Tutorial 1

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Tutorial 1: Units, Measurements and Vectors

PART A – Units and Measurements

The seven base units:

- **LENGTH (meter)**
- **MASS (kilogram)**
- **TIME (second)**
- **ELECTRIC CURRENT (ampere)**
- **TEMPERATURE (kelvin)**
- **AMOUNT OF SUBSTANCE (mole)**
- **BRIGHTNESS (candela)**

Definitions:

The three fundamental physical quantities of mechanics are ....**length**...., ...**mass**...., and ....**time**...., which in the SI system have the units ....**meter**.... (m), ...**kilogram**.... (kg), and ....**second**.... (s), respectively.

The fundamental quantities ....**cannot**.... be defined in term of more basic quantities.

The ....**density**.... of a substance is defined as its *mass per unit volume*.

Problem (Conversion of Units):

1) A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm^3. From these data, calculate the density of lead in SI units (kilogram per cubic meter).

We know that: $1\ kg = 1000\ g \ \rightarrow\ 23.94\ g = (23.94\ g)\left(\frac{1\ kg}{1000\ g}\right) = 23.94 \times 10^{-3}\ kg$

We know that: $1\ m^3 = 10^6\ cm^3 \ \rightarrow\ 2.10\ cm^3 = (2.10\ cm^3)\left(\frac{1\ m^3}{10^6\ cm^3}\right) = 2.10 \times 10^{-6}\ m^3$

$$\rho = \frac{m}{V} = \frac{23.9 \times 10^{-3}\ kg}{2.10 \times 10^{-6}\ m^3} = 1.14 \times 10^4\ kg/m^3$$

2) One gallon of paint (volume = 3.78 × 10^{-3} m^3) covers an area of 25.0 m^2. What is the thickness of the fresh paint on the wall?

$$V = A \times d \ \rightarrow\ d = \frac{V}{A} = \frac{3.78 \times 10^{-3}\ m^3}{25.0\ m^2} = 1.51 \times 10^{-4}\ m$$

Area of the wall

Thickness of the paint on the wall

3) An auditorium measures 40.0 m × 20.0 m × 12.0 m. The density of air is 1.20 kg/m^3. What is the volume of the room in cubic feet?

We know that: $1\ m = 3.281\ ft$

$$V = (40.0\ m)(20.0\ m)(12.0\ m) = 9.60 \times 10^3\ m^3$$

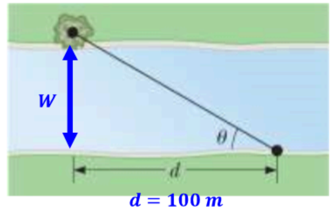
$$V = (9.60 \times 10^3\ m^3)\left(\frac{3.281\ ft}{1\ m}\right)^3 = 3.39 \times 10^5\ ft^3$$

Problem (Modeling):

A surveyor measures the distance across a straight river by the following method: Starting directly across from a tree on the opposite bank, she walks $d = 100\ m$ along the riverbank to establish a baseline. Ten she sights across to the tree. The angle from her baseline to the tree is $\theta = 35.0°$. How wide is the river?

$$\tan\ \theta = \frac{W}{d} \quad \rightarrow \quad W = d \times tan\theta$$

$$W = (100\ m) \times tan(35.0°) = 70.0\ m$$



Problem (Dimensional Analysis):

Assume the equation $x = \frac{1}{2}At^3 + Bt$ describes the motion of a particular object, with x having the dimension of length and t having the dimension of time.

a) Determine the dimension of the constants A and B.

We show dimension of length with symbol L, and dimension of time with symbol T

$$x = \frac{1}{2} A\, t^3 + B\, t$$

$$L \qquad \frac{L}{T^3} \qquad T^3 \qquad \frac{L}{T} \qquad T$$

b) Determine the dimensions of the derivative $\frac{dx}{dt} = \frac{3}{2}At^2 + B$.

$$\frac{dx}{dt} = \frac{3}{2} A\, t^2 + B$$

$$\frac{L}{T} \qquad \frac{L}{T^3} \qquad T^2 \qquad \frac{L}{T}$$

Activity (Breaths in a Lifetime):

Estimate the number of breaths taken during an average human lifetime. Start by guessing the typical human lifetime. Then, think about the average number of breaths that a person takes in 1 min. (This number varies depending on whether the person exercising, sleeping, angry and so forth.)

a) First find the approximate number of minutes in a year

$$1\ yr = (1\ yr)\left(\frac{365\ days}{1\ yr}\right)\left(\frac{24\ h}{1\ day}\right)\left(\frac{60\ min}{1\ h}\right) = 5.26 \times 10^5\ min$$

b) Find the approximate number of minutes in the average human lifetime

Assume that the average human life is between 70 to 80 years. Here, we consider it as 70 years.

$$70\ years = (70\ yr)\left(\frac{5.26 \times 10^5\ min}{1\ yr}\right) = 36.8 \times 10^6\ min$$

c) Approximate number of breaths in a lifetime.

Assume that the average number of breaths in a minute is between 10 to 15. Here we consider 15 breaths.

number of breaths in one year $= (36.8 \times 10^6\ min)\left(\frac{15\ breaths}{1\ min}\right) = 5.52 \times 10^8\ breaths$

d) Compare your results with other students. How the estimated average lifetime affects your final estimate?

You may consider the average human life 75 years instead of 70 years. In this case the number of breaths in a lifetime will be as below:

$$75\ years = (75\ yr)\left(\frac{5.26\times10^{5}\ min}{1\ yr}\right) = 39.5\times10^{6}\ min$$

$$\text{number of breaths} = (39.5\times10^{6}\ min)\left(\frac{15\ breath}{1\ min}\right) = 5.93\times10^{8}\ breaths$$

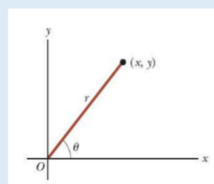
PART B – Vectors and Vector Operations

Coordinate Systems:

1) Cartesian Coordinate System
2) Polar Coordinate System

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$\tan\theta = \frac{y}{x} \qquad r = \sqrt{x^2+y^2}$$



Vector and Scaler Quantities:

A vector is a quantity that has both magnitude and direction.

Vector Operations:

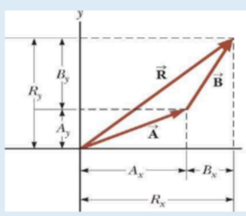
$\vec{\mathbf{A}}+\vec{\mathbf{B}}=\vec{\mathbf{B}}+\vec{\mathbf{A}}=\vec{\mathbf{R}}$

$\vec{\mathbf{A}}-\vec{\mathbf{B}}=\vec{\mathbf{A}}+(-\vec{\mathbf{B}})$

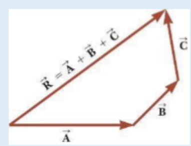
$\vec{\mathbf{A}}+(-\vec{\mathbf{A}})=0$

$$\vec{\mathbf{R}}=\vec{\mathbf{A}}+\vec{\mathbf{B}}=\left(A_x\hat{\mathbf{i}}+A_y\hat{\mathbf{j}}\right)+\left(B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{R}}=\left(A_x+B_x\right)\hat{\mathbf{i}}+\left(A_y+B_y\right)\hat{\mathbf{j}}$$



Component Method



Head-to-Tail Method

Definitions:

Scalar quantities are those have only a numerical value and no associated direction.

Vector quantities have both **magnitude** and **direction** and obey the laws of **vector** addition.

The magnitude of a vector is always a **positive** number.

Problems (Coordinate Systems):

1) Two points in the xy plane have Cartesian coordinates (2.00, -4.00) m and (-3.00, 3.00) m.

a) Determine the distance between these points.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-3.00-2.00)^2+(3.00-(-4.00))^2}$$

$$d=\sqrt{25.0+49.0}=8.60\ m$$

b) Determine their polar coordinates.

$$r_1=\sqrt{x_1^2+y_1^2}=\sqrt{(2.00)^2+(-4.00)^2}=\sqrt{20.0}=4.47\ m\ ,\qquad \theta_1=tan^{-1}\left(\frac{y_1}{x_1}\right)=tan^{-1}\left(\frac{-4.00}{2.00}\right)=296.6°$$

$$r_2=\sqrt{x_2^2+y_2^2}=\sqrt{(-3.00)^2+(3.00)^2}=\sqrt{18.0}=4.24\ m\ ,\qquad \theta_2=tan^{-1}\left(\frac{y_2}{x_2}\right)=tan^{-1}\left(\frac{3.00}{-3.00}\right)=135°$$

$(r_1,\theta_1)=(4.47\ m, 296.6°)$, $(r_2,\theta_2)=(4.24\ m, 135°)$, **Angles are measured from the $+x$ axis**

2) Two points in a plane have polar coordinates (2.50 m, 30.0°) and (3.80 m, 120.0°).

a) Determine the Cartesian coordinates of these points

$x_1 = r_1 Cos\ \theta_1 = (2.50\ m)cos(30.0°) = 2.17\ m$ $(x_1, y_1) = (2.17, 1.25)m$

$y_1 = r_1 Sin\ \theta_1 = (2.50\ m)sin(30.0°) = 1.25\ m$

$x_2 = r_2 Cos\ \theta_2 = (3.80\ m)cos(120.0°) = -1.90\ m$ $(x_2, y_2) = (-1.90, 3.29)m$

$y_2 = r_2 Sin\ \theta_2 = (3.80\ m)sin(120.0°) = 3.29\ m$

b) Determine the distance between them.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1.90 - 2.17)^2 + (3.29 - 1.25)^2}$$

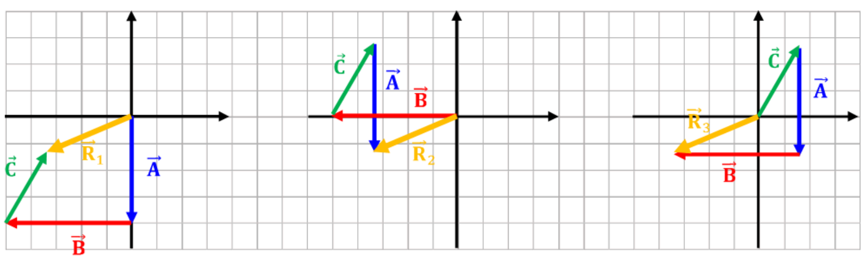
$$d = \sqrt{(-4.07)^2 + (2.04)^2} = 4.55\ m$$

Problems (Vector Arithmetic):

1) Three displacements are $\vec{A} = 200\ m$ due south, $\vec{B} = 250\ m$ due west, and $\vec{C} = 150\ m$ at 30.0° east of north.

a) Construct a separate diagram for each of the following possible ways of adding these vectors:

$\vec{R}_1 = \vec{A} + \vec{B} + \vec{C}$, $\vec{R}_2 = \vec{B} + \vec{C} + \vec{A}$, $\vec{R}_3 = \vec{C} + \vec{A} + \vec{B}$



Scale = 50 m each unit

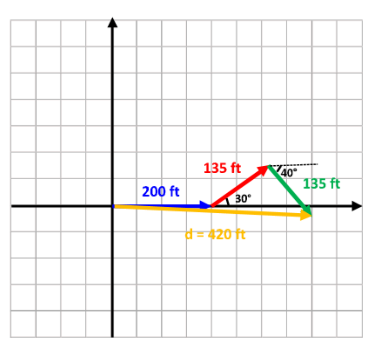
b) Explain what you can conclude from comparing the diagrams.

Comparing the diagrams show that order of vectors in addition is irrelevant.

2) A roller-coaster car moves 200 ft horizontally and then rises 135 ft at an angle of 30.0° above the horizontal. It next travels 135 ft at an angle of 40.0° downward. What is its displacement from its starting point? Use graphical techniques.

The magnitude and direction of displacement from starting point are obtained by measuring the length of the total displacement vector and the angle and applying the scale factor used in drawing. The results should be:

$d = 420\ ft$ **and** $\theta = -3°\ or + 357°$



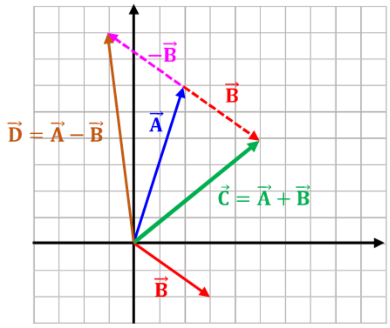
Scale = 50 ft each unit

Scale = 50 ft each unit

Problems (Components of a Vector and Unit Vectors):

1) Given the vectors $\vec{\mathbf{A}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$.

a) Draw the vector sum $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ and the vector difference $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$. Use Head-to-Tail method.



Here, we applied the Head-to-Tail Method to draw the vector sum and difference.

b) Calculate $\vec{\mathbf{C}}$ and $\vec{\mathbf{D}}$ in terms of unit vectors.

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}) + (3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}) = \underbrace{5.00}_{C_x}\hat{\mathbf{i}} + \underbrace{4.00}_{C_y}\hat{\mathbf{j}}$$

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = (2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}) - (3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}) = \underbrace{-1.00}_{D_x}\hat{\mathbf{i}} + \underbrace{8.00}_{D_y}\hat{\mathbf{j}}$$

c) Calculate $\vec{\mathbf{C}}$ and $\vec{\mathbf{D}}$ in terms of polar coordinates.

$\vec{\mathbf{C}} = C_x\hat{\mathbf{i}} + C_y\hat{\mathbf{j}}$

Magnitude: $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(5.00)^2 + (4.00)^2} = \sqrt{25.0 + 16.0} = 6.40$

Angle: $\theta = tan^{-1}\left(\frac{C_y}{C_x}\right) = tan^{-1}\left(\frac{4.00}{5.00}\right) = 38.7°$

$\vec{\mathbf{D}} = D_x\hat{\mathbf{i}} + D_y\hat{\mathbf{j}}$

Magnitude: $D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-1.00)^2 + (8.00)^2} = \sqrt{1.00 + 64.0} = 8.06$

Angle: $\theta = tan^{-1}\left(\frac{D_y}{D_x}\right) = tan^{-1}\left(\frac{8.00}{-1.00}\right) = 97.2°$

Activity:

You are working at a radar station for the Coast Guard. While everyone else is out to lunch, you hear a distress call from a sinking ship. The ship is located at a distance of 51.2 km from the station, at a bearing of 36° west of north. On your radar screen, you see the locations of four other ships as follows:

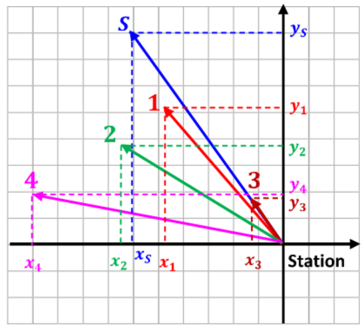
Ship Number	Distance from Station (km)	Bearing	Maximum Speed (km/h)
1	36.1	42° W of N	30.0
2	37.3	61° W of N	38.0
3	10.2	36° W of N	32.0
4	51.2	79° W of N	45.0

Which ship do you contact to help the sinking ship? Which ship will get there in the shortest time interval? Assume that each ship would accelerate quickly to its maximum speed and then maintain that interval? Assume that each ship would accelerate quickly to its maximum speed and then maintain that constant speed in a straight line for the entire trip to the sinking ship.

Hint: You need to find the x and y components of the four ships, and the sinking ship.
Next, find the distance between the sinking ship and each of the other ships.
Finally, find the required time interval for each ship to reach the sinking ship and compare.

First, find the x and y components of the four ships, and the sinking ship.

Ship	$x\ (km)$	$y\ (km)$
1	-24.2	26.8
2	-32.6	18.1
3	-6.00	8.25
4	-50.3	9.77
Sinking	-30.1	41.4



Scale = 5 km each unit

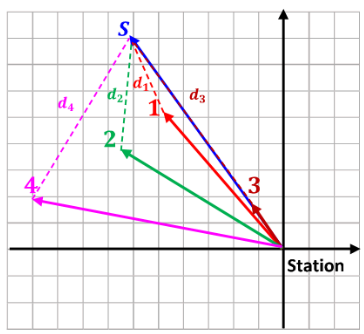
$$x_1 = (36.1\ km)\cos(132°) = -24.2\ km$$
$$y_1 = (36.1\ km)\sin(132°) = 26.8\ km$$

$$x_2 = (37.3\ km)\cos(151°) = -32.6\ km$$
$$y_2 = (37.3\ km)\sin(151°) = 18.1\ km$$

$$x_3 = (10.2\ km)\cos(126°) = -6.00\ km$$
$$y_3 = (10.2\ km)\sin(126°) = 8.25\ km$$

$$x_4 = (51.2\ km)\cos(169°) = -50.3\ km$$
$$y_4 = (51.2\ km)\sin(169°) = 9.77\ km$$

$$x_S = (51.2\ km)\cos(126°) = -30.1\ km$$
$$y_S = (51.2\ km)\sin(126°) = 41.4\ km$$



Next, find the distance between the sinking ship and each of the other ships.

Ship	Distance from sinking ship $d\ (km)$
1	15.8
2	23.5
3	41.0
4	37.6

$$d_1 = \sqrt{(x_1 - x_S)^2 + (y_1 - y_S)^2} = \sqrt{\left(-24.2 - (-30.1)\right)^2 + (26.8 - 41.4)^2} = \sqrt{(5.94)^2 + (14.6)^2} = 15.8\ km$$

$$d_2 = \sqrt{(x_2 - x_S)^2 + (y_2 - y_S)^2} = \sqrt{\left(-32.6 - (-30.1)\right)^2 + (18.1 - 41.4)^2} = \sqrt{(-2.53)^2 + (-23.3)^2} = 23.5\ km$$

$$d_3 = \sqrt{(x_3 - x_S)^2 + (y_3 - y_S)^2} = \sqrt{\left(-6.00 - (-30.1)\right)^2 + (8.25 - 41.4)^2} = \sqrt{(24.1)^2 + (-33.2)^2} = 41.0\ km$$

$$d_4 = \sqrt{(x_4 - x_S)^2 + (y_4 - y_S)^2} = \sqrt{\left(-50.3 - (-30.1)\right)^2 + (9.77 - 41.4)^2} = \sqrt{(-20.2)^2 + (-31.7)^2} = 37.6\ km$$

Finally, use the speed, assumed constant, to find the time interval required for the ship to reach the sinking ship:

Ship	Distance from sinking ship	Maximum Speed	Time interval	Time interval

	$d\ (km)$	$v\ (km/h)$	$t\ (hr)$	$t\ (min)$
1	15.8	30.0	0.53	31.8
2	23.5	38.0	0.62	37.2
3	41.0	32.0	1.28	76.8
4	37.6	45.0	0.83	49.8

$$t_1 = \frac{d_1}{v_1} = \frac{15.8\ km}{30.0\ km/h} = 0.53\ hr = 31.8\ min$$

$$t_2 = \frac{d_2}{v_2} = \frac{23.5\ km}{38.0\ km/h} = 0.62\ hr = 37.2\ min$$

$$t_3 = \frac{d_3}{v_3} = \frac{41.0\ km}{32.0\ km/h} = 1.28\ hr = 76.8\ min$$

$$t_4 = \frac{d_4}{v_4} = \frac{37.6\ km}{45.0\ km/h} = 0.83\ hr = 49.8\ min$$

You need to alert Ship 1. Even though it has the lowest speed, it reaches in less time.