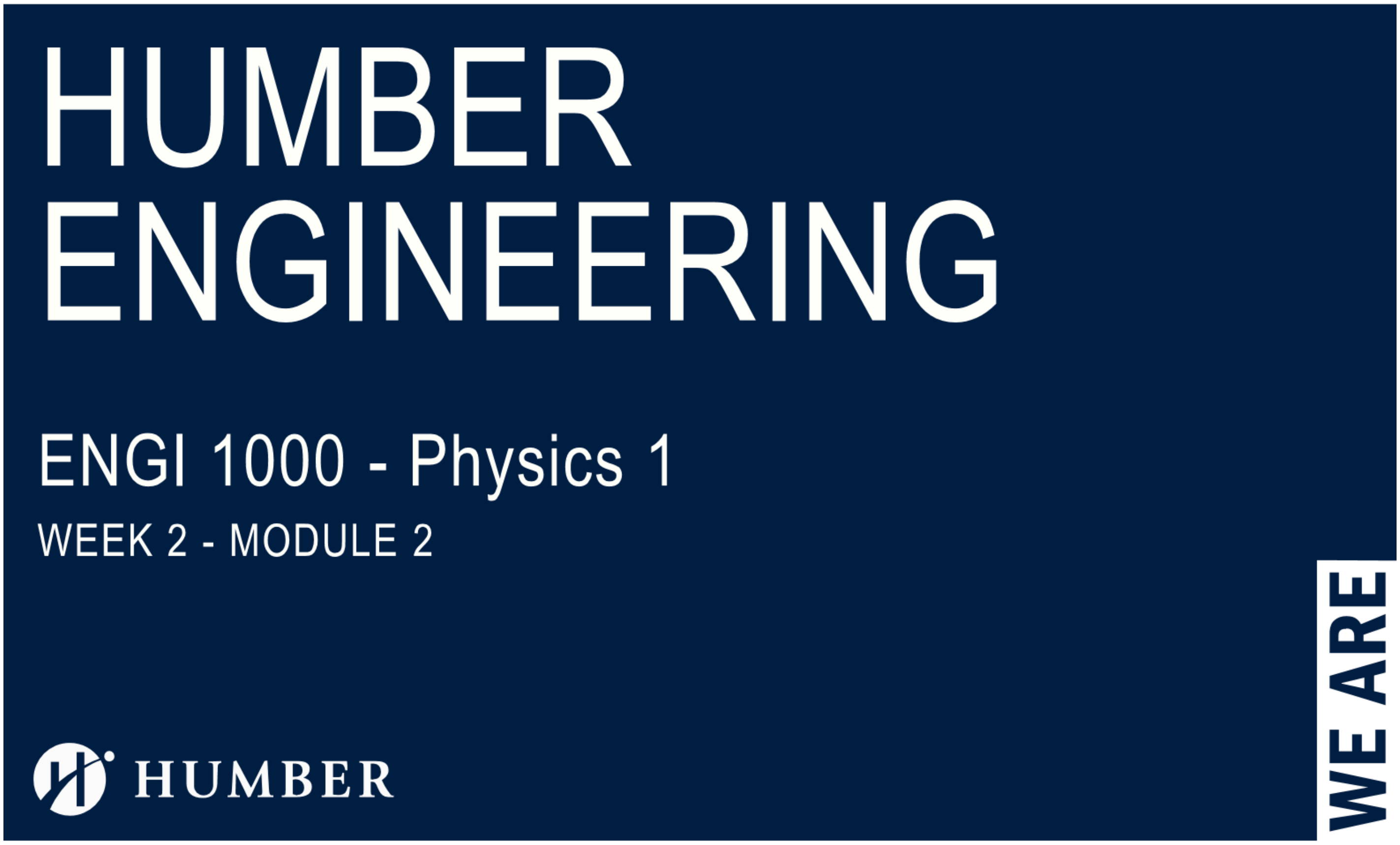
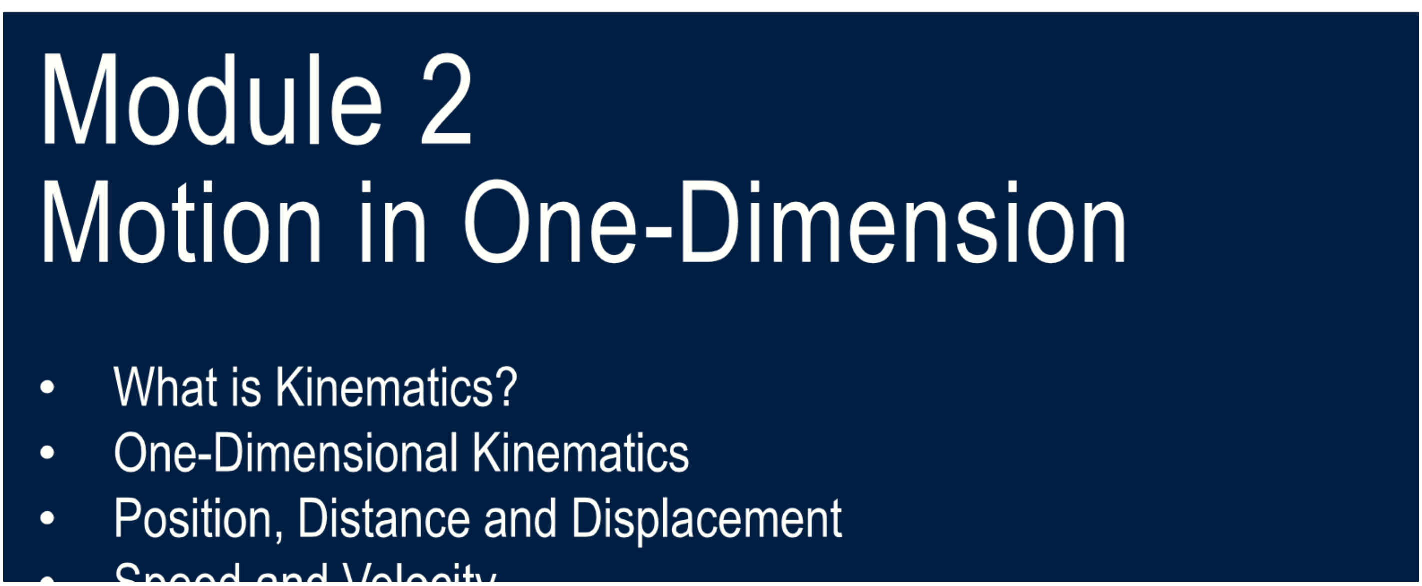
Motion in 1 Dimension

September 14, 2022 2:33 PM

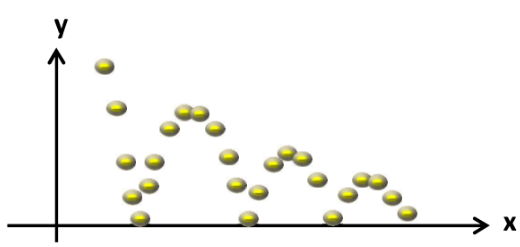


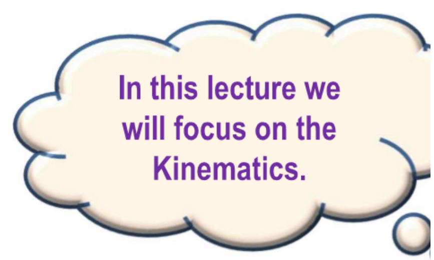


- Speed and Velocity
- Average and Instantaneous Velocity
- Average and Instantaneous Acceleration
- Kinematics Equations

What is Kinematics and Dynamics?

- What is Kinematics? What is Dynamics?
- Kinematics is the study of <u>geometry</u> of motion through properties such as position, velocity and acceleration.
  - Kinematics shows **<u>how</u>** the object moves.
- Dynamics is the study of the <u>cause</u> behind the changes of motion using the Newton's Laws of Motion.
  - Dynamics shows **<u>why</u>** the object moves in a particular way.





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Kinematics

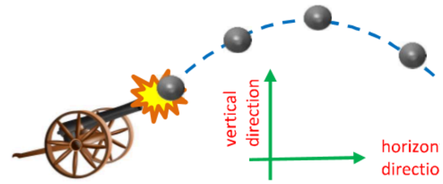
- The kinematic motion of an object can be:
  - **One-Dimensional:** The object is moving in a **<u>straight line</u>**, either forwards or backwards, up or down, left or right.

either forwards or backwards, up or down, left or right.

- Climbing a rope, Dropping a ball, Motion of a car on a straight road

- **Two-Dimensional:** The object is moving in a **plane** in both horizontal and vertical directions.
  - Throwing a rock, Kicking a ball, Cannonball's motion

- **Three-Dimensional:** The object is moving in **3D space**.
  - Birds flying, Fish swimming, Motion of aircraft, Free-fall parachuting



One-Dimensional Kinematics

- Kinematics is the branch of mechanics that mathematically describes the motion of objects witho discussing what causes the motion.
- One-Dimensional Kinematics refers to motion in a straight line.
- The motion can be in the horizontal (left or right) or vertical (up or down) direction.

Motion of a car on a straight road

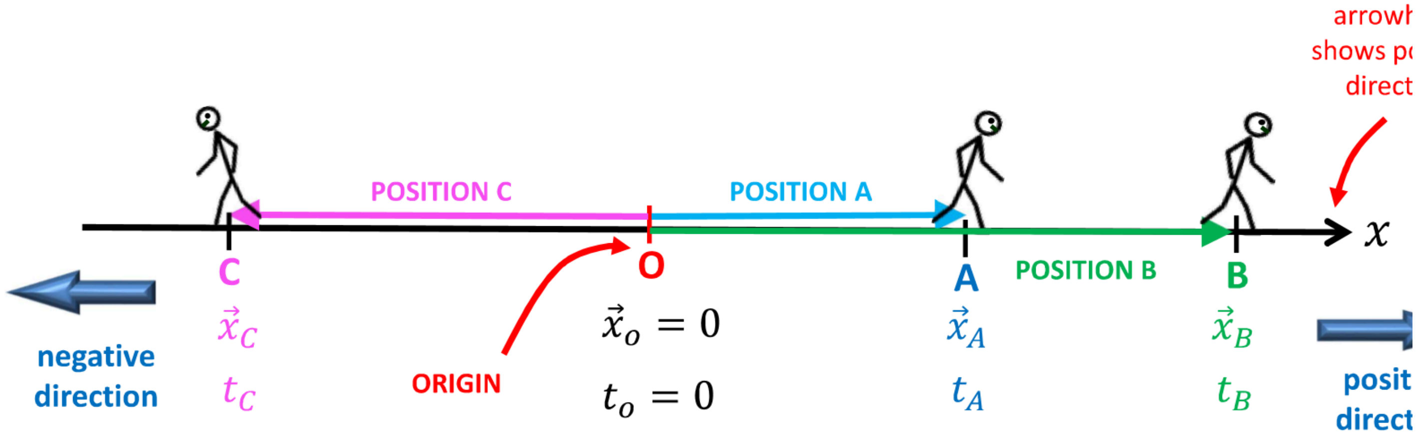
Dropping a ball

- Before describing the motion of an object, you have to determine the location or position of the object.

Position

Position

- Position (x): Location of object with respect to chosen reference point at a particular time.
- We use vector in a coordinate system to show the location of objects.



Alternative Representations

- There are alternative representations to show the information of the motion of an object.
  - Pictorial representation
  - Tabular representation
  - Graphical representation

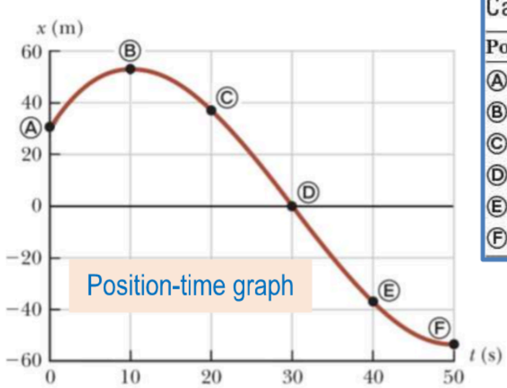
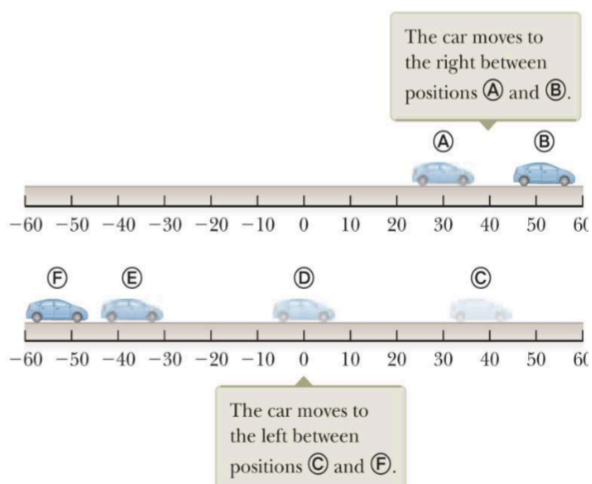


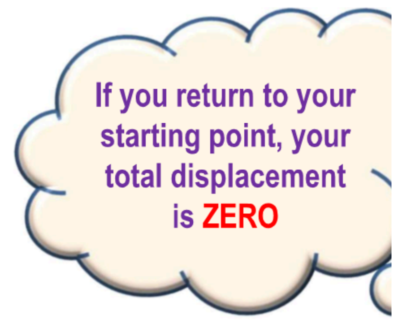
TABLE 2.1 Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



Displacement

- Displacement is the change in the position of an object.
  - It is a vector quantity and has a magnitude and a direction.
  - It is represented by a vector joining the initial and final positions.
  - It can be a positive, negative or zero number depending on direction.
  - The SI unit of displacement is the meter (m).
  - Displacement is calculated in a coordinate system.



Displacement (Change in position) = Final position – Initial position

$$\boxed{\Delta\vec{x} = \vec{x}_f - \vec{x}_i}$$

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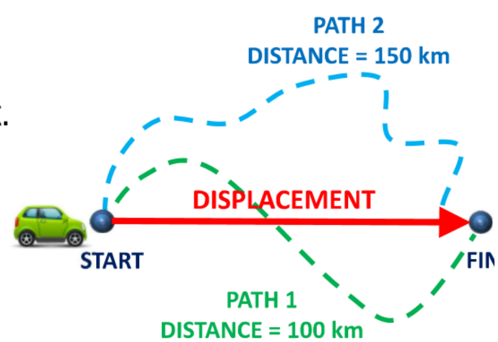
Distance



- Distance is the total length of the path traveled by an object.
  - It is a scalar quantity and has no direction.
  - It is always a positive number.
  - The SI unit of distance is the meter (m).
  - It is measured by the odometer in your car.
- The distance between two point depends on the chosen path.
- From Path 1, the total distance you have traveled is 100 km.

$$\boxed{d_1 = 100\ \mathrm{km}}$$

- From Path 2, the total distance will be 150 km.

$$d_2 = 150\ \text{km}$$

- If the path taken is straight, the distance is equal to the displacement.

Average Velocity

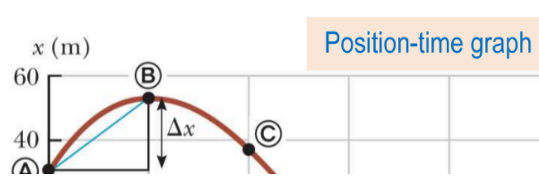
- Average Velocity is the rate at which an object changes its position.
  - It describes how fast the displacement is changing.
  - It is a vector quantity and has a magnitude and a direction.
  - It can be a positive or negative depending on direction.
  - The SI unit of velocity is m/s
  - Velocity is calculated in a coordinate system.

$$average\ velocity \equiv \frac{total\ displacement}{total\ elapsed\ time}$$

$$\vec{v}_{x,avg} \equiv \frac{\Delta \vec{x}}{\Delta t}$$

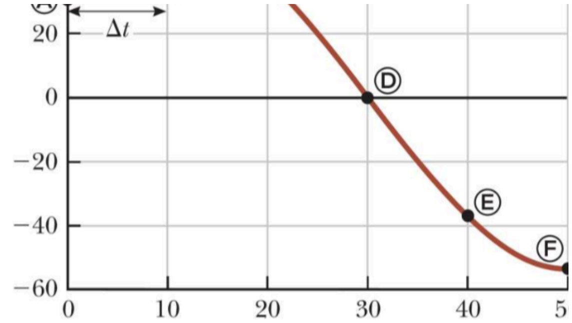
Geometric Interpretation of Average Velocity

- Average velocity can be interpreted geometrically by drawing a straight line between any two points on the position-time graph.
- The slope of this line shows the average velocity:



$$Slope \quad \Delta x$$

$$Slope = \frac{\quad}{\Delta t}$$



- For example, the line between positions A and B has a slope equal to the average velocity of the car between those two times:

$$\text{Example:} \quad \frac{x_B - x_A}{t_B - t_A} = \frac{52\ \text{m} - 30\ \text{m}}{10\ \text{s} - 0} = 2.2\ \text{m/s}$$

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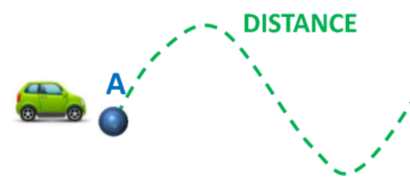
Average Speed

- Average Speed is the rate at which the distance is travelled.
  - It describes how fast an object is moving
  - It is a scalar quantity and has no direction
  - It is always a positive number
  - The SI unit of speed is m/s



$$average\ speed \equiv \frac{total\ distance\ traveled}{total\ elapsed\ time}$$

$$v_{avg} \equiv \frac{d}{\Delta t}$$

The magnitude of the average velocity is NOT the average speed.

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Quick Quiz 1

- Under which of the following conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed over some time interval?

  a) A particle moves in the $+x$ direction without reversing
  b) A particle moves in the $-x$ direction without reversing
  c) A particle moves in the $+x$ direction and then reverse the direction of its motion
  d) There are no conditions for which this is true

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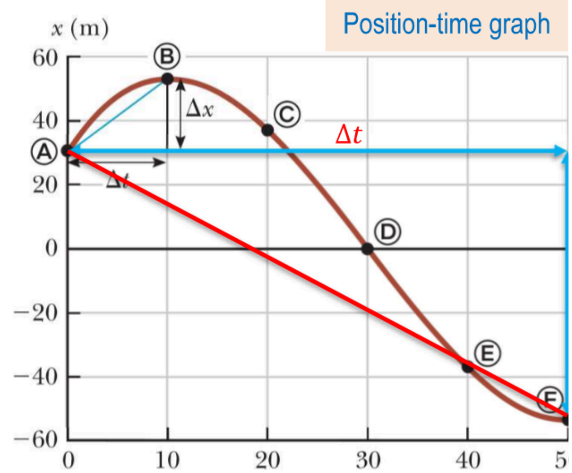
Calculating the Average Velocity and Spee

Example 1: Find the displacement, average velocity, and average speed of the car in the figure between positions A and F.

Position-time graph



Displacement:
$$\begin{aligned}\Delta x &= x_F - x_A \\ &= -53\text{ m} - 30\text{ m} \\ &= -83\text{ m}\end{aligned}$$

Average velocity:
$$\begin{aligned}v_{x,\text{avg}} &= \frac{x_F - x_A}{t_F - t_A} \\ &= \frac{-53\text{ m} - 30\text{ m}}{50\text{ s} - 0\text{ s}} \\ &= \frac{-83\text{ m}}{50\text{ s}} = \boxed{-1.7\text{ m/s}}\end{aligned}$$

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Calculating the Average Velocity and Spee

Example 1: Find the displacement, average velocity, and average speed of the car in the figure between positions A and F.

Average speed: $v_{avg} = \dfrac{127\ \text{m}}{50.0\ \text{s}} = \boxed{2.54\ \text{m/s}}$

$$\boxed{v_{avg} \equiv \frac{d}{\Delta t}}$$

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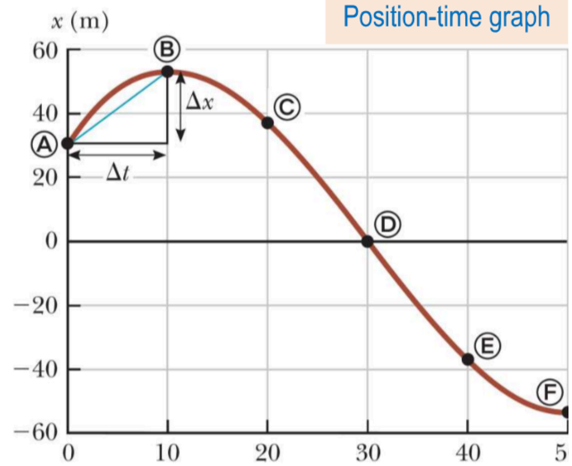
Position-time graph



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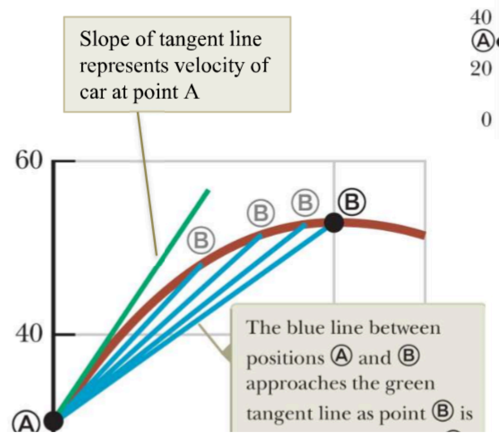
Instantaneous Velocity and Speed

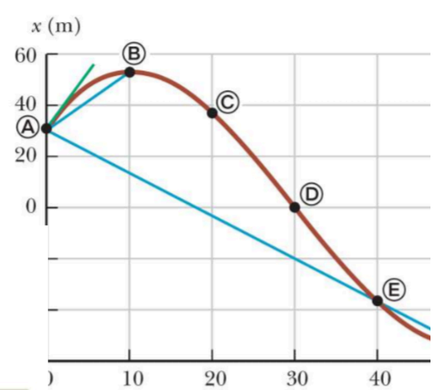
- Instantaneous velocity shows velocity of a particle at a particular instant in time.
- The instantaneous velocity is defined by the following limit:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Derivative of the position with respect to time.

- The instantaneous speed is defined as the magnitude of its instantaneous velocity.





moved closer to point Ⓐ.

Quick Quiz 2

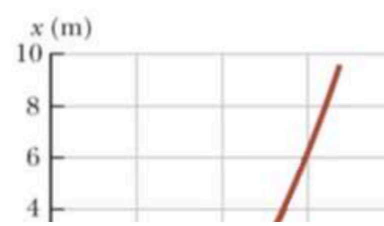
- Are members of the highway patrol more interest in ………….. as you drive.

  a) Your average speed

  b) Your instantaneous speed

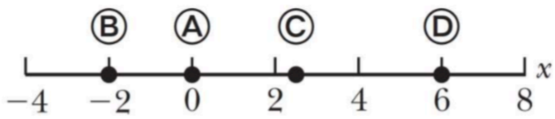
Average and Instantaneous Velocity

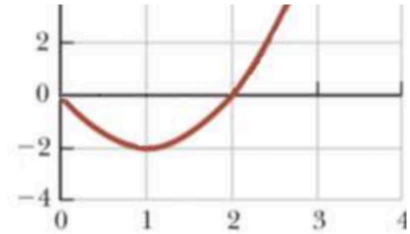
Example 2: A particle moves along the x-axis. Its position varies with time according to the express
$x = -4t + 2t^2$, where x is in meters and t is in seconds. The position–time graph for this motion
shown in the figure.



Because the position of the particle is given by a mathematical function, the motion of the particle is known at all times.

Notice that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1s$ and moves in the positive x direction at times $t > 1s$.





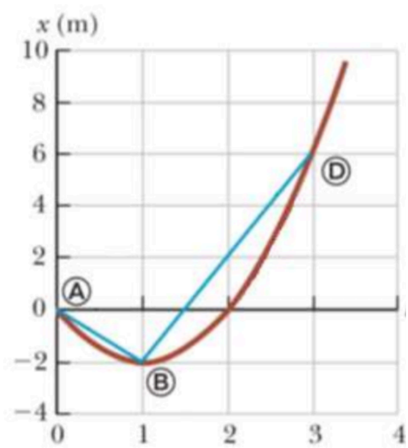
Average and Instantaneous Velocity

Example 2: A particle moves along the x-axis. Its position varies with time according to the express $x = -4t + 2t^2$, where x is in meters and t is in seconds. The position–time graph for this motion shown in the figure.

(a) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1s$ and $t = 1s$ to $t = 3s$.

$$\begin{aligned}\Delta x_{A \to B} &= x_f - x_i = x_B - x_A \\ &= \left[-4(1) + 2(1)^2\right] - \left[-4(0) + 2(0)^2\right] \\ &= \boxed{-2\text{ m}}\end{aligned}$$

$$\begin{aligned}\Delta x_{B \to D} &= x_f - x_i = x_D - x_B \\ &= \left[-4(3) + 2(3)^2\right] - \left[-4(1) + 2(1)^2\right] \\ &= \boxed{+8\text{ m}}\end{aligned}$$



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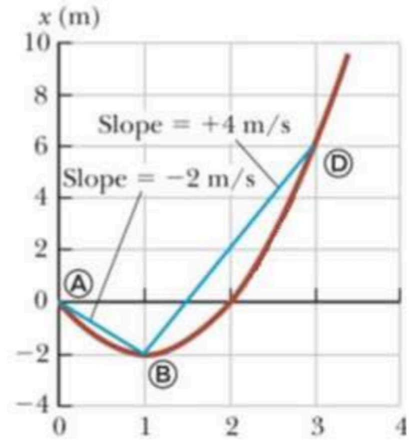
Average and Instantaneous Velocity

Example 2: A particle moves along the x-axis. Its position varies with time according to the express
$x = -4t + 2t^2$, where x is in meters and t is in seconds. The position–time graph for this motion

shown in the figure.

(b) Calculate the average velocity during these two time-intervals.



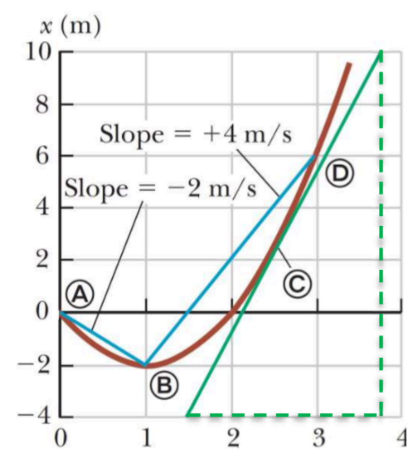
$$v_{x,\text{avg}(A\to B)} = \frac{\Delta x_{A\to B}}{\Delta t} = \frac{x_B - x_A}{t_B - t_A} = \frac{-2\text{m} - 0\text{m}}{1\text{s} - 0\text{s}} = \frac{-2\text{ m}}{1\text{ s}} = \boxed{-2\text{ m/s}}$$

$$v_{x,\text{avg}(B\to D)} = \frac{\Delta x_{B\to D}}{\Delta t} = \frac{x_D - x_B}{t_D - t_B} = \frac{6\text{m} - (-2\text{m})}{3\text{s} - 1\text{s}} = \frac{8\text{ m}}{2\text{ s}} = \boxed{+4\text{ m/s}}$$

Average and Instantaneous Velocity

Example 2: A particle moves along the x-axis. Its position varies with time according to the express $x = -4t + 2t^2$, where x is in meters and t is in seconds. The position–time graph for this motion shown in the figure.

(c) Find the instantaneous velocity of the particle at $t = 2.5s$.



First draw the tangent line at $t = 2.5\ s$, which is the green line in the graph. Then calculate the slope of the line.

$$v_x = \frac{\Delta x}{\Delta t} = \frac{10\text{ m} - (-4\text{ m})}{3.8\text{ s} - 1.5\text{ s}} = \frac{14\text{ m}}{5.3\text{ s}} = \boxed{+6\text{ m/s}}$$

Analysis Model

- **Analysis model**: represents common situation when solving physics problems
- When you encounter new problem:
  - Identify fundamental details,
  - Ignore details not important,
  - Attempt to recognize which of the situations you have already seen that might be used as model for new problem

- Imagine car moving along straight freeway at constant speed
  - Is it important that it is an automobile?
  - Is it important that it is a freeway?
- We can model the car as *particle under constant velocity*

Analysis Model: Particle Under Constant Velocity

- Model of **particle under constant velocity** can be applied in *any* situation in which entity that can be modeled as particle moving with constant velocity
- In this case, the instantaneous velocity is the same as the average velocity

$$v_{x,\text{avg}} \equiv \frac{\Delta x}{\Delta t} \quad \rightarrow \quad v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta x = x_f - x_i \quad \rightarrow \quad v_x = \frac{x_f - x_i}{\Delta t} \quad \rightarrow \quad x_f = x_i + v_x \Delta t$$

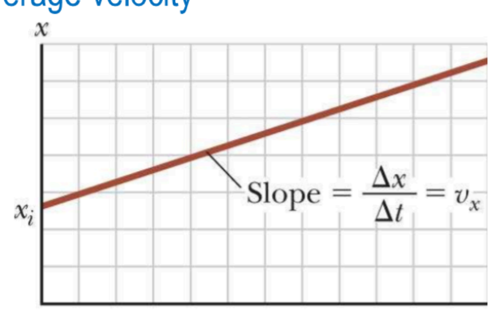
$$\boxed{x_f = x_i + v_x t}$$

Position at time t

Initial position

Constant velocity

Time



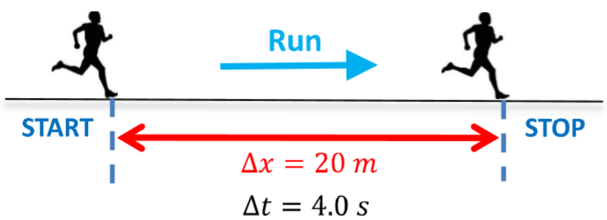
Position-time ($x - t$) graph for a particle under constant velocity.

Analysis Model: Particle Under Constant Velocity

Example 3 (Modeling a Runner as a Particle): A kinesiologist is studying the biomechanics of the human body. (*Kinesiology* is the study of the movement of the human body. Notice the connection to the word *kinematics*.)

She determines the velocity of an experimental subject while he runs along a straight line at a cons rate.

The kinesiologist starts the stopwatch at the moment the runner passes a given point and stops it a the runner has passed another point 20 m away. The time interval indicated on the stopwatch is 4.0



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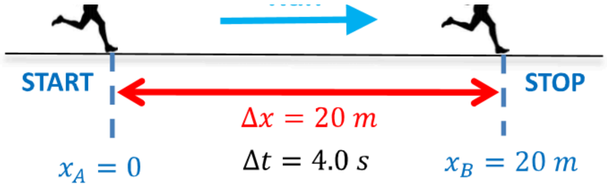
Analysis Model: Particle Under Constant Velocity

Example 3 (Modeling a Runner as a Particle): A kinesiologist is studying the biomechanics of the human body. (*Kinesiology* is the study of the movement of the human body. Notice the connection to the word *kinematics*.)

(a) What is the runner's velocity?

Because the runner runs "at a constant rate" we can model him as a *particle under constant velocity.*

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_B - x_A}{\Delta t} = \frac{20\ \text{m} - 0}{4.0\ \text{s}} = 5.0\ \text{m/s}$$

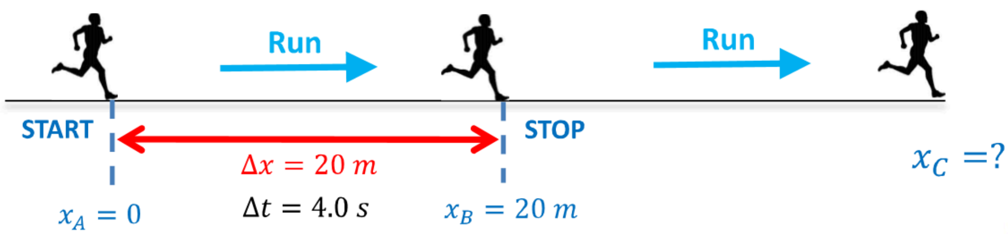


Analysis Model: Particle Under Constant Velocity

Example 3 (Modeling a Runner as a Particle): A kinesiologist is studying the biomechanics of the human body. (*Kinesiology* is the study of the movement of the human body. Notice the connection to the word *kinematics*.)

(b) If the runner continues his motion after the stopwatch is stopped, what is his position after 10 s have passed?

$$x_C = x_A + v_x t = 0 + (5.0\ \mathrm{m/s})(10\ \mathrm{s}) = \boxed{50\ \mathrm{m}}$$



Analysis Model: Particle Under Constant Velocity

- Imagine a moving object that can be modeled as a particle. If it moves at a constant speed throu displacement Δx in a straight line in a time interval Δt, its constant velocity is:

$$v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta t$$

- The position of the particle as a function of time is given by:

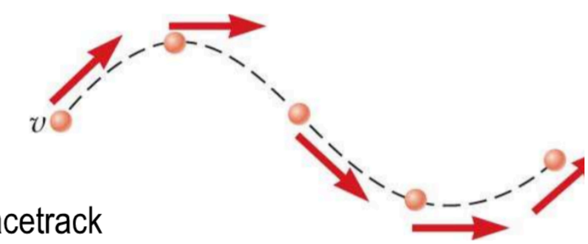
$$\boxed{x_f = x_i + v_x t}$$

Examples:
- A meteoroid traveling through gravity-free space
- A car traveling at a constant speed on a straight highway
- A runner traveling at constant speed on a perfectly straight path

Analysis Model: Particle Under Constant Speed

- Imagine a moving object that can be modeled as a particle. If it moves at a constant speed throu distance d along a straight line or a curved path in a time interval Δt, its constant speed is:

$$\boxed{v = \frac{d}{\Delta t}}$$

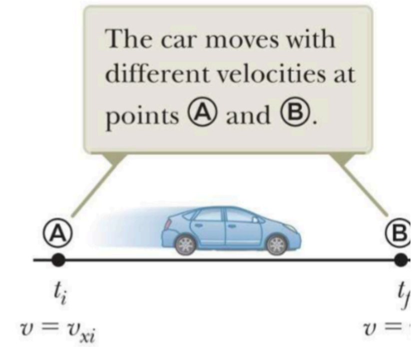


Examples:
- A planet traveling around a perfectly circular orbit
- A car traveling at a constant speed on a curved racetrack
- A runner traveling at constant speed on a curved path

Acceleration

- When the velocity of a particle changes with time, the particle is said to be accelerating.
- For example, magnitude of car's velocity increases when you step on gas and decreases when apply brakes, which result in an acceleration of the car.

  - Acceleration is a vector quantity, and it has a magnitude and a direction.
  - It can be a positive or negative depends on the direction.
  - The sign of acceleration indicates its direction.
  - Acceleration can be speeding up or slowing down.
  - The SI unit of acceleration is m/s^2

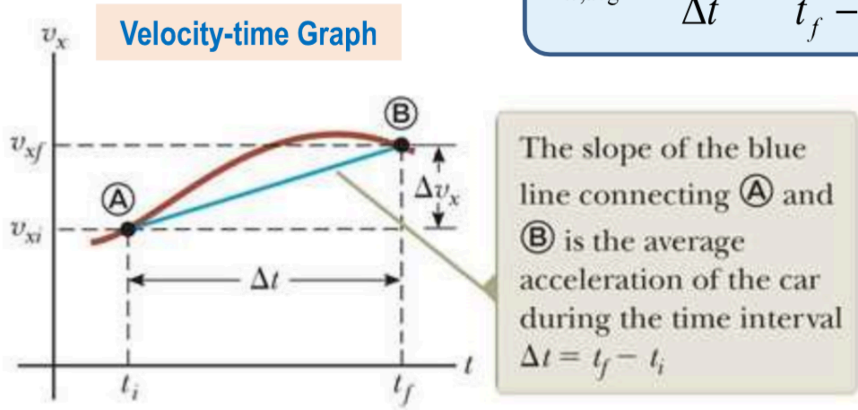


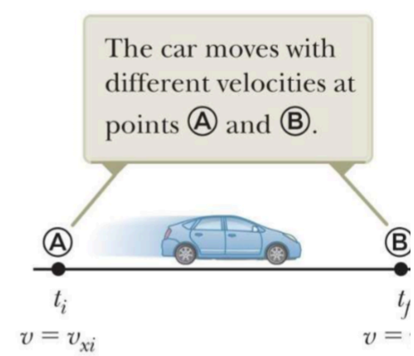
Average Acceleration

- The average acceleration $a_{x,\text{avg}}$ of the particle is defined as the change in velocity Δv_x divided the time interval Δt during which that changes occurs:

$$a_{x,\text{avg}} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

Velocity-time Graph





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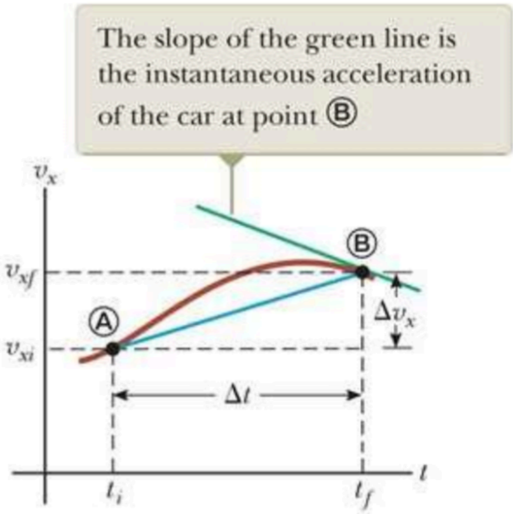
Instantaneous Acceleration

- The instantaneous acceleration a_x of the particle is defined as the limit of the average accelerat as Δt approaches zero.



$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

Derivative of the veloc with respect to time.

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

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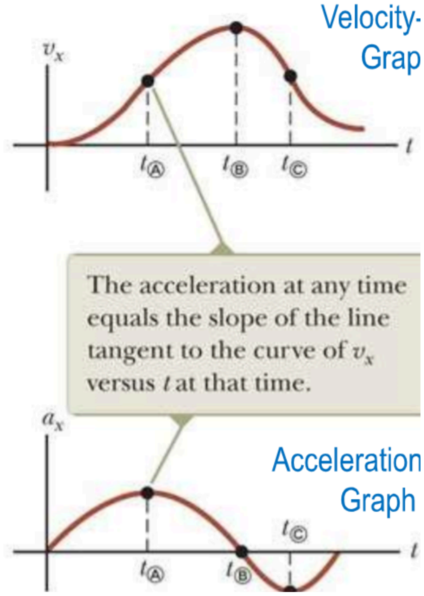
Acceleration vs. Time Graph

- Figure shows how an acceleration-time graph is related to a velocity-time graph.
  - The acceleration at any time is the slope of the velocity-time graph at that time.
  - The acceleration is positive when the velocity is increasing in the positive x direction, like point A.
  - The acceleration is negative when the velocity is decreasing in the positive x direction, like point C.
  - The acceleration is zero when the velocity is maximum.

Velocity-Grap

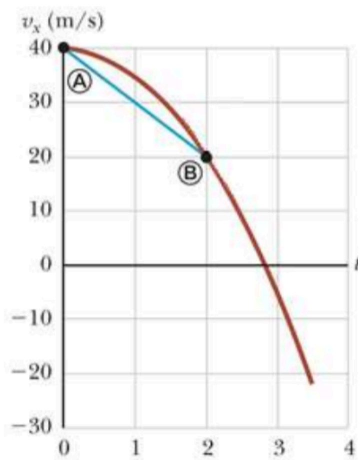


Acceleration Graph

like point B.

Average and Instantaneous Acceleration

Example 4: The velocity of a particle moving along the x-axis varies according to the expression $v_x = 40 - 5t^2$, where v_x is in m/s and t is in seconds.



(a) Find the average acceleration in the time interval $t = 0$ to $t = 2.0\ s$.

$$v_{xA} = 40 - 5t_A^{\,2} = 40 - 5(0)^2 = +40 \text{ m/s}$$

$$v_{xB} = 40 - 5t_B^{\,2} = 40 - 5(2.0)^2 = +20 \text{ m/s}$$

$$a_{x,\text{avg}} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{20 \text{ m/s} - 40 \text{ m/s}}{2.0 \text{ s} - 0 \text{ s}}$$

$$= \boxed{-10 \text{ m/s}^2}$$

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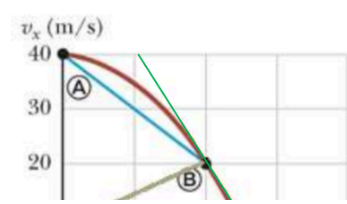
Average and Instantaneous Acceleration

Example 4: The velocity of a particle moving along the x-axis varies according to the expression $v_x = 40 - 5t^2$, where v_x is in m/s and t is in seconds.



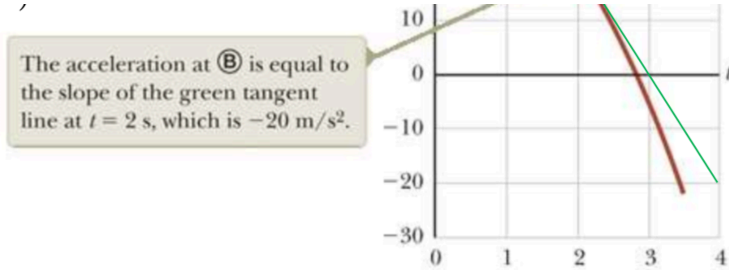
(b) Determine the acceleration at $t = 2.0\ s$.

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\Delta t - 5(\Delta t)^2$$

$$\Delta v_x = v_{xf} - v_{xi} = -10t\Delta t - 5(\Delta t)^2$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t$$

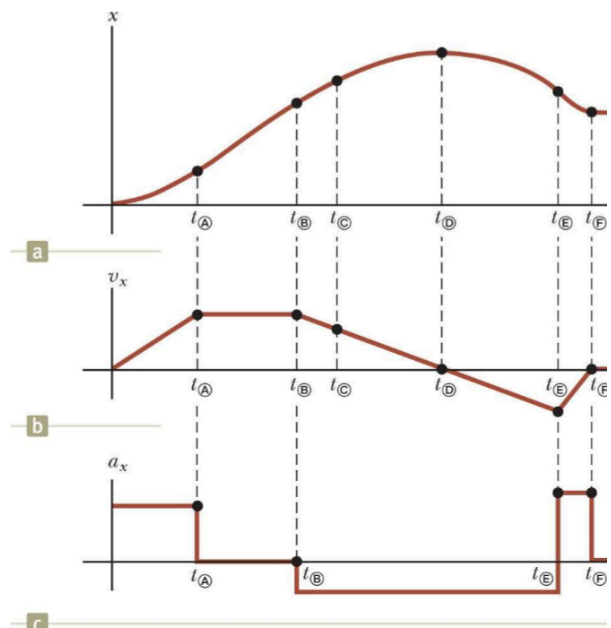
$$a_x = (-10)(2.0)\ \text{m/s}^2 = \boxed{-20\ \text{m/s}^2}$$



At $t = 2\ s$: $v_x > 0$ and $a_x < 0$ $\rightarrow$ The particle is slowing down

Graphical Relation Between x, v_x and a_x

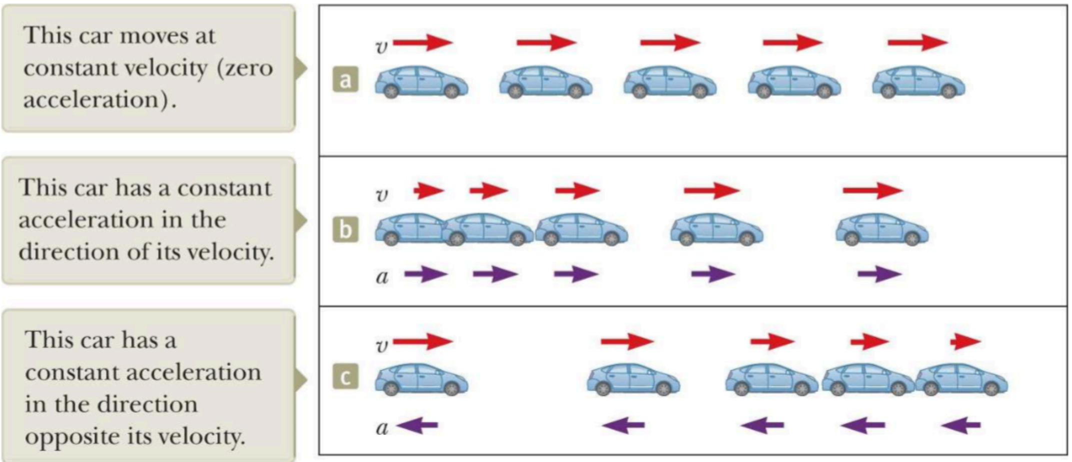
- Figure (a): Position-time $(x - t)$ graph for an object moving along the x-axis.
- Figure (b): The velocity-time $(v_x - t)$ graph for the object is obtained by measuring the slope of the $(x - t)$ graph at each instant.
- Figure (c): The acceleration-time $(a_x - t)$ graph for the object is obtained by measuring the slope of the $(v_x - t)$ graph at each instant.



Motion Diagrams

- Motion diagram is a pictorial representation of a moving object

- Motion diagram is a pictorial representation of a moving object.
- It is useful to describe the velocity and acceleration while an object is in motion



Constant velocity
No acceleration

The car speeds up

The car slows down

Force and Acceleration

- The acceleration is caused by force and the force on an object is proportional to the acceleration the object

$$\boxed{F_x \propto a_x}$$

- Force and acceleration are a vector quantities and both are in the same direction.
- If the force and the velocity are in the same direction, then the object speeds up!
- If the force and the velocity are in the opposite directions, then the object slows down!

Quick Quiz 3

- If a car is traveling eastward and slowing down, what is the direction of the force on the car that causes it to slow down?

  a) Eastward
  b) Westward
  c) Neither eastward nor westward

Quick Quiz 4

- Which of the following statements is true?

  a) If a car is traveling eastward, its acceleration must be eastward
  b) If a car is slowing down, its acceleration must be negative
  c) A particle with constant acceleration can never stop and stay stopped

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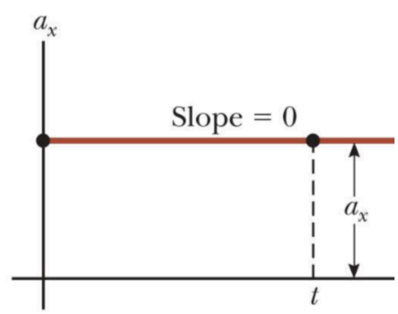
Analysis Model: Particle Under Constant Acceleratic

- In the constant acceleration motion, the average acceleration $a_{x,\text{avg}}$ over any time-interval is numerically equal to the instantaneous acceleration a_x at any instant within the interval.

$$a_{x,\text{avg}} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \xrightarrow[t_i = 0\,,\, t_f = t]{a_{x,\text{avg}} = a_x} a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

$$\boxed{v_{xf} = v_{xi} + a_x t}$$

Velocity at time t — Initial velocity — Constant acceleration — Time

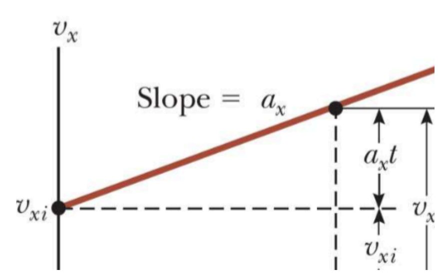


Acceleration-time ($a_x - t$) graph fo particle under constant acceleratic

Analysis Model: Particle Under Constant Acceleratic

- The velocity-time graph is a straight line, and the slope is constant that equals to a_x.

$$\boxed{v_{xf} = v_{xi} + a_x t}$$



- Since velocity varies linearly at time, the average velocity can be expressed as the mean of the initial and final velocity.

$$v_{x,\text{avg}} = \frac{v_{xi} + v_{xf}}{2} \quad (\text{for constant } a_x)$$

Velocity-time ($v_x - t$) graph for a
particle under constant acceleratio

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Analysis Model: Particle Under Constant Acceleratic

- The position of object can be derived as a function of time as below:

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_f - x_i}{t - 0} = \frac{x_f - x_i}{t}$$

$$v_{x,avg} = \frac{v_{xi} + v_{xf}}{2}$$

$$x_f - x_i = v_{x,\text{avg}} t = \frac{1}{2}\left(v_{xi} + v_{xf}\right) t$$

- This equation provides the final position of the particle at time t in terms of the initial position, initial and final velocities.

$$x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xf}\right) t \quad (\text{for constant } a_x)$$

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Analysis Model: Particle Under Constant Acceleratic

- Another useful expression for the position of a particle under constant acceleration is obtained:

$$v_{xf} = v_{xi} + a_x t$$

$$x_f = x_i + \frac{1}{2}(v_{xi} - v_{xf})t \qquad x_f = x_i + \frac{1}{2}\left[v_{xi} + (v_{xi} + a_x t)\right]t$$

- This equation provides the final position of the particle at time t in terms of the initial position, initial velocity and the constant acceleration.

$$\boxed{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \quad \text{(for constant } a_x\text{)}}$$

Position at time t — Initial position — Initial velocity — Time — Constant acceleration — Time

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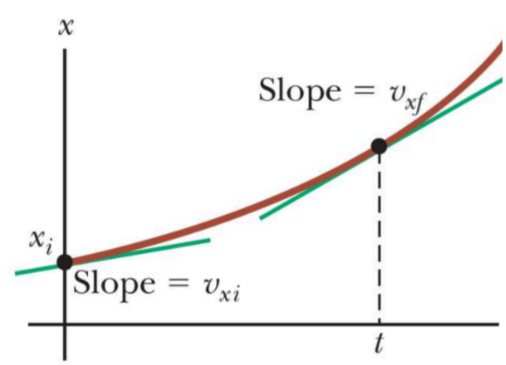
Analysis Model: Particle Under Constant Acceleratio

- The position-time graph is a parabola, the slope of the tangent line at any time t equals the velocity at that time v_{xf}.

$$\boxed{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$$

- This slope of tangent line to this curve at $t = 0$ equals the initial velocity v_{xi}.



Position-time ($x - t$) graph for a particle under constant acceleratio

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Analysis Model: Particle Under Constant Acceleratio

Analysis Model: Particle Under Constant Acceleratio

- We can also obtain an expression for the final velocity that <u>does not contain time</u> as a variable.

$$v_{xf} = v_{xi} + a_x t \quad \rightarrow \quad t = \frac{v_{xf} - v_{xi}}{a_x}$$

$$x_f = x_i + \frac{1}{2}(v_{xi} - v_{xf})t$$

$$x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xf}\right)\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = x_i + \frac{v_{xf}^2 -}{2a}$$

- This equation provides the final velocity in terms of the initial velocity, the constant acceleration, and the position of the particle.

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right) \quad (\text{for constant } a_x)}$$

Kinematic Equations (Constant Acceleration)

- Summary of the One-dimensional Kinematic Equations for a particle under constant acceleratior

$$v_{xf} = v_{xi} + a_x t \qquad x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2$$

$$v_{x,avg} = \frac{v_{xi} + v_{xf}}{2} \qquad x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Analysis Model: Particle Under Constant Acceleratic

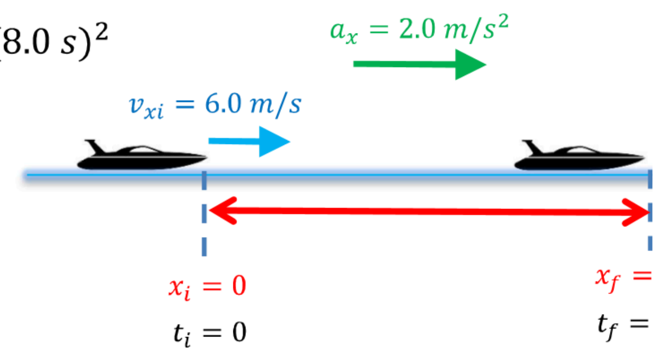
Example 5: A speedboat has a constant acceleration of 2.0 m/s². If the initial velocity of the boat is 6.0 m/s, find displacement after 8.0 seconds.

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$x_f = 0 + (6.0\ m/s)(8.0\ s) + \frac{1}{2}(2.0\ m/s^2)(8.0\ s)^2$$

$$x_f = 0 + 48\ m + 64\ m = \boxed{112\ m}$$

Displacement



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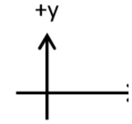
Freely Falling Objects

- Free fall is the motion of an object falling freely under the influence of gravity.
  - In free fall the only force on the object is gravity and air resistance or any form of friction is ignored.
  - Gravity accelerates the object toward the earth.
  - The acceleration due to gravity near the surface of the Earth has a magnitude of $g = 9.8\ \frac{m}{s}$ and the direction of gravity acceleration is downward.
  - Free fall is a motion with constant acceleration of $g = 9.8\ \frac{m}{s^2}$ downward.

Free fall downward

Throw upward and fall back downward

Freely Falling Objects



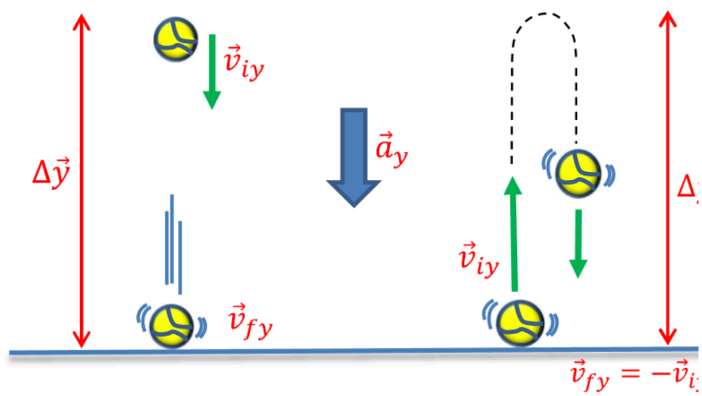
- The constant acceleration equations for free-fall motion is given as below
- Considering the upward as the $+y$ direction, the acceleration is $a_y = -g = -9.8\frac{m}{s^2}$

$$\vec{v}_{yf} = \vec{v}_{yi} + \vec{a}_y t$$

$$\vec{y}_f = \vec{y}_i + \tfrac{1}{2}(\vec{v}_{yf} + \vec{v}_{yi})t$$

$$\vec{v}^2_{yf} = \vec{v}^2_{yi} + 2\vec{a}_y(\vec{y}_f - \vec{y}_i)$$

$$\vec{y}_f = \vec{y}_i + \vec{v}_{yi}t + \tfrac{1}{2}\vec{a}_y t^2$$



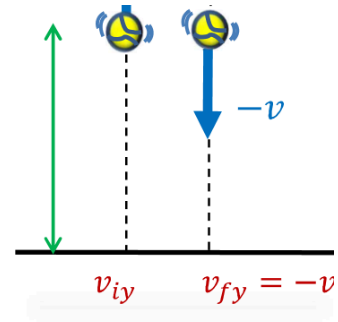
$$\Delta\vec{y} = \vec{y}_f - \vec{y}_i$$

Freely Falling Objects

- **Symmetry in free fall:** When something is thrown straight upward under the influence of gravity, and then returns to the thrower, this is very symmetric.
- The object spends half its time travelling up; half traveling down.

- Velocity when it returns to the ground is the opposite of the velocity it was thrown upward with.

- Acceleration is $a_y = -g = -9.8\frac{m}{s^2}$ (directed down) the entire time the object is in the air.



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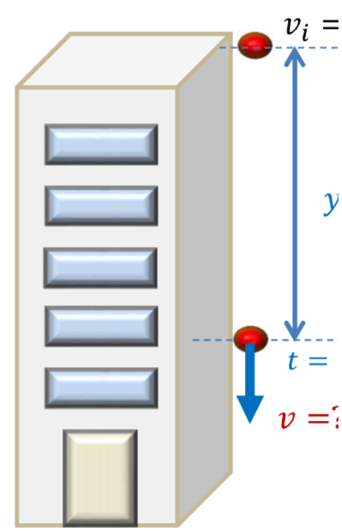
Freely Falling Objects

Example 6: A ball is dropped from rest from top of a tall building. After 3.00 s of free-fall, what is the displacement and velocity of the ball?



Displacement: $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$y_f = 0 + (0\ m/s)(3.00\ s) + \tfrac{1}{2}(-9.8\ m/s^2)(3.00\ s)^2$$

$$y_f = \boxed{-44.1\ m}$$

Velocity: $v_{yf} = v_{yi} + a_y t$

$$v_{yf} = (0\ m/s) + (-9.8\ m/s^2)(3.00\ s)$$

$$v_{yf} = \boxed{-29.4\ m/s}$$

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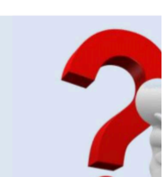
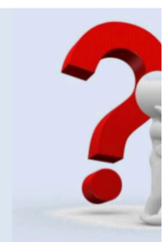
Quick Quiz 5



- What happens to the acceleration of a ball after it is thrown upward into the air? (Neglect air resistance.)

  (a) increases
  (b) decreases
  (c) increases and then decreases
  (d) decreases and then increases
  (e) remains the same

Quick Quiz 6



- What happens to the speed of a ball after it is thrown upward into the air? (Neglect air resistance.)

  (a) increases
  (b) decreases
  (c) increases and then decreases
  (d) decreases and then increases
  (e) remains the same

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Kinematic Equations Derived from Calcul

- The One-dimensional Kinematic Equations for a particle under constant acceleration can also be derived from calculus:
- The defining equation for acceleration may be written in terms of an integral as:

$$a_x = \frac{dv_x}{dt} \quad \rightarrow \quad dv_x = a_x dt \quad \rightarrow \quad v_{xf} - v_{xi} = \int_0^t a_x dt$$

- In the constant acceleration motion, a_x can be removed from the integral to give the following equation:

$$v_{xf} - v_{xi} = a_x \int_0^t dt = a_x(t-0) = a_x t \longrightarrow \boxed{v_{xf} = v_{xi} + a_x t}$$

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Kinematic Equations Derived from Calcul

- The One-dimensional Kinematic Equations for a particle under constant acceleration can also be derived from calculus:

- The defining equation for velocity may be written in terms of an integral as:

$$v_x = \frac{dx}{dt} \quad \rightarrow \quad dx = v_x dt \quad \rightarrow \quad x_f - x_i = \int_0^t v_x dt$$

- Because $v_x = v_{xf} = v_{xi} + a_x t$, this expression becomes:

$$x_f - x_i = \int_0^t (v_{xi} + a_x t)dt = \int_0^t v_{xi} dt + a_x \int_0^t t dt = v_{xi}(t-0) + a_x(\frac{t^2}{2} - 0)$$

$$\boxed{x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2}$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

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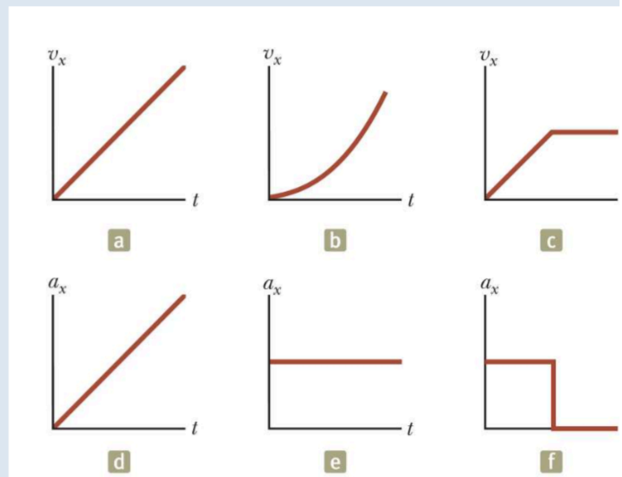
Quick Quiz 7

- Match each velocity-time graph with the acceleration-time graph on the bottom that best describes the motion.



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THANK YOU

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