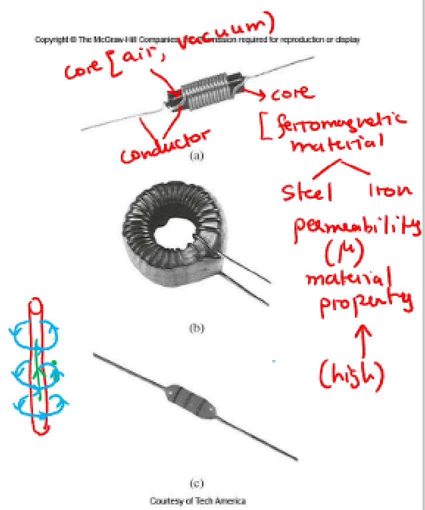
Lecture 5

November 9, 2023 8:32 PM

First Order RL Circuits

Inductors

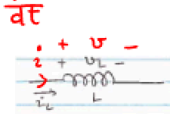
- An inductor is a passive element that stores energy in its magnetic field
- They have applications in power supplies, transformers, radios, TVs, radars, and electric motors
- Any conductor has inductance, but the effect is typically enhanced by coiling the wire up



Inductors

- If a current is passed through an inductor, the voltage across it is directly proportional to the time rate of change in current $v \propto \frac{di}{dt}$

$$v = L\frac{di}{dt}$$



$$H = \frac{v}{di/dt} = \frac{V}{A/sec}$$

- Where, L, is the unit of inductance, measured in

Henries, H

$1H = \frac{1V}{A/sec} = \frac{1V\text{-}sec}{A}$

- One Henry is 1 volt-second per ampere
- The voltage developed tends to oppose a changing flow of current.

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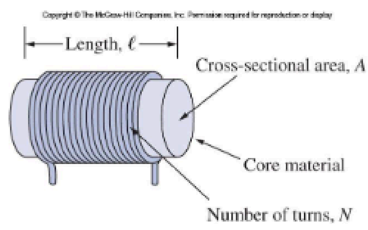
Inductors

- Calculating the inductance depends on the geometry:
- For example, for a solenoid the inductance is: permeability

$$L = \frac{N^2 \mu A}{l}$$

$\mu \to$ permeability, $A \to m^2$, $l \to m$, $L \to H$

- Where N is the number of turns of the wire around the core of cross sectional area A and length l
- The material used for the core has a magnetic property called the permeability, μ



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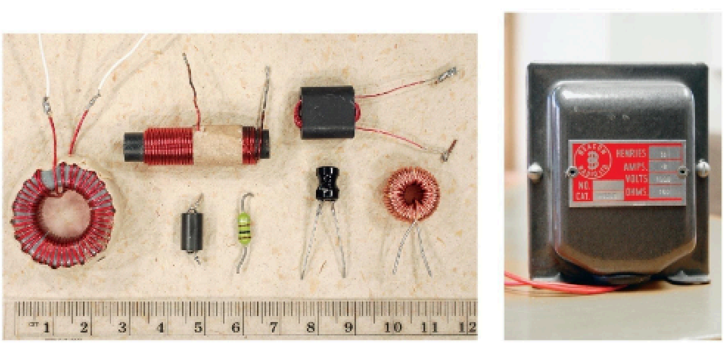
Example Inductors

Inductors can be bulky, with typical values ranging from μH to H



(a) (b)

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Current in an Inductor

$v = L\frac{di}{dt}$

i + v − L

- The current voltage relationship for an inductor is:

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

$\int_{t_0}^{t} \frac{1}{L} v\,dt = \int_{i(t_0)}^{i(t)} di$

$\frac{1}{L}\int_{t_0}^{t} v\,dt = i(t) - i(t_0)$

- The power delivered to the inductor is: $\frac{1}{L}\int v\,dt + i(t_0) = i(t)$

$$p = vi = \left(L\frac{di}{dt}\right)i = Li\frac{di}{dt}$$

$i(t)^{t_0}$

- The energy stored is: $W = \int_0^t p\,dt = \int_0^{i(t)}\left(Li\frac{di}{\cancel{dt}}\right)\cancel{dt} = L\frac{i^2}{2}\Big|_0^{i(t)} = \frac{1}{2}Li^2$

$$w = \frac{1}{2}Li^2$$

$w \to$ J, $L \to$ H, $i \to$ A

$W = \frac{1}{2}Li^2$

Properties of Inductors

2A + v − L

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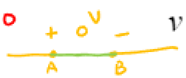
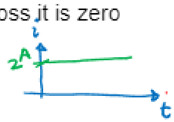
- If the current through an inductor is constant, the voltage across it is zero
- (2) Thus an inductor acts like a short for DC

$v = L\frac{d}{dt}(2) = L(0) = 0$

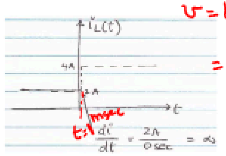
Fully charged → short



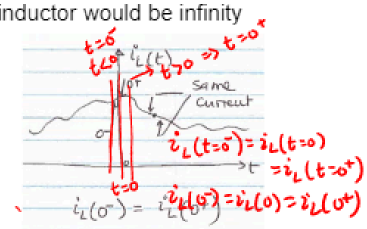
$$v = L\frac{di}{dt}$$



- (3) The current through an inductor cannot change instantaneously
- If this did happen, the voltage across the inductor would be infinity



$v = L\frac{(4-2)}{(1-1)\text{msec}} = L\frac{2}{0} \Rightarrow \infty$



- This is an important consideration if an inductor is to be turned off abruptly; it will produce a high voltage
- (4) An ideal inductor does not dissipate energy, the stored energy can be returned to the circuit later

Series Inductors

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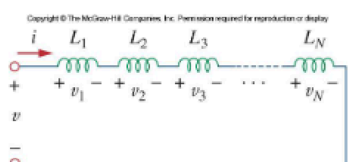
- Series connected inductors
- Applying KVL to the loop:

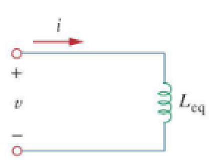
$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt}$$

$$= \left(\sum_{k=1}^{N} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$



(a)



(b)

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

Series connected inductors have the same behavior as series connected resistors

Parallel Inductors

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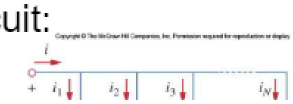
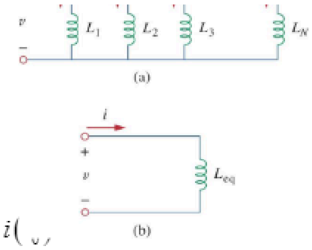
- Applying KCL to the circuit:

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$



- When the current voltage relationship is considered, we have:



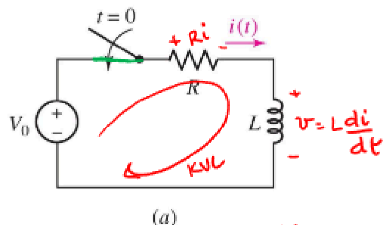
$$i=\left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt+\sum_{k=1}^{N} i_k(t_0)=\frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt+i(t_0)$$

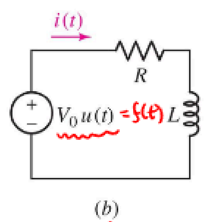
$$\frac{1}{L_{eq}}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}+\cdots+\frac{1}{L_N}$$

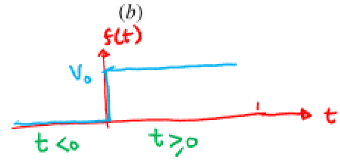
$$L_{eq}=\frac{1}{\frac{1}{L_1}+\frac{1}{L_2}+\cdots+\frac{1}{L_N}}$$

Parallel combination of Inductors resembles to the parallel combination of the resistors

First order RL Circuit







$t\geq 0 \quad V_0-Ri-L\frac{di}{dt}=0$

$$-V_0+Ri+L\frac{di}{dt}=0$$

$$i(t)=\frac{V_0}{R}+\left[i(t_0)-\frac{V_0}{R}\right]e^{-\frac{t}{L/R}}$$

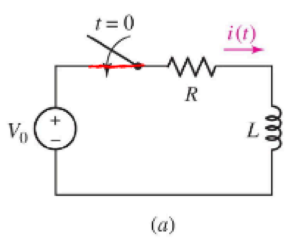
V_0 → source voltage; R → resistance in the circuit; t → time; L → inductance in the circuit

$f(t)=0 \qquad f(t)=V_0$

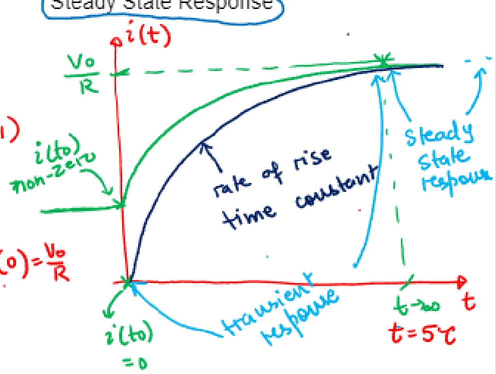
$$\cancel{i(t)=i(\infty)+[i(0^+)-i(\infty)]e^{-\frac{t}{\tau}}}$$

$i(t_0)\rightarrow i(t)$ at time $t=t_0$

First order RL Circuit



$$i(t)=\underbrace{\frac{V_0}{R}}_{\text{Steady State Response}}+\underbrace{\left[i(t_0)-\frac{V_0}{R}\right]e^{-\frac{t}{L/R}}}_{\text{Transient Response}}$$



$t=0$

$i(t=0)=\frac{V_0}{R}+\left[i(t_0)-\frac{V_0}{R}\right](1)$

$i(t=0)=i(t_0)$

$t\to\infty$

$i(t\to\infty)=\frac{V_0}{R}+\left[i(t_0)-\frac{V_0}{R}\right](0)=\frac{V_0}{R}$

Time Constant

Tau, $\tau=\frac{L}{R}$ (L → Henry, R → Ω, τ → sec)

First order RL Circuit

$t\geq 0$ $t=0^+$

$i(t=0)=i(t_0)$ zero

non-zero

$i(t=5\tau) = i(t \to \infty)$

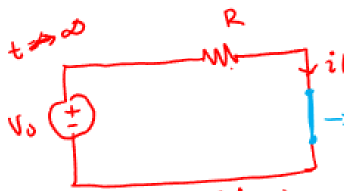
$= \frac{V_o}{R}$

$$i(t) = \frac{V_o}{R} + \left[i(t_o) - \frac{V_o}{R}\right]e^{-\frac{t}{\tau}}$$

$-(t - \cancel{t_0})$ 0 sec

$-\frac{t}{\tau}$

@ $t \to \infty$



$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-\frac{t}{\tau}} \quad t \geq 0$$

Fully Charged Inductor – Short Circuit

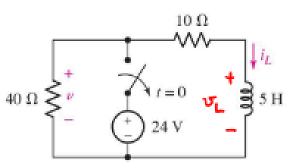
First order RL Circuit

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-\frac{t}{\tau}}$$

Response of First-order R-L Circuit : Algorithm (6)

1. Draw the circuit with initial position of the switch
2. Find the current through the inductor, $i_L(0^-)$
3. $i_L(0^+) = i_L(0^-)$
4. Draw the circuit with final position of the switch
5. Find $i_L(\infty)$ — circuit $t \geq 0$
6. Calculate $\tau = \frac{L}{R_{eq}}$, where R_{eq} is the equivalent resistance at inductor terminals with all independent sources turned OFF — voltage sources OFF → short; current sources OFF → open
7. $i_L(t) = i_L(\infty) + \left\{\left[i_L(0^+) - i_L(\infty)\right]e^{-\frac{t}{\tau}}\right\}$

Exercise - First order RL Circuit

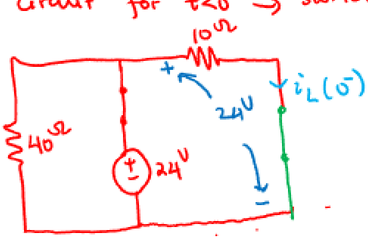


The switch stays in the closed position for a long time before it is opened at t = 0.

1. Determine an expression for $i_L(t)$ for t>0
2. Show that the voltage $v(t)$ is -12.99 V at t = 200 ms.
3. Determine an expression for $v_L(t)$ for t>0

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-\frac{t}{\tau}}$$

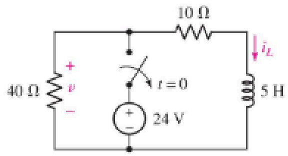
① circuit for $t<0$ → switch is closed



② $i_L(0^-) = \frac{24V}{10\Omega} = 2.4A$

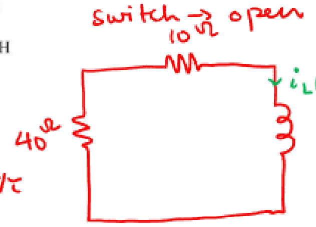
③ $i_L(0^+) = i_L(0^-) = 2.4A$

Exercise - First order RL Circuit



④ circuit for $t \geq 0$

switch → open



⑦

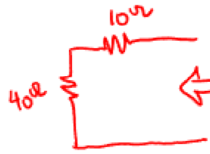
$i_L(t) = 0 + [2.4 - 0]e^{-t/\tau}$

$i_L(t) = 2.4\, e^{-\frac{t}{0.1}}$ A for $t \geq 0$

⑤ $t \to \infty \Rightarrow$ inductor is fully discharged

$i_L(\infty) = 0$

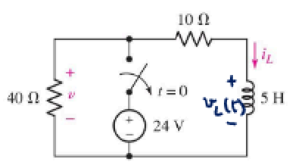
⑥

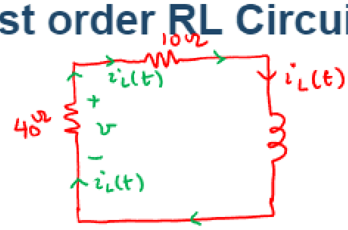


⇐ $R_{eq} = 50\Omega$

$\tau = \frac{L}{R_{eq}} = \frac{5}{50} = 0.1$ sec

Exercise - First order RL Circuit





③

$v_L(t) = L\frac{di_L}{dt}$

$= 5\frac{d}{dt}\left[2.4e^{-t/0.1}\right]$

$= 5 \times 2.4 \times \left(-\frac{1}{0.1}\right)e^{-t/0.1}$

$v_L(t) = -120e^{-\frac{t}{0.1}}$ V for $t \geq 0$

$i_L(t) = 2.4e^{-t/0.1}$

②

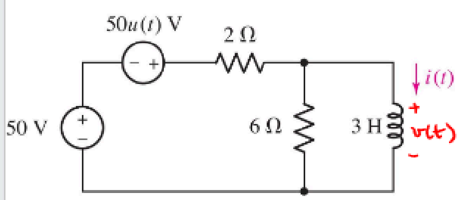
$v(t) = -40 \times i_L(t)$

$= -40\left[2.4e^{-t/0.1}\right]$

$v(t) = -96e^{-t/0.1}$ Volts

$v(t = 200^{msec}) = -96e^{-\frac{0.2}{0.1}} = -12.99V$

Exercise - First order RL Circuit

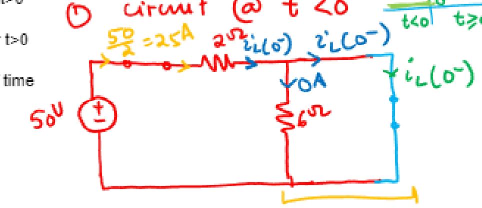


$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-\frac{t}{\tau}}$

$50u(t)$ — $0 \to t < 0$; $50 \to t \geq 0$

1. Determine an expression for *i(t)* for t>0
2. Determine an expression for *v(t)* for t>0
3. Draw a graph of *i(t)* as a function of time

① circuit @ $t < 0$



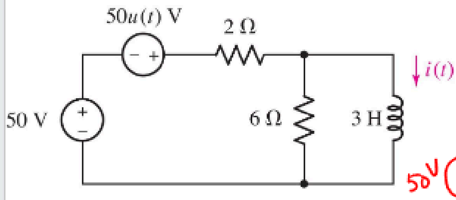
$= \frac{6 \times 0}{6+0} = 0$

③ $i(0^+) = i(0^-)$

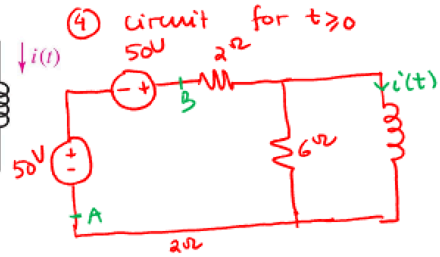
$= 25$A

② $i_L(0^-) = 25$A

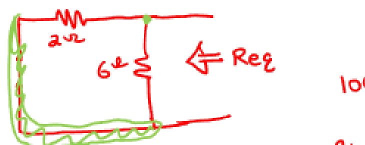
Exercise - First order RL Circuit

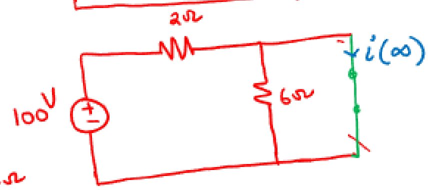


④ circuit for $t \geq 0$



⑥ $t \geq 0$





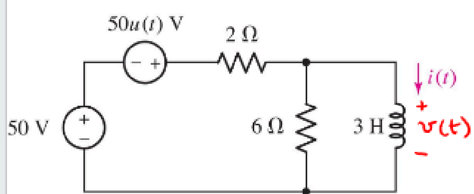
$R_{eq} = 2 /\!/ 6 = \frac{12}{8} = \frac{3}{2} = 1.5\Omega$

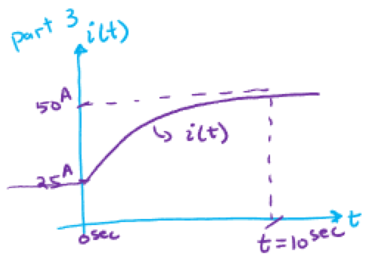
$\tau = \frac{L}{R_{eq}} = \frac{3 \times 2}{3} = 2\,sec$

⑤ $i(\infty) = \frac{100}{2} = 50A$

~~$i(t) = 25 + 25\left(1 - e^{-t/2}\right)u(t)$ A~~

Exercise - First order RL Circuit





⑦ $i(t) = 50 + [25 - 50]e^{-t/2}$

$i(t) = 50 - 25e^{-t/2}$ A for $t \geq 0$

part 2

$v(t) = L\frac{di}{dt} = 3\frac{d}{dt}\left[50 - 25e^{-t/2}\right]$

$= 3(-25)\left(-\frac{1}{2}\right)e^{-t/2} = 37.5e^{-t/2}$ Volt for $t \geq 0$