Motion in 2 dimension

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HUMBER ENGINEERING

ENGI 1000 - Physics 1

WEEK 3 - MODULE 3

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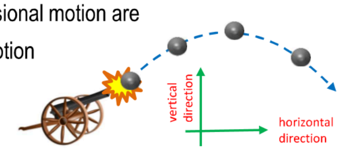
Module 3
Motion in Two-Dimensions

- Two-Dimensional Kinematics
- Motion in Plane: Displacement, Velocity and Acceleration
- 2D Motion with Constant Acceleration
- Projectile Motion
- Relative Velocity

Kinematics

- Kinematics is the branch of mechanics that mathematically describes the motion of objects without discussing what causes the motion.
- **One-Dimensional Motion:** The object is moving in a **straight line**, either forwards or backwards, up or down, left or right.
- Important types of one-dimensional motion are
  - Constant Velocity Motion
  - Constant Acceleration Motion
  - Free-fall Motion

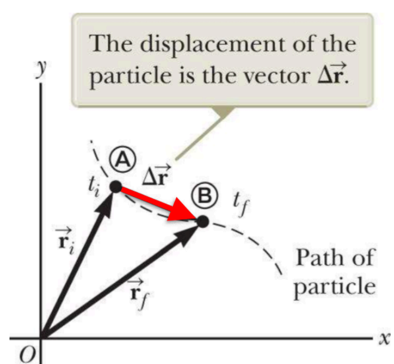
- **Two-Dimensional:** The object is moving in a **plane** in both horizontal and vertical directions.
- Important types of two-dimensional motion are
  - Constant Acceleration Motion
  - Projectile Motion
  - Circular Motion



Position Vector

- In two-dimensional motion the position of a particle is described by its position vector $\vec{\mathbf{r}}$.
- The position vector is drawn from the origin to the location of the particle in the xy plane.
- The particle moves from point A to B in the time interval $\Delta t = t_f - t_i$.
- The displacement vector is defined as the difference between the final position vector $\vec{\mathbf{r}}_f$ and initial position vector $\vec{\mathbf{r}}_i$.

$$\boxed{\Delta\vec{\mathbf{r}} \equiv \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i}$$



Velocity Vector

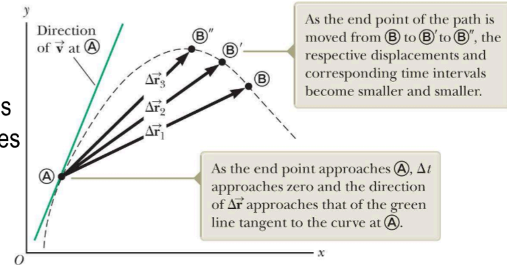
- The average velocity vector $\vec{\mathbf{v}}_{\mathbf{avg}}$ of a particle during the time interval Δt is defined as the displacement vector $\overrightarrow{\Delta\mathbf{r}}$ of the particle divided by the time interval.

displacement vector $\Delta\vec{\mathbf{r}}$ of the particle divided by the time interval.

$$\vec{\mathbf{v}}_{\text{avg}} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

- The instantaneous velocity vector $\vec{\mathbf{v}}$ is defined as the limit of the average velocity as Δt approaches zero:

$$\vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$



- The magnitude of the instantaneous velocity vector is called the speed of the particle.

$$v = |\vec{\mathbf{v}}|$$

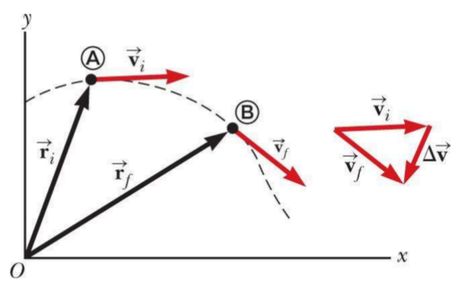
Acceleration Vector

- The average acceleration vector $\vec{\mathbf{a}}_{\mathbf{avg}}$ of a particle is defined as the change in its instantaneous velocity vector $\vec{\mathbf{v}}$ divided by the time interval Δt.

$$\vec{\mathbf{a}}_{\text{avg}} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i}$$

- The instantaneous acceleration vector $\vec{\mathbf{a}}$ is defined as the limit of the average acceleration as Δt approaches zero:

$$\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}$$



Quick Quiz 1

- Suppose you are driving due east, traveling a distance of 1500 m in 2 minutes. You then turn due north and travel the same distance in the same time. What can be said about the average speeds and the average velocities for the two segments of the trip?

  a) The average speeds are the same, and the average velocities are the same
  b) The average speeds are the same, but the average velocities are different
  c) The average speeds are different, but the average velocities are the same

e) The average speeds are different, but the average velocities are the same

Quick Quiz 2

- Consider the following controls in an automobile in motion: gas pedal, brake, steering wheel. What are the controls in this list that cause a change in acceleration of the car?

  a) All three controls
  b) The gas pedal and the brake
  c) Only the brake
  d) Only the gas pedal
  e) Only the steering wheel

Two-Dimensional Motion with Constant Acceleration

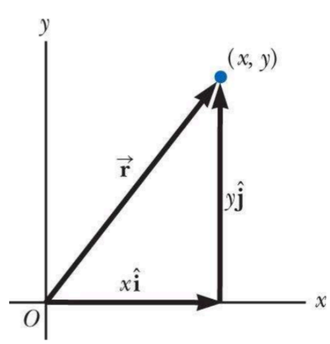
- The position vector of a particle in xy plane can be written as:

$$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

- As the particle moves x, y and $\vec{\mathbf{r}}$ change with the time but unit vectors remain constant.

- Knowing the position vector, the velocity of the particle can also be obtained as:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$



Two-Dimensional Motion with Constant Acceleration

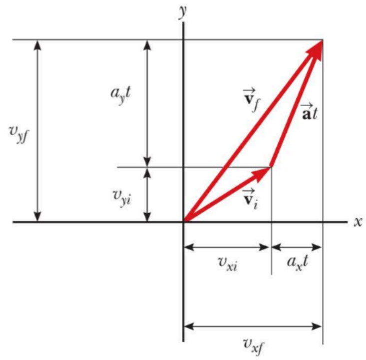
- Since the acceleration is constant, we can model the particle independently in each of the x and y directions separately and find the final velocity equation:

$$\vec{\mathbf{v}}_f = v_{xf}\hat{\mathbf{i}} + v_{yf}\hat{\mathbf{j}}$$

$$v_{xf} = v_{xi} + a_x t \quad \text{and} \quad v_{yf} = v_{yi} + a_y t$$

$$\vec{\mathbf{v}}_f = (v_{xi} + a_x t)\hat{\mathbf{i}} + (v_{yi} + a_y t)\hat{\mathbf{j}} = \underbrace{(v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}})}_{\vec{\mathbf{v}}_i} + \underbrace{(a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})}_{\vec{\mathbf{a}}} t$$

$$\boxed{\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t \quad (\text{for constant } \vec{\mathbf{a}})}$$



Two-Dimensional Motion with Constant Acceleration

- Since the acceleration is constant, the x and y coordinates of a particle are:

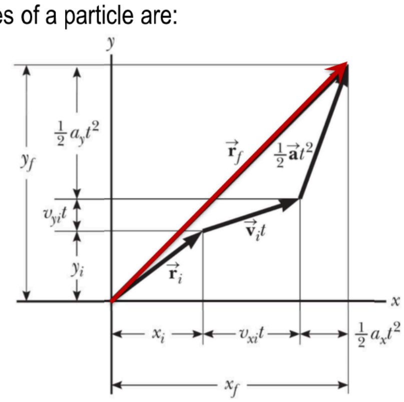
$$\vec{\mathbf{r}}_f = x_f\hat{\mathbf{i}} + y_f\hat{\mathbf{j}}$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \quad \text{and} \quad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$\vec{\mathbf{r}}_f = \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\hat{\mathbf{i}} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\hat{\mathbf{j}}$$

$$= \underbrace{(x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}})}_{\vec{\mathbf{r}}_i} + \underbrace{(v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}})}_{\vec{\mathbf{v}}_i} t + \frac{1}{2}\underbrace{(a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})}_{\vec{\mathbf{a}}} t^2$$

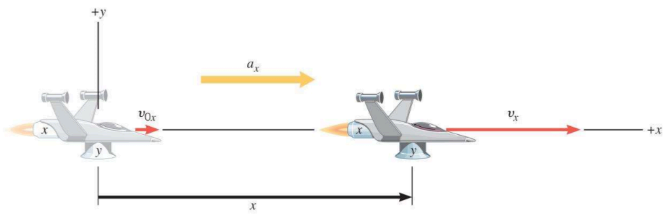
$$\boxed{\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2 \quad (\text{for constant } \vec{\mathbf{a}})}$$



Two-Dimensional Motion with Constant Acceleration

Two-Dimensional Motion with Constant Acceleration

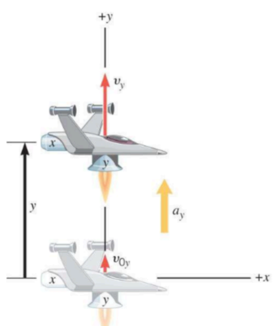
- Consider a spacecraft with two engines that are mounted perpendicular to each other, and these engines produce the only forces that the spacecraft experiences.
- First, assume only the engine oriented along the x direction is firing, and the y engine is turned off.
- If the spacecraft has a constant acceleration a_x along the x direction, the equations of one-dimensional motion in x direction can be used.



$$v_{xf} = v_{xi} + a_x t$$
$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$$
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Two-Dimensional Motion with Constant Acceleration

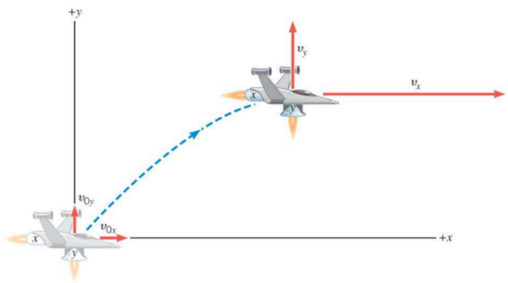
- Next, assume only the engine oriented along the y direction is firing, and the x engine is turned off.
- If the spacecraft has a constant acceleration a_y along the y direction, the equations of one-dimensional motion in y direction can be used.



$$v_{yf} = v_{yi} + a_y t$$
$$y_f = y_i + \frac{1}{2}(v_{yi} + v_{yf})t$$
$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

Two-Dimensional Motion with Constant Acceleration

- If both engines of the spacecraft are firing at the same time, the resulting motion takes place in part along the x axis and in part along the y axis, which causes the two-dimensional motion.
- Note that any influence in the y direction does not affect the motion in the x direction and vice versa



- Motion in two dimensions can be modeled as two independent motions in each of the two perpendicular directions associated with the x and y axes.

Two-Dimensional Motion with Constant Acceleration

Example 1 (Motion of a Spacecraft): A spacecraft moves in the xy plane, starting from the origin at $t = 0$ with an initial velocity having an x component of $v_{xi} = 22\ m/s$ and a y component of $v_{yi} = 14\ m/s$. The particle experiences an acceleration in both x and y directions, given by $a_x = 24.0\ m/s^2$ and $a_y = 12.0\ m/s^2$

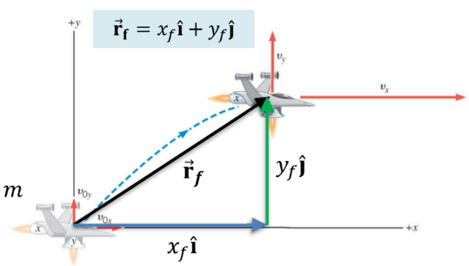
(a) Determine the x and y components of the spacecraft's displacement at time $t = 7.0\ s$.

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$x_f = 0 + \left(22\frac{m}{s}\right)(7.0\ s) + \frac{1}{2}\left(24.0\frac{m}{s^2}\right)(7.0\ s)^2 = 742\ m$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_f = 0 + \left(14\frac{m}{s}\right)(7.0\ s) + \frac{1}{2}\left(12.0\frac{m}{s^2}\right)(7.0\ s)^2 = 392\ m$$



Two-Dimensional Motion with Constant Acceleration

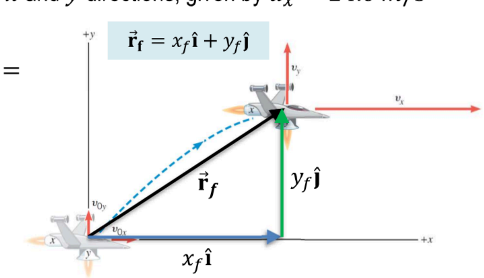
Example 1 (Motion of a Spacecraft): A spacecraft moves in the xy plane, starting from the origin at $t = 0$ with an initial velocity having an x component of $v_{xi} = 22\ m/s$ and a y component of $v_{yi} = 14\ m/s$. The particle experiences an acceleration in both x and y directions, given by $a_x = 24.0\ m/s^2$ and $a_y = 12.0\ m/s^2$

(b) Determine the final position of the spacecraft at time $t = 7.0\ s$ (vector, magnitude and direction).

The position vector in xy plane can be written as below

$$\vec{\mathbf{r}}_\mathbf{f} = x_f\hat{\mathbf{i}} + y_f\hat{\mathbf{j}} = 742\hat{\mathbf{i}} + 392\hat{\mathbf{j}}\ m$$

$$\theta = \tan^{-1}\left(\frac{y_f}{x_f}\right) = \tan^{-1}\left(\frac{392\ \text{m}}{742\ \text{m}}\right) = 27.8°$$

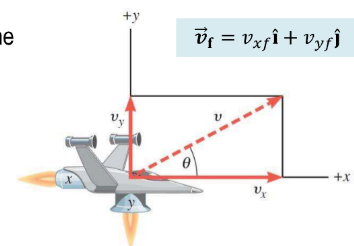


$$r_f = |\vec{\mathbf{r}}_\mathbf{f}| = \sqrt{{x_f}^2 + {y_f}^2} = \sqrt{(742)^2 + (392)^2} = \boxed{839\ \text{m}}$$

Two-Dimensional Motion with Constant Acceleration

Example 1 (Motion of a Spacecraft): A spacecraft moves in the xy plane, starting from the origin at $t = 0$ with an initial velocity having an x component of $v_{xi} = 22\ m/s$ and a y component of $v_{yi} = 14\ m/s$. The particle experiences an acceleration in both x and y directions, given by $a_x = 24.0\ m/s^2$ and $a_y = 12.0\ m/s^2$

(c) Determine the final velocity (vector, magnitude and direction) of the spacecraft at time $t = 7.0\ s$.



The x and y components of the final velocity vector

$$v_{xf} = v_{xi} + a_x t$$

$$v_{xf} = (22\ m/s) + (24.0\ m/s^2)(7.0\ s) = 190\ m/s$$

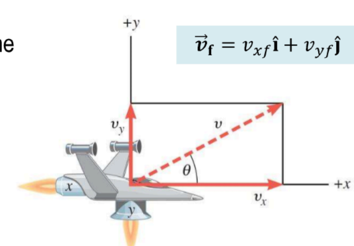
$$v_{yf} = v_{yi} + a_y t$$

$$v_{yf} = (14\ m/s) + (12.0\ m/s^2)(7.0\ s) = 98\ m/s$$

Two-Dimensional Motion with Constant Acceleration

Example 1 (Motion of a Spacecraft): A spacecraft moves in the xy plane, starting from the origin at $t = 0$ with an initial velocity having an x component of $v_{xi} = 22\ m/s$ and a y component of $v_{yi} = 14\ m/s$. The particle experiences an acceleration in both x and y directions, given by $a_x = 24.0\ m/s^2$ and $a_y = 12.0\ m/s^2$

(c) Determine the final velocity (vector, magnitude and direction) of the spacecraft at time $t = 7.0\ s$.



The velocity vector in xy plane can be written as below

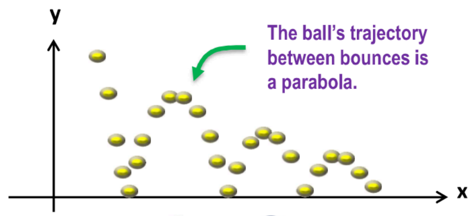
$$\vec{\mathbf{v}}_\mathbf{f} = v_{xf}\hat{\mathbf{i}} + v_{yf}\hat{\mathbf{j}} = 190\hat{\mathbf{i}} + 98\hat{\mathbf{j}}\ \ m/s$$

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{98\ \text{m/s}}{190\ \text{m/s}}\right) = 27.3°$$

$$v_f = |\vec{\mathbf{v}}_\mathbf{f}| = \sqrt{{v_{xf}}^2 + {v_{yf}}^2} = \sqrt{(190)^2 + (98)^2} = \boxed{213.8\ \text{m/s}}$$

Projectile Motion

- Projectile Motion is one of the important real-world applications of two-dimensional motions, which the object moves under the influence of *only* gravity.
- It is any sort of free fall motion that has a horizontal component of velocity.
- In projectile motion, the object moves on a parabolic trajectory, by considering two assumptions:
  - The effect of air resistance negligible
  - The downward free-fall acceleration is constant over the range of motion

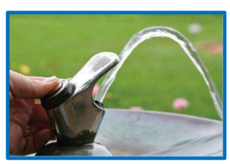


Projectile Motion

- Some examples of projectile motion:
  - Jumping off a diving board
  - Kicking a ball
  - Throwing a rock


Sparks caused by the welder

The water in the park fountain



The water in the drinking fountain

Diving teenager


Projectile Motion

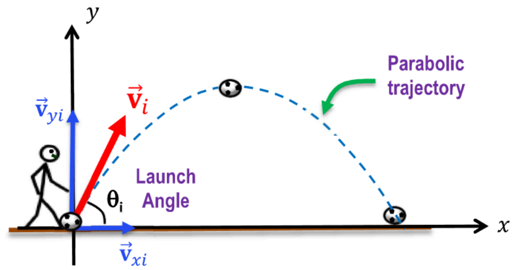
- The start of a projectile's motion is called launch.
- The angle θ_i of the initial velocity vector $\vec{\mathbf{v}}_i$ above the x-axis is called the launch angle.
- The initial velocity vector can be shown into x and y components:

$$\vec{\mathbf{v}}_{\mathbf{i}} = \vec{\mathbf{v}}_{xi} + \vec{\mathbf{v}}_{yi} \quad \rightarrow \quad \vec{\mathbf{v}}_{\mathbf{i}} = v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}}$$

$$\boxed{\begin{aligned} v_{xi} &= v_i \cos\theta_i \\ v_{yi} &= v_i \sin\theta_i \end{aligned}}$$

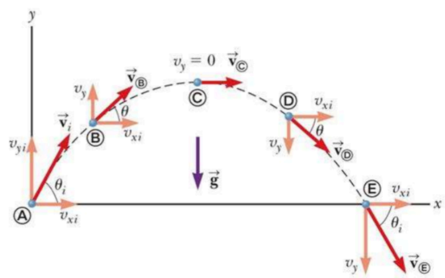
where v_i is the initial speed

- If the launch angle is zero, $\theta_i = 0$, it is called a horizontally launched projectile.



Projectile Motion

- The motion of a projectile can be analyzed by looking at the horizontal and vertical components separately, by ignoring the effect of air resistance.
- Each direction is independent of the other:
- The horizontal component follows constant velocity motion
  - Constant velocity, $\vec{v}_x = constant$
  - No acceleration, $\vec{a}_x = 0$
- The vertical component follows constant acceleration motion
  - Velocity $\vec{v}_y$ is NOT constant
  - Constant acceleration downward
  - $a_y = -g = -9.8\frac{m}{s^2}$
- Time links the horizontal and vertical motions.



Projectile Motion

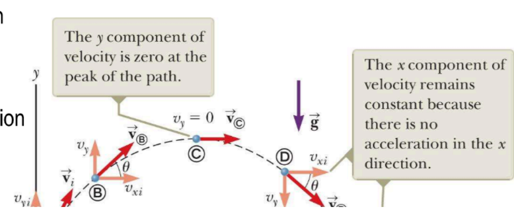
- Therefore, when solving projectile motion problems, use two analysis models:
  1) Constant velocity motion in the horizontal direction

$$\boxed{x_f = x_i + v_{xi}t}$$

  2) Constant acceleration motion in the vertical direction

$$v_{yf} = v_{yi} + a_y t$$

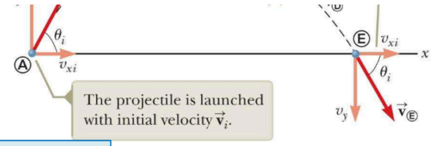
1



$$y_f = y_i + \frac{1}{2}(v_{yi} + v_{yf})t$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

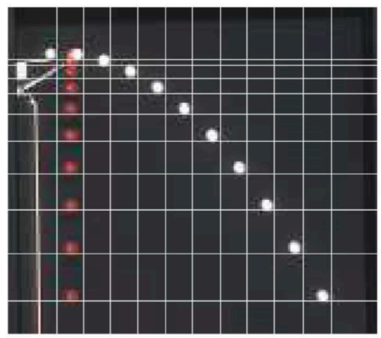
$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$



$$a_y = -g = -9.8\frac{m}{s^2}$$

Projectile Motion

- Assume that a ball is launched horizontally at height h above a horizontal field. A second ball is simply dropped from height h simultaneously.
- The time interval between samples are constant.
- Vertical Motion:
  - The vertical components of the displacement increase by the same amount for each ball.
  - Both balls experiences constant downward acceleration due to gravity.
- Horizontal Motion:
  - The projected ball travels a constant horizontal displacement in each time interval.
  - The horizontal component of a projectile's velocity is constant.

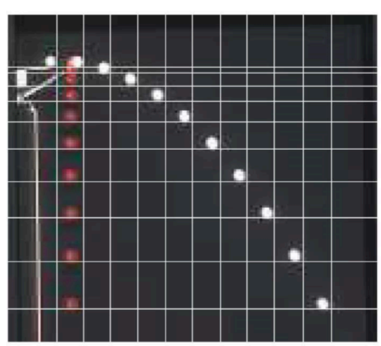


Projectile Motion

- Assume that a ball is launched horizontally at height h above a horizontal field. A second ball is simply dropped from height h simultaneously.

Which ball hits the ground first?

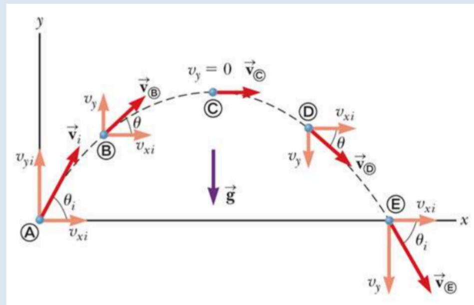
- If air resistance is neglected, the balls hit the ground simultaneously.
- The initial horizontal velocity of the first ball has no influence over its vertical motion.
- Neither ball has any initial vertical motion, so <u>both fall the distance h in the same amount of time.</u>



Quick Quiz 3

- As a projectile thrown at an upward angle moves in its parabolic path (such as in the figure), at what point along its path are the velocity and acceleration vectors for the projectile perpendicular to each other?

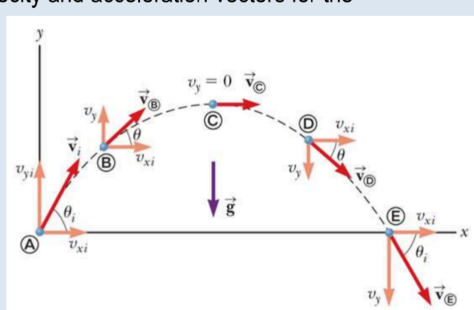
  a) Nowhere
  b) The highest point
  c) The launch point



Quick Quiz 4

- As a projectile thrown at an upward angle moves in its parabolic path (such as in the figure), at what point along its path are the velocity and acceleration vectors for the projectile parallel to each other?

  a) Nowhere
  b) The highest point
  c) The launch point



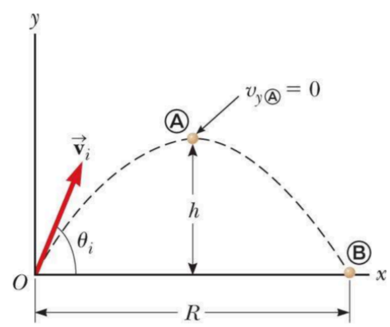
Quick Quiz 5

- An astronaut hits a golf ball on the Moon. Which of the following quantities, if any, remain constant as a ball travels through the vacuum there?

  a) Speed
  b) Acceleration
  c) Horizontal component of velocity
  d) Vertical component of velocity
  e) Velocity

Horizontal Range and Maximum Height of a Projectile

- Assume a projectile is launched from the origin at $t_i = 0$ with a positive v_{yi} component and returns to the same horizontal level.
- This situation is common in sports, for example soccer, golf and baseball.
- In this motion, two points are interesting to analyze:
  - **The Maximum Height (h)**
  - **The Horizontal Range (R)**



Height of a Projectile

- The height of a projectile is the peak point at point A, which has Cartesian coordinates of $(\frac{R}{2}, h)$
- The distance h is called maximum height, which can be determined as below
- First, find the time at which the projectile reaches the peak:

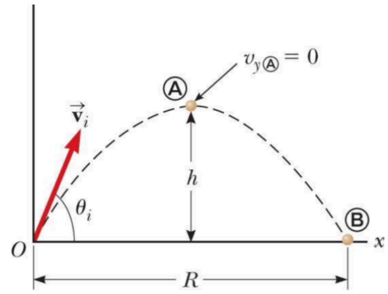
- First, find the time at which the projectile reaches the peak.

$$v_{yf} = v_{yi} + a_y t \Rightarrow 0 = v_i \sin\theta_i - g t_A \Rightarrow \boxed{t_A = \frac{v_i \sin\theta_i}{g}}$$

- Substitute the peak-time t_A and the maximum height $y_f = y_A = h$:

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2 \Rightarrow h = (v_i \sin\theta_i)\frac{v_i \sin\theta_i}{g} - \frac{1}{2} g \left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$\boxed{h = \frac{v_i^2 \sin^2\theta_i}{2g}}$$ Maximum Height



Horizontal Range of a Projectile

- The range of a projectile at point B, which has Cartesian coordinates of $(R, 0)$ has a time that is twice of the peak-time:

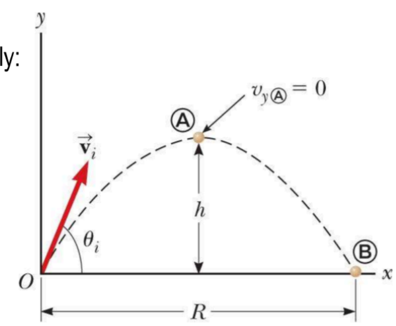
$$t_B = 2t_A = \frac{2v_i \sin\theta_i}{g}$$

- Having the particle under constant velocity motion horizontally:

$$x_f = x_i + v_{xi} t \Rightarrow R = v_{xi} t_B = (v_i \cos\theta) 2t_A$$

$$R = (v_i \cos\theta_i)\frac{2v_i \sin\theta_i}{g} = \frac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$$

$$\boxed{2\sin\theta_i \cos\theta_i = \sin 2\theta_i} \Rightarrow \boxed{R = \frac{v_i^2 \sin 2\theta_i}{g}}$$ Horizontal Range



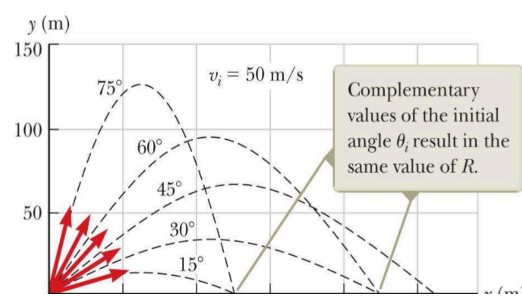
Trajectories of a Projectile

- Figure shows various trajectories for a projectile having a given initial speed but launched at different angles. Recall the horizontal range formula:

$$R = \frac{v_i^2 \sin 2\theta_i}{g}$$

- The range is maximum for $\theta_i = 45°$

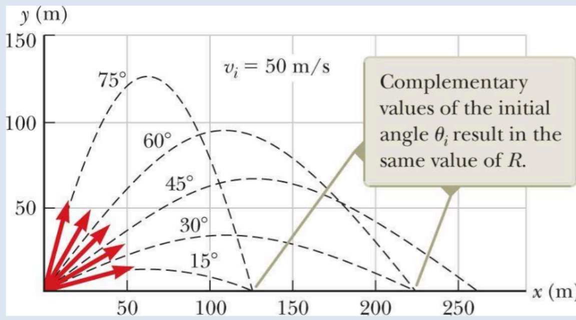
$$\boxed{R_{max} = \frac{v_i^2}{g}}$$ Maximum Range



- Launch angles of θ_i and $90° - \theta_i$ give the same range. For example, $15°$ and $75°$.

Quick Quiz 6

- Rank the launch angles for the five paths in the figure with respect to time of flight from the shortest time of flight to the longest.



Projectile Motion

Example 2 (The Long Jump): A long jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

(a) How far does he jump in the horizontal direction?

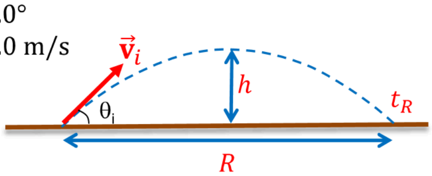
(b) What is the maximum height reached?

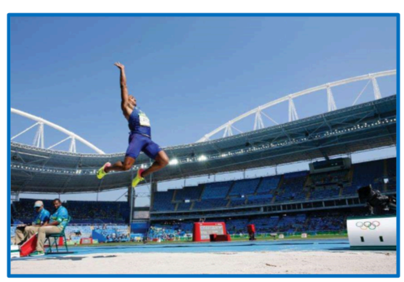
(c) How long does it take the jumper to reach the ground?

Given information:

$\theta_i = 20.0°$

$v_i = 11.0\ \text{m/s}$





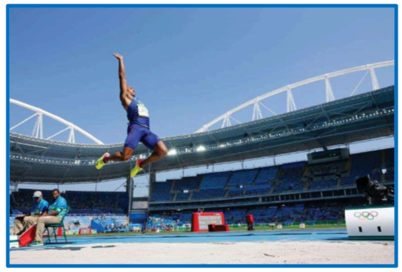
Projectile Motion

Example 2 (The Long Jump): A long jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

(a) How far does he jump in the horizontal direction?

Find the range of the jump

$$R = \frac{v_i{}^2 \sin 2\,\theta_i}{g}$$
$$= \frac{(11.0\ \mathrm{m/s})^2 \sin(2 \cdot 20.0°)}{9.80\ \mathrm{m/s^2}}$$
$$= \boxed{7.94\ \mathrm{m}}$$



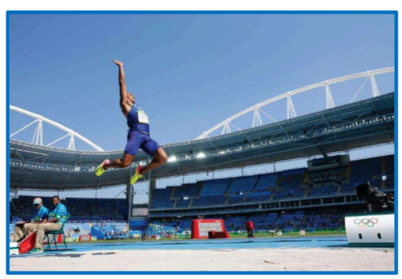
Projectile Motion

Example 2 (The Long Jump): A long jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

(b) What is the maximum height reached?

Find the maximum height of the jump

$$h = \frac{v_i{}^2 \sin^2 \theta_i}{2g}$$
$$= \frac{(11.0\ \mathrm{m/s})^2 (\sin 20.0°)^2}{2(9.80\ \mathrm{m/s^2})}$$
$$= \boxed{0.722\ \mathrm{m}}$$



Projectile Motion

Example 2 (The Long Jump): A long jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

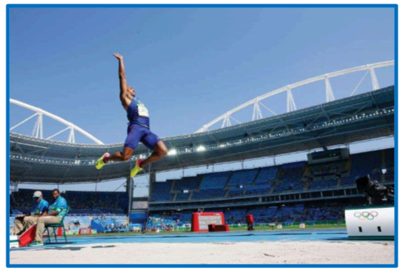
(c) How long does it take the jumper to reach the ground?

Find the range time, which is twice of the peak time

$$t_R = \frac{2v_i \sin\theta_i}{g}$$

$$= \frac{2(11.0\ \mathrm{m/s})(\sin 20.0°)}{(9.80\ \mathrm{m/s^2})}$$

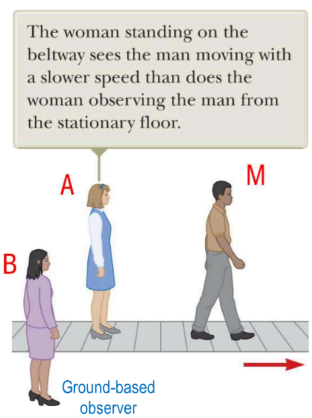
$$= \boxed{0.78\ s}$$



Relative Velocity

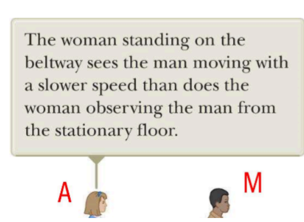
- Consider two observers watching man walking on moving beltway at an airport
  - Woman A is standing on moving beltway
  - Woman B is observing from stationary floor
- Both observers look at same man

Do they see the man moving with the same speed?



Relative Velocity

- Consider two observers watching man walking on moving beltway at an airport
  - Woman A sees man moving at normal walking speed
  - Woman B sees man moving with higher speed because beltway speed combines with his walking speed



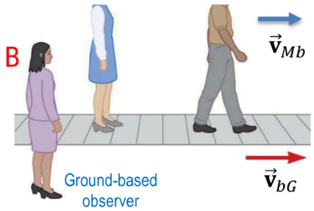
- Both observers look at same man and arrive at different values for his speed. Why? Which one is correct?

Velocity of the beltway relative to the Ground: $\vec{\mathbf{v}}_{bG} = +4\ m/s$

Velocity of the Man relative to the beltway: $\vec{\mathbf{v}}_{Mb} = +2\ m/s$

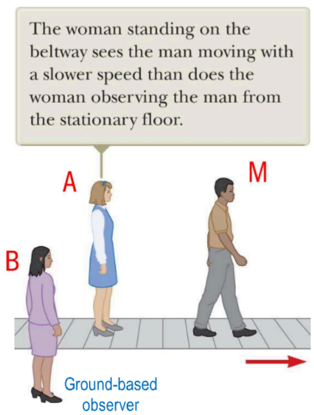
Velocity of the Man relative to the Ground: $\vec{\mathbf{v}}_{MG} = +6\ m/s$

$$\boxed{\vec{\mathbf{v}}_{MG} = \vec{\mathbf{v}}_{Mb} + \vec{\mathbf{v}}_{bG}}$$



Relative Velocity

- Consider two observers watching man walking on moving beltway at an airport
  - Woman A sees man moving at normal walking speed
  - Woman B sees man moving with higher speed because beltway speed combines with his walking speed
- Both observers look at same man and arrive at different values for his speed. Why? Which one is correct?
- Both are correct
  - Difference in their measurements results from relative velocity of their frames of reference



Relative Velocity

Example 3 (A Boat Crossing a River): A boat crossing a wide river moves with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth.

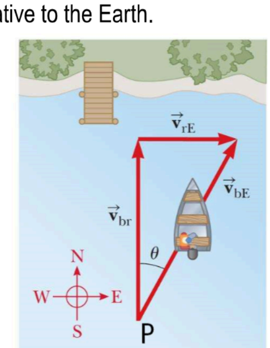
(a) If the boat heads due north, determine the velocity of the boat relative to an observer standing on either bank.

First, analyze the given information:

$\vec{\mathbf{v}}_{br} = 10.0\frac{km}{h}$ (North) $\rightarrow$ velocity of the boat relative to the river

$\vec{\mathbf{v}}_{rE} = 5.0\frac{km}{h}$ (East) $\rightarrow$ velocity of the river relative to the Earth

$\vec{\mathbf{v}}_{bE} = ?$ $\rightarrow$ velocity of the boat relative to the Earth

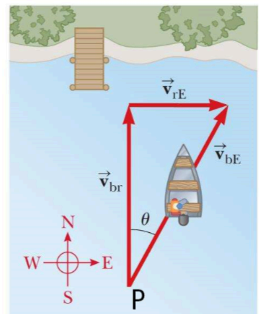


$$\boxed{\vec{\mathbf{v}}_{bE} = \vec{\mathbf{v}}_{br} + \vec{\mathbf{v}}_{rE}}$$

Relative Velocity

Example 3 (A Boat Crossing a River): A boat crossing a wide river moves with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth.

(a) If the boat heads due north, determine the velocity of the boat relative to an observer standing on either bank.



Find the magnitude and direction of $\vec{\mathbf{v}}_{bE}$ using the Pythagorean theorem

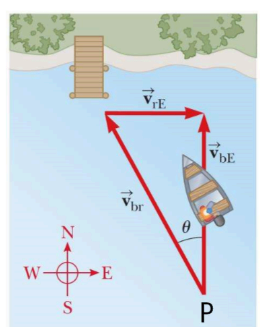
$$v_{bE} = \sqrt{v_{br}^2 + v_{rE}^2} = \sqrt{(10.0\text{ km/h})^2 + (5.00\text{ km/h})^2} = \boxed{11.2\text{ km/h}}$$

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{br}}\right) = \tan^{-1}\left(\frac{5.00}{10.0}\right) = \boxed{26.6°}$$ Northeast

Relative Velocity

Example 3 (A Boat Crossing a River): A boat crossing a wide river moves with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth.

(b) If the boat travels with the same speed of 10.0 km/h relative to the river and is to travel due north as shown in the figure, what should its heading be?



$$v_{bE} = \sqrt{v_{br}^2 - v_{rE}^2} = \sqrt{(10.0\text{ km/h})^2 - (5.00\text{ km/h})^2} = \boxed{8.66\text{ km/h}}$$

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{bE}}\right) = \tan^{-1}\left(\frac{5.00}{8.66}\right) = \boxed{30.0°}$$ Northwest

THANK YOU

HUMBER

WE ARE HUMBER