CHARACTERISTICS OF DRY FRICTION & PROBLEMS INVOLVING DRY FRICTION

ENGI 1510 - ENGINEERING DESIGN
Winter 2022

CHARACTERISTICS OF DRY FRICTION & PROBLEMS INVOLVING DRY FRICTION

<u>Today's Objective</u>:

Students will be able to:

a) Understand the characteristics of dry friction

b) Draw a FBD including friction.

c) Solve problems involving friction.

<u>In-Class Activities</u>:

- Check Homework, if any
- Reading Quiz
- Applications
- Characteristics of Dry Friction
- Problems involving Dry Friction
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. A friction force always acts _____ to the contact surface.

   A) Normal B) At 45°

   C) Parallel D) At the angle of static friction

2. If a block is stationary, then the friction force acting on it is ________ .

   A) $\leq \mu_s N$ B) $= \mu_s N$

   C) $\geq \mu_s N$ D) $= \mu_k N$

APPLICATIONS

In designing a brake system for a bicycle, car, or any other vehicle, it is important to understand the frictional forces involved.

For an applied force on the bike tire brake pads, how can we determine the magnitude and direction of the resulting friction force?

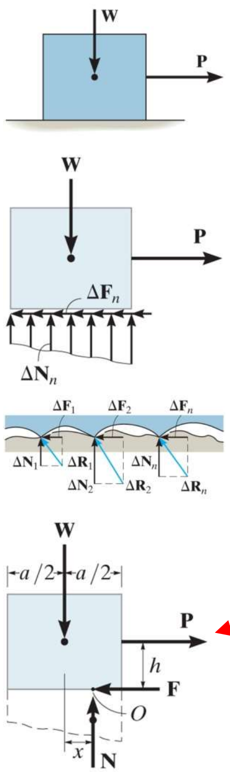
APPLICATIONS (continued)

The rope is used to tow the refrigerator.

In order to move the refrigerator, is it best to pull up as shown, pull horizontally, or pull downwards on the rope?

What physical factors affect the answer to this question?

CHARACTERISTICS OF DRY FRICTION (Section 8.1)



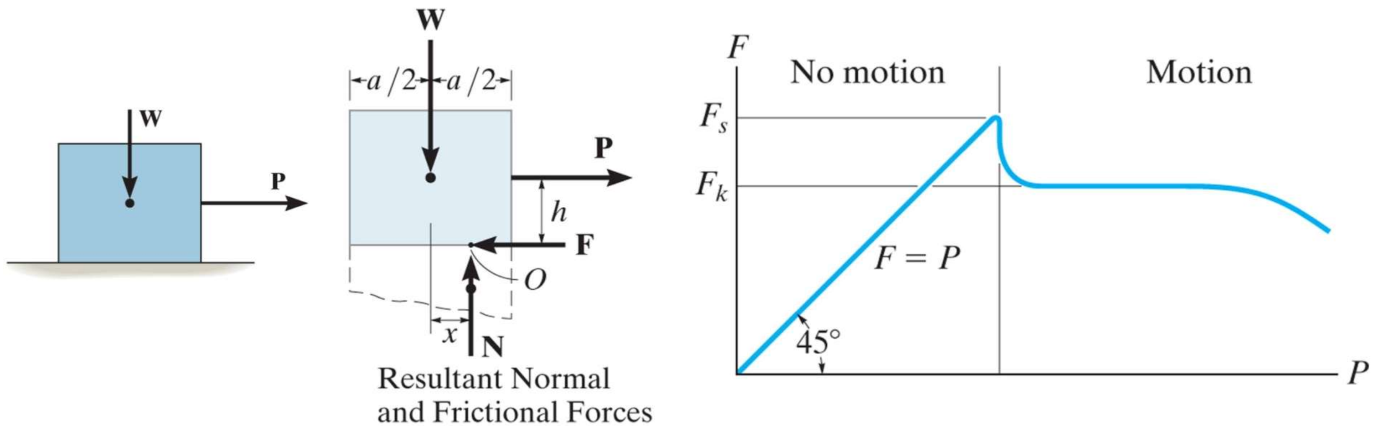
Resultant Normal and Frictional Forces

Friction is defined as a force of resistance acting on a body which prevents or resists the slipping of a body relative to a second body.

Experiments show that frictional forces act tangent (parallel) to the contacting surface in a direction opposing the relative motion or tendency for motion.

For the body shown in the figure to be in equilibrium, the following must be true:
F = P,   N = W, and W*x = P*h.

CHARACTERISTICS OF DRY FRICTION (continued)

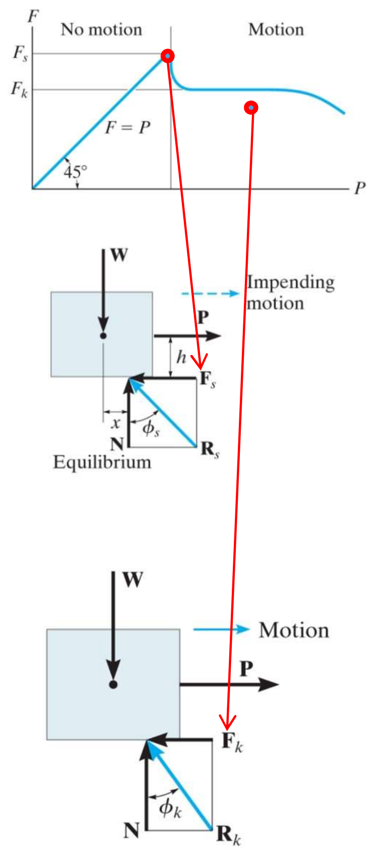


To study the characteristics of the friction force F, let us assume that tipping does not occur (i.e., “h” is small or “a” is large).

Then we gradually increase the magnitude of the force P.

Typically, experiments show that the friction force F varies with P, as shown in the right figure above.

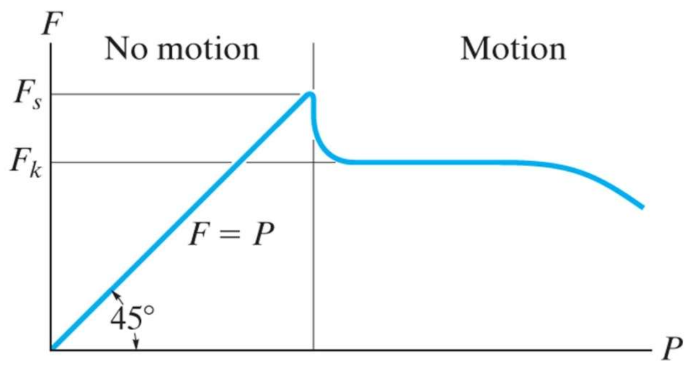
CHARACTERISTICS OF DRY FRICTION (continued)



The maximum friction force is attained just before the block begins to move (a situation that is called "impending motion"). The value of the force is found using $F_s = \mu_s N$, where μ_s is called the coefficient of static friction. The value of μ_s depends on the two materials in contact.

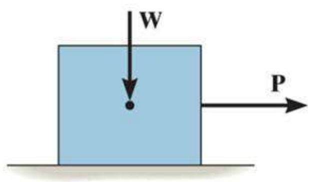
Once the block begins to move, the frictional force typically drops and is given by $F_k = \mu_k N$. The value of μ_k (coefficient of kinetic friction) is less than μ_s .

CHARACTERISTICS OF DRY FRICTION (continued)

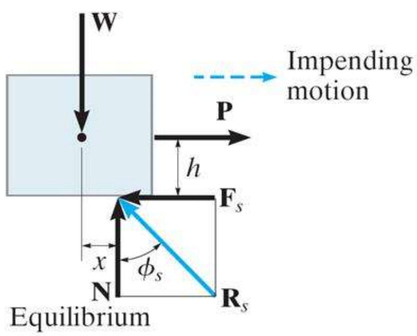


It is also very important to note that the friction force may be less than the maximum friction force. So, just because the object is not moving, <u>don't assume</u> the friction force is at its maximum of $F_s = \mu_s N$ unless you are told or know motion is impending!

DETERMENING μ_s EXPERIMENTALLY



If the block just begins to slip, the maximum friction force is $F_s = \mu_s N$, where μ_s is the coefficient of static friction.

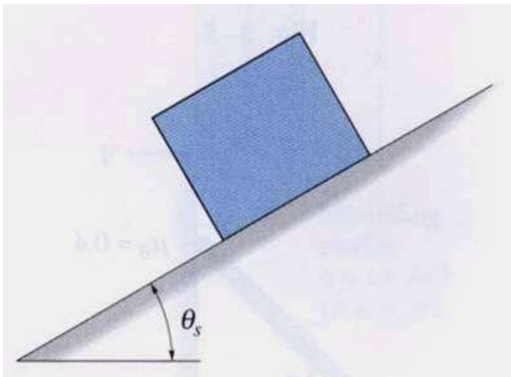


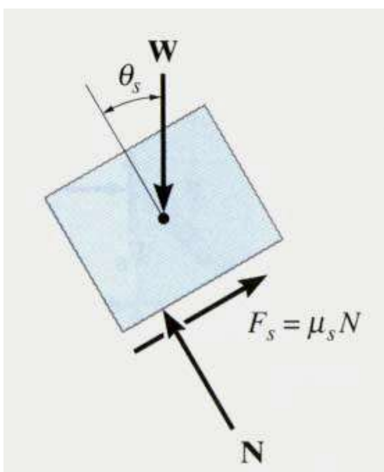
Thus, when the block is on the verge of sliding, the normal force N and frictional force F_s combine to create a resultant R_s.

From the figure,

$\tan \phi_s = (F_s / N) = (\mu_s N / N) = \mu_s$

DETERMING μ_s EXPERIMENTALLY (continued)





A block with weight w is placed on an inclined plane. The plane is slowly tilted until the block just begins to slip.

The inclination, θ_s, is noted. Analysis of the block just before it begins to move gives (using $F_s = \mu_s N$):

$$\nwarrow + \quad \sum F_y \;=\; N \;-\; W \cos \theta_s \;=\; 0$$

$$\nearrow + \quad \sum F_X \;=\; \mu_S N \;-\; W \sin \theta_s \;=\; 0$$

Using these two equations, we get
$\mu_s \;=\; (W \sin \theta_s\,) \,/\, (W \cos \theta_s\,) \;=\; \tan \theta_s$
This simple experiment allows us to find the μ_S between two materials in contact.

PROBLEMS INVOLVING DRY FRICTION
(Section 8.2)

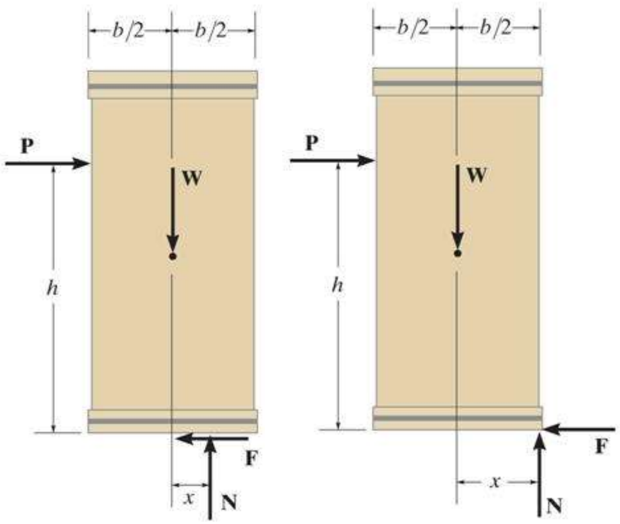
Steps for solving equilibrium problems involving dry friction:

1. Draw necessary free body diagrams. Make sure that you show the friction force in the correct direction (it always opposes the motion or impending motion).

2. Determine the number of unknowns. Do not assume that $F = \mu_S N$ unless the impending motion condition is given.

3. Apply the equations of equilibrium and appropriate frictional equations to solve for the unknowns.

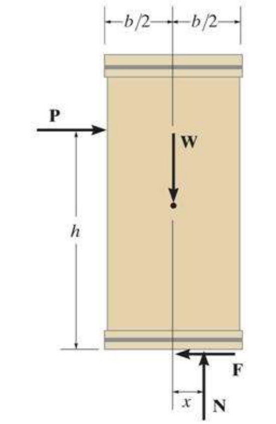
IMPENDING TIPPING versus SLIPPING



For a given W and h of the box, how can we determine if the block will slide or tip first? In this case, we have four unknowns (F, N, x, and P) and only the three E-of-E.

Hence, we have to make an assumption to give us another equation (the friction equation!). Then we can solve for the unknowns using the three E-of-E. Finally, we need to check if our assumption was correct.

IMPENDING TIPPING versus SLIPPING (continued)



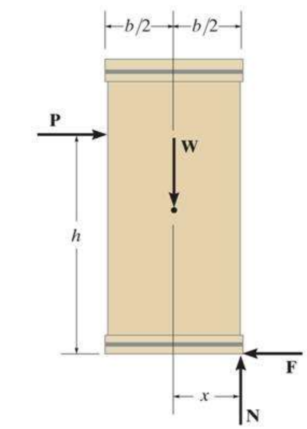
Assume: Slipping occurs

Known: F = μ_s N

Solve: x, P, and N

Check: $0 \leq x \leq b/2$

Or



Assume: Tipping occurs

Known: x = b/2

Solve: P, N, and F

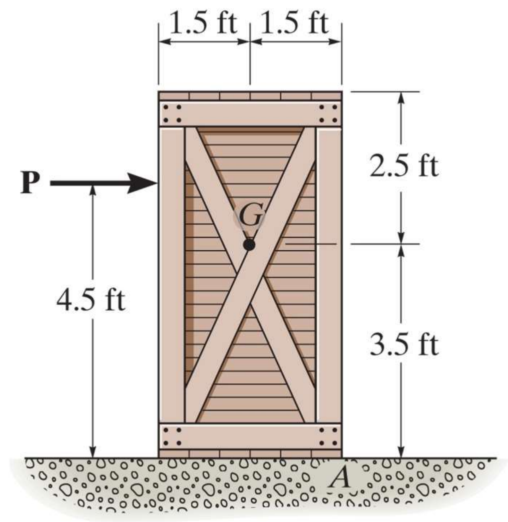
Check: $F \leq \mu_s N$

EXAMPLE

Given: Crate weight = 250 lb and $\mu_s = 0.4$

Find: The maximum force P that can be applied without causing movement of the crate.

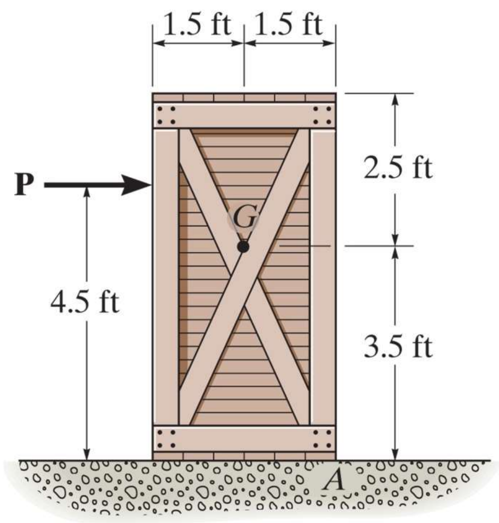
Plan: ??

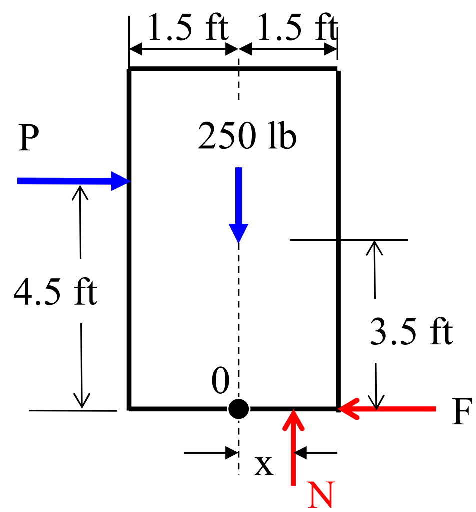


a) Draw a FBD of the box.
b) Determine the unknowns.
c) Make your friction assumptions.
d) Apply E-of-E (and friction equations, if appropriate) to solve for the unknowns.
e) Check assumptions, if required.

EXAMPLE (continued)

Solution:



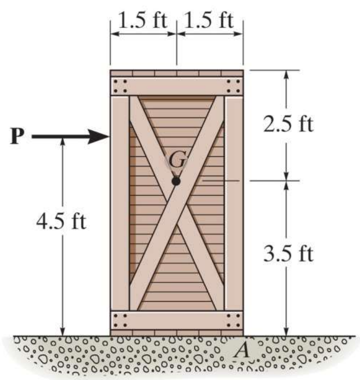


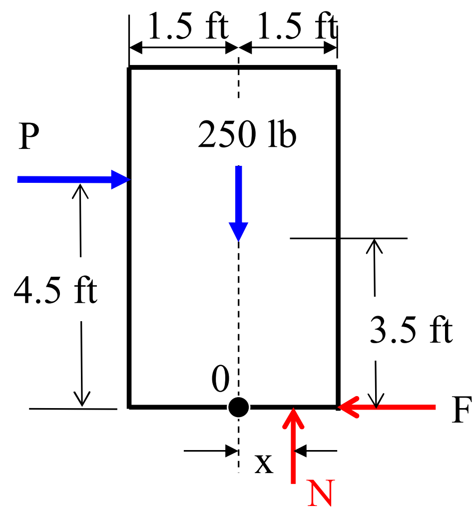
FBD of the crate

There are four unknowns: P, N, F and x.

First, let's assume the crate slips. Then the friction equation is $F = \mu_s N = 0.4 N$.

EXAMPLE (continued)





FBD of the crate

$$+ \rightarrow \sum F_X = P - 0.4\,N = 0$$

$$+ \uparrow \sum F_Y = N - 250 = 0$$

Solving these two equations gives:

$P = 100$ lb and $N = 250$ lb

$$+ \sum M_O = -100\,(4.5) + 250\,(x) = 0$$

Check: $x = 1.8 \geq 1.5$: No slipping will occur since $x > 1.5$

EXAMPLE (continued)

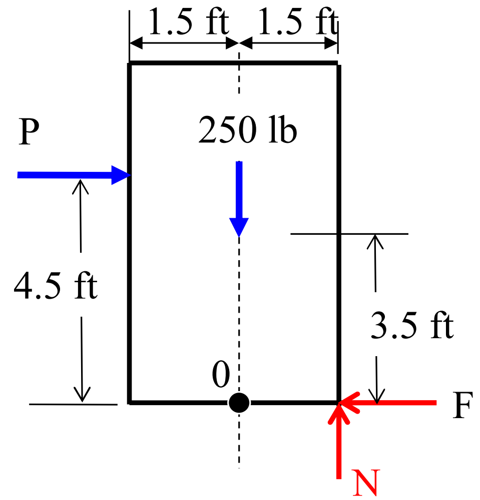
Since tipping occurs, here is the correct FBD:

$$+ \rightarrow \sum F_X = P - F = 0$$

$$+ \uparrow \sum F_Y = N - 250 = 0$$

These two equations give:

$P = F$ and $N = 250$ lb



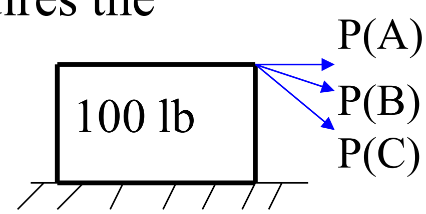
FBD of the crate

$$\circlearrowright + \sum M_O = -P\,(4.5) + 250\,(1.5) = 0$$

$P = 83.3$ lb, and $F = 83.3 \text{ lb} < \mu_s N = 100$ lb

CONCEPT QUIZ

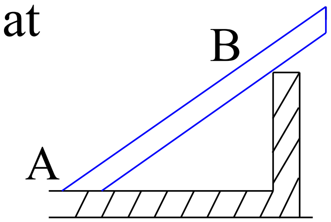
1. A 100 lb box with a wide base is pulled by a force P and $\mu_s = 0.4$. Which force orientation requires the least force to begin sliding?



   A) P(A)   B) P(B)

   C) P(C)   D) Can not be determined

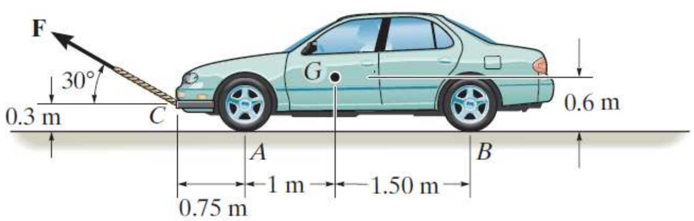
2. A ladder is positioned as shown. Please indicate the direction of the friction force on the ladder at B.



   A) ↑   B) ↓

   C) ↗   D) ↙

GROUP PROBLEM SOLVING



Given: Automobile has a mass of 2000 kg and $\mu_s = 0.3$.

Find: The smallest magnitude of **F** required to move the car if the back brakes are locked and the front wheels are free to roll.

Plan: a) Draw FBDs of the car.

b) Determine the unknowns.

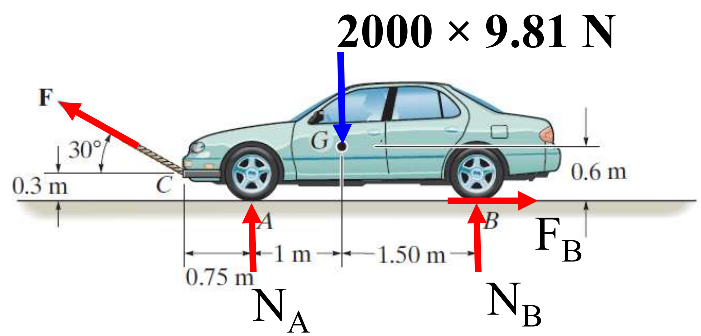
d) Apply the E-of-E and friction equations to solve for the unknowns.

GROUP PROBLEM SOLVING (continued)

Here is the correct FBD:

Note that there are four unknowns: F, N_A, N_B, and F_B.

FBD of the car



Equations of Equilibrium:

$$+\rightarrow \sum F_X = F_B - F(\cos 30°) = 0 \qquad (1)$$

$$+\uparrow \sum F_Y = N_A + N_B + F(\sin 30°) - 19620 = 0 \qquad (2)$$

$$\circlearrowleft + \sum M_A = F\cos 30°(0.3) - F\sin 30°(0.75) + N_B(2.5) - 19620(1) = 0 \qquad (3)$$

Assume that the rear wheels are on the verge of slip. Thus

$$F_B = \mu_s N_B = 0.3\, N_B \qquad (4)$$

GROUP PROBLEM SOLVING (continued)

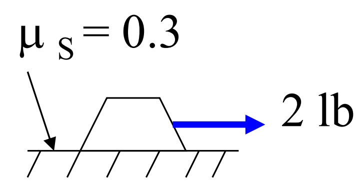
Solving Equations (1) to (4),

$\underline{F = 2762\ N}$

and $\underline{N_A = 10263\ N}$, $\underline{N_B = 7975\ N}$, $\underline{F_B = 2393\ N}$.

ATTENTION QUIZ

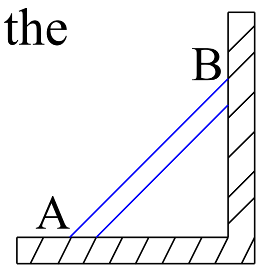
1. A 10 lb block is in equilibrium. What is the magnitude of the friction force between this block and the surface?



   A) 0 lb B) 1 lb

   C) 2 lb D) 3 lb

2. The ladder AB is positioned as shown. What is the direction of the friction force on the ladder at B.



   A) ↗ B) ↙

   C) ← D) ↑

WEDGES AND FRICTIONAL FORCES ON FLAT BELTS

Today's Objectives:

Students will be able to:

a) Determine the forces on a wedge.

b) Determine tension in a belt.

In-Class Activities:

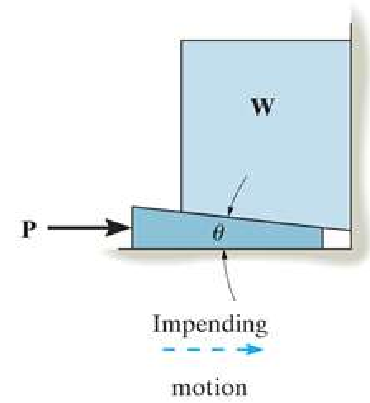
- Check Homework, if any
- Reading Quiz
- Applications
- Analysis of a Wedge
- Analysis of a Belt
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. A wedge allows a ______ force P to lift a _________ weight W.

   A) (large, large)   B) (small, large)

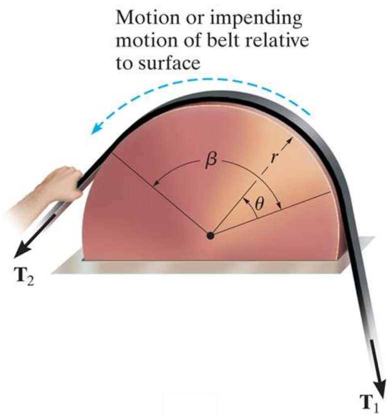
   C) (small, small)   D) (large, small)



2. Considering friction forces and the indicated motion of the belt, how are belt tensions T_1 and T_2 related?

   A) $T_1 > T_2$ B) $T_1 = T_2$

   C) $T_1 < T_2$ D) $T_1 = T_2 e^{\mu}$



APPLICATIONS

Wedges are used to adjust the elevation or provide stability for heavy objects such as this large steel pipe.

How can we determine the force required to pull the wedge out?

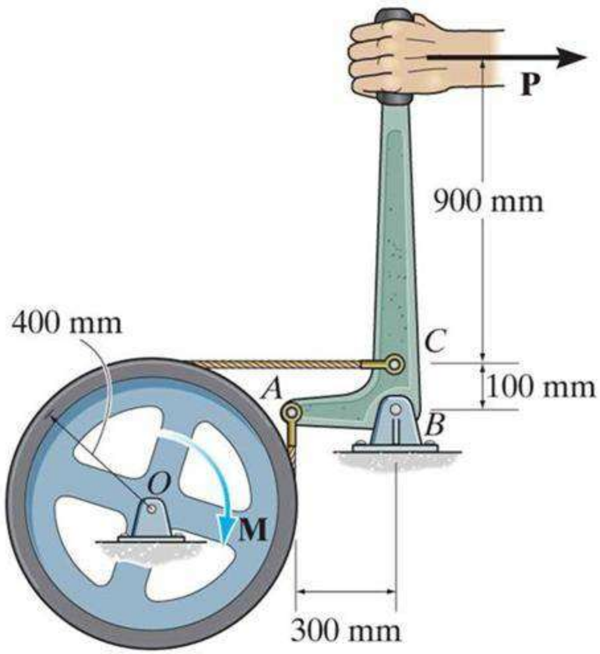
When there are no applied forces on the wedge, will it stay in place (i.e., be self-locking) or will it come out on its own? Under what physical conditions will it come out?

APPLICATIONS (continued)

Belt drives are commonly used for transmitting the torque developed by a motor to a wheel attached to a pump, fan or blower.

How can we decide if the belts will function properly, i.e., without slipping or breaking?

APPLICATIONS (continued)

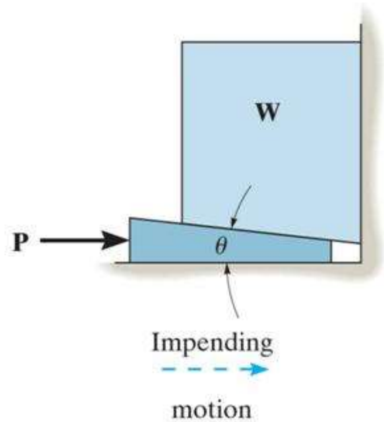


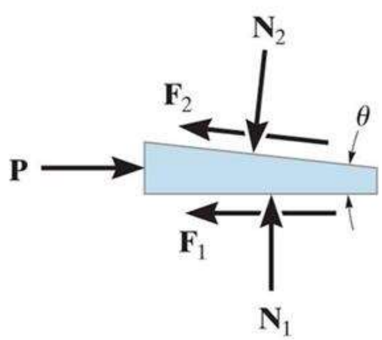
In the design of a band brake, it is essential to analyze the frictional forces acting on the band (which acts like a belt).

How can you determine the tension in the cable pulling on the band?

Also from a design perspective, how are the belt tension, the applied force P and the torque M, related?

ANALYSIS OF A WEDGE





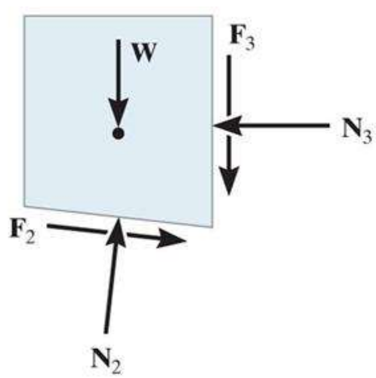
A wedge is a simple machine in which a small force P is used to lift a large weight W.

To determine the force required to push the wedge in or out, it is necessary to draw FBDs of the wedge and the object on top of it.

It is easier to start with a FBD of the wedge since you know the direction of its motion.

Note that:
a) the friction forces are always in the direction opposite to the motion, or impending motion, of the wedge;
b) the friction forces are along the contacting surfaces; and,
c) the normal forces are perpendicular to the contacting surfaces.

ANALYSIS OF A WEDGE (continued)



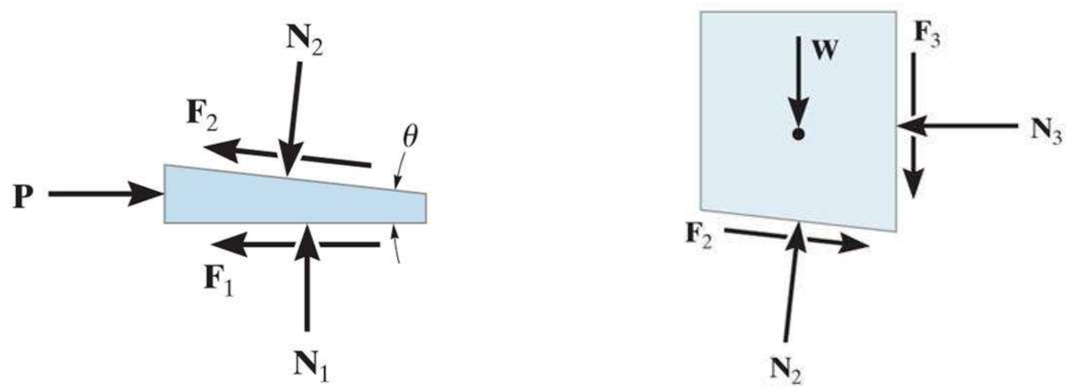
Next, a FBD of the object on top of the wedge is drawn. Please note that:

a) at the contacting surfaces between the wedge and the object, the forces are equal in magnitude and opposite in direction to those on the wedge; and,

b) all other forces acting on the object should be shown.

To determine the unknowns, we must apply E-of-E, $\sum F_x = 0$ and $\sum F_y = 0$, to the wedge and the object as well as the impending motion frictional equation, $F = \mu_S N$.

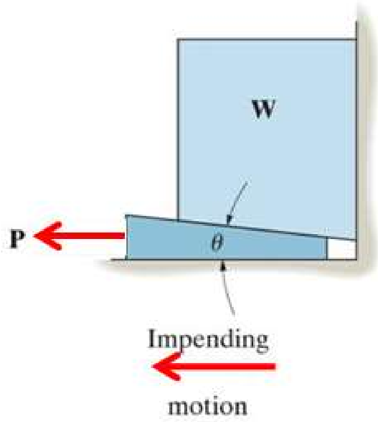
ANALYSIS OF A WEDGE (continued)



Now of the two FBDs, which one should we start analyzing first?

We should start analyzing the FBD in which the number of unknowns are less than or equal to the number of E-of-E and frictional equations.

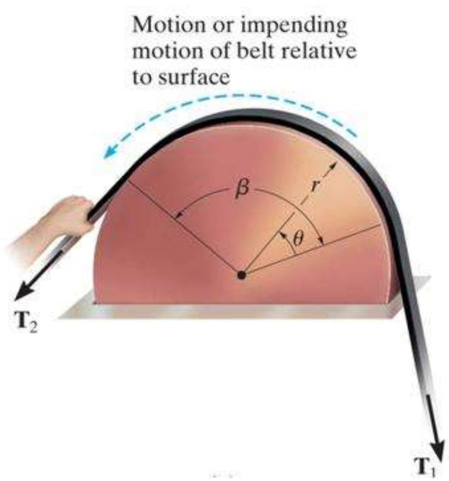
ANALYSIS OF A WEDGE (continued)



NOTE:

If the object is to be lowered, then the wedge needs to be pulled out. If the value of the force P needed to remove the wedge is positive, then the wedge is self-locking, i.e., it will not come out on its own.

BELT ANALYSIS

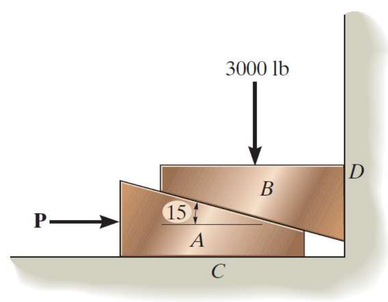


Consider a flat belt passing over a fixed curved surface with the total angle of contact equal to β radians.

If the belt slips or is just about to slip, then T_2 must be larger than T_1 and the motion resisting friction forces. Hence, T_2 must be greater than T_1.

Detailed analysis (please refer to your textbook) shows that $T_2 = T_1 \, e^{\mu \beta}$ where μ is the coefficient of static friction between the belt and the surface. Be sure to use radians when using this formula!!

EXAMPLE



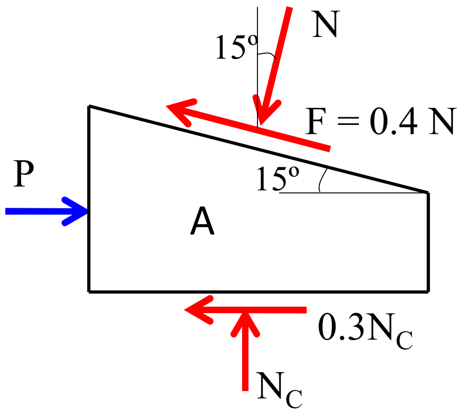
Given: The 3000-lb load is applied to wedge B. The coefficient of static friction between A and C and between B and D is 0.3, and between A and B it is 0.4. Assume the wedges have negligible weight.

Find: The smallest force P needed to lift 3000 lb load.

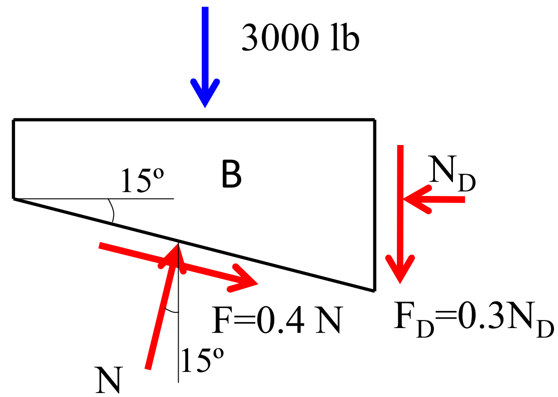
Plan:

1. Draw FBDs of wedge A and wedge B.
2. Apply the E-of-E to wedge B. Why do wedge B first?
3. Apply the E-of-E to wedge A.

EXAMPLE (continued)



FBD of Wedge A



FBD of Wedge B

Applying the E-of-E to wedge B, we get

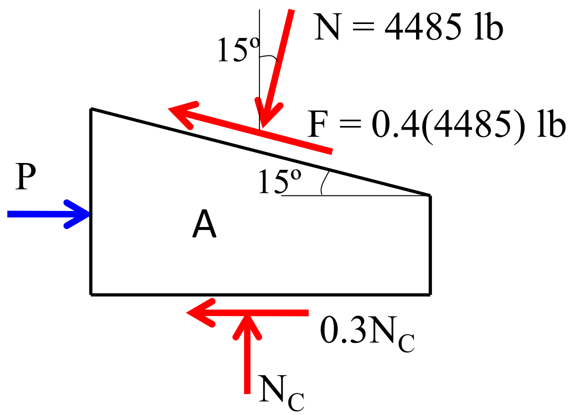
$\rightarrow+ \;\sum F_X = N \sin 15° + 0.4\ N \cos 15° - N_D = 0$

$\uparrow+ \;\sum F_Y = N \cos 15° - 0.4\ N \sin 15° - 0.3\ N_D - 3000 = 0$

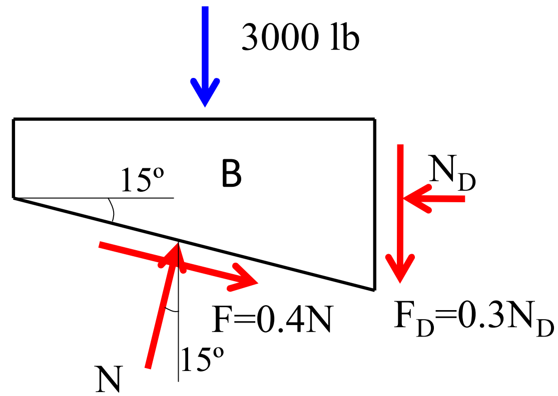
Solving these two equations, we get

$N = 4485$ lb, $N_D = 2894$ lb

EXAMPLE (continued)



FBD of Wedge A



FBD of Wedge B

Applying the E-of-E to wedge A, we get

$\uparrow + \sum F_Y = N_C + 0.4(4485) \sin 15° - 4485 \cos 15° = 0;$

$N_C = 3868$ lb

$\rightarrow + \sum F_X = P - 0.3(3868) - 4485 \sin 15° - 1794 \cos 15° = 0;$

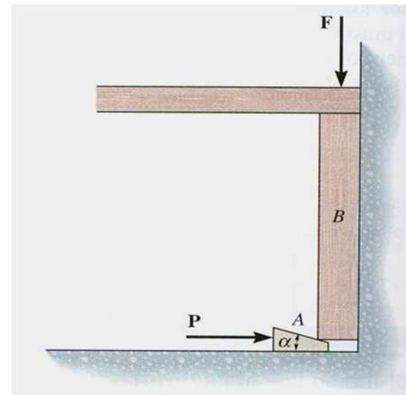
$P = 4054$ lb

CONCEPT QUIZ

1. Determine the direction of the friction force on object B at the contact point between A and B.

   A) ↘ B) ←

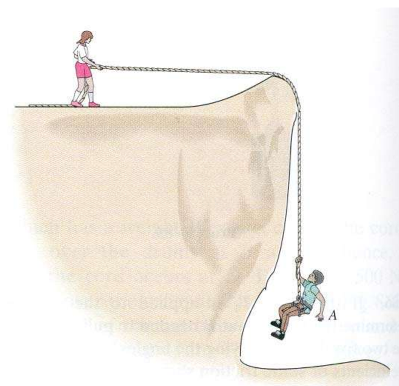
   C) → D) ↖



2. The boy (hanging) in the picture weighs 100 lb and the woman weighs 150 lb. The coefficient of static friction between her shoes and the ground is 0.6. The boy will ______ ?

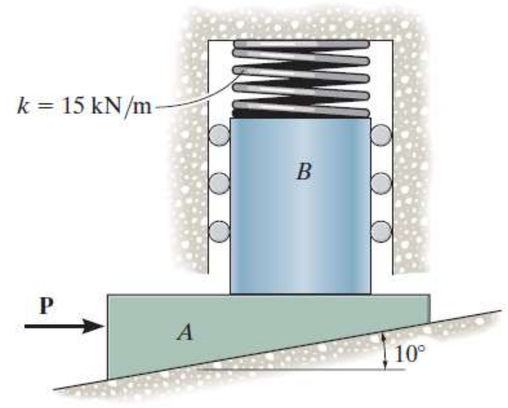
   A) Be lifted up B) Slide down

   C) Not be lifted up D) Not slide down



GROUP PROBLEM SOLVING

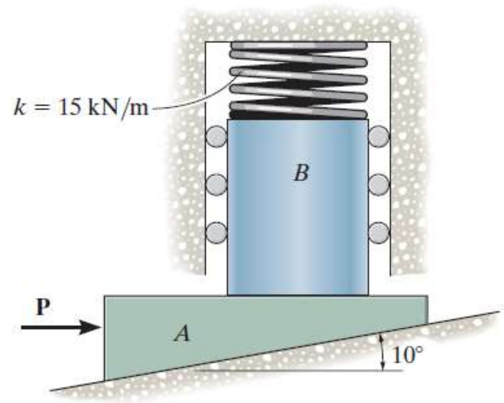
Given: A force **P** is applied to move wedge A to the right. The spring is compressed a distance of 175 mm. The static friction coefficient is $\mu_S = 0.35$ for all contacting surfaces. Neglect the weight of A and B.



Find: The smallest force P needed to move wedge A.

Plan:

GROUP PROBLEM SOLVING (continued)



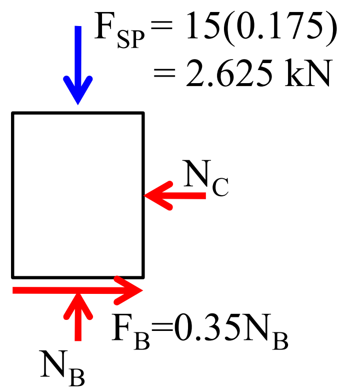
Plan:

1. Draw FBDs of block B and wedge A.

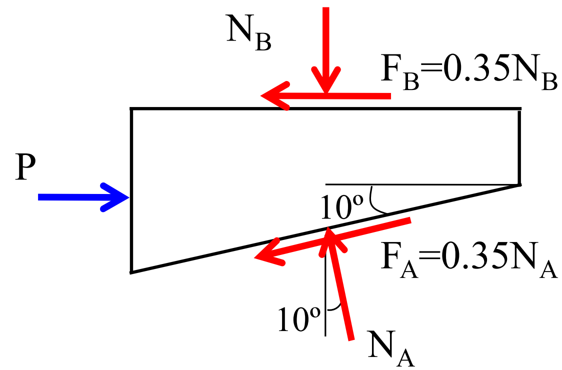
2. Apply the E-of-E to block B to find the friction force when the wedge is on the verge of moving.

3. Apply the E-of-E to wedge A to find the smallest force needed to cause sliding.

GROUP PROBLEM SOLVING (continued)



FBD of Block B



FBD of Wedge A

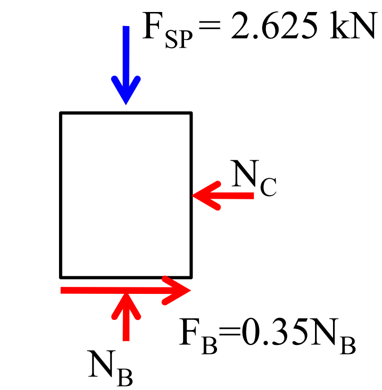
Using the spring formula:

F_{sp} = K x = (15 kN/m) (0.175m) = 2.625 kN

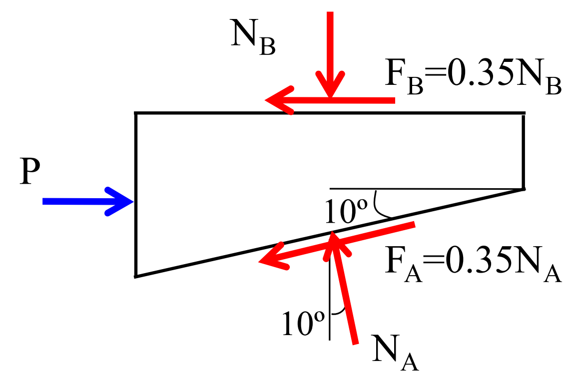
If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces.

Thus, $F_A = \mu_S N_A = 0.35\ N_A$ and $F_B = 0.35\ N_B$.

GROUP PROBLEM SOLVING (continued)



FBD of Block B



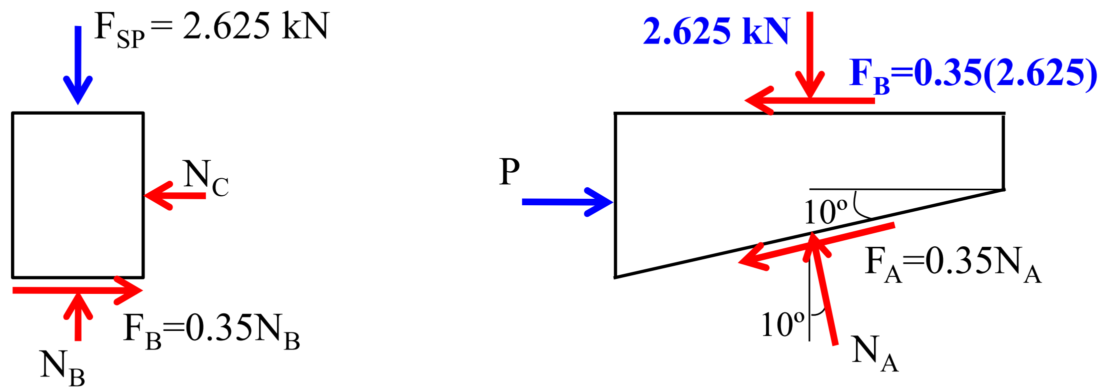
FBD of Wedge A

Applying the E-of-E to the Block B, we get:

$$\uparrow + \sum F_Y = N_B - 2.625 = 0$$

$N_B = 2.625$ kN

GROUP PROBLEM SOLVING (continued)



FBD of Block B

FBD of Wedge A

Applying the E-of-E to Wedge A:

$\uparrow + \sum F_Y = N_A \cos 10° - 0.35 N_A \sin 10° - 2.625 = 0$

$N_A = 2.841$ kN

$\rightarrow + \sum F_X = P - 0.35(2.625) - 0.35(2.841)\cos 10° - 2.841 \sin 10° = 0$

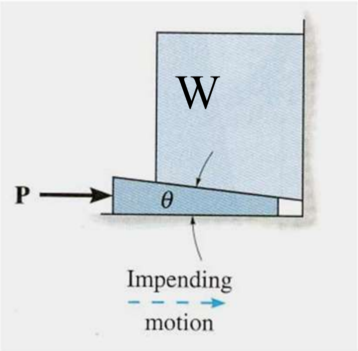
$\underline{P = 2.39 \text{ kN}}$

ATTENTION QUIZ

1. When determining the force P needed to lift the block of weight W, it is easier to draw a FBD of ______ first.

   A) The wedge B) The block

   C) The horizontal ground D) The vertical wall



2. In the analysis of frictional forces on a flat belt, $T_2 = T_1 e^{\mu \beta}$. In this equation, β equals ______ .

   A) Angle of contact in degrees B) Angle of contact in radians

   C) Coefficient of static friction D) Coefficient of kinetic friction

End of the Lecture

Let Learning Continue