Lecture 2

November 9, 2023 7:42 PM

Basic Circuit Laws

Nodes, Paths, Loops, Branches

\# of elements: 5
① independent current source
④ → resistors

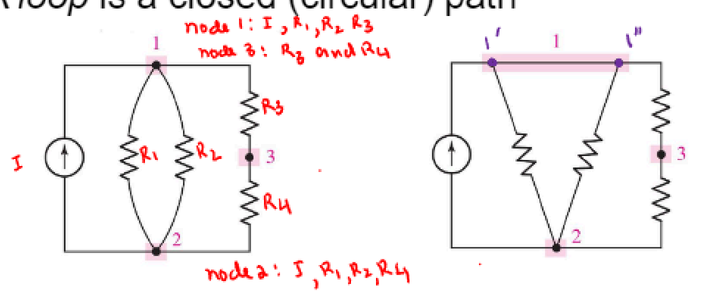
These two networks are equivalent

There are three *nodes* and five *branches*

Node : pt. @ which 2 or more branches (elements) meet.

A *path* is a sequence of nodes

A *loop* is a closed (circular) path



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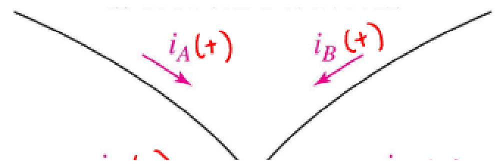
Kirchhoff's Current Law (KCL)

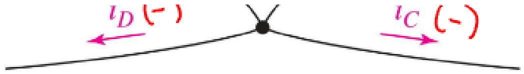
conservation of charge

applicable to a node

KCL: Algebraic sum of currents entering any node is zero.

entering → + ve
leaving → − ve





$$+i_A + i_B + (-i_C) + (-i_D) = 0$$

$i_A + i_B = i_C + i_D$ [Σ of currents entering the node = Σ of currents leaving the node]

KCL: Alternative Forms

Current *IN* is zero:

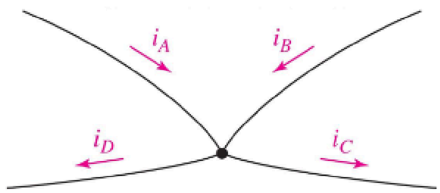
$$i_A + i_B + (-i_C) + (-i_D) = 0$$

Current *OUT* is zero:

$$(-i_A) + (-i_B) + i_C + i_D = 0$$

Current *IN* = *OUT*:

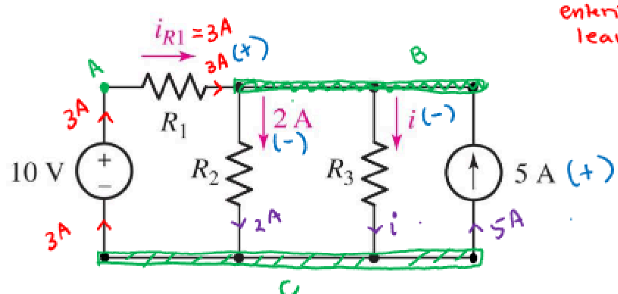
$$i_A + i_B = i_C + i_D$$



Example: KCL Application

Find the current through resistor R_3 if it is known that the voltage source supplies a current of 3 A.

entering → +ve
leaving → −ve



KCL @ node B

$+3^A - 2^A - i + 5^A = 0$

$i = 6^A$

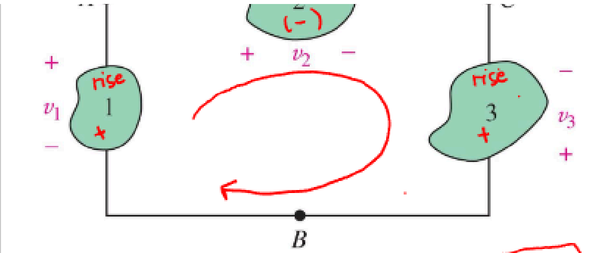
KCL @ node C ⇒ $-3^A + 2^A + i - 5^A = 0$ ⇒ $i = 6^A$

Kirchhoff's Voltage Law (KVL)

Conservation of Energy

KVL: Algebraic sum of voltages around any closed path is zero.

Voltage rise → +
Voltage drop → −



5^V

B to A → voltage rise
A to B → voltage drop

$+v_1 - v_2 + v_3 = 0 \Rightarrow \boxed{v_1+v_3} = v_2$

KVL: Alternative Forms

Sum of *RISES* is zero (clockwise from B):

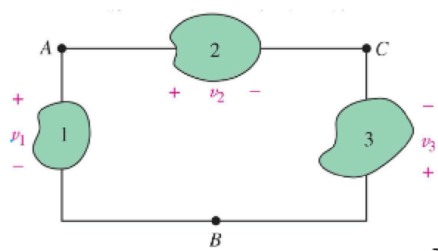
$$v_1 + (-v_2) + v_3 = 0$$

Sum of *DROPS* is zero (clockwise from B):

$$(-v_1) + v_2 + (-v_3) = 0$$

Two paths, same voltage (A to B):

$$v_1 = (-v_3) + v_2$$

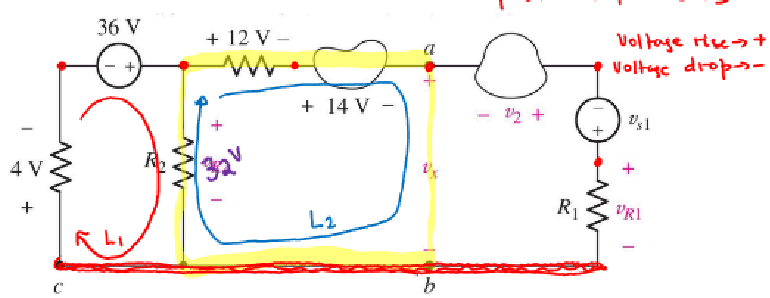


Example: Applying KVL

Find v_{R2} and the voltage v_x.

# of elements : 8
# of closed paths : 3

Voltage rise → +
Voltage drop → −



KVL @ L_1

$-4 + 36 - v_{R_2} = 0 \Rightarrow v_{R_2} = 32^V$

KVL @ L_2

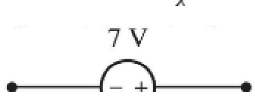
$+32 - 12 - 14 - v_x = 0 \Rightarrow v_x = 6^V$

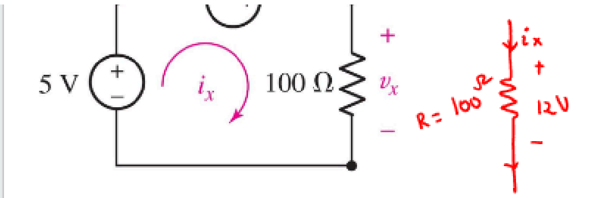
Applying KCL, KVL, Ohm's Law[1]

Find the current i_x and the voltage v_x

@ KVL

$+5^V + 7^V - v_x = 0 \Rightarrow v_x = 12^V$





passive sign convention

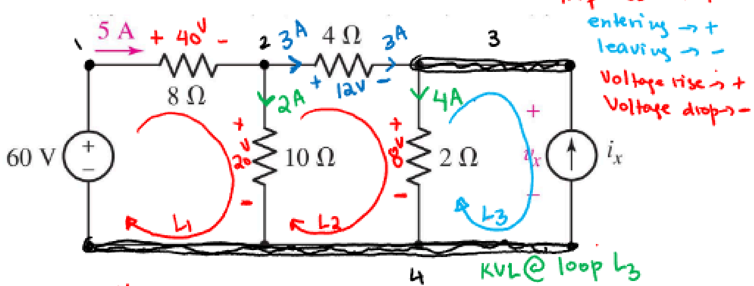
$i_x = \frac{v}{R} = \frac{12V}{100\Omega}$

$i_x = 0.12\ A$

Applying KCL, KVL, Ohm's Law₂

Solve for the voltage v_x and the current i_x

# of elements: 6
# of nodes: 4
entering → +
leaving → −
Voltage rise → +
Voltage drop → −



$+60^V - 40^V - ? = 0$
$? = 20^V$

KVL @ loop L_3
$+8^V - v_x = 0$
$V_x = 8^V$

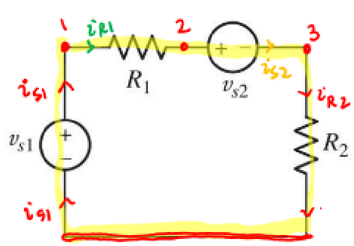
KCL @ node 2 ⇒ $+5^A - 2^A - ? = 0$ ⇒ $? = 3^A$

KVL @ loop L_2 ⇒ $+20^V - 12^V - ? = 0$ ⇒ $? = 8^V$

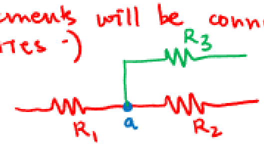
KCL @ node 3 ⇒ $+3^A - 4^A + i_x = 0$ ⇒ $i_x = 1^A$

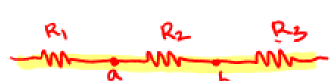
Circuit Topologies - Series Connections

All the elements in a circuit that carry the same current are said to be connected in **series.**



one node is shared by two elements only → the two elements will be connected in series :)





KCL @ node 1

$+i_{s1} - i_{R1} = 0$ ⇒ $i_{s1} = i_{R1}$

$\boxed{i_{s1} = i_{R1} = i_{s2} = i_{R2}}$

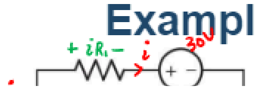
KCL @ node 2

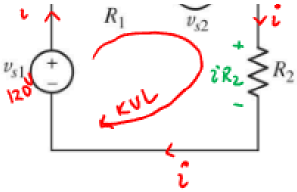
$+i_{R1} - i_{s2} = 0$ ⇒ $i_{R1} = i_{s2}$

KCL @ node 3 ⇒ $+i_{s2} - i_{R2} = 0$ ⇒ $i_{s2} = i_{R2}$

KCL @ node 4 ⇒ $+i_{R2} - i_{s1} = 0$ ⇒ $i_{R2} = i_{s1}$

Example - Series Connections





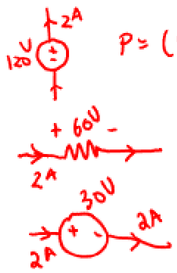
PRACTICE

3.5 In the circuit of Fig. 3.12b, $v_{s1} = 120$ V, $v_{s2} = 30$ V, $R_1 = 30\ \Omega$, and $R_2 = 15\ \Omega$. Compute the power absorbed by each element. absorbed/supplied

@ KVL

$+120^V - (i\,30) - 30^V - (i \times 15) = 0$

$90 - (i \times 45) = 0 \Rightarrow i = \frac{90}{45} = 2A$



$P = (120 \times 2) = -240^W\ (S)$

$\Rightarrow p = 60 \times 2 = +120^W\ (A)$

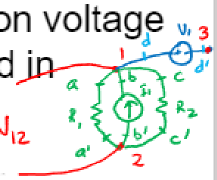
$\Rightarrow p = (30 \times 2) = 60^W\ (A)$

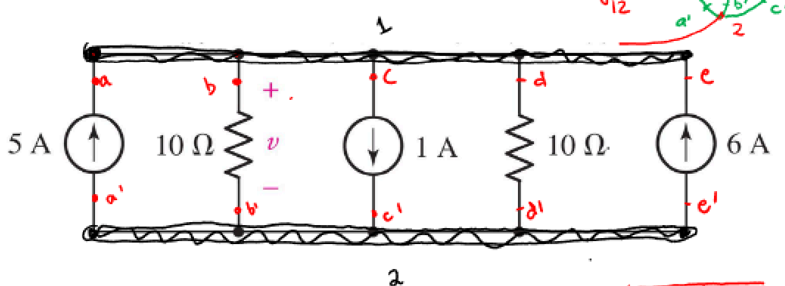
$p = 60^W\ (A)$

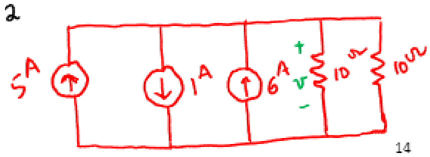
$\Sigma P = -240 + 120 + 60 + 60 = 0$

Circuit Topologies - Parallel Connections

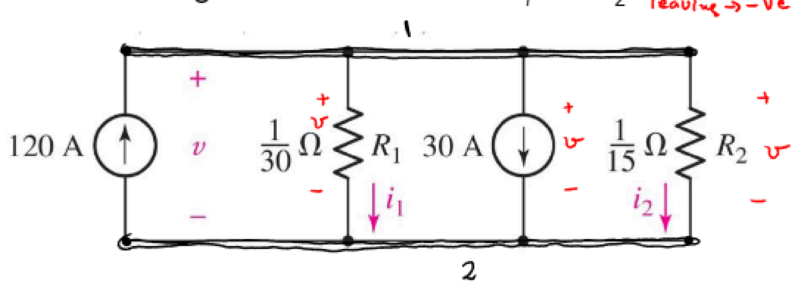
Elements in a circuit having a common voltage across them are said to be connected in **parallel.**







Find the voltage v and the currents i_1 and i_2. entering → +ve leaving → −ve



KCL @ node 1: $+120^A - i_1 - 30^A - i_2 = 0$

$120 - \left(\frac{v}{R_1}\right) - 30 - \left(\frac{v}{R_2}\right) = 0$

$120 - (30v) - 30 - (15v) = 0$

$45v = 90$

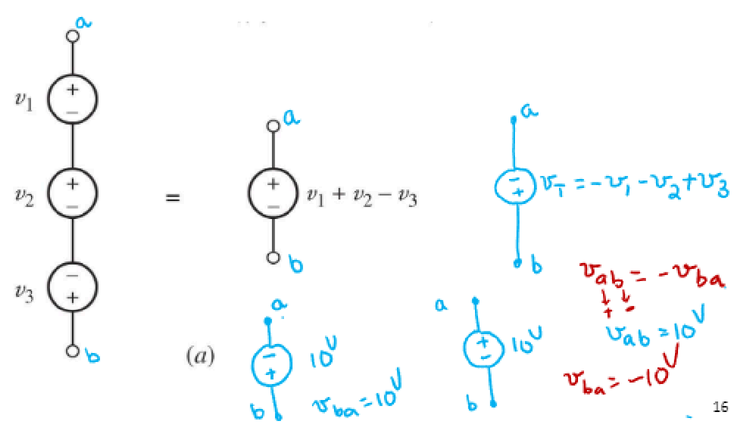
$v = 2^V$

$i_1 = \frac{2}{(1/30)} = 60A$

$i_2 = 15 \times v = 30^A$

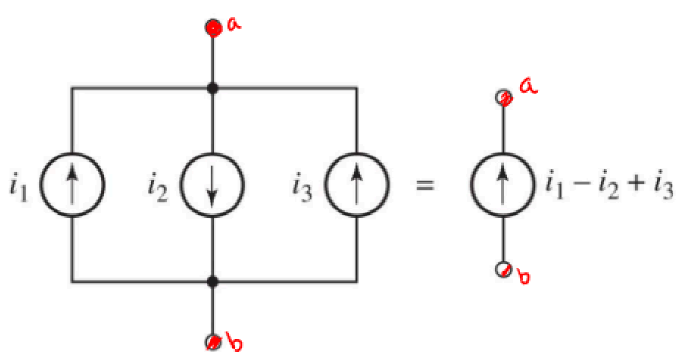
Series and Parallel Sources

Voltage sources connected in series can be combined into an equivalent voltage source:

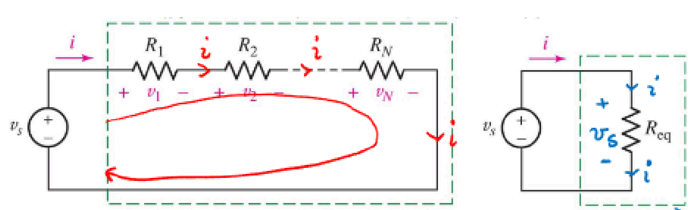


Series and Parallel Sources

Current sources connected in parallel can be combined into an equivalent current source:



Resistors in Series



@ KVL $\Rightarrow + v_s - v_1 - v_2 - \cdots - v_N = 0$

$v_1 = iR_1 \quad v_2 = iR_2 \quad v_N = iR_N$

Ohm's law

$\boxed{v_s = i R_{eq}}$

$v_s = iR_1 + iR_2 + \cdots + iR_N$

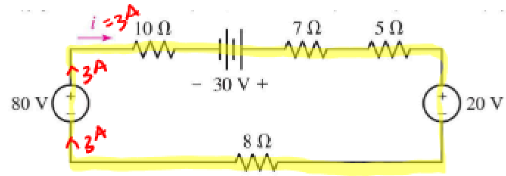
$\boxed{v_s = i(R_1 + R_2 + \cdots + R_N)}$

$R_{eq} = R_1 + R_2 + \cdots + R_N$

$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

Example: Circuit Simplifying

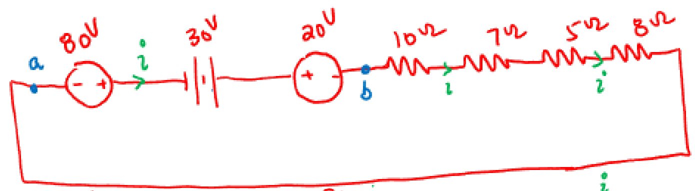
Find *i* and the power supplied by the 80 V source.

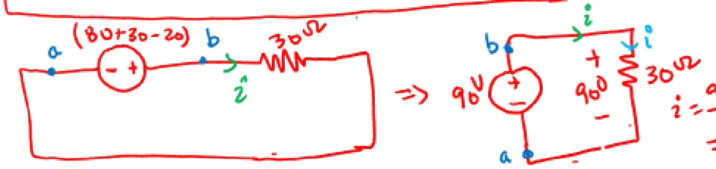


$P_{80V} = VI = 80^V \times 3^A$

$= -240^W$

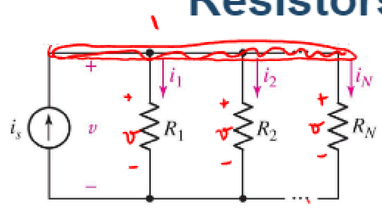
(supplied)

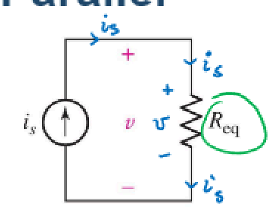




$i = \frac{90}{30}$
$= 3^A$

Resistors in Parallel





KCL @ node 1

$+i_s - i_1 - i_2 - \cdots - i_N = 0$

$i_s = \frac{v}{R_1} + \frac{v}{R_2} + \cdots + \frac{v}{R_N}$

$i_s = v\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)$

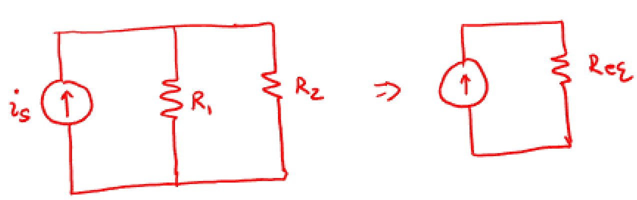
$i_s = \frac{v}{R_{eq}} = v\left(\frac{1}{R_{eq}}\right)$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$

$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

Special case - Two Resistors in Parallel

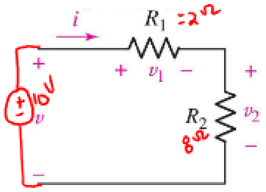


$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{R_2 + R_1}{R_1 R_2}} = \frac{R_1 R_2}{R_1 + R_2}$

$$\cancel{R_{eq} = \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3}}$$

Voltage Division

Resistors in series "share" the applied voltage.



$$i = \frac{v}{R_1 + R_2} = \frac{v}{R_T}$$

$R_T = R_1 + R_2$

$v = v_1 + v_2 \leftarrow$ KVL

$v_1 = iR_1 \qquad v_2 = iR_2$

$$v_1 = \left(\frac{v}{R_T}\right)R_1 \qquad v_2 = \left(\frac{v}{R_T}\right)R_2$$

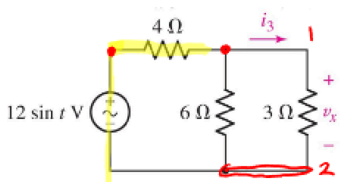
$$v_1 = \frac{R_1}{R_T}v \qquad v_2 = \frac{R_2}{R_T}(v)$$

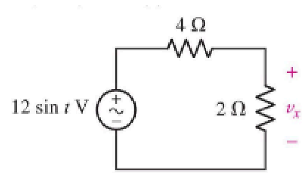
$$v_N = \frac{R_1}{(R_1 + R_2 + \cdots + R_N)}v$$

$$v_1 = \frac{R_1}{R_1 + R_2}v \qquad v_2 = \frac{R_2}{R_1 + R_2}v$$

Example: Voltage Division and Circuit Simplification

Find v_x





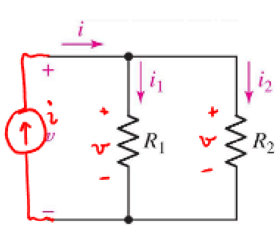
$$6 \,//\, 3 \Rightarrow \frac{6 \times 3}{6+3} = \frac{1}{\frac{1}{6} + \frac{1}{3}} = 2\Omega$$

$$v_x = \frac{2}{(2+4)} 12 \sin t = \frac{1}{3}(12 \sin t)$$

$$v_x = 4 \sin t \text{ V}$$

Current Division

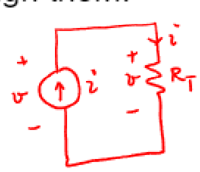
Resistors in parallel "share" current through them.



$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$i_1 = \frac{v}{R_1} = \frac{i R_T}{R_1}$$

$$i_1 = \frac{R_T}{R_1}(i)$$



$v = iR_T$

$$\boxed{i_N = \frac{R_T}{R_N}(i)}$$

$$\boxed{i_1 = i\frac{R_2}{R_1+R_2}}$$

$$\boxed{i_2 = i\frac{R_1}{R_1+R_2}}$$

special case of 2 resistors ONLY

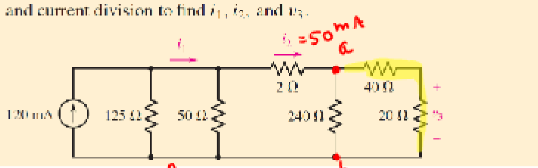
$$i_1 = \frac{\left(\frac{\cancel{R_1}R_2}{R_1+R_2}\right)}{\cancel{R_1}}(i) = \frac{R_2}{(R_1+R_2)}(i)$$

$$i_2 = \frac{R_1}{(R_1+R_2)}(i)$$

Example: Circuit Simplification and Current Division

PRACTICE

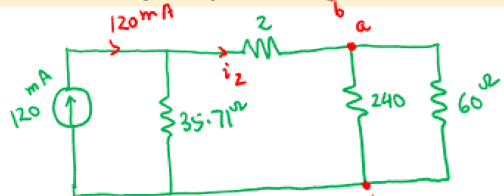
3.16 In the circuit of Fig. 3.39, use resistance combination methods and current division to find i_1, i_2, and v_3.



$$125 // 50 = \frac{125 \times 50}{175}$$

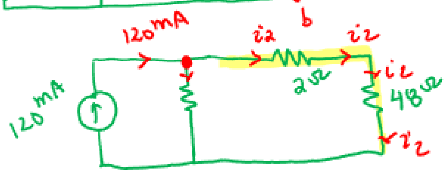
$$240 // 60 = \frac{240 \times 60}{300}$$

$$= 48\,\Omega$$



$$i_2 = \frac{35.71}{(35.71+50)}(120)$$

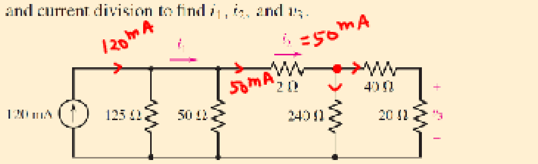
$$i_2 = 50\text{ mA}$$

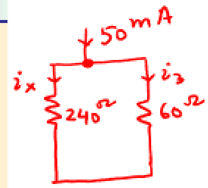


Example: Circuit Simplification and Current Division

PRACTICE

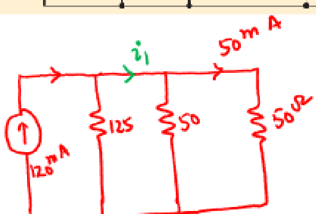
3.16 In the circuit of Fig. 3.39, use resistance combination methods and current division to find i_1, i_2, and v_3.

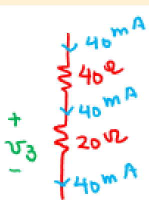




$$i_3 = \frac{240}{(240+60)}(50)\text{ mA}$$

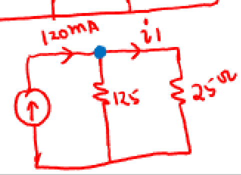
$$i_3 = 40\text{ mA}$$





$$v_3 = (20\,\Omega \times 40\text{ mA})$$

$$v_3 = 0.8\text{ V} = 800\text{ mV}$$



$$i_1 = \frac{125}{(125+25)} \times (120\text{ mA}) = 100\text{ mA}$$