Topic 1

January 30, 2023 3:25 PM

TOPIC 1 – FORCES & VECTORS

ENGI 1510 - ENGINEERING DESIGN

WINTER 2023

1
DEFINING MECHANICS

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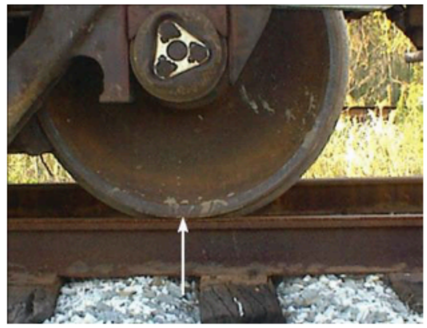
01_COC01

Large cranes such as this one are required to lift extremely large loads. Their design is based on the basic principles of statics and dynamics, which form the subject matter of engineering mechanics.

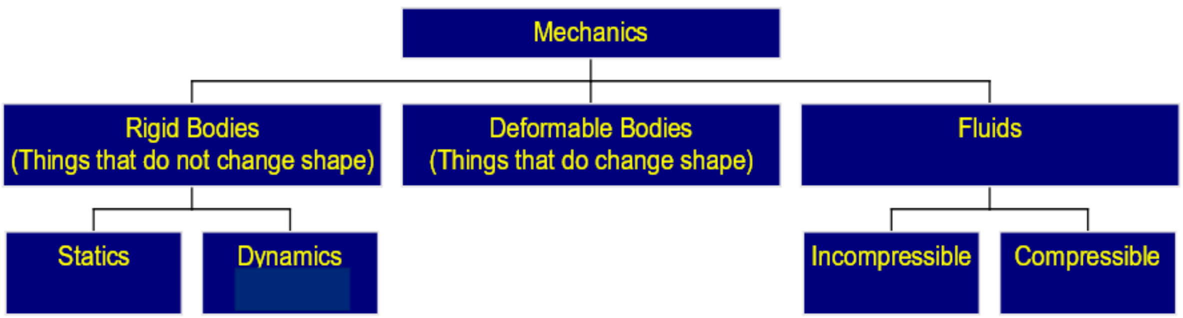
WHAT IS MECHANICS?

Study of what happens to a "thing" (the technical name is "BODY") when FORCES are applied to it.

Either the body or forces can be large or small.



CLASSIFICATION OF MECHANICS



Statics

Statics *is concerned with forces associated* with the equilibrium of bodies that are either at rest or move with a constant velocity.

The forces on the person are balanced.

the person.

Dynamics

Dynamics *is concerned with forces associated the accelerated or decelerated motion of bodies.*

Speeding up Train

Quiz

1. The subject of mechanics deals with what happens to a body when ______ is/are applied to it.

a. a magnetic field b. heat c. forces d. neutrons e. lasers

Answer:

2. In statics, the equilibrium condition of a body under the effect external forces implies that the body is:

a. Moving b. accelerating c. stationary d. Moving at constant velocity e. slowing down

Answer:

3. The dynamics of a body under the effect external forces suggests that the body is:

a. Moving b. accelerating .c stationary d. Moving at constant velocity e. slowing down

b. Answer:

4. Can statics and dynamics be used in the same time to study body motion under the effect external forces :

a. True b. Fulse

GENERAL MATH, GEOMETRY AND TRIGONOMETRY

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DEFINITIONS

- **Force** is an action that tends to change or change the state of motion of the body upon which it acts.

- There are many type of forces: push, pull, gravity, friction, magnetic, drag, lift, etc.

- Forces can be external or internal.
- In statics, we study external forces only.

REVIEW

TABLE 1–1 Systems of Units

Name	Length	Time	Mass	Force
International System of Units SI	meter m	second s	kilogram kg	$\boxed{\text{newton*}}$ N $\left(\frac{kg \cdot m}{s^2}\right)$
U.S. Customary FPS	foot ft	second s	$\boxed{\text{slug*}}$ $\left(\frac{lb \cdot s^2}{ft}\right)$	pound lb

*Derived unit.

➢ Other than newton and slug, the rest are called base or basic units.

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REVIEW

Newton's three laws of motion.

- Engineering mechanics is formulated on the basis of Newton's laws.
- Newton 1st law: a body stays in static equilibrium if the external forces acting on it are balance.

- Most statics calculations are based on it.

- Newton 2nd law: a body acted upon by an unbalance force experiences an acceleration or deceleration.

- Newton 3rd law: there are always mutual action and reaction forces between bodies. These action and reaction forces are equal in magnitude, opposite in direction and collinear.

REVIEW

- A **particle** has a mass but a size that can be neglected.
- A **rigid body** does not deform under load. Its mass and size are considered.
- **Concentrated forces** are assumed to act at a point on a body.

ALGEBRAIC EQUATIONS

SOLVING FOR A SINGLE UNKNOWN

Example: Solve for x in the equation below

$$3.45 + \frac{3x}{5} = \frac{7x}{4}$$

Solution:

Multiply each side by 20. Why 20?

Because 20 is the common denominator of 3x/5 and 7x/4.

$(20)(3.45) + 20\left(\frac{3}{5}x\right) = (20)\left(\frac{7}{4}x\right)$

This yields to

69 + 12x = 35 x

EXAMPLE 1: (cont.)

Solve for x in the equation below

$$3.45 + \frac{3x}{5} = \frac{7x}{4}$$

Solution: (cont.)

Subtract 12x from each side

69 + 12 x -12 x = 35 x – 12 x

This yields to

69 = 23 x

Divide by 23

x = 3 (RESULT)

Check for answer x =3

Substitute in equation x =3

3.45 + (3)(3)/5 = (7)(3)/4

3.45 + 1.8 = 5.25

RHS = 5.25
LHS = 5.25

correct

Practice

Exercise 1: Solve for x in the equation below:

$$4 + \frac{3(6 + x)}{2} = 16$$

Answer: $\underline{x = 2}$

Practice

Exercise 2: Solve for x in the equation below:

$$\frac{2x+1}{8} = \frac{7}{12}$$

$\frac{2x+1}{8} = \frac{7}{12}$

$\frac{2x}{8} = \frac{7}{12} - 1$

$\frac{2x}{2} = \frac{(\frac{7}{12} - 1) \cdot 8}{2}$

$x =$

Answer: x = 1.83

Practice

Exercise 3: Solve for x in the equation below:

$$\frac{1}{8}(2x + 1) = \frac{7}{5}$$

Answer: x = 5.10

SIMULTANEOUS EQUATIONS

SOLVING FOR MORE THAN ONE UNKOWN

Example: Solve for x and y in the equations below

$3x + 4y = 8$ .... (1)

$6x + 2y = 10$ .... (2)

Solution: (Method A)

Multiply Equation (1) by -2

$$-6x - 8y = -16$$

Add Equation (2)

$$6x + 2y = 10$$

$$0 - 6y = -6$$

Divide both sides of the equation by -6

$$y = 1$$

EXAMPLE (cont.)

Substitute y=1 in either equation 1 or 2. Using equation 1 in this case

$$3x + (4)(1) = 8$$

$$3x = 8 - 4$$

$$x = 1.33$$

EXAMPLE

Exercise 1: Solve for x and y in the equations below

$2x - y = 14$ .... (1)

$3x + 2y = 70$ .... (2)

Answer: x = 14.0 , y = 14.0

Solution: (Method B)

EXAMPLE

Exercise 2: Solve for x and y in the equations below

$y = 9x + 15$ ……………… (1)

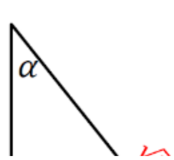
$y = 2x + 1$ ……………… (2)

Answer: x = - 2.0 , y = -3.0

TRIGONOMETRY

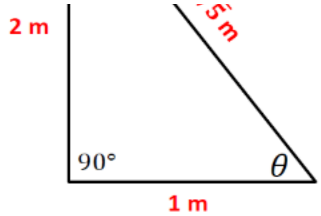
BASIC TRIGONOMETRY FUNCTIONS

Basic Trigonometry functions: (Apply only to right- angle triangles)



$$\sin\theta = \frac{side\ opposite}{hypotenuse} = \frac{O}{H} = \frac{2\,m}{\sqrt{5}\,m}$$

$$\cos\theta = \frac{side\ adjacent}{hypotenuse} = \frac{A}{H} = \frac{1\,m}{\sqrt{5}\,m}$$



$$\tan\theta = \frac{side\ oposite}{side\ adjacent} = \frac{O}{A} = \frac{2\ m}{1\ m}$$

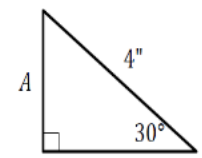
$$\sin\alpha = \frac{O}{H} = \frac{1\ m}{\sqrt{5}\ m}$$

$$\cos\alpha = \frac{A}{H} = \frac{2\ m}{\sqrt{5}\ m}$$

$$\tan\alpha = \frac{O}{A} = \frac{1\ m}{2\ m}$$

Exercise 1

Determine the value of length (*A*)



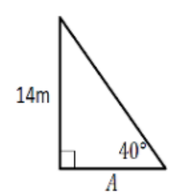
Ans.

A= 2"

EXAMPLE # 4

Exercise 2

Determine the value of length (*A*)



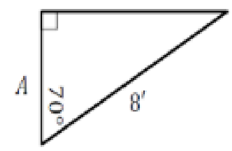
Ans.

A= 16.7 m

EXAMPLE # 4

Exercise 3

Determine the value of length (A)



Ans.

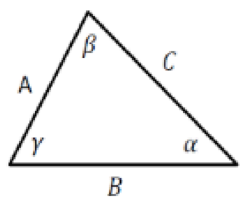
$A = 2.74$ ft

SINE AND COSINE LAWS

SINE AND COSINE LAWS

- Used for all triangles, whether right-angle or not

Sine Law:

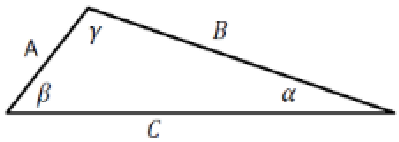


$$\frac{A}{Sin\ \alpha} = \frac{B}{Sin\ \beta} = \frac{C}{Sin\ \gamma}$$

- 2 sides and one opposite angle

- 2 angles and one opposite side

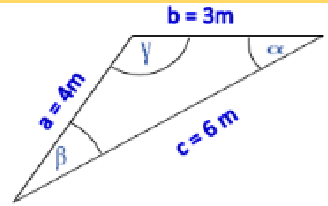
Cosine Law:



$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

- 2 sides and one included angle known
- 3 sides

Example: Determine the interior angles of the triangle shown below.



Solution:

Using the cosine law: $c^2 = a^2 + b^2 - 2ab\cos\gamma$

$$(6m)^2 = (4m)^2 + (3m)^2 - 2(4m)(3m)\cos\gamma$$

$$(6m)^2 = (4m)^2 + (3m)^2 - 2(4m)(3m)\cos\gamma$$

$$36\cancel{m^2} = 16\cancel{m^2} + 9\cancel{m^2} - 24\cancel{m^2}\cos\gamma$$

$$36 = 16 + 9 - 24\cos\gamma$$

$$36 - 16 - 9 = -24\cos\gamma$$

$$11 = -24\cos\gamma$$

$$\cos\gamma = \frac{11}{-24} = -0.4583$$

$$\gamma = \cos^{-1}(-0.4583)$$

$$\boxed{\gamma = 117.3°}$$

Or , we can do it this way:

If $\cos\gamma = +0.4583$

then $\gamma = 62.7°$

But $\cos\gamma = -0.4583$

then $\gamma = 180° - 62.7°$

$$\boxed{\gamma = 117.3°}$$

Solution: (cont.)

Using the sin law now yields

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\frac{4m}{\sin\alpha} = \frac{6m}{\sin\gamma}$$

$$\sin\alpha = \frac{4(\sin 117°)}{6}$$

$$= 0.5924$$

$$\boxed{\alpha = 36.3^0}$$

We know that:
$\beta + \alpha + \gamma = 180°$

Then:

$$\beta = 180^0 - \alpha - \gamma$$

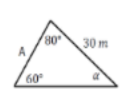
$$\beta = 180^0 - 117.3° - 36.3°$$

$$\boxed{\beta = 26.4°}$$

EXERCISE

Exercise 1

Determine the value of unknown side or angle



Ans.

A= 22.3

SPECIAL CASE OF COSINE LAW

It is a special case of cosine law when $\gamma = 90°$ (right-angle triangle)

$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

If $\gamma = 90°$ $\longrightarrow$ $Cos\,\gamma = 0$

Then,

$$C^2 = A^2 + B^2$$

This is used to find any side of a right-angle triangle if you know the other two.

INVERSE FUNCTIONS

$\sin\theta = A \longrightarrow \theta = sin^{-1}\ A$

ex: $\sin\theta = 0.5 \longrightarrow \theta = sin^{-1}\ 0.5 = 30°$

$\cos\theta = B \longrightarrow \theta = cos^{-1}\ B$

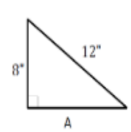
ex: $\cos\theta = 0.5 \longrightarrow \theta = cos^{-1}\ 0.5 = 60°$

$\tan\theta = C \longrightarrow \theta = tan^{-1}\ C$

ex: $\tan\theta = 1 \longrightarrow \theta = tan^{-1}\ 1 = 45°$

Exercise 2

Determine the value of unknown side or angle

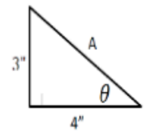


Ans.

A= 8.9"

Exercise 3

Determine the value of unknown side or angle



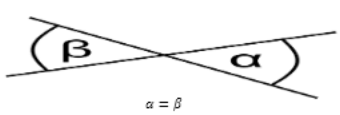
Ans.

$\boxed{A = 5''}$

GEOMETRY

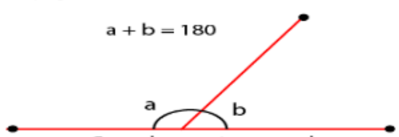
BASIC RULES

a) Opposite angles are equal when two straight lines intersect



$\alpha = \beta$

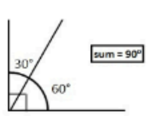
b) Supplementary angles total 180°



Supplementary angle

BASIC RULES

c) Complementary angles total 90°

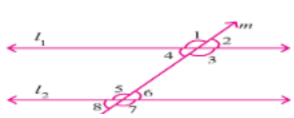


complementary angles

d) Straight line intersecting two parallel lines produces the following equal angles

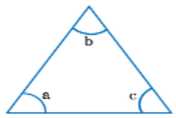
$\angle 2 = \angle 6 = \angle 4 = \angle 8$

$\angle 3 = \angle 7 = \angle 1 = \angle 5$



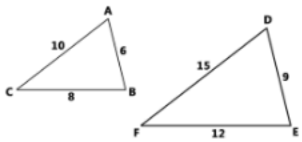
BASIC RULES

e) The sum of the interior angles of any triangle equals 180°



$a + b + c = 180°$

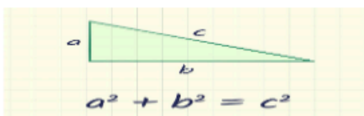
f) Triangles with same angles have proportional side lengths



$$\frac{DF}{DE} = \frac{AC}{AB}$$

BASIC RULES

g) Pythagoras Theorem



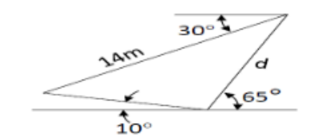
h) Circle Equations

Circumference= $2\pi r = \pi D$

Area = $\frac{\pi D^2}{4} = \Pi r^2$

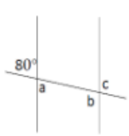
EXERCISES

1. Determine distance d in the figure below

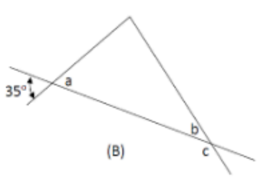


EXERCISES

2. Determine angles a, b and c in the figures below



(A)



(B)

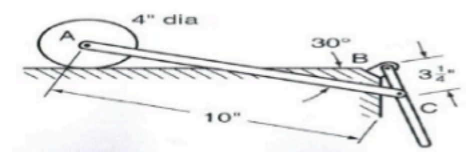
EXERCISES

EXERCISES

3. Determine x and y in the figure below

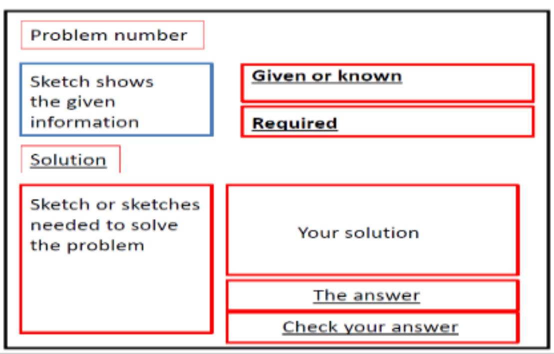
EXERCISES

4. The roller and lever mechanism is in the position shown below.
Determine the horizontal distance between A and B.



PROBLEM SOLVING METHOD

The following diagram shows a typical problem solving methodology



VECTORS

SCALAR AND VECTOR QUANTITIES

- Scalar quantity is a quantity that can be completely described by its magnitude and associated unit.

Temperature *ex: $T=30°C$* *Length* *ex: $L= 2m$* *Area* *ex: $A=4m^2$*

Other examples: time, speed, density, etc.

- Vector quantity is a quantity which require a magnitude, associated unit and direction to be completely defined.

Velocity 90km/h east *Moment M=50N-m clockwise* *Displacement D= 2km north*

Other examples: Acceleration, Force, position vector, etc.

SCALAR Vs. VECTOR QUANTITIES

	Scalars	*Vectors*
Examples:	Mass, Volume	Force, Velocity
Characteristics:	It has a magnitude only	It has a magnitude and direction
Operation rules:	Simple arithmetic's	Graphical & Analytical
Special Notation:	None	Arrow

Quiz

- Which of the following is not a scalar quantity.
  a. time b. mass c. volume d. density e. acceleration
- Which of the following is not a vector quantity.
  a. area b. velocity c. acceleration d. force e. moment
- Which of the following statement is true?
  a. A scalar is any physical quantity that can be completely specified by its magnitude
  b. A vector is any positive or negative physical quantity that can be completely specified by its magnitude
  c. A scalar is any physical quantity that requires both a magnitude and a direction for its complete description
  d. A scalar is any physical quantity that can be completely specified by its direction

Quiz

- Scaler quantities can only have one direction.
  a. True b. False
- Vector quantities can only have positive direction.
  a. True b. False
- Friction is a scaler quantity.
  a. True b. False
- Vector quantity can be defined by its direction only.
  a. True b. False
- Scaler quantities can only have positive value.
  a. True b. False
- Speed is a vector quantity.
  a. True b. False
- Temperature is not a vector quantity.
  a. True b. False
- Pressure is a scaler quantity.
  a. True b. False

VECTOR PRESENTATION

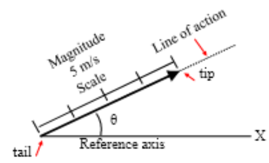
- Graphical representation: Arrow with a tip and tail.
- Magnitude: Length of the arrow drawn to scale.
- Direction: An angle θ between the vector "line of action" and a reference axis.
- Sense: Tip of the arrow indicate the sense of direction.

Note: Precise vector length and angle measurement are only required in graphical representation.

- In text representation, it is often convenient to denote a vector quantity by simply:

1. drawing an arrow above it, $\vec{V}$, or,
2. use bold letter **V**

Velocity Vector



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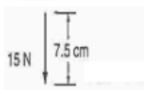
DRAWING VECTORS TO SCALE

Scale: 1 cm = 2 N

$15\ N = (15\ \cancel{N})\ [\frac{1\ cm}{2\ \cancel{N}}]$

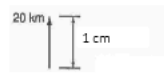
$15\ N = [\frac{(15)(1)}{2}]\ cm = 7.5\ cm$

Force Vector



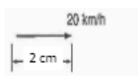
Scale: 10 cm = 200 km

_______ Vector!



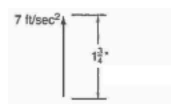
Scale: 10 cm = 100 km/h

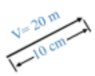
_______Vector!



Scale: 1in. = 4 ft/s²

_______Vector!





Scale: x cm = y m

$20\ m\ [\frac{x\ cm}{y\ m}] = 10\ cm$

$[\frac{x\ cm}{y\ m}] = \frac{10\ cm}{20\ m} = \frac{1\ cm}{2\ m}$

Scale: 1 cm = 2 m

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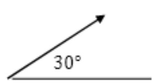
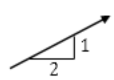
VECTOR DIRECTION

- Horizontal 
- Vertical 
- At an angle or slope



Angle

OR



rise and run

55

Quiz

The direction of a vector is always determined by an angle θ! a. True b. False

The direction of a vector is determined by the angle between its line of action and a defined reference axis. a. True b. False

The sense of a vector is not as important as its magnitude! a. True b. False

What is the length in centimeter of a 20 N force vector drawn to scale: 2cm = 5N

Answer: 8 cm

What is the length in inch of a 6 m/s velocity vector drawn to scale: 1 in = 1.5 m/s

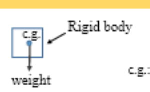
Answer: 4 inch

If a 3 m/s^2 acceleration vector drawn to a length of 9 cm, the scale is _______?

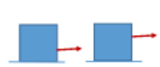
Answer: 3 cm = 1 m/s^2

VECTOR TYPES

- **Fixed or bound** vectors have well defined points of application that cannot be changed without affecting an analysis.



- **Free** vectors have certain magnitude, direction and sense, may be freely moved in space without changing their effect on an analysis.



Pulling Force
In certain condition
(e.g.no chance of tipping)

- **Sliding** vectors may be applied anywhere along their line of action without affecting an analysis.

Friction force

- **Equal vectors** have the same magnitude

- **Equal** vectors have the same magnitude, direction and sense.

- **Negative** vector of a given vector has the same magnitude and direction but opposite sense.

Quiz

Free vectors can only be freely moved along their line of action!

a. True b. False

Sliding vectors can be freely moved in space without changing their effect on a system.

a. True b. False

Vectors that have identical magnitude, direction and sense are equal.

a. True b. False

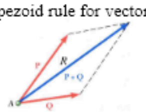
A negative vector of a given vector has the same direction but a negative magnitude!

a. True b. False

VECTOR OPERATIONS

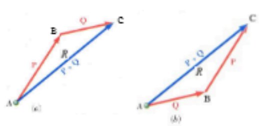
Vector Addition: $\vec{R} = \vec{P} + \vec{Q}$

- Trapezoid rule for vector addition

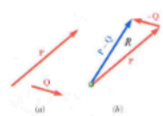


- Triangle rule for vector addition
- Vector addition is commutative:

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$



Vector Subtraction: $\vec{R} = \vec{P} - \vec{Q}$



- We use the laws of cosines & sines to do vectors addition and subtraction
- Law of cosines

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$

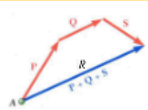
- Law of sines

$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{P}$$

VECTOR OPERATIONS

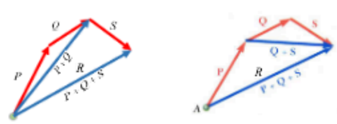
Vector Addition: $\vec{R} = \vec{P} + \vec{Q} + \vec{S}$

- The polygon rule for the addition of three or more vectors.



- Addition of three or more vectors through repeated application of the triangle rule.
- Vector addition is associative:

$$\vec{P} + \vec{Q} + \vec{S} = (\vec{P} + \vec{Q}) + \vec{S} = \vec{P} + (\vec{Q} + \vec{S})$$



Quiz

The cosine law can only be used for vector addition whilst the sine law are used for both vector addition and subtraction. a. True b. False

When using the trapezoidal rule, vectors are added tip to tail! a. True b. False

When using the triangle rule, vectors are added tip to tail! a. True b. False

Vector subtraction is commutative. a. True b. False

Vector addition is associative and also commutative. a. True b. False

The polygon rule is a repeated application of triangle rule.   a. True   b. False

FORCE VECTORS

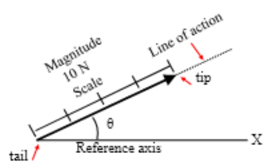
Force

- Force is an action that tends to change or change the state of motion of the body upon which it acts.
- A force is a vector. It is completely described by its characteristics:

1. Magnitude
2. Direction
3. Sense
4. Point of application

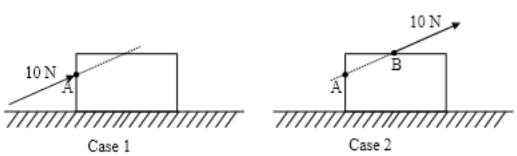
- A body is any object or any part of an object, which may be considered separately.

Force Vector



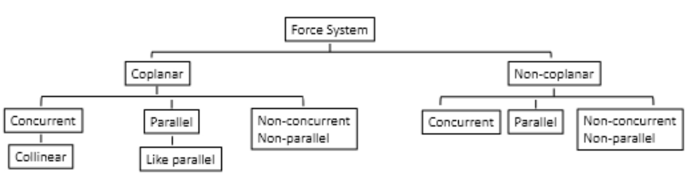
FORCE TRANSMISSIBILITY

A force acting on an object can be applied anywhere along the line of action of the force, this called **principle of transmissibility.**



In both cases (1 and 2), the force (10N) has the same effect on the body

TYPES OF FORCE SYSTEMS



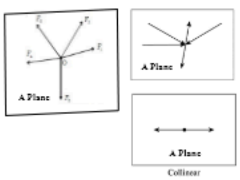
DEFINITIONS OF FORCE SYSTEMS

- **Coplanar Force system:** all forces of the system lie in one plane. Conversely, when these forces do not lie in one plane, the system is **noncoplanar**.
- **Concurrent Force system:** the line of action of all forces intersect at a common point. Conversely, when the line of action of all forces do not intersect at a common point, the system is **nonconcurrent**.
- **Parallel Force system:** all forces in the system are parallel. When all forces in a parallel system acts along a single line of action, the system is called **collinear**. This a special case of coplanar, concurrent force system.

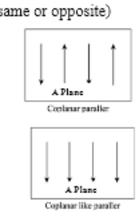
TYPES OF FORCE SYSTEMS

Force systems are classified into six groups:

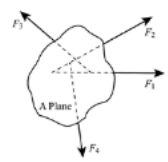
1. Coplanar, Concurrent
(line of actions of forces intersect at one point)



2. Coplanar, Parallel
(line of actions of forces are parallel but force sense could be the same or opposite)



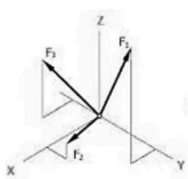
3. Coplanar, Non-Concurrent
(line of actions of forces do not intersect at one single point)



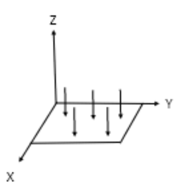
TYPES OF FORCE SYSTEMS

Force systems are classified into six groups:

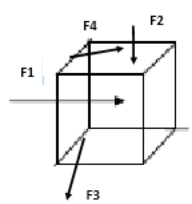
4. Non-Coplanar, Concurrent
(line of action of forces do not lie in the same plane but they intersect at one point)



5. Non-Coplanar, Parallel
(line of action of forces do not lie in one plane but they are parallel)

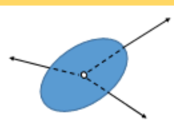


6. Non-Coplanar, Non-Concurrent
(line of action of forces do not lie in one plane and they do not intersect at one single point)

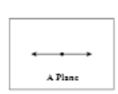


Quiz

The figure indicate ------------ force system.
a. Coplanar
b. Non-Concurrent
c. Concurrent, Coplanar
d. Concurrent



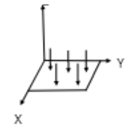
The figure indicate ------------ force system.
a. Concurrent, Coplanar
b. Coplanar collinear
b. Non-Concurrent
d. Non-Coplanar



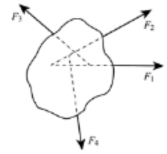
Quiz

The figure indicate ------------ force system.

a. Coplanar, Parallel
b. Non-Concurrent, Non -Coplanar
c. Non-Coplanar, Parallel
d. Concurrent



The figure indicate ------------ force system.
a. Coplanar, Parallel
b. Non-Concurrent, Non -Coplanar
c. Non-Coplanar, Concurrent
d. Concurrent



Quiz

- Forces are called concurrent when their lines of action meet in
  a. one point b. two points c. plane d. perpendicular planes
  e. different planes

- Forces are called coplanar when all of them acting on body lie in
  a. one point b. one plane c. different planes d. perpendicular planes
  e. different points

- A force acting on a body may
  a. introduce internal stresses b. balance the other forces acting on it c. retard its motion
  d. change its motion e. all choices.

- All vectors quantities obey:
  a. Parallelogram law of addition
  b. Parallelogram law of multiplication
  c. Parallelogram law of addition of square root of their magnitudes
  d. Parallelogram law of addition of square of their magnitudes

Quiz

- According to principle of transmissibility of forces, the effect of a force on a body is
  a. maximum when it acts at the center of gravity of a body
  b. different at different points in its line of action
  c. the same at every point in its line of action
  d. minimum when it acts at the C.G. of the body

For two vectors defined by an arrow with a head and a tail. The length of each vector and the angle between them represents:
a. Their magnitude's square and direction of the line of action respectively
b. Their magnitude and direction of the line of action respectively
c. Magnitude's square root and direction of the line of action respectively
d. Magnitude's square and the ratio of their lengths respectively

A force is completely defined when we specify
a. magnitude b. direction c. sense d. all choices

Quiz

- Forces are called concurrent when their lines of action meet in a plane.
  a. True b. False
- Forces are called concurrent when their lines of action meet in perpendicular planes.
  a. True b. False
- Forces are called concurrent when their lines of action meet in one point.
  a. True b. False
- Forces are called coplanar when their lines of action meet in one point.
  a. True b. False
- Forces are called coplanar when their lines of action lie in a plane.
  a. True b. False

RESULTANT OF SYSTEM OF FORCES
Coplanar – Concurrent

RESULTANT OF SYSTEM OF FORCES
Coplanar – Concurrent

- The Resultant Force is the vectorial sum of two or more forces and has the same effect of all forces combined.
- There are two ways to add vectors:
  - Graphical methods: not our focus
  - Analytical or mathematical methods

GRAPHICAL METHODS

Disadvantage:
- Needs graphic equipment
- Not accurate

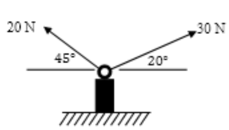
There are three methods for finding the resultant of forces graphically.

- Triangle method
- Parallelogram method
- Vector polygon method

GRAPHICAL METHODS

Exercise:

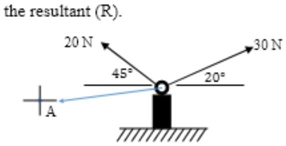
Determine the resultant of the forces acting on the eye bolt shown in the figure using the:
- Triangle graphical method
- Parallelogram graphical method



TRIANGLE METHOD - GRAPHICAL

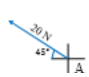
- We are going to construct a triangle of forces to determine the resultant (R).

Given: Forces 20N and 30 N with direction shown.
Determine: the resultant R.



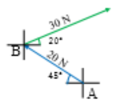
Step 1: locate a point (e.g. A) on the drawing sheet to represent the center of the eye bolt shown. On this point Draw a small x,y coordinate system to use it as reference.

Step 2: From point A, Draw the 20 N force vector tail to tip to a proper scale at 45° angle as



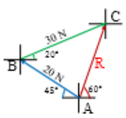
proper scale at 45° angle as shown.

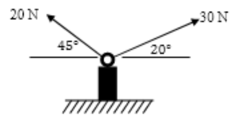
Step 3: On the 20 N force vector tip point (B), draw the small x,y coordinate system. Draw the 30 N force vector tail to tip to the same scale at 20° angle as shown.



TRIANGLE METHOD - GRAPHICAL

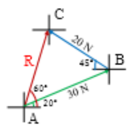
Step 4: On the 30 N force vector tip point (C), draw the small x,y coordinate system. Draw the Resultant force vector from point A (tail) to point (C) tip.





Step 5: Measure the resultant vector length and the direction angle as shown. Use the resultant force R length along with the same scale used before to find the resultant force value.
R = 28.20 N at angle of 60° Don't forget the direction

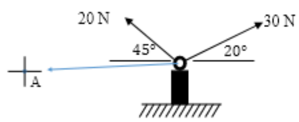
Note: You can the repeat the same procedure starting by drawing the 30 N force. You should get the same results.



PARALLELOGRAM METHOD - GRAPHICAL

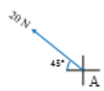
- We are going to construct a parallelogram of forces to determine the resultant (R).

Given: Forces 20N and 30 N with direction shown.
Determine: the resultant R.

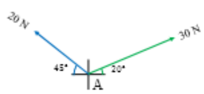


Step 1: locate a point (e.g. A) on the drawing sheet to represent the center of the eye bolt shown. On this point Draw a small x,y coordinate system to use it as reference.

Step 2: From point A, Draw the 20 N force vector tail to tip to a proper scale at 45° angle as shown.

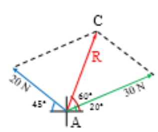


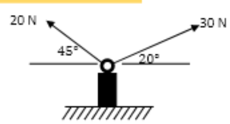
Step 3: From point A, draw the 30 N force vector tail to tip using the same scale at 20° angle as shown.



PARALLELOGRAM METHOD - GRAPHICAL

Step 4: From the tip of the 20 N force, draw a line that is parallel to the line of action of the 30 n force. Do similar procedure with the 30 N force. The two lines intersect at point (C). Draw the Resultant force vector from point A (tail) to point (C) tip.



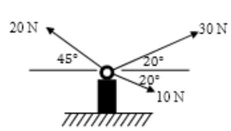


Step 5: Measure the resultant vector length and the direction angle as shown. Use the resultant force R length along with the same scale used before to find the resultant force value.
R = 28.30 N at angle of 60° Another way of writing the angle

POLYGON METHOD - GRAPHICAL

Determine the resultant of the forces acting on the eye bolt shown in the figure using the:
- Polygon graphical method

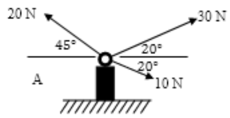
- The polygon method is used to determine the resultant of more than two forces by drawing vectors tip to tail.



POLYGON METHOD - GRAPHICAL

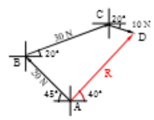
- We are going to construct a polygon of forces to determine the resultant (R).

Given: Forces 20N,30 N and 10 N with directions shown.
Determine: Resultant R.



Solution:
- Follow the triangle graphical method steps and procedure.
- Add one more one step to draw the 10 N force.
- Measure the resultant force and direction.

R = 31.5 N at angle of 40°



Quiz

- If two equal vector forces are mutually perpendicular then the resultant force is acting at which angle as compared to one of the vector?
  a. 45 degree b. 90 degree c. 180 degree d. 0 degree

- The resultant of three equal vectors having mutual angles being 120 degrees and being originated from a single point is zero.
  a. True b. False

RESULTANT OF SYSTEM OF FORCES
Coplanar – Concurrent

ANALYTICAL METHOD

ANALYTICAL METHOD

- **Triangle method:** Construct a triangle and solve mathematically (two forces).
  - Construct a triangle tip to tail (or parallelogram) and use simple trigonometry functions or Cosine & Sine law to solve the problem.
- **Polygon method** (more than two forces): Successive application of the triangle method
- **Component method:** Addition of the Components of forces (usually more than two forces). This will be detailed after being introduced to Resolving Forces into two Components.

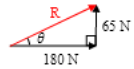
TRIANGLE METHOD - ANALYTICAL

Exercise 1: Determine the resultant of forces shown in the figure below:

Given: Forces 180N east and 65 N north
Find: Resultant (R)

Solution Steps:

1. Sketch forces 180 N then 65 N tip to tail.
2. Draw a line from the 180 N force tail to the 65 n tip. This is the resultant force.
3. Apply Pythagoras rule to determine the resultant force magnitude.



$$R^2 = (180N)^2 + (65N)^2$$
$$R^2 = 36{,}625\ N^2$$
$$R = \sqrt{36{,}625}\ N = 191.4\ N$$

4. Apply trigonometric functions to determine the direction of the resultant force.

$$\tan\theta = \frac{O}{A} = \frac{65}{180} = 0.361$$
$$\therefore \theta = \tan^{-1} 0.361 = 19.8°$$

$\boxed{\text{Ans: } R = 191.4\text{ N } \angle 19.8°}$

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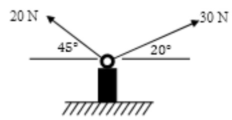
TRIANGLE METHOD - ANALYTICAL

Exercise 2: Determine the resultant of the forces acting on the eye bolt shown in the figure.
Given: Forces 20N and 30 N
Find: Resultant (R)



Solution Steps:

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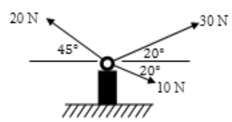
POLYGON METHOD - ANALYTICAL

Exercise 2: Determine the resultant of the forces acting on the eye bolt shown in the figure.
Given: Forces 20N, 30 N and 10 N
Find: Resultant (R)

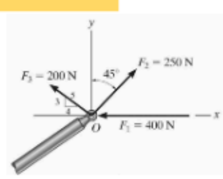


Solution Steps:

89

POLYGON METHOD - ANALYTICAL

Exercise 3: Determine the resultant of the forces acting on the end of the boom O shown in the figure.
Given: Forces 400N, 200 N and 250 N
Find: Resultant (R)



Solution Steps:

90

COMPONENT METHOD - ANALYTICAL

This method is useful to find the resultant of more than two forces analytically. To do so, follow the following **steps:**

1. Resolve forces into two perpendicular components.

2. Add the vertical and horizontal components separately:

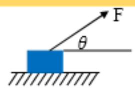
$$R_x = \sum F_x \qquad R_y = \sum F_y$$

3. Find the resultant by combining the vertical and the horizontal components (R_x and R_y) as follow:

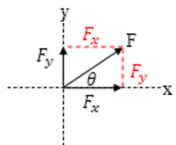
$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

FORCE RESOLUTION

Let assume a force is exerted on a body as shown in the figure:



Force F can be resolved (broken up) into two components as shown :

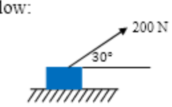


Where,

$F_x = F\cos\theta$
$F_y = F\sin\theta$ } From Simple trigonometry

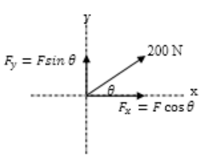
EXAMPLE

Find the vertical and horizontal components for the figure bellow:



Given: Force 200 N, $\theta = 30°$
Find: F_x and F_y

Solution:



$F_x = F\cos\theta = 200N\cos 30$

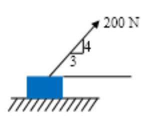
$\boxed{F_x = 173N \longrightarrow}$

$F_y = F\sin\theta = 200N\sin 30$

$\boxed{F_y = 100\ N \uparrow}$

EXAMPLE

Find the vertical and horizontal components for the figure bellow:

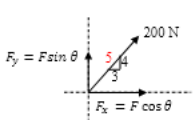


Given: Force 200 N
Find: F_x and F_y

Solution:

Find the hypotenuse for the small triangle

$$h = \sqrt{3^2 + 4^2} = 5$$

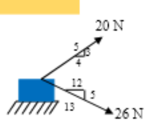


$F_x = F\cos\theta = 200N\frac{3}{5}$ $\boxed{F_x = 120N \longrightarrow}$

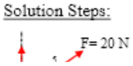
$F_y = F\sin\theta = 200N\frac{4}{5}$ $\boxed{F_y = 160\ N \uparrow}$

EXAMPLE

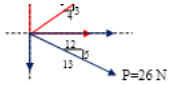
Find the resultant of forces for the figure shown using components method.



Given: Forces F (20N) and P (26 N)
Find: Resultant
Solution Steps:



1. Resolve forces into two perpendicular components:

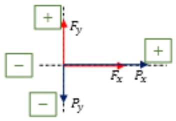


F= 20 N

$F_x = F\cos\theta = 20N(\frac{4}{5}) = +16\text{ N}$ $\quad F_y = F\sin\theta = 20N(\frac{3}{5}) = +12\text{ N}$

P = 26 N

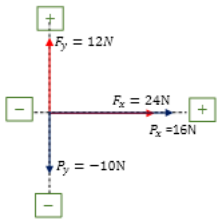
$P_x = P\cos\theta = 26N(\frac{12}{13}) = +24\text{ N}$ $\quad P_y = P\sin\theta = -26N(\frac{5}{13}) = -10\text{ N}$



2. Add the vertical and horizontal components separately:

$R_y = 12N - 10\,N = +2\,N$

$R_x = 16N + 24\,N = +40\,N$



3. Find the resultant by combining the vertical and the horizontal components (R_x and R_y) as follow:

Magnitude of (R)

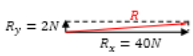
$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$R = \sqrt{(40N)^2 + (2N)^2} = 40.1\,N$$

Direction of (R)

$$\tan\theta = \frac{O}{A} = \frac{2}{40} = 0.05$$

$$\therefore \theta = \tan^{-1} 0.05 = 2.9°$$



$\boxed{Ans: 40.1\,N \nearrow 2.9°}$