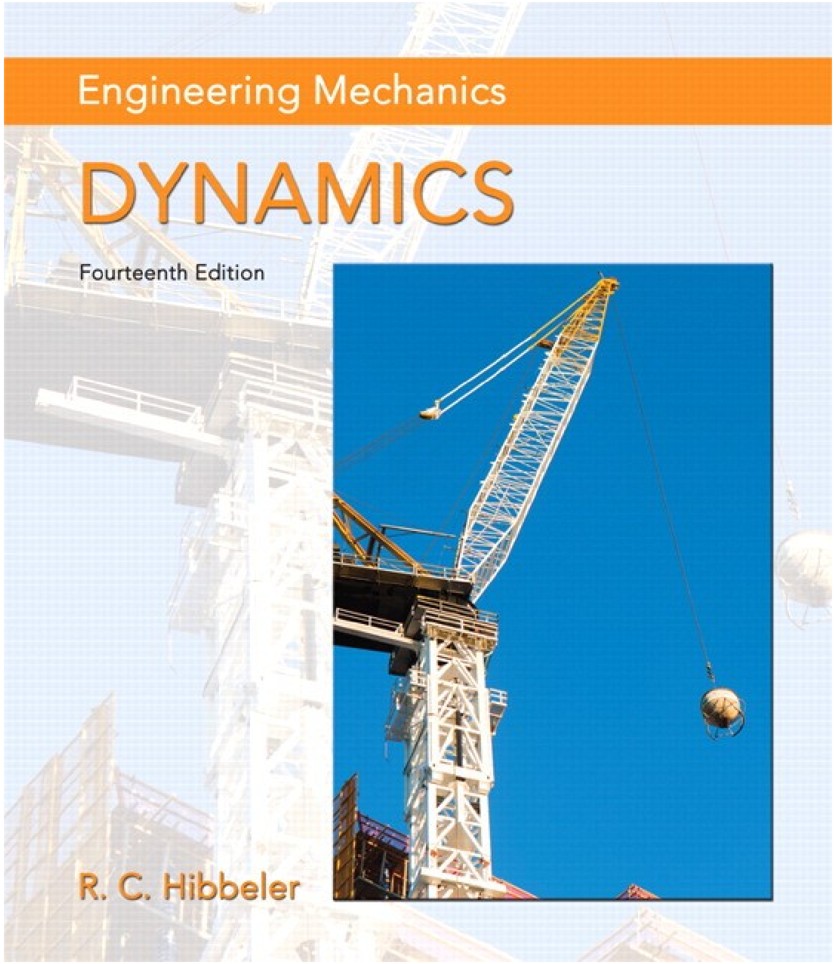
Engineering Mechanics: Dynamics

Fourteenth Edition



Chapter 17

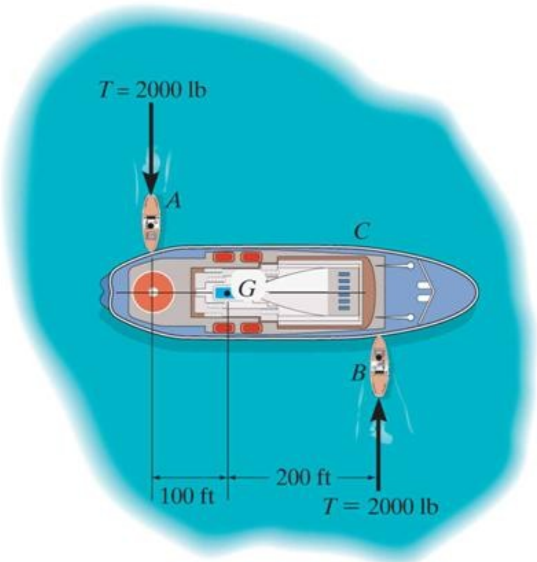
Planar Kinetics of a Rigid Body: Force and Acceleration

Equations of Motion: General Plane Motion 1 (1 of 2)

Today's Objectives:

Students will be able to:

1. Analyze the planar kinetics of a rigid body undergoing general plane motion.



Equations of Motion: General Plane Motion1 (2 of 2)

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- **Equations of Motion**
- **Frictional Rolling Problems**
- Concept Quiz
- Group Problem Solving
- Attention Quiz

Reading Quiz

1. If a disk rolls on a rough surface without slipping, the acceleration of the center of gravity (G) will __________ and the friction force will be __________.

   A) not be equal to $\alpha\ r$; less than $\mu_s N$

   B) be equal to $\alpha\ r$; equal to $\mu_k N$

   C) **be equal to $\boldsymbol{\alpha\ r}$; less than $\boldsymbol{\mu_s N}$**

   D) None of the above

2. If a rigid body experiences general plane motion, the sum of the moments of external forces acting on the body about any point P is equal to __________.

   A) $\boldsymbol{I_P \alpha}$

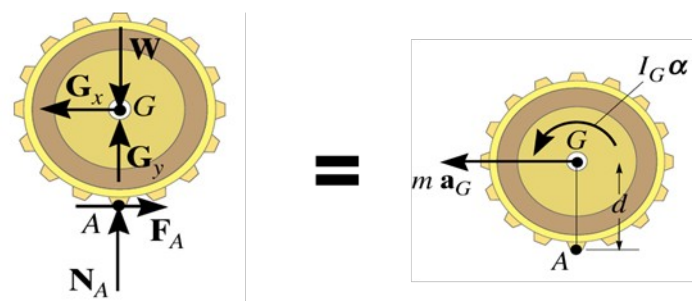
   B) $I_P \alpha + m a_P$

   C) $m a_G$

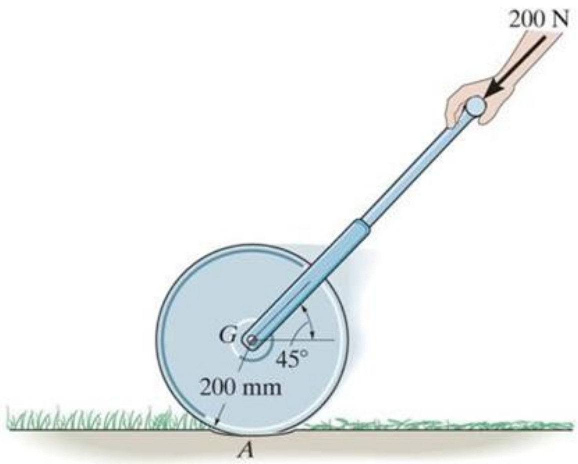
   D) $I_G \alpha + r_{GP} \times m a_P$

Applications (1 of 3)

As the soil compactor accelerates forward, the front roller experiences general plane motion (both translation and rotation). How would you find the loads experienced by the roller shaft or its bearings? The forces shown on the roller shaft's FBD cause the accelerations shown on the kinetic diagram. Is point A the IC?



Applications (2 of 3)

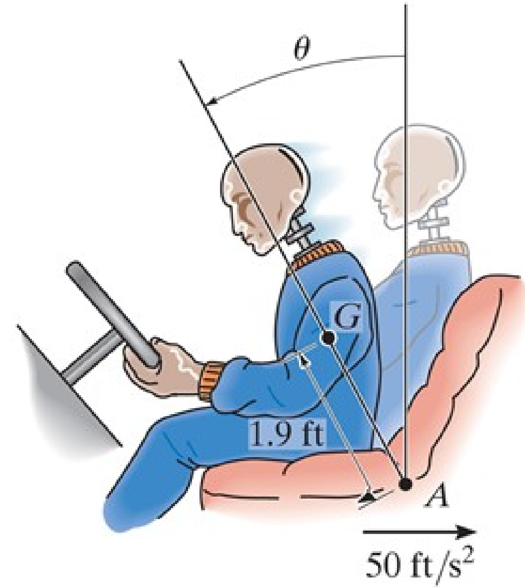


The lawn roller is pushed forward with a force of 200 N when the handle is held at 45°. How can we determine its translational acceleration and angular acceleration?

Does the total acceleration depend on the coefficient's of static and kinetic friction?

Applications (3 of 3)

During an impact, the center of gravity G of this crash dummy will decelerate with the vehicle, but also experience another acceleration due to its rotation about point A.



Why?

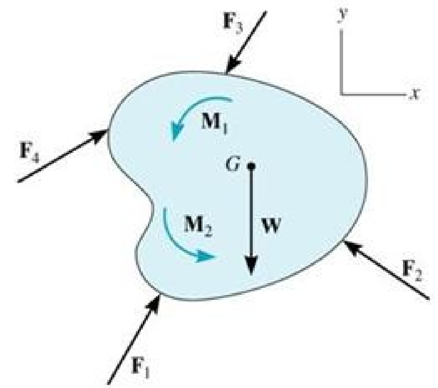
How can engineers use this information to determine the forces exerted by the seat belt on a passenger during a crash? How would these accelerations impact the design of the seat belt itself?

Section 17.5

Equations of Motion: General Plane Motion

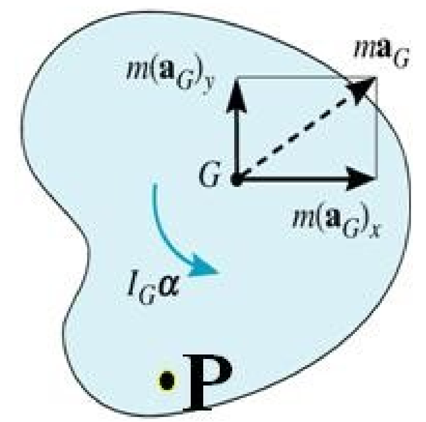
Equations of Motion: General Plane Motion (1 of 2)

When a rigid body is subjected to external forces and couple-moments, it can undergo both translational motion and rotational motion. This combination is called **general plane motion.**



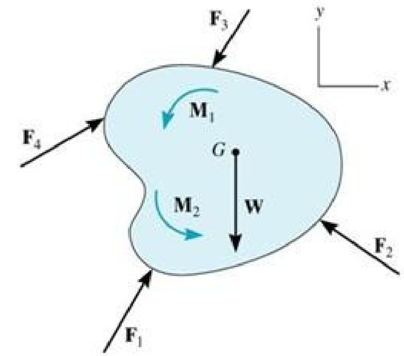
Using an x-y inertial coordinate system, the scalar equations of motions about the center of mass, G, may be written as:

$$\sum F_x = m\,(a_G)_x$$
$$\sum F_y = m\,(a_G)_y$$
$$\sum M_G = I_G \alpha$$



Equations of Motion: General Plane Motion (2 of 2)

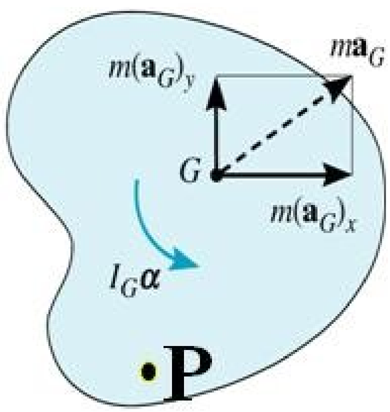
Sometimes, it may be convenient to write the moment equation about a point P, rather than G. Then the equations of motion are written as follows:



$$\Sigma F_x = m(a_G)_x$$

$$\Sigma F_y = m\,(a_G)_y$$

$$\Sigma M_P = \Sigma(M_k)_P$$

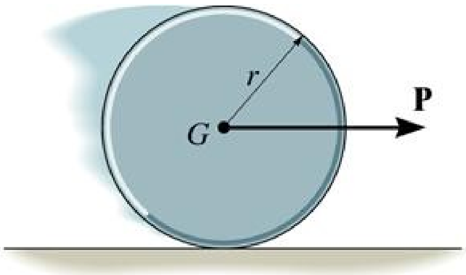


In this case, $\Sigma(M_k)_P$ represents the sum of the moments of $I_G\alpha$ and ma_G about point P.

Frictional Rolling Problems (1 of 2)

When analyzing the rolling motion of wheels, cylinders, or disks, it may not be known if the body rolls without slipping or if it slips/slides as it rolls.

For example, consider a disk with mass m and radius r, subjected to a known force P.

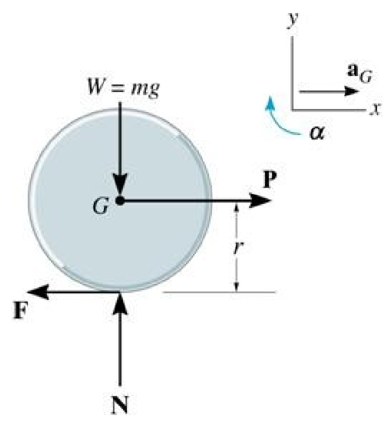


The **equations of motion** will be:

$$\sum F_x = m\,(a_G)_x \Rightarrow P - F = m\,a_G$$

$$\sum F_y = m\,(a_G)_y \Rightarrow N - mg = 0$$

$$\sum M_G = I_G\alpha \Rightarrow F\,r = I_G\alpha$$

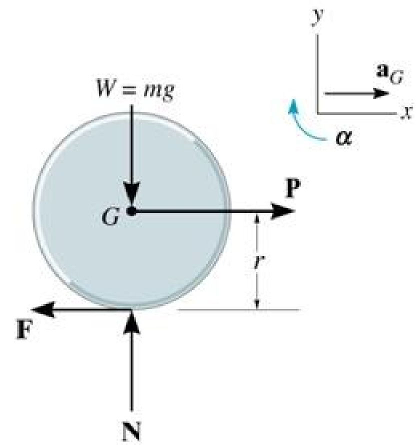


There are **4 unknowns** $(F, N, \alpha,$ and $a_G)$ in these three equations.

Frictional Rolling Problems (2 of 2)

Hence, we have to make an assumption to provide another equation. Then, we can solve for the unknowns.

The 4^{th} equation can be obtained from **the slip or non-slip condition of the disk**.



Case 1:
Assume **no slipping** and use $a_G = \alpha r$ as the 4^{th} equation and **Do Not use** $F_f = \mu_s N$. After solving, you will need to verify that the assumption was correct by checking if $F_f \leq \mu_s N$.

Case 2:
Assume **slipping** and use $F_f = \mu_k N$ as the 4^{th} equation.
In this case, $a_G \neq \alpha r$.

Procedure For Analysis (1 of 2)

Problems involving the kinetics of a rigid body undergoing general plane motion can be solved using the following procedure.

1. **Establish** the x-y inertial coordinate system. Draw both the free body diagram and kinetic diagram for the body.

2. **Specify** the direction and sense of the acceleration of the mass center, a_G, and the angular acceleration *α* of the body. If necessary, compute the body's mass moment of inertia I_G.

3. If the moment equation $\Sigma M_p = \Sigma (M_k)_p$ is used, use the kinetic diagram to help visualize the moments developed by the components $m(a_G)_x$, $m(a_G)_y$, and $I_G \alpha$.

4. Apply **the three equations of motion.**

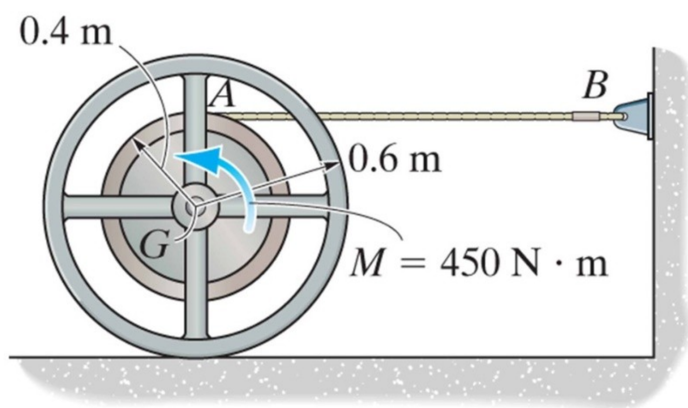
Procedure For Analysis (2 of 2)

5. **Identify the unknowns.** If necessary (i.e., there are four unknowns), make your slip-no slip assumption (typically no slipping, or the use of $a_G = \alpha r$, is assumed first).
6. Use kinematic equations as necessary to complete the solution.
7. If a slip-no slip assumption was made, **check its validity**!!!

Key points to consider:

1. Be **consistent** in using the assumed directions. The direction of a_G must be consistent with $\boldsymbol{\alpha}$.
2. If $F_f = \mu_k N$ is used, F_f **must oppose the motion**. As a test, assume no friction and observe the resulting motion. This may help visualize the correct direction of F_f.

Example (1 of 3)



Given: A spool has a mass of 200 kg and a radius of gyration (k_G) of 0.3 m. The coefficient of kinetic friction between the spool and the ground is $\mu_k = 0.1$.

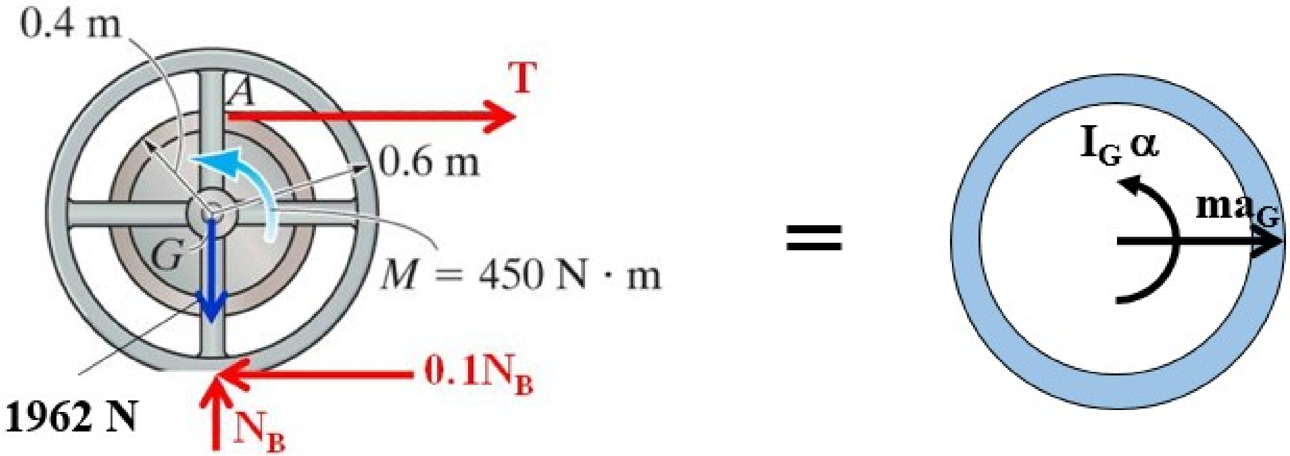
Find: The angular acceleration (a) of the spool and the tension in the cable.

Plan: Focus on the spool. Follow the solution procedure (draw a FBD, etc.) and identify the unknowns.

Example (2 of 2)

Solution:

The **free-body diagram** and **kinetic diagram** for the body are:

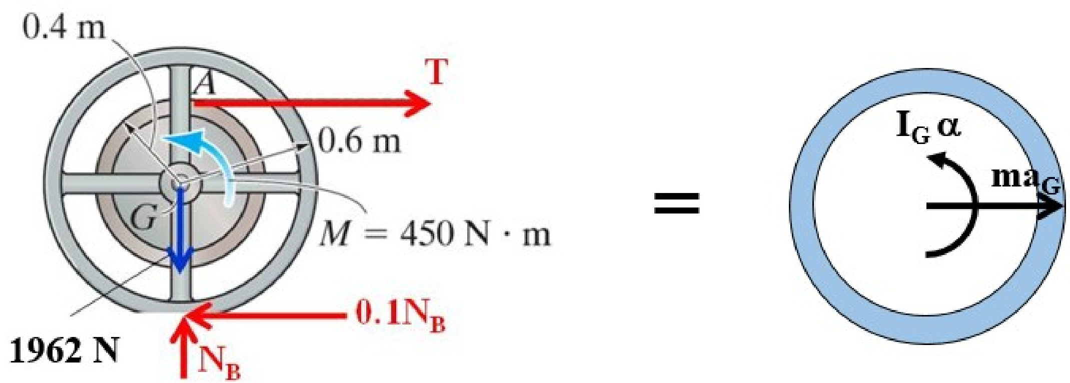


Equation of motion in the y-direction (do first since there is only one unknown):

$$+\uparrow \sum F_y = m\,(a_G)_y : N_B - 1962 = 0$$

$$\Rightarrow N_B = 1962\ N$$

Example (3 of 3)



Note that $a_G = (0.4)\alpha.$ Why?

$$+\rightarrow \sum F_x = m\,(a_G)_x : T - 0.1\,N_B = 200 a_G = 200\,(0.4)\alpha$$

$$\Rightarrow T - 196.2 = 80\alpha$$

$$\circlearrowleft + \sum M_G = I_G\alpha : \; 450 - T(0.4) - 0.1\,N_B(0.6) = 20\,(0.3)^2\,\alpha$$

$$\Rightarrow 450 - T(0.4) - 196.2\,(0.6) = 1.8\alpha$$

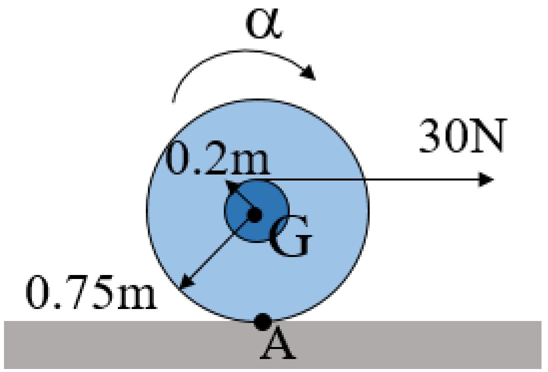
Solving these **two** equations, we get

$$\alpha = 7.50\; rad/s^2,\; T = 797\; N$$

Concept Quiz

1. An 80 kg spool $(k_G = 0.3\ m)$ is on a rough surface and a cable exerts a 30 N load to the right. The friction force at A acts to the ___________ and the a_G should be directed to the ___________

   A) right, left B) left, right
   C) right, right D) left, left



2. For the situation above, the moment equation about G is?

   A) $0.75\,(F_{fA}) - 0.2(30) = -(80)(0.3^2)\alpha$

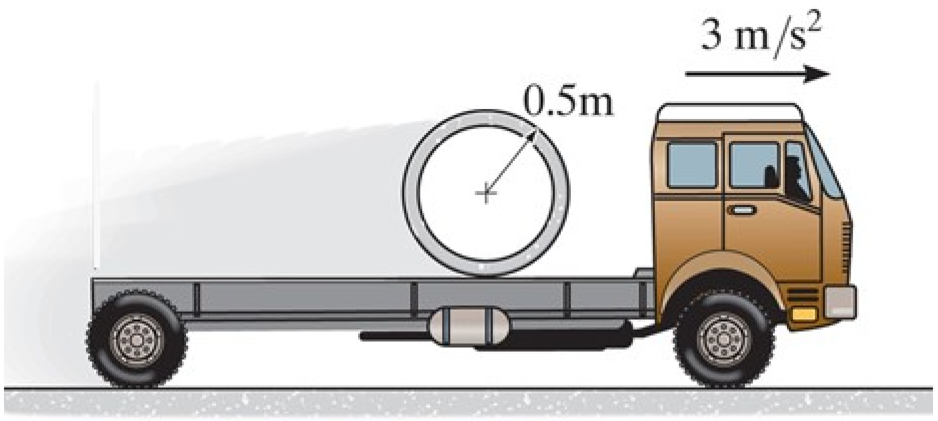
   B) $-0.2(30) = -(80)(0.3^2)\alpha$

   C) $0.75\,(F_{fA}) - 0.2(30) = -(80)(0.3^2)\alpha + 80a_G$

   D) None of the above

Group Problem Solving (1 of 4)

Given: The 500-kg concrete culvert has a mean radius of 0.5 m. Assume the culvert does not slip on the truck bed but can roll, and you can neglect its thickness.



Find: The culvert's angular acceleration when the truck has an acceleration of $3 m/s^2$

Plan: Follow the problem-solving procedure.

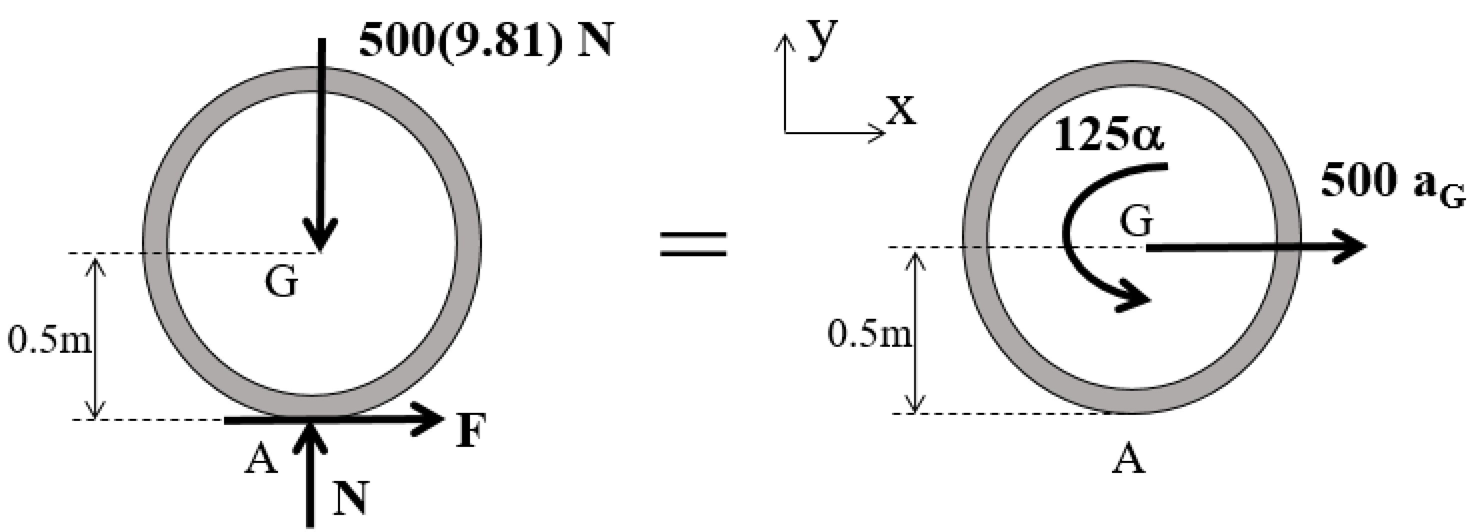
Group Problem Solving (2 of 4)

Solution:

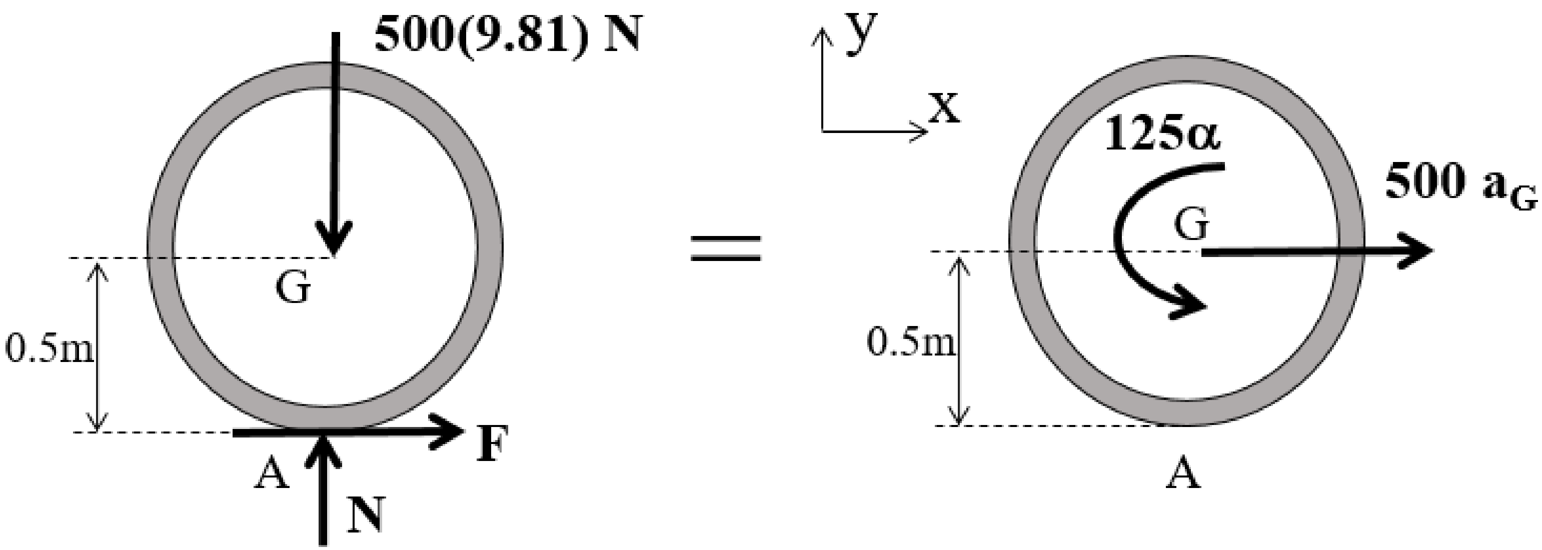
The moment of inertia of the culvert about G is

$$I_G = m(r)^2 = (500)(0.5)^2 = 125\ kg{\cdot}m^2$$

Draw the **FBD** and **Kinetic Diagram.**



Group Problem Solving (3 of 4)



Equations of motion: the moment equation of motion about A

$$\circlearrowleft + \sum M_A = \sum (M_k)_A$$

$$0 = 125\alpha - 500 a_G (0.5) \qquad (1)$$

Group Problem Solving (4 of 4)

Since the culvert does not slip at A,

$$a_A = 3m/s^2.$$

Apply the relative acceleration equation to find α and a_G.

$$a_G = a_A + \alpha \times r_{G/A} - \omega^2 r_{G/A}$$

$$a_G i = 3i + \alpha k \times 0.5j - \omega^2(0.5)j$$

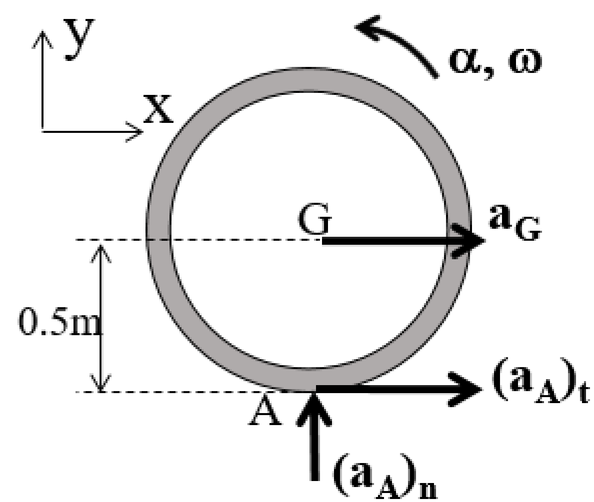
$$= (3 - 0.5\alpha)i - \omega^2(0.5)j$$

Equating the i components,

$$a_G = (3 - 0.5\alpha) \qquad (2)$$

Solving Equations (1) and (2) yields

$\alpha = 3\ rad/s2$ and $a_G = 1.5\ m/s^2 \rightarrow$



Attention Quiz

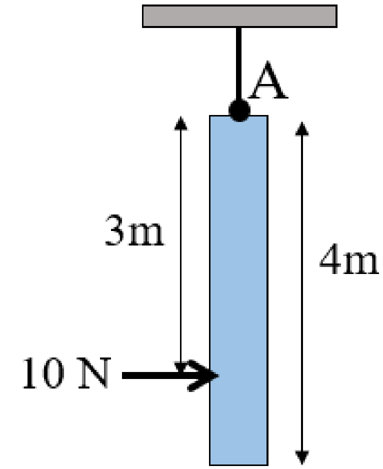
1. A slender 100 kg beam is suspended by a cable. The moment equation about point A is?

   A) $3(10) = 1/12(100)(4^2)\ \alpha$

   B) $3(10) = 1/3(100)(4^2)\ \alpha$

   C) $3(10) = 1/12(100)(4^2)\alpha + (100a_{Gx})(2)$

   D) None of the above



2. Select the equation that best represents the "no-slip" assumption.

   A) $F_f = \mu_s N$

   B) $a_G = r\alpha$

   C) $F_f = \mu_k N$

   D) None of the above

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