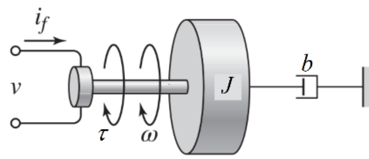
MENG 3020 – Quiz 2 Solution – Fall 2024

Question: A certain rotational system has an inertia $J = 100\text{kg.m}^2$ and a viscous damping constant $b = 20\text{Ns/m}$. The torque $\tau(t)$ is applied by an electric motor.



The equation of motion of the mechanical subsystem is:

$$100\frac{d\omega(t)}{dt} + 20\omega(t) = \tau(t)$$

The voltage $v(t)$ is applied to the motor. The model of the motor's field current $i_f(t)$ in amperes is:

$$0.001\frac{di_f(t)}{dt} + 2i_f(t) = v(t)$$

The torque-current relationship is $\tau(t) = 36i_f(t)$.

a) [5 Marks] Determine the transfer function model of the mechanical and electrical subsystems and complete the block diagram model of the system. Show your work.

Transfer functions of the mechanical and electrical subsystems are determined by taking Laplace transform.

Mechanical Subsystem:

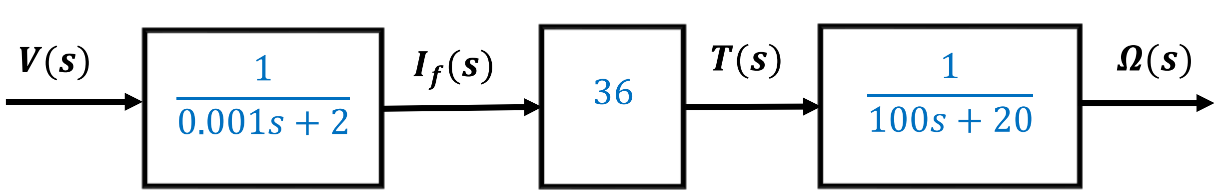
$$100\frac{d\omega(t)}{dt} + 20\omega(t) = \tau(t) \quad \rightarrow \quad 100s\Omega(s) + 20\Omega(s) = T(s) \quad \rightarrow \quad \frac{\Omega(s)}{T(s)} = \frac{1}{100s + 20}$$

Electrical Subsystem:

$$0.001\frac{di_f(t)}{dt} + 2i_f(t) = v(t) \quad \rightarrow \quad 0.001sI_f(s) + 2I_f\Omega(s) = V(s) \quad \rightarrow \quad \frac{I_f(s)}{V(s)} = \frac{1}{0.001s + 2}$$

The torque-current relationship is:

$$\tau(t) = 36i_f(t) \quad \rightarrow \quad T(s) = 36I_f(s)$$



b) [4 marks] Determine the time-constant of the mechanical and electrical subsystems. Which one has a faster response? Which time-constant is the dominant time-constant of the overall system? Show your work and justify your answer.

Time-constant of the mechanical subsystem:

$$\frac{\Omega(s)}{T(s)} = \frac{1}{100s + 20} \quad \rightarrow \quad \tau = \frac{50}{10} = 5\ sec$$

Time-constant of the electrical subsystem:

$$\frac{I_f(s)}{V(s)} = \frac{1}{0.001s + 2} \quad \rightarrow \quad \tau = \frac{0.001}{2} = 5 \times 10^{-4}\ sec$$

The electrical subsystem (motor) has a smaller time-constant, which means it has a much faster response than the rotational mechanical subsystem.

The mechanical subsystem has a greater time-constant, which means it is the dominant time-constant of the overall system.

c) [6 marks] Suppose the applied voltage is $v(t) = 10V$. Find the steady-state speed of the inertia and the steady-state value of the current using the **Final-value Theorem**. Determine the steady-state value of the torque. Estimate the required time to reach the steady-state speed based on the dominant time-constant. Show your work and justify your answer.

Final-value Theorem:
$$f_{ss} = f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

First find the overall transfer function:

$$\frac{\Omega(s)}{V(s)} = \frac{36}{(0.001s + 2)(100s + 20)}$$

The steady-state speed is obtained using the Final-Value theorem:

$$\lim_{t\to\infty} \omega(t) = \lim_{s\to 0} s\Omega(s) \quad \rightarrow \quad \omega(\infty) = \lim_{s\to 0} s\left(\frac{36}{(0.001s + 2)(100s + 20)}\right)\left(\frac{10}{s}\right) = 9\ rad/s$$

The steady-state value of the current is obtained using the Final-Value theorem:

$$\lim_{t\to\infty} i_f(t) = \lim_{s\to 0} sI_f(s) \quad \rightarrow \quad i_f(\infty) = \lim_{s\to 0} s\left(\frac{1}{0.001s + 2}\right)\left(\frac{10}{s}\right) = 5\ A$$

The steady-state value of the torque:

$$\tau(t) = 36 i_f(t) \quad \rightarrow \quad \tau(\infty) = 36(5A) = 180\ N.m$$

The estimated time to reach that speed is determined by the dominant time-constant, which is about

$$4\tau = 4(5) = 20\ sec.$$