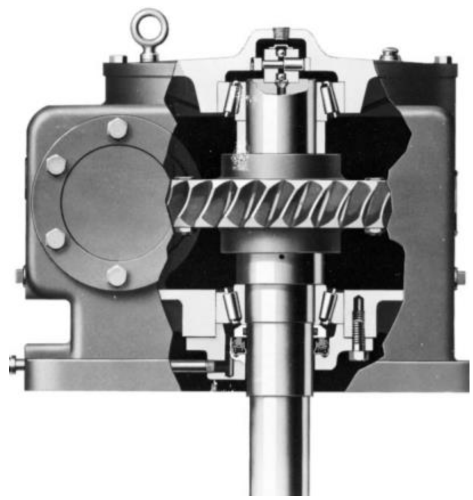
Lesson 05: Shafts,

Shaft:

- Is a rotating member,
- usually of circular cross section,
- Used to transmit power or motion.
- It provides the axis of rotation, or oscillation, of elements such as gears, pulleys, flywheels, cranks, sprockets, and the like and controls the geometry of their motion.



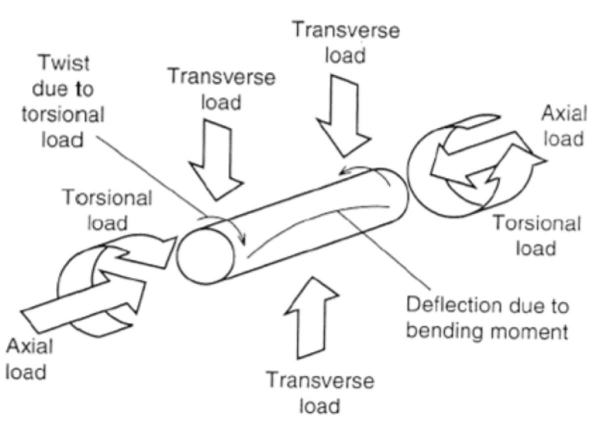
Axle:

- Is a nonrotating member that carries no torque and is used to support rotating wheels, pulleys.

Consideration in designing a shaft:

- Material selection
- Geometric layout
- Stress and strength
    - Static strength
    - Fatigue strength
- Deflection and rigidity
    - Bending deflection
    - Torsional deflection
    - Shear deflection due to transverse loading of short shafts
- Vibration due to natural frequency

Typical shaft loading:



Shaft Materials:

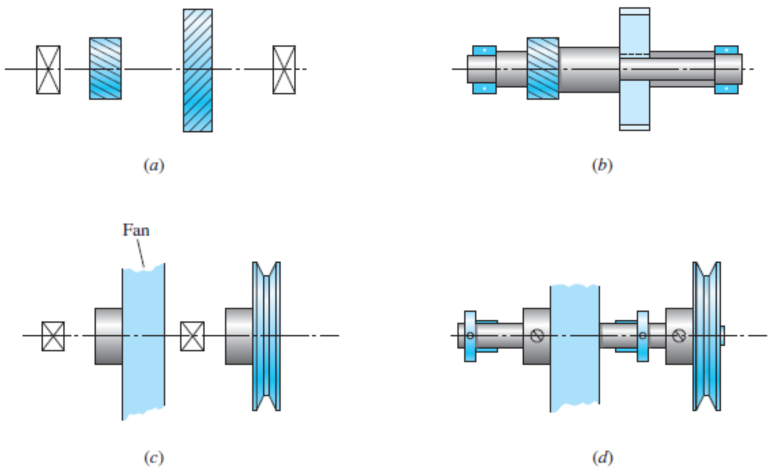
- Deflection is affected by stiffness as represented by the modulus of elasticity, which is essentially constant for all steels.

  Deflection = PL/AE

- For that reason, rigidity cannot be controlled by material decisions only but by geometric decisions as well.
- Necessary strength to resist loading stresses affects the choice of materials and their treatments.
- Many shafts are made from low carbon, cold-drawn or hot-rolled steel, such as AISI 1020-1050 steels.
- Shafts usually don’t need to be surface hardened unless they serve as the actual journal of a bearing surface.

Geometric layout:

- The general layout of a shaft to accommodate shaft elements, e.g., gears, bearings, and pulleys, must be specified early in the design process.
- That helps to perform a free body force analysis and to obtain shear-moment diagrams.
- The geometry of a shaft is generally that of a stepped cylinder.
- The use of shaft shoulders is an excellent means of axially locating the shaft elements and to carry any thrust loads.
- An example of a stepped shaft supporting the gear of a worm-gear speed reducer. Each shoulder in the shaft serves a specific purpose, which you should attempt to determine by observation.



- This problem is illustrated by the two examples in the above figure.
- In Fig. a, a geared countershaft is to be supported by two bearings. In Fig. c a fan shaft is to be configured.
- The solutions shown in Fig. b and d are not necessarily the best ones, but they do illustrate how the shaft-mounted devices are fixed and located in the axial direction, and how provision is made for torque transfer from one element to another.
- Pulleys and sprockets often need to be mounted outboard for ease of installation of the belt or chain.
- The length of the cantilever should be kept short to minimize the deflection.

Shaft Deflection Consideration:

- Shafts must be designed so that deflections are within acceptable levels.
- Too much deflection:
  - degrades gear performance.
  - cause noise and vibration
  - Allowable deflections will depend on many factors. As a rough guideline, typical ranges for maximum slopes and transverse deflections of the shaft centerline are given in the table:

Slopes	
Tapered roller	0.0005–0.0012 rad
Cylindrical roller	0.0008–0.0012 rad
Deep-groove ball	0.001–0.003 rad
Spherical ball	0.026–0.052 rad
Self-align ball	0.026–0.052 rad
Uncrowned spur gear	$<$0.0005 rad
Transverse Deflections	
Spur gears with $P < 10$ teeth/in	0.010 in
Spur gears with $11 < P < 19$	0.005 in
Spur gears with $20 < P < 50$	0.003 in

Critical Speeds:

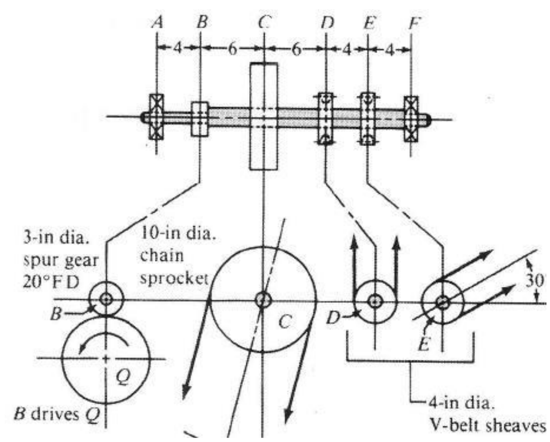
- Sufficient lateral rigidity so that the lowest critical speed is significantly above the range of operation.
- Sufficient torsional stiffness so that the shaft lowest natural frequency is much higher than the highest torsional input frequency.
- Shafts should be designed to avoid operation at or near critical speeds. This is usually achieved by providing:
- When geometry is simple, as in a shaft of uniform diameter, simply supported, the task is easy. It can be expressed as

$$\omega_1 = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{m}} = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{gEI}{A\gamma}}$$

where m is the mass per unit length,
A: the cross-sectional area,
γ the specific weight.

Forces exerted on shafts by machine elements:

Gears, belt sheaves, chain sprockets, etc, typically carried by shafts exert forces on the shaft that cause bending moments.



Spur Gears Forces:

T = 63 000 P / n

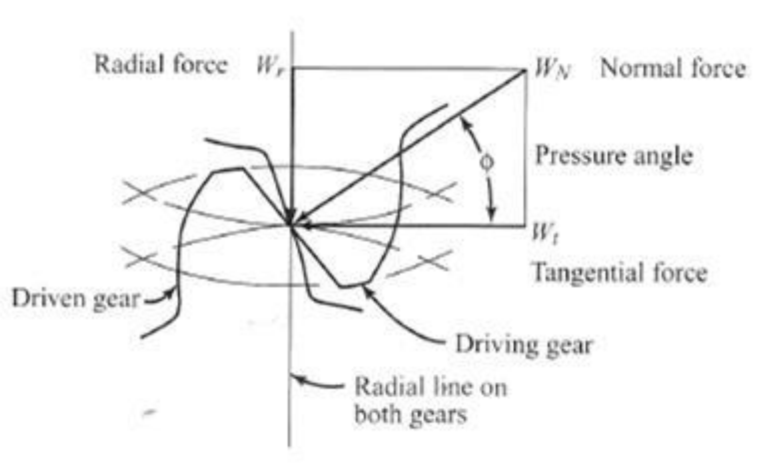
T = torque on the gear (lb. in)
P = power being transmitted (hp)
n = rotational speed (rpm)

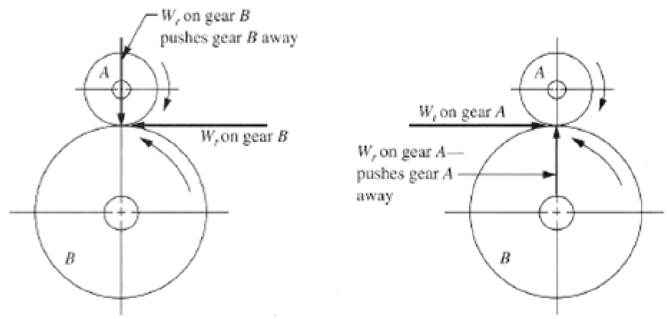
$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

T = Twisting moment in N-m, and
N = Speed of the shaft in r.p.m.

Wt = T / (D/2)
Wr = Wt tanϕ

Wt = Tangential Force (lb)
Wr = Radial Forces (lb)
D = pitch diameter of the gear (in)
ϕ = Pressure angle (deg.)





(a) Forces exerted on gear B by gear A. Action forces—gear A drives gear B.

(b) Forces exerted on gear A by gear B. Reaction forces.

Chain Drive Forces

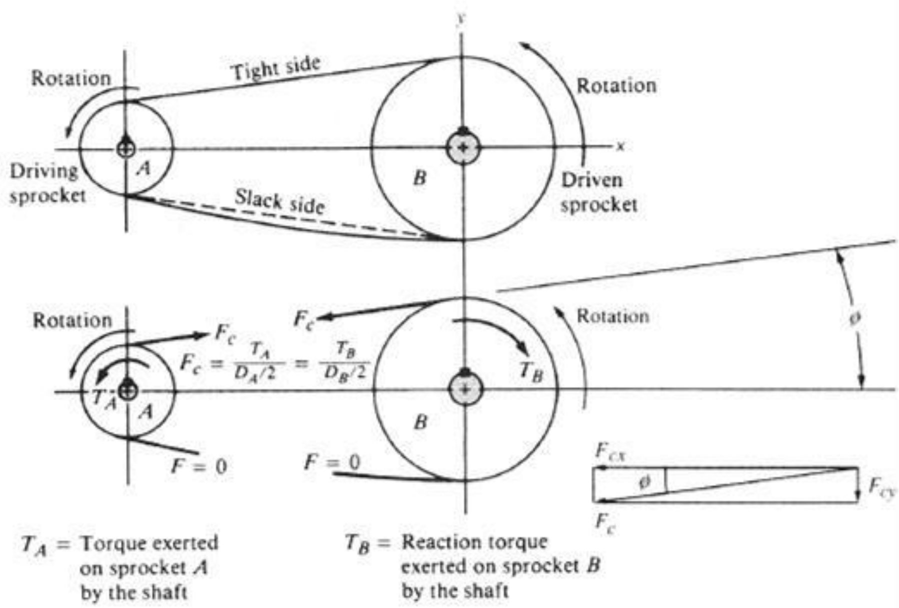
T = 63 000 P / n

T = torque on the sprocket (lb. in)
P = power being transmitted (hp)
n = rotational speed (rpm)

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

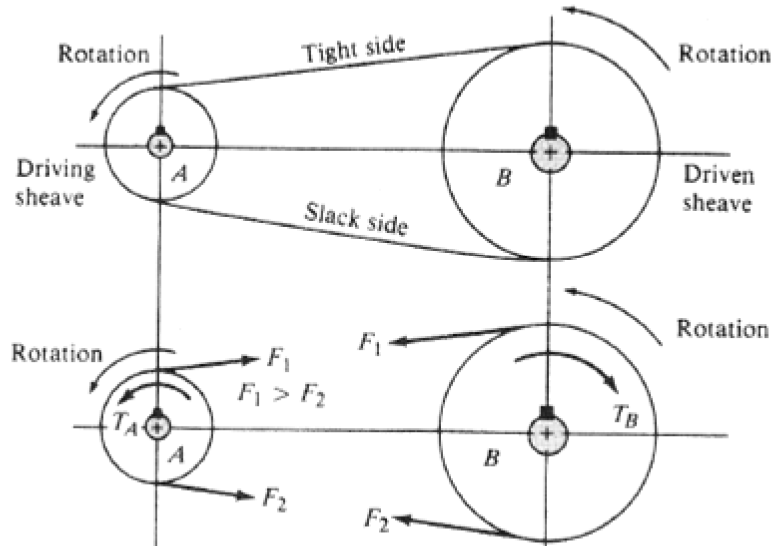
T = Twisting moment in N-m, and
N = Speed of the shaft in r.p.m.

Fc = T/(D/2)
Fcx = Fc cos ϕ
Fcy = Fc sin ϕ



Fc = Chain force
D = Pitch diameter of sprocket
ϕ = the angle of inclination of the tight side of the chain with respect to the x-direction

Belt Drive Forces



T = 63 000 P / n
T = torque on the sheave (lb. in)
P = power being transmitted (hp)
n = rotational speed (rpm)

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

T = Twisting moment in N-m, and
N = Speed of the shaft in r.p.m.

FN = T /(D/2)
FN = F1 - F2
FB = F1+ F2

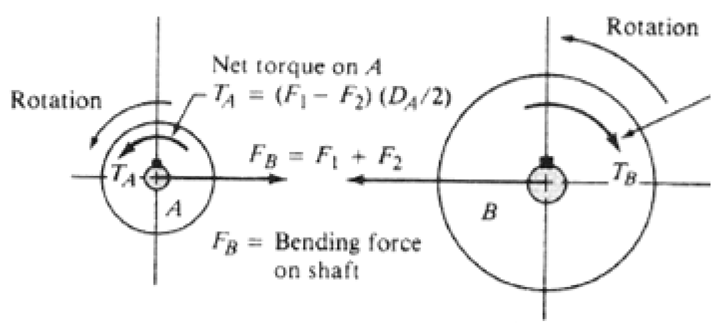
FN = Net Driving Force
FB = Bending force on the shaft
The general form of the relation between F1 & F2 is :

$$Ln\left(\frac{F_1}{F_2}\right) = \mu.\theta$$

$$\left(\frac{F_1}{F_2}\right) = e^{\mu.\theta}$$

For V-belt drives, the typical force ratio is:

F1 / F2 = 5



Relating the bending and net driving forces:

If:

C = FB/ FN

Then:

$$C = \frac{F_B}{F_N} = \frac{F_1 + F_2}{F_1 - F_2}$$

$$C = \frac{F_1 + F_2}{F_1 - F_2} = \frac{5F_2 + F_2}{5F_2 - F_2} = \frac{6F_2}{4F_2} = 1.5$$

$F_B = 1.5\ F_N = 1.5\ T / (D/2)$

For Flat Belt drives, the typical force ratio is:

$F1 / F2 = 3$

$F_B = 2.0 F_N = 2.0\ T / (D/2)$

Design a shaft subjected to twisting moment or torque only:

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation.

$$\frac{T}{J} = \frac{\tau}{r}$$

T = Twisting moment (or torque) acting upon the shaft,
J = Polar moment of inertia of the shaft about the axis of rotation,
τ = Torsional shear stress, and
r = radius

for round solid shaft, polar moment of inertia:

$$J = \frac{\pi}{32} \times d^4$$

for hollow shaft

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

do and di = Outside and inside diameter of the shaft

$k = di / do$

Shafts Subjected to Bending Moment Only:

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation.

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

M = Bending moment,
I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation
σ_b = Bending stress, and
y = Distance from neutral axis to the outer-most fiber.

for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

for a hollow shaft, moment of inertia

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]$$

$$y = d_o / 2$$

Shafts Subjected to Combined Twisting Moment and Bending Moment:

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Where: .

τ = Shear stress induced due to twisting moment, and
σ_b = Bending stress (tensile or compressive) induced due to bending moment.

Substituting the values of τ and σ_b: <u>(assume solid shaft)</u>

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as equivalent twisting moment and is denoted by T_e.

Hence:

$$T_e = \frac{\pi}{16} \times \tau_{max} \times d^3$$

According to maximum normal stress theory, the maximum normal stress in the shaft:

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σb:

$$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$

Or

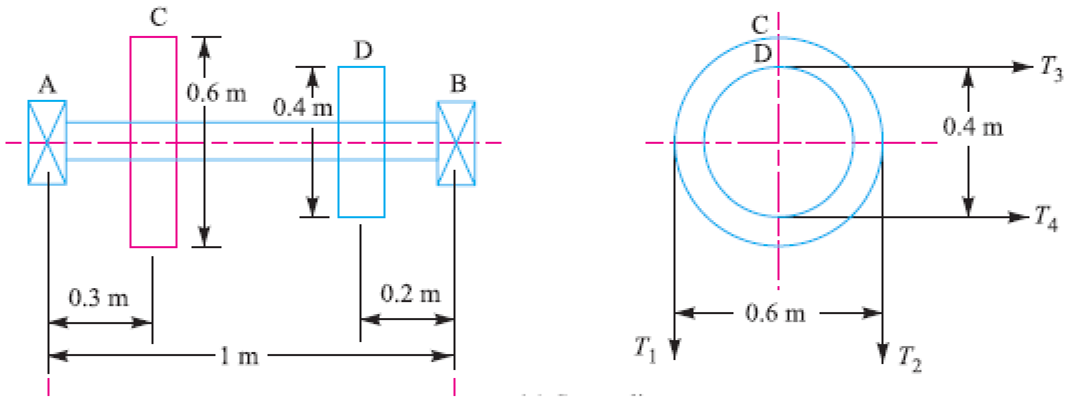
$$M_e = \frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3$$

Where Me: is the equivalent Bending moment:

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

Exercise:
A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

Answer:



T1 = Tension in the tight side of the belt on pulley C = 2250 N
T2 = Tension in the slack side of the belt on pulley C.

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.24 \times \pi = 0.754$$

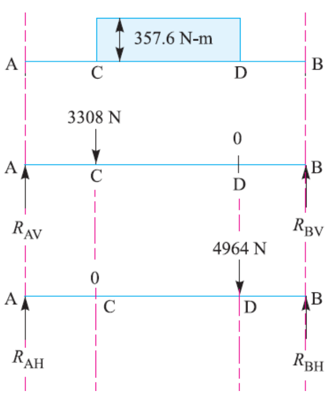
$$\log \left(\frac{T_1}{T_2}\right) = \frac{0.754}{2.3} = 0.3278 \quad \text{or} \quad \frac{T_1}{T_2} = 2.127$$

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

Vertical load acting on the shaft at C,

W_C = T1 + T2 = 2250 + 1058 = 3308 N
and vertical load on the shaft at D = 0

Torque acting on the pulley C,
T = (T1 – T2) R_C = (2250 – 1058) 0.3 = 357.6 N-m



(*b*) Torque diagram.
(*c*) Vertical load diagram.
(*d*) Horizontal load diagram.

T3 = Tension in the tight side of the belt on pulley D, and
T4 = Tension in the slack side of the belt on pulley D.

Since the torque on both the pulleys (i.e. C and D) is same, therefore

$$(T_3 - T_4)\, R_D = T = 357.6 \text{ N-m} \;\text{ or }\; T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N}$$

Since belt type-material-and sizes are same then :

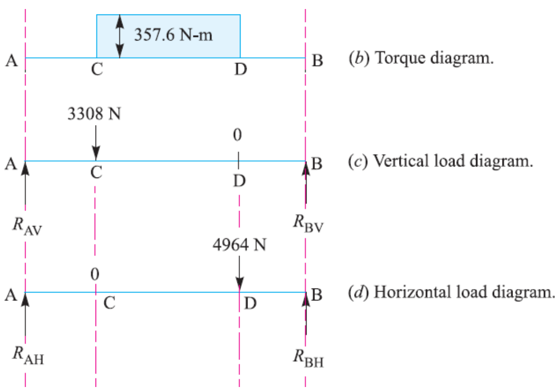
$$\frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \;\text{ or }\; T_3 = 2.127\, T_4$$

T_3 = 3376 N, and T_4 = 1588 N

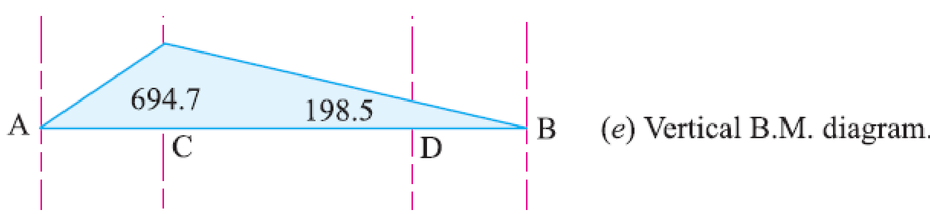
Horizontal load acting on the shaft at D:

$W_D = T_3 + T_4$ = 3376 + 1588 = 4964 N

and horizontal load on the shaft at C = 0



R_{AV} and R_{BV} be the reactions at the bearings A and B respectively.
$R_{AV} + R_{BV}$ = 3308 N
Taking moments about A,
$R_{BV} \times 1 = 3308 \times 0.3$ or R_{BV} = 992.4 N
and R_{AV} = 3308 – 992.4 = 2315.6 N
We know that B.M. at A and B,
M_{AV} = MBV = 0
B.M. at C, $M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3$ = 694.7 N-m



B.M. at D, $M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2$ = 198.5 N-m

Now considering horizontal loading at D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that:

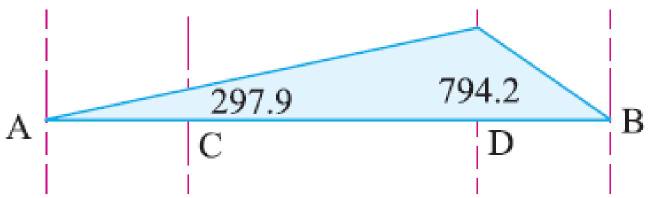
$R_{AH} + R_{BH} = 4964$ N

Taking moments about A,

$R_{BH} \times 1 = 4964 \times 0.8$ or $R_{BH} = 3971$ N
$R_{AH} = 4964 - 3971 = 993$ N

B.M. at A and B

$M_{AH} = M_{BH} = 0$
B.M. at C, $M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9$ N-m
B.M. at D, $M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2$ N-m

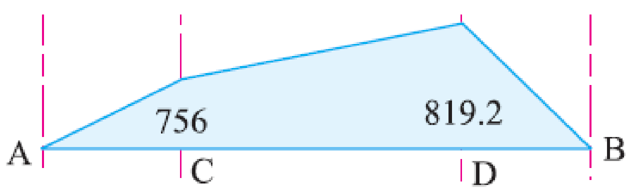


(*f*) Horizontal B.M. diagram.

Resultant B.M. at C and D:

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$



(*g*) Resultant B.M. diagram.

Maximum bending moment, to be considered is:

$M = M_D = 819.2$ N-m

The equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$
$$= 894 \times 10^3 \text{ N-mm}$$

The equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$\therefore\ d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3$ or $d = 47.6$ mm

The equivalent bending moment,

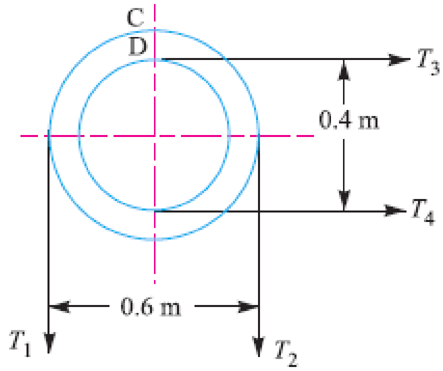
$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}\,(M + T_e)$$
$$= \tfrac{1}{2}\,(819.2 + 894) = 856.6 \text{ N-m} = 856.6 \times 10^3 \text{ N-mm}$$

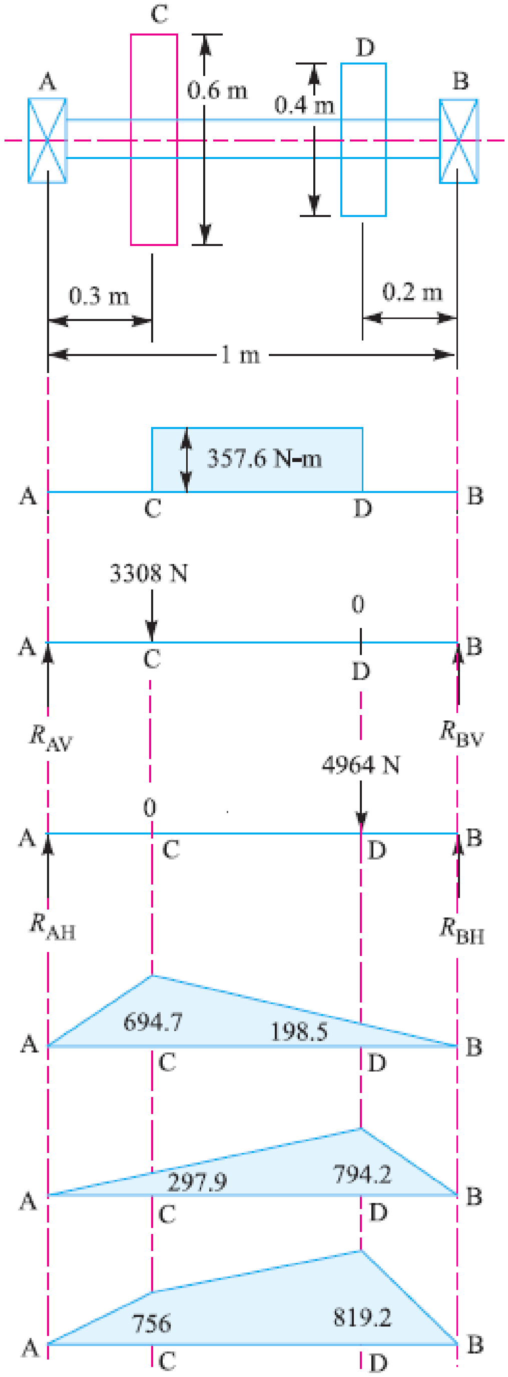
The equivalent bending moment (M_e),

$$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2\, d^3$$
$$d^3 = 856.6 \times 10^3/6.2 = 138.2 \times 10^3 \text{ or } d = 51.7 \text{ mm}$$

Taking larger of the two values,

$$d = 51.7 \text{ select } 55 \text{ mm}$$





(a) Space diagram.

(b) Torque diagram.

(c) Vertical load diagram.

(d) Horizontal load diagram.

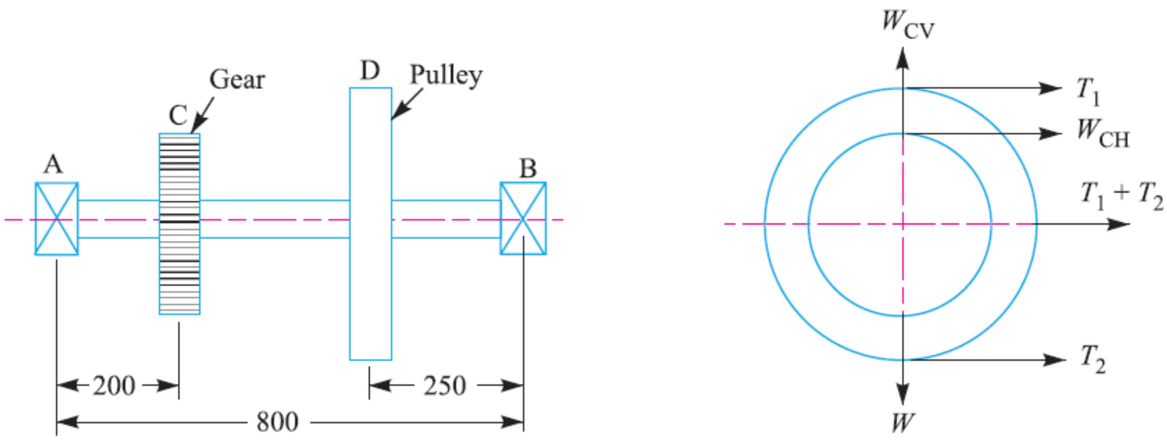
(e) Vertical B.M. diagram.

(f) Horizontal B.M. diagram.

(g) Resultant B.M. diagram.

Assignment 01:

A shaft is supported on bearings A and B, 800 mm between centers. A 20° straight tooth spur gear having 600 mm pitch diameter, is located 200 mm to the right of the left-hand bearing A, and a 700 mm diameter pulley is mounted 250 mm towards the left of bearing B. The gear is driven by a pinion with a downward tangential force while the pulley drives a horizontal belt having 180° angle of wrap. The pulley also serves as a flywheel and weighs 2000 N. The maximum belt tension is 3000 N and the tension ratio is 3 : 1. Determine the maximum bending moment and the necessary shaft diameter if the allowable shear stress of the material is 40 MPa and allowable working stress is 63 MPa



All dimensions in mm.
(*a*) Space diagram.

the torque acting on the shaft at D:

$$T = (T_1 - T_2)\,R_D = T_1\left(1 - \frac{T_2}{T_1}\right)R_D$$

Tangential force on gear:

$$F_{tc} = \frac{T}{R_C}$$

normal load acting on the tooth of gear C

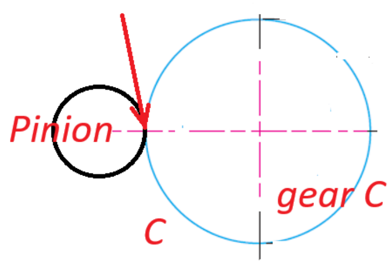
$$W_C = \frac{F_{tc}}{\cos \alpha_C}$$

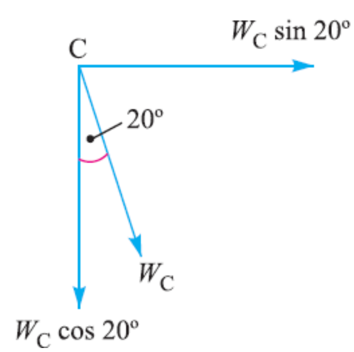
horizontal load acting on the shaft at C:

$$W_{CH} = W_C \sin 20°$$

Horizontal load acting on the shaft at D,
W_{DH} = T1 + T2

and vertical load acting on the shaft at D,





W_{DV} = W

bearings A and B respectively.
R_{AV} + R_{BV}

find vertical and horizontal B.M.s at critical points then find the resultant:

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2}$$

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2}$$

use Maximum bending moment to find diameters from the maximum shear and BM theories:

Solution of assignment o1:

Solution. Given : AB = 800 mm ; α_C = 20° ; D_C = 600 mm or R_C = 300 mm ; AC = 200 mm ; D_D = 700 mm or R_D = 350 mm ; DB = 250 mm ; θ = 180° = π rad ; W = 2000 N ; T_1 = 3000 N ; $T_1/T_2 = 3$; τ = 40 MPa = 40 N/mm²

The space diagram of the shaft is shown in Fig. 14.6 (*a*).

We know that the torque acting on the shaft at D,

$$T = (T_1 - T_2)\, R_D = T_1 \left(1 - \frac{T_2}{T_1}\right) R_D$$

$$= 3000 \left(1 - \frac{1}{3}\right) 350 = 700 \times 10^3 \text{ N-mm} \qquad \ldots(\because T_1/T_2 = 3)$$

The torque diagram is shown in Fig. 14.6 (*b*).

Assuming that the torque at D is equal to the torque at C, therefore the tangential force acting on the gear C,

$$F_{tc} = \frac{T}{R_C} = \frac{700 \times 10^3}{300} = 2333 \text{ N}$$

and the normal load acting on the tooth of gear C,

$$W_C = \frac{F_{tc}}{\cos \alpha_C} = \frac{2333}{\cos 20°} = \frac{2333}{0.9397} = 2483 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. 14.7. Resolving the normal load vertically and horizontally, we get

Vertical component of W_C *i.e.* the vertical load acting on the shaft at C,

$$W_{CV} = W_C \cos 20°$$
$$= 2483 \times 0.9397 = 2333 \text{ N}$$

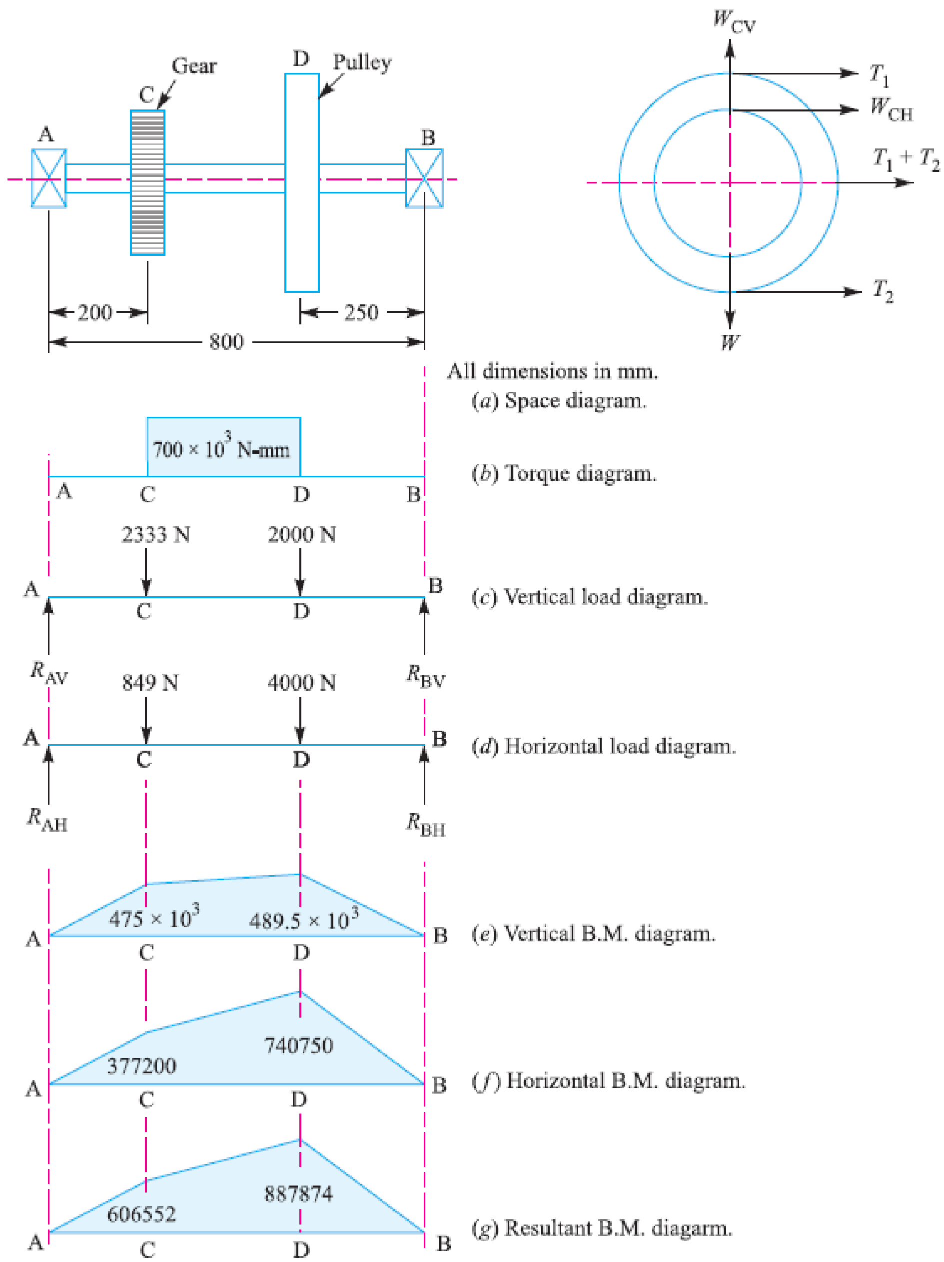
and horizontal component of W_C *i.e.* the horizontal load acting on the shaft at C,

$$W_{CH} = W_C \sin 20°$$
$$= 2483 \times 0.342 = 849 \text{ N}$$

Since $T_1 / T_2 = 3$ and T_1 = 3000 N, therefore

$$T_2 = T_1 / 3 = 3000 / 3 = 1000 \text{ N}$$

Camshaft



∴ Horizontal load acting on the shaft at D,

$$W_{DH} = T_1 + T_2 = 3000 + 1000 = 4000 \text{ N}$$

and vertical load acting on the shaft at D,

$$W_{DV} = W = 2000 \text{ N}$$

The vertical and horizontal load diagram at C and D is shown in Fig. 14.6 (c) and (d) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all considering the vertical loading at C and D. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 2333 + 2000 = 4333 \text{ N}$$

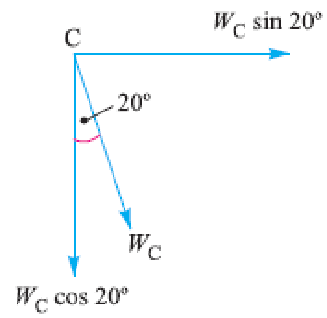


Fig. 14.7

Taking moments about A, we get

$$R_{BV} \times 800 = 2000\,(800 - 250) + 2333 \times 200$$
$$= 1\,566\,600$$

$$\therefore \quad R_{BV} = 1\,566\,600 / 800 = 1958 \text{ N}$$

and $$R_{AV} = 4333 - 1958 = 2375 \text{ N}$$

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $$M_{CV} = R_{AV} \times 200 = 2375 \times 200$$
$$= 475 \times 10^3 \text{ N-mm}$$

B.M. at D, $$M_{DV} = R_{BV} \times 250 = 1958 \times 250 = 489.5 \times 10^3 \text{ N-mm}$$

The bending moment diagram for vertical loading is shown in Fig. 14.6 (e).

Now consider the horizontal loading at C and D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 849 + 4000 = 4849 \text{ N}$$

Taking moments about A, we get

$$R_{BH} \times 800 = 4000\,(800 - 250) + 849 \times 200 = 2\,369\,800$$

$$\therefore \quad R_{BH} = 2\,369\,800 / 800 = 2963 \text{ N}$$

and $$R_{AH} = 4849 - 2963 = 1886 \text{ N}$$

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $$M_{CH} = R_{AH} \times 200 = 1886 \times 200 = 377\,200 \text{ N-mm}$$

B.M. at D, $$M_{DH} = R_{BH} \times 250 = 2963 \times 250 = 740\,750 \text{ N-mm}$$

The bending moment diagram for horizontal loading is shown in Fig. 14.6 (f).

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(475 \times 10^3)^2 + (377\,200)^2}$$
$$= 606\,552 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(489.5 \times 10^3)^2 + (740\,750)^2}$$
$$= 887\,874 \text{ N-mm}$$

$= 887\ 874$ N-mm

Maximum bending moment

The resultant B.M. diagram is shown in Fig. 14.6 (*g*). We see that the bending moment is maximum at *D*, therefore

Maximum B.M., $M = M_D = 887\ 874$ N-mm **Ans.**

Diameter of the shaft

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(887\ 874)^2 + (700 \times 10^3)^2} = 1131 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

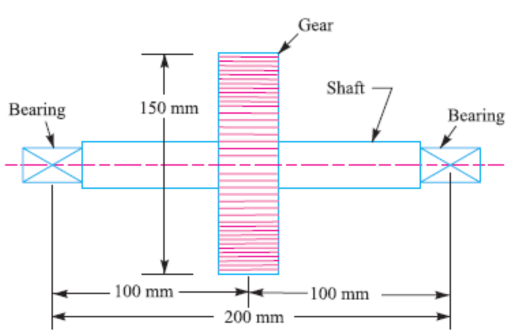
$$1131 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 1131 \times 10^3 / 7.86 = 144 \times 10^3$ or $d = 52.4$ say 55 mm **Ans.**

Assignment 02:

A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm. The distances between the center line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted? The pressure angle of the gear may be taken as 20°.

Ans. d = 32 say 35 mm



Solution for Assignment 02:

Solution. Given : $P = 7.5$ kW $= 7500$ W ; $N = 300$ r.p.m. ; $D = 150$ mm $= 0.15$ m ; $L = 200$ mm $= 0.2$ m ; $\tau = 45$ MPa $= 45$ N/mm^2 ; $\alpha = 20°$

Fig. 14.2 shows a shaft with a gear mounted on the bearings.

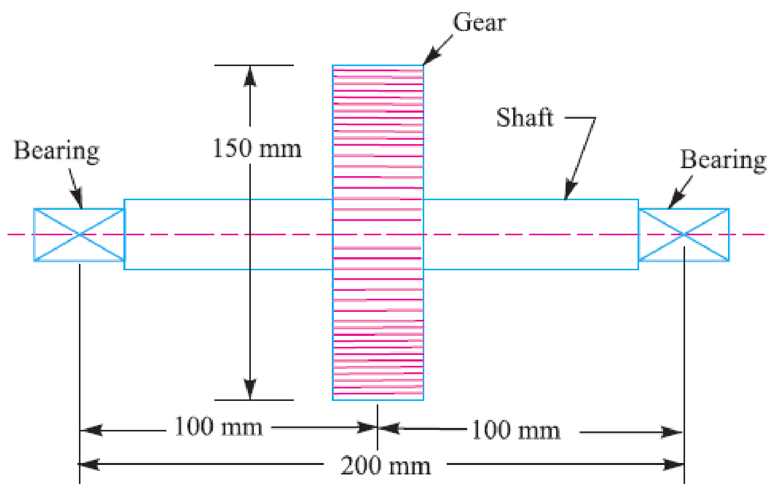


Fig. 14.2

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{7500 \times 60}{2\pi \times 300} = 238.7 \text{ N-m}$$

$\therefore$ Tangential force on the gear,

$$F_t = \frac{2T}{D} = \frac{2 \times 238.7}{0.15} = 3182.7 \text{ N}$$

and the normal load acting on the tooth of the gear,

$$W = \frac{F_t}{\cos\alpha} = \frac{3182.7}{\cos 20^\circ} = \frac{3182.7}{0.9397} = 3387 \text{ N}$$

Since the gear is mounted at the middle of the shaft, therefore maximum bending moment at the centre of the gear,

$$M = \frac{W.L}{4} = \frac{3387 \times 0.2}{4} = 169.4 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(169.4)^2 + (238.7)^2} = 292.7 \text{ N-m}$$
$$= 292.7 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$292.7 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

$$\therefore \quad d^3 = 292.7 \times 10^3 / 8.84 = 33 \times 10^3 \text{ or } d = 32 \text{ say } 35 \text{ mm}$$ **Ans.**

Assignment 03:
A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.

Ans. d = 48.7 to choose 50 mm

Assignment 04:
solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.

Ans. d = 159.4 say 160 mm

Assignment 05:
A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

ANS. d = 86 mm due to max shear, = 86 say 90 mm due to Max BM theory