Signal Processing (MENG3520)

Module 1

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Course Outline & Critical Path

Class Structure

- **Lectures:** Tuesdays 9:50 am to 12:30 pm (room J202)
- **Labs:** Thursdays 3:20 pm to 5:05 pm (room J217)
- **Consultations:** TBD

Evaluation Plan

Laboratory Assignments (7x)	40%
Course Project	20%
Midterm Exam	20%
Final Exam	20%
Total	100%

Minimum passing grade: overall above 60%

Expectations

- Expectations of you
- Expectations of me
- Expectations of classes
- Expectations of labs
- What to do when you miss an evaluation.
- Consultation hours
- Any questions or issues, make an appointment and discuss!

Module 1

Introduction to Signals and Systems

MODULE OUTLINE

1.0 Overview of signals and systems

1.1 Models of signals

1.2 Properties of signals

1.3 Continuous-time signals

- 1.3.1 Commonly used continuous-time functions
- 1.3.2 Independent- and dependent- variable transformation of CT signals

1.4 Discrete-time signals

- 1.4.1 Sampling and discrete-time signals
- 1.4.2 Common discrete-time sequences
- 1.4.3 Independent- and dependent- variable transformation of DT signals
- 1.4.4 Differencing and accumulation of DT signals

ROADMAP

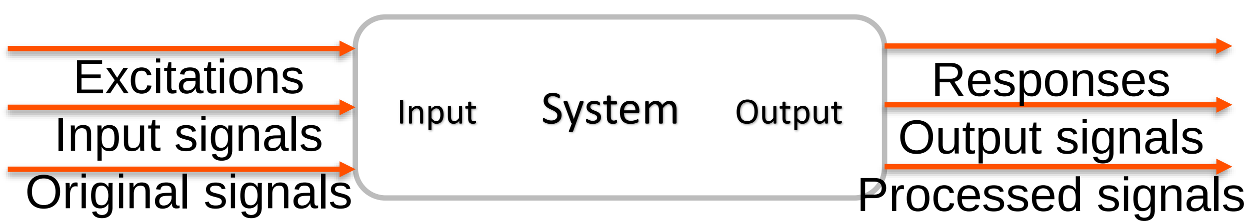
Signals: any time-varying phenomenon that contains information that can be conveyed from one point to another.
They may be generated by electronic means or by other means such as:

- Talking
- Heart beats
- Earthquakes
- Stock trading
- Temperature fluctuations
- ...

ROADMAP

System: process signals to produce a modified or transformed version of the original signal.

- The complexity of transformation varies, may be as simple analog to digital conversion or as sophisticated as recognizing a running car through digital camera signals.

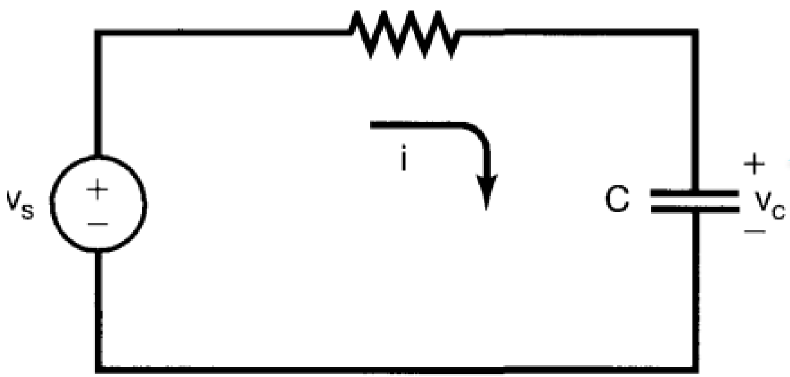


ROADMAP

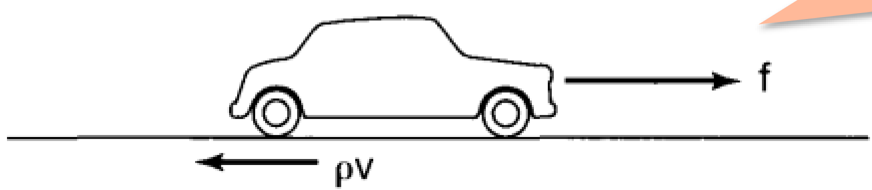
Signals and systems can be of many physical forms. For this course, both signals and systems are primarily those of an electrical nature.

- signals and systems means the subject on mathematical modeling of the signals and systems, in order to design and develop electrical devices.
- If other physical signals are used, a transducer is needed convert such non-electrical signals into electrical signals so that the electrical systems will be able to operate on them.

EXAMPLES OF SIGNALS

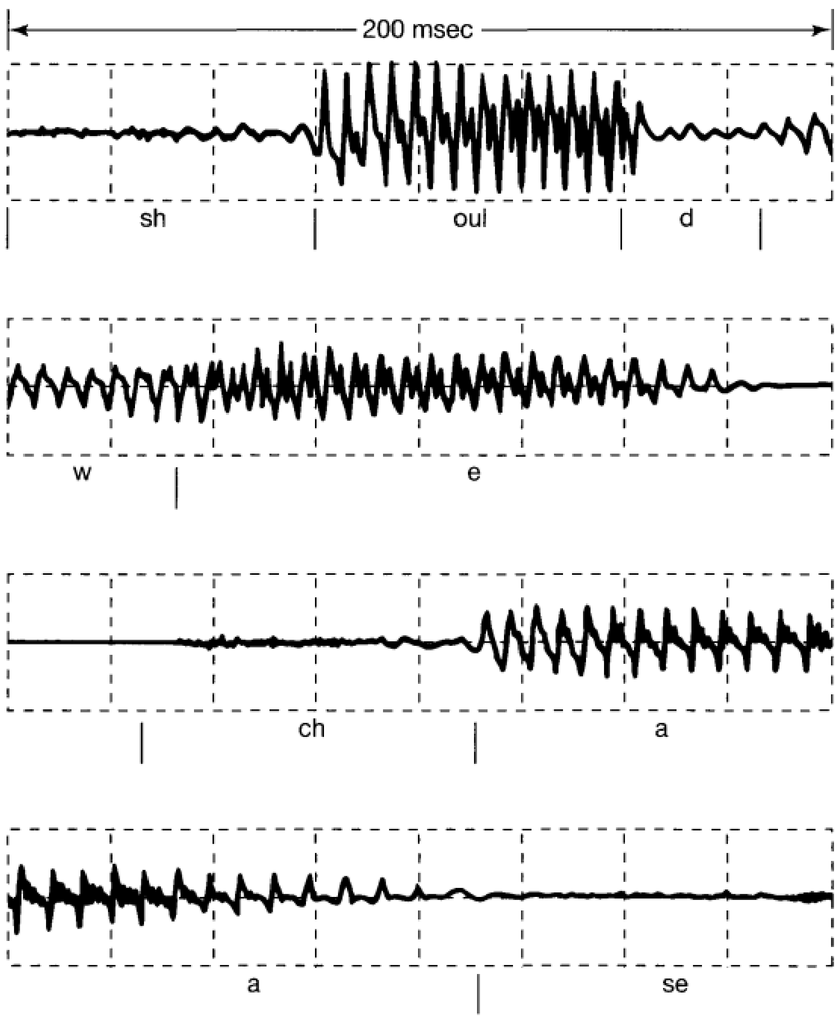


The pattern of variation over time in the source and capacitor voltage *Vs*, *Vc*, are examples of signals



The pattern of variation over time of the applied engine force *f* and the friction force *pv* and the resulting care velocity v are signals.

EXAMPLES OF SIGNALS



The pattern of variation over time in the source and capacitor voltage *Vs*, *Vc*, are examples of signals

The pattern of variation over time of the applied engine force *f* and the friction force *pv* and the resulting care velocity v are signals.

Other Examples of Signals and Systems

OTHER EXAMPLES OF SIGNALS AND SYSTEMS

Image from jotrin.de

1.1

MODELING OF SIGNALS

Mathematical Modeling of Signals

Signals are usually represented mathematically using functions of one or more independent variables.

Commonly used independent variables can be time, spatial coordinates or spectrum component depending on which domain the signals are represented in.

$$s1 = x(t)$$

$$s2 = y[n]$$

$$s3 = Image[row, col]$$

Mathematical Modeling of Signals

Dimensionality of signals:

- Signals with one independent variable are considered one dimensional.
- Signals with more than one independent variables are considered multi-dimensional.

- Common examples:
  - Hourly temperature recording
  - Images and videos
  - GPS (global positioning system)

MATHEMATICAL MODELING OF SIGNALS

Types of independent variables.

In particular, two types of independent variables are commonly seen in signals:

- Continuous-time (CT) signals: if the independent variables are continuous.
- Discrete-time (DT) signals: if the independent variables are discrete.
- Please note that the term “time” is a legacy since non-time related independent variables can also be referred to as CT or DT signals.

Mathematical Modeling of Signals

Types of dependent variables.

In particular, two types of dependent variables are commonly seen in signals:

- Continuous-valued signals: if the dependent variable is continuous.
- Discrete-valued signals: if the dependent variable is discrete.

Analog signals are continuous-valued CT signals.

- With an infinite and uncountable sequence of numbers.
- Also, infinite and uncountable as are the possible values each number can have.
- Analog signal cannot be stored, or processed, in a computer.
- Analog signals must be digitized to produce a finite set of numbers for computers to use.

Digital signals are discrete valued DT signals.

- Digital signals can only take discrete values within the range of signal value.
- Digital signals are created by quantizing DT signal amplitude into limited number of values.
- Digital signals are the only acceptable format for computers to process, analyze and store.

Mathematical Modeling of Signals

To distinguish between continuous-time and discrete-time signals, we apply the following rules in this course:

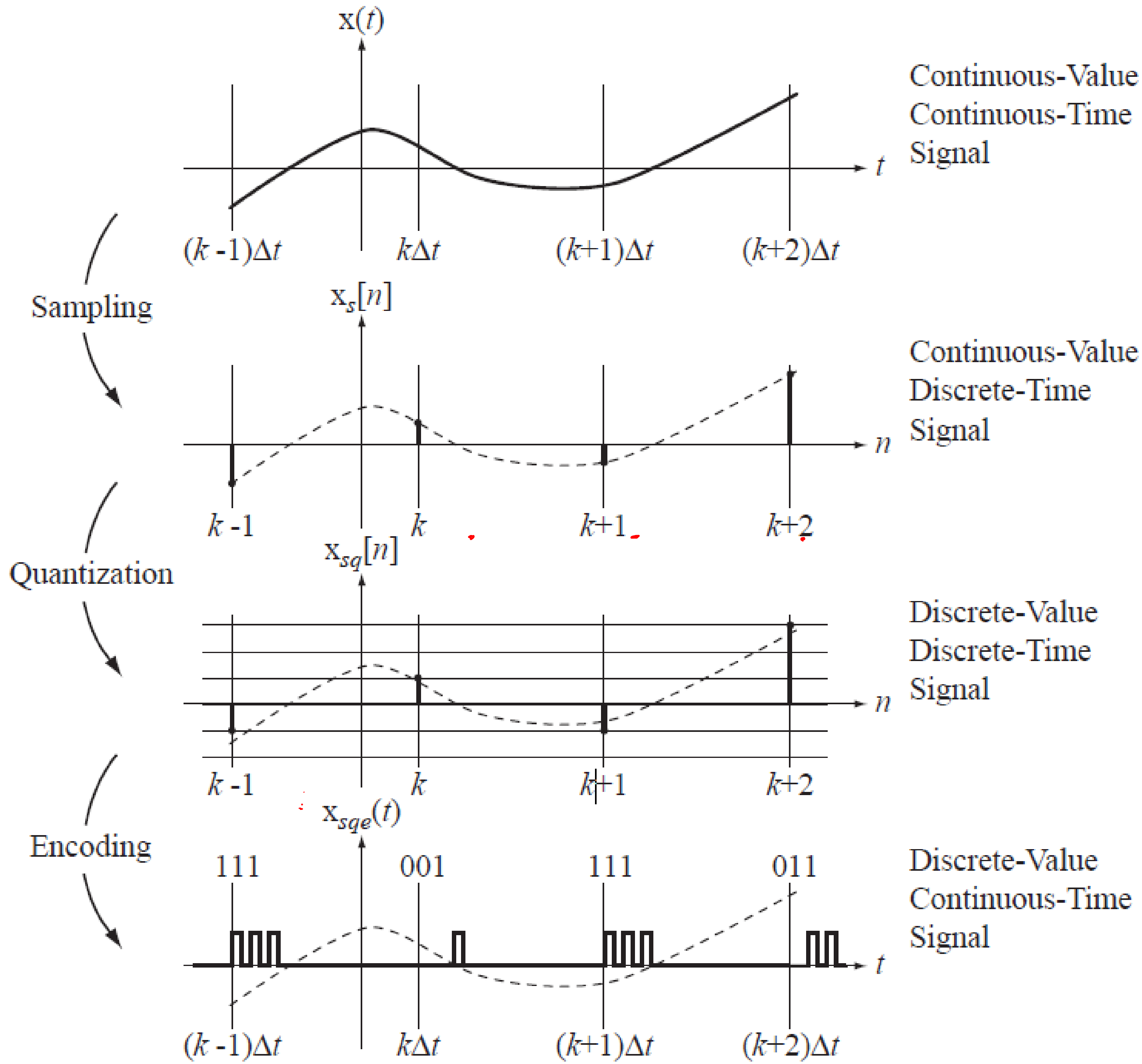
- Use t as the continuous independent variable and use n as the discrete independent variable.
- Use parenthesis (.) for CT signals and use brackets [.] for DT signals.
- A CT signals is called a **function**. A DT signals is called a **sequence**.

MATHEMATICAL MODELING OF SIGNALS

Exercise:

$x(t)$ is ________________ time signal.

$x[n]$ is ________________ time signal.


x(t)
Continuous-Value
Continuous-Time
Signal
t
(k -1)Δt
kΔt
(k+1)Δt
(k+2)Δt
Sampling
x_s[n]
Continuous-Value
Discrete-Time
Signal
n
k -1
k
k+1
k+2
Quantization
x_sq[n]
Discrete-Value
Discrete-Time
Signal
n
k -1
k
k+1
k+2
Encoding
x_sqe(t)
111
001
111
011
Discrete-Value
Continuous-Time
Signal
t
(k -1)Δt
kΔt
(k+1)Δt
(k+2)Δt

Activity: draw the relation between DT, CT, digital and analog signals:

1.2

Properties of Signals

Classification of Signals

Based on the different properties, signals can be classified differently:

- Continuous-time and discrete-time signals
- Analog and digital signals
- Deterministic and probabilistic signals
- Even and odd signals
- Periodic and aperiodic signals
- Energy and power signals

CLASSIFICATION OF SIGNALS

- Continuous-time and discrete-time signals
- Analog and digital signals
- Deterministic and probabilistic signals
- Even and odd signals
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- Energy and power signals

- Deterministic signal:
  - any signal that can be uniquely described by an explicit mathematical expression, a table of data, or a well-defined rule.
  - past, present and future values of the signal are known precisely without any uncertainty.
- Random signal:
  - any signal that lacks a unique and explicit mathematical expression and thus evolves in time in an unpredictable manner.
  - it may not be possible to accurately describe the signal.
  - the deterministic model of the signal may be too complicated to be of use.

Classification of Signals

- Continuous-time and discrete-time signals
- Analog and digital signals
- Deterministic and probabilistic signals
- Even and odd signals
- Periodic and aperiodic signals
- Energy and power signals

Even Signals

- A function x is even if:
- $x(t) = x(-t)$ for any t

- A sequence x is even if:
- $x[n] = x[-n]$ for any n

- Geometrically, an even signal is symmetrical about the vertical axis.

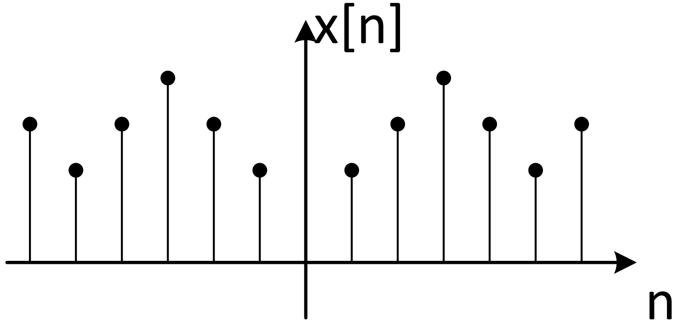
ODD SIGNALS

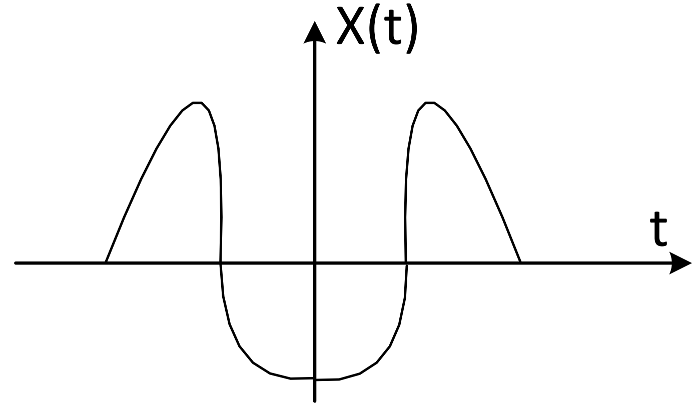
- A function x is odd if:
- $x(t) = -x(-t)$ for any t

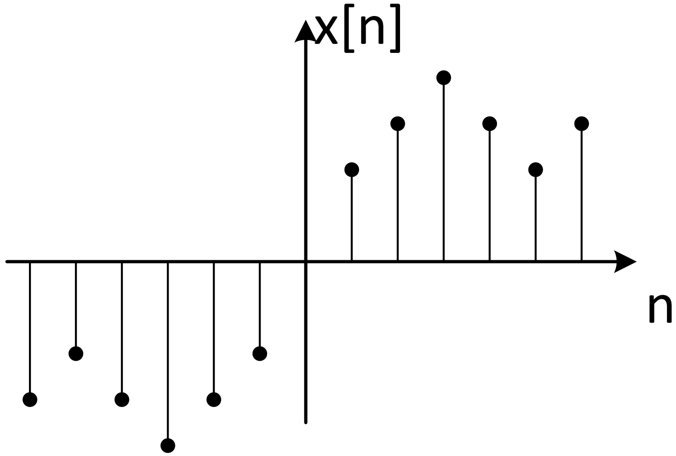
- A sequence x is odd if:
- $x[n] = -x[-n]$ for any n

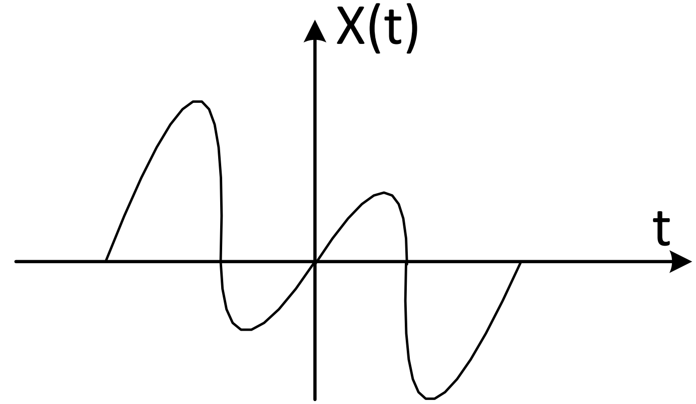
- Geometrically, an odd signal is anti-symmetrical about the origin – why?

ODD? EVEN?









EVAN AND ODD COMPONENTS OF ANY SIGNAL

Any CT function $x(t)$ can be broken into the sum of an even signal and an odd signal:

$$x(t) = \underbrace{\frac{1}{2}[x(t) + x(-t)]}_{even} + \underbrace{\frac{1}{2}[x(t) - x(-t)]}_{odd}$$

EVAN AND ODD COMPONENTS OF ANY SIGNAL

Any CT function $x(t)$ can be broken into the sum of an even signal and an odd signal:

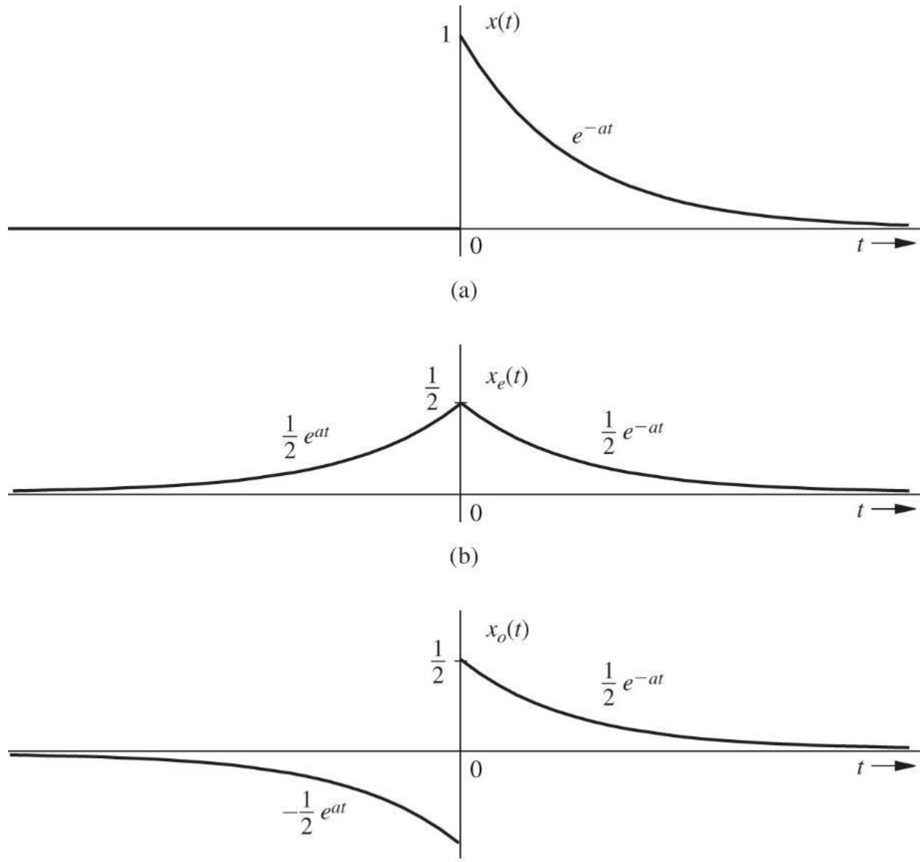


Figure 1.24 Finding even and odd components of a signal.

Similarly, any DT sequence $x[n]$ can also be broken into the sum of an even signal and an odd signal:

$$x[n] = \underbrace{\frac{1}{2}(x[n] + \mathrm{x}[-\mathrm{n}])}_{even} + \underbrace{\frac{1}{2}(x[n] - x[-n])}_{odd}$$

Even and odd functions have the following properties:

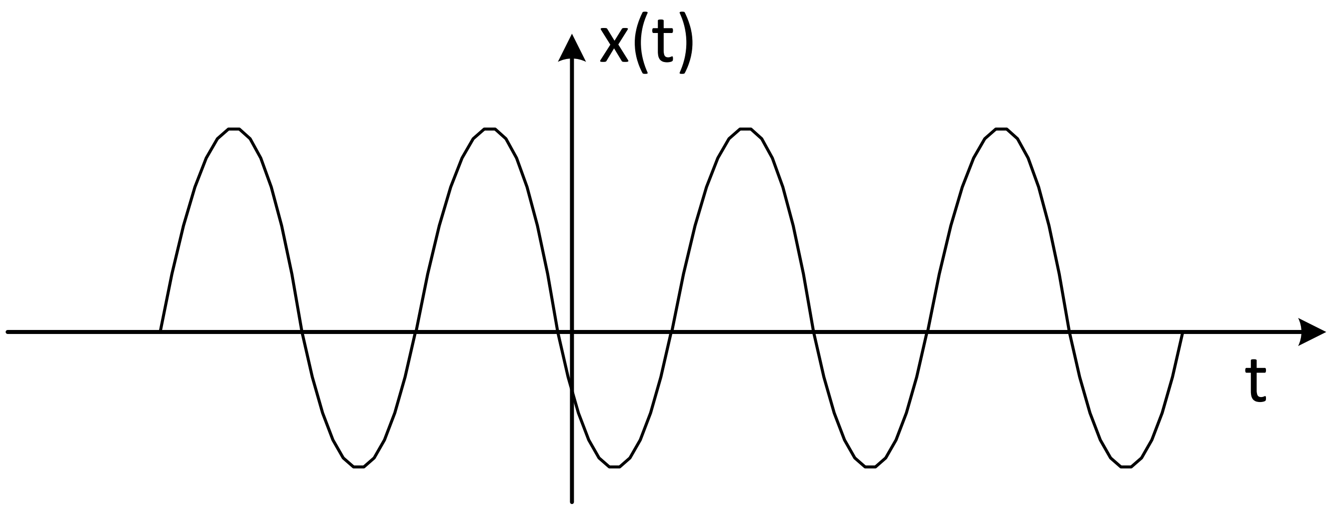
- even function $\times$ odd function = odd function
- odd function $\times$ odd function = odd function
- even function $\times$ even function = even function

CLASSIFICATION OF SIGNALS

- Continuous-time and discrete-time signals
- Analog and digital signals
- Deterministic and probabilistic signals
- Even and odd signals
- Periodic and aperiodic signals
- Energy and power signals

PERIODIC SIGNALS

- A function x is periodic with period T if:
- $x(t) = x(t + T)$ for any t



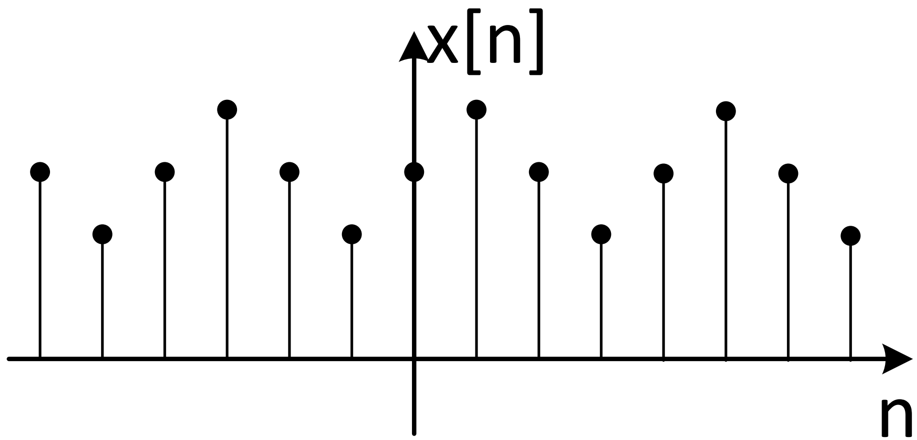
PERIODIC SIGNALS

- A sequence x is periodic with period N if:
- $x[n] = x[n + N]$ for any n

- Other signals are called aperiodic signals.

PERIODIC SIGNALS

- A sequence x is periodic with period N if:
- $x[n] = x[n + N]$ for any n



Classification of Signals

- Continuous-time and discrete-time signals
- Analog and digital signals
- Deterministic and probabilistic signals
- Even and odd signals
- Periodic and aperiodic signals
- Energy and power signals

CT SIGNAL ENERGY

- Often CT signals that we are working on represents some physical quantities that are related to energy transfer.
- CT signal energy over the time interval $t_1 \leq t \leq t_2$ of any signal $x(t)$ is defined as:

$$E_x = \int_{t_1}^{t_2} |x(t)|^2 dt$$

CT SIGNAL ENERGY

- CT signal energy over an infinite time interval of a signal $x(t)$ is defined as total signal energy:

$$E_\infty \triangleq \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

CT SIGNAL ENERGY

- Example: compute signal energy of the following signal:

$$x(t) = \begin{cases} 3(1 - |t|), |t| < 1 \\ 0, otherwise \end{cases}$$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-1}^{1} |x(t)|^2 dt$$

$$= 18 \int_0^1 (1 - 2t + t^2) dt = 6$$

CT Periodic Signal Power

- For many signals, signal energy does not converge, in this case, average signal power is used:

$$P_\infty \triangleq \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} E_\infty$$

$$P_\infty = P_T \triangleq \frac{1}{T}\int_{t}^{t+T} |x(t)|^2 dt = \frac{1}{T}\int_{T} |x(t)|^2 dt$$

DT Signal Energy

- Similar to its CT counterpart, signal energy over an infinite time interval of a DT signal $x[n]$ is defined as total signal energy:

$$E_\infty \triangleq \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

DT SIGNAL POWER

- Similarly, the average DT signal power is:

$$P_\infty \triangleq \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

THREE CLASSES OF SIGNALS

- Class 1. Energy signal: signals with finite total energy $E_\infty < \infty$, and zero average power $P_\infty = 0$.
- Class 2. Power signal: signals with finite average power $P_\infty < \infty$, and infinite total energy E_∞.
- Class 3. Signals with infinite average power P_∞, and infinite total energy E_∞.

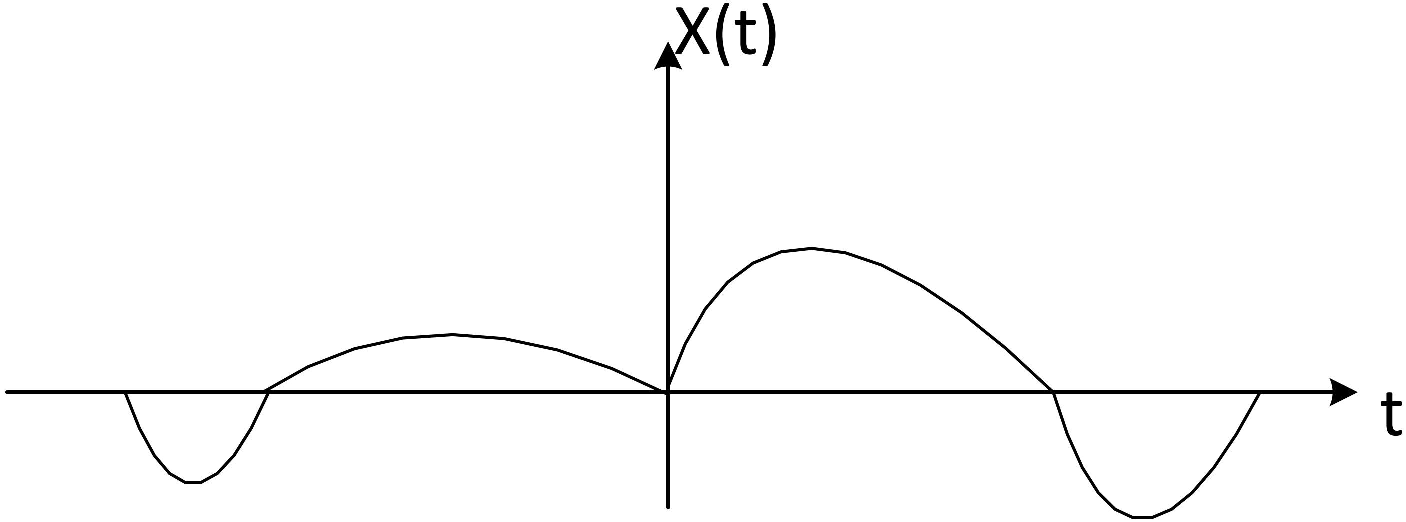
1.3

Continuous-Time Signals

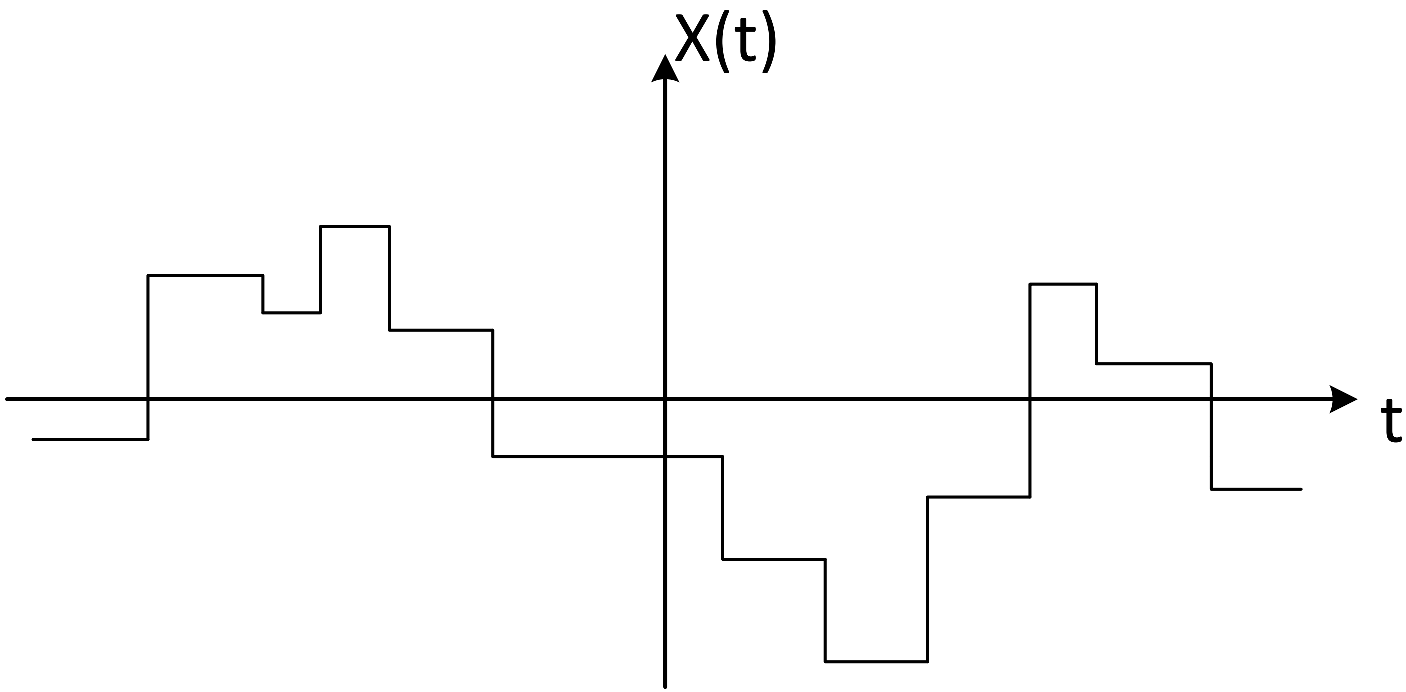
1.3.1

Commonly Used Continuous-Time Functions

- The difference between CT functions and Continuous functions of Time?



- The difference between CT functions and Continuous functions of Time?



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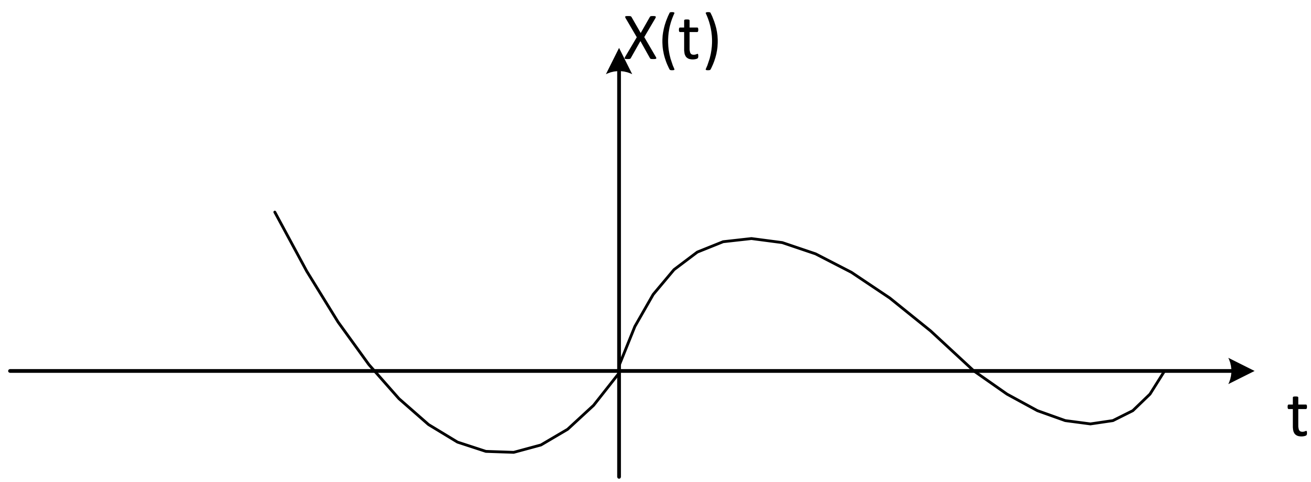


Most commonly used CT functions:

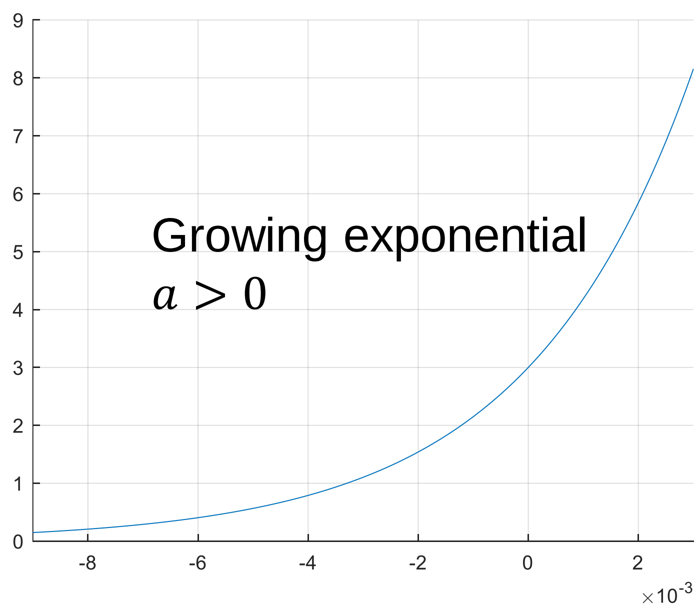
- Exponentials and sinusoids
- Unit step function
- Unit impulse function

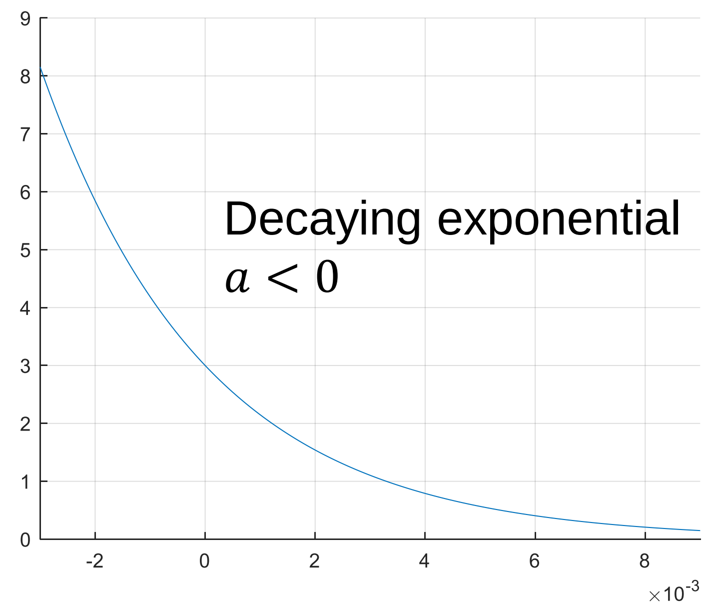
CT Complex Exponentials

- General format: $x(t) = Ce^{at}$, where both C, a are in general complex.
- Different cases:
  - Case 1: C real, a real
  - Case 2: C real, a purely imaginary
  - Case 3: C complex, a complex

CT Complex Exponentials

- Case 1: $x(t) = Ce^{at}$, where both C, a are both real.





CT PERIODIC COMPLEX EXPONENTIALS

- Case 2: $x(t) = Ce^{at}$, where C is real, and a is purely imaginary.
- This signal is periodic: why?

CT Periodic Complex Exponentials

- Consider a simple case where $C = 1, a = j\omega_0$. Here ω_0 is real.
- This signal becomes: $x(t) = e^{j\omega_0 t}$
- If there exist a non-zero period T such that:

$e^{j\omega_0 T} = 1$, then

$$x(t) = x(t)e^{j\omega_0 T} = e^{j\omega_0 (t+T)} = x(t+T)$$

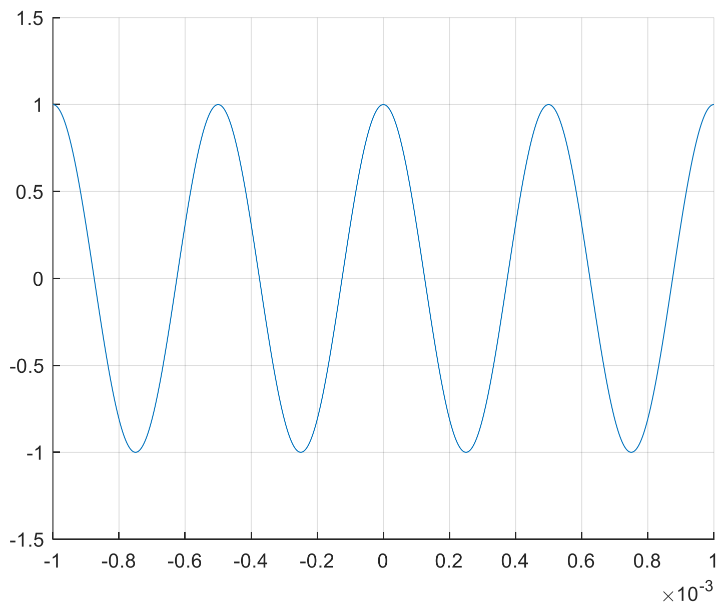
- Using Euler's relation:
- $e^{j\omega_0 T} = \cos(\omega_0 T) + j\sin(\omega_0 T)$ = 1, then:
- $T = \frac{2k\pi}{\omega_0}, k = 0, \pm 1, \pm 2, \pm 3, \ldots$

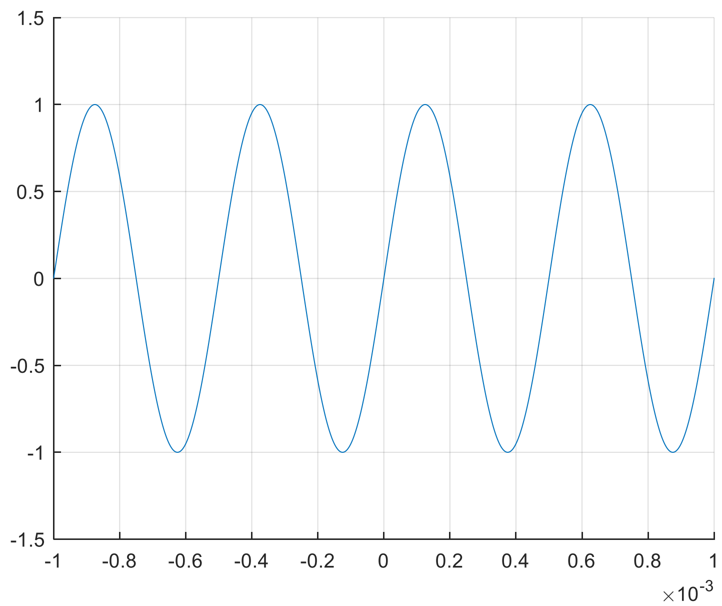
CT Periodic Complex Exponential and CT Sinusoidal Signals

- CT periodic complex exponential signals and CT sinusoidal signals are closely related to each other.
- Follow the previous example, using Euler's relation: a CT period complex exponential can be written as follows:
- $x(t) = Ae^{j\omega_0 t} = \mathrm{A}\left(\cos(\omega_0 t) + j\sin(\omega_0 t)\right)$
- $Re\{x(t)\} = A\cos(\omega_0 t),\ Im\{x(t)\} = A\sin(\omega_0 t)$
- So, the real and imaginary parts of a CT periodic exponential signal are both sinusoidal.

CT Sinusoidal Signals

- Case 2: $x(t) = Ce^{at}$, where C is real, and a is purely imaginary.
- $Re\{x(t)\} = A\cos(\omega_0 t),\ Im\{x(t)\} = A\sin(\omega_0 t)$





CT Complex Exponentials

- Case 3: $x(t) = Ce^{at}$, where C and a are both complex.
- Express C in polar format and a in rectangular format: $C = |C|e^{j\theta}$ and $a = r + j\omega_0$
- Thus:
- $x(t) = Ce^{at} = |C|e^{j\theta}e^{(r+j\omega_0)t} = \boxed{|C|e^{rt}}e^{j(\omega_0 t+\theta)}$

$|C|e^{rt}$: case 1, decaying or growing exponential

CT Complex Exponentials

- Case 3: $x(t) = Ce^{at}$, where C and a are both complex.
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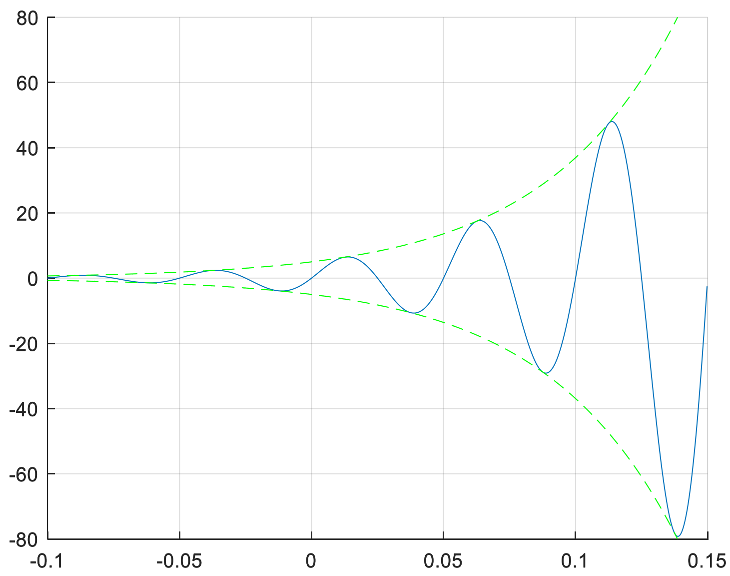
- Thus:
- $x(t) = Ce^{at} = |C|e^{j\theta}e^{(r+j\omega_0)t} = \boxed{|C|e^{rt}}\boxed{e^{j(\omega_0 t+\theta)}}$

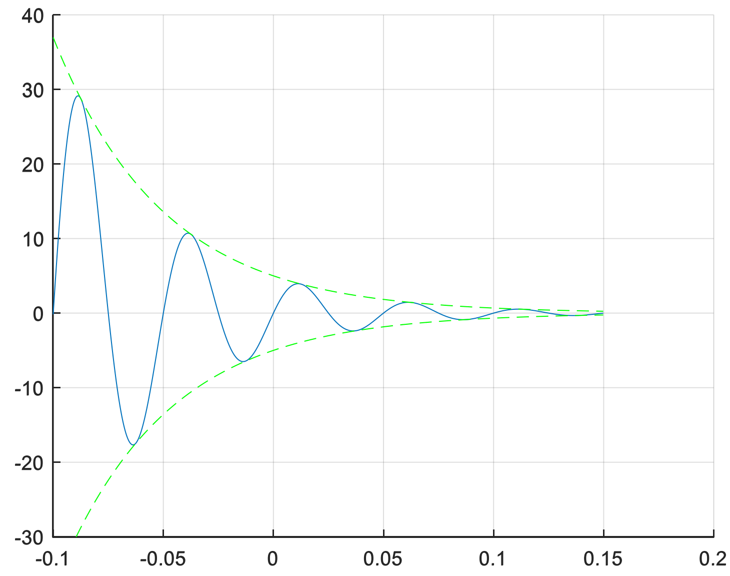
$|C|e^{rt}$: case 1, decaying or growing exponential

$e^{j(\omega_0 t+\theta)}$: case 2, periodic sinusoids in real and imaginary planes.

CT COMPLEX EXPONENTIALS

- Case 3: $x(t) = Ce^{at}$, where C and a are both complex.



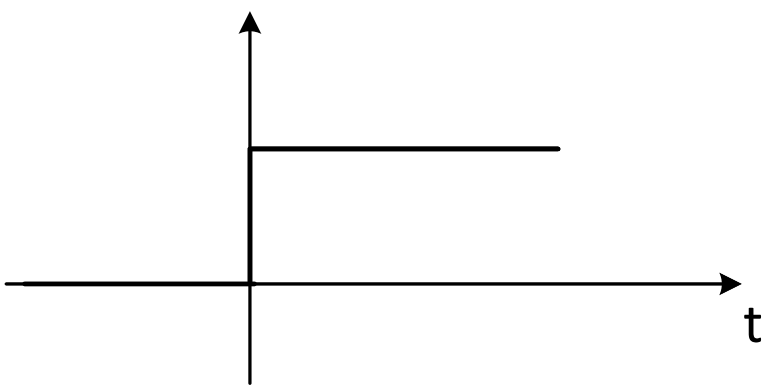


Most commonly used CT functions:

- Exponentials and sinusoids
- Unit step function
- Unit impulse function

Unit step function (Heaviside function):

$$u(t) = \begin{cases} 0, t < 0 \\ \dfrac{1}{2}, t = 0 \\ 1, t > 0 \end{cases}$$



Is this a continuous function of time?

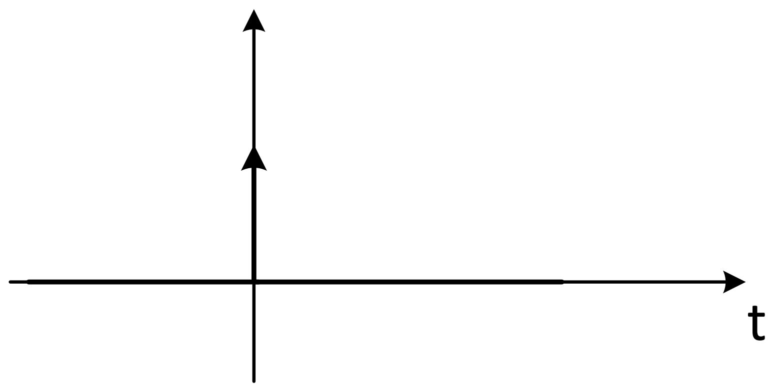
Is this a continuous-time function?

Most commonly used CT functions:

- Exponentials and sinusoids
- Unit step function
- Unit impulse function

Unit impulse function:

$$\delta(t) = \frac{du(t)}{dt} \quad \Longleftrightarrow \quad u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$



Unit impulse function (Dirac / Delta function):

- Very useful in signal and system analysis
- It is a mathematical idealization.
- Defined through its property rather than values at each point, thus strictly speaking, not a function.

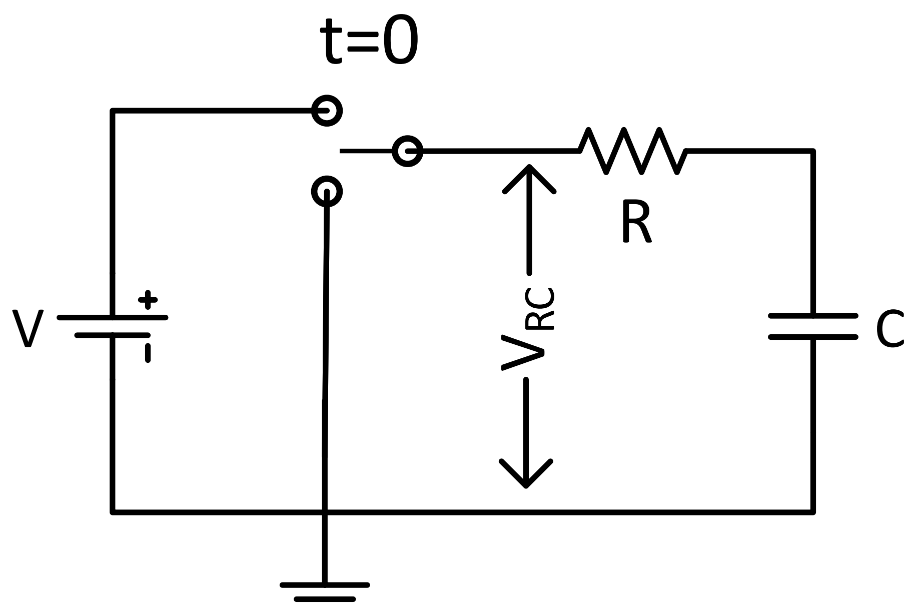
Additional features of the Dirac function:

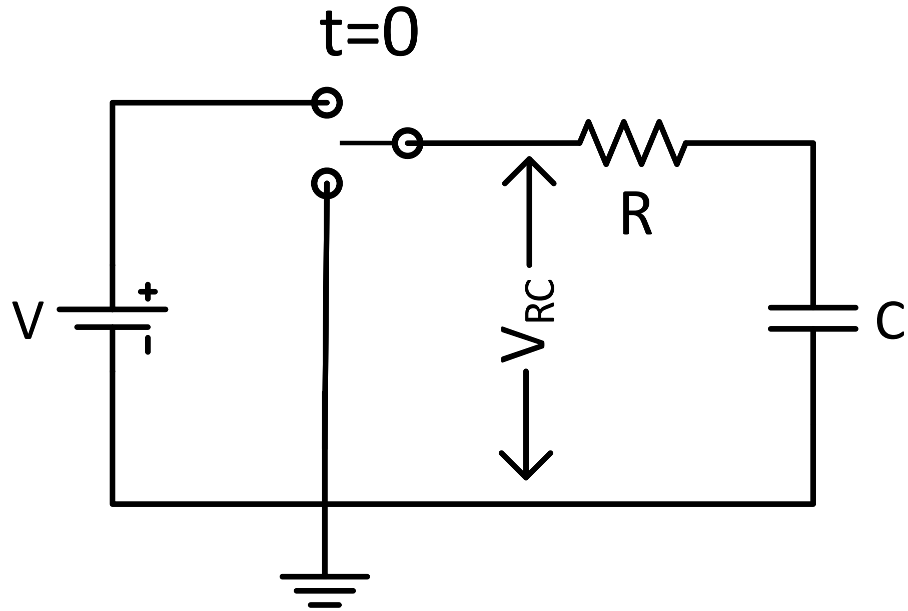
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$x(t)\delta(t) = x(0)\delta(t)$$
$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)dt$$
$$= \int_{-\infty}^{\infty} x(t_0)\delta(t - t_0)dt = x(t_0)$$

Activity: this circuit below is controlled by a SPDT switch, at time t=0, the switch is switched from the bottom to the up position. Please use CT functions to illustrate the voltage and the current V_{RC} and I_{RC}.

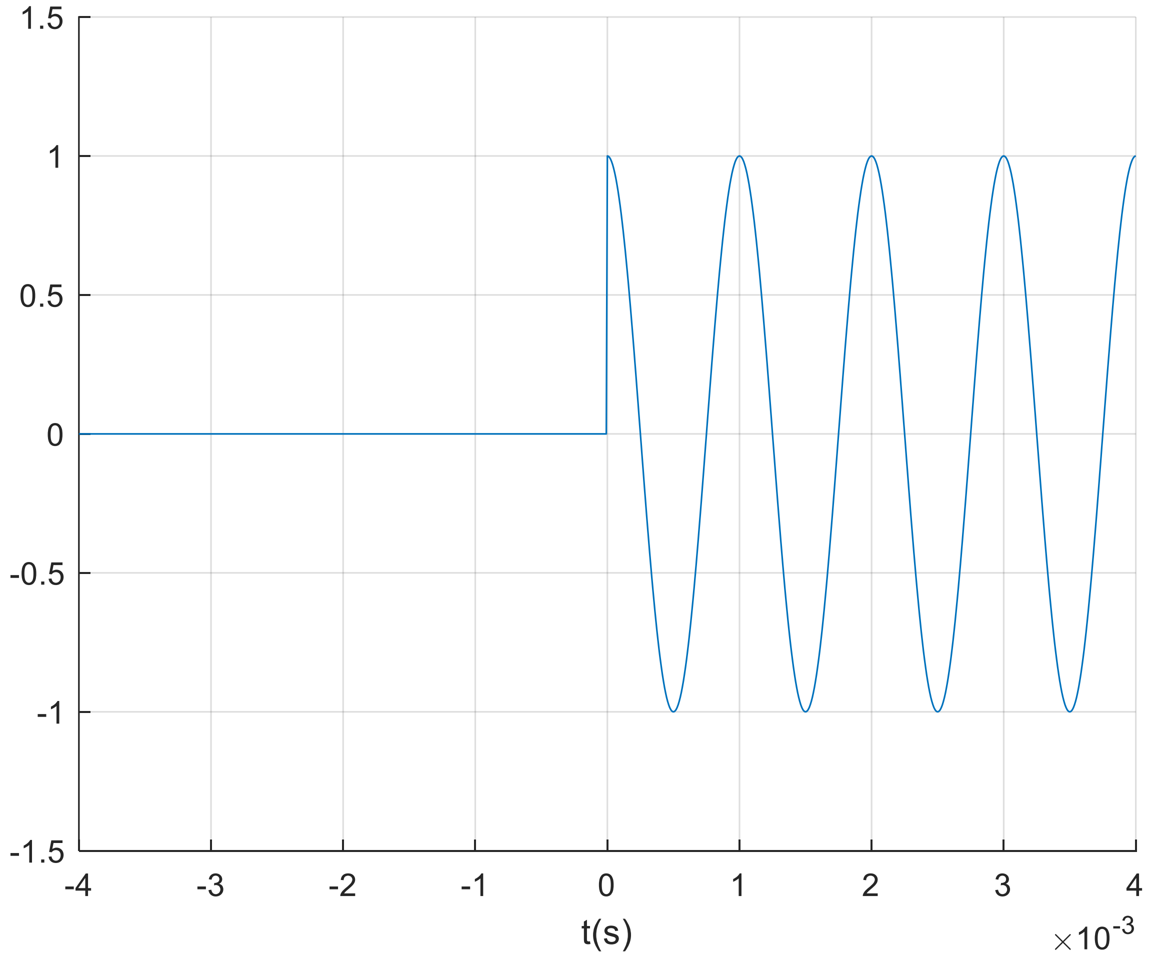




$$V_{RC}(t) = Vu(t)$$

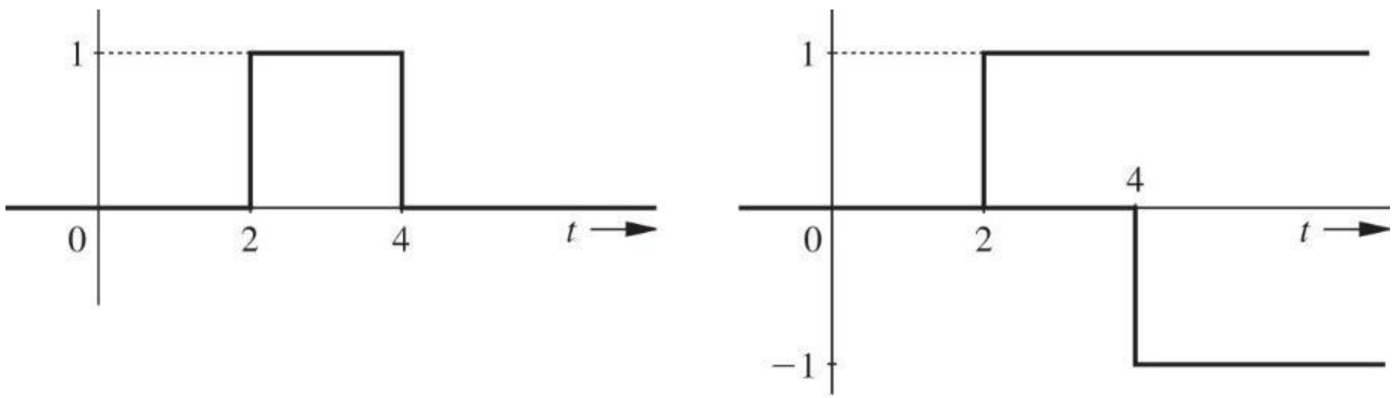
$$I_{RC}(t) = \frac{V}{R}e^{-t/RC}u(t)$$

Activity: write the corresponding CT function of the signal below:



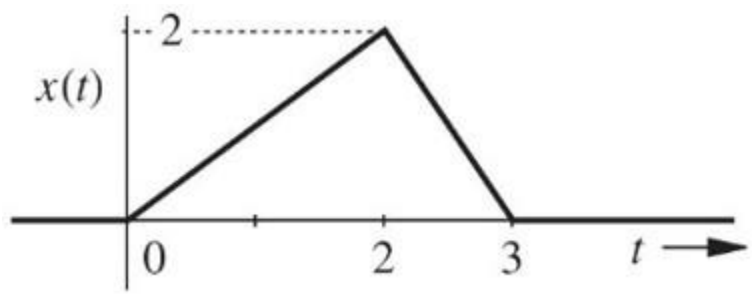
Activity: Representation of a rectangular pulse by unit step function

$$x(t) = u(t-2) - u(t-4)$$



(a) (b)

Activity: Representation of the following pulse by unit step function

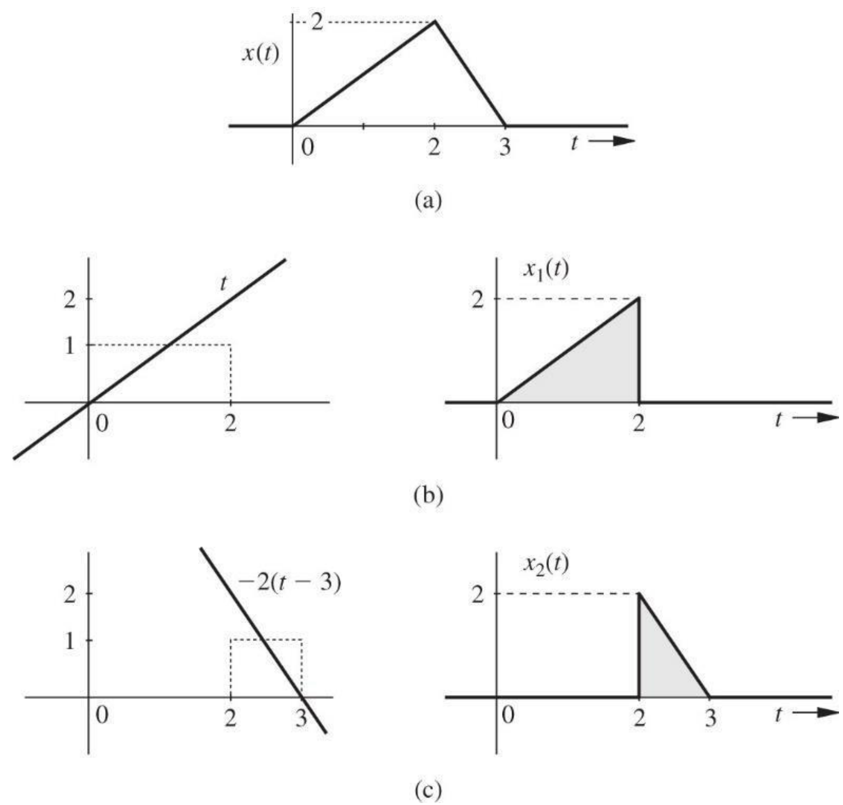


(a)

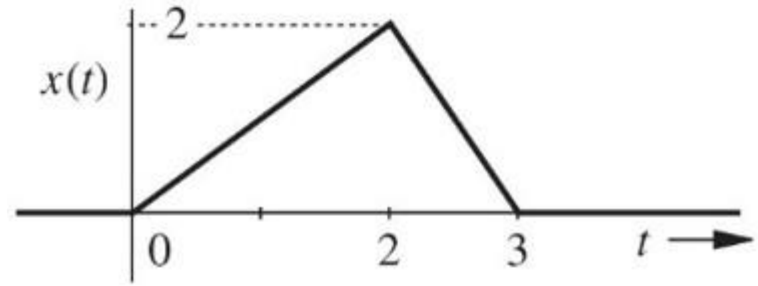
Solution:

$$x_1(t) = t[u(t) - u(t-2)]$$

$$x_2(t) = -2(t-3)[u(t-2) - u(t-3)]$$



Solution(contd.)



(a)

$$x(t) = x_1(t) + x_2(\mathrm{t})$$

$$x(t) = t[u(t) - u(t-2)] - 2(t-3)[u(t-2) - u(t-3)]$$

$$x(t) = tu(t) - tu(t-2) - 2(t-3)u(t-2) + 2(t-3)u(t-3)$$

$$x(t) = tu(t) - 3(t-2)u(t-2) + 2(t-3)u(t-3)$$

1.3.2

Independent- and Dependent-Variable Transformation of CT Signals

BASIC SIGNAL TRANSFORMATION

$$f = y(t)$$

Dependent variable transformation:

- Shifting: amplitude translation
- Scaling: amplitude scaling

Independent variable transformation:

- Shifting: time translation/time shifting
- Scaling: time scaling.

BASIC SIGNAL TRANSFORMATION

Dependent variable transformation:

- Shifting: amplitude translation
- Scaling: amplitude scaling

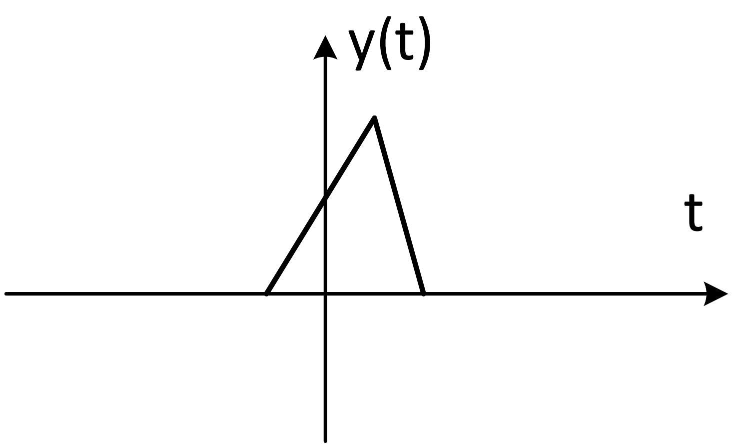
Independent variable transformation:

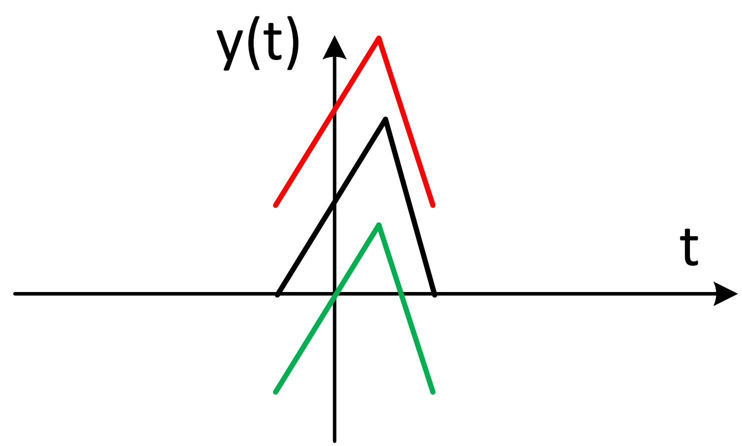
- Shifting: time translation/time shifting
- Scaling: time scaling.

Basic Signal Transformation

Dependent variable transformation:

- Shifting: amplitude translation
- $y(t) \longrightarrow y(t) + D$

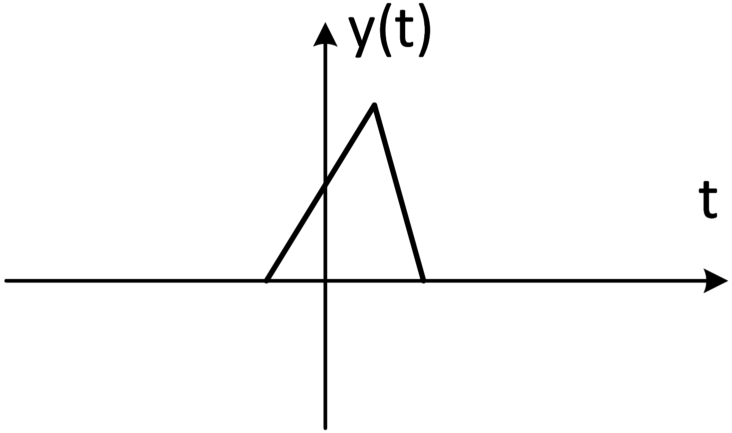


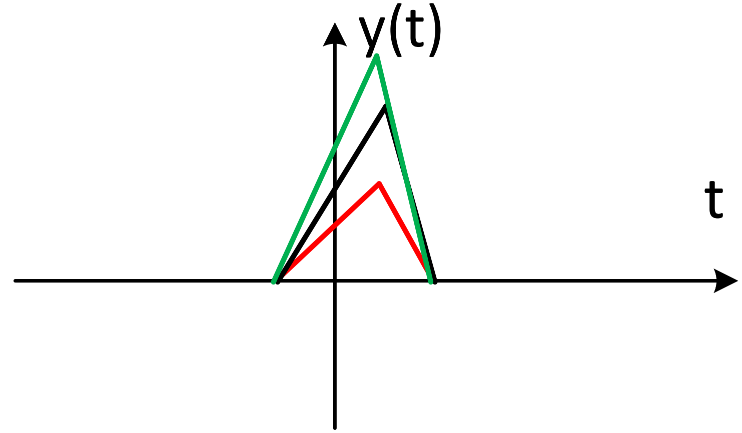


BASIC SIGNAL TRANSFORMATION

Dependent variable transformation:

- Scaling: amplitude scaling
- $y(t) \longrightarrow Ay(t)$

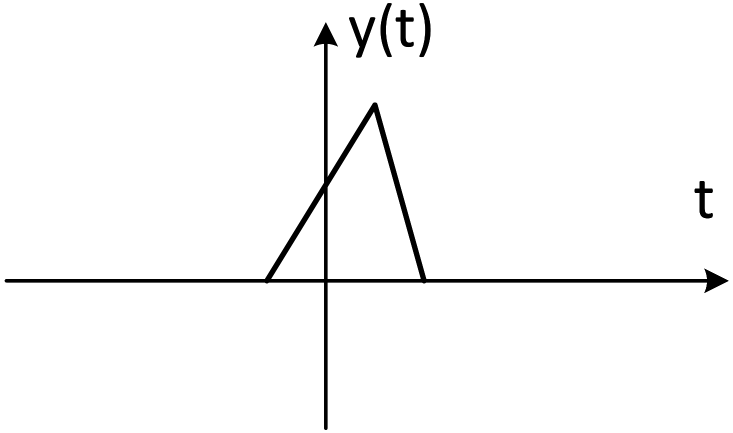


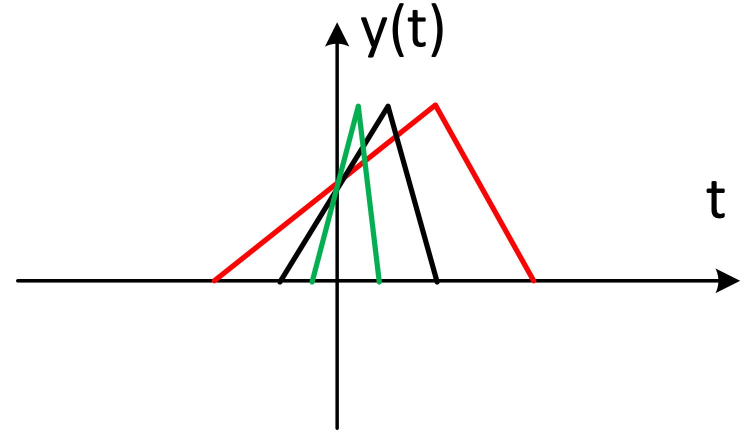


BASIC SIGNAL TRANSFORMATION

Independent variable transformation:

- Scaling: time scaling
- $y(t) \longrightarrow y(at)$

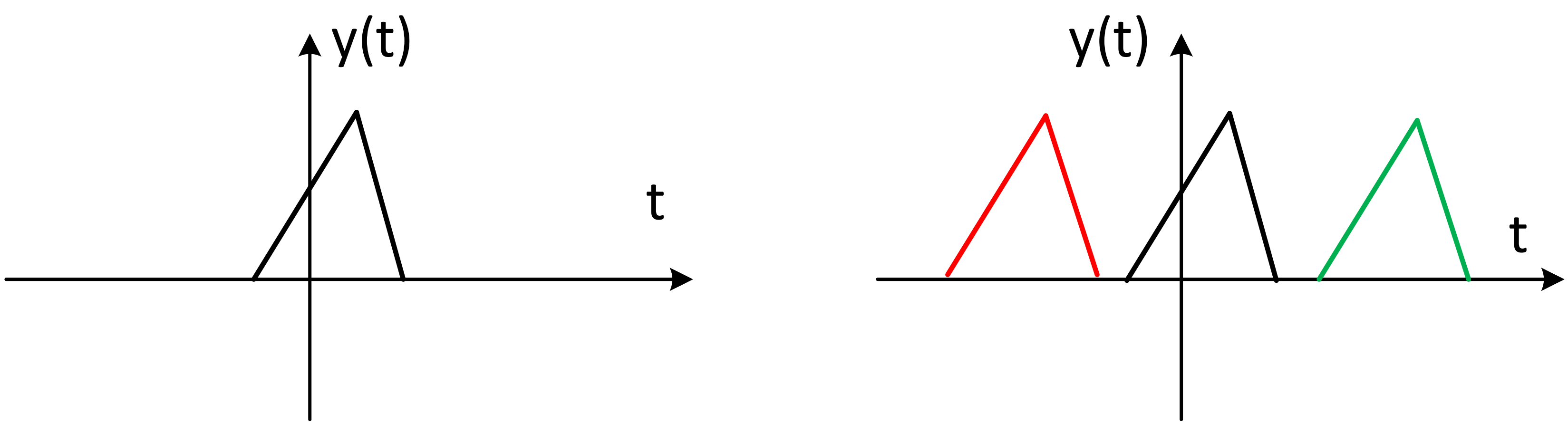




Basic Signal Transformation

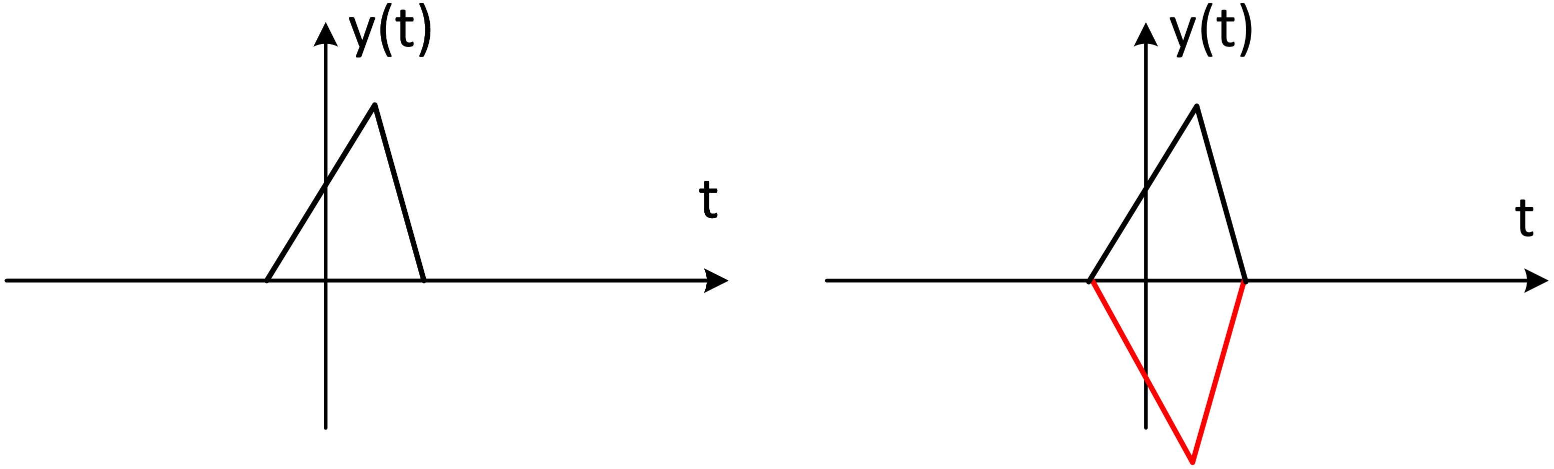
Independent variable transformation:

- Shifting: time translation
- $y(t) \longrightarrow y(t - t_0)$



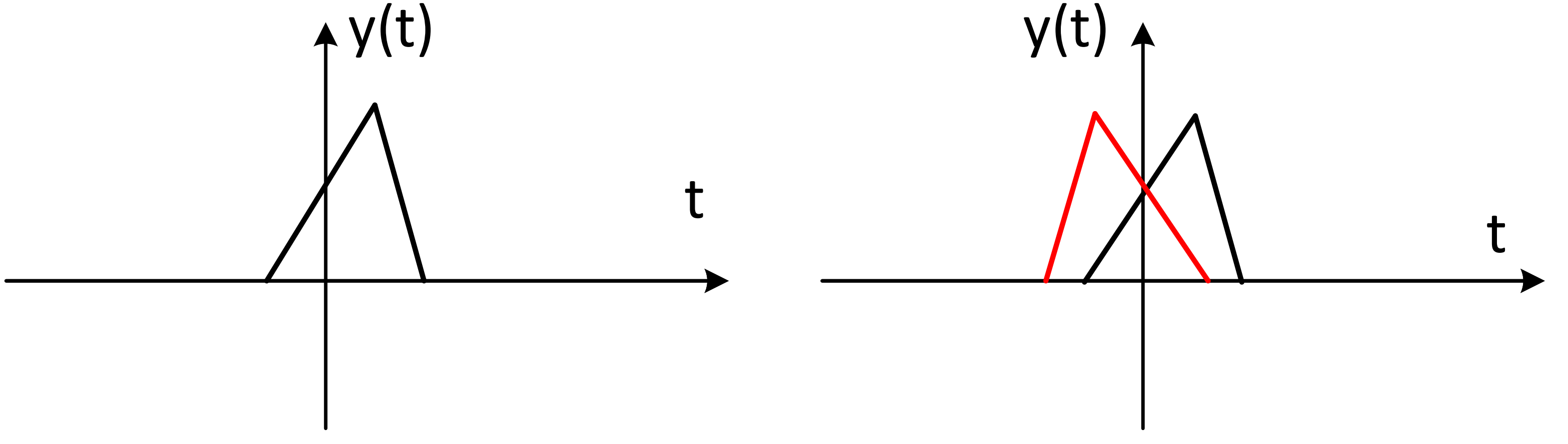
Basic Signal Transformation

Activity: what type of transformation?



Basic Signal Transformation

Activity: what type of transformation?



BASIC SIGNAL TRANSFORMATION

Activity: what type of transformation?