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EQUILIBRIUM OF RIGID BODIES AND FREE BODY DIAGRAM

ENGI 1510 - ENGINEERING DESIGN

Winter 2023

EQUILIBRIUM OF A RIGID BODY & FREE-BODY DIAGRAMS

Today's Objectives:

Students will be able to:

a) Identify support reactions, and,

b) Draw a free-body diagram.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Support Reactions
- Free-Body Diagrams
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Answers:
1. B
2. D

READING QUIZ

1. If a support prevents translation of a body, then the support exerts a ___________ on the body.

   A) Couple moment

   B) Force

   C) Both A and B.

   D) None of the above

2. Internal forces are _________ shown on the free body diagram of a whole body.

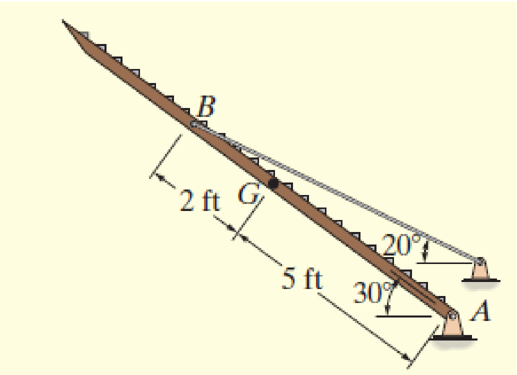
   A) Always

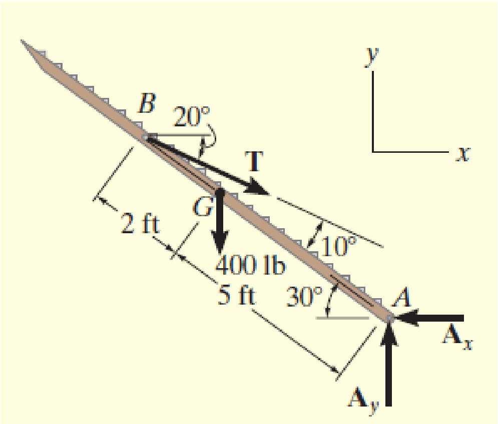
   B) Often

   C) Rarely

   D) Never

APPLICATIONS



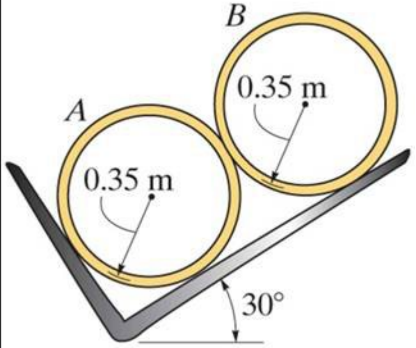


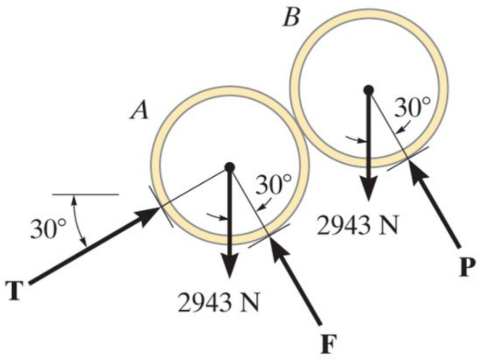
The truck ramps have a weight of 400 lb each.
Each ramp is pinned to the body of the truck and held in the position by a cable. How can we determine the cable tension and support reactions?

How are the idealized model and the free body diagram used to do this?

Which diagram above is the idealized model?

APPLICATIONS (continued)



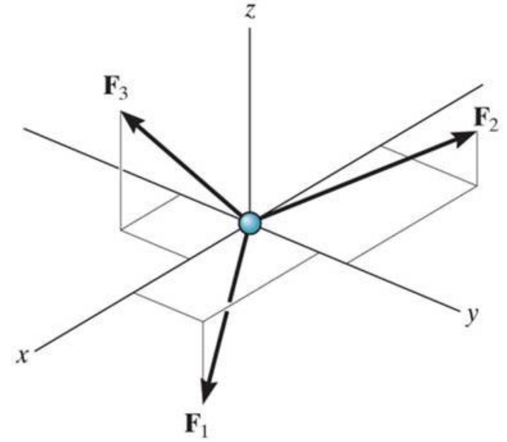


Two smooth pipes, each having a mass of 300 kg, are supported by the tines of the loader's fork attachment.

How can we determine all the reactive forces?

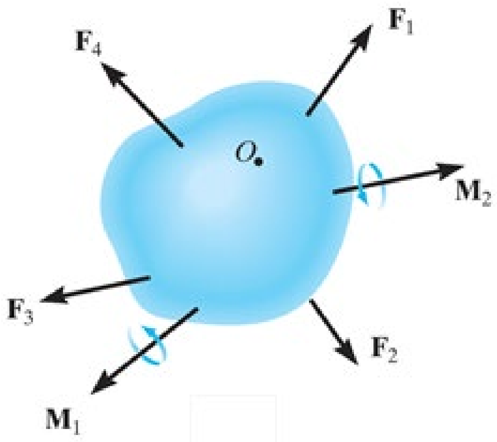
Again, how can we make use of an idealized model and a free body diagram to answer this question?

CONDITIONS FOR RIGID-BODY EQUILIBRIUM (Section 5.1)



Forces on a particle

In contrast to the forces on a particle, the forces on a rigid-body are not usually concurrent and may cause rotation of the body (due to moments created by the forces).



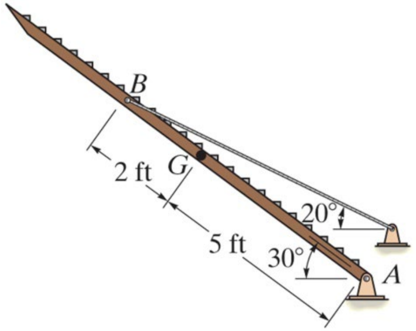
Forces on a rigid body

For a rigid body to be in equilibrium, the net force as well as the net moment about any arbitrary point O must be equal to zero.

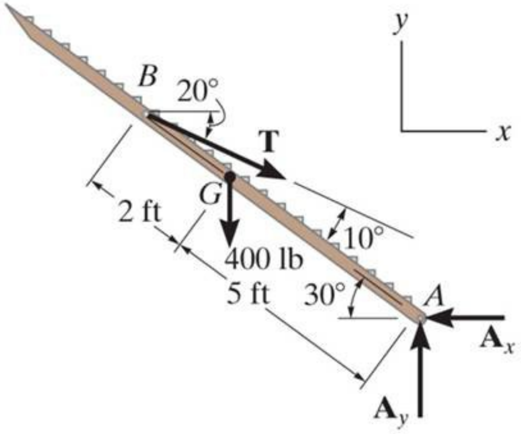
$\sum \boldsymbol{F} = 0$ (no translation)

and $\sum \boldsymbol{M_O} = 0$ (no rotation)

THE PROCESS OF SOLVING RIGID BODY EQUILIBRIUM PROBLEMS



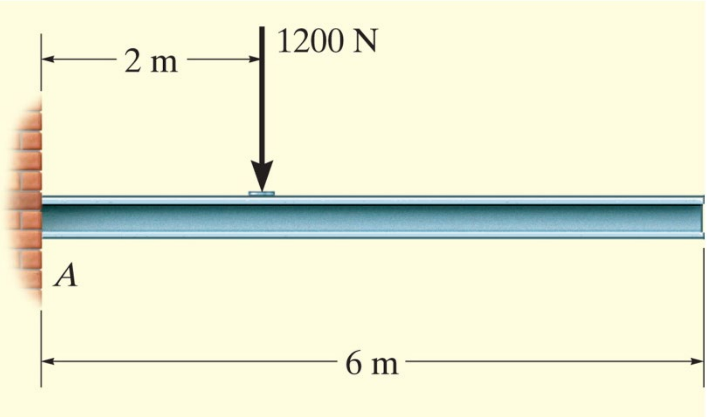
For analyzing an actual physical system, first we need to create an idealized model (above right).



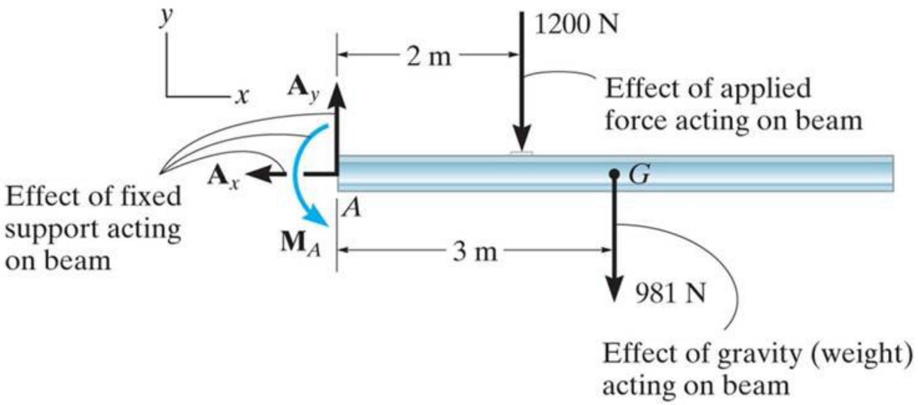
Then we need to draw a free-body diagram (FBD) showing all the external (active and reactive) forces.

Finally, we need to apply the equations of equilibrium to solve for any unknowns.

FREE-BODY DIAGRAMS (Section 5.2)



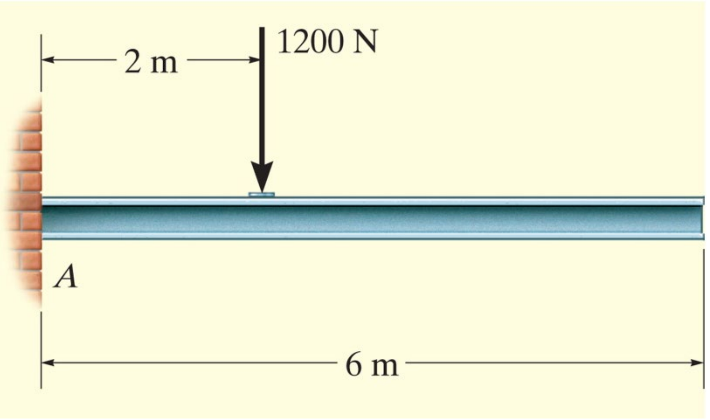
Idealized model



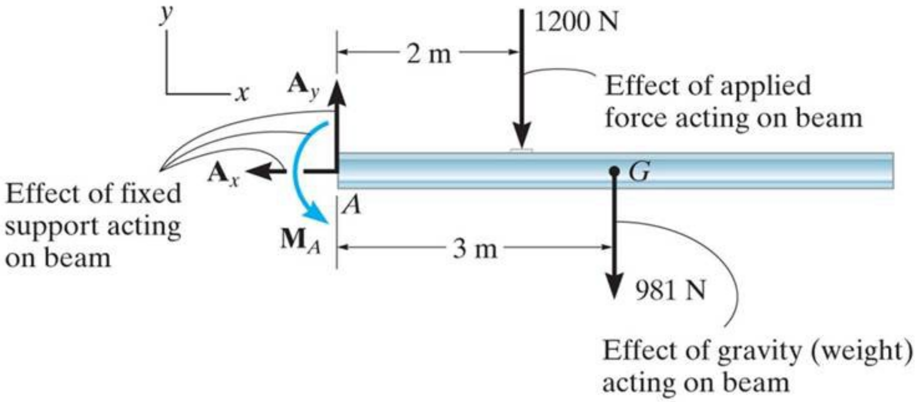
Free-body diagram (FBD)

1. Draw an outlined shape. Imagine the body to be isolated or cut "free" from its constraints and draw its outlined shape.
2. Show all the external forces and couple moments. These typically include: a) applied loads, b) support reactions, and, c) the weight of the body.

FREE-BODY DIAGRAMS (continued)



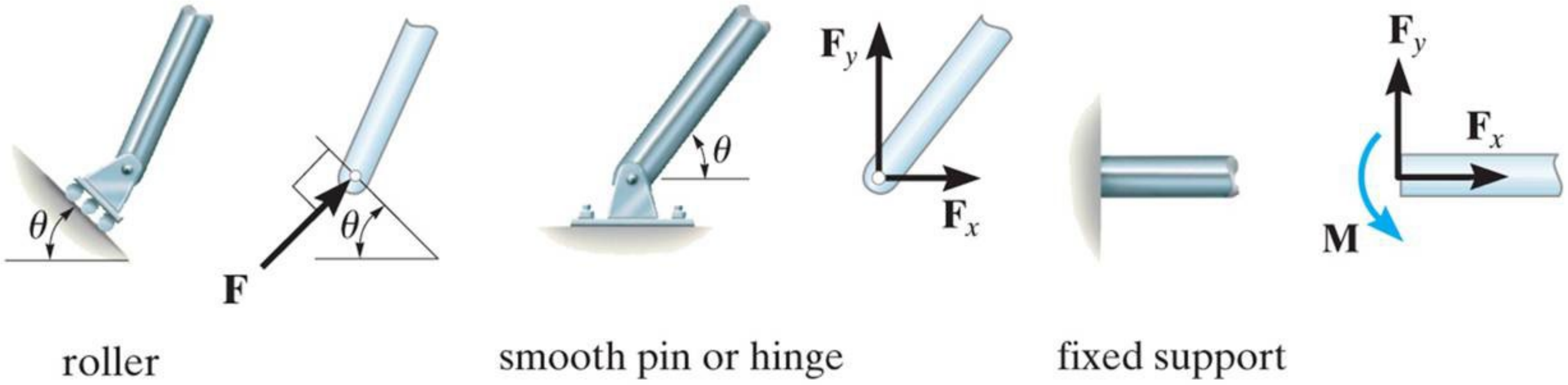
Idealized model



Free-body diagram

3. Label loads and dimensions on the FBD: All known forces and couple moments should be labeled with their magnitudes and directions. For the unknown forces and couple moments, use letters like A_x, A_y, M_A. Indicate any necessary dimensions.

SUPPORT REACTIONS IN 2-D



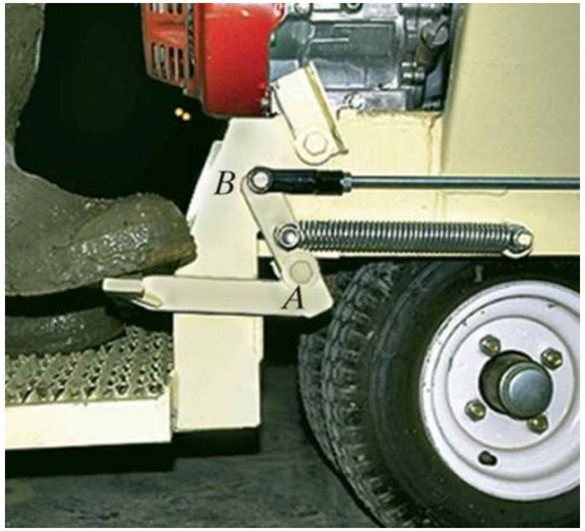
A few example sets of diagrams s are shown above. Other support reactions are given in your textbook (Table 5-1).

As a general rule, if a support prevents translation of a body in a given direction, then a force is developed on the body in the opposite direction.

Similarly, if rotation is prevented, a couple moment is exerted on the body in the opposite direction.

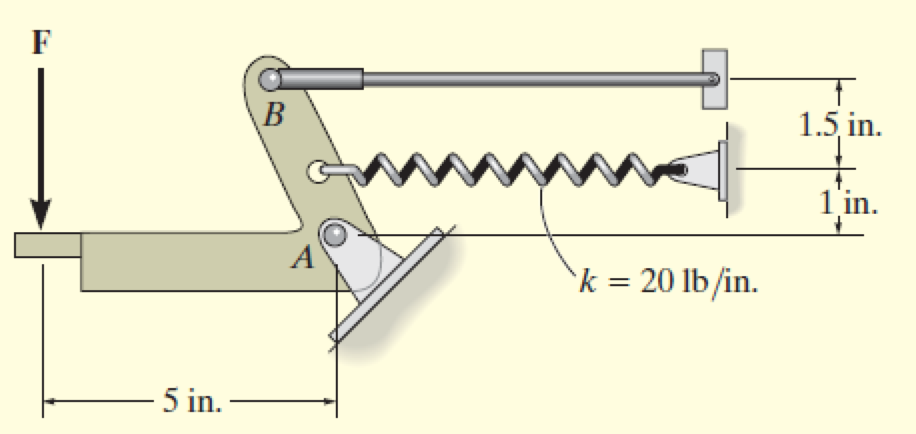
Source : Example 5.2

EXAMPLE I

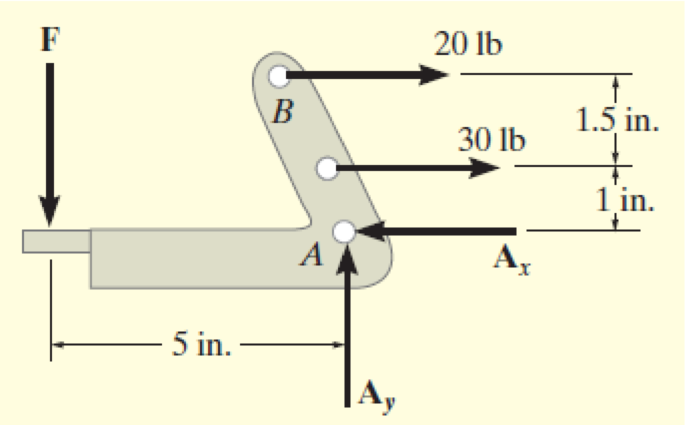


Given: The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at *B is 20 lb*.

Draw: An idealized model and free-body diagram of the foot pedal.



The idealized model



The free-body diagram

Source : Example 5.4

EXAMPLE II

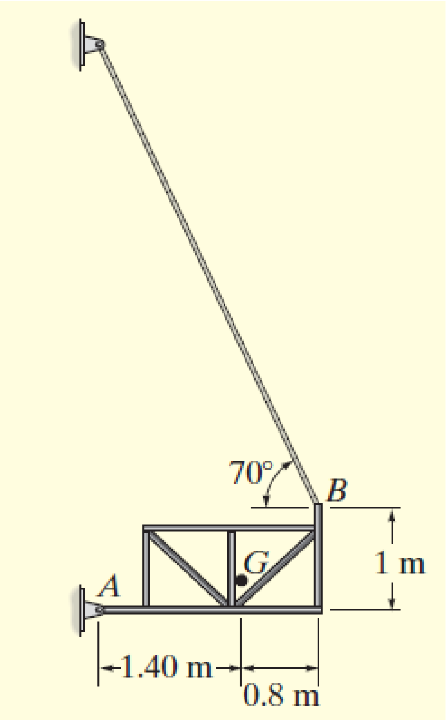
Given: The unloaded platform is suspended off the edge of the oil rig. The platform has a mass of 200 kg.

Draw: An idealized model and free-body diagram of the platform.

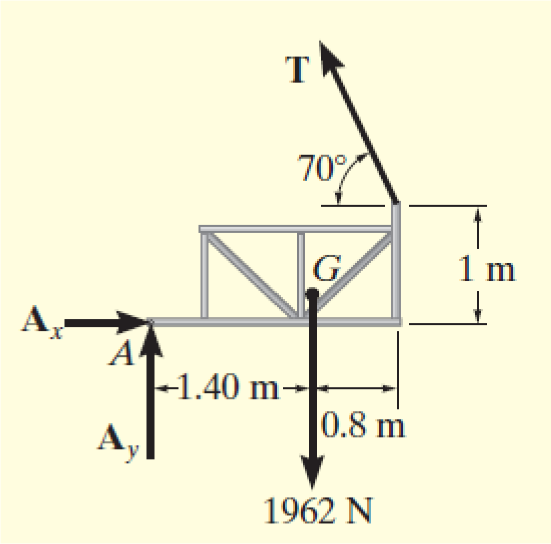
The idealized model of the platform is considered in two dimensions because the loading and the dimensions are all symmetrical about a vertical plane passing through its center.

EXAMPLE II (continued)

The connection at A is treated as a pin, and the cable supports the platform at B. Note the assumed directions of the forces! The point G is the center of gravity of the platform.



The idealized model



The free-body diagram

Answers :
1. C

CONCEPT QUIZ

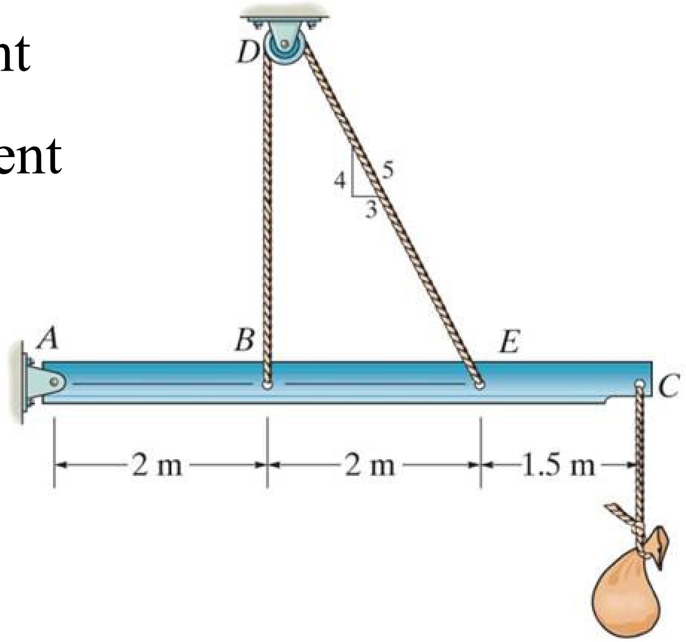
1. The beam and the cable (with a frictionless pulley at D) support an 80 kg load at C. In a FBD of only the beam, there are how many unknowns?

   A) Two forces and one couple moment

   B) Three forces and one couple moment

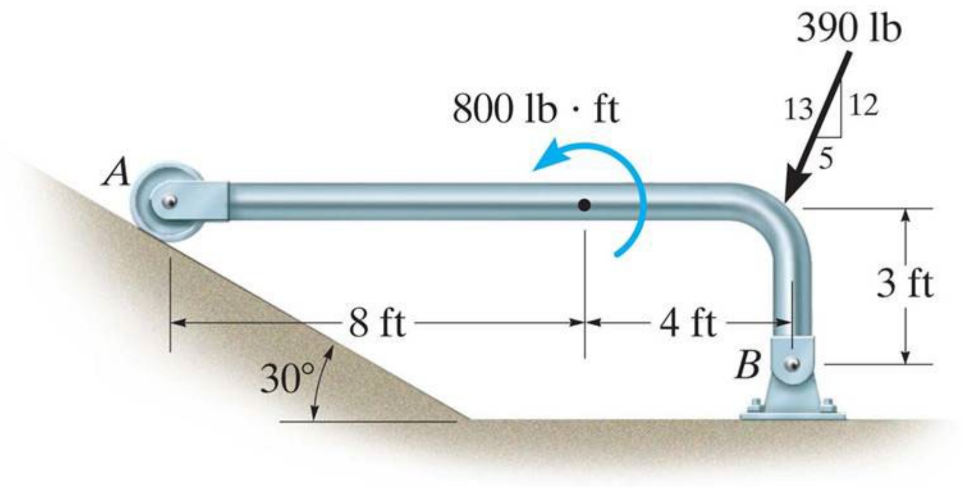
   C) Three forces

   D) Four forces



Answer
2. A

CONCEPT QUIZ (continued)

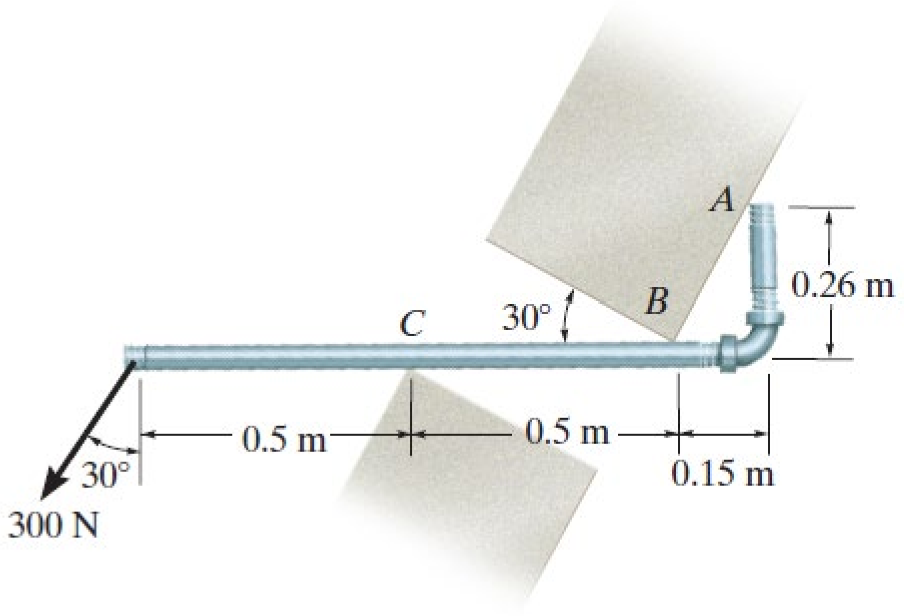


2. If the directions of the force and the couple moments are both reversed, what will happen to the beam?

   A) The beam will lift from A.
   B) The beam will lift at B.
   C) The beam will be restrained.
   D) The beam will break.

GROUP PROBLEM SOLVING I

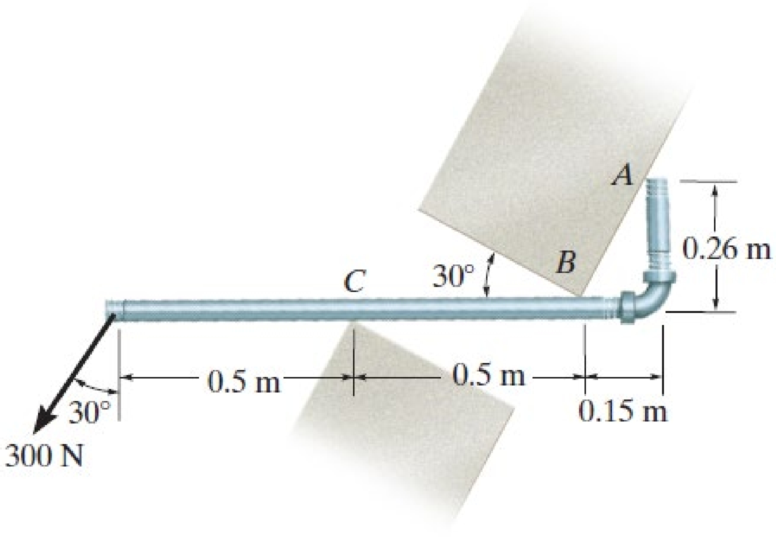
Given:



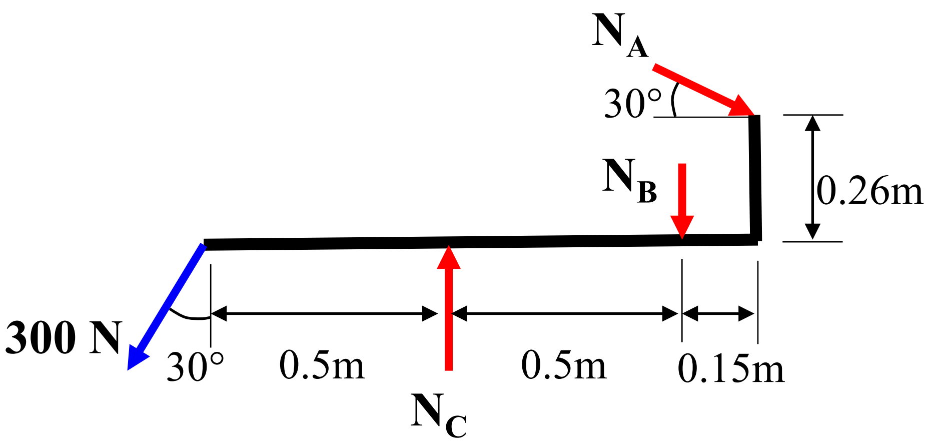
Draw:

A FBD of the smooth pipe which rests against the opening at the points of contact *A*, *B*, and *C*.

GROUP PROBLEM SOLVING I (continued)



The idealized model

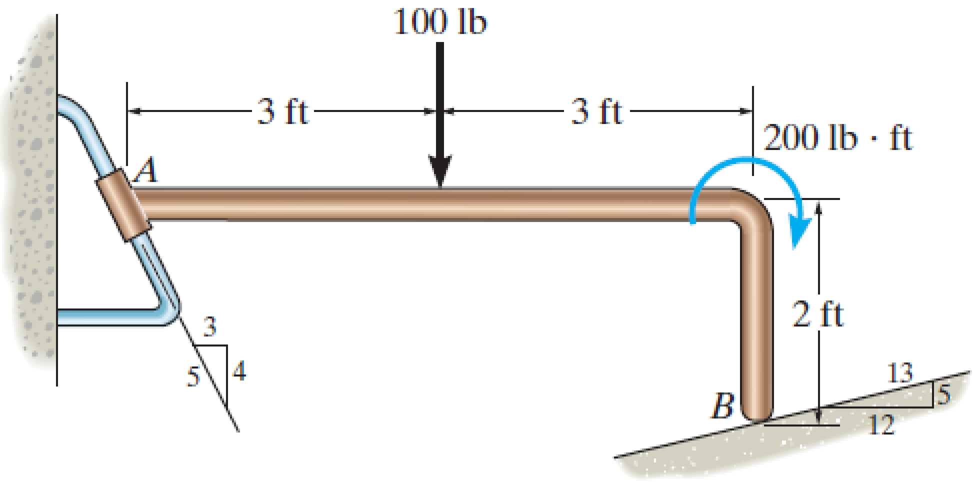


The free body diagram

Source : P5-25

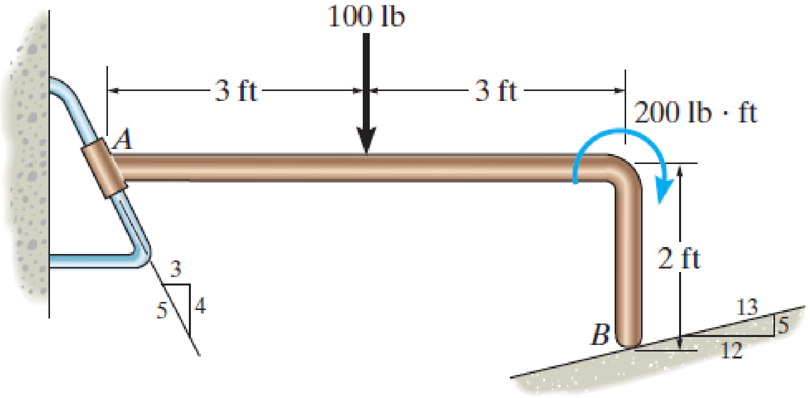
GROUP PROBLEM SOLVING II

Given:

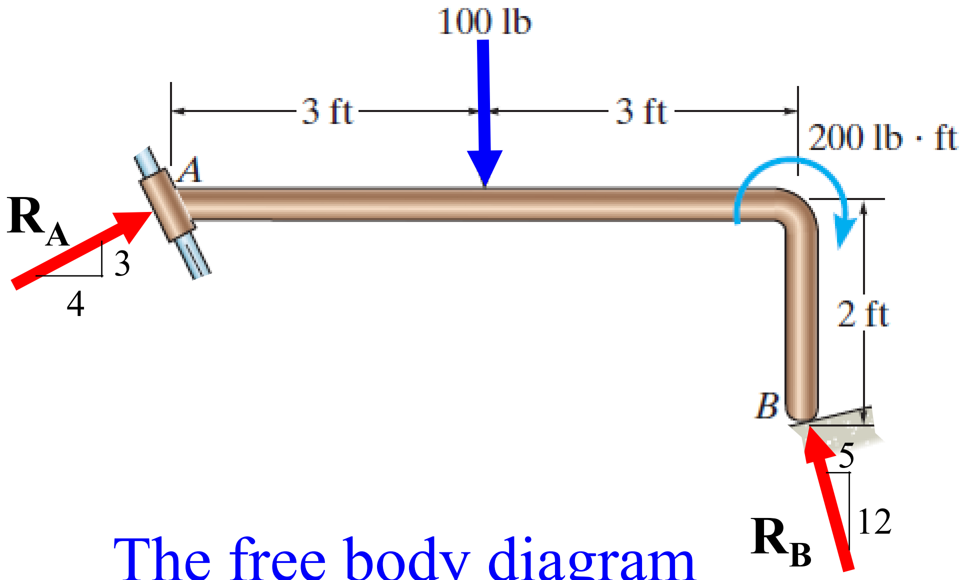


Draw: Draw a FBD of the bent rod supported by a smooth surface at *B* and by a collar at *A*, which is fixed to the rod and is free to slide over the fixed inclined rod.

GROUP PROBLEM SOLVING II (continued)



The idealized model



The free body diagram

Answers:
1.B
2.D

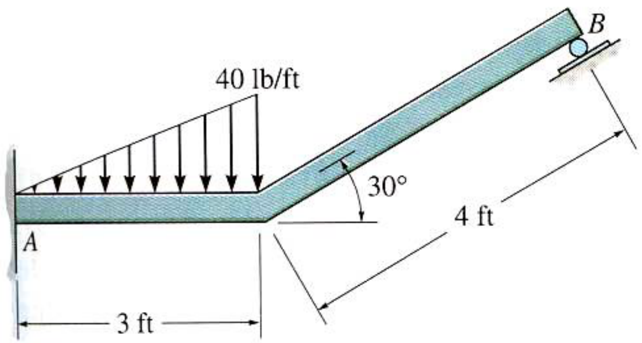
ATTENTION QUIZ

1. Internal forces are not shown on a free-body diagram because the internal forces are _____. (Choose the most appropriate answer.)

A) Equal to zero
B) Equal and opposite and they do not affect the calculations
C) Negligibly small
D) Not important

2. How many unknown support reactions are there in this problem?

A) Two forces and two couple moments
B) One force and two couple moments
C) Three forces
D) Three forces and one couple moment



EQUATIONS OF EQUILIBRIUM & TWO- AND THREE-FORCE MEMEBERS

Today's Objectives:

Students will be able to:

a) Apply equations of equilibrium to solve for unknowns, and,

b) Recognize two-force members.

In-Class Activities:

- Check Homework, if any
- Reading Quiz
- Applications
- Equations of Equilibrium
- Two-Force Members
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Answers:
1. C
2. B

READING QUIZ

1. The three scalar equations, $\sum F_X = \sum F_Y = \sum M_O = 0$, are ____ equations of equilibrium in two dimensions.

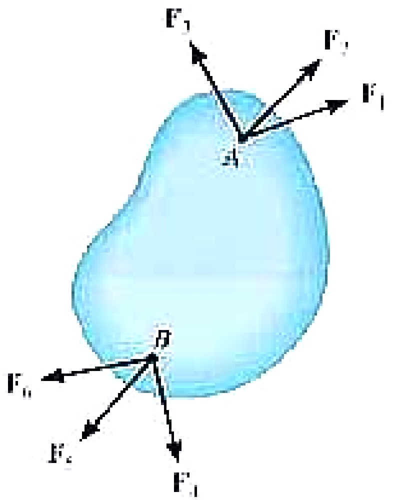
   A) Incorrect B) The only correct

   C) The most commonly used D) Not sufficient

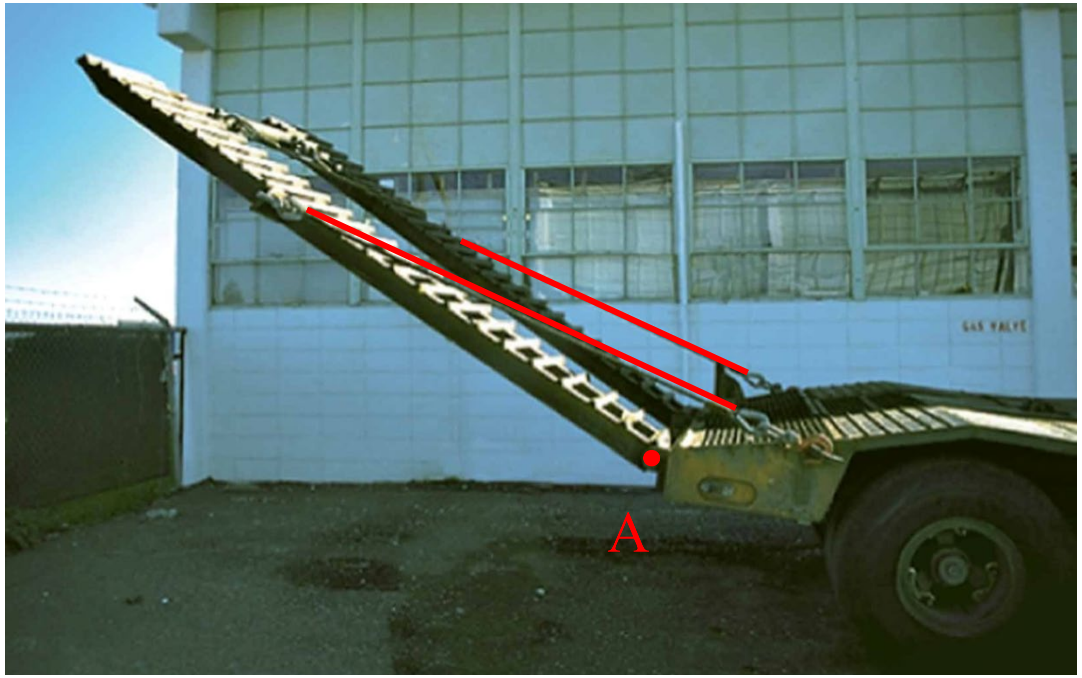
2. A rigid body is subjected to forces as shown. This body can be considered as a ______ member.

   A) Single-force B) Two-force

   C) Three-force D) Six-force



APPLICATIONS

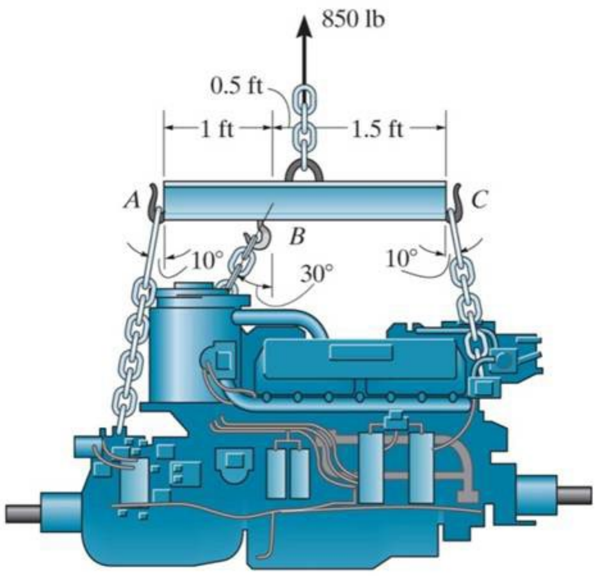


The uniform truck ramp has a weight of 400 lb.

The ramp is pinned at A and held in the position by the cables.

How can we determine the forces acting at the pin A and the force in the cables?

APPLICATIONS (continued)

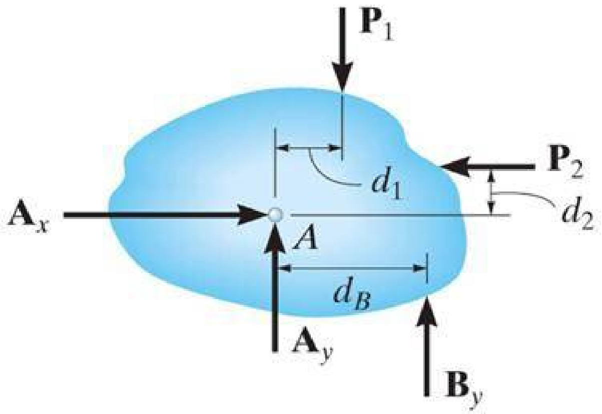


An 850 lb engine is supported by three chains, which are attached to the spreader bar of a hoist.

You need to check to see if the breaking strength of any of the chains is going to be exceeded. How can you determine the force acting in each of the chains?

EQUATIONS OF EQUILIBRIUM (Section 5.3)

A body is subjected to a system of forces that lie in the x-y plane. When in equilibrium, the net force and net moment acting on the body are zero (as discussed earlier in Section 5.1). This 2-D condition can be represented by the three scalar equations:



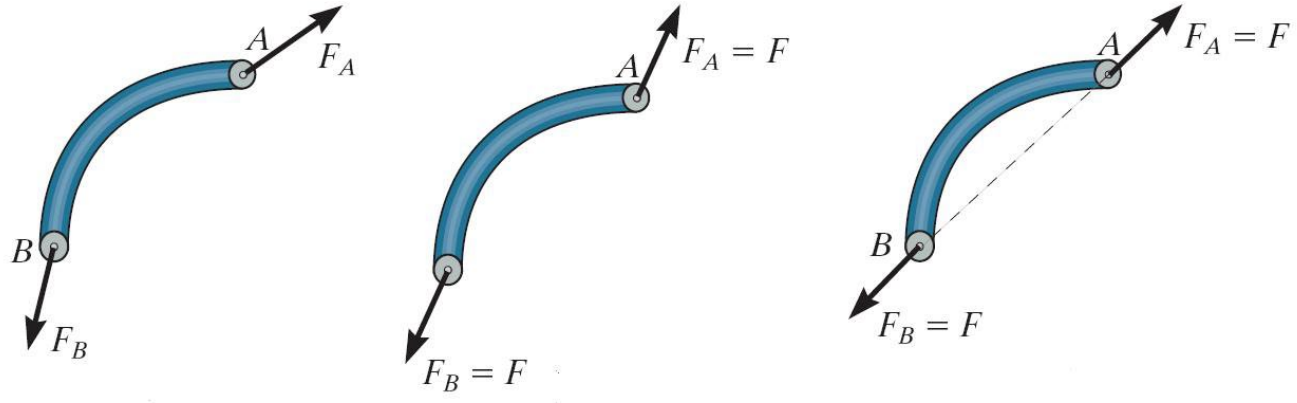
$$\sum F_x = 0 \qquad \sum F_y = 0 \sum M_O = 0$$

where point O is any arbitrary point.

Please note that these equations are the ones most commonly used for solving 2-D equilibrium problems. There are two other sets of equilibrium equations that are rarely used. For your reference, they are described in the textbook.

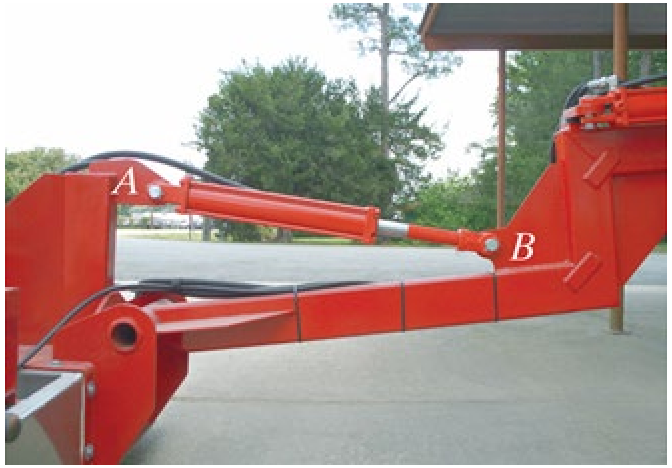
TWO-FORCE MEMBERS & THREE FORCE-MEMBERS (Section 5.4)

The solution to some equilibrium problems can be simplified if we recognize members that are subjected to forces at only two points (e.g., at points A and B in the figure below).

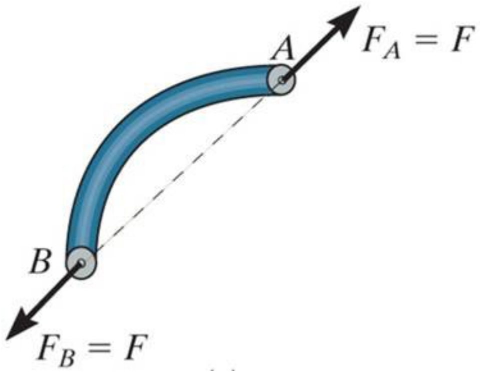


If we apply the equations of equilibrium to such a member, we can quickly determine that the resultant forces at A and B must be equal in magnitude and act in the opposite directions along the line joining points A and B.

EXAMPLES OF TWO-FORCE MEMBERS



In the cases above, members AB can be considered as two-force members, provided that their weight is neglected.



This fact simplifies the equilibrium analysis of some rigid bodies since the directions of the resultant forces at A and B are thus known (along the line joining points A and B).

STEPS FOR SOLVING 2-D EQUILIBRIUM PROBLEMS

1. If not given, establish a suitable x - y coordinate system.

2. Draw a free-body diagram (FBD) of the object under analysis.

3. Apply the three equations of equilibrium (E-of-E) to solve for the unknowns.

IMPORTANT NOTES

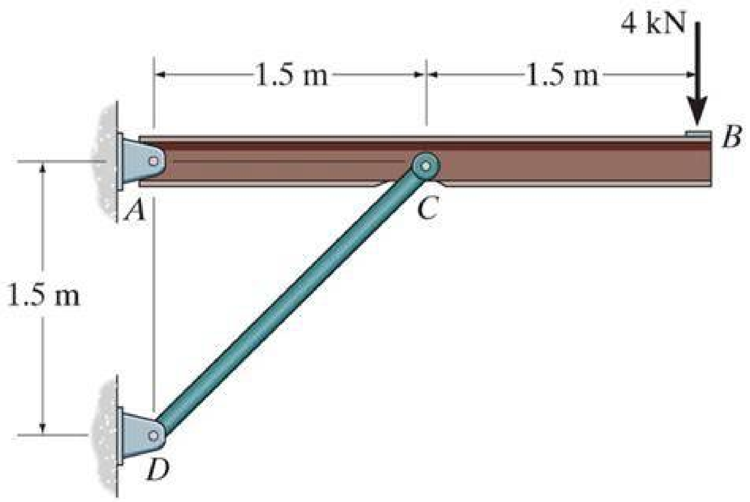
1. If there are more unknowns than the number of independent equations, then we have a statically indeterminate situation. We cannot solve these problems using just statics.

2. The order in which we apply equations may affect the simplicity of the solution. For example, if we have two unknown vertical forces and one unknown horizontal force, then solving $\sum F_X = 0$ first allows us to find the horizontal unknown quickly.

3. If the answer for an unknown comes out as negative number, then the sense (direction) of the unknown force is opposite to that assumed when starting the problem.

Source : F5-2

EXAMPLE



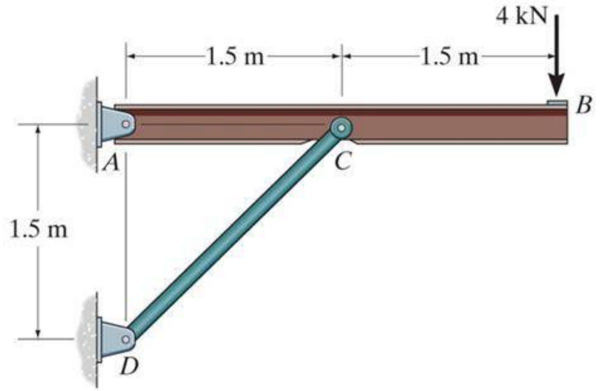
Given: The 4kN load at B of the beam is supported by pins at A and C.

Find: The support reactions at A and C.

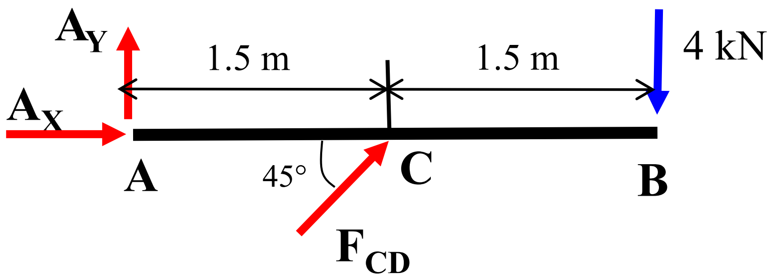
Plan:

1. Put the x and y-axes in the horizontal and vertical directions, respectively.
2. Determine if there are any two-force members.
3. Draw a complete FBD of the boom.
4. Apply the E-of-E to solve for the unknowns.

EXAMPLE (continued)



FBD of the beam:



Note: Upon recognizing CD as a two-force member, the number of unknowns at C is reduced from two to one. Now, using E-o-f E, we get,

$$\circlearrowleft + \ \Sigma M_A = F_{CD} \sin 45° \times 1.5 - 4 \times 3 = 0$$

$$F_{CD} = 11.31 \text{ kN} \ \text{or} \ \underline{11.3 \text{ kN}}$$

$$\rightarrow + \Sigma F_X = A_X + 11.31 \cos 45° = 0; \quad \underline{A_X = -8.00 \text{ kN}}$$

$$\uparrow + \Sigma F_Y = A_Y + 11.31 \sin 45° - 4 = 0; \quad \underline{A_Y = -4.00 \text{ kN}}$$

Note that the negative signs means that the reactions have the opposite directions to that assumed (as originally shown on FBD).

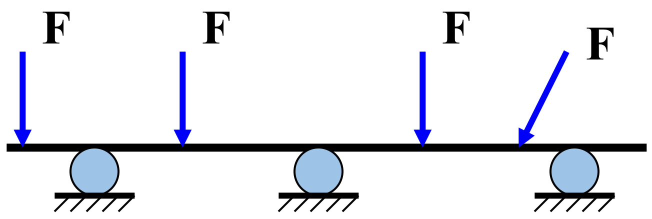
Answers:
1. D
2. B

CONCEPT QUIZ

1. For this beam, how many support reactions are there and is the problem statically determinate?

   A) (2, Yes) B) (2, No)

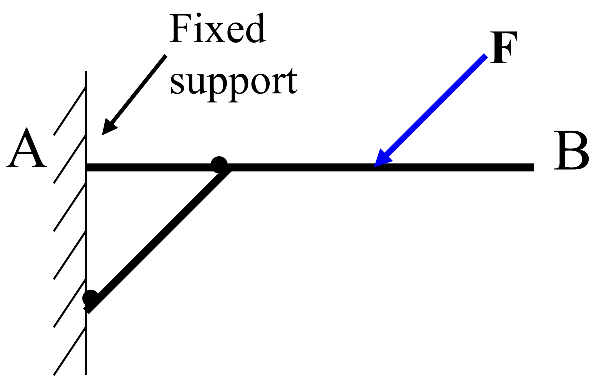
   C) (3, Yes) D) (3, No)



2. The beam AB is loaded and supported as shown: a) how many support reactions are there on the beam, b) is this problem statically determinate, and c) is the structure stable?

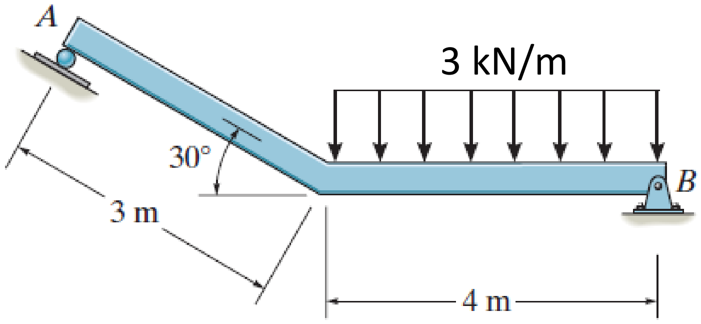
   A) (4, Yes, No) B) (4, No, Yes)

   C) (5, Yes, No) D) (5, No, Yes)



Source : P5-22

GROUP PROBLEM SOLVING



Given: The beam is supported by the roller at A and a pin at B.

Find: The reactions at points A and B on the beam.

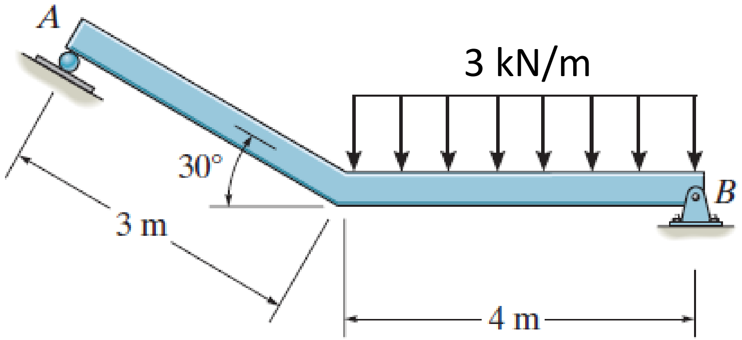
Plan:

a) Establish the x–y axis system.

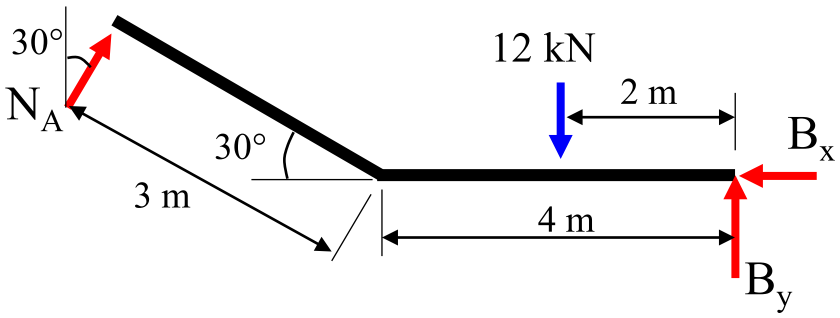
b) Draw a complete FBD of the beam.

c) Apply the E-of-E to solve for the unknowns.

GROUP PROBLEM SOLVING (continued)



FBD of the beam



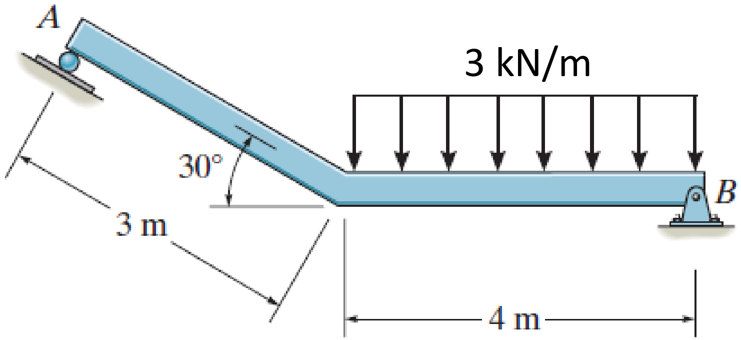
Note that the distributed load has been reduced to a single force.

First, write a moment equation about point B. Why point B?

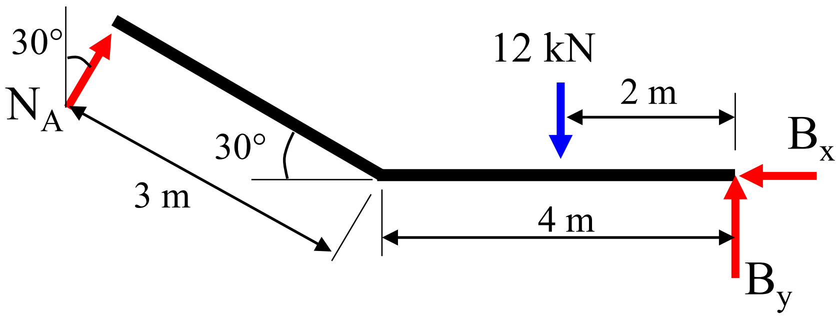
$$\circlearrowleft + \sum M_B = -(N_A \cos 30°) \times (4 + 3 \cos 30°) - (N_A \sin 30°) \times (3 \sin 30°) + 12 \times 2 = 0$$

$N_A = 3.713 = 3.71$ kN

GROUP PROBLEM SOLVING (continued)



FBD of the beam



Recall N_A = 3.713 =3.71 kN

Now write the $\sum F_X = \sum F_Y = 0$ equations.

$\rightarrow + \sum F_X = 3.713 \sin 30° - B_x = 0$

$\uparrow + \sum F_Y = 3.713 \cos 30° - 12 + B_y = 0$

Solving these two equations, we get

$\underline{B_x = 1.86 \text{ kN}}$ $\leftarrow$

$\underline{B_y = 8.78 \text{ kN}}$ $\uparrow$

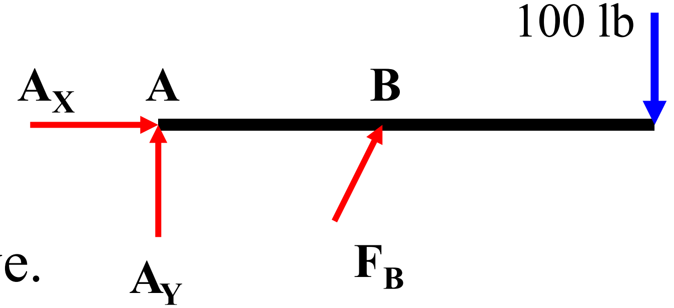
Answers:
1. C
2. B

ATTENTION QUIZ

1. Which equation of equilibrium allows you to determine F_B right away?

   A) $\sum F_X = 0$ B) $\sum F_Y = 0$

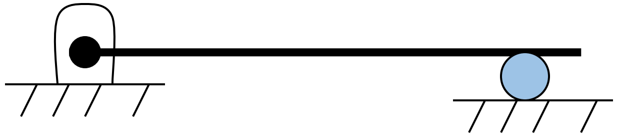
   C) $\sum M_A = 0$ D) Any one of the above.



2. A beam is supported by a pin joint and a roller. How many support reactions are there and is the structure stable for all types of loadings?

   A) (3, Yes) B) (3, No)

   C) (4, Yes) D) (4, No)



3-D FREE-BODY DIAGRAMS, EQUILIBRIUM EQUATIONS, CONSTRAINTS AND STATICAL DETERMINACY

Today's Objective:

Students will be able to:

a) Identify support reactions in 3-D and draw a free-body diagram, and,

b) Apply the equations of equilibrium.

In-Class Activities:

- Check Homework, if any
- Reading Quiz
- Applications
- Support Reactions in 3-D
- Equations of Equilibrium
- Concept Quiz
- Group Problem Solving
- Attention quiz

ANSWERS:
1. A
2. D

READING QUIZ

1. If a support prevents rotation of a body about an axis, then the support exerts a _________ on the body about that axis.

   A) Couple moment B) Force

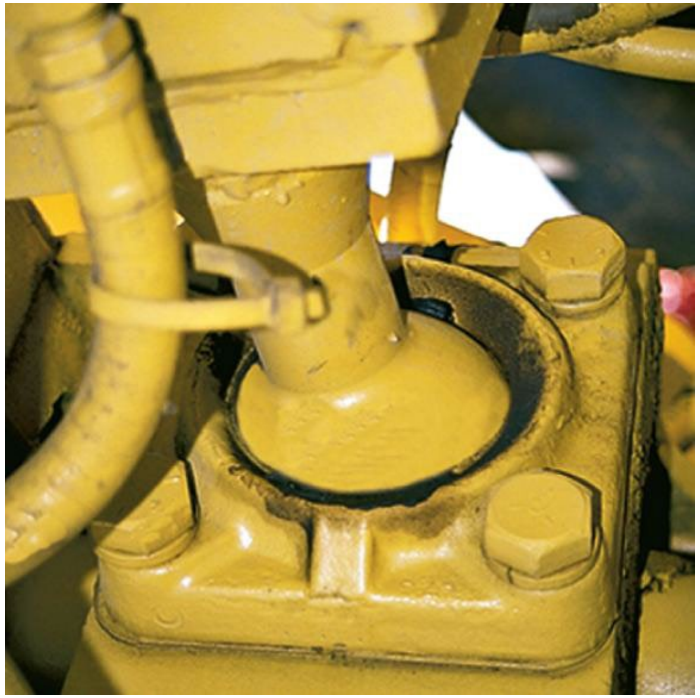
   C) Both A and B. D) None of the above.

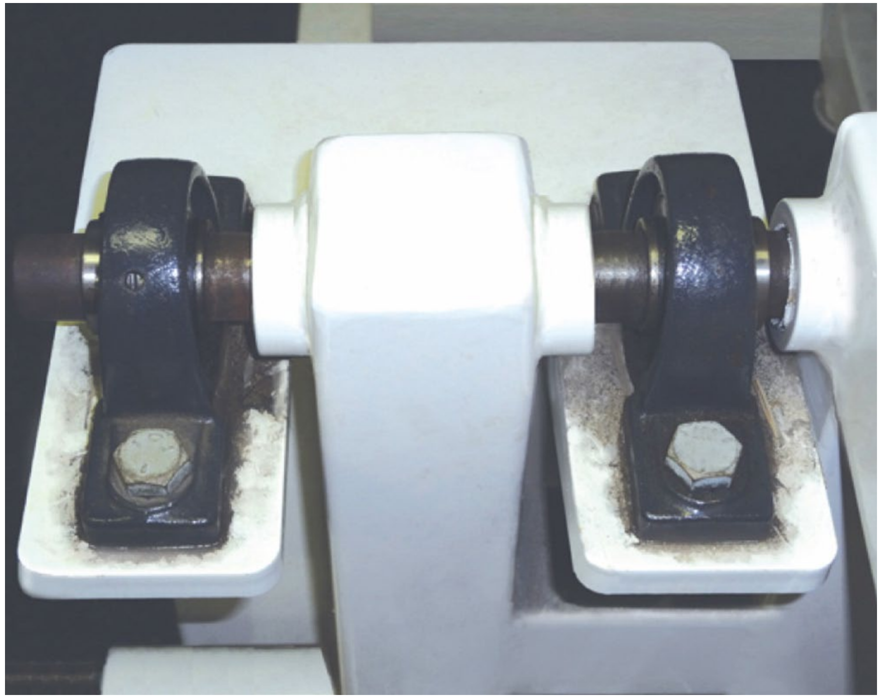
2. When doing a 3-D problem analysis, you have _________ scalar equations of equilibrium.

   A) 3 B) 4

   C) 5 D) 6

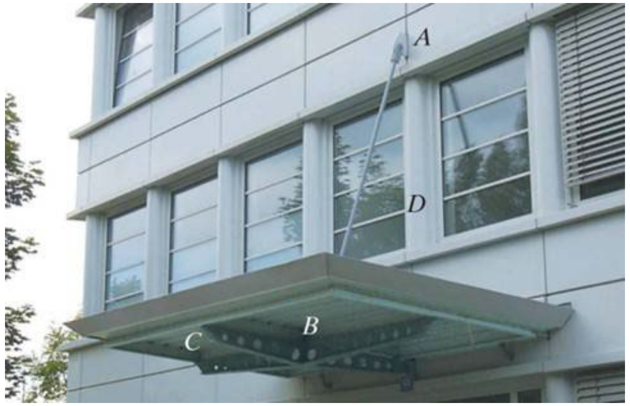
APPLICATIONS





Ball-and-socket joints and journal bearings are often used in mechanical systems. To design the joints or bearings, the support reactions at these joints and the loads must be determined.

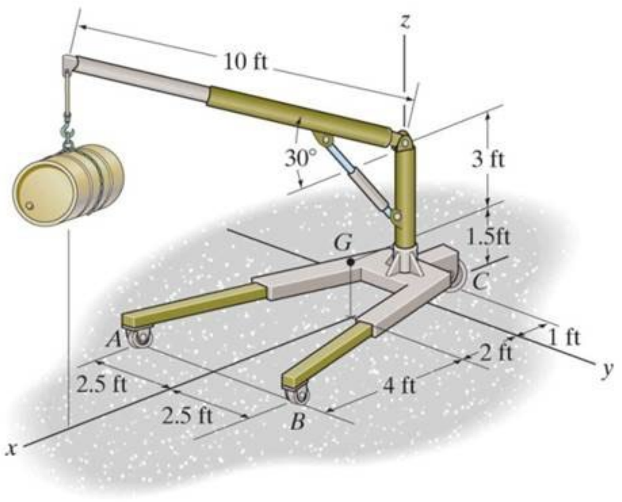
APPLICATIONS (continued)



The tie rod from point A is used to support the overhang at the entrance of a building. It is pin connected to the wall at A and to the center of the overhang B.

If A is moved to a lower position D, will the force in the rod change or remain the same? By making such a change without understanding if there is a change in forces, failure might occur.

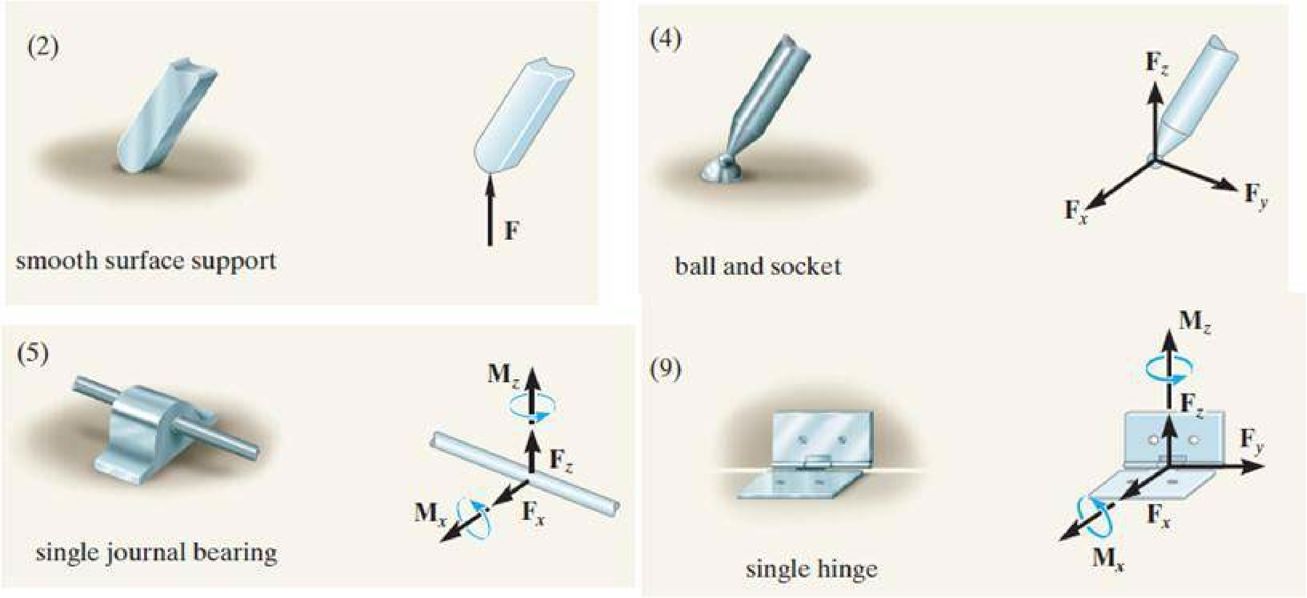
APPLICATIONS (continued)



The floor crane, which weighs 350 lb, is supporting a oil drum.

How do you determine the largest oil drum weight that the crane can support without overturning?

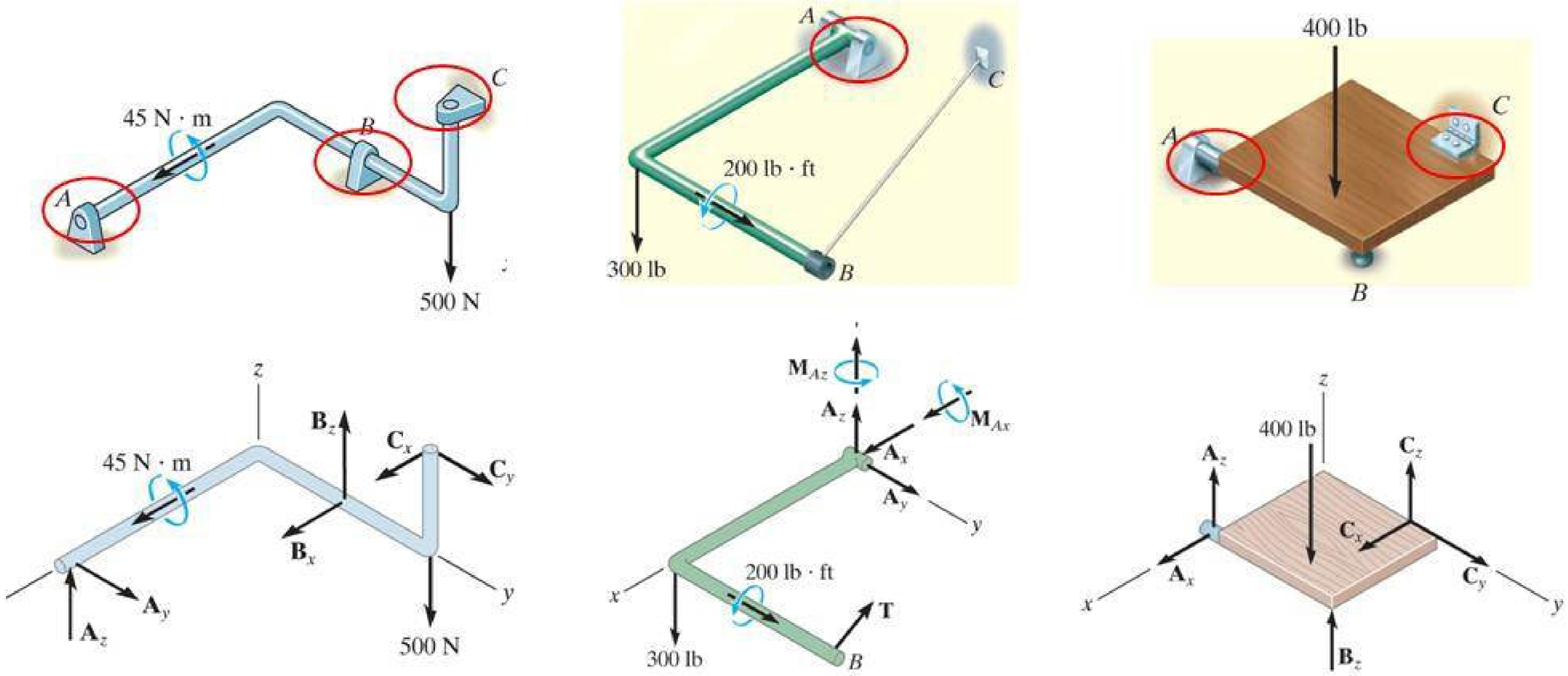
SUPPORT REACTIONS IN 3-D (Table 5-2)



A few examples of supports are shown above. Other support reactions are given in your textbook (Table 5-2).

As a general rule, if a support prevents translation of a body in a given direction, then a reaction force acting in the opposite direction is developed on the body. Similarly, if rotation is prevented, a couple moment is exerted on the body by the support.

IMPORTANT NOTE



A single bearing or hinge can prevent rotation by providing a resistive couple moment. However, it is usually preferred to use two or more properly aligned bearings or hinges. In these cases, only force reactions are generated and no moment reactions are created.

EQUATIONS OF EQUILIBRIUM (Section 5.6)

As stated earlier, when a body is in equilibrium, the net force and the net moment equal zero, i.e., $\sum \boldsymbol{F} = 0$ and $\sum \boldsymbol{M_O} = 0$.

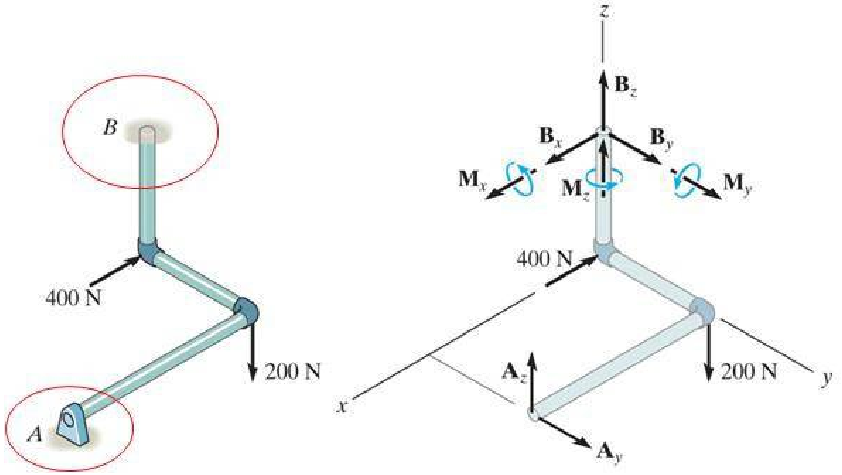
These two vector equations can be written as six scalar equations of equilibrium (E-of-E). These are

$$\sum F_X = \sum F_Y = \sum F_Z = 0$$

$$\sum M_X = \sum M_Y = \sum M_Z = 0$$

The moment equations can be determined about any point. Usually, choosing the point where the maximum number of unknown forces are present simplifies the solution. Any forces passing through the point where moments are taken do not appear in the moment equation.

CONSTRAINTS AND STATICAL DETERMINACY
(Section 5.7)

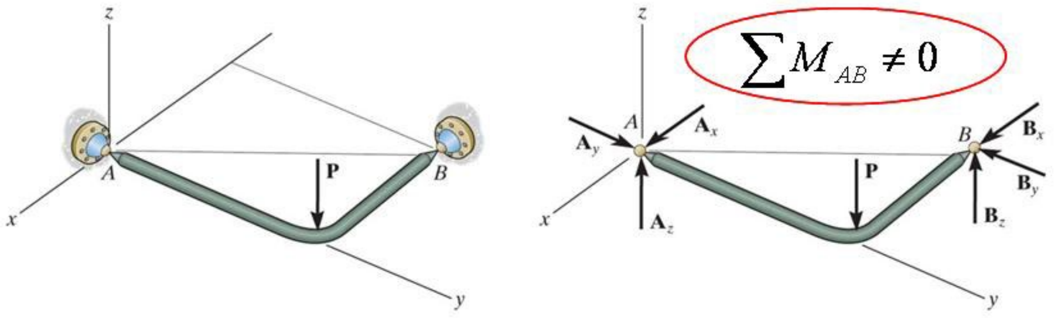


Redundant Constraints: When a body has more supports than necessary to hold it in equilibrium, it becomes statically indeterminate.

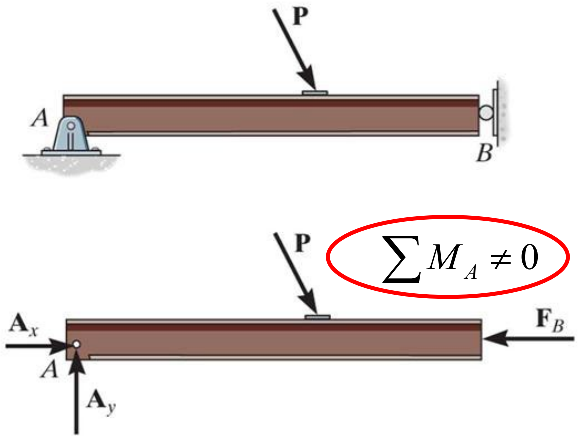
A problem that is statically indeterminate has more unknowns than equations of equilibrium.

Are statically indeterminate structures used in practice? Why or why not?

IMPROPER CONSTRAINTS



Here, while we have 6 unknowns, there is nothing restricting rotation about the AB axis!

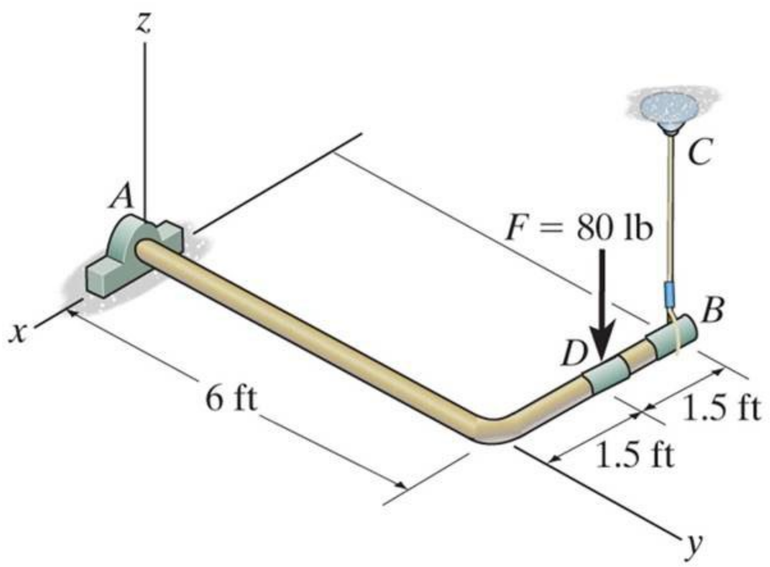


In some cases, there may be as many unknown reactions as there are equations of equilibrium.

However, if the supports are not properly constrained, the body may become unstable for some loading cases.

Source : F5-12

EXAMPLE I



Given: The rod, supported by thrust bearing at A and cable BC, is subjected to an 80 lb force.

Find: Reactions at the thrust bearing A and cable BC.

Plan:

a) Use the established x, y and z-axes.

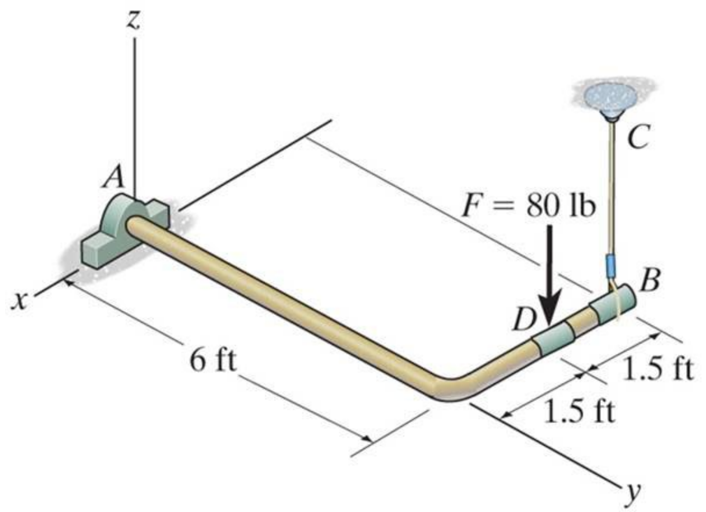
b) Draw a FBD of the rod.

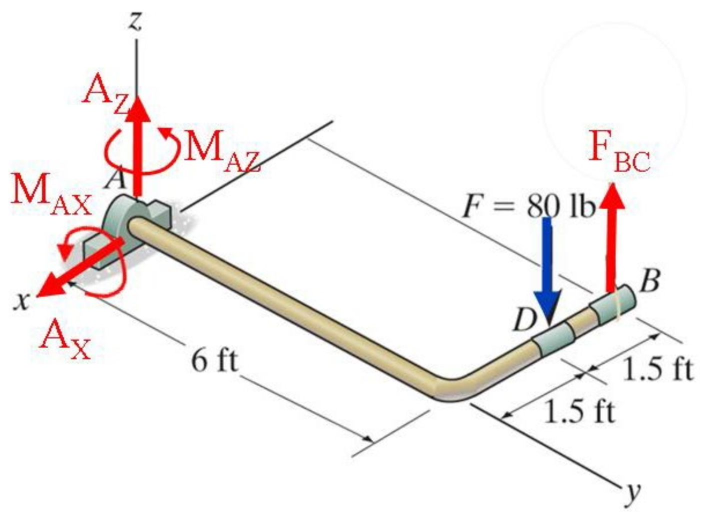
c) Write the forces using scalar equations.

d) Apply scalar equations of equilibrium to solve for the unknown forces.

EXAMPLE I (continued)

FBD of the rod:





Applying scalar equations of equilibrium in appropriate order, we get

$\sum F_X = A_X = 0;$ $\underline{A_X = 0}$

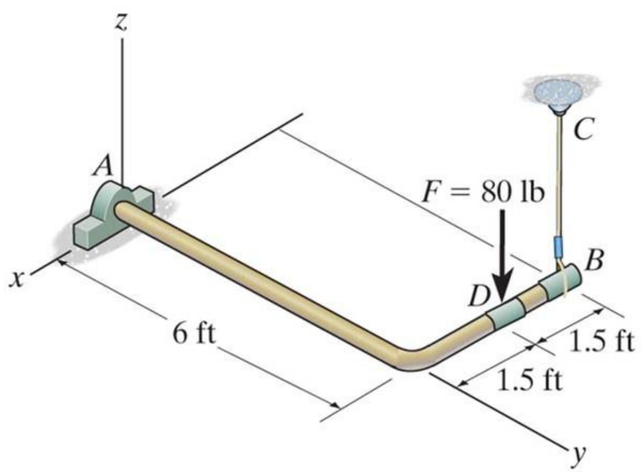
$\sum F_Z = A_Z + F_{BC} - 80 = 0;$

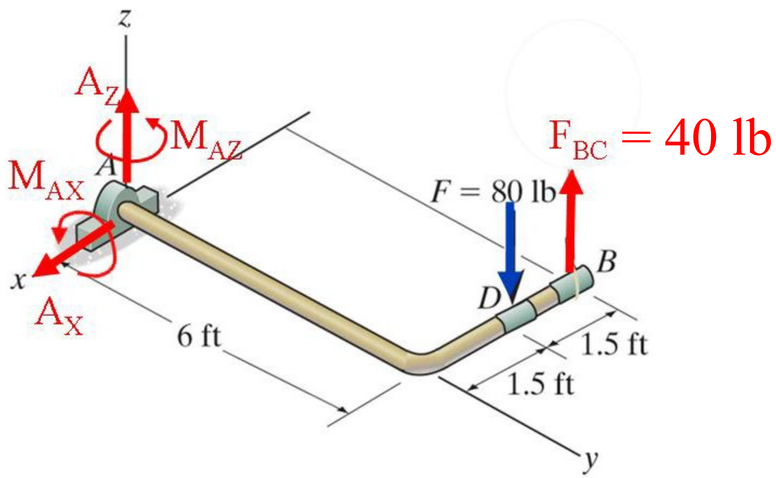
$\sum M_Y = -80\,(1.5) + F_{BC}\,(3.0) = 0;$

Solving the last two equations: $\underline{F_{BC} = 40 \text{ lb}}$, $\underline{A_Z = 40 \text{ lb}}$

EXAMPLE I (continued)

FBD of the rod





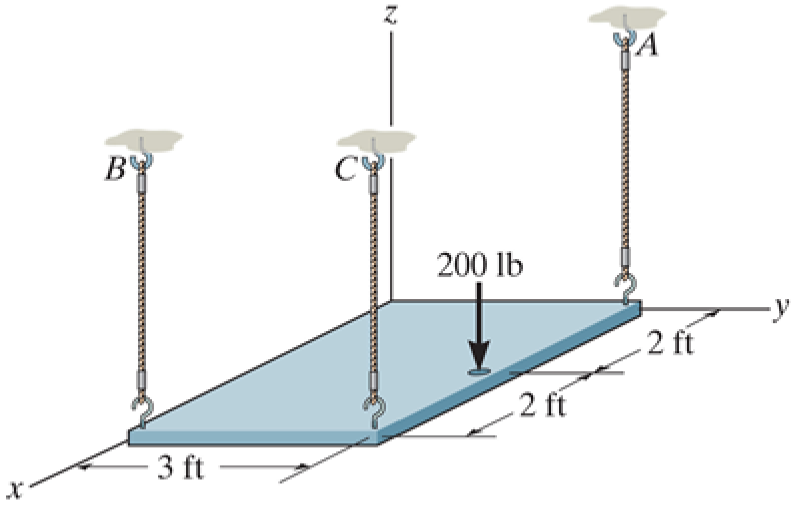
Now write scalar moment equations about what point? Point A!

$M_X = (M_A)_X + 40\,(6) - 80\,(6) = 0$; $(M_A)_X = 240$ lb ft CCW

$\sum M_Z = (M_A)_Z = 0$; $(M_A)_Z = 0$

Source : F5-7

EXAMPLE II



Given: The uniform plate has a weight of 500 lb, supported by three cables.

Find: The tension in each of the supporting cables.

Plan:

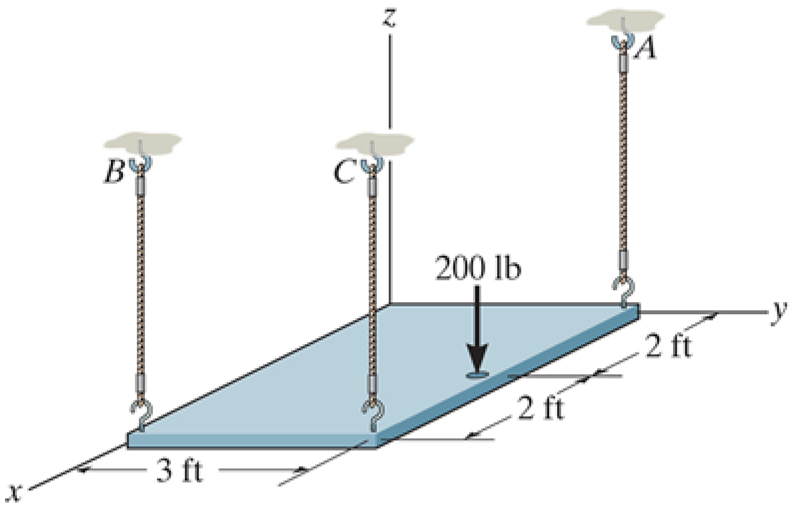
a) Use established x, y and z-axes.

b) Draw a FBD of the plate.

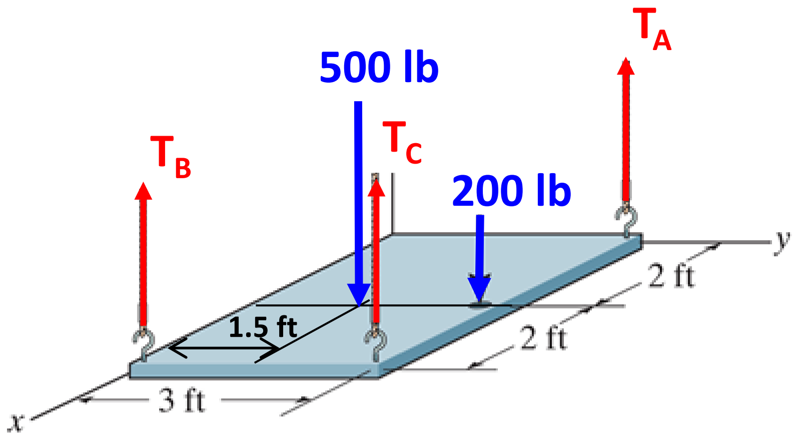
c) Write the forces using scalar equations.

d) Apply scalar equations of equilibrium to solve for the unknown forces.

EXAMPLE II (continued)



FBD of the plate:



Applying scalar equations of equilibrium:

$$\Sigma F_z = T_A + T_B + T_C - 200 - 500 = 0 \qquad (1)$$

$$\Sigma M_x = T_A (3) + T_C (3) - 500 (1.5) - 200 (3) = 0 \qquad (2)$$

$$\Sigma M_y = - T_B (4) - T_C (4) + 500 (2) + 200 (2) = 0 \qquad (3)$$

EXAMPLE II (continued)

$$\Sigma F_z = T_A + T_B + T_C - 200 - 500 = 0 \qquad (1)$$

$$\Sigma M_x = T_A\,(3) + T_C\,(3) - 500\,(1.5) - 200\,(3) = 0 \qquad (2)$$

$$\Sigma M_y = -T_B\,(4) - T_C\,(4) + 500\,(2) + 200\,(2) = 0 \qquad (3)$$

Using Eqs. (2) and (3), express T_A and T_B in terms of T_C:

Eq. (2) $\Rightarrow$ $T_A = 450 - T_C$

Eq. (3) $\Rightarrow$ $T_B = 350 - T_C$

Substituting the results into Eq. (1) & solving for T_C

Eq. (1) $\Rightarrow$ $(450 - T_C) + (350 - T_C) + T_C - 200 - 500 = 0$

$\underline{T_C = 100 \text{ lb}}$ $\uparrow$

$\underline{T_A = 350 \text{ lb}}$ $\uparrow$ and $\underline{T_A = 250 \text{ lb}}$ $\uparrow$

ANSWERS :
1. B

CONCEPT QUIZ

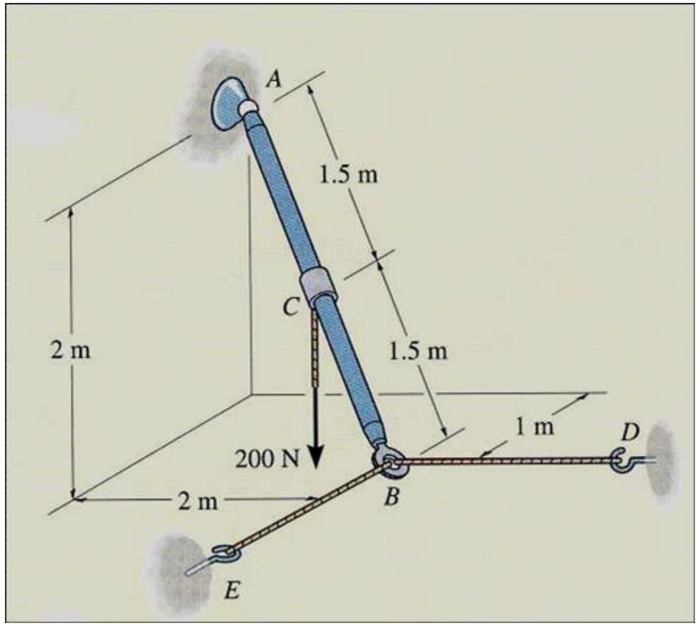
1. The rod AB is supported using two cables at B and a ball-and-socket joint at A. How many unknown support reactions exist in this problem?

   A) Five force and one moment reaction

   B) Five force reactions

   C) Three force and three moment reactions

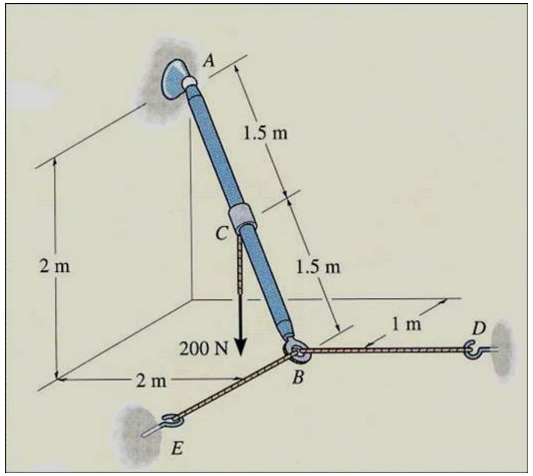
   D) Four force and two moment reactions



ANSWERS:
2. D

CONCEPT QUIZ (continued)

2. If an additional couple moment in the vertical direction is applied to rod AB at point C, then what will happen to the rod?



A) The rod remains in equilibrium as the cables provide the necessary support reactions.

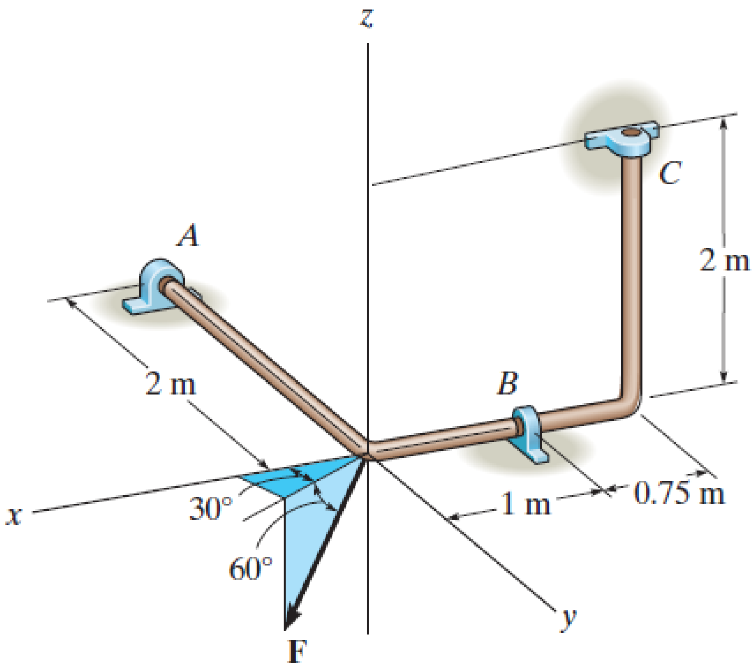
B) The rod remains in equilibrium as the ball-and-socket joint will provide the necessary resistive reactions.

C) The rod becomes unstable as the cables cannot support compressive forces.

D) The rod becomes unstable since a moment about AB cannot be restricted.

Source : P5-73

GROUP PROBLEM SOLVING



Given: A bent rod is supported by smooth journal bearings at A, B, and C. F=800 N. Assume the rod is properly aligned.

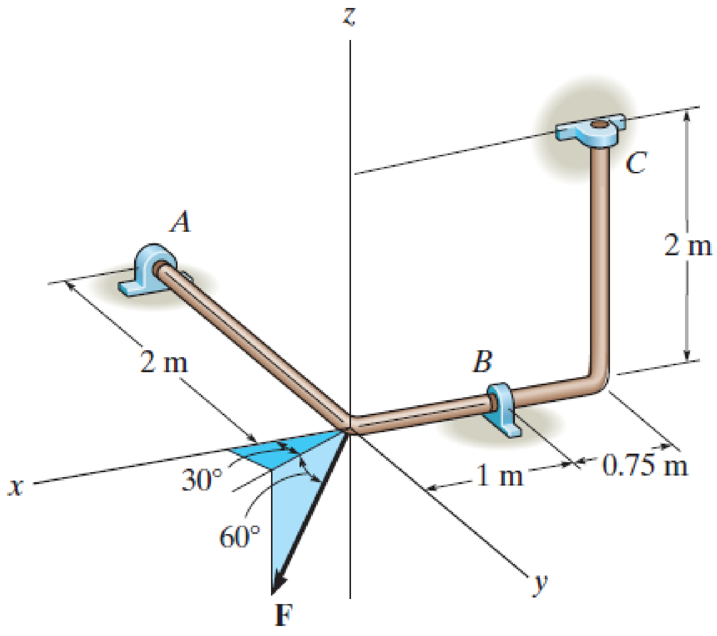
Find: The reactions at all the supports.

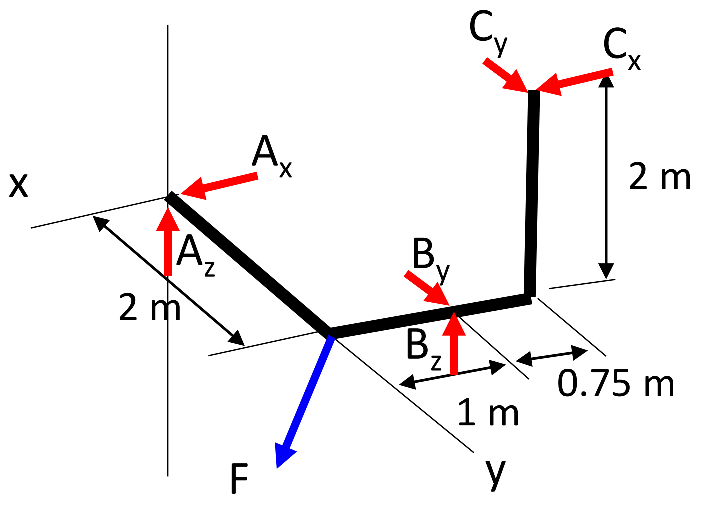
Plan:

a) Draw a FBD of the rod.

b) Apply scalar equations of equilibrium to solve for the unknowns.

GROUP PROBLEM SOLVING (continued)





The x, y and z components of force F are

$F_x = (800 \cos 60°) \cos 30° = 346.4$ N

$F_y = (800 \cos 60°) \sin 30° = 200$ N

$F_z = 800 \sin 60° = 692.8$ N

$\boldsymbol{F} = 346.4\, \boldsymbol{i} + 200\, \boldsymbol{j} + 692.8\, \boldsymbol{k}$

GROUP PROBLEM SOLVING (continued)

Applying scalar equations of equilibrium, we get

$$\Sigma F_x = A_x + C_x + 346.4 = 0 \qquad (1)$$

$$\Sigma F_y = 200 + B_y + C_y = 0 \qquad (2)$$

$$\Sigma F_z = A_z + B_z - 692.8 = 0 \qquad (3)$$

$$\Sigma M_x = -C_y(2) + B_z(2) - 692.8(2) = 0 \qquad (4)$$

$$\Sigma M_y = B_z(1) + C_x(2) = 0 \qquad (5)$$

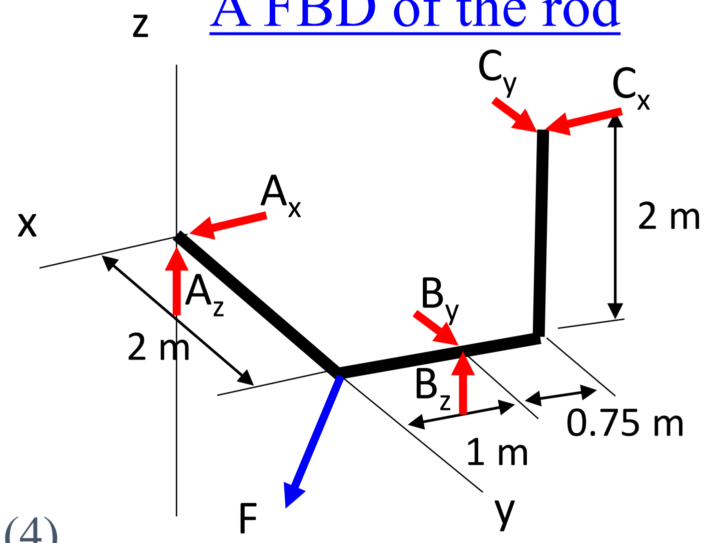
$$\Sigma M_z = -C_y(1.75) - C_x(2) - B_y(1) - 346.4(2) = 0 \qquad (6)$$

Solving Eqs. (1) to (6),

$A_x = 400$ N, $B_y = 600$ N, $C_x = 53.6$ N

$A_z = 800$ N, $B_z = -107$ N, $C_y = 800$ N

A FBD of the rod



Recall

$\boldsymbol{F} = 346.4\,\boldsymbol{i} + 200\,\boldsymbol{j} + 692.8\,\boldsymbol{k}$

ANSWER:
1. C

ATTENTION QUIZ

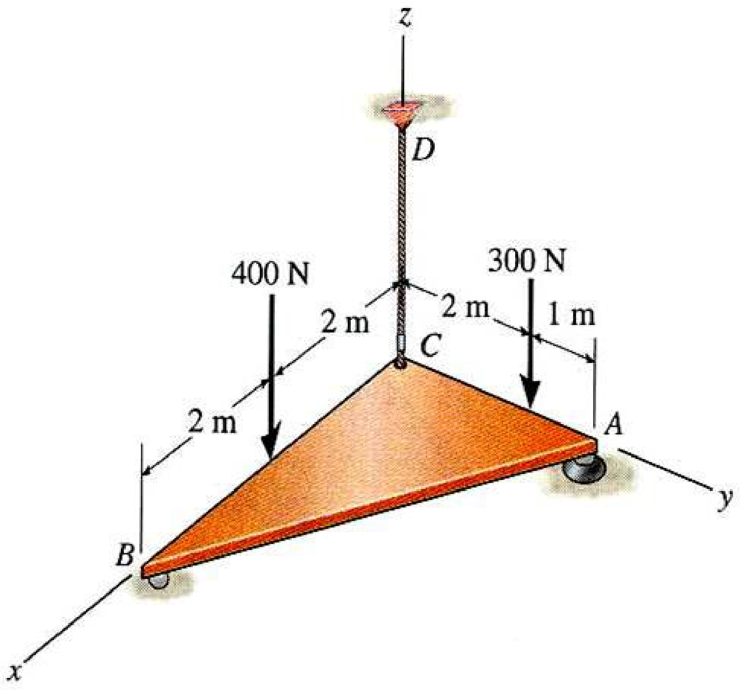
1. A plate is supported by a ball-and socket joint at A, a roller joint at] and a cable at C. How many unknown support reactions are th in this problem?

   A) Four forces and two moments

   B) Six forces

   C) Five forces

   D) Four forces and one moment



ANSWERS:
2. D

ATTENTION QUIZ

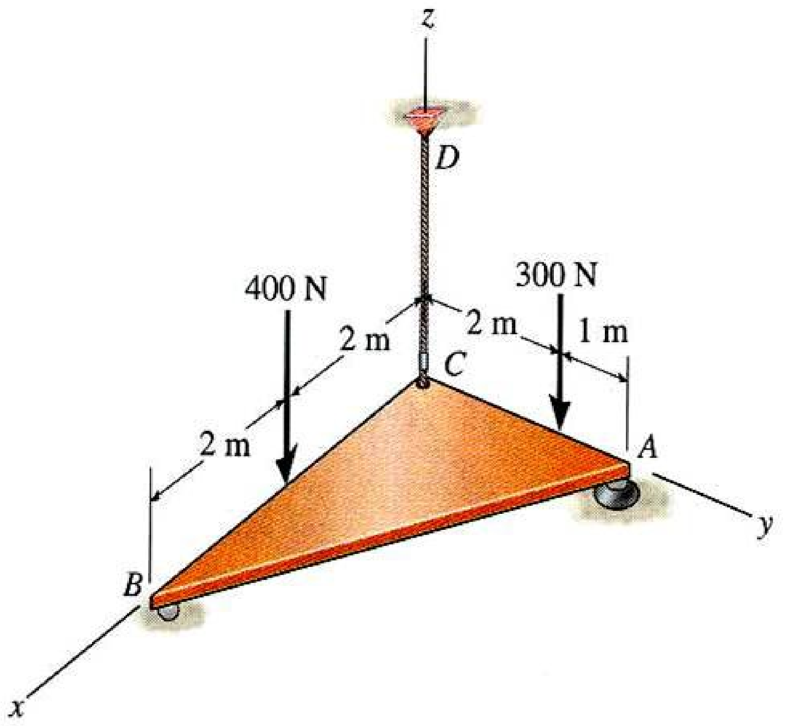
2. What will be the easiest way to determine the force reaction B_Z ?

   A) Scalar equation $\sum F_Z = 0$

   B) Vector equation $\sum \boldsymbol{M_A} = 0$

   C) Scalar equation $\sum M_Z = 0$

   D) Scalar equation $\sum M_Y = 0$



End of the Lecture

Let Learning Continue