HUMBER ENGINEERING

ENGI 1000 - Physics 1

WEEK 10 - MODULE 7

HUMBER

WE ARE HUMBER

Module 7
Linear Momentum & Collisions

- Linear Momentum and Applications
- Momentum in Isolated Systems
- Momentum in Non-Isolated Systems
- Elastic and Inelastic Collisions in One-Dimension
- Elastic and Inelastic Collisions in Two-Dimensions
- The Center of Mass

Introduction

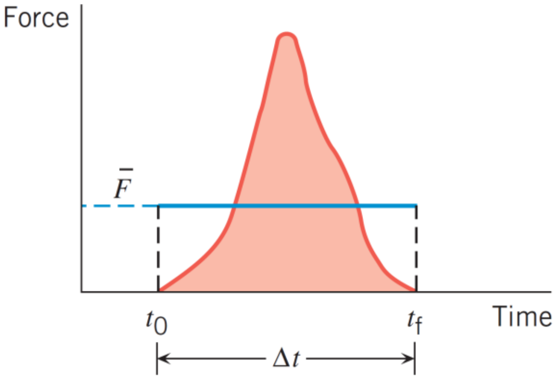
- In many situations the force acting on an object is not constant but varies with time.
- For example, when a baseball being hit, the force applied to the ball by the bat changes during the time of contact.
- Consider these examples. In each of these situations:
  - The force applied to the ball varies with time
  - The time of contact is very short , less than millisecond
  - The maximum force is very large, thousands of newtons

- To describe how a time-varying force affects the motion of an object during the collisions, we will introduce two new ideas:
  - **The impulse of a force**
  - **The linear momentum of an object**

Impulse of a Force

- In the baseball hitting example, the collision time is very short, but the force is quite large.
  - Before collision → The force magnitude is zero
  - At the strike time → The force magnitude rises to a maximum in a short time
  - After collision → The force magnitude returns to zero

- If a baseball is to be hit well, both the magnitude of the force and the time of contact are important.



- To describe the situation, we define the ***impulse of the force*** as the product of the average force and the time of contact.

Impulse of a Force

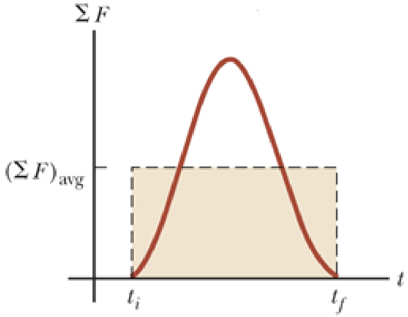
- The **impulse** $\vec{\mathrm{I}}$ of a force is defined as the product of the average net force $(\sum\vec{\mathrm{F}})_{\mathrm{avg}}$ and the time interval $\Delta t = t_f - t_i$ during the force acts:

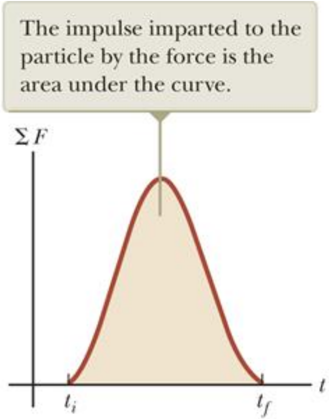
$$\boxed{\vec{\mathrm{I}} \equiv (\sum\vec{\mathrm{F}})_{avg}\,\Delta t}$$

  - Impulse is a vector quantity that points in the same direction as the net force.
  - The SI unit of impulse is newton.second $(\mathrm{N}.\,s)$.

- The magnitude of the impulse of a force can also be determined as the area under the force-time curve.

$$\boxed{\vec{\mathrm{I}} \equiv \int_{t_i}^{t_f} \sum \vec{\mathrm{F}} dt}$$





Response to Impulse of a Force

- When a ball is hit, it responds to the value of the impulse.
- The response depends on the mass and velocity of the ball.
  - More massive the ball, the less velocity it has after leaving the bat.

- Both mass and velocity play a role in how an object <u>responds to a given impulse</u>.
- Effect of each of them is included in the concept of **linear momentum**.

- The linear momentum shows that the effort required to bring a moving object to the rest, depends not only on <u>its mass</u> but also on <u>how fast it is moving</u>

Linear Momentum

- Consider an object of mass m that is moving with velocity $\vec{\mathrm{v}}$ with respect to some fixed reference frame.
- The **linear momentum $\vec{\mathrm{p}}$** of the object is defined as the product of the object's mass m and velocity $\vec{\mathrm{v}}$:

$$\boxed{\vec{\mathrm{p}} = m\vec{\mathrm{v}}}$$

  - Linear momentum is a vector quantity that points in the same direction as the velocity.
  - The SI unit of linear momentum is kilogram.meter/second $(kg.m/s)$.
- Linear momentum is also called the quantity of motion.

Linear Momentum and Impulse

- Newton's second law of motion can be used to show the relation between impulse and linear momentum. Assume that the mass of the object does not change.

$$\left(\sum \vec{F}\right)_{avg} = m\vec{a}_{avg} = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right) = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$$

$$\left(\sum \vec{F}\right)_{avg}\Delta t = \Delta\vec{p} \quad \rightarrow \quad \boxed{\vec{I} = \Delta\vec{p}}$$

- **Newton's Second Law** can also be rewritten in more general form based on momentum.

$$\vec{F}_{\mathrm{NET}} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} \quad \rightarrow \quad \boxed{\vec{F}_{\mathrm{NET}} = \frac{d\vec{p}}{dt}}$$

- The formula can be used even if mass changes with time.

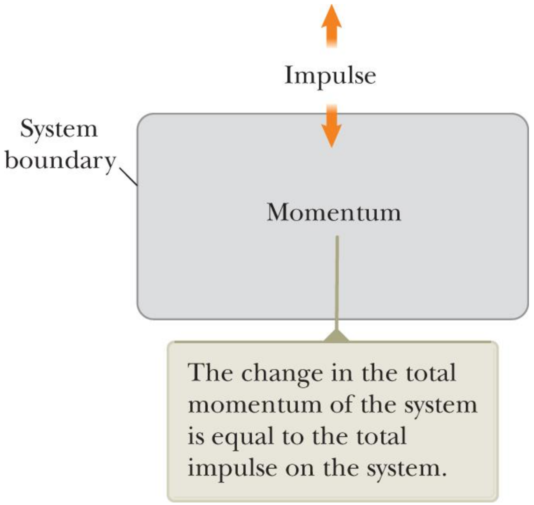
Linear Momentum and Impulse

- **Impulse-Momentum Theorem:**

The change in the momentum of a particle (or a system) is equal to the impulse of the net external force acting on the particle (or the system)

$$\Delta\vec{p} = \vec{I} \quad \rightarrow \quad \vec{p}_f - \vec{p}_i = \vec{I}$$

Final momentum — Initial momentum — Impulse of net external force



Change in momentum = Impulse

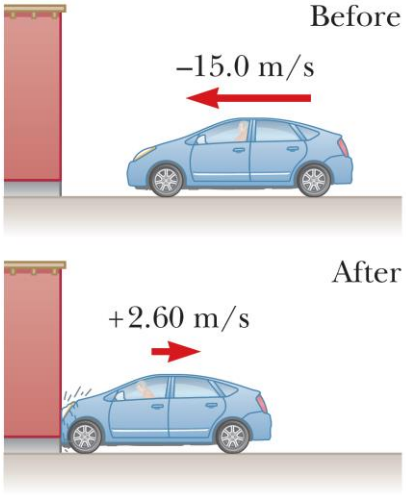
$$m\vec{v}_f - m\vec{v}_i = \left(\sum \vec{F}\right)_{avg} \Delta t$$

Linear Momentum and Impulse

Example 1 (How Good Are the Bumpers?): In a particular crash test, a car of mass 1500 kg collides with a wall. The initial velocity of the car is v_i = -15.0 m/s.

(a) If the final velocity of the car is v_f = +2.60 m/s, and the collision lasts 0.150 s, find the impulse on the car during the collision and the average net force exerted on the car.

According to the impulse-momentum theorem.



$$\vec{\mathrm{I}} = \Delta\vec{\mathrm{p}} = \vec{\mathrm{p}}_f - \vec{\mathrm{p}}_i = m\vec{\mathrm{v}}_f - m\vec{\mathrm{v}}_i = m(\vec{\mathrm{v}}_f - \vec{\mathrm{v}}_i)$$

$$= (1500\ \mathrm{kg})[2.60\ \mathrm{m/s} - (-15.0\ \mathrm{m/s})]$$

$$= \boxed{2.64 \times 10^4\ \mathrm{kg \cdot m/s}}$$

$$\left(\sum \vec{\mathrm{F}}\right)_{\mathrm{avg}} = \frac{\vec{\mathrm{I}}}{\Delta t} = \frac{2.64 \times 10^4\ \mathrm{kg \cdot m/s}}{0.150\ \mathrm{s}} = 1.76 \times 10^5\ \mathrm{N}$$

Linear Momentum and Impulse

Example 1 (How Good Are the Bumpers?): In a particular crash test, a car of mass 1500 kg collides with a wall. The initial velocity of the car is v_i = -15.0 m/s.

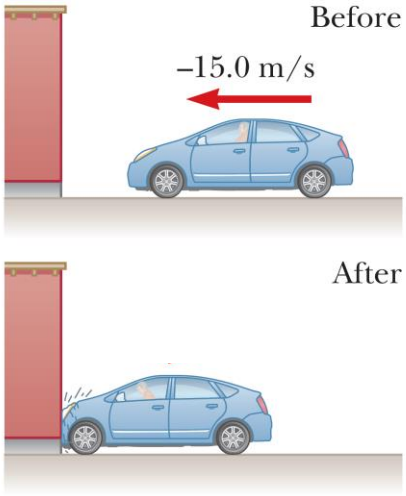
(b) Suppose the final velocity of the car is zero and the time interval of the collision remains at 0.150 s. Would that represent a larger or a smaller net force on the car?

According to the impulse-momentum theorem.

$$\vec{\mathrm{I}} = \Delta\vec{\mathrm{p}} = \vec{\mathrm{p}}_f - \vec{\mathrm{p}}_i = m\vec{\mathrm{v}}_f - m\vec{\mathrm{v}}_i = m(\vec{\mathrm{v}}_f - \vec{\mathrm{v}}_i)$$

$$= (1500\ \mathrm{kg})[0 - (-15.0\ \mathrm{m/s})]$$

$$= \boxed{2.25 \times 10^4\ \mathrm{kg \cdot m/s}}$$

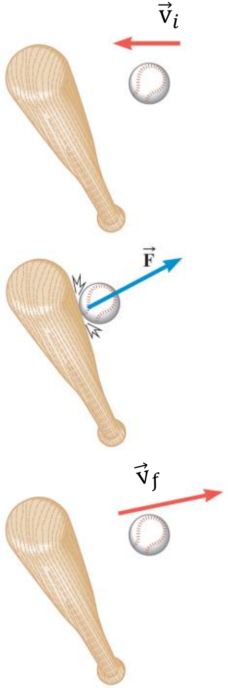


$$\left(\sum \vec{\mathrm{F}}\right)_{\mathrm{avg}} = \frac{\vec{\mathrm{I}}}{\Delta t} = \frac{2.25 \times 10^4\ \mathrm{kg \cdot m/s}}{0.150\ \mathrm{s}} = 1.50 \times 10^5\ \mathrm{N}$$

smaller net force

Linear Momentum and Impulse

Example 2 (A Well-Hit Ball): A baseball of *m* = 0.14 kg has an initial velocity of -38 m/s as it approaches a bat. We have chosen the direction of approach as the negative direction. The bat applies an average force $\vec{F}$ that is much larger than the weight of the ball, and the ball departs from the bat with a final velocity of +58 m/s.



(a) Determine the impulse applied to the ball by the bat.

According to the impulse-momentum theorem.

$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = (0.14\,kg)\left(+58\frac{m}{s}\right) - (0.14\,kg)\left(-38\frac{m}{s}\right) = +13.4\,kg.m/s$$

(b) Assuming that the time of contact is $\Delta t = 1.6 \times 10^{-3}$ s, find the average force exerted on the ball by the bat.

$$\vec{I} = (\sum\vec{F})_{avg}\Delta t \quad \rightarrow \quad (\sum\vec{F})_{avg} = \frac{\vec{I}}{\Delta t} = \frac{+13.4\,kg.m/s}{1.6 \times 10^{-3}\,s} = +8400\,N$$

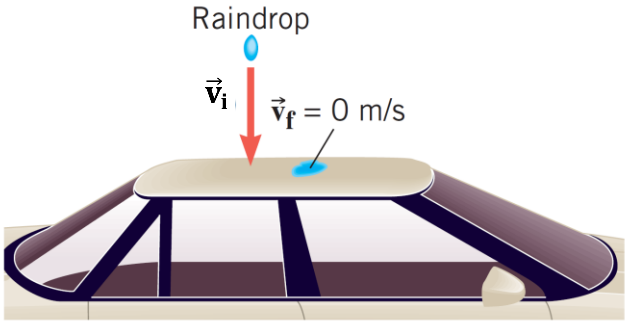
Linear Momentum and Impulse

Example 3 (A Rainstorm): During a storm, rain comes straight down with a velocity of -15 m/s and hits the roof of a car perpendicularly. The mass of rain per second that strikes the car roof is 0.060 kg/s. Assuming that the rain comes to rest upon striking the car, find the average force exerted by the rain on the roof.

The average force needed to reduce the raindrop velocity from -15 m/s to 0 m/s is obtained as

$$\left(\sum \vec{F}\right)_{avg} = \frac{\vec{I}}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = -\frac{m}{\Delta t}\vec{v}_i = -(0.060\,kg/s)(-15\,m/s)$$

$$= +0.90\,N$$



The force is in the positive or upward direction, since the roof exert upward force on each falling drop to bring it to rest.

According to the Newton's third law (action-reaction law) the force exerted on the roof by the rain is -0.90 N.

Quick Quiz 1

- Two objects have equal kinetic energies. How do the magnitudes of their momenta compare?

a) $p_1 < p_2$

b) $p_1 = p_2$

c) $p_1 > p_2$

d) not enough information to tell

Quick Quiz 2

- Your physical education teacher throws a baseball to you at a certain speed, and you catch it. The teacher is next going to throw you a medicine ball whose mass is ten times the mass of the baseball. You are given the following choices:

  You can have the medicine ball thrown with ……………………………. as the baseball.

- Rank these choices from easiest to hardest to catch.

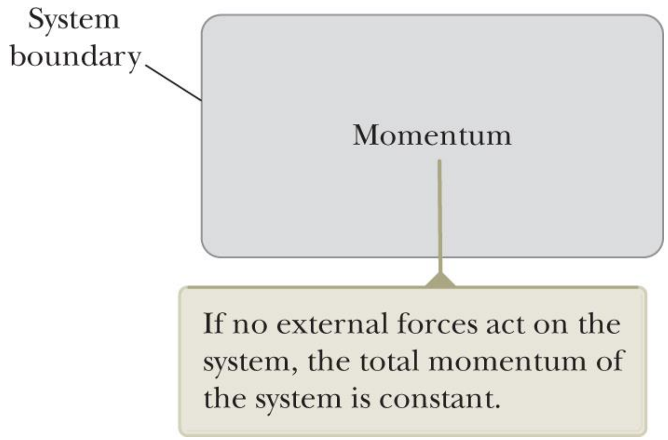
  a) the same speed
  b) the same momentum
  c) the same kinetic energy

Conservation of Linear Momentum

- **Principle of Conservation of Linear Momentum:**

Whenever two or more particles in an isolated system interact, the total momentum of the system does not change.

$$\Delta\vec{p}_{tot} = 0 \quad \rightarrow \quad \vec{p}_f = \vec{p}_i$$



- For *energy*: The system is isolated if there are <u>no transfers of energy</u> across boundary of the system
- For *momentum*: There must be <u>no external forces</u> applied on the system

Conservation of Linear Momentum

- Consider an isolated system of two particles with masses m_1 and m_2 approaching each other with velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$ at instant of time.
- They interact during the collision and then depart with the final velocities $\vec{v}_{1f}$ and $\vec{v}_{2f}$.
- Applying the impulse-momentum theorem to each ball:

$$\boxed{\Delta\vec{p} = \vec{I}} \Rightarrow$$

$$Ball\ 1 \rightarrow \qquad m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = \vec{F}_{1NET}\,\Delta t$$

$$Ball\ 2 \rightarrow \qquad m_2\vec{v}_{2f} - m_2\vec{v}_{2i} = \vec{F}_{2NET}\,\Delta t$$

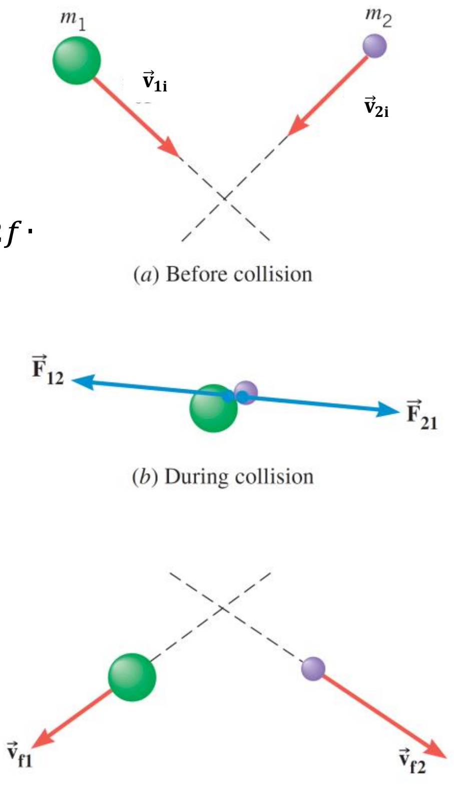
- Since system is isolated, sum of the external forces is zero. The only force on one particle during the collision is the action-reaction forces.

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

$$\left(m_1\vec{v}_{1f} - m_1\vec{v}_{1i}\right) + \left(m_2\vec{v}_{2f} - m_2\vec{v}_{2i}\right) = 0$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \Rightarrow \boxed{\vec{p}_i = \vec{p}_f}$$



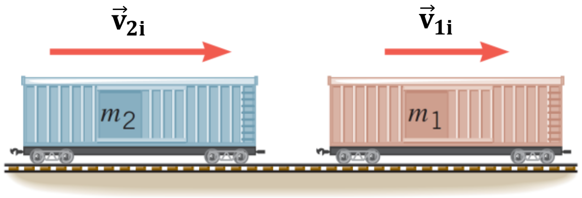
(*a*) Before collision

(*b*) During collision

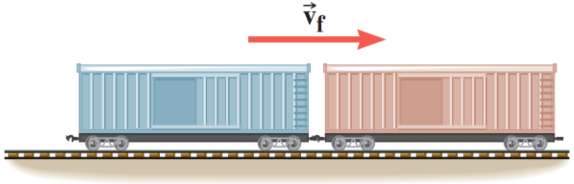
(*c*) After collision

Conservation of Linear Momentum

Example 4 (Assembling a Freight Train): A freight train is being assembled in a switching yard. Car 1 has a mass of $m_1 = 65 \times 10^3$ kg and moves at a velocity of $v_{1i} = +0.80$ m/s. Car 2, with a mass of $m_2 = 92 \times 10^3$ kg and a velocity of $v_{2i} = +1.3$ m/s, overtakes car 1 and couples to it. Neglecting friction, find the common velocity v_f of the cars after they become coupled.



(*a*) Before coupling



(*b*) After coupling

The principle of conservation of momentum:

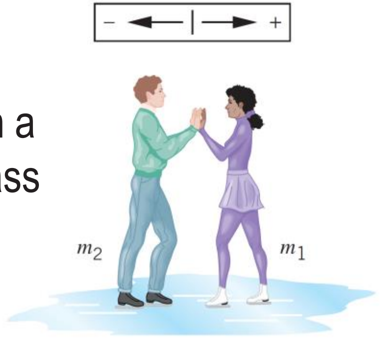
$$\boxed{\vec{p}_f = \vec{p}_i} \quad \rightarrow \quad \underbrace{m_1\vec{v}_f + m_2\vec{v}_f}_{\text{Total momentum after collision}} = \underbrace{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}_{\text{Total momentum before collision}}$$

$$\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2}$$

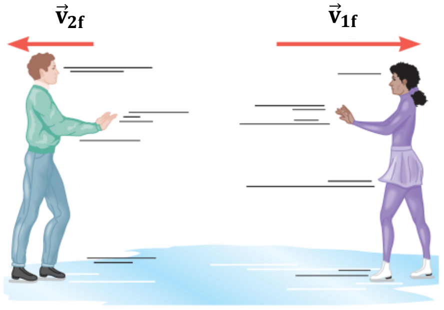
$$\vec{v}_f = \frac{(65 \times 10^3 kg)(0.80\, m/s) + (92 \times 10^3\, kg)(1.3\, m/s)}{65 \times 10^3 kg + 92 \times 10^3 kg} = +1.1 \text{ m/s}$$

Conservation of Linear Momentum

Example 5 (Ice Skaters): Starting from rest, two skaters push off against each other on smooth level ice, where friction is negligible. The woman moves away with a velocity of v_{1f} = +2.5 m/s. Find the “recoil” velocity v_{2f} of the man. (The woman’s mass is m_1 = 54 kg, and the man’s mass is m_2 = 88 kg)



(*a*) Before pushoff



(*b*) After pushoff

The principle of conservation of momentum:

$$\boxed{\vec{p}_f = \vec{p}_i} \quad \rightarrow \quad \underbrace{m_1\vec{v}_{1f} + m_2\vec{v}_{2f}}_{\text{Total momentum after pushing}} = \underbrace{0}_{\text{Total momentum before pushing}}$$

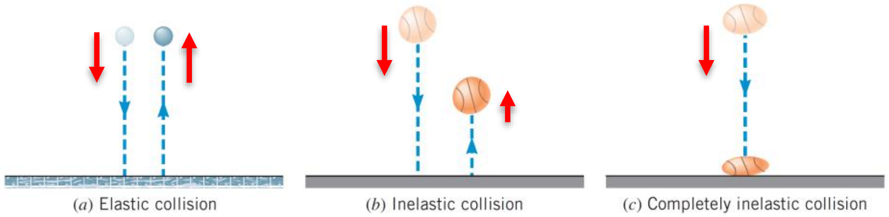
$$\vec{v}_{2f} = \frac{-m_1\vec{v}_{1i}}{m_2} = \frac{-(54\,kg)(+2.5\,m/s)}{88\,kg} = -1.5\,m/s$$

The minus sign indicates that the man moves to the left.

Types of Collisions

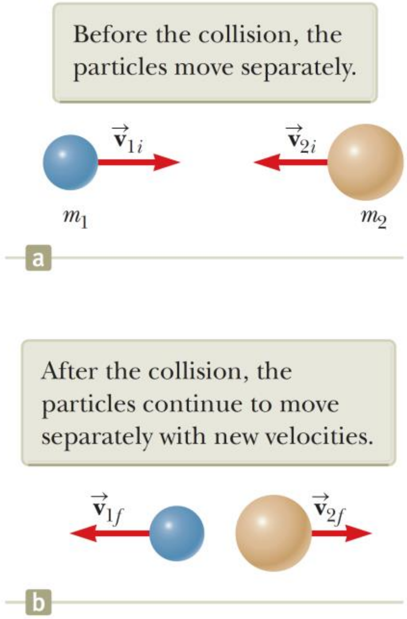
- The total linear momentum is conserved when two objects collide in an isolated system.
- The total kinetic energy of the system is conserved only in atomic and subatomic scale collisions.
- In macroscopic objects, such as two cars, the total kinetic energy of the system is not conserved. The kinetic energy is lost mainly in two ways:
  1) Converted to heat because of friction.
  2) Spent in structural change because of the permanent distortion or damage in the collision

- Collisions are often classified according to whether the total kinetic energy changes during the collision:
  - **Elastic Collisions**
  - **Inelastic Collisions**
  - **Perfectly Inelastic Collisions**



Elastic Collisions in One-Dimension

- In **elastic collisions**, both momentum and kinetic energy of system are conserved.
- The total kinetic energy, as well as the total momentum, of the system is the same before and after the collision.
- Two particles, masses m_1 and m_2 moving with initial velocities $\mathbf{v}_{1i}$ and $\mathbf{v}_{2i}$ along the same straight line collide head-on then leave the collision site with different velocities, $\mathbf{v}_{1f}$ and $\mathbf{v}_{2f}$



$$\boxed{\vec{p}_i = \vec{p}_f \quad \rightarrow \quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}}$$

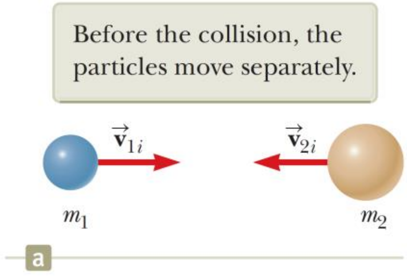
$$\boxed{K_i = K_f \quad \rightarrow \quad \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2}$$

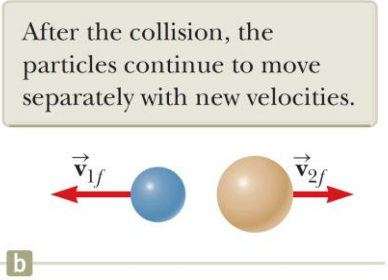
Elastic Collisions in One-Dimension

- In **elastic collisions**, both momentum and kinetic energy of system are conserved.
- The total kinetic energy, as well as the total momentum, of the system is the same before and after the collision.
- Two particles, masses m_1 and m_2 moving with initial velocities $\mathbf{v}_{1i}$ and $\mathbf{v}_{2i}$ along the same straight line collide head-on then leave the collision site with different velocities, $\mathbf{v}_{1f}$ and $\mathbf{v}_{2f}$
- In elastic collisions, if the masses and the initial velocities are known, the final velocities after the collision can be obtained as below:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$



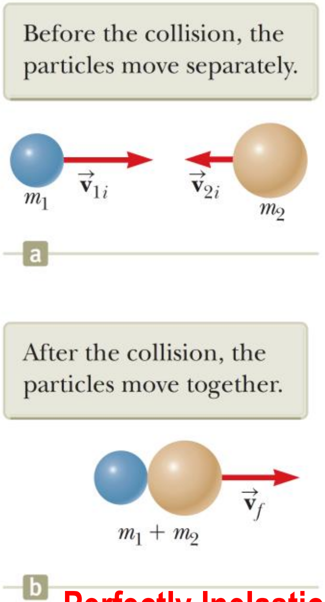


Inelastic Collisions in One-Dimension

- In **inelastic collisions**, momentum is <u>conserved</u>, but kinetic energy of system is **<u>not</u>** <u>conserved</u>.

$$\boxed{\vec{p}_i = \vec{p}_f \quad \rightarrow \quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}}$$

$$\boxed{K_i \neq K_f}$$



Perfectly Inelastic Collision

- If the objects stick together after colliding, the collision is called **perfectly inelastic.**
- Two particles, masses m_1 and m_2 moving with initial velocities $\mathbf{v}_{1i}$ and $\mathbf{v}_{2i}$ along the same straight line collide head-on, stick together, then move with common velocity $\mathbf{v}_f$

$$\boxed{\vec{p}_i = \vec{p}_f \quad \rightarrow \quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f}$$

Collisions in One-Dimension

Example 6 (A Head-On Collision): One ball has a mass of m_1 = 0.250 kg and an initial velocity of v_{1i} = +5.00 m/s. The other has a mass of m_2 = 0.800 kg and is initially at rest. No external forces act on the balls. What are the velocities of the balls after the collision?

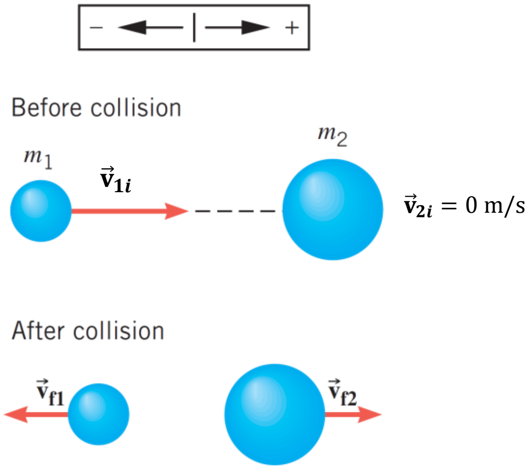
An elastic collision both kinetic energy and momentum are conserved:

$$\boxed{\vec{p}_i = \vec{p}_f} \rightarrow \quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$\boxed{K_i = K_f} \rightarrow \quad \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$



Collisions in One-Dimension

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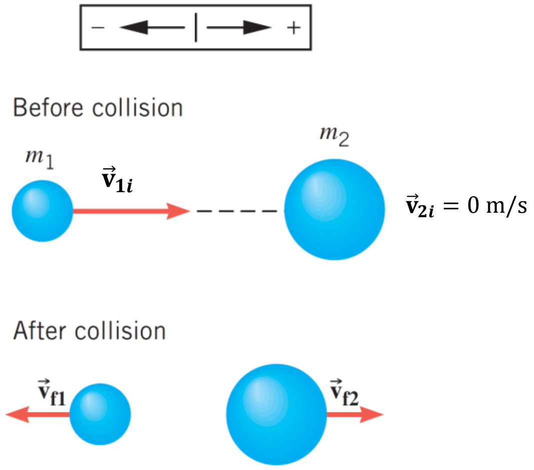
Solve equations for v_{2f} and v_{1f} :

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = \left(\frac{0.250\,kg - 0.800\,kg}{0.250\,kg + 0.800\,k}\right)\left(+5.00\frac{m}{s}\right) = -2.62\,m/s$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} = \left(\frac{2(0.250\,kg)}{0.250\,kg + 0.800\,kg}\right)\left(+5.00\frac{m}{s}\right) = +2.38\,m/s$$



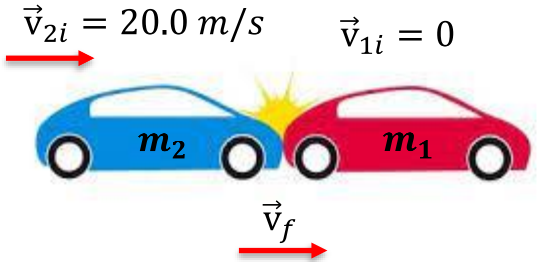
Collisions in One-Dimension

Example 7 (Car Collision): An m_1 = 1800 kg car stopped at a traffic light is struck from the rear by an m_2 = 900 kg car. The two cars become entangled, moving along the same path as that of the originally moving car. If the smaller car were moving at 20.0 m/s before the collision, what is the velocity of the entangled cars after the collision?

A perfectly inelastic collision:

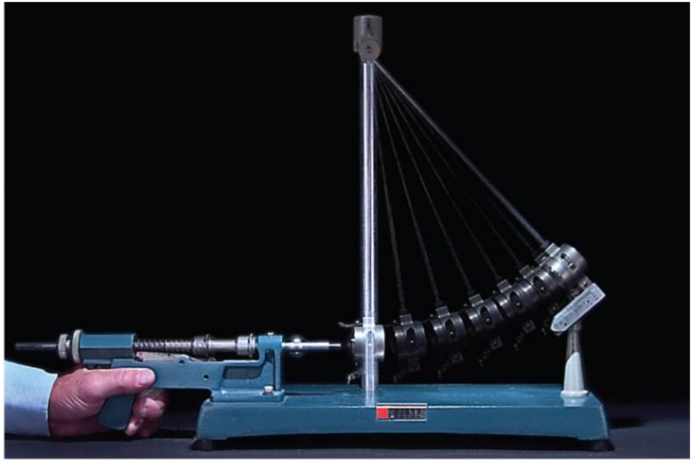
$$\boxed{\vec{p}_i = \vec{p}_f} \quad \rightarrow \quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$v_f = \frac{m_2 v_{2i}}{m_1 + m_2} = \frac{(900\ \mathrm{kg})(20.0\ \mathrm{m/s})}{1800\ \mathrm{kg} + 900\ \mathrm{kg}} = \boxed{6.67\ \mathrm{m/s}}$$

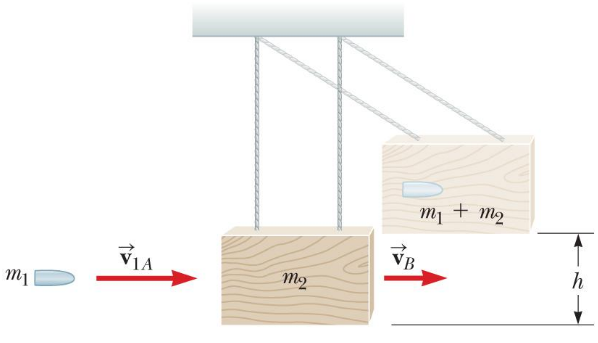


Collisions in One-Dimension

Example 8 (The Ballistic Pendulum): The ballistic pendulum an apparatus used to measure the speed of a fast-moving projectile such as a bullet. A projectile of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The projectile embeds in the block, and the entire system swings through a height *h.* How can we determine the speed of the projectile from a measurement of *h*?



b



a

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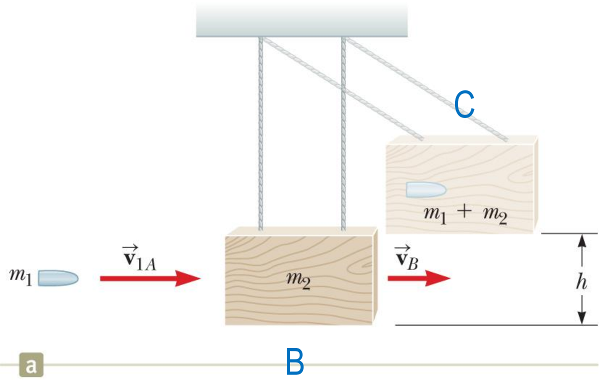
A perfectly inelastic collision just after the bullet collides with the block:

$$\boxed{\vec{p}_i = \vec{p}_f} \rightarrow \quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$m_1\vec{v}_{1A} + 0 = (m_1 + m_2)\vec{v}_B \quad \rightarrow \quad v_{1A} = \frac{(m_1 + m_2)v_B}{m_1}$$



Conservation of mechanical energy in the swing from position B to position C:

$$\boxed{E_{mB} = E_{mC}} \quad \rightarrow \quad K_B + U_B = K_C + U_C$$

$$\frac{1}{2}(m_1 + m_2)v_B^2 + 0 = 0 + (m_1 + m_2)gh$$

$$v_B = \sqrt{2gh}$$

$$\boxed{v_{1A} = \frac{(m_1 + m_2)\sqrt{2gh}}{m_1}}$$

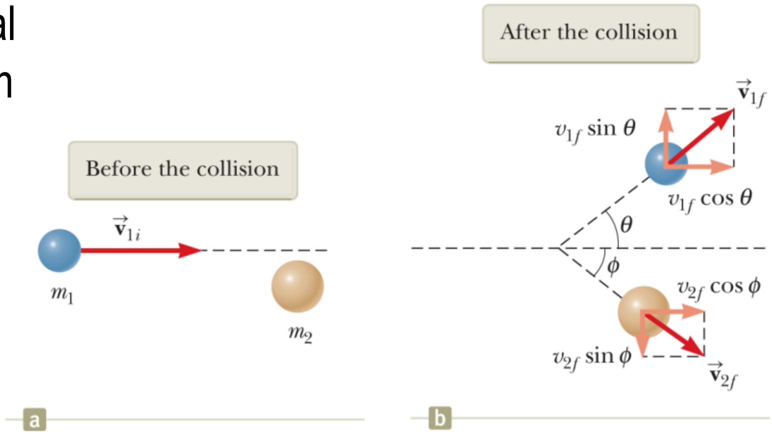
Quick Quiz 3

- In a perfectly inelastic one-dimensional collision between two moving objects, what condition alone is necessary so that the final kinetic energy of the system is zero after the collision?

  a) The objects must have initial momenta with the same magnitude but opposite directions.
  b) The objects must have the same mass.
  c) The objects must have the same initial velocity.
  d) The objects must have the same initial speed, with velocity vectors in opposite directions.

Collisions in Two-Dimensions

- The collisions discussed so far have been one -dimensional. The velocities of the objects all point along a single line before and after contact.
- However, collisions often occur in two or three dimensions.
- Assume an isolated two-ball system. The total momentum of the system is conserved.
- Since momentum is a vector quantity, in two-dimensional collision the x and y components of the total momentum are conserved separately.



$$\vec{p}_{xi} = \vec{p}_{xf} \quad \rightarrow \quad m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$\vec{p}_{yi} = \vec{p}_{yf} \quad \rightarrow \quad m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Collisions in Two-Dimensions

Example 9 (Collision at an Intersection): A 1500 kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2500 kg truck traveling north at a speed of 20.0 m/s as shown in the figure. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming the vehicles stick together after the collision.

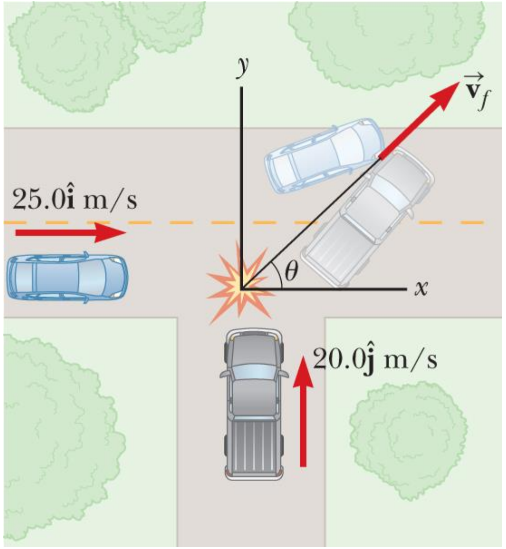
A two-dimensions perfectly inelastic collision, only momentum is conserved:

$$\boxed{\vec{p}_{xi} = \vec{p}_{xf}} \quad \rightarrow \quad m_1 v_{1ix} + m_2 v_{2ix} = (m_1 + m_2) v_{fx}$$

$$\boxed{m_1 v_{1ix} = (m_1 + m_2) v_f \cos\theta}$$

$$\boxed{\vec{p}_{yi} = \vec{p}_{yf}} \quad \rightarrow \quad m_1 v_{1iy} + m_2 v_{2iy} = (m_1 + m_2) v_{fy}$$

$$\boxed{m_2 v_{2iy} = (m_1 + m_2) v_f \sin\theta}$$



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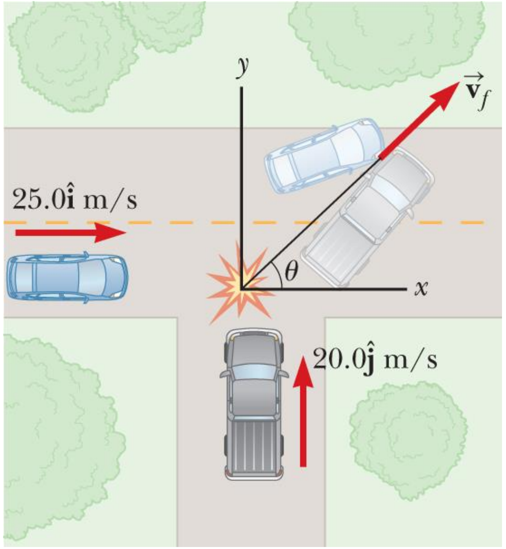
A two-dimensions perfectly inelastic collision, only momentum is conserved:

$$m_1 v_{1ix} = (m_1 + m_2) v_f \cos\theta$$

$$m_2 v_{2iy} = (m_1 + m_2) v_f \sin\theta \quad \Rightarrow \quad v_f = \frac{m_2 v_{2iy}}{(m_1 + m_2)\sin\theta}$$

$$\frac{m_2 v_{2iy}}{m_1 v_{1ix}} = \frac{\sin\theta}{\cos\theta} = \tan\theta \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{m_2 v_{2iy}}{m_1 v_{1ix}}\right)$$



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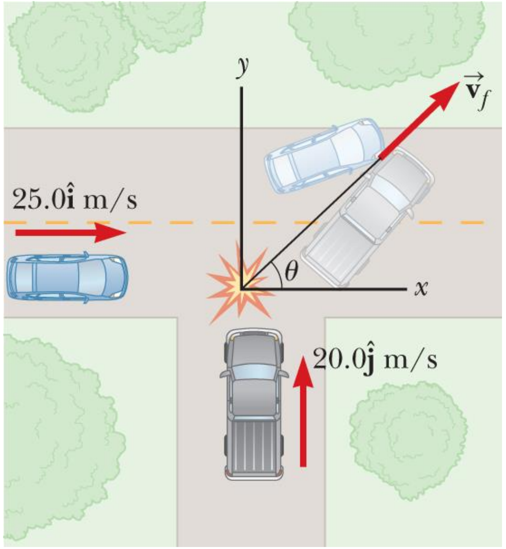
A two-dimensions perfectly inelastic collision, only momentum is conserved:

$$\theta = \tan^{-1}\left(\frac{m_2 v_{2iy}}{m_1 v_{1ix}}\right) = \tan^{-1}\left[\frac{(2500\text{ kg})(20.0\text{ m/s})}{(1500\text{ kg})(25.0\text{ m/s})}\right] = \boxed{53.1°}$$

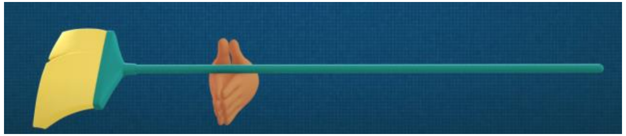
$$v_f = \frac{m_2 v_{2iy}}{(m_1 + m_2)\sin\theta} = \frac{(2500\text{ kg})(20.0\text{ m/s})}{(1500\text{ kg} + 2500\text{ kg})\sin 53.1°} = \boxed{15.6\text{ m/s}}$$



The Center of Mass

- When a system consist of several objects, the mass of the system is located in several places, and the various objects move relative to each other before, after, and even during the interaction.
- It is possible, however, to speak of a kind of average location for the total mass by introducing a concept known as the ***center of mass.***
- Also, when analyzing the motion of an extended object, we can treat the entire object as if its mass were contained in a single point, known as the object's ***center of mass.***

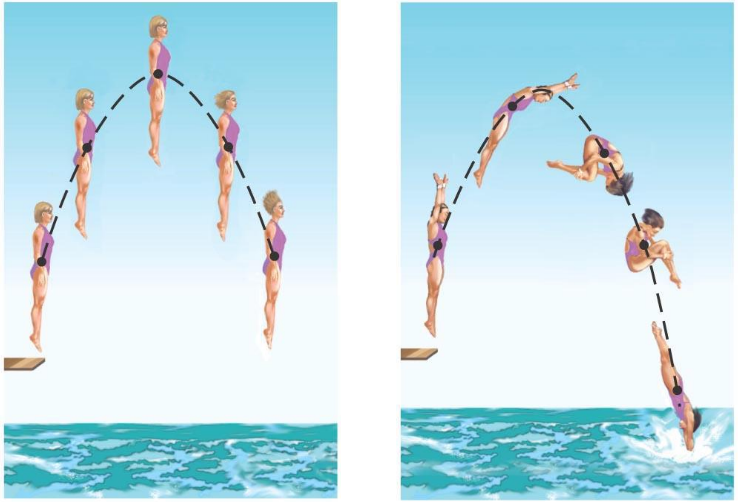
The **center of mass (CM)** is a point that represents the average location for the total mass of a system or a solid object



The Center of Mass and Motion

- Until now, we have assumed all objects behave like a particle that undergoes only translational motion.
- However, real objects also rotate and move in other ways that are not just translational – it is called general motion.

- But there is one point in the object that despite the general motion, that one point only has translational motion like a particle.
- The center of mass is the point in an object that behaves like a particle, and it moves according to the net force.



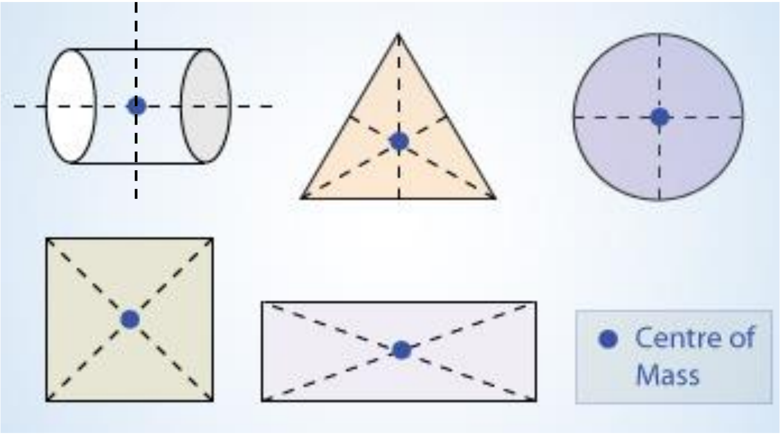
(a) (b)

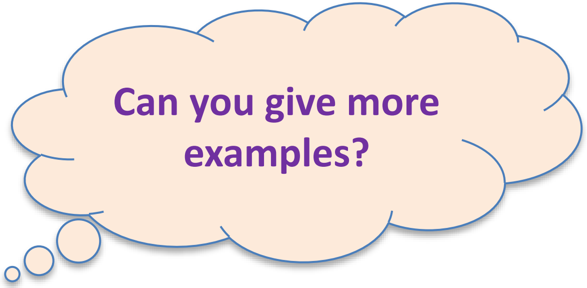
The Center of Mass and Symmetry

- If an object has a point, line or plane of symmetry, the center-of-mass point must lie on that point, on that line or on that plane.
- The center of mass need **not** be within the actual object. It can be outside the object.


Boomerang

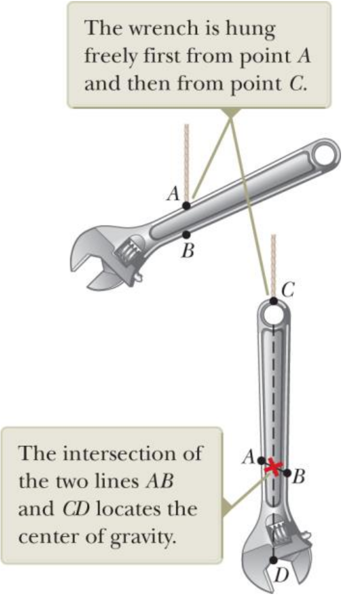

Donut





The Center of Gravity

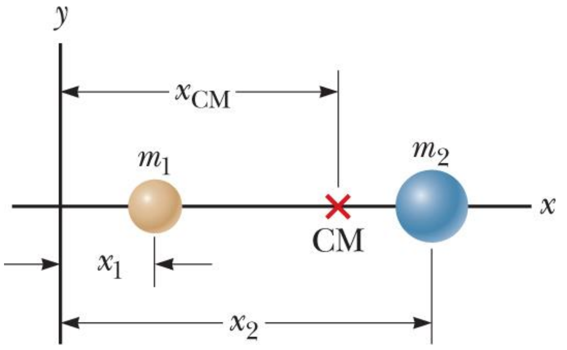
- The center of gravity is the point where the gravitational force can be considered to act.
- It is same as the center of mass as long as the gravitational force does not vary among different parts of the object.
- The center of gravity can be found experimentally by suspending an object from different points.
- For example, the wrench is hung freely from different points.
- The intersection of the lines indicates the center of gravity.



How do you find the center of gravity of a mug?

The Center of Mass of Two-Particle System

- Assume that two particles of mass m_1 and m_2 that are located on the x axis at the positions x_1 and x_2, respectively.
- The position x_{CM} of the center-of-mass point from the origin is defined to be



$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

- If the two masses are equal, the center-of-mass point is the midway between the particles.

$$m_1 = m_2 \quad \rightarrow \quad x_{CM} = \frac{x_1 + x_2}{2}$$

- Suppose that $m_1 = 5.0\ kg$ and $x_1 = 2.0\ m$, while $m_2 = 12\ kg$, and $x_2 = 6.0\ m$. Then the center-of-mass point will be closer to particle 2, since it is more massive.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(5.0\ kg)(2.0\ m) + (12\ kg)(6.0\ m)}{5.0\ kg + 12\ kg} = 4.8\ m$$

The Center of Mass of Many Particle System

- We can extend this concept to a system of many particles with masses m_i in a two- or three-dimensional space.

$$x_{\text{CM}} \equiv \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{m_1 + m_2 + \cdots + m_n} = \frac{1}{M}\sum_i m_ix_i$$

Total mass

- The *y* and *z* coordinates of the center of mass can be similarly defined.

$$y_{\text{CM}} \equiv \frac{1}{M}\sum_i m_iy_i \quad \text{and} \quad z_{\text{CM}} \equiv \frac{1}{M}\sum_i m_iz_i$$

- The position vector of center-of-mass point is obtained as:

$$\vec{r}_{\text{CM}} = x_{\text{CM}}\hat{\imath} + y_{\text{CM}}\hat{\jmath} + z_{\text{CM}}\hat{k}$$

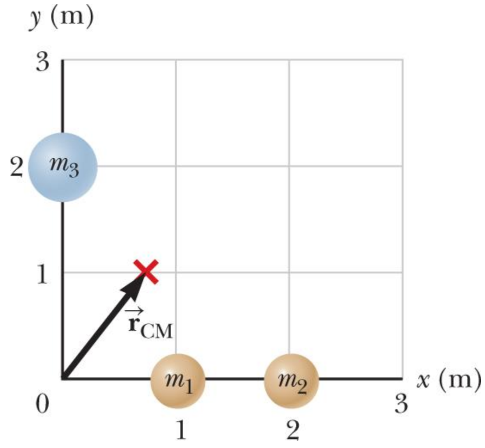
The Center of Mass

Example 10 (Center of Mass of Three Particles): A system consists of three particles located as shown in the figure. Find the center of mass of the system. The masses of the particles are $m_1 = m_2 = 1.0$ kg and $m_3 = 2.0$ kg.

$$\boxed{x_{\mathrm{CM}} = \frac{1}{M}\sum_i m_i x_i = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}}$$

$$= \frac{(1.0\text{ kg})(1.0\text{ m}) + (1.0\text{ kg})(2.0\text{ m}) + (2.0\text{ kg})(0)}{1.0\text{ kg} + 1.0\text{ kg} + 2.0\text{ kg}}$$

$$= \frac{3.0\text{ kg}\cdot\text{m}}{4.0\text{ kg}} = 0.75\text{ m}$$



$$\boxed{y_{\mathrm{CM}} = \frac{1}{M}\sum_i m_i y_i = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}}$$

$$r_{\mathrm{CM}} \equiv x_{\mathrm{CM}}\hat{\imath} + y_{\mathrm{CM}}\hat{\jmath} = \boxed{(0.75\hat{\imath} + 1.0\hat{\jmath})\text{ m}}$$

$$= \frac{(1.0\text{ kg})(0) + (1.0\text{ kg})(0) + (2.0\text{ kg})(2.0\text{ m})}{1.0\text{ kg} + 1.0\text{ kg} + 2.0\text{ kg}}$$

$$= \frac{4.0\text{ kg}\cdot\text{m}}{4.0\text{ kg}} = 1.0\text{ m}$$

The Center of Mass and Momentum

- The concept of center of mass enables us to gain additional insight into the linear momentum.
- The total linear momentum of a system of particles is equal to the product of the total mass and the velocity of the center of mass.

$$\boxed{\vec{p}_{total} = M\vec{v}_{CM}}$$

Total mass

Velocity of CM

- The velocity of the center of mass of the system is obtained as:

$$\boxed{\vec{v}_{CM} = \frac{1}{M}\sum_i m_i\vec{v}_i}$$

velocity of *i*th particle

The Center of Mass of Solid Objects

- To obtain the center of mass for a solid, continuous object, we can consider the solid object as a system containing a large number of small mass elements.
- By dividing the object into elements of mass Δm_i with coordinates x_i, y_i, z_i.
- The x coordinate of the center of mass is obtained as

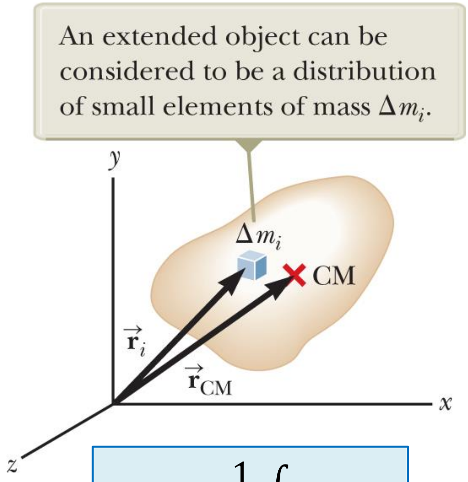
$$x_{\text{CM}} \approx \frac{1}{M}\sum_i x_i \Delta m_i$$

- If the number of elements approach infinity, the size of each element approaches zero.

$$\boxed{x_{\text{CM}} = \lim_{\Delta m_i \to 0} \frac{1}{M}\sum_i x_i \Delta m_i = \frac{1}{M}\int x\,dm}$$

- Likewise, for y_{CM} and z_{CM} we obtain:

$$\boxed{y_{\text{CM}} = \frac{1}{M}\int y\,dm \quad \text{and} \quad z_{\text{CM}} = \frac{1}{M}\int z\,dm}$$

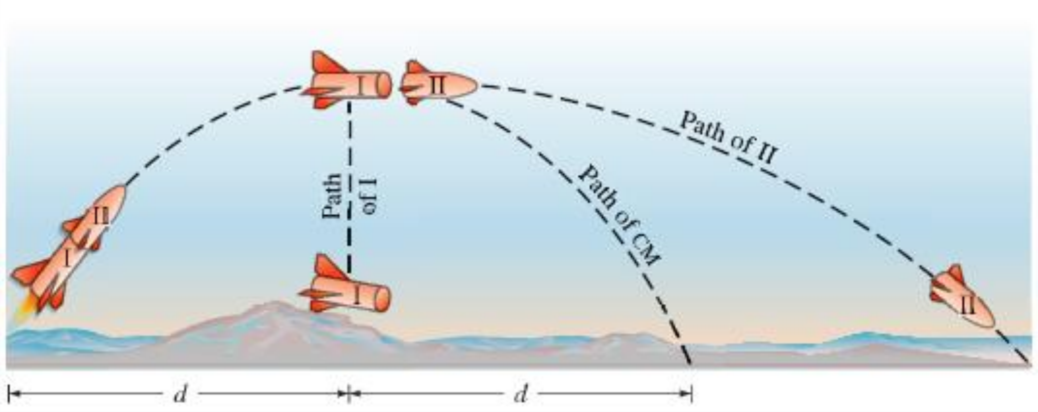


$$\boxed{\vec{r}_{\text{CM}} = \frac{1}{M}\int \vec{r}\,dm}$$

The vector position of the center of mass

Quick Quiz 4

- A rocket is shot into the air. At the moment the rocket reaches its highest point, a horizontal distance $d = 10.0\ m$ from its starting point, a prearranged explosion separates it into two parts of equal mass. Part I is stopped in midair by the explosion and falls vertically to Earth. Where does part II land compared to the starting point?



THANK YOU

HUMBER

WE ARE HUMBER