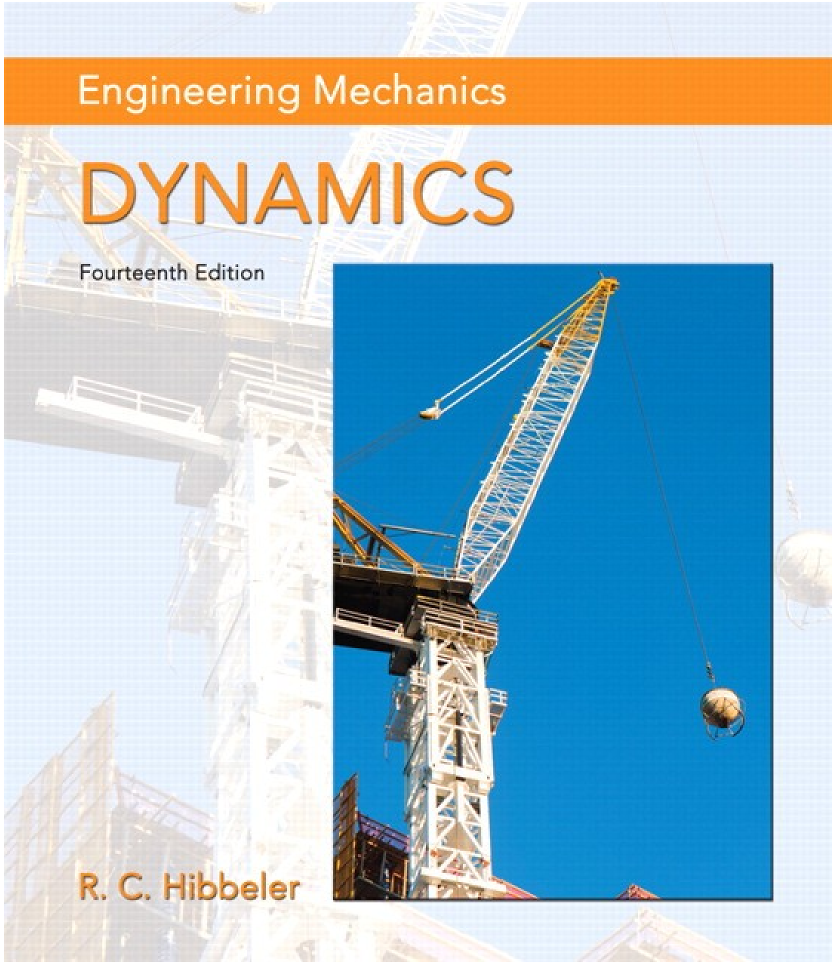
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 13

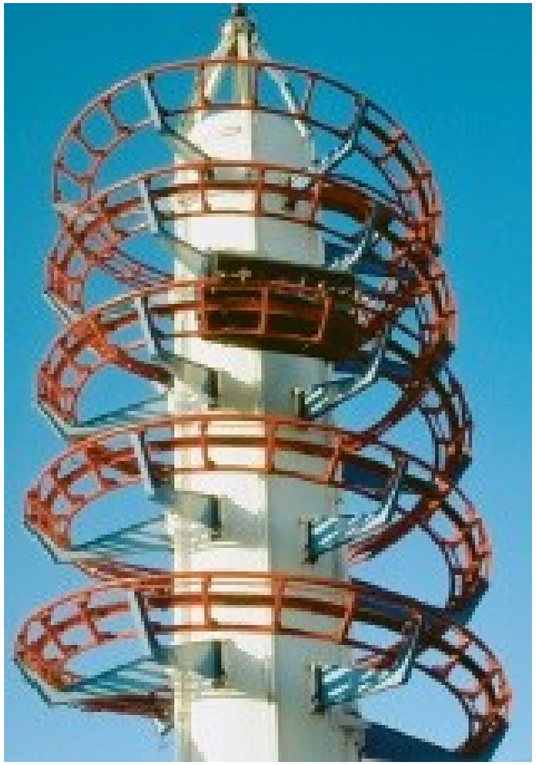
Kinetics of a Particle:
Force and Acceleration

Pearson

Equations of Motion: Cylindrical Coordinates (1 of 2)

Today's Objectives:
Students will be able to:

1. Analyze the kinetics of a particle using cylindrical coordinates.



Equations of Motion: Cylindrical Coordinates (2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- **Equations of Motion using Cylindrical Coordinates**
- **Angle between Radial and Tangential Directions**
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1. The normal force which the path exerts on a particle is always perpendicular to the __________

   A) radial line.
   B) transvers direction.
   C) tangent to the path.
   D) None of the above.

2. When the forces acting on a particle are resolved into cylindrical components, friction forces always act in the ___________ direction.

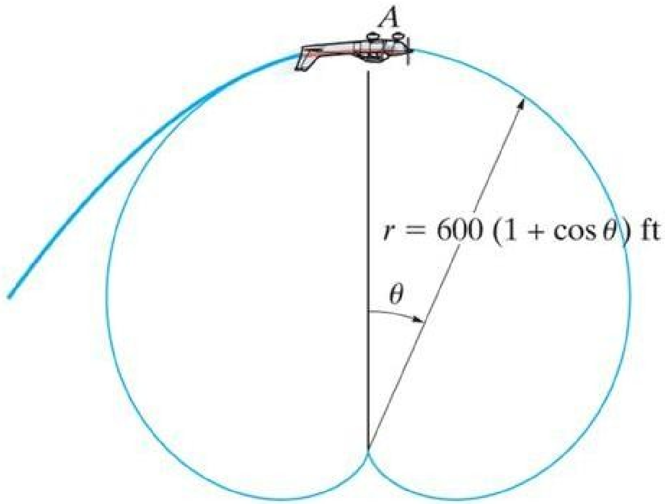
   A) radial
   B) tangential
   C) transverse
   D) None of the above.

Applications (1 of 2)

The forces acting on the 100-lb boy can be analyzed using the cylindrical coordinate system.

How would you write the equation describing the frictional force on the boy as he slides down this helical slide?

Applications (2 of 2)



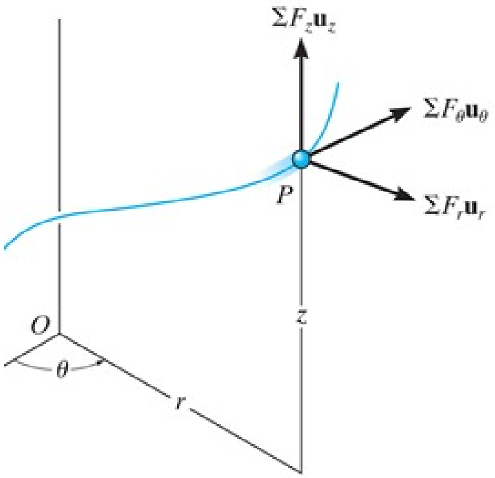
When an airplane executes the vertical loop shown above, the centrifugal force causes the normal force (apparent weight) on the pilot to be smaller than her actual weight.

How would you calculate the velocity necessary for the pilot to experience weightlessness at A?

Section 13.6

Cylindrical Coordinates

Cylindrical Coordinates (1 of 2)



This approach to solving problems has some external similarity to the normal & tangential method just studied. However, the path may be more complex or the problem may have other attributes that make it desirable to use cylindrical coordinates.

Equilibrium equations or "Equations of Motion" in cylindrical coordinates (using r,θ, and z coordinates) may be expressed in scalar form as:

$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} - 2\dot{r}\dot{\theta})$$

$$\Sigma F_z = ma_z = m\ddot{z}$$

Cylindrical Coordinates (2 of 2)

If the particle is constrained to move only in the r-θ plane (i.e., the z coordinate is constant), then only the first two equations are used (as shown below). The coordinate system in such a case becomes a polar coordinate system. In this case, the path is only a function of θ.

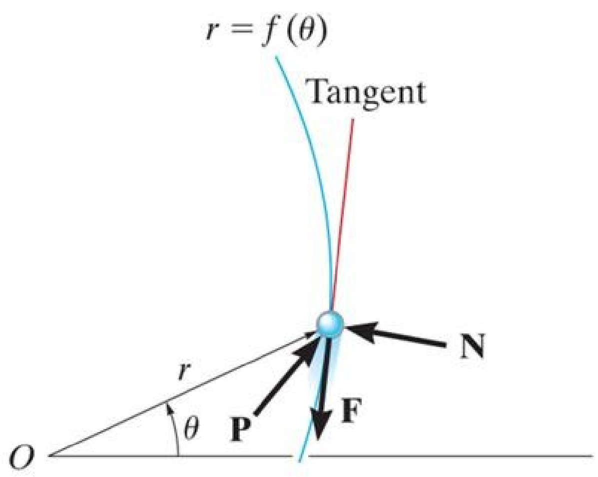
$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$$

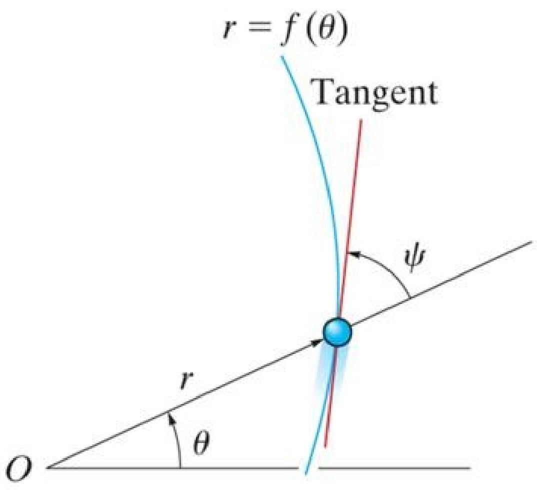
$$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} - 2\dot{r}\dot{\theta})$$

Note that a fixed coordinate system is used, not a “body-centered” system as used in the n - t approach.

Tangential and Normal Forces

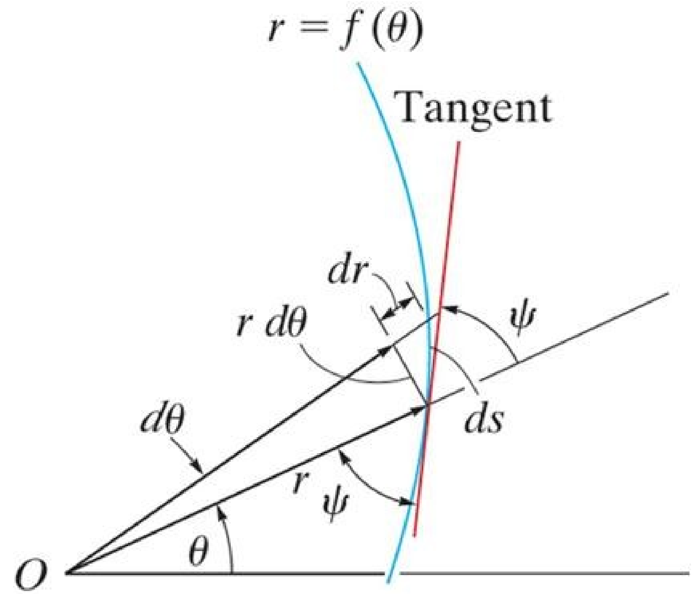
If a force **P** causes the particle to move along a path defined by $r = f(\theta)$, the normal force **N** exerted by the path on the particle is always perpendicular to the path's tangent. The frictional force **F** always acts along the tangent in the opposite direction of motion. The directions of **N** and **F** can be specified relative to the radial coordinate by using angle ψ





Determination of Angle ψ

The angle ψ, defined as the angle between the extended radial line and the tangent to the curve, can be required to solve some problems.
It can be determined from the following relationship.

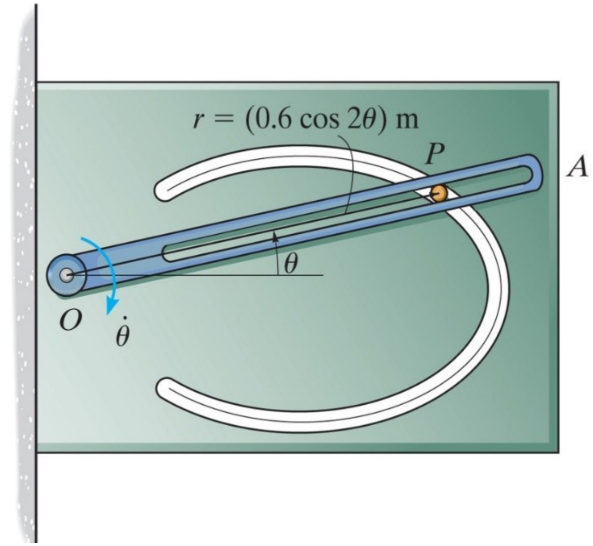


$$\tan\Psi = \frac{r\ d\theta}{dr} = \frac{r}{dr/d\theta}$$

If ψ is positive, it is measured counterclockwise from the radial line to the tangent. If it is negative, it is measured clockwise.

Example (1 of 5)

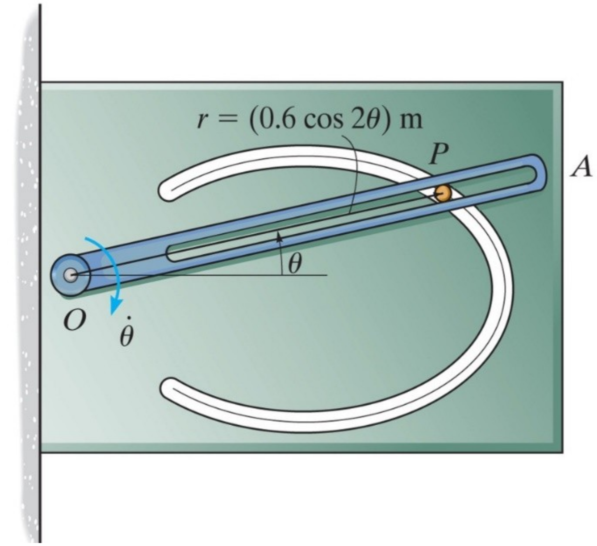
Given: The 0.2 kg pin (P) is constrained to move in the smooth curved slot, defined by $r = (0.6 \cos 2\theta)$ m. The slotted arm OA has a constant angular velocity of $\dot{\theta} = -3$ rad/s. Motion is in the vertical plane.



Find: Force of the arm OA on the pin P when $\theta = 0°$

Example (2 of 5)

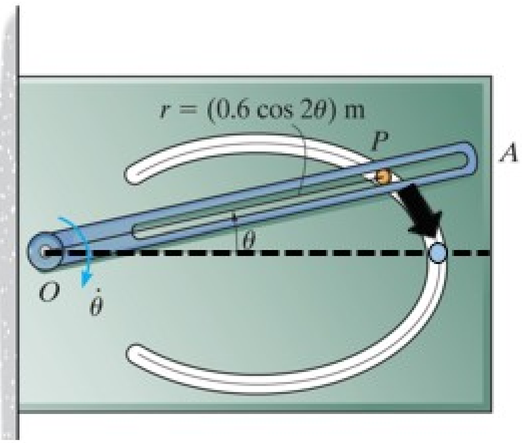
Given: The 0.2 kg pin (P) is constrained to move in the smooth curved slot, defined by $r = (0.6 \cos 2\theta)$ m. The slotted arm OA has a constant angular velocity of $\dot{\theta} = 3$ rad/s. Motion is in the vertical plane.



Find: Force of the arm OA on the pin P when $\theta = 0°$

Plan:
1. Draw the FBD and kinetic diagrams.
2. Develop the kinematic equations using cylindrical coordinates.
3. Apply the equation of motion to find the force.

Example (3 of 5)

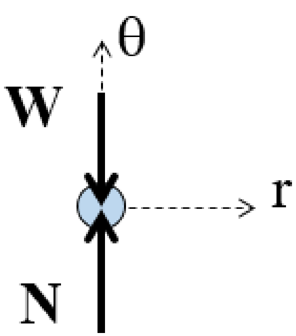


Solution:

1. **Free Body and Kinetic Diagrams:**

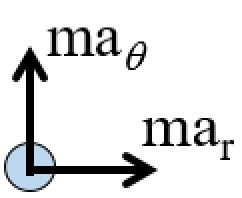
   Establish the r,θ coordinate system when $\theta = 0^\circ$, and draw the free body and kinetic diagrams.

Free-body diagram



=

Kinetic diagram



Example (4 of 5)

2. Notice that $r = 0.6\cos(2\theta)$, therefore:

$$\dot{r} = -1.2\sin(2\theta)\dot{\theta}$$

$$\ddot{r} = -2.4\cos(2\theta)\dot{\theta}^2 - 1.2\sin(2\theta)\ddot{\theta}$$

Kinematics: at $\theta = 0^\circ$, $\dot{\theta} = -3$ rad/s, $\ddot{\theta} = 0$ rad/s^2.

$$r = 0.6\cos(0) = 0.6\text{m}$$

$$\dot{r} = -1.2\sin(0)(-3) = 0 \text{ m/s}$$

$$\ddot{r} = -2.4\cos(0)(-3)^2 - 1.2\sin(0)(0) = -21.6\text{m/s}^2$$

Acceleration components are

$$a_r = \ddot{r} - r\dot{\theta}^2 = -21.6 - (0.6)(-3)^2 = -27 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.6)(0) + 2(0)(-3) = 0\text{m/s}^2$$

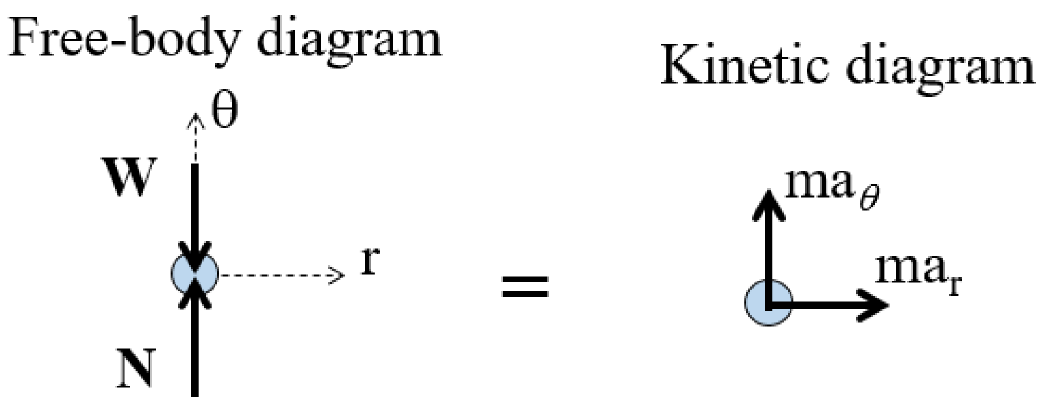
Example (5 of 5)

3. Equation of motion: θ direction

$$(+\uparrow)\,\Sigma F_\theta = ma_\theta$$

$$N - 0.2(9.81) = 0.2(0)$$

$$N = 1.96\,N \uparrow$$

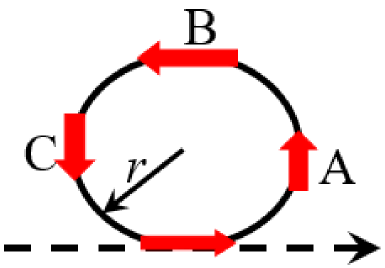


$a_r = -27\ \text{m/s}^2$

$a_\theta = 0\ \text{m/s}^2$

Concept Quiz

1. When a pilot flies an airplane in a vertical loop of constant radius *r* at constant speed *v*, his apparent weight is maximum at



   A) Point A B) Point B (top of the loop)

   C) Point C D) Point D (bottom of the loop)

2. If needing to solve a problem involving the pilot's weight at Point C, select the approach that would be best.

   A) Equations of Motion: Cylindrical Coordinates

   B) Equations of Motion: Normal & Tangential Coordinates

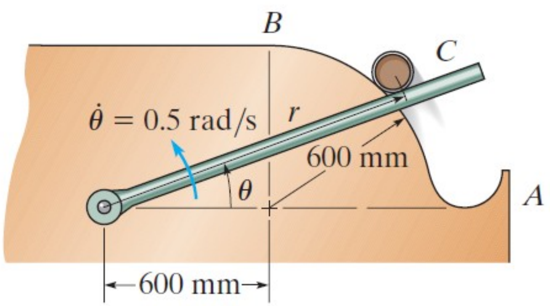
   C) Equations of Motion: Polar Coordinates

   D) No real difference – all are bad.

   E) Toss up between B and C.

Group Problem Solving 1 (1 of 3)

Given: The smooth can C is lifted from A to B by a rotating rod. The mass of can is 3 kg. Neglect the effects of friction in the calculation and the size of the can so that $r = (1.2\cos\theta)\, m.$



Find: Forces of the rod on the can when $\theta = 30^\circ$ and $\dot{\theta} = 0.5$ rad/s, which is constant.

Plan:
1. Find the acceleration components using the kinematic equations.
2. Draw free body diagram & kinetic diagram.
3. Apply the equation of motion to find the forces.

Group Problem Solving 1

1. Kinematics: $r = 1.2(\cos\theta)$

$$\dot{r} = -1.2(\sin\theta)\dot{\theta}$$

$$\ddot{r} = -1.2\cos(\theta)\dot{\theta}^2 - 1.2\sin(\theta)\ddot{\theta}$$

When $\theta = 30^\circ$, $\dot{\theta} = 0.5$ rad/s, $\ddot{\theta} = 0$ rad/s^2.

$$r = 1.039\text{m}$$

$$\dot{r} = -0.3\text{ m/s}$$

$$\ddot{r} = -0.2598\text{m/s}^2$$

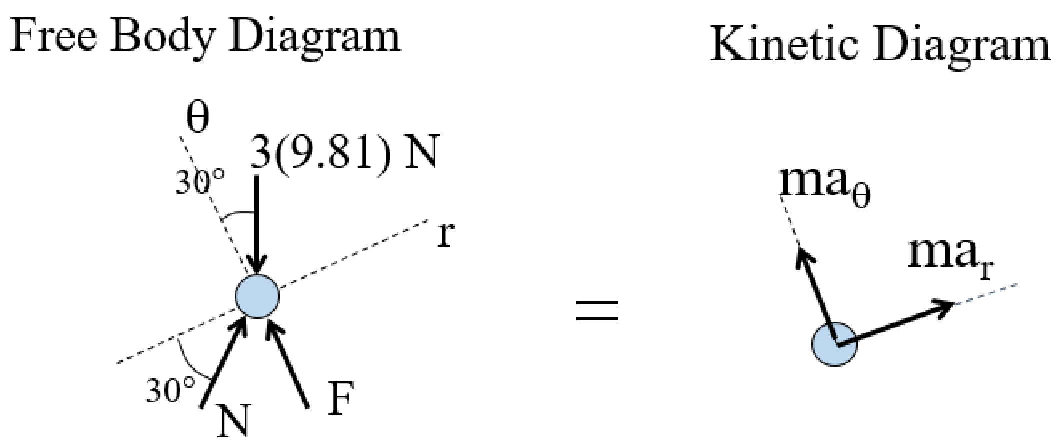
Acceleration

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.2598 - (1.039)0.5^2 = -0.5196\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1.039)0 + 2(-0.3)0.5 = -0.3\text{ m/s}^2$$

Group Problem Solving 1 (3 of 3)

2.



3. Apply equation of motion:

$$\Sigma F_r = ma_r \Rightarrow -3(9.81)\sin 30^\circ + N\cos 30^\circ = 3(-0.5196)$$

$$\Sigma F_\theta = ma_\theta \Rightarrow F + N\sin 30^\circ - 3(9.81)\cos 30^\circ = 3(-0.3)$$

N = 15.2 N, F = 17.0 N

Attention Quiz

1. For the path defined by $r = \theta^2$, the angle ψ at θ= 0.5 rad is

A) 10° B) 14°

C) 26° D) 75°

2. If $r = \theta^2$ and $\theta = 2t$ find the magnitude of $\dot{r}$ and $\ddot{\theta}$ when t = 2 seconds.

A) 4 cm/sec, 2 rad/sec^2 B) 4 cm/sec, 0 rad/sec^2

C) 8 cm/sec, 16 rad/sec^2 D) 16 cm/sec, 0 rad/sec^2

Pearson

Copyright

