ENGI-1500
Physics -2

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Humber Institute of Technology and Advanced Learning

Winter 2023

HUMBER

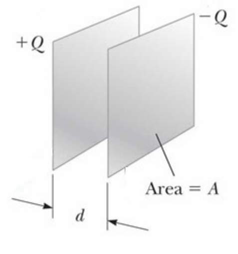
Reminder of the previous week

Capacitance and Dielectrics

- Definition

$Q = C\Delta V$

$C = \dfrac{\varepsilon_0 A}{d}$



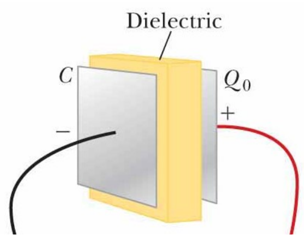
$U_E = \dfrac{Q^2}{2C} = \dfrac{1}{2}Q\Delta V = \dfrac{1}{2}C(\Delta V)^2$

- With Dielectric Slab

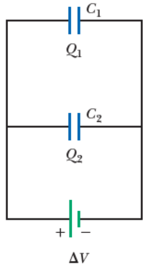
$C = \kappa C_0$

$C = \varepsilon_r C_0$

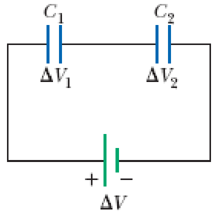
$C = \kappa \dfrac{\varepsilon_0 A}{d}$



- Combination of Capacitors



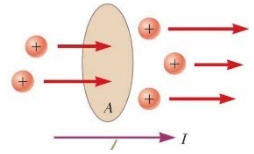
$C_{eq} = C_1 + C_2 + C_3 + \cdots$ (parallel combination)



$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \cdots$ (series combination)

Current and Resistance

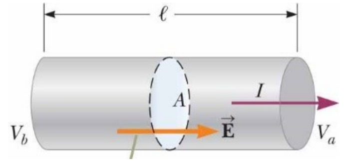
- Electric current



$I \equiv \dfrac{dQ}{dt}$

$1\ \mathrm{A} = 1\ \mathrm{C/s}$

- Resistance



$R = \rho \dfrac{\ell}{A}$

$R \equiv \dfrac{\Delta V}{I}$

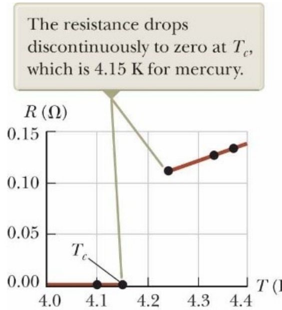
- Resistivities

TABLE 26.2 Resistivities and Temperature Coefficients of Resistivity for Various Materials

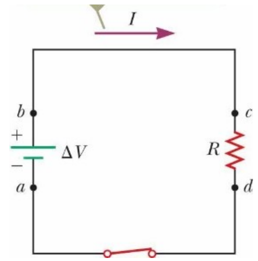
Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] α [(°C)$^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 26.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and $1.50 \times 10^{-6}\ \Omega \cdot m$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

- Superconductors



- Electrical Power

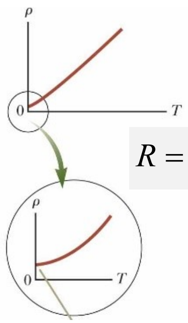


$P = IV$

$P = I^2 R$

$P = \dfrac{V^2}{R}$

- Resistance and Temperature



$R = R_0\left[1 + \alpha\left(T - T_0\right)\right]$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Week 4 / Class 4

- Direct Current Circuits (Ch. 27)

Outline of Week 4 / Class 4

- Reminder of the previous week

- Direct Current Circuits (Ch. 27)
  - Electromotive Force
  - Resistors in Series and Parallel
  - Kirchhoff’s Rules
  - RC Circuits
  - Household Wiring and Electrical Safety [Reading from textbook]

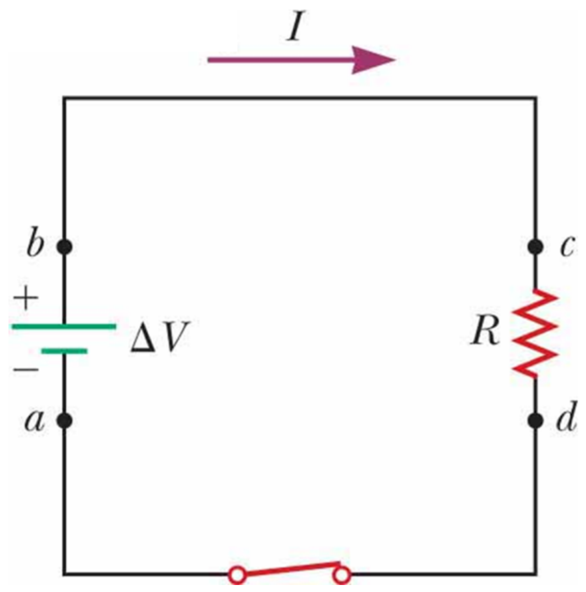
- Examples
- Next week’s topic

Direct Current Circuits (Ch. 27)

Electromotive Force

Electromotive Force

- Earlier, we discussed a circuit in which a ***battery*** produces a current.
- We will generally use a battery as a ***source of energy*** for circuits in our discussion.
- The potential difference at the battery terminals is constant → it creates a current with ***constant magnitude and direction*** which is called ***direct current*** (with resistive load as shown in figure).
- A battery is called either a ***source of electromotive force*** or, more commonly, a source of ***emf***. (The phrase electromotive force is an unfortunate historical term, describing not a force, but rather a potential difference in volts.)
- The ***emf (Ɛ)*** of a battery is the ***maximum possible voltage*** the battery can provide between its terminals.



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Electromotive Force

- We will generally assume:
  - connecting wires in circuit have no resistance
  - positive terminal of battery at higher potential than negative terminal

- In a real battery, there is resistance to flow of charge within the battery:
  - ***Internal resistance r***

- Idealized battery is assumed with zero internal resistance:
  - Potential difference across battery ***(terminal voltage) = emf***

- We can model a real battery as shown: ideal source and an internal resistance connected in series.
  - In a real battery, terminal voltage is not equal to its emf – there is voltage drop on the internal resistance

- Total power output can be calculated as ***Iℰ*** (delivered to the external load ***R*** and the internal resistance ***r***)

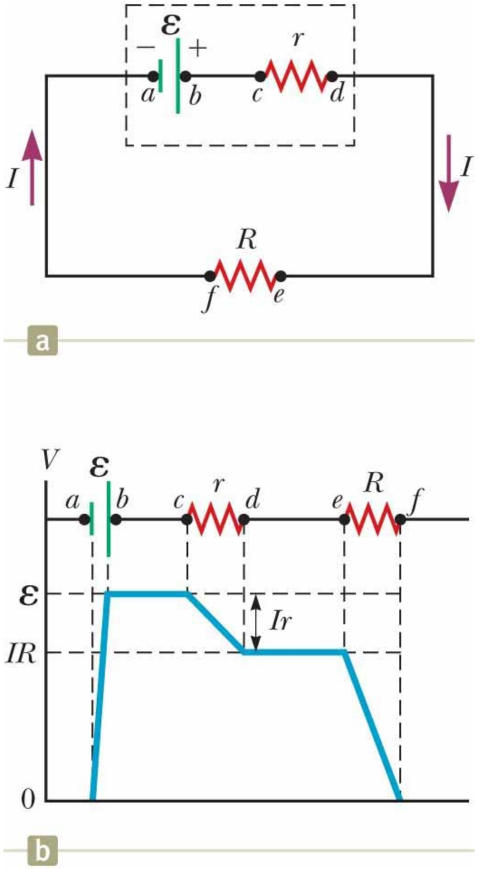
- In real world circuits, ***r*** is usually negligible compared to ***R***.

$$\Delta V = \varepsilon - Ir$$

$$\varepsilon = IR + Ir$$

$$I = \frac{\varepsilon}{R + r}$$

$$I\varepsilon = I^2 R + I^2 r$$



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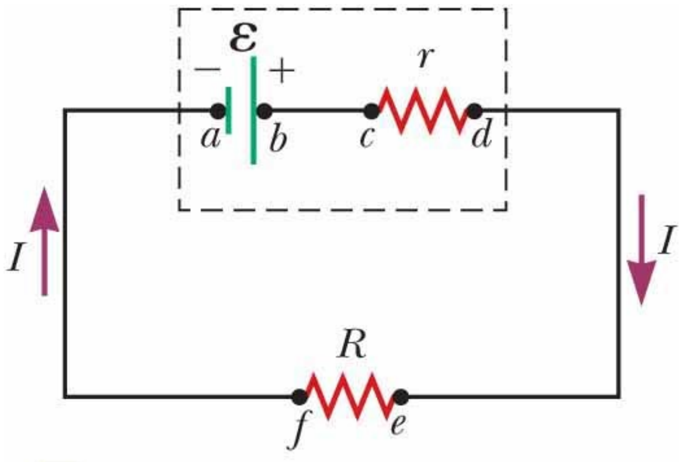
Terminal Voltage of a Battery

Example 27.1

A battery has an emf of **12.0 V** and an internal resistance of **0.05 Ω**. Its terminals are connected to a load resistance of **3.00 Ω**.

(A) Find the current in the circuit and the terminal voltage of the battery.
(B) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.



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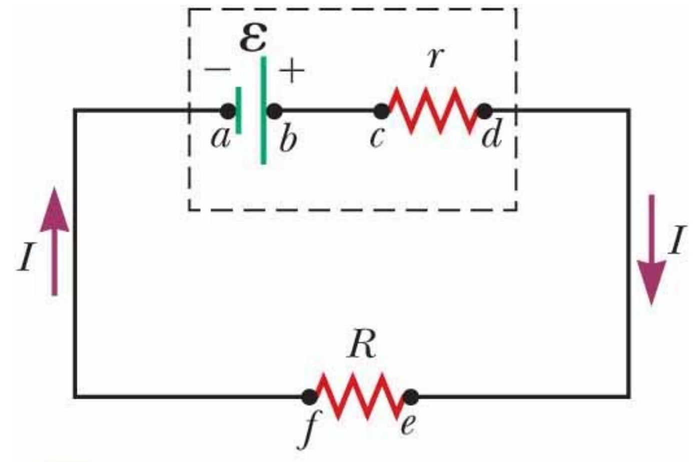

Solution-A

We can find the current first:

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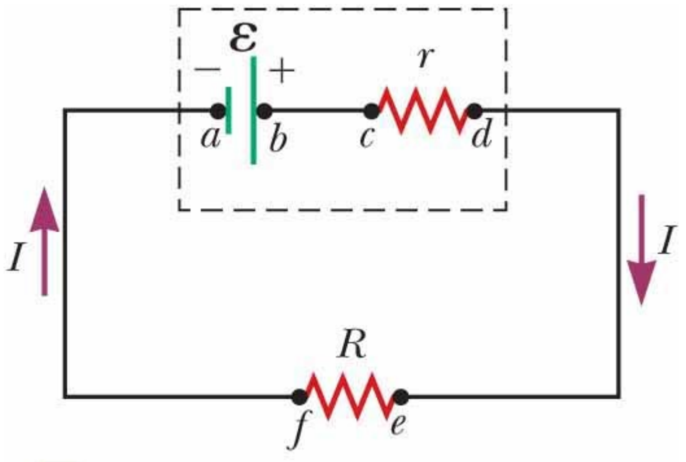

Solution-A

We can find the current first:

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0\ \mathrm{V}}{3.00\ \Omega\ + 0.050\ 0\ \Omega} = \boxed{3.93\ \mathrm{A}}$$

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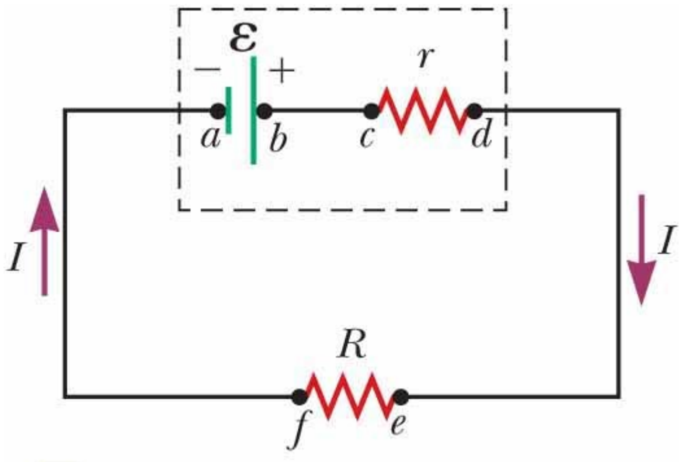

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We can find the voltage drop and then the terminal voltage:

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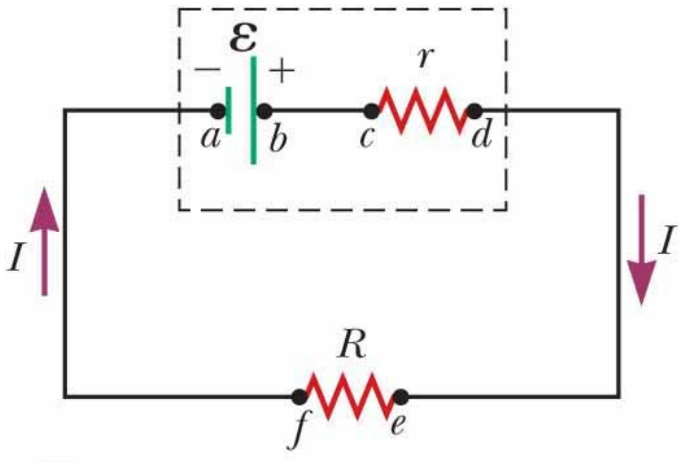

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$$\Delta V = \mathcal{E} - Ir = 12.0\text{ V} - (3.93\text{ A})(0.050\,0\ \Omega) = \boxed{11.8\text{ V}}$$

$$\Delta V = IR = (3.93\text{ A})(3.00\ \Omega) = 11.8\text{ V}$$

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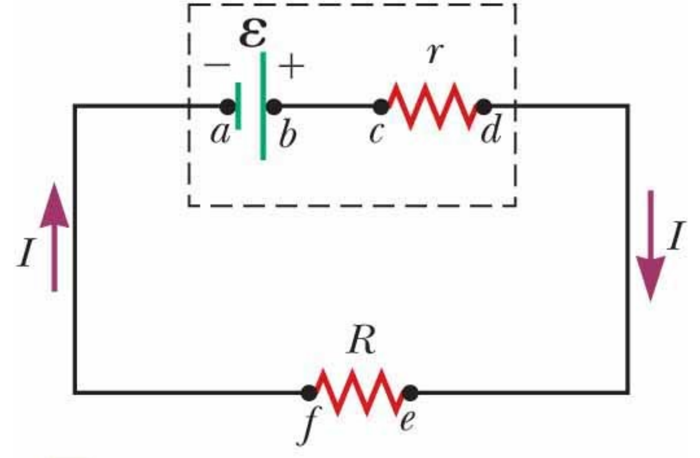
Solution-B

Let's find the power delivered to the load resistor R:



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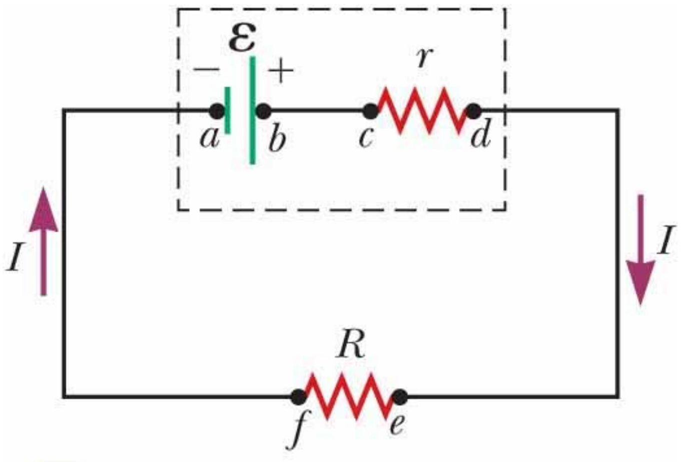

Solution-B

Let's find the power delivered to the load resistor R:

$$P_R = I^2R = (3.93\ \text{A})^2(3.00\ \Omega) = \boxed{46.3\ \text{W}}$$

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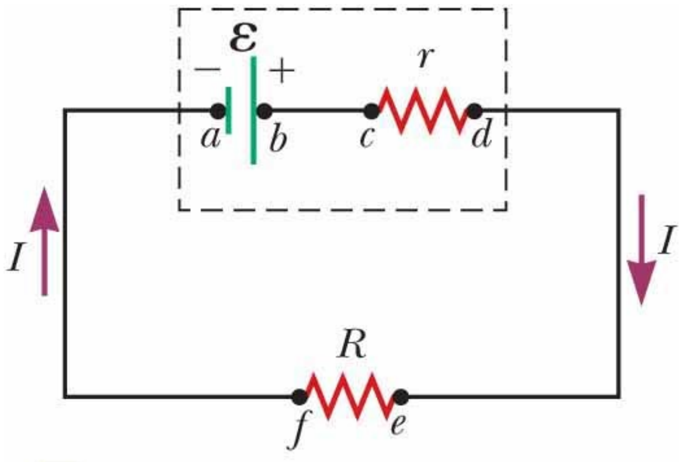

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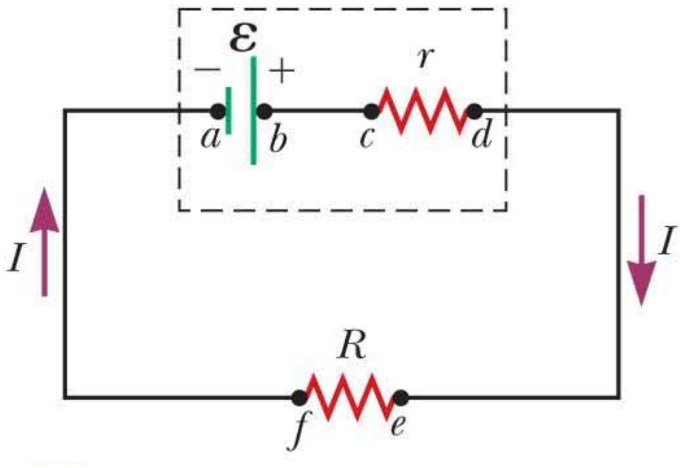

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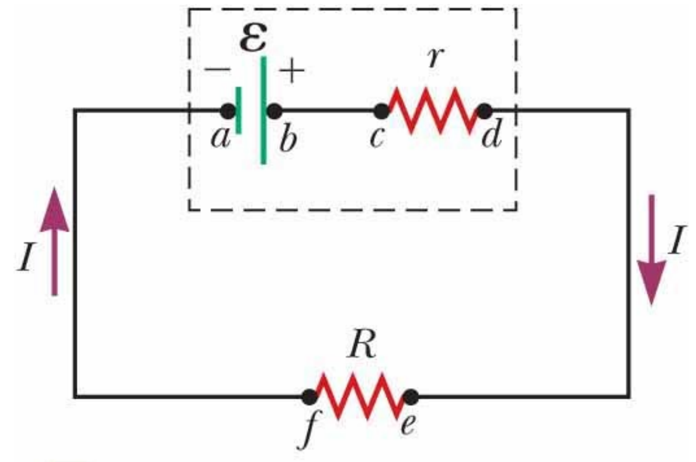

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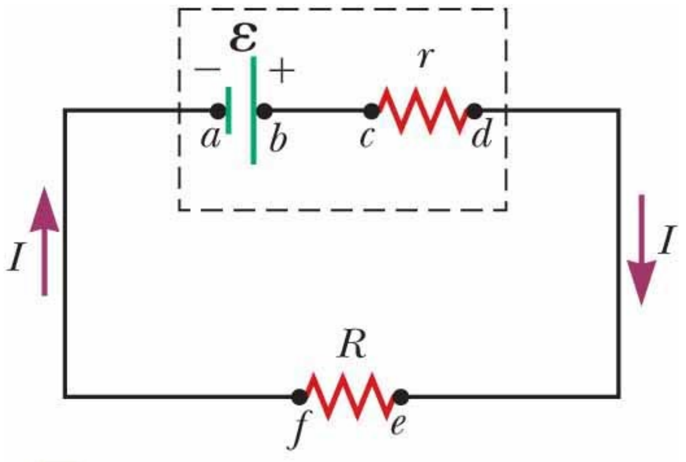

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$$P = P_R + P_r = 46.3\text{ W} + 0.772\text{ W} = \boxed{47.1\text{ W}}$$

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Direct Current Circuits (Ch. 27)

Resistors in Series and Parallel

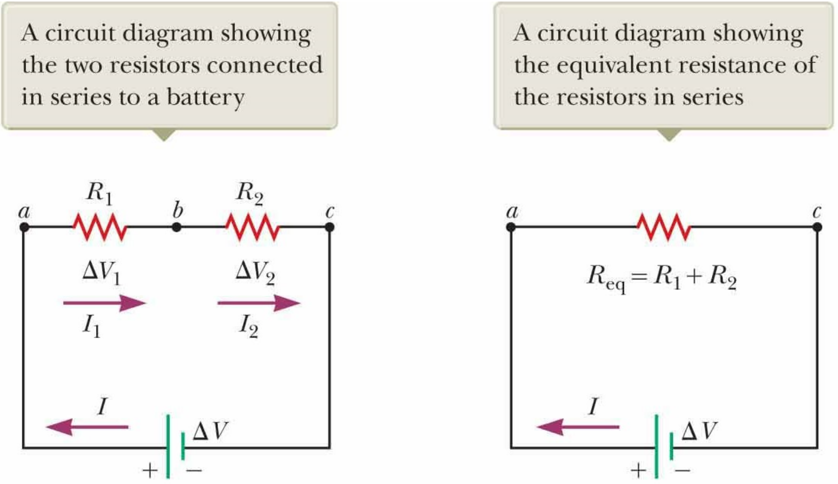
Resistors in Series and Parallel

Series Combination

- When two or more resistors are connected together, sometimes it is easier to analyze the circuit by finding the ***equivalent resistance*** first. Equivalent resistor can replace the complex combination and draw the same current from the battery.

- Figure has two resistors in ***series combination***. Same current flows through both resistors.

$$I = I_1 = I_2$$



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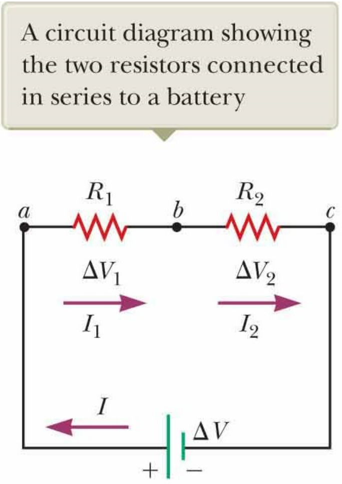
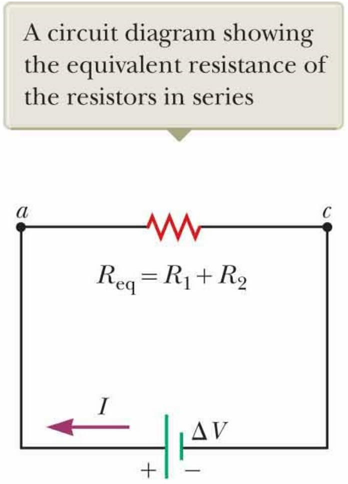
- Potential difference applied across series combination of resistors divides between resistors:

$$\Delta V = \Delta V_1 + \Delta V_2 = IR_1 + IR_2 = I(R_1+R_2)$$

$$R_{eq} = R_1 + R_2$$





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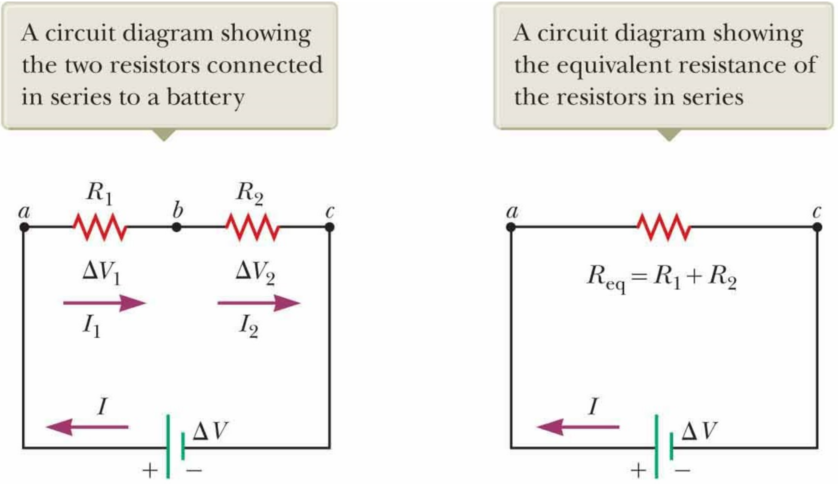
$$R_{eq} = R_1 + R_2$$

- We can generalize the above approach for ***n*** number of resistors in series:

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n$$



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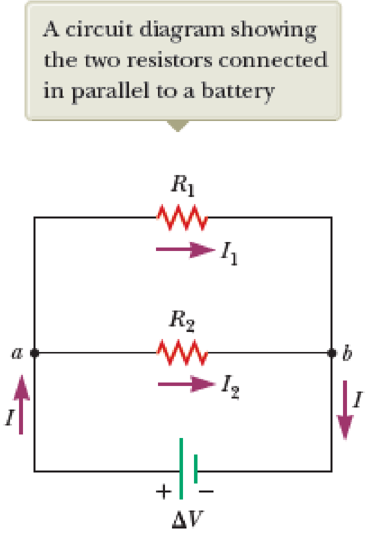
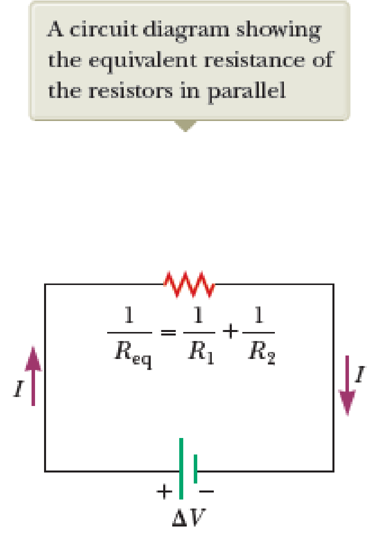
Parallel Combination

- Figure has two resistors in ***parallel combination***.
- We are looking for the value of a single resistor that could replace the combination and draw the same current from the battery (i.e. ***equivalent resistor***).
- Both resistors are connected directly across the terminals of the battery. Therefore, the potential difference across the resistors are the same:

$$\Delta V = \Delta V_1 = \Delta V_2$$





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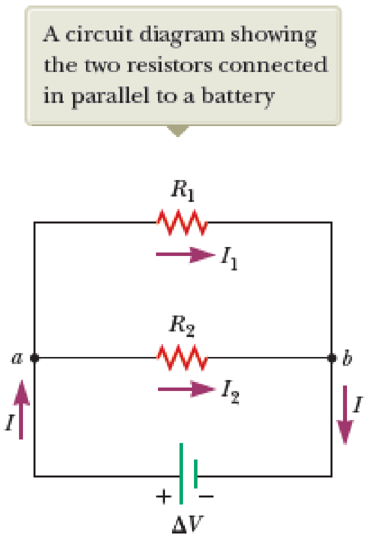
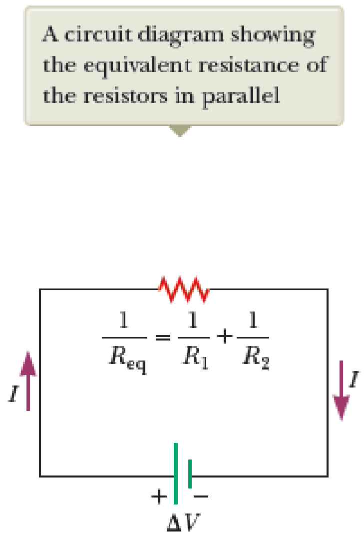
- At point ***a***, since the charges split between $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$, the current splits into two branches:

$$I = I_1 + I_2 = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \qquad \frac{\Delta V}{R_{eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$





Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Resistors in Series and Parallel

Parallel Combination

- Figure has two resistors in ***parallel combination***.
- We are looking for the value of a single resistor that could replace the combination and draw the same current from the battery (i.e. ***equivalent resistor***).
- Both resistors are connected directly across the terminals of the battery. Therefore, the potential difference across the resistors are the same:

$$\underbrace{\Delta V = \Delta V_1 = \Delta V_2}$$

- At point ***a***, since the charges split between $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$, the current splits into two branches:

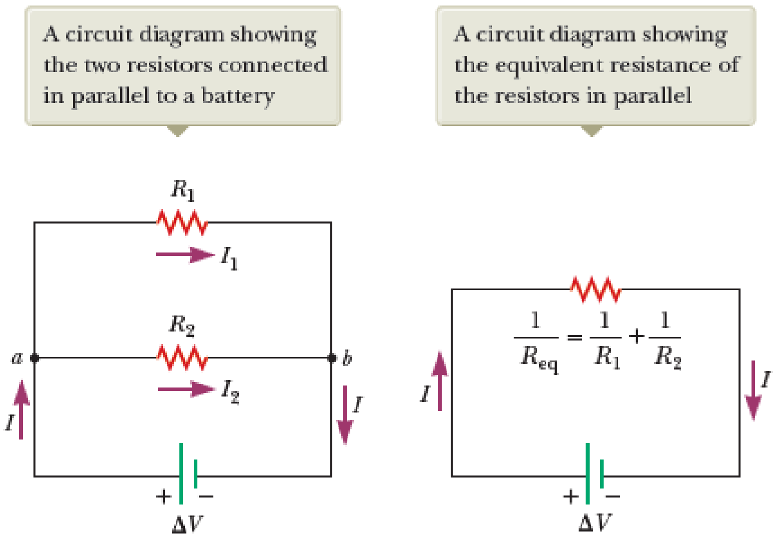
$$I = I_1 + I_2 = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2}$$

$$\frac{\Delta V}{R_{eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

- We can generalize the above approach for ***n*** number of resistors in series:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$



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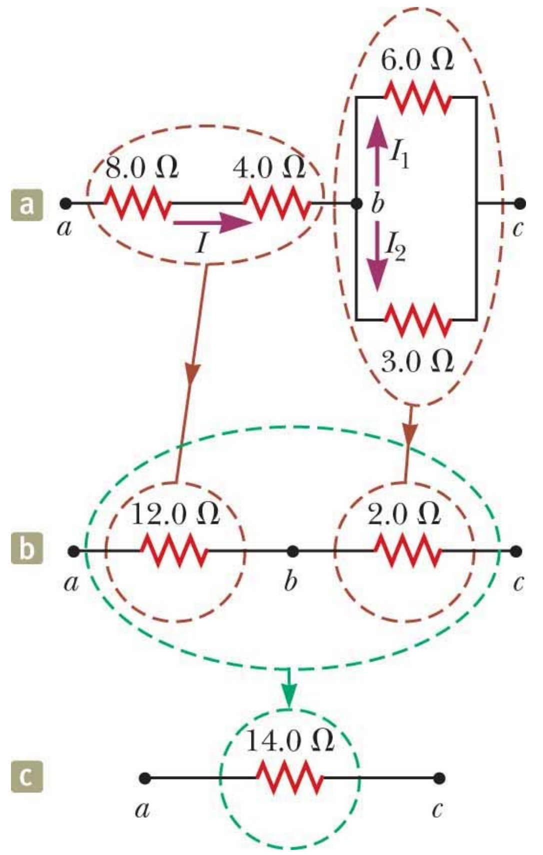
Find the Equivalent Resistance

Example 27.4

Four resistors are connected as shown in the figure.

(A) Find the equivalent resistance between points a and c.
(B) What is the current in each resistor if a potential difference of 42 V is maintained between a and c?



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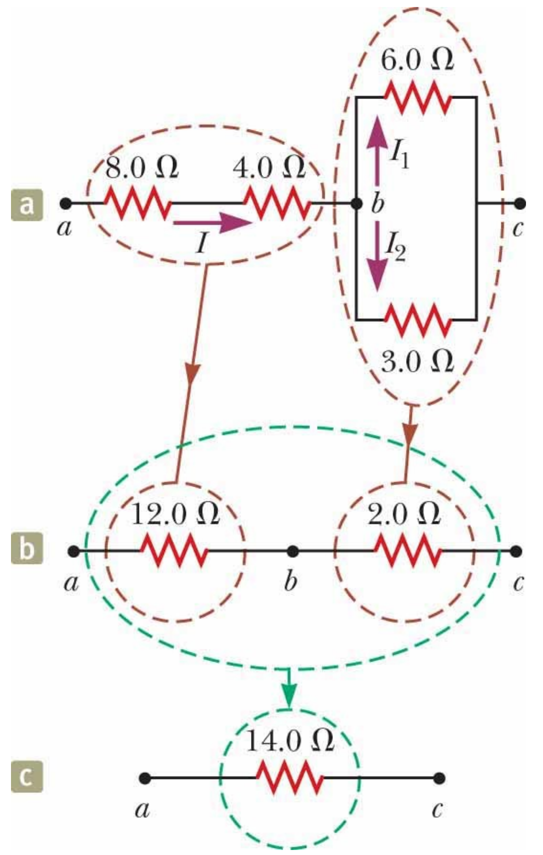
Solution-A

Find the equivalent resistance between a and b of the $8.0\text{-}\Omega$ and $4.0\text{-}\Omega$ resistors, which are in series (left-hand red-brown circles):



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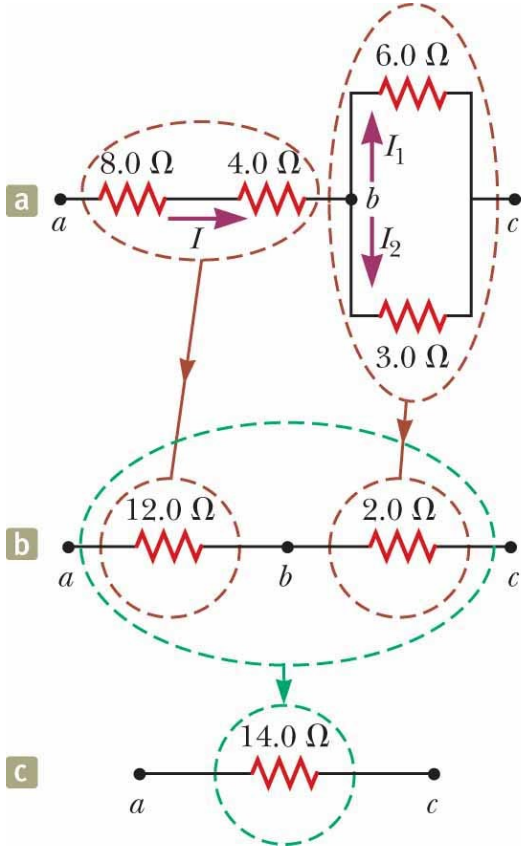
Find the equivalent resistance between a and b of the 8.0-Ω and 4.0-Ω resistors, which are in series (left-hand red-brown circles):

$$R_{eq} = 8.0\ \Omega + 4.0\ \Omega = 12.0\ \Omega$$



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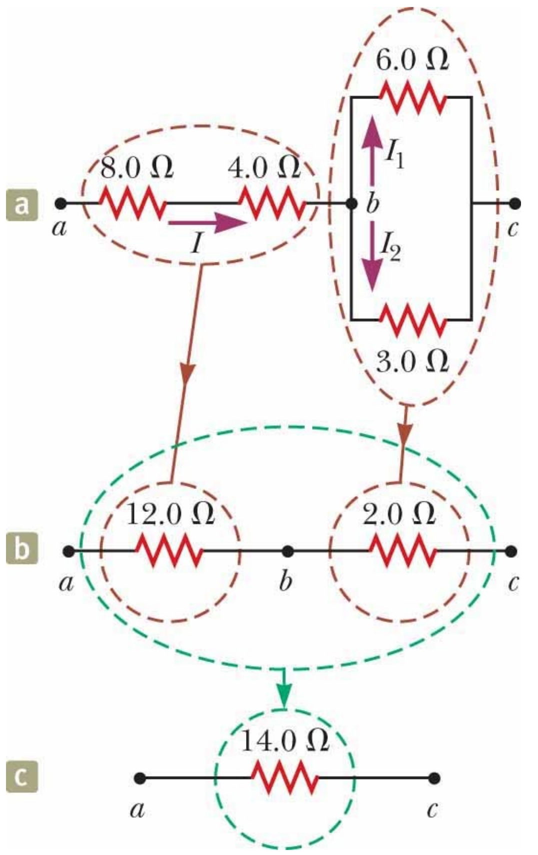

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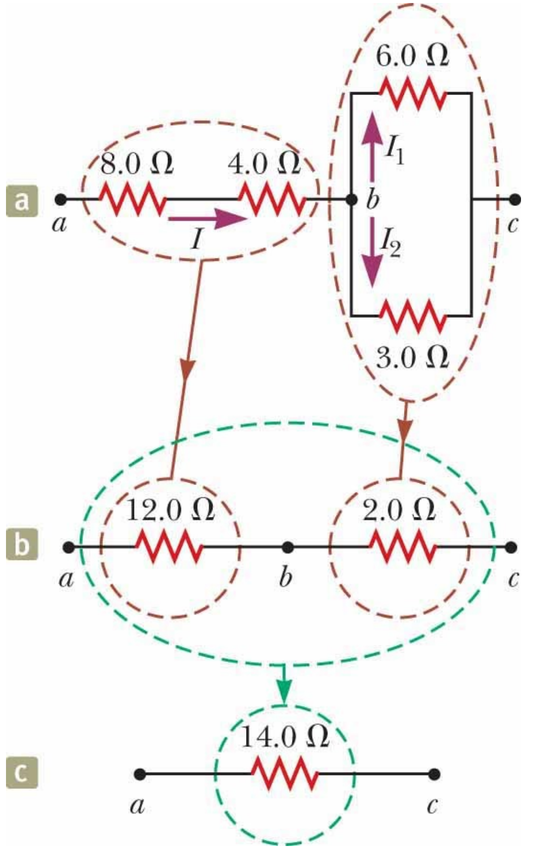
Find the equivalent resistance between b and c of the 6.0-Ω and 3.0-Ω resistors, which are in parallel (right-hand red-brown circles):

$$\frac{1}{R_{eq}} = \frac{1}{6.0\ \Omega} + \frac{1}{3.0\ \Omega} = \frac{3}{6.0\ \Omega}$$

$$R_{eq} = \frac{6.0\ \Omega}{3} = 2.0\ \Omega$$



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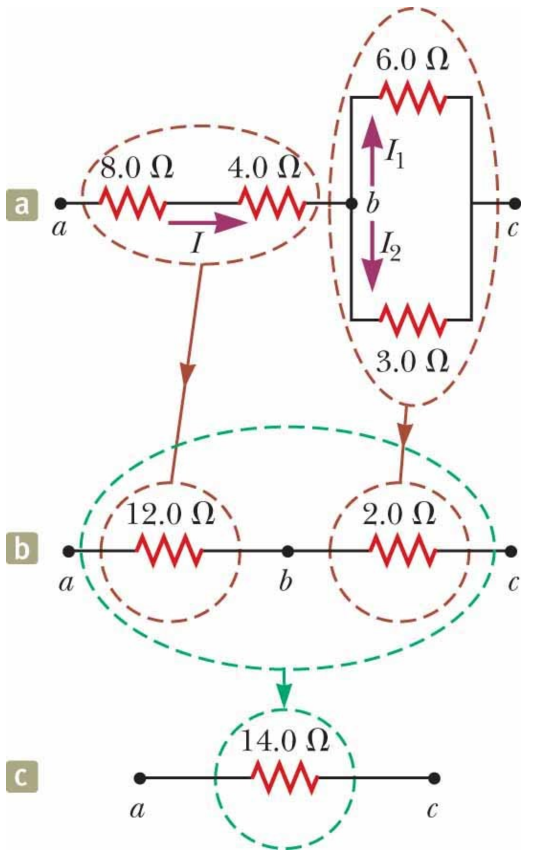
The circuit of equivalent resistances now looks like Figure 28.10b. The 12.0-Ω and 2.0-Ω resistors are in series (green circles). Find the equivalent resistance from a to c:



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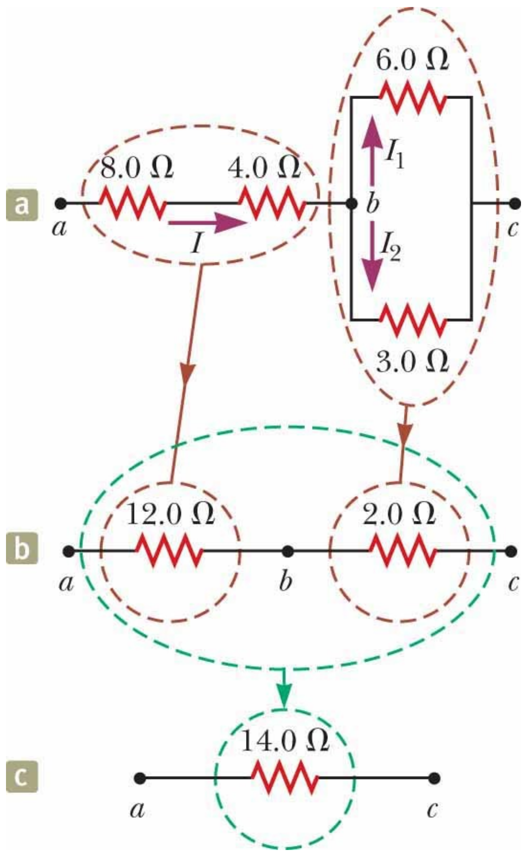

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Solution-A

Find the equivalent resistance between a and b of the 8.0-Ω and 4.0-Ω resistors, which are in series (left-hand red-brown circles):

$$R_{eq} = 8.0\ \Omega + 4.0\ \Omega = 12.0\ \Omega$$

Find the equivalent resistance between b and c of the 6.0-Ω and 3.0-Ω resistors, which are in parallel (right-hand red-brown circles):

$$\frac{1}{R_{eq}} = \frac{1}{6.0\ \Omega} + \frac{1}{3.0\ \Omega} = \frac{3}{6.0\ \Omega}$$

$$R_{eq} = \frac{6.0\ \Omega}{3} = 2.0\ \Omega$$

The circuit of equivalent resistances now looks like Figure 28.10b. The 12.0-Ω and 2.0-Ω resistors are in series (green circles). Find the equivalent resistance from a to c:

$$R_{eq} = 12.0\ \Omega + 2.0\ \Omega = \boxed{14.0\ \Omega}$$

Find the Equivalent Resistance

Example 27.4

Four resistors are connected as shown in the figure.

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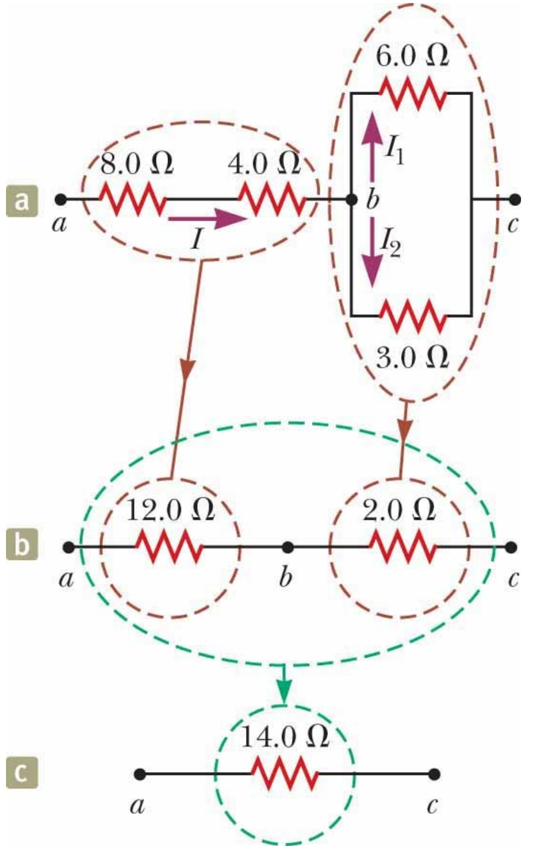
(B) What is the current in each resistor if a potential difference of 42 V is maintained between a and c?



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Solution-B

Let's find the main current ***I*** first:

$$I = \frac{\Delta V_{ac}}{R_{\text{eq}}} = \frac{42\text{ V}}{14.0\ \Omega} = \boxed{3.0\text{ A}}$$

Find the Equivalent Resistance

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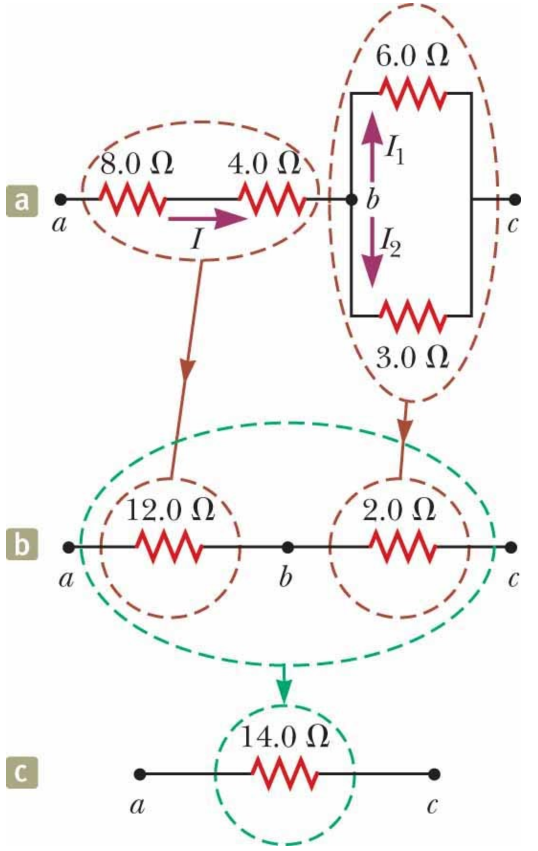

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Let's find the main current ***I*** first:

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\text{ V}}{14.0\ \Omega} = \boxed{3.0\text{ A}}$$

The voltage across 6Ω and 3Ω should be equal:

$$\Delta V_1 = \Delta V_2 \rightarrow (6.0\ \Omega)I_1 = (3.0\ \Omega)I_2 \rightarrow I_2 = 2I_1$$

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Four resistors are connected as shown in the figure.

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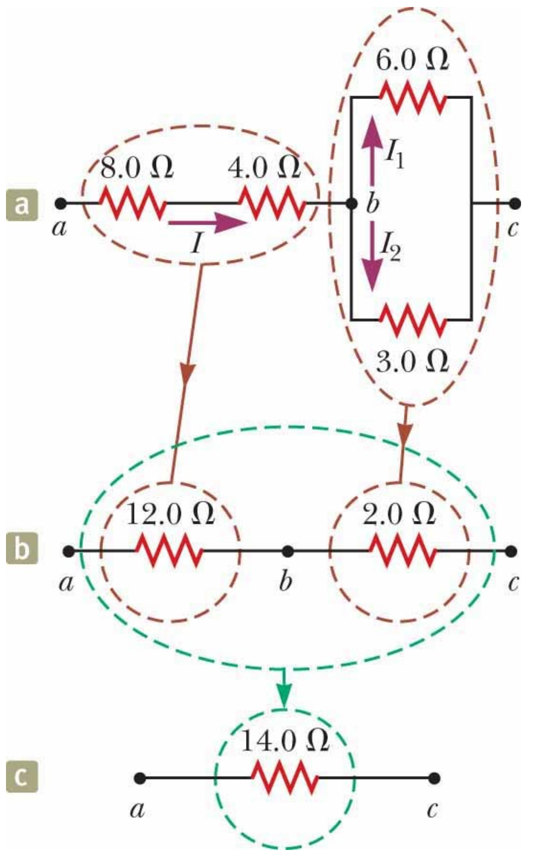
We know $I_1 + I_2$ is equal to the main current 3A:

$$I_1 + I_2 = 3.0\ \text{A} \rightarrow I_1 + 2I_1 = 3.0\ \text{A} \rightarrow I_1 = \boxed{1.0\ \text{A}}$$



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The voltage across 6Ω and 3Ω should be equal: $\Delta V_1 = \Delta V_2 \rightarrow (6.0\ \Omega)I_1 = (3.0\ \Omega)I_2 \rightarrow I_2 = 2I_1$

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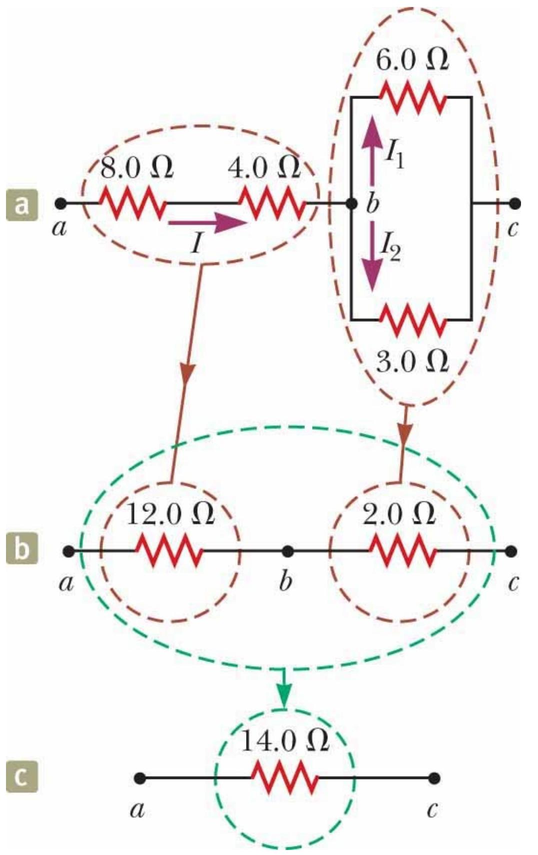
Now we can find I_2: $I_2 = 2I_1 = 2(1.0\text{ A}) = \boxed{2.0\text{ A}}$



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Direct Current Circuits (Ch. 27)

Kirchhoff’s Rules

Kirchhoff's Rules

The procedure for analyzing more complex circuits is made possible by using the following two principles, called ***Kirchhoff's rules***.

1. Junction rule (***Kirchhoff's Current Rule***). At any junction, the sum of the currents must equal zero:

$$\sum_{\text{junction}} I = 0$$

2. Loop rule (***Kirchhoff's Voltage Rule***). The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\text{closed loop}} \Delta V = 0$$

Kirchhoff’s Current (Junction) Rule

- Kirchhoff’s first rule is about ***conservation of electric charge***. All charges that enter a given point in a circuit must leave that point because charge cannot build up or disappear at a point.

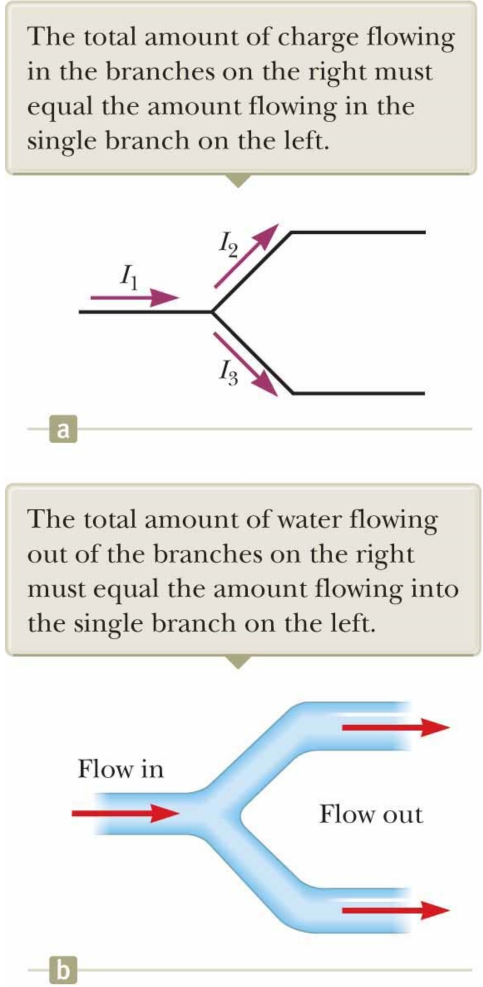
- **Current / Junction rule**: At any junction, the sum of the currents must equal zero:
    - Currents directed into junction → +I
    - Currents directed out of junction → -I

$$\sum_{\text{junction}} I = 0$$

- For the figure on the right;

$$I_1 - I_2 - I_3 = 0$$

$$I_1 = I_2 + I_3$$



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Kirchhoff’s Voltage (Loop) Rule

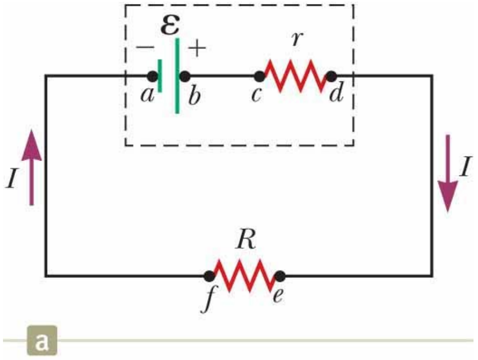
- Kirchhoff’s second rule follows the law of ***conservation of energy*** for an isolated system.

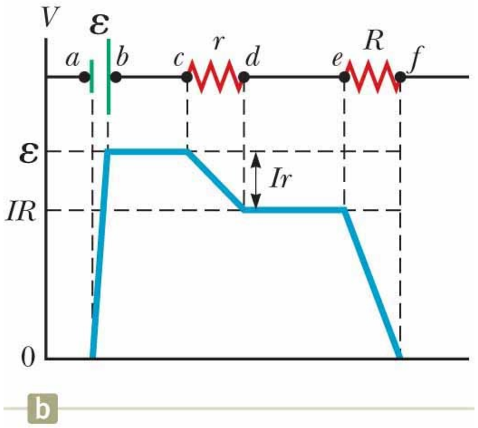
- Let’s imagine moving a charge around a closed loop of a circuit. When the charge returns to the starting point, the charge–circuit system must have the same total energy as it had before the charge was moved. The sum of the increases in energy as the charge passes through some circuit elements must equal the sum of the decreases in energy as it passes through other elements.

- **Voltage / Loop rule:** The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\text{closed loop}} \Delta V = 0$$

- For the figure on the right;
  - The potential energy of the system decreases whenever the charge moves through a potential drop ***-IR*** across a resistor.
  - The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal.





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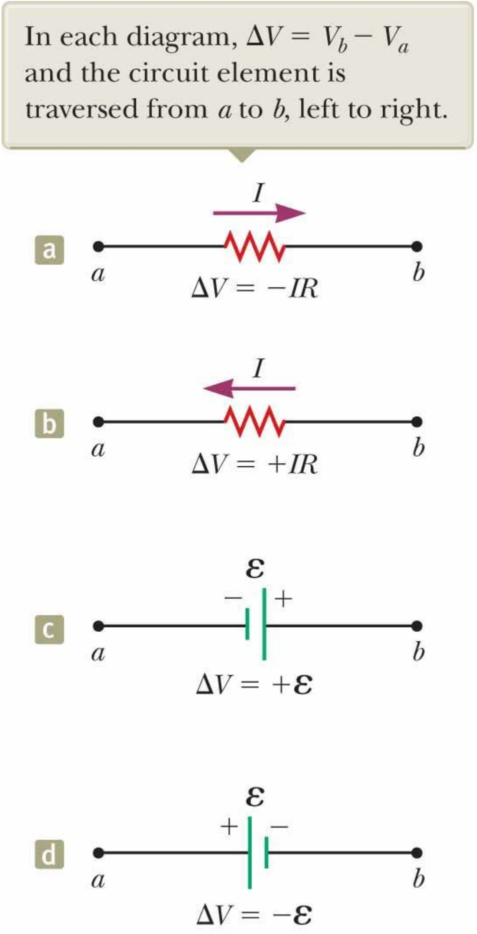
Kirchhoff’s Voltage (Loop) Rule

- When applying Kirchhoff’s Voltage rule, imagine traveling around the loop and consider changes in electric potential.

- Charges move from the high-potential end of a resistor toward the low-potential end:
  - If the resistor is traversed in the direction of the current → potential difference across resistor $\Delta V = -IR$ (figure (a))
  - If the resistor is traversed in the direction ***opposite*** of the current → $\Delta V = +IR$ (figure (b))

- If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from negative to positive) →
  - Potential difference **$\Delta V = +\mathcal{E}$** (figure (c))

- If a source of emf (assumed to have zero internal resistance) is traversed in the direction ***opposite*** of the emf (from positive to negative) →
  - Potential difference **$\Delta V = -\mathcal{E}$** (figure (d))



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Guidelines for Using Kirchhoff’s Rules

Conceptualize
- Identify all circuit elements
- Identify polarity of each battery

Categorize
- Determine whether circuit can be reduced by combining series and parallel resistors
- If not → apply Kirchhoff’s rules

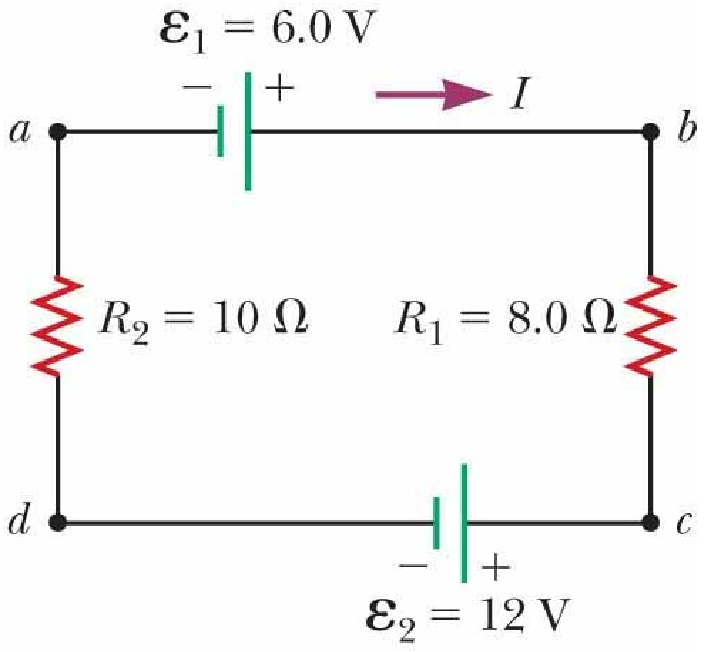
Analyze
- Assign labels to all known quantities and symbols to all unknown quantities
- Assign directions to currents in each part of circuit
- Although assignment of current directions is arbitrary →
    - ***Must adhere rigorously to directions you assign*** when you apply Kirchhoff’s rules
- Apply Kirchhoff’s Current and Voltage rules to obtain as many equations as there are unknowns
- Choose a direction to travel around loop (clockwise or counter clockwise) and correctly identify change in potential as you cross each element
- Be careful with signs!

Finalize
- Check numerical answers for consistency
- ***Do not be alarmed if resulting currents have negative value*** →
    - Means you have guessed the direction of that current incorrectly, but its magnitude will be correct

A Single Loop Circuit

Example 27.6

A single-loop circuit contains two resistors and two batteries as shown in the figure. (Neglect the internal resistances of the batteries.) Find the current in the circuit.



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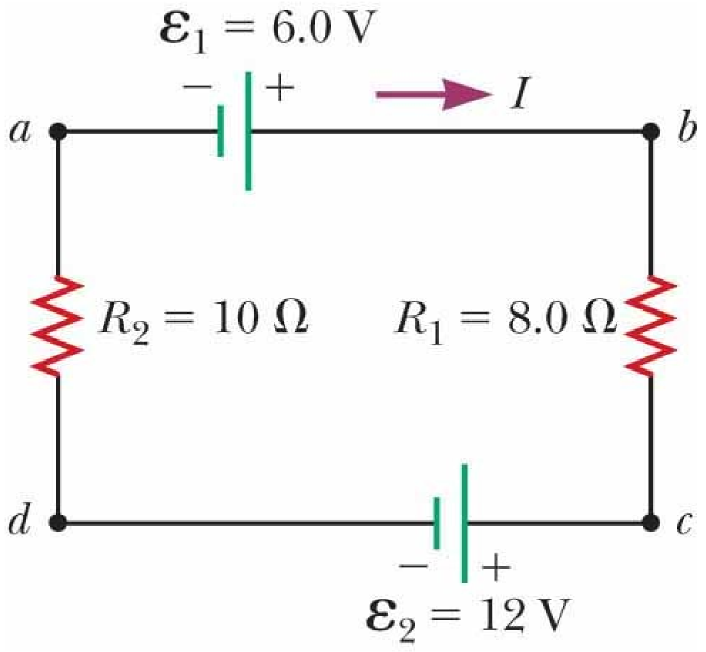

Solution

Conceptualize

The figure shows the polarities of the batteries and a guess at the direction of the current. The 12-V battery is the stronger of the two, so the current should be counter clockwise. Therefore, we expect our guess for the direction of the current to be wrong, but we will continue and see how this incorrect guess is represented by our final answer.

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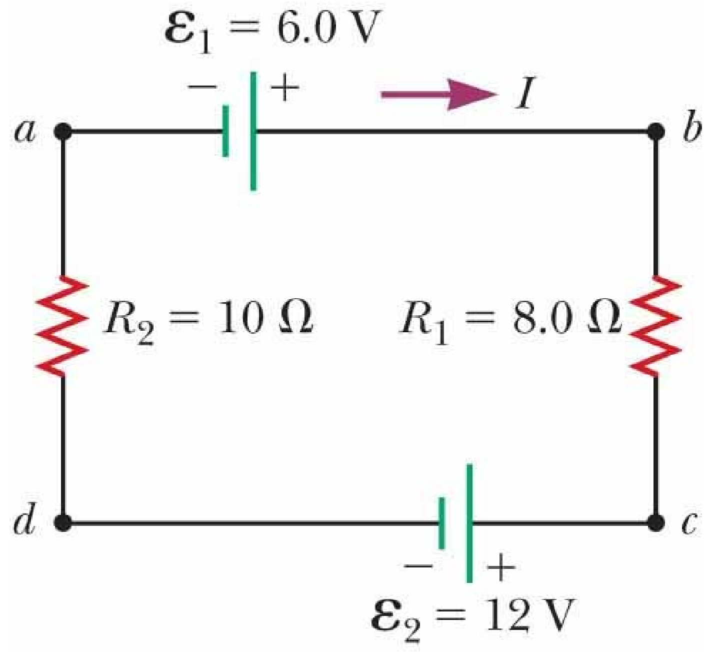

Solution

Categorize

We do not need Kirchhoff's rules to analyze this simple circuit, but let's use them anyway simply to see how they are applied. There are no junctions in this single-loop circuit; therefore, the current is the same in all elements.

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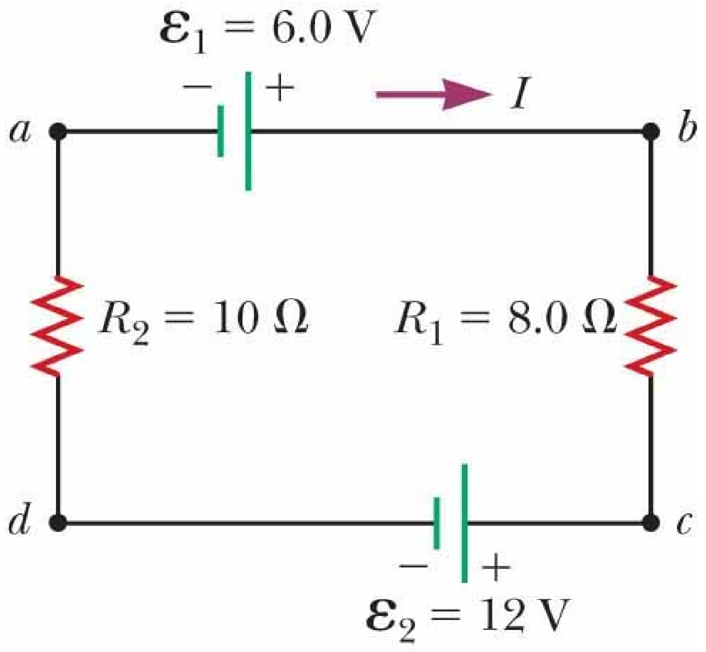

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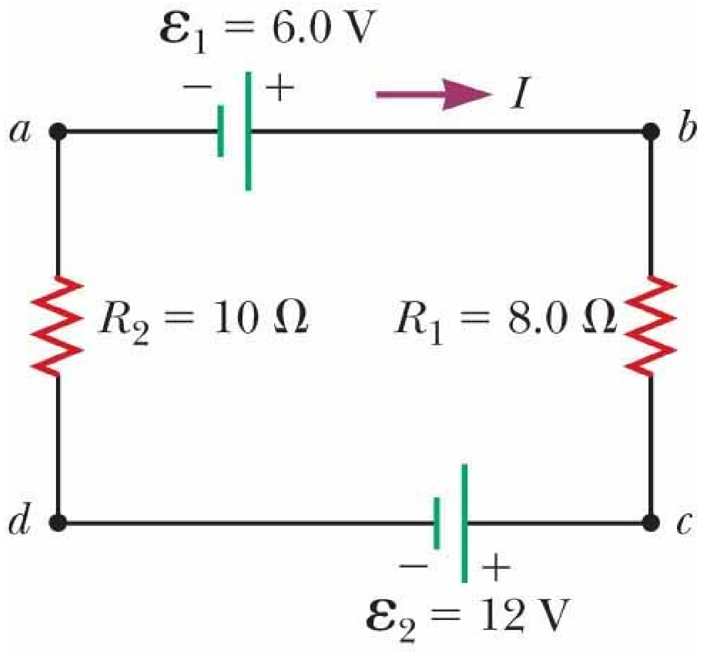

Solution

Analyze

Let's apply Kirchhoff's Voltage rule, traversing the circuit in the clockwise direction (chosen current direction) starting from point ***a***.

$$\sum \Delta V = 0 \Rightarrow \varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$$

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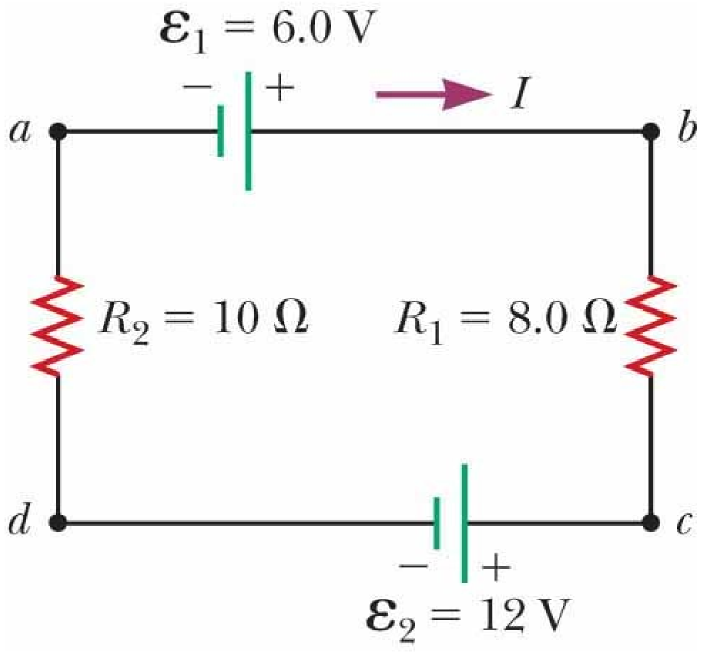

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$$\sum \Delta V = 0 \Rightarrow \varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$$

Now we can solve for the current I:

$$I = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{6.0\ \text{V} - 12\ \text{V}}{8.0\ \Omega + 10\ \Omega} = \boxed{-0.33\ \text{A}}$$

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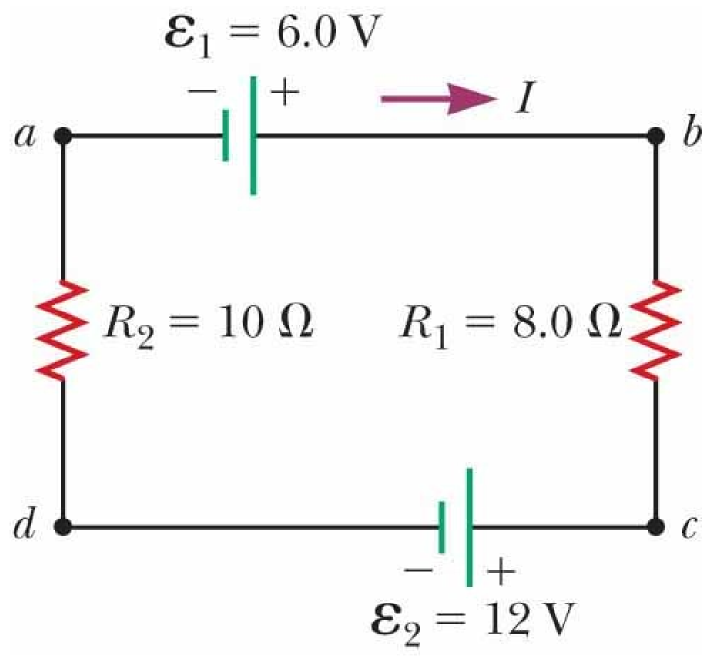

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$$\sum \Delta V = 0 \Rightarrow \varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$$

Now we can solve for the current I:

$$I = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{6.0\ \text{V} - 12\ \text{V}}{8.0\ \Omega + 10\ \Omega} = \boxed{-0.33\ \text{A}}$$

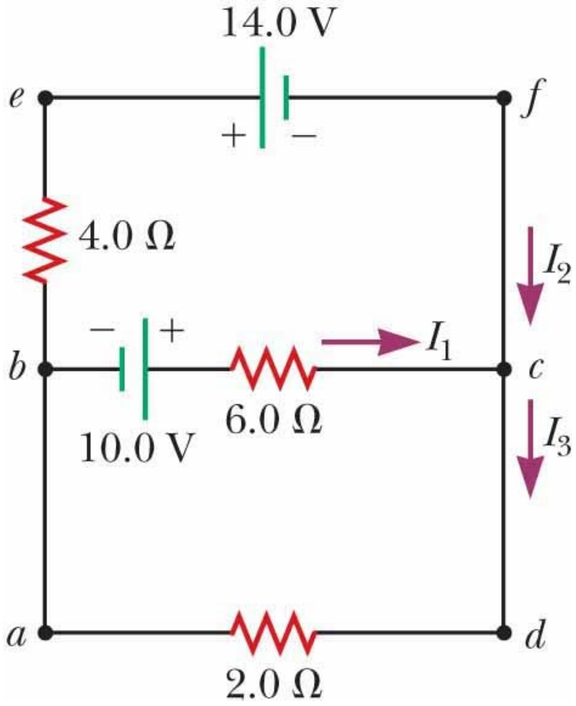
Finalize

The negative sign for ***I*** indicates that the direction of the current is opposite of the assumed direction.

A Multi Loop Circuit

Example 27.7

Find the currents I_1, I_2, and I_3 in the circuit shown in the figure.



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

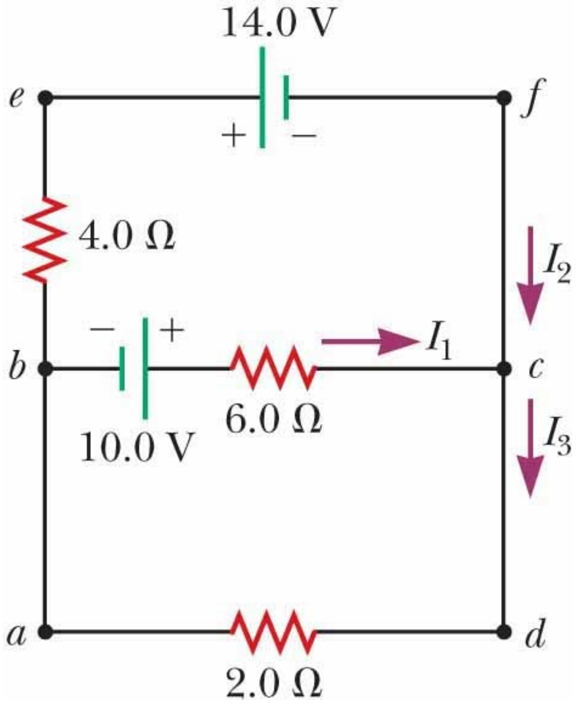
A Multi Loop Circuit

Example 27.7

Find the currents I_1, I_2, and I_3 in the circuit shown in the figure.

Solution

<u>Conceptualize</u>
Imagine physically rearranging the circuit while keeping it electrically the same. Can you rearrange it so that it consists of simple series or parallel combinations of resistors? You should find that you cannot.



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A Multi Loop Circuit

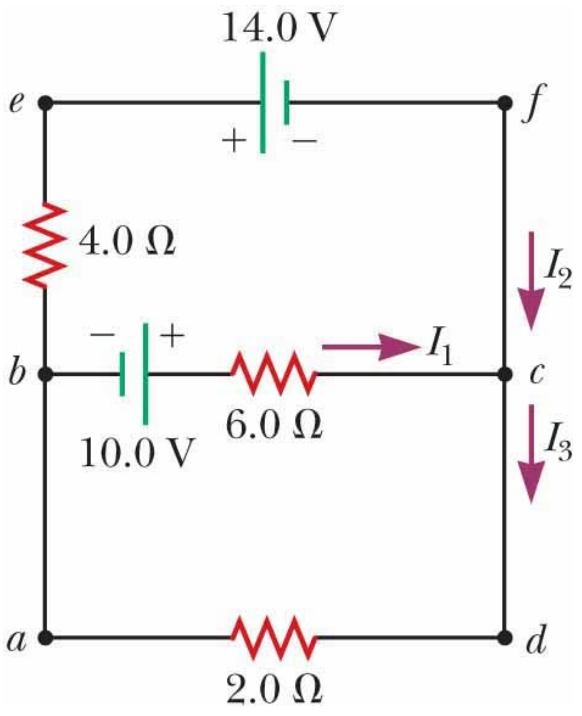
Example 27.7

Find the currents I_1, I_2, and I_3 in the circuit shown in the figure.

Solution

Categorize

We cannot simplify the circuit by the rules associated with combining resistances in series and in parallel. Therefore, this problem is one in which we must use Kirchhoff's rules.



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A Multi Loop Circuit

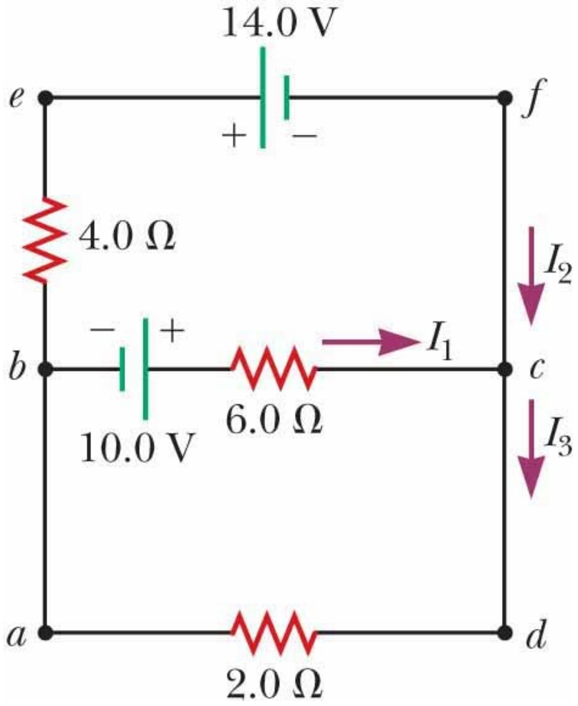
Example 27.7

Find the currents I_1, I_2, and I_3 in the circuit shown in the figure.

Solution

Analyze

We identify three different currents and arbitrarily choose their directions as labeled in the figure.
Let's now apply Kirchhoff's rules to find I_1, I_2, and I_3.



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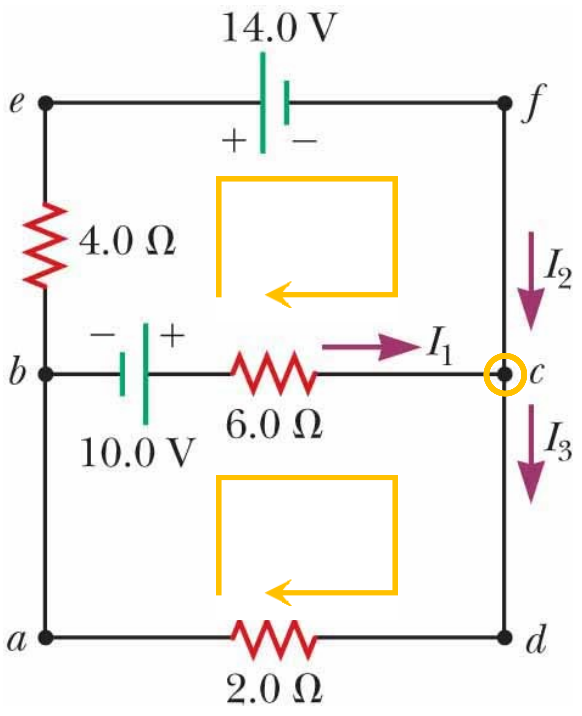

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Solution

Apply Kirchhoff's Current rule at junction ***c***:

$$I_1 + I_2 - I_3 = 0$$

Apply Kirchhoff's Voltage rule on path ***abcda***:

$$10.0\text{ V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)I_3 = 0$$

Apply Kirchhoff's Voltage rule on path ***befcb***:

$$-(4.0\ \Omega)I_2 - 14.0\text{ V} + (6.0\ \Omega)I_1 - 10.0\text{ V} = 0$$

A Multi Loop Circuit

Example 27.7

Find the currents I_1, I_2, and I_3 in the circuit shown in the figure.

Solution

Apply Kirchhoff's Current rule at junction ***c***:

(1) $$I_1 + I_2 - I_3 = 0$$

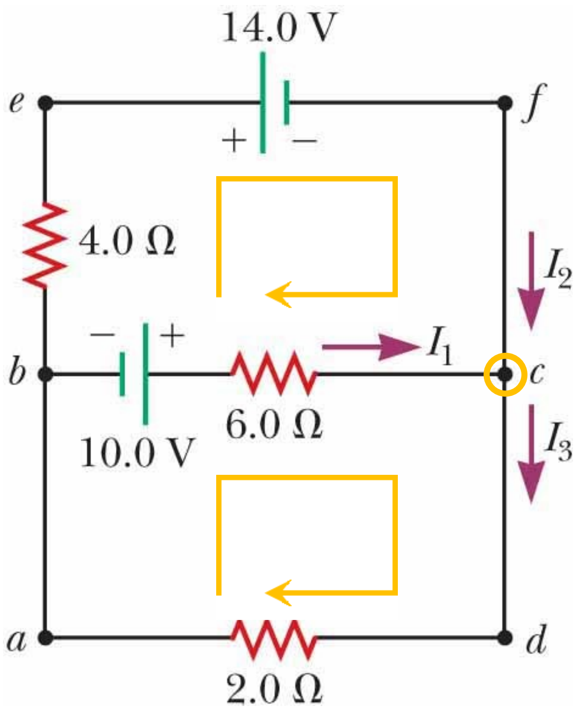
Apply Kirchhoff's Voltage rule on path ***abcda***:

(2) $$10.0\ \text{V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)I_3 = 0$$

Apply Kirchhoff's Voltage rule on path ***befcb***:

(3) $$-(4.0\ \Omega)I_2 - 14.0\ \text{V} + (6.0\ \Omega)I_1 - 10.0\ \text{V} = 0$$

We have
3 unknowns
3 equations



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A Multi Loop Circuit

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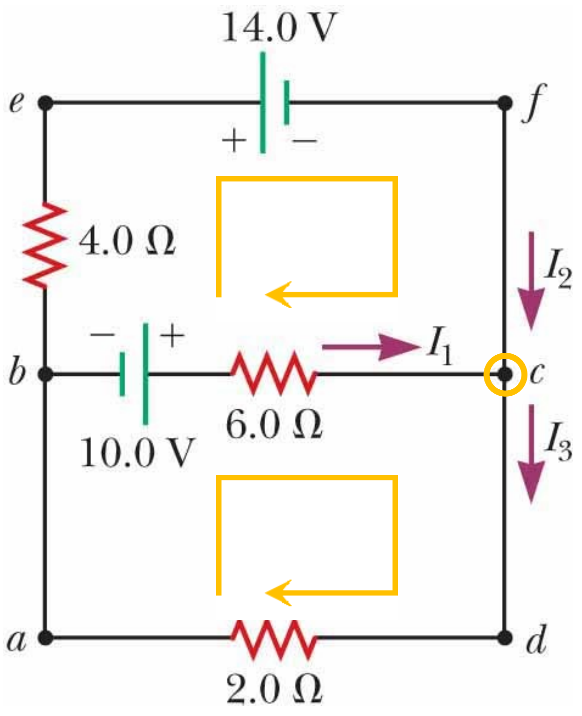
We have
3 unknowns
3 equations

Solving we get:
$I_1 = 2.0\text{A}$
$I_2 = -3.0\text{A}$
$I_3 = -1.0\text{A}$



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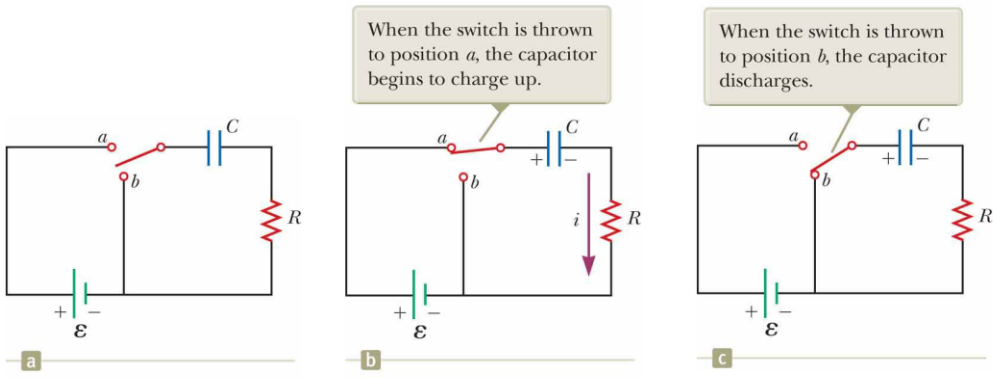
Direct Current Circuits (Ch. 27)

RC Circuits

RC Circuits

So far, we have analyzed direct-current (DC) circuits in which the current is constant.
In ***DC circuits containing capacitors***, the ***current may vary in magnitude*** at different times. A circuit containing a series combination of a resistor and a capacitor is called an ***RC circuit***.



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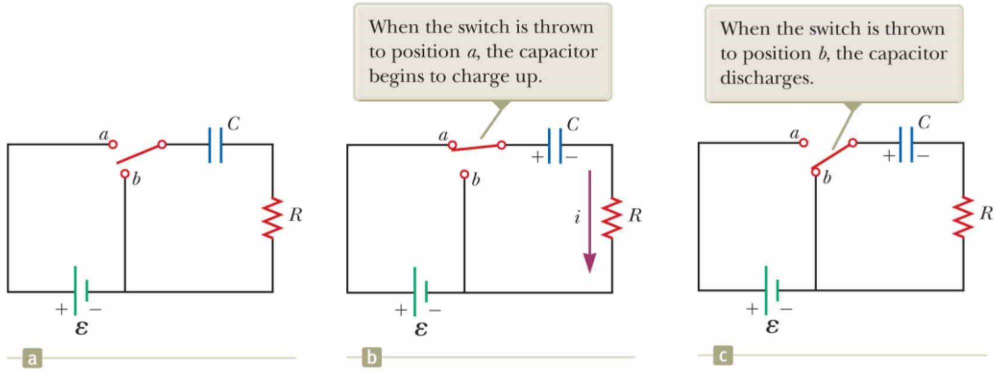
In ***DC circuits containing capacitors***, the ***current may vary in magnitude*** at different times. A circuit containing a series combination of a resistor and a capacitor is called an ***RC circuit***.

- Assume capacitor is initially uncharged
- No current while switch open (figure (a))
- If switch thrown to position a at t = 0 (figure (b)) →
  - Charge begins to flow
  - Capacitor begins to charge

- During charging, charges do not jump across capacitor plates because gap between plates represents open circuit to DC
- Charge is transferred between each plate and its connecting wires due to electric field established in the wires by battery until capacitor is fully charged
- As plates charge → potential difference across capacitor increases
- Once maximum charge reached → current in circuit = 0
- Potential difference across capacitor matches that supplied by battery



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RC Circuits

- Let's analyze the circuit by applying Kirchhoff's Voltage rule when the switch is on position ***a***:
  - The potential difference across the capacitor is q/C
  - The potential difference across the resistor is iR

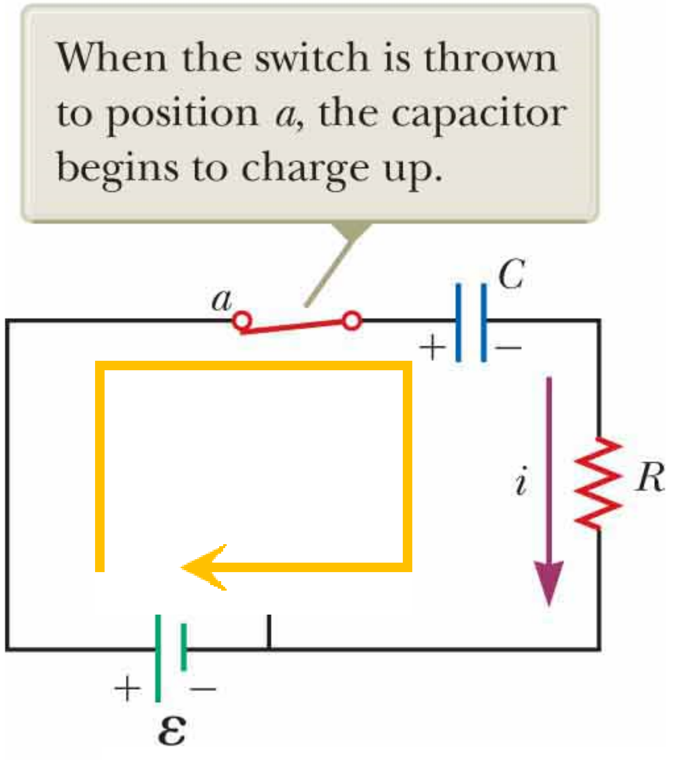
$$\varepsilon - \frac{q}{C} - iR = 0$$

- Initial current (when the capacitor has zero charge → q=0):

$$I_i = \frac{\varepsilon}{R} \quad (\text{current at } t = 0)$$

- When ***C*** is fully charged to Q_{max}, charge flow stops (I=0).

$$Q_{\max} = C\varepsilon \quad (\text{maximum charge})$$



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RC Circuits

- To solve for the time dependence of charge and current, we need to solve the following equation (as a differential equation):

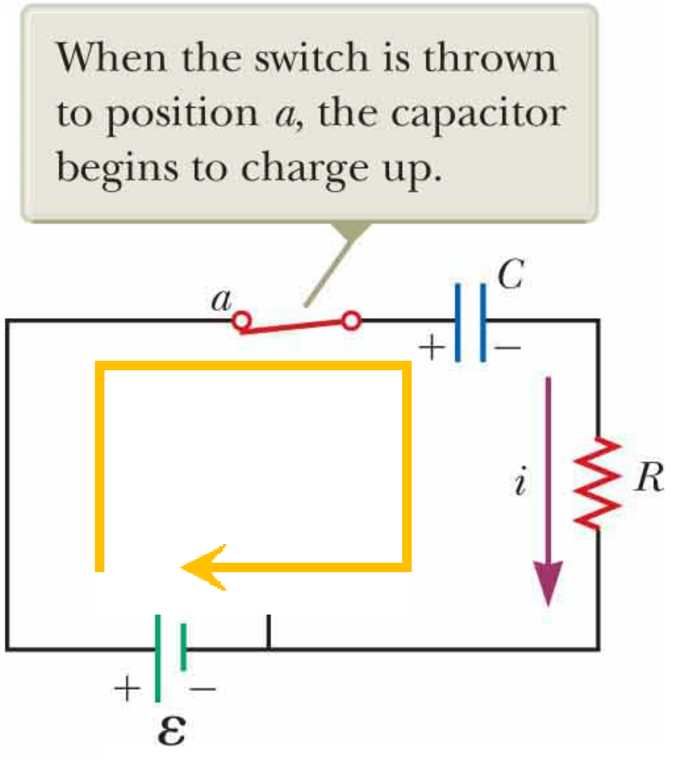
$$\varepsilon - \frac{q}{C} - iR = 0$$

The current through the resistor and the capacitor is the same: ***i=dq/dt***

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC}$$

$$\frac{dq}{dt} = \frac{C\varepsilon}{RC} - \frac{q}{RC} = -\frac{q - C\varepsilon}{RC}$$

$$\frac{dq}{q - C\varepsilon} = -\frac{1}{RC}dt$$



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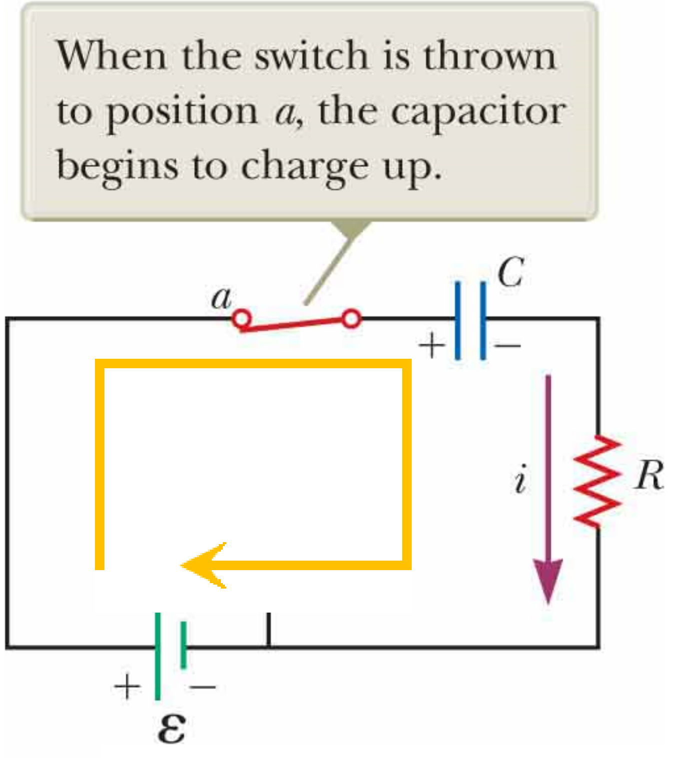
Integrate over 0→t, 0→q

$$\int_0^q \frac{dq}{q - C\varepsilon} = -\frac{1}{RC}\int_0^t dt \Rightarrow \ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$



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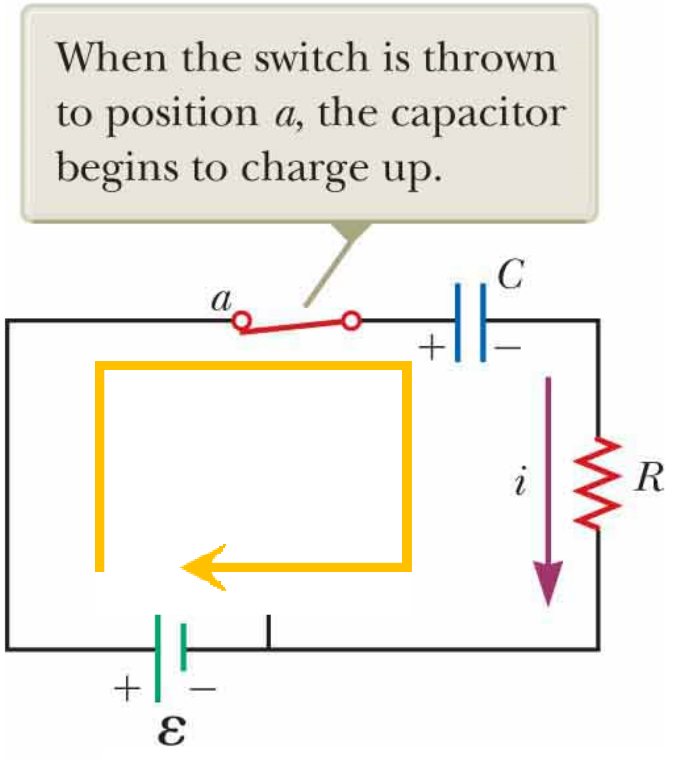
$$q(t) = C\varepsilon\left(1 - e^{-t/RC}\right)$$
$$= Q_{\max}\left(1 - e^{-t/RC}\right)$$

i = dq/dt

$$i(t) = \frac{\varepsilon}{R}e^{-t/RC}$$



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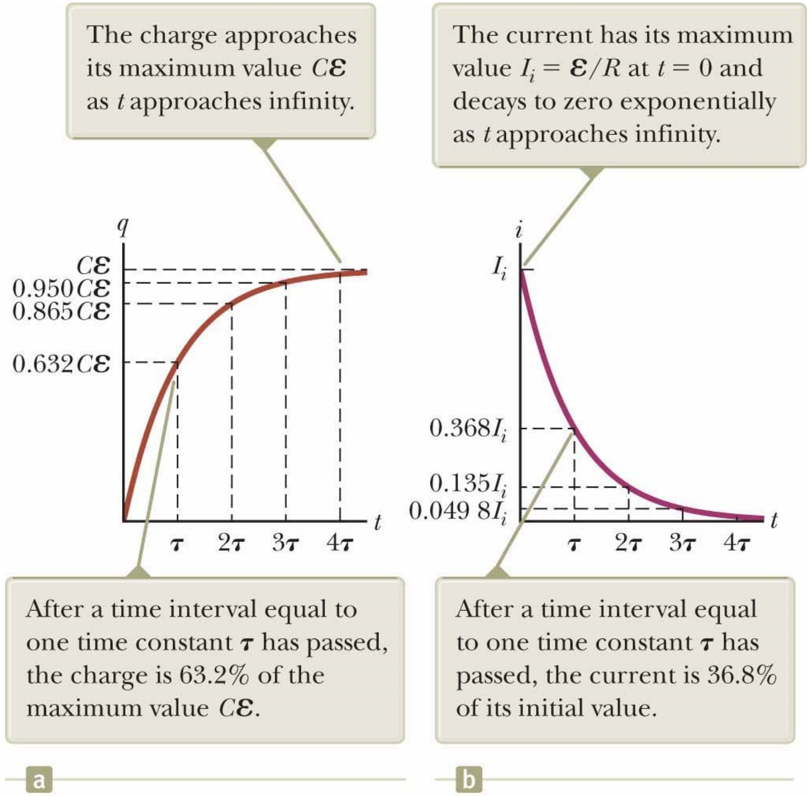
RC Circuits

- Let's take a closer look at how the charge and the circuit current changes with time:

$$q(t) = C\varepsilon\left(1 - e^{-t/RC}\right) = Q_{\max}\left(1 - e^{-t/RC}\right)$$

$$i(t) = \frac{\varepsilon}{R} e^{-t/RC}$$

- Observations for the charge of the capacitor:
  - ***t=0, q=0***
  - ***t→∞, q→Cℇ***
- Observations for the current:
  - ***t=0, I_i=ℇ/R***
  - ***t→∞, I→0*** *(decays exponentially to zero)*



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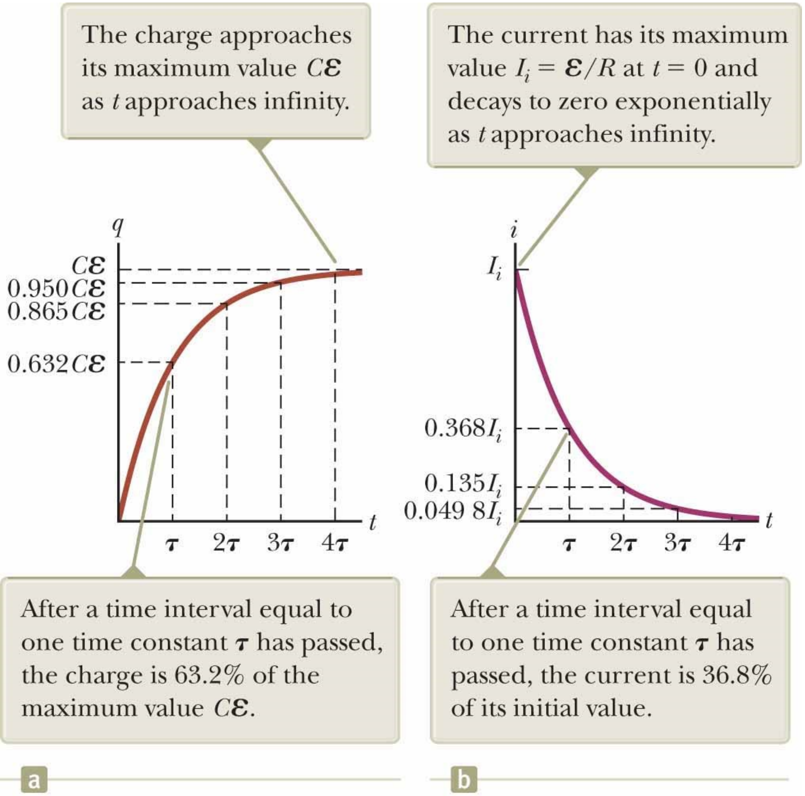
- Observations for the charge of the capacitor:
  - ***t=0, q=0***
  - ***t→∞, q→Cℰ***
- Observations for the current:
  - ***t=0, I_i=ℰ/R***
  - ***t→∞, I→0*** *(decays exponentially to zero)*
- The quantity ***RC*** (exponent in the equations for ***q*** and ***i***) is called the ***time constant τ*** of the circuit:

$$\tau = RC$$

$$i = e^{-1} I_i = 0.368 I_i$$ (After one time constant, current decreases to its 36.8%)

$$i = e^{-2} I_i = 0.135 I_i$$ (After two time constants, current decreases to its 13.5%)



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RC Circuits

Time Constant

If we do a dimensional analysis, we find that the time constant $\boldsymbol{\tau}$ has units of time:

$$\tau = RC$$

$$[\tau] = [RC] = \left[\left(\frac{\Delta V}{I}\right)\left(\frac{Q}{\Delta V}\right)\right] = \left[\frac{Q}{Q/\Delta t}\right] = [\Delta t] = \mathrm{T}$$

RC Circuits

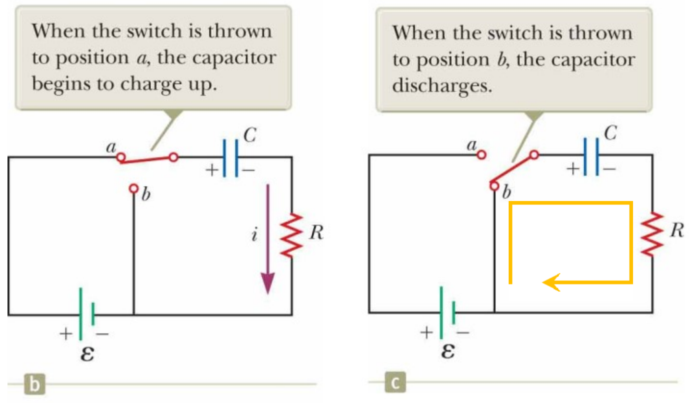
Discharging a Capacitor

- Imagine the capacitor in figure (b) is completely charged.
  - Initial potential difference Q_i/C exists across the capacitor.
  - Zero potential difference across the resistor because $i = 0$
- If switch is now thrown to position ***b*** at $t = 0$ (figure (c)):
  - The capacitor begins to discharge through the resistor
  - At some time ***t*** during the discharge →
    - current in the circuit = ***i*** and the charge on the capacitor = ***q***
- Applying Kirchhoff's Voltage rule for circuit in figure (c):

$$-\frac{q}{C} - iR = 0$$

Substitute $i = dq/dt$

$$-R\frac{dq}{dt} = \frac{q}{C} \Rightarrow \frac{dq}{q} = -\frac{1}{RC}dt$$



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RC Circuits

Discharging a Capacitor

$$-R\frac{dq}{dt}=\frac{q}{C}\Rightarrow\frac{dq}{q}=-\frac{1}{RC}dt$$

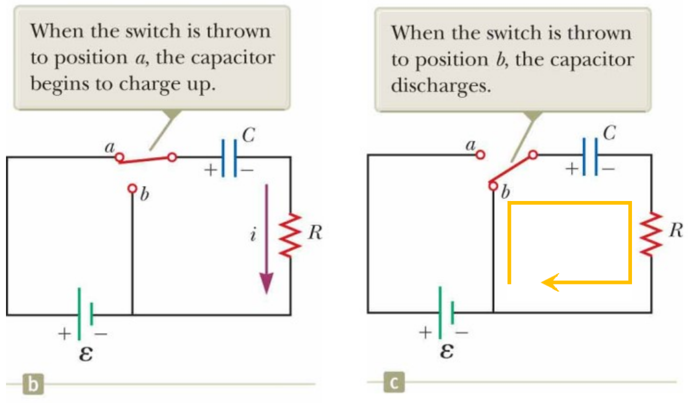
integrating over ***0→t, Q_i→q***

$$\int_{Q_i}^{q}\frac{dq}{q}=-\frac{1}{RC}\int_0^t dt\Rightarrow\ln\left(\frac{q}{Q_i}\right)=-\frac{t}{RC}$$

$$q(t)=Q_i e^{-t/RC}$$

differentiating with respect to time (i=dq/dt) gives us the instantaneous current

$$i(t)=-\frac{Q_i}{RC}e^{-t/RC}$$



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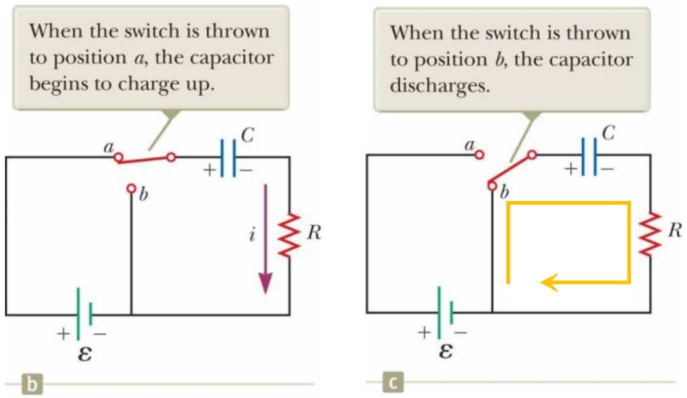

Figure (b) shows the current in the resistor in downward direction. Equation shows that the current in discharging capacitor is negative → meaning the current is upward direction in the resistor in figure (c).

Both capacitor charging and current decaying happens exponentially at a rate characterized by the time constant ***τ = RC***

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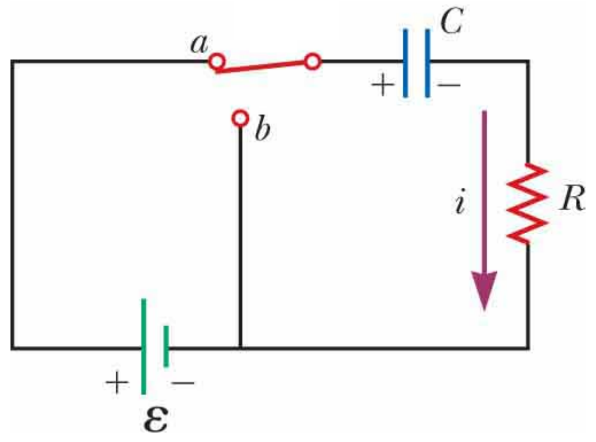
Charging a Capacitor in an RC Circuit

Example 27.9

An uncharged capacitor and a resistor are connected in series to a battery as shown in the figure, where **$\mathcal{E}$ = 12.0 V**, **C = 5.00 μF**, and **R = 8.00 x 10^5 Ω**. The switch is thrown to position **a**. Find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as functions of time.



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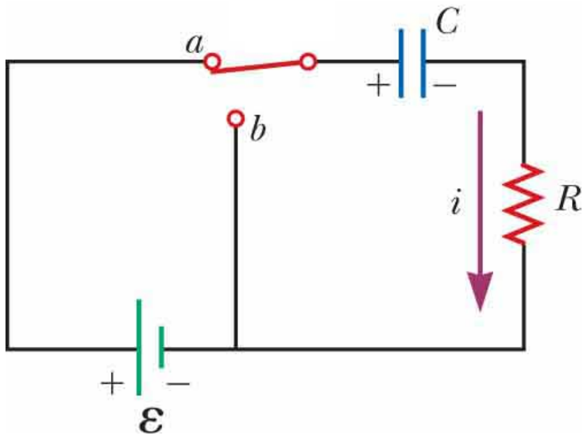

Solution

Evaluate the time constant of the circuit $\tau = RC$

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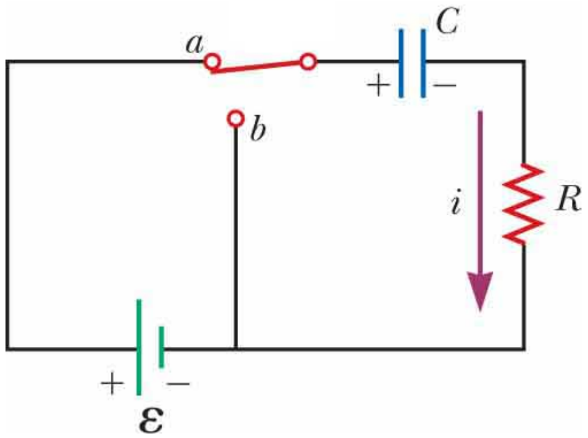

Solution

Evaluate the time constant of the circuit $\tau = RC = (8.00 \times 10^5\ \Omega)(5.00 \times 10^{-6}\ \text{F}) = \boxed{4.00\ \text{s}}$

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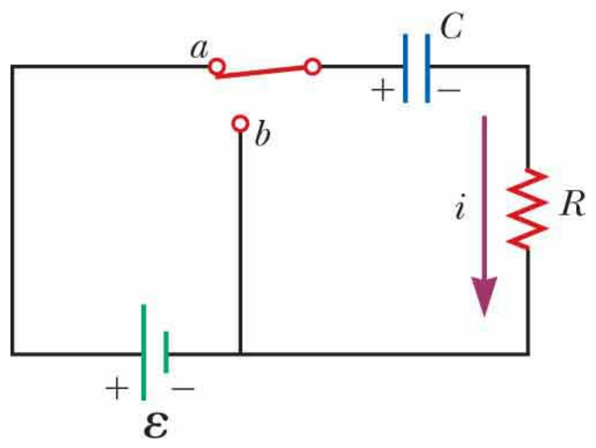

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Evaluate the maximum charge on the capacitor $Q_{\max} = C\mathcal{E}$

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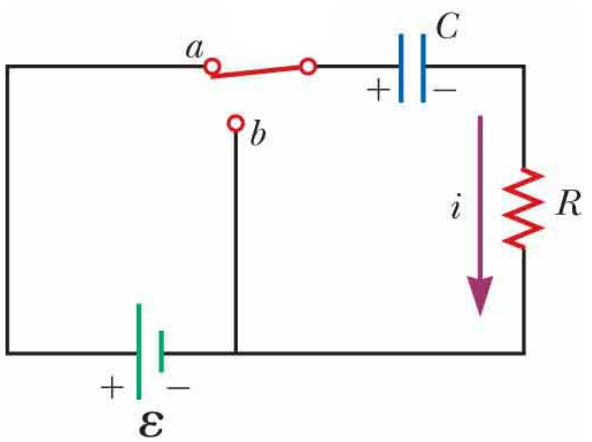

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Evaluate the maximum charge on the capacitor $Q_{\text{max}} = C\mathcal{E} = (5.00\ \mu\text{F})(12.0\ \text{V}) = \boxed{60.0\ \mu\text{C}}$

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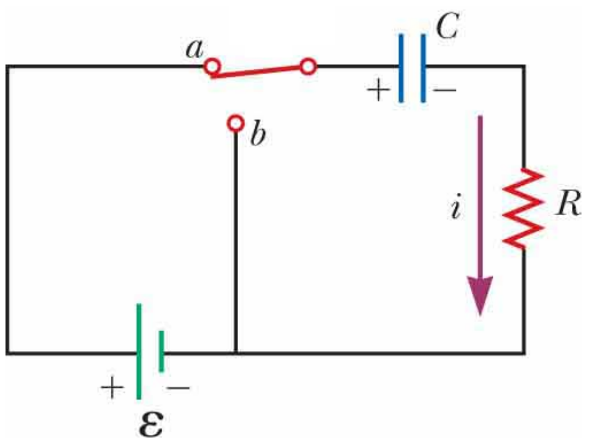

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Evaluate the maximum current in the circuit $I_i = \dfrac{\mathcal{E}}{R}$

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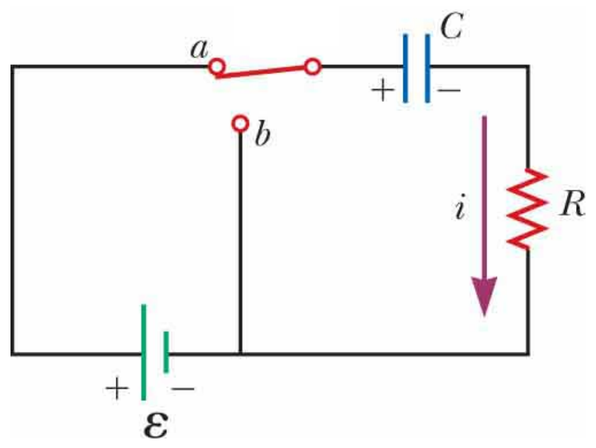
Example 27.9

An uncharged capacitor and a resistor are connected in series to a battery as shown in the figure, where **$\mathcal{E}$ = 12.0 V**, **C = 5.00 μF**, and **R = 8.00 x 10^5 Ω**. The switch is thrown to position **a**. Find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as functions of time.



Solution

Evaluate the time constant of the circuit

$$\tau = RC = (8.00 \times 10^5\ \Omega)(5.00 \times 10^{-6}\ \text{F}) = \boxed{4.00\ \text{s}}$$

Evaluate the maximum charge on the capacitor

$$Q_{\max} = C\mathcal{E} = (5.00\ \mu\text{F})(12.0\ \text{V}) = \boxed{60.0\ \mu\text{C}}$$

Evaluate the maximum current in the circuit

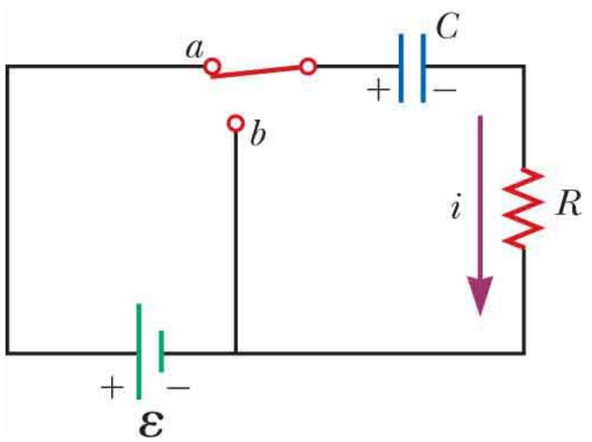
$$I_i = \frac{\mathcal{E}}{R} = \frac{12.0\ \text{V}}{8.00 \times 10^5\ \Omega} = \boxed{15.0\ \mu\text{A}}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Charging a Capacitor in an RC Circuit

Example 27.9

An uncharged capacitor and a resistor are connected in series to a battery as shown in the figure, where **Ɛ = 12.0 V**, **C = 5.00 μF**, and **R = 8.00 x 10^{5} Ω**. The switch is thrown to position **a**. Find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as functions of time.



Solution

Evaluate the time constant of the circuit $\tau = RC = (8.00 \times 10^{5}\ \Omega)(5.00 \times 10^{-6}\ \text{F}) = \boxed{4.00\ \text{s}}$

Evaluate the maximum charge on the capacitor $Q_{\max} = C\mathcal{E} = (5.00\ \mu\text{F})(12.0\ \text{V}) = \boxed{60.0\ \mu\text{C}}$

Evaluate the maximum current in the circuit $I_i = \dfrac{\mathcal{E}}{R} = \dfrac{12.0\ \text{V}}{8.00 \times 10^{5}\ \Omega} = \boxed{15.0\ \mu\text{A}}$

Use these values to find the charge and current as functions of time:

$$q(t) = Q_{\max}\left(1 - e^{-t/RC}\right)$$

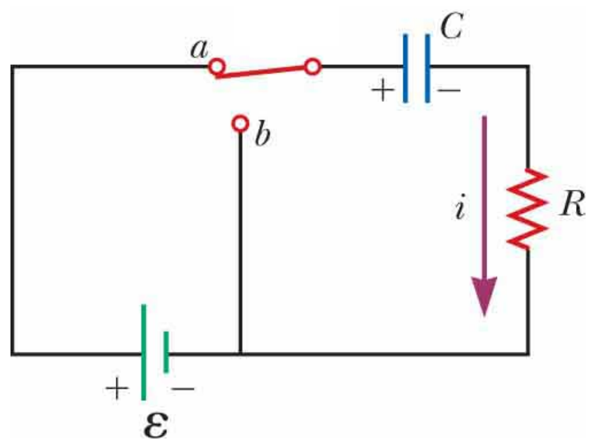
$$i(t) = \frac{\varepsilon}{R} e^{-t/RC}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Charging a Capacitor in an RC Circuit

Example 27.9

An uncharged capacitor and a resistor are connected in series to a battery as shown in the figure, where **$\mathcal{E}$ = 12.0 V**, **C = 5.00 μF**, and **R = 8.00 x 10^5 Ω**. The switch is thrown to position **a**. Find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as functions of time.



Solution

Evaluate the time constant of the circuit

$$\tau = RC = (8.00 \times 10^5\ \Omega)(5.00 \times 10^{-6}\ \text{F}) = \boxed{4.00\ \text{s}}$$

Evaluate the maximum charge on the capacitor

$$Q_{\max} = C\mathcal{E} = (5.00\ \mu\text{F})(12.0\ \text{V}) = \boxed{60.0\ \mu\text{C}}$$

Evaluate the maximum current in the circuit

$$I_i = \frac{\mathcal{E}}{R} = \frac{12.0\ \text{V}}{8.00 \times 10^5\ \Omega} = \boxed{15.0\ \mu\text{A}}$$

Use these values to find the charge and current as functions of time:

$$q(t) = Q_{\max}\left(1 - e^{-t/RC}\right) \qquad (1)\quad q(t) = \boxed{60.0\left(1 - e^{-t/4.00}\right)}$$

$$i(t) = \frac{\mathcal{E}}{R}e^{-t/RC} \qquad (2)\quad i(t) = \boxed{15.0e^{-t/4.00}}$$

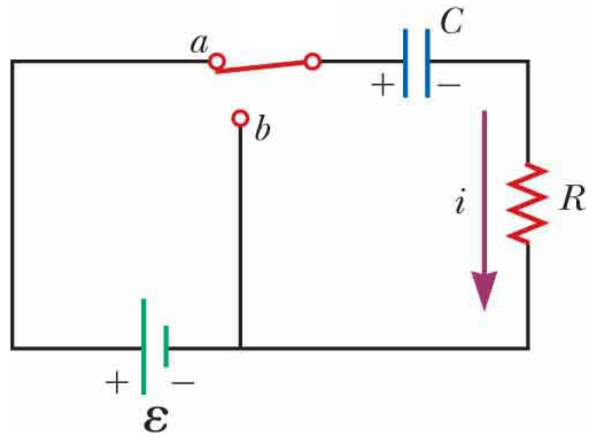
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Discharging a Capacitor in an RC Circuit

Example 27.10

Consider a capacitor of capacitance **C** that is being discharged through a resistor of resistance **R** as shown in the figure.

(A) After how many time constants is the charge on the capacitor one-fourth (1/4) its initial value?
(B) The energy stored in the capacitor decreases with time as the capacitor discharges. After how many time constants is this stored energy one-fourth (1/4) its initial value?



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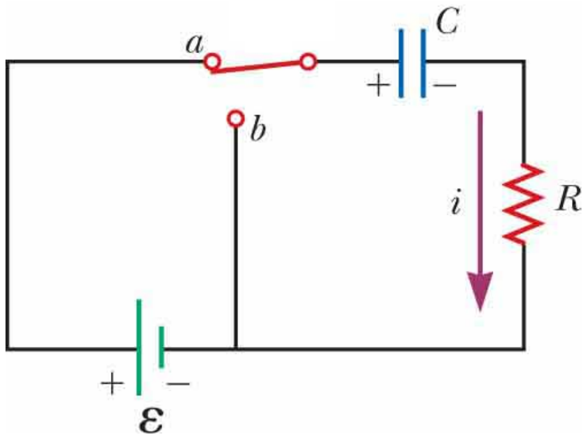

Solution A

Substitute q(t) = $Q_i/4$ into → $q(t) = Q_i e^{-t/RC}$

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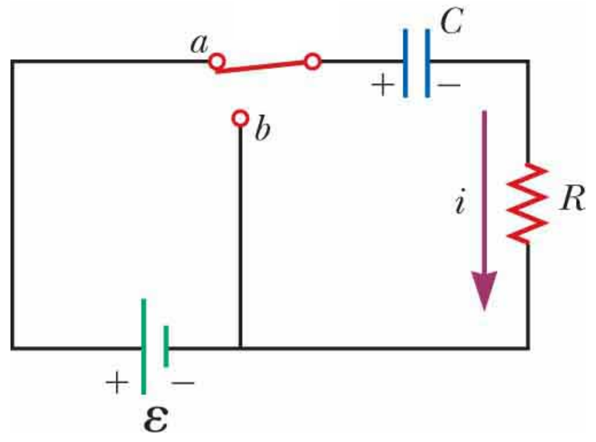

Solution A

Substitute $q(t) = Q_i/4$ into → $q(t) = Q_i e^{-t/RC}$

$$\frac{Q_i}{4} = Q_i e^{-t/RC}$$

$$\frac{1}{4} = e^{-t/RC}$$

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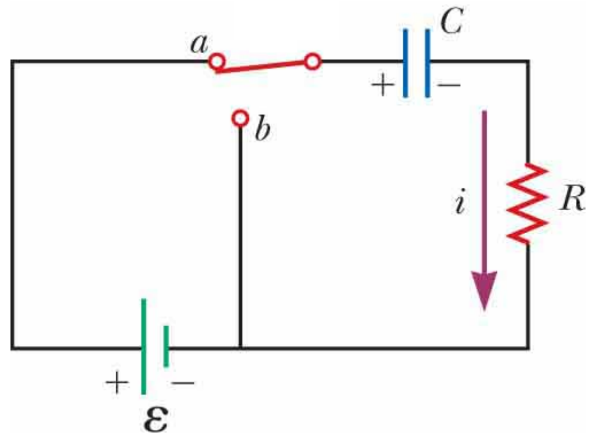

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Substitute $q(t) = Q_i/4$ into → $q(t) = Q_i e^{-t/RC}$

$$\frac{Q_i}{4} = Q_i e^{-t/RC}$$

$$\frac{1}{4} = e^{-t/RC}$$

Take the logarithm of both sides of the equation and solve for t →

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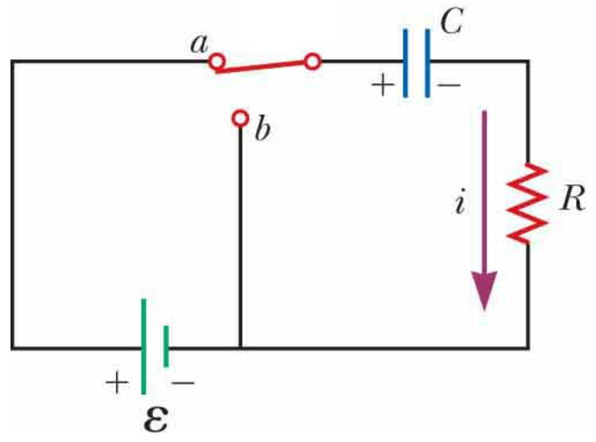

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$$\frac{Q_i}{4} = Q_i e^{-t/RC}$$

$$\frac{1}{4} = e^{-t/RC}$$

Take the logarithm of both sides of the equation and solve for t →

$$-\ln 4 = -\frac{t}{RC}$$

$$t = RC \ln 4 = 1.39RC = 1.39\tau$$

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Discharging a Capacitor in an RC Circuit

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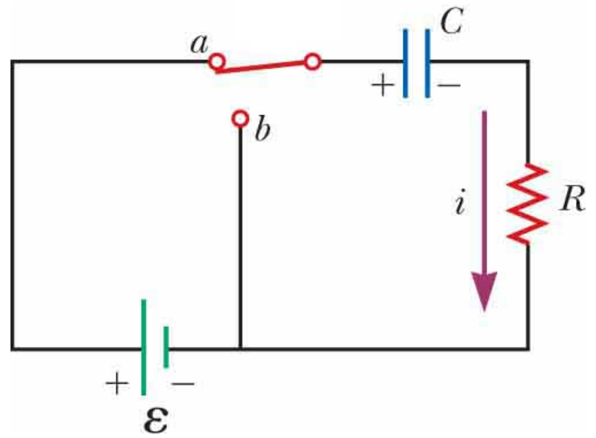

Solution B

Express the energy stored in the capacitor at any time ***t***:

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Discharging a Capacitor in an RC Circuit

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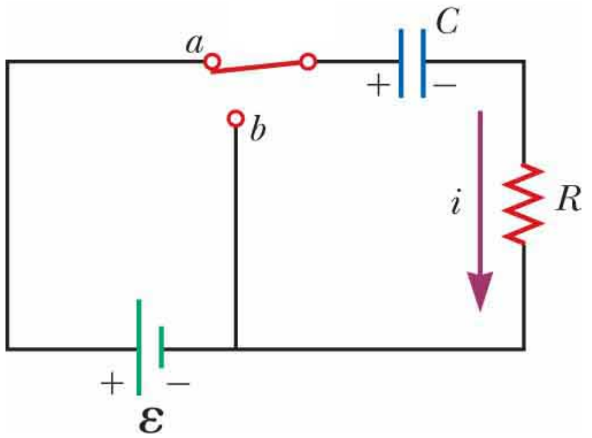

Solution B

Express the energy stored in the capacitor at any time ***t***:

$$U(t) = \frac{q^2}{2C} = \frac{Q_i^2}{2C} e^{-2t/RC}$$

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Discharging a Capacitor in an RC Circuit

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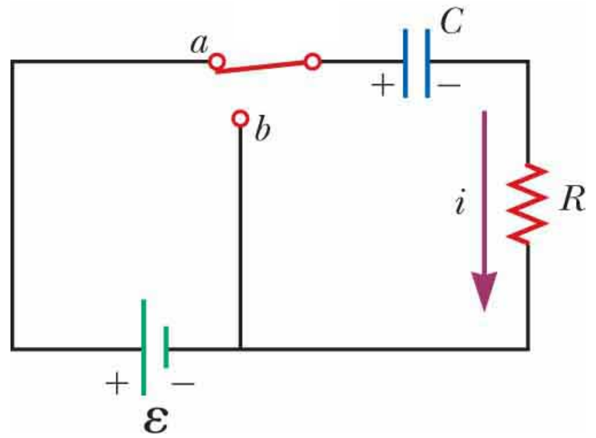

Solution B

Express the energy stored in the capacitor at any time ***t***:

$$U(t) = \frac{q^2}{2C} = \frac{Q_i^2}{2C} e^{-2t/RC}$$

Substitute $U(t) = \frac{1}{4}(Q_i^2/2C) \rightarrow$

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Discharging a Capacitor in an RC Circuit

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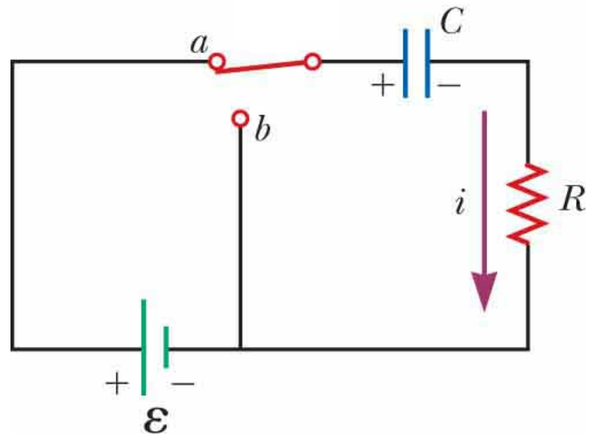

Solution B

Express the energy stored in the capacitor at any time ***t***:

$$U(t) = \frac{q^2}{2C} = \frac{Q_i^2}{2C} e^{-2t/RC}$$

Substitute $U(t) = ¼ (Q_i^2/2C)$ →

$$\frac{1}{4}\frac{Q_i^2}{2C} = \frac{Q_i^2}{2C} e^{-2t/RC}$$

$$\frac{1}{4} = e^{-2t/RC}$$

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Discharging a Capacitor in an RC Circuit

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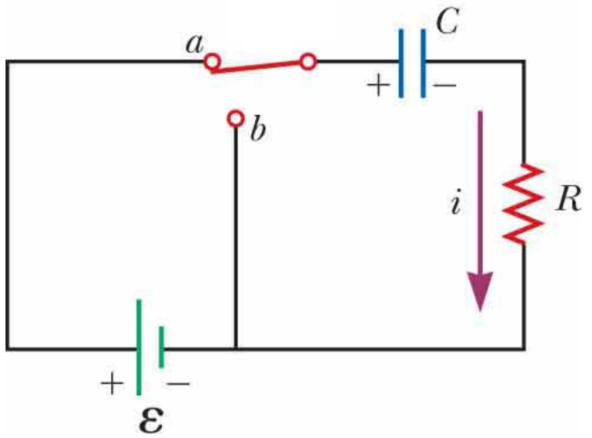

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$$\frac{1}{4} = e^{-2t/RC}$$

Take the logarithm of both sides of the equation and solve for t →

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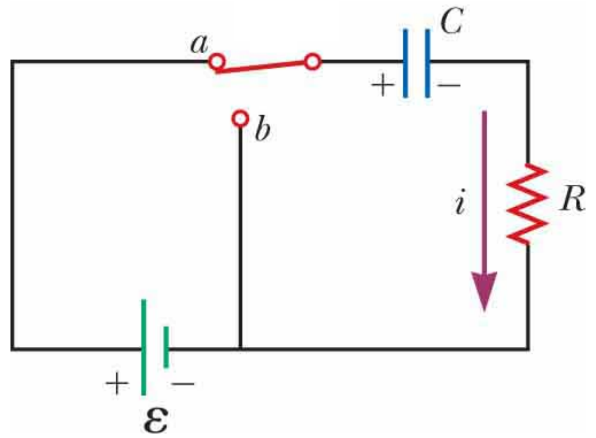

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Solution B

Express the energy stored in the capacitor at any time ***t***:

$$U(t) = \frac{q^2}{2C} = \frac{Q_i^{\,2}}{2C}\, e^{-2t/RC}$$

Substitute U(t) = ¼ (Q_i^2/2C) →

$$\frac{1}{4}\frac{Q_i^{\,2}}{2C} = \frac{Q_i^{\,2}}{2C}\, e^{-2t/RC}$$

$$\frac{1}{4} = e^{-2t/RC}$$

Take the logarithm of both sides of the equation and solve for t →

$$-\ln 4 = -\frac{2t}{RC}$$

$$t = \tfrac{1}{2}RC\ln 4 = 0.693RC = \boxed{0.693\tau}$$

Direct Current Circuits (Ch. 27)

Household Wiring and Electrical Safety [Reading from textbook]

Household Wiring and Electrical Safety

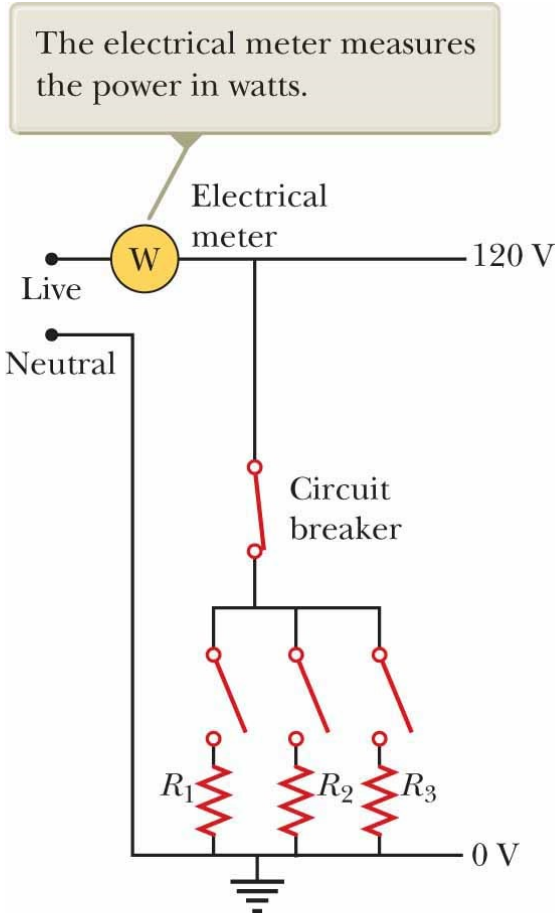
$$P = IV \rightarrow I = \frac{P}{V}$$

$$I_{\text{toaster}} = \frac{1000\text{ W}}{120\text{ V}} = 8.33\text{ A}$$

$$I_{\text{microwave}} = \frac{1300\text{ W}}{120\text{ V}} = 10.8\text{ A}$$

$$I_{\text{coffee maker}} = \frac{800\text{ W}}{120\text{ V}} = 6.67\text{ A}$$

$$I_{\text{total}} = 8.33\text{ A} + 10.8\text{ A} + 6.67\text{ A} = 25.8\text{ A}$$



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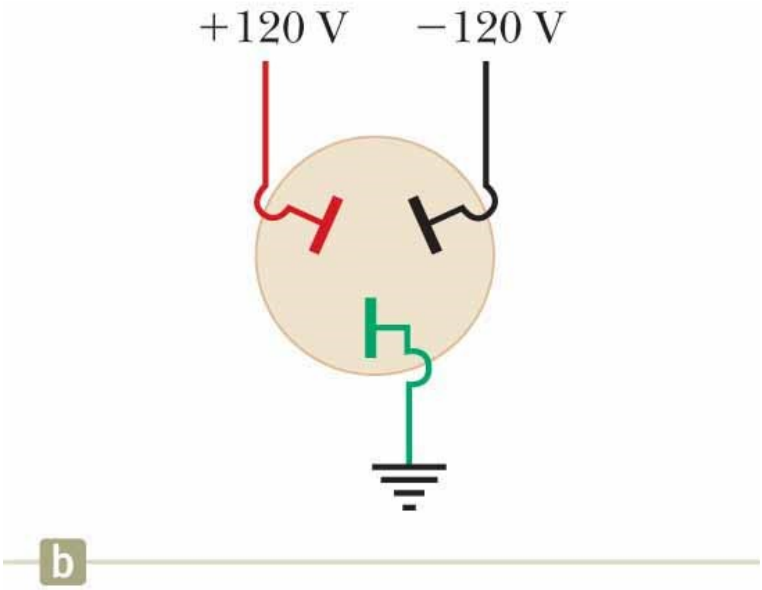
Household Wiring and Electrical Safety



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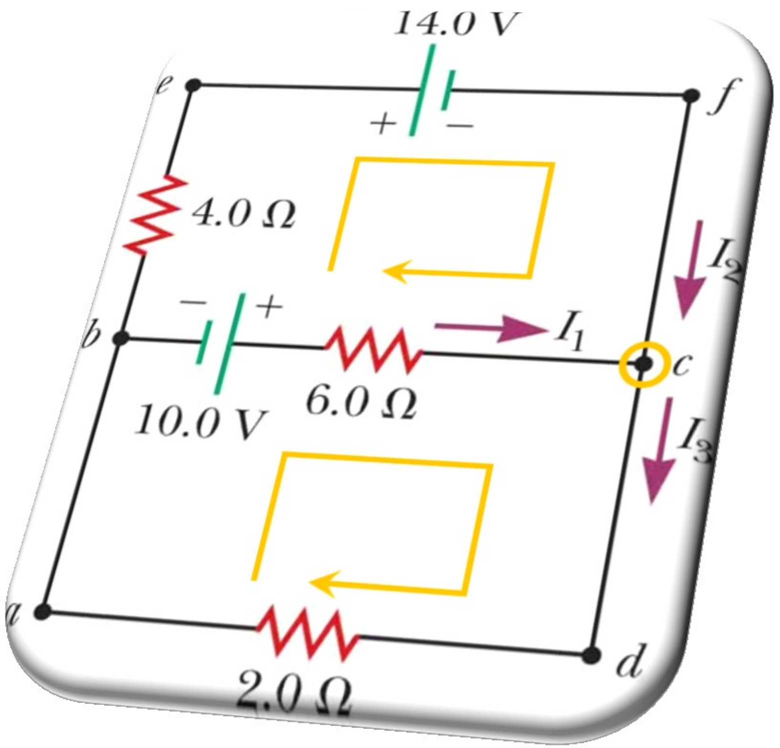
Summary of Week 4, Class 4

- Reminder of the previous week

- Direct Current Circuits (Ch. 27)
  - Electromotive Force
  - Resistors in Series and Parallel
  - Kirchhoff's Rules
  - RC Circuits
  - Household Wiring and Electrical Safety [Reading from textbook]

- Examples
- Next week's topic



Reading / Preparation for Next Week

Topics for next week: Magnetic Fields (Ch.28)