ABSOLUTE DEPENDENT MOTION ANALYSIS OF TWO PARTICLES

Today's Objectives:

Students will be able to:

1. Relate positions, velocities, and accelerations of particles undergoing dependent motion.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Define Dependent Motion
- Develop Position, Velocity, and Acceleration Relationships
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. When particles are interconnected by a cable, the motions of the particles are ______

   A) always independent.

   B) always dependent.

   C) not always dependent.

   D) None of the above.

2. If the motion of one particle is dependent on that of another particle, each coordinate axis system for the particles _______

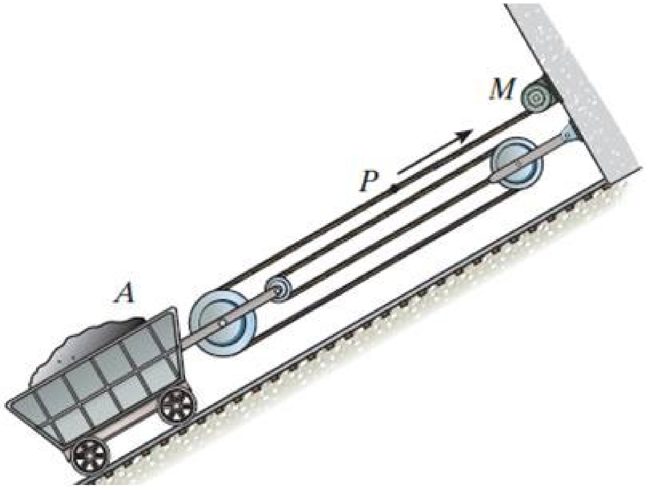
   A) should be directed along the path of motion.

   B) can be directed anywhere.

   C) should have the same origin.

   D) None of the above.

APPLICATIONS

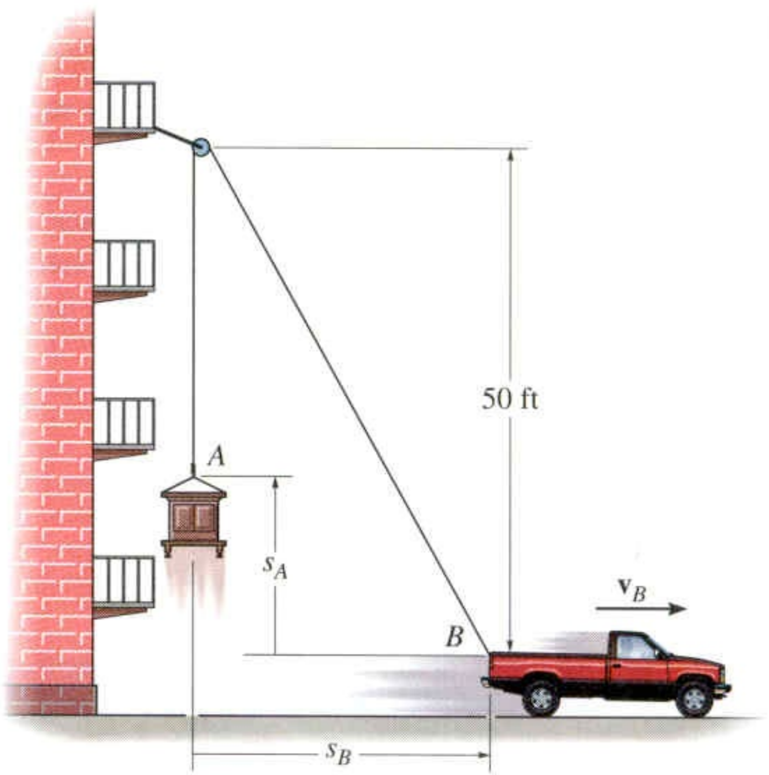


The cable and pulley system shown can be used to modify the speed of the mine car, A, relative to the speed of the motor, M.

It is important to establish the relationships between the various motions in order to determine the power requirements for the motor and the tension in the cable.

For instance, if the speed of the cable (P) is known because we know the motor characteristics, how can we determine the speed of the mine car? Will the slope of the track have any impact on the answer?

APPLICATIONS (continued)

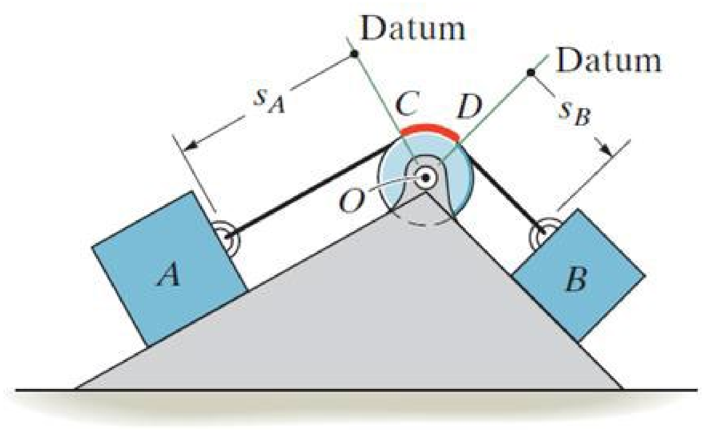


Rope and pulley arrangements are often used to assist in lifting heavy objects. The total lifting force required from the truck depends on both the weight and the acceleration of the cabinet.

How can we determine the acceleration and velocity of the cabinet if the acceleration of the truck is known?

DEPENDENT MOTION (Section 12.9)

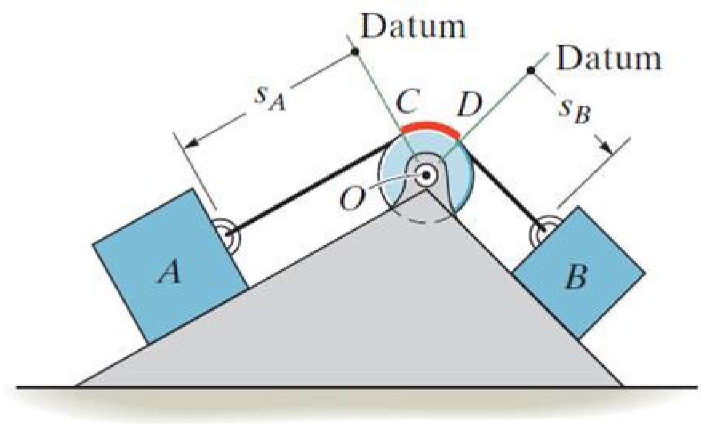
In many kinematics problems, the motion of one object will depend on the motion of another object.



The blocks in this figure are connected by an inextensible cord wrapped around a pulley.
If block A moves downward along the inclined plane, block B will move up the other incline.

The motion of each block can be related mathematically by defining position coordinates, s_A and s_B. Each coordinate axis is defined from a fixed point or datum line, measured positive along each plane in the direction of motion of each block.

DEPENDENT MOTION (continued)



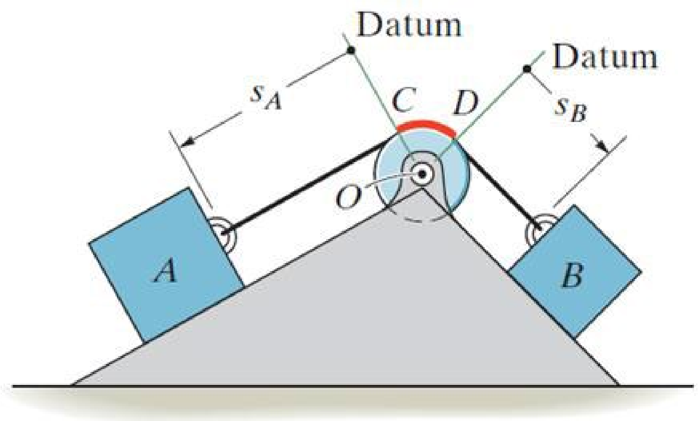
In this example, position coordinates s_A and s_B can be defined from fixed datum lines extending from the center of the pulley along each incline to blocks A and B.

If the cord has a fixed length, the position coordinates s_A and s_B are related mathematically by the equation

$$s_A + l_{CD} + s_B = l_T$$

Here l_T is the total cord length and l_{CD} is the length of cord passing over the arc CD on the pulley.

DEPENDENT MOTION (continued)



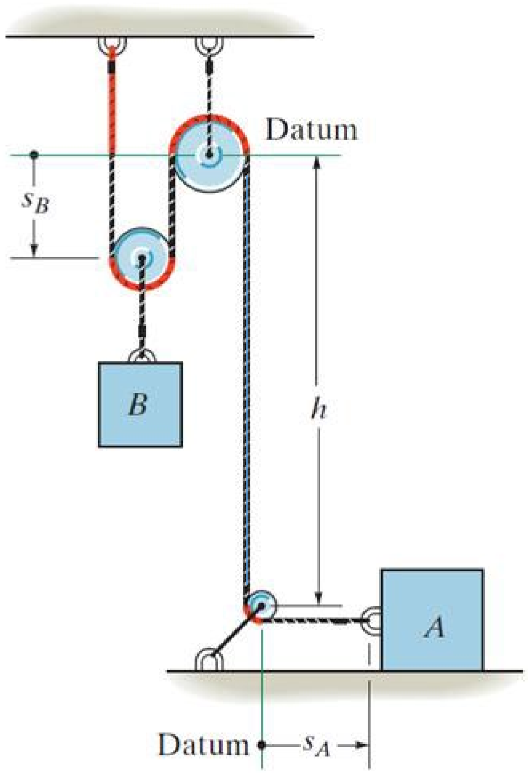
The velocities of blocks A and B can be related by differentiating the position equation. Note that l_{CD} and l_T remain constant, so $dl_{CD}/dt = dl_T/dt = 0$

$$ds_A/dt + ds_B/dt = 0 \quad \Rightarrow \quad v_B = -v_A$$

The negative sign indicates that as A moves down the incline (positive s_A direction), B moves up the incline (negative s_B direction).

Accelerations can be found by differentiating the velocity expression. Prove to yourself that $a_B = -a_A$.

DEPENDENT MOTION EXAMPLE

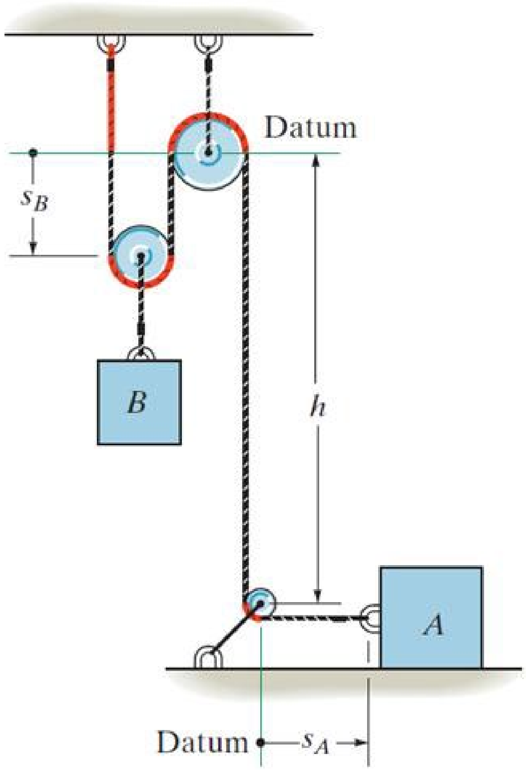


Consider a more complicated example. Position coordinates (s_A and s_B) are defined from fixed datum lines, measured along the direction of motion of each block.

Note that s_B is only defined to the center of the pulley above block B, since this block moves with the pulley. Also, *h* is a constant.

The red-colored segments of the cord remain constant in length during motion of the blocks.

DEPENDENT MOTION EXAMPLE (continued)



The position coordinates are related by the equation

$$2s_B + h + s_A = l_T$$

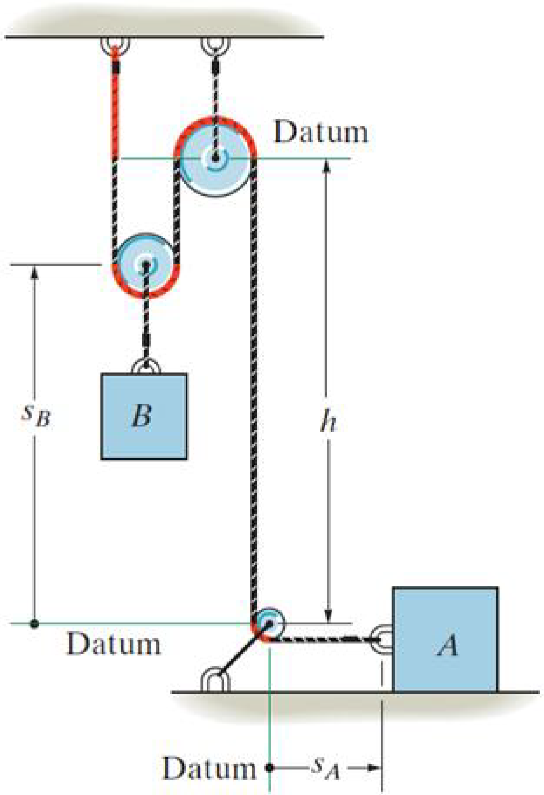
Where l_T is the total cord length minus the lengths of the red segments.

Since l_T and *h* remain constant during the motion, the velocities and accelerations can be related by two successive time derivatives:

$$2v_B = -v_A \quad \text{and} \quad 2a_B = -a_A$$

When block B moves downward ($+s_B$), block A moves to the left ($-s_A$). Remember to be consistent with your sign convention!

DEPENDENT MOTION EXAMPLE (continued)



This example can also be worked by defining the position coordinate for B (s_B) from the bottom pulley instead of the top pulley.

The position, velocity, and acceleration relations then become

$$2(h - s_B) + h + s_A = l_T$$

and $2v_B = v_A$ $\qquad 2a_B = a_A$

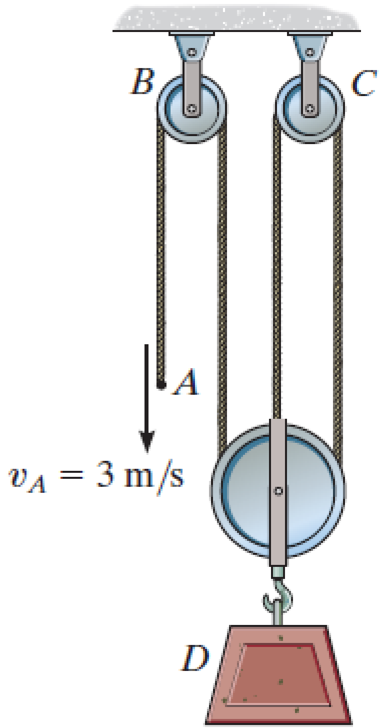
Prove to yourself that the results are the same, even if the sign conventions are different than the previous formulation.

DEPENDENT MOTION: PROCEDURES

These procedures can be used to relate the dependent motion of particles moving along rectilinear paths (only the magnitudes of velocity and acceleration change, not their line of direction).

1. Define position coordinates from fixed datum lines, along the path of each particle. Different datum lines can be used for each particle.
2. Relate the position coordinates to the cord length. Segments of cord that do not change in length during the motion may be left out.
3. If a system contains more than one cord, relate the position of a point on one cord to a point on another cord. Separate equations are written for each cord.
4. Differentiate the position coordinate equation(s) to relate velocities and accelerations. Keep track of signs!

EXAMPLE



Given: In the figure on the left, the cord at A is pulled down with a speed of 3 m/s.

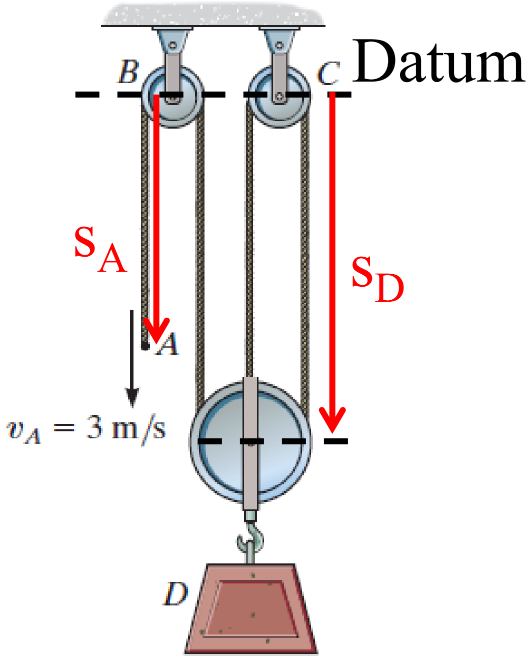
Find: The speed of block D.

Plan: There is only one cord involved in the motion, so only one position/length equation is required. Define position coordinates for block D and cable lengths that change, write the position relation and then differentiate it to find the relationship between the two velocities.

EXAMPLE (continued)

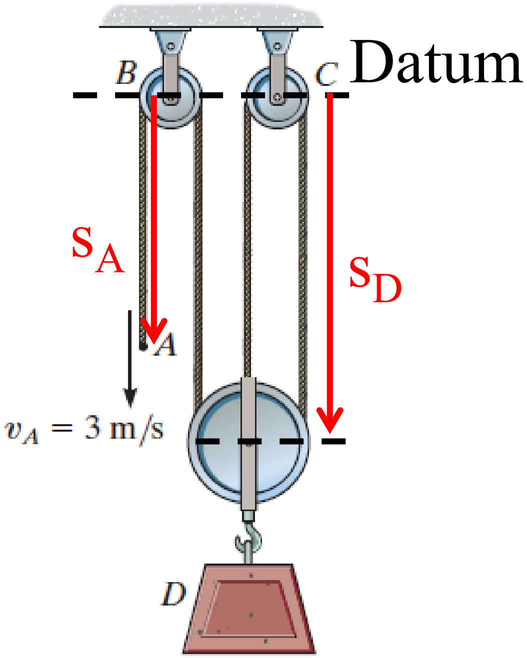
Solution:

1) A datum line can be drawn through the upper, fixed pulleys. Two coordinates must be defined: one for block D (s_D) and one for the changing cable length (s_A).



- s_A can be defined to the point A.
- s_D can be defined to the center of the pulley above D.
- All coordinates are defined as positive down and along the direction of motion of each point/object.

EXAMPLE (continued)



2) Write position/length equations for the cord. Define l_T as the length of the cord, minus any segments of constant length.

$$s_A + 3s_D = l_T$$

3) Differentiate to find the velocity relationship:

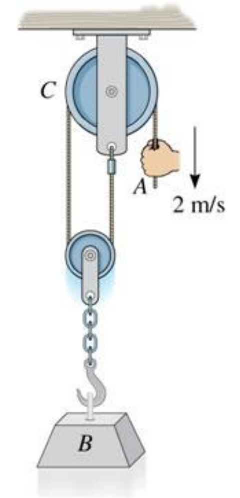
$$v_A + 3v_D = 0$$

Since the cord at A is pulled down with a speed of 3 m/s,

$3 + 3v_D = 0 \;\Rightarrow\; v_D = -1$ m/s = 1 m/s ↑

CONCEPT QUIZ

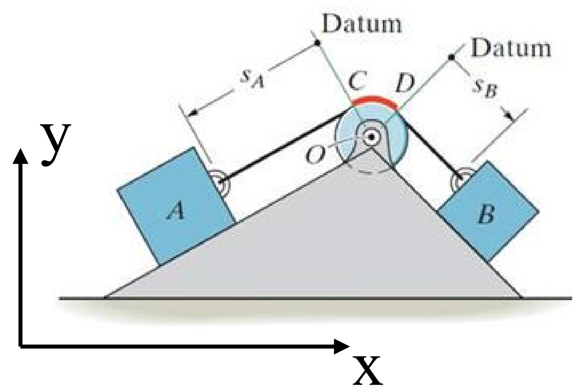
1. Determine the speed of block B.



A) 1 m/s B) 2 m/s

C) 4 m/s D) None of the above.

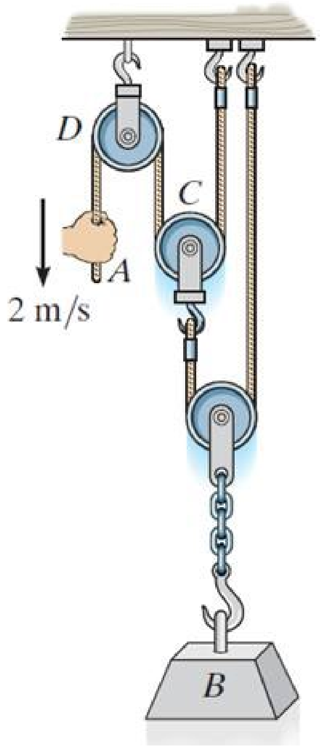
2. Two blocks are interconnected by a cable. Which of the following is correct?



A) $(v_x)_A = -(v_x)_B$ B) $v_A = -v_B$

C) $(v_y)_A = -(v_y)_B$ D) All of the above.

GROUP PROBLEM SOLVING I



Given: In the figure on the left, the cord at A is pulled down with a speed of 2 m/s.

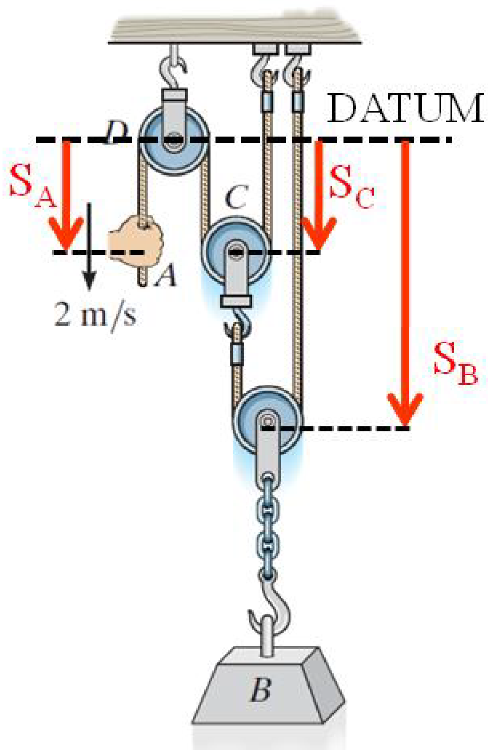
Find: The speed of block B.

Plan: There are two cords involved in the motion in this example. There will be two position equations (one for each cord). Write these two equations, combine them, and then differentiate them.

GROUP PROBLEM SOLVING I (continued)

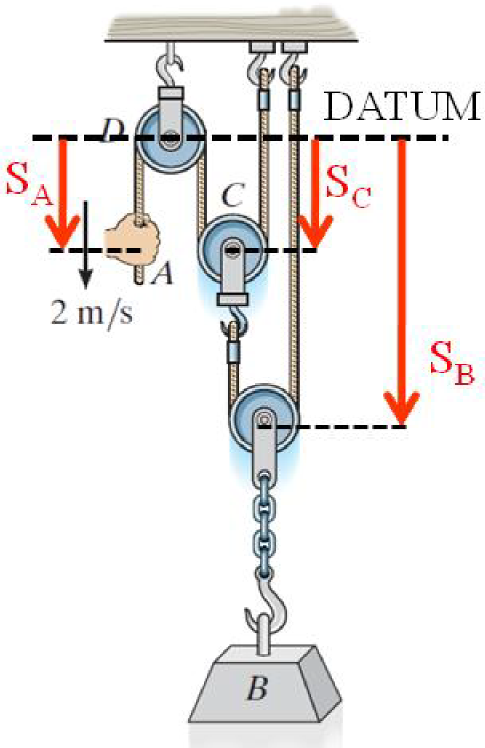
Solution:

1) Define the position coordinates from a fixed datum line. Three coordinates must be defined: one for point A (s_A), one for block B (s_B), and one for block C (s_C).



- Define the datum line through the top pulley (which has a fixed position).
- s_A can be defined to the point A.
- s_B can be defined to the center of the pulley above B.
- s_C is defined to the center of pulley C.
- All coordinates are defined as positive down and along the direction of motion of each point/object.

GROUP PROBLEM SOLVING I (continued)



2) Write position/length equations for each cord. Define l_1 as the length of the first cord, minus any segments of constant length. Define l_2 in a similar manner for the second cord:

$$\text{Cord 1:} \quad s_A + 2s_C = l_1$$
$$\text{Cord 2:} \quad s_B + (s_B - s_C) = l_2$$

3) Eliminating s_C between the two equations, we get

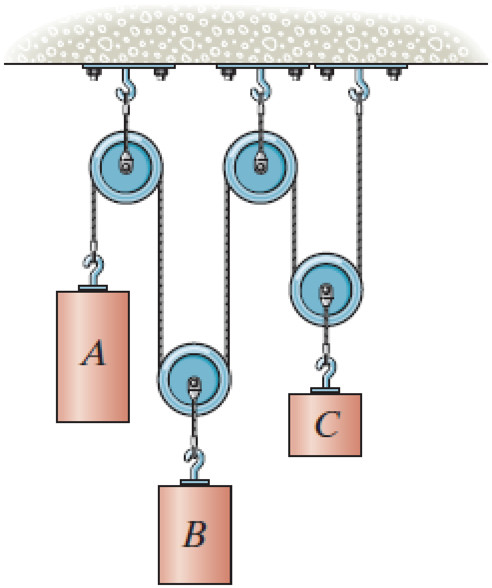
$$s_A + 4s_B = l_1 + 2l_2$$

4) Relate velocities by differentiating this expression. Note that l_1 and l_2 are constant lengths.

$$v_A + 4v_B = 0 \quad \Rightarrow \quad v_B = -0.25v_A = -0.25(2) = \underline{-0.5 \text{ m/s}}$$

The velocity of block B is 0.5 m/s up (negative s_B direction).

GROUP PROBLEM SOLVING II



Given: In this pulley system, block A is moving downward with a speed of 6 ft/s while block C is moving down at 18 ft/s.

Find: The speed of block B.

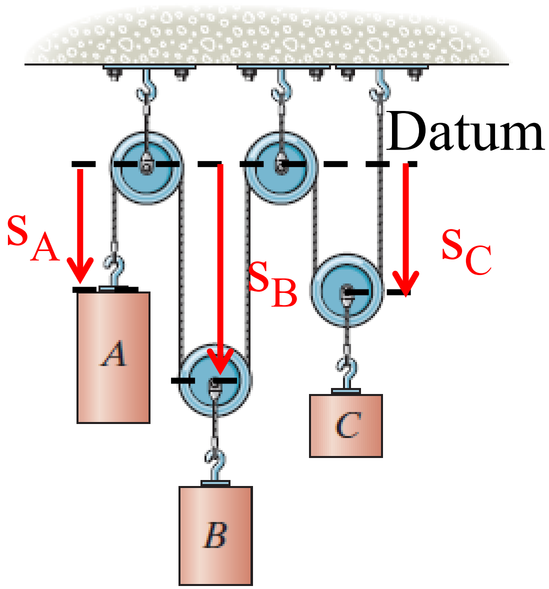
Plan:

All blocks are connected to a single cable, so only one position/length equation will be required. Define position coordinates for each block, write out the position relation, and then differentiate it to relate the velocities.

GROUP PROBLEM SOLVING II (continued)

Solution:

1) A datum line can be drawn through the upper, fixed, pulleys and position coordinates defined from this line to each block (or the pulley above the block).



2) Defining s_A, s_B, and s_C as shown, the position relation can be written:

$$s_A + 2s_B + 2\,s_C = l_T$$

3) Differentiate to relate velocities:

$$v_A + 2v_B + 2\,v_C = 0$$

$$6 + 2v_B + 2(18) = 0$$

$$v_B = -\,21\text{ ft/s} = \underline{21\text{ ft/s}}\uparrow$$

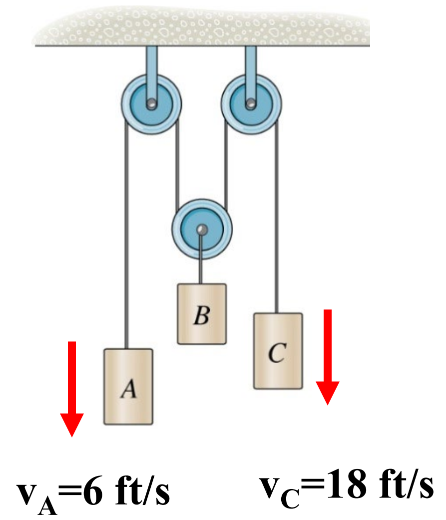
The velocity of block B is 21 ft/s up.

ATTENTION QUIZ

1. Determine the speed of block B when block A is moving down at 6 ft/s while block C is moving down at 18 ft/s .

   A) 24 ft/s　　B) 3 ft/s

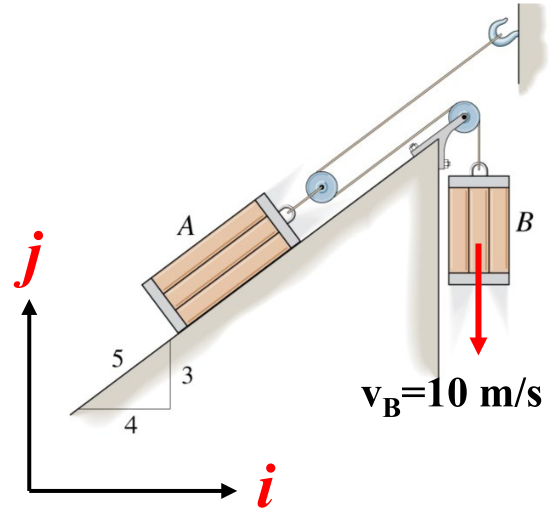
   C) 12 ft/s　　D) 9 ft/s



2. Determine the velocity vector of block A when block B is moving downward with a speed of 10 m/s.

   A) (8*i* + 6*j*) m/s　　B) (4*i* + 3*j*) m/s

   C) (-8*i* - 6*j*) m/s　　D) (3*i* + 4*j*) m/s



PEARSON

End of the Lecture

Let Learning Continue