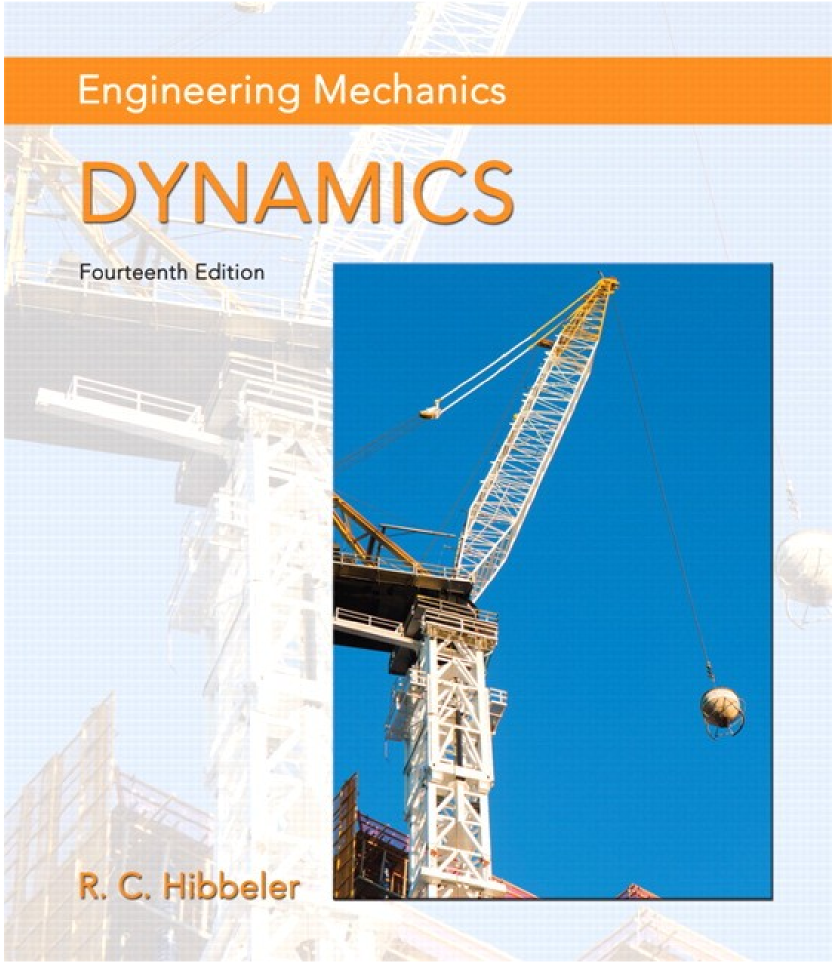
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 16

Planar Kinematics of a Rigid Body

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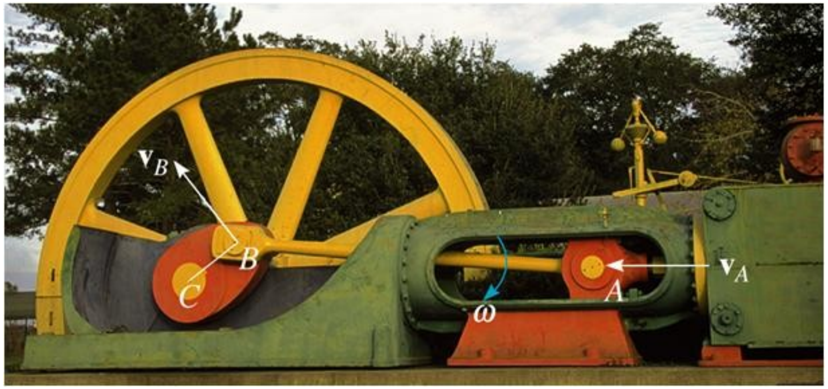
Relative Motion Analysis: Velocity (1 of 5)

Today's Objectives:

Students will be able to:

1. Describe the velocity of a rigid body in terms of translation and rotation components.

2. Perform a relative-motion velocity analysis of a point on the body.



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Relative Motion Analysis: Velocity (2 of 5)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Translation and Rotation Components of Velocity
- Relative Velocity Analysis
- Concept Quiz
- Group Problem Solving
- Attention Quiz

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Reading Quiz

1. When a relative-motion analysis involving two sets of coordinate axes is used, the x’ - y’ coordinate system will

   A) be attached to the selected point for analysis.

   B) rotate with the body.

   C) not be allowed to translate with respect to the fixed frame.

   D) None of the above.

2. In the relative velocity equation, $v_{B/A}$ is

   A) the relative velocity of B with respect to A.

   B) due to the rotational motion.

   C) $\omega \times r_{B/A}$.

   D) All of the above.

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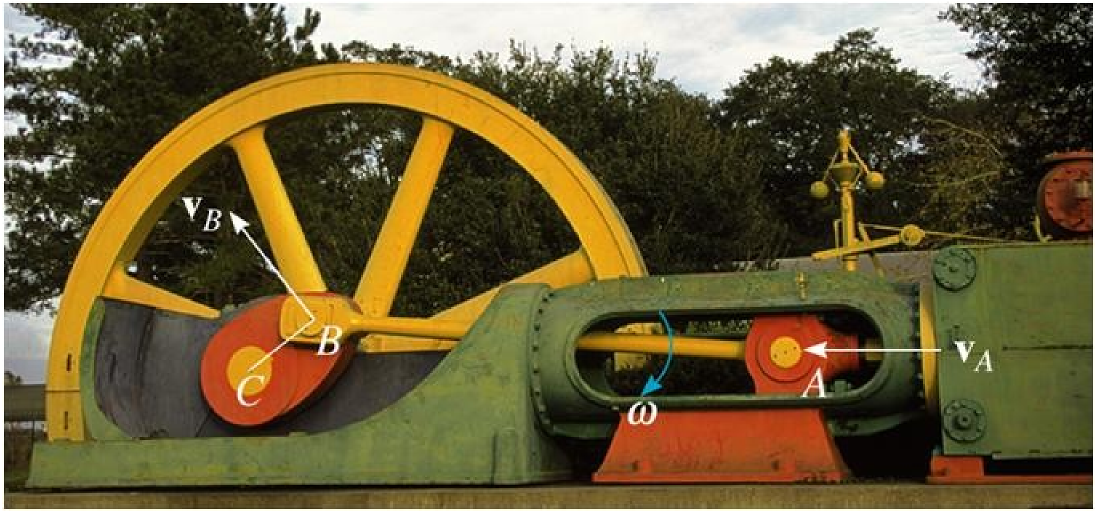
Applications (1 of 2)



As the slider block A moves horizontally to the left with v_A, it causes the link CB to rotate counterclockwise. Thus v_B is directed tangent to its circular path.

Which link is undergoing general plane motion? Link AB or link BC?

How can the angular velocity, ω, of link AB be found?

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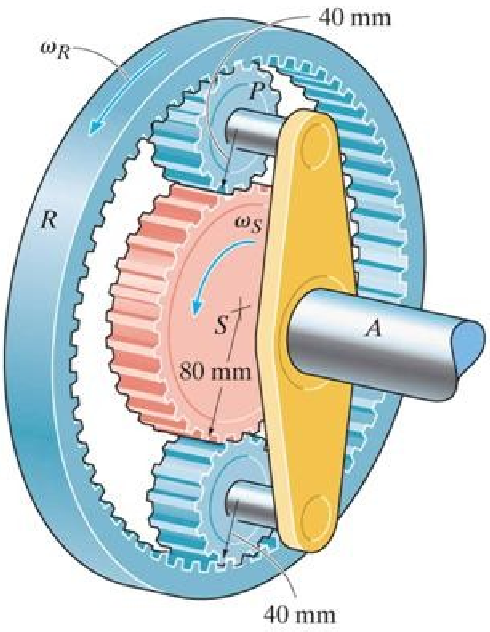
Applications (2 of 2)

Planetary gear systems are used in many automobile automatic transmissions. By locking or releasing different gears, this system can operate the car at different speeds.



How can we relate the angular velocities of the various gears in the system?

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Section 16.5

Relative Motion Analysis

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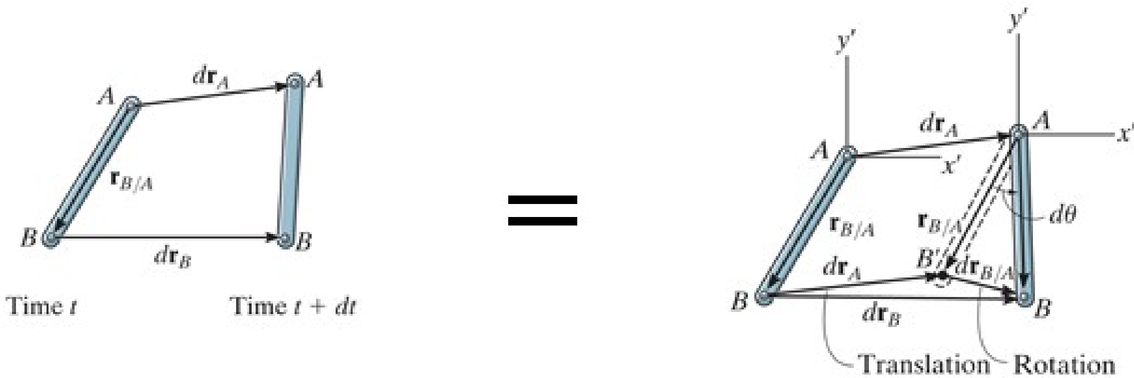
Relative Motion Analysis

When a body is subjected to general plane motion, it undergoes a combination of translation and rotation.



Point A is called the base point in this analysis. It generally has a known motion. The x'- y' frame translates with the body, but does not rotate. The displacement of point B can be written:

Disp. due to translation

$$dr_B = dr_A + dr_{B/A}$$

Disp. due to translation and rotation

Disp. due to rotation

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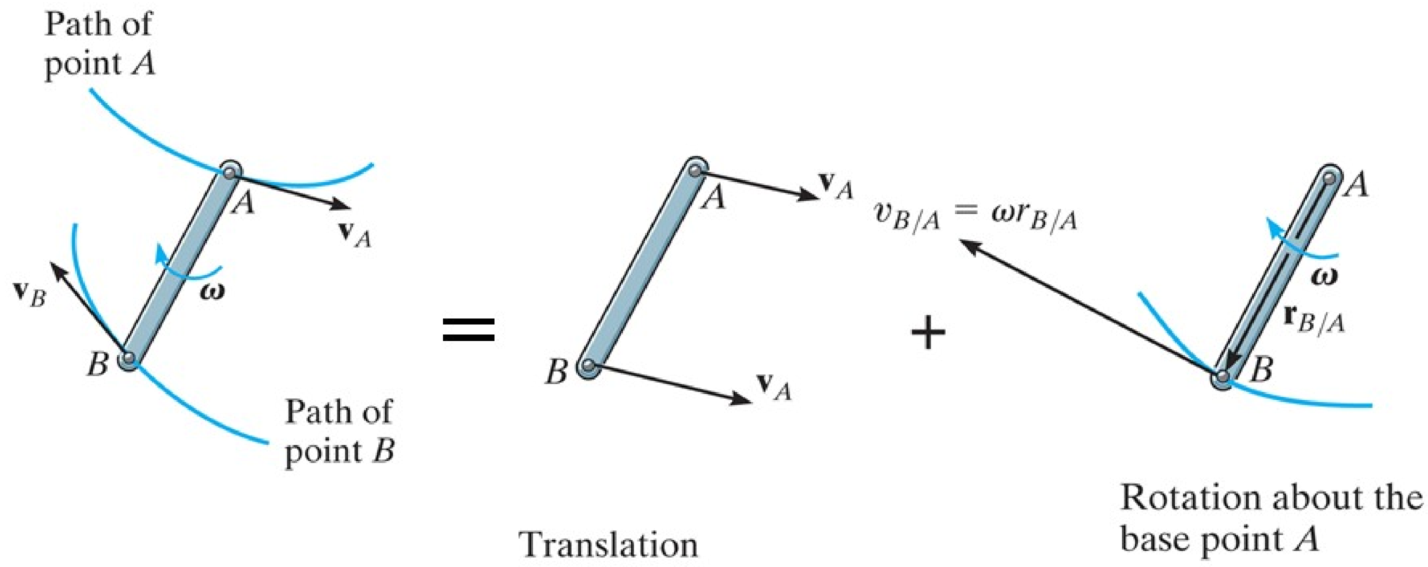
Relative Motion Analysis: Velocity (3 of 5)



The velocity at B is given as : $(d\mathbf{r}_B / dt) = (d\mathbf{r}_A / dt) + (d\mathbf{r}_{B/A} / dt)$ or

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

Since the body is taken as rotating about A,

$$\mathbf{v}_{B/A} = d\mathbf{r}_{B/A} / dt = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

Here ω will only have a k component since the axis of rotation is perpendicular to the plane of translation.

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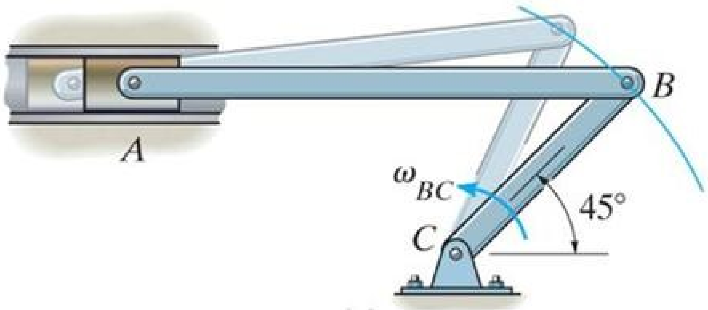
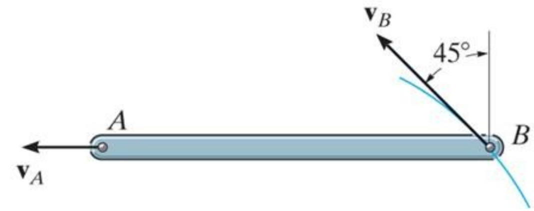
Relative Motion Analysis: Velocity (4 of 5)

$$v_B = v_A + \omega \times r_{B/A}$$



When using the relative velocity equation, points A and B should generally be points on the body with a known motion. Often these points are pin connections in linkages.

For example, point A on link AB must move along a horizontal path, whereas point B moves on a circular path.



The directions of v_A and v_B are known since they are always tangent to their paths of motion.

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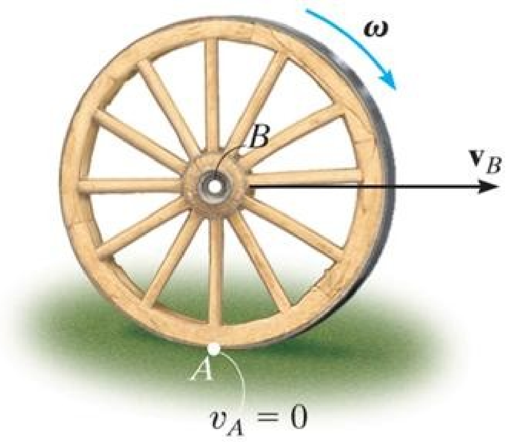
Relative Motion Analysis: Velocity (5 of 5)

$$v_B = v_A + \omega \times r_{B/A}$$



When a wheel rolls without slipping, point A is often selected to be at the point of contact with the ground.

Since there is no slipping, point A has zero velocity.

Furthermore, point B at the center of the wheel moves along a horizontal path. Thus, v_B has a known direction, e.g., parallel to the surface.

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Procedure For Analysis (1 of 2)

The relative velocity equation can be applied using scalar x and y component equations or via a Cartesian vector analysis.

Scalar Analysis:

1. Establish the fixed x-y coordinate directions and draw a kinematic diagram for the body. Then establish the magnitude and direction of the relative velocity vector $v_{B/A}$
2. Write the equation $v_B = v_A + v_{B/A}$. In the kinematic diagram, represent the vectors graphically by showing their magnitudes and directions underneath each term.
3. Write the scalar equations from the x and y components of these graphical representations of the vectors. Solve for the unknowns.

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Procedure For Analysis (2 of 2)

Vector Analysis:

1. Establish the fixed x - y coordinate directions and draw the kinematic diagram of the body, showing the vectors $v_A, v_B, r_{B/A}$ and ω. If the magnitudes are unknown, the sense of direction may be assumed.

2. Express the vectors in Cartesian vector form (CVN) and substitute them into $v_B = v_A + \omega \times r_{B/A}$. Evaluate the cross product and equate respective ***i*** and ***j*** components to obtain two scalar equations.

3. If the solution yields a negative answer, the sense of direction of the vector is opposite to that assumed.

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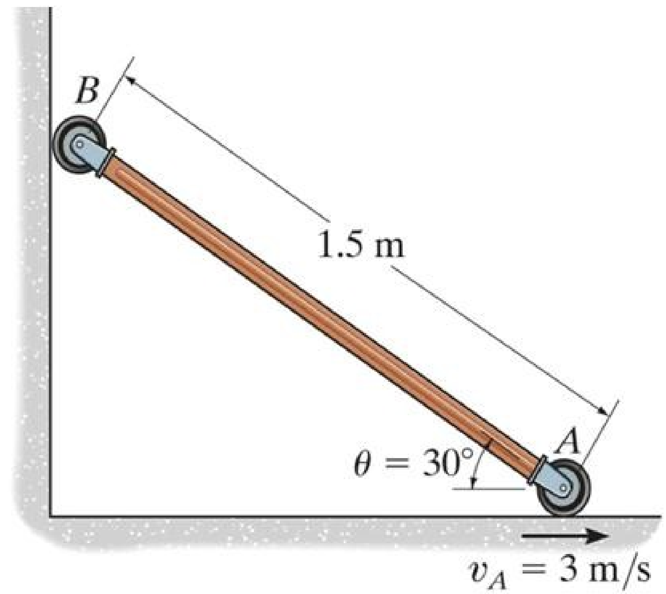
Example 1 (1 of 2)

Given: Roller A is moving to the right at 3 m/s.



Find: The velocity of B at the instant $\theta = 30°$.

Plan:

1. Establish the fixed x - y directions and draw a kinematic diagram of the bar and rollers.
2. Express each of the velocity vectors for A and B in terms of their ***i, j, k*** components and solve $v_B = v_A + \omega \times r_{B/A}$.

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Example 1 (2 of 2)

<u>Solution:</u>

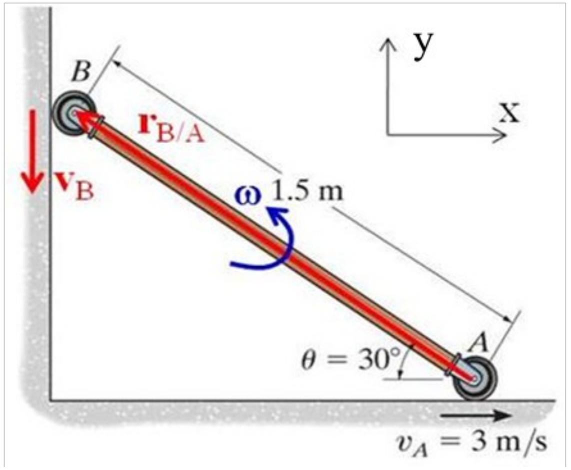
Express the velocity vectors in CVN

$$v_B = v_A + \omega \times r_{B/A}$$

$$-v_B j = 3i + [\omega \times (-1.5\cos 30 i + 1.5 \sin 30 j)]$$

$$-v_B j = 3i - 1.299\omega j - 0.75\omega i$$

Kinematic diagram:



Equating the *i* and *j* components gives:

$$0 = 3 - 0.75\omega$$

$$-v_B = -1.299\omega$$

Solving:

$\omega = 4\ rad/s$ or $\omega = 4\ rad/s\ k$

$v_B = 5.2\ m/s$ or $v_B = -5.2\ m/s\ j$

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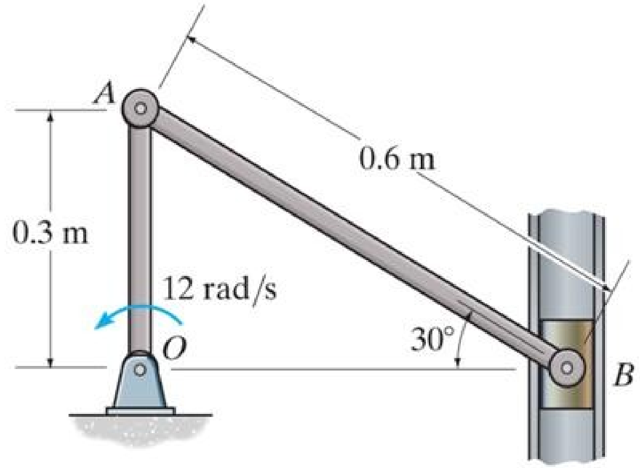
Example 2 (1 of 2)

Given: Crank rotates OA with an angular velocity of 12 rad/s

Find: The velocity of piston B and the angular velocity of rod AB.



Plan: Notice that point A moves on a circular path. The directions of v_A is tangent to its path of motion. Draw A kinematic diagram of rod AB and use

$$v_B = v_A + \omega_{AB} \times r_{B/A}.$$

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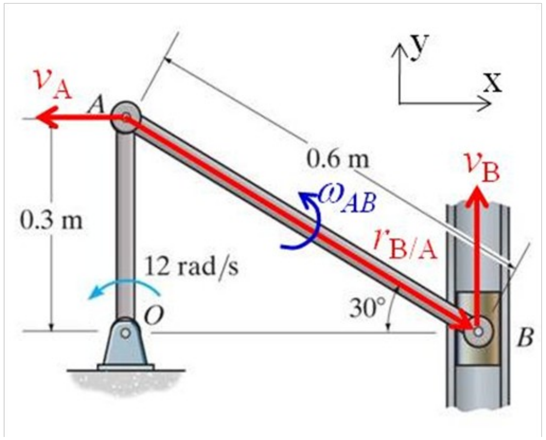
Example 2 (2 of 2)

Solution:

Kinematic diagram of AB:



Since crack OA rotates with an angular velocity of 12 rad/s, the velocity at A will be:

$$v_A = -0.3(12)i = -3.6i\ m/s$$

Rod AB. Write the relative-velocity equation:

$$v_B = v_A + \omega_{AB} \times r_{B/A}.$$

$$-v_B j = -3.6i + \omega_{AB} k \times (0.6\cos 30 i - 0.6 \sin 30 j)]$$

$$v_B j = -3.6i + 0.5196\omega_{AB} j + 0.3\omega_{AB} i$$

By comparing the *i*, *j* components:

$$i{:}\ 0 = -3.6 + 0.3\ \omega_{AB} \Rightarrow \omega_{AB} = 12\ rad/s$$

$$j{:}\ v_B = 0.5196\ \omega_{AB} \Rightarrow v_B = 6.24\ m/s$$

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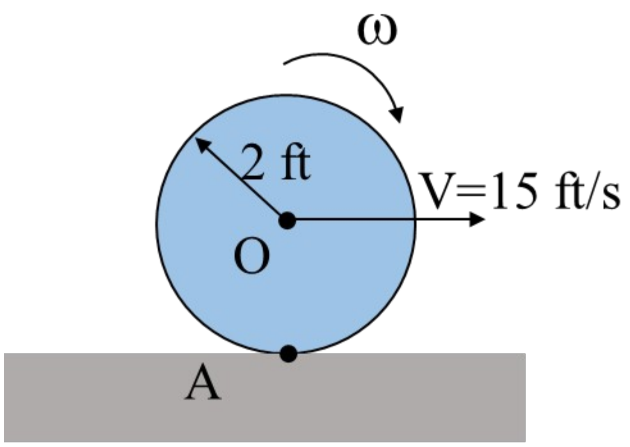
Concept Quiz

1. If the disk is moving with a velocity at point O of 15 ft/s and ω =2 rad/s Determine the velocity of A.

   A) 0 ft/s B) 4 ft/s

   C) 15 ft/s D) 11 ft/s



2. If the velocity at A is zero, then determine the angular velocity ω

   A) 30 rad/s B) 0 rad/s

   C) 7.5 rad/s D) 15 rad/s

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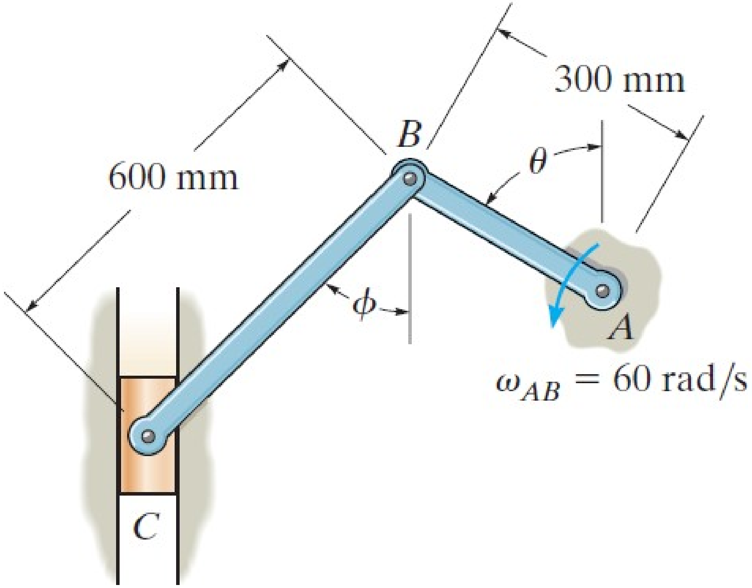
Group Problem Solving (1 of 3)

Given: Rod AB is rotating wit an angular velocity of $\omega_{AB} = 60\ rad/s$.

Find: The velocity of the slider block C when $\theta = 60^\circ$ **and** $\phi = 45^\circ$.



Plan: Notice that rod AB rotates about a fixed point A. The directions of v_B is tangent to its path of motion. Draw a kinematic diagram of rod BC. Then, apply the relative velocity equations to the rod and solve for unknowns.

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Group Problem Solving (2 of 3)

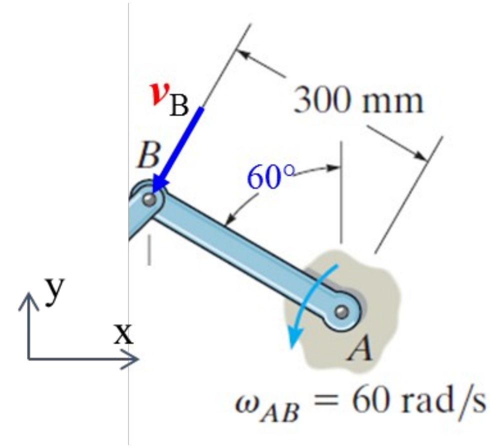
Solution:

Link AB: Since link AB is rotating at $\omega_{AB} = 60\ rad/s$, the velocity at point B will be:

$$v_B = \omega_{AB} \times r_{AB}$$

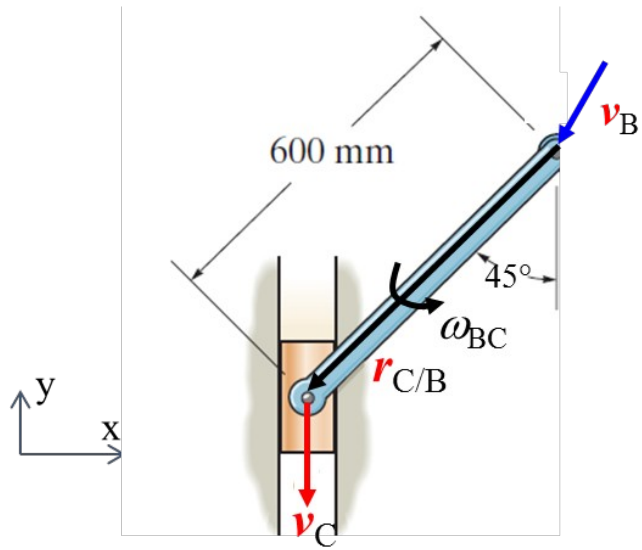
At $\theta = 60^{\circ}$,

$$v_B = 60k \times (-0.3 \sin 60 i + 0.3 \cos 60 j)$$
$$= (-9i - 15.59 j)\ m/s$$



Rod BC : Draw a kinematic diagram of rod BC.

Notice that the slider block C has a vertical motion.



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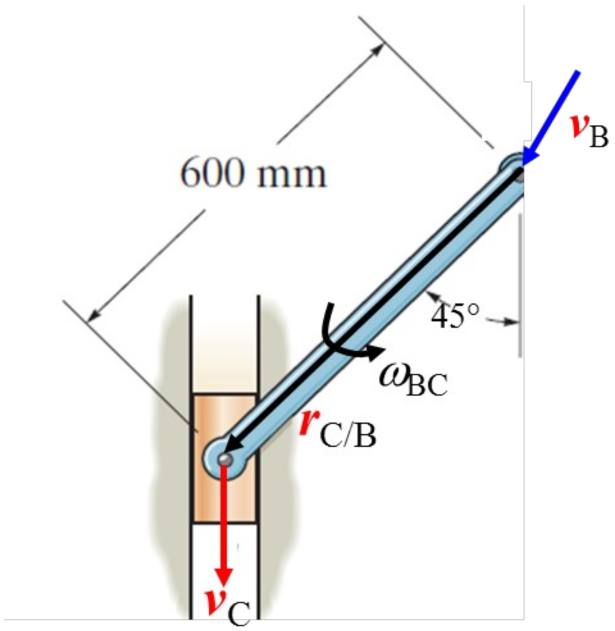
Group Problem Solving (3 of 3)

Apply the relative velocity equation in order to find the velocity at C.

$$\boldsymbol{v}_C = \boldsymbol{v}_B + \boldsymbol{\omega}_{BC} \times \boldsymbol{r}_{C/B}$$

$$-v_C j = (-9i - 15.59j) + \omega_{BC} k \times (-0.6\sin 45 i - 0.6\cos 45 j)$$



Equating the ***i*** and ***j*** components yields:

$$0 = -9 + \omega_{BC}(0.6)\cos 45$$

$$-v_C = -15.59 - \omega_{BC}(0.6)\sin 45$$

$$\omega_{BC} = 21.2\ rad/s$$

$$v_C = 24.59\ m/s = 24.6\ m/s \downarrow$$

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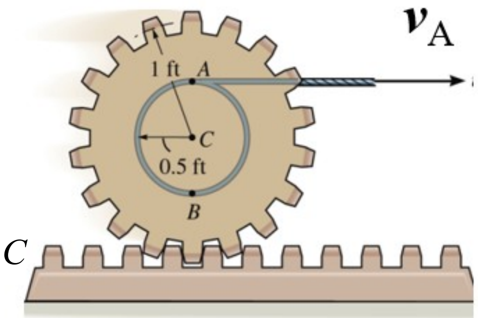
Attention Quiz

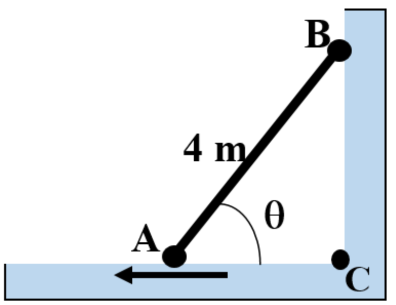
1. Which equation could be used to find the velocity of the center of the gear, C, if the velocity v_A is known?



A) $v_B = v_A + \omega_{gear} \times r_{B/A}$ B) $v_A = v_C + \omega_{gear} \times r_{A/C}$

C) $v_B = v_C + \omega_{gear} \times r_{C/B}$ D) $v_A = v_C + \omega_{gear} \times r_{C/A}$

2. If the bar's velocity at A is 3 m/s what "base" point (first term on the RHS of the velocity equation) would be best used to simplify finding the bar's angular velocity when



A) A B) B

C) C D) No difference.

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