Part 1. Find the Jacobian matrix for the 2 joint robot shown in Figure 1 (the first 2 joints of a Scara robot). Show your calculation and submit a scan of it.

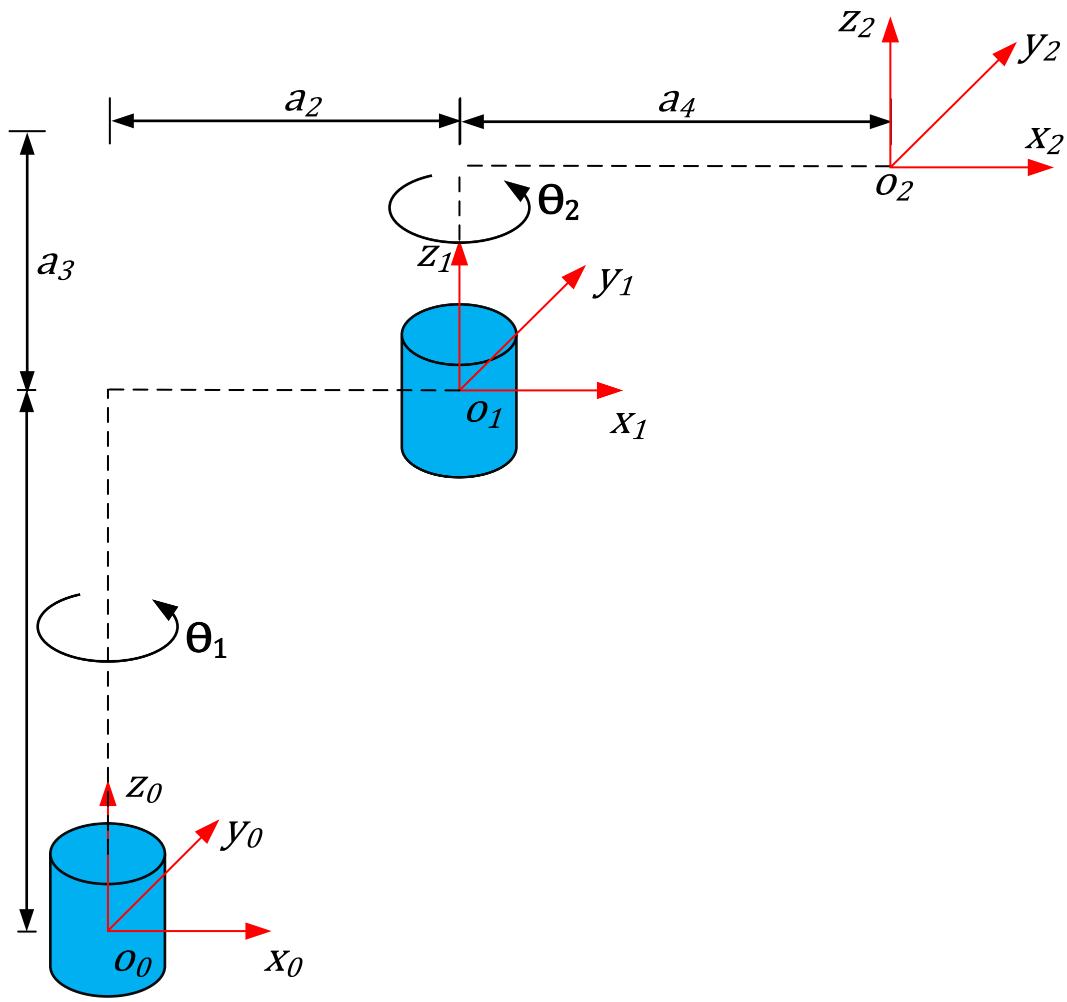

Figure1. A kinematic diagram for Part 1.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} R^0_{1-1}\begin{bmatrix}0\\0\\1\end{bmatrix} & R^0_{2-1}\begin{bmatrix}0\\0\\1\end{bmatrix} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

θ = Revolute

α = rotation

d = Prismatic

r = distance

$$R^0_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R^0_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R^0_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x = a_2 \cos(\theta_1) + a_3 \cos(\theta_1 + \theta_2)$$

$$y = a_2 \sin(\theta_1) + a_3 \sin(\theta_1 + \theta_2)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} -a_2 \sin(\theta_1) - a_3 \sin(\theta_1 + \theta_2) & -a_3 \sin(\theta_1 + \theta_2) \\ a_2 \cos(\theta_1) + a_3 \cos(\theta_1 + \theta_2) & a_3 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

Part 2: In class we found θ_1, θ_2, and d_3, for the following robot based on x_3^0, y_3^0, and z_3^0 end-effector position (x_3^0, y_3^0, and z_3^0) using geometric approach in Elbow-up configuration. Do the same thing to find θ_1, θ_2, and d_3, based on (x_3^0, y_3^0, and z_3^0) in Elbow-down configuration. Show your calculation and submit a scan of it.

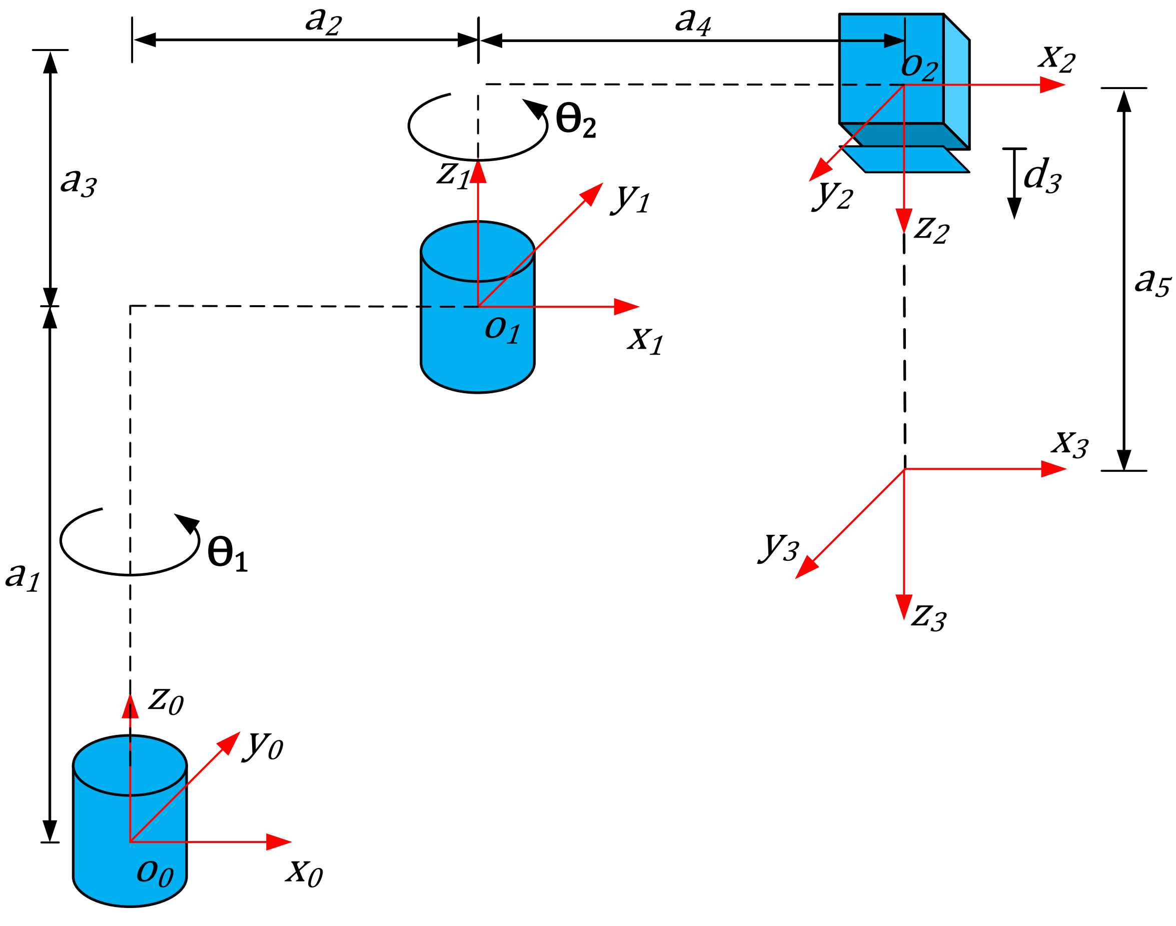

Figure 2. A kinematic diagram for a Scara robot.

Find θ_1

$$K_1 = a_1 + a_2 \cos(\theta_2)$$
$$K_2 = a_2 \sin(\theta_2)$$

Then $\theta_1 = \operatorname{atan2}(y, x) - \operatorname{atan2}(K_2, K_1)$

Find θ_2

$$\theta_2 = \operatorname{atan2}\left(-\sqrt{1 - \cos^2(\theta_2)}, \cos(\theta_2)\right)$$

Find d_3

$$d_3 = d_3 + a_5$$

$$r = \sqrt{x^2 + y^2}$$

Find θ_2

$$\cos(\theta_2) = \frac{r^2 - a_2^2 - a_3^2}{2 a_2 a_3}$$

For Elbow Down

$$\theta = \cos^{-1}\left(\frac{r^2 - a_2^2 - a_3^2}{2 a_2 a_3}\right)$$

Find d_3

$$d_3 = d_3 + a_5$$

Find θ_1

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$
$$\psi = \tan^{-1}\left(\frac{a_3 \sin(\theta_2)}{a_2 + a_3 \cos(\theta_2)}\right)$$
$$\theta_1 = \phi + \psi$$