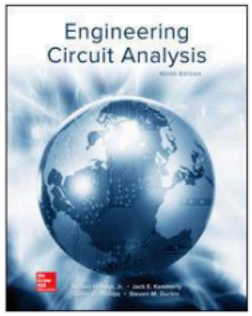
Lecture 9

November 24, 2023 12:18 AM

W9

AC Steady State Analysis



Sinusoid-Phasor Transformation

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TABLE 9.1

Sinusoid-phasor transformation.

Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$ $= V_m \cos(\omega t + (\phi - 90°))$	$V_m \angle \phi - 90°$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90°$

$$\cos(\omega t \pm 90^\circ) = \mp \sin \omega t$$

Summary: Phasor Voltage/Current Relationships

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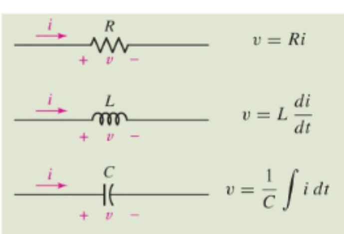
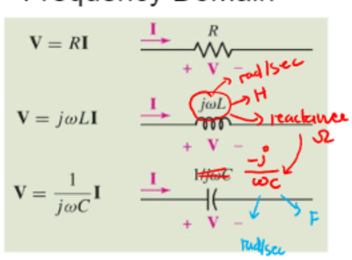
Time Domain



Frequency Domain



Calculus (hard but real)

Algebra (easy but complex)

Kirchhoff's Laws for Phasors

Applying KVL in time implies KVL for phasors:

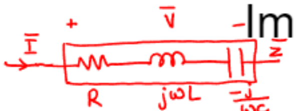
complex #s

$$\bar{\mathbf{V}}_1 + \bar{\mathbf{V}}_2 + \ldots + \bar{\mathbf{V}}_N = \mathbf{0} \qquad \bar{V}_1 = |\bar{V}_1| \angle \pm\theta$$

Applying KCL in time implies KCL for phasors:

$$\bar{\mathbf{I}}_1 + \bar{\mathbf{I}}_2 + \ldots + \bar{\mathbf{I}}_N = \mathbf{0}$$

Impedance



Ohm's law $\Rightarrow \frac{\bar{V}}{\bar{I}} = \bar{Z}$

Define impedance as $\bar{Z} = \bar{\mathbf{V}} / \bar{\mathbf{I}}$, i.e. $\bar{\mathbf{V}} = \bar{\mathbf{I}}\bar{Z}$

(Ω) (Volt) (Amp)

$\bar{Z} = R + j\omega L - j\left(\frac{1}{\omega C}\right) = R + j\left(\underset{(X_L)}{\omega L} - \underset{(X_c)}{\frac{1}{\omega C}}\right)$

$$Z_R = R \qquad Z_L = j\omega L \qquad Z_C = 1/\, j\omega C$$

$\bar{Z} = R + jX \quad (X_L > X_c)$

$\bar{Z} = R - jX \quad (X_L < X_c)$

$\Rightarrow \quad \bar{Z} = R \pm jX$

Impedance is the equivalent of resistance in the frequency domain.

$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$

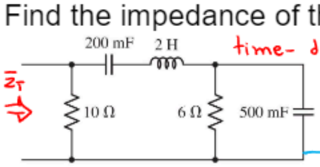
Impedance is a complex number (unit ohm). $\bar{Z}_T = \frac{1}{\frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2}}$

Impedances in series or parallel can be combined using "resistor rules."

$R_T = R_1 + R_2 \qquad \bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$

Example: Equivalent Impedance

Find the impedance of the network at 5 rad/s.



time-domain

$2^H \rightarrow j\omega L = j(5)(2) = j10^{\Omega}$

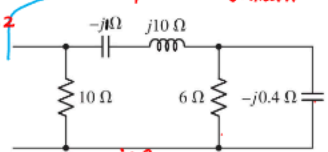
$200^{mF} \rightarrow \frac{-j}{\omega C} = \frac{-j}{(5)(200\times10^{-3})} = -j1$

$500^{mF} \rightarrow \frac{-j}{(5)(500\times10^{-3})} = -j0.4$

$6 \,//\, (-j0.4) = \frac{\bar{Z}_1\bar{Z}_2}{(\bar{Z}_1+\bar{Z}_2)} = \frac{(6)(-j0.4)}{6-j0.4} = 0.027 - j0.398$

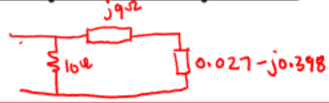
Phasor-domain

$j9 + (0.027 - j0.398) = 0.027 + j8.602$



$\bar{Z}_T = 10 \,//\, (0.027 + j8.602)$

$= \frac{(10)(0.027+j8.602)}{(10.027+j8.602)}$

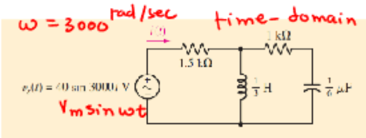


$\bar{Z}_T = 4.255 + j4.93$

Example: Equivalent Impedance and Ohm's Law

Find the current *i(t)* in the following circuit.

$\omega = 3000$ rad/sec time-domain

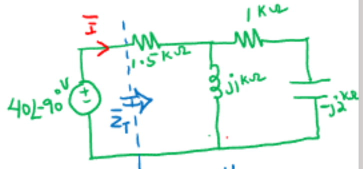


$\frac{1}{3}^{H} \rightarrow j(3000)(\frac{1}{3}) = j1000\,\Omega = j1\,k\Omega$

$\frac{1}{6}^{\mu F} \rightarrow \frac{-j}{(3000 \times \frac{1}{6} \times 10^{-6})} = -j2000\,\Omega = -j2\,k\Omega$

$v_s(t) = 40\cos(3000t - 90^\circ)$

$\bar{V}_s = 40\angle{-90^\circ}$ V



$\bar{Z}_T = 1.5 + [(j1) \,//\, (1 - j2)]$

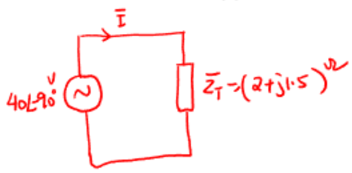
$= 1.5 + \left[\frac{(j1)(1-j2)}{(1-j1)}\right]$

$\bar{Z}_T = (2 + j1.5)\,\Omega$

$\bar{I} = \frac{40\angle{-90^\circ}\,V}{\bar{Z}_T}$

Example: Equivalent Impedance and Ohm's Law

Find the current *i(t)* in the following circuit.



$\bar{I} = \frac{40\angle{-90^\circ}\,V}{(2+j1.5)\,k\Omega} = 16\angle{-126.87^\circ}$ mA

$i(t) = 16\cos(3000t - 126.87^\circ)$ mA

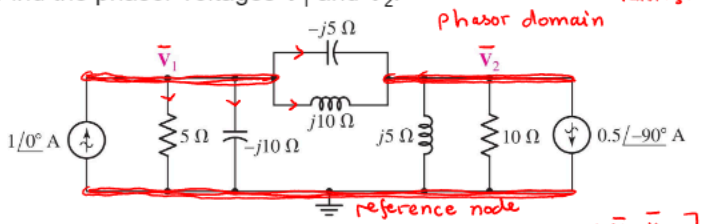
$i(t) = 16\cos(3000t - 126.9^\circ)$ mA

Example: Nodal Analysis

Find the phasor voltages $\mathbf{V}_1$ and $\mathbf{V}_2$.

entering → +ve
leaving → −ve

Phasor domain



$(j10) \times \left\{ +(1\angle 0^\circ) - \left(\frac{\bar{V}_1 - 0}{5}\right) - \left[\frac{\bar{V}_1 - 0}{(-j10)}\right] - \left[\frac{\bar{V}_1 - \bar{V}_2}{j10}\right] - \left[\frac{\bar{V}_1 - \bar{V}_2}{(-j5)}\right] = 0 \right.$

$j10 - j2\bar{V}_1 - [-\bar{V}_1] - (\bar{V}_1 - \bar{V}_2) - (-2(\bar{V}_1 - \bar{V}_2)) = 0$

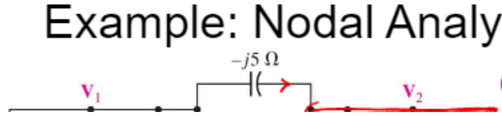
$j10 - j2\bar{V}_1 + \cancel{\bar{V}_1} - \cancel{\bar{V}_1} + \bar{V}_2 + 2(\bar{V}_1 - \bar{V}_2) = 0$

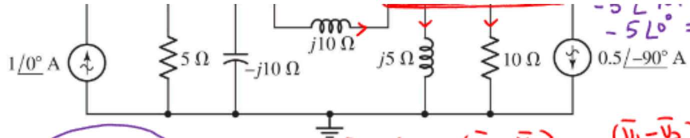
$-(2 - j2)\bar{V}_1 + \bar{V}_2 = +j10 \quad \text{--- } (1)$

Example: Nodal Analysis

entering +ve

$(-0.5\angle{-90^\circ})(10\angle 90^\circ)$





$$-5\angle 0° = -5$$

$$-0.5\angle -90° - \frac{(\bar{V}_2 - 0)}{10} - \frac{(\bar{V}_2 - 0)}{j5} + \frac{(\bar{V}_1 - \bar{V}_2)}{j10} + \frac{(\bar{V}_1 - \bar{V}_2)}{(-j5)} = 0 \quad \times j10$$

$$-5 \quad -j\bar{V}_2 - 2\bar{V}_2 + (\bar{V}_1 - \bar{V}_2) - 2(\bar{V}_1 - \bar{V}_2) = 0$$

$$-\bar{V}_1 - (1+j1)\bar{V}_2 = 5 \quad --- ② \Rightarrow \bar{V}_1 = -5 - (1+j1)\bar{V}_2$$

$$(-2+j2)\bar{V}_1 + \bar{V}_2 = j10 \quad --- ①$$

$$(-2+j2)[-5-(1+j1)\bar{V}_2] + \bar{V}_2 = j10$$

$$-5(-2+j2) + (1-(1+j1)(-2+j2))\bar{V}_2 = j10$$

$$\bar{V}_2 = \frac{j10 + 5(-2+j2)}{[1-(1+j1)(-2+j2)]}$$

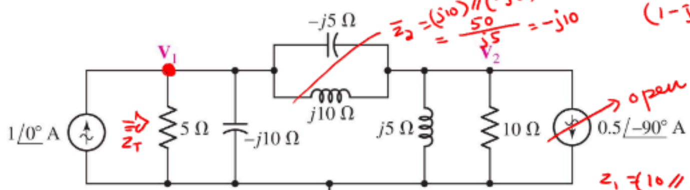
$$\bar{V}_2 = (-2+j4)^V$$

V$_1$=1−*j*2 V and **V**$_2$=−2+*j*4 V

Superposition Example

The superposition principle applies to phasors; use it to find **V**$_1$.

$$(1-j2)^{\Omega}$$



$$\bar{Z}_2 = (j10) // (-j5) = \frac{50}{j5} = -j10$$

$$Z_1 = (10 // j5) = \frac{j50}{(10+j5)}$$

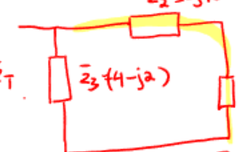
consider $1\angle 0°$ A source only

$$\bar{V}_1 = (1\angle 0°)(\bar{Z}_T)$$

$$\bar{Z}_3 = 5 // (-j10) = \frac{-j50}{(5-j10)}$$

$$\bar{Z}_T = (2-j2)^{\Omega}$$

$$\bar{V}_1' = (2-j2)^V$$

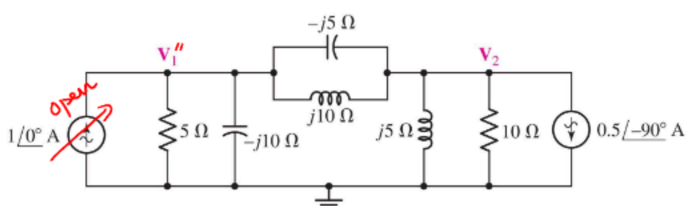


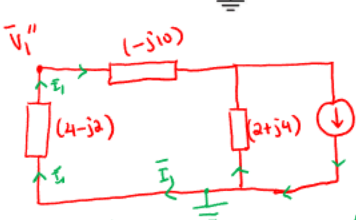
$$\bar{Z}_{12} = \bar{Z}_1 + \bar{Z}_2 = (2-j6)$$

$$\bar{Z}_T = \bar{Z}_{12} // \bar{Z}_3 = \frac{(2-j6)(4-j2)}{(6-j8)}$$

Superposition Example

The superposition principle applies to phasors; use it to find **V**$_1$.





$$\bar{I}_1 = \frac{0 - V_1''}{(4-j2)}$$

$$-\bar{V}_1'' = \bar{I}_1(4-j2)$$

current division

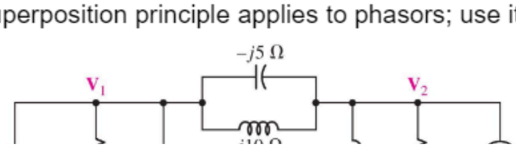
$$\bar{I}_1 = \frac{(2+j4)\,(0.5\angle -90°)}{(2+j4+4-j2-j10)}$$

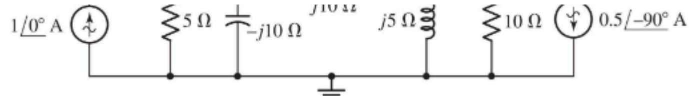
$$\bar{V}_1'' = -(0.2+j0.1)(4-j2) = -1^V$$

$$(0.2+j0.1)^A = \bar{I}_1 = \frac{(2+j4)(0.5\angle -90°)}{(6-j8)}$$

Superposition Example

The superposition principle applies to phasors; use it to find **V**$_1$.





$\bar{V}_1' = (2-j2)^V$ $\quad$ $\bar{V}_1'' = -1^V$

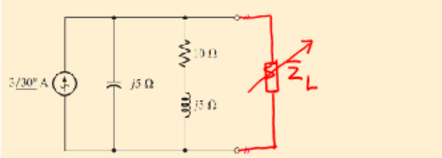
$\bar{V}_1 = \bar{V}_1' + \bar{V}_1'' = (2-j2) - 1 = (1-j2)^V$

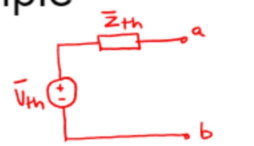
$V_1 = 1-j2$ V

Thévenin Example

10.15 For the circuit of Fig. 10.32, find the (a) open-circuit voltage $\mathbf{V}_{ab}$ $\bar{V}_{th}$; (b) downward current in a short circuit between a and b; (c) Thévenin equivalent impedance $\mathbf{Z}_{ab}$ $\bar{Z}_{th}$ in parallel with the current source.

Ans: $16.77\angle{-33.4^\circ}$ V; $2.60 + j1.500$ A; $2.5 - j5\ \Omega$.

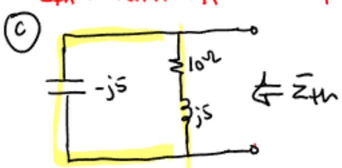




Thevenin's Equivalent

$\bar{Z}_{th}$: turn off all independent sources, $3\angle 30^\circ A \rightarrow$ open

(c)



$$\bar{Z}_{th} = (10+j5) \,//\, (-j5) = \frac{(10+j5)(-j5)}{10} = \frac{-j50+25}{10} = (2.5-j5)^{\Omega}$$

Thévenin Example

10.15 For the circuit of Fig. 10.32, find the (a) open-circuit voltage $\mathbf{V}_{ab}$ $\bar{V}_{th}$; (b) downward current in a short circuit between a and b; (c) Thévenin equivalent impedance $\mathbf{Z}_{ab}$ $\bar{Z}_{th}$ in parallel with the current source.

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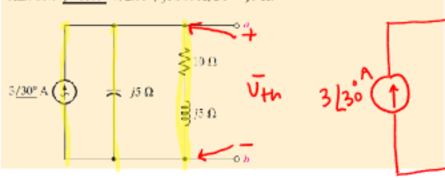
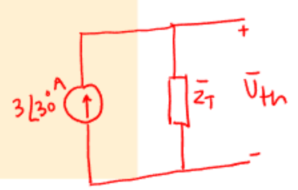




(a)

$\bar{Z}_T = (-j5) \,//\, (10+j5) = 2.5-j5$

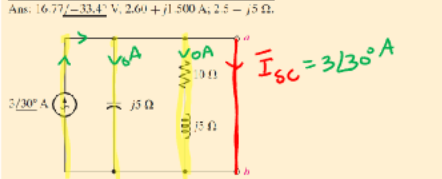
Ohm's law

$\bar{V}_{th} = (3\angle 30^\circ)(\bar{Z}_T) = (3\angle 30^\circ)(2.5-j5) = 16.77\angle{-33.43^\circ}\,V$

Thévenin Example

10.15 For the circuit of Fig. 10.32, find the (a) open-circuit voltage $\mathbf{V}_{ab}$; (b) downward current in a short circuit between a and b; (c) Thévenin equivalent impedance $\mathbf{Z}_{ab}$ in parallel with the current source.

Ans: $16.77\angle{-33.4^\circ}$ V; $2.60 + j1.500$ A; $2.5 - j5\ \Omega$.



$\bar{I}_{SC} = 3\angle 30^\circ A$

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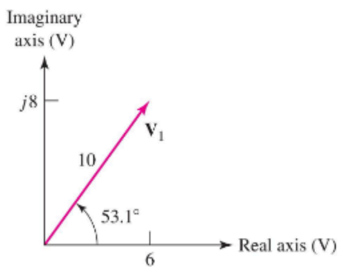
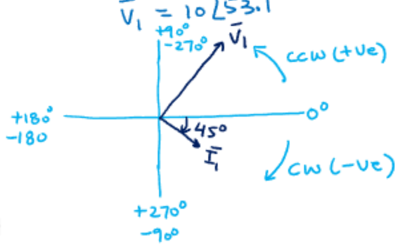
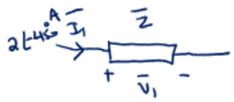
Phasor Diagrams



$\bar{V}_1 = 10\angle 53.1°$ V





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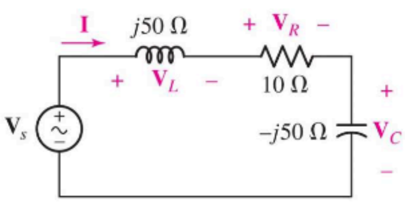
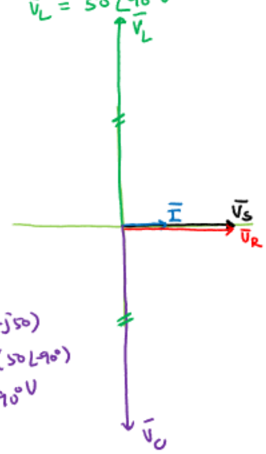
Example Phasor Diagram

If we assume $\mathbf{I} = 1\angle 0°$ A



$\bar{V}_L = \bar{I}\,(j50) = (1\angle 0°)(50\angle 90°)$

$\bar{V}_L = 50\angle 90°$ V



$\bar{V}_S = \bar{I}\,\bar{Z}_T = (1\angle 0°)(10 + \cancel{j50} - \cancel{j50})$

$\bar{V}_S = 10\angle 0°$ V

$\bar{V}_S$ and $\bar{I}$ are in phase

$\bar{V}_R = R\bar{I} = 10\angle 0°$ V

$\bar{V}_C = \bar{I}\,(-j50)$

$(1\angle 0°)(50\angle -90°)$

$\bar{V}_C = 50\angle -90°$ V

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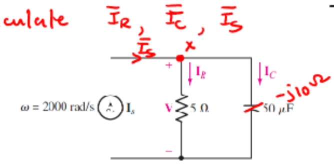
Phasor Diagram: Parallel RC

Assume $\mathbf{V} = 1\,\underline{/0°}\ V$

$50\mu F \rightarrow \dfrac{-j}{(2000 \times 50\times 10^{-6})} = -j10$

Calculate $\bar{I}_R$, $\bar{I}_C$, $\bar{I}_S$



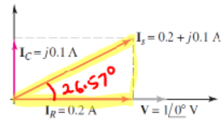
$\bar{I}_R = \dfrac{\bar{V}}{5} = \dfrac{1\angle 0°}{5} = 0.2\angle 0°$ A

$\bar{I}_C = \dfrac{\bar{V}}{(-j10)} = \dfrac{1\angle 0°}{10\angle -90°} = 0.1\angle 90°$ A

KCL @ node x

$\bar{I}_S = \bar{I}_R + \bar{I}_C = (0.2 + j0.1)$

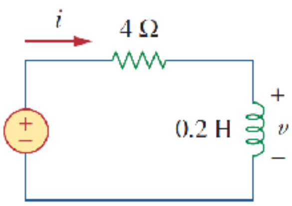
$\bar{I}_S = 0.224 \angle 26.57^\circ$ A



AC Circuits – Problem Solving

Determine *v(t)* and *i(t)*

$v_s = 20 \sin(10t + 30^\circ)$ V



$0.2H \Rightarrow j(10)(0.2) = j2\,\Omega$

$0.2H \Rightarrow j(10)(0.2) = j2\Omega$

$j\omega L$

$V_m \sin(\omega t + \theta)$

$20\cos(10t + 30^\circ - 90^\circ) \Rightarrow 20\cos(10t - 60^\circ)$

$\vec{V}_S = (20\angle -60^\circ)$

$\bar{Z}_T = 4\,\Omega + [j2\,\Omega]$

$\bar{Z}_T = (4 + j2)\,\Omega$

$i(t) = \frac{20\angle -60^\circ V}{(4+j2)\,\Omega} = 2\sqrt{5} \angle -86.56^\circ$

$i(t) = 2\sqrt{5} \cos(10t - 86.56^\circ)$