HUMBER ENGINEERING

MENG 3510 – Control Systems
LECTURE 7

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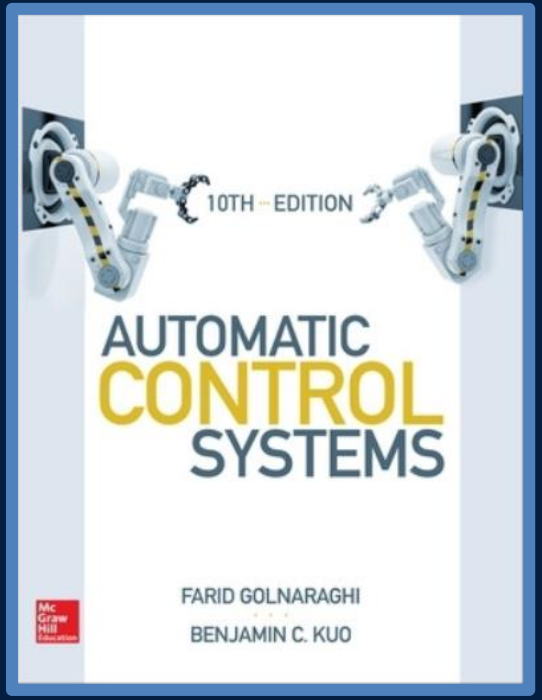
LECTURE 7
Root Locus Design

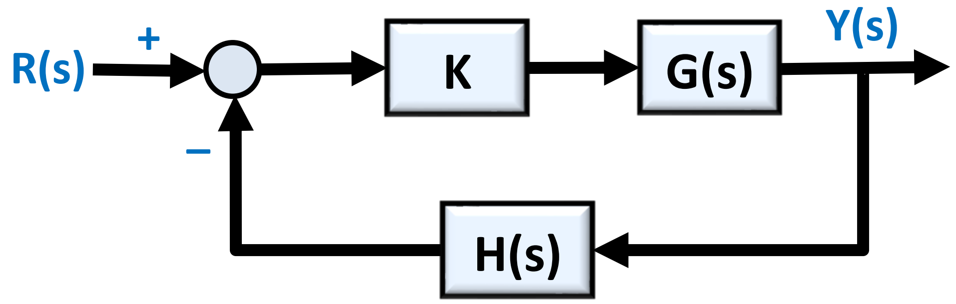
- Properties of Root-Locus
  - Magnitude and Angle Conditions
- Control System Design via Root-Locus
  - Static Feedback Design
  - Effect of Adding a Pole/Zero to Root-Locus
  - Dynamic Compensator Design
    - Lead & Lag Compensators
    - PD & PI Controller



Chapter 9 & 11

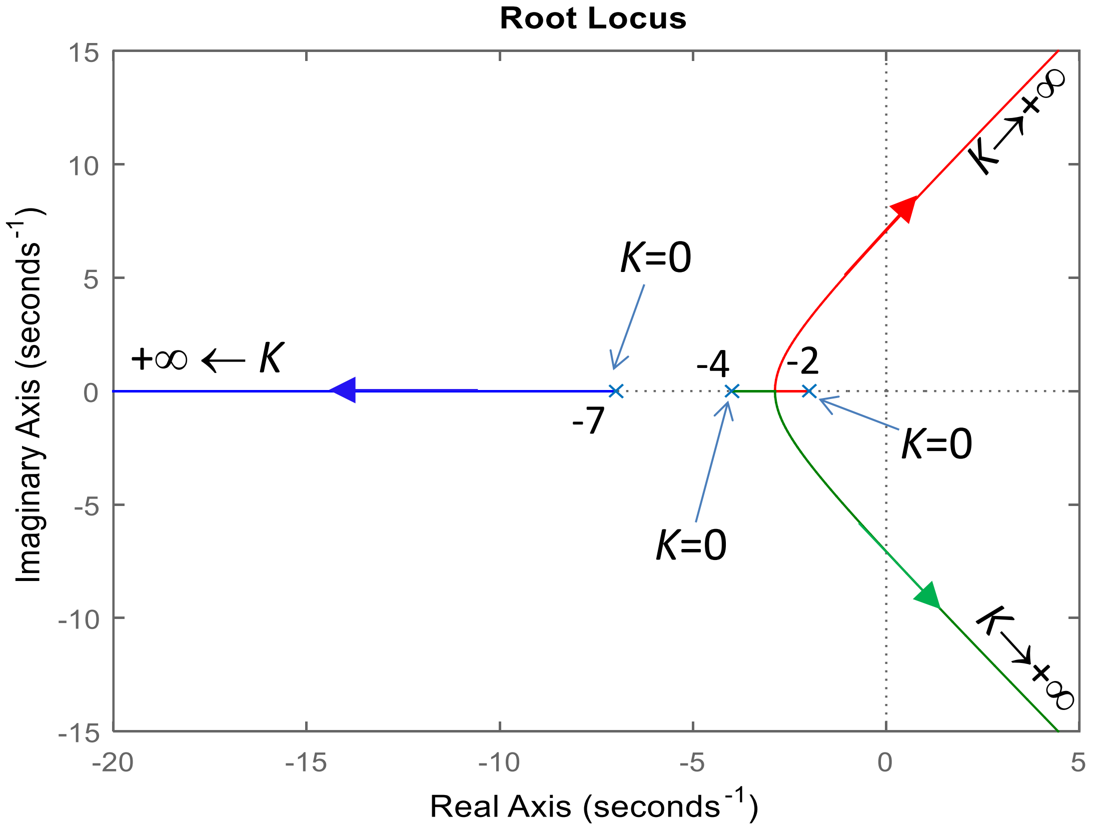
Properties of Root Locus

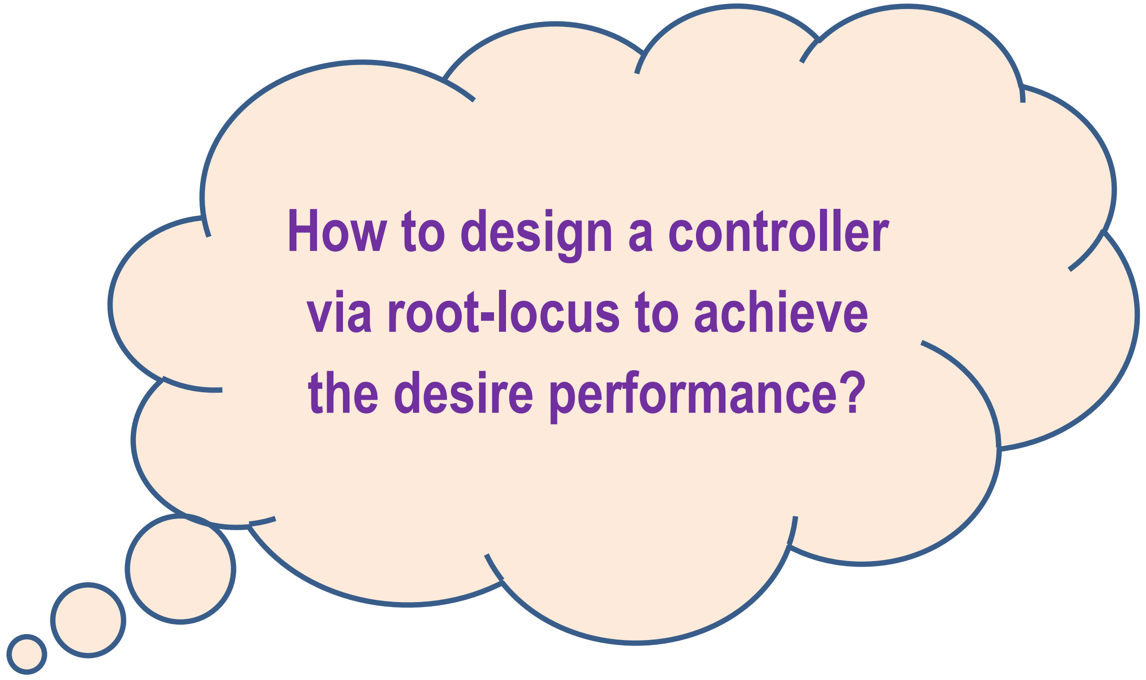
- **Root-locus** is a graphical technique to show the closed-loop pole locations by variation of a certain parameter, such as loop-gain K in the following closed-loop system:



$$T(s) = \frac{Y(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)H(s)}$$

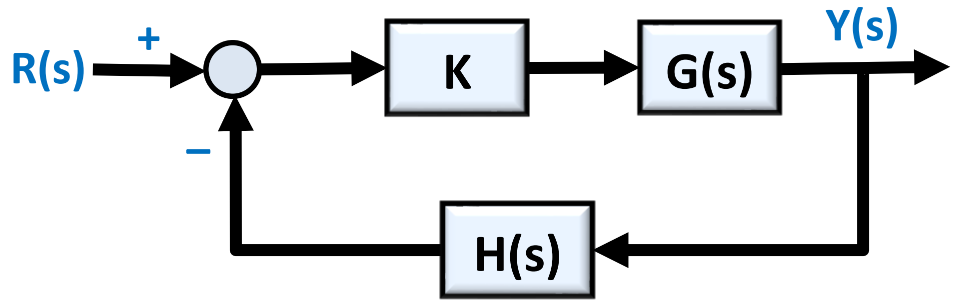
- Stability and performance of the closed-loop system depends on the closed-loop poles location.





Properties of Root Locus

- Consider the following closed-loop system with adjustable gain K



$$\boxed{T(s) = \frac{Y(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)H(s)}}$$

- The closed-loop characteristic equation is

$$1 + KG(s)H(s) = 0 \quad \rightarrow \quad KG(s)H(s) = -1$$

- Since $KG(s)H(s)$ is a complex quantity, it can be written as

$$KG(s)H(s) = \underbrace{|KG(s)H(s)|}_{magnitude} \underbrace{\angle\big(KG(s)H(s)\big)}_{angle} = -1$$

- Therefore, the closed-loop poles must satisfy the following magnitude and angle conditions:

Magnitude and Angle Conditions for $K \in [0 \ +\infty)$:

$$\boxed{|KG(s)H(s)| = 1}$$

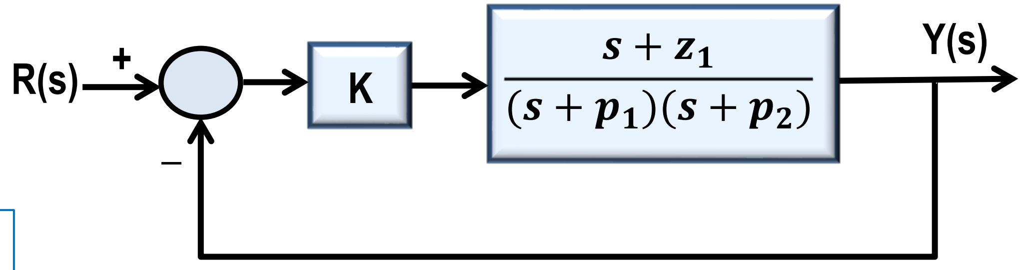
$$\boxed{\angle\big(KG(s)H(s)\big) = \pm(2i+1)180^\circ \ , \qquad i = 0, 1, 2, \cdots}$$

The values of s that fulfill the magnitude and angle conditions are poles of the closed-loop system and located on the root-locus.

Properties of Root Locus - Example

Example 1

Consider the following system with arbitrary poles and zeros at $-p_1,\ -p_2$ and $-z_1$



Assume the point *A* as an arbitrary point in the s-plane.

Angle Condition →
$$\boxed{\angle\left(KG(s)H(s)\right) = \pm(2i+1)180^\circ}$$

Check the angle condition for point A.

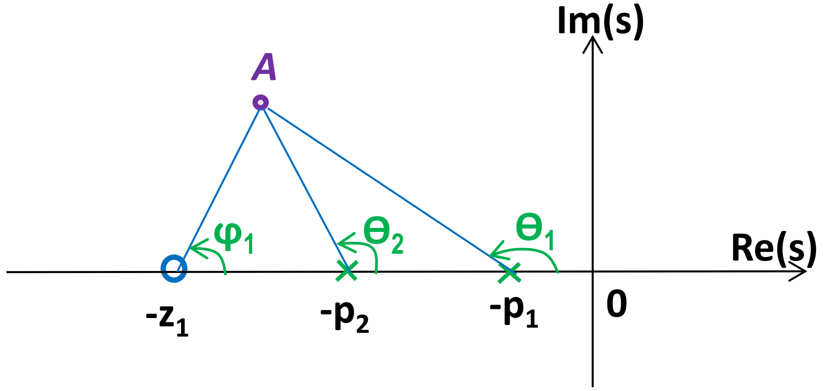
$$\angle\left(K\frac{s+z_1}{(s+p_1)(s+p_2)}\right) = \left(\angle K + \angle(s+z_1)\right) - \left(\angle(s+p_1) + \angle(s+p_2)\right) = \pm(2i+1)180^\circ$$

Method 1: Calculation the angles by evaluation at point A

$$\left(tan^{-1}\left(\frac{\mathrm{Im}[K]}{\mathrm{Re}[K]}\right) + tan^{-1}\left(\frac{\mathrm{Im}[s_A+z_1]}{\mathrm{Re}[s_A+z_1]}\right)\right) - \left(tan^{-1}\left(\frac{\mathrm{Im}[s_A+p_1]}{\mathrm{Re}[s_A+p_1]}\right) + tan^{-1}\left(\frac{\mathrm{Im}[s_A+p_2]}{\mathrm{Re}[s_A+p_2]}\right)\right) = \pm(2i+1)180^\circ$$

Method 2: Geometrically by measuring the angles

$$(0+\varphi_1) - (\theta_1+\theta_2) = \pm(2i+1)180^\circ$$

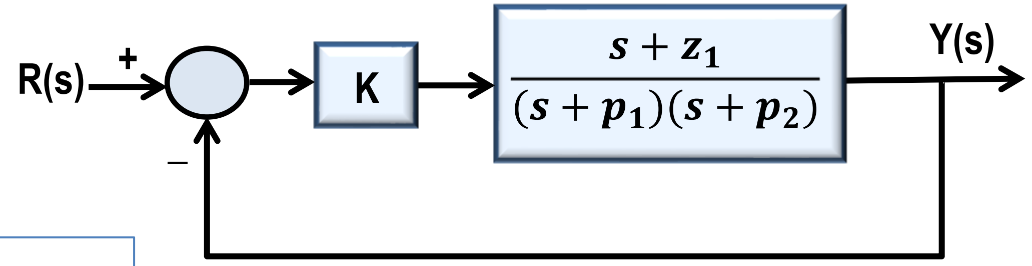


If point *A* satisfies the angle condition it means it is located on the root-locus of this system.

Properties of Root Locus - Example

Example 1

Consider the following system with arbitrary poles and zeros at $-p_1,\ -p_2$ and $-z_1$



Next, we can use the magnitude condition to find the magnitude of K at point A.

Magnitude Condition → $|KG(s)H(s)| = 1 \quad \rightarrow \quad |K| = \dfrac{1}{|G(s)H(s)|}$

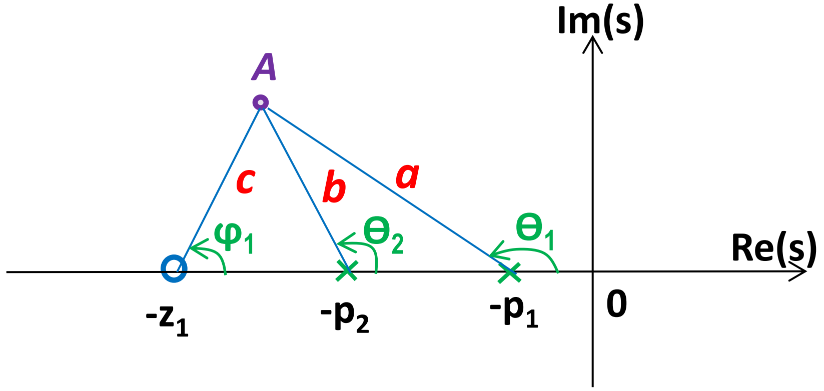
Method 1: Calculation the gain by evaluation at point A

$$|K| = \frac{1}{|G(s)H(s)|} = \left.\frac{|s+p_1||s+p_2|}{|s+z_1|}\right|_{at\ point\ A} = \frac{|s_A+p_1||s_A+p_2|}{|s_A+z_1|}$$

Method 2: Geometrically by measuring length of the vectors

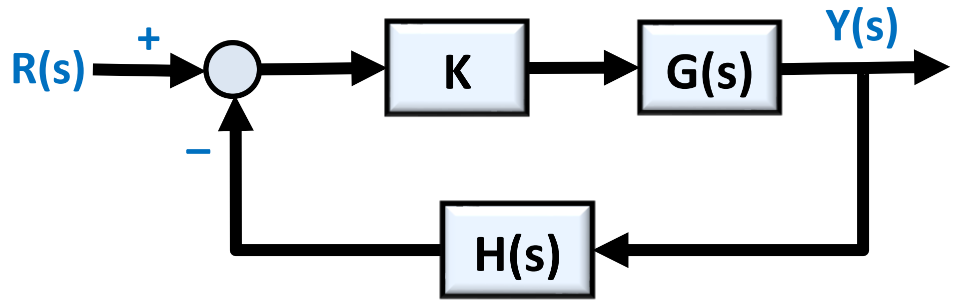
$$|K| = \frac{1}{|G(s)H(s)|} = \left.\frac{|s+p_1||s+p_2|}{|s+z_1|}\right|_{at\ point\ A}$$

$$|K| = \frac{a \times b}{c}$$



Control System Design via Root-Locus

- Consider the following closed-loop system with adjustable gain K



$$\boxed{T(s) = \frac{Y(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)H(s)}}$$

- There are two approaches to design a control system by using the Root-locus diagram:

❑ Static Feedback Design

- Selecting the value of K from root-locus in order to place the closed-loop poles at the desired locations and satisfy the desired performance criteria.
- The technique is similar to the **Proportional Controller** design. Since the control signal $u(t)$ is proportional to the error signal $e(t)$ via the static gain K.

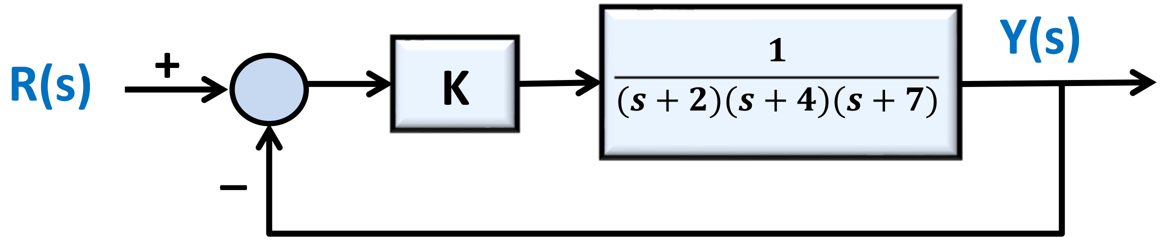
❑ Dynamic Compensator Design

- If the desired performance criteria cannot be obtained by adjusting the gain K *only*, then we need to reshape the root-locus by adding some poles/zeros as a compensator, such as:
  - **Lead & Lag Compensators**
  - **PD & PI Controllers**

Static Feedback Design via Root Locus

Example 2

Consider the following third-order system. Determine the K value so that the maximum overshoot of unit-step response is 10%.



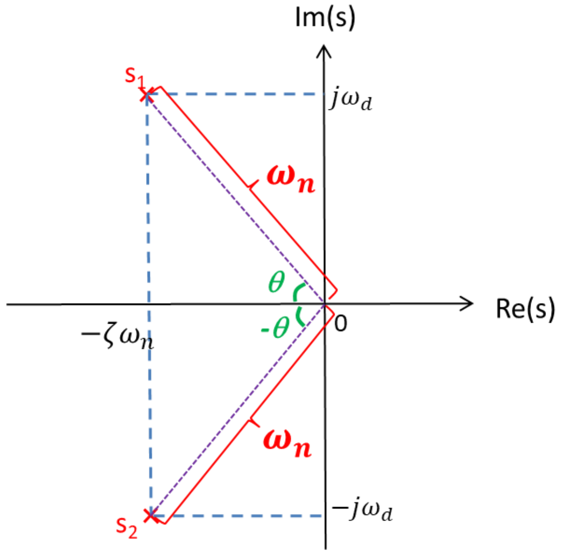
First, calculate the desired damping ratio from the desired maximum overshoot value,

$$\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}} \rightarrow \zeta = \frac{-\ln(0.1)}{\sqrt{\pi^2 + \ln^2(0.1)}} \rightarrow \boxed{\zeta = 0.5912}$$

Desired Damping Ratio

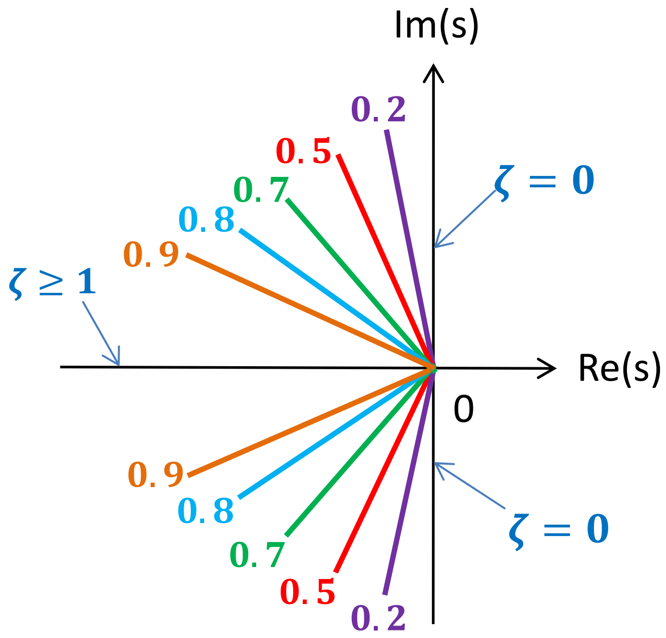
Next, plot the root-locus for this system, and sketch the constant-damping-ratio lines of $\zeta = 0.5912$.

The intersection of the lines with root-locus will be the desired pole locations.



- Recall that the constant-damping-ratio ζ loci in the s-plane are radial lines passing through the origin.

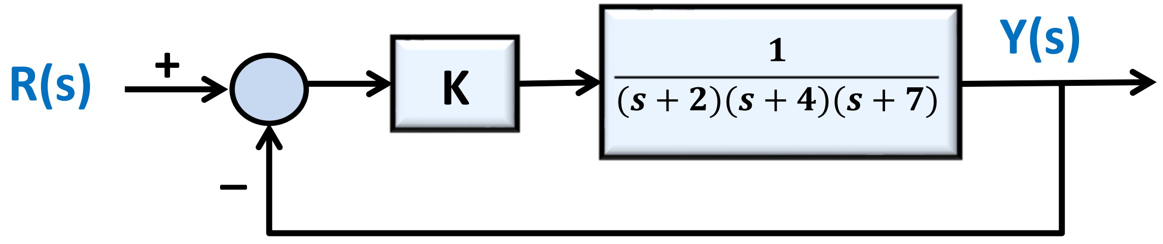
$$\boxed{\zeta = \cos\theta \quad \rightarrow \quad \theta = \cos^{-1}(\zeta)}$$



$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

Static Feedback Design via Root Locus

Example 2 Consider the following third-order system. Determine the K value so that the maximum overshoot of unit-step response is 10%.



$$\text{poles} \rightarrow \ p_1 = -2, p_2 = -4, p_3 = -7$$

$$\text{zeros} \rightarrow \text{No finite zeros, three zeros at infinity}$$

$$\boxed{\zeta = 0.5914} \rightarrow \ \theta = \cos^{-1}(\zeta) = 53.76^{\circ} \approx 54^{\circ}$$

From the graph the desired pole locations are: $s_d = -2.1 \pm j3$

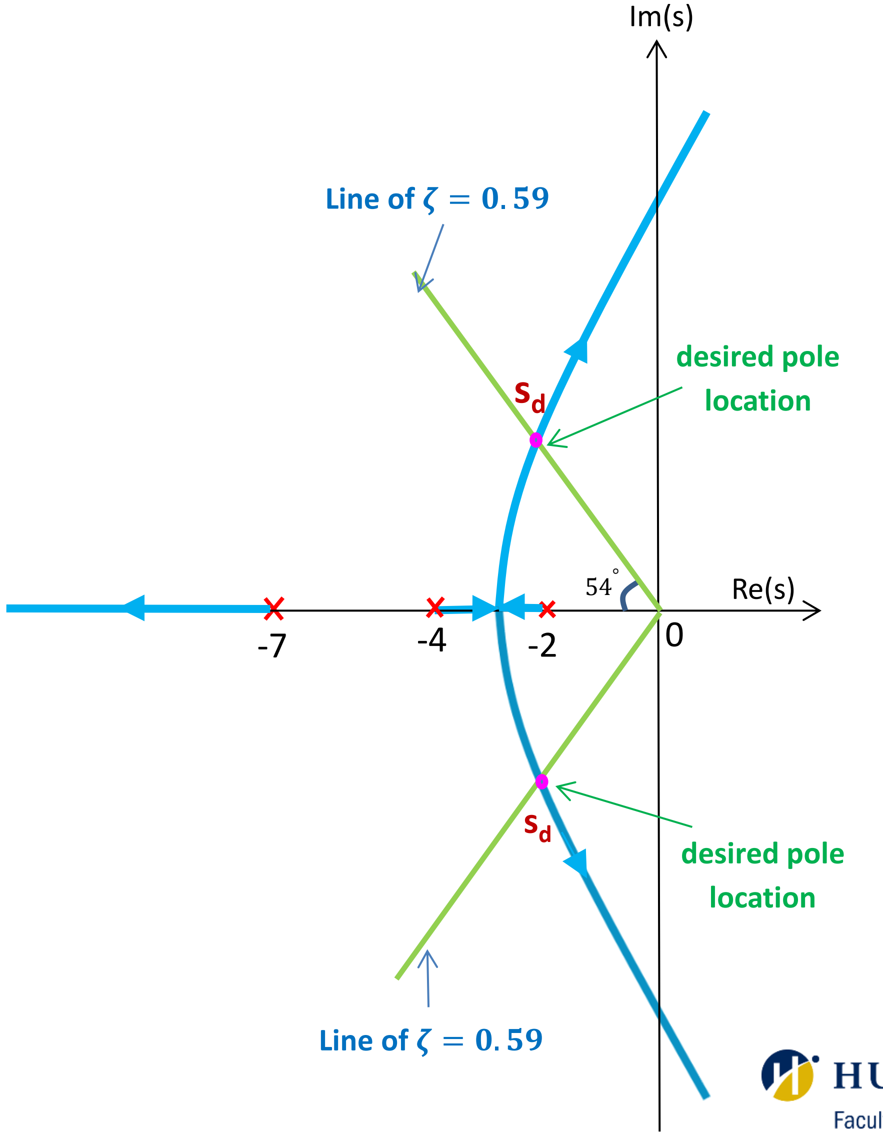
Find the gain K at the desired pole locations by using the magnitude condition:

$$\boxed{|KG(s)H(s)|_{s=s_d} = 1 \quad \rightarrow \quad |K| = \frac{1}{|G(s_d)H(s_d)|}}$$

$$|KG(s)H(s)| = 1 \quad \rightarrow \quad \left|\frac{K}{(s+2)(s+4)(s+7)}\right| = 1$$

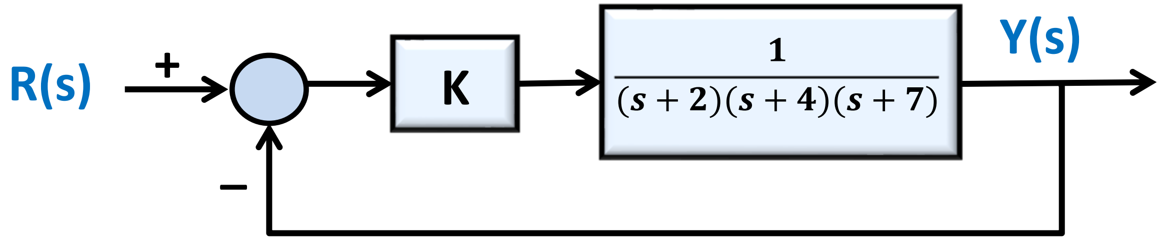
$$|K| = |(s+2)(s+4)(s+7)|_{s=s_d}$$

$$|K| = |s_d+2||s_d+4||s_d+7|$$



Static Feedback Design via Root Locus

Example 2 Consider the following third-order system. Determine the K value so that the maximum overshoot of unit-step response is 10%.



poles $\rightarrow \; p_1 = -2, p_2 = -4, p_3 = -7$

zeros $\rightarrow$ No finite zeros, three zeros at infinity

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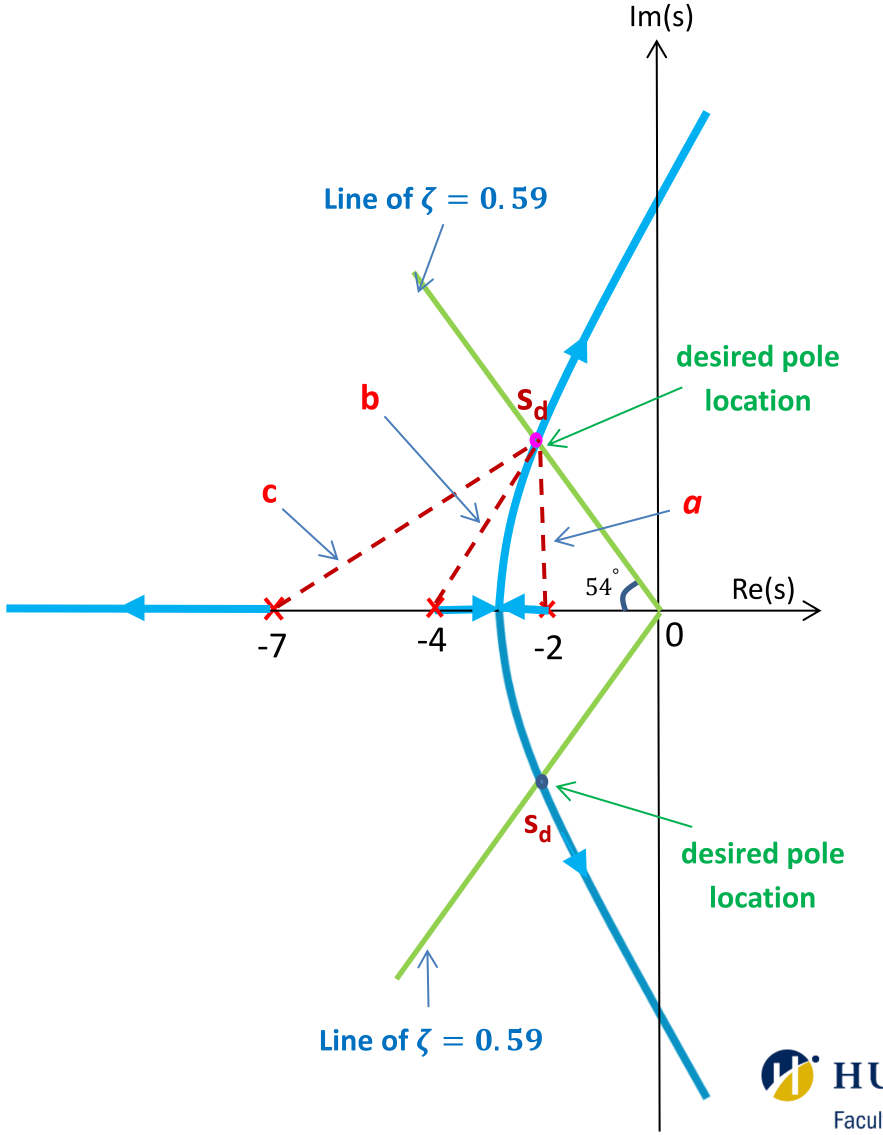


Method 1: Calculation the gain by evaluation at point s_d

$$|K| = |s_d + 2||s_d + 4||s_d + 7|$$

$$|K| = |-0.1 + j3||1.9 + j3||4.9 + j3| = 3 \times 3.6 \times 5.7 = 61.56$$

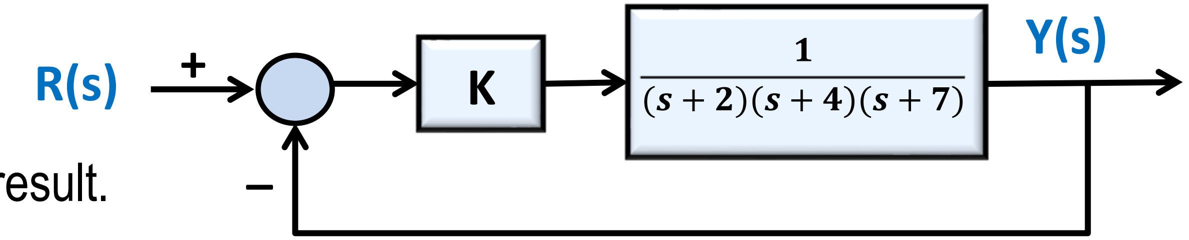
Method 2: Geometrically by measuring length of the vectors

$$|K| = |s_d + 2||s_d + 4||s_d + 7|$$

$$|K| = a \times b \times c = (3.1)(3.5)(5.7) = 61.87$$

Static Feedback Design via Root Locus

Example 2 Consider the following third-order system. Determine the K value so that the maximum overshoot of unit-step response is 10%.

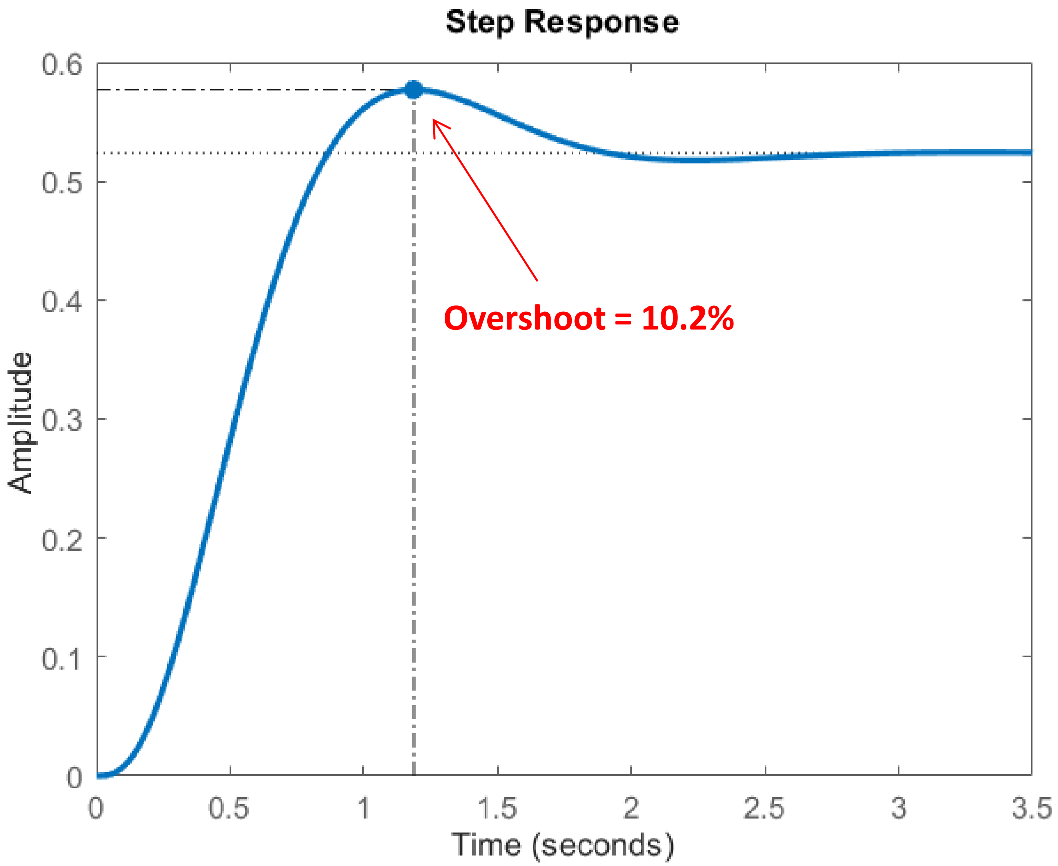


We can also plot the unit-step response of the closed-loop system in MATLAB to check the result.

The open-loop transfer function for $K = 61.56$ is

$$KG(s)H(s) = \frac{61.56}{s^3 + 13s^2 + 50s + 56}$$

```
num = [61.56];
den = [1  13  50  56];
sys_OL = tf(num,den);
sys_CL = feedback(sys_OL,1);
step(sys_CL)
```



The closed-loop transfer function, and the closed-loop poles are:

$$\frac{Y(s)}{R(s)} = \frac{61.56}{s^3 + 13s^2 + 50s + 117.6}$$

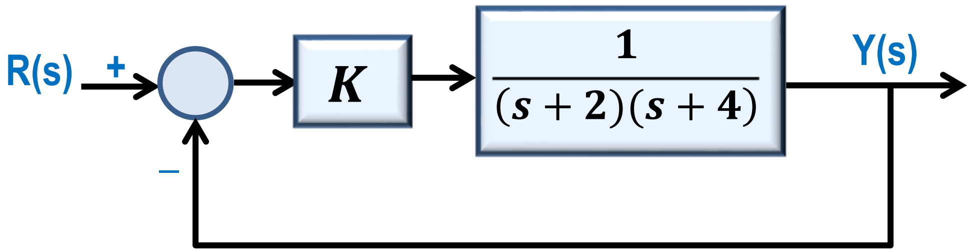
$$\boxed{s_{1,2} = -2.071 \pm j2.9965, \qquad s_3 = -8.8518}$$

Dominant poles

Design Aspects of Root Locus

- The general problem of controller design in control systems may be treated as an investigation of the effects to the root loci when **poles** and **zeros** are added to the **open-loop transfer function G(s)H(s)**.

Consider the following second-order system.



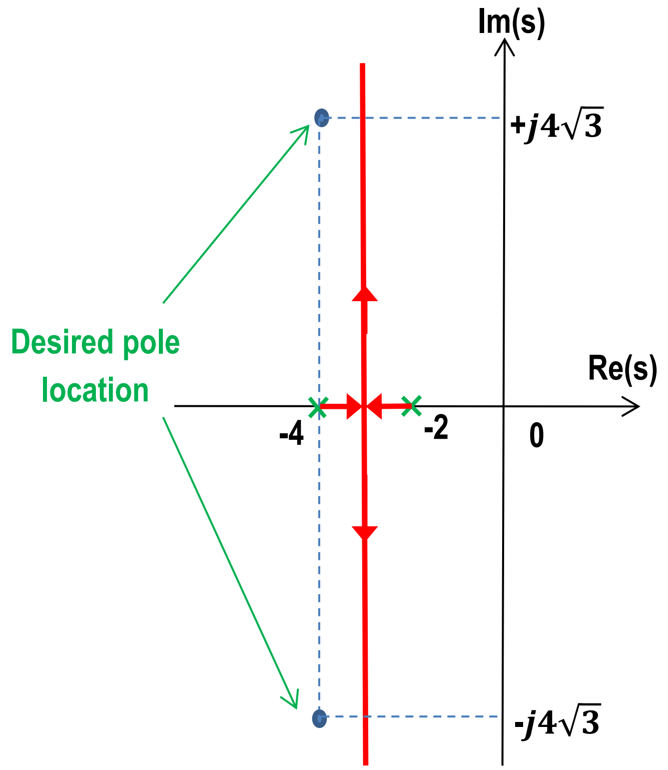
The goal is to find the K value so that the desired closed-loop poles have a damping-ratio of 0.5 and undamped natural frequency of 8 rad/sec.

First, calculate the desired pole locations based on the given damping ratio and the undamped natural frequency values.

$\boxed{\zeta = 0.5}$

$\boxed{\omega_n = 8}$

$$\Rightarrow \quad s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \quad \rightarrow \quad \boldsymbol{s = -4 \pm j4\sqrt{3}}$$ **Desired Poles**



Next, plot the root-locus for this system, and locate the desired poles.

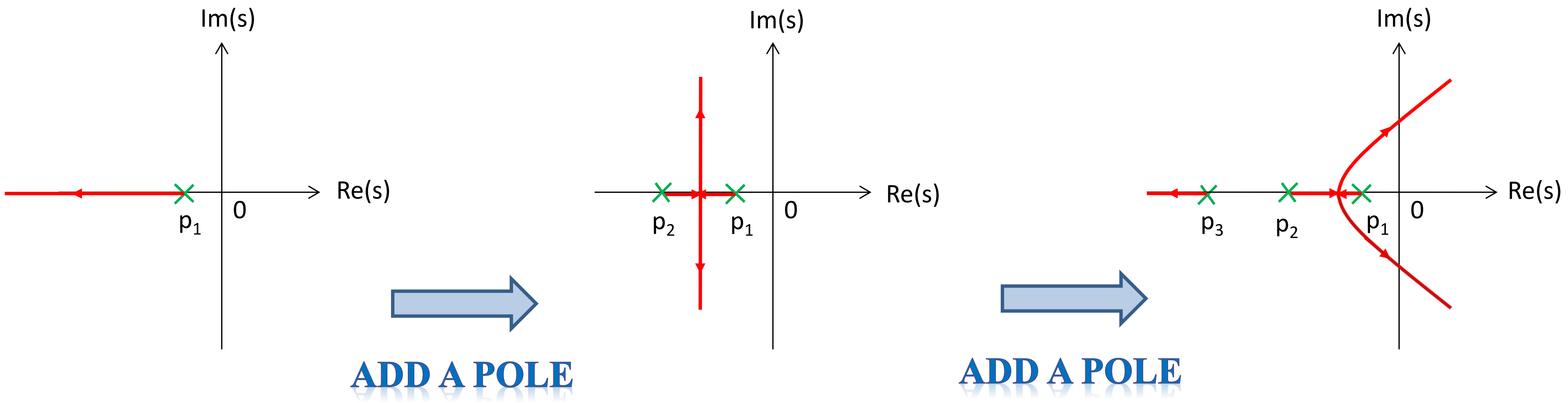
- The root-locus does not pass the desired pole locations.
- The desired characteristics are not achievable by only adjusting the gain K value.

Reshape the root-locus by adding poles and zeros.

Design Aspects of Root Locus

- The general problem of controller design in control systems may be treated as an investigation of the effects to the root loci when **poles** and **zeros** are added to the **open-loop transfer function G(s)H(s)**.

❑ Effect of Adding Poles on Root-Locus

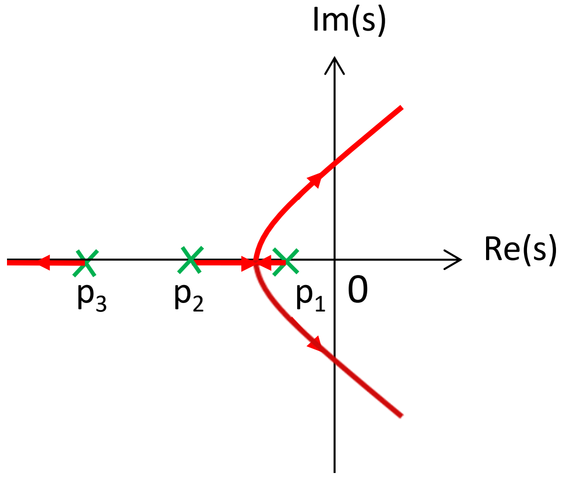


- Pulling the root-locus to the right
- Decreasing the relative stability of closed-loop system
- Increasing the overshoot of closed-loop response
- Slow down the settling of the response

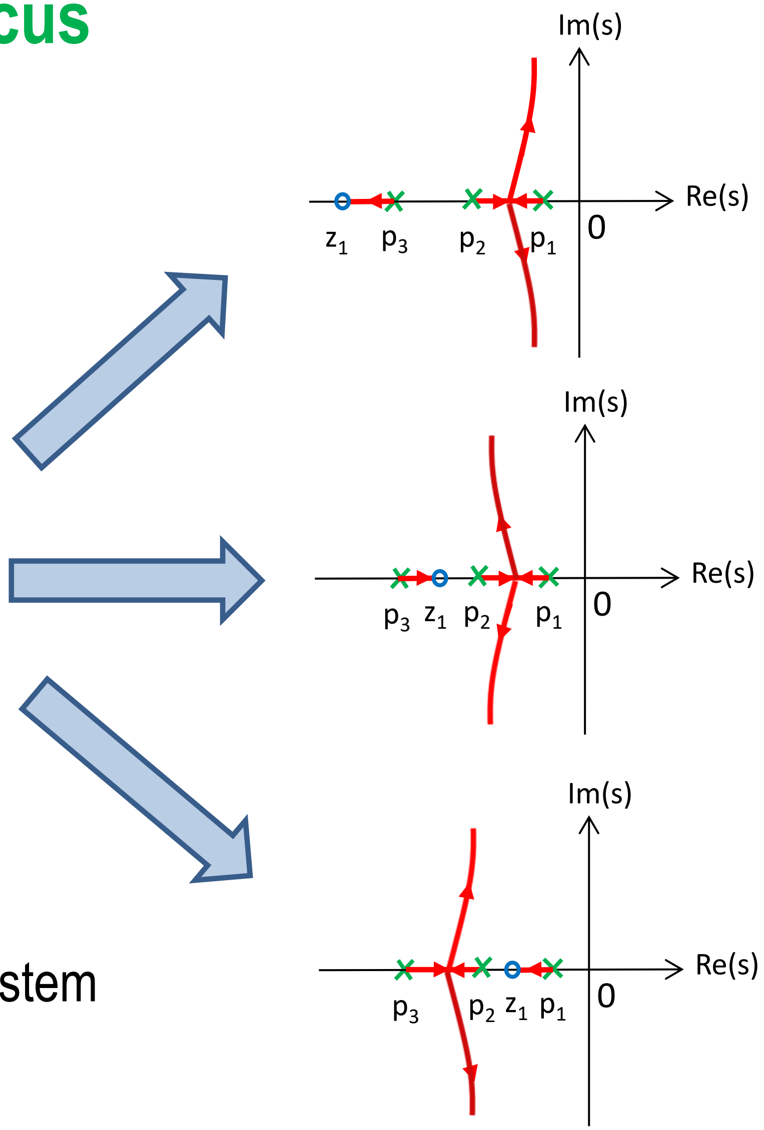
Design Aspects of Root Locus

- The general problem of controller design in control systems may be treated as an investigation of the effects to the root loci when **poles** and **zeros** are added to the **open-loop transfer function G(s)H(s)**.

❑ Effect of Adding Zeros on Root-Locus



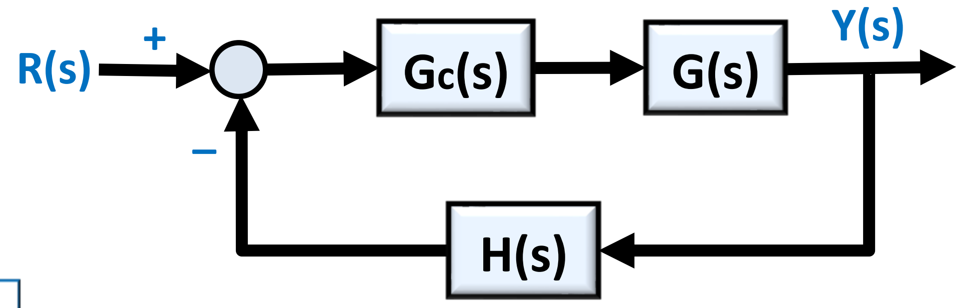
ADD A ZERO



- Pulling the root-locus to the left
- Increasing the relative stability of closed-loop system
- Speed up the settling time of the response
- Amplifies the high-frequency noise

- Based to the effect of adding poles/zeros on root-locus:
- Adding only zero is often problematic, because it amplifies the high-frequency noise.
- Adding only pole generates a less stable system by moving the root-locus (closed-loop poles) to the right.
- Therefore, we need to **add both zero** and **pole** to design a ***compensator***.

Dynamic Compensator Design

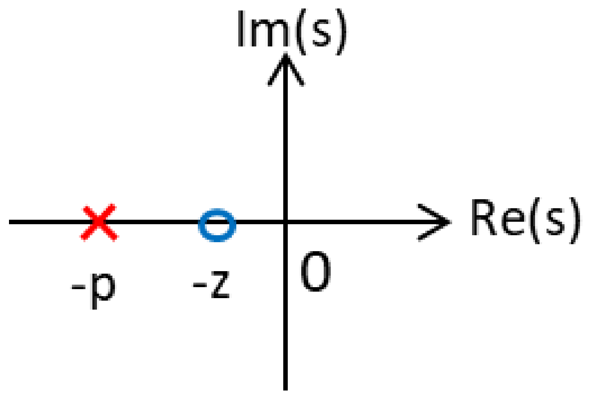


❑ Lead Compensator & Lag Compensator

- In general, a **compensator** has the following transfer function

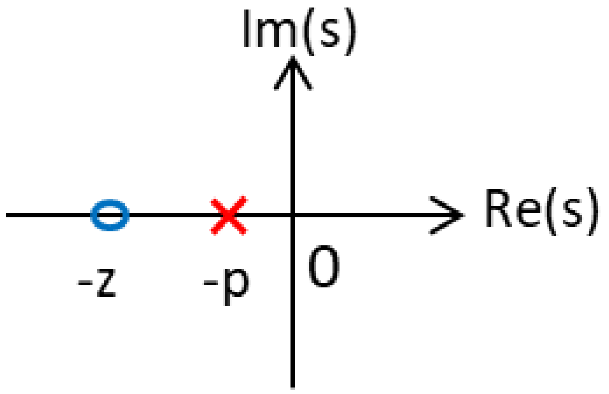
$$G_c(s) = K_c \frac{s+z}{s+p}, \qquad z > 0, \qquad p > 0$$

- According to the pole/zero locations we have the following structures:



Lead Compensator

- To improve transient response and stability.
- Similar to PD controller.
- Positive angle contribution.



Lag Compensator

- To improve steady-state error.
- Similar to PI controller.
- Negative angle contribution.

- Combined **lead-lag compensator** can improve both transient response and steady-state response, similar to a PID controller.

Lead Compensator Design via Root Locus

Step 1: Determine desired location of the dominant closed-loop poles.

Step 2: Plot the axes of s-plane and mark open-loop poles/zeros and the desired closed-loop poles.

Step 3: Find the sum of the angles at the desired location of the dominant closed-loop pole.

Step 4: Determine the angle deficiency, ϕ, which is necessary to satisfy the angle condition.

Step 5: Design a lead compensator to compensate the angle deficiency by determining the locations of the pole/zero as below:

$$\boxed{G_c(s) = K_c \frac{s+z}{s+p}}$$

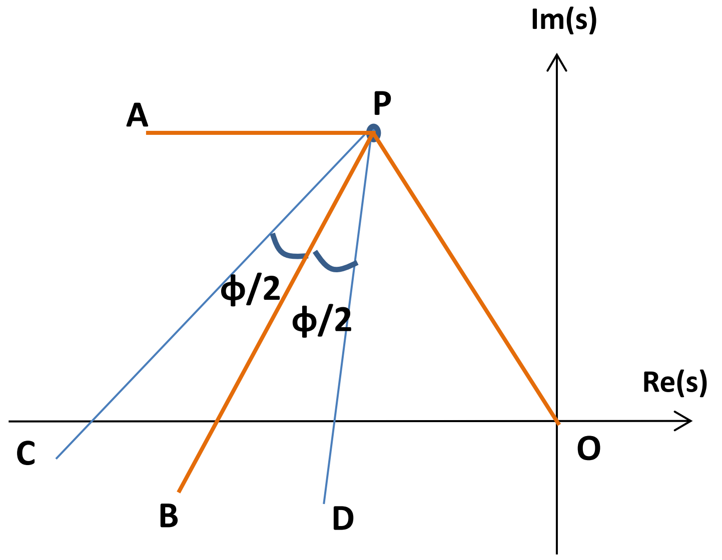
- P is the desired pole location
- Draw lines PA and PO
- Draw bisector line PB

$$\angle APB = \angle BPO = \frac{\angle APO}{2}$$

- Draw lines PC and PD so that

$$\angle CPB = \angle BPD = \phi/2$$

- Pole and zero are the intersections of PC and PD with real axis



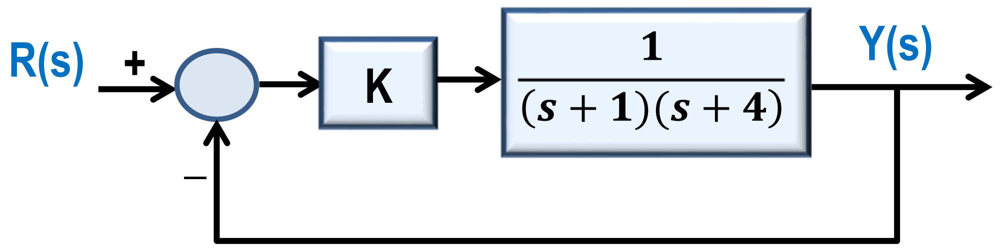
Step 6: Determine gain of the compensator by using the root-locus magnitude condition or the given steady-state error condition.

Lead Compensator Design via Root Locus

Example 3

Consider the following second-order system

Design a lead compensator to speed up unit-step response with maximum overshoot of about 10% and settling time of less than $1sec$.



Step 1: Determine the desired dominant closed-loop pole locations

First, determine the desired damping ratio and undamped natural frequency based on the desired maximum overshoot and settling time value:

$$\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}} \rightarrow \zeta = \frac{-\ln(0.1)}{\sqrt{\pi^2 + \ln^2(0.1)}} \rightarrow \boxed{\zeta = 0.5912}$$

Desired Damping Ratio

$$t_s \approx \frac{4}{\zeta\omega_n} \rightarrow 1 = \frac{4}{0.6\omega_n} \rightarrow \boxed{\omega_n = 6.7659}$$

Desired Natural Frequency

The **desired closed-loop poles** location ➡ $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \rightarrow \boxed{s_d = -4 \pm j5.5}$

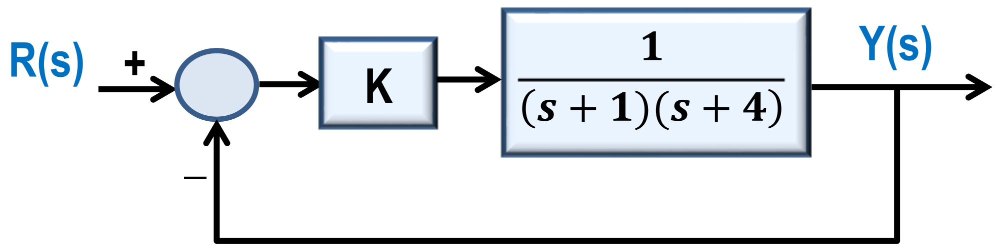
Desired Closed-loop Poles

Lead Compensator Design via Root Locus

Example 3

Consider the following second-order system

Design a lead compensator to speed up unit-step response with maximum overshoot of about 10% and settling time of less than $1sec$.



Step 2: Plot the axes of s-plane and mark open-loop poles/zeros and the desired closed-loop poles.

poles $\rightarrow$ $p_1 = -1, p_2 = -4$ zeros $\rightarrow$ No finite zeros

Desired closed-loop poles $\Rightarrow$ $\boxed{s_d = -4 \pm j5.5}$

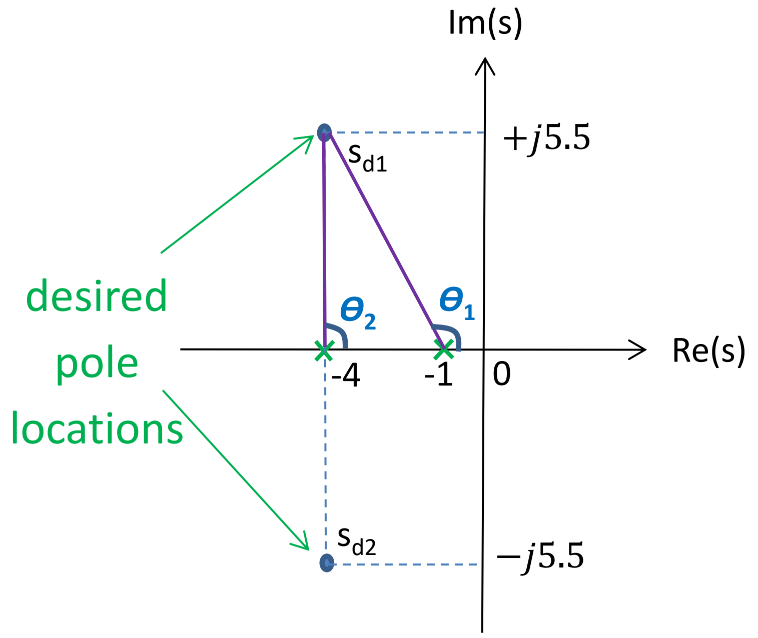
Step 3: Find sum of the angles at the desired closed-loop poles location.

- Apply the angle condition to check that if the desired poles are on the root-locus or not

$$\angle\left(\frac{\mathrm{K}}{(s+1)(s+4)}\right)\Bigg|_{s=s_{d1}} = \angle \mathrm{K} - \angle(s+1) - \angle(s+4)\Big|_{s=-4+j5.5}$$

$$= 0 - \angle\theta_1 - \angle\theta_2$$

$$= 0 - 120^\circ - 90^\circ = -210^\circ$$



$$\boxed{-210^\circ \neq \pm(2i+1)180^\circ}$$

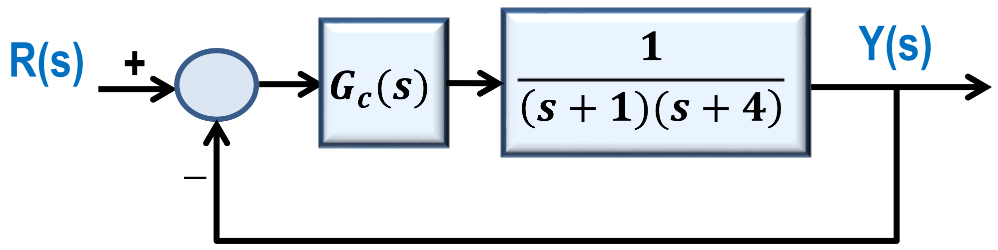
The angle condition is not satisfied $\Rightarrow$ **Desired poles are not on the root-locus**

Lead Compensator Design via Root Locus

Example 3

Consider the following second-order system

Design a lead compensator to speed up unit-step response with maximum overshoot of about 10% and settling time of less than $1sec$.



$$G_c(s) = K_c\frac{s+z}{s+p}$$

Step 4: Determine the required angle deficiency to satisfy the root-locus angle condition.

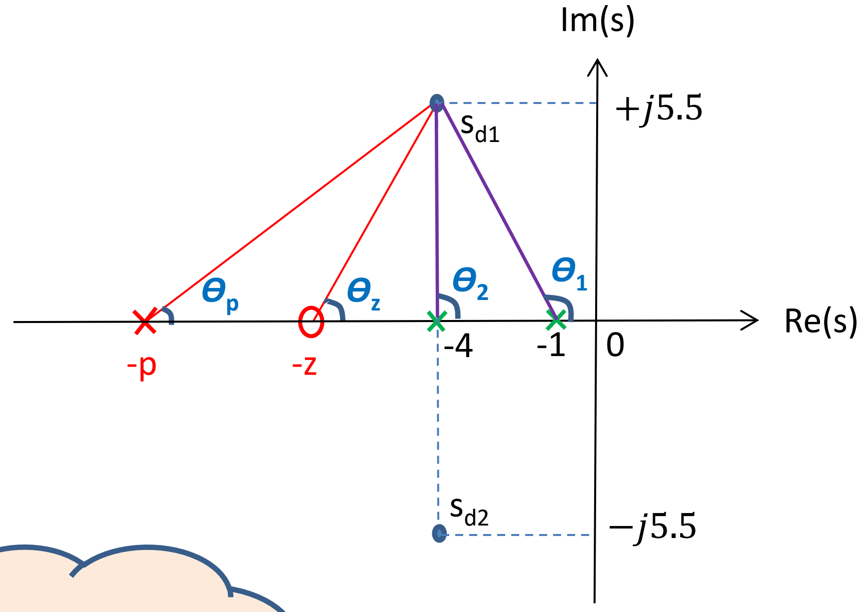
The angle deficiency is calculated as

$$-210^\circ + \phi = -180^\circ \quad \longrightarrow \quad \phi = 210^\circ - 180^\circ = 30^\circ$$

Design a **lead compensator** to contribute the angle of $\phi = 30^\circ$ at the desired pole locations.

$$\angle G_c(s)G(s)\Big|_{s=s_{d1}} = \angle\left(\frac{K_c(s+z)}{s+p}\cdot\frac{1}{(s+1)(s+4)}\right)\Bigg|_{s=s_{d1}}$$

$$= \underbrace{\angle K_c}_{0^\circ} + \underbrace{\angle\theta_z - \angle\theta_p}_{30^\circ} \underbrace{-\angle\theta_1 - \angle\theta_2}_{-210^\circ} = -180^\circ$$



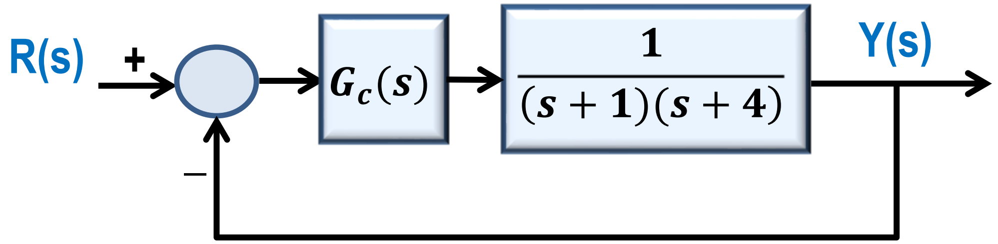
How to select the pole/zero locations?

Lead Compensator Design via Root Locus

Example 3

Consider the following second-order system

Design a lead compensator to speed up unit-step response with maximum overshoot of about 10% and settling time of less than $1sec$.



Step 5: Determine pole/zero locations of the lead compensator to compensate the angle deficiency.

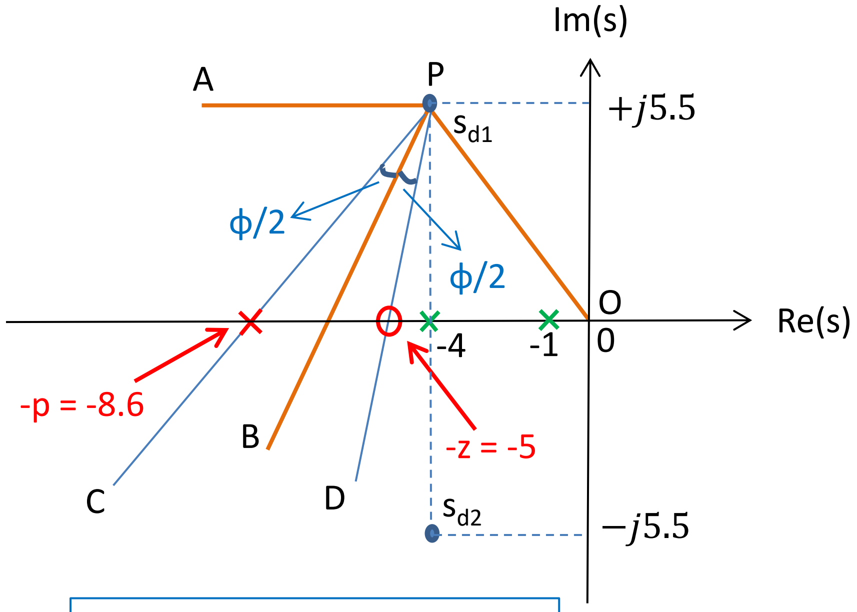
$$G_c(s) = K_c\frac{s+z}{s+p}$$

- Draw lines PA and PO
- Draw bisector line PB $\qquad \angle APB = \angle BPO = \dfrac{\angle APO}{2}$
- Draw lines PC and PD so that

$$\angle CPB = \angle BPD = \frac{\phi}{2} = \frac{30°}{2} = 15^{\circ}$$

- Pole and zero are the intersections of PC and PD with real axis

$$\boxed{G_c(s) = K_c\frac{s+5}{s+8.6}}$$



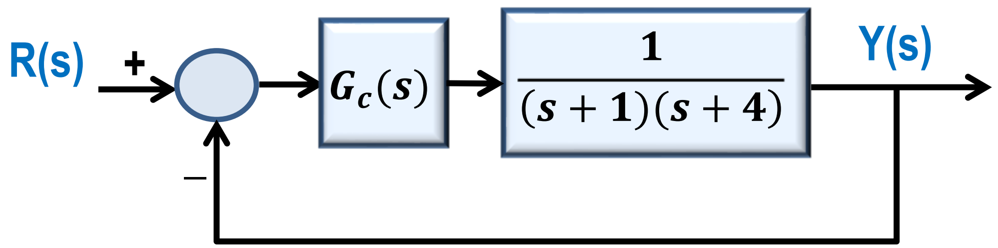
$$\boxed{z = 5, \qquad p = 8.6}$$

Lead Compensator Design via Root Locus

Example 3

Consider the following second-order system

Design a lead compensator to speed up unit-step response with maximum overshoot of about 10% and settling time of less than $1sec$.



$$G_c(s) = K_c \frac{s+z}{s+p}$$

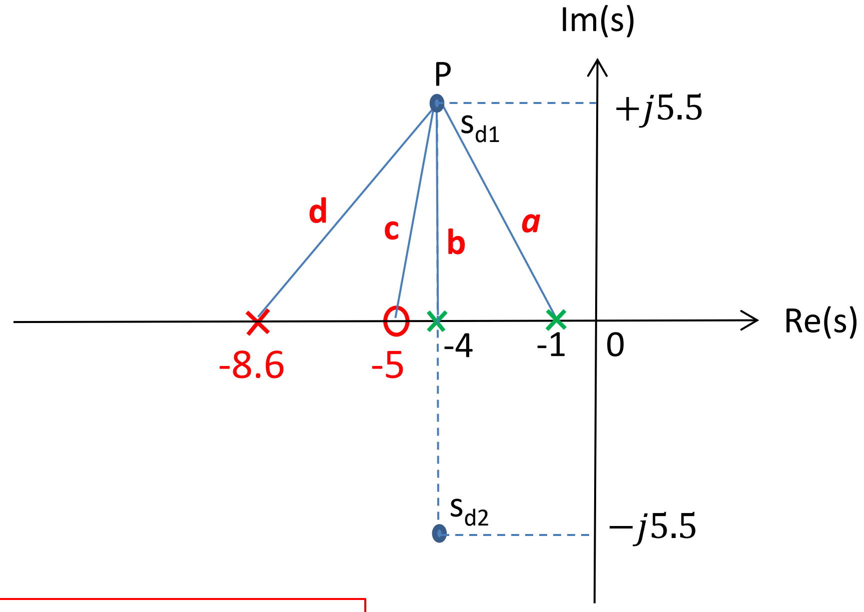
Step 6: Determine gain of the lead compensator from magnitude condition.

Next, calculate the gain K_c using the magnitude condition $\boxed{|G_c(s)G(s)|_{s=s_d} = 1}$

$$\left| K_c \frac{s+5}{s+8.6} \cdot \frac{1}{(s+1)(s+4)} \right|_{s=-4+j5.5} = 1$$

$$|K_c| = \frac{|-3+j5.5||j5.5||4.6+j5.5|}{|1+j5.5|} = \frac{\sqrt{(-3)^2+(5.5)^2} \times 5.5 \times \sqrt{(4.6)^2+(5.5)^2}}{\sqrt{(1)^2+(5.5)^2}} = 44.32$$

$$\boxed{K_c = 44.32}$$



We can also find the gain K_c by measuring the length of the vectors connected from the desired pole to the other poles and zeros.

$$K_c = \frac{a \times b \times d}{c}$$

$$\boxed{G_c(s) = K_c \frac{s+z}{s+p} = 44.32 \frac{s+5}{s+8.6}}$$

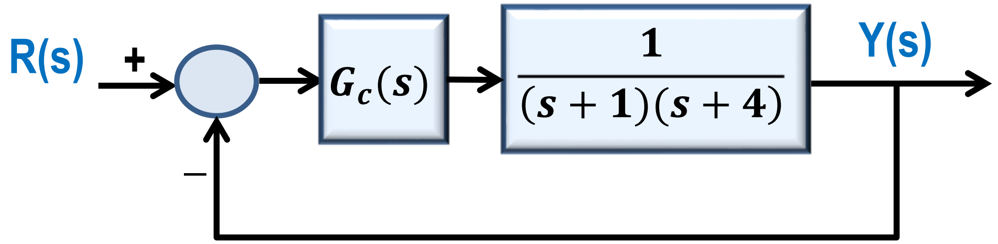
The designed lead compensator

Lead Compensator Design via Root Locus

Example 3

Consider the following second-order system

Design a lead compensator to speed up unit-step response with maximum overshoot of about 10% and settling time of less than $1sec$.



Step 7: Analyze and verify the designed compensator.

$$\boxed{G_c(s) = 44.32\frac{s+5}{s+8.6}}$$

Determine closed-loop transfer function of the compensated system and check the pole locations

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} \longrightarrow T(s) = \frac{44.32(s+5)}{s^3+13.6s^2+91.32s+256}$$

The closed-loop poles are located at

$$\boxed{s_{1,2} = -4.07 \pm j5.51, \qquad s_3 = -5.46}$$

Dominant Poles **Close to the Zero**

Desired Closed-loop Poles

$$\boxed{s = -4 \pm j5.5}$$

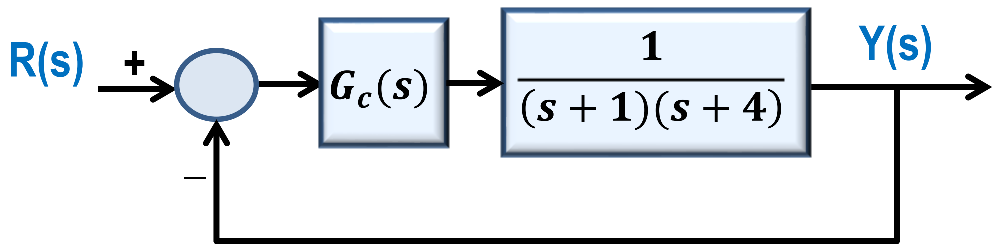
- The dominant closed-loop poles are almost located at the desired places in the s-plane.
- The third pole at $s_3 = -5.46$ is very close to the added zero at $s = -5$. Therefore, the effect of this pole on the transient response is negligible.

Lead Compensator Design via Root Locus

Example 3

Consider the following second-order system

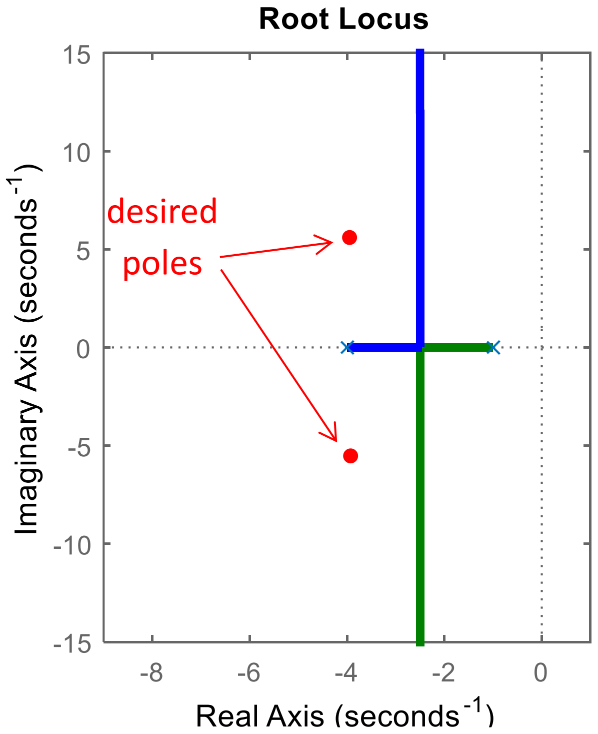
Design a lead compensator to speed up unit-step response with maximum overshoot of about 10% and settling time of less than $1sec$.



Step 7: **Analyze and verify the designed compensator.**

Uncompensated System

$$G(s) = \frac{1}{(s+1)(s+4)}$$



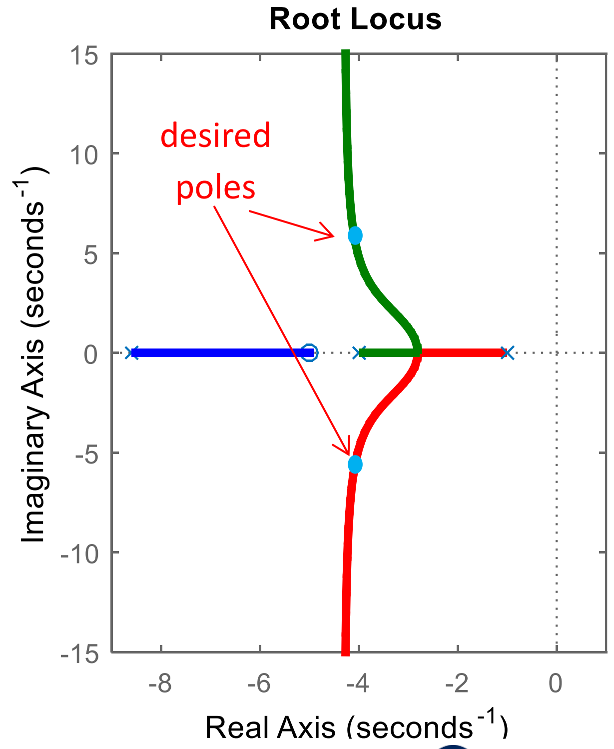
Desired Closed-loop Poles

$$\boxed{s = -4 \pm j5.5}$$

Lead compensator pulls the root-locus to the left and improves the relative stability of the system.

Compensated System

$$G_c(s)G(s) = \frac{44.32(s+5)}{(s+8.6)(s+1)(s+4)}$$

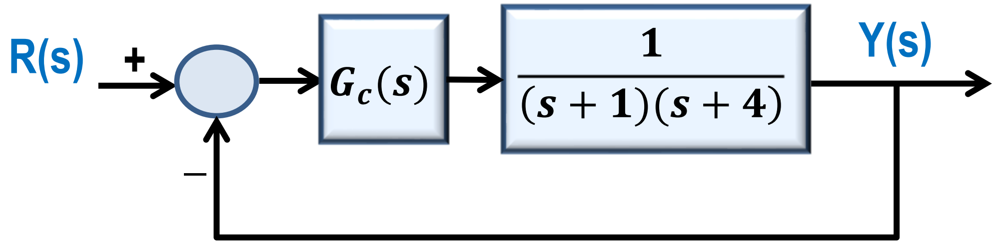


Lead Compensator Design via Root Locus

Example 3

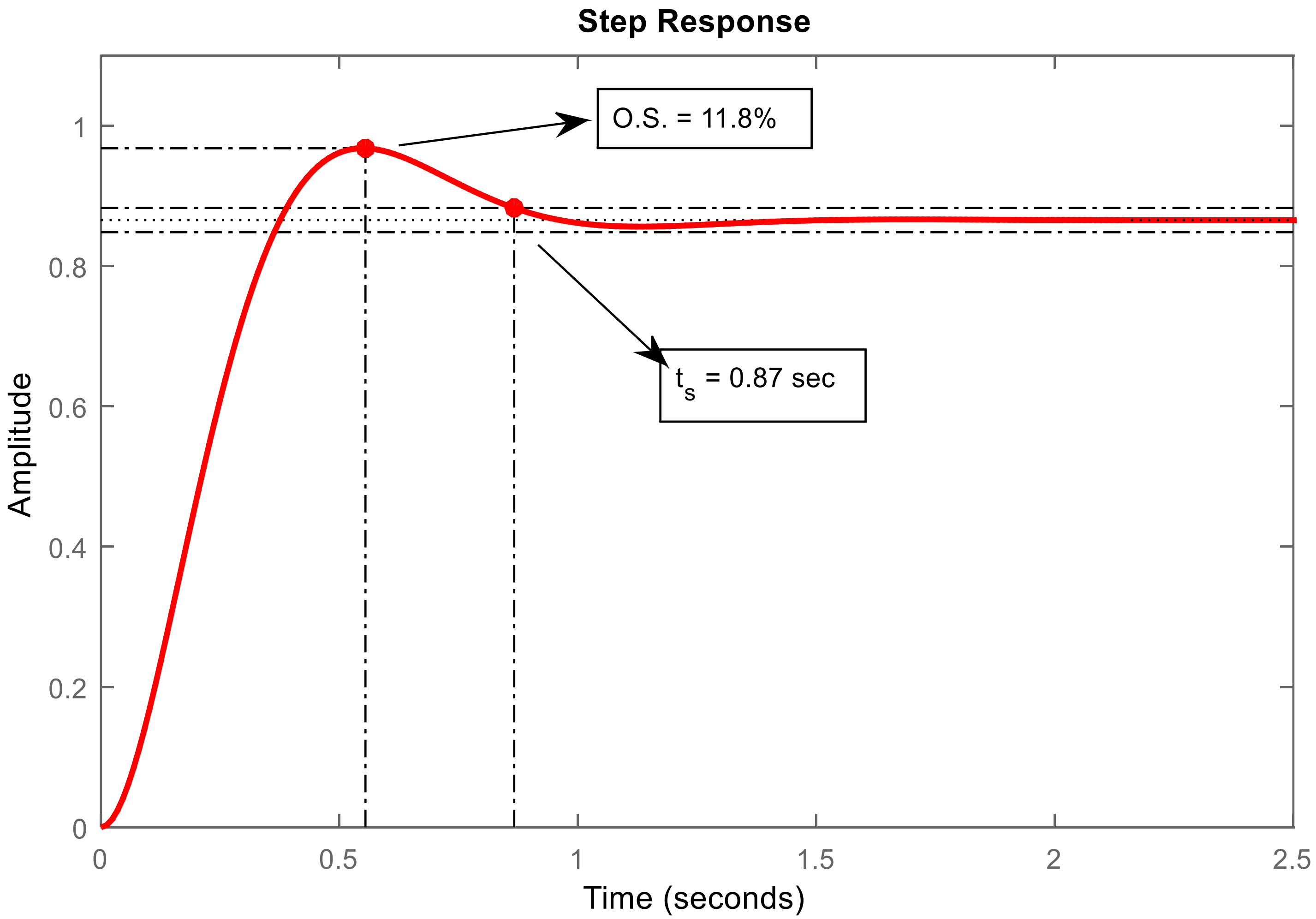
Consider the following second-order system

Design a lead compensator to speed up unit-step response with maximum overshoot of about 10% and settling time of less than $1sec$.



Step 7: Analyze and verify the designed compensator.

- Step responses of the compensated closed-loop system has the overshoot of about 11.8% and settling time of 0.87sec.



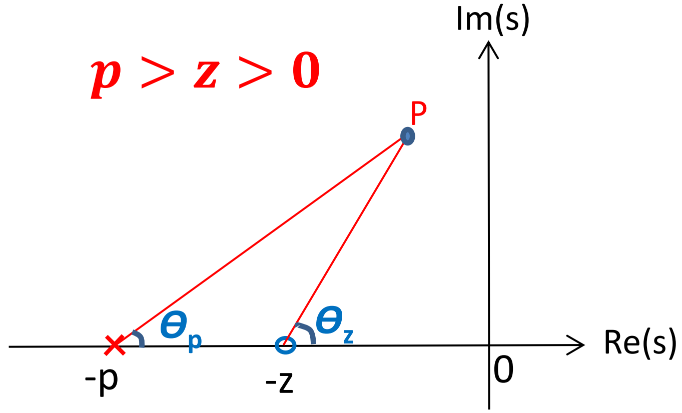
PD Controller Design via Root Locus

- In designing the lead compensator, the pole-zero locations of the compensator are determined to compensate the angle deficiency of the desired closed-loop pole location.

$$\boxed{G_c(s) = K_c \frac{s+z}{s+p}}$$

$$\boxed{\phi = \angle\theta_z - \angle\theta_p}$$

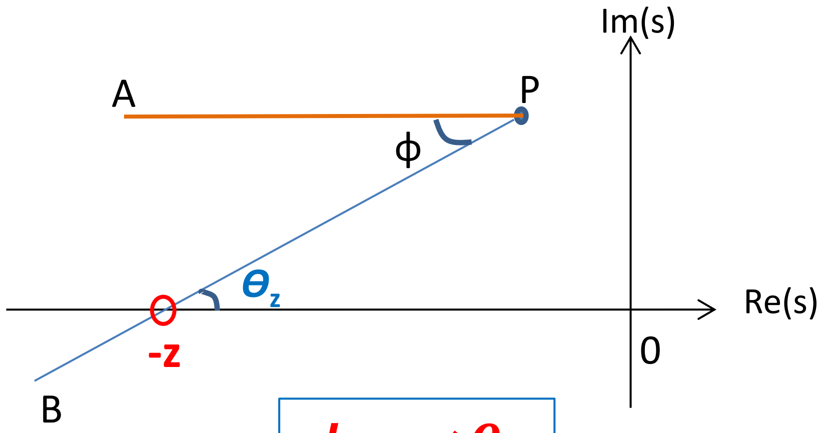
Angle Deficiency



- The angle deficiency, ϕ, can also be compensated by adding an only zero without pole.

$$\boxed{G_c(s) = K_c(s+z)}$$

  - P is the desired pole location
  - Draw line PA
  - Draw line PB such that $\angle APB = \phi$
  - The zero is located at the intersection of PB with real axis
  - Determine K_c by using the magnitude condition



$$\boxed{\phi = \angle\theta_z}$$

- The lead compensator without pole can also be shown as a **PD controller**:

$$G_c(s) = K_c(s+z) = K_c z\left(\frac{s}{z} + 1\right) \longrightarrow \boxed{G_c(s) = K_p(T_d s + 1)}$$

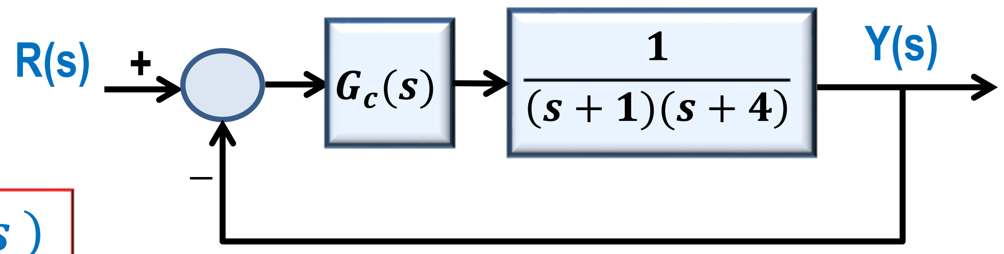
where, the proportional gain K_p and the derivative time-constant T_d are defined as $\boxed{T_d = \frac{1}{z}}$ $\boxed{K_p = K_c z}$

PD Controller Design via Root Locus

Example 4

Consider the following closed-loop system with PD controller. Design PD controller to achieve a percentage of overshoot about 10% and settling time less than $\mathbf{1}\boldsymbol{sec}$.

$$\boxed{G_c(s) = K_p(1 + T_d s)}$$



Step 1: Determine the desired dominant closed-loop pole locations

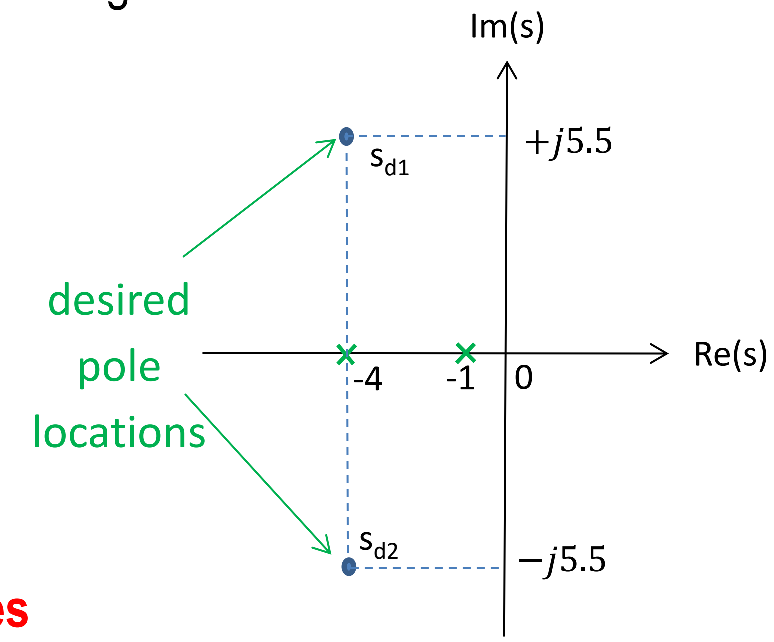
First, find the desired damping ratio and undamped natural frequency based on the desired overshoot and the settling time.

$$\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}} \rightarrow \zeta = \frac{-\ln(0.1)}{\sqrt{\pi^2 + \ln^2(0.1)}} \rightarrow \boxed{\zeta = 0.5912}$$ **Desired Damping Ratio**

$$t_s \approx \frac{4}{\zeta\omega_n} \rightarrow 1 = \frac{4}{0.6\omega_n} \rightarrow \boxed{\omega_n = 6.7659}$$ **Desired Natural Frequency**

The **desired closed-loop poles** location ➡ $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \rightarrow \boxed{s_d = -4 \pm j5.5}$

Desired Closed-loop Poles



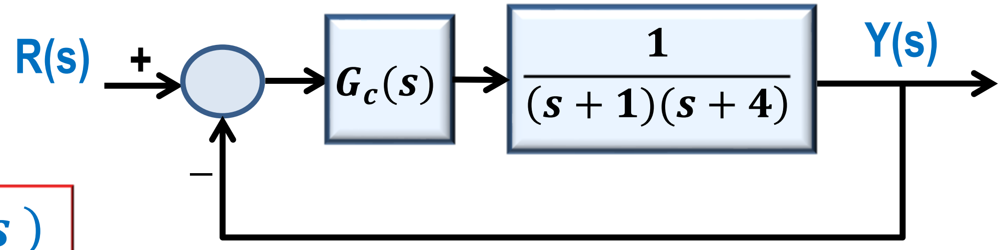
Step 2: Plot the axes of s-plane and mark open-loop poles/zeros and the desired closed-loop poles.

$\text{poles} \rightarrow p_1 = -1, p_2 = -4$ $\qquad$ $\text{zeros} \rightarrow \text{No finite zeros}$

PD Controller Design via Root Locus

Example 4

Consider the following closed-loop system with PD controller. Design PD controller to achieve a percentage of overshoot about 10% and settling time less than $\mathbf{1sec}$.



$$\boxed{G_c(s) = K_p(1 + T_d s)}$$

Step 3: Find sum of the angles at the desired closed-loop poles location

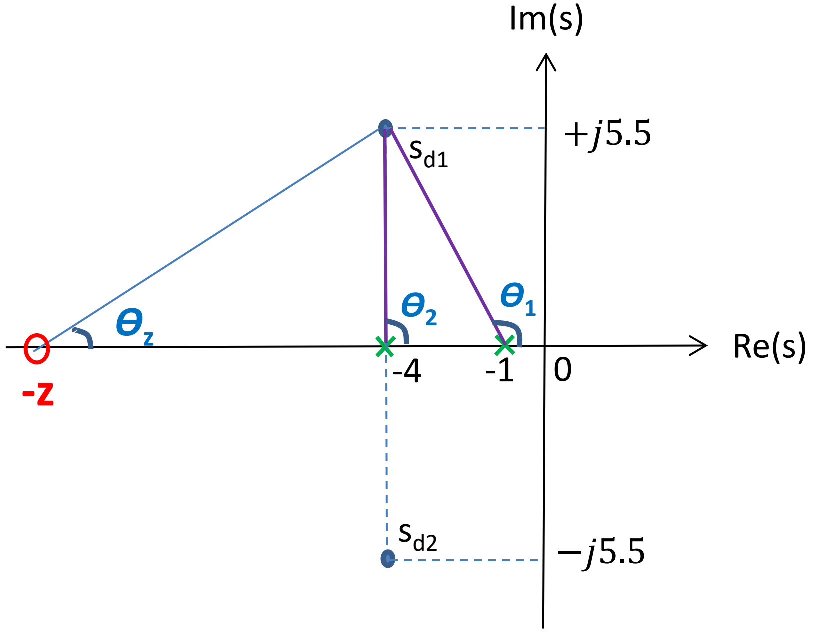
- Apply the angle condition to check that if the desired poles are on the root-locus or not

$$\angle\left(\frac{K}{(s+1)(s+4)}\right)\Bigg|_{s=s_{d1}} = \angle K - \angle(s+1) - \angle(s+4)\Big|_{s=-4+j5.5}$$

$$= 0 - \angle\theta_1 - \angle\theta_2 = 0 - 120^\circ - 90^\circ = \mathbf{-210^\circ}$$

The angle condition is not satisfied

Desired poles are not on the root-locus



Step 4: Determine the required angle deficiency to satisfy the root-locus angle condition.

The angle deficiency is calculated as

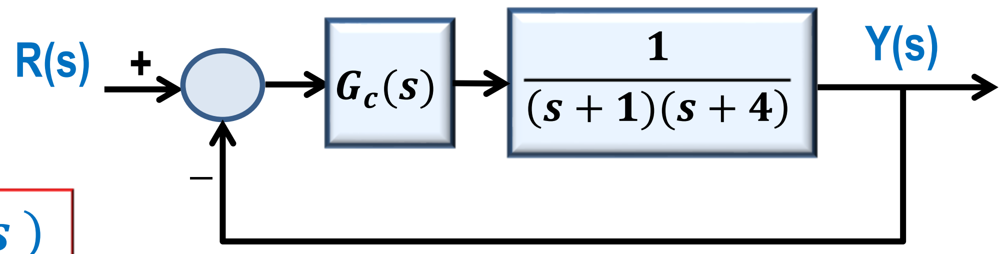
$$-210^\circ + \phi = -180^\circ \quad \longrightarrow \quad \phi = 210^\circ - 180^\circ = \mathbf{30^\circ}$$

The **PD controller** must contribute the angle of $\phi = 30^\circ$ at the desired pole locations.

PD Controller Design via Root Locus

Example 4

Consider the following closed-loop system with PD controller. Design PD controller to achieve a percentage of overshoot about 10% and settling time less than $\mathbf{1}\boldsymbol{sec}$.



$$\boxed{G_c(s) = K_p(1 + T_d s)}$$

Step 5: Determine zero location of the PD controller to compensate the angle deficiency

- P is the desired pole location
- Draw line PA
- Draw line PB such that

$$\angle APB = \phi = 30°$$

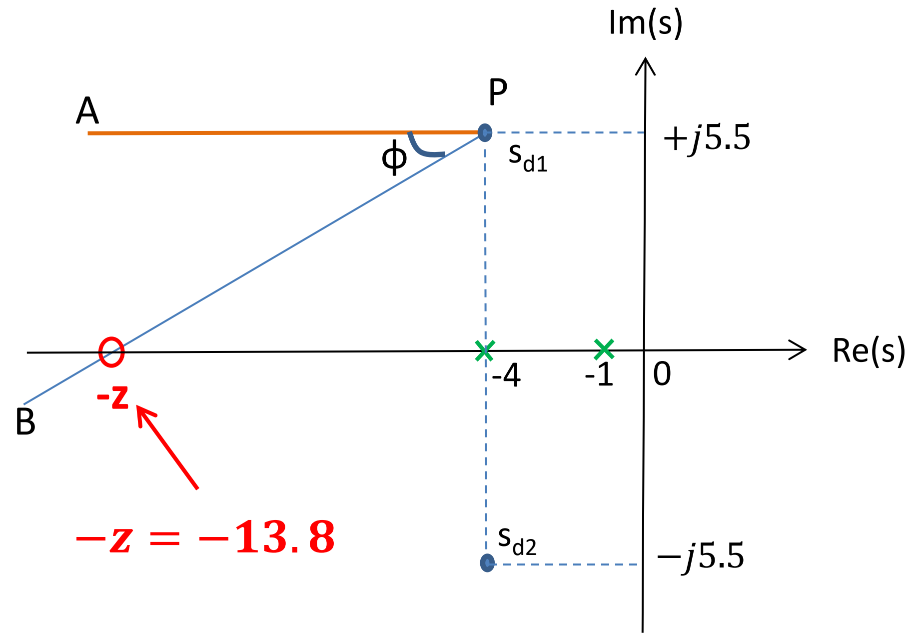
- The zero is located at the intersection of PB with real axis

$$\boxed{z = 13.8}$$

- Determine the derivative time-constant T_d

$$\boxed{T_d = \frac{1}{z}} \rightarrow T_d = \frac{1}{13.8} = 0.074 \rightarrow \boxed{T_d = 0.074}$$

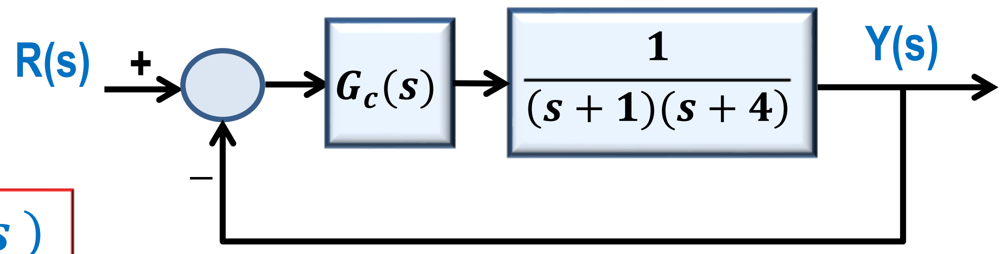
$$\boxed{G_c(s) = K_p(1 + 0.074s)}$$



PD Controller Design via Root Locus

Example 4

Consider the following closed-loop system with PD controller. Design PD controller to achieve a percentage of overshoot about 10% and settling time less than $1sec$.



$$\boxed{G_c(s) = K_p(1 + T_d s)}$$

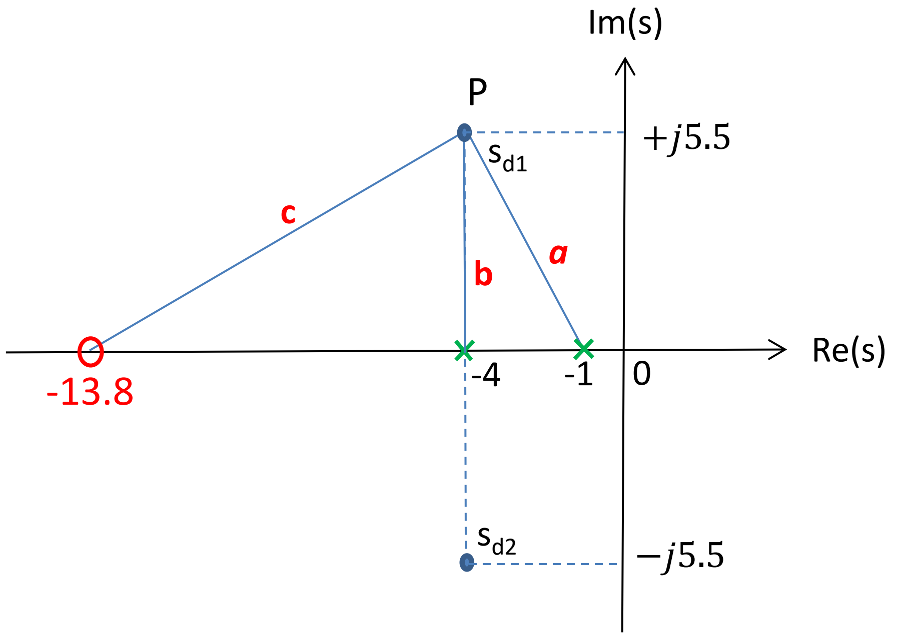
Step 6: Determine proportional gain K_p from the magnitude condition.

Next, calculate the K_p using the magnitude condition $\boxed{|G_c(s)G(s)|_{s=s_d} = 1}$

$$\left| K_p(1+0.074s) \cdot \frac{1}{(s+1)(s+4)} \right|_{s=-4+j5.5} = 1$$

$$|K_p| = \frac{|-3+j5.5||j5.5|}{|0.704+j0.407|} = \frac{6.26 \times 5.5}{0.81} = 42.51$$

$$\boxed{K_p = 42.51}$$



We can also find the gain K_p by measuring the length of the vectors connected from the desired pole to the other poles and zeros.

$$K_c = \frac{a \times b}{c} \quad \rightarrow \quad K_p = K_c z$$

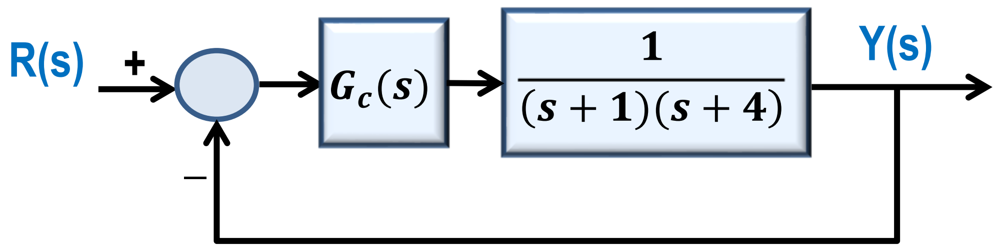
$$\boxed{G_c(s) = 42.51(1 + 0.074s)}$$

The designed PD controller

PD Controller Design via Root Locus

Example 4

Consider the following closed-loop system with PD controller. Design PD controller to achieve a percentage of overshoot about 10% and settling time less than $\mathbf{1}\boldsymbol{sec}$.



Step 7: Analyze and verify the designed compensator

$$\boldsymbol{G_c(s) = 42.51(1 + 0.074s)}$$

Determine closed-loop transfer function of the compensated system and check the pole locations

$$\boldsymbol{T(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}} \longrightarrow \boldsymbol{T(s) = \frac{42.51(1 + 0.074s)}{s^2 + 8.146s + 46.51}}$$

The closed-loop poles are located at

$$\boldsymbol{s_{1,2} = -4.073 \pm j5.47,}$$

Dominant Poles

Desired Closed-loop Poles

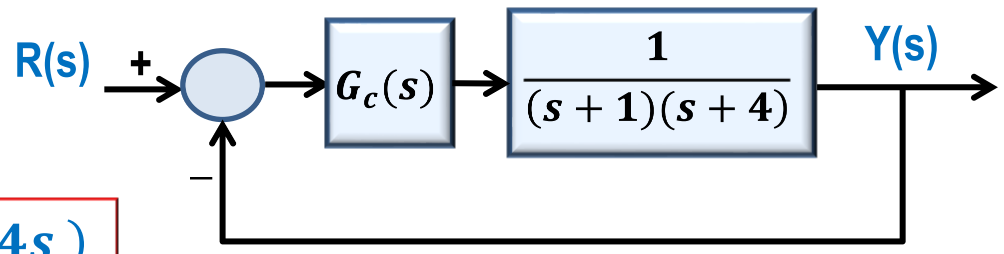
$$\boldsymbol{s = -4 \pm j5.5}$$

- The dominant closed-loop poles are almost located at the desired places in the s-plane.

PD Controller Design via Root Locus

Example 4

Consider the following closed-loop system with PD controller. Design PD controller to achieve a percentage of overshoot about 10% and settling time less than $1sec$.



$$G_c(s) = 42.51(1 + 0.074s)$$

Step 7: Analyze and verify the designed compensator

Uncompensated System

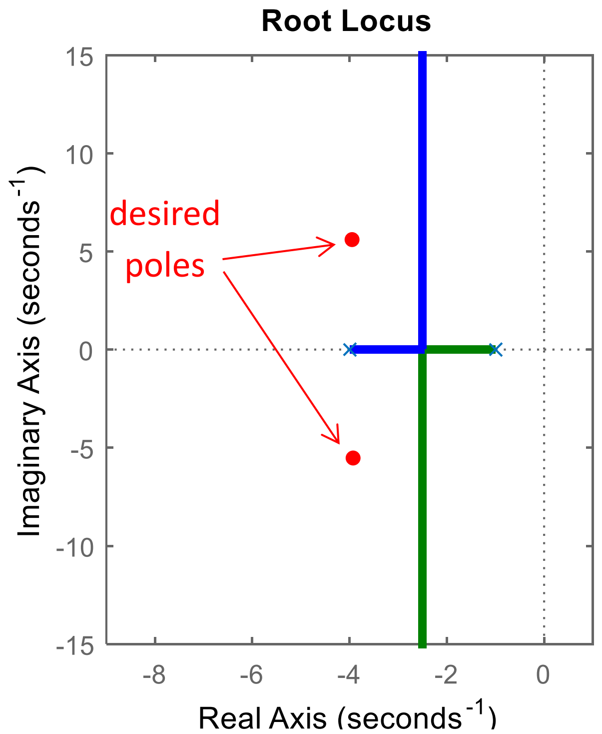
$$G(s) = \frac{1}{(s+1)(s+4)}$$

Compensated System

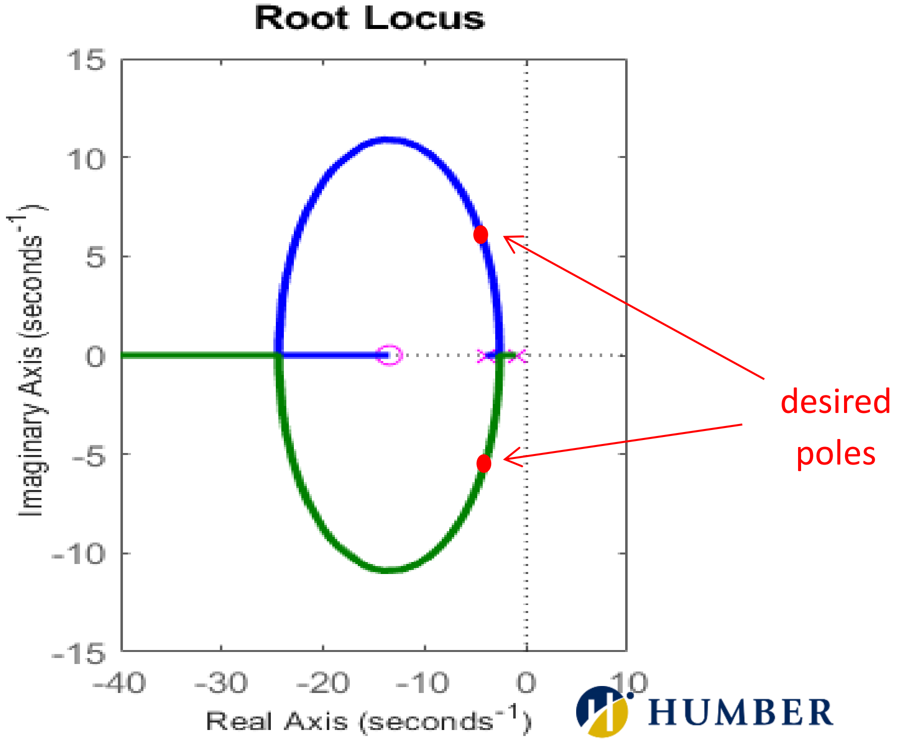
$$G_c(s)G(s) = \frac{42.51(1+0.074s)}{(s+1)(s+4)}$$

Desired Closed-loop Poles

$$s = -4 \pm j5.5$$



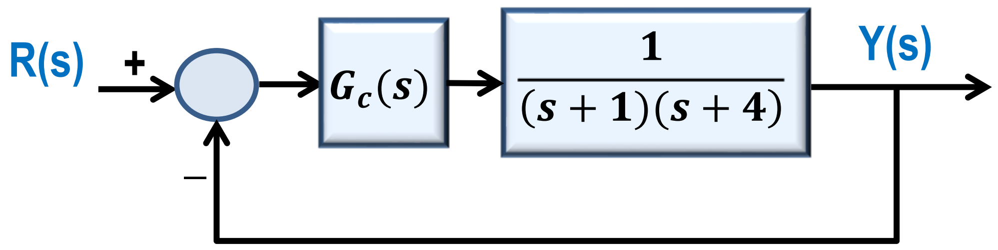
PD controller pulls the root-locus to the left and improves the relative stability of the system.



PD Controller Design via Root Locus

Example 4

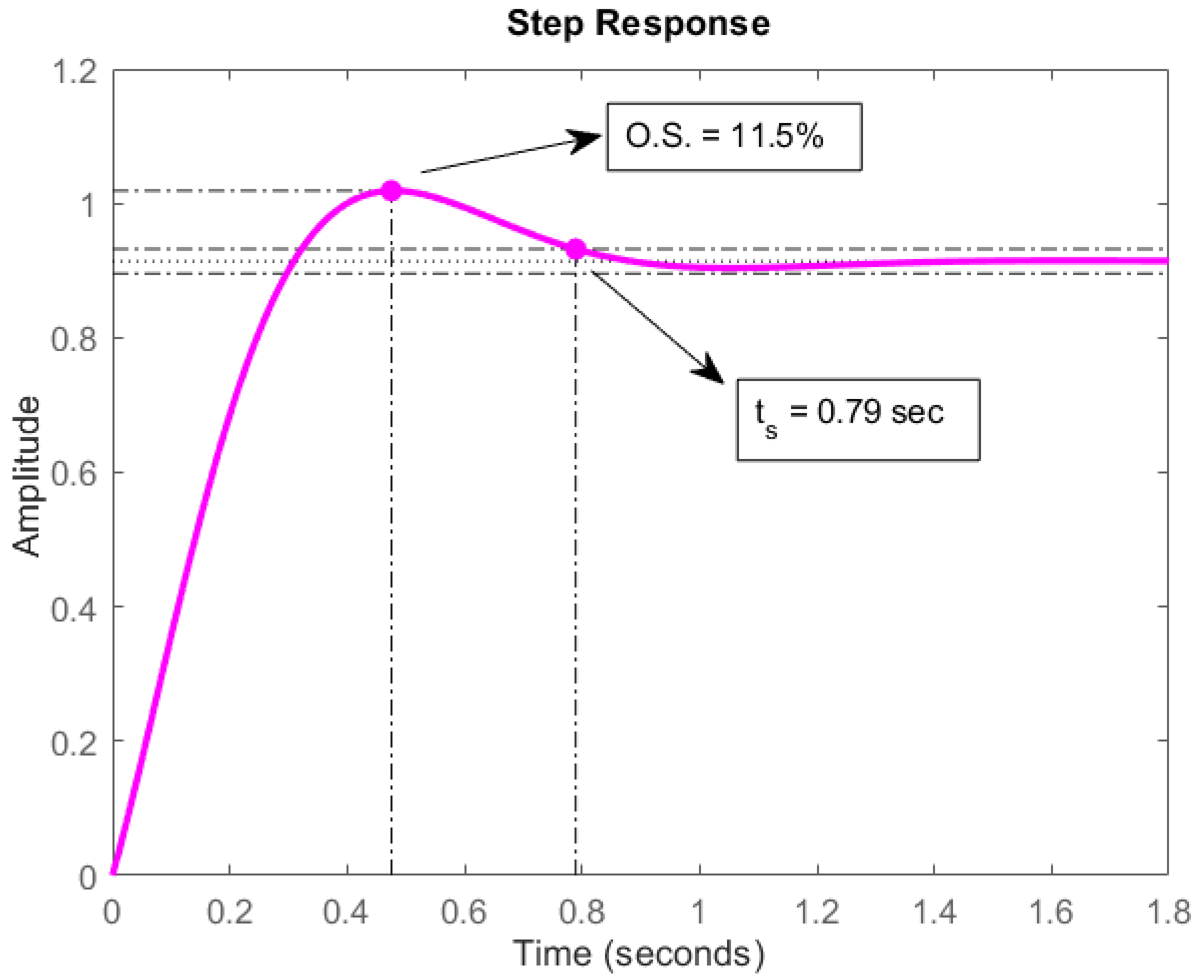
Consider the following closed-loop system with PD controller. Design PD controller to achieve a percentage of overshoot about 10% and settling time less than $\mathbf{1}\boldsymbol{sec}$.



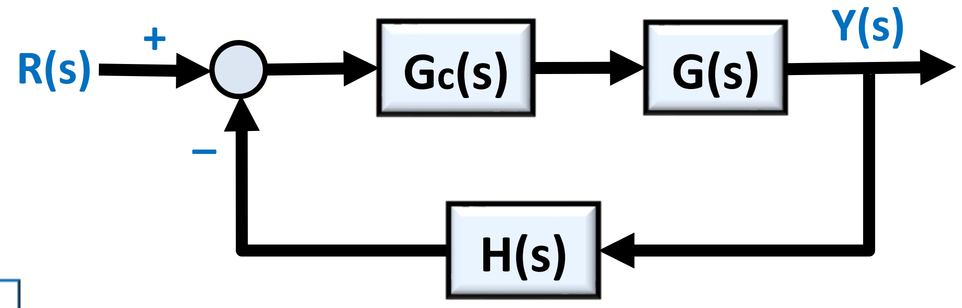
Step 7: Analyze and verify the designed compensator

$$\boldsymbol{G_c(s) = 42.36(1 + 0.074s\,)}$$

- Step responses of the compensated closed-loop system has the overshoot of about 11.5% and settling time of 0.79sec.



Dynamic Compensator Design



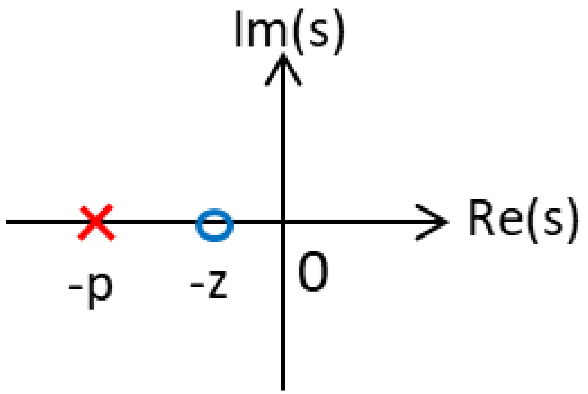
❑ Lead Compensator & Lag Compensator

- In general, a **compensator** has the following transfer function

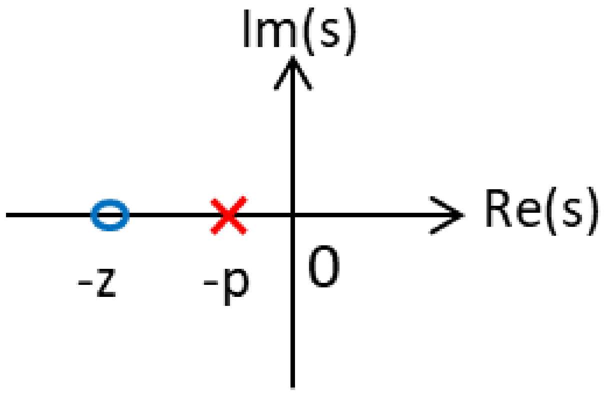
$$\boxed{G_c(s) = K_c \frac{s+z}{s+p}\,, \qquad z > 0, \qquad p > 0}$$

- According to the pole/zero locations we have the following structures:

- To improve transient response and stability.
- Similar to PD controller.
- Positive angle contribution.



Lead Compensator



Lag Compensator

- To improve steady-state error.
- Similar to PI controller.
- Negative angle contribution.

- Combined **lead-lag compensator** can improve both transient response and steady-state response, similar to a PID controller.

Lag Compensator Design via Root Locus

Assume that the uncompensated system meets the desired transient response specifications by simple gain adjustment.

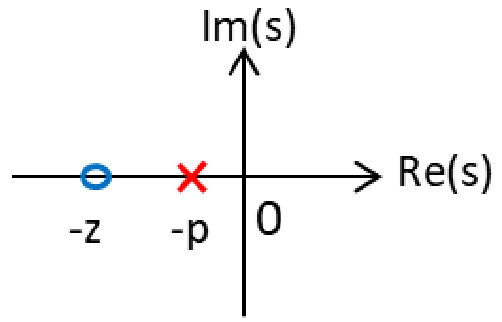
Step 1: Determine desired location of the dominant closed-loop poles. If requires determine the open-loop gain at the location of the closed-loop poles from magnitude condition.

Step 2: Calculate the desired error-constant (k_p, k_v or k_a), based on the desired steady-state error.

$$\boxed{k_p = \lim_{s\to 0} G(s)H(s)} \qquad \boxed{k_v = \lim_{s\to 0} sG(s)H(s)} \qquad \boxed{k_a = \lim_{s\to 0} s^2G(s)H(s)}$$

Step 3: Design a lag compensator to increase the error constant (k_p, k_v or k_a) to the desired value without significantly altering the original root-locus and the dominant pole locations.

$$\boxed{G_c(s) = K_c \frac{s+z}{s+p}}$$

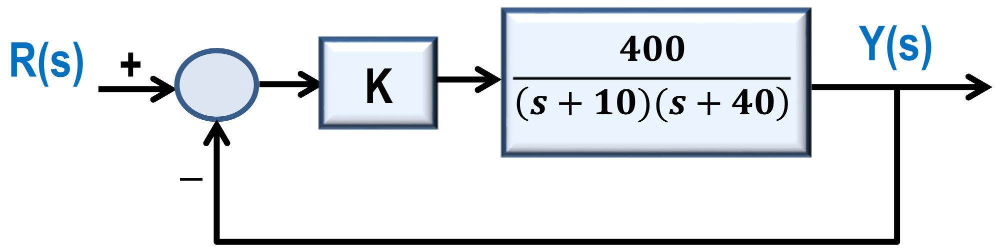


Step 4: Verify your design by comparing the dominant closed-loop pole locations with the desired poles by calculation or plotting the root-locus. If needs, adjust open-loop gain of the compensated system from the root-locus magnitude condition.

Lag Compensator Design via Root Locus

Example 5

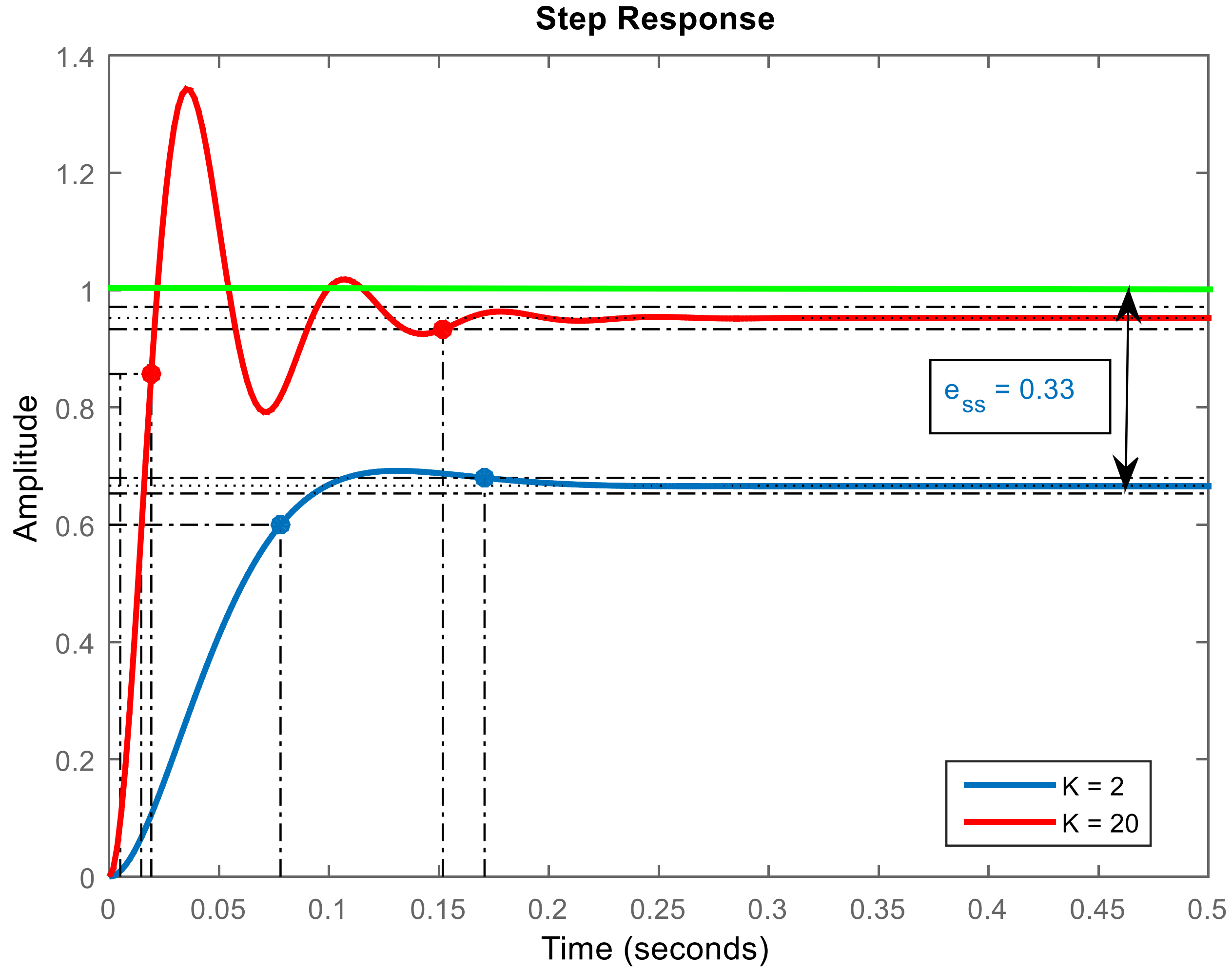
Consider the following second-order system with gain $K = 2$.
It is desired to decrease the steady-state error (to achieve $e_{ss} = 0.03$) without altering the transient response with $K = 2$.



Pre-design Performance Study

- The graph shows that $G(s)$ is a fast system.

$$K = 2 \quad \begin{cases} O.S. = 3.8\% \\ t_s = 0.17 \text{ sec} \\ t_r = 0.063 \text{ sec} \\ e_{ss} = 0.33 = 33\% \end{cases}$$



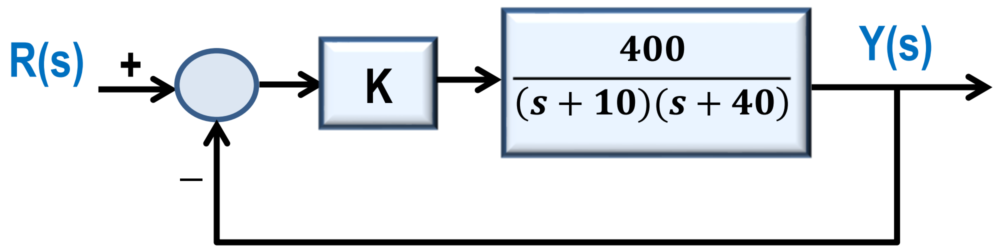
- The transient response specifications are good, but the steady-state error of 33% is not acceptable.
- The **goal** is to decrease the e_{ss} to **3%** without altering the transient response specifications.

Lag Compensator Design via Root Locus

Example 5

Consider the following second-order system with gain $K = 2$.

It is desired to decrease the steady-state error (to achieve $e_{ss} = 0.03$) without altering the transient response with $K = 2$.



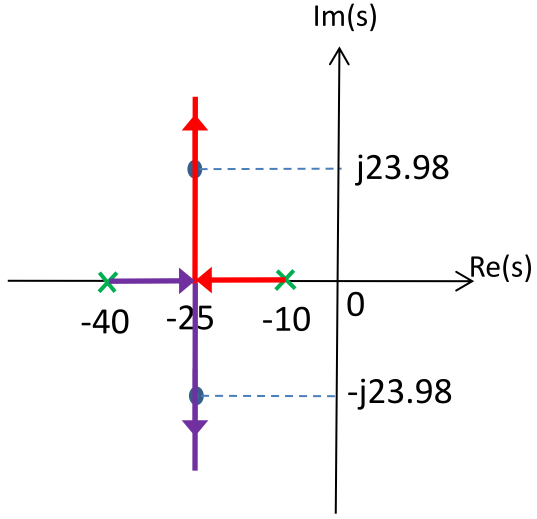
Step 1: Determine desired dominant closed-loop pole locations and the corresponding open-loop gain K

First, determine location of the dominant poles for closed-loop system with $\boldsymbol{K = 2}$.

$$T(s) = \frac{Y(s)}{R(s)} = \frac{KG(s)}{1+KG(s)} \rightarrow T(s) = \frac{800}{s^2+50s+1200}$$

$$\boxed{s = -25 \pm j23.98}$$

Desired Closed-loop Poles



$\boldsymbol{O.S. = 3.8\%}$
$\boldsymbol{t_s = 0.17}$ sec
$\boldsymbol{t_r = 0.063}$ sec

Find the corresponding open-loop gain from magnitude condition:

$$|KG(s)H(s)| = 1 \rightarrow |K| = \left.\frac{|s+10||s+40|}{400}\right|_{s=s_d} = \frac{|-15+j23.98||15+j23.98|}{400}$$

$$K = \frac{\sqrt{(15)^2+(23.98)^2} \times \sqrt{(15)^2+(23.98)^2}}{400}$$

$$\boxed{K = 2}$$

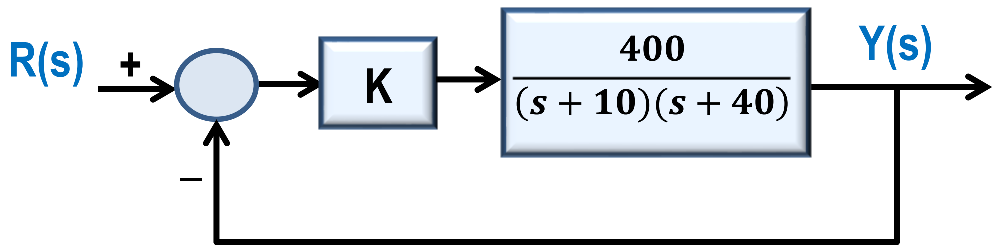
Open-loop gain

Lag Compensator Design via Root Locus

Example 5

Consider the following second-order system with gain $K = 2$.

It is desired to decrease the steady-state error (to achieve $e_{ss} = 0.03$) without altering the transient response with $K = 2$.



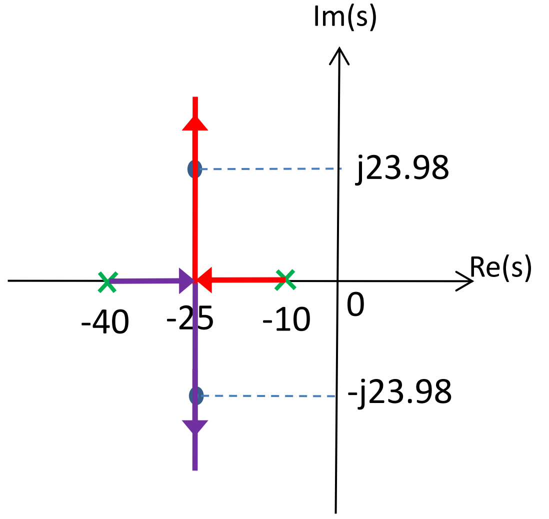
Step 2: Calculate the desired error-constant, from the given e_{ss}.

Calculate the desired step-error constant

$$\boxed{e_{ss} = 0.03} \rightarrow \quad e_{ss} = \frac{1}{1+k_p} = 0.03 \quad \Rightarrow \quad \boxed{k_p = 32.3}$$

Desired Steady-state Error

Desired Step-error Constant



$O.S. = 3.8\%$

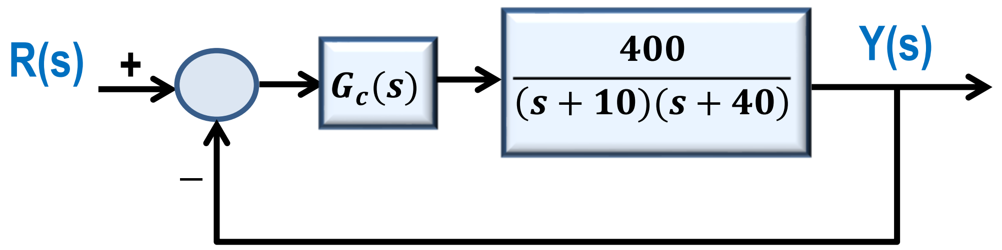
$t_s = 0.17$ sec

$t_r = 0.063$ sec

Lag Compensator Design via Root Locus

Example 5

Consider the following second-order system with gain $K = 2$.
It is desired to decrease the steady-state error (to achieve $e_{ss} = 0.03$) without altering the transient response with $K = 2$.

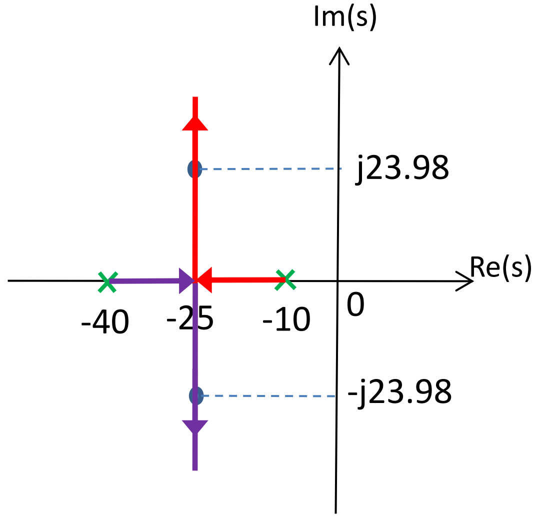


Step 3: Design a lag compensator to achieve the desired error value without altering the dominant poles.

$$G_c(s) = K_c \frac{s+z}{s+p}$$

To not change the location of the dominant closed-loop poles, the compensator's gain must be selected equal to $K = 2$

$$\boxed{K_c = K = 2}$$



The step-error constant for the compensated system is,

$$k_p = \lim_{s\to 0} G_c(s)G(s) = \lim_{s\to 0} K_c \frac{s+z}{s+p} \cdot \frac{400}{(s+10)(s+40)} = K_c \frac{z}{p}$$

$$\boxed{k_p = 32.3}$$

$$\boxed{K_c = 2}$$

$$\Rightarrow \quad 32.3 = 2 \times \frac{z}{p} \rightarrow \boxed{z \approx 16p}$$

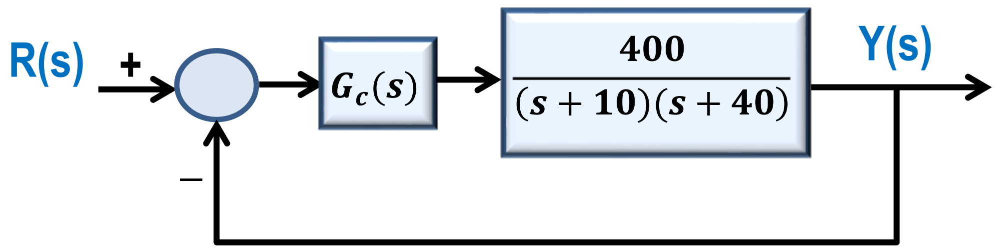
$O.S. = 3.8\%$
$t_s = 0.17$ sec
$t_r = 0.063$ sec

- **Pole/zero of lag compensator must be selected far enough from the dominant closed-loop poles and close to the origin. However, settling time increases by selecting them too close to the origin.**

Lag Compensator Design via Root Locus

Example 5

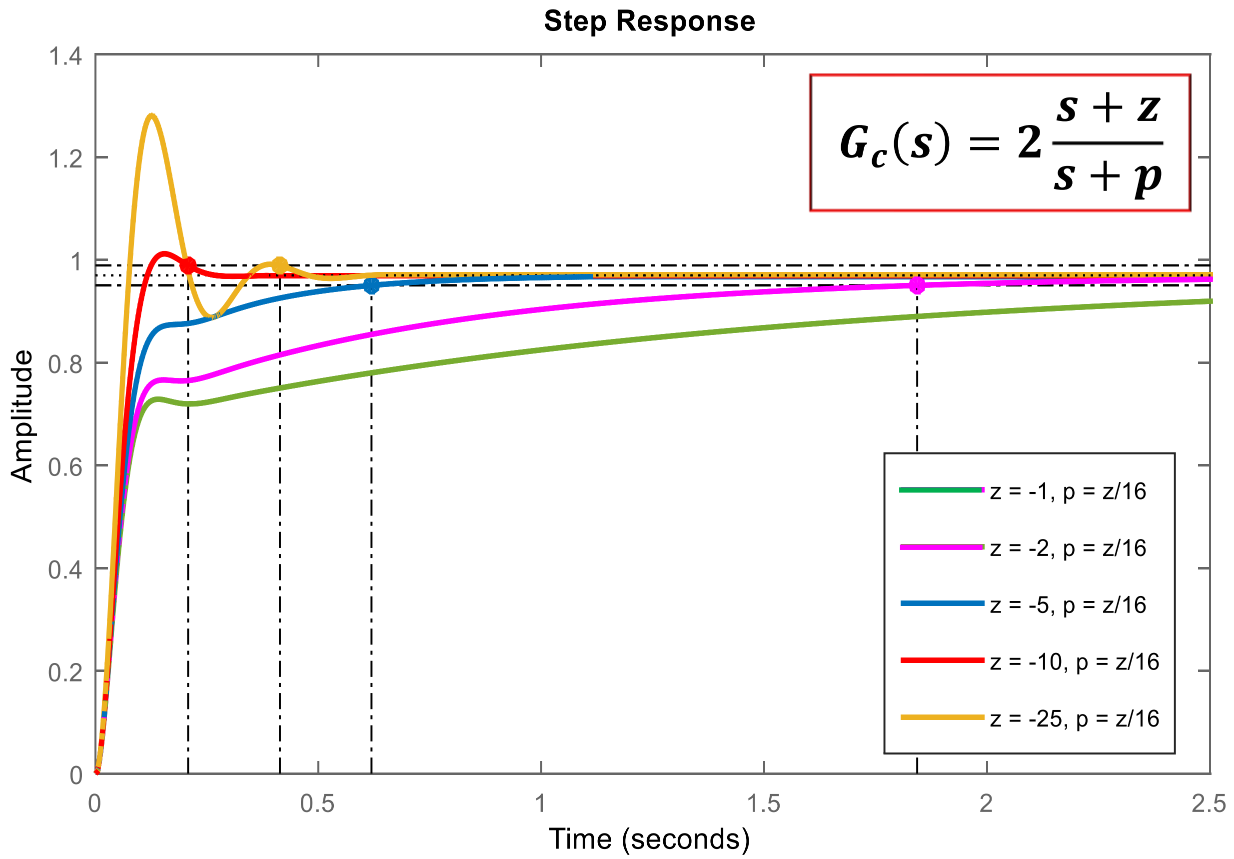
Consider the following second-order system with gain $K = 2$.
It is desired to decrease the steady-state error (to achieve $e_{ss} = 0.03$) without altering the transient response with $K = 2$.



Step 3: Design a lag compensator to achieve the desired error value without altering the dominant poles.

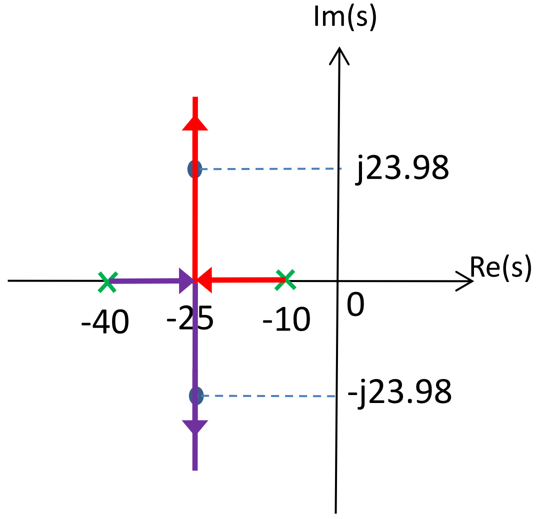
$$G_c(s) = K_c \frac{s+z}{s+p}$$

- We can compare the effect of selecting different pole/zero locations for the lag compensator and fine tune the compensator.



$$\textbf{If } z = 10 \rightarrow p = \frac{10}{16} = 0.625$$

$$G_c(s) = 2\frac{s+10}{s+0.625}$$



$O.S. = 3.8\%$
$t_s = 0.17$ sec
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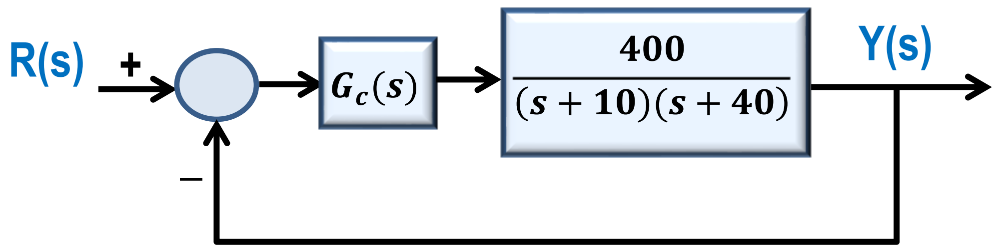
Lag Compensator Design via Root Locus

Example 5

Consider the following second-order system with gain $K = 2$.
It is desired to decrease the steady-state error (to achieve $e_{ss} = 0.03$) without altering the transient response with $K = 2$.

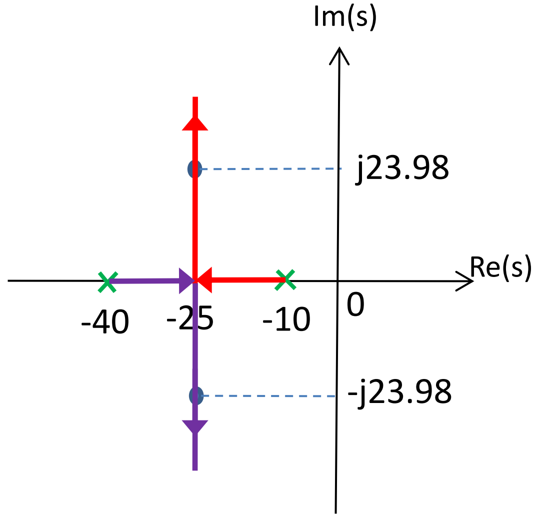


Step 4: **Analyze and verify the designed compensator.**

$$G_c(s) = 2\frac{s+10}{s+0.625}$$

Determine closed-loop transfer function of the compensated system and check the pole locations

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} \longrightarrow T(s) = \frac{800(s+10)}{s^3+50.62s^2+1231s+8250}$$



The closed-loop poles are located at

$$\boxed{s_{1,2} = -20.3125 \pm j20.3077, \qquad s_3 = -10}$$

Dominant Poles | Close to zero location

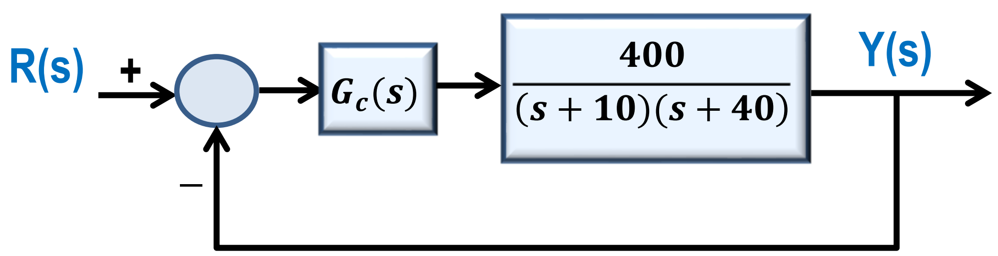
$O.S. = 3.8\%$
$t_s = 0.17$ sec
$t_r = 0.063$ sec

- The dominant closed-loop poles are located at the desired places in the s-plane.
- The third pole at $s_3 = -10$ will be cancelled out with the zero at $s = -10$.

Lag Compensator Design via Root Locus

Example 5

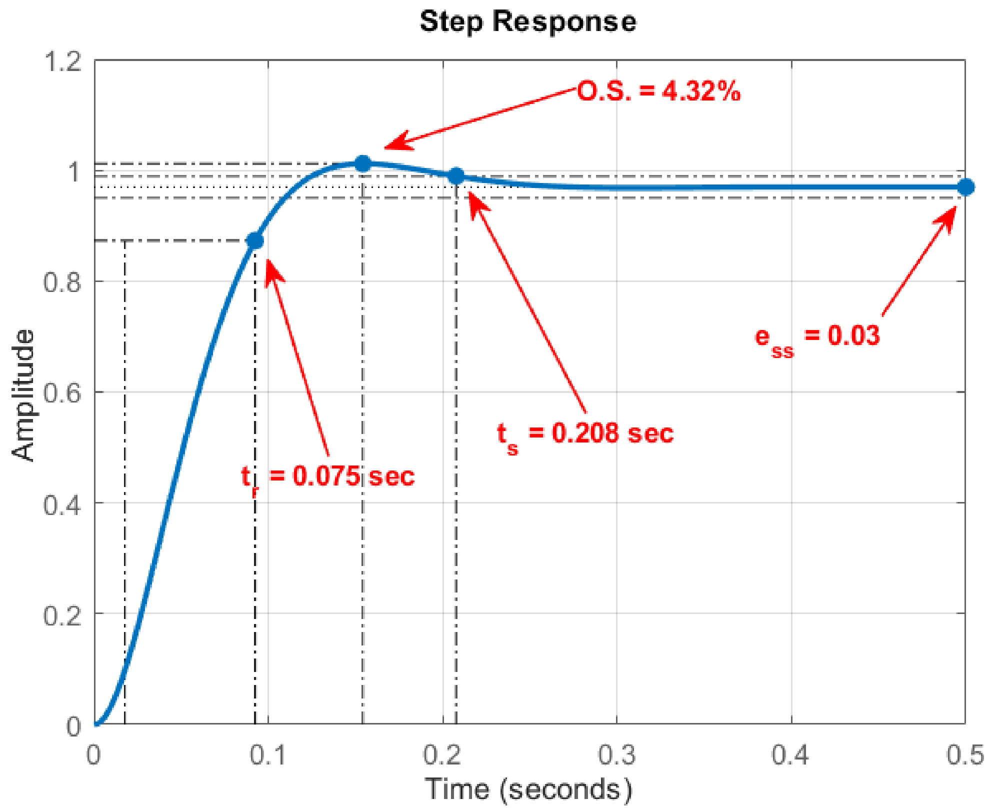
Consider the following second-order system with gain $K = 2$.
It is desired to decrease the steady-state error (to achieve $e_{ss} = 0.03$) without altering the transient response with $K = 2$.

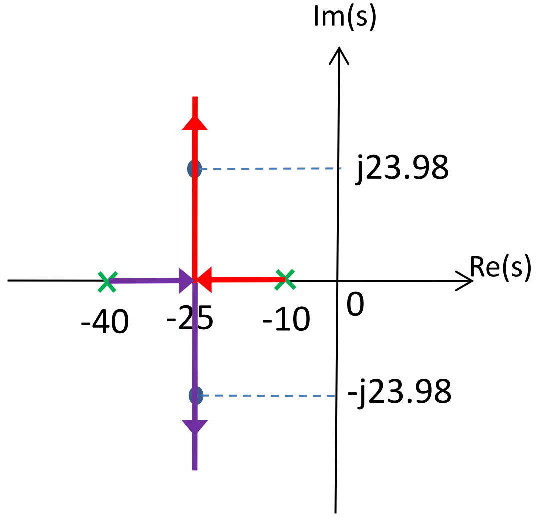


$$G_c(s) = 2\,\frac{s+10}{s+0.625}$$

Step 4: **Analyze and verify the designed compensator.**

- Transient responses are close to the desired values.
- Steady-state error decreases to the desired value of $e_{ss} = 0.03$.



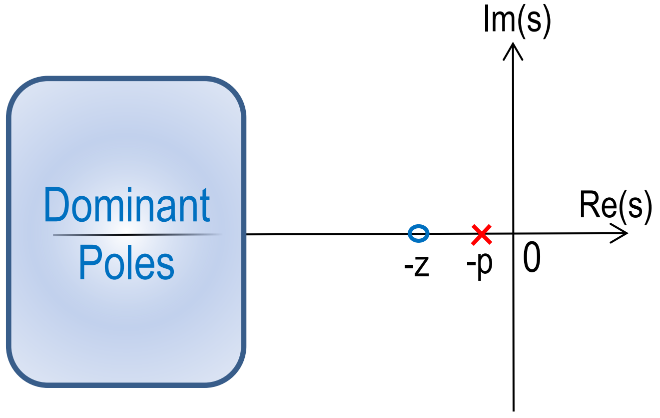


$O.S. = 3.8\%$
$t_s = 0.17$ sec
$t_r = 0.063$ sec

PI Controller Design via Root Locus

- In lag compensator design, the pole and zero have to be located far enough from the dominant poles and close to the origin.

$$G_c(s) = K_c \frac{s+z}{s+p}$$



- The lag compensator can also be designed by placing the pole exactly at the origin

$$p = 0 \rightarrow G_c(s) = K_c \frac{s+z}{s}$$

- In this case the steady-state error of the compensated closed-loop system for unit-step input will be zero:

$$k_p = \lim_{s\to 0} G_c(s)G(s) = \lim_{s\to 0} K_c \frac{s+z}{s} G(s) = \infty \rightarrow e_{ss} = \frac{1}{1+k_p} = 0$$

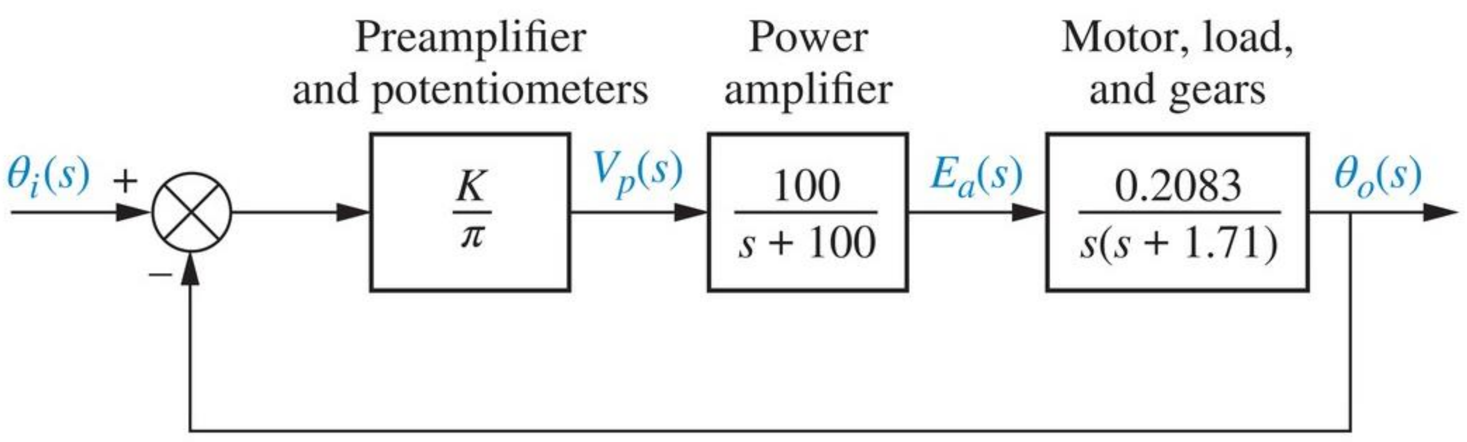
- The compensator's zero must be selected far enough from the dominant poles, and close to the origin.
- The gain, K_c is selected to achieve the desired performance of transient response.
- The lag compensator with a pole at the origin can also be shown as a **PI controller**:

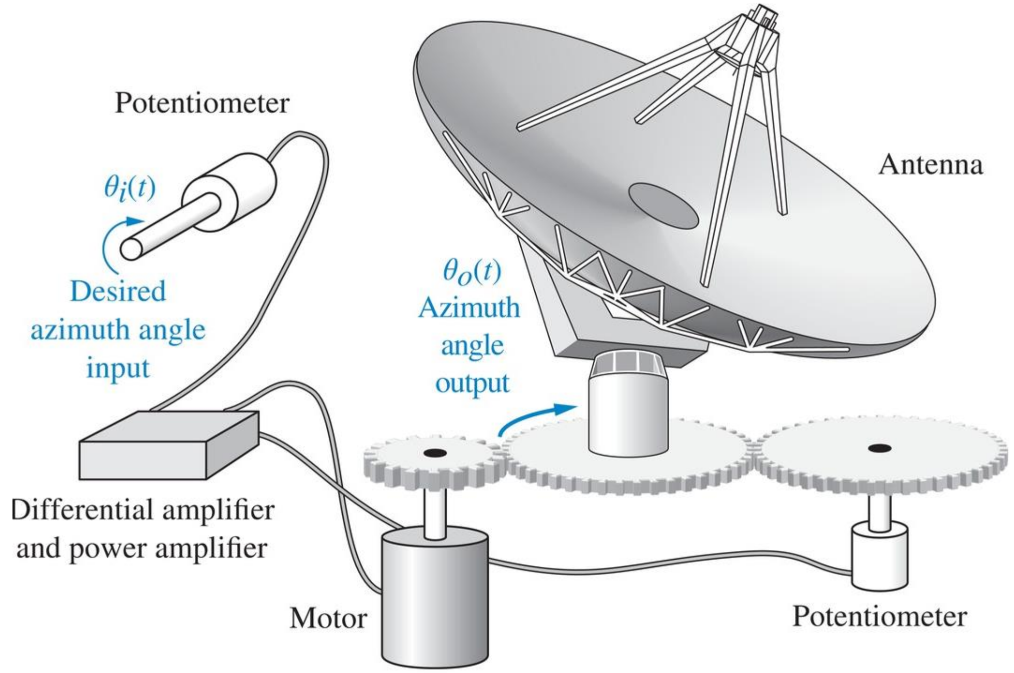
$$G_c(s) = K_c \frac{s+z}{s} = K_c\left(1+\frac{z}{s}\right) \rightarrow G_c(s) = K_p\left(1+\frac{1}{T_i s}\right)$$

where, the proportional gain K_p and the integral time-constant T_i are defined as $T_i = \frac{1}{z}$ $\quad K_p = K_c$

Case Study: Antenna Control System

- Consider the ***motor-driven antenna azimuth position control system*** example from Lecture 1.
- We determined the block diagram of the control system as below:





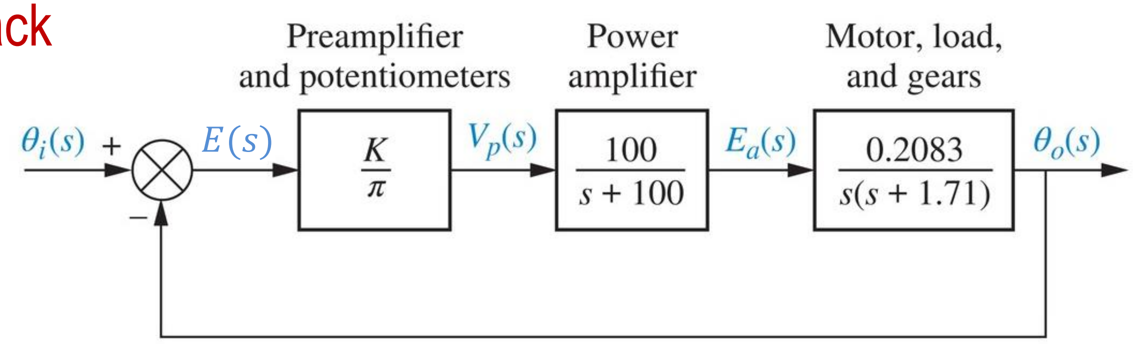
- In this part, we are interested in determining the value of required gain K and design a lead-lag compensator to meet time response requirements, such as percent overshoot, settling time, peak time and the steady-state error.

- The following case study emphasizes this design procedure, using the root locus.

Case Study: Antenna Control System

For the antenna azimuth position control system with the given simplified unity-feedback block diagram model:

a) Find the preamplifier gain K required for 25% overshoot via root-locus.



Find the overall open-loop transfer function.

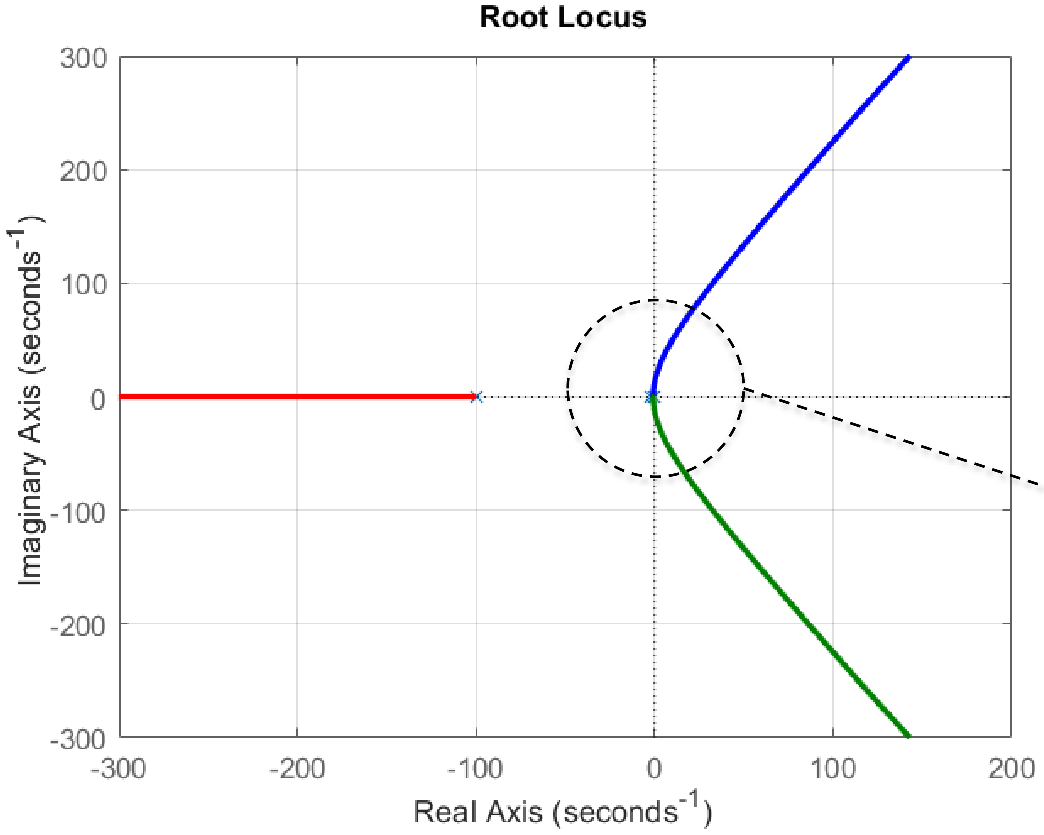
$$G(s) = \frac{6.63K}{s(s+100)(s+1.71)}$$

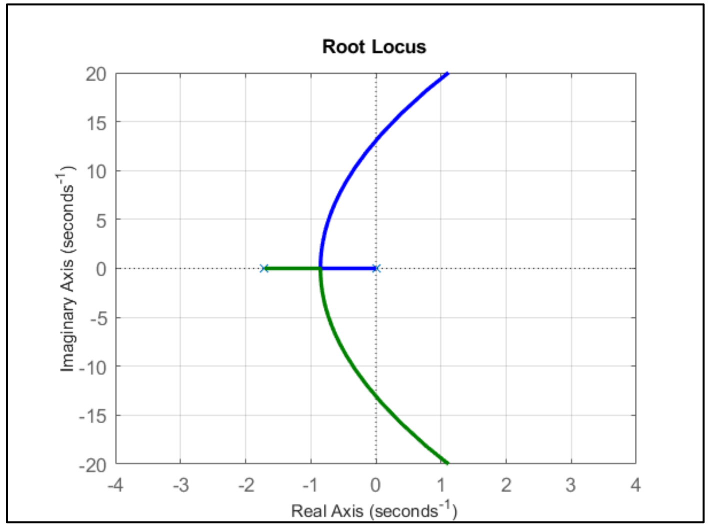
Plot the root-locus of the system.

Poles: $s_1 = 0, \quad s_2 = -1.71, \quad s_3 = -100$

Zeros: No finite zero. Three zeros at infinity.

Since the pole at -100 is too far from the other two poles, the poles at 0 and -1.71 are the dominant poles.





Case Study: Antenna Control System

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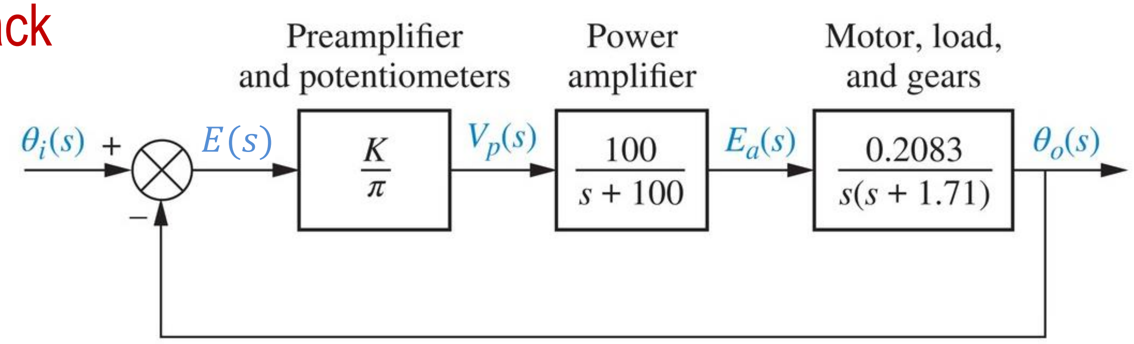


Find the damping ratio correspond to 25% overshoot.

$$\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}} = \frac{-\ln(0.25)}{\sqrt{\pi^2 + \ln^2(0.25)}} \rightarrow \zeta = 0.404$$

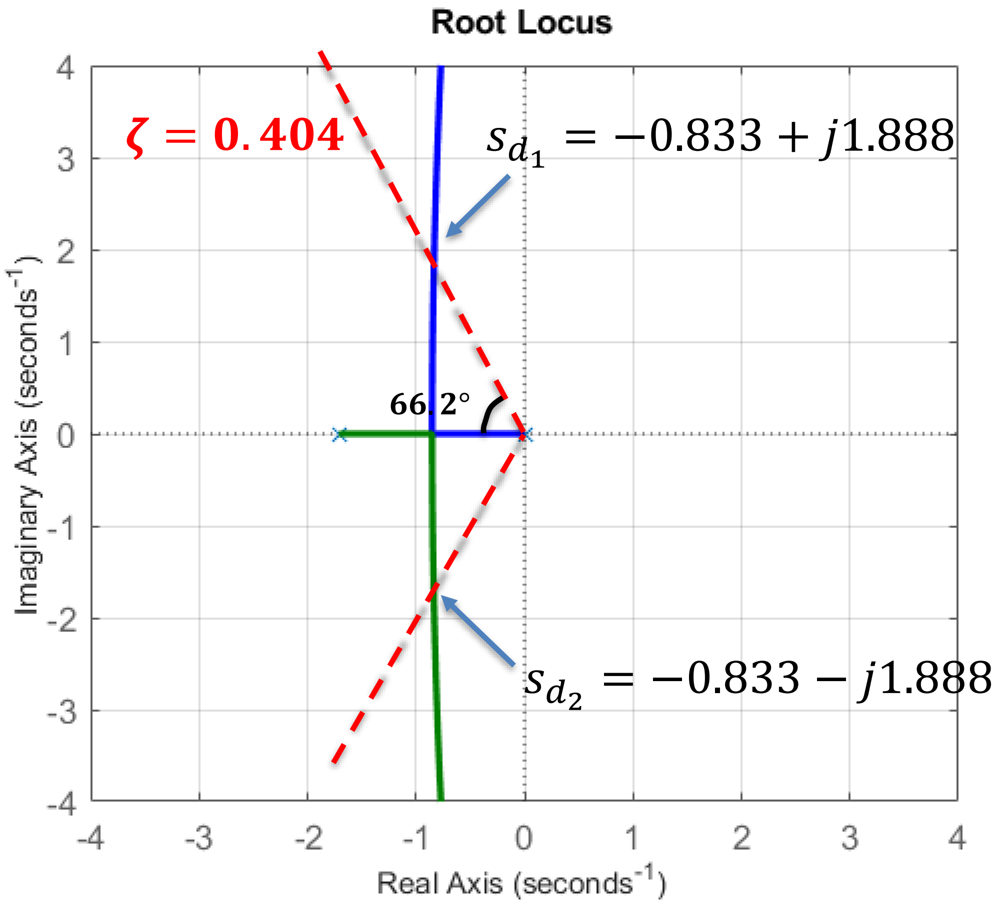
Draw a radial line from the origin at an angle of $\theta = \cos^{-1}\zeta$

$$\theta = \cos^{-1} 0.404 = 66.2°$$

The intersection of this line with the root locus locates the systems dominant closed-loop poles to have a 25% overshoot.

From the graph the dominant poles are at:

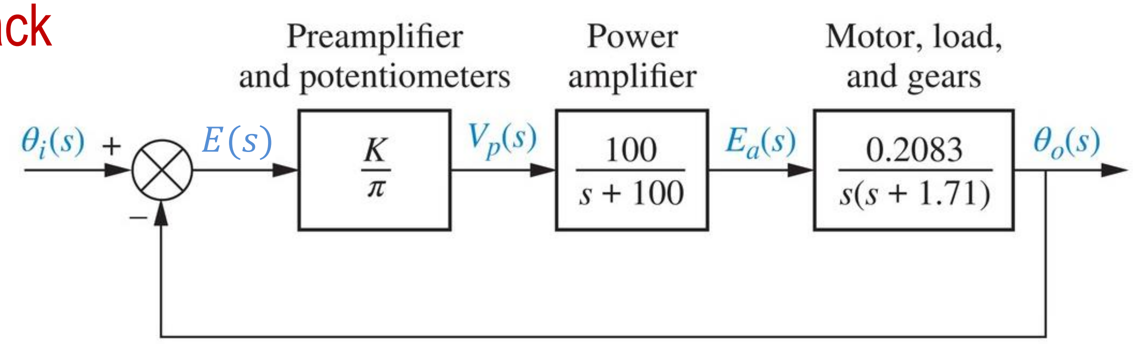
$$\boxed{s_{d_{1,2}} = -0.833 \pm j1.888}$$



Case Study: Antenna Control System

For the antenna azimuth position control system with the given simplified unity-feedback block diagram model:

a) Find the preamplifier gain K required for 25% overshoot via root-locus.



The corresponding gain K is obtained from the root-locus gain condition:

$$|\boldsymbol{KG(s)H(s)}| = \mathbf{1} \quad \rightarrow \quad \left|\frac{6.63K}{s(s+100)(s+1.71)}\right| = 1$$

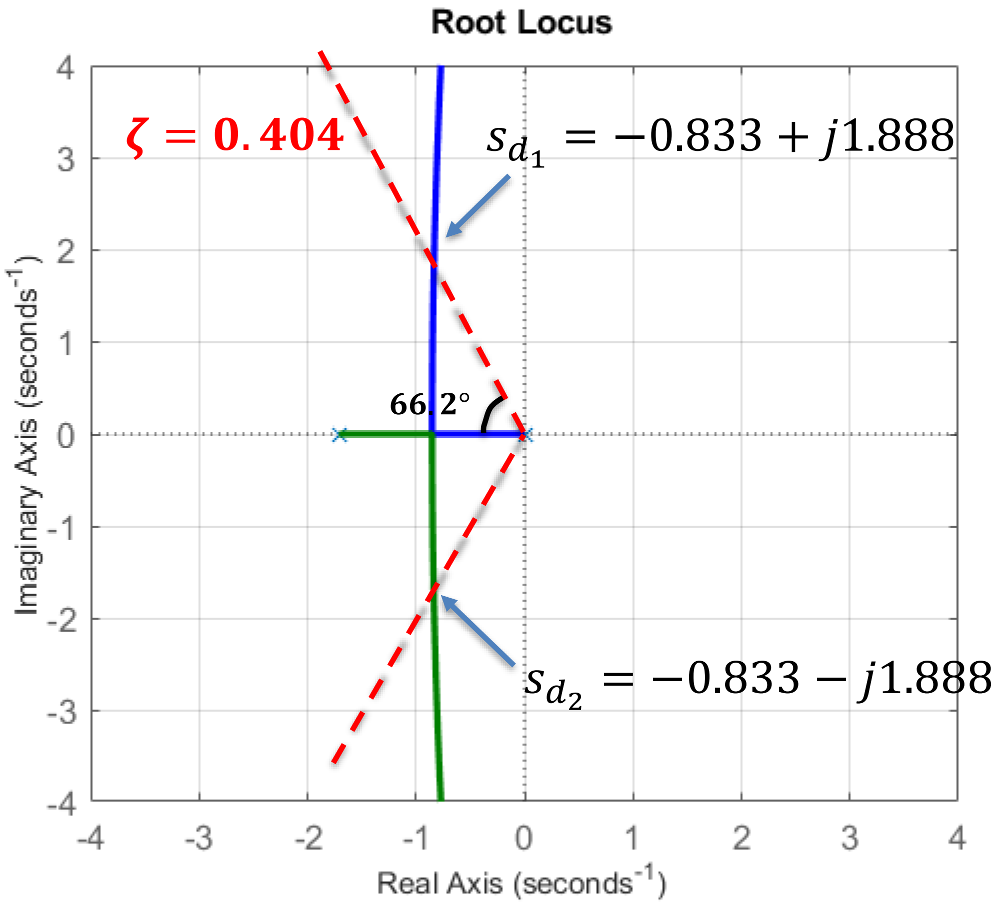
$$|6.63K| = |s(s+100)(s+1.71)|_{s=s_{d_1}}$$

$$|6.63K| = |s_d||s_d+100||s_d+1.71|$$

$$|6.63K| = |-0.833+j1.888||99.167+j1.888||0.877+j1.888|$$

$$|6.63K| = (2.064)(99.185)(2.082) = 426.22$$

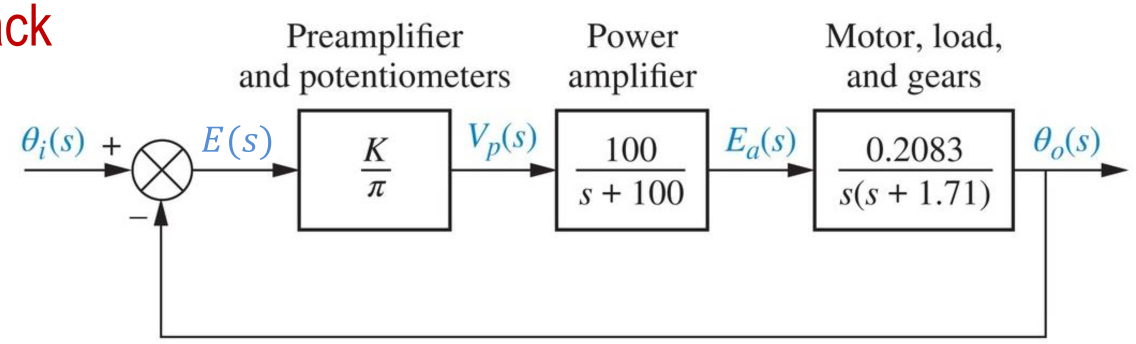
$$K = \frac{426.22}{6.63} \quad \rightarrow \quad \boxed{\boldsymbol{K = 64.29}}$$



Case Study: Antenna Control System

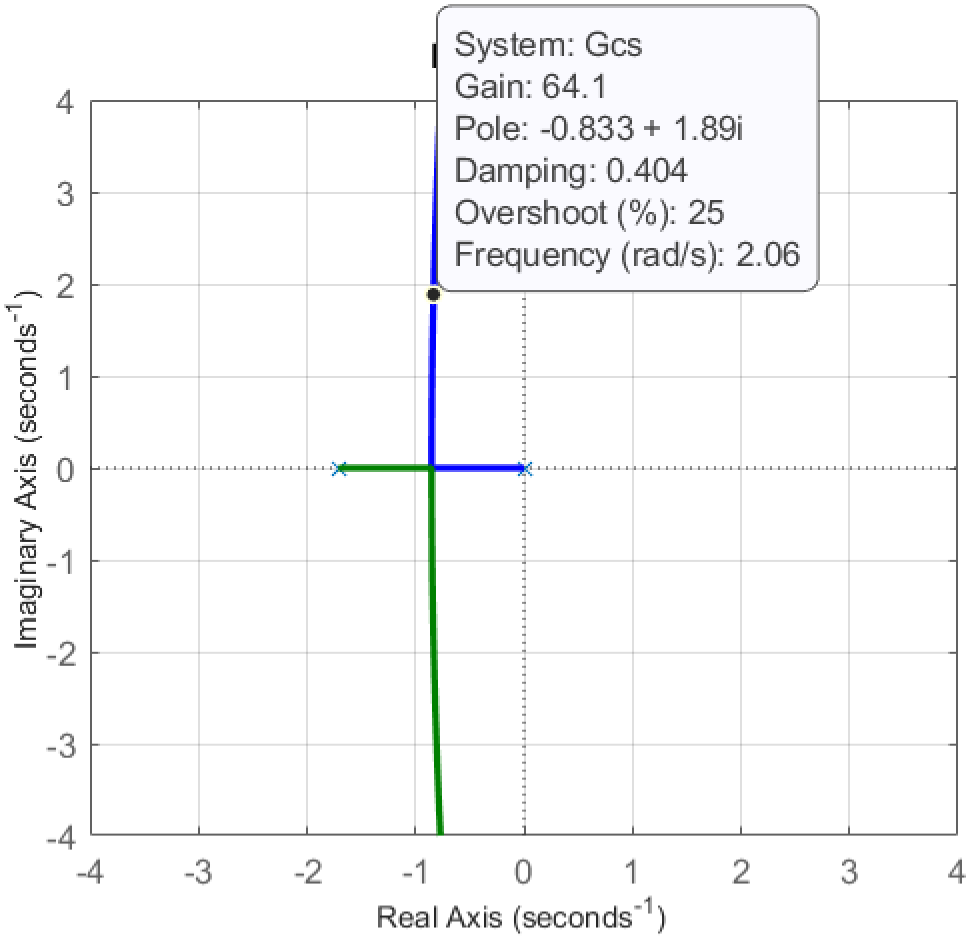
For the antenna azimuth position control system with the given simplified unity-feedback block diagram model:

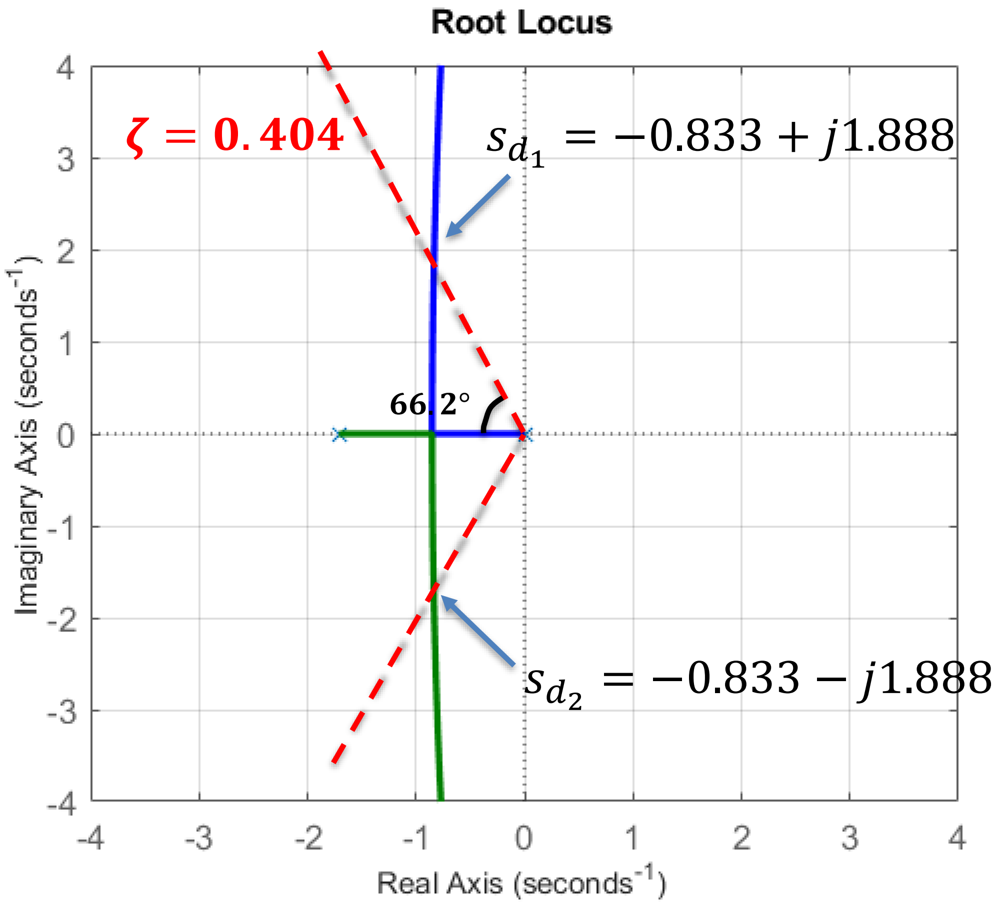
a) Find the preamplifier gain K required for 25% overshoot via root-locus.



We can also determine the required gain K by inspecting the root-locus plot in MATLAB

Gain = 64.1
Pole = -0.833+j1.89
Damping ration = 0.404
Overshoot = 25%
Natural freq. = 2.06

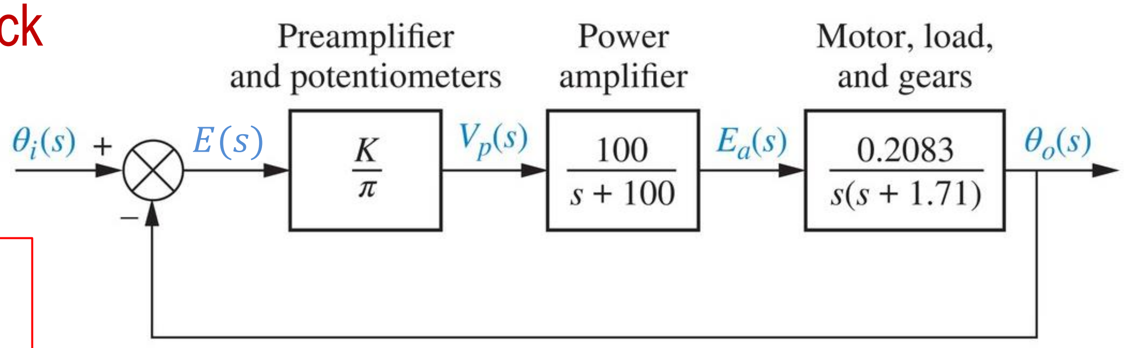




Case Study: Antenna Control System

For the antenna azimuth position control system with the given simplified unity-feedback block diagram model:

b) Design a lead compensator for settling-time of 2 sec and 25% overshoot.



$$G_c(s) = K_c \frac{s+z}{s+p}$$

Find the damping ratio correspond to 25% overshoot.

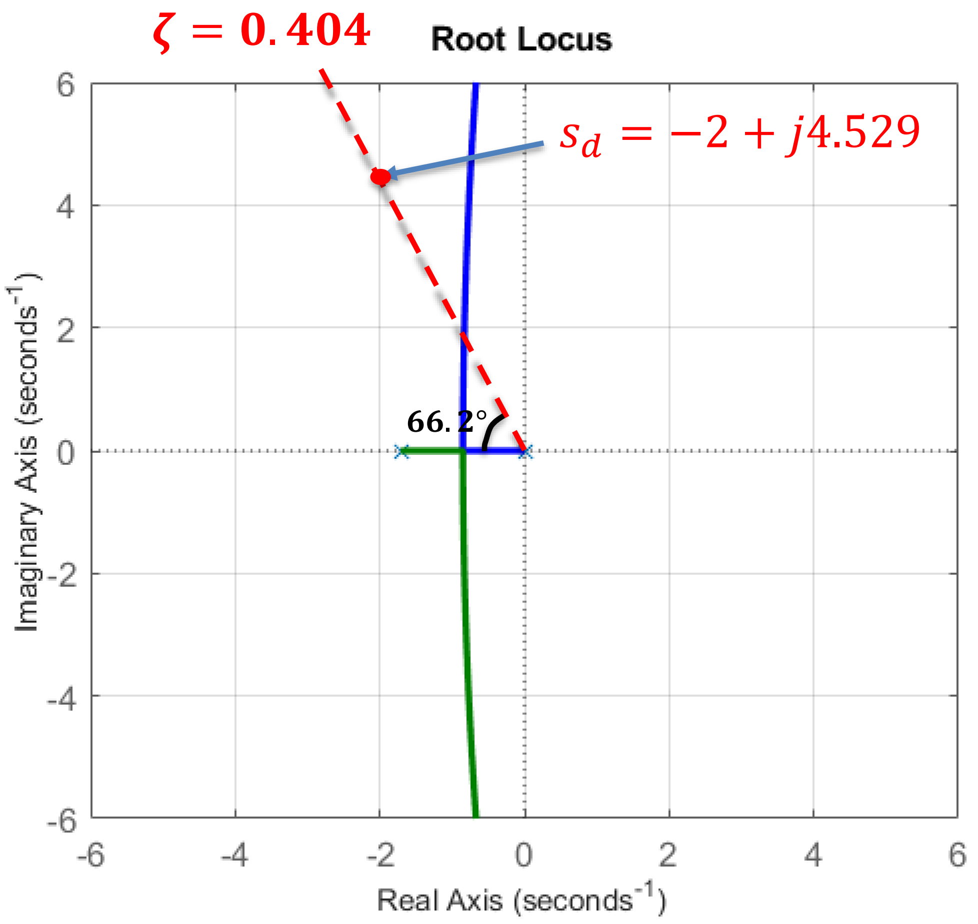
$$\zeta = \frac{-\ln(O.S.)}{\sqrt{\pi^2 + \ln^2(O.S.)}} = \frac{-\ln(0.25)}{\sqrt{\pi^2 + \ln^2(0.25)}} \quad \rightarrow \quad \zeta = 0.404$$

Determine the required natural frequency to have settling-time of 2 sec.

$$t_s = \frac{4}{\zeta\omega_n} \quad \rightarrow \quad \omega_n = \frac{4}{t_s\zeta} = \frac{4}{2(0.404)} \quad \rightarrow \quad \omega_n = 4.95 \quad rad/s$$

Therefore, the desired dominant poles will be at:

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \quad \rightarrow \quad s_{1,2} = -2 \pm j4.529$$

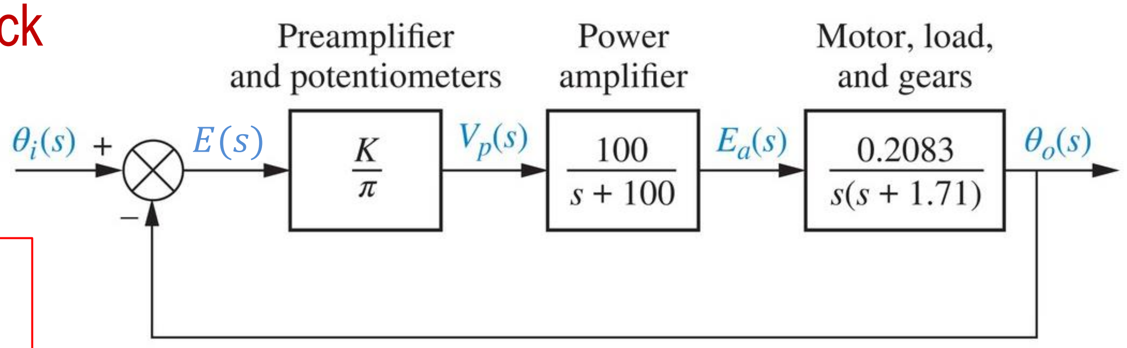


The desired poles are **not** on the root-locus of the system, the desired transient response characteristics are not achievable by a simple gain tuning.

Case Study: Antenna Control System

For the antenna azimuth position control system with the given simplified unity-feedback block diagram model:

b) Design a lead compensator for settling-time of 2 sec and 25% overshoot.

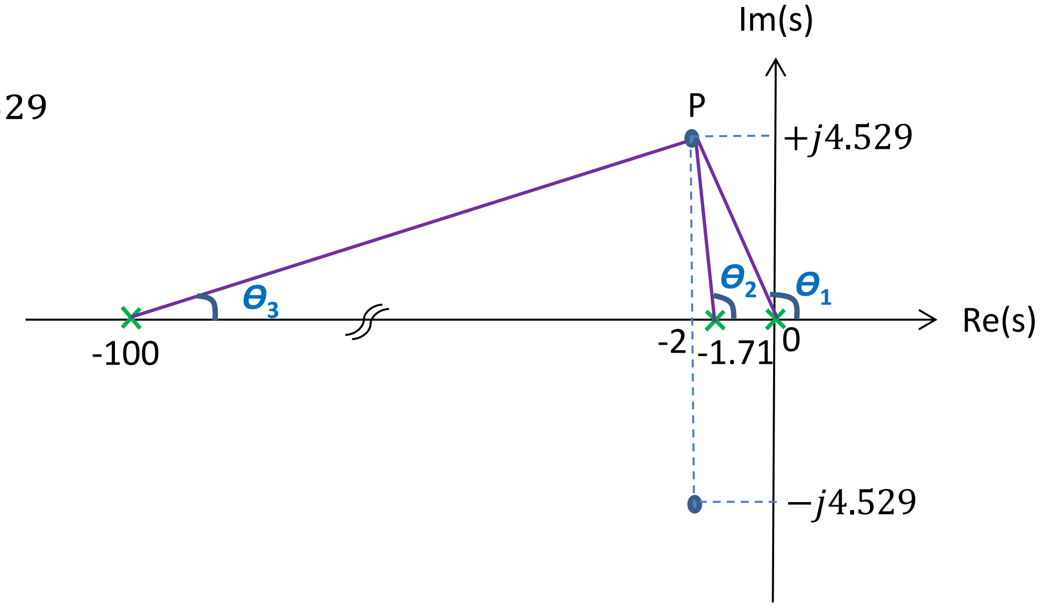


$$G_c(s) = K_c \frac{s+z}{s+p}$$

Find sum of the angles at the desired closed-loop poles location and determine the angle deficiency.

$$\angle\left(\frac{6.63K}{s\,(s+1.71)(s+100)}\right)\Bigg|_{s=s_{d1}} = \angle 6.63\text{K} - \angle(s) - \angle(s+1.71) - \angle(s+100)\Big|_{s=-2+j4.529}$$

$$= 0 - \angle\theta_1 - \angle\theta_2 - \angle\theta_3 \;= 0 - 113.83^{\circ} - 93.66^{\circ} - 2.64^{\circ} = \mathbf{-210.13^{\circ}}$$



The angle deficiency is calculated as

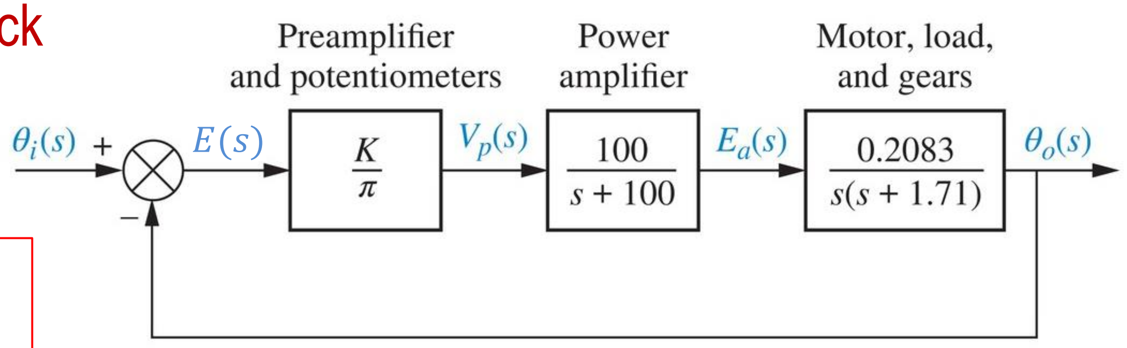
$$-210.13^{\circ} + \boldsymbol{\phi} = -180^{\circ} \;\rightarrow\; \boldsymbol{\phi} = 210.13^{\circ} - 180^{\circ} = \mathbf{30.13^{\circ}}$$

The **lead compensator** must contribute the angle of $\boldsymbol{\phi = 30.13^{\circ}}$ at the desired pole locations.

Case Study: Antenna Control System

For the antenna azimuth position control system with the given simplified unity-feedback block diagram model:

b) Design a lead compensator for settling-time of 2 sec and 25% overshoot.



$$G_c(s) = K_c \frac{s+z}{s+p}$$

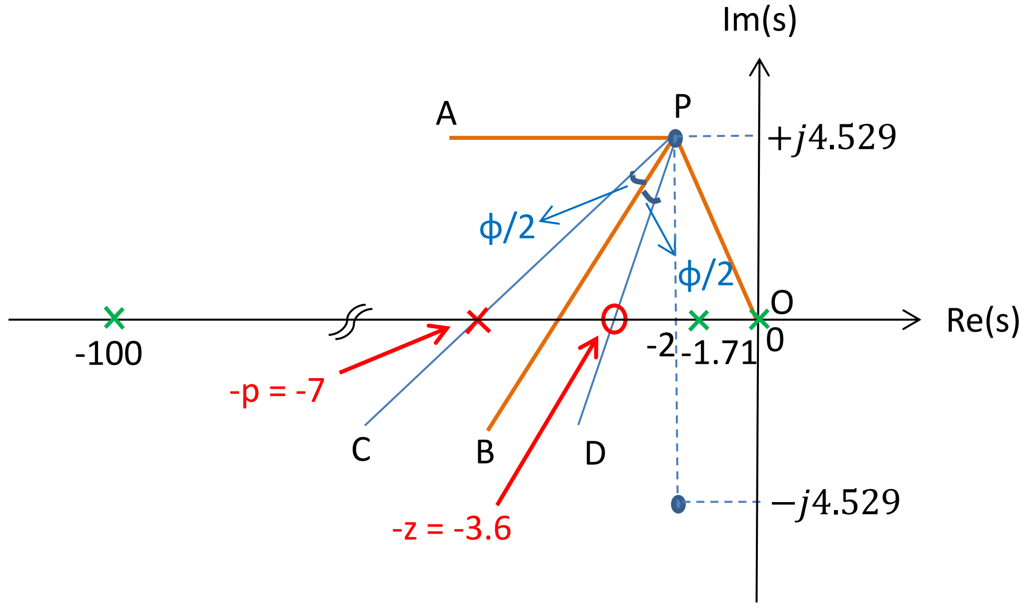
Determine pole/zero locations of the lead compensator to compensate the angle deficiency

- Draw lines PA and PO
- Draw bisector line PB $\angle APB = \angle BPO = \dfrac{\angle APO}{2}$
- Draw lines PC and PD so that

$$\angle CPB = \angle BPD = \frac{\phi}{2} = \frac{30.13°}{2} = 15.065^{\circ}$$

- Pole and zero are the intersections of PC and PD with real axis

$$\boldsymbol{G_c(s) = K_c \frac{s+3.6}{s+7}}$$

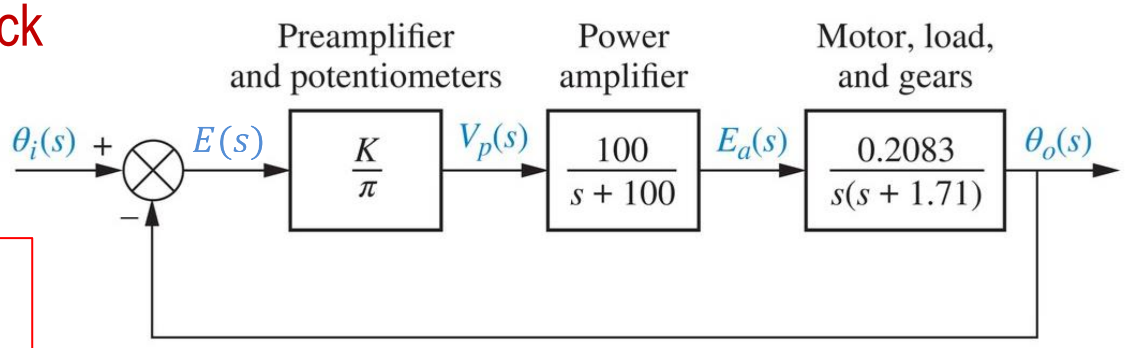


$$\boldsymbol{z = 3.6, \quad p = 7}$$

Case Study: Antenna Control System

For the antenna azimuth position control system with the given simplified unity-feedback block diagram model:

b) Design a lead compensator for settling-time of 2 sec and 25% overshoot.



$$G_c(s) = K_c \frac{s+z}{s+p}$$

Next, calculate the gain K_c using the magnitude condition

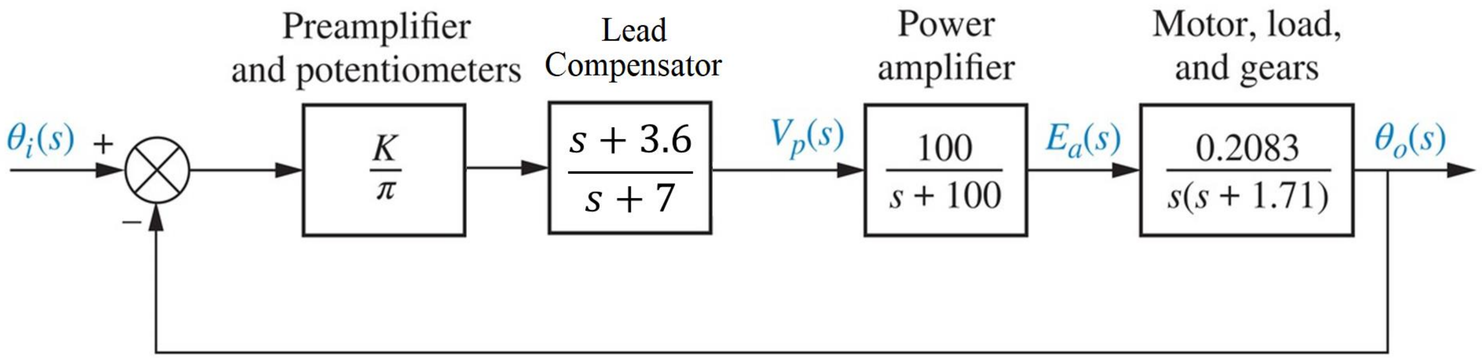
$$\left| K_c \frac{s+3.6}{s+7} \cdot \frac{6.63}{s(s+100)(s+1.71)} \right|_{s=-2+j4.529} = 1$$

$$|K_c| = \left| \frac{|s||s+100||s+1.71||s+7|}{|6.63||s+3.6|} \right|_{s=-2+j4.529} = \frac{|-2+j4.529||98+j4.529||-0.29+j4.529||5+j4.529|}{|6.63||1.6+j4.529|}$$

$$|K_c| = \frac{(4.95)(98.1)(4.54)(6.75)}{(6.63)(4.80)} = 433.6$$

$$\boxed{G_c(s) = 433.6 \frac{s+3.6}{s+7}}$$

Lead compensator

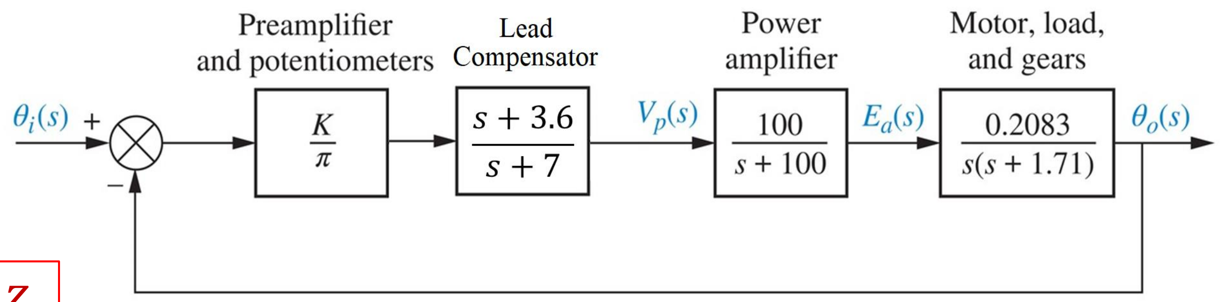


The preamplifier gain should be selected as: $K = 433.6$

Case Study: Antenna Control System

For the antenna azimuth position control system with the given simplified unity-feedback block diagram model:

c) Design a lag compensator to have a steady-state error of 0.05 for unit-ramp input without changing the transient response characteristics.



$$\boxed{G_c(s) = \frac{s+z}{s+p}}$$

Calculate the desired ramp error-constant

$$\boldsymbol{e_{ss} = \frac{1}{k_v}} \quad \rightarrow \quad 0.05 = \frac{1}{k_v} \quad \rightarrow \quad \boldsymbol{k_v = 20}$$

The ramp error-constant for the lead-lag compensated system is,

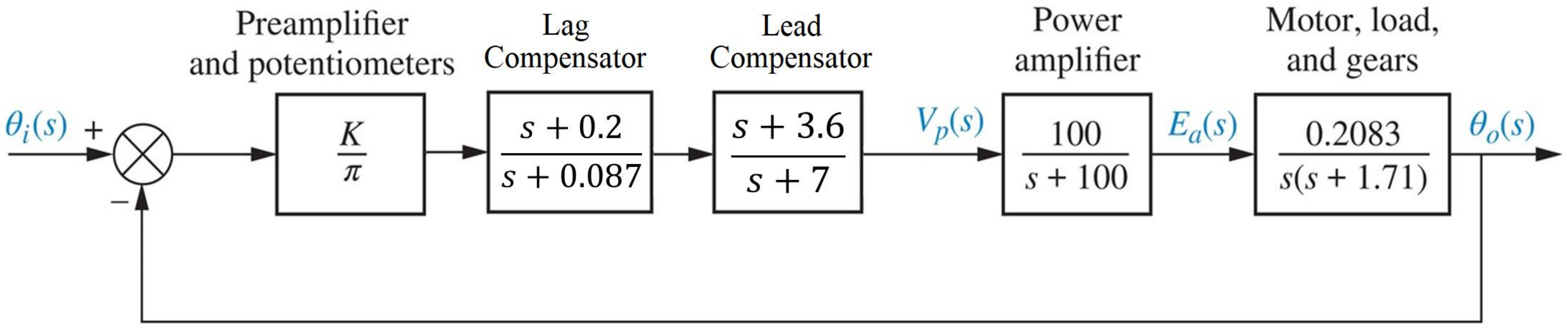
$$\boldsymbol{k_v = \lim_{s\to 0} sG_c(s)G(s)} = \lim_{s\to 0} s.\frac{s+z}{s+p}\cdot\frac{6.63(433.6)(s+3.6)}{s(s+100)(s+1.71)(s+7)} \quad \rightarrow \quad 20 = 8.65\times\frac{z}{p} \quad \rightarrow \quad \boldsymbol{z \approx 2.31p}$$

Since the desired dominant poles are at $\boldsymbol{s_{1,2} = -2 \pm j4.529}$, we can select the pole and zero of the lag compensator as:

$$z = 0.2 \rightarrow p = \frac{0.2}{2.31} = 0.087$$

$$\boxed{\boldsymbol{G_c(s) = \frac{s+0.2}{s+0.087}}}$$

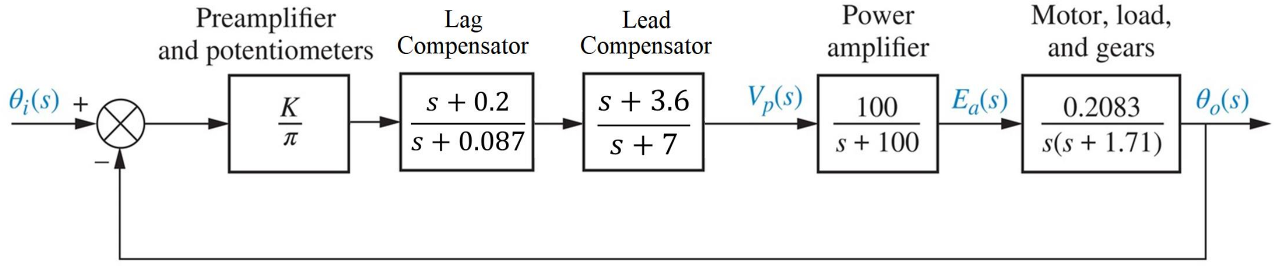
Lag compensator



Case Study: Antenna Control System

For the antenna azimuth position control system with the given simplified unity-feedback block diagram model:

c) Design a lag compensator to have a steady-state error of 0.05 for unit-ramp input without changing the transient response characteristics.

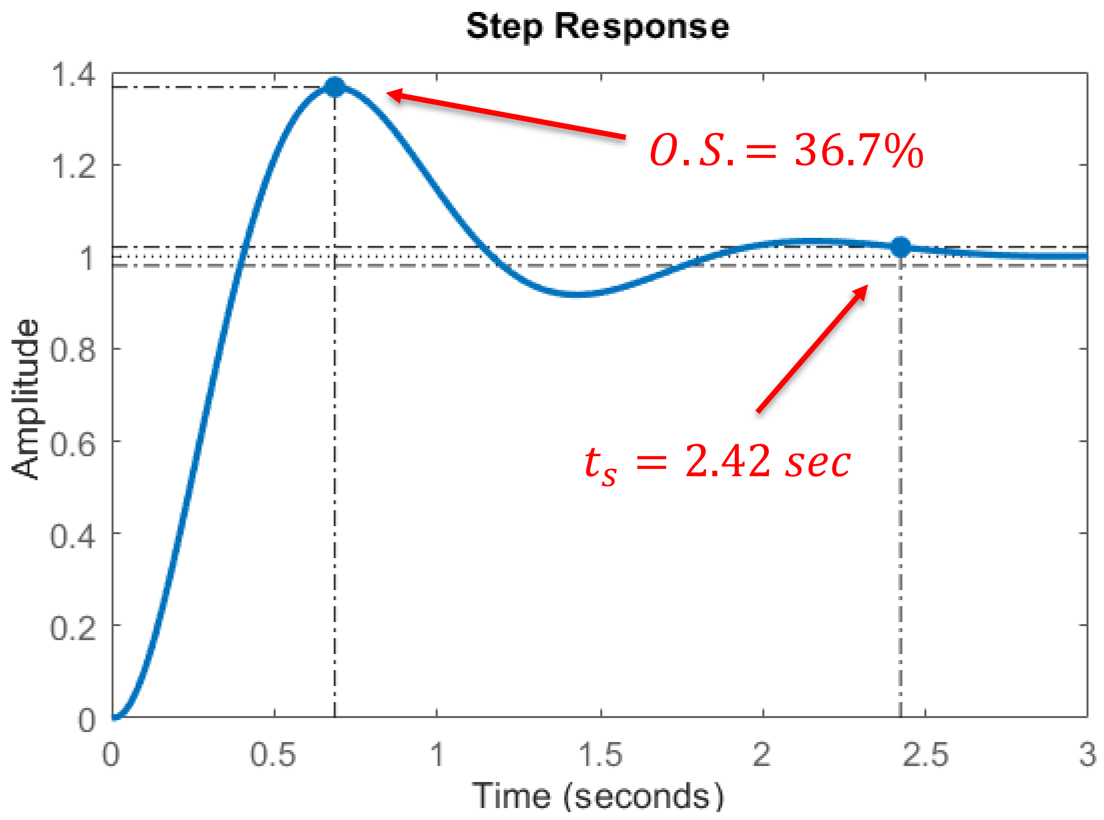


The complete lead-lag compensated open-loop system transfer function is

$$G(s) = \frac{6.63K(s+0.2)(s+3.6)}{s(s+100)(s+1.71)(s+0.087)(s+7)}$$

We can plot the step response of closed-loop system for $K = 433.6$ and check for the settling-time and the overshoot.

$\boxed{O.S. = 36.7\%}$ $\boxed{t_s = 2.42\ sec}$

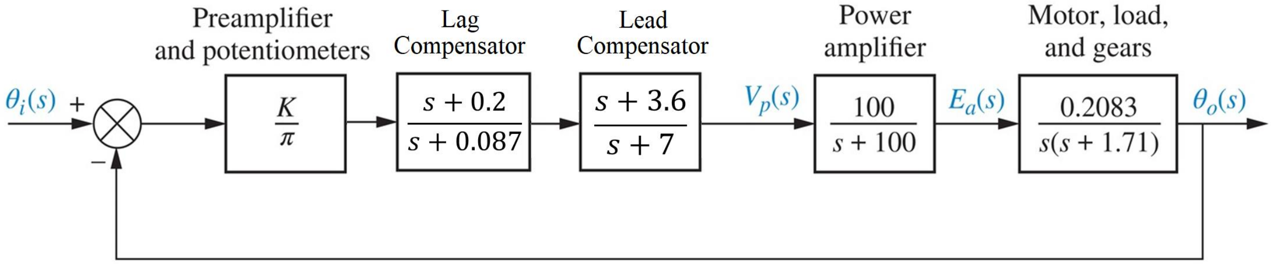


We can plot root-locus of the lead-lag compensated system and fine tune the gain K to achieve the desired transient response of $t_s = 2\ sec$, $O.S. = 25\%$.

Case Study: Antenna Control System

For the antenna azimuth position control system with the given simplified unity-feedback block diagram model:

c) Design a lag compensator to have a steady-state error of 0.05 for unit-ramp input without changing the transient response characteristics.



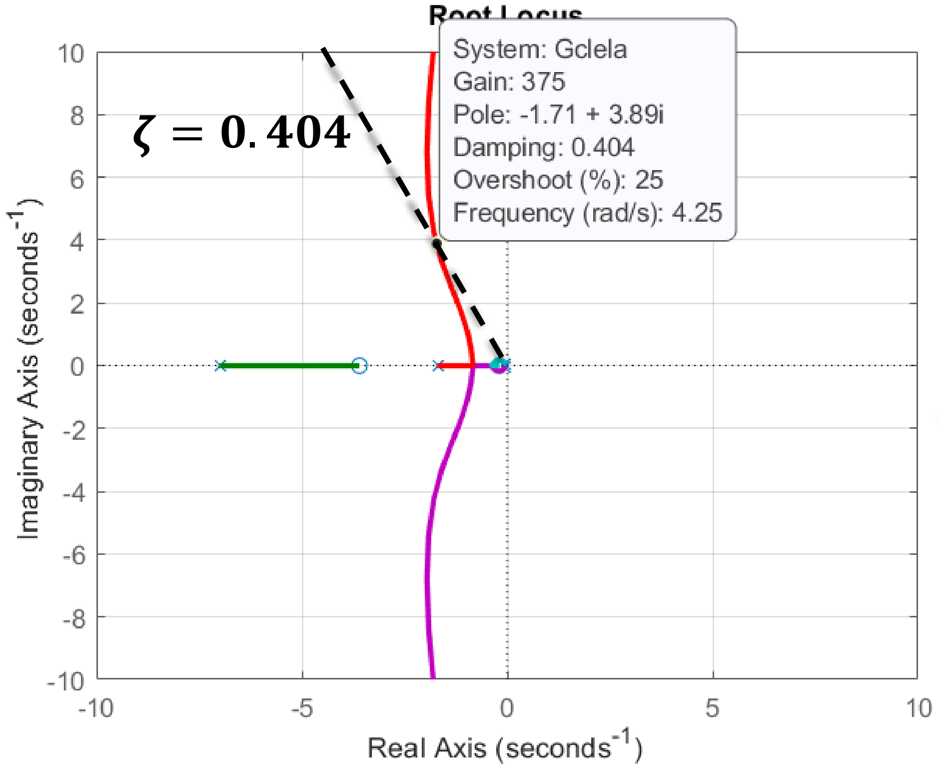
The complete lead-lag compensated open-loop system transfer function is

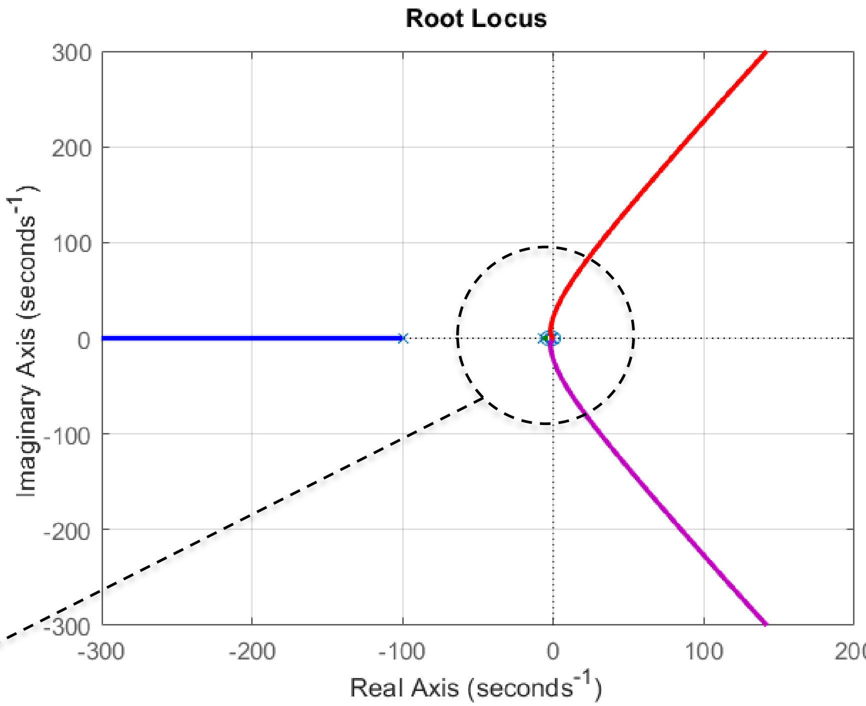
$$G(s) = \frac{6.63K(s+0.2)(s+3.6)}{s(s+100)(s+1.71)(s+0.087)(s+7)}$$

We can plot the root-locus of the lead-lag compensated system and search for the damping ratio of $\zeta = 0.404$, which represent $O.S. = 25\%$.

Then, find the desired poles and calculate the required gain K for it.

$$\boxed{K = 375}$$





THANK YOU

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