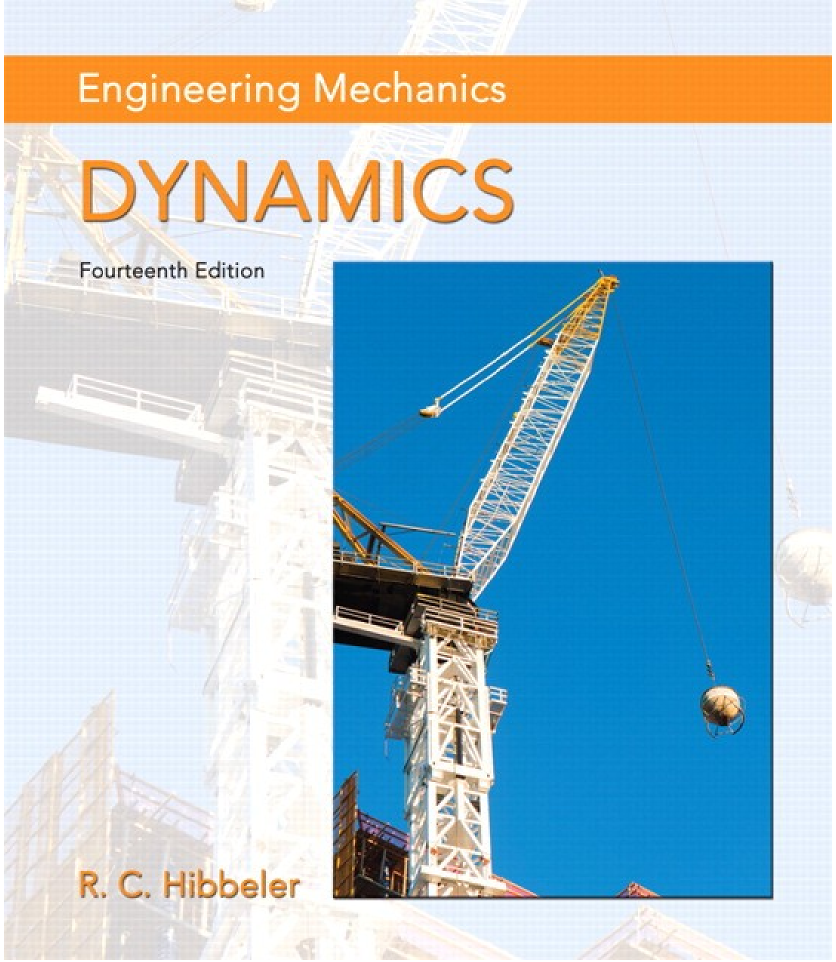
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 14

Kinetics of a Particle:
Work and Energy

The Work of A Force, The Principle of Work And Energy & Systems of Particles (1 of 2)

Today's Objectives:

Students will be able to:

1. Calculate the work of a force.
2. Apply the principle of work and energy to a particle or system of particles.

The Work of A Force, The Principle of Work And Energy & Systems of Particles (2 of 2)

In-Class Activities:

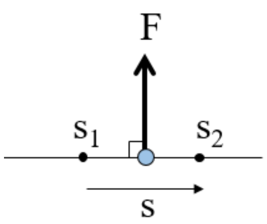
- Check Homework
- Reading Quiz
- Applications
- **Work of a Force**
- **Principle of Work and Energy**
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1. What is the work done by the force F?

   A) F s B) –F s

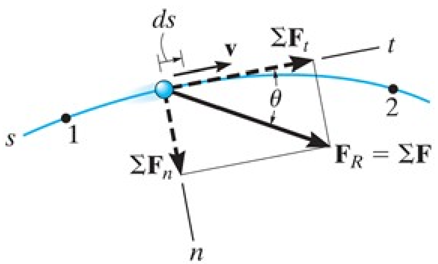
   C) Zero D) None of the above.



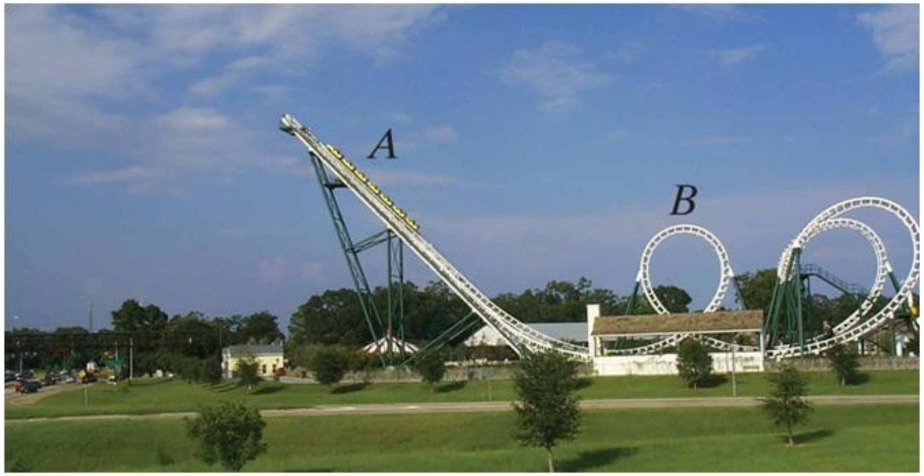
2. If a particle is moved from 1 to 2, the work done on the particle by the force, F_R will be

   A) $\int_{s_1}^{s_2} \sum F_t \, ds$ B) $\int_{s_1}^{s_2} \sum F_t \, ds$

   C) $\int_{s_1}^{s_2} \sum F_n \, ds$ D) $\int_{s_1}^{s_2} \sum F_n \, ds$



Applications (1 of 2)



A roller coaster makes use of gravitational forces to assist the cars in reaching high speeds in the "valleys" of the track.

How can we design the track (e.g., the height, h, and the radius of curvature, r) to control the forces experienced by the passengers?

Applications (2 of 2)

Crash barrels are often used along roadways in front of barriers for crash protection.

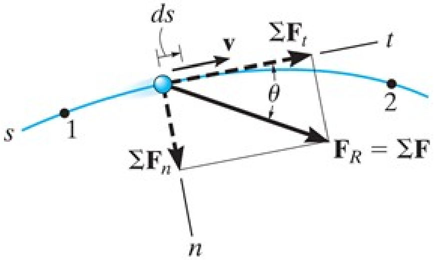
The barrels absorb the car’s kinetic energy by deforming.

If we know the velocity of an oncoming car and the amount of energy that can be absorbed by each barrel, how can we design a crash cushion?

Work And Energy

Another equation for working kinetics problems involving particles can be derived by **integrating** the **equation of motion (F = ma) with respect to displacement.**

By substituting $a_t = v\ (dv/ds)$ into $F_t = ma_t$, the result is integrated to yield an equation known as the **principle of work and energy.**



This principle is useful for solving problems that involve **force, velocity,** and **displacement**. It can also be used to explore the concept of **power**.

To use this principle, we must first understand how to calculate the **work of a force**.

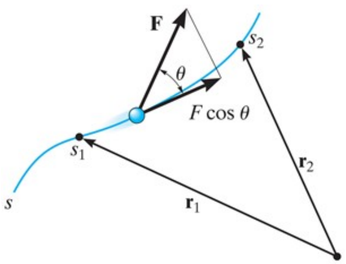
Section 14.1

Work of A Force

Work of A Force (1 of 2)

A **force does work on a particle when the particle undergoes a displacement along the line of action of the force.**

Work is defined as the **product** of **force** and **displacement components** acting in the **same direction**. So, if the angle between the force and displacement vector is *q*, the increment of work dU done by the force is



$$dU = E\, ds \cos\theta$$

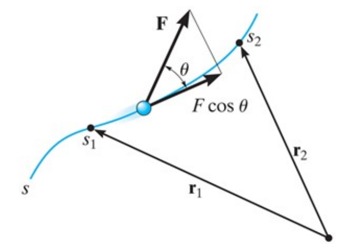
By using the definition of the **dot product** and integrating, the total work can be written as

$$U_{1-2} = \int_{r_1}^{r_2} F.dr$$

Work of A Force (2 of 2)

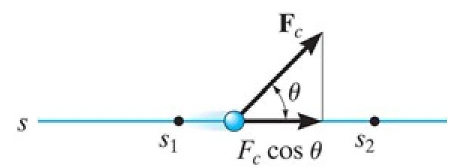
If F is a function of position (a common case) this becomes

$$U_{1-2} = \int_{S_1}^{S_2} F\cos\theta \, ds$$



If both F and *q* are constant $(F = F_C)$, this equation further simplifies to

$$U_{1-2} = F_c \cos\theta (s_2 - s_1)$$



Work is **positive** if the force and the movement are in the **same direction**. If they are **opposing**, then the work is **negative**. If the force and the displacement directions are **perpendicular**, the work is **zero**.

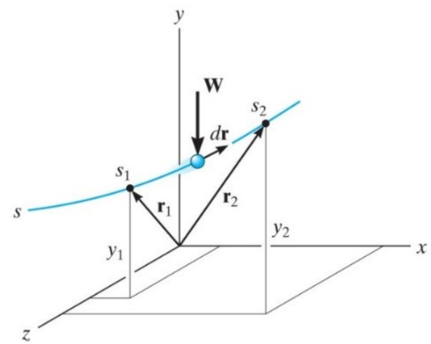
Pearson

Work of A Weight

The work done by the gravitational force acting on a particle (or **weight of an object**) can be calculated by using

$$U_{1-2} = \int_{y_1}^{y_2} -W\,dv$$

$$U_{1-2} = -W(y_2 - y_1) = -W\Delta v$$



The work of a weight is the product of the magnitude of the particle’s weight and its vertical displacement. If Dy is **upward**, the work is **negative** since the weight force always acts downward.

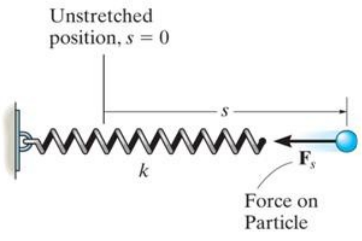
Section 14.2 & Section 14.3

Principle Of Work And Energy

Work of A Spring Force

When stretched, a **linear elastic spring** develops a force of magnitude $\mathbf{F_s = ks}$, where k is the **spring stiffness** and s is the **displacement from the unstretched position**.

The work of the spring force moving from position s_1 to position s_2 is



$$U_{1-2} = \int_{s_1}^{s_2} F_s \, ds = \int_{s_1}^{s_2} k \, s \, ds = 0.5k(s_2)^2 - 0.5k(s_1)^2$$

If a particle is attached to the spring, the $\mathbf{F_s}$ force exerted **on the particle is opposite** to that exerted on the spring. Thus, the work done on the particle by the spring force will be **negative** or

$$U_{1-2} = -[0.5k(s_2)^2 - 0.5k(s_1)^2].$$

Spring Forces

It is important to note the following about spring forces

1. The equations above are for **linear** springs only! Recall that a linear spring develops a force according to

   F = ks (essentially the equation of a line).

2. The work of a spring is **not** just spring force times distance at some point, i.e., $(ks_i)(s_i)$. **Beware**, this is a trap that students often fall into!

3. Always **double check** the sign of the spring work after calculating it. It is positive work if the force on the object by the spring and the movement are in the same direction.

Principle Of Work And Energy (1 of 2)

By integrating the equation of motion, $\Sigma F_t = ma_t = mv(dv/ds)$, the **principle of work and energy** can be written as

$$\sum U_{1-2} = 0.5m(v_2)^2 \quad or \quad T_1 + \sum U_{1-2} = T_2$$

ΣU_{1-2} is the **work done by all the forces** acting on the particle as it moves from point 1 to point 2. Work can be either a **positive or negative** scalar.

T_1 and T_2 are the **kinetic energies** of the particle at the initial and final position, respectively. Thus, $T_1 = 0.5m(v_1)^2$ and $T_2 = 0.5m(v_2)^2$. The kinetic energy is always a **positive scalar** (velocity is squared!).

So, the particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to final position is equal to the particle's final kinetic energy.

Principle Of Work And Energy (2 of 2)

Note that the principle of work and energy $\left(T_1 + \Sigma U_{1-2} = T_2\right)$ is **not a vector equation**! Each term results in a scalar value.

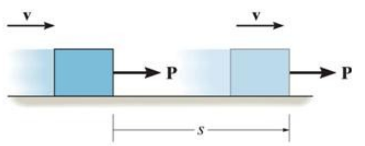
Both kinetic energy and work have the same units, that of energy! In the SI system, the unit for energy is called a **joule** (J), where $1\,J = 1\,N \cdot m$. In the FPS system, units are $\mathbf{ft \cdot lb}$

The principle of work and energy **cannot** be used, in general, to determine forces directed **normal** to the path, since these forces do no work.

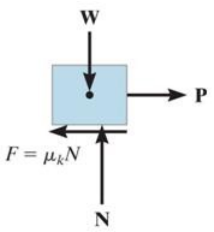
The principle of work and energy can also be applied to a **system of particles** by summing the kinetic energies of all particles in the system and the work due to all forces acting on the system.

Work of Friction Caused By Sliding

The case of a body sliding over a **rough surface** merits special consideration.



Consider a block which is moving over a rough surface. If the applied force **P** just balances the resultant **frictional force** $\mu_k N$, a constant velocity v would be maintained.



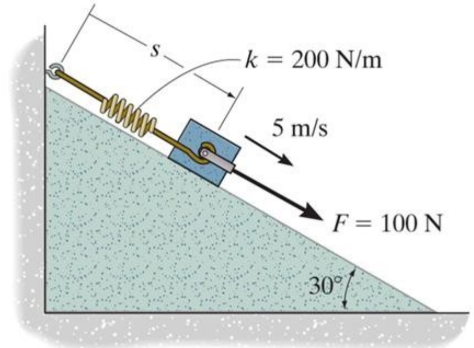
The principle of work and energy would be applied as

$$0.5m(v)^2 + Ps - (\mu_k N)s = 0.5m(v)^2$$

This equation is satisfied if $P = \mu_k N$. However, we know from experience that friction generates **heat**, a form of energy that does not seem to be accounted for in this equation. It can be shown that the work term $(\mu_k N)s$ represents **both** the **external work** of the friction force and the **internal work** that is converted into heat.

Example (1 of 3)

Given: When $s = 0.6$ m, the spring is not stretched or compressed, and the 10 kg block, which is subjected to a. force of 100 N, has a speed of 5 m/s down the smooth plane.



Find: The distance s when the block stops.

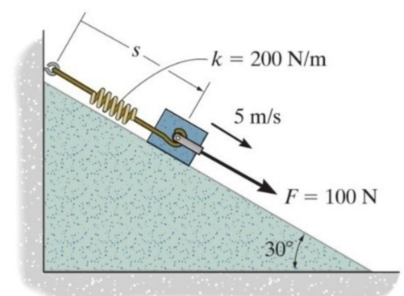
Path Since this problem involves forces, velocity and displacement, apply the principle of work and energy to determine s.

Example (2 of 3)

Solution

Apply the principle of work and energy between position 1 $(s_1 = 0.6\text{m})$ and position 2 (s_2). Note that the normal force (**N**) does no work since it is always perpendicular to the displacement.

$$T_1 + \sum U_{1-2} = T_2$$



There is work done by three different forces;

1) work of a the force F =100 N;

$$U_F = 100(s_2 - s_1) = 100(s_2 - 0.6)$$

2) work of the block weight;

$$U_W = 10(9.81)(s_2 - s_1)\sin 30^\circ = 49.05(s_2 - 0.6)$$

3) and, work of the spring force.

$$U_S = -0.5(200)(s_2 - 0.6)^2 = -100(s_2 - 0.6)^2$$

Example (3 of 3)

The work and energy equation will be

$$T_1 + \sum U_{1-2} = T_2$$

$$0.5(10)5^2 + 100(s_2 - 0.6) + 49.05(s_2 - 0.6) - 100(s_2 - 0.6)^2 = 0$$

$$\Rightarrow 125 + 149.05(s_2 - 0.6) - 100(s_2 - 0.6)^2 = 0$$

Solving for $(s_2 - 0.6)$,

$$(s_2 - 0.6) = \{-149.05 \pm (149.05^2 - 4 \times (-100) \times 125)^{0.5}\} / 2(-100)$$

Selecting the positive root, indicating a positive spring deflection,

$$(s_2 - 0.6) = 2.09m$$

Therefore, $s_2 = 2.69m$

Concept Quiz

1. A spring with an unstretched length of 5 inches expands from a length of 2 inches to a length of 4 inches. The work done **on** the spring is __________ $in \cdot lb$.

   A) $-[0.5\,k\,(4\text{ in})^2 - 0.5\,k\,(2\text{ in})^2]$ B) $0.5\,k\,(2\text{ in})^2$

   C) $-[0.5\,k\,(3\text{ in})^2 - 0.5\,k\,(1\text{ in})^2]$ D) $0.5\,k\,(3\text{ in})^2 - 0.5\,k\,(1\text{ in})^2$

2. If a spring force is $\mathbf{F = 5s^3\ N/m}$ and the spring is **compressed by $s = 0.5\,m$,** the work done on a particle attached to the spring will be

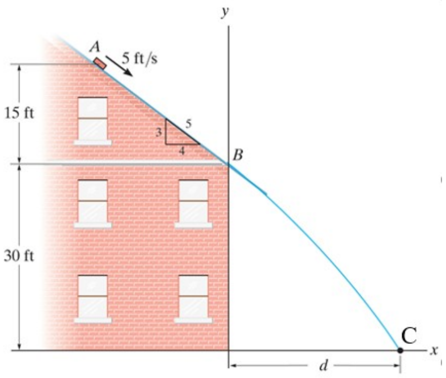
   A) $0.625\ N \cdot m$ B) $-0.625\ N \cdot m$

   C) $0.0781\ N \cdot m$ D) $-0.0781\ N \cdot m$

Group Problem Solving I (1 of 3)

Given: The 2 lb brick slides down a smooth roof, with $v_A = 5$ ft/s.

Find: The speed at B, the distance d from the wall to where the brick strikes the ground, and its speed at C.



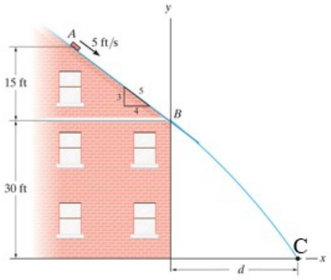
Plan:

1. Apply the principle of work and energy to the brick, and determine the speeds at B and C.
2. Apply the kinematic relations in x and y-directions.

Group Problem Solving I (2 of 3)

Solution

1. Apply the principle of work and energy



$$\sum T_A + \sum U_{A-B} = \sum T_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)5^2 + 2(15) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_B)^2$$

Solving for the unknown velocity yields $v_B = 31.48\, ft/s$

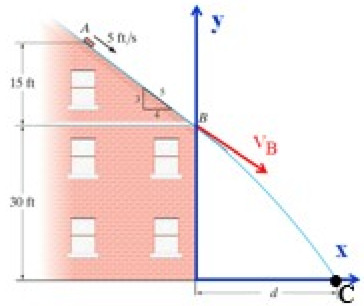
Similarly, apply the work and energy principle between A and C

$$\sum T_A + \sum U_{A-C} = \sum T_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)5^2 + 2(45) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_C)^2$$

$$v_C = 54.1\, ft/s$$

Group Problem Solving I (3 of 3)



2. Apply the kinematic relations in x and y-directions:

Equation for horizontal motion

$$+\rightarrow \; x_C = x_B + v_{BX} t_{BC}$$

$$d = 0 + 31.48(4/5) t_{BC}$$

$$\Rightarrow \; d = 6.996 t_{BC}$$

Equation for vertical motion

$$+\uparrow \; v_C = v_B + v_{By} t_{BC} - 0.5 g t_{BC}^2$$

$$\Rightarrow \; -30 = 0 + (-31.48)(3/5) t_{BC} - 0.5(32.2) t_{BC}^2$$

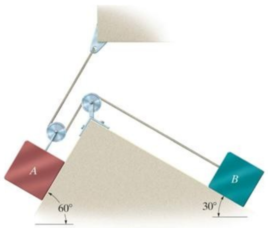
Solving for the positive t_{BC} yields $t_{BC} = 0.899 s$

$$\Rightarrow \; d = 6.996\, t_{BC} = 6.996(0.899) = 22.6 ft$$

Pearson

Group Problem Solving II (1 of 5)

Given: Block A has a weight of 60 lb and block B has a weight of 40 lb. The coefficient of kinetic friction between the blocks and the incline is $m_k = 0.1$. Neglect the mass of the cord and pulleys.



Find: The speed of block A after block B **moves 2 ft** up the plane, starting from rest.

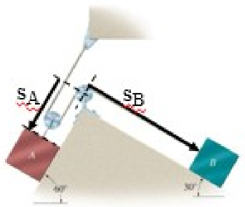
Plan:

1. Define the kinematic relationships between the blocks.
2. Draw the FBD of each block.
3. Apply the principle of work and energy to the system of blocks. Why choose this method?

Group Problem Solving II (2 of 5)

Solution

1. **The kinematic relationships** can be determined by defining position coordinates s_A and s_B, and then differentiating.



Since the cable length is constant:

$$2s_A + s_B = l$$

$$2\Delta s_A + \Delta s_B = 0$$

When $\Delta s_B = -2\,ft \Rightarrow \Delta s_A = 1\,ft$

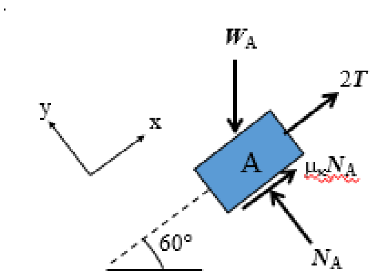
$$2v_A + v_B = 0$$

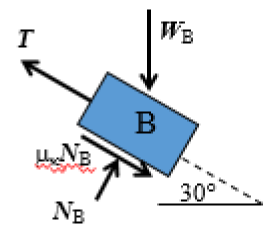
$$\Rightarrow v_B = -2v_A$$

Note that, by this definition of s_A and s_B, **positive motion** for each block is defined as downwards.

Group Problem Solving II (3 of 5)

1. **Draw the FBD of each block.**





Sum forces in the y-direction for block A (note that there is no motion in y-direction):

$$\sum F_y = 0: \; N_A - W_A \cos 60^\circ = 0$$

$$N_A = W_A \cos 60^\circ$$

Similarly, for block B:

$$N_B = W_B \cos 30^\circ$$

Group Problem Solving II (4 of 5)

3. **Apply the principle of work and energy** to the system (the blocks start from rest).

$$\sum T_1 + \sum U_{1-2} = \sum T_2$$

$$[0.5m_A(v_{A1})^2 + .5m_B(v_{B1})^2] + [W_A \sin 60^\circ - 2T - \mu_k N_A]\Delta s_A$$

$$+[W_B \sin 30^\circ - T + \mu_k N_B]\Delta s_B = [0.5m_A(v_{A2})^2 + 0.5m_B(v_{B2})^2]$$

Where $v_{A1} = v_{B1} = 0, \Delta s_A = 1\,ft,$

$\Delta s_B = -2\,ft, v_B = -2v_A,$

$N_A = W_A \cos 60^\circ, N_B = W_B \cos 30^\circ$

$$\Rightarrow [0+0] + [60 \sin 60^\circ - 2T - 0.1(60 \cos 60^\circ)](1)$$

$$+[40 \sin 30^\circ - T + 0.1(40 \cos 30^\circ)](-2)$$

$$= [0.5(60/32.2)(v_{A2})^2 + 0.5(40/32.2)(-2v_{A2})^2]$$

Group Problem Solving II (5 of 5)

Again, the Principal of Work and Energy equation is:

$$\Rightarrow [0+0]+[60\sin 60^{\circ} - 2T - 0.1(60\cos 60^{\circ})](1)$$

$$+[40\sin 30^{\circ} - T + 0.1(40\cos 30^{\circ})](-2)$$

$$=[0.5(60/32.2)(v_{A2})^2 + 0.5(40/32.2)(-2v_{A2})^2]$$

Solving for the unknown velocity yields

$$\Rightarrow v_{A2} = 0.771\, ft/s$$

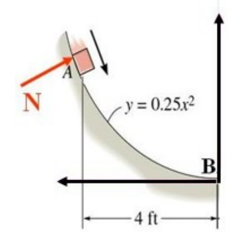
Note that the work being done due to the cable tension force on each block cancels each other (add to zero).

Attention Quiz

1. What is the work done by the normal force N if a **10 lb** box is moved from A to B ?

   A) - 1.24 lb ·ft B) 0 lb ·ft

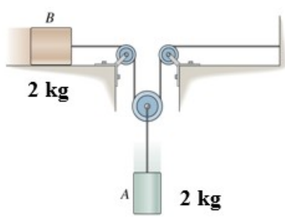
   C) 1.24 lb ·ft D) 2.48 lb ·ft



2. Two blocks are initially at rest. How many equations would be needed to determine the velocity of block A after block B moves 4 meters horizontally on the smooth surface?

   A)One B)Two

   C)Three D)Four



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