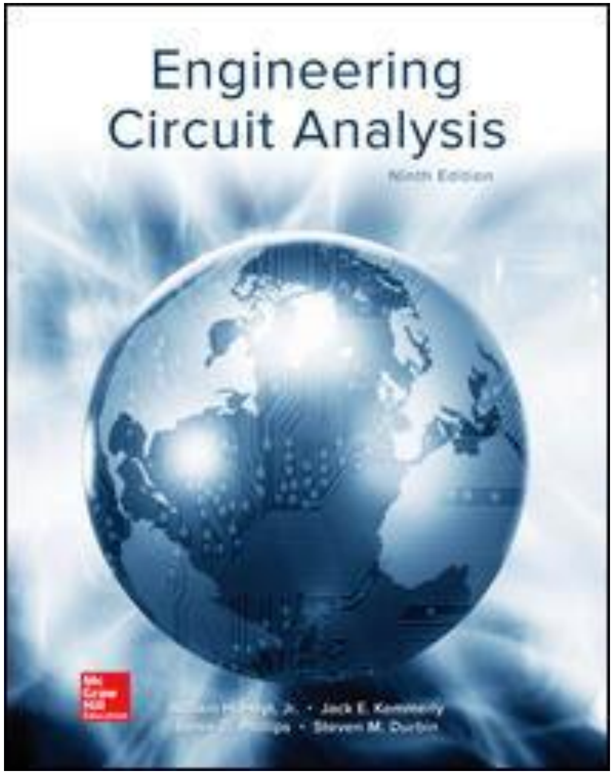
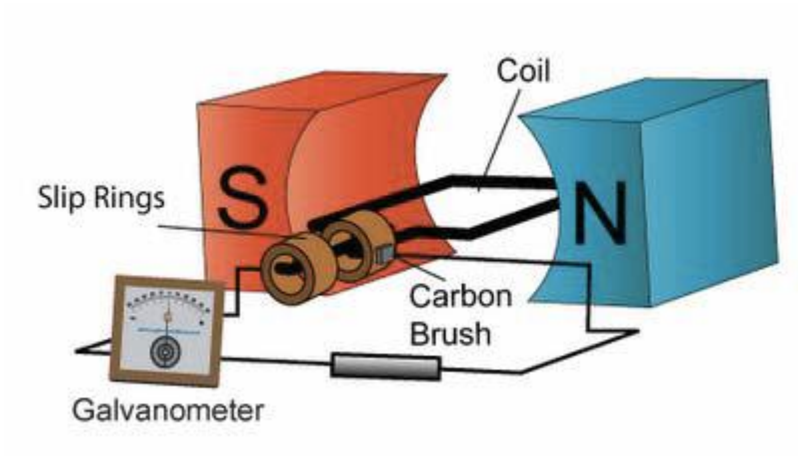
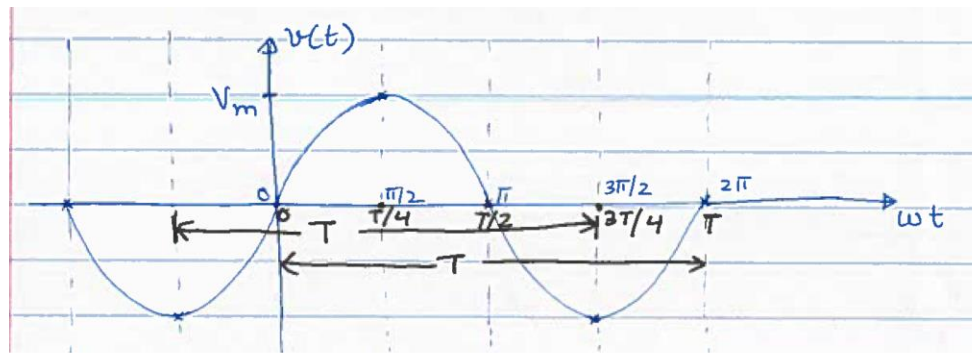
Polyphase Circuits and Transformers

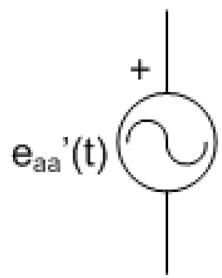


Polyphase Circuits

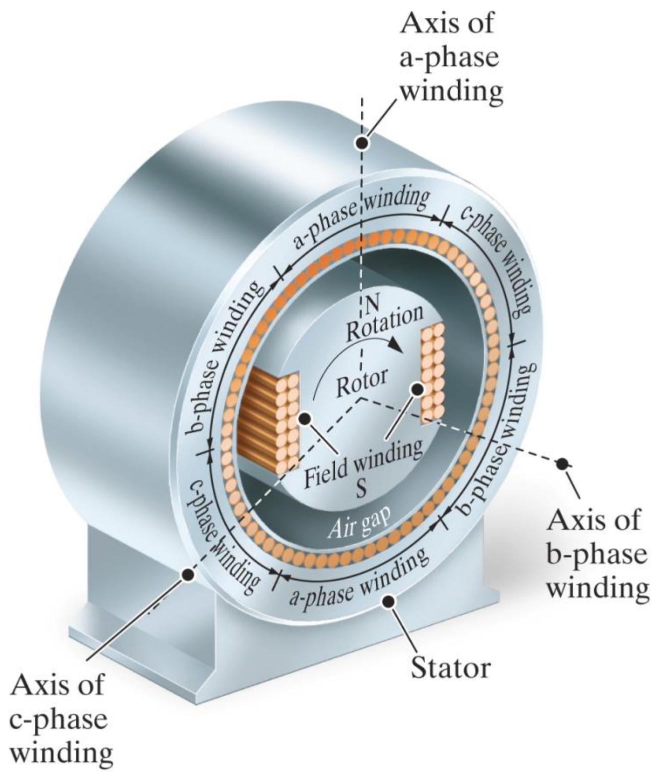




$v(t) = V_m \cos \omega\, t$



Static magnets, one rotating coil, single output voltage

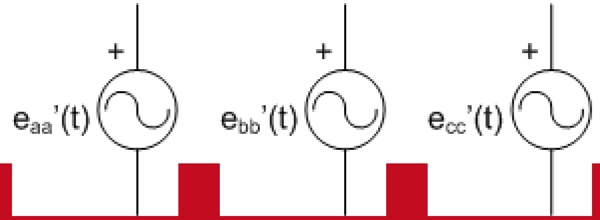


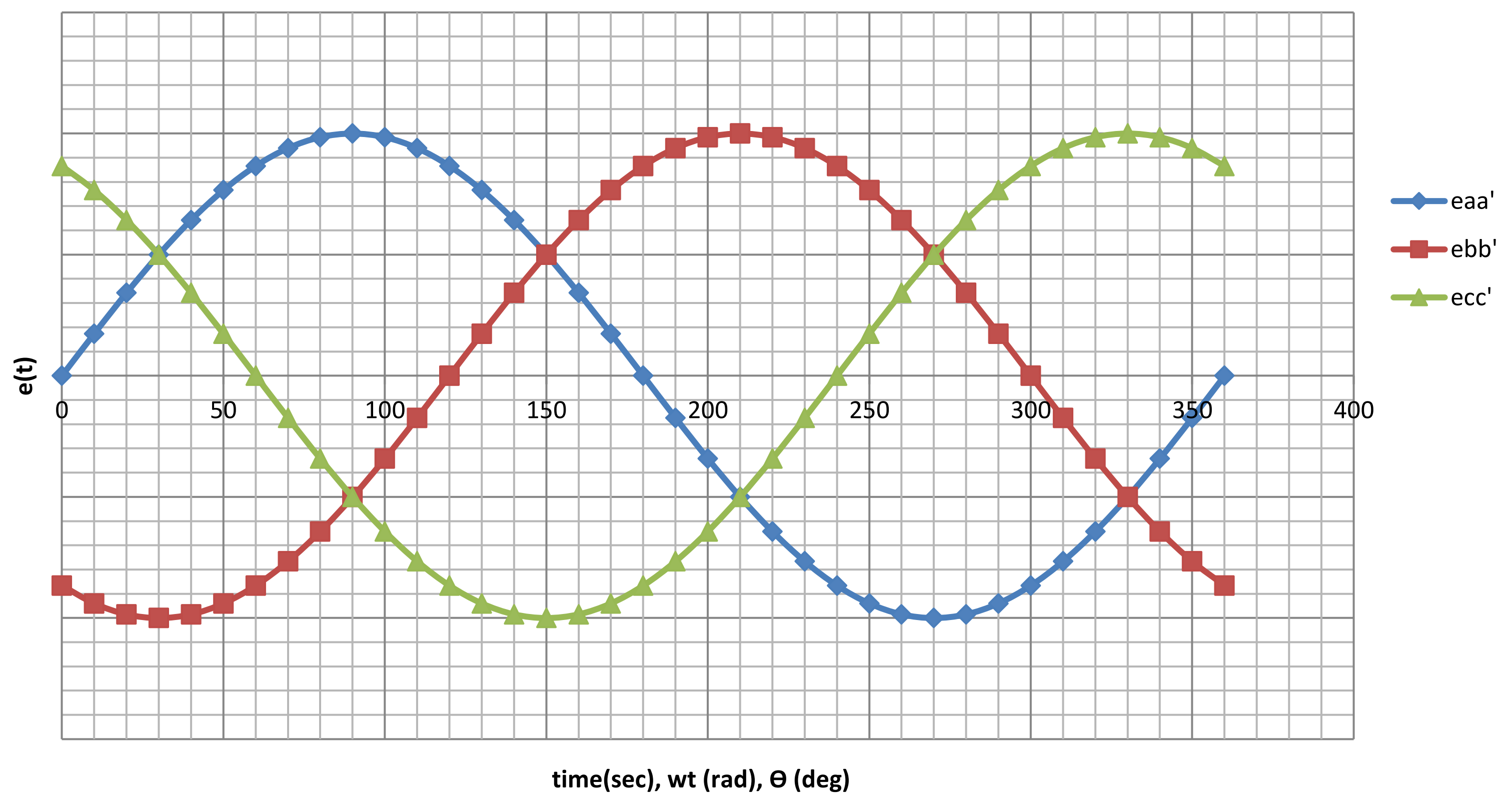
Three static coils, rotating magnets, three output voltages

$$e_{aa}'(t) = \sqrt{2}E\cos\omega t$$

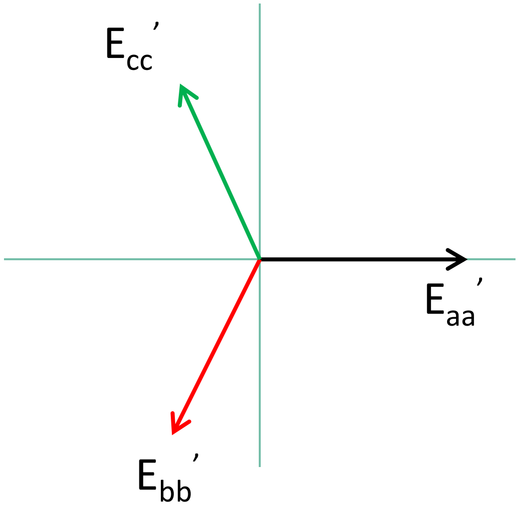
$$e_{bb}'(t) = \sqrt{2}E\cos\left(\omega t - 120^0\right)$$

$$e_{cc}'(t) = \sqrt{2}E\cos\left(\omega t + 120^0\right)$$





e(t)
0
50
100
150
200
250
300
350
400
eaa'
ebb'
ecc'
time(sec), wt (rad), ϴ (deg)



- Positive sequence:
  - abc ; bca ; cab (same)

Rotor is rotating CCW
Eaa' is followed by Ebb' which is followed by Ecc'
In polar form

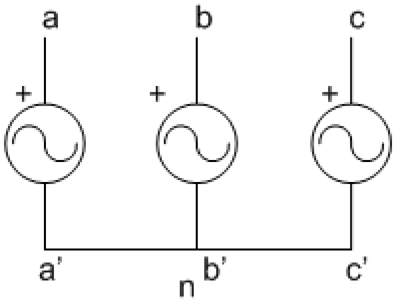
$$\vec{E}_{aa}{}' = E\angle 0^0$$

$$\vec{E}_{bb}{}' = E\angle -120^0$$

$$\vec{E}_{cc}{}' = E\angle 120^0$$

Y - Connection of Three Phase AC Systems

Y-connected sources

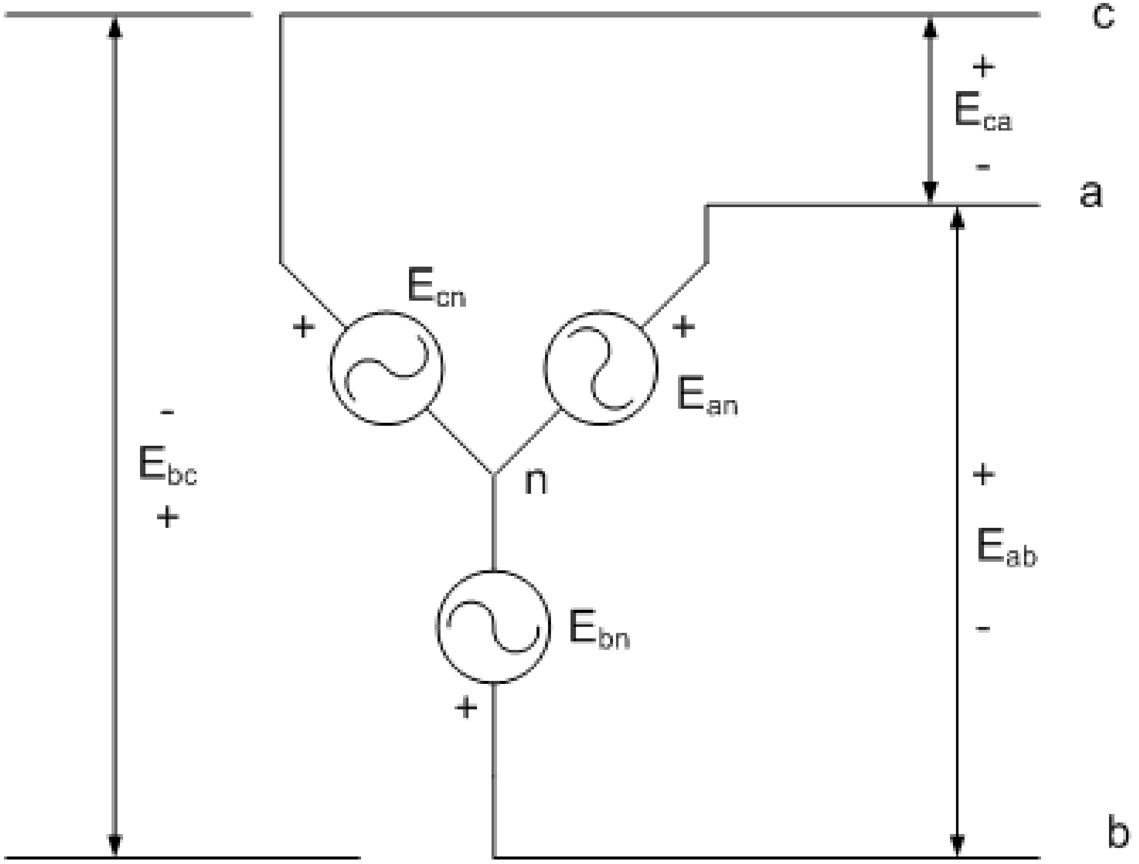


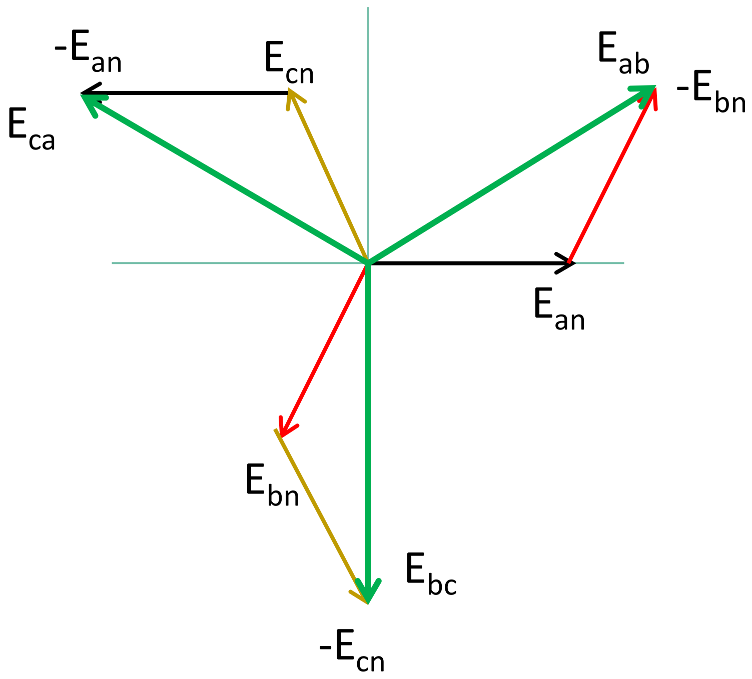
A balanced three phase source has

$|E_{an}| = |E_{bn}| = |E_{cn}|$

And

$E_{an} + E_{bn} + E_{cn} = 0$





$$\vec{E}_{an} = E\angle 0^0$$
$$\vec{E}_{bn} = E\angle -120^0$$
$$\vec{E}_{cn} = E\angle 120^0$$

The *abc* (positive) sequence

$$E_L = \sqrt{3}E_{ph}\angle 30^0$$

$$and \quad E_{ph} = \frac{E_L}{\sqrt{3}\angle 30^0}$$

Y-connected sources

Voltages from terminals to neutral are called "PHASE VOLTAGES"; E_{an}, E_{bn}, E_{cn}

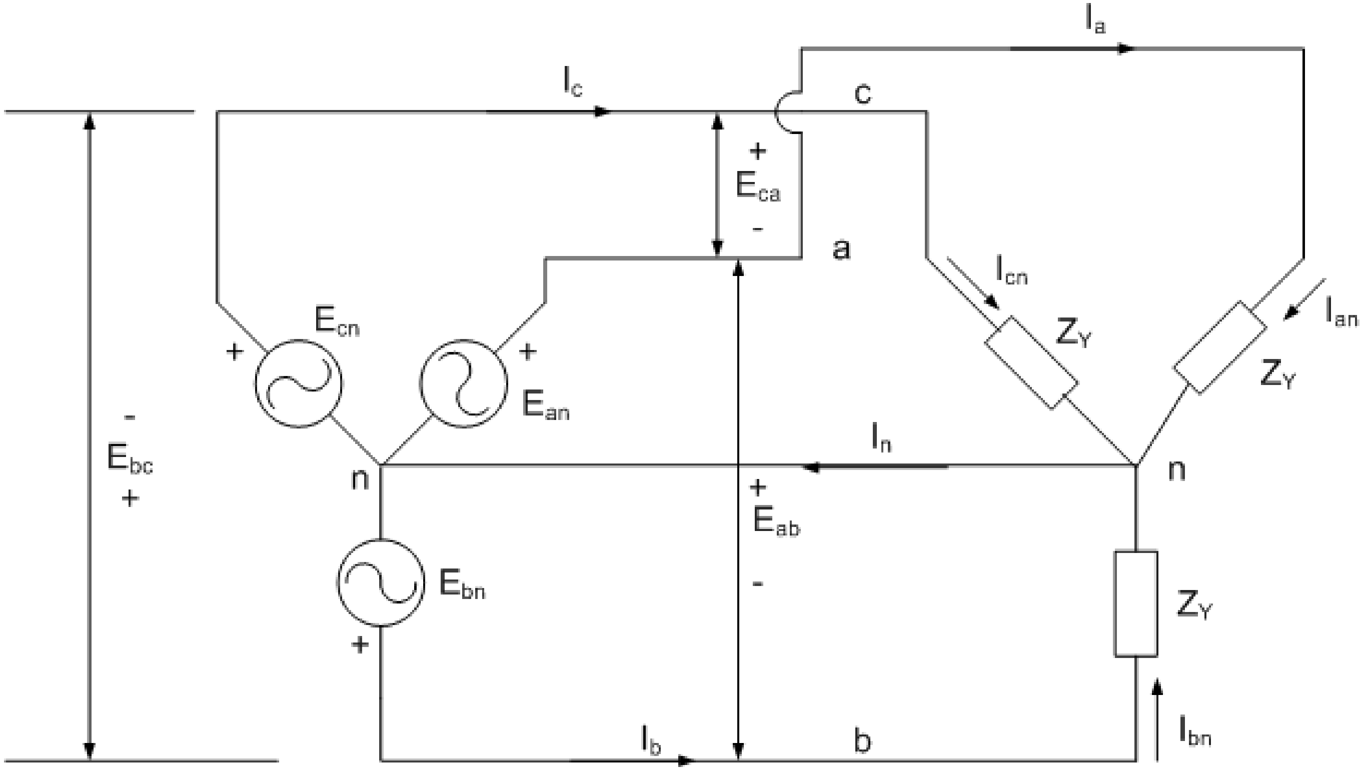
Voltages between terminals are called "LINE VOLTAGES"; E_{ab}, E_{bc}, E_{ca}

$$E_L = \sqrt{3} E_{ph} \angle 30^0 \qquad and \qquad E_{ph} = \frac{E_L}{\sqrt{3} \angle 30^0}$$

$$I_L = I_{ph}$$

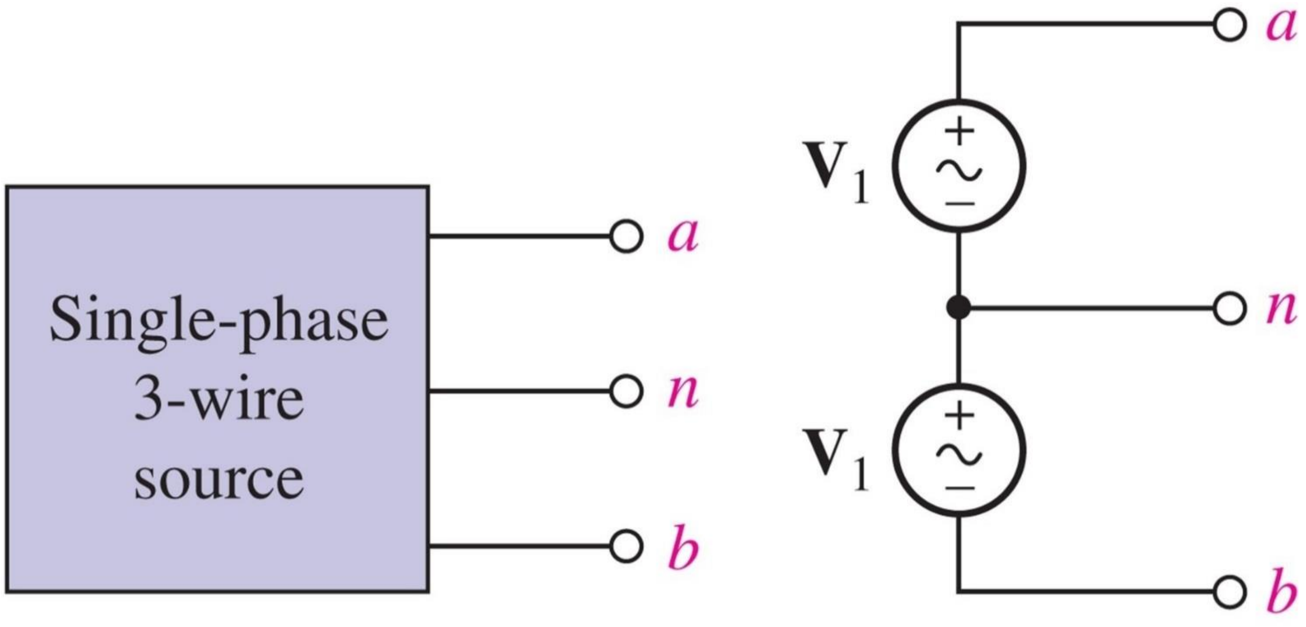
Line voltage of corresponding phases are $\sqrt{3}$ times larger than the phase voltages and lead by 30^0

3-phase balanced load: all impedances in 3- phases are equal



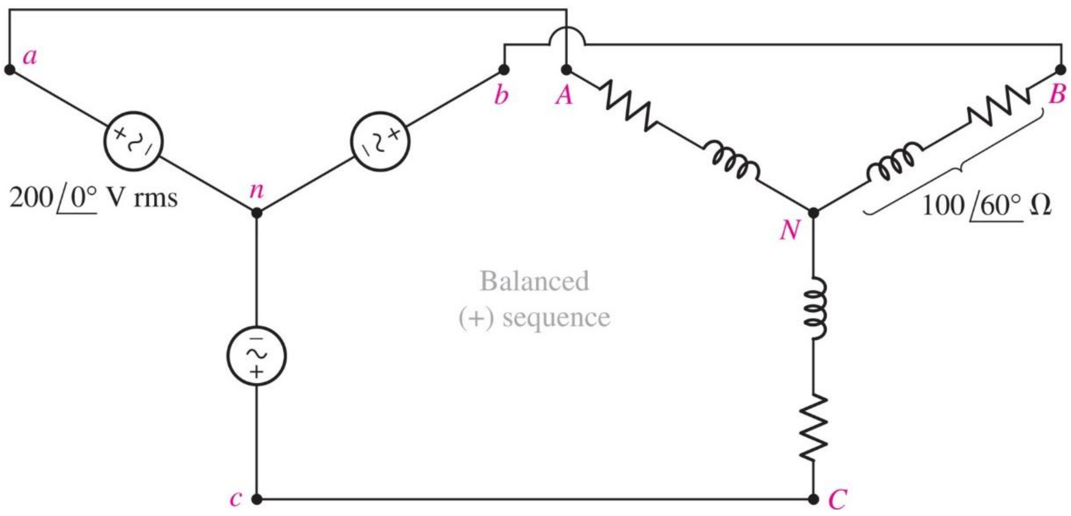
Household Power: Not Polyphase

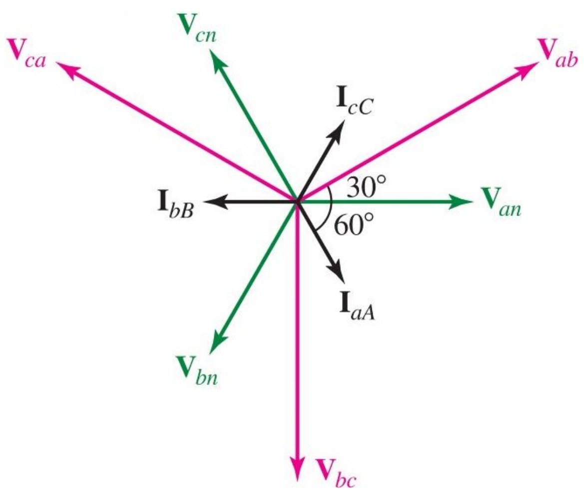
The typical *North American* household is provided single-phase 3-wire power, where $V_1 \cong 110$ V and $V_{ab} = 2V_1$.



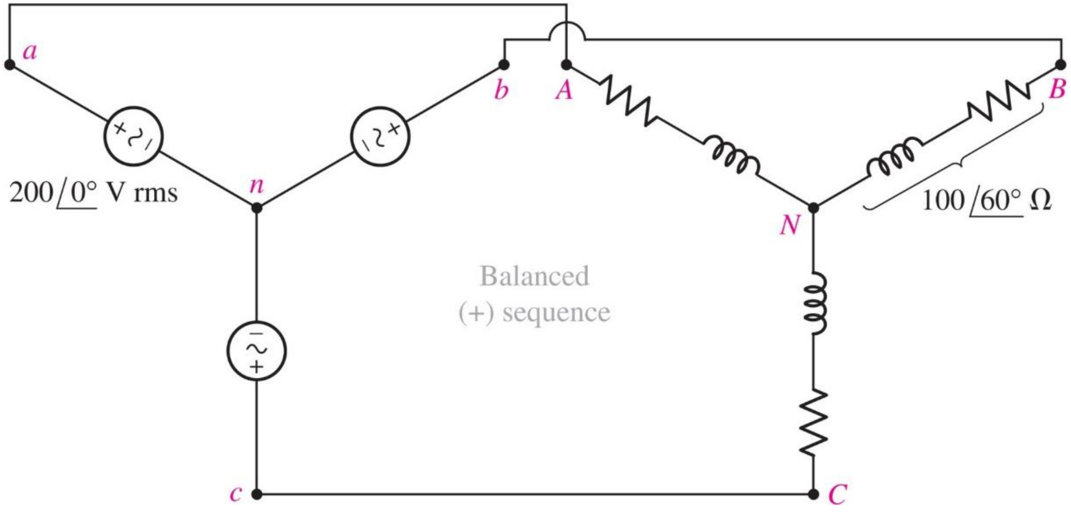
Example: Y-Y Connection

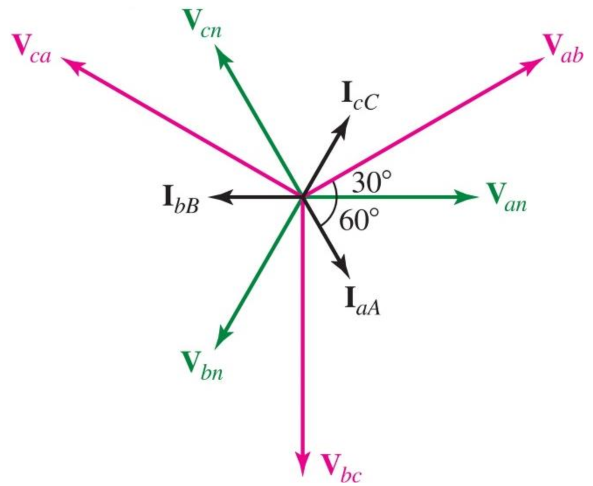
For the circuit of Fig. 12.15, find both the phase and line currents, and the phase and line voltages throughout the circuit; then calculate the total power dissipated in the load.





Example: Y-Y Connection





Answer: P=600 W

Example: Per-phase Analysis

A balanced three-phase system with a line voltage of 300 V is supplying a balanced Y-connected load with 1200 W at a leading PF of 0.8.

Find the line current and the per-phase load impedance.

Example: Per-phase Analysis

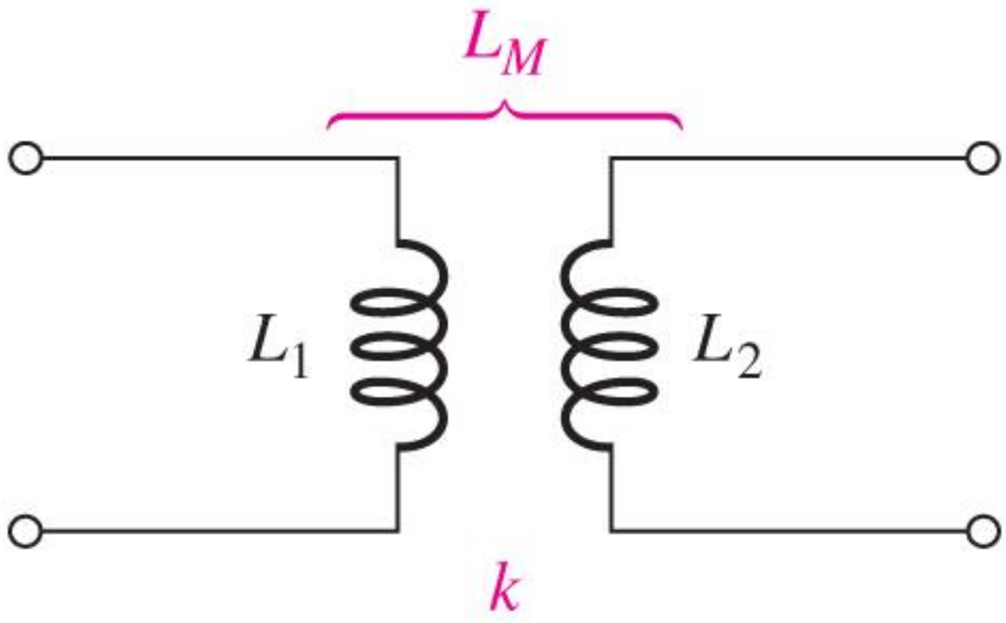
Answer: $I_L = 2.89\ A$, $Z_p = 60$ *angle* $(-36.9^0)\Omega$

Transformers

Mutual Inductance

When two coils are placed close to each other, a changing flux in one coil will cause an induced voltage in the second coil. The coils are said to have **mutual inductance** (L_M), which can either add or subtract from the total inductance depending on if the fields are aiding or opposing.

The coefficient of coupling is a measure of how well the coils are linked; it is a number between 0 (no coupling) and 1 (maximum coupling).



Mutual Inductance

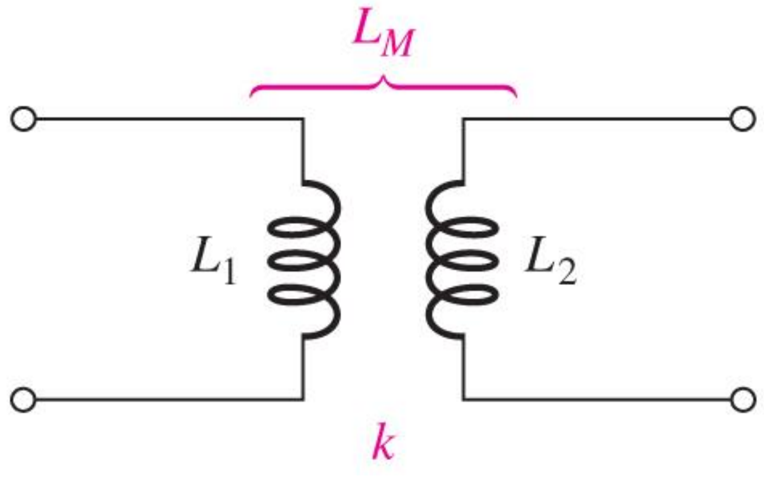
The formula for mutual inductance is $L_M = k\sqrt{L_1 L_2}$

Where

k = the coefficient of coupling (dimensionless)

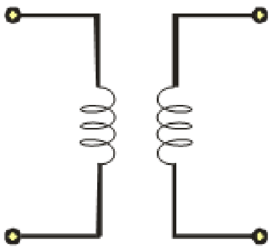
L_1, L_2 = inductance of each coil (H)

k depends on factors such as the orientation of the coils to each other, their proximity, and if they are on a common core.

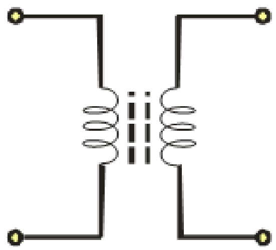


Basic Transformer

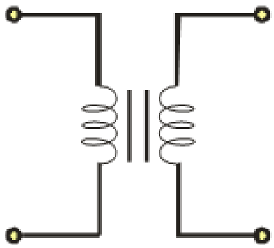
The basic transformer is formed from two coils that are usually wound on a common core to provide a path for the magnetic field lines. Schematic symbols indicate the type of core.



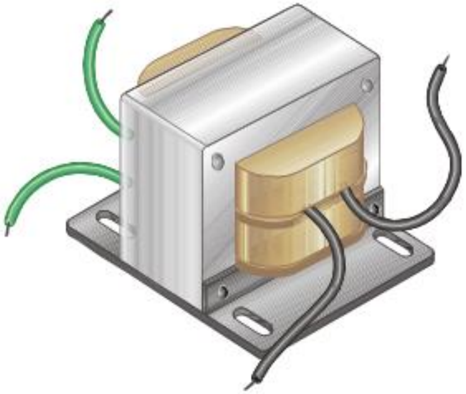
Air core



Ferrite core



Iron core



Small power transformer

Turns ratio

A useful parameter for ideal transformers is the turns ratio, which is defined* as

$$n = \frac{N_{sec}}{N_{pri}}$$

N_{sec} = number of secondary windings

N_{pri} = number of primary windings

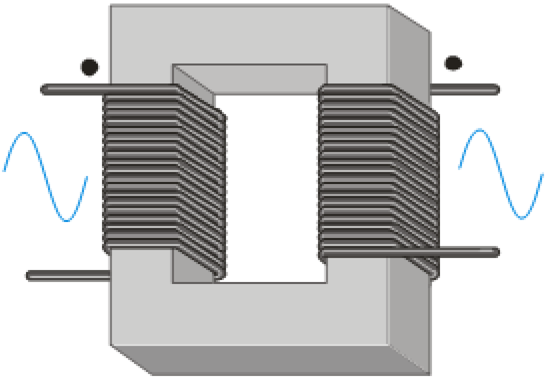
* Based on the IEEE dictionary definition for electronics power transformers. Most transformers are not marked with turns ratio, however it is a useful parameter for understanding transformer operation.

Example

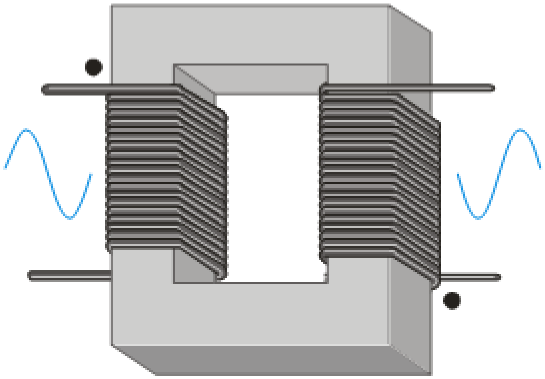
A transformer has 800 turns on the primary and a turns ratio of 0.25. How many turns are on the secondary? 200

Direction of windings

The direction of the windings determines the polarity of the voltage across the secondary winding with respect to the voltage across the primary. Phase dots are sometimes used to indicate polarities.



In phase



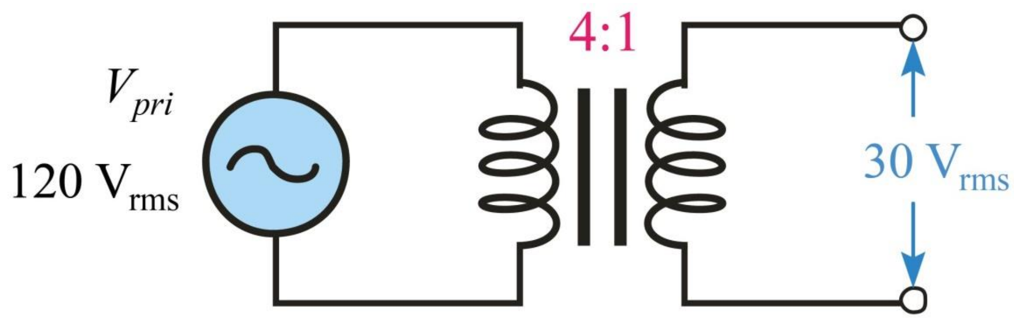
Out of phase

Step-up and step-down transformers

In a **step-up transformer**, the secondary voltage is greater than the primary voltage and $n > 1$.
In a **step-down transformer**, the secondary voltage is less than the primary voltage and $n < 1$.

Example

What is the secondary voltage?

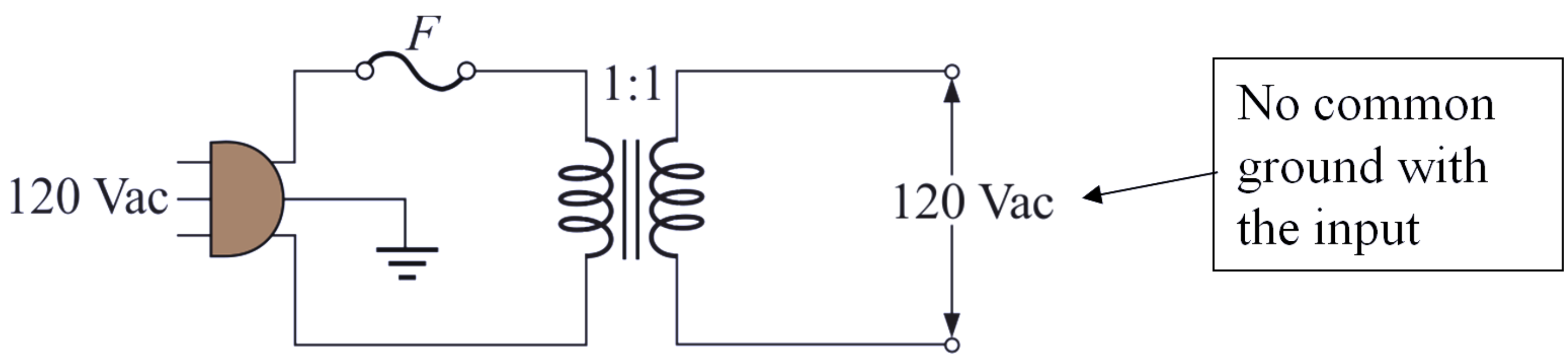


What is the turns ratio? 0.25

Isolation transformers

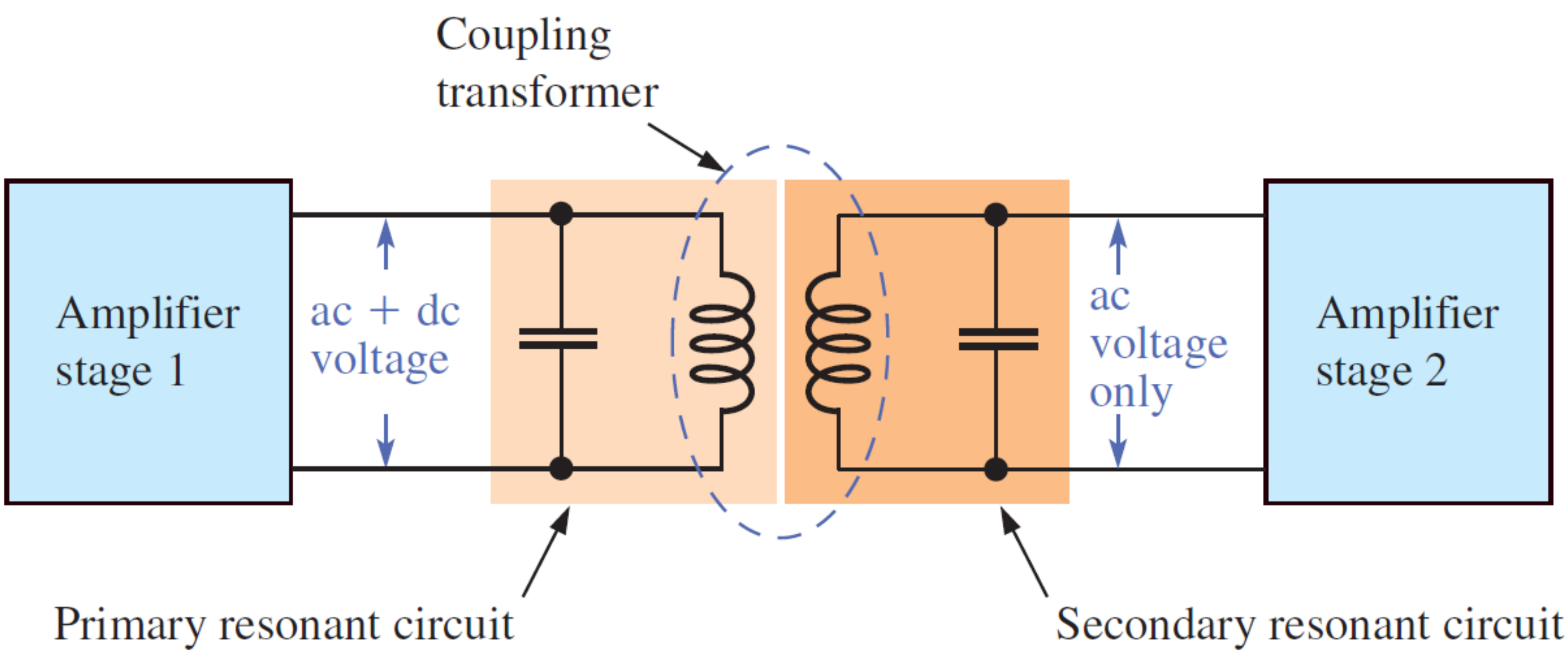
A special transformer with a turns ratio of 1 is called an **isolation transformer**. Because the turns ratio is 1, the secondary voltage is the same as the primary voltage, hence ac is passed from one circuit to another.

The purpose of an isolation transformer is to break a dc path between two circuits while maintaining the ac path. The dc is blocked by the transformer, because the magnetic flux for dc is not changing.



Coupling transformers

Another important transformer type is the **coupling transformer**. A coupling transformer typically isolates dc and passes a select band of frequencies to the next stage or an output speaker.



Current transformation

Transformers cannot increase power. If the secondary voltage is higher than the primary voltage, then the secondary current must be lower than the primary current and vice-versa.
The ideal transformer turns ratio equation for current is

$$\boxed{n = \frac{I_{pri}}{I_{sec}}}$$

Notice that the primary current is in the numerator.

Power

The ideal transformer does not dissipate power. Power delivered from the source is passed on to the load by the ideal transformer. This important idea can be summarized as

$$P_{pri} = P_{sec}$$

$$V_{pri} I_{pri} = V_{sec} I_{sec}$$

$$\frac{V_{sec}}{V_{pri}} = \frac{I_{pri}}{I_{sec}}$$

← These last ratios are the turns ratio, n.

All practical transformers do dissipate power. Power transformers are designed to pass only the utility frequency, so tend to be closer to ideal than other transformer types.

Reflected resistance

A transformer changes both the voltage and current on the primary side to different values on the secondary side. This makes a load resistance appear to have a different value on the primary side.

From Ohm's law, $$R_{pri} = \frac{V_{pri}}{I_{pri}} \text{ and } R_L = \frac{V_{sec}}{I_{sec}}$$

Taking the ratio of R_{pri} to R_L,

$$\frac{R_{pri}}{R_L} = \left(\frac{V_{pri}}{V_{sec}}\right)\left(\frac{I_{sec}}{I_{pri}}\right) = \left(\frac{1}{n}\right)\left(\frac{1}{n}\right) = \frac{1}{n^2}$$

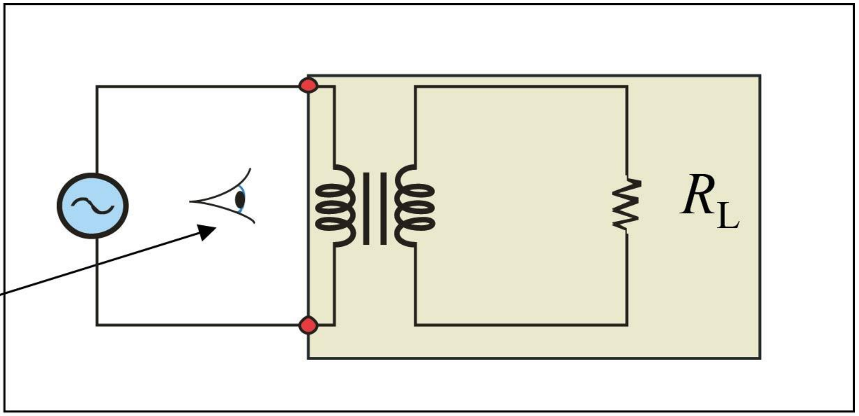
Reflected resistance

The resistance "seen" on the primary side is called the **reflected resistance**.

$$R_{pri} = \left(\frac{1}{n}\right)^2 R_L$$

If you "look" into the primary side of the circuit, you see an effective load that is changed by the reciprocal of the turns ratio squared.

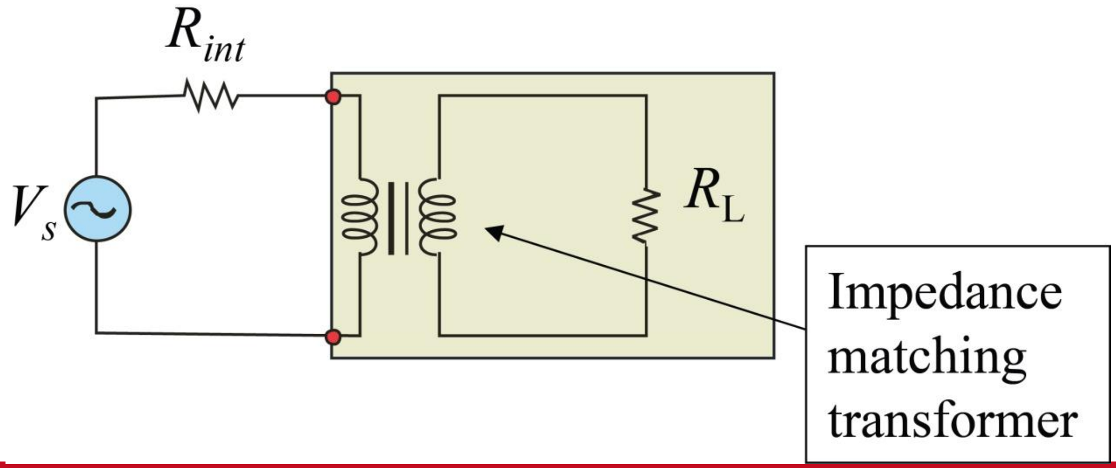
The primary voltage divided by the primary current is the resistance from the perspective of the primary side. Thus, the load resistance is effectively changed on the primary side.



Impedance matching

The word *impedance* is used in ac work to take into account resistance and reactance effects. To match a load resistance to the internal source resistance (and hence transfer maximum power to the load), a special impedance matching transformer is used.

Impedance matching transformers are designed for a wider range of frequencies than power transformers, hence tend to be non-ideal.



Practical transformers

An ideal transformer has no power loss; all power applied to the primary is all delivered to the load. Actual transformers depart from this ideal model. Some loss mechanisms are:

Winding resistance (causing power to be dissipated in the windings)
Hysteresis loss (due to the continuous reversal of the magnetic field)
Core losses due to circulating current in the core (eddy currents)
Flux leakage flux from the primary that does not link to the secondary
Winding capacitance that has a bypassing effect for the windings

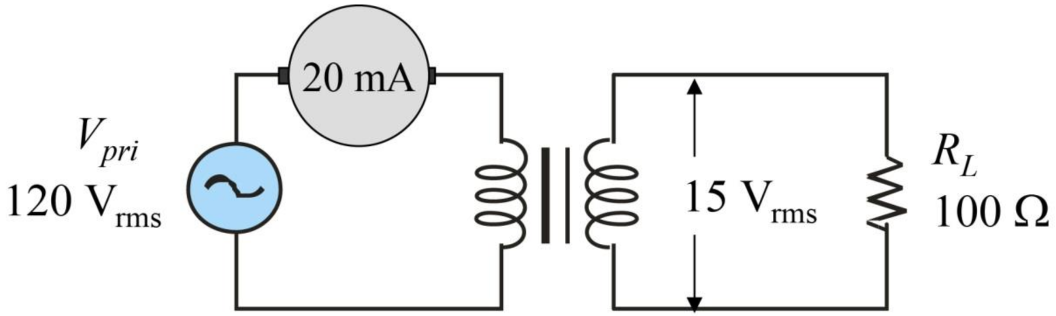
Transformer efficiency

The efficiency of a transformer is the ratio of power delivered to the load (P_{out}) to the power delivered to the primary (P_{in}):

$$\eta = \left(\frac{P_{out}}{P_{in}}\right)100\%$$

Example

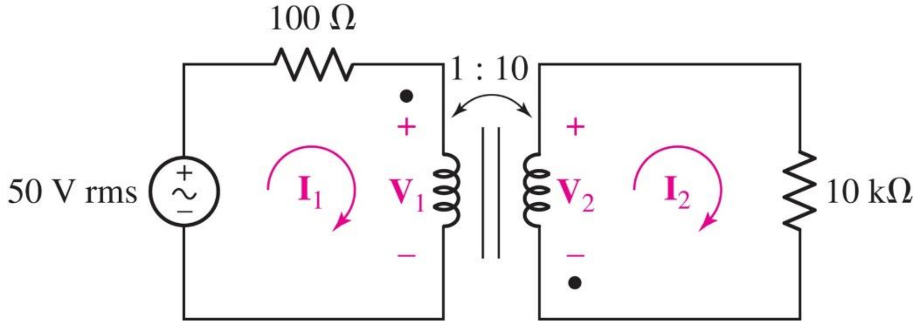
What is the efficiency of the transformer?



$$\eta = \left(\frac{P_{out}}{P_{in}}\right)100\% = \left(\frac{V_L^2 / R_L}{(V_{pri})(I_{pri})}\right)100\% = \left(\frac{15\ \text{V}^2 / 100\ \Omega}{(120\ \text{V})(0.020\ \text{A})}\right)100\% = 94\%$$

Transformer Calculations

Determine the average power dissipated in the 10 kΩ resistor.



Answer: 6.25 W