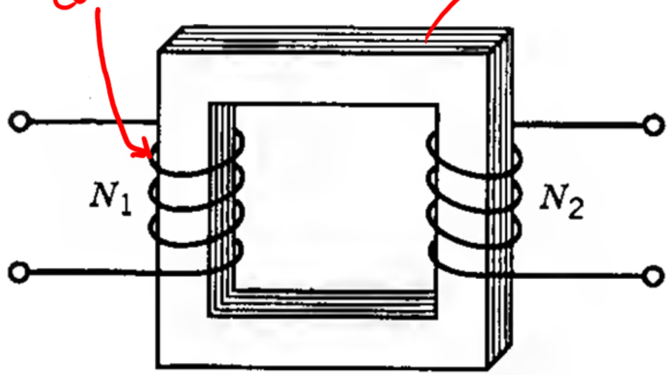
Transformers

Introduction

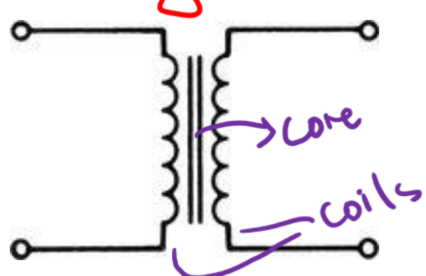
machine

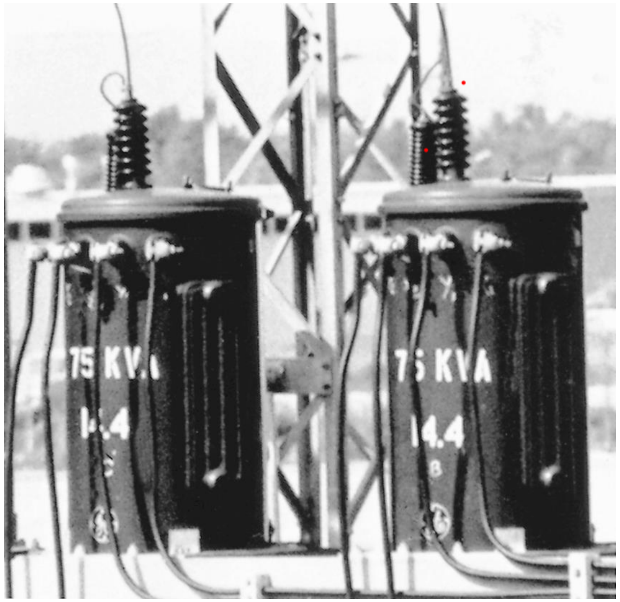
- A transformer is a static machine (no rotating parts)
- A transformer consists of two or more windings coupled by a mutual magnetic field
  coils
- Ferromagnetic cores are used to provide high magnetic coupling and high flux densities
  → good magnetic material
- Iron Core (High Power) Vs Air core (low power electronic circuits) Transformers



N_1 => # of turns

Symbol





Introduction

$v \to V$, $i \to A$, $p \to W$

$v \to V$, $i \to A$, $\bar{S}$ VA, $\bar{S} \to P \pm Qj$

increase / decrease

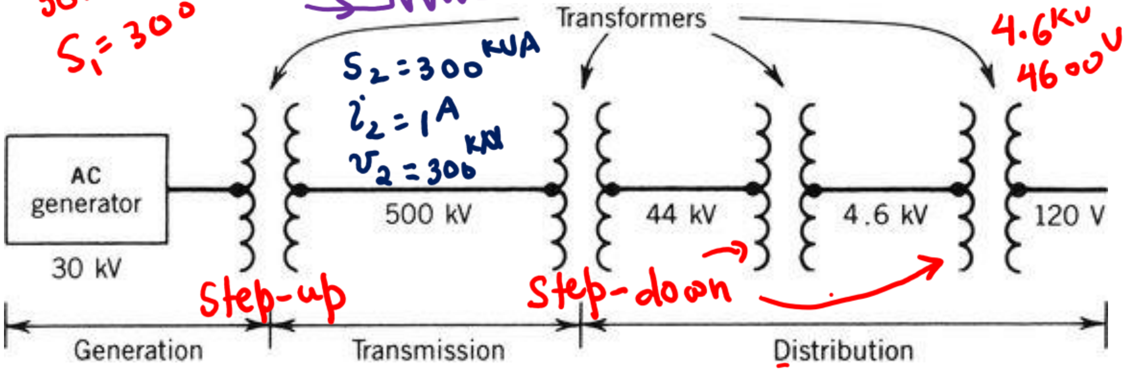
- Transformers have widespread use
- The primary function of a transformer is to change the voltage level
- Transformers re used to step up and step down voltage at various stages of power transmission

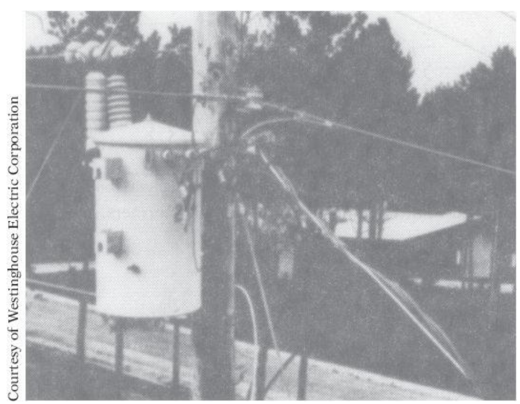
$i_1 = 10^A$, $30kV = v_1$, $S_1 = 300^{kVA}$

10^A → loss $= (10)^2 R$

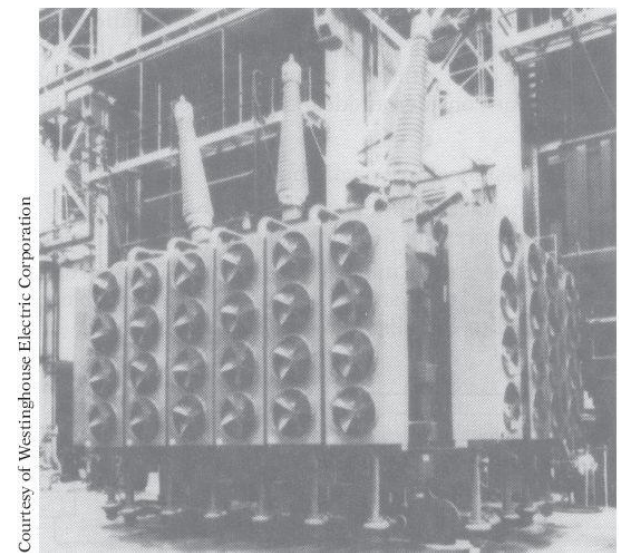
1^A → loss $= (1)^2 R$

user: Industrial 4.6^{kV} 4600^V; Commercial 600^V; Residential $110^V/120^V$





Distribution Transformer to step down the voltage from 4.6 kV to 120 V



Power Transformer to step up the voltage from 24 kV to 345 kV

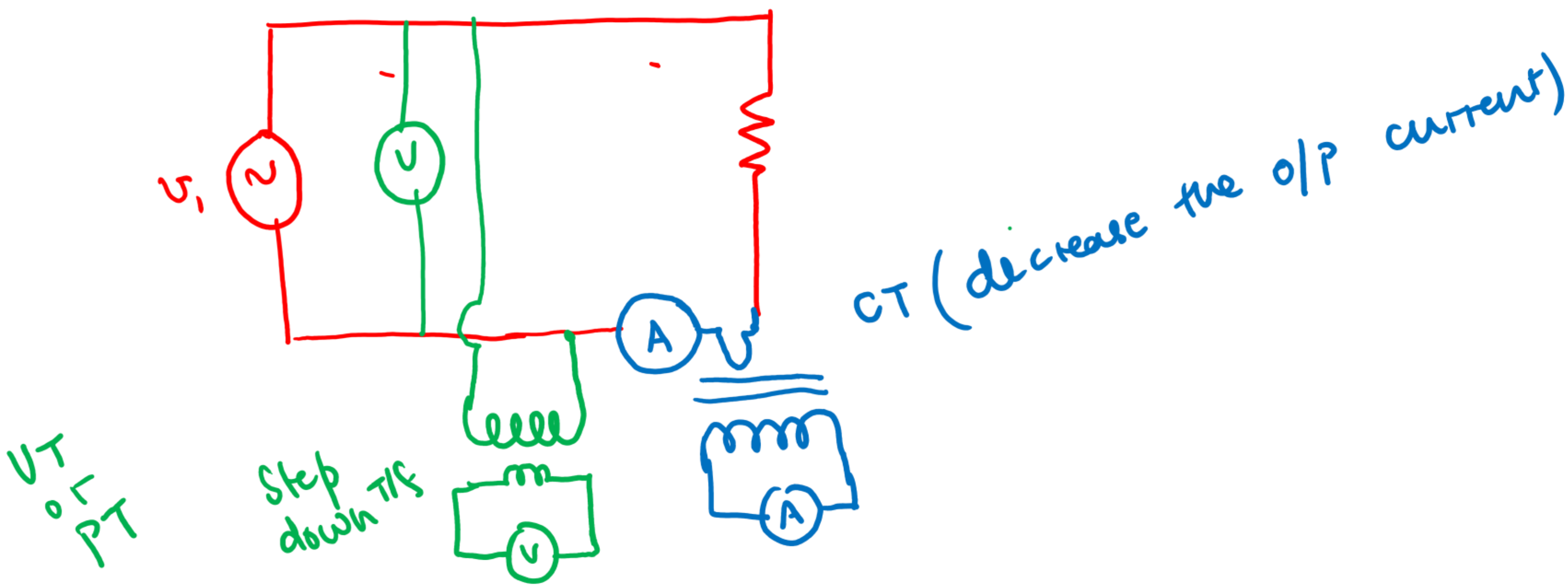
Introduction

- Transformers are also used in low power applications

- Low power electronic or control circuits to isolate one circuit from the other or to match the impedance of a source with its load for maximum power transfer

potential

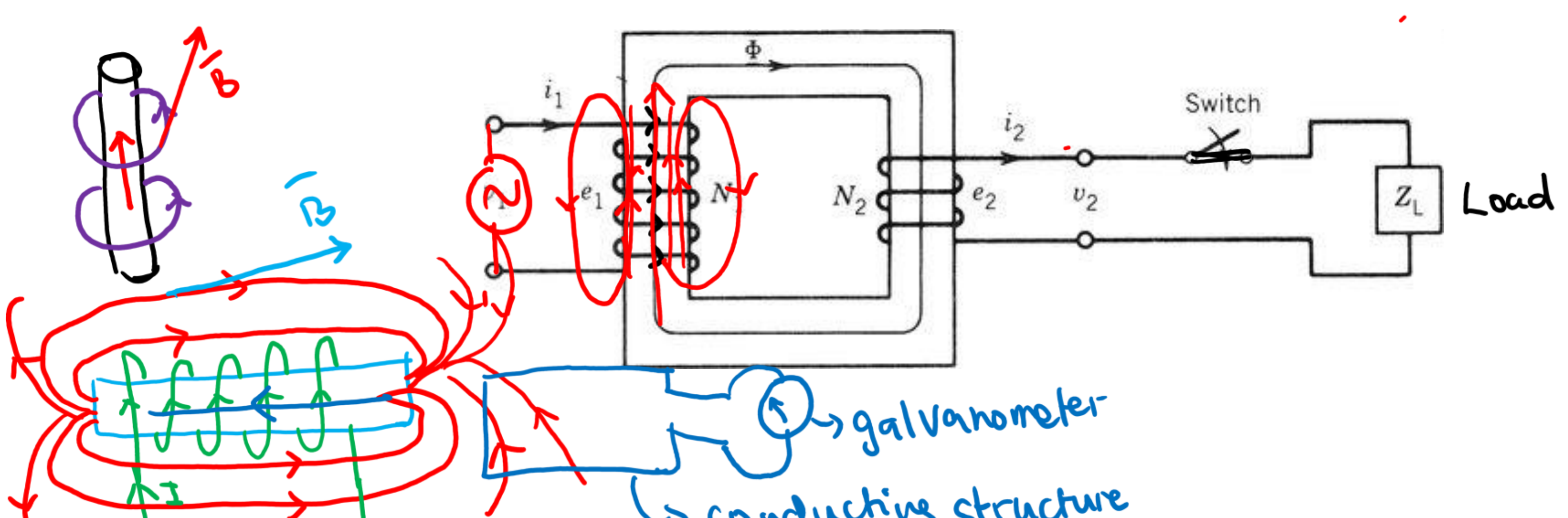
- Transformers are also used for voltage and current measurements – Instrument Transformers



Ideal Transformer

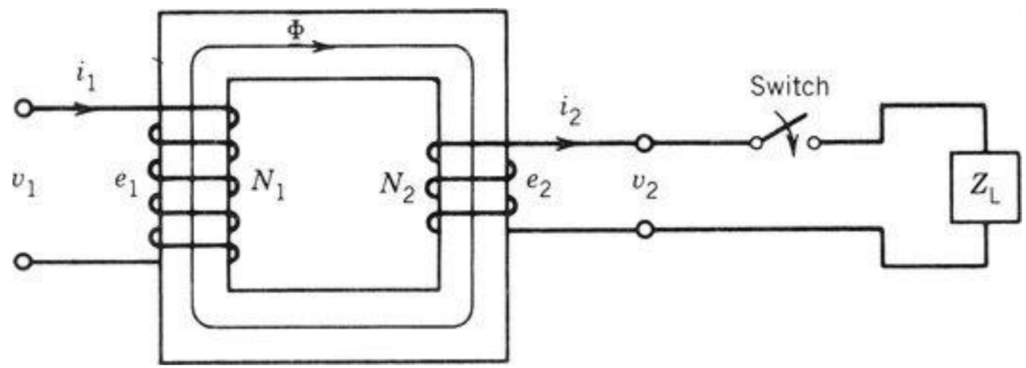
magnetic field $< \bar{\Phi}, \bar{B}, \bar{H}$

1. The winding resistance is negligible
2. All fluxes are confined to the core and link both windings; that is, no leakage fluxes are present
3. Permeability (μ) of the core is infinite. Therefore, the exciting current required to establish flux in the core is negligible.
4. No eddy current and hysteresis losses



- The winding connected to an AC supply – Primary winding has N_1 turns
- The winding connected to an electrical load – Secondary winding has N_2 turns

Ideal Transformer



Faraday's law:

$v_1 = e_1 = N_1 \frac{d\phi}{dt}$

$v_2 = e_2 = N_2 \frac{d\phi}{dt}$

$$\frac{v_1}{v_2} = \frac{N_1 \cancel{\frac{d\phi}{dt}}}{N_2 \cancel{\frac{d\phi}{dt}}} = \frac{N_1}{N_2}$$

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2} = a}$$

$\frac{v_1}{v_2} = \frac{N_1}{N_2}$

$N_1 > N_2$

10 5

$\frac{v_1}{v_2} = \frac{10}{5}$

$v_1 = \frac{10}{5} v_2$

$v_1 > v_2$

step-down T/f

$N_1 < N_2$

5 10

$v_1 = \frac{5}{10} v_2 = \frac{1}{2} v_2$

$v_2 > v_1$

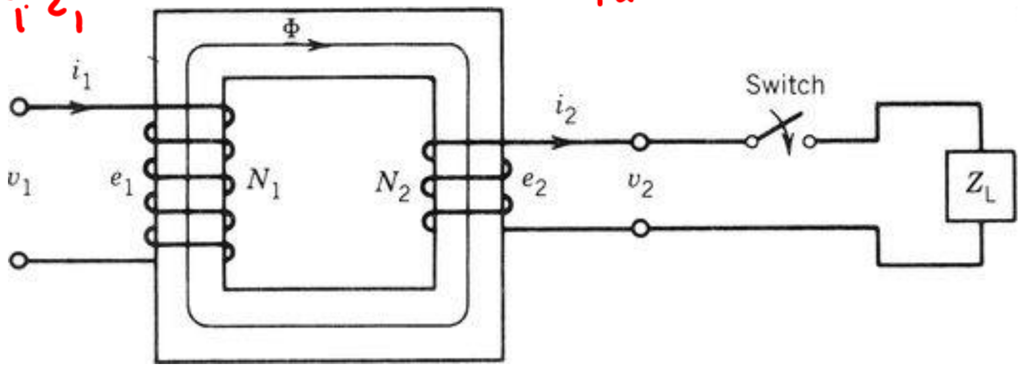
step-up T/f.

a turns ratio

Ideal Transformer

$p_1 = v_1 i_1$

$p_2 = v_2 i_2$



No power losses:

$$p_1 = p_2$$

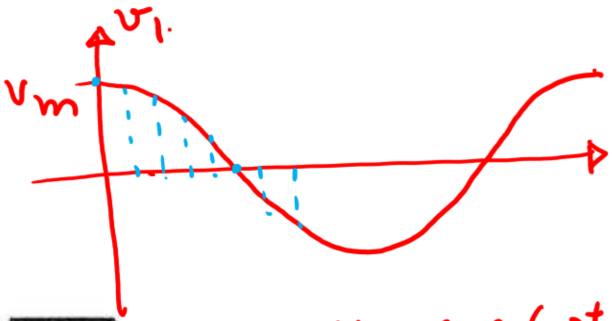
$$\frac{v_1 i_1}{v_2 i_1} = \frac{v_2 i_2}{v_2 i_1}$$

$$\frac{N_1}{N_2} = \frac{v_1}{v_2} = \frac{i_2}{i_1}$$

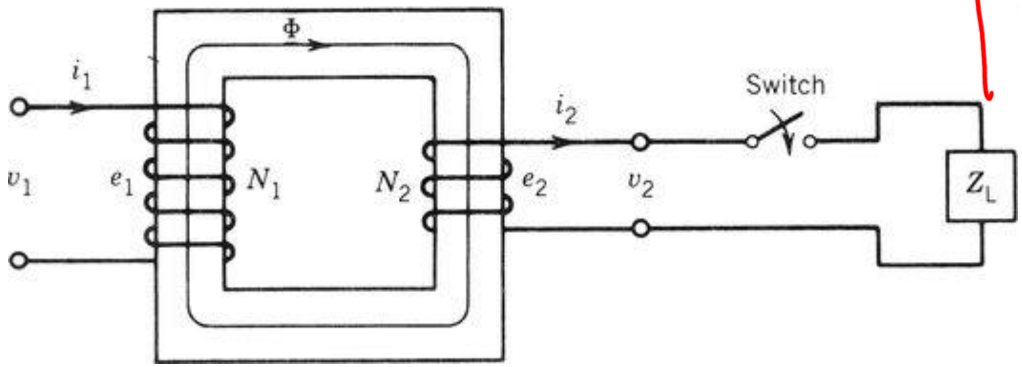
$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2} = \frac{i_2}{i_1}}$$

a turns ratio

Ideal Transformer



$v_1 = V_m \cos(\omega t \pm \phi)$



For Sinusoidal supply voltage v_1, in terms of RMS values of the quantities

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

$$V_1 I_1 = V_2 I_2$$

Ideal Transformer: Example

$\omega = 377 \; rad/sec$

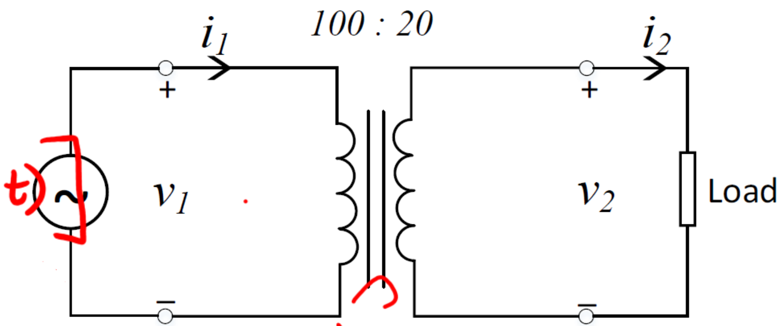
$\omega \rightarrow$ angular freq.

$\omega = 2\pi f$

$f = \frac{377}{2\pi} = 60 \, Hz$

If $v_1 = 120\sqrt{2}\cos(377t)\,V$, $i_2 = 10\sqrt{2}\cos\left(377t - \frac{\pi}{6}\right)\,A$,
calculate v_2 and i_1

$N_1 \quad N_2$



step-down T/S

$\frac{v_1}{v_2} = \frac{N_1}{N_2}$

$v_2 = \frac{N_2}{N_1} v_1 = \frac{2\cancel{0}}{10\cancel{0}}\left[12\cancel{0}\sqrt{2}\cos(377t)\right]$

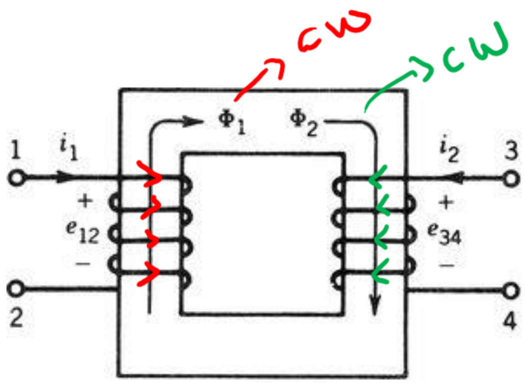
$v_2 = 24\sqrt{2}\cos(377t)\,V$

$\frac{i_2}{i_1} = \frac{N_1}{N_2}$

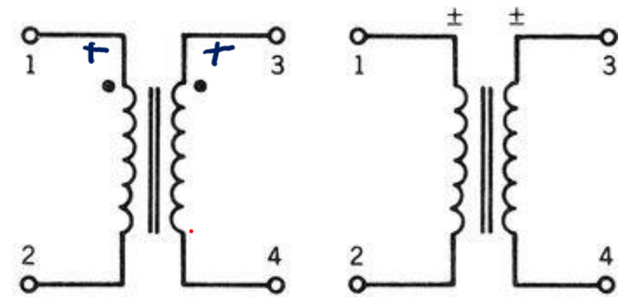
$i_1 = \frac{N_2}{N_1} i_2 = \frac{2\cancel{0}}{10\cancel{0}}\left[1\cancel{0}\sqrt{2}\cos\left(377t - \frac{\pi}{6}\right)\right]$

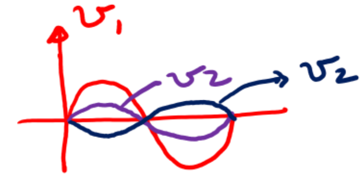
$i_1 = 2\sqrt{2}\cos\left(377t - \frac{\pi}{6}\right)\,A$

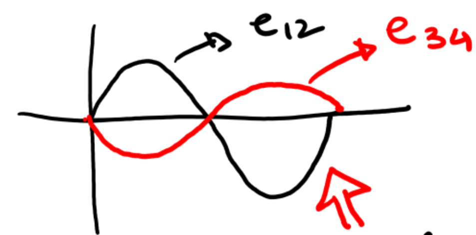
Ideal Transformer: Polarity dots

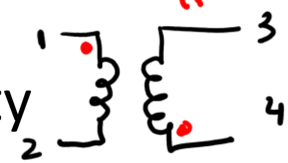


$e_{34} = -e_{43}$

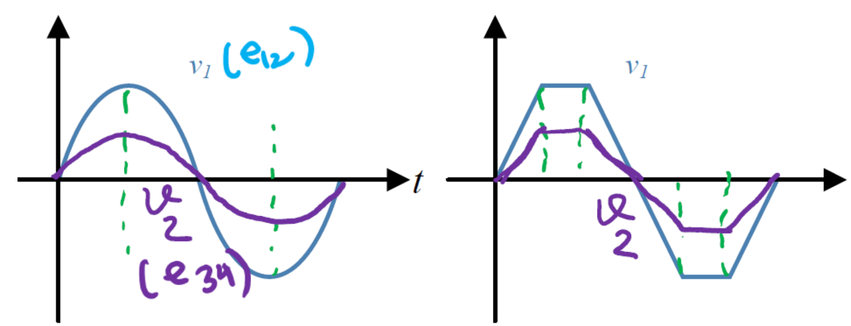








- Windings are marked to indicate terminals of like polarity
- Terminals 1 and 3 are identical because currents entering these terminals produce fluxes in the same direction in the core that forms a common path
- If the two windings are linked by a common time varying flux, voltages will be induced in these windings such that at a particular instant the potential of terminal 1 is positive w.r.t terminal 2, then at the same instant the potential of terminal 3 will be positive w.r.t terminal 4. The induced voltage e_{12} and e_{34} are in phase



Ideal Transformer: Impedance Transfer $Z = \frac{V}{I}$

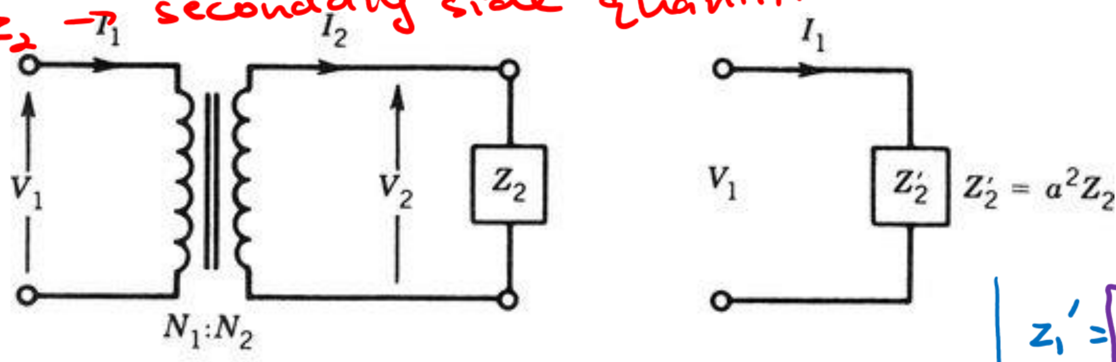
$V_1, I_1 \rightarrow$ primary side quantities

$V_2, I_2, Z_2 \rightarrow$ secondary side quantities

$\frac{N_1}{N_2} = \frac{V_1}{V_2} = a$

$\boxed{N_1, N_2, Z_2}$

$a = \frac{I_2}{I_1}$



(a) (b)

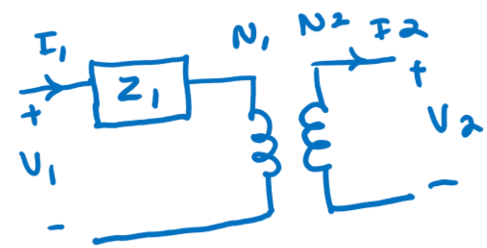
$Z_1' = \boxed{Z_2} = \frac{V_2}{I_2} = \frac{V_1/a}{a I_1}$

$= \frac{V_1}{a^2 I_1} = \frac{1}{a^2}\left(\frac{V_1}{I_1}\right)$

$Z_1' = \frac{1}{a^2} Z_1$

transfer secondary side impedance Z_2 to the primary side

$Z_2' = \boxed{Z_1} = \frac{V_1}{I_1} = \frac{a V_2}{I_2 / a} = \frac{a^2 V_2}{I_2} = a^2\left(\frac{V_2}{I_2}\right) = a^2 Z_2$



transferred / reflected / referred impedance to the primary side

$$Z_2' = a^2 Z_2$$

$$Z_1' = \frac{1}{a^2} Z_1$$

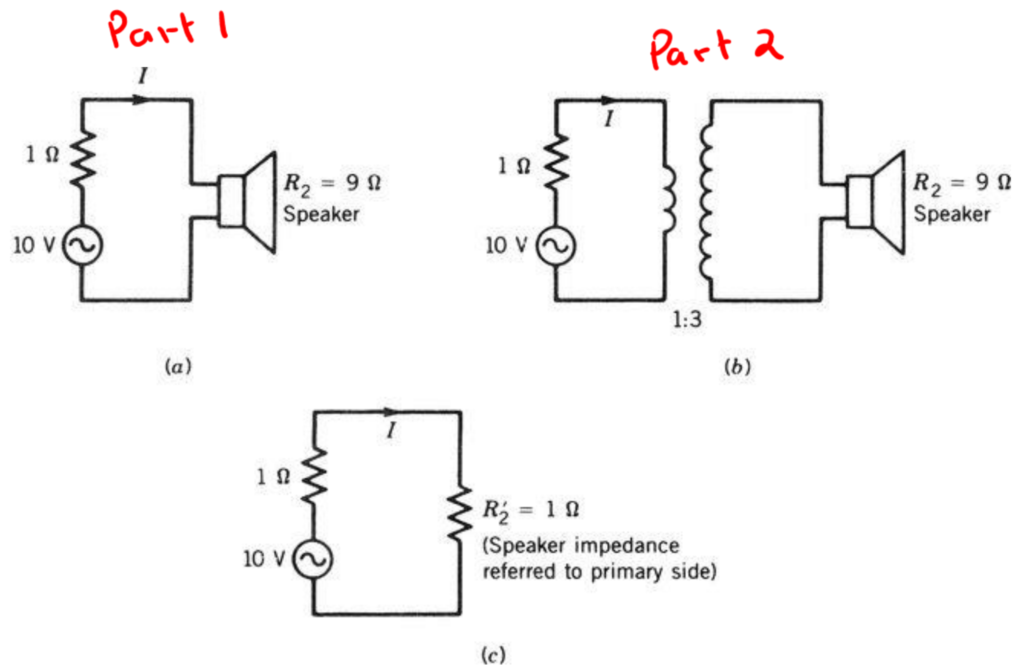
An impedance Z_2 connected in the secondary will appear as an impedance Z_2' on the primary side

An impedance Z_1 can be transferred from primary side to the secondary side as Z_1'

Ideal Transformer: Impedance Transfer: Example

A speaker of 9 Ω, resistive impedance is connected to supply of 10 V with internal resistive impedance of 1 Ω.

1. Calculate the power absorbed by the speaker
2. To maximize the power transfer to the speaker, a transformer of 1:3 turns ratio is used between source and speaker. Determine the power taken by the speaker

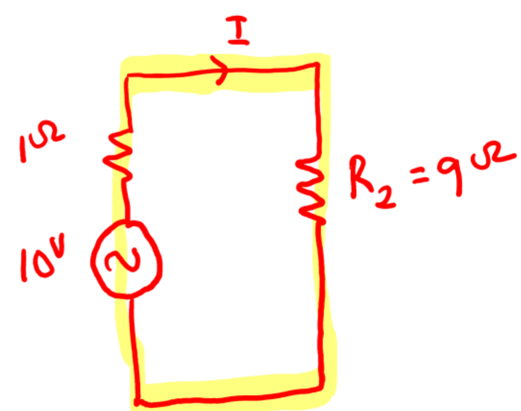


$$R_2' = a^2 R_2$$

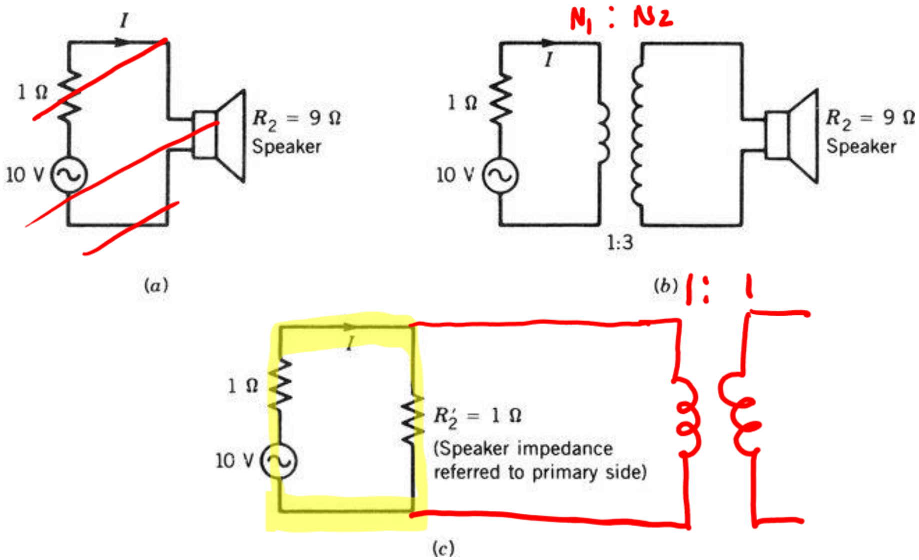
① $P_{speaker} = P_{R2} = V_{R2} I_{R2} = (I_{R2})^2 R_2 = (I^2) R_2$

$I = \frac{10^V}{(1+9)} = 1^A$

$P_{speaker} = 1^2 \times 9 = 9^W$



Ideal Transformer: Impedance Transfer: Example



$$R_2' = a^2 R_2$$

$$a = \frac{N_1}{N_2} = \frac{1}{3}$$

$$R_2' = \left(\frac{1}{3}\right)^2 \times 9 = \frac{1}{9} \times 9 = 1\ \Omega$$

$$I = \frac{10}{1+1} = \frac{10}{2} = 5\ A$$

$$P_{R_2'} = P_{speaker} = (I)^2 \times 1 = 5^2 \times 1 = 25\ W$$

Practical Transformer

1. The winding has resistance – wires have resistance (R₁ and R₂)
2. All fluxes are essentially confined to the core. However, a small amount of the flux known as leakage flux, φ_l links only one winding and does not link the other winding. The leakage path is in air.

φ_{L1} L_{L1} $\Rightarrow$ X_{L1}

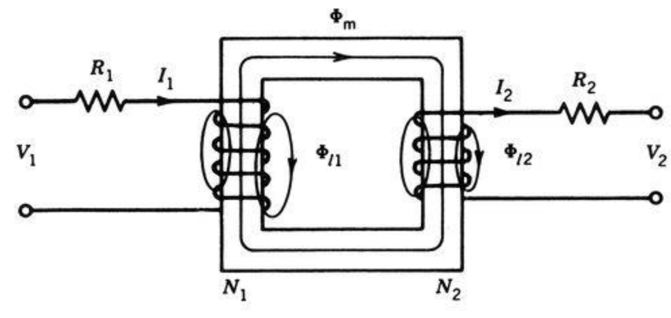
φ_{L2} L_{L2} $\Rightarrow$ X_{L2}

$X = 2\pi f L$

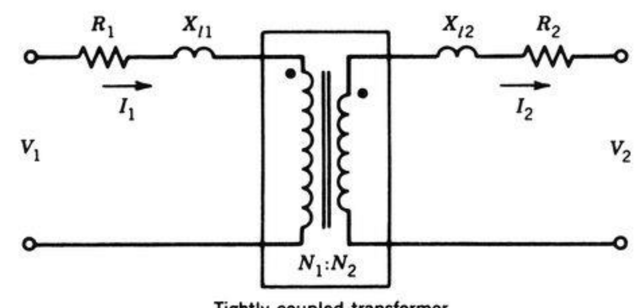
3. Permeability of the core is finite. A magnetizing current I_m is required to establish flux in the core. The $\boxed{\text{core losses}}$ are represented by R_c

$\rightarrow$ L_m $\Rightarrow$ X_m $\rightarrow$ magnetizing reactance

$$I_{\varphi 1} = I_{m1} + I_{c1}$$

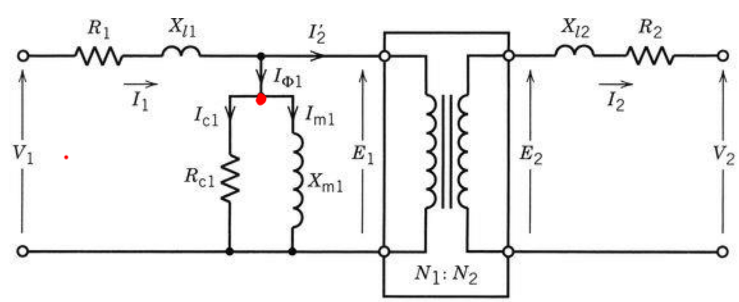


(a)



Tightly coupled transformer

(b)



Ideal transformer

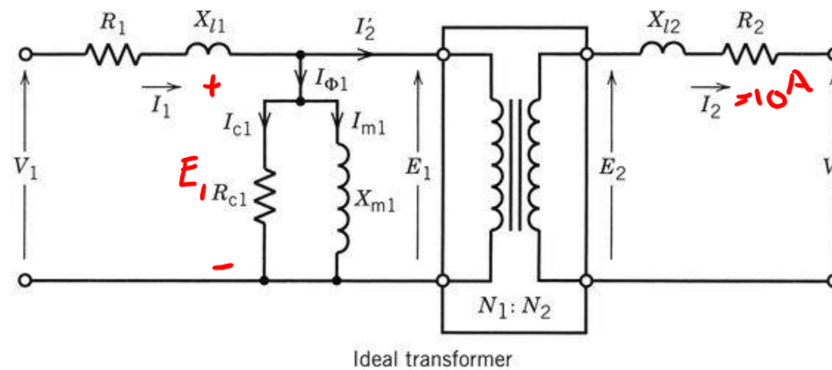
Practical Transformer eq. circuit

Practical Transformer: Equivalent circuit

The ideal transformer can be moved to the right or to the left by referring all quantities to the primary or secondary side, respectively.

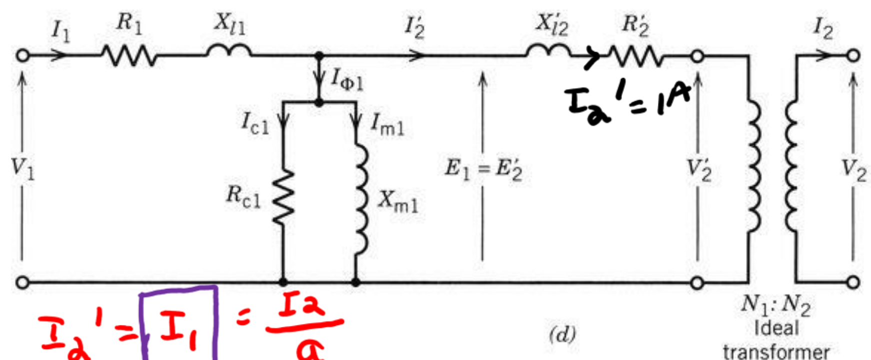
diagram (d): ideal transformer moved to the right

diagram (e): ideal transformer is not shown, all quantities referred to one side



(c)

if $a = 10$



(d)

$E_1 = E_2' = aE_2$
$V_2' = aV_2$
$I_2' = I_2/a$
$X_{l2}' = a^2 x_{l2}$
$R_2' = a^2 R_2$

$a = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$

$I_2' = \boxed{I_1} = \frac{I_2}{a}$

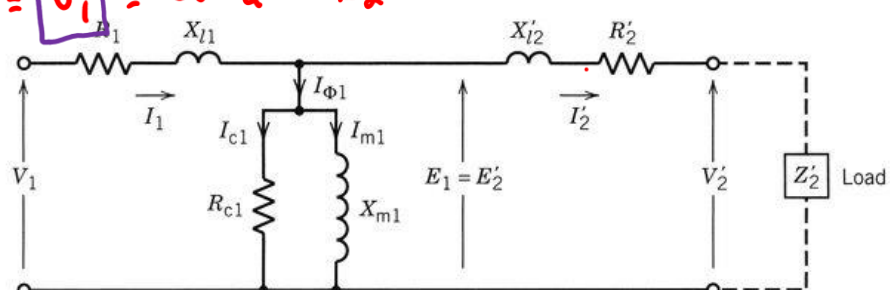
$V_2' = \boxed{V_1} = aV_2$ $\quad E_2' = aE_2$

$R_2 \rightarrow$ secondary resistance

$R_2' \rightarrow$ secondary resistance reflected to primary side

$R_2' = a^2 R_2$

$X_{l2}' = a^2 X_{l2}$



$Z_2' = a^2 Z_2$

(e)

Practical Transformer: Approximate Equivalent circuit

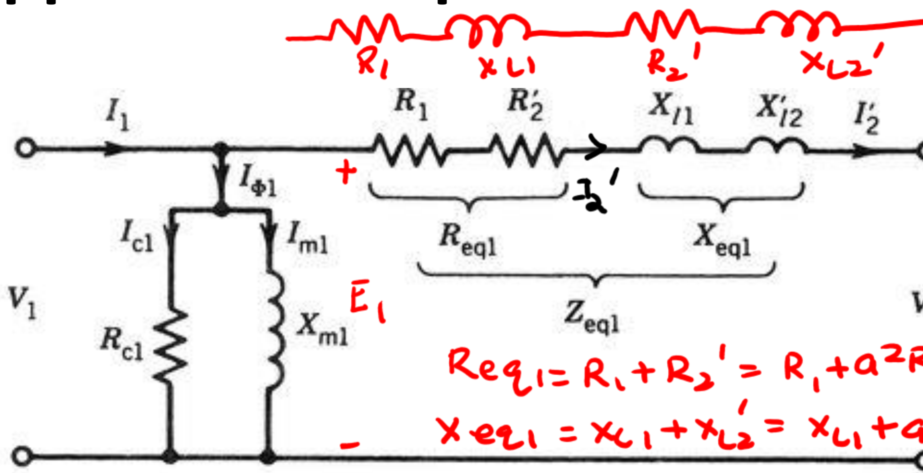
The voltage drop I_1R_1 and I_1X_{l1} are normally small and $V_1 \simeq E1$, then the shunt branch can be moved to the supply terminals diagram (a)

diagram (b) and (c), as the exciting current I_ϕ is small, the excitation (shunt) branch can be completely neglected

$I_1 = I_{\phi 1} + I_2'$

$I_{\phi 1} \rightarrow$ small

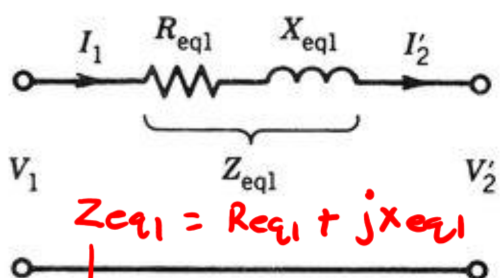
$I_{\phi 1} \simeq 0$



$R_{eq1} = R_1 + R_2' = R_1 + a^2R_2$

$X_{eq1} = X_{L1} + X_{L2}' = X_{L1} + a^2X_{L2}$

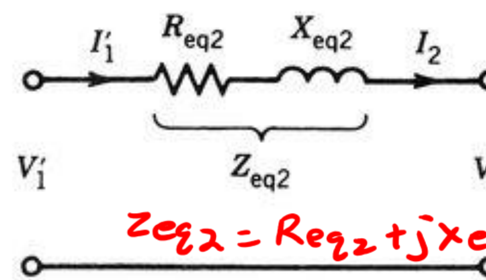
(a) $V_2' = aV_2, \quad I_2' = I_2/a$



$Z_{eq1} = R_{eq1} + jX_{eq1}$

(b) Referred to side 1, $Z_{eq1} = R_{eq1} + jX_{eq1}$

complex quantity



$Z_{eq2} = R_{eq2} + jX_{eq2}$

(c) Referred to side 2, $Z_{eq2} = R_{eq2} + jX_{eq2}$

$$R_{eq2} = \frac{R_{eq1}}{a^2} = R_2 + R_1' = R_2 + \frac{R_1}{a^2}$$

$$X_{L2} + X_{L1}' = X_{eq2} = \frac{X_{eq1}}{a^2} = X_{l2} + X_{l1}'$$

$$X_{L2} + \frac{X_{L1}}{a^2}$$

$$V_1' = \frac{V_1}{a}, \quad I_1' = I_2 = aI_1$$

Practical Transformer: Rating

- Rating: the condition under which a transformer is designed to operate continuously
- Determining factors:
  1. the voltage across its magnetizing inductance (the maximum flux density before saturation of the magnetic material of the core
  2. the maximum current that a winding can tolerate without overheating (temperature limit of insulation and the cooling methods)
  - Apparent power: S_{rated}
  - Nameplate: S_{rated}, V_{1rated}/V_{2rated}, frequency

Practical Transformer Ratings: Example

HV → 1
LV → 2

The low voltage winding of a 500 kVA, 69/4.16 kV, 60 Hz transformer has 25 turns of the wire. Calculate:

S_{rated} V_1 V_2 (HV) (LV) N_2

1. The rated current of the transformer
2. The number of turns of the high voltage winding N_1
3. The maximum flux within the core, if the transformer operates off its rated voltage

① $$I_{1,rated} = \frac{S_{rated}}{V_{1,rated}} = \frac{500\ \text{kVA}}{69\ \text{kV}} = 7.25\ \text{A}$$ rated current of the HV side

$$I_{2,rated} = \frac{S_{rated}}{V_{2,rated}} = \frac{500\ \text{kVA}}{4.16\ \text{kV}} = 120\ \text{A}$$ rated current of LV side

② $$\frac{N_1}{N_2} = \frac{V_1}{V_2} \Rightarrow N_1 = N_2\left(\frac{V_1}{V_2}\right) = 25\left(\frac{69}{4.16}\right)$$
$$= 415\ \text{turns}$$

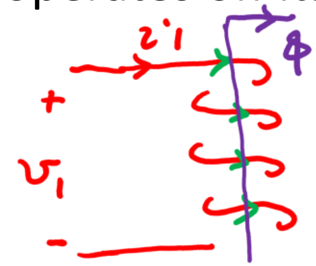
high voltage winding turns

Practical Transformer Ratings: Example

rms

The low voltage winding of a 500 kVA, $\boxed{69/4.16 \text{ kV}}$, 60 Hz transformer has 25 turns of the wire. Calculate:

1. The rated current of the transformer
2. The number of turns of the high voltage winding
3. The maximum flux within the core, if the transformer operates off its rated voltage ϕ_m



$$\phi = \phi_m \sin \omega t$$

$$v_1 = N_1 \frac{d\phi}{dt} = N_1 \frac{d}{dt}\left[\phi_m \sin \omega t\right]$$

$$= N_1 \phi_m \cos(\omega t)\ (\omega)$$

$$v_1 = (N_1 \phi_m \omega) \cos(\omega t)$$

↳ peak of v_1 ⇒ $V_{1,peak}$

$$f = F_m \cos(\omega t \pm \theta)$$

$$V_{1,peak} = N_1 \phi_m (2\pi f)$$

$$\phi_m = \frac{V_{1,peak}}{N_1 (2\pi f)} = \frac{\sqrt{2} \times 69 \times 10^3}{415 \times (2\pi \times 60)} = 0.624 \text{ Weber (Wb)}$$

① Practical Transformer Voltage Regulation and ② Efficiency

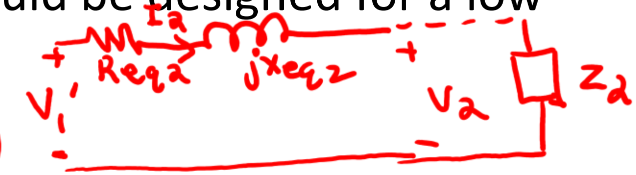
p.f. = $\cos\theta_2$

$\bar{I}_2 = \left|\frac{S}{V_2}\right| \angle -\theta_2$ $\quad \bar{V}_2 = V_2 \angle 0^\circ$

nature of load < lagging p.f. load / leading p.f. load

$\bar{I}_2 = I_2 \angle \theta_2$

Voltage Regulation: most loads connected to the secondary of a transformer are designed to operate at essentially constant voltage. However, as the current is drawn through the transformer, the load terminal voltage changes because of the voltage drop in the internal impedance of the transformer. The load terminal voltage may go up or down depending on the nature of the load. The large voltage change is not desirable for many loads. For example, as more and more light bulbs are connected to the transformer secondary and the voltage decreases appreciably, the bulbs will glow with diminished illumination. To reduce the magnitude of the voltage change, the transformer should be designed for a low value of the internal impedance Z_{eq}.



$$\bar{V}_1' = \bar{V}_2 + \bar{I}_2 (R_{eq2} + j x_{eq2})$$

Voltage regulation is used to identify voltage change in the transformer with loading

$$V_1 = V_2' + I_2' [R_{eq1} + j x_{eq1}]$$

$$Voltage\ Regulation = \frac{|\cancel{V_2}|_{NL} - |\cancel{V_2}|_L}{|V_2|_L}$$

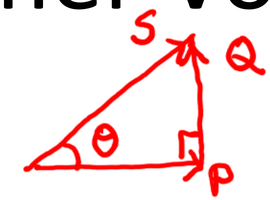
NL: V_1' ; L: $V_{2,rated}$

< 0 / +ve / -ve

$V_1' > V_2$ (lagging p.f. load)

$V_1' < V_2$ (leading p.f. load)

Practical Transformer Voltage Regulation and Efficiency



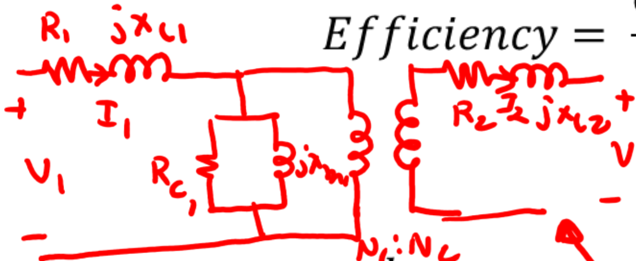
$\bar{S} = P \pm jQ = |\bar{S}| \angle\theta$

↑ apparent power

$P = S\cos\theta$

↘ kW ↘ kVA

Efficiency: losses in the transformer are small because the transformer is a static device and there are no rotational losses such as windage and friction losses in a rotating machine. In a well designed transformer the efficiency can be as high as 99%.



$$Efficiency = \frac{Output\ Power\ (P_{out})}{Input\ Power\ (P_{in})} = \frac{P_{out}}{P_{out} + losses} = \frac{P_{in} - losses}{P_{in}}$$

$$Pout = V_2 I_2 cos\theta_2$$

→ phase angle b/w V_2 and I_2

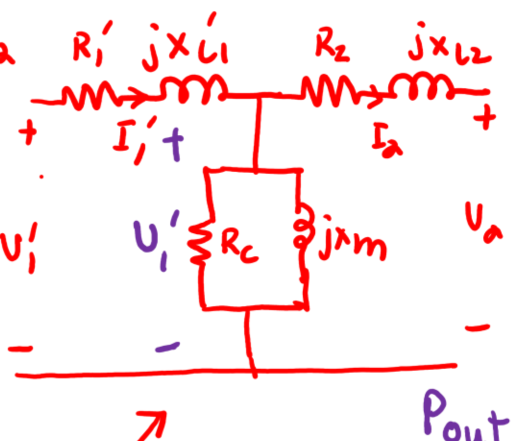
$$losses = core\ loss\ (P_c) + Copper\ losses\ (P_{cu})$$

$$P_{core\ loss} = \frac{(V_1)^2}{R_c}$$

$$P_{cu} = I_1^2 R_1 + I_2^2 R_2$$

$$P_{cu} = I_1'^2 R_{eq1} = I_1'^2 [R_1 + a^2 R_2]$$

$$P_{cu} = I_2^2 R_{eq2} = I_2^2 \left[R_2 + \frac{R_1}{a^2}\right]$$



Equivalent circuit, Voltage regulation and Efficiency

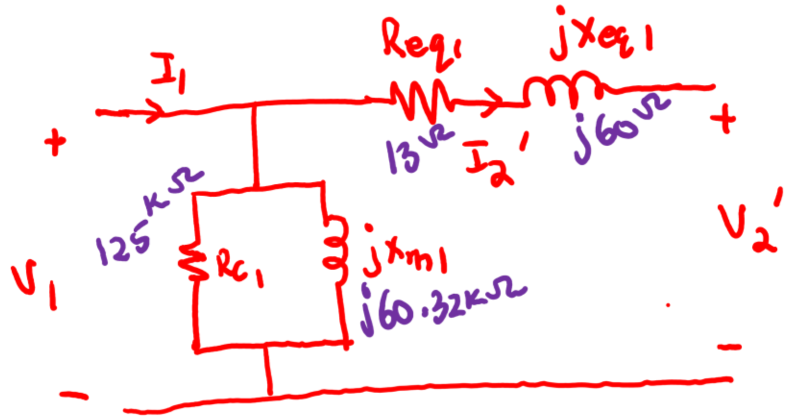
HV → 1
LV → 2

A 1-phase, 100 kVA, 11000/2200 V, 60 Hz transformer has the following parameters:

$R_{HV} = 6\ \Omega$, $L_{HV} = 0.08$ H, $L_{m(HV)} = 160$ H, $R_{C(HV)} = 125\ k\Omega$, $R_{LV} = 0.28\ \Omega$, LLV = 0.0032 H. Obtain an equivalent circuit of the transformer

1. Referred to the high voltage side
2. Referred to the low voltage side

Slide #16



$V_2' = aV_2 = 5 \times 2200 = 11000^V$

$I_1 = \dfrac{100\,kVA}{11\,kV} = 9.09^A = I_2'$

$R_{eq1} = R_1 + a^2 R_2$

$a = \dfrac{N_1}{N_2} = \dfrac{V_1}{V_2} = \dfrac{11000^V}{2200^V}$

$a = 5$

$R_{eq1} = 6 + [(5^2) \times 0.28]$

$= 13\ \Omega$

$X_{eq1} = X_1 + a^2 X_2$

$= 30.16 + [5^2 \times 1.21]$

$= 60\ \Omega$

$R_1 = 6\ \Omega$

$R_2 = 0.28\ \Omega$

$X_1 = 2\pi(60)(0.08) = 30.16\ \Omega$

$L_1 = 0.08$ H

$X_2 = 1.21\ \Omega$

$L_2 = 0.0032$ H

$L_{m1} = 160$ H

$R_{c1} = 125\ k\Omega$

$X_{m1} = 2\pi(60)(160)$

$= 60319\ \Omega$

$= 60\ k\Omega$

Equivalent circuit, Voltage regulation and Efficiency

A 1-phase, 100 kVA, 11000/2200 V, 60 Hz transformer has the following parameters:

R_{HV} = 6 Ω, L_{HV} = 0.08 H, $L_{m(HV)}$ = 160 H, $R_{C(HV)}$ = 125 kΩ, R_{LV} = 0.28 Ω, LLV = 0.0032 H. Obtain an equivalent circuit of the transformer

1. Referred to the high voltage side
2. Referred to the low voltage side

Equivalent circuit, Voltage regulation and Efficiency

1 2

S HV LV

$R_{eq2} = \frac{R_1}{a^2} + R_2 = 0.104\,\Omega$

$X_{eq2} = 0.313\,\Omega$

A 1-phase, 10 kVA, 2200/220 V, 60 Hz transformer has the following parameters:

R_{eq} = 0.104 Ω, X_{eq} = 0.313 Ω referred to low voltage side. If the transformer supplies a 10 kVA, 220 V load whose power factor is 0.85 (lagging), determine

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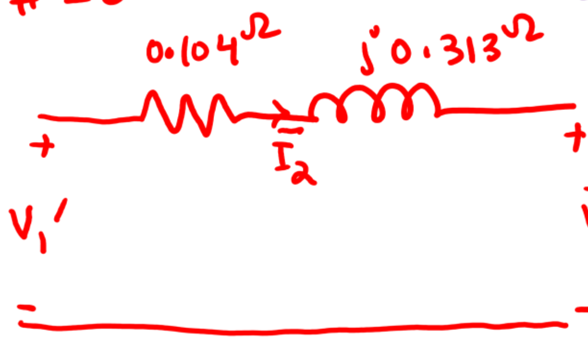
$\cos\theta = 0.85$

$\theta = 31.79°$

1. The Voltage Regulation



$$\%\,VR = \frac{|V_1'| - |V_2|}{|V_2|} \times 100\%$$

$|\bar{V}_2|$ = rated LV side voltage = 220 V

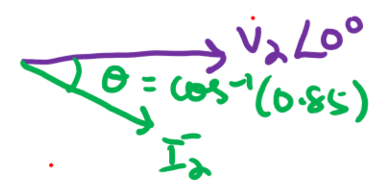
$$\bar{I}_2 = \frac{10\text{ kVA}}{220\text{ V}}\angle -31.79°$$

$$\bar{I}_2 = 45.4\angle -31.79°\text{ A}$$

$$\bar{V}_1' = \bar{V}_2 + [\bar{I}_2(0.104 + j0.313)]$$

$$= 220\angle 0° + [(45.4\angle -31.79°)(0.104 + j0.313)]$$

$$\bar{V}_1' = 231.7\angle 2.37°\text{ V}$$



$$\%\,VR = \frac{231.7 - 220}{220} \times 100\% = 5.3\%$$

Equivalent circuit, Voltage regulation and Efficiency

$R_{eq1} = 10.4\,\Omega$
$X_{eq1} = 31.3\,\Omega$

A 1-phase, 10 kVA, 2200/220 V, 60 Hz transformer has the following parameters:
S — HV (1) — LV (2)

$P_{core} = 100\,W$

R_{eq} = 10.4 Ω, X_{eq} = 31.3 Ω referred to high voltage side. The core loss is 100 W at rated terminal voltage. The transformer is providing 75% of the full load with load power factor 0.6 (lagging), determine

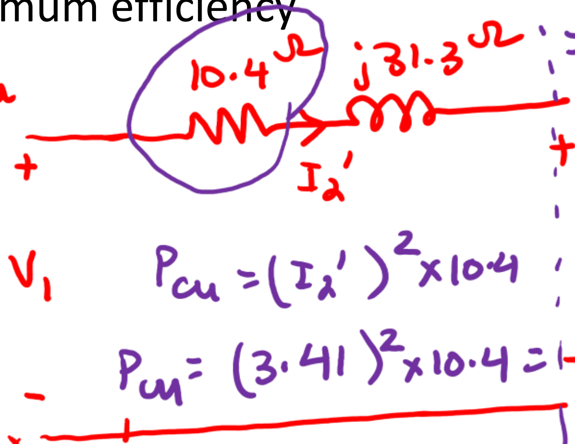
1. The Efficiency
2. The value of the maximum efficiency
3. The transformer power output at the maximum efficiency

$$a = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{2200}{220} = 10$$

$$V_2' = aV_2 = 2200 \qquad I_2' = \frac{I_2}{a}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$P_{in} = P_{out} + P_{core} + P_{cu}$$



$$P_{cu} = (I_2')^2 \times 10.4$$

$$P_{cu} = (3.41)^2 \times 10.4 = 121\,W$$

$$P_{out} = V_2' I_2' \cos\theta_2$$

$$\eta = \frac{2200 \times \left(0.75 \times \frac{45.4}{10}\right) \times 0.6}{(P_{out} + P_{core} + P_{cu})} \times 100\%$$

$$0.75 \times \frac{45.4}{10} = 3.41\,A$$

$$\eta = \frac{(2200 \times 3.41 \times 0.6)}{(2200 \times 3.41 \times 0.6) + 100 + 121} \times 100\% = 95.32\%$$

Equivalent circuit, Voltage regulation and Efficiency

A 1-phase, 10 kVA, 2200/220 V, 60 Hz transformer has the following parameters:

R_{eq} = 10.4 Ω, X_{eq} = 31.3 Ω referred to high voltage side. The core loss is 100 W at rated terminal voltage. The transformer is providing 75% of the full load with load power factor 0.6 (lagging), determine

The value of the maximum efficiency

η_{max} occurs @ $P_{cu} = P_{core}$, $\cos\theta_2 = 1$

$P_{cu} = 100^W$

$(I_2'')^2 \times 10.4 = 100$

$I_2'' = \sqrt{\frac{100}{10.4}}$

$= 3.1^A$

$$\eta_{max} = \frac{V_2' I_2'' \cos\theta_2}{[V_2' I_2'' \cos\theta_2] + P_{core} + P_{cu}} \times 100\%$$

$$= \frac{2200 \times 3.1 \times 1}{[2200 \times 3.1 \times 1] + 100 + 100} \times 100\% = 97\%$$

$P_{out} = 6820^W$

@ unity p.f.

to deliver max. η, the transformer should be loaded at $\frac{6820}{10000} = 68.2\%$

Equivalent circuit, Voltage regulation and Efficiency

A 1-phase, 10 kVA, 2200/220 V, 60 Hz transformer has the following parameters:

R_{eq} = 10.4 Ω, X_{eq} = 31.3 Ω referred to high voltage side. The core loss is 100 W at rated terminal voltage. The transformer is providing 75% of the full load with load power factor 0.6 (lagging), determine

The transformer power output at the maximum efficiency

$$P_{out}\Big|_{\eta=\eta_{max}} = 2200 \times 3.1 \times 1 = 6820 \text{ W}$$