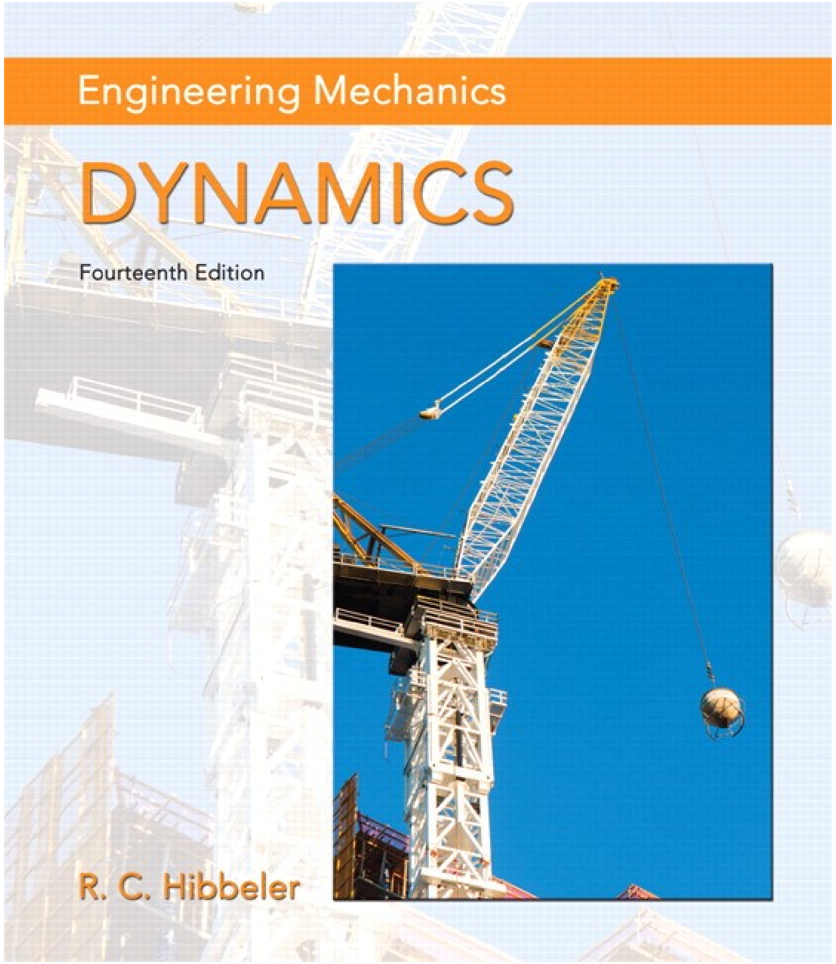
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 17

Planar Kinetics of a Rigid Body: Force and Acceleration

Planar Kinetic Equations of Motion: Translation (1 of 2)

Today's Objectives:

Students will be able to:

1. Apply the three equations of motion for a rigid body in planar motion.
2. Analyze problems involving translational motion.

Planar Kinetic Equations of Motion: Translation (2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- **FBD of Rigid Bodies**
- **EOM for Rigid Bodies**
- **Translational Motion**
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1. When a rigid body undergoes translational motion due to external forces, the translational equations of motion (EoM) can be expressed for _______________.

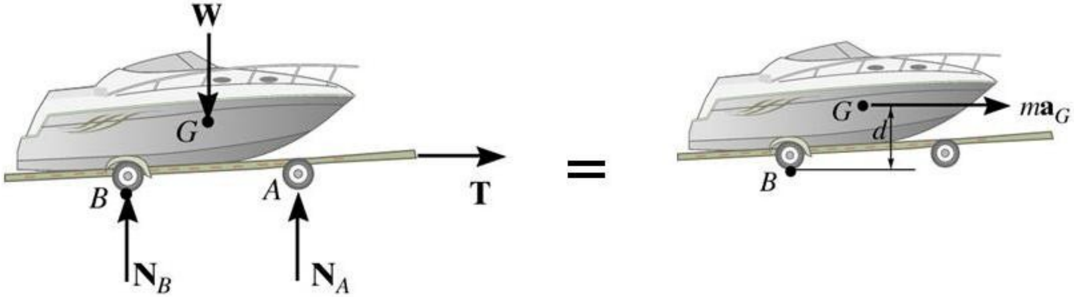
   A) the center of rotation
   B) the center of mass
   C) any arbitrary point
   D) All of the above

2. The rotational EoM about the mass center of the rigid body indicates that the sum of moments due to the external loads equals _______________.

   A) $I_G \alpha$
   B) $m a_G$
   C) $I_G \alpha + m a_G$
   D) None of the above.

Applications (1 of 2)

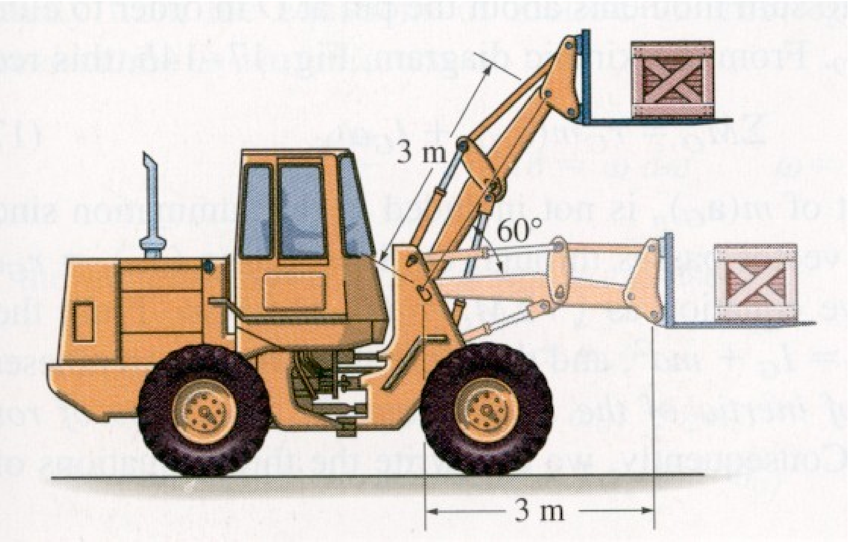
The boat and trailer undergo rectilinear motion. In order to find the reactions at the trailer wheels and the acceleration of the boat, we need to draw the FBD and kinetic diagram for the boat and trailer.



How many equations of motion do we need to solve this problem? What are they?

Applications (2 of 2)

As the tractor raises the load, the crate will undergo curvilinear translation if the forks do not rotate.



If the load is raised too quickly, will the crate slide to the left or right?

How fast can we raise the load before the crate will slide?

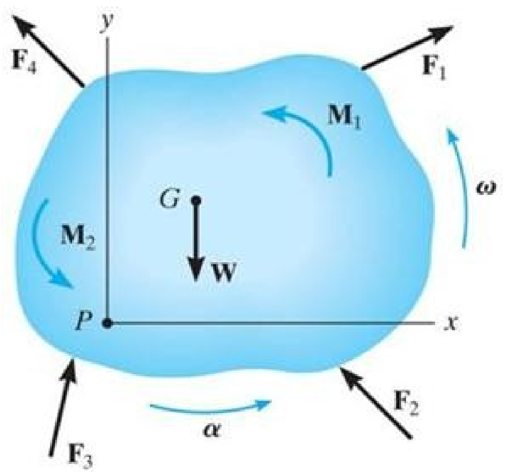
Section 17.2

Planar Kinetic Equations of Motion

Planar Kinetic Equations of Motion

- We will limit our study of **planar kinetics** to rigid bodies that are symmetric with respect to a fixed reference plane.
- As discussed in Chapter 16, when a body is subjected to general plane motion, it undergoes a combination of **translation** and **rotation**.
- First, a coordinate system with its origin at an arbitrary point P is established.

The x−y axes should not rotate **but** can either be fixed or translate with constant velocity.

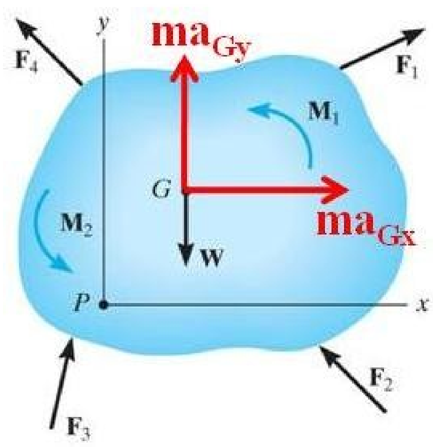


Equations of Translational Motion

- If a body undergoes **translational motion**, the **equation of motion** is $\Sigma \boldsymbol{F} = m\boldsymbol{a}_G$. This can also be written in scalar form as

$$\Sigma F_x = m(a_G)_x \quad \text{and} \quad \Sigma F_y = m(a_G)_y$$

In words: the sum of all the external forces acting on the body is equal to the body's mass times the acceleration of it's mass center.



Equations of Rotational Motion (1 of 2)

We need to determine the effects caused by the moments of an external force system.

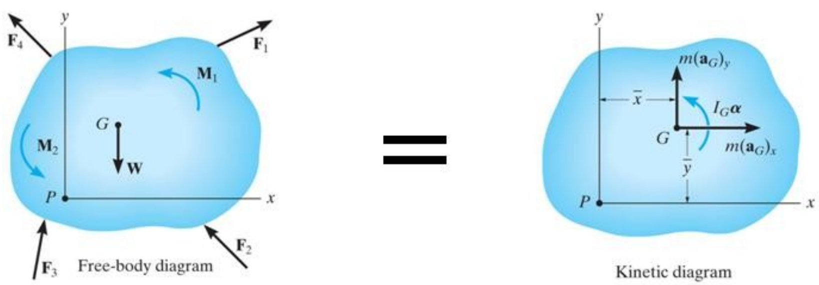
The moment about point P can be written as:

$$\Sigma(r_i \times F_i) + \Sigma M_i = \bar{r} \times m a_G + I_G \alpha$$

and $$\Sigma M_p = \Sigma(M_k)_p$$

where $\bar{r} = \bar{x}i + \bar{y}j$ and ΣM_p is the resultant moment about P due to all the external forces.

The **term** $\Sigma(M_k)_p$ **is called the kinetic moment** about point P.



Equations of Rotational Motion (2 of 2)

If point P coincides with the mass center G, this equation reduces to the **scalar equation** of $\Sigma M_G = I_G \alpha$.

In words: the resultant (summation) moment about the mass center due to all the external forces is equal to the moment of inertia about G times the angular acceleration of the body.

Thus, **three** independent **scalar** equations of motion may be used to describe the general planar motion of a rigid body. These equations are:

$$\Sigma F_x = m(a_G)_x$$

$$\Sigma F_y = m(a_G)_y$$

and $\Sigma M_G = I_G \alpha$ or $\Sigma M_p = \Sigma (M_k)_p$

Equations of Motion: Translation (Section 17.3)

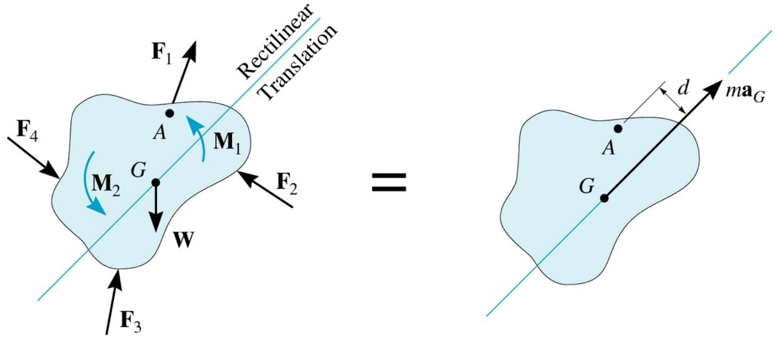
When a rigid body undergoes **only translation**, all the particles of the body have the same acceleration so $a_G = a$ and $\alpha = 0$.

The equations of motion become:

$$\sum F_x = m(a_G)_x$$

$$\sum F_y = m(a_G)_y$$

$$\sum M_G = 0$$



Note that, if it makes the problem easier, the moment equation can be applied about another point instead of the mass center. For example, if point A is chosen,

$$\sum M_A = (m\, a_G)d.$$

Equations of Motion: Translation

When a rigid body is subjected to **curvilinear translation**, it is best to use an **n-t coordinate system.**

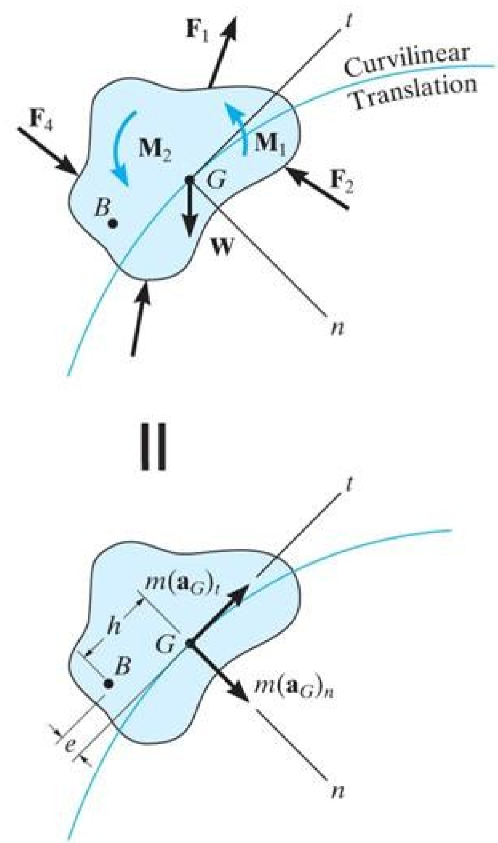
Then apply the equations of motion, as written below, for n-t coordinates.

$$\sum F_n = m(a_G)_n$$

$$\sum F_t = m(a_G)_t$$

$$\sum M_G = 0 \text{ or}$$

$$\sum M_B = e\left[m(a_G)_t\right] - h\left[m(a_G)_n\right]$$



Procedure For Analysis

Problems involving kinetics of a rigid body in only translation should be solved using the following procedure:

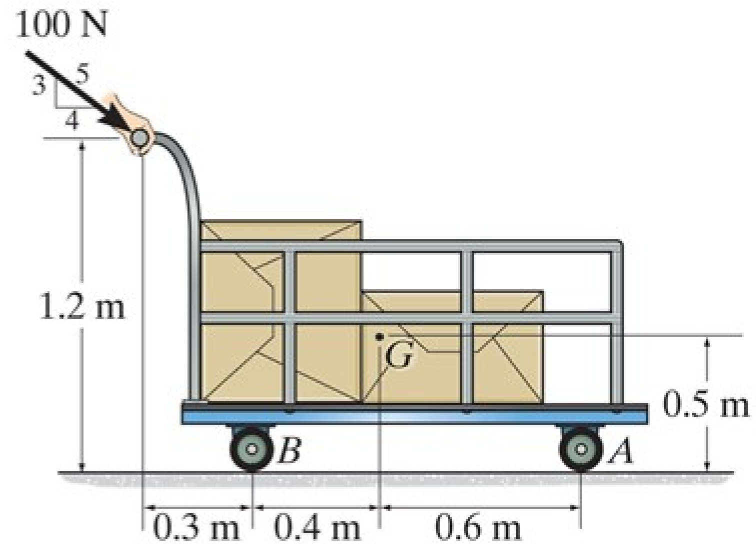
1. Establish an $(x - y)$ or $(n - t)$ inertial coordinate system and specify the sense and direction of acceleration of the mass center, $\mathbf{a}_G$.
2. Draw a FBD and kinetic diagram showing all external forces, couples and the **inertia forces and couples.**
3. Identify the unknowns.
4. Apply the **three equations of motion** (one set or the other):

$\sum F_x = m(a_G)_x \quad \sum F_y = m(a_G)_y$	$\sum F_n = m(a_G)_n \quad \sum F_t = m(a_G)_t$
$\sum M_G = 0$ or $\sum M_P = \sum(M_k)_P$	$\sum M_G = 0$ or $\sum M_P = \sum(M_k)_P$

5. Remember, friction forces always act on the body **opposing** the motion of the body.

Example I (1 of 3)

Given: The cart and its load have a total mass of 100 kg and center of mass at *G.* A force of P = 100 N is applied to the handle. Neglect the mass of the wheels.

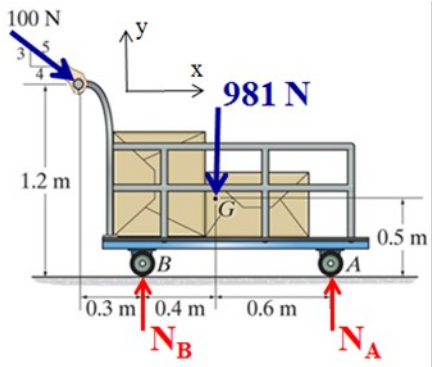


Find: The normal reactions at each of the two wheels at A and B.

Plan: Follow the procedure for analysis.

Example I (2 of 3)

Solution: The cart will move along a rectilinear path. Draw the **FBD** and **kinetic diagram.**



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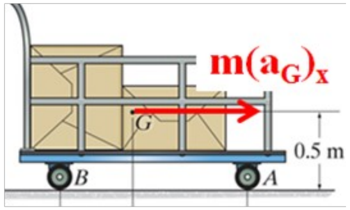


Apply the **equation of motion in the x-direction** first:

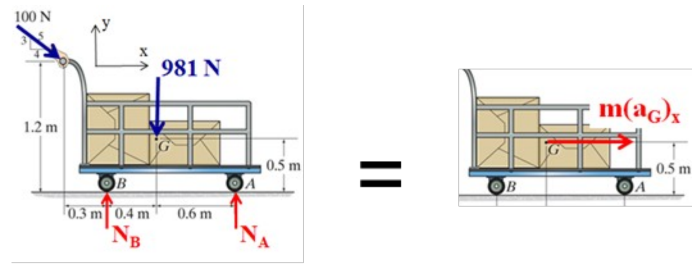
$$\xrightarrow{+} \sum F_x = m(a_G)_x$$

$$100\,(4/5) = 100 a_G$$

$$a_G = 0.8 m/s^2$$

Example I (3 of 3)

Then apply the **equation of motion in the y-direction and sum moments about G.**



$$+\uparrow \sum F_y = 0 \Rightarrow N_A + N_B - 981 - 100\left(3/5\right) = 0$$

$$N_A + N_B = 1041\ N \qquad (1)$$

$$\curvearrowright + \sum M_G = 0$$

$$\Rightarrow N_A\left(0.6\right) - N_B\left(0.4\right) + 100\left(3/5\right)\ 0.7 - 100\left(4/5\right)\left(1.2 - 0.5\right) = 0$$

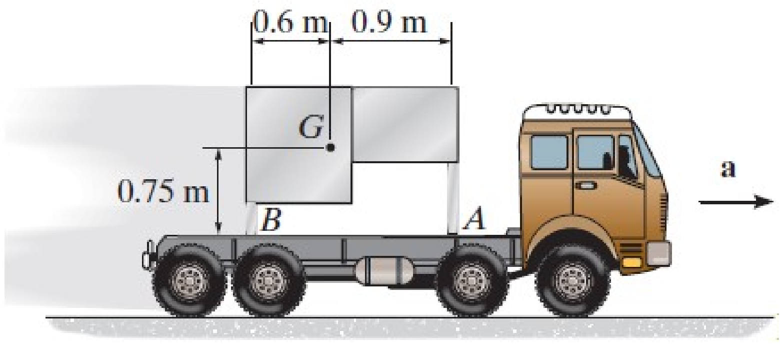
$$0.6\ N_A - 0.4 N_B = 14\ \ N m \qquad (2)$$

Using Equations (1) and (2), solve for the reactions, N_A and N_B

$$N_A = 430\ N \text{ and } N_B = 611 N$$

Example II (1 of 3)

Given: The 100 kg table has a mass center at *G* and the coefficient of static friction between the legs of the table and the bed of the truck is $\mu_s = 0.2$.



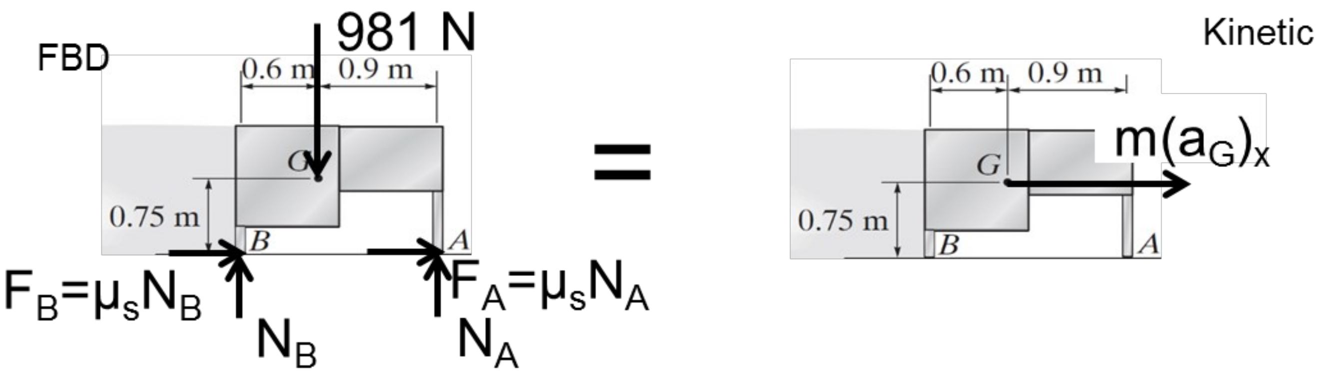
Find: The maximum acceleration of the truck possible without causing the table to move relative to the truck, and the corresponding normal reactions on legs A and B.

Plan: Follow the procedure for analysis.

Example II (2 of 3)

Solution:

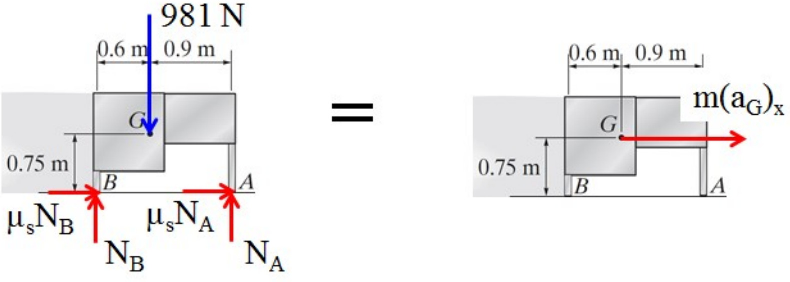
The table will have a rectilinear motion.
Draw the **FBD** and **kinetic diagram**.



Notice that $F_A = \mu_s N_A$ and $F_B = \mu_s N_B$ when the table is about to slide.

Example II (3 of 3)

Apply the **equations of motion.**



$$+\rightarrow \sum F_x = m(a_G)_x$$
$$\Rightarrow 0.2\,N_A + 0.2\,N_B = 100a_G \quad (1)$$
$$+\uparrow \sum F_y = 0 \Rightarrow N_A + N_B - 981 = 0 \quad (2)$$
$$\circlearrowleft + \sum M_G = 0$$
$$\Rightarrow 0.2\,N_A(0.75) + 0.2\,N_B(0.75) + N_A(0.9) - N_B(0.6) = 0 \quad (3)$$

Using Equations (2) and (3), solve for the reactions, N_A and N_B

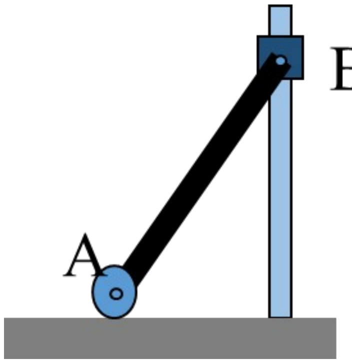
$$N_A = 294\ N,\ N_B = 687\ N$$

The maximum acceleration a_G can be found from equation (1).

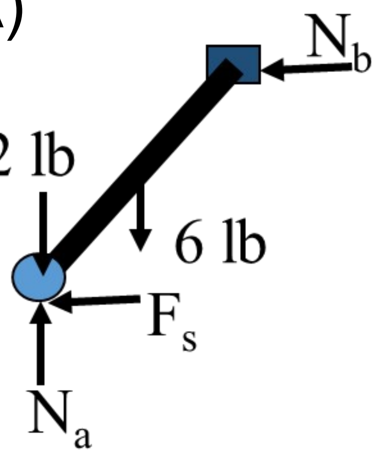
$$a_G = (0.2N_A + 0.2N_B)/100 = 1.96\ m/s^2$$

Concept Quiz (1 of 2)

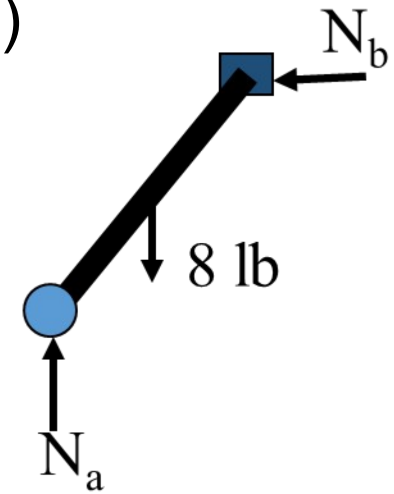
1. A 2 lb disk is attached to a uniform 6 lb rod AB with a frictionless collar at B. If the disk rolls **without** slipping, select the correct FBD.



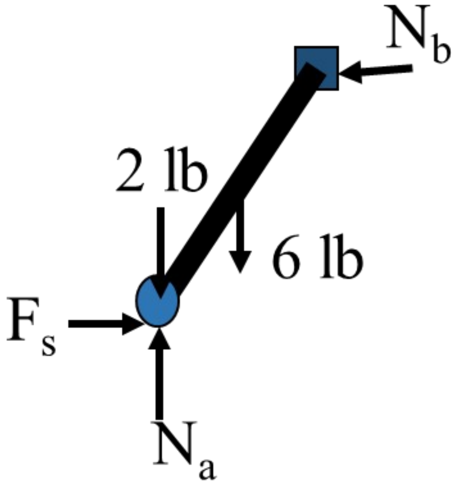
A)



B)

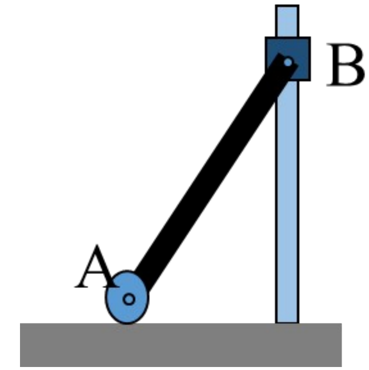


C)

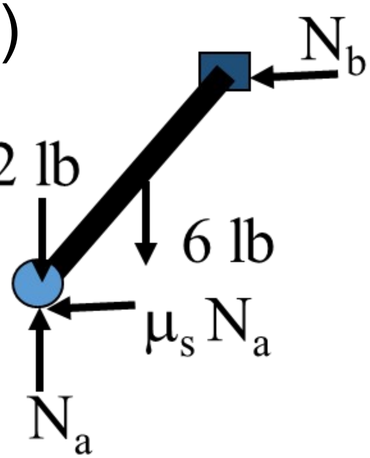


Concept Quiz (2 of 2)

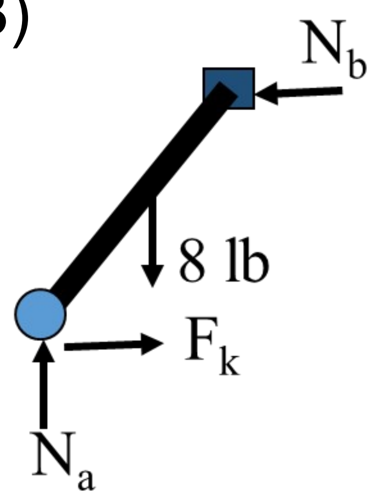
2. A 2 lb disk is attached to a uniform 6 lb rod AB with a frictionless collar at B. If the disk rolls **with** slipping, select the correct FBD.



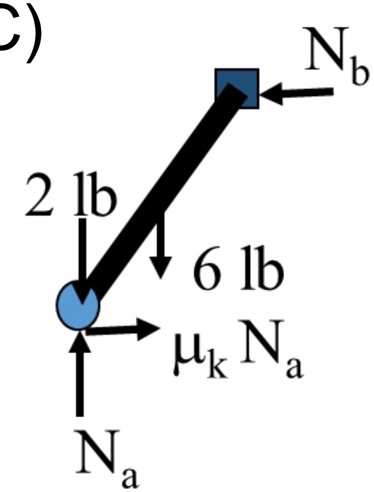
A)



B)



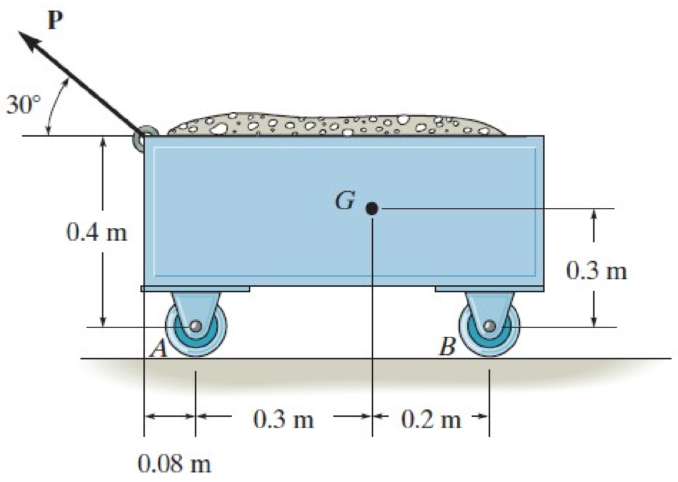
C)



Group Problem Solving (1 of 3)

Given: A force of P = 300 N is applied to the 60-kg cart. The mass center of the cart is at *G*.

Find: The normal reactions at both the wheels at A and both the wheels at B.



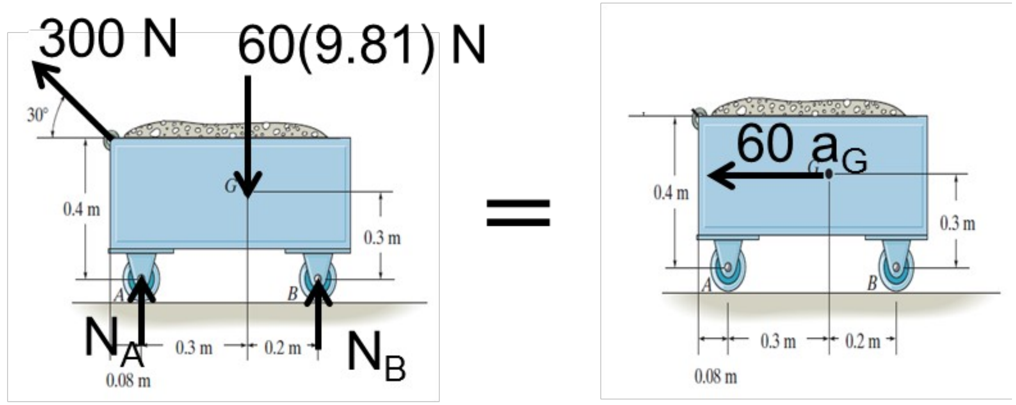
Plan: Follow the procedure for analysis.

Pearson

Group Problem Solving (2 of 3)

Solution:

Draw **FBD** and **kinetic diagram.**



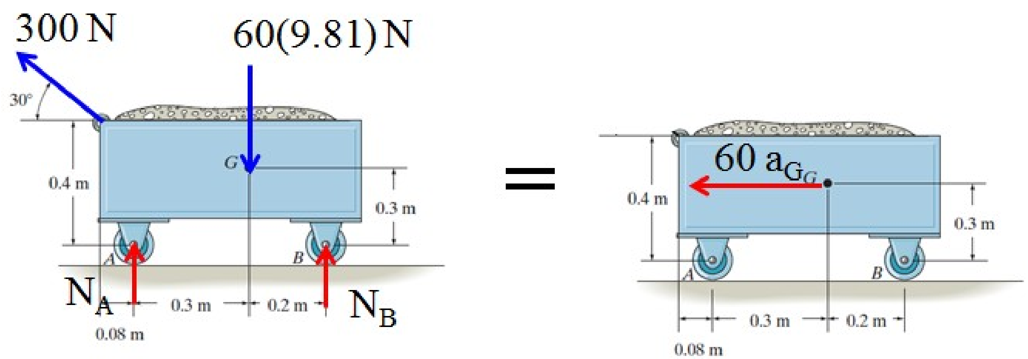
Applying the **equations of motion:**

$$+\leftarrow \sum F_x = m\left(a_G\right)_x$$

$$\Rightarrow 300 \cos 30° = 60\, a_G$$

$$a_G = 4.33\, m/s^2$$

Group Problem Solving (3 of 3)



$$+\uparrow \sum F_y = m(a_G)_y$$

$$N_A + N_B - 60\,(9.81) + 300 \sin 30^\circ = 0 \qquad (1)$$

$$\circlearrowleft + \sum M_G = 0$$

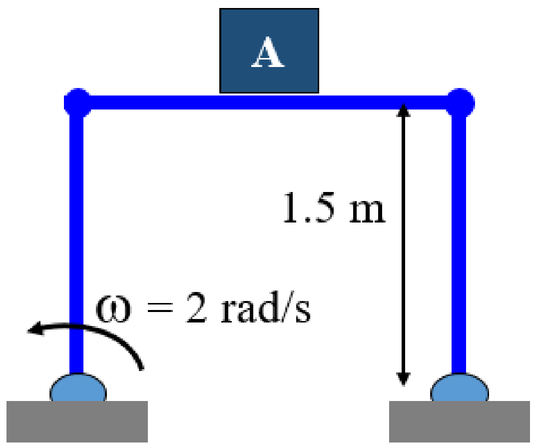
$$0.2 N_B - 0.3\,N_A - (0.1)300 \cos 30^\circ - (0.3)300 \sin 30^\circ = 0 \qquad (2)$$

Using Equations (1) and (2), solve for the reactions, N_A and N_B

$$N_A = 113\,N, N_B = 325\,N$$

Attention Quiz

1. As the linkage rotates, box A undergoes ___________.
   A) general plane motion
   B) pure rotation
   C) linear translation
   D) curvilinear translation



2. How many independent scalar equations of motion can be applied to box A?

   A) One B) Two
   C) Three D) Four

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