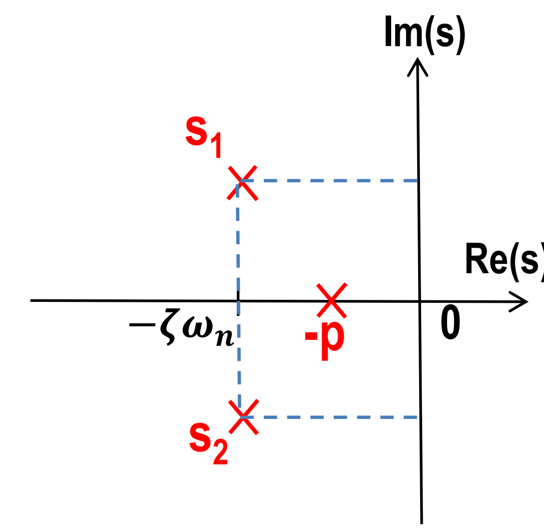
$$4\zeta\omega_n > p > \zeta\omega_n$$




$$0 < p < \zeta\omega_n$$



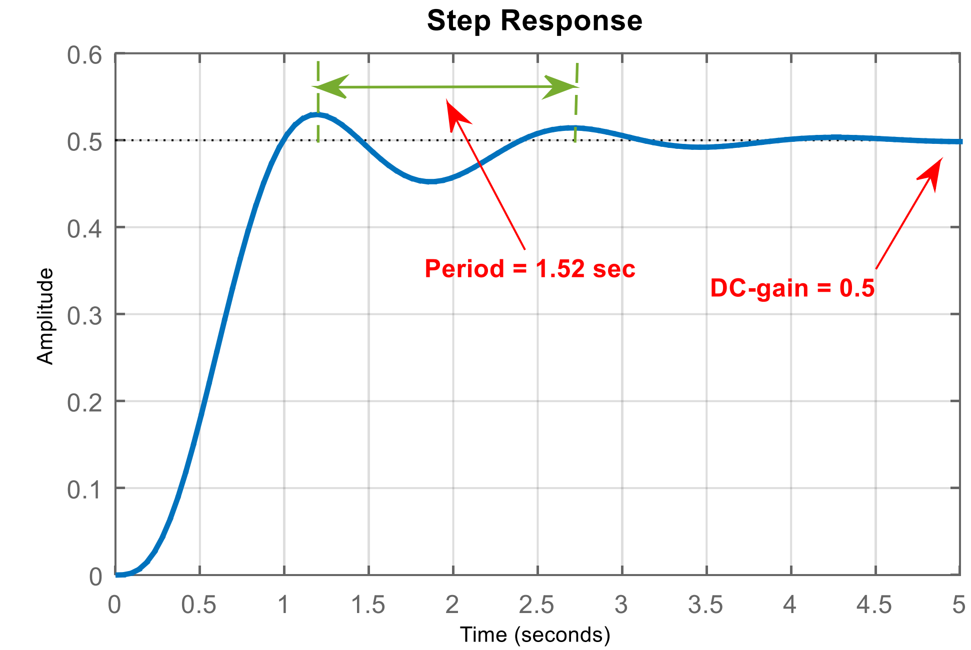

- Effects of the stable real pole at $s = -p$ on the unit-step response is:
  - Reducing the maximum overshoot
  - Increasing the settling time

# System Modeling via Step Response

**Example 6** Determine TF model of the system based on the given step response.

$$G(s) = K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{p}{s+p}$$

$$\boxed{4\zeta\omega_n > p > \zeta\omega_n}$$




From the unit-step response graph we have the DC-gain:

$$\boxed{K = 0.5}$$

The damped natural frequency ($\omega_d$) can be obtained from the period of the oscillations in step response

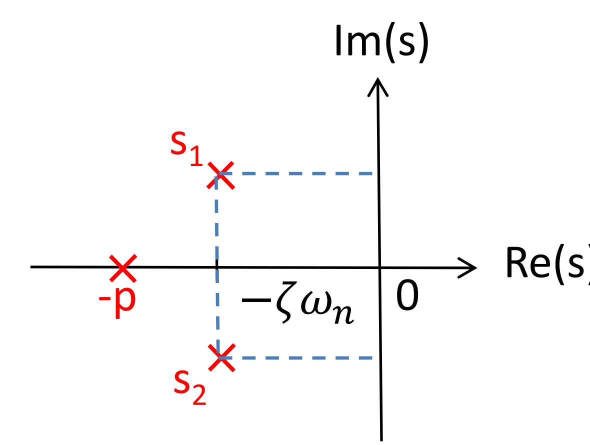
$$\omega_d = \frac{2\pi}{\text{period}} = \frac{2\pi}{1.52} = 4.13 \text{rad/sec}$$

The relationship between the $\omega_d, \omega_n$ and $\zeta$ $\rightarrow$ $\boxed{\omega_d = \omega_n\sqrt{1-\zeta^2}}$

Since the step response is oscillatory, we can estimate the damping ratio as $\zeta = 0.2$ and determine the undamped natural frequency as:

$\boxed{\zeta = 0.2}$ ➡ $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{4.13}{\sqrt{1-0.2^2}} \rightarrow \boxed{\omega_n = 4.22 \text{ rad/sec}}$$



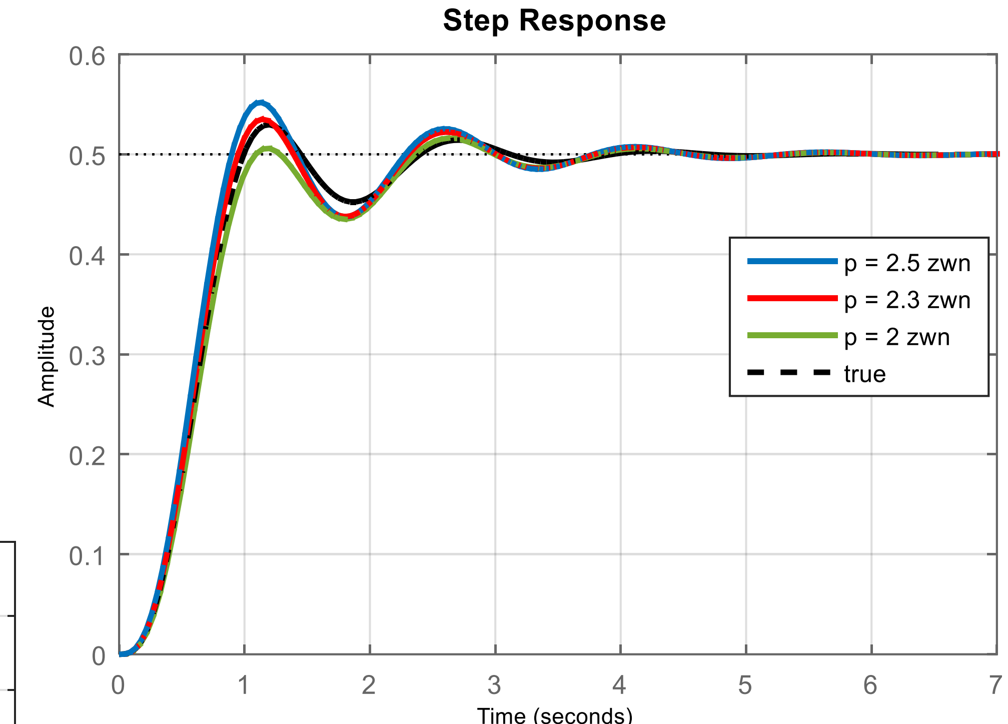
# System Modeling via Step Response

**Example 6** Determine TF model of the system based on the given step response.

$$G(s) = K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\cdot\frac{p}{s+p}$$

$\boxed{4\zeta\omega_n > p > \zeta\omega_n}$

$\boxed{K = 5}$ $\boxed{\zeta = 0.2}$ $\boxed{\omega_n = 4.11}$




Therefore, having the $\zeta$ and the $\omega_n$ the complex-conjugate poles location can be determined

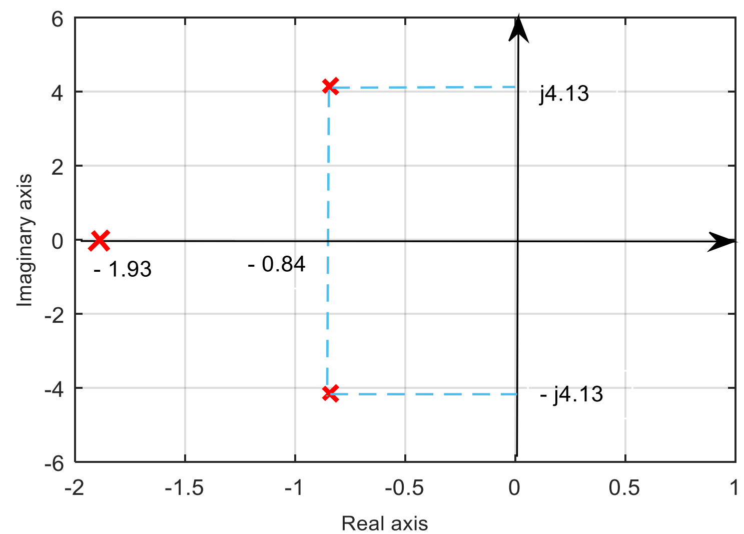
**Poles** $\rightarrow$ $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -0.84 \pm j4.13$

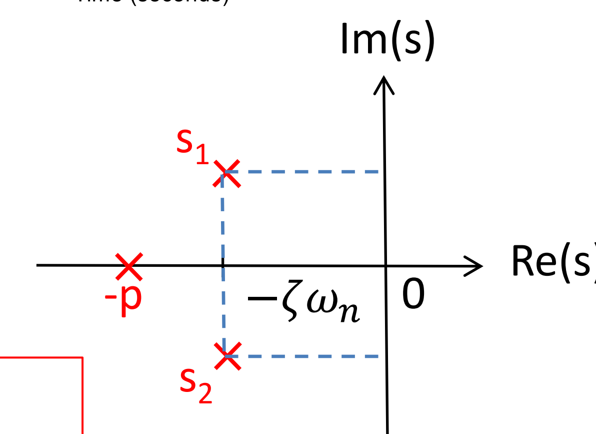
Location of the additional real pole $(s+p)$ must be selected between the complex conjugate poles and imaginary axis.

$$\boxed{4\zeta\omega_n > p > \zeta\omega_n \quad\rightarrow\quad 3.36 > p > 0.84}$$

$$p = 2.3\zeta\omega_n = 2.3 \times 0.84$$

$\boxed{p = 1.93}$ **The real pole location**



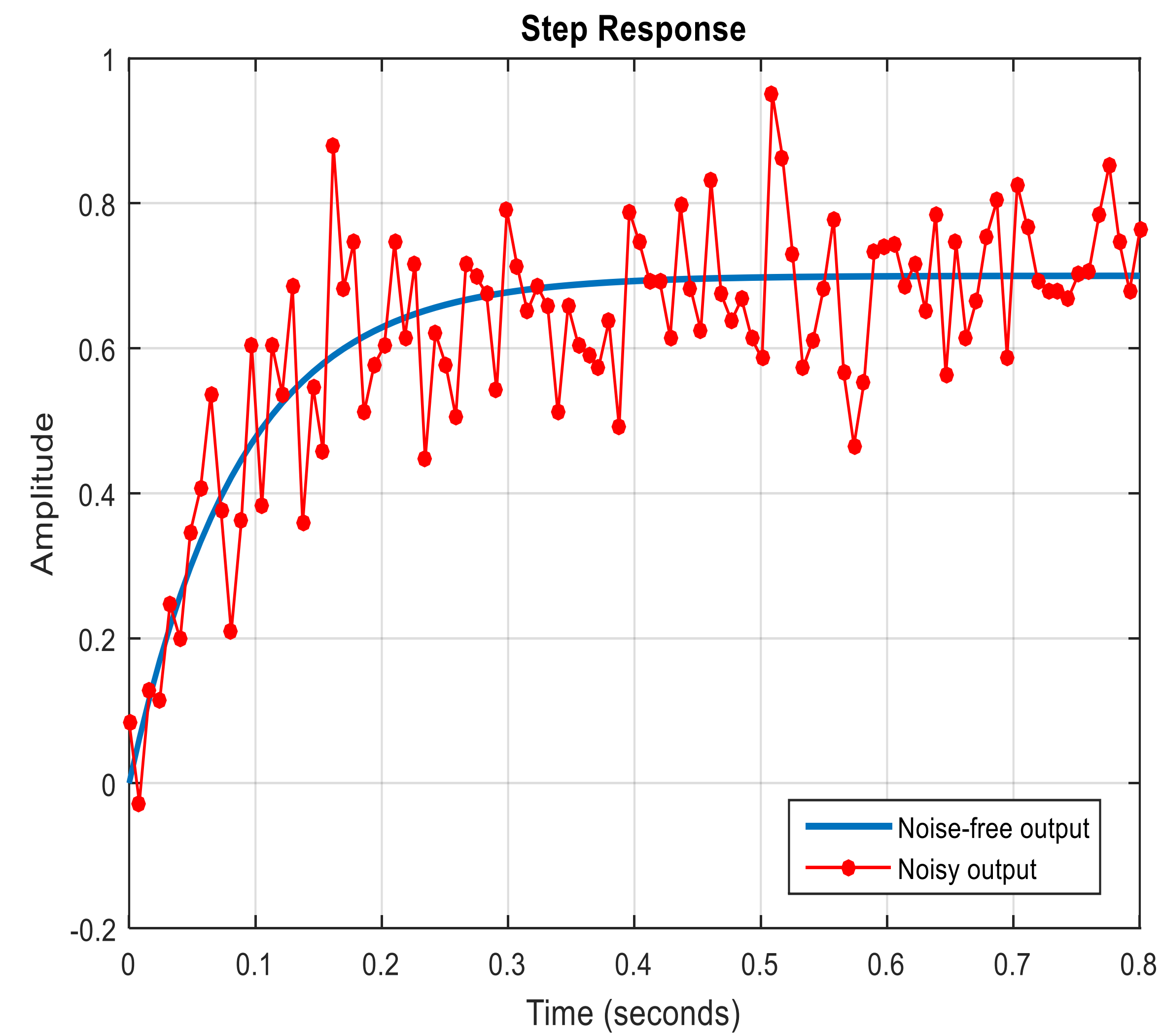

$$G(s) = 0.5\frac{4.22^2}{s^2 + 2\times 0.84s + 4.22^2}\cdot\frac{1.93}{s+1.93} \rightarrow \boxed{G(s) = \frac{17.18}{s^3 + 3.618s^2 + 21.06s + 34.35}}$$

# System Modeling via Step Response

## ❑ Disadvantages of Transient Response Modeling

- Transient-response analysis is very sensitive to noise.
- If measurement of the output signal contaminated by a considerable noise level, it will be hard to assess the system properties by a single measurement.

We can estimate a transfer function model using the input/output data and applying System Identification methods.




$$G(s) = ?$$

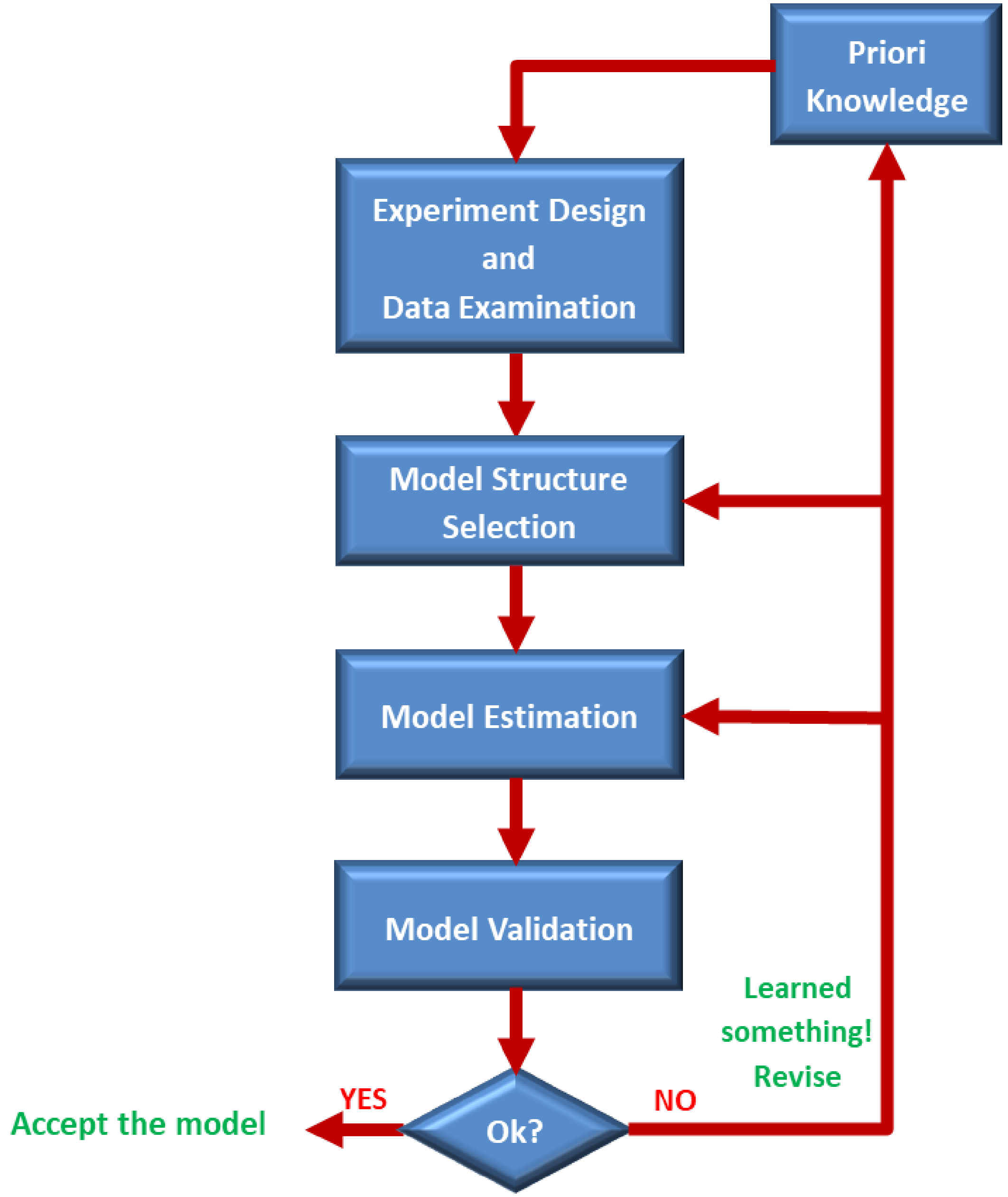
# System Identification

- Three modeling approaches are common in the field of system modeling and identification:
  - **White-Box Modeling**:
    - The system is entirely known.
    - The model order, model parameters and model structure are known.
    - The system can be modeled by differential equations derived from first-principles.
  - **Grey-Box Modeling**:
    - The system is not entirely known.
    - A certain model is constructed based on both insight into the system and experimental data.
    - The model still have some unknown parameters to be estimated using system identification.
  - **Black-Box Modeling**:
    - No prior model is available. Most of the system identification algorithms are of this type.

- The field of System Identification uses ***statistical methods*** to build mathematical models of dynamical systems from measured experimental data.
- System identification also includes the optimal design of experiments for efficiently generating informative data for fitting such models as well as model reduction.
- A much more common approach is therefore to start from measurements of the behavior of the system (output response) and the external influences (inputs to the system) and try to determine a mathematical relation between them without going into the details of what is actually happening inside the system.
- This approach is called System Identification.

# System Identification Procedure

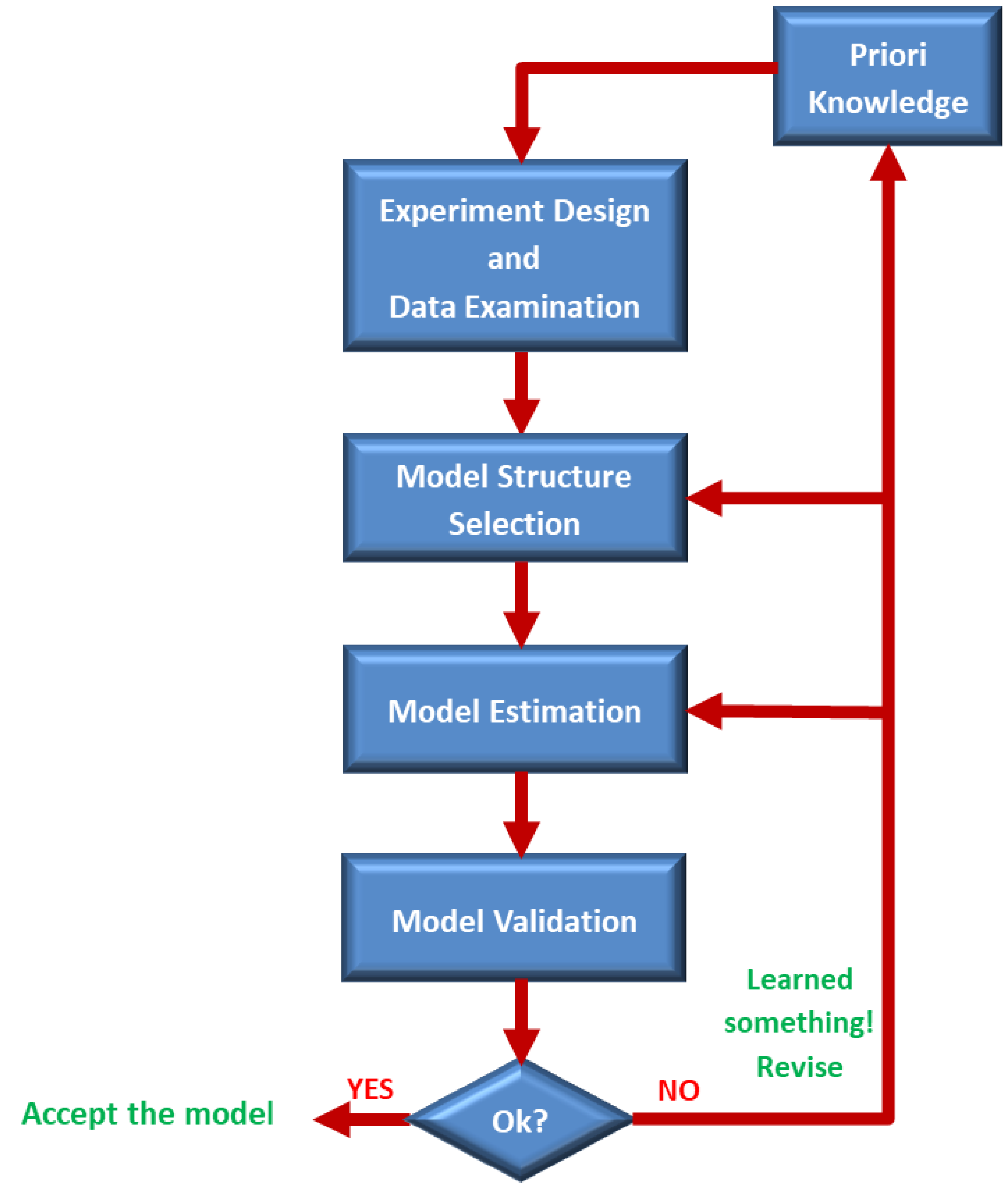
- System Identification is an **iterative** procedure, and it is often necessary to go back and repeat earlier steps.
  - **Prior Knowledge**
    - Purpose of Modeling
      - Control System Design
    - Grey-box Identification
      - Some part of the system is known
        - Model Order, Dominant Pole Locations, An Integrator, ….
    - Black-box Identification
      - No prior knowledge about the system
  - **Experiment Design & Data Examination**
    - Choice of Input Signal and I/O Data Collection
    - I/O Data Examination
      - Aliasing, Outliers and Trends, Noise Filtering
    - Preliminary Diagnostic Experiments
      - Frequency Response Analysis
        - Bode Diagrams
      - Time Response Analysis
        - Impulse Response
        - Step Response



# System Identification Procedure

- System Identification is an **iterative** procedure, and it is often necessary to go back and repeat earlier steps.

  - **Parametric Model Structure Selection**
    - Continuous Time Models
      - Transfer Function Model
      - State Space Model
    - Discrete Time Models
      - Transfer Function Models
        - Box-Jenkins (BJ) Model
        - Output-Error (OE) Model
        - ARMAX Model
        - ARX Model
      - State Space Models
      - Time Series Models
        - AR Model
        - MA Model
        - ARMA Model



# System Identification Procedure

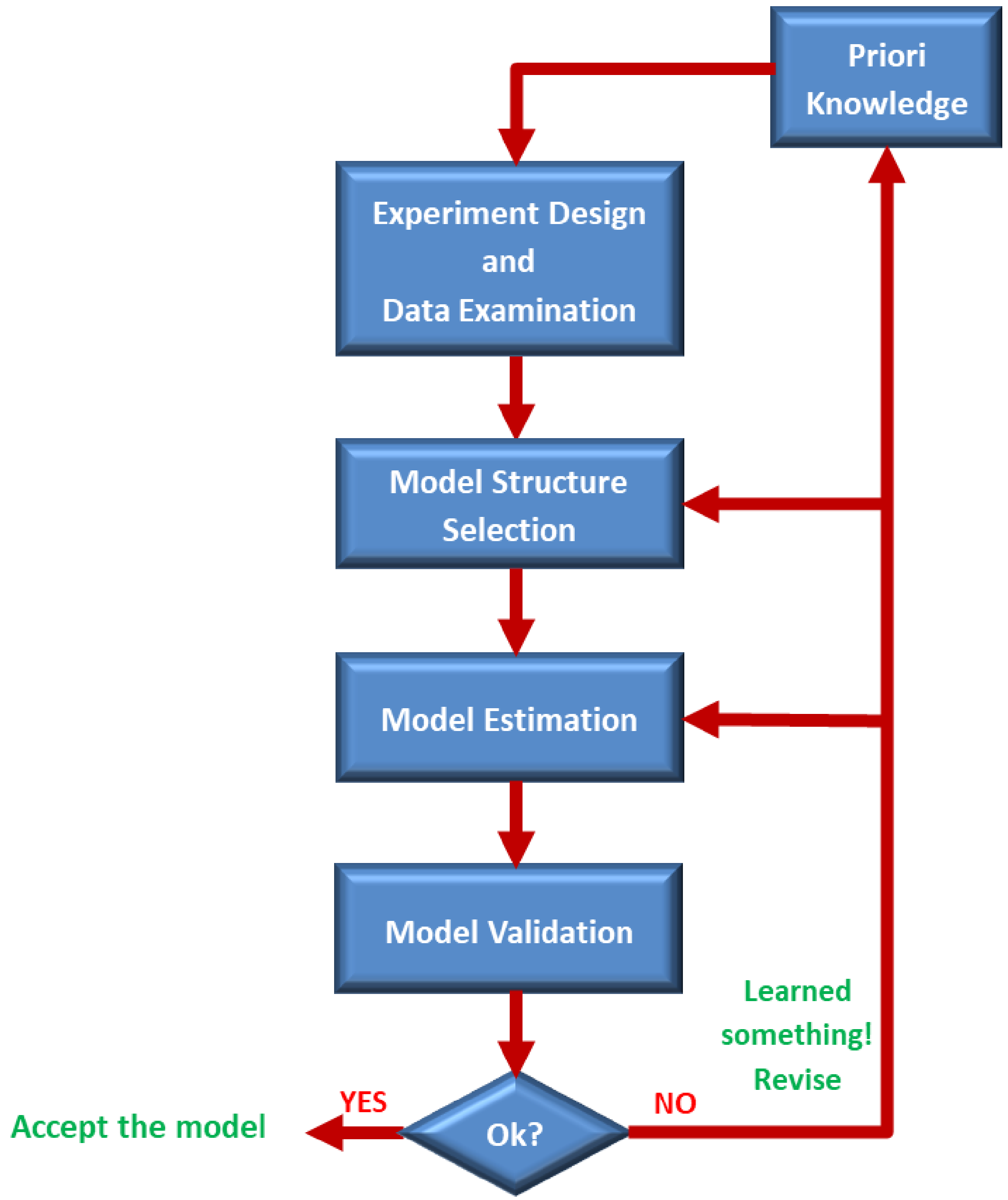
- System Identification is an **iterative** procedure, and it is often necessary to go back and repeat earlier steps.

## Model Estimation Techniques

- Nonparametric Methods
  - Spectral Analysis
  - Correlation Analysis
- Parametric Methods
  - Least Squares Method

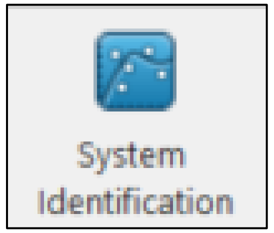
## Model Validation Techniques

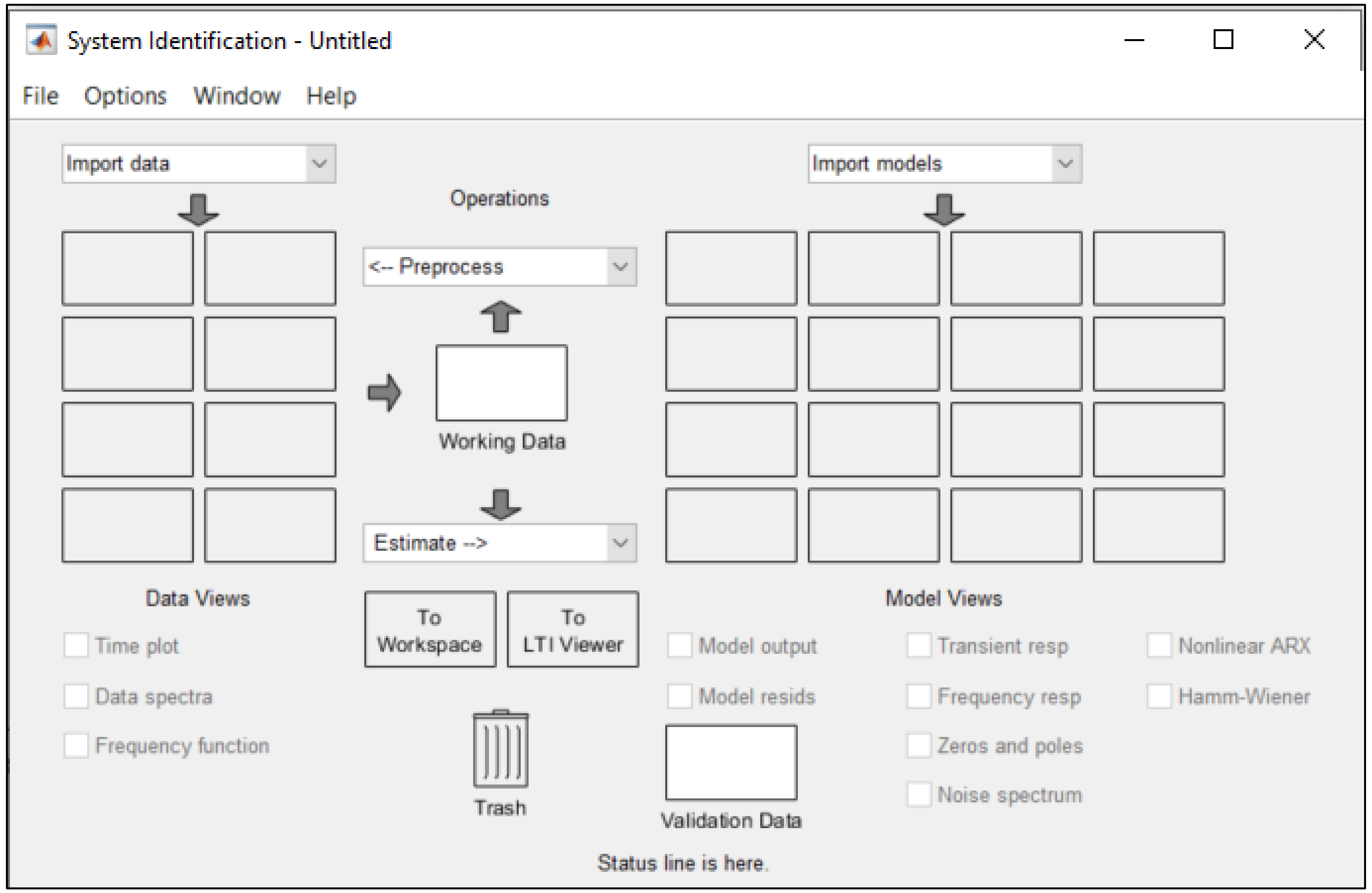
- Simulation
- Cross-Validation
- Model Validity Criterion
  - Mean-Squares Error (MSE)
  - Akaike's Final Prediction Error (FPE)
- Pole-Zero Plots
- Bode Diagram
- Residual Analysis
  - Auto-correlation Analysis
  - Cross-correlation Analysis



# Identification of Transfer Function Models

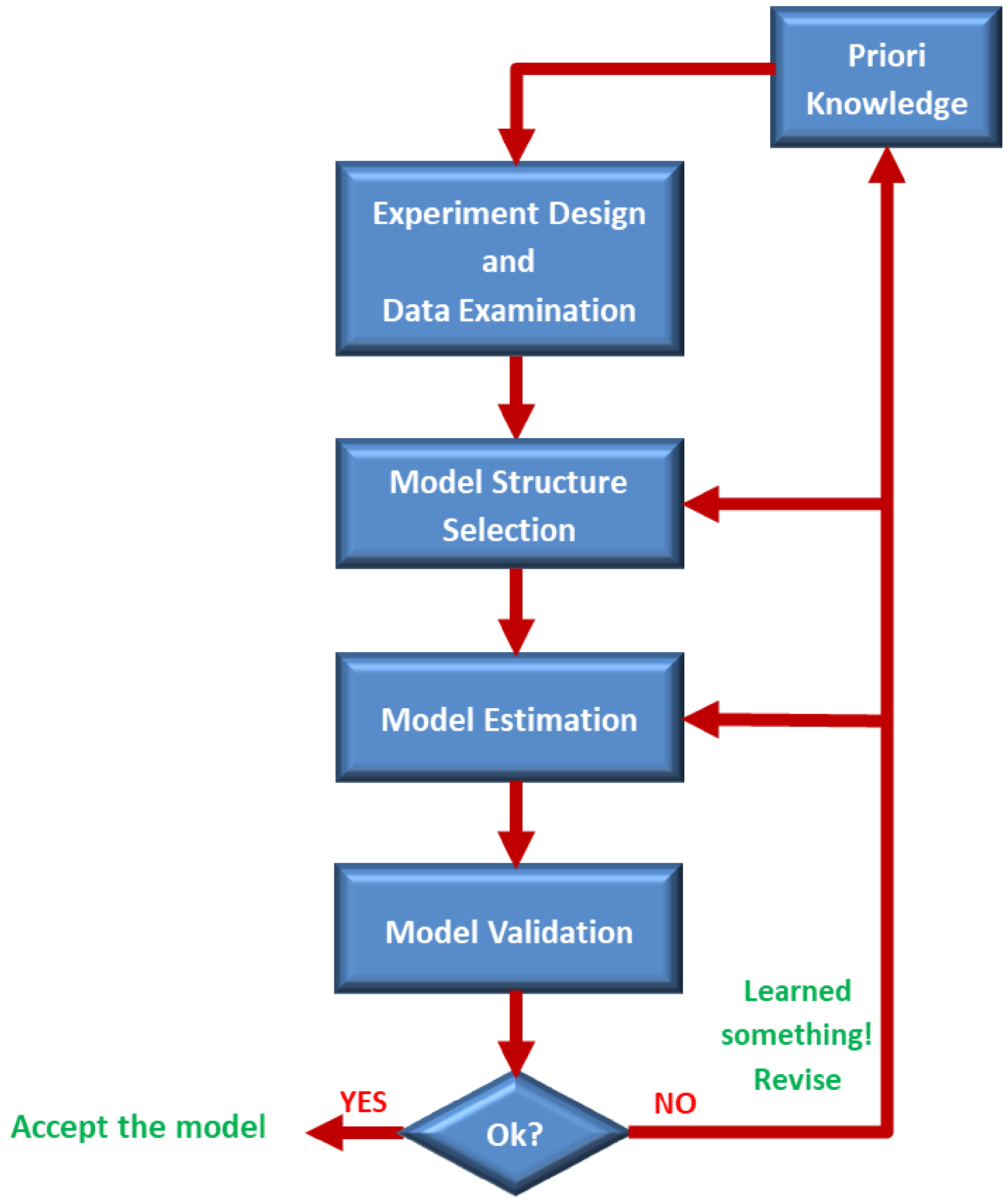
- **MATLAB System Identification Toolbox** provides line commands and an app for constructing mathematical models of dynamic systems from measured input-output data.
- It enables us to create and use models of dynamic systems that are not easily be modeled from first principles or specifications.
- We can use time-domain and frequency-domain input-output data to identify continuous-time and discrete-time transfer functions, process models, and state-space models.
- System Identification Toolbox enables us to create models from measured input-output data.
  - Analyze and process the input-output data
  - Select suitable model structure and order
  - Estimate model parameters
  - Validate the model accuracy



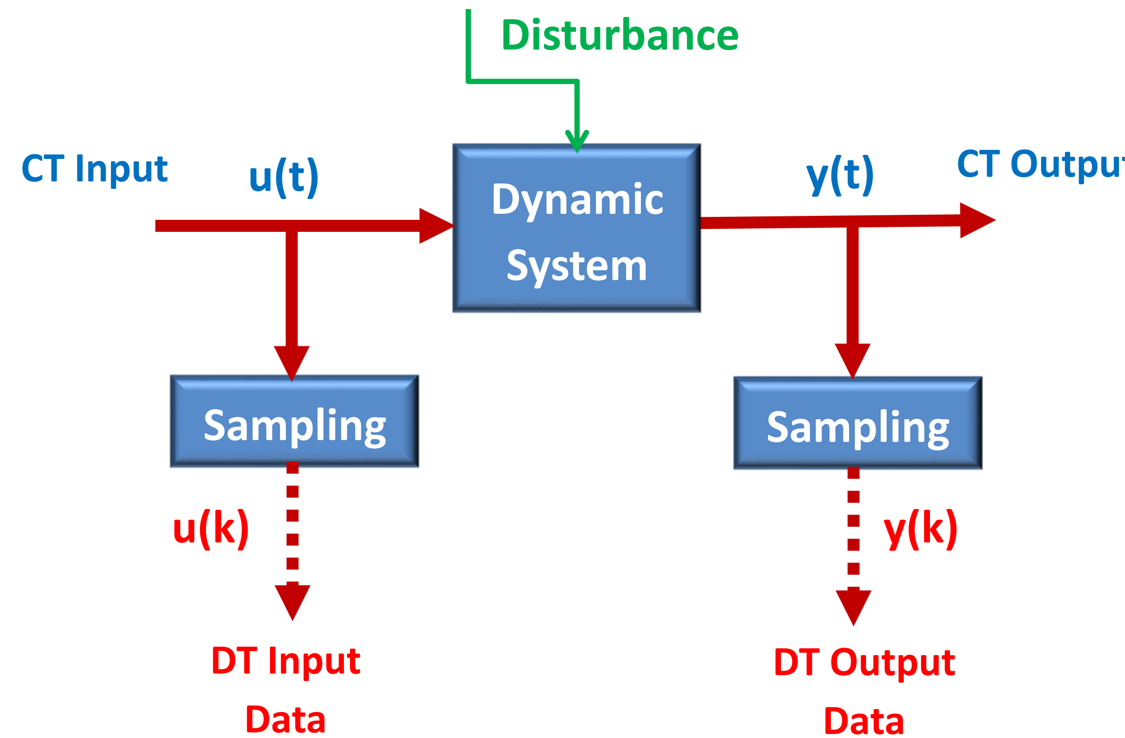
# System Identification Procedure

- **Experiment Design & Data Examination**
  - Choice of Input Signal and I/O Data Collection
  - I/O Data Examination
    - Aliasing, Outliers and Trends, Noise Filtering
  - Preliminary Diagnostic Experiments
    - Frequency Response Analysis
      - Bode Diagrams
    - Time Response Analysis
      - Impulse Response
      - Step Response



# Choice of Input Signal and Data Collection

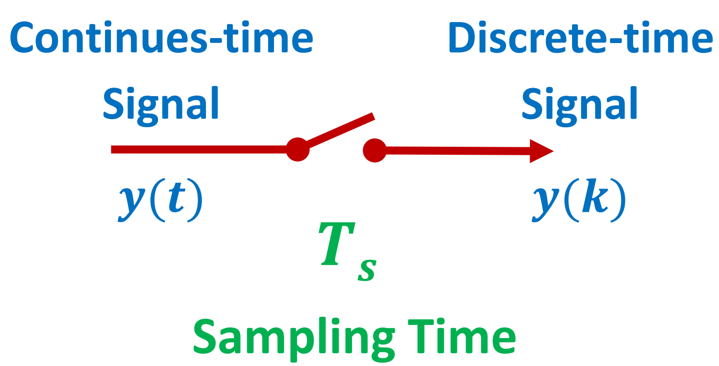
- In system identification, models are built from experimental data.
- Such data is obtained by exciting the system with an input and observing its response at regular intervals.
- This means we are dealing with **samples of discrete-time data.**

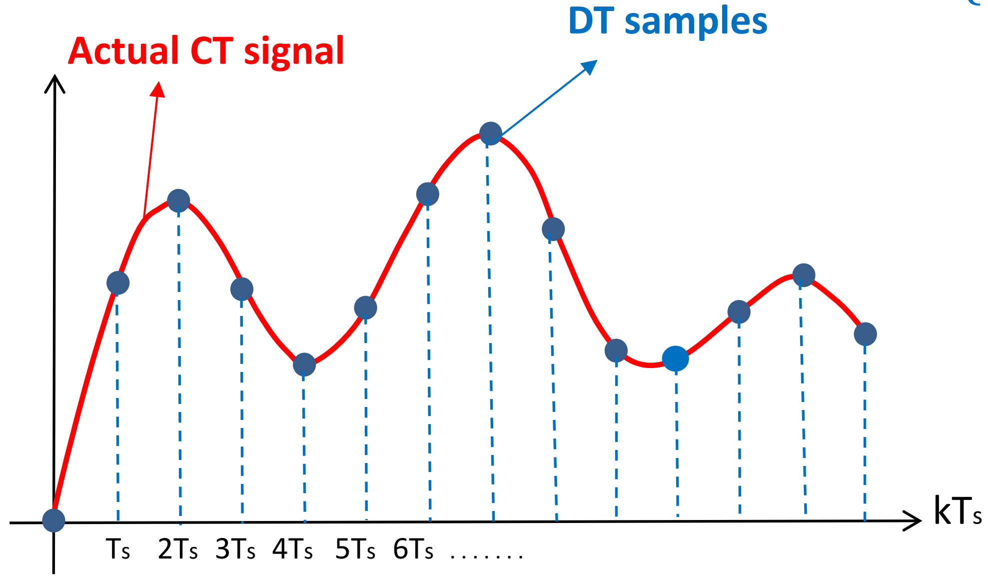



## ❑ Sampling

- Sampling is the process of converting the continues-time signal to a discrete-time signal

$$\{u(k), y(k) | k = 1, 2, \cdots, N\}$$

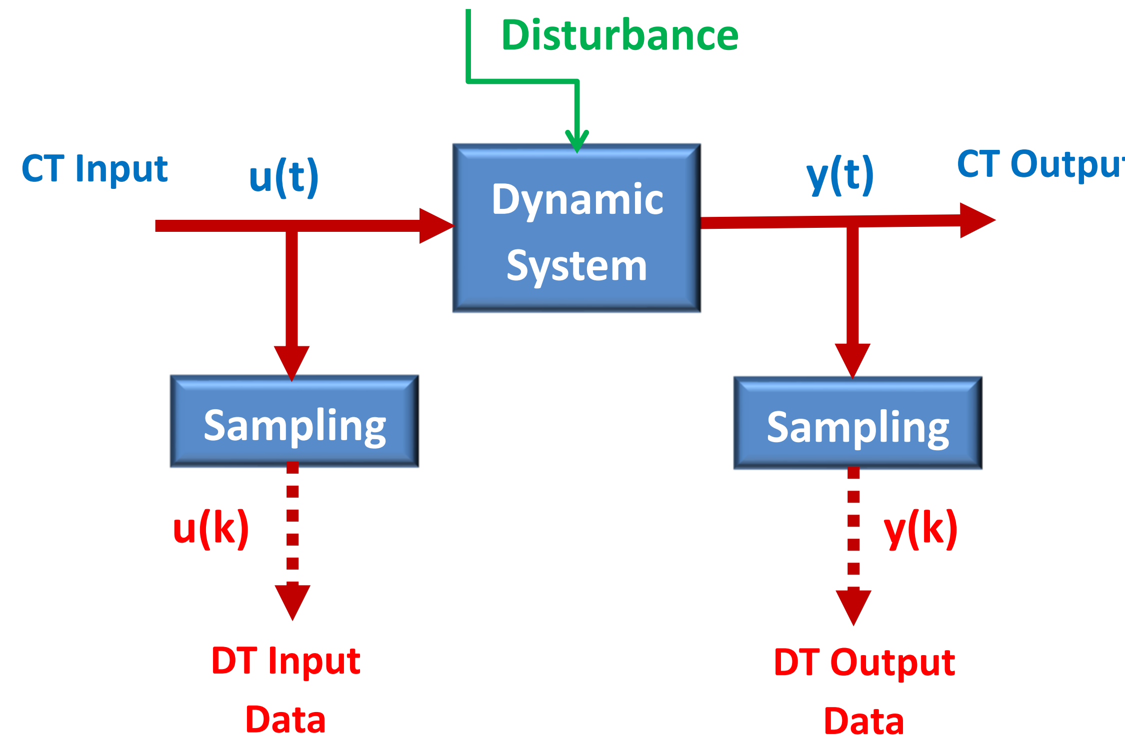


# Choice of Input Signal and Data Collection

## ❑ Sampling Rate

- Selecting the correct sampling rate is essential, when we convert a continues-time signal to a discrete-time signal.




- **Sampling Theorem** $\rightarrow$ The sampling rate $f_s$ must be at least twice faster than the highest frequency contained in the continuous-time signal.

$$\boxed{f_s \geq 2f_{max}}$$

$$\{u(k), y(k) \mid k = 1, 2, \cdots, N\}$$

- The minimum required sampling rate is called **Nyquist Rate:**

$$\boxed{\text{Nyquist Rate} = 2f_{max}}$$

## ❑ Aliasing Effect

- If the selected sampling rate does not satisfy the Sampling Theorem condition, then reconstructed signal from its sampled version leads to a distortion. This effect is called frequency-folding or aliasing.
- If there are aliasing effects in data, the sampling rate should be increased by considering the Sampling Theorem condition.

# Choice of Input Signal and Data Collection

## Example 7

Consider the continues-time signal is a sinusoid with the frequency of

$$f = 60Hz$$

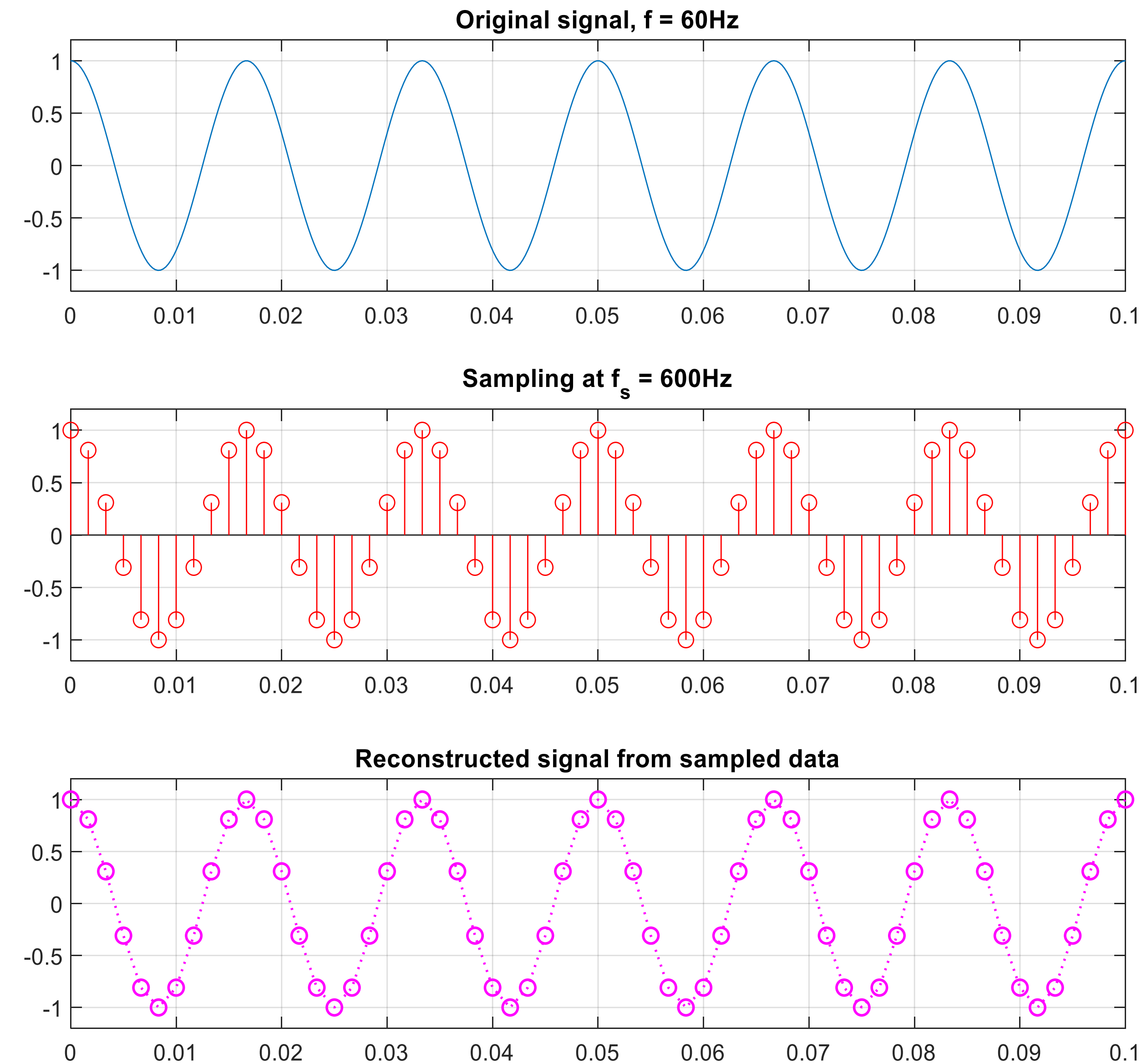
From the Sampling Theorem the sampling has to be selected as

$$f_s \geq 120Hz$$

Here, the selected sampling frequency is

$$f_s = 600Hz$$

**Good Sampling**




The selected sampling frequency is well above the Nyquist rate,120Hz.

Therefore, the samples can capture the oscillation of the original signal. Reconstructed signal is same as the original signal.

# Choice of Input Signal and Data Collection

**Example 7**

Consider the continues-time signal is a sinusoid with the frequency of

$$f = 60Hz$$

From the Sampling Theorem the sampling has to be selected as

$$f_s \geq 120Hz$$

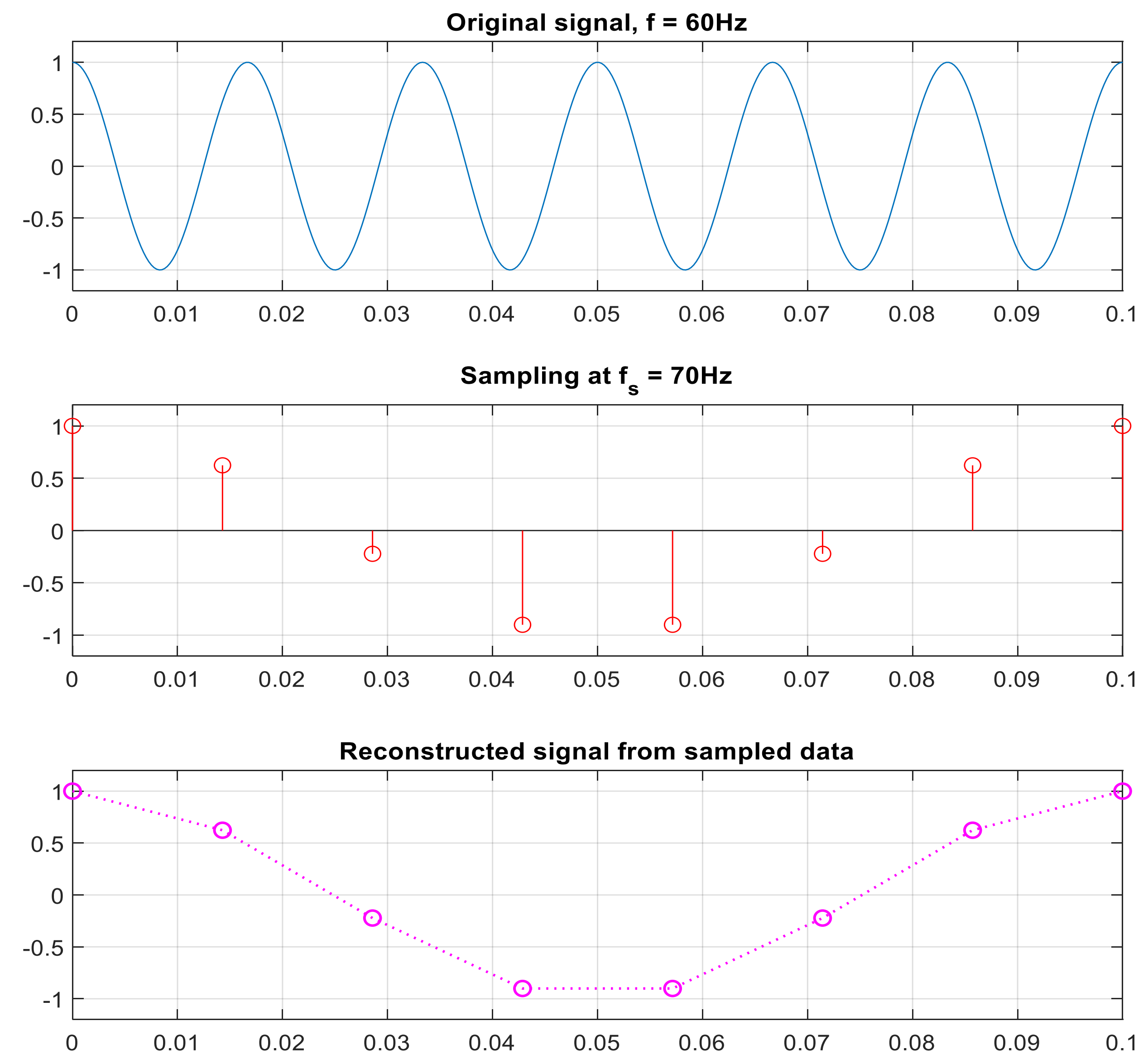
Here, the selected sampling frequency is

$f_s = 70Hz$ → **Aliasing**

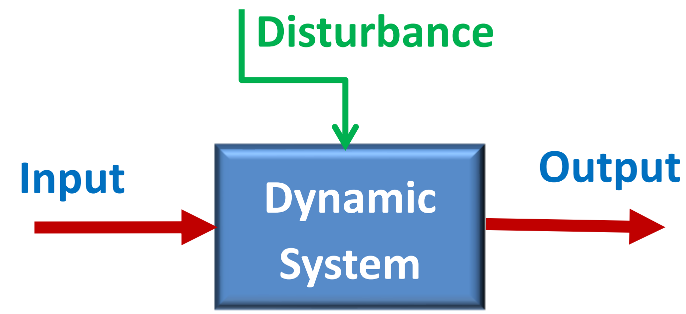
The selected sampling frequency is too low.

Therefore, the samples cannot capture the oscillation of the original signal, and aliasing happens.

The reconstructed signal looked like a low frequency sinusoid signal.



# Choice of Input Signal and Data Collection




- The input signal of the process plays an important role in system used in a system identification.
- The input signal is the only possibility to influence the system to collect information about its dynamic behavior.

❑ **The most often used input signals in practice for system identification**

- **Step/Impulse Signal** → Time-domain Identification
- **Sinusoid Signal** → Frequency-domain Identification

⇒ **Non-parametric Methods**

- **Pseudo-Random Binary Sequence (PRBS)** → Time-domain Identification
  Frequency-domain Identification

⇒ **Parametric Methods**

In this course, we use **PRBS** signal for system identification

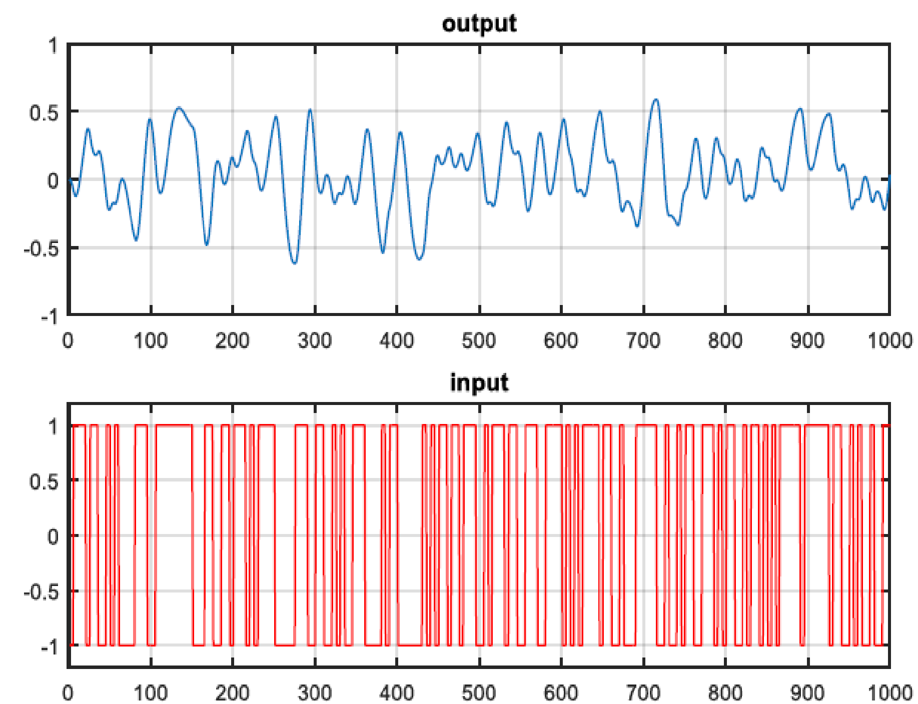
# Choice of Input Signal and Data Collection

## ❑ Pseudo-Random Binary Sequence (PRBS)

- A Pseudo-random binary sequence (PRBS) is a common choice of input signal, since it has a large energy content in a large frequency range.
- PRBS is a square wave (sum of sinusoids) that randomly changes between +1 and -1
- It is pseudo random because we can control when the switch may occur.
- The **`idinput`** function from System Identification Toolbox is available to generate PRBS signal.

```
u = A*idinput(N,'prbs',band,range)
```

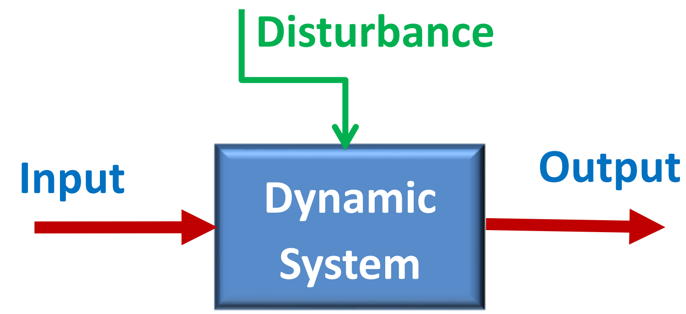
- u: **Pseudo random binary sequence**
- A: **Amplitude of signal**
- N: **Number of sample points**
- band: **Frequency band of signal**
- range: **Amplitude range of signal**




## ❑ Important Characteristics of Input Signal

- Amplitude
- Frequency Range $\rightarrow$ Good excitation in the frequency range (Persistent Excitation)
- Duration $\rightarrow$ Number of Samples

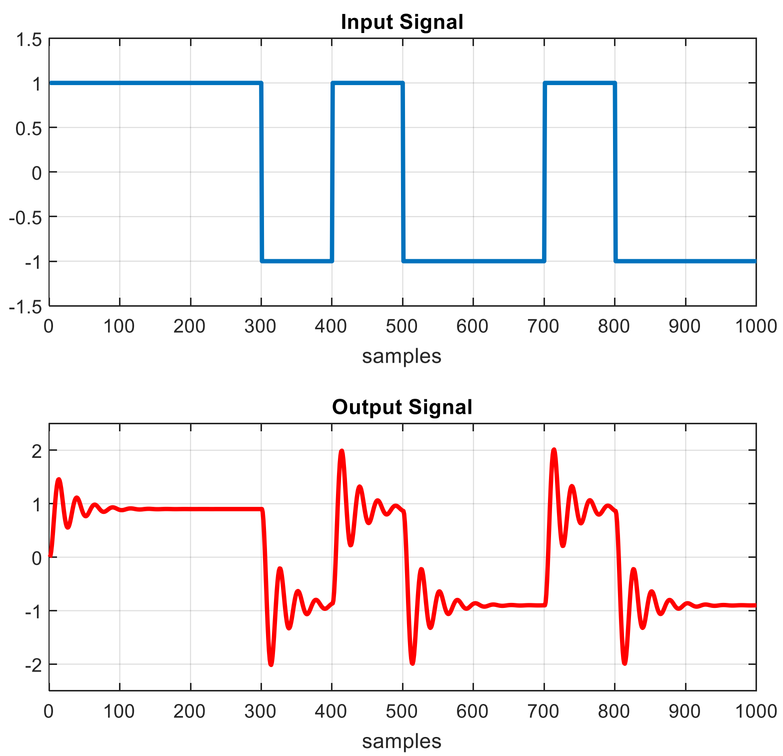
# Choice of Input Signal and Data Collection



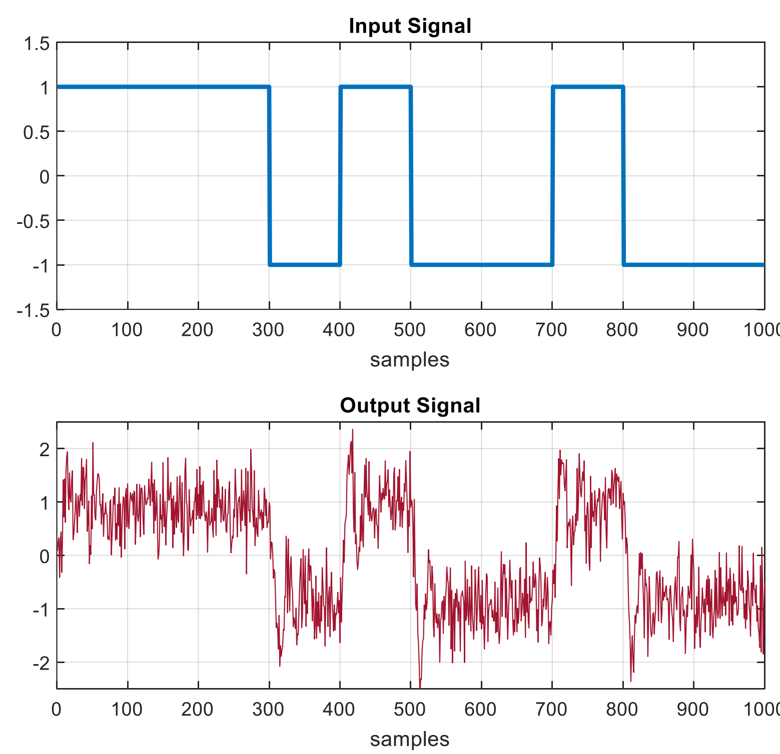

## ❑ Amplitude

- Amplitude of the input signal should be selected appropriately
  - To achieve a good signal to noise ratio (SNR)
  - To overcome friction and dead-zone issues
  - To avoid saturation and non-linearity range

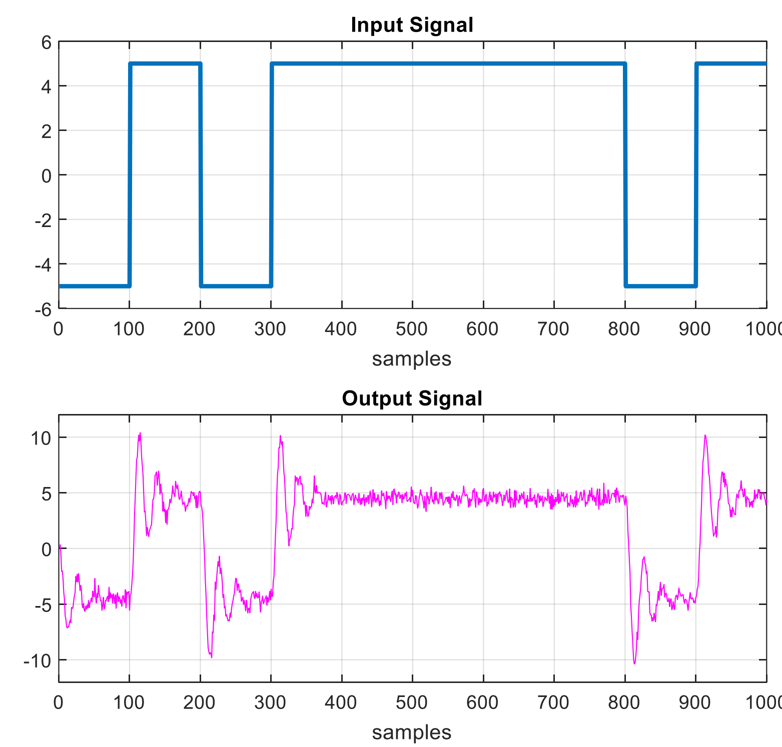
**Noise-free System**




**Low SNR Noisy System**




**High SNR Noisy System**



# Choice of Input Signal and Data Collection

## ❑ Amplitude

- Amplitude of the input signal should be selected appropriately
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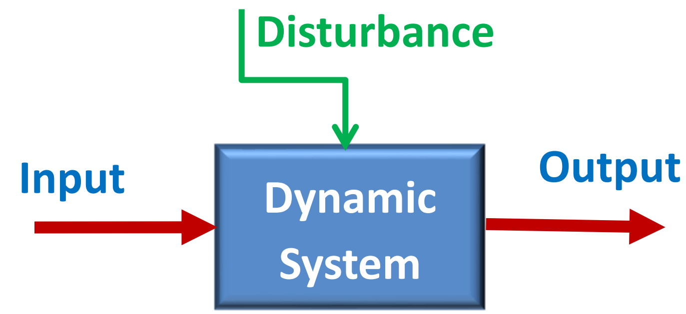
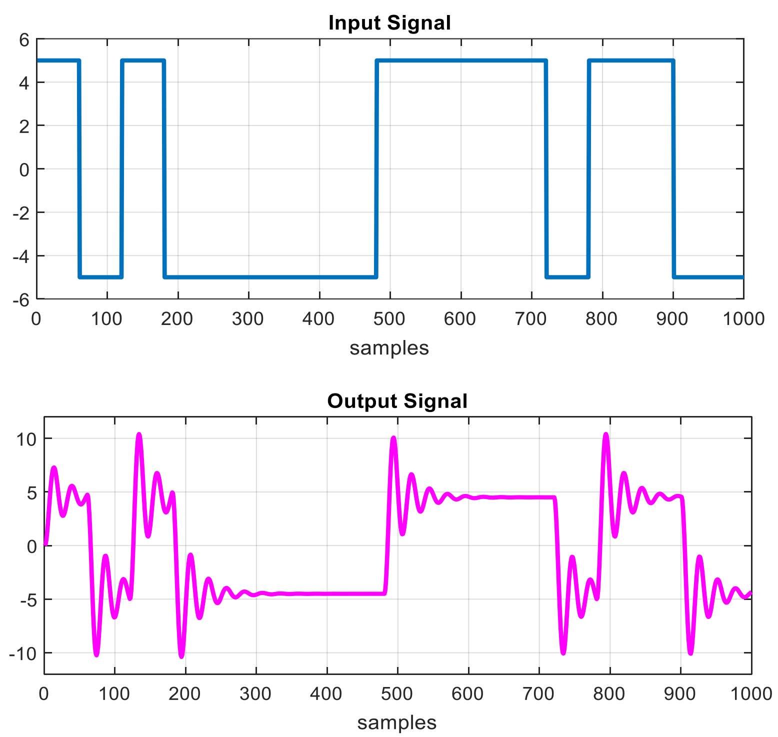
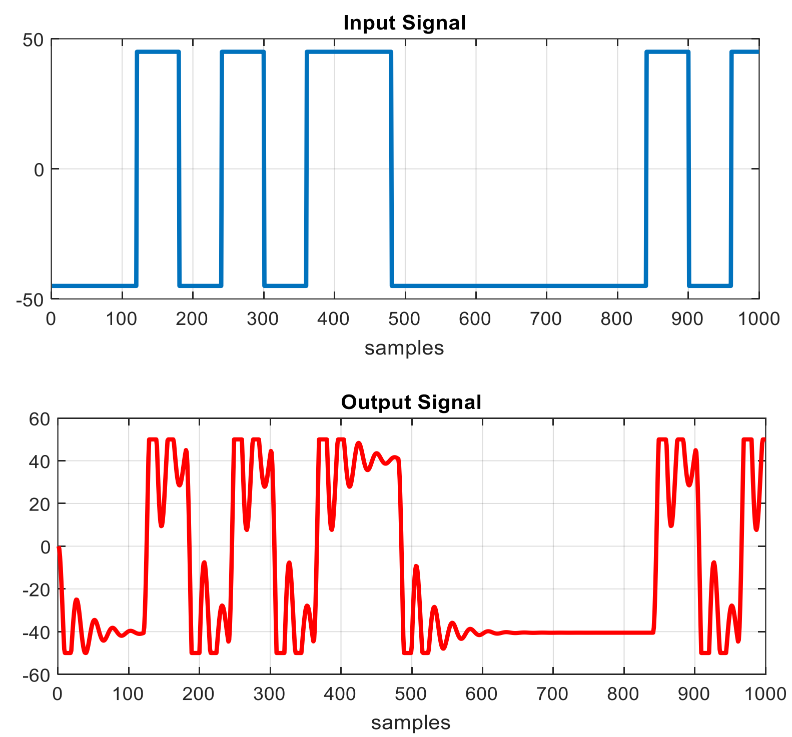


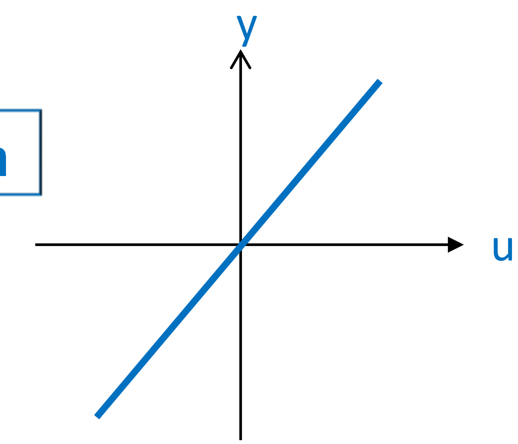

**No Saturation**



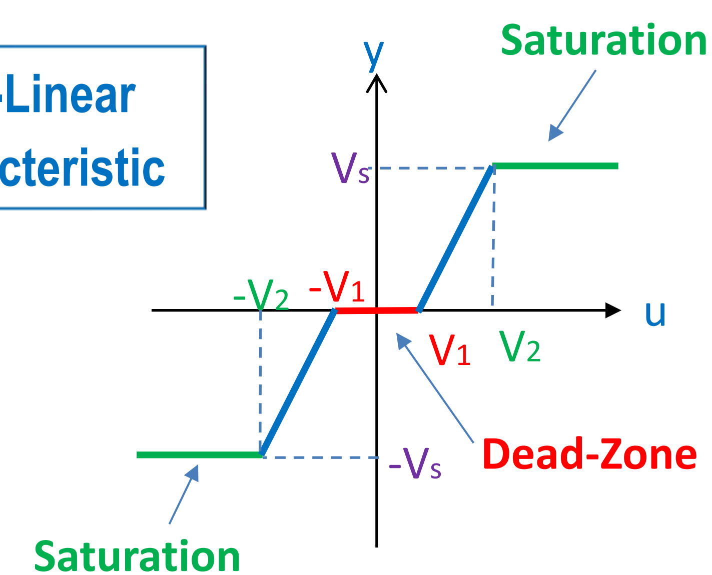

**Saturation at ±50**




**Linear System**



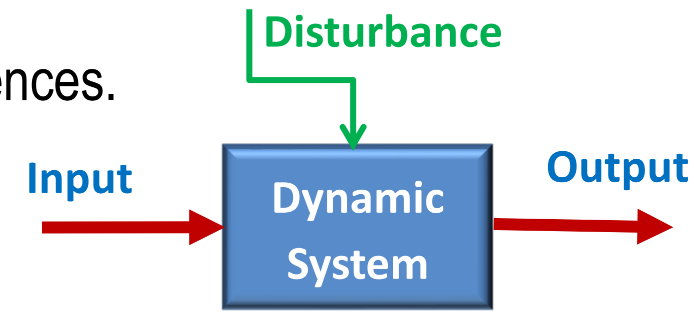

**Non-Linear Characteristic**




- The amplitude may not be chosen larger than the range in which the linearity assumption holds.

# Input-Output Data Examination

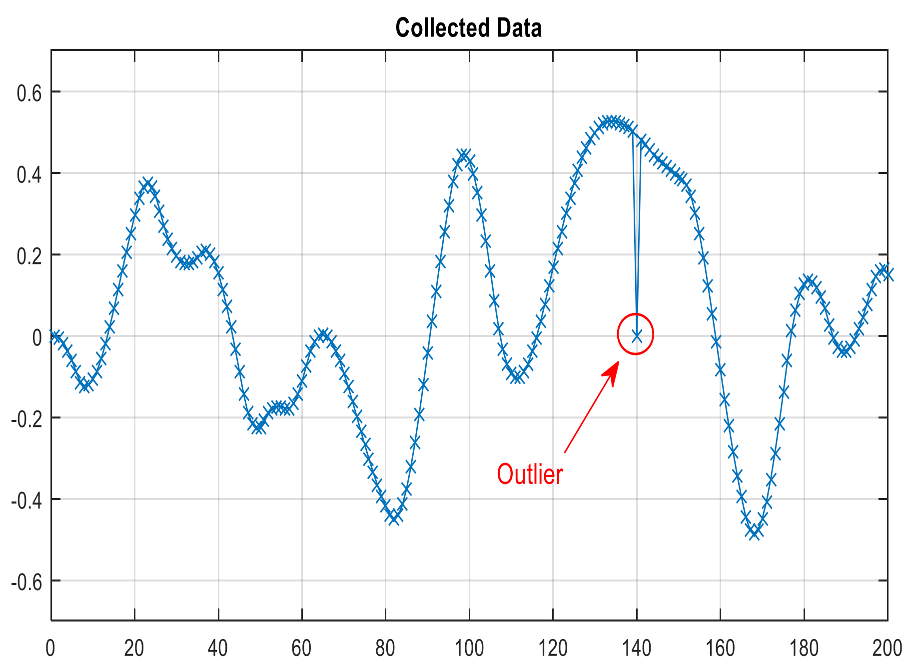
- Assume that an experiment has been performed, and we have the input and output sequences.
- Check the data manually via plots and look for:
  - Aliasing Effect
  - Outliers
  - Trends or DC-offset

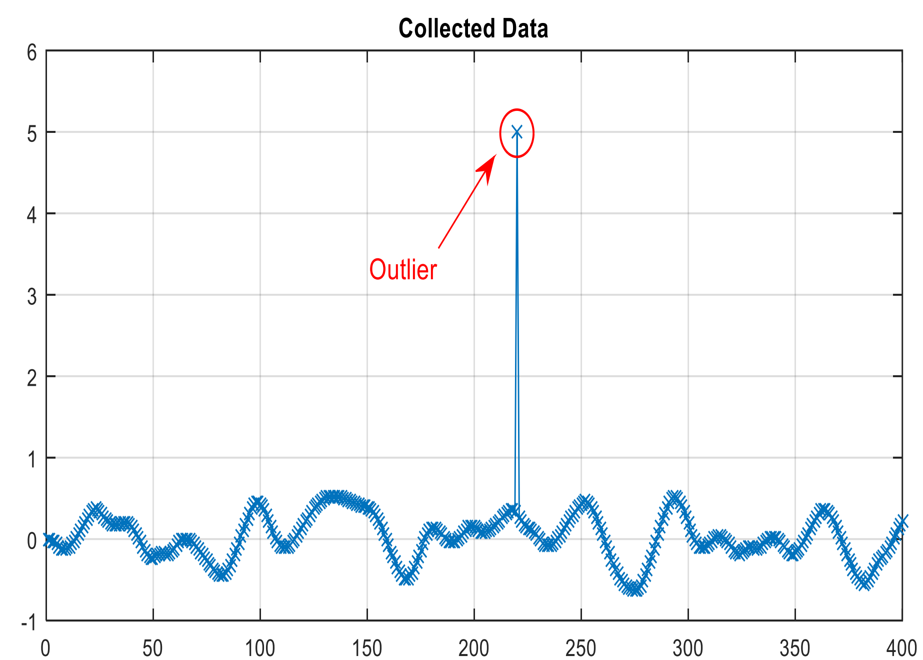



$$\{u(k), y(k) | k = 1, 2, \cdots, N\}$$

## ❑ Outliers

- In practice, the data acquisition equipment is not perfect.
- It may be that certain measured values are in obvious error due to measurement failures.
- Such bad values are often called outliers.



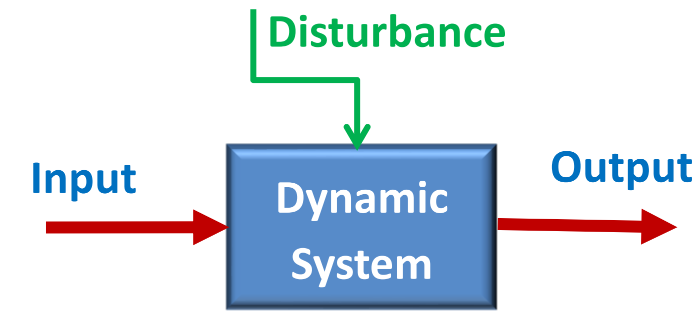




Outliers may have a substantial negative effect on the estimation and should simply be removed from the data.

# Input-Output Data Examination

## ❑ Trends or DC-offset

- Measured signals often show low-frequency drifts or non-zero means, which may have a bad effect on identification results if they are not specifically accounted for.

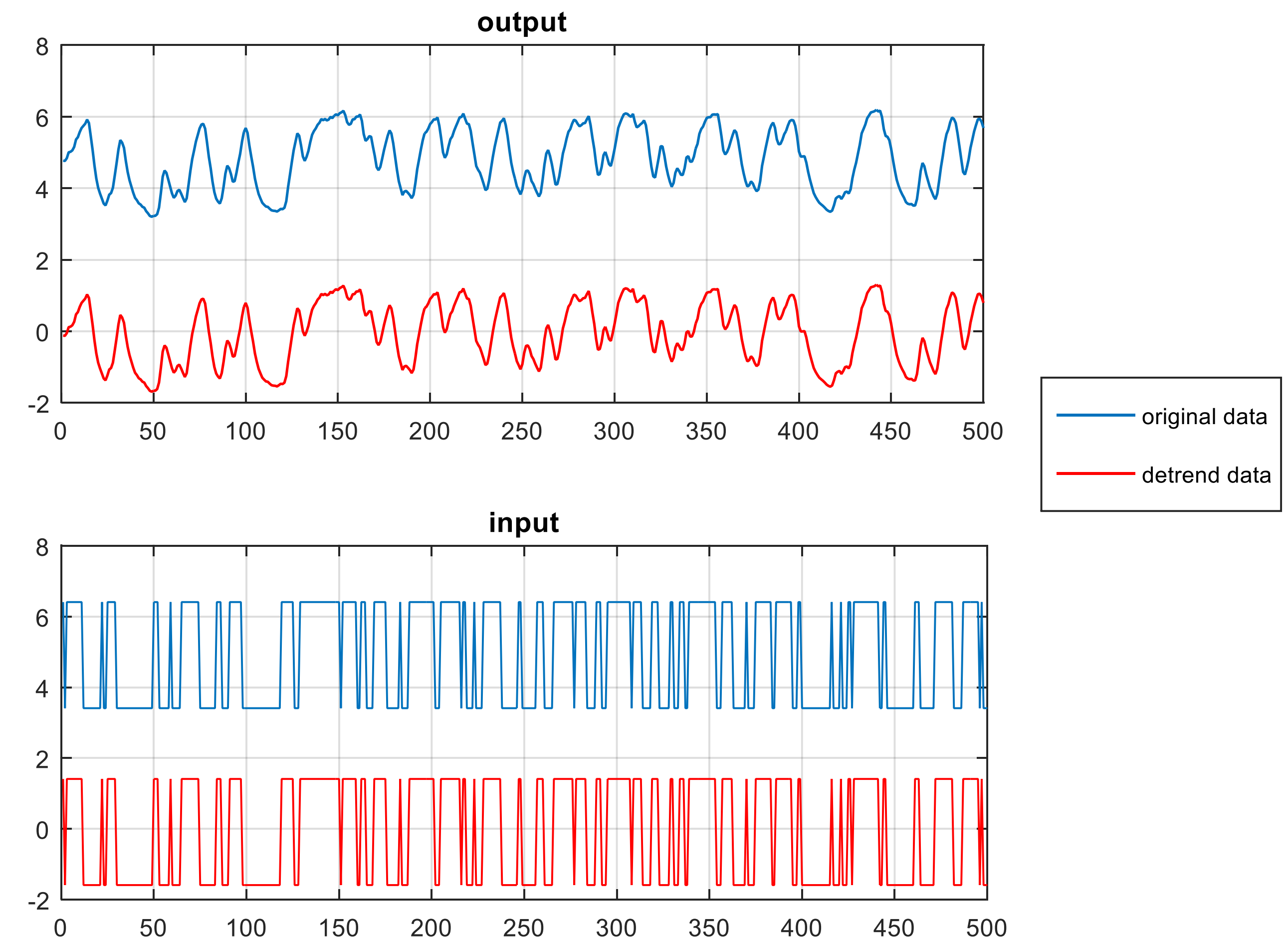



$$\{u(k)\,, y(k)|\; k = 1, 2, \cdots, N\}$$

- Linear trends in data, such as low-frequency drifts or non-zero means, should be removed from the I/O data.

- A linear trend can be removed by **`dtrend`** command in MATLAB

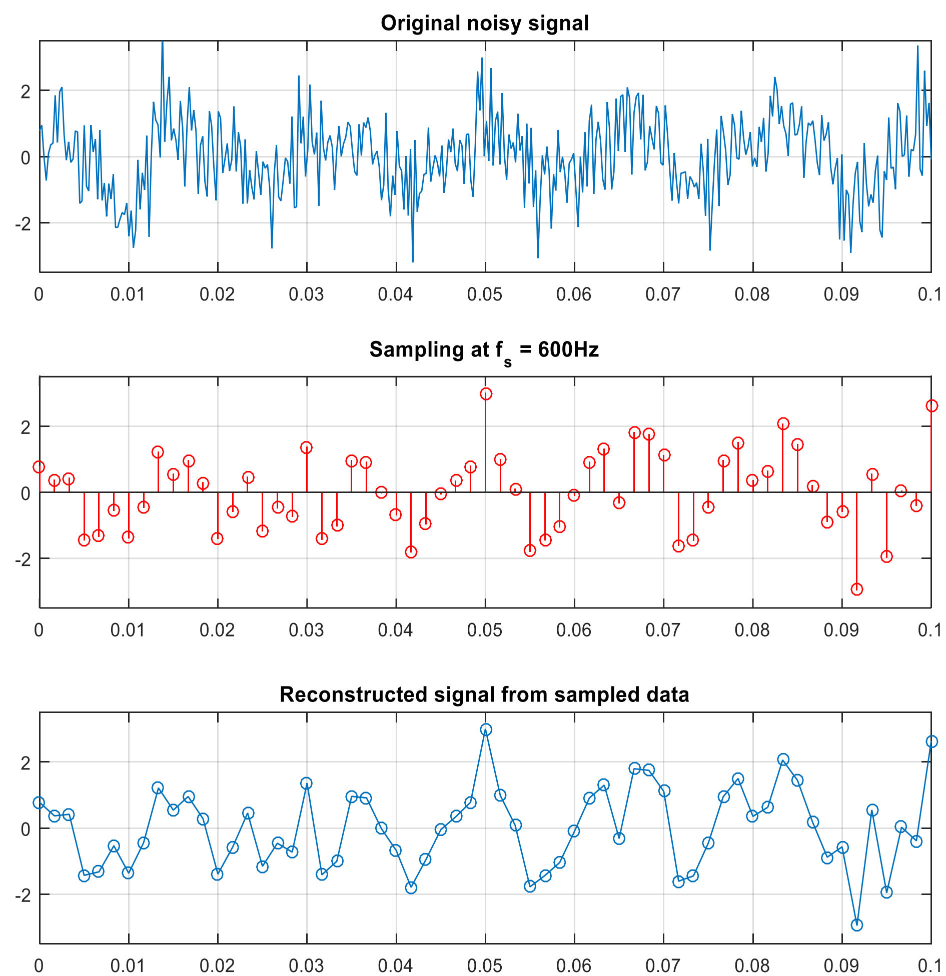
```
z = [y u];
z = dtrend(z);
```

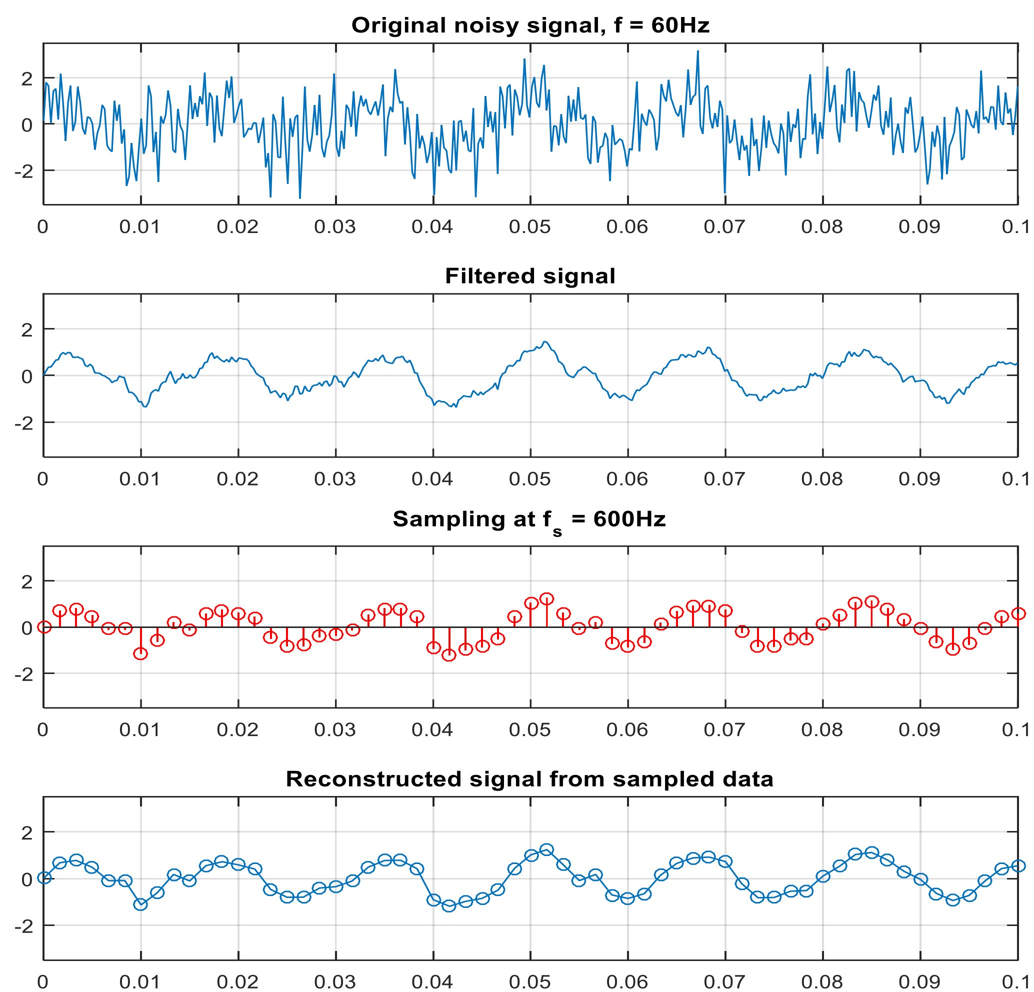


# Input-Output Data Examination

## ❑ Noise Filtering

- High-frequency measurement noise can cause trouble if it is not filtered out before sampling the signals.
- The remedy is to use analog low-pass filters before the signals are sampled.
- Filtering concentrates the identification to the frequency range of interest by increasing the SNR.
- This figures show sampling of noisy data with and without filtering.

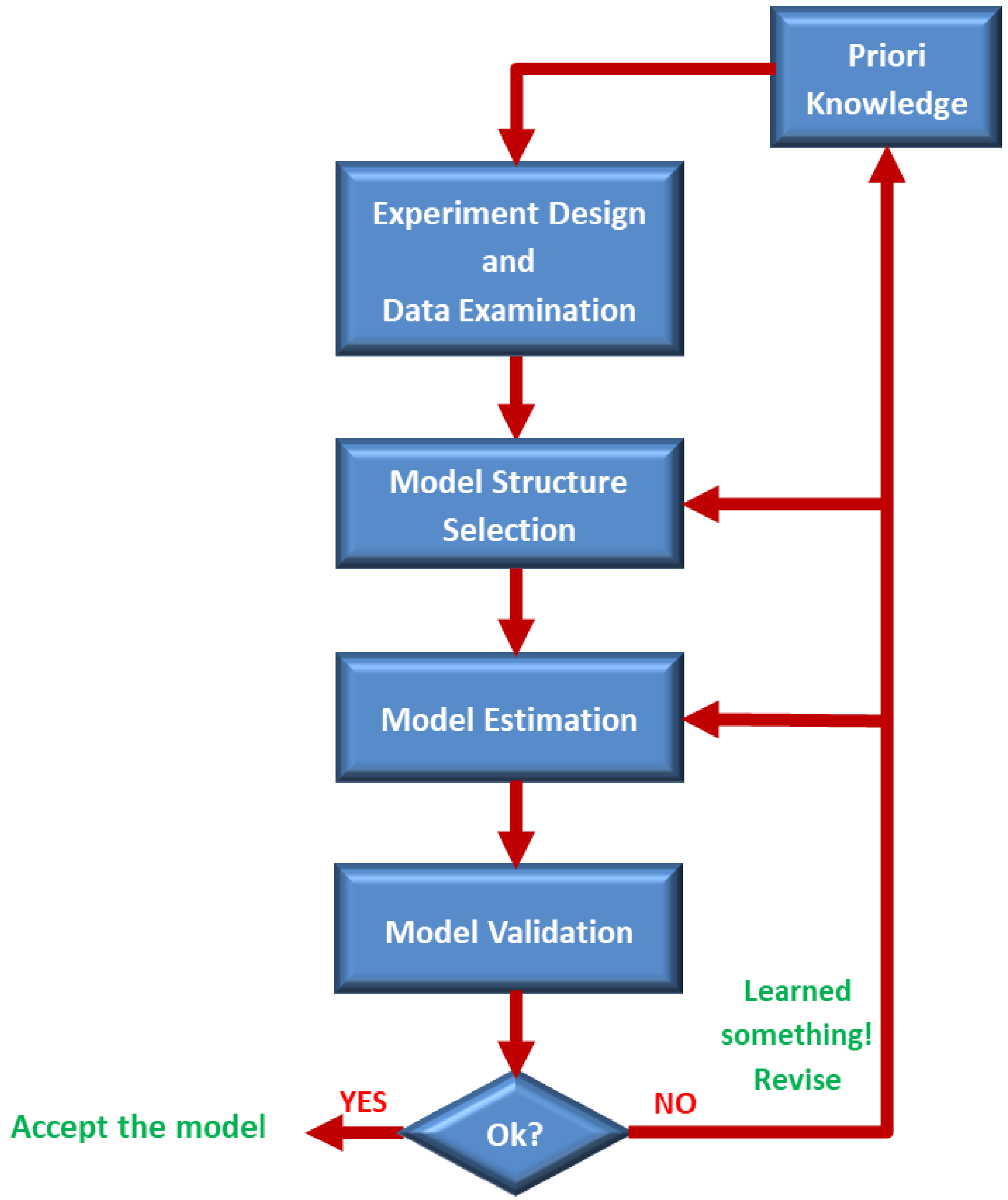



Note that filtering of data may affect the noise model

# System Identification Procedure

- **Parametric Model Structure Selection**
  - Continuous Time Models
    - Transfer Function Model
    - State Space Model
- **Model Estimation Techniques**
  - Nonparametric Methods
    - Spectral Analysis
    - Correlation Analysis
  - Parametric Methods
    - Least Squares Method
- **Model Validation Techniques**
  - Simulation
  - Cross-Validation
  - Model Validity Criterion
  - Pole-Zero Plots
  - Bode Diagram
  - Residual Analysis



# THANK YOU

HUMBER

WE ARE HUMBER