HUMBER ENGINEERING

ENGI 1000 - Physics 1

WEEK 8 - MODULE 6

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Module 6
Energy of a System – Part 1

- Energy and Work
- Systems and Surrounding
- Work
  - Work Done by a Constant Force & Variable Force
  - Work Done by a Spring
  - Work Done by Gravitational Force
- Energy
  - Kinetic Energy & Potential Energy
  - Work-Energy Theorem
  - Gravitational Potential Energy
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Energy and Work

What do you think of when you hear the words "work" and "energy"?

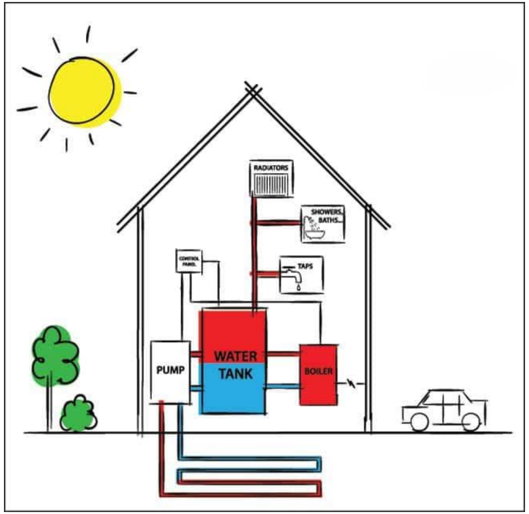
Energy and Work

- **Energy** is simply defined as the ability of an object to do work. It is a property of an object, like age or height or mass.
- Types of energy include:
  - Mechanical energy: Energy of movement and position
  - Chemical Energy: Energy stored in chemical bonds of molecules
  - Thermal Energy or Heat Energy: Energy stored in materials at a certain temperature
  - Nuclear Energy: Energy produced from splitting of atoms
  - Radiant Energy: Energy traveling the form of electromagnetic waves
  - Electric Energy: Energy traveling as the flow of electrons.
- **Work** is done when a task produces a change or transfer in energy.


Energy transferred $\Rightarrow$ Work done

Systems and Surroundings

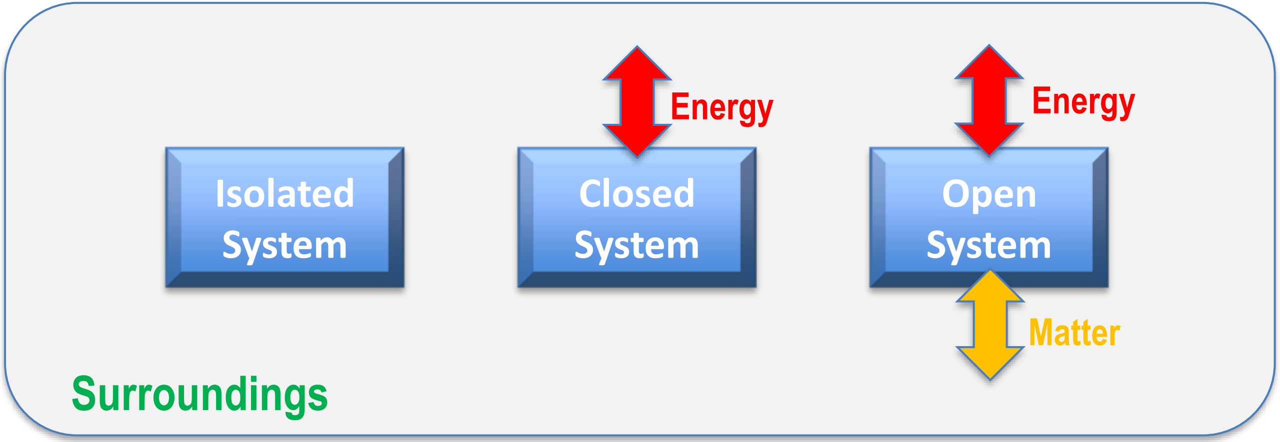
- A **system**, as it is defined in physics or chemistry as a collection of objects or smaller systems that can be identified.
- The **surrounding** is everything else that is **not** the system defined.
- For example, if the system being studied is a house, the surrounding would be everything else that is not the house, such as other houses, the neighborhood, the general environment around the house, etc.



- We may interest in studying the home heating and cooling system to avoid the loss of energy and increasing the efficiency of the system.

Systems and Surroundings

- Systems can be described in three different ways:
  - **Isolated System:** A system in which no matter or energy is being exchanged with the surroundings.
  - **Closed System:** A system in which only energy is being exchanged with the surroundings.
  - **Open System:** A system in which both matter and energy is being exchanged with the surroundings.



Give an example for each type of system

Work Done by a Constant Force

- Work is the measure of energy transfer when a force moves an object through a displacement.
- The amount of work done when a constant force $\vec{F}$ acts on an object depends on two things:

  1) The magnitude of the force acting on the object
  2) The displacement through which the force causes the object to move in the direction of the force

  - Work is a scalar quantity; it has no direction.
  - The SI unit of work is the joules (J). (1 J = 1 kg.m^2/s^2 = 1 N.m)

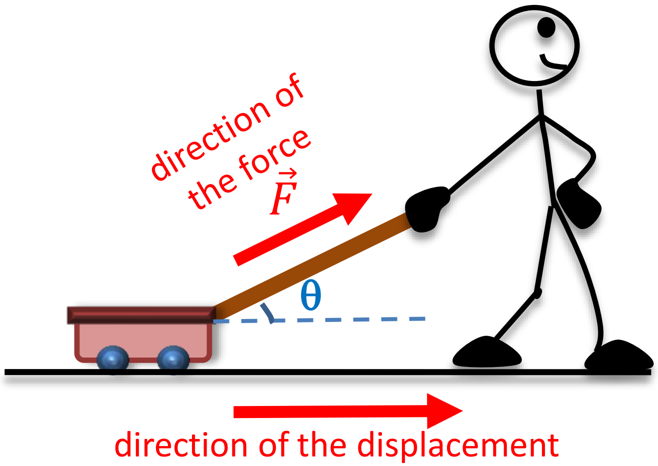
$$\boxed{W = (F\cos\theta)\Delta x}$$

W : the work done on an object (J)

F : magnitude of force (N)

Δx : the magnitude of the displacement (m)

θ : the angle between the force and the displacement (deg)



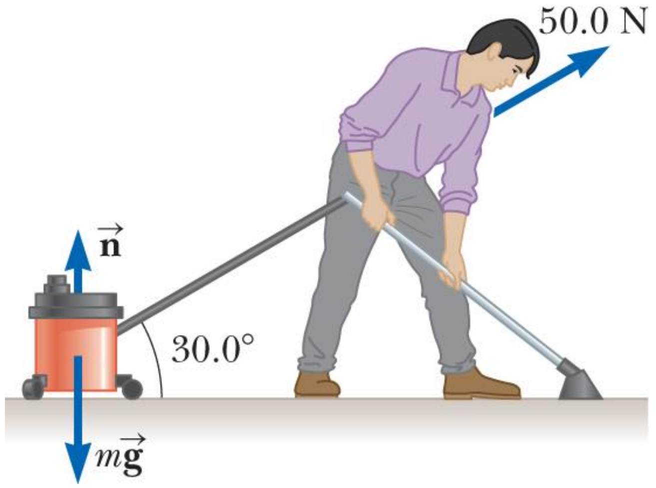
Work Done by a Constant Force

Example 1 (Mr. Clean): A man cleaning a floor pulls a vacuum cleaner with a force of magnitude *F* = 50.0 N at an angle of 30.0° with the horizontal. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

We identify the vacuum cleaner as the system and determine the applied force on it.

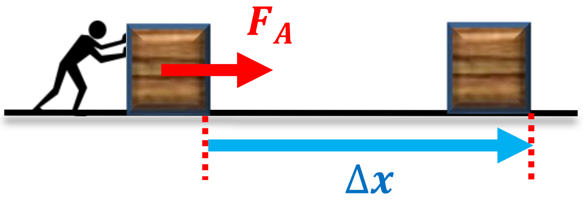
We have to distinguished which forces do work on the system and which do no work on the system.

$$\begin{aligned} W &= F\cos\theta\,\Delta x \\ &= (50.0\text{ N})(\cos 30.0°)(3.00\text{ m}) \\ &= \boxed{130\text{ J}} \end{aligned}$$

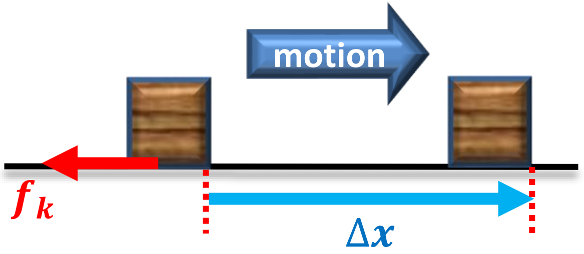


Positive, Negative and Zero Work

- **Positive work** is done if the force acting on an object is in the same direction as displacement.
  - Positive work means the object gains energy.
  - For example:
    - The work done when pushing a crate across the floor
    - The work done when lifting up a barbell



- **Negative work** is done if the force acting on an object is in the opposite direction of displacement.
  - Negative work means the object loses energy.
  - For example:
    - The work done by friction force on a sliding object
    - The work done when lowering down a barbell



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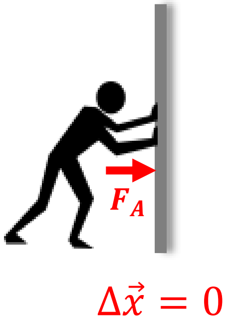
Positive, Negative and Zero Work

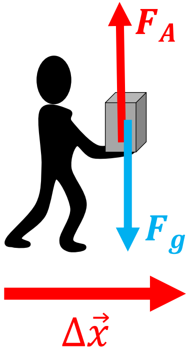
- **Zero work** is done if any of the elements of the work formula becomes zero:

$$W = (F\cos\theta)\Delta x$$

1) No work is done if the object does not move ($\Delta x = 0$)
   - For example, pushing a wall, the force acts on the wall, but no displacement.

2) No work is done if the force and displacement are perpendicular ($\theta = 90°$)
   - For example, hold an object and walk, the force acts in a downward direction, but displacement is in the forward direction.





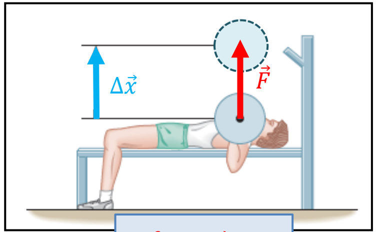
Zero work means the force is not changing the energy of the object

Positive, Negative and Zero Work

Example 2 (Weight Lifting): A weight lifter is lifting up a barbell whose weight is 710 N. First, he raises the barbell a distance of 0.65 m above his chest, and then he lowers it the same distance. If the weight is raised and lowered at a constant velocity, determine the work done on the barbell by the weight lifter at each phase.

During the lifting phase, the force $\vec{F}$ and displacement $\Delta\vec{x}$ are in the same direction. So, the angle between them is $\theta = 0°$.
The work done by the force $\vec{F}$ is

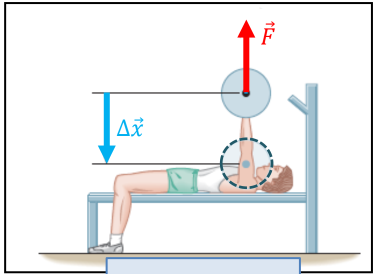
$$W = (F\cos\theta)\Delta x = [(710\ N)\cos(0°)](0.65\ m) = 461.5\ J$$



Lifting phase

When the barbell is lowered, the force $\vec{F}$ and displacement $\Delta\vec{x}$ are in opposite direction. So, the angle between them is $\theta = 180°$.
The work done by the force $\vec{F}$ is

$$W = (F\cos\theta)\Delta x = [(710\ N)\cos(180°)](0.65\ m) = -461.5\ J$$



Lowering phase

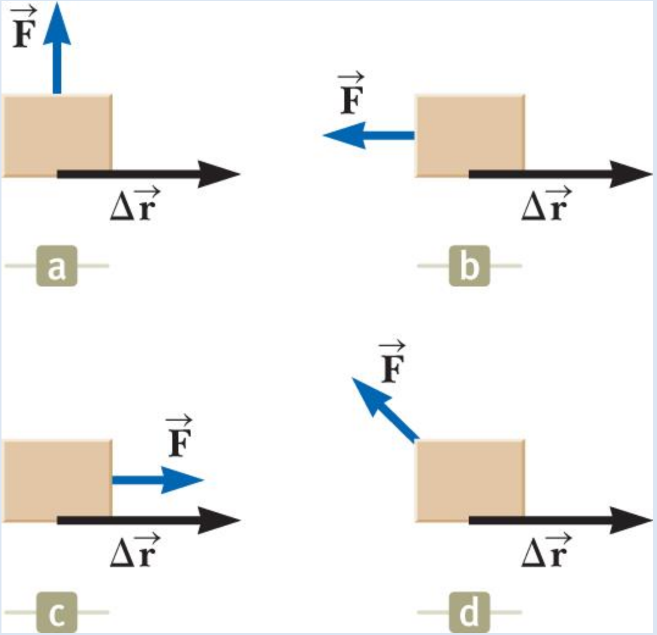
Quick Quiz 1

- As a simple pendulum swings back and forth, the forces acting on the suspended object are (a) the gravitational force, (b) the tension in the supporting cord, and (c) air resistance.
  - Which of these forces, if any, does **no work** on the pendulum at any time?
  - Which of these forces does **negative work** on the pendulum at all times during its motion?

a) gravitational force
b) the tension in the supporting cord
c) air resistance
d) None of the above

Quick Quiz 2

- The figure shows four situations in which a force is applied to an object. In all four cases, the force has the same magnitude, and the displacement of the object is to the right and of the same magnitude. Rank the situations in order of the work done by the force on the object, from most positive to most negative.

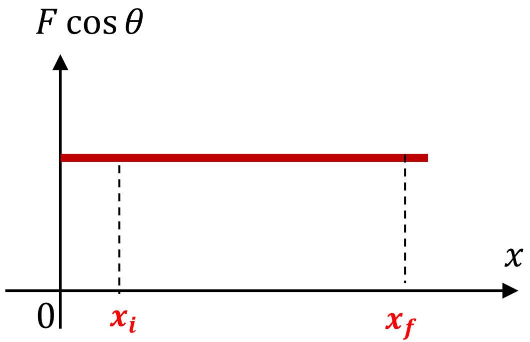


Work Done by a Variable Force

- The formula for work done by a constant force is valid only when the force is constant in both <u>magnitude</u> and <u>direction</u>.

$$\boxed{W = \underbrace{(F\cos\theta)}_{F_x}\Delta x}$$ Work done by a constant force

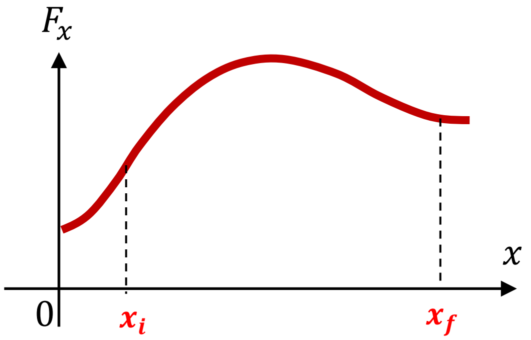
$F_x \rightarrow$ The component of force in the x direction



- There are some situations that the force is not constant but changes with the displacement of the object.

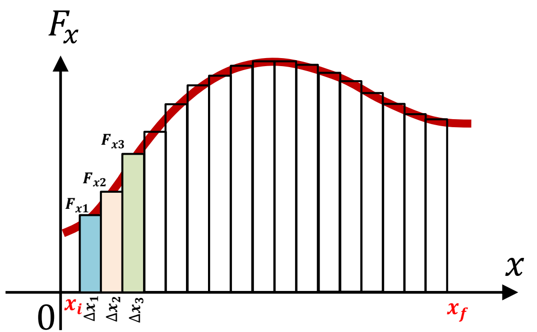
- When the force varies with the displacement, we cannot use the above formula to find the work.

- We can use a graphical method and introduce a new formula.



Work Done by a Variable Force

- Consider particle displaced along *x*-axis under action of force that *varies* with displacement.
- In this case we can divide the total displacement into very small segments $\Delta x_1, \Delta x_2, \ldots$
- For each segment, the average value of force components is indicated by a vertical line.
- We can use this average value as the constant-force component and determine an approximate value for the work done during each segment $\Delta x_1, \Delta x_2, \ldots$



$$\Delta W_1 = \underbrace{F_{x1}\Delta x_1}_{\text{Area of first rectangle}}, \quad \Delta W_2 = \underbrace{F_{x2}\Delta x_2}_{\text{Area of second rectangle}}, \quad \Delta W_3 = \underbrace{F_{x3}\Delta x_3}_{\text{Area of third rectangle}}, \quad \cdots$$

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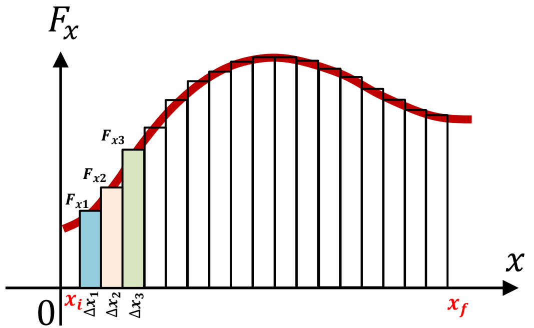
Work Done by a Variable Force

- Then the total work done by the variable force for the displacement from x_i to x_f is approximately equal to the sum of the areas of all the rectangular.

$$W \approx \Delta W_1 + \Delta W_2 + \Delta W_3 + \cdots \quad \Rightarrow \quad W \approx \sum_{x_i}^{x_f} F_x \Delta x$$



$$\Delta W_1 = F_{x1}\Delta x_1$$

$$\Delta W_2 = F_{x2}\Delta x_2$$

$$\Delta W_3 = F_{x3}\Delta x_3$$

- If the rectangles are made narrower and narrower by decreasing each Δx, the sum can be express as integral.

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx \quad \Rightarrow \quad \boxed{W = \int_{x_i}^{x_f} F_x dx}$$

Work Done by a Variable Force

Example 3 (Calculating Total Work Done from Graph): A force acting on a particle varies with *x* as shown in the figure. Calculate the work done by the force on the particle as it moves from *x* = 0 to *x* = 6.0 m.

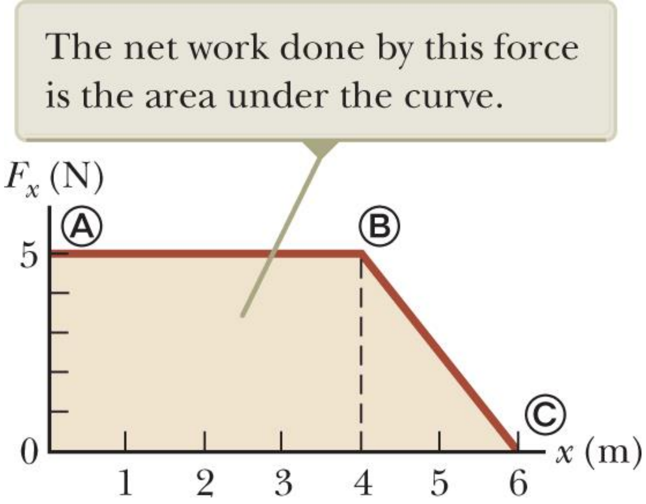
The work done by the force is equal to the area under the curve from point A to pint C.

This area is equal to the area of the rectangular section from A to B plus the area of the triangular section from B to C.

$$W_{\text{A to B}} = (5.0\text{ N})(4.0\text{ m}) = 20\text{ J}$$

$$W_{\text{B to C}} = \frac{1}{2}(5.0\text{ N})(2.0\text{ m}) = 5.0\text{ J}$$

$$W_{\text{A to C}} = W_{\text{A to B}} + W_{\text{B to C}} = 20\text{ J} + 5.0\text{ J} = \boxed{25\text{ J}}$$



Work Done by a Spring

- Consider a Mass-Spring system on a frictionless horizontal surface.
- **Hooke's Law:** The magnitude of the force exerted by a spring is directly proportional to the distance the spring has moved from equilibrium.

$$\boxed{F_s = -kx}$$

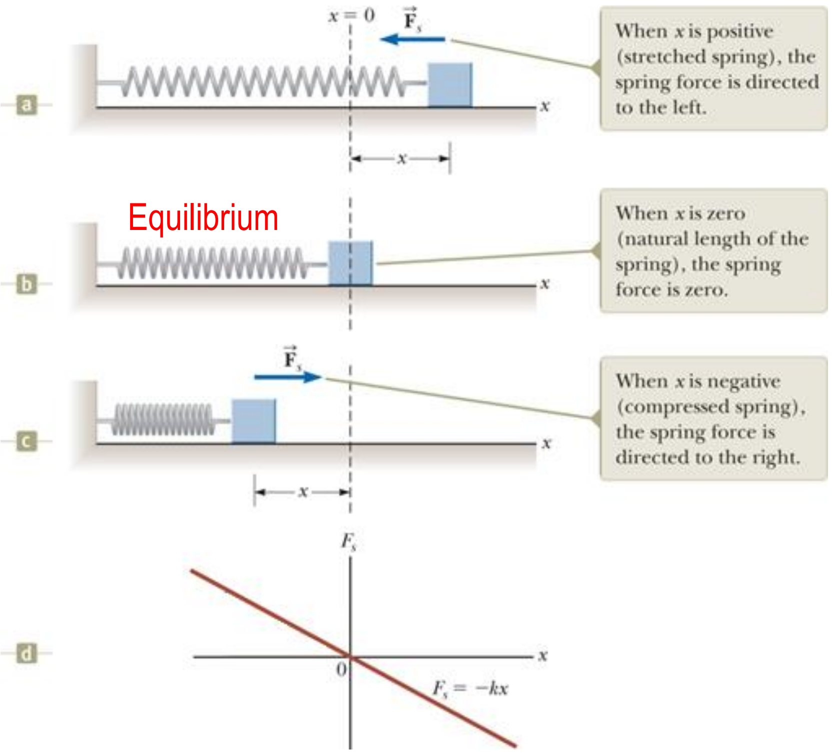
Spring Force (N)

Spring Constant (N/m)

Displacement (m)

- From the Newton's third law, the force applied to the spring to stretch or compress it to position x is

$$\boxed{F_{app} = +kx}$$



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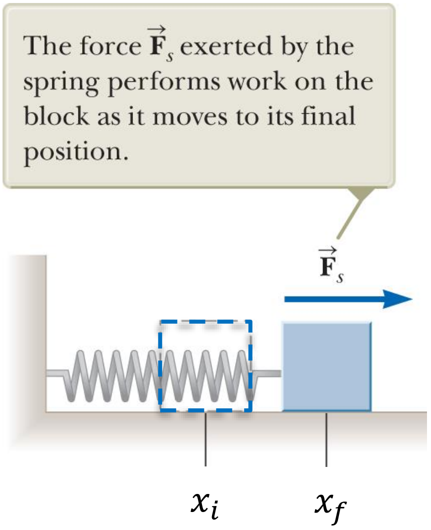
Work Done by a Spring

- If the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$, the work done by the spring force on the block is:

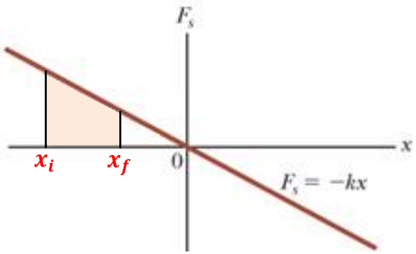
$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{x_i}^{x_f} (-kx) dx$$

$$\boxed{W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2}$$

The work done by the spring on the block



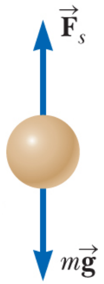
- The work done by the spring force on the block as it moves from x_i to x_f is the area of the shaded part.

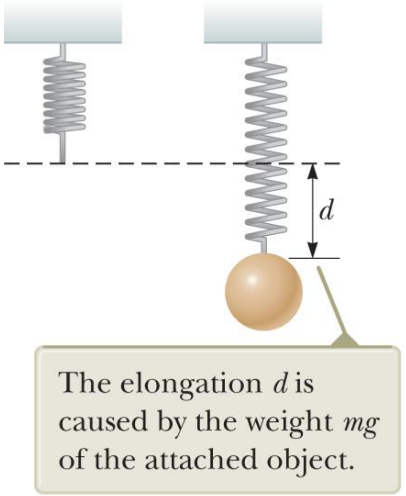


Work Done by a Spring

Example 4 (Measuring Spring Constant): A common technique used to measure the spring constant is demonstrated by the setup in the figure. The spring is hung vertically, and an object of mass *m* is attached to its lower end. Under the action of the "load" *mg*, the spring stretches a distance *d* from its equilibrium position.

(a) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the spring constant?





First, draw the free body diagram of the mass

Apply the Newton's second law on the mass in x and y directions.

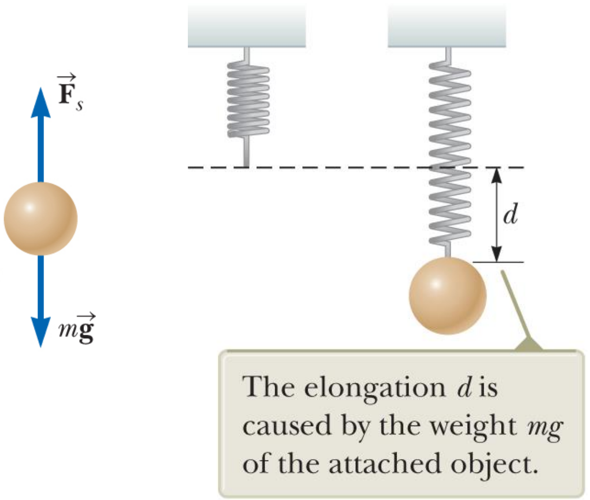
$$\sum F_y = 0 \quad \rightarrow \quad F_s - F_g = 0 \quad \rightarrow \quad F_s = mg$$

$$-kx = mg \quad \rightarrow \quad k = \frac{mg}{d} = \frac{(0.55\text{ kg})(9.80\text{ m/s}^2)}{2.0 \times 10^{-2}\text{ m}} = \boxed{2.7 \times 10^2\text{ N/m}}$$

Work Done by a Spring

Example 4 (Measuring Spring Constant): A common technique used to measure the spring constant is demonstrated by the setup in the figure. The spring is hung vertically, and an object of mass *m* is attached to its lower end. Under the action of the “load” *mg*, the spring stretches a distance *d* from its equilibrium position.

(b) How much work is done by the spring on the object as it stretches through this distance?



Find the work done by the spring on the object

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$W_s = 0 - \frac{1}{2}kd^2 = -\frac{1}{2}(2.7 \times 10^2\ \mathrm{N/m})(2.0 \times 10^{-2}\ \mathrm{m})^2$$

$$W_s = -5.4 \times 10^{-2}\ \mathrm{J}$$

The work is <u>negative</u> because the spring force acts upward on the object, but displacement is downward.

Quick Quiz 3

- A dart is inserted into a spring-loaded dart gun by pushing the spring in by a distance *x*. For the next loading, the spring is compressed a distance 2*x*. How much work is required to load the second dart compared with that required to load the first?

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

a) four times as much
b) two times as much
c) the same
d) half as much
e) one-fourth as much

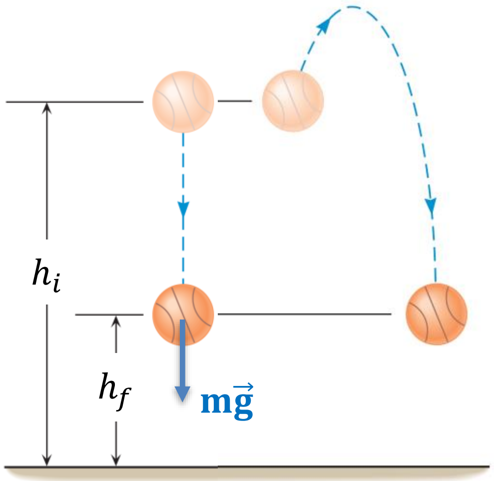
Work Done by Gravitational Force

- Consider a basketball of mass m moving vertically downward under the free-fell motion. The initial height and final height of the ball from the earth's surface are h_i and h_f.
- The work done by the force of gravity is obtained as

$$W = (F\cos\theta)\Delta y \quad \Rightarrow \quad W_{gravity} = (mg\cos 0°)\left(h_i - h_f\right)$$

$$\boxed{W_{gravity} = mg(h_i - h_f)}$$

The work done by the gravitational force



- The work done by the force of gravity is independent from the selected path. It depends on the vertical distance.

- The object can move along different paths in going from the initial height of h_i to a final height of h_f. In each case, the work done by the force of gravity is the same.

Work Done by Gravitational Force

Example 5: A spring is hung vertically, and an object of mass *m = 0.55 kg* is attached to its lower end. Under the action of the "load" *mg*, the spring stretches a distance *d = 2.0 cm* from its equilibrium position.

How much work is done by the gravitational force on the object as it stretches through this distance?

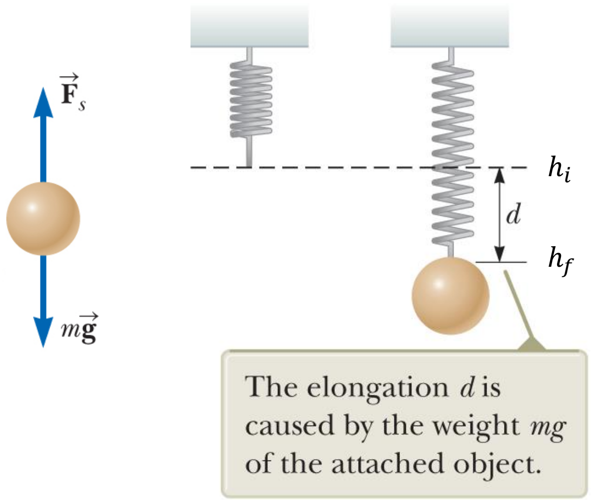
Evaluate the work done by the gravitational force on the object.

$$W_{gravity} = mg(h_i - h_f)$$

$$W_{gravity} = mgd = (0.55\ kg)(9.8 m/s^2)(2.0 \times 10^{-2}\ m)$$

$$W_{gravity} = 1.1 \times 10^{-1}\ J$$

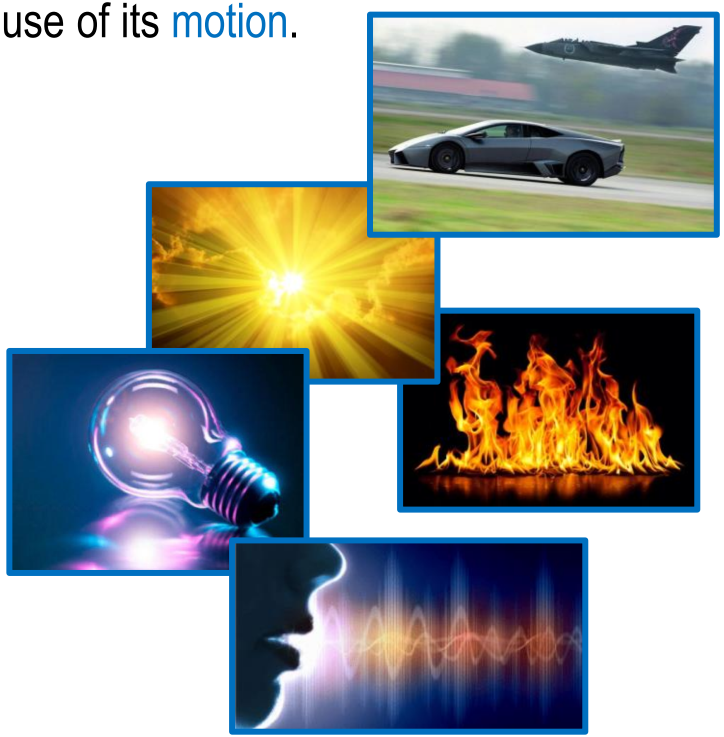
The work is positive because the gravitational force acts downward on the object, and displacement is also downward.



Kinetic Energy

- Kinetic energy (K) is the energy a moving object has because of its motion.

- Some forms of kinetic energy include:
    - Thermal Energy or Heat Energy: The energy an object has as a result of random motion of its molecules
    - Radiant Energy: The energy that travels as electromagnetic waves, such as light and radio waves
    - Electrical Energy: The energy possessed by electrons moving through a circuit
    - Sound Energy: The energy carried from molecule to molecule by vibration



Kinetic Energy

- Kinetic energy (K) is the energy a moving object has because of its motion.
- The kinetic energy (K) of an object with mass m and speed v is given by

$$\boxed{K \equiv \frac{1}{2}mv^2}$$

- Energy is a scalar quantity; it has no direction.
- The SI unit of both energy and work are the joules (J).
- 1 J = 1 $kg.m^2/s^2$ = 1 N.m

Work-Energy Theorem

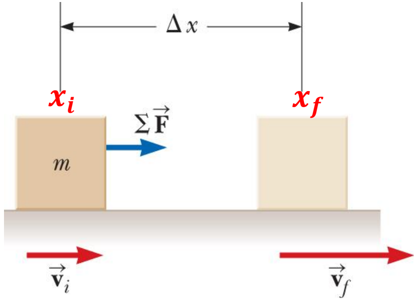
- The Work-Energy Theorem states that the work applied to a system is equal to the change in kinetic energy of that system.

$$\boxed{W_{ext} = \Delta K = K_f - K_i = \underbrace{\tfrac{1}{2}mv_f^2}_{\text{Final kinetic energy}} - \underbrace{\tfrac{1}{2}mv_i^2}_{\text{Initial kinetic energy}}}$$

- Consider a block of mass m moving through displacement directed to the right under action of net force, which is also directed to the right.
- According to Newton's second law, the net force produces an acceleration:

$$\sum F = ma$$

- Work done by the net external force is:



$$W_{ext} = \int_{x_i}^{x_f} \sum F dx = \int_{x_i}^{x_f} ma\, dx = \int_{x_i}^{x_f} m\frac{dv}{dt}\, dx = \int_{x_i}^{x_f} m\frac{dx}{dt}\, dv = \int_{v_i}^{v_f} mv\, dv = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Work-Energy Theorem

Example 6 (A Block Pulled on a Frictionless Surface): A 6.0-kg block initially at rest is pulled to the right along a frictionless, horizontal surface by a constant horizontal force of magnitude 12 N. Find the block's speed after it has moved through a horizontal distance of 3.0 m.

Try to solve it by Newton's Laws approach and compare.

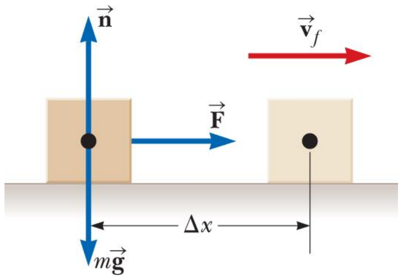
Draw the forces applied on the block and find the net external force applied on the block.

The net external force acting on the block is the horizontal 12-N force.

Use the work–kinetic energy theorem for the block and solve for v_f

$$W_{\text{ext}} = \Delta K = K_f - K_i = \frac{1}{2}m{v_f}^2 - 0 = \frac{1}{2}m{v_f}^2$$

$$v_f = \sqrt{\frac{2W_{\text{ext}}}{m}} = \sqrt{\frac{2F\cos\theta\,\Delta x}{m}} = \sqrt{\frac{2(12\text{ N})(\cos 0°)(3.0\text{ m})}{6.0\text{ kg}}} = \boxed{3.5\text{ m/s}}$$



Quick Quiz 4

- A dart is inserted into a spring-loaded dart gun by pushing the spring in by a distance *x*. For the next loading, the spring is compressed a distance 2*x*. How much faster does the second dart leave the gun compared with the first?

a) four times as fast
b) two times as fast
c) the same
d) half as fast
e) one-fourth as fast

$$W_{\text{ext}} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

$$W_{\text{ext}} = \frac{1}{2}m{v_f}^2 - \frac{1}{2}m{v_i}^2$$

Potential Energy

- Potential energy is the energy stored in an object, which gives the object the ability or potential to do work at a later time.

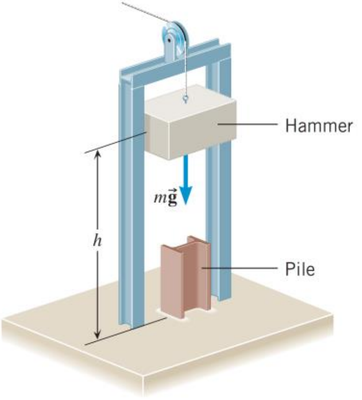
- Some forms of potential energy include:
  - Chemical potential energy: The energy stored in molecules
  - Nuclear potential energy: The energy stored in the nucleus of an atom
  - Gravitational potential energy: The potential energy an object has due to its height relative to a reference point, and therefore has an ability to fall.
  - Elastic potential energy: The energy stored in an elastic material when it is forced out of its normal shape.




Gravitational Potential Energy

- Gravitational potential energy (U_g) is the energy stored in an object due to its height relative to some other surface.
  - It is measured relative to a zero level such as the ground.
- The amount of gravitational potential energy possessed by an object is proportional to its mass m, its height h and the gravitational acceleration g.

$$\boxed{U_g \equiv mgh}$$



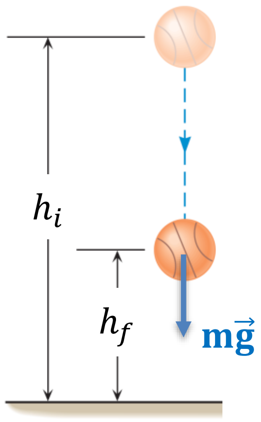
- Energy is a scalar quantity; it has no direction.
- The SI unit of both energy and work are the joules (J).
- 1 J = 1 kg.m^2/s^2 = 1 N.m

Gravitational Potential Energy

- Recall the formula for the work done by the gravitational force on an object when moves from the height of h_i to the height of h_f

$$\boxed{W_{gravity} = mg(h_i - h_f) = \underbrace{mgh_i}_{\text{Initial gravitational potential energy}} - \underbrace{mgh_f}_{\text{Final gravitational potential energy}} = U_i - U_f = \Delta U_g}$$

The work done by the gravitational force



- The work done by the force of gravity is defined as the change in the gravitational potential energy of the object.

Gravitational Potential Energy

Example 7 (The Proud Athlete and The Sore Toe): A trophy being shown off by a careless athlete slips from the athlete's hands and drops on his foot. Assume $m_{trophy} \approx 2kg$, the top of foot is about $0.05\ m$ above the floor, and trophy falls from a height $h = 1.4m$.

(a) Choosing floor level as the $y = 0$ point of your coordinate system, estimate the change in gravitational potential energy of the trophy–Earth system as the trophy falls.

$$U_i = mgy_i = (2\text{ kg})(9.80\text{ m/s}^2)(1.4\text{ m}) = 27.4\text{ J}$$

$$U_f = mgy_f = (2\text{ kg})(9.80\text{ m/s}^2)(0.05\text{ m}) = 0.98\text{ J}$$

$$\Delta U_g = U_f - U_i = 0.98\text{ J} - 27.4\text{ J} = -26.4\text{ J}$$

(b) Determine the work done by the gravitational force on the object.

$$W_{gravity} = U_i - U_f = +26.4\text{ J}$$

Quick Quiz 5

- Choose the correct answer.

  The gravitational potential energy of a system …………………………………

  a) is always positive
  b) is always negative
  c) can be negative or positive

Quick Quiz 6

- Rank the gravitational potential energies (of the object–Earth system) for the following four objects, largest first, taking $y = 0$ at the floor.

  A) a 2-kg object 5 cm above the floor,
  B) a 2-kg object 120 cm above the floor,
  C) a 3-kg object 120 cm above the floor, and
  D) a 3-kg object 80 cm above the floor.

a) C > B = D > A

b) A = B = C = D

c) C = D > A = B

d) D > A = C > B

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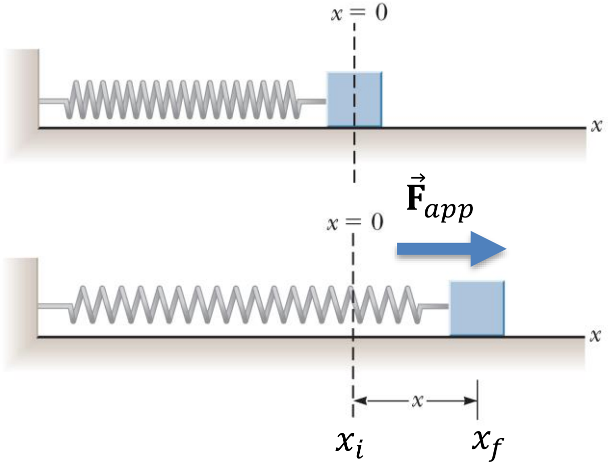
Elastic Potential Energy

- Elastic potential energy is energy that is stored in a compressed or stretched elastic object, which is equal to the work done to stretch or compressed the elastic object.
- Assume a mass-spring system on a frictionless horizontal surface. The elastic potential energy function associated with the mass-spring system depends on the spring constant and the distance stretched, which is defined as

$$\boxed{U_s \equiv \frac{1}{2}kx^2}$$

- The external work done to stretch the spring from x_i to x_f is

$$\boxed{W_{ext} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 = U_f - U_i = \Delta U_s}$$



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Quick Quiz 7

- A ball is connected to a light spring suspended vertically as shown in the figure. When pulled downward from its equilibrium position and released, the ball oscillates up and down.
- In the system of *the ball, the spring, and the Earth,* what forms of energy are there during the motion?

a) kinetic and elastic potential
b) kinetic and gravitational potential
c) kinetic, elastic potential, and gravitational potential
d) elastic potential and gravitational potential

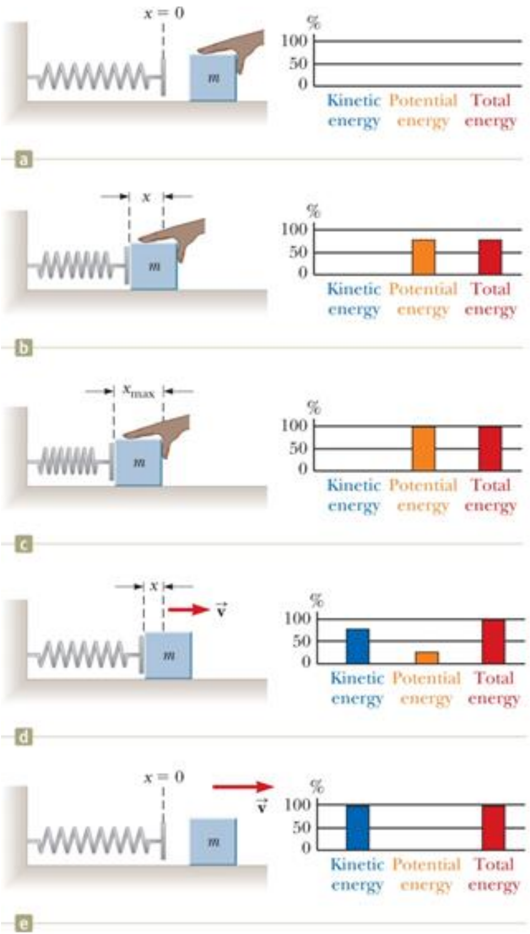
Quick Quiz 8

- A ball is connected to a light spring suspended vertically as shown in the figure. When pulled downward from its equilibrium position and released, the ball oscillates up and down.
- In the system of *the ball and the spring,* what forms of energy are there during the motion?

a) kinetic and elastic potential
b) kinetic and gravitational potential
c) kinetic, elastic potential, and gravitational potential
d) elastic potential and gravitational potential

Energy Bar Charts

- An energy bar chart is a graphical representation of information related to energy of the system.
  - The horizontal axis shows the types of energy in the system.
  - The vertical axis represents the amount of energy of a given type in the system.
- Consider the mass-spring system on a frictionless horizontal surface.
- External force is applied to push the mass against the spring to the left.
  - Figure (a): Before the spring is compressed, there is no energy in the mass-spring system.
  - Figure (b): When the spring is partially compressed, the total energy of the system is elastic potential energy.
  - Figure (c): The spring is compressed by a maximum amount, and the block is held steady; there is elastic potential energy in the system and no kinetic energy.
  - Figure (d): After the block is released, the elastic potential energy in the system decreases and the kinetic energy increases.
  - Figure (e): After the block loses contact with the spring, the total energy of the system is kinetic energy.



THANK YOU

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