# ENGI-1500
# Physics -2

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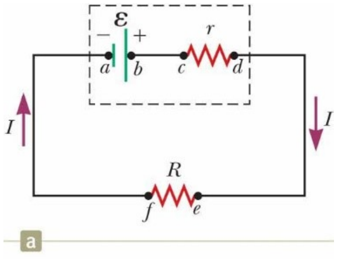
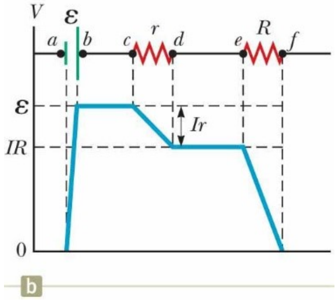
Winter 2023

# Reminder of the previous week

## Electromotive Force (EMF)





$$\Delta V = \varepsilon - Ir$$

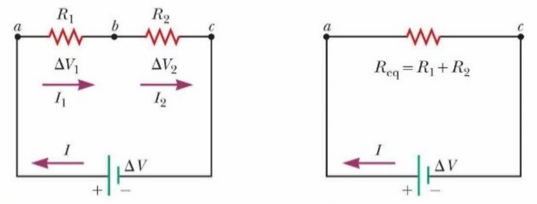
$$\varepsilon = IR + Ir$$

$$I = \frac{\varepsilon}{R + r}$$

$$I\varepsilon = I^2 R + I^2 r$$

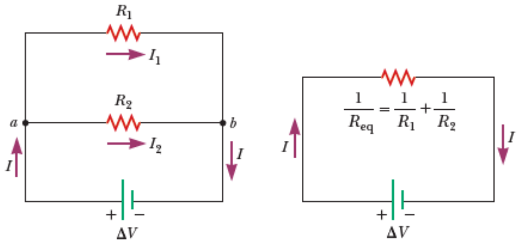
## Resistor Combinations

### Series Combination




$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n$$

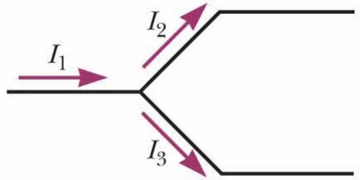
### Parallel Combination




$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

## Kirchhoff's Rules

### Kirchhoff's Current (Junction) Rule

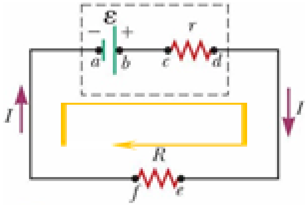


$$\sum_{\text{junction}} I = 0$$

$$I_1 - I_2 - I_3 = 0$$

$$I_1 = I_2 + I_3$$

### Kirchhoff's Voltage (Loop) Rule



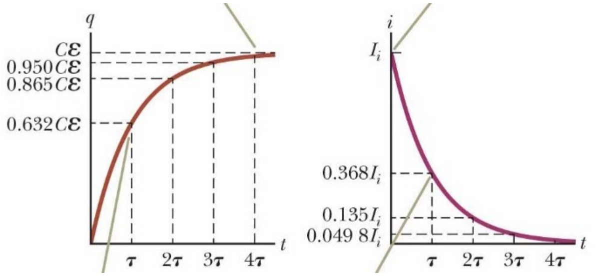
$$\sum_{\text{closed loop}} \Delta V = 0$$

## RC Circuits

$$q(t) = C\varepsilon\left(1 - e^{-t/RC}\right) = Q_{\max}\left(1 - e^{-t/RC}\right)$$

$$i(t) = \frac{\varepsilon}{R} e^{-t/RC}$$

$$\tau = RC$$






Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

# Week 5 / Class 5

Magnetic Fields (Ch. 28)

# Outline of Week 5 / Class 5

- Reminder of the previous week

- Magnetic Fields (Ch. 28)
  - Analysis Model: Particle in a Magnetic Field
  - Motion of a Charged Particle in a Uniform Magnetic Field
  - Applications Involving Charged Particles Moving in a Magnetic Field
  - Magnetic Force Acting on a Current Carrying Conductor
  - Torque on a Current Loop In a Uniform Magnetic Field
  - The Hall Effect

- Examples
- Next week's topic

# Magnetic Fields (Ch. 28)

## Analysis Model: Particle in a Magnetic Field

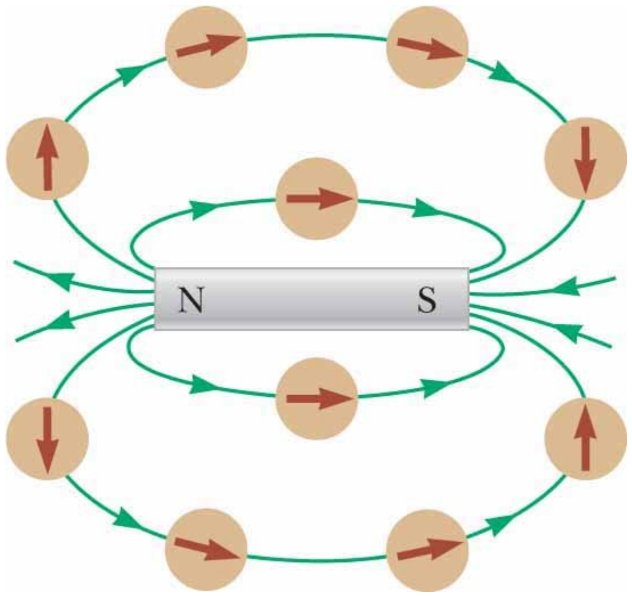
# Magnetic Field

- Earlier, we described the electric field surrounding electric charges.
- Moving electric charges also create a ***magnetic field*** around them.
- A ***magnetic field*** also surrounds a magnetic substance making up a permanent magnet.

- Source of any magnetic field has two poles (north and south)
- If bar magnet is suspended from its midpoint and can swing freely in horizontal plane →
  - Will rotate until its magnetic north pole points toward Earth's geographic North Pole and its magnetic south pole points toward Earth's geographic South Pole

- Poles of magnet have similarities to electric charges:
  - Experiments show magnetic poles exert attractive or repulsive forces on each other
  - These forces vary as inverse square of distance between interacting poles

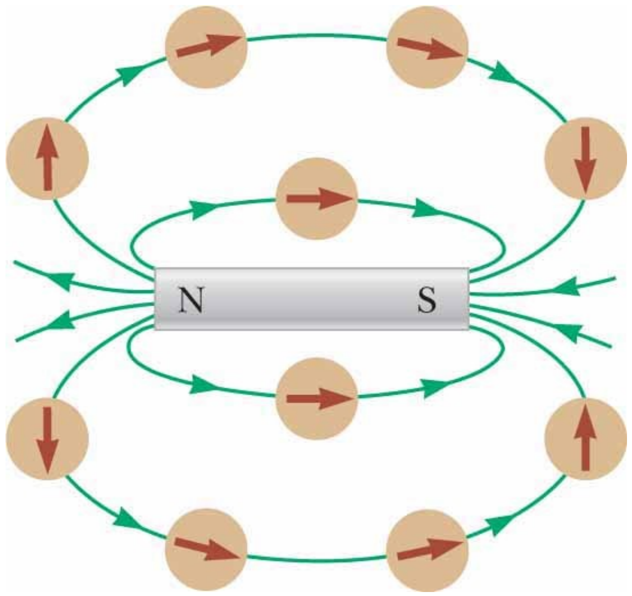
- Major differences between electric charges and magnetic poles:
  - Electric charges can be isolated (e.g., electron and proton)
  - Single magnetic pole has never been isolated →
    - Magnetic poles are always found in pairs
    - No matter how many times a permanent magnet is cut in two; each piece always has north and south pole




Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

# Magnetic Field

- Historically, symbol $\vec{\mathbf{B}}$ is used to represent the magnetic field
- Direction of the magnetic field **B** at any location is the direction in which north pole of compass needle points at that location
  - Magnetic field is represented with magnetic field lines
- Figure: shows how the magnetic field lines of a bar magnet can be traced using a compass
- **Note**: magnetic field lines outside the magnet point away from the north pole and toward the south pole

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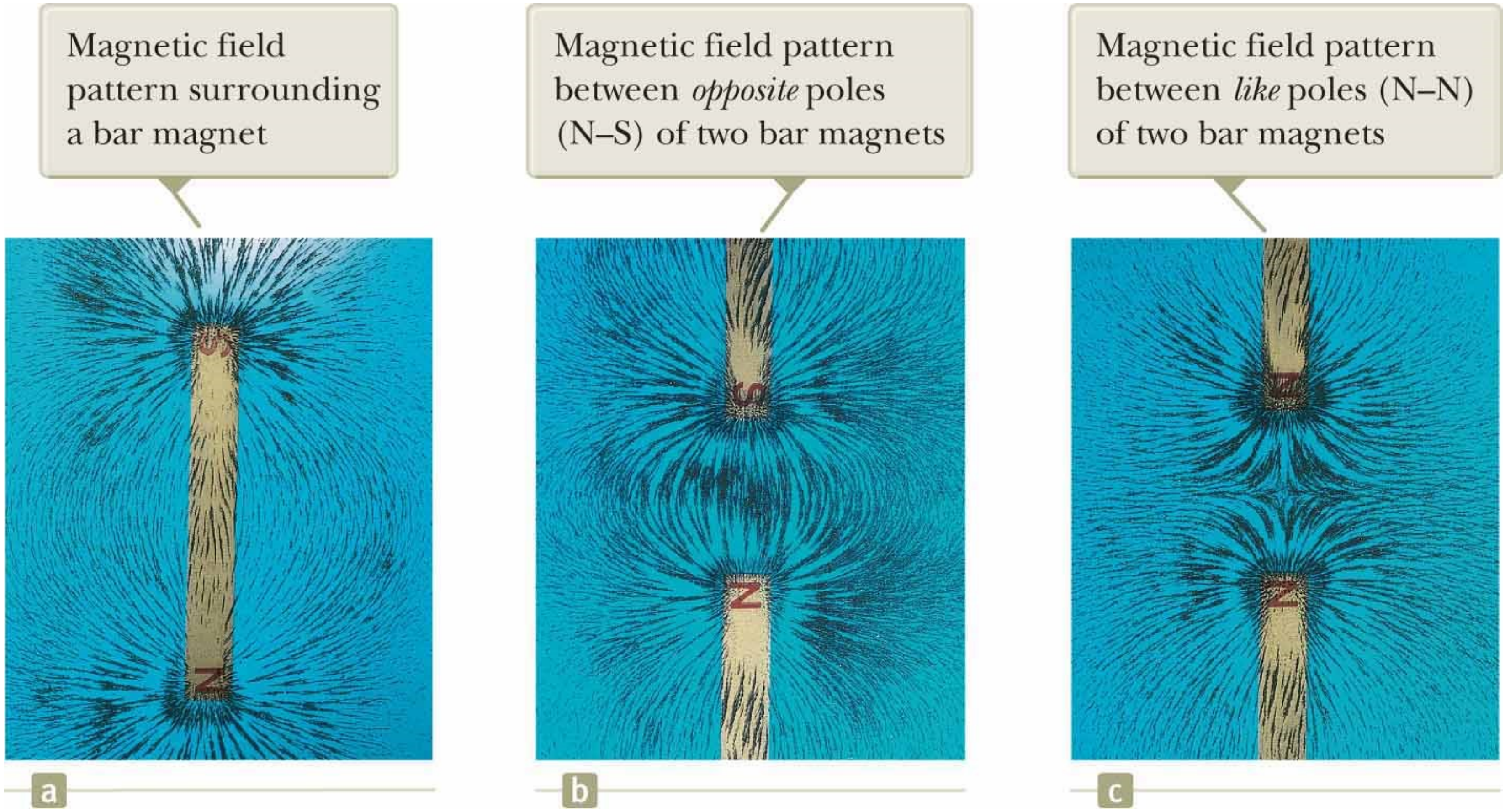
# Magnetic Field

Magnetic field patterns can be displayed with iron filings sprinkled on paper near magnets.

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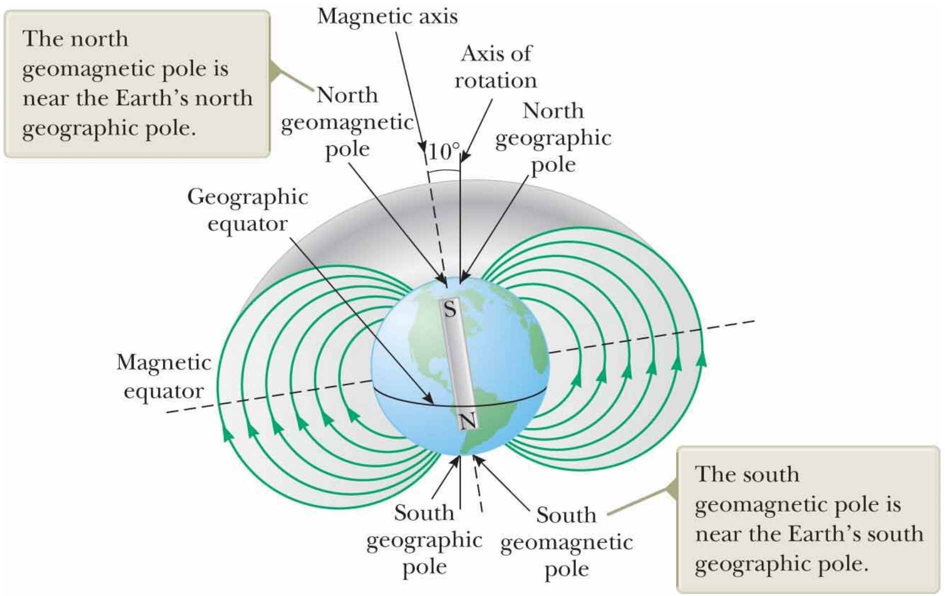
# Magnetic Field

- Earth's magnetic field is similar to a bar magnet (figure)
- Earth's south magnetic pole is located near the north geographic pole, and vs – north magnetic pole is located near the south geographic pole
- The position of Earth's magnetic poles have been slightly moving over time - they moved over hundreds of miles since they were first discovered

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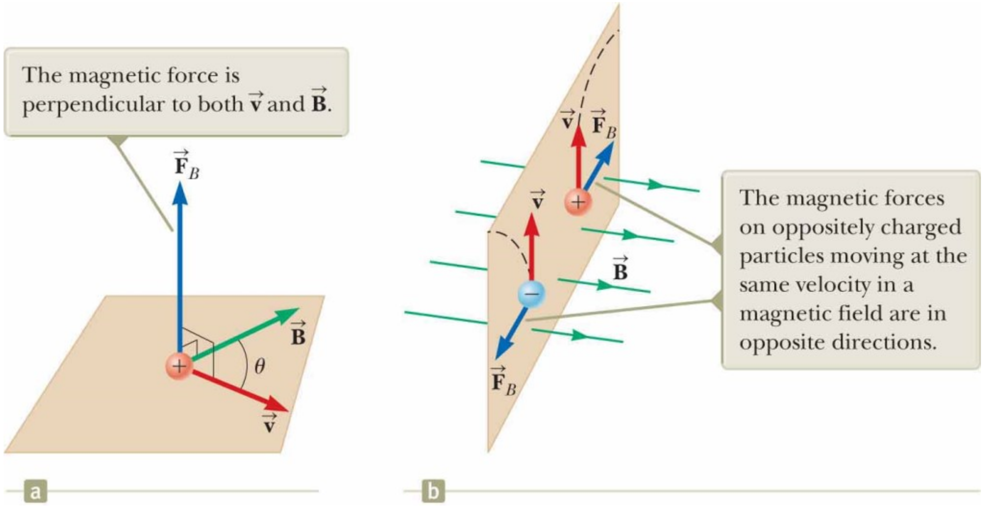
# Magnetic Field

- We can quantify the magnetic field $\vec{B}$ by using the particle in a field model:

- The existence of a magnetic field at some point in space can be determined by measuring the magnetic force $\vec{F}_B$ exerted on an appropriate test particle placed at that point.

- If we perform such an experiment with a particle with charge ***q***, we would find:
  - Magnetic force is proportional to the charge ***q*** of the particle.
  - Magnetic force on a negative charge is directed opposite to the force on a positive charge moving in same direction.
  - Magnetic force is proportional to the magnitude of the magnetic field vector **B**.

- Above findings are similar to the findings on electrical forces.

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# Magnetic Field

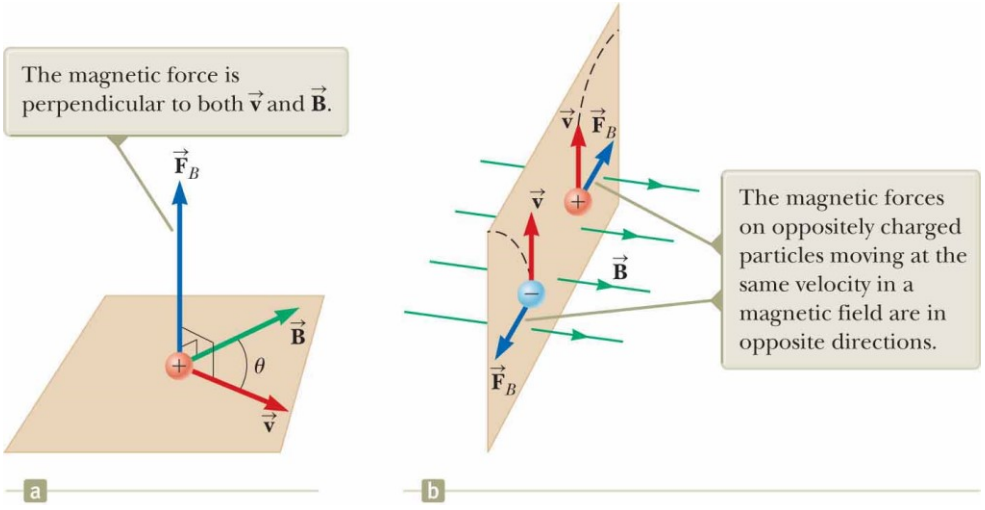
- We also find the following results, which are ***totally different*** from those for experiments on electric forces:

    - The magnetic force is proportional to the speed (***v)*** of the particle.

    - If the velocity vector makes an angle ***Θ*** with the magnetic field, the magnitude of the magnetic force is proportional to ***sinΘ***.

    - When a charged particle moves ***parallel*** to the magnetic field vector, the magnetic force on the charge is zero.

    - When a charged particle moves in a direction ***not parallel*** to the magnetic field vector, the magnetic force acts in a direction perpendicular to both $\vec{v}$ and $\vec{B}$; that is, the magnetic force is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$

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# Magnetic Field

- The magnetic force is distinctive because it depends on the velocity of the particle and because its direction is perpendicular to both $\vec{v}$ and $\vec{B}$

- These observations can be summarized in a compact way by writing the magnetic force in the form:

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

- By definition of the cross product, the force is perpendicular to both $\vec{v}$ and $\vec{B}$

- If $\vec{v}$ and $\vec{B}$ are not perpendicular (with ϴ as the smaller angle between them);

$$F_B = |q| vB \sin\theta$$

# Magnetic Field

- SI unit of magnetic field is ***Tesla (T)***

$$1\ \mathrm{T} = 1\frac{\mathrm{N}}{\mathrm{C}\cdot\mathrm{m/s}} \qquad 1\ \mathrm{T} = 1\frac{\mathrm{N}}{\mathrm{A}\cdot\mathrm{m}}$$

- Non-SI magnetic-field unit in common use is ***Gauss (G)***
  - Related to Tesla through conversion

$$1\ \mathrm{T} = 10^4\ \mathrm{G}$$

- Table: shows typical values of magnetic fields

**TABLE 28.1** Some Approximate Magnetic Field Magnitudes

| Source of Field | Field Magnitude (T) |
|---|---|
| Strong superconducting laboratory magnet | 30 |
| Strong conventional laboratory magnet | 2 |
| Medical MRI unit | 1.5 |
| Bar magnet | $10^{-2}$ |
| Surface of the Sun | $10^{-2}$ |
| Surface of the Earth | $5 \times 10^{-5}$ |
| Inside human brain (due to nerve impulses) | $10^{-13}$ |

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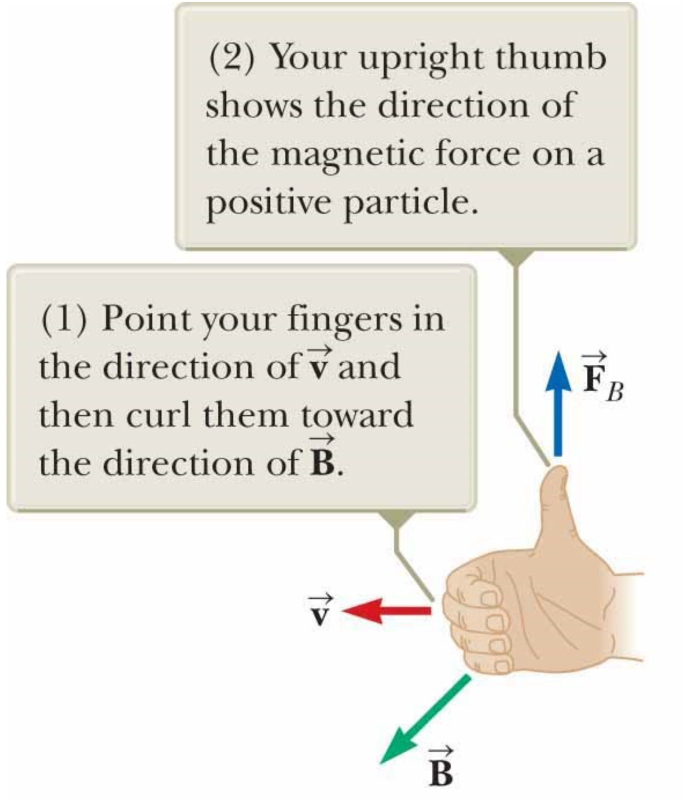
# Magnetic Field

## Right-Hand Rule

- Figure: right-hand rule for determining the direction of cross product $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ and $\vec{\mathbf{F}}_B$
- Point four fingers of your right hand along the direction of $\vec{\mathbf{v}}$ with your palm facing $\vec{\mathbf{B}}$
- Curl them toward $\vec{\mathbf{B}}$
- Your extended thumb (at right angle to your fingers) points in the direction of $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$
- $\vec{\mathbf{F}}_B$ is in the direction of your thumb if $q$ is positive
- $\vec{\mathbf{F}}_B$ is in the opposite direction of your thumb if $q$ is negative

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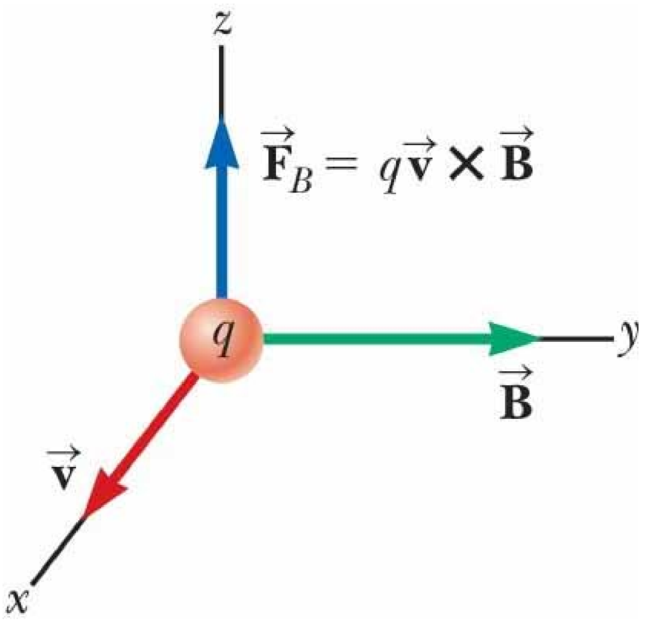
# Magnetic Field

## Comparison of Electric and Magnetic Particle in Field Models

| | |
|---|---|
| • Electric force vector along direction of electric field | • Magnetic force vector perpendicular to magnetic field |
| • Electric force acts on charged particle regardless of whether particle is moving | • Magnetic force acts on charged particle only when particle is in motion |
| • Electric force does work in displacing a charged particle | • Magnetic force associated with steady magnetic field does no work when a particle is displaced → force perpendicular to displacement of its point of application |

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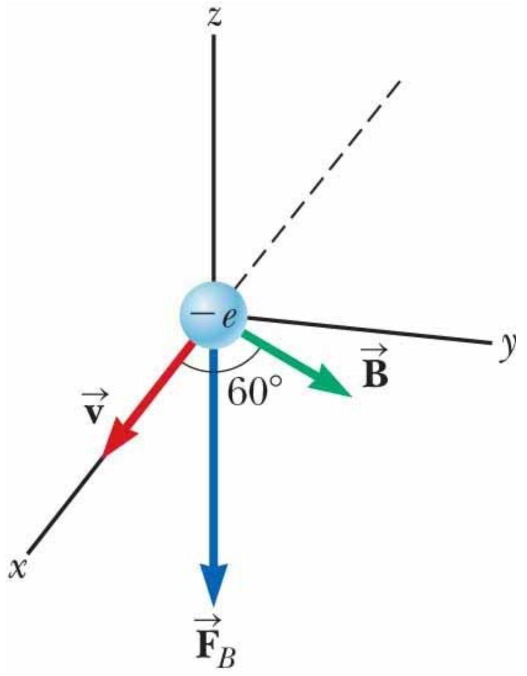
# An Electron Moving in a Magnetic Field

Example 28.1

An electron moves through space as a cosmic ray with a speed of ***v= 8.0 x 10⁶ m/s*** along the ***x axis***. At its location, the magnetic field of the Earth has a magnitude ***B = 0.050 mT***, and is directed at an angle of ***Θ = 60°*** to the x axis, lying in the xy plane. Calculate the magnetic force on the electron.

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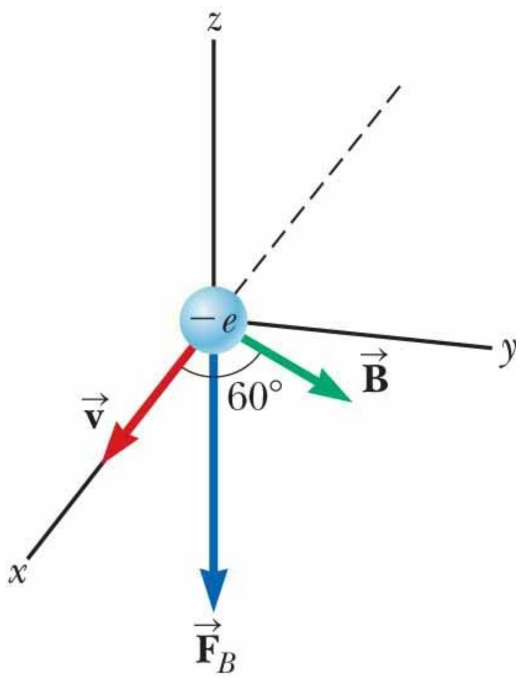
## Solution

First, use the right-hand rule to find the direction of the magnetic force.

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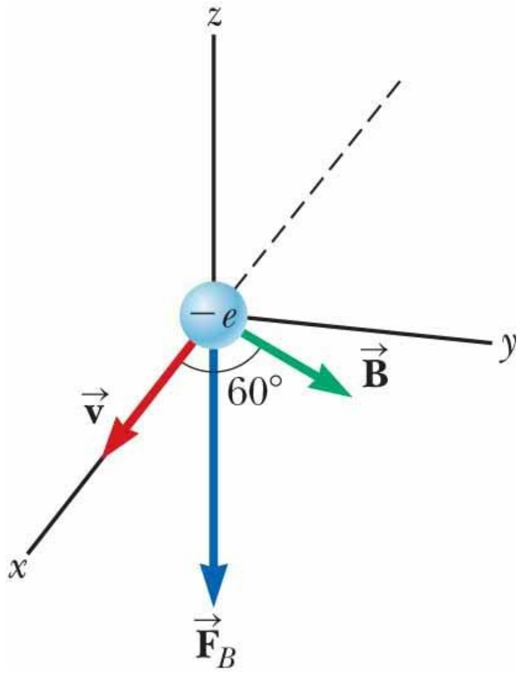
# An Electron Moving in a Magnetic Field

## Example 28.1

An electron moves through space as a cosmic ray with a speed of ***v= 8.0 x 10^6^ m/s*** along the ***x axis***. At its location, the magnetic field of the Earth has a magnitude ***B = 0.050 mT***, and is directed at an angle of ***Θ = 60°*** to the x axis, lying in the xy plane. Calculate the magnetic force on the electron.




## Solution

First, use the right-hand rule to find the direction of the magnetic force.

When there is an angle between **v** and **B**, recall the force calculation formula;

$$F_B = |q| vB \sin\theta$$

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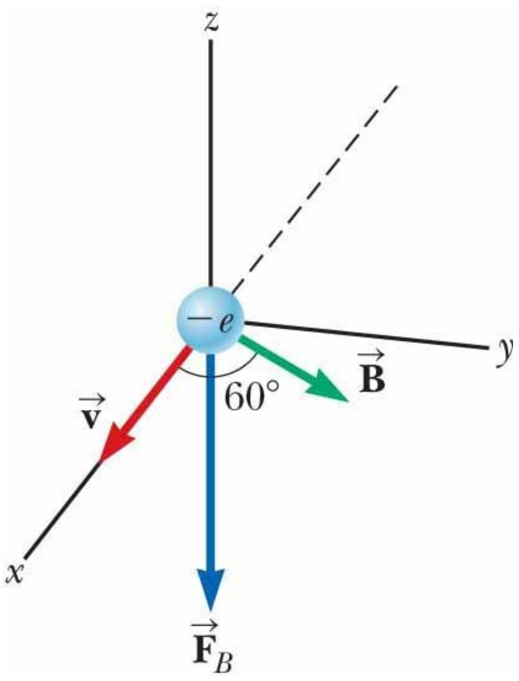


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## Solution

First, use the right-hand rule to find the direction of the magnetic force.

When there is an angle between **v** and **B**, recall the force calculation formula;

$$F_B = |q| vB \sin\theta$$

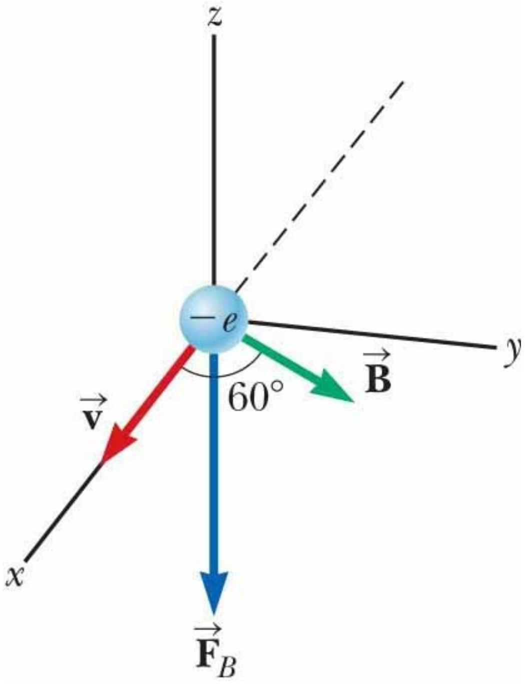
Applying the known parameters;

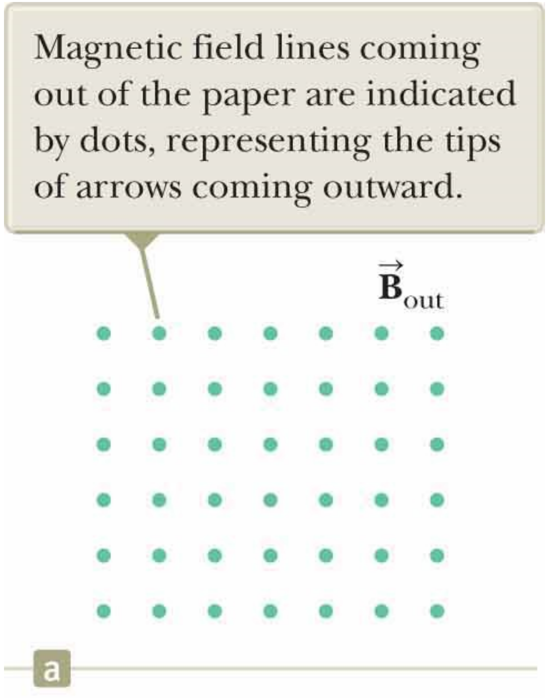
$$\begin{aligned} F_B &= |q| vB \sin\theta \\ &= \left(1.6\times10^{-19}\ \text{C}\right)\left(8.0\times10^{6}\ \text{m/s}\right)\left(5.0\times10^{-5}\ \text{T}\right)\left(\sin 60°\right) \\ &= 5.5\times10^{-17}\ \text{N} \end{aligned}$$

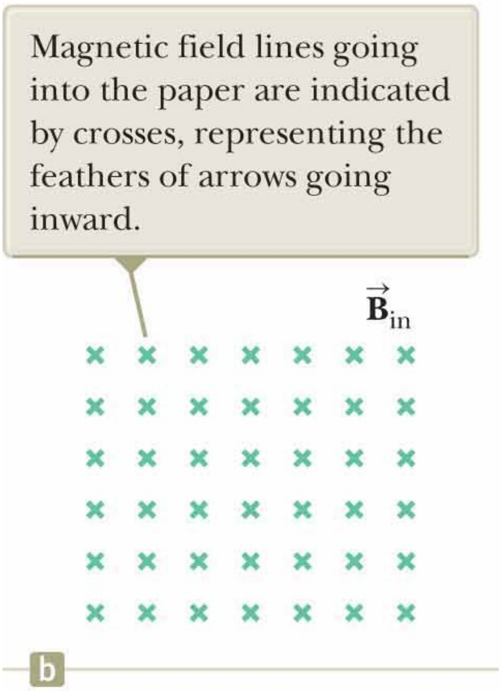
# Magnetic Fields (Ch. 28)

## Motion of a Charged Particle in a Uniform Magnetic Field

# Representation of Magnetic Field

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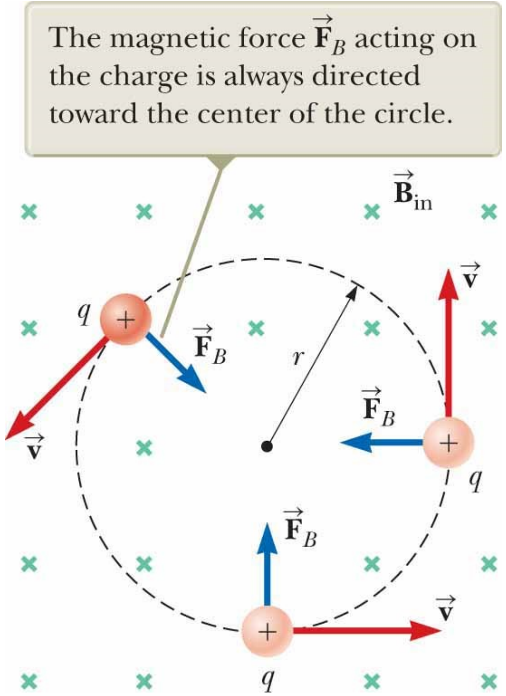
# Motion of a Charged Particle in a Uniform Magnetic Field

- Consider a special case of a positively charged particle moving in a uniform magnetic field
  - Initial velocity vector of particle perpendicular to field
  - Assume direction of magnetic field into the page (figure)
- Magnetic force on the particle is perpendicular to both the magnetic field lines and the velocity of the particle
- As the particle changes direction of its velocity in response to the magnetic force →
  - Magnetic force remains perpendicular to the velocity
  - Path of the particle is a ***circle***
- Rotation is counter-clockwise for a positive charge in a magnetic field directed into the page
  - If ***q*** were negative → clockwise rotation.

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# Motion of a Charged Particle in a Uniform Magnetic Field

- Use ‘particle under net force model’ to write ***Newton’s second law*** for particle:

$$\sum F = F_B = ma$$

- Because the particle is in a uniform circular motion, we can replace the acceleration with ***centripetal acceleration***:

$$F_B = qvB = \frac{mv^2}{r}$$

- Solving for the radius:

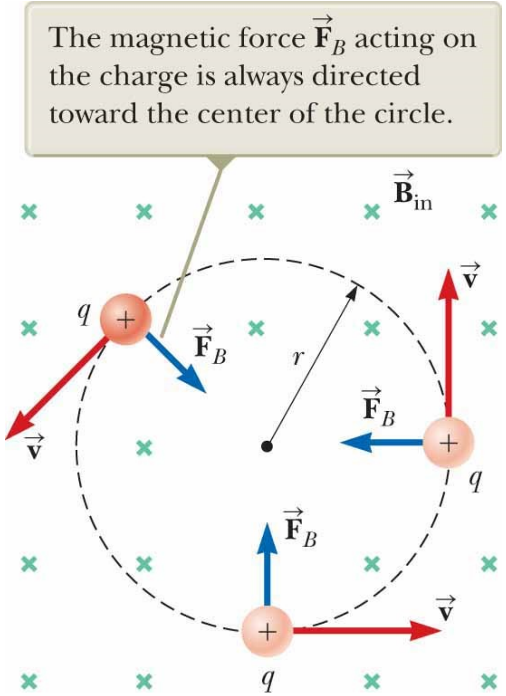
$$r = \frac{mv}{qB}$$

- Angular speed of the particle is:

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- Period of motion (time interval particle requires to complete one revolution):

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

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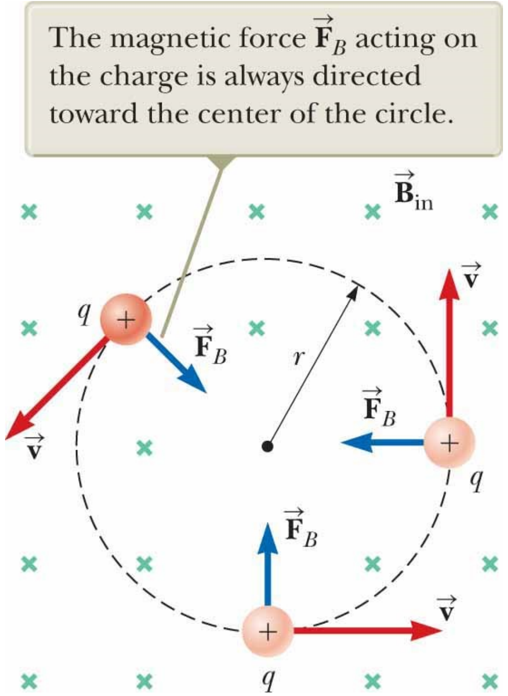
- Period of motion (time interval particle requires to complete one revolution):

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

- Angular speed of the particle and the period of circular motion do not depend on speed of the particle or on the radius of the orbit.
- ***Angular speed ω: cyclotron frequency***
- Charged particles circulate at this angular frequency in the type of accelerator called ***cyclotron***

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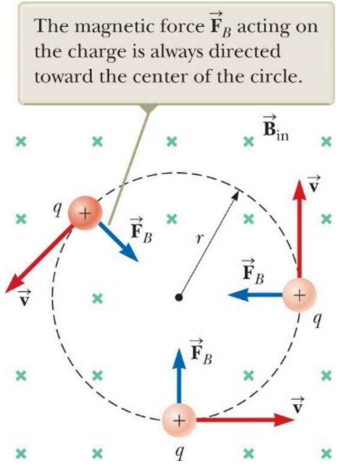
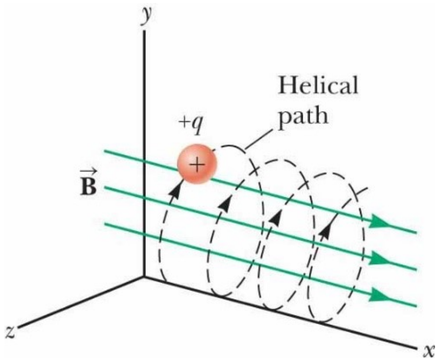
# Motion of a Charged Particle in a Uniform Magnetic Field

- Top figure: charged particle's velocity vector ***v*** is perpendicular to the magnetic field ***B***

- If charged particle moves in a uniform magnetic field with a velocity at some arbitrary angle with respect to B →
  - The path becomes a ***helix*** (bottom figure)

- Bottom figure: Magnetic field ***B*** is directed in x-direction
  - Result: $a_x = 0$
  - x component of the velocity of the particle remains constant
  - Charged particle is a particle in equilibrium in this direction

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# A Proton Moving Perpendicular to a Uniform Magnetic Field

## Example 28.2

A proton is moving in a circular orbit of radius ***r = 14cm*** in a uniform ***B = 0.35T*** magnetic field perpendicular to the velocity of the proton. Find the speed of the proton (***v***).

$$r = \frac{mv}{qB}$$

**TABLE 22.1** Charge and Mass of the Electron, Proton, and Neutron

| Particle | Charge (C) | Mass (kg) |
|---|---|---|
| Electron (e) | $-1.602\,176\,5 \times 10^{-19}$ | $9.109\,4 \times 10^{-31}$ |
| Proton (p) | $+1.602\,176\,5 \times 10^{-19}$ | $1.672\,62 \times 10^{-27}$ |
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## Solution

Re-organizing the '***r***' equation to find the speed of the particle (***v***):

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$$v = \frac{qBr}{m_p}$$

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## Solution

Re-organizing the '***r***' equation to find the speed of the particle (***v***):

$$v = \frac{qBr}{m_p}$$

Solving with the provided numerical values:

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# A Proton Moving Perpendicular to a Uniform Magnetic Field

## Example 28.2

A proton is moving in a circular orbit of radius ***r = 14cm*** in a uniform ***B = 0.35T*** magnetic field perpendicular to the velocity of the proton. Find the speed of the proton (***v***).

$$r = \frac{mv}{qB}$$

**TABLE 22.1** Charge and Mass of the Electron, Proton, and Neutron

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$$= \boxed{4.7 \times 10^{6}\,\mathrm{m/s}}$$

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**Example 28.3**

In an experiment designed to measure the magnitude of a uniform magnetic field (***B***), electrons are accelerated from rest through a potential difference of ***350V*** and then enter a uniform magnetic field that is perpendicular to the velocity vector of the electrons. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be ***r = 7.5cm***. (Such a curved beam of electrons is shown in the figure)

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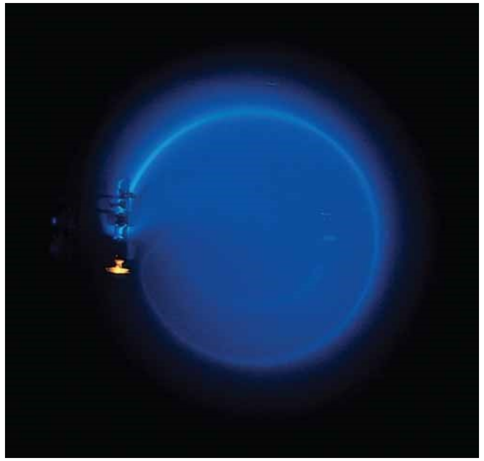

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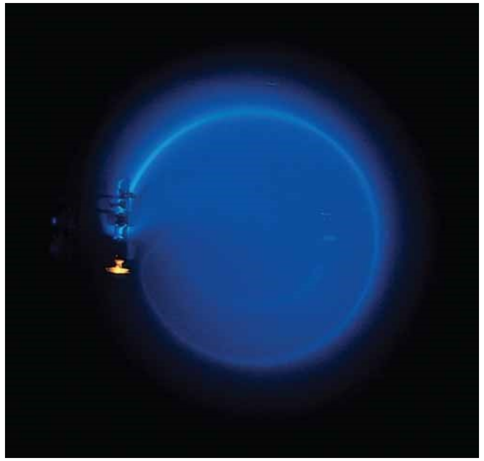

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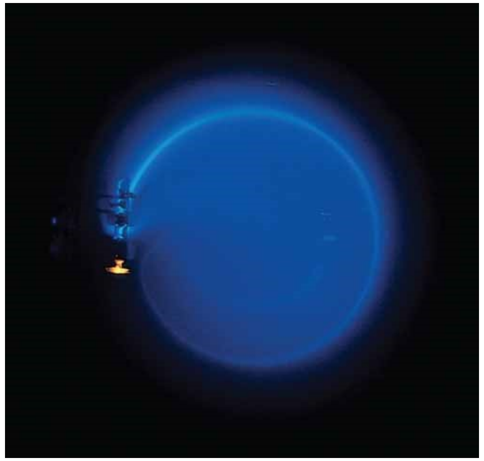

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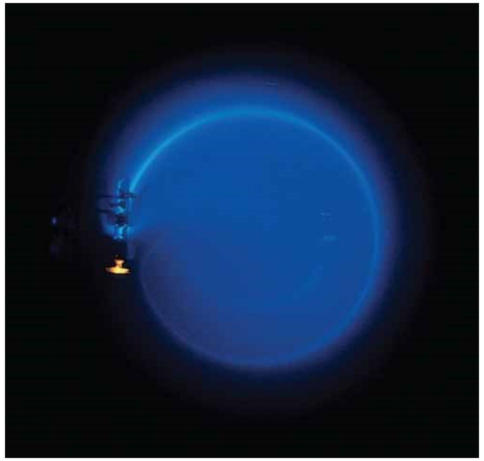

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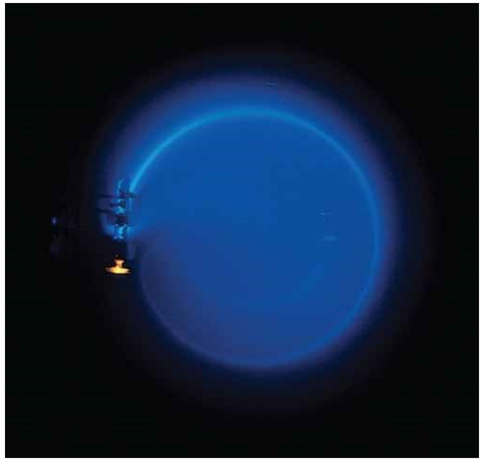
$$\Delta K + \Delta U = 0$$

$$\left(\tfrac{1}{2}m_e v^2 - 0\right) + (q\,\Delta V) = 0$$

$$v = \sqrt{\frac{-2q\,\Delta V}{m_e}}$$

$$v = \sqrt{\frac{-2(-1.60 \times 10^{-19}\ \mathrm{C})(350\ \mathrm{V})}{9.11 \times 10^{-31}\ \mathrm{kg}}} = 1.11 \times 10^7\ \mathrm{m/s}$$



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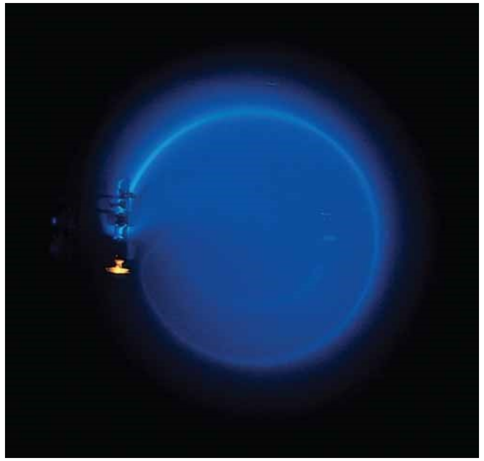

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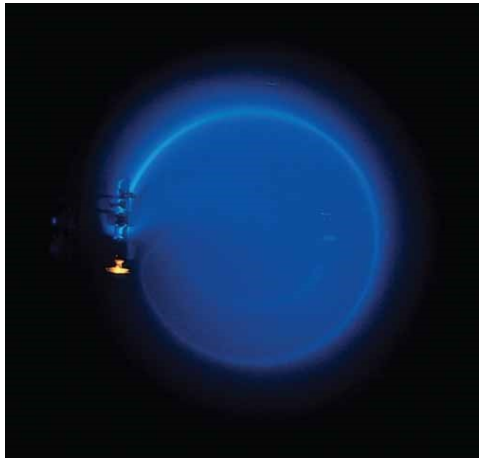

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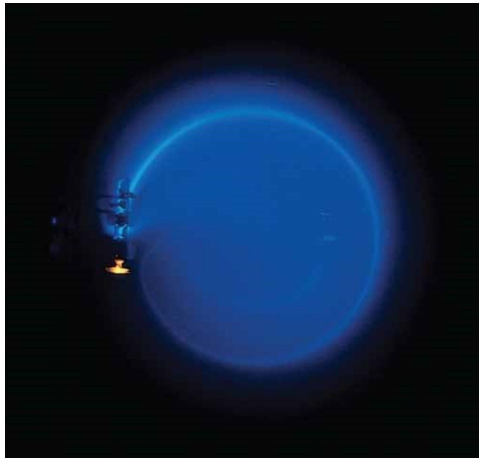

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## Solution – a

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$$B = \frac{m_e v}{er}$$

$$B = \frac{(9.11 \times 10^{-31}\ \mathrm{kg})(1.11 \times 10^{7}\ \mathrm{m/s})}{(1.60 \times 10^{-19}\ \mathrm{C})(0.075\ \mathrm{m})}$$

$$B = \boxed{8.4 \times 10^{-4}\ \mathrm{T}}$$

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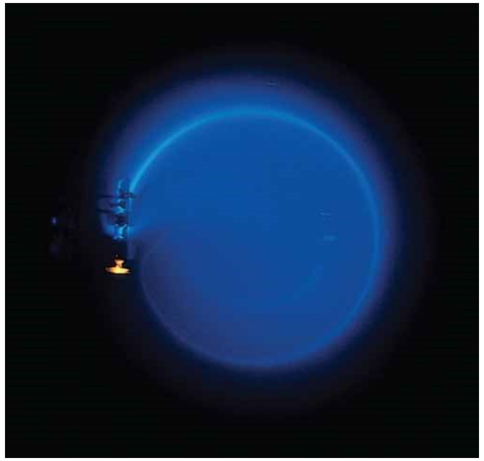
## Solution – b

We know the speed v, we know the radius r, we can simply use the angular speed definition to find ***w***:

$$\omega = \frac{v}{r}$$



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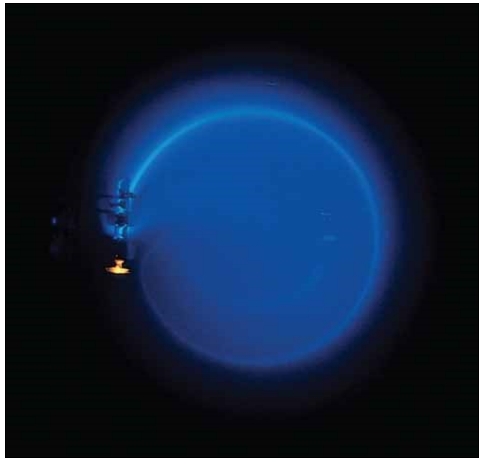
$$\omega = \frac{v}{r} = \frac{1.11 \times 10^7 \text{ m/s}}{0.075 \text{ m}} = \boxed{1.5 \times 10^8 \text{ rad/s}}$$



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# Magnetic Fields (Ch. 28)

Applications Involving Charged Particles Moving in a Magnetic Field

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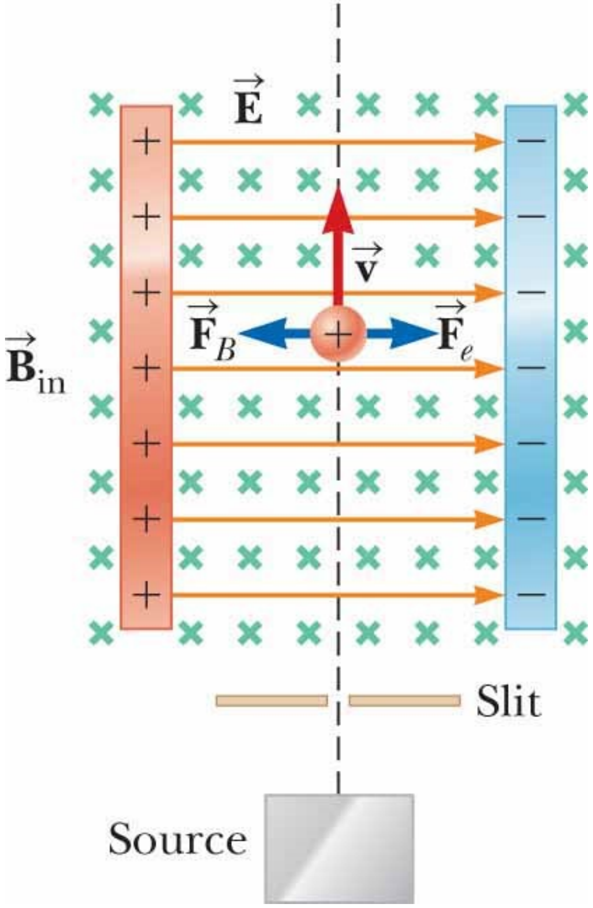
- A charge moving with a velocity $\vec{v}$ in the presence of both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ experiences both an electric force $q\vec{E}$ and a magnetic force $q\vec{v} \times \vec{B}$.

- The total force (called the Lorentz force) acting on the charge is:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

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# Applications Involving Charged Particles Moving in a Magnetic Field

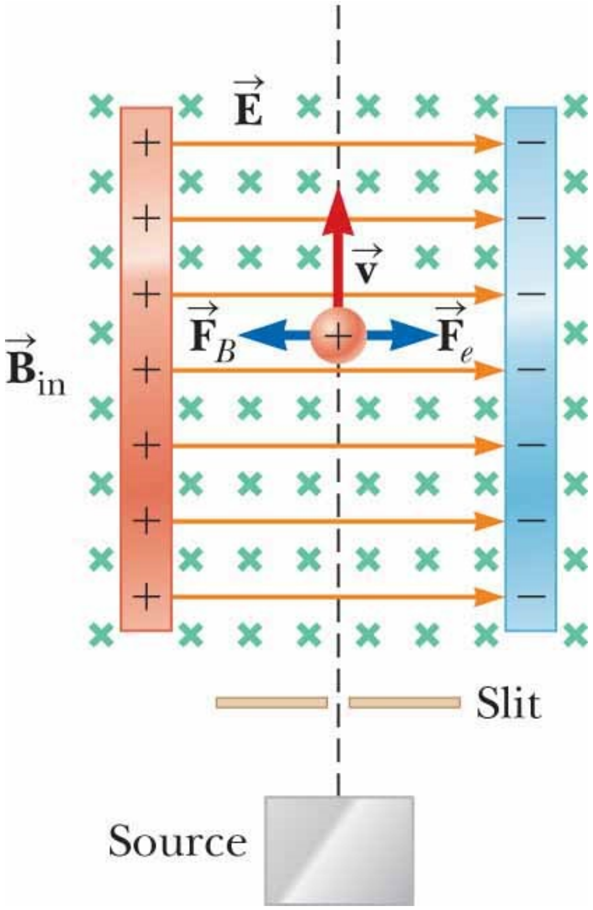
## Velocity Selector

- In some experiments, the speed of the charged particles is very important. To make sure all particles move with the same velocity, a ***velocity selector*** can be used.

- For a positively charged particle ***q***, if the ***E*** field and the ***B*** field are chosen to satisfy ***qE = qvB***, the forces cancel and the charged particle moves in equilibrium in a straight line.

- In the equilibrium: $$v = \frac{E}{B}$$

- Particles with a ***speed > v are pulled to the left***.
- Particles with a ***speed < v are pulled to the right***.

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# Applications Involving Charged Particles Moving in a Magnetic Field

## The Mass Spectrometer

- A mass spectrometer separates ions according to their ***mass-to-charge ratio***.

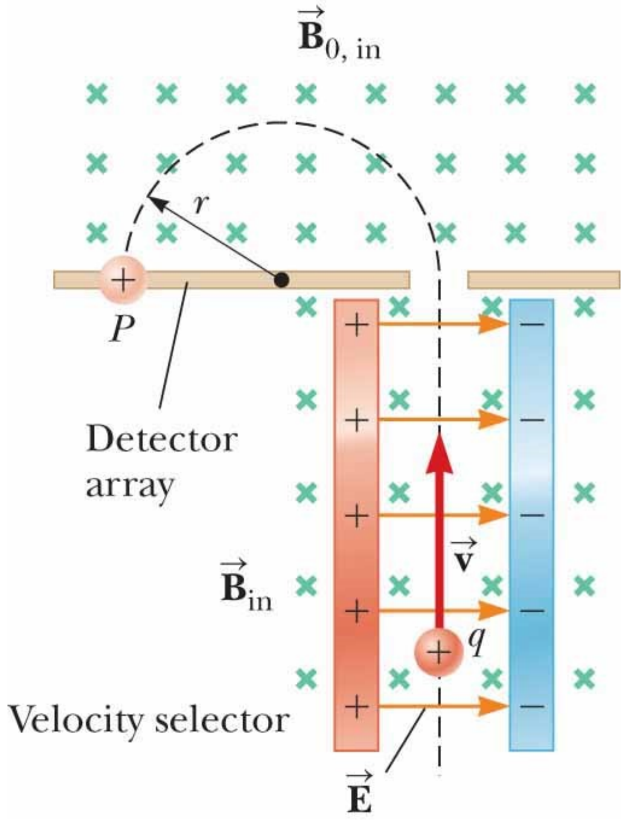
- In one version of this device, known as the ***Bainbridge mass spectrometer***, a beam of ions first passes through a velocity selector and then enters a second uniform magnetic field $\boldsymbol{B_0}$ that has the same direction as the magnetic field in the selector.

- Upon entering the second magnetic field, the ions move in a semicircle of radius ***r*** before striking a detector array at ***P***.

- Using the previously provided radius equation: $$r = \frac{mv}{qB}$$

- We can solve for the ratio of mass (***m***) over charge (***q***) as: $$\frac{m}{q} = \frac{rB_0}{v} \rightarrow \frac{m}{q} = \frac{rB_0 B}{E}$$

- Therefore, measuring ***r*** gives us the ***m/q*** ratio. In practice, one usually measures the masses of various ***isotopes of a given ion***, with the ions all carrying the same charge q. In this way, the mass ratios can be determined even if q is unknown.

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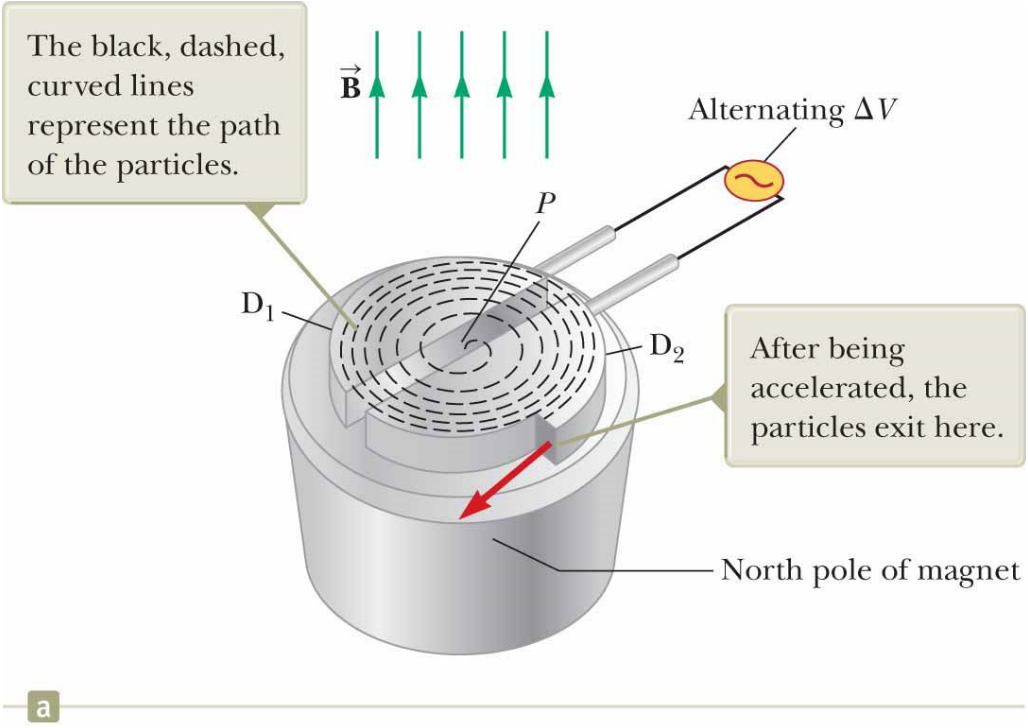
# Applications Involving Charged Particles Moving in a Magnetic Field

## The Cyclotron

- A ***cyclotron*** is a device that can accelerate charged particles to very high speeds.
- The energetic particles produced can be used to produce nuclear reactions of interest to researchers.
- Both electric and magnetic forces play key roles in the operation of a cyclotron.
- The charges move inside two semi-circular containers D1 and D2, referred to as ***dees*** because of their shape like the letter D.





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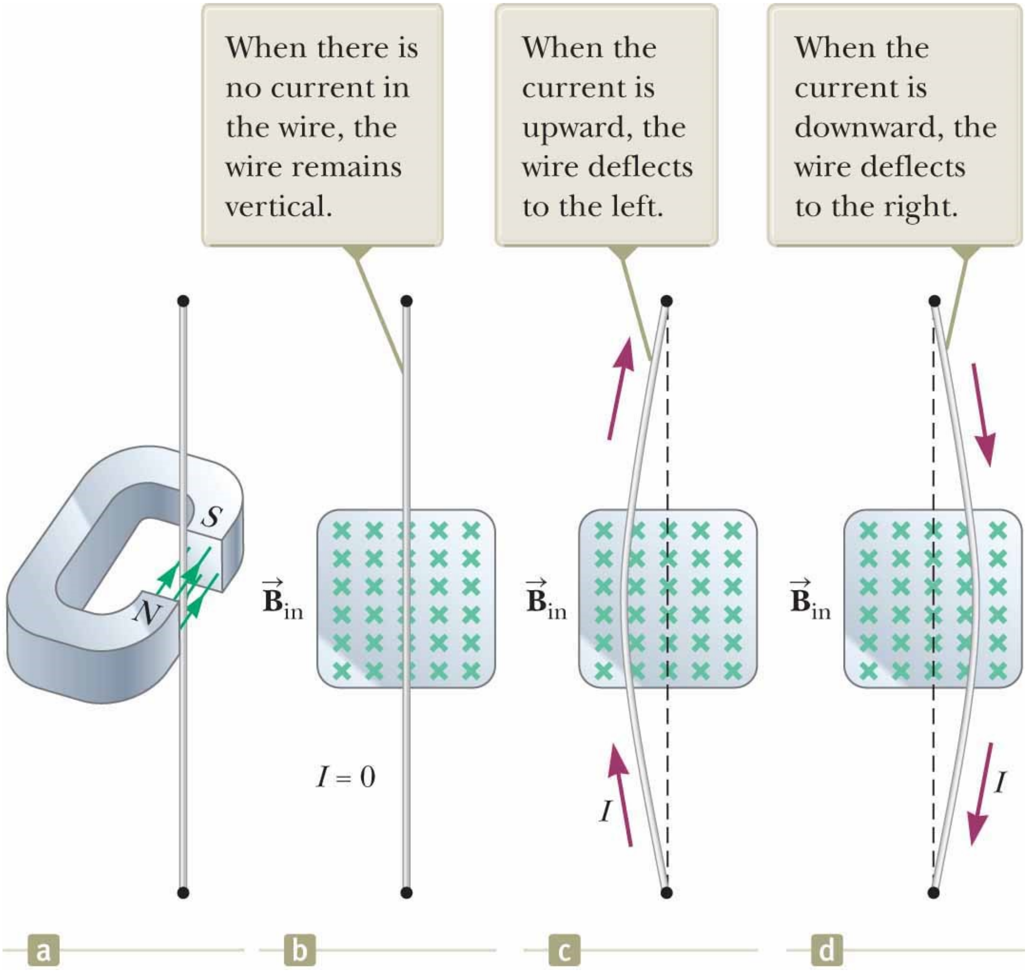
# Magnetic Fields (Ch. 28)

## Magnetic Force Acting on a Current-Carrying Conductor

# Magnetic Force Acting on a Current-Carrying Conductor

- So far, we've learned that a magnetic force is exerted on a ***charged particle*** when the particle is moving through a magnetic field.
- Early on, we've also learned that electric ***current*** can be described as collection of many charged particles in motion.
- Therefore, it shouldn't be surprising that a ***current-carrying wire also experiences a force*** when the wire is placed in a magnetic field.
- We can visualize the magnetic force acting on a current-carrying conductor by hanging a wire between the poles of a magnet.




Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

# Magnetic Force Acting on a Current-Carrying Conductor

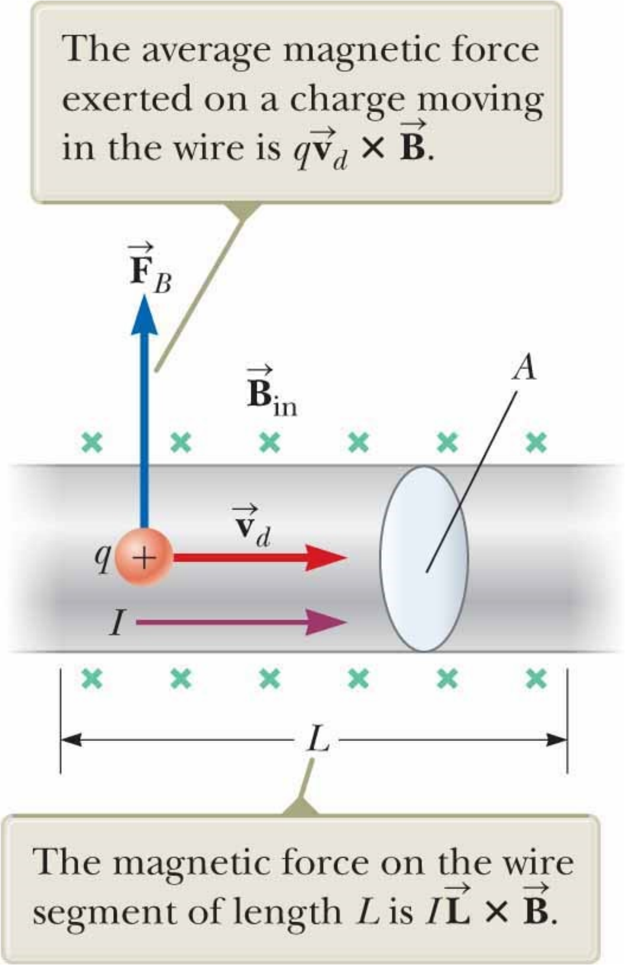
- To quantify the force, let's consider a straight segment of wire of length ***L*** and cross-sectional area ***A*** carrying a current ***I*** in a uniform magnetic field $\vec{B}$.
- The magnetic force exerted on a charge ***q*** moving with a drift velocity $\vec{v}_d$ is $q\vec{v}_d \times \vec{B}$
- To find the total force acting on the wire, we multiply the force exerted on one charge by the number of charges in the segment.
- The volume of the segment is ***AL*** and the number of charges in the segment is ***nAL*** – where ***n*** is the number of mobile charges per unit volume.
- Hence, the total magnetic force on the segment of wire of length *L* is:

$$\vec{\mathbf{F}}_B = \left(q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}\right)nAL$$

The current in the wire was earlier expressed as $I = nqv_dA$

$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$

***L*** is a vector that points in the direction of the current ***I*** and has a magnitude equal to the length ***L*** of the segment

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# Magnetic Force Acting on a Current-Carrying Conductor

- Now let's consider an arbitrarily shaped wire segment of uniform cross section in a magnetic field as shown in the figure.

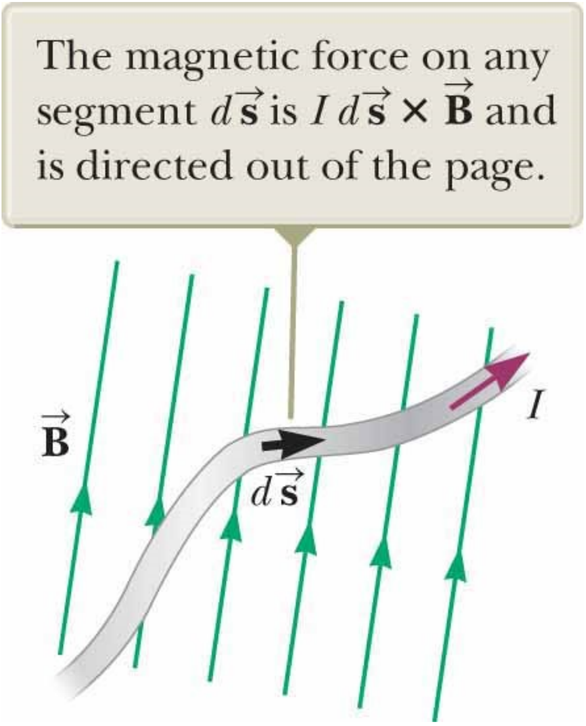
- The magnetic force exerted on a small segment of vector length $\vec{ds}$ in the presence of a field $\vec{B}$ is:

$$d\vec{\mathbf{F}}_B = I d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

- To calculate the total force, we integrate the above equation over the length of the wire:

$$\vec{\mathbf{F}}_B = I\int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

***a*** and ***b*** are the start and end points of the wire in the field, respectively.

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# Force on a Semi-circular Conductor

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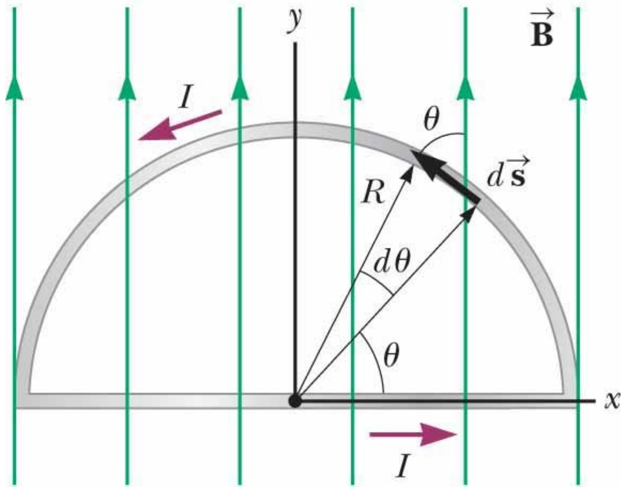
A wire bent into a semicircle of radius ***R*** forms a closed circuit and carries a current ***I***. The wire lies in the ***xy plane***, and a uniform magnetic field is directed along the ***positive y axis*** as in the figure. Find the magnitude and direction of the magnetic force acting on the straight portion of the wire and on the curved portion.

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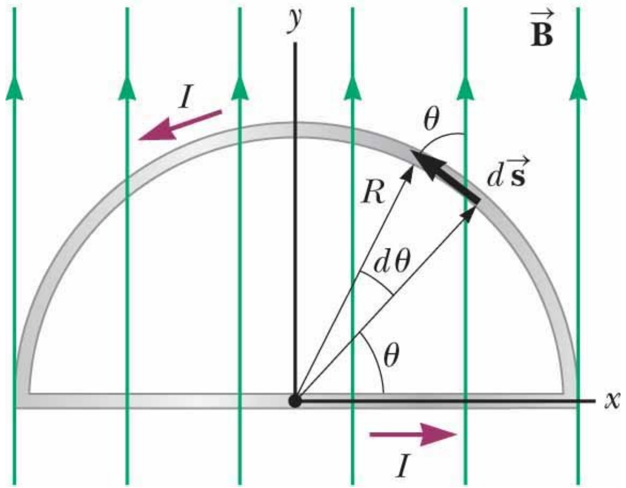
Solution

**Conceptualize**
Using the right-hand rule for cross products, we see that the force $\mathbf{F}_1$ on the straight portion of the wire is out of the page and the force $\mathbf{F}_2$ on the curved portion is into the page.

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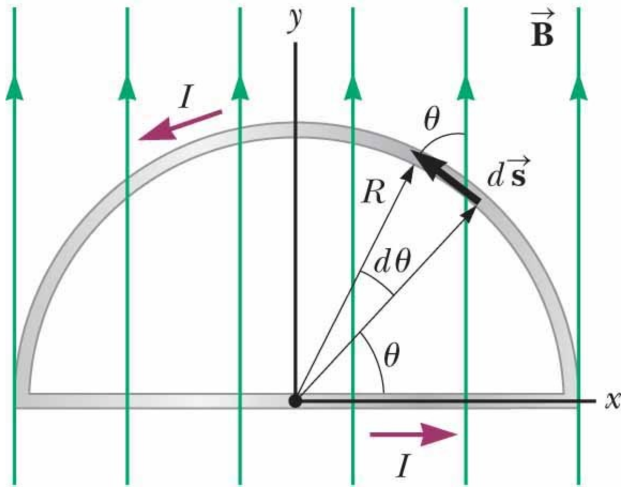
Using the right-hand rule for cross products, we see that the force $\mathbf{F_1}$ on the straight portion of the wire is out of the page and the force $\mathbf{F_2}$ on the curved portion is into the page.

**Categorize**
Because we are dealing with a current-carrying wire in a magnetic field rather than a single charged particle, we must use integration to find the total force on each portion of the wire.

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Solution

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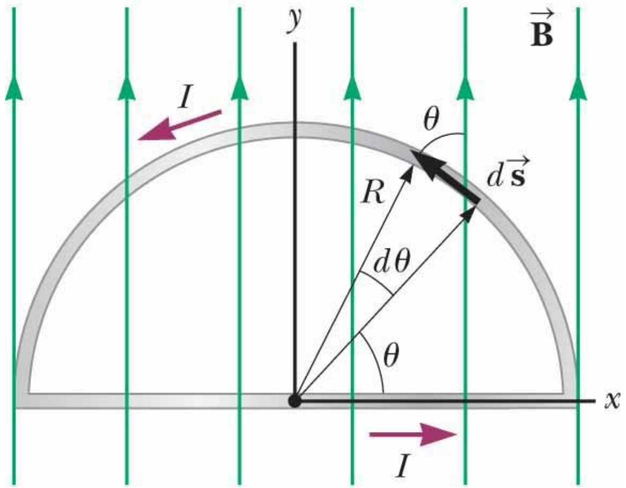


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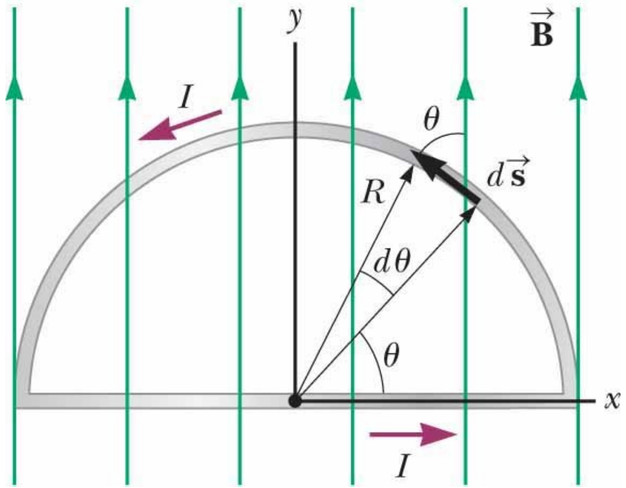


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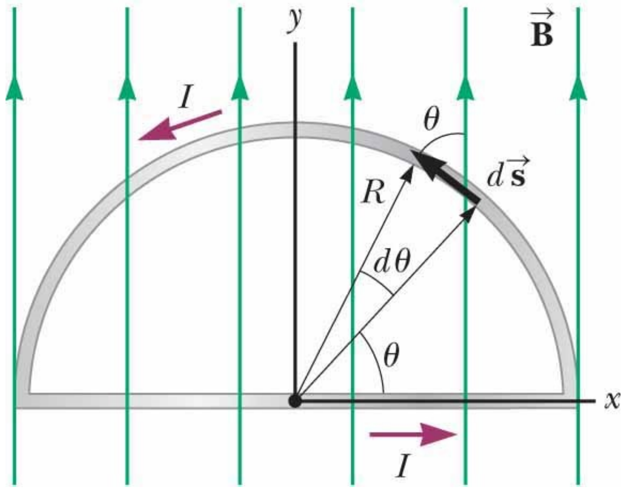


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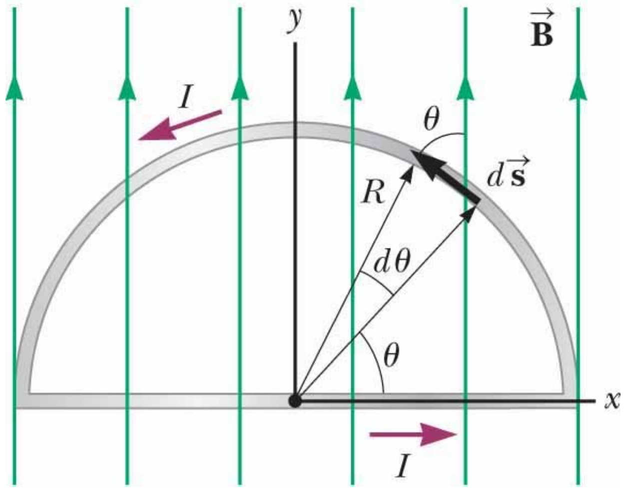


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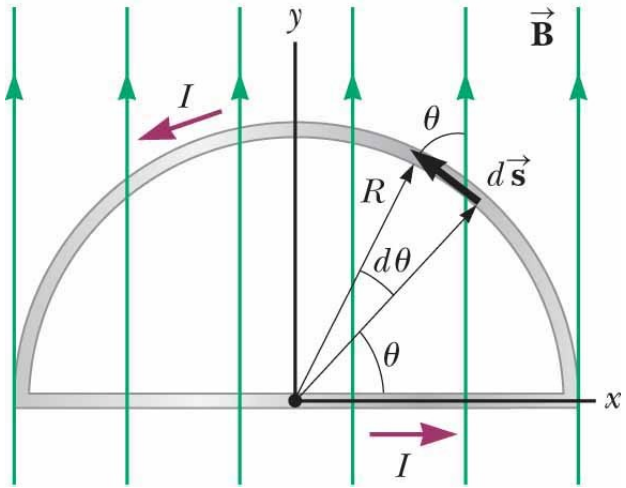
$$d\vec{\mathbf{F}}_2 = I d\vec{\mathbf{s}} \times \vec{\mathbf{B}} = -IB\sin\theta\,ds\,\hat{\mathbf{k}}$$

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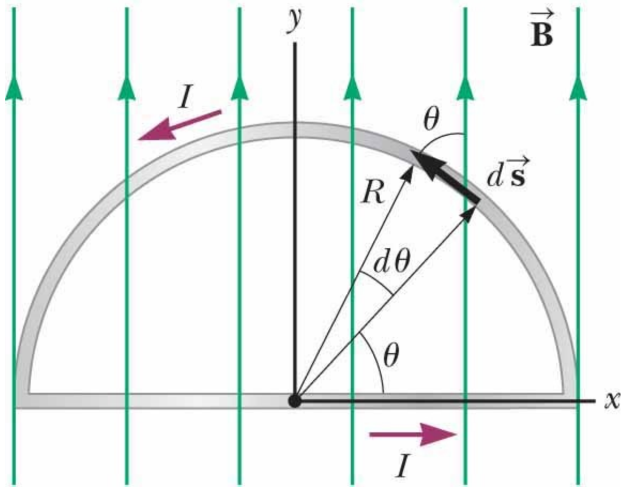


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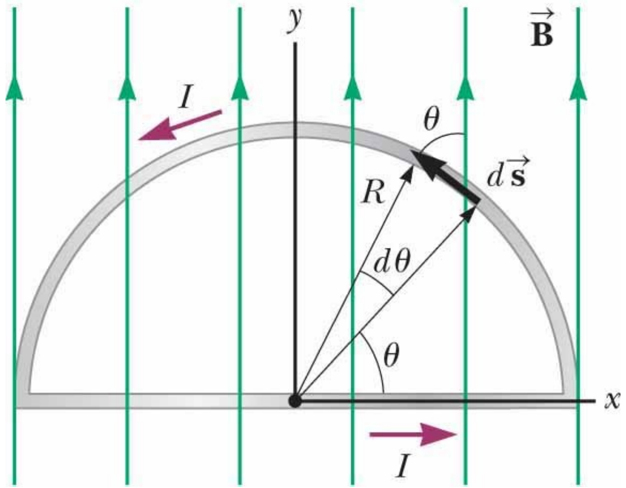


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Now from the geometry, let's write an expression for ***dS*** in terms of ***Θ***:

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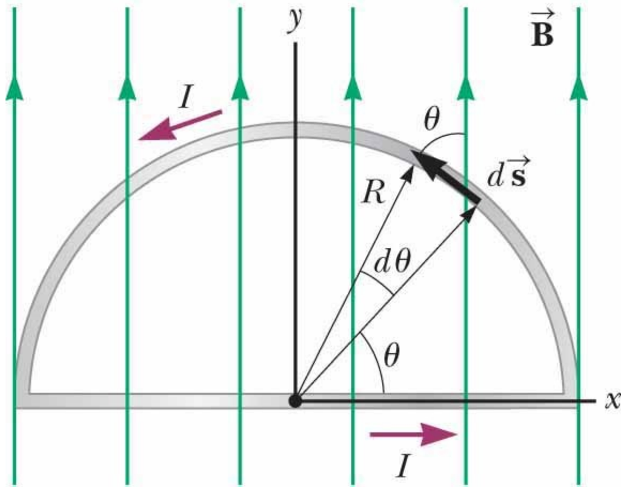
Now substituting in the $dF_2$ equation and integrating:

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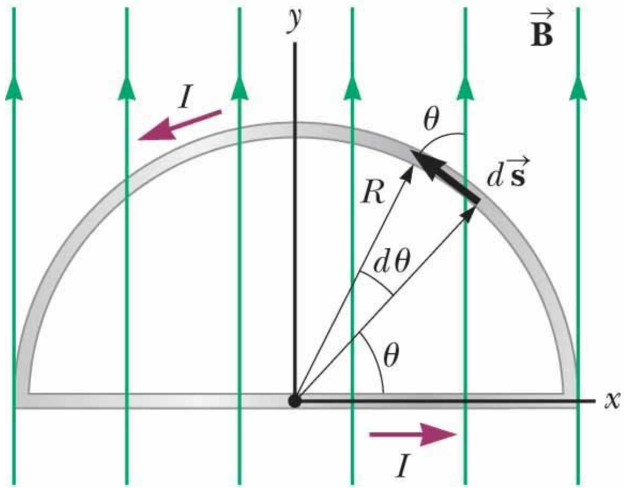
Now substituting in the ***dF*₂** equation and integrating:

$$\vec{\mathbf{F}}_2 = -\int_0^{\pi} IRB\sin\theta\,d\theta\,\hat{\mathbf{k}} = -IRB\int_0^{\pi}\sin\theta\,d\theta\,\hat{\mathbf{k}} = -IRB\left[-\cos\theta\right]_0^{\pi}\,\hat{\mathbf{k}}$$

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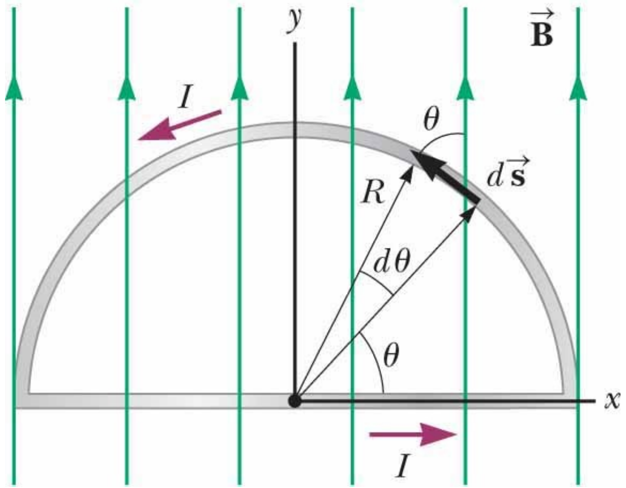
$$= IRB\left(\cos\pi - \cos 0\right)\hat{\mathbf{k}} = IRB\left(-1-1\right)\hat{\mathbf{k}} = \boxed{-2IRB\hat{\mathbf{k}}}$$

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## Solution

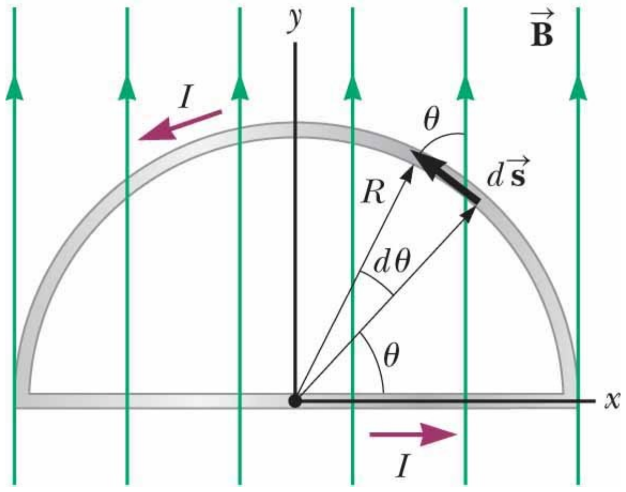
**$F_1 = 2IRB\,\hat{k}$ $F_2 = -2IRB\,\hat{k}$**

Observations:

1. The force on the curved portion is the same in magnitude as the force on the straight wire between the same two points ($|F_1| = |F_2|$)

2. This can be generalized: the magnetic force on a curved current-carrying wire in a uniform magnetic field is equal to that on a straight wire connecting the endpoints and carrying the same current. Only the perpendicular portion is contributing.

3. The net magnetic force acting on any closed current loop in a uniform magnetic field is zero ($\vec{F}_1 + \vec{F}_2 = 0$)

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# Magnetic Fields (Ch. 28)

## Torque on a Current Loop in a Uniform Magnetic Field

# Torque on a Current Loop in a Uniform Magnetic Field

- Previous example showed the net total magnetic force to be zero. But different forces were exerted on different part of the current carrying loop.

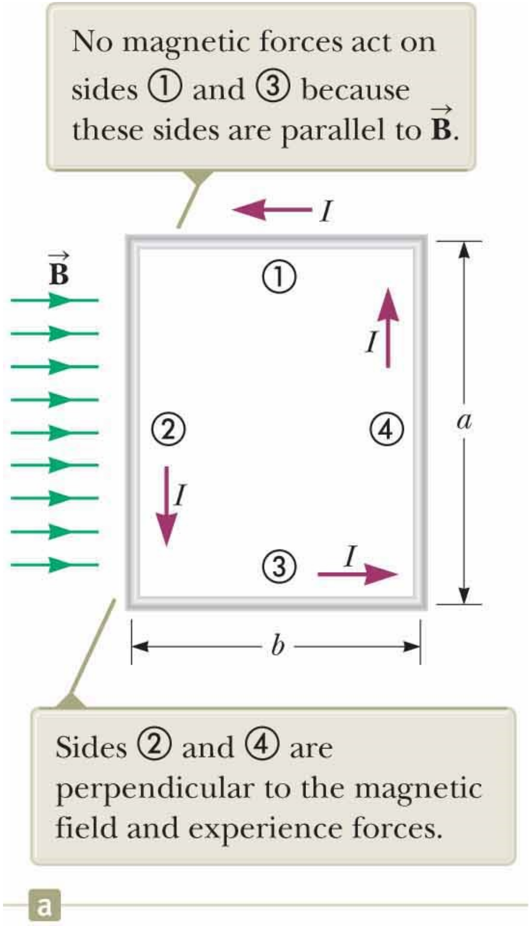
- Consider a rectangular loop carrying a current ***I*** in the presence of a uniform magnetic field directed parallel to the plane of the loop as shown in the figure.

- No magnetic forces act on sides ***(1)*** and ***(3)*** because these wires are parallel to the field; hence, $\vec{L} \times \vec{B} = 0$ for these sides.

- Magnetic forces act on sides ***(2)*** and ***(4)*** because these sides are oriented perpendicular to the field.

- The magnitude of these forces is:

$$F_2 = F_4 = IaB$$

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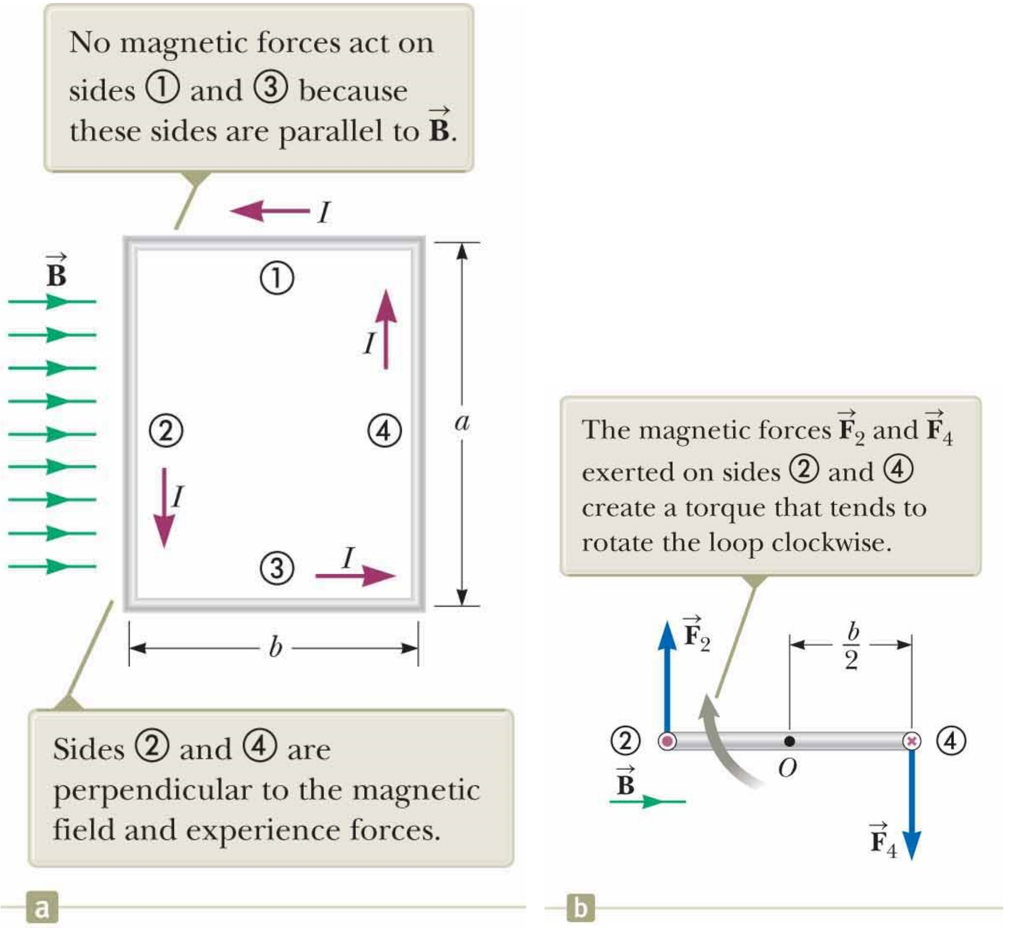
# Torque on a Current Loop in a Uniform Magnetic Field

- The direction of $\vec{\boldsymbol{F}}_2$, the magnetic force exerted on wire (2), is out of the page.
- The direction of $\vec{\boldsymbol{F}}_4$, the magnetic force exerted on wire (4), is into the page.
- If we view the loop from side (3), the two magnetic forces $\vec{\boldsymbol{F}}_2$ and $\vec{\boldsymbol{F}}_4$ would not be directed along the same line of action.
- If the loop is pivoted so that it can rotate about point ***O***, these two forces produce a torque that rotates the loop clockwise. The magnitude of this torque $\tau_{max}$ is:

$$\begin{aligned}\tau_{\max} &= F_2\frac{b}{2} + F_4\frac{b}{2} \\ &= (IaB)\frac{b}{2} + (IaB)\frac{b}{2} \\ &= IabB\end{aligned}$$

$$\tau_{\max} = IAB$$

- Moment arm is ***b/2.***
- The area of the loop is ***A=ab.***

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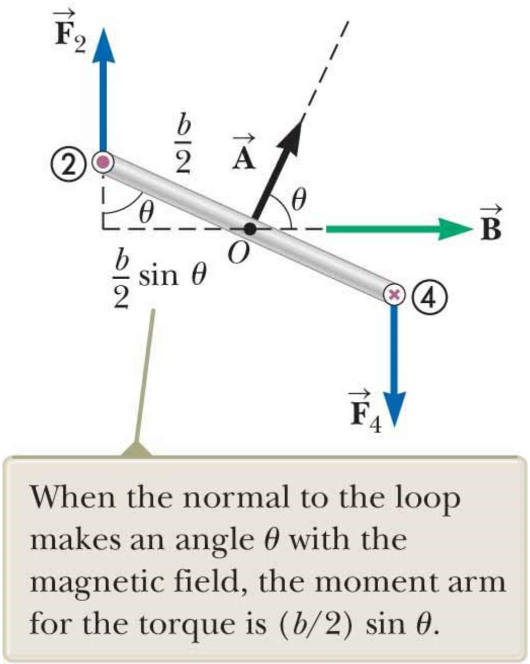
# Torque on a Current Loop in a Uniform Magnetic Field

- Now suppose the uniform magnetic field makes an angle ***Θ < 90°*** with a line perpendicular to the plane of the loop.
- ***B*** is still perpendicular to the sides ***(2)*** and ***(4)***.
- In this case, momentum arms are ***(b/2)sinΘ***.
- The magnitude of the net torque about point ***O*** is:

$$
\begin{aligned}
\tau &= F_2 \frac{b}{2}\sin\theta + F_4 \frac{b}{2}\sin\theta \\
&= IaB\left(\frac{b}{2}\sin\theta\right) + IaB\left(\frac{b}{2}\sin\theta\right) = IabB\sin\theta \\
&= IAB\sin\theta
\end{aligned}
$$

- Torque is maximum when Θ = 90°.
- Torque is zero when Θ = 0°.

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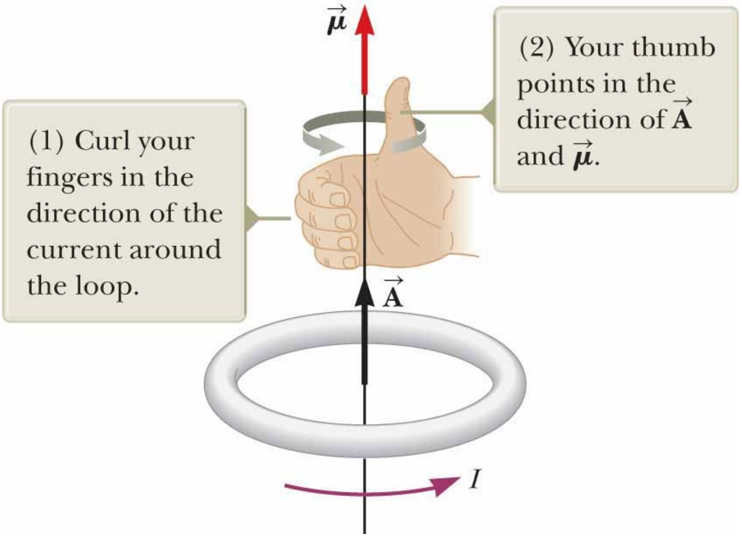
# Torque on a Current Loop in a Uniform Magnetic Field

- A convenient vector expression for the torque exerted on a loop placed in a uniform magnetic field $\vec{B}$ is:

$$\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

- $\vec{A}$ is perpendicular to the plane of the loop and has a magnitude equal to the area of the loop.
    - To determine the direction of $\vec{A}$, use the right-hand rule as shown in the figure




- The product $I\vec{A}$ is defined to be the ***magnetic dipole moment*** $\vec{\mu}$ (often simply called the "***magnetic moment***") of the loop.
    - The SI unit of magnetic dipole moment is the ampere-meter$^2$ (A · m$^2$).
    - If a coil of wire contains N loops of the same area, the magnetic moment of the coil is:

$$\vec{\boldsymbol{\mu}} \equiv I\vec{\mathbf{A}}$$

$$\vec{\boldsymbol{\mu}}_{\text{coil}} = NI\vec{\mathbf{A}}$$

- The torque exerted on a current-carrying loop in a magnetic field $\vec{B}$ is:

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

# Torque on a Current Loop in a Uniform Magnetic Field

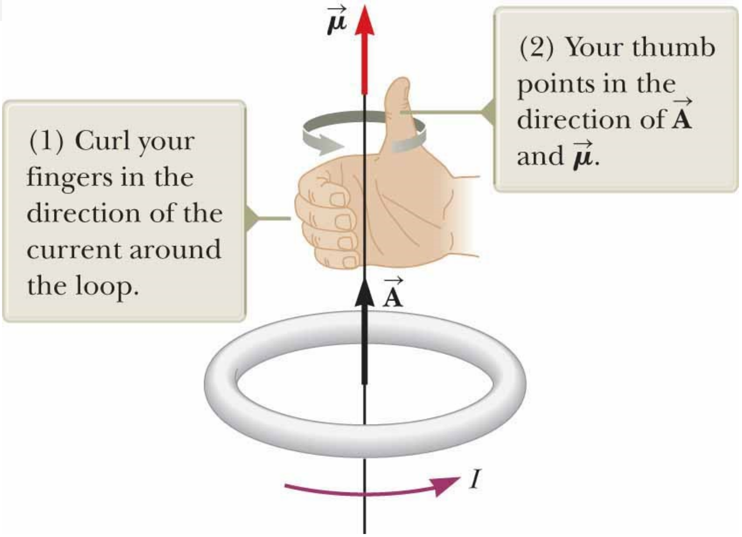
- The torque exerted on a current-carrying loop in a magnetic field $\vec{B}$ is analogous to the torque exerted on an electric dipole in the presence of an electric field $\vec{E}$, where $\vec{p}$ is the electric dipole moment.

- We derived the torque expression for a rectangular loop, but the result is valid for a loop of any shape.

- The potential energy of a system of a magnetic dipole in a magnetic field depends on the orientation of the dipole in the magnetic field and is given by:

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}} \qquad \vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$

$$U_B = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$$

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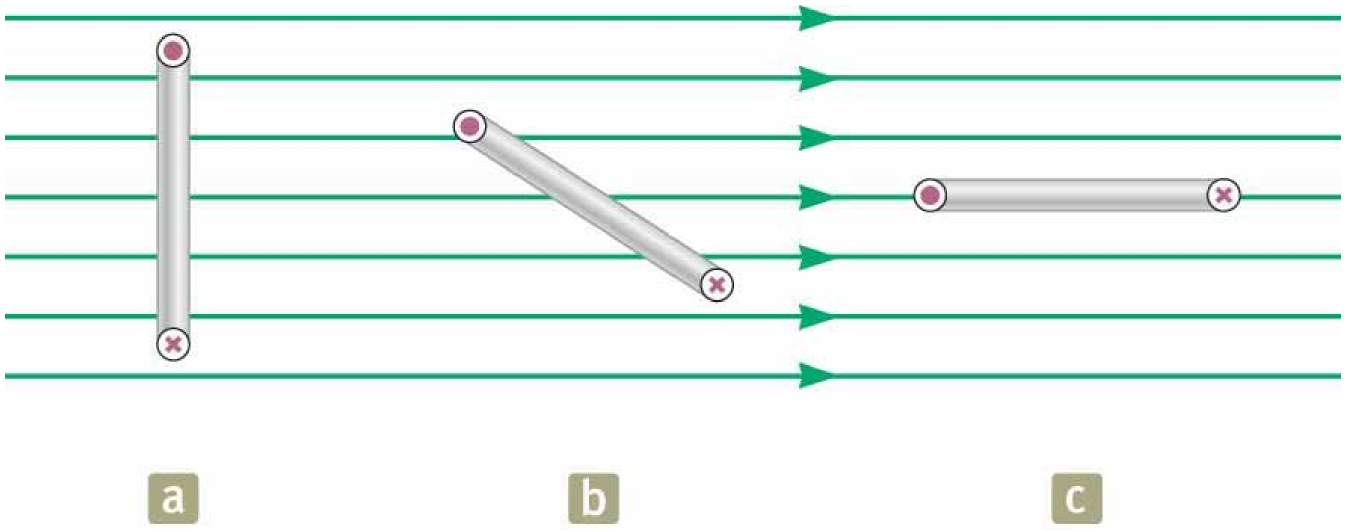
# Torque on a Current Loop in a Uniform Magnetic Field

**Quick Quiz**

Rank the magnitudes of the torques acting on the rectangular loops shown in the figure from highest to lowest.

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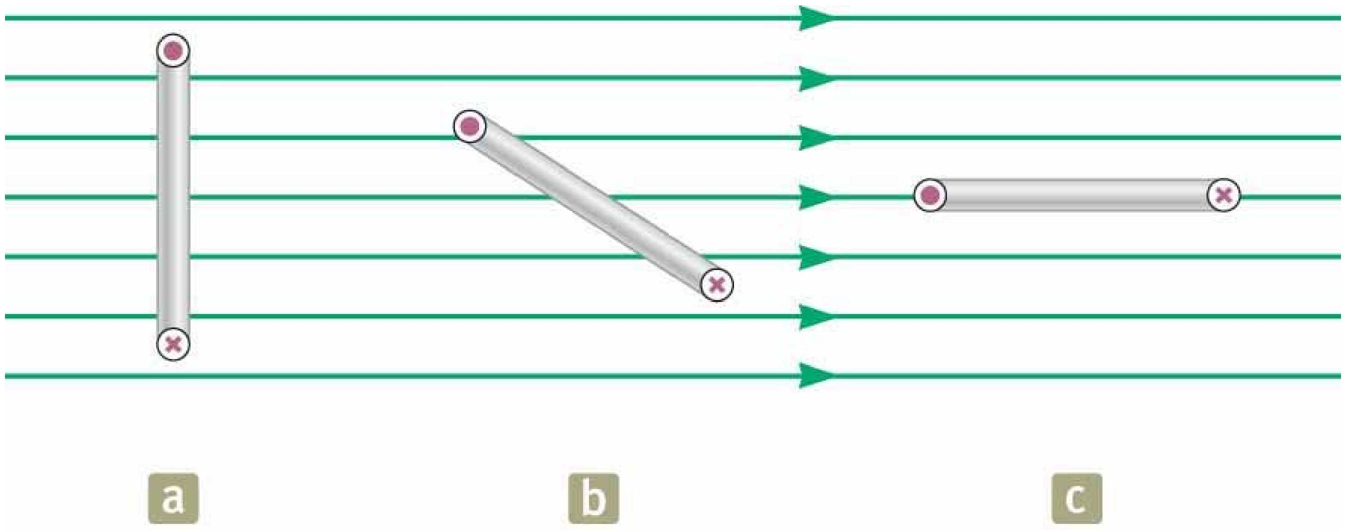


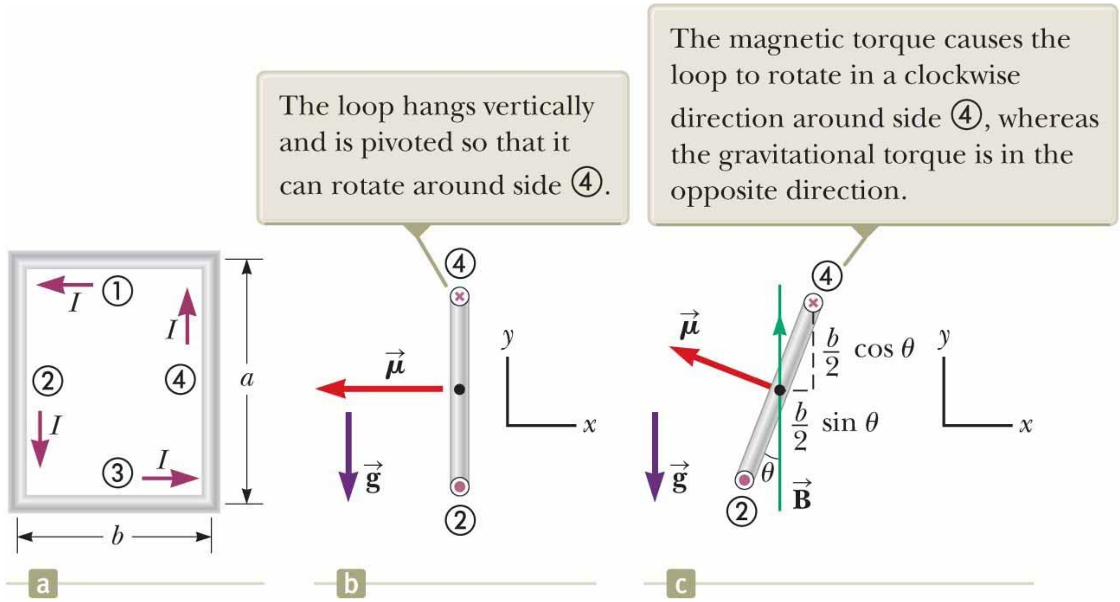

Solution c > b > a

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

# Rotating a Coil

Example 28.5

Consider the loop of wire in figure (a). Imagine it is pivoted along side 4, which is parallel to the ***z axis*** and fastened so that side 4 remains fixed and the rest of the loop hangs vertically in the gravitational field of the Earth but can rotate around side 4 (figure (b)). The mass of the loop is ***m = 50.0 g***, and the sides are of lengths ***a = 0.200 m*** and ***b = 0.100 m***. The loop carries a current of ***I = 3.50 A*** and is immersed in a vertical uniform magnetic field of magnitude ***B = 0.010 0 T*** in the positive y direction (figure (c)). What angle does the plane of the loop make with the vertical?

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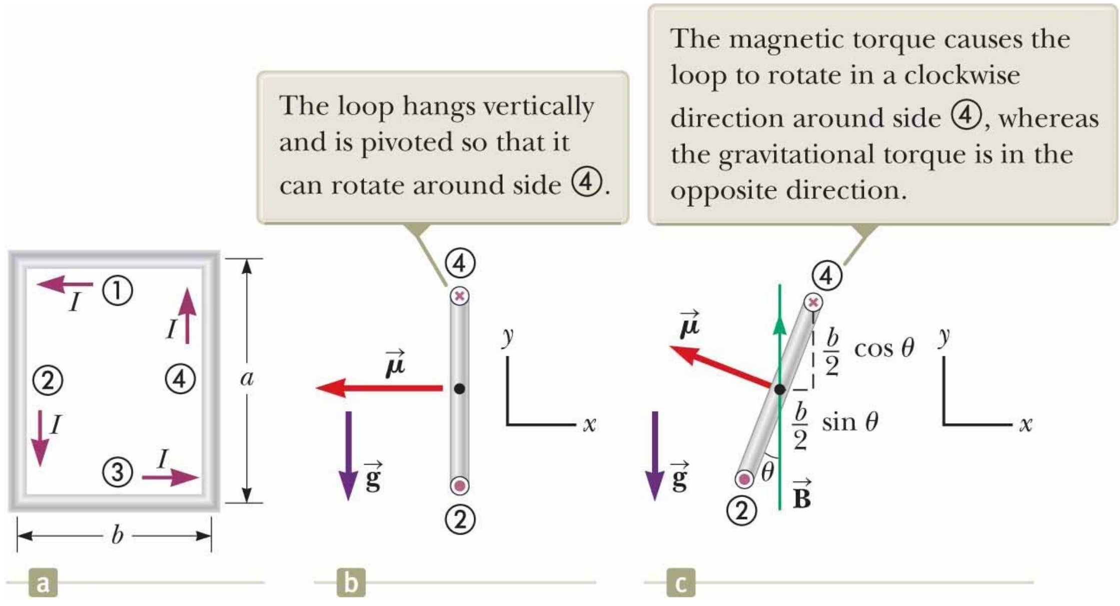
## Solution

**Conceptualize**

- When there is an angle, the magnetic forces on side (1) and (3) are not zero anymore but equal in magnitude and in opposite direction – hence, cancel each other out.
- Magnetic force on the side (2) results in a magnetic torque on the loop to rotate clockwise.
- The gravitational force on the loop exerts a torque to rotate counter-clockwise.

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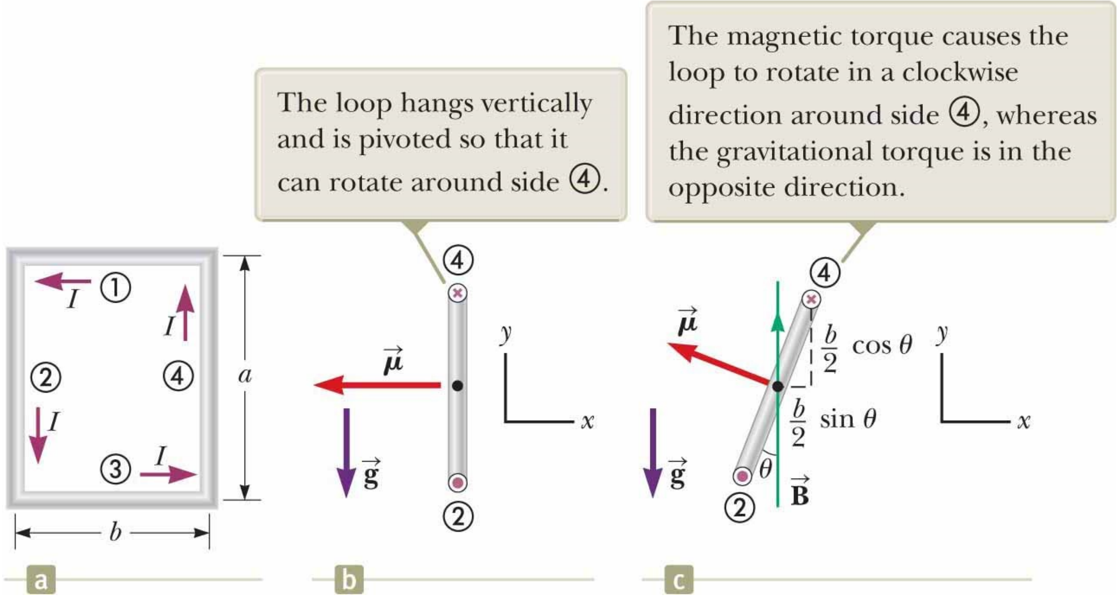
## Solution

**Categorize**

- At some angle Ɵ, the magnetic torque and the gravitational torque cancels each other out and the object remains in equilibrium.

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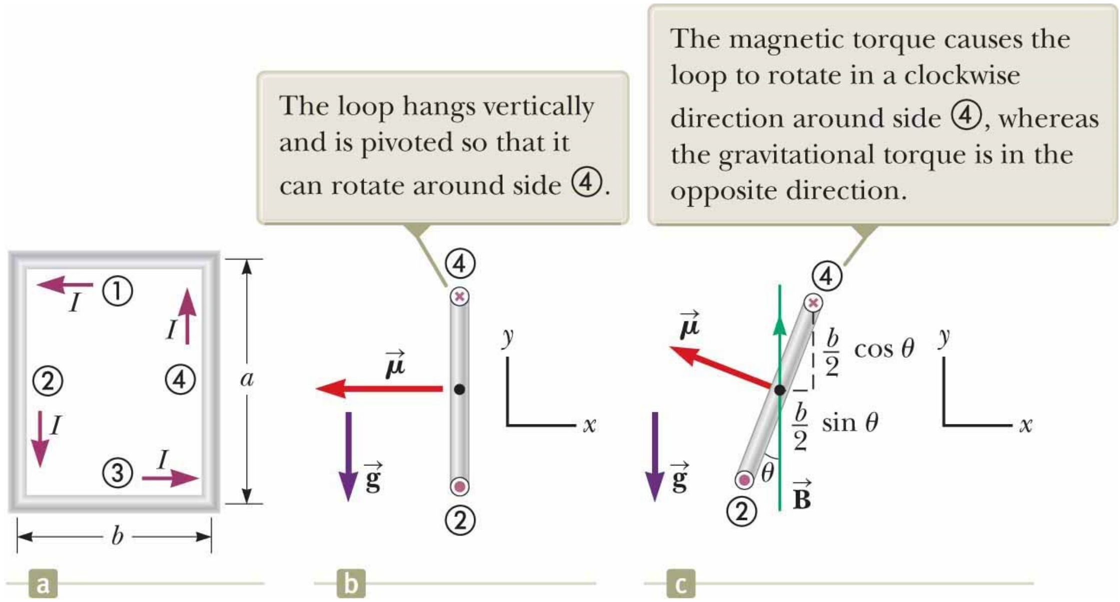
## Solution

Let's find the magnetic torque first:

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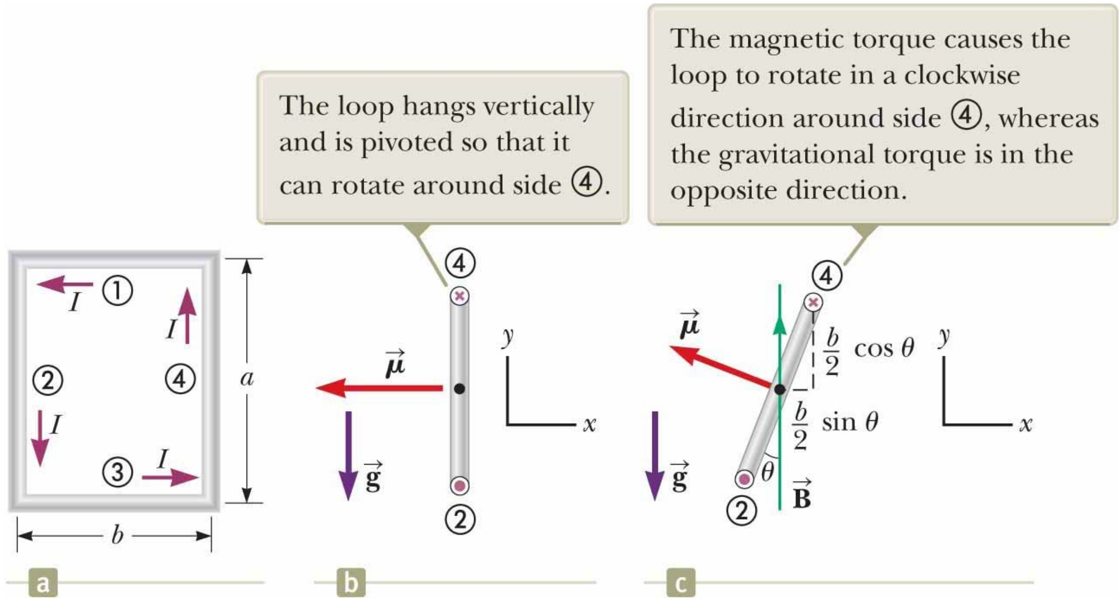
## Solution

Let's find the magnetic torque first: $F_M = IaB$

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## Solution

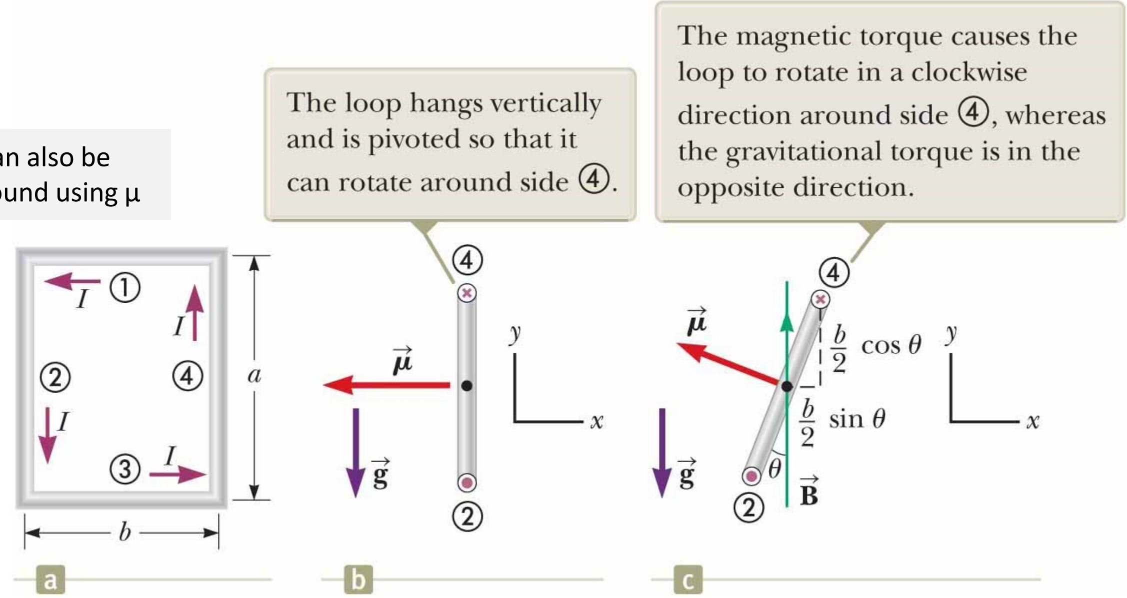
Let's find the magnetic torque first:

$$F_M = IaB$$

$$\tau_M = IaBb\cos\theta$$

clockwise

can also be found using μ

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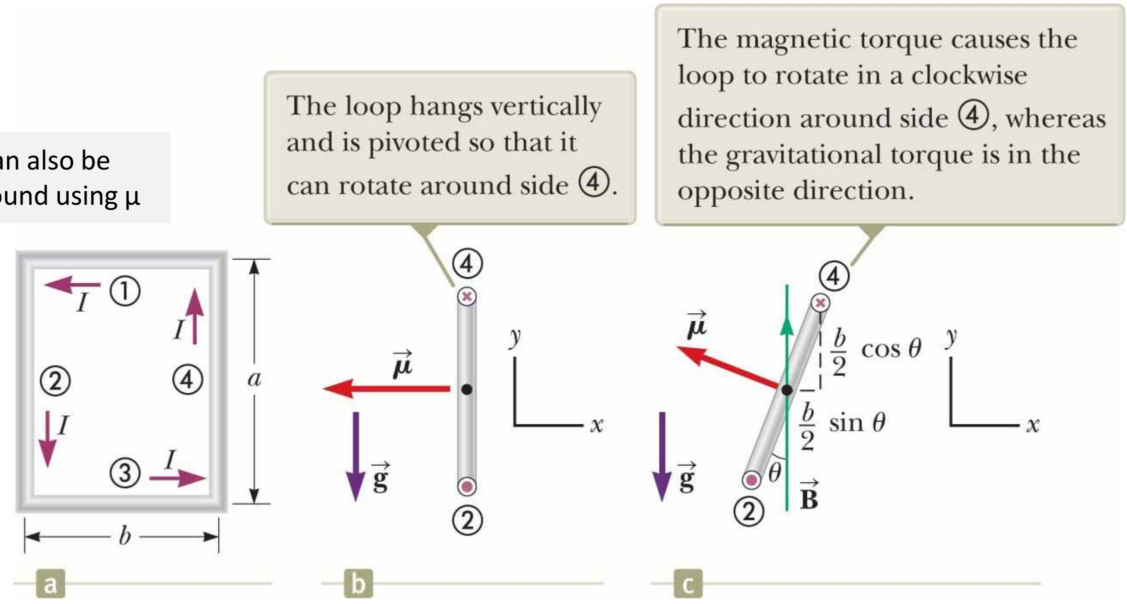


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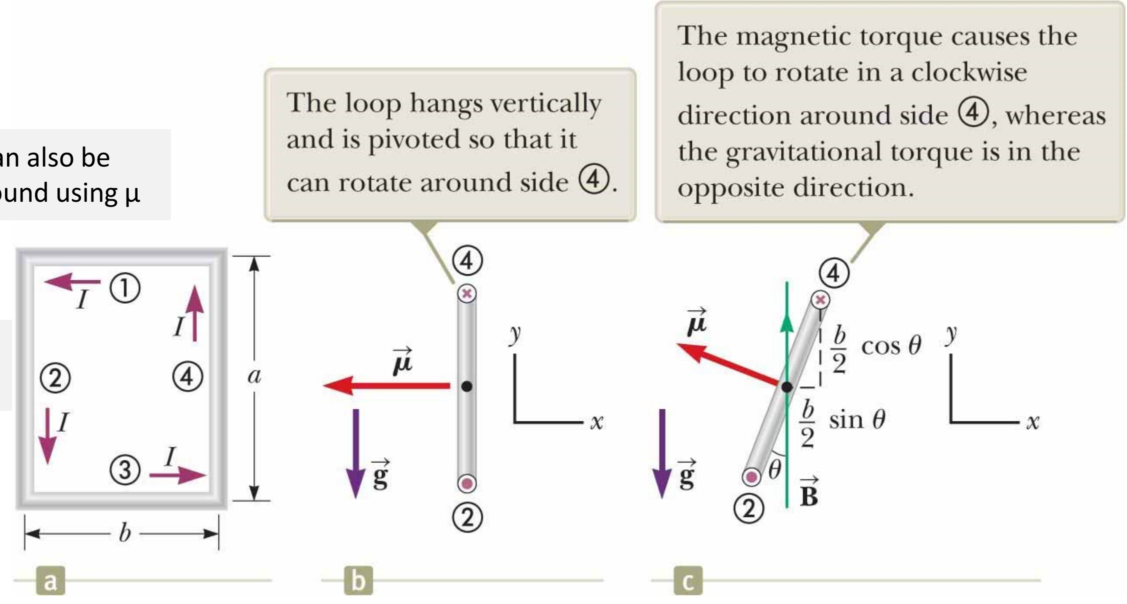
Now let's find the gravitational torque:

$$F_G = mg$$

$$\tau_G = mg(b/2)\sin\theta$$

counter-clockwise

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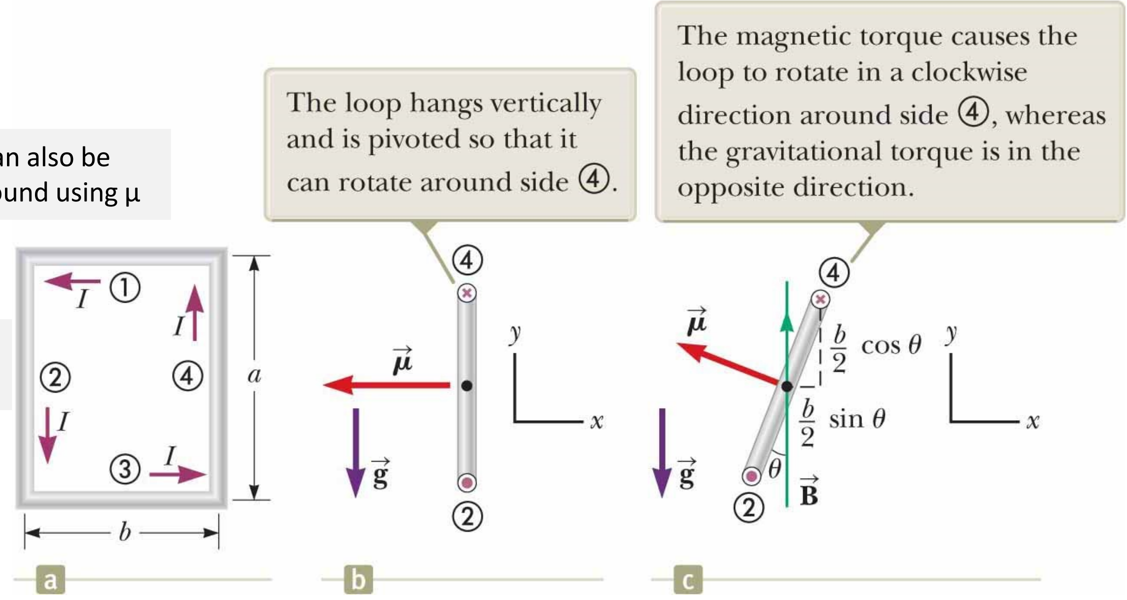
At the equilibrium, the torques must cancel:

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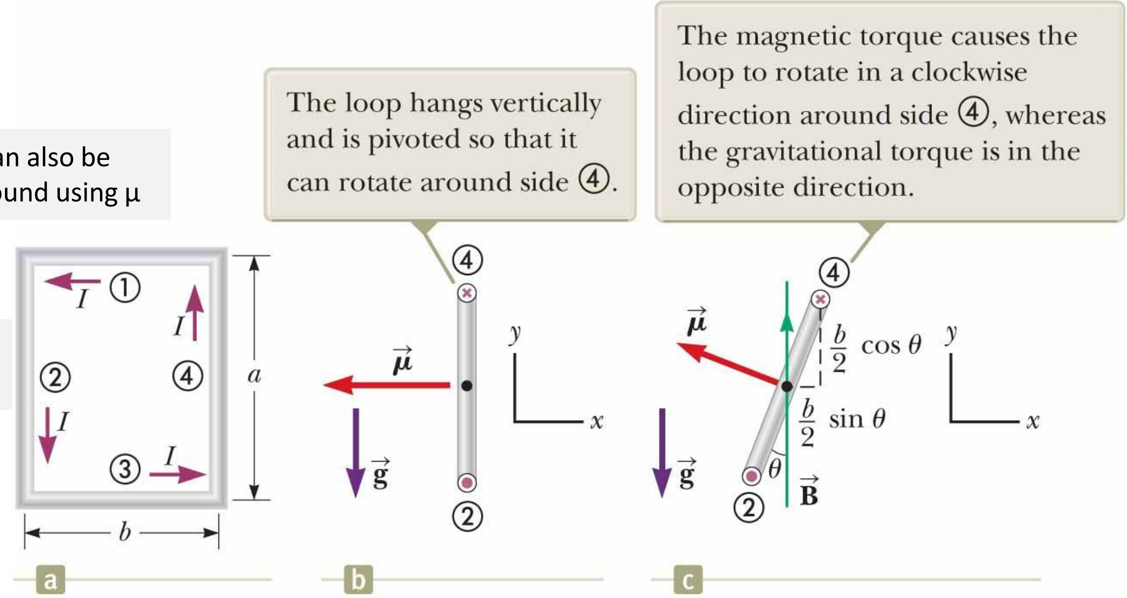
At the equilibrium, the torques must cancel:

$$\tau_M = \tau_G$$

$$IaBb\cos\theta = mg(b/2)\sin\theta$$

$$\tan\theta = \frac{2IaBb}{mgb} = \frac{2IaB}{mg}$$

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## Solution

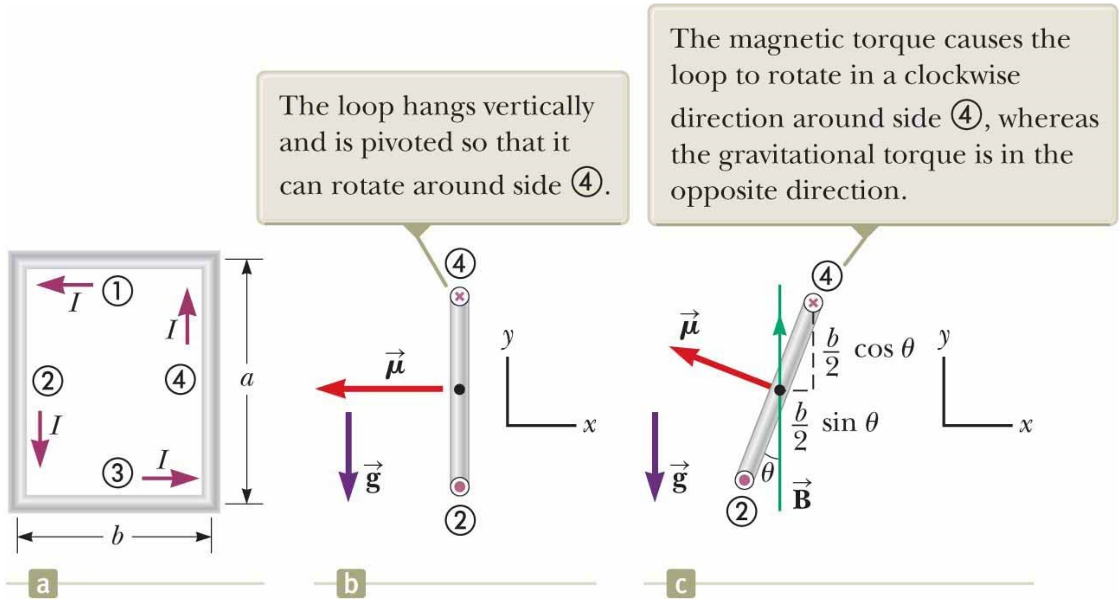
At the equilibrium, the torques must cancel:

$$\tau_{M2} = \tau_{G2}$$

$$tan\theta = \frac{2IaBb}{mgb} = \frac{2IaB}{mg}$$

$$tan\theta = \frac{2(3.5A)(0.2m)(0.01T)}{(0.05kg)(9.80m/s^2)} = 0.02857$$

$$\theta = 1.64°$$

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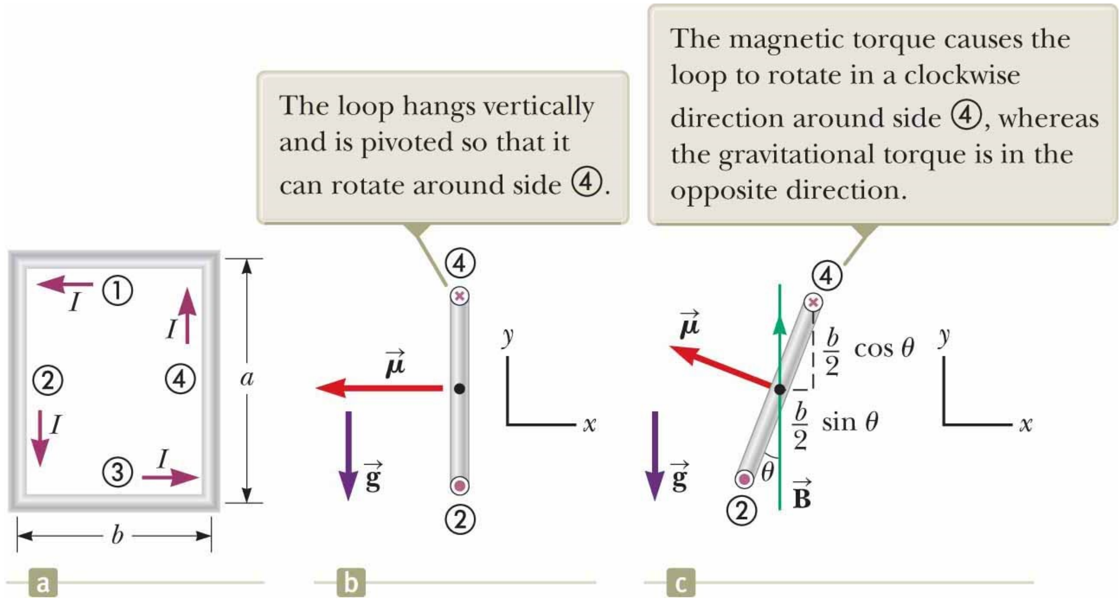
$$tan\theta = \frac{2(3.5A)(0.2m)(0.01T)}{(0.05kg)(9.80m/s^2)} = 0.02857$$

$$\theta = 1.64°$$

The angle is relatively small, so the loop still hangs almost vertically. If the current or the magnetic field is increased, however, the angle increases as the magnetic torque becomes stronger.

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# Magnetic Fields (Ch. 28)

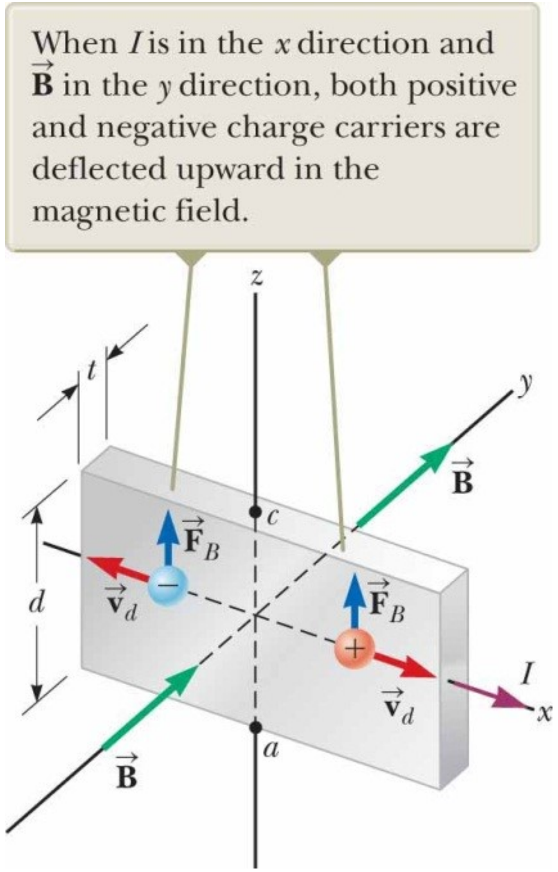
## The Hall Effect

# The Hall Effect

- When a current-carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field.

- This phenomenon, first observed by Edwin Hall (1855–1938) in 1879, is known as the ***Hall effect***.

- To observe the Hall effect, a magnetic field is applied to a current-carrying conductor. The Hall voltage is measured between points ***a*** and ***c*** (in the figure).

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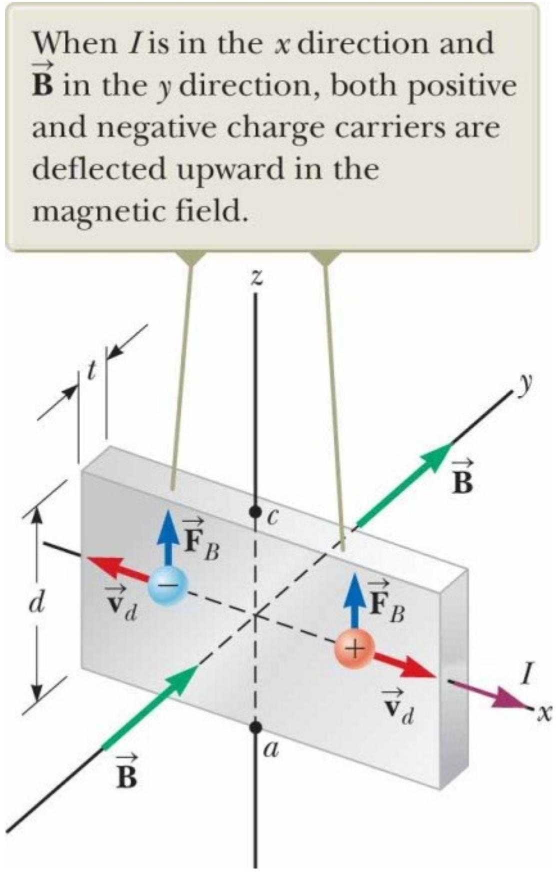
# The Hall Effect

- If the charge carriers are ***electrons*** moving in the negative ***x*** direction with a drift velocity $\vec{v}_d$, they experience an upward magnetic force $\vec{F}_B = q\vec{v}_d \times \vec{B}$, are deflected upward, and accumulate at the upper edge of the flat conductor, leaving an excess of positive charge at the lower edge.

- This accumulation of charge at the edges establishes an ***electric field*** in the conductor and increases until the ***electric force*** on carriers remaining in the bulk of the conductor ***balances the magnetic force*** acting on the carriers.

- A sensitive voltmeter can measure the potential difference, known as the ***Hall voltage*** $\Delta V_H$, generated across the conductor.




Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

# The Hall Effect

- Let's try to derive the expression for the Hall voltage.

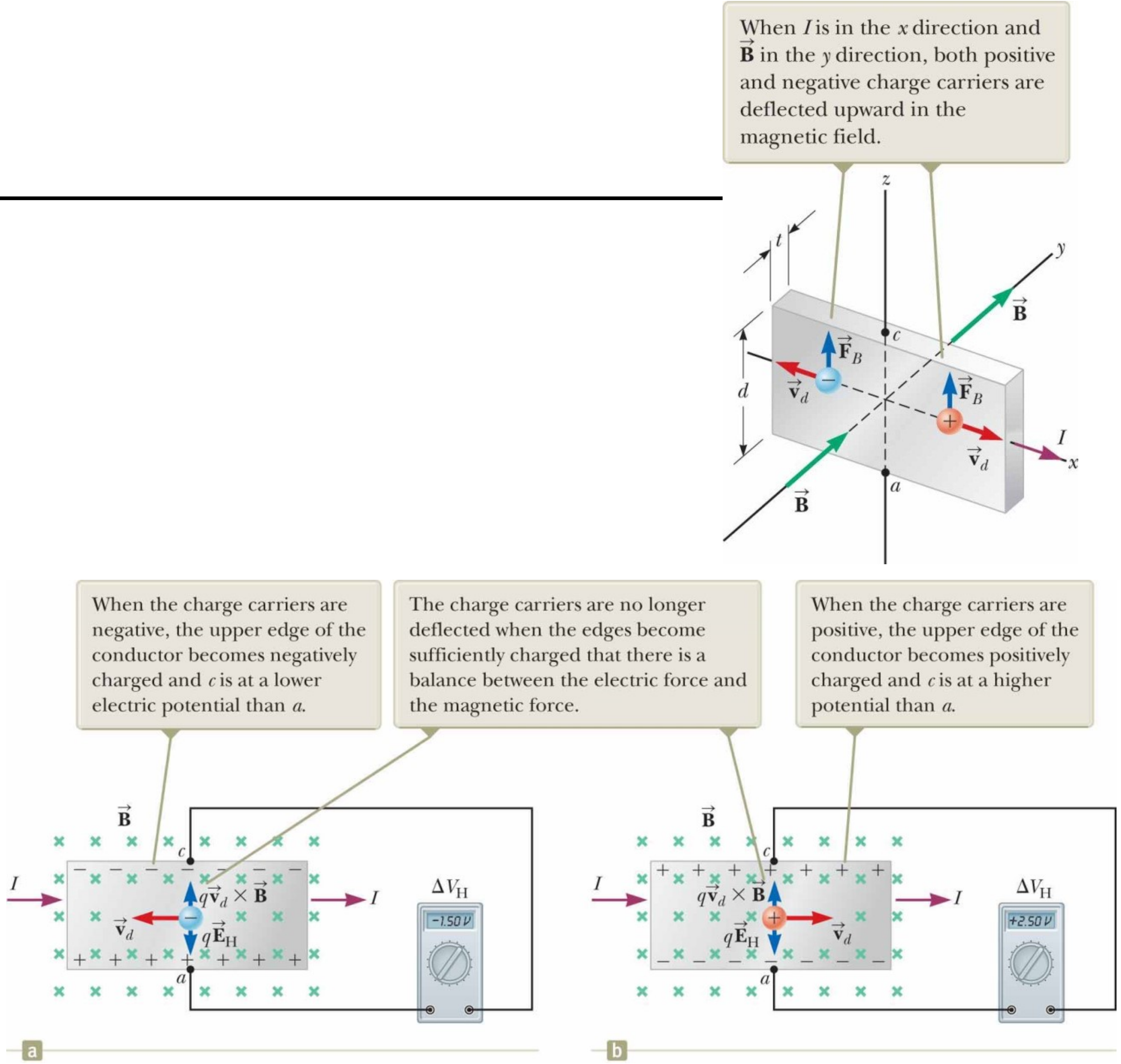
- The magnetic force exerted on the carriers has magnitude $qv_dB$.

- The electric force $qE_H$, where $E_H$ is the magnitude of the electric field due to the charge separation (sometimes referred to as the ***Hall field***).

- In the equilibrium;

$$qv_d B = qE_{\mathrm{H}} \Rightarrow E_H = v_d B$$

$$\Delta V_{\mathrm{H}} = E_{\mathrm{H}} d = v_d B d$$

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# The Hall Effect

- If we know the B and d, the measured Hall voltage gives a value for the drift speed of the charge carriers:

$$\Delta V_H = E_H d = v_d B d$$

- We can obtain the charge-carrier density ***n*** by measuring the current:

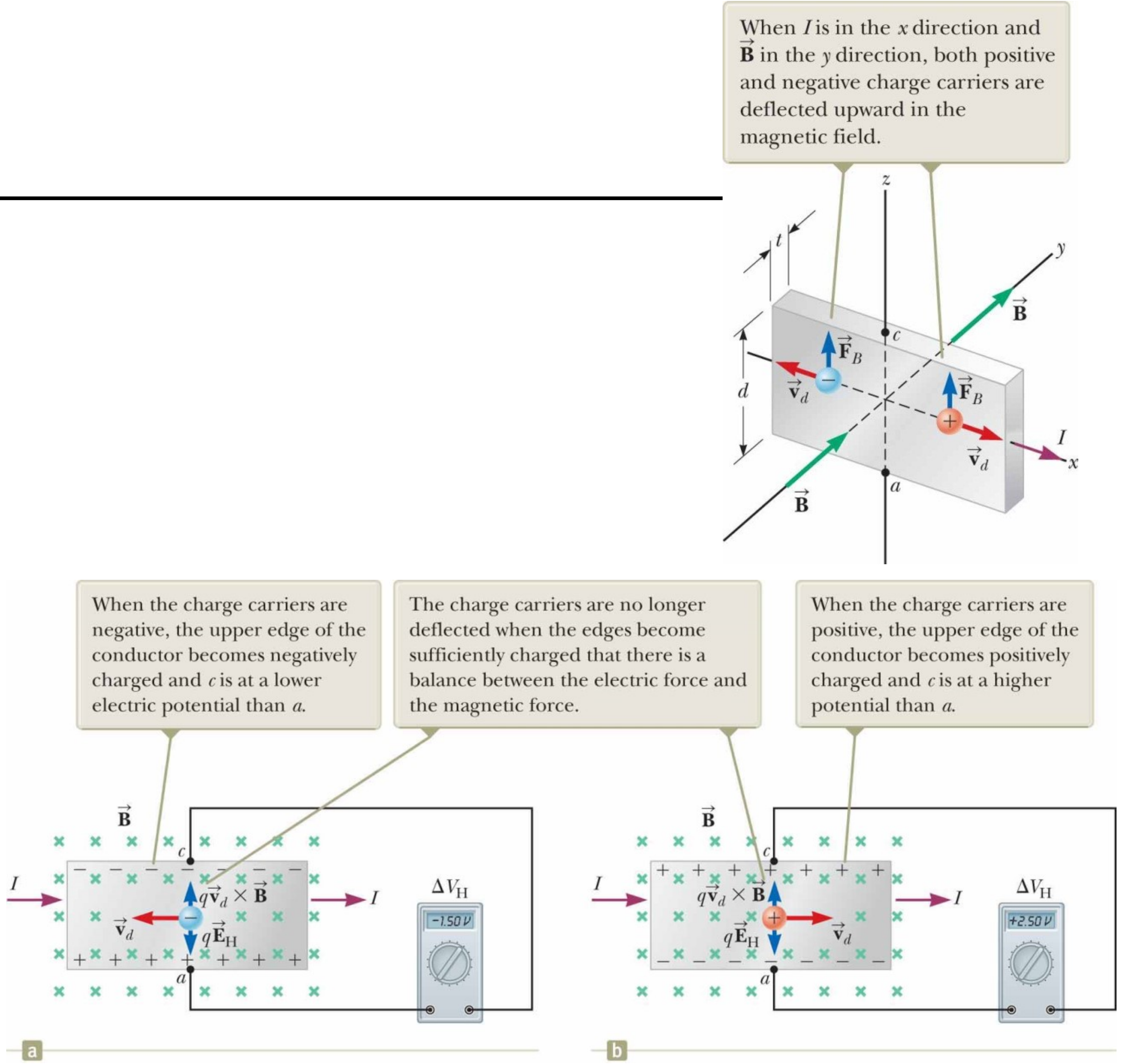
$$v_d = \frac{I}{nqA} \longrightarrow \Delta V_H = \frac{IBd}{nqA}$$

- Cross-sectional area ***A =td*** :

$$\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$$

$$R_H = \frac{1}{nq}$$ (Hall coefficient)

All quantities other than the ***nq*** can be measured. Hence, a value for the ***Hall coefficient*** is readily obtainable.

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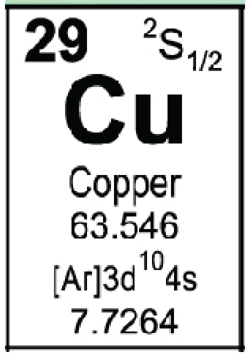
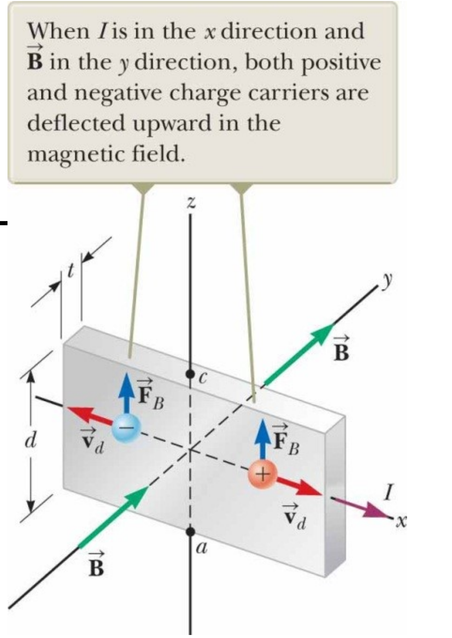
# Rotating a Coil

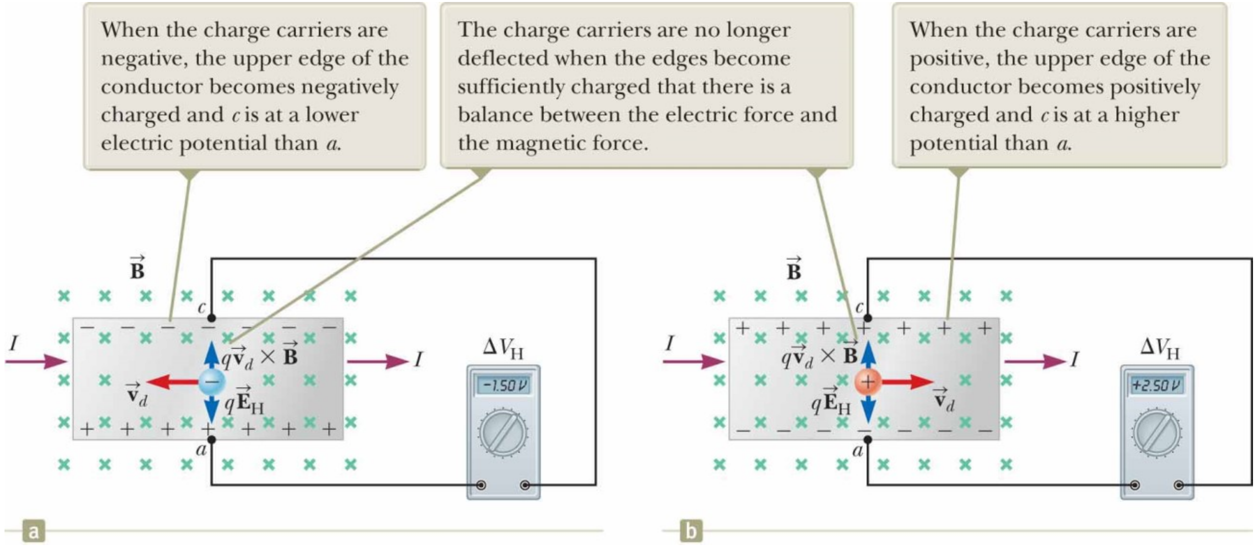
Example 28.6

A rectangular copper strip ***1.5 cm wide*** and ***0.10 cm thick*** carries a current of ***I = 5.0 A***. Find the ***Hall voltage*** for a ***T = 1.2-T*** magnetic field applied in a direction perpendicular to the strip.

**<u>Note:</u>** The periodic table of the elements shows that the molar mass of copper is ***M = 63.5 g/mol.*** Recall that 1 mol of any substance contains Avogadro's number of atoms (***$N_A$ = 6.02 x $10^{23}$ mol$^{-1}$***). Assume each copper atom contributes one free electron to the current. The density of copper is ***ρ = 8.92 g/cm3***.

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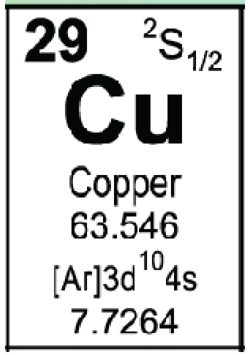
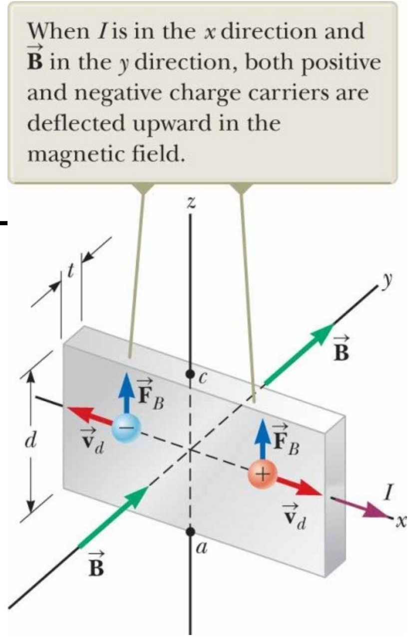
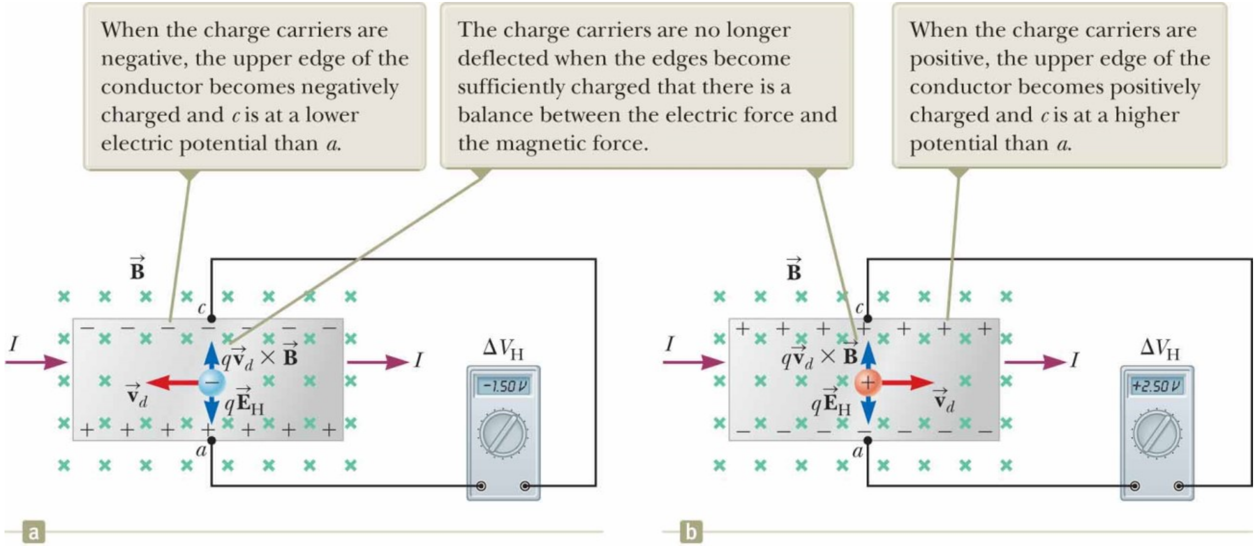



## Solution

Assuming one electron per atom is available for conduction, find the charge-carrier density in terms of the molar mass $M$ and density $\rho$ of copper:



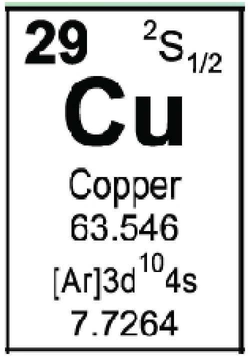

*scientists and engineers*. 10th Edition. Cengage learning, 2018.

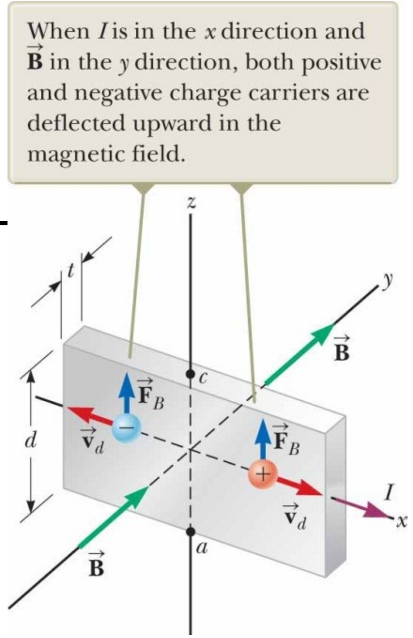
# Rotating a Coil

## Example 28.6

A rectangular copper strip ***1.5 cm wide*** and ***0.10 cm thick*** carries a current of ***I = 5.0 A***. Find the ***Hall voltage*** for a ***T = 1.2-T*** magnetic field applied in a direction perpendicular to the strip.

**Note:** The periodic table of the elements shows that the molar mass of copper is ***M = 63.5 g/mol.*** Recall that 1 mol of any substance contains Avogadro's number of atoms (***$N_A$ = 6.02 x $10^{23}$ $mol^{-1}$***). Assume each copper atom contributes one free electron to the current. The density of copper is ***ρ = 8.92 g/cm3***.

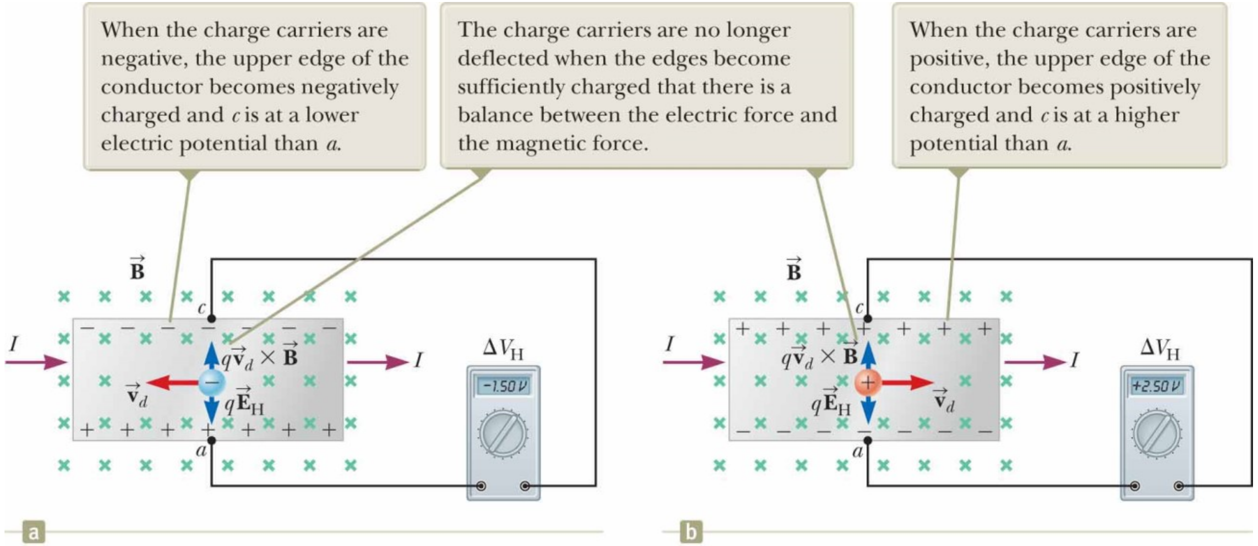



## Solution

Assuming one electron per atom is available for conduction, find the charge-carrier density in terms of the molar mass $M$ and density $\rho$ of copper:

$$n = \frac{N_A}{V} = \frac{N_A \rho}{M}$$

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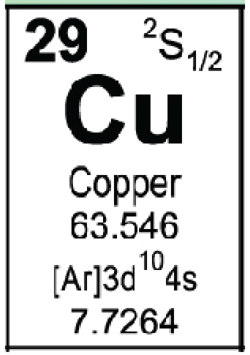


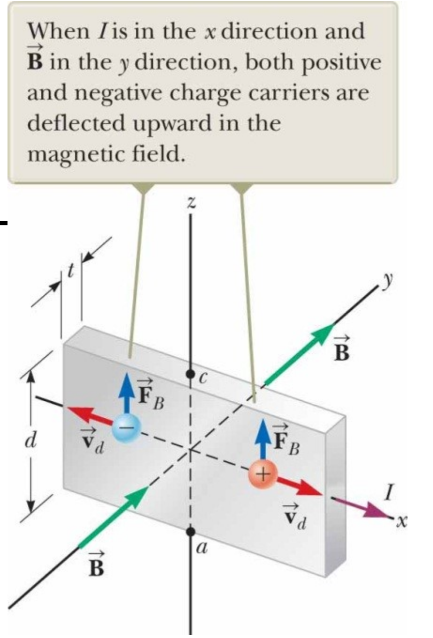


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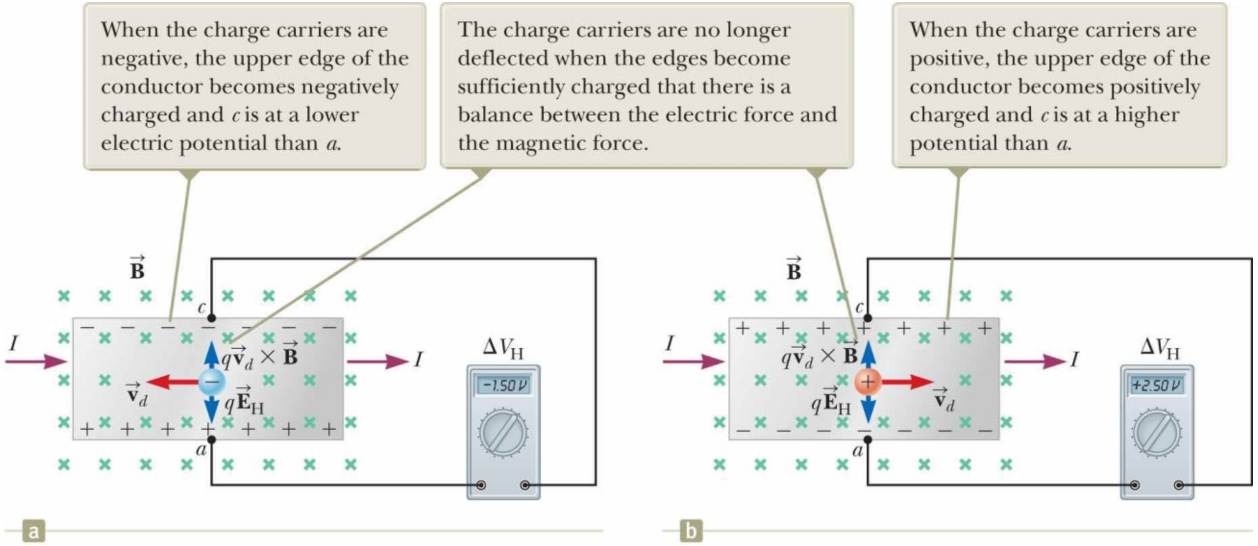
$$n = \frac{N_A}{V} = \frac{N_A\rho}{M}$$

Substitute this result

$$\Delta V_H = \frac{IB}{nqt} = \frac{MIB}{N_A\rho qt}$$

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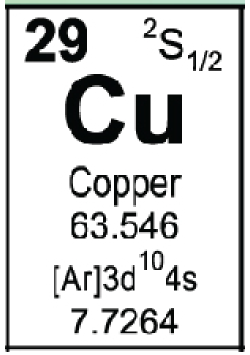


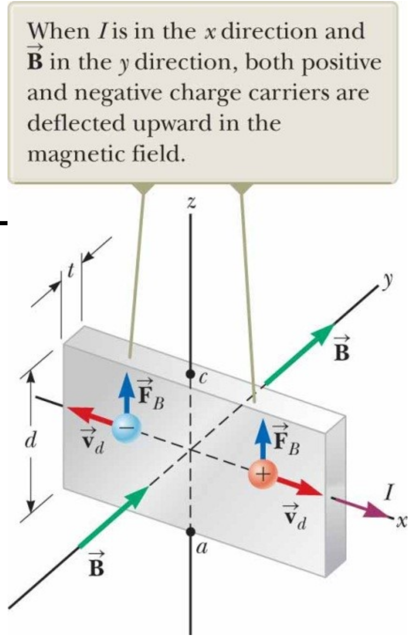


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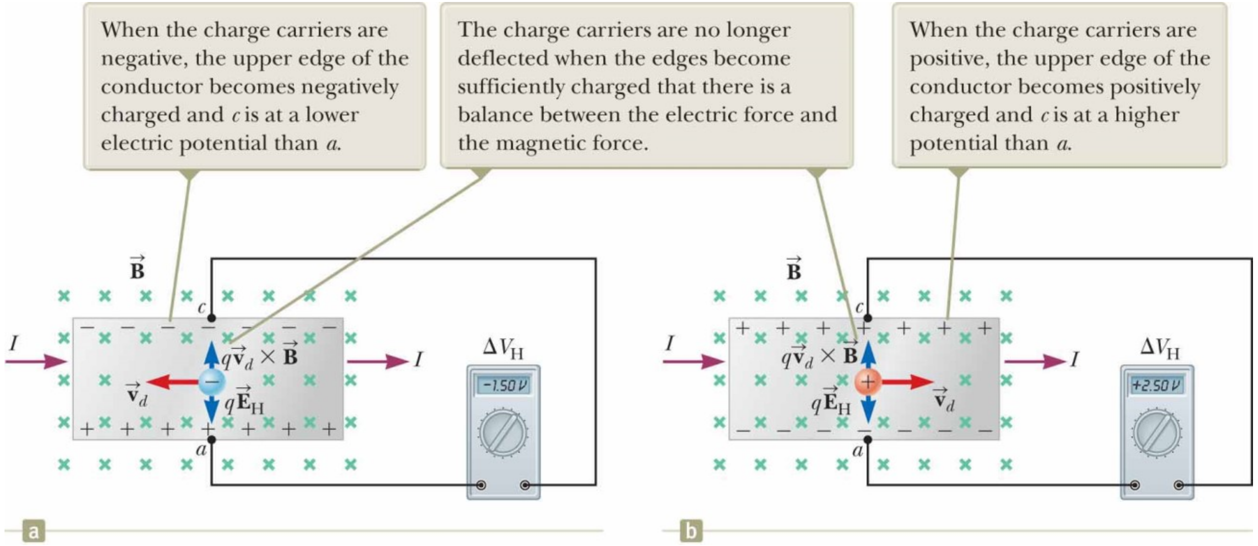
$$\Delta V_H = \frac{IB}{nqt} = \frac{MIB}{N_A \rho q t}$$

$$\Delta V_H = \frac{(0.063\,5\ \text{kg/mol})(5.0\ \text{A})(1.2\ \text{T})}{(6.02 \times 10^{23}\ \text{mol}^{-1})(8\,920\ \text{kg/m}^3)(1.60 \times 10^{-19}\ \text{C})(0.001\,0\ \text{m})}$$

$$= \boxed{0.44\ \mu\text{V}}$$

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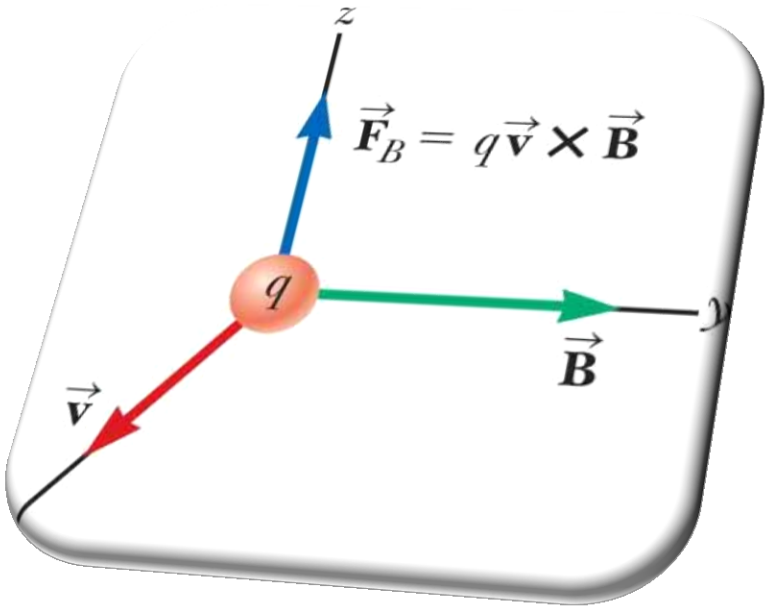
# Summary of Week 5, Class 5

- Reminder of the previous week

- Magnetic Fields (Ch. 28)
  - Analysis Model: Particle in a Magnetic Field
  - Motion of a Charged Particle in a Uniform Magnetic Field
  - Applications Involving Charged Particles Moving in a Magnetic Fi
  - Magnetic Force Acting on a Current Carrying Conductor
  - Torque on a Current Loop In a Uniform Magnetic Field
  - The Hall Effect




- Examples
- Next week's topic

# Reading / Preparation for Next Week

Topics for next week:

- Sources of Magnetic Fields (Ch. 29)