5.3

December 15, 2022 10:09 PM

5.3 Integrating Rational Functions by Partial Fraction Decomposition (PFD) Method.

Introduction.

Simple Fractions: add them up	**Complex Fraction:** break it up into simple(partial) fractions
$\frac{5}{x+1}+\frac{6}{x-2}=$	$\frac{11x-4}{x^2-x-2}=$

Example 1. Integrate:

$$\int \frac{11x-4}{x^2-x-2}dx =$$

Polynomial Functions: $P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0 x^0$,
where all coefficients a_i are real numbers.

$P_n(x)$, where $n \geq 0$ is any integer, the degree of a polynomial

BKM

5.3

The following rule e

REPEATED LINEAR F

For each linear repe

where $A_1, A_2, \ldots, A_n$

EXAMPLE 3 Genera

a. $\frac{10}{(x-1)(x+5)^3}$ =

b. (Self-check)

EXAMPLE 4. Integra

Solution:

Finding the PFD for

$\frac{x^2+2x+4}{(x+1)^3} = \frac{A}{(x+1)}+$

The following rule c

denominator of a ra

QUADRATIC FACTO

For each quadratic
fraction of the form

$P_0(x) = any\ \#$, constant, e.g. $P_0(x) = 5x^0 = 5$; Degree: x^0

$P_1(x) = ax + b$, linear, e.g. $P_1(x) = 11x - 4$; Degree: x^1

$P_2(x) = ax^2 + bx + c$, quadratic, $P_2(x) = x^2 - x - 2$ or $P_2(x) = x^2 + 5$; Degree: x^2

And so on…

Factorization of polynomials: Every polynomial $P_n(x)$ can be factored into a product of *prime* polynomials.

Polynomials that are not factorable are called *prime* or *irreducible*

such as $P_0(x)$, $P_1(x)$, some quadratics such as $x^2 + 1$ or $x^2 + 2x + 8$

Rational Functions

$Q(x) = \frac{P_n(x)}{P_m(x)}$; e.g. $Q(x) = \frac{Linear}{Quadratic} = \frac{11x-4}{x^2-x-2}$

Partial Fraction Decomposition of rational functions:

Every rational function $Q(x)$can be broken down (decomposed) into a sum of partial fractions of the form

$$\frac{A}{(ax+b)^k} \text{ or } \frac{Ax+B}{(ax^2+bx+c)^k}$$

where A, B are con

For each quadratic fractions

where $A_1, A_2, \ldots, A_n$

EXAMPLE 5 Write o integrati

Solution:

Use PFD (partial fraction decomposition) with undetermined coefficients to integrate the following:

Example 1. $\int \frac{11x-4}{x^2-x-2}\,dx$

Example 2. $\int \frac{3x+1}{x^2-x-6}\,dx$

EXAMPLE 1. Integrate by PFD with undetermined coefficients:

$$\int \frac{11x-4}{x^2-x-2}\,dx$$

Solution:

LINEAR FACTOR RULE

For each linear non-repeated factor $ax + b$, the PFD contains the partial fraction of the form

$$\frac{A}{ax+b}$$

where A is a constant to be determined.

EXAMPLE 6. Integra

Solution:

Start by making the

Examine factors pre

where A, B, C are c