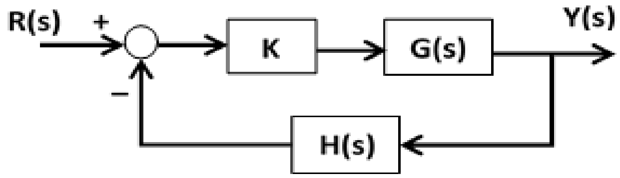
Characteristics of Root-Locus Diagram



Rule 1: Number of loci is equal to the order of the characteristic equation.

Rule 2: By variation of K from 0 to infinity, each locus starts at an open-loop pole (when $K = 0$) and finishes at an open-loop zero, (including the ones at infinity), when $K \to \infty$.

Rule 3: Loci either run along the real axis or occur as complex conjugate pairs. The root-locus diagram will be symmetrical about the real axis.

Rule 4: A locus will never cross over its own path.

Guidelines of Root-Locus Plotting for $K \geq 0$

Step 1: Draw the axes of the s-plane

- The open-loop system has n poles and m zeros. Mark poles $\times$ and zeros o of the open-loop system in s-plane.

Step 2: Draw the root-locus on the real axis

- A point on the real axis is part of a locus if the number of poles and zeros to the right of that point is ODD. (Here, 0 is considered as an even number)

Step 3: Draw the asymptote lines for large *K* values

- Away from the open-loop poles and zeros the loci become asymptotic to asymptote lines.
- The number of asymptote lines is equal to the relative degree of the open-loop transfer function.

Relative degree = $n - m$

- The asymptote lines intersect the real axis at point α, which is called centroid of the asymptotes, and determined as

$$\text{centroid} \quad \to \quad \alpha = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i}{n-m}$$

- Angles of asymptote lines with real axis are determined as

$$\varphi_i = \frac{180^\circ}{n-m}(2i+1) \quad , \quad i = 0, 1, 2, \cdots$$

Step 4: Calculate the points where root-locus cross the imaginary axis

- Assume $s = j\omega$ and compute the cross points and *K* value from characteristic equation.
- The corresponding *K* value may also be found by the Routh-Hurwitz criterion.

Step 5: Calculate locations of breakaway (or break in) points on the real axis

- The breakaway (or break in) point between two poles on the real axis is the point where two or more branches meet.
- The breakaway (or break in) point is the location of multiple roots.
- At the breakaway (or break in) point the parameter K is at a maximum (or minimum) along the real axis.
- The breakaway (or break in) point can be determined as follows

$$1 + KG(s)H(s) = 0 \quad \rightarrow \quad K = \frac{-1}{G(s)H(s)} \quad \rightarrow \quad \frac{dK}{ds} = 0$$

Solving this equation gives the breakaway (or break in) point and the maximum (or minimum) value of K.

Step 6: Calculate angle of departure from complex poles or arrival to complex zeros of the root-loci

- This calculation is useful to determine the departure angle from complex poles or arrival angles to complex zeros.
- For systems with complex open-loop poles the angle of departure of the root-locus from those poles is determined by:

$$\begin{aligned}\text{Angle of depurture from the complex pole} \\ &= 180^{\circ} - (\text{sum of the angles of vectors drawn to this pole from other poles}) \\ &+ (\text{sum of the angles of vectors drawn to this pole from zeros})\end{aligned}$$

- For systems with complex open-loop zeros the angle of arrival of the root-locus at these zeros is determined by

$$\begin{aligned}\text{Angle of arrival to the complex zero} \\ &= 180^{\circ} - (\text{sum of the angles of vectors drawn to this zero from other zeros}) \\ &+ (\text{sum of the angles of vectors drawn to this zero from poles})\end{aligned}$$

Step 7: Complete the plotting by considering the characteristics of the root-locus diagram

- Root-locus is symmetric with respect to the real axis.
- Root-locus started from the poles of open-loop transfer function and terminates at the zeros of the open-loop transfer function or goes to infinity approaching the asymptote lines.
- For K = 0 the root-loci is at the open-loop poles including those at $s = \infty$.
- For $K = \infty$ the root-loci is at the open-loop zeros including those at $s = \infty$.
- The number of separate root-locus is equal to the order of characteristic equation.