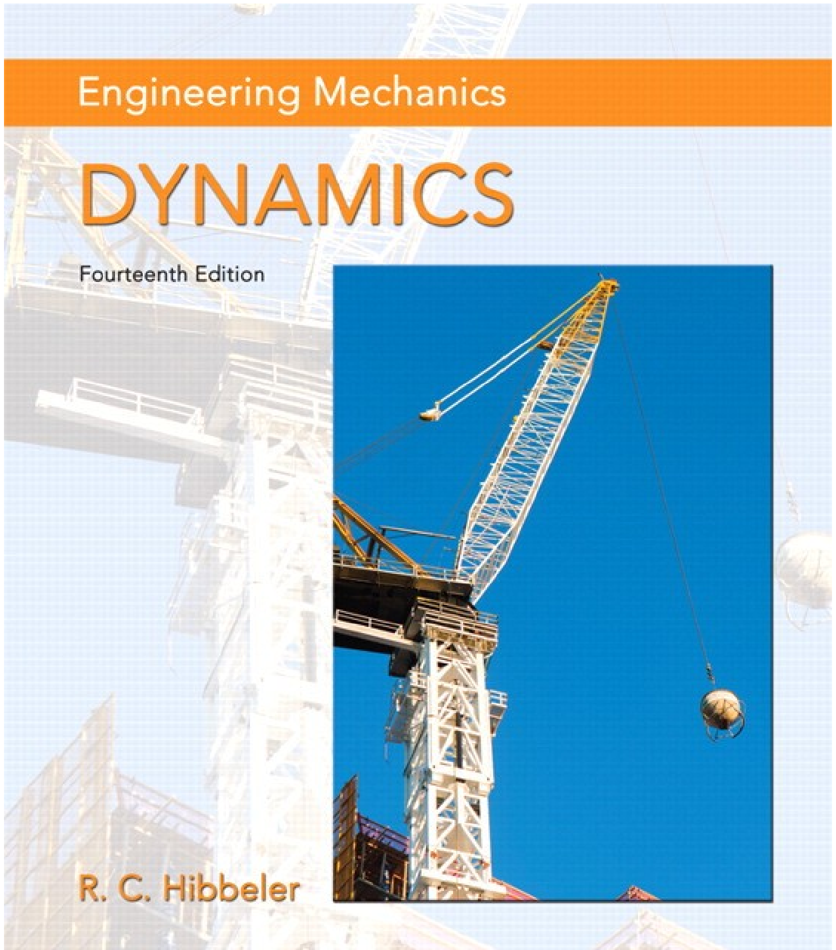
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 17

Planar Kinetics of a Rigid Body: Force and Acceleration

Pearson

Equations of Motion: Rotation About A Fixed Axis (1 of 2)

Today's Objectives:

Students will be able to:

1. Analyze the planar kinetics of a rigid body undergoing rotational motion.

Equations of Motion: Rotation About A Fixed Axis (2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- **Rotation about an Axis**
- **Equations of Motion**
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1. In rotational motion, the normal component of acceleration at the body's center of gravity (G) is always ____________.

   A) zero

   B) tangent to the path of motion of G

   C) directed from G toward the center of rotation

   D) directed from the center of rotation toward G

2. If a rigid body rotates about point O, the sum of the moments of the external forces acting on the body about point O equals which of the following?

   A) $I_G\alpha$ B) $I_O\alpha$

   C) $m\, a_G$ D) $m\, a_O$

Applications (1 of 2)

The crank on the oil-pump rig undergoes rotation about a fixed axis, caused by the driving torque, M, from a motor. As the crank turns, a dynamic reaction is produced at the pin. This reaction is a function of angular velocity, angular acceleration, and the orientation of the crank. If the motor exerts a constant torque M on the crank, does the crank turn at a constant angular velocity? Is this desirable for such a machine?

Pin at the center of rotation.

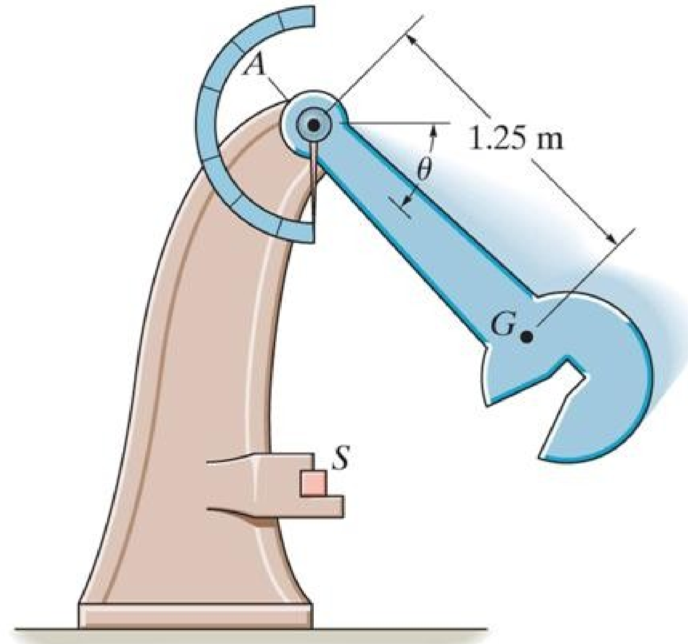
Applications (2 of 2)

The pendulum of the Charpy impact machine is released from rest when $\theta = 0^{\circ}$. Its angular velocity (ω) begins to increase.

Can we determine the angular velocity when it is in vertical position?

On which property (P) of the pendulum does the angular acceleration (α) depend?

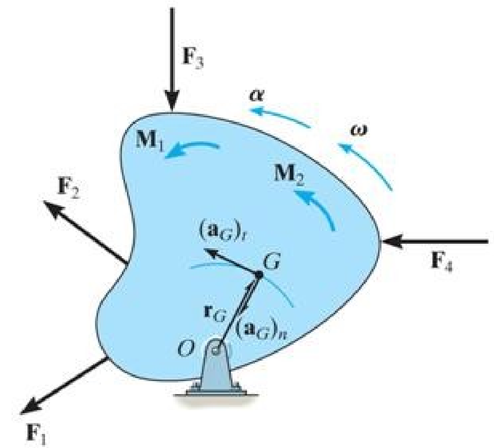
What is the relationship between P and α?



Section 17.4

Equations of Motion: Rotation About A Fixed Axis

Equations of Motion: Rotation About A Fixed Axis



When a rigid body rotates about a fixed axis perpendicular to the plane of the body at point O, the body's center of gravity G moves in a circular path of radius r_G. Thus, the **acceleration of point G** can be represented by a **tangential** component $(a_G)_t = r_G\alpha$ and a **normal** component $(a_G)_n = r_G\omega^2$.

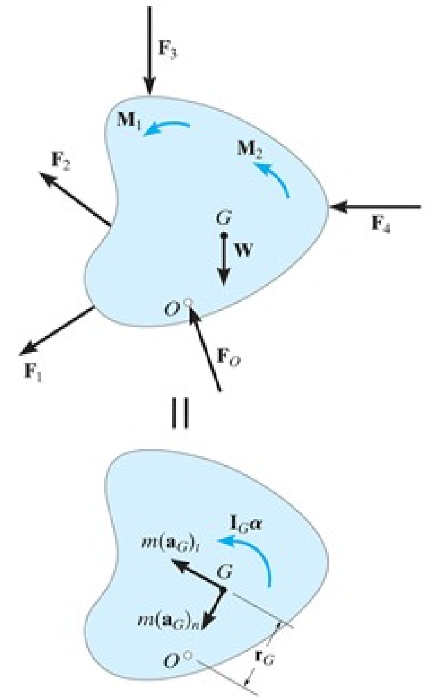
Since the body experiences an angular acceleration, its inertia creates a moment of magnitude, $I_g\alpha$, equal to the moment of the external forces about point G. Thus, the **scalar equations of motion** can be stated as:

$$\sum F_n = m\,(a_G)_n = m\,r_G\omega^2$$
$$\sum F_t = m\,(a_G)_t = m\,r_G\alpha$$
$$\sum M_G = I_G\alpha$$

Equations of Motion

Note that the $\sum M_G$ moment equation may be replaced by a moment summation about any arbitrary point. Summing the moment about the center of rotation O yields

$$\sum M_O = I_G \alpha + r_G m\,(a_G)_t = \left[I_G + m(r_G)^2 \right] \alpha$$



From the parallel axis theorem, $I_O = I_G + m(r_G)^2$, therefore the term in parentheses represents I_O.

Consequently, we can write the **three equations of motion** for the body as:

$$\sum F_n = m(a_G)_n = m\, r_G \omega^2$$

$$\sum F_t = m\,(a_G)_t = m\, r_G \alpha$$

$$\sum M_O = I_O \alpha$$

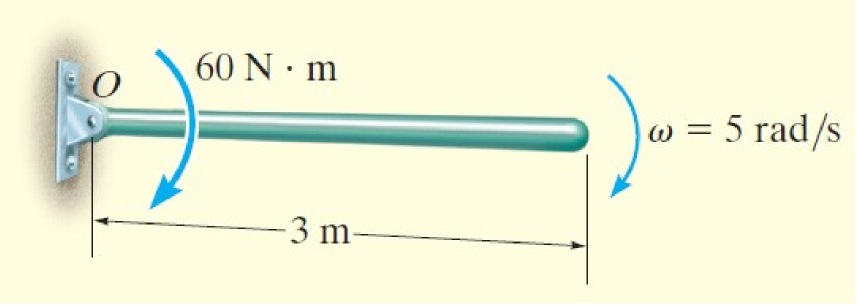
Procedure For Analysis

Problems involving the kinetics of a rigid body rotating about a fixed axis can be solved using the following process.

1. Establish an inertial coordinate system and specify the sign and direction of $(a_G)_n$ and $(a_G)_t$.
2. Draw a free body diagram accounting for all external forces and couples. Show the resulting **inertia forces and couple** (typically on a separate kinetic diagram).
3. Compute the mass moment of inertia I_G or I_O.
4. Write the **three equations of motion** and identify the unknowns. Solve for the unknowns.
5. **Use kinematics if there are more than three unknowns** (since the equations of motion allow for only three unknowns).

Example I (1 of 3)

Given: A rod with mass of 20 kg is rotating at $5\ \mathbf{rad/s}$ at the instant shown. A moment of $60\ N{\cdot}m$ is applied to the rod.



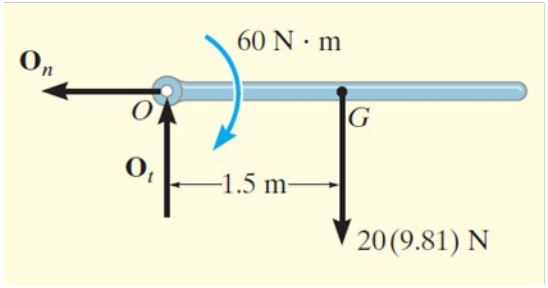
Find: The angular acceleration α and the reaction at pin O when the rod is in the horizontal position.

Plan: Since the mass center moves in a circle of radius 1.5 m, it's acceleration has a normal component toward O and a tangential component acting downward and perpendicular to r_G. Apply the problem solving procedure.

Example I (2 of 3)

Solution:

FBD



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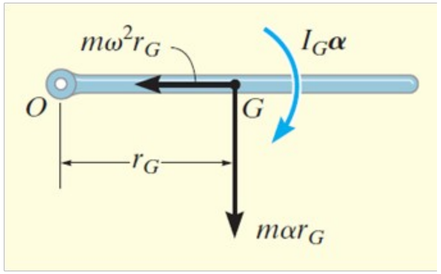
Kinetic Diagram



Equations of motion:

$$+ \rightarrow \sum F_n = m\, a_n = m\, r_G \omega^2$$

$$O_n = 20(1.5)(5)^2 = 750\ N$$

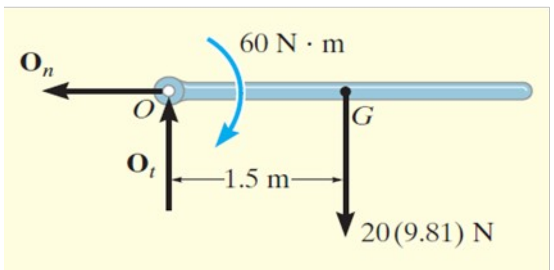
$$+ \downarrow \sum F_t = m\, a_t = m\, r_G \alpha$$

$$-O_t + 20(9.81) = 20(1.5)\alpha$$

Example I (3 of 3)

Solution:

FBD



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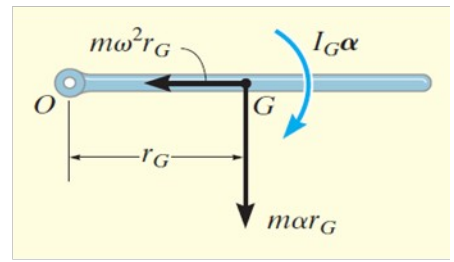
Kinetic Diagram



$$\circlearrowright \Sigma M_O = I_G a + m\, r_G \alpha (r_G)$$

$$\Rightarrow 0.15\ (15)9.81\ =\ I_G \alpha + m(r_G)^2 \alpha$$

Using $I_G = (ml^2)/12$ and $r_G = (0.5)(l)$, we can write:

$\Sigma M_O = \left[(ml^2/12) + (ml^2/4)\right] \alpha = (ml^2/3)\alpha$ where $(ml^2/3) = I_O$.

After **substituting:**

$$60 + 20(9.81)(1.5) = 20(3^2/3)\alpha$$

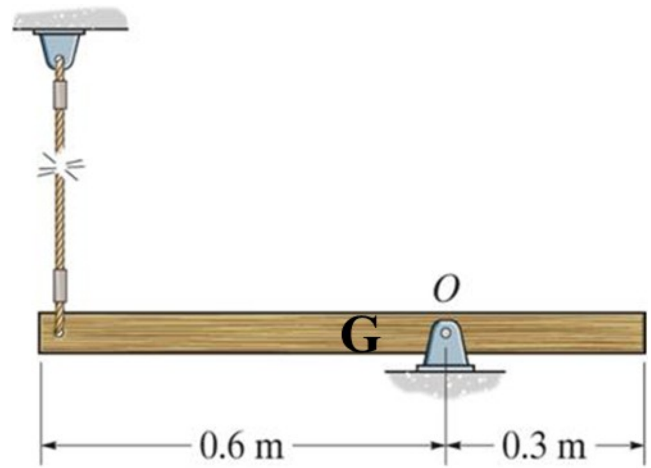
Solving: $\alpha = 5.9\ rad/s^2$

$O_t = 19\ N$

Example II (1 of 3)

Given: The uniform slender rod has a mass of 15 kg and its mass center is at point G.

Find: The reactions at the pin O and the angular acceleration of the rod just after the cord is cut.

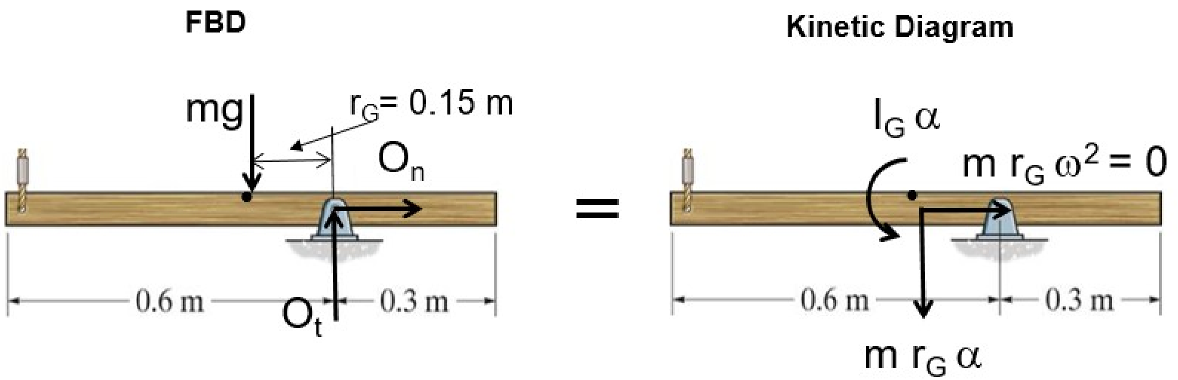


Plan: Since the mass center, G, moves in a circle of radius **0.15 m**, it's acceleration has a normal component toward O and a tangential component acting downward and perpendicular to r_G.

Apply the problem solving procedure.

Example II (2 of 3)

Solution:



Equations of motion:

$$\xrightarrow{+} \sum Fn = m\ a_n = m\ r_G\omega^2 \qquad \Rightarrow O_n = 0N$$

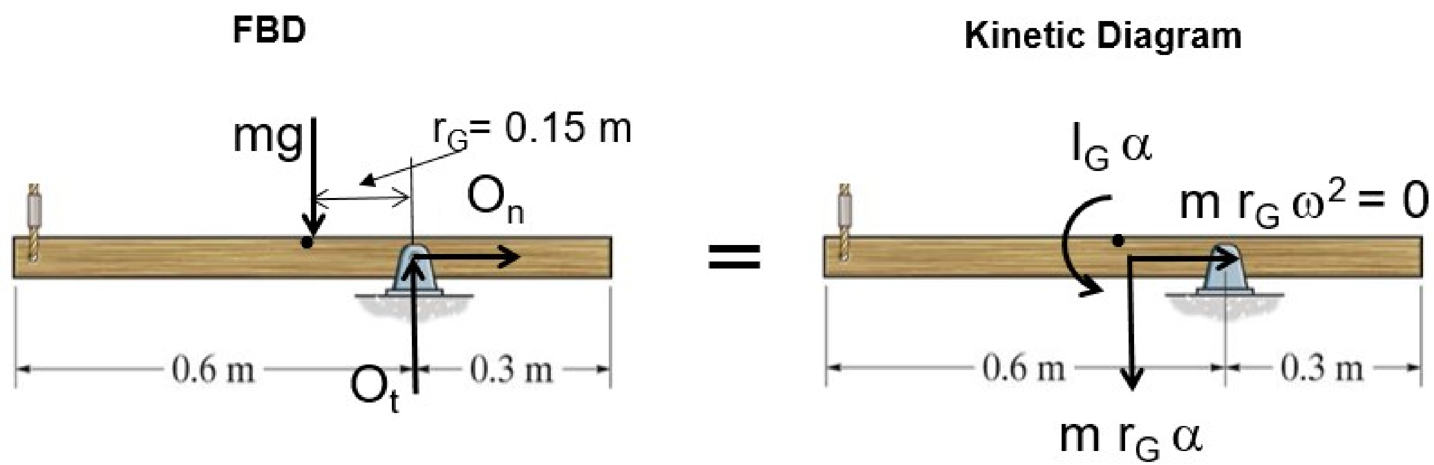
$$+\downarrow \sum Ft = m\ a_t = m\ r_G\alpha \qquad \Rightarrow -O_t + 15(9.81) = 15\ (0.15)\alpha \quad (1)$$

$$\curvearrowright + \sum M_O = I_G\alpha + m\ r_G\alpha(r_G) \qquad \Rightarrow 0.15\ (15)\ 9.81 = I_G\alpha + m(r_G)^2\alpha$$

Using $I_G = (ml^2)/12$ and $r_G = (0.15)$, we can write:

$$I_G\alpha + m(r_G)^2\alpha = \left[(15 \times 0.9^2)/12 + 15(0.15)^2\right]\alpha = 1.35\alpha$$

Example II (3 of 3)



After **substituting:**

$$22.07 = 1.35\alpha \Rightarrow \alpha = 16.4\,rad/s^2$$

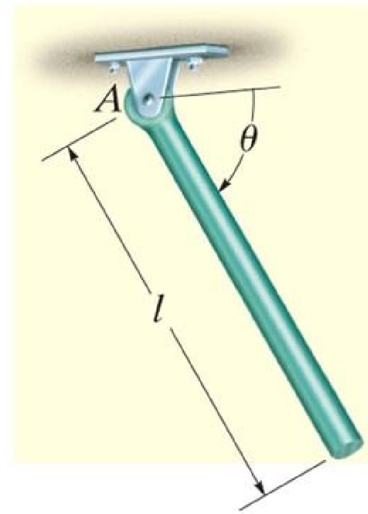
From Eq

$$(1)\; O_t + 15(9.81) = 15(0.15)\alpha$$

$$\Rightarrow O_t = 15(9.81) - 15(0.15)16.4 = 110\,N$$

Concept Quiz

1. If a rigid bar of length l (above) is released from rest in the horizontal position (θ = 0), the magnitude of its angular acceleration is at maximum when



A) $\theta = 0$

B) $\theta = 90^\circ$

C) $\theta = 180^\circ$

D) $\theta = 0^\circ$ and 180°

2. In the above problem, when $\theta = 90^\circ$, the horizontal component of the reaction at pin O is __________.

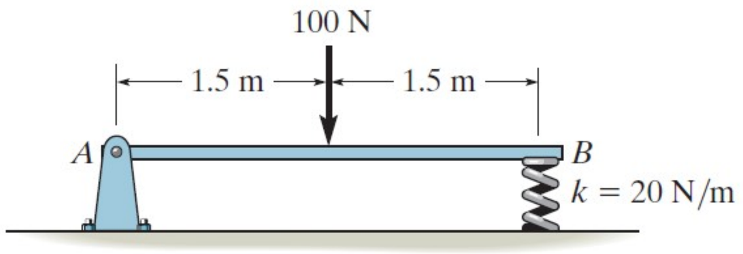
A) zero

B) m g

C) $m\,(l/2)\omega^2$

D) None of the above

Group Problem Solving I (1 of 3)



Given: The 4-kg slender rod is initially supported horizontally by a spring at *B* and pin at *A*.

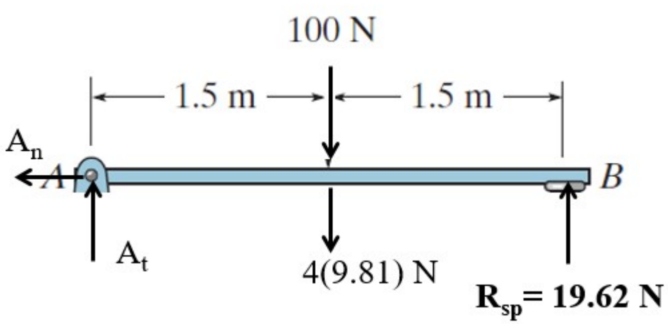
Find: The angular acceleration of the rod and the acceleration of the rod's mass center at the instant the 100-N force is applied.

Plan: Find the spring reaction force before the 100 N is applied. Draw the free body diagram and kinetic diagram of the rod. Then apply the equations of motion.

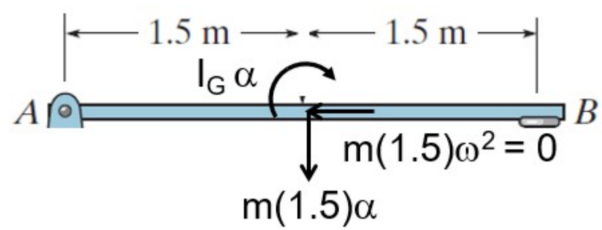
Group Problem Solving I (2 of 3)

Solution:

FBD



Kinetic Diagram



Notice that the spring force, R_{sp} developed before the application of the 100 N force is half of the rod weight:

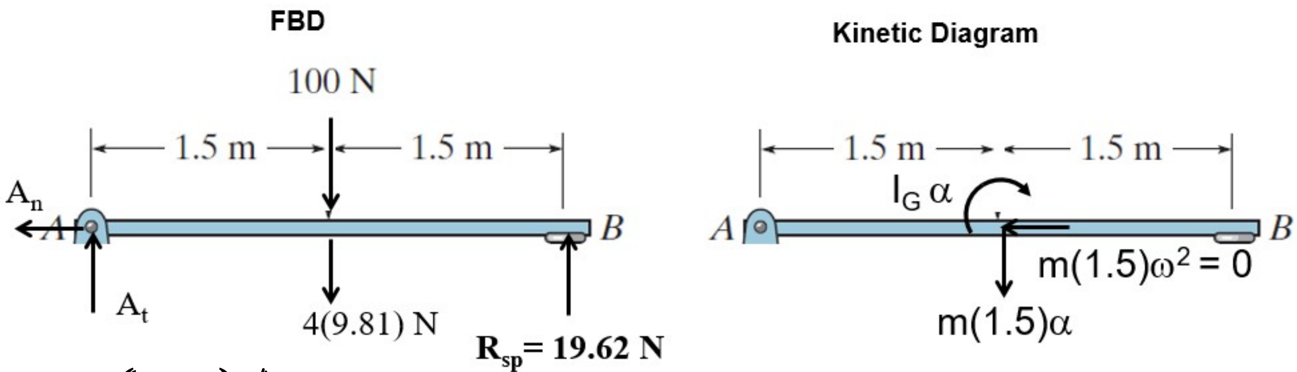
$$R_{sp} = 4(9.81)/2 = 19.62\ N$$

Equation of motion:

$$\circlearrowleft + \sum M_A = I_G\,\alpha + m\,r_G\alpha\left(r_G\right)$$

$$\Rightarrow -19.62(3) + 100(1.5) + 4(9.81)(1.5) = I_G\alpha + m(r_G)^2\alpha$$

Group Problem Solving I (3 of 3)



Using $I_G = \left(ml^2\right)/12$ and $r_G = (1.5)$, we can write:

$$I_G\alpha + m(r_G)^2\alpha = \left[\left(4\times 3^2\right)/12 + 4(1.5)^2\right]\alpha = 12\alpha$$

After **substituting**:

$$150 = 12\alpha \Rightarrow \alpha = 12.5\ rad/s^2$$

The acceleration of the rod's mass center is :

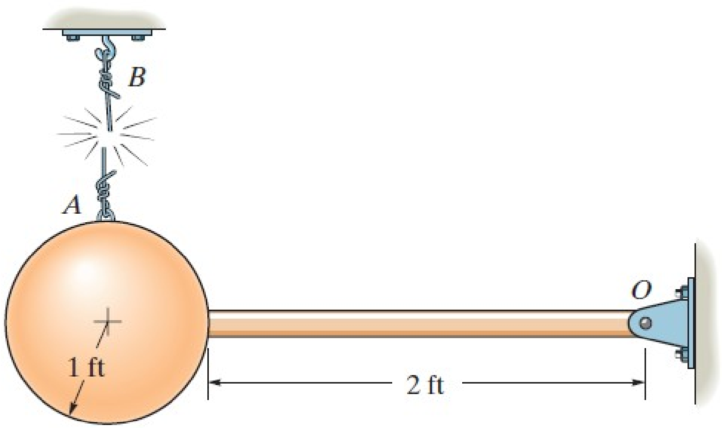
$$a_n = r_G\omega^2 = 0\,m/s^2$$

$$a_t = r_G\alpha = 18.8\ m/s^2 \downarrow$$

Group Problem Solving II (1 of 3)

Given: The pendulum consists of a 30-lb sphere and a 10-lb slender rod.

Find: The reaction at the pin O just after the cord AB is cut.

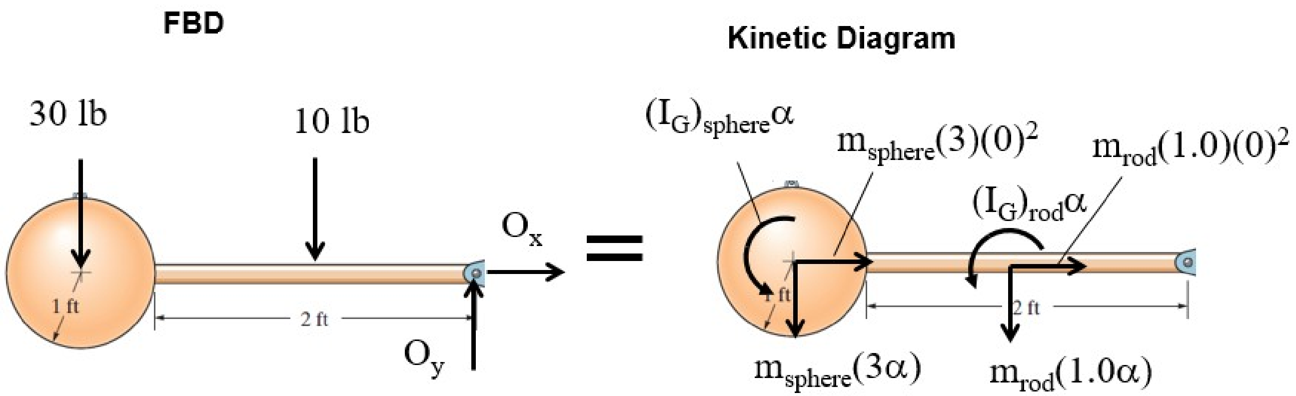


Plan: Draw the free body diagram and kinetic diagram of the rod and sphere as one unit.

Then apply the equations of motion.

Group Problem Solving II (2 of 3)

Solution:



Equations of motion:

$$+\rightarrow \sum F_n = m(a_G)_n$$

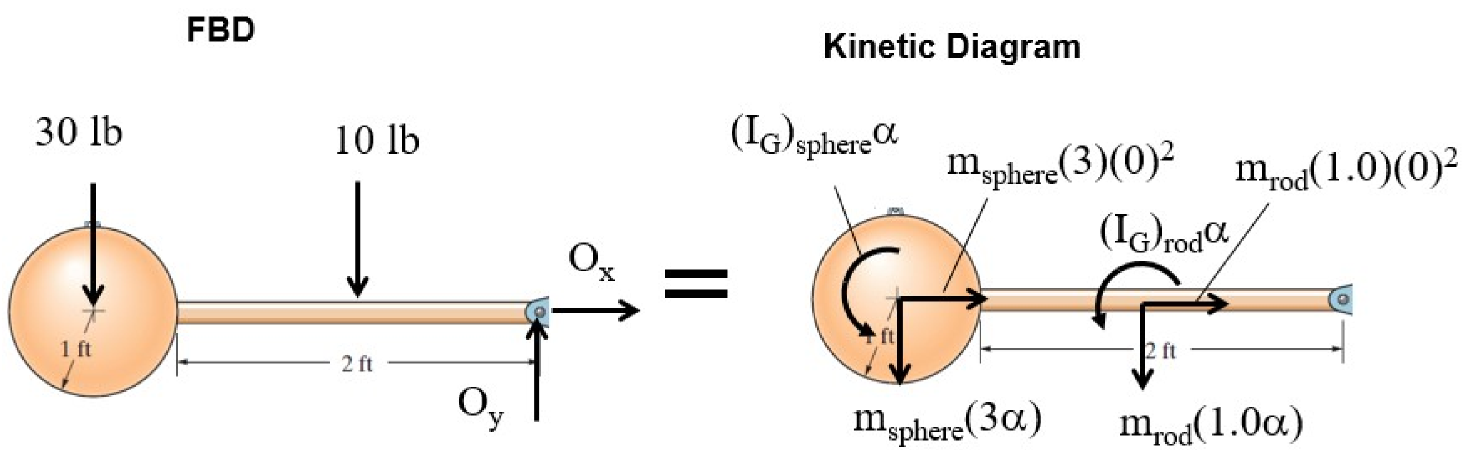
$$O_x = (30/32.2)(3)(0)^2 + (10/32.2)(1.0)(0)^2 \;\Rightarrow O_x = 0\ lb$$

$$+\downarrow \sum F_t = m(a_G)_t$$

$$-O_y + 30 + 10 \;=\; (30/32.2)(3\alpha) + (10/32.2)(1.0\alpha)$$

$$\Rightarrow O_y \;=\; 40 - 3.106\alpha$$

Group Problem Solving II (3 of 3)



$$+\sum M_O = I_o\alpha:$$

$$30(3.0) + 10(1.0) = \left[0.4(30/32.2)(1)^2 + (30/32.2)(3)^2\right]_{sphere}\alpha$$
$$+ \left[(1/12)(10/32.2)\ (2)^2 + \ (10/32.2)\ (1)^2\right]_{rod}\alpha$$
$$\Rightarrow 100 = 9.172\ \alpha$$

Therefore, $\alpha = 10.9\ rad/s^2,\ O_y = 6.14 lb$

Attention Quiz

1. A drum of mass m is set into motion in two ways: (a) by a constant 40 N force, and, (b) by a block of weight 40 N. If α_a and α_b represent the angular acceleration of the drum in each case, select the true statement.

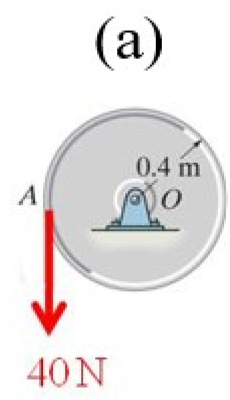
   A) $\alpha_a > \alpha_b$ B) $\alpha_a < \alpha_b$

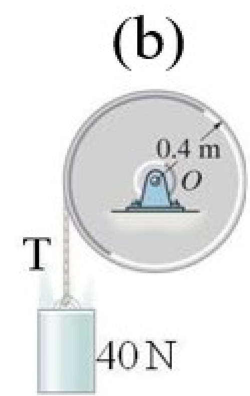
   C) $\alpha_a = \alpha_b$ D) None of the above

2. In case (b), what is the tension T in the cable?

   A) T = 40 N B) T < 40 N

   C) T > 40 N D) None of the above





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