



MENG 3050 - Power Transmission Components

## Lesson 01- A- Kinematics and Kinetics (review)

**Displacement, velocity and acceleration, force, Moment, Torque, work and power**

### Displacement, velocity and acceleration:

The following will apply only if constant acceleration were applied:

$$v = v_0 + a \cdot t$$

$$v^2 = v_0^2 + 2 \cdot a \cdot (s - s_0)$$

$$s = s_0 + v_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2$$

where:

a= acceleration

V= velocity

t = time

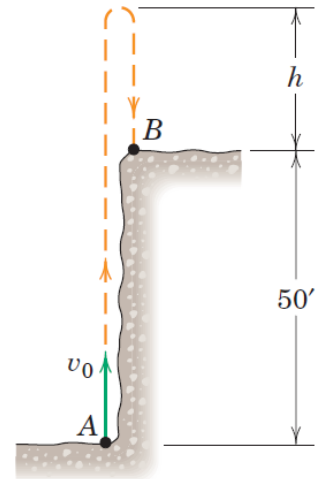
S = displacement at time t

$V_0$ = initial velocity at time  $t_0$

$S_0$ = initial displacement at time  $t_0$

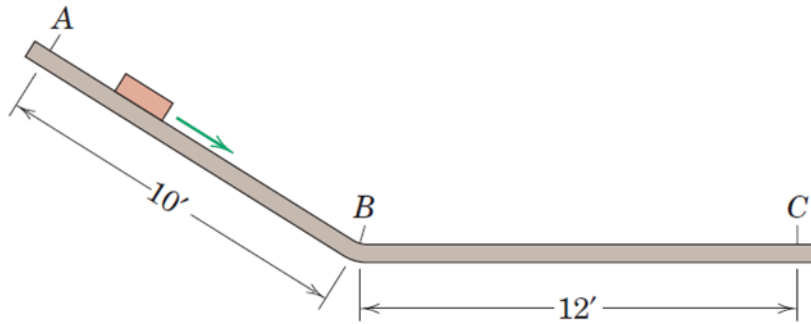
### In class Assignment:

ball is thrown vertically upward with an initial speed of 80 ft/sec from the base A of a 50-ft cliff. Determine the distance h by which the ball clears the top of the cliff and the time t after release for the ball to land at B. Also, calculate the impact velocity  $v_B$ . Neglect air resistance and the small horizontal motion of the ball. Ans.  $h=49.4$  ft,  $t=4.24$ s,  $v_B=56.4$  ft/s down.

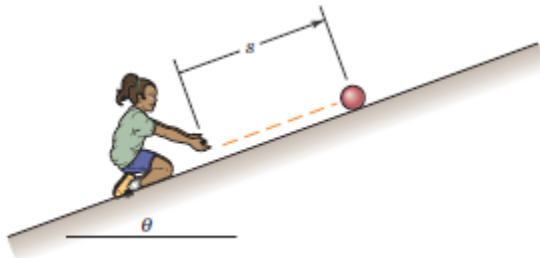


Packages enter the 10-ft chute at A with a speed of 4 ft/sec and have a  $0.3g$  acceleration from A to B. If the packages come to rest at C, calculate the constant acceleration a of the packages from B to C. Also find the time required for the packages to go from A to C.

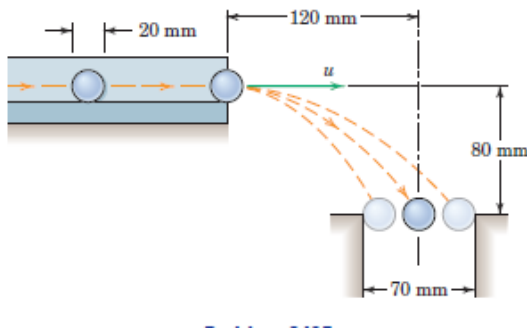
Ans a =  $-8.72$  ft/sec<sup>2</sup>, t = 2.74 sec



A girl rolls a ball up an incline and allows it to return to her. For the angle and ball involved, the acceleration of the ball along the incline is constant at  $0.25g$ , directed down the incline. If the ball is released with a speed of  $4 \text{ m/s}$ , determine the distance  $s$  it moves up the incline before reversing its direction and the total time  $t$  required for the ball to return to the child's hand



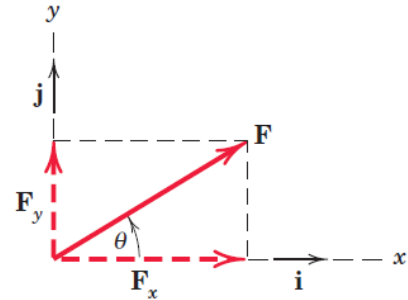
Ball bearings leave the horizontal trough with a velocity of magnitude  $u$  and fall through the 70-mm diameter hole as shown. Calculate the permissible range of  $u$  which will enable the balls to enter the hole. Take the dashed positions to represent the limiting conditions. Ans.  $\max = 1.135 \text{ m/s}$ ,  $\min = 0.744 \text{ m/s}$



## Force in statics:

### Newton' s 1<sup>st</sup> law

Law 1. A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.



### The Free-Body Diagram

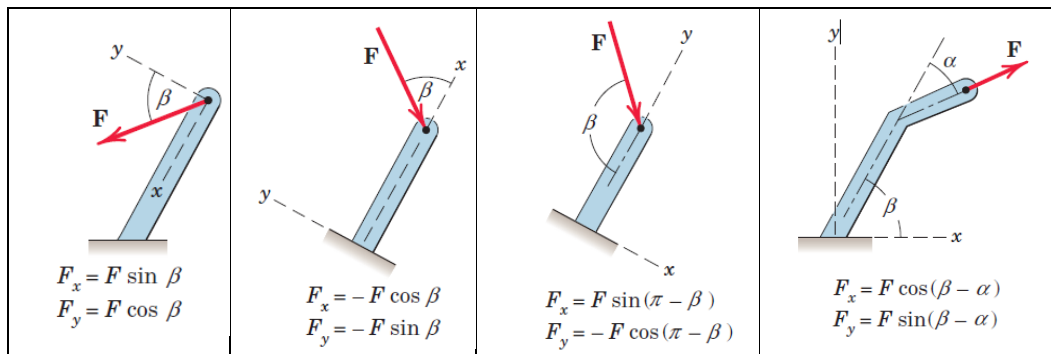
The free-body-diagram method is the key to the understanding of mechanics. This is so because the isolation of a body is the tool by which cause and effect are clearly separated, and by which our attention is clearly focused on the literal application of a principle of mechanics

## Force:

### Force systems:

- External and Internal Effects: We can separate the action of a force on a body into two effects, external and internal.
- Forces external to a body can be either applied forces or reactive forces,
- The effects of force internal to the bracket are the resulting internal forces such as weight, elastic stresses.

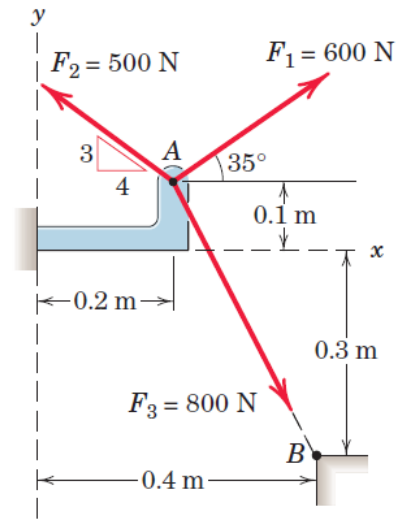
## Exercise:



Example:

The forces  $F_1$ ,  $F_2$ , and  $F_3$ , all of which act on point  $A$  of the bracket, are specified in three different ways. Determine the  $x$  and  $y$  scalar components of each of the three forces. Then find the resultant (scalar value and direction)

Solution:



The scalar components of  $F_1$ , from Fig. a, are;

$$F_{1x} = 600 \cos 35 = 491 \text{ N}$$

$$F_{1y} = 600 \sin 35 = 344 \text{ N}$$

The scalar components of  $F_2$ , from Fig. b, are ;

$$F_{2x} = -500 \cdot (4/5) = -400 \text{ N}$$

$$F_{2y} = 500 \cdot (3/5) = 300 \text{ N}$$

The scalar components of  $F_3$  can be obtained by first computing the angle  $\alpha$ :

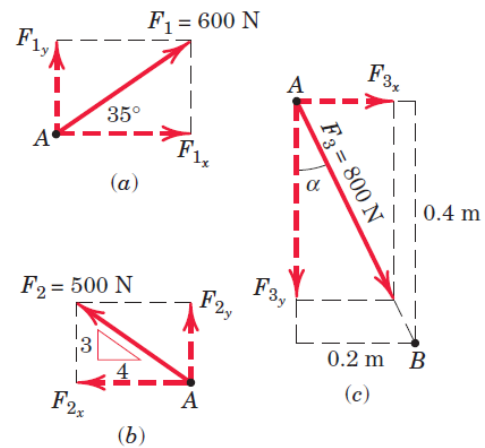
Then apply  $\tan^{-1} \alpha$ . Lead to :

$$F_{3x} = 358 \text{ N}$$

$$F_{3y} = -716 \text{ N}$$

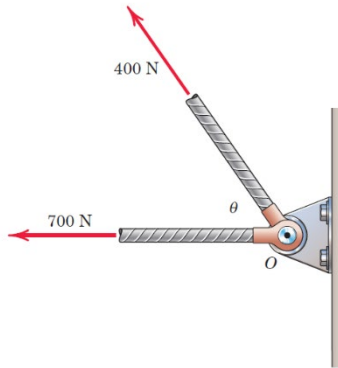
To find resultant :|( use vector format)

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

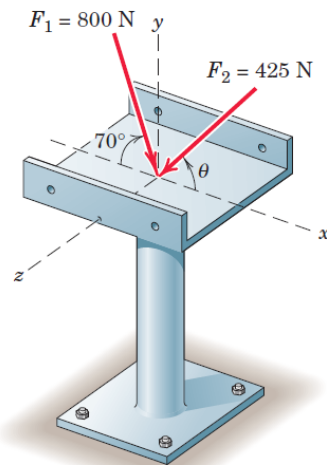


In class Assignments:

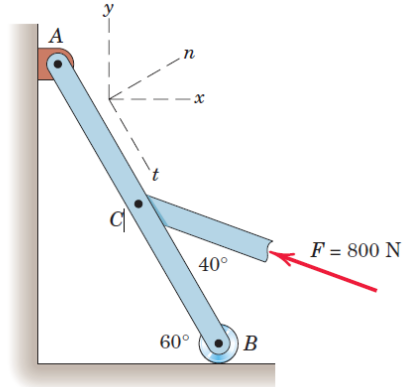
At what angle  $\theta$  must the 400-N force be applied in order that the resultant  $R$  of the two forces have a magnitude of 1000 N? For this condition what will be the angle  $\beta$  between  $R$  and the horizontal? Ans.  $\theta = 51.3^\circ$ ,  $\beta = 18.19^\circ$



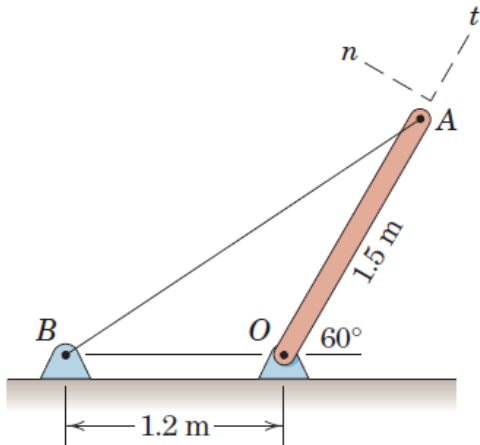
Two forces are applied to the construction bracket as shown. Determine the angle  $\theta$  which makes the resultant of the two forces vertical. Determine the magnitude  $R$  of the resultant



A force  $F$  of magnitude 800 N is applied to point  $C$  of the bar  $AB$  as shown. Determine both the  $x$ - $y$  and the  $n$ - $t$  components of  $F$ .



The cable  $AB$  prevents bar  $OA$  from rotating clockwise about the pivot  $O$ . If the cable tension is 750 N, determine the  $n$ - and  $t$ -components of this force acting on point  $A$  of the bar.



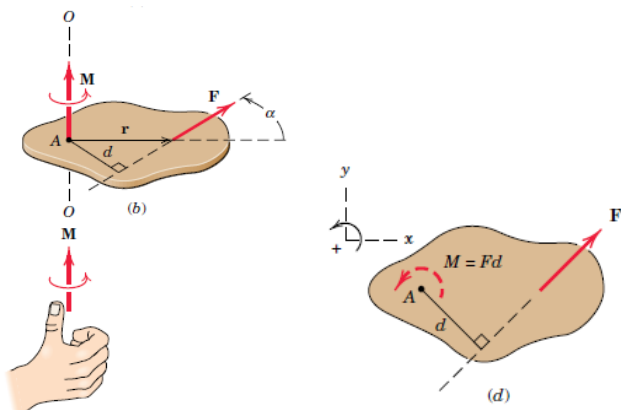
### Moment (Statics):

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the *moment*  $\mathbf{M}$  of the force. Moment is also referred to as *torque*.

### Moment about a Point:

a two-dimensional body acted on by a force  $\mathbf{F}$  in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis  $O-O$  perpendicular to the plane of the body is proportional both to the magnitude of the force and to the *moment arm*  $d$ , which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as

$$M = d \cdot F$$



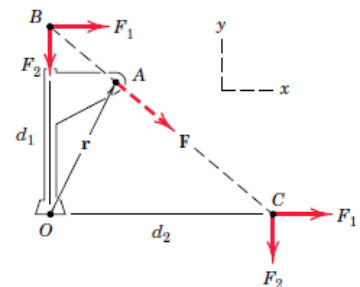
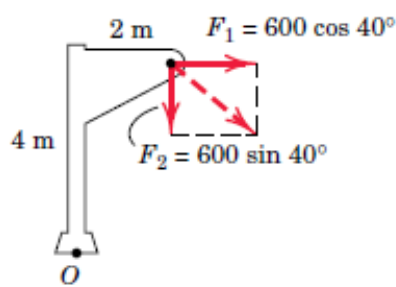
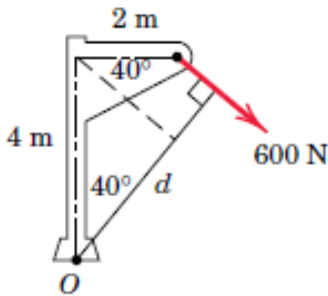
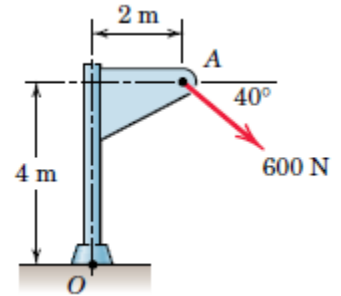
The moment  $\mathbf{M}$  obeys all the rules of vector combination and may be considered a sliding vector with a line of action coinciding with the moment axis. The basic units of moment in SI units are newton-meters (N.m) and in the U.S. customary system are pound-feet (lb-ft).

Moment directions may be accounted for by using a stated sign convention, such as a plus sign (+) for counter-clockwise moments and a minus sign (-) for clockwise moments, or vice versa.

Example:

Calculate the magnitude of the moment about the base point  $O$  of the 600-N force in five different ways

Solution:



**Solution.** (I) The moment arm to the 600-N force is

$$d = 4 \cos 40^\circ + 2 \sin 40^\circ = 4.35 \text{ m}$$

① By  $M = Fd$  the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N}\cdot\text{m} \quad \text{Ans.}$$

(II) Replace the force by its rectangular components at A

$$F_1 = 600 \cos 40^\circ = 460 \text{ N}, \quad F_2 = 600 \sin 40^\circ = 386 \text{ N}$$

By Varignon's theorem, the moment becomes

$$\textcircled{2} \quad M_O = 460(4) + 386(2) = 2610 \text{ N}\cdot\text{m} \quad \text{Ans.}$$

(III) By the principle of transmissibility, move the 600-N force along its line of action to point B, which eliminates the moment of the component  $F_2$ . The moment arm of  $F_1$  becomes

$$d_1 = 4 + 2 \tan 40^\circ = 5.68 \text{ m}$$

and the moment is

$$M_O = 460(5.68) = 2610 \text{ N}\cdot\text{m} \quad \text{Ans.}$$

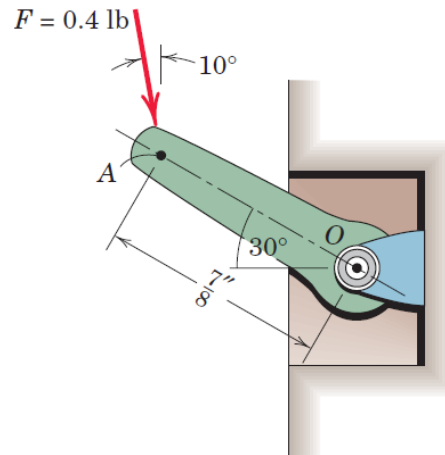


In class Assignment

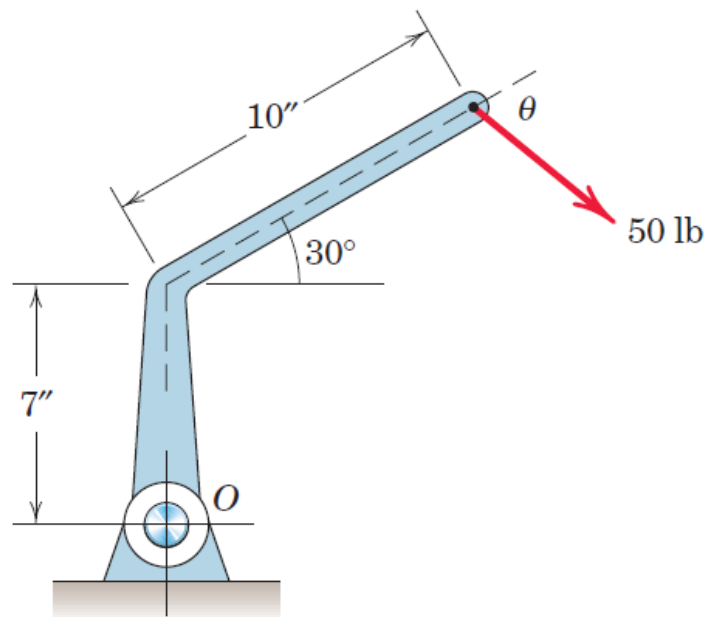
- Compute the moment of the 0.4-lb force about the pivot O of the wall-switch toggle.

Answer:

$$M_O = 0.268 \text{ lb-in. CCW}$$



- Determine the angle  $\theta$  which will maximize the moment  $M_O$  of the 50-lb force about the shaft axis at O. Also compute  $M_O$ , Answer  $\theta = 65.8^\circ$ ,  $M_O = 740 \text{ lb-in. CW}$



## Equation of Motion (Dynamics):

### Newton's Second Law of Motion:

When a body is acted upon by a net force (Not in equilibrium), the body's acceleration multiplied by its mass is equal to the net force

When a particle of mass  $m$  in a rectilinear motion is subjected to the action of concurrent forces  $F_1, F_2, F_3, \dots$  whose sum is  $\Sigma F$ , Then:

$$\Sigma F_x = ma_x$$

$$\Sigma F_y = ma_y$$

$$\Sigma F_z = ma_z$$

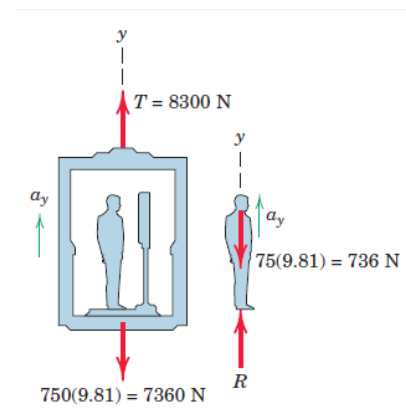
### Example:

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension  $T$  in the hoisting cable is 8300 N. Find the reading  $R$  of the scale in newtons during this interval and the upward velocity  $v$  of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg.

$$[\Sigma F_y = m \cdot a_y]$$

$$8300 - 7360 = 750 \cdot a_y$$

$$a_y = 1.257 \text{ m/s}^2$$



The scale reads the downward force exerted on it by the man's feet. The equal and opposite reaction  $R$  to this action is shown on the free-body diagram of the man alone together with his weight, and the equation of motion for him gives

$$[F_y = ma_y]$$

$$R - 736 = 75(1.257)$$

$$R = 830 \text{ N}$$

For constant acceleration :

$$a = v/t,$$

$$v = s/t$$

where  $a$  = acceleration,  $v$  = velocity,  $s$  = displacement,  $t$  = time

hence: The velocity reached at the end of the 3 seconds is:

$$v = a \cdot t, \rightarrow v = 1.257 \times 3 = 3.77 \text{ m/s}$$

### Example:

The 250-lb concrete block A is released from rest in the position shown and pulls the 400-lb log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at B.

Solution:

The motions of the log and the block A are clearly dependent. Although by now it should be evident that the acceleration of the log up the incline is half the downward acceleration of A, The constant total length of the cable is:

$$L = 2s_C + s_A = \text{constant},$$

Hence:

$$2s_C = s_A$$

$$2V_C = V_A$$

$$2a_C = a_A$$

The free-body diagram of the log shows the

friction force  $= 0.5N$  ( $N$  = normal force)

for motion up the plane.

Equilibrium of the log in the y-direction gives

$$[\Sigma F_y = 0]$$

$$N - 400 \cos 30^\circ = 0$$

$$N = 346 \text{ lb}$$

and its equation of motion in the x-direction gives

$$[\Sigma F_x = m \cdot a_x]$$

$$0.5(346) - 2T + 400 \sin 30^\circ = (400/32.2) \cdot a_C$$

For the block in the positive downward direction, we have

$$[\Sigma F_y = m a_y]$$

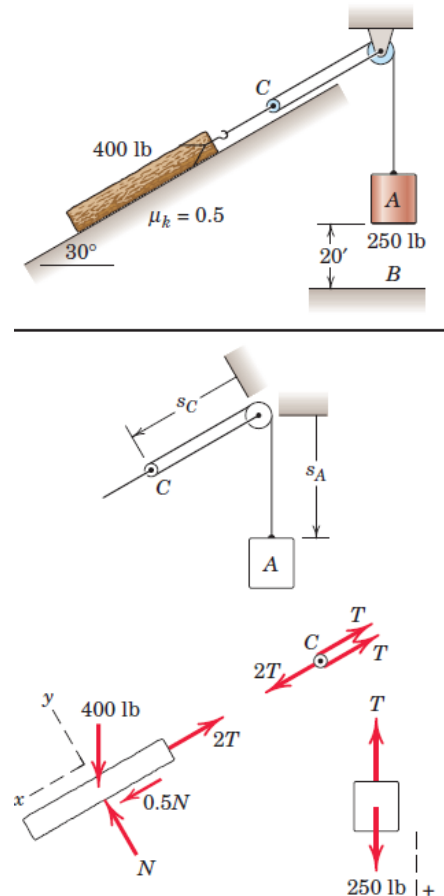
$$250 - T = (250/32.2) \cdot a_A$$

Solving the three equations in  $a_C$ ,  $a_A$ , and  $T$  gives us

$$a_A = 5.83 \text{ ft/sec}^2$$

$$a_C = -2.92 \text{ ft/sec}^2$$

$$T = 205 \text{ lb}$$



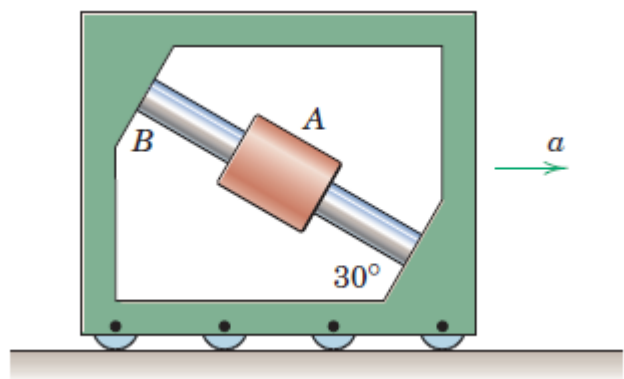
For the 20-ft drop with constant acceleration, the block acquires a velocity;

$$[v^2 = 2a \cdot x]$$

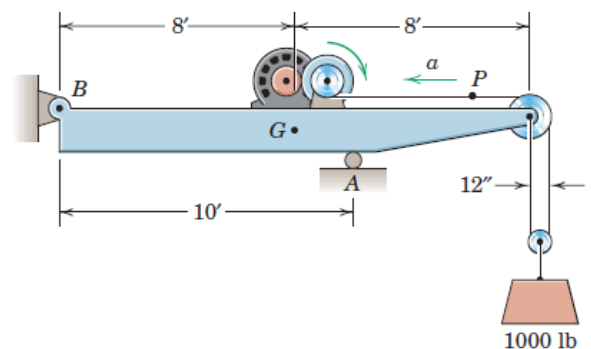
$$\Rightarrow v_a = 15.27 \text{ ft/Sec}$$

### In Class Assignment:

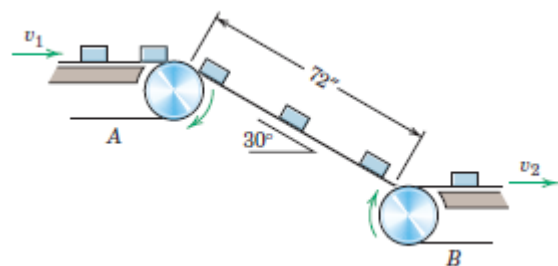
The collar A is free to slide along the smooth shaft B mounted in the frame. The plane of the frame is vertical. Determine the horizontal acceleration  $a$  of the frame necessary to maintain the collar in a fixed position on the shaft, ans.  $A = 5.66 \text{ m/s}^2$



The beam and attached hoisting mechanism together weigh 2400 lb with center of gravity at G. If the initial acceleration  $a$  of point P on the hoisting cable is 20 ft/sec<sup>2</sup>, calculate the corresponding reaction at the support A. ans;  $F_A = 4080 \text{ lb}$

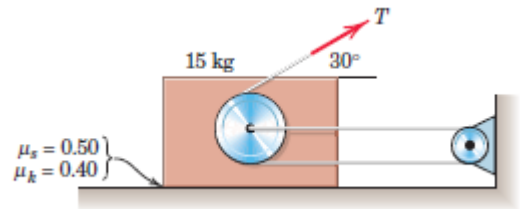


Small objects are delivered to the 72-in. inclined chute by a conveyor belt A which moves at a speed  $v_1 = 1.2 \text{ ft/sec}$ . If the conveyor belt B has a speed  $v_2 = 3.0 \text{ ft/sec}$  and the objects are delivered to this belt with no slipping, calculate the coefficient of friction between the objects and the chute.



Answer: 0.555

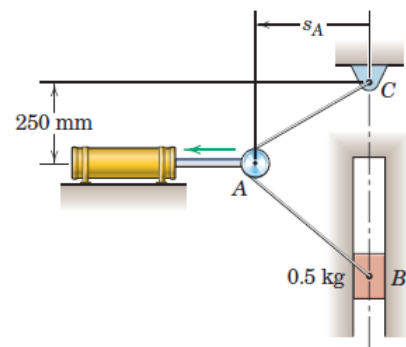
Determine the initial acceleration of the 15-kg block if (a)  $T=23\text{N}$  and (b)  $T=26\text{N}$ . The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible, answer: a=0, b)  $a = 1.390\text{ m/s}^2$



The rod of the fixed hydraulic cylinder is moving to the left with a speed of  $100\text{ mm/s}$  and this speed is momentarily increasing at a rate of  $400\text{ mm/s}$  each second at the instant when  $s_A = 425\text{ mm}$ . Determine the tension in the cord at that instant. The mass of slider B is  $0.5\text{ kg}$ , the length of the cord is  $1050\text{ mm}$ , and the effects of the radius and friction of the small pulley at A are negligible. Find results for cases

- (a) negligible friction at slider B and (b)  $\mu_k = 0.40$  at slider B. The action is in a vertical plane

Ans:  $T = 8.52\text{ N}$ ,  $T = 16.14\text{ N}$



## Work and Power

linear work:

Work:

$$W = F \cdot d$$

Where

- W: work (jule) or J or kJ, lb.in
- F: force , N, KN, lb
- d: Distance at force direction, m, mm , in , ft,

angular work

$$\text{or } W = T \cdot \Theta$$

- W: work (jule) or J or kJ, lb.in
- T: Torque , m.N, m, kN, in.lb..
- $\Theta$ : angular distance or angle in (rad)
- 

$$\pi \text{ rad} = 180^\circ$$

$$2 \pi \text{ rad} = 360^\circ = 1 \text{ rev}$$

## Work = Energy

### Power = work / time

$$P = F \cdot d / t = F \cdot v \quad , \text{ watt (w) or (kw), lb.ft/s}$$

Where:

- V: velocity

Or :

$$P = T \cdot \Theta / t = T \cdot \omega \quad , \text{ watt (w) or (kw), lb.ft/s}$$

Where:

- $\omega$  : angular speed (rad/sec)

$$2 \pi \text{ rad / sec} = 1 \text{ rev/sec}$$

$$(2 \pi / 60) \text{ rad / sec} = 1 \text{ rev/min} = 1 \text{ rpm}$$

**Conversion from linear speed to angular speed is:**

$$v = \omega \cdot r$$

Where v: linear speed

$\omega$ : angular speed (rad/sec)

r: radius