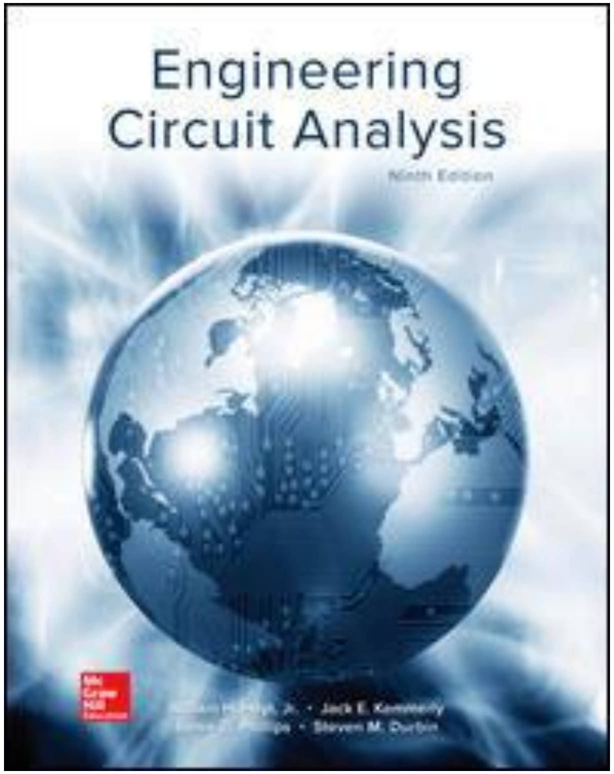


Frequency Response

Frequency Response

Sinusoidal Circuit Analysis – finding voltage and current in circuits with constant frequency source

Frequency response – with the amplitude of the sinusoidal source constant and by varying the frequency

Frequency response of a circuit is the variation in its behavior with change in signal frequency

Sinusoidal steady-state frequency response of circuits has applications in communications and control systems

Application – Filters: that block or eliminate signals with unwanted frequencies and pass signals of the desired frequencies

Filters are used in TV, radio, telephone systems to separate one broadcast frequency from another

Frequency Response

DC – zero frequency

Circuits defined for specific frequency – radio station, TV, cell phone, microwave (frequencies of each device is isolated from one another

Resonant or Tuned circuits – resonance

Resonance condition – when a fixed amplitude sinusoidal forcing function produces a response of maximum amplitude

Resonant system may be **electrical**, mechanical, hydraulic, acoustic etc.

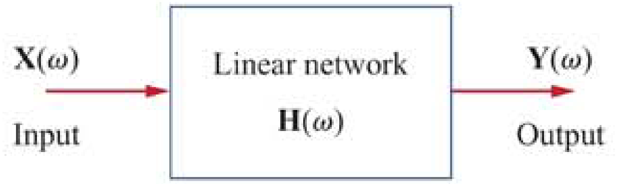
Electrical – in a two-terminal electrical network containing at least one inductor and one capacitor, resonance exists when input impedance of the network is purely resistive (voltage and current at the network input terminals are in-phase)

Transfer Function

Transfer function **H**(ω) is a useful analytical tool for finding the frequency response of a circuit

Frequency response – plot of circuit's transfer function versus ω with varying from $\omega = 0$ to $\omega = \infty$

$$\mathbf{H}(j\omega) = \frac{\mathbf{V}_{out}(j\omega)}{\mathbf{V}_{in}(j\omega)} = \text{Voltage gain}$$



$$\mathbf{H}(j\omega) = \frac{\mathbf{I}_{out}(j\omega)}{\mathbf{I}_{in}(j\omega)} = \text{Current gain}$$

$$\mathbf{H}(j\omega) = \frac{\mathbf{V}_{out}(j\omega)}{\mathbf{I}_{in}(j\omega)} = \text{Transfer impedance}$$

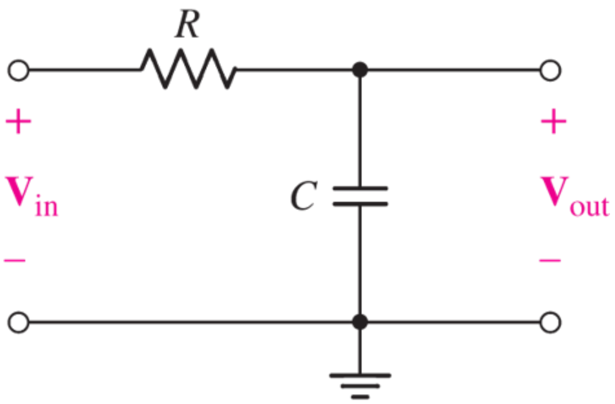
$$\mathbf{H}(j\omega) = \frac{\mathbf{I}_{out}(j\omega)}{\mathbf{V}_{in}(j\omega)} = \text{Transfer admittance}$$

Represent in phasor form: $\mathbf{H}(j\omega) = |H(j\omega)| \angle \phi(j\omega)$

Transfer Function

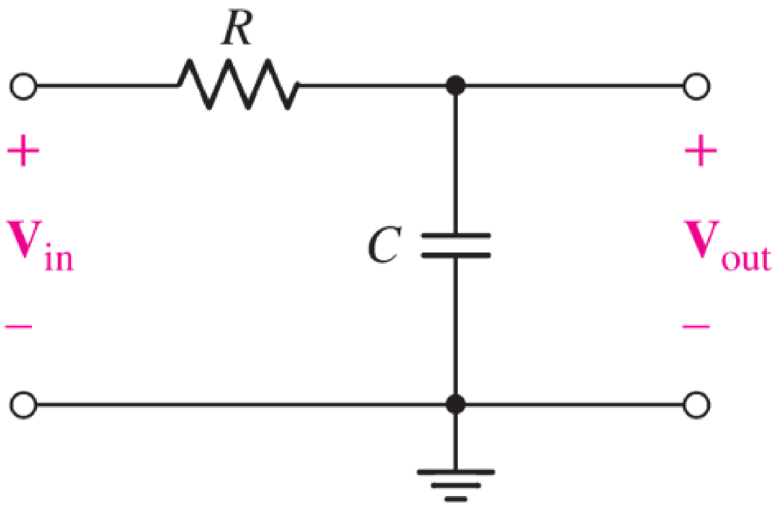
To determine the Transfer function **H**(ω), the circuit should be in frequency domain with resistors, inductors and capacitors represented by their resistance and reactance values

Frequency response is determined by plotting the magnitude and phase of the transfer function as the frequency varies



TF Example: *RC* Circuit

Determine $\mathbf{H(j\omega)=V}_{out}/\mathbf{V}_{in}$ and plot magnitude and phase as a function of frequency



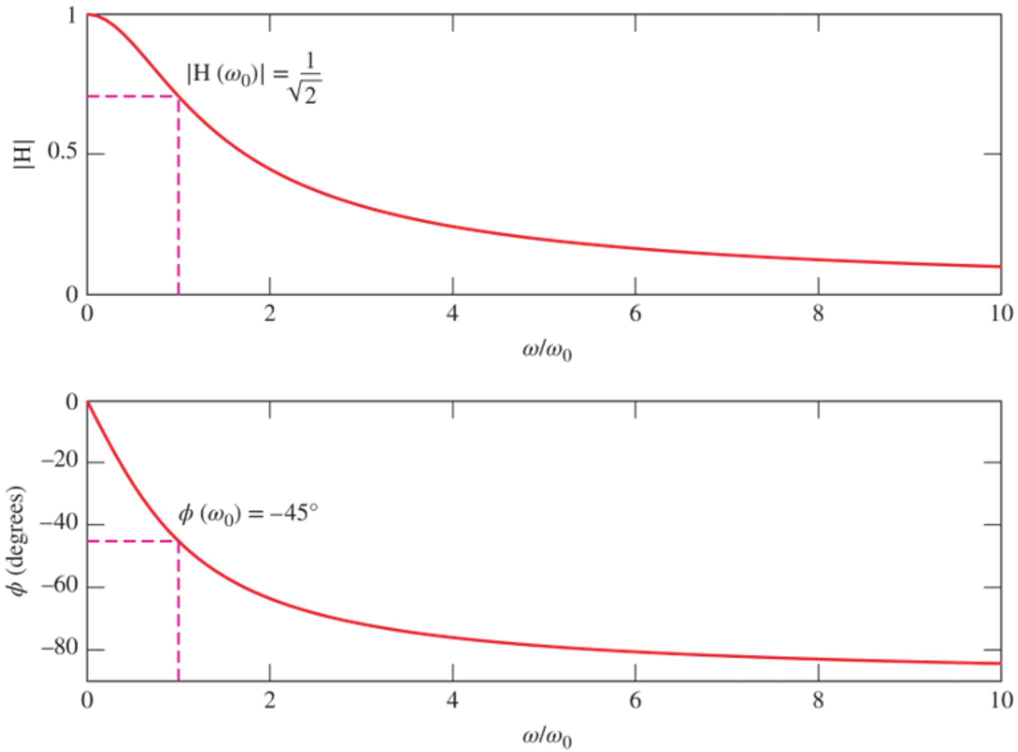
$$\mathbf{H}(j\omega) = \frac{\mathbf{V}out}{\mathbf{V}in} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega CR}$$

TF Example: *RC* Circuit

$$\mathbf{H}(j\omega) = \frac{\mathbf{V}out}{\mathbf{V}in} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega CR}$$

TF Example: *RC* Circuit

Determine $\mathbf{H(j\omega)=V}_{out}\mathbf{/V}_{in}$ and plot magnitude and phase as a function of frequency



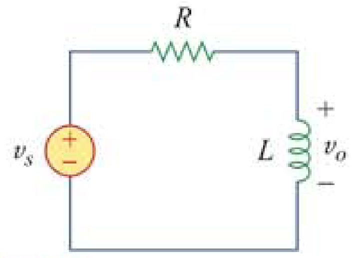
$$H = \frac{1}{\sqrt{1+(\omega/\omega_0)^2}}$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$\omega_0 = \frac{1}{RC}$$

TF Practice Problem: *RL* Circuit

Determine $\mathbf{H(j\omega)=V_{out}/V_{in}}$ and plot magnitude and phase as a function of frequency



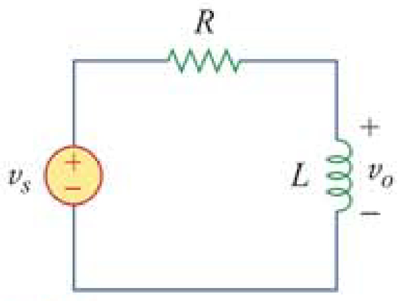
$$\mathrm{H}(j\omega) = \frac{\mathrm{V}out}{\mathrm{V}in} = \frac{j\omega L}{R + j\omega L}$$

TF Practice Problem: *RL* Circuit

$$\mathrm{H}(j\omega) = \frac{\mathrm{V}out}{\mathrm{V}in} = \frac{j\omega L}{R + j\omega L}$$

TF Practice Problem: *RL* Circuit

Plot magnitude and phase as a function of frequency



Bode Diagrams

A Bode diagram or Bode plot is a useful tool for visualizing transfer functions and frequency responses.

A Bode plot shows either magnitude or phase on a logarithmic scale for frequency ω.

Magnitude is shown on a decibel (dB) scale defined as:

$$H_{dB} = 20\,\log_{10}|\,H(\,j\omega)\,|$$

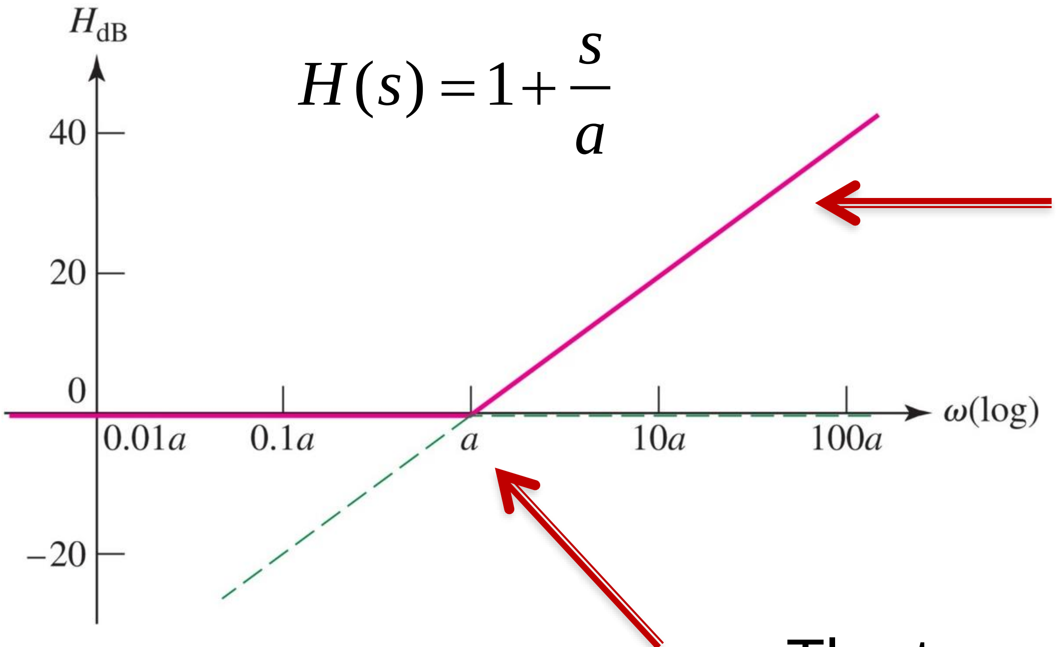
Decibel

$$H_{dB} = 20\ \log_{10} |H(j\omega)|$$

16.10 Calculate H_{dB} at $\omega = 146$ rad/s if $\mathbf{H}(\mathbf{s})$ equals (*a*) $20/(\mathbf{s}+100)$; (*b*) $20(\mathbf{s}+100)$; (*c*) $20\mathbf{s}$. Calculate $|\mathbf{H}(j\omega)|$ if H_{dB} equals (*d*) 29.2 dB; (*e*) −15.6 dB; (*f*) −0.318 dB.

Ans: −18.94 dB; 71.0 dB; 69.3 dB; 28.8; 0.1660; 0.964.

Determining Asymptotes for Bode Plots



A single zero results in 20 dB/dec increase at the zero frequency

The two asymptotes intersect at ω = *a*, the frequency of the zero.

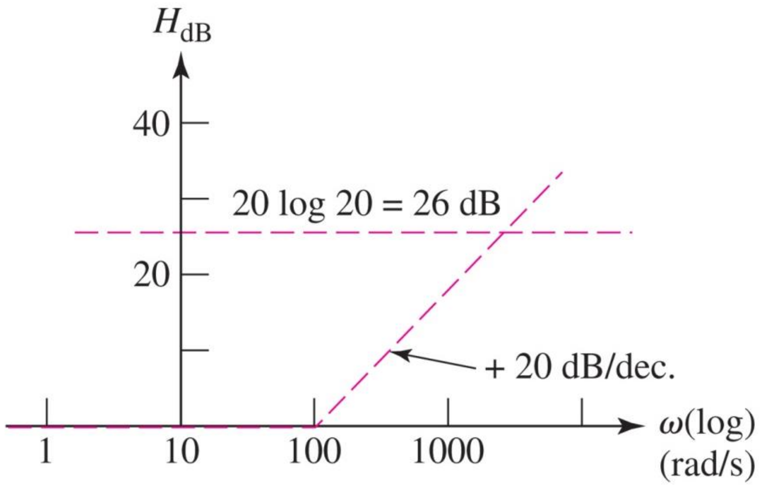
This frequency is also described as the corner, break, 3 dB, or half-power frequency.

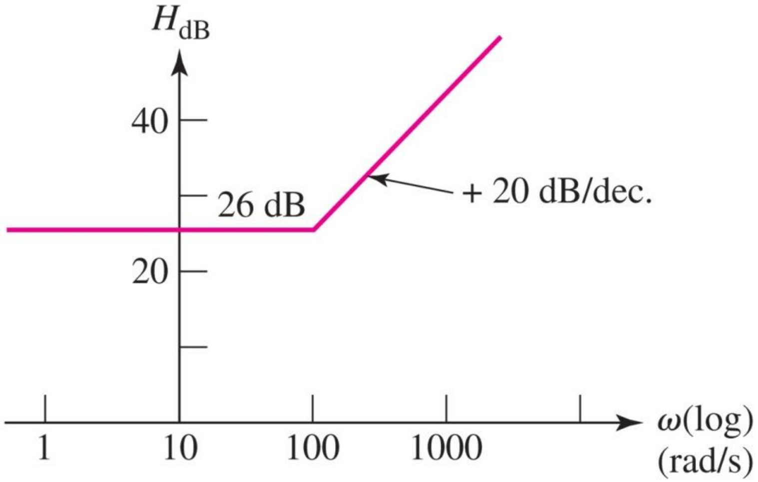
$$|\mathbf{H}(j\omega)| = \left|1 + \frac{j\omega}{a}\right| = \sqrt{1 + \frac{\omega^2}{a^2}}$$

$$H_{dB} = 20\log\left|1 + \frac{j\omega}{a}\right| = 20\log\sqrt{1 + \frac{\omega^2}{a^2}}$$

Bode Plots: Multiple Terms

$$\mathbf{H}(s) = 20 + 0.2\mathbf{s} = 20(1 + \mathbf{s}/100)$$

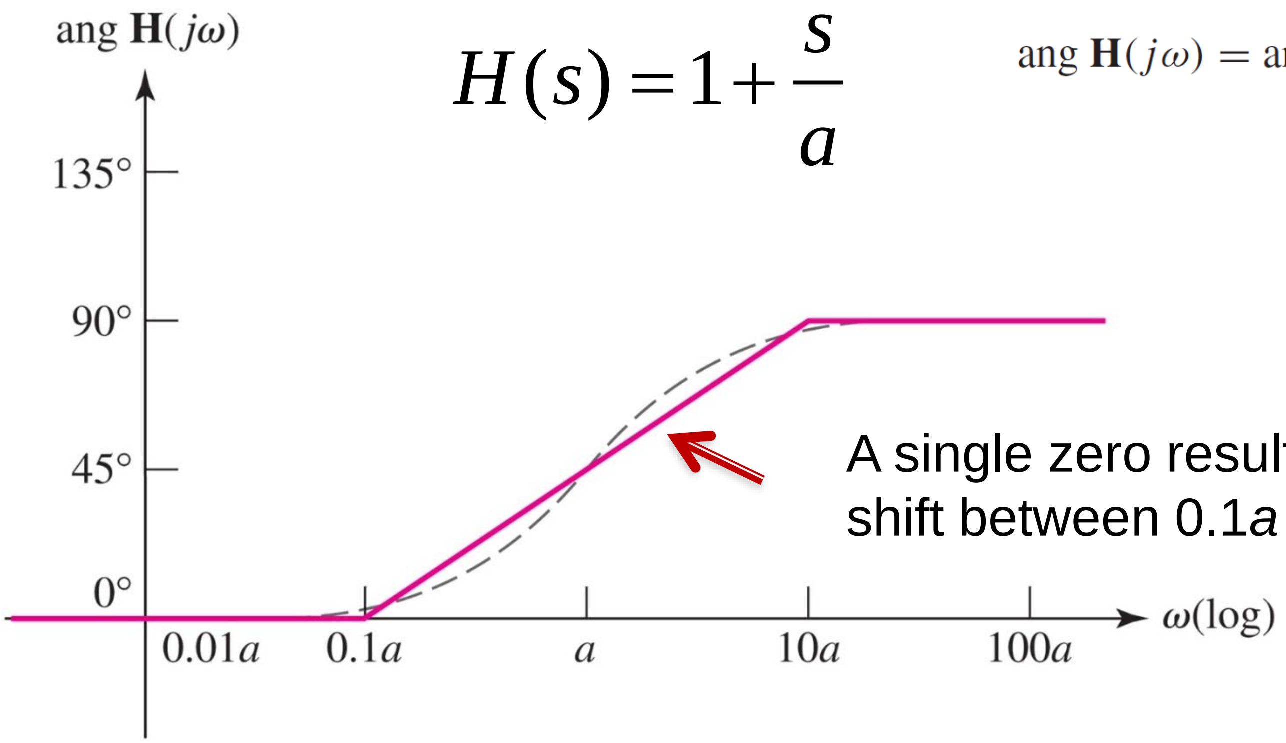




Bode Plot: Phase Response

$$H(s) = 1 + \frac{s}{a}$$

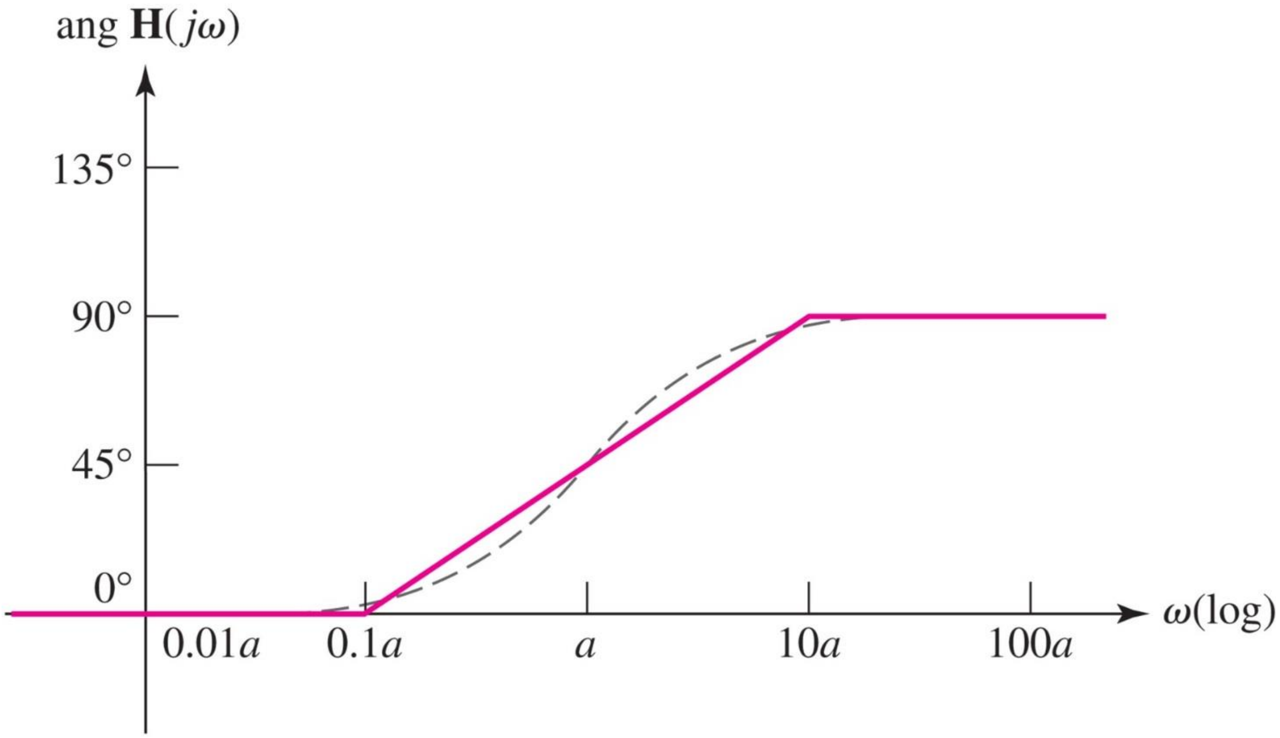
$$\text{ang } \mathbf{H}(j\omega) = \text{ang}\left(1 + \frac{j\omega}{a}\right) = \tan^{-1}\frac{\omega}{a}$$



A single zero results in 90° phase shift between 0.1a and 10a

Bode Plots: Phase Response Multiple Terms

$$\mathbf{H}(s) = 20 + 0.2\mathbf{s} = 20(1 + \mathbf{s}/100)$$



Complete Bode Magnitude Plot

16.13 Construct a Bode magnitude plot for $\mathbf{H}(\mathbf{s})$ equal to
(*a*) $50/(\mathbf{s}+100)$; (*b*) $(\mathbf{s}+10)/(\mathbf{s}+100)$; (*c*) $(\mathbf{s}+10)/\mathbf{s}$.

Ans: (*a*) -6 dB, $\omega < 100$; -20 dB/decade, $\omega > 100$; (*b*) -20 dB, $\omega < 10$; $+20$ dB/decade, $10 < \omega < 100$; 0 dB, $\omega > 100$; (*c*) 0 dB, $\omega > 10$; -20 dB/decade, $\omega < 10$.

Complete Bode Phase Plot

16.14 Draw the Bode phase plot for $\mathbf{H}(\mathbf{s})$ equal to (*a*) $50/(\mathbf{s}+100)$; (*b*) $(\mathbf{s}+10)/(\mathbf{s}+100)$; (*c*) $(\mathbf{s}+10)/\mathbf{s}$.

Ans: (*a*) 0°, $\omega < 10$; -45°/decade, $10 < \omega < 1000$; -90°, $\omega > 1000$; (*b*) 0°, $\omega < 1$; $+45^\circ$/decade, $1 < \omega < 10$; 45°, $10 < \omega < 100$; -45°/decade, $100 < \omega < 1000$; 0°, $\omega > 1000$; (*c*) -90°, $\omega < 1$; $+45^\circ$/decade, $1 < \omega < 100$; 0°, $\omega > 100$.

Resonance

The most common feature of the frequency response of a circuit may be the sharp peak shown in its amplitude characteristics – Resonant peak.

Resonance is the cause of oscillations of stored energy from one form to another.

Resonance phenomenon allows the frequency discrimination in communication network.

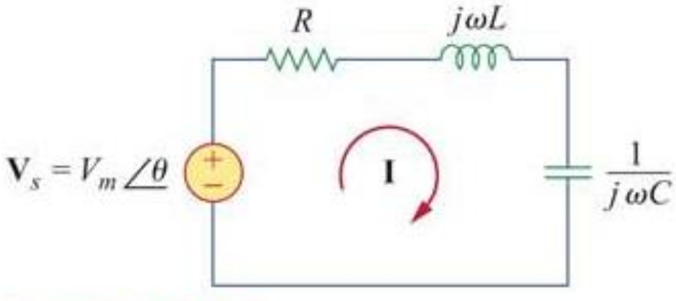
Resonance occurs in any circuit that has at least one inductor and one capacitor

Resonance is a condition in an RLC circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance

Resonant circuits (series or parallel) are useful for constructing filters, as their transfer function can be highly frequency selective. They are used in many applications such as selecting the desired stations in radio and TV receivers

Resonance: Series *RLC*

A network is in *resonance* when the voltage and current at the network input terminals are in phase.



$$Z = H(\omega) = \frac{V_s}{I} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

At resonance: Im{Z} = 0

The value of ω that satisfies this condition is called the resonance

The resonant frequency is $$\omega_o = \frac{1}{\sqrt{LC}}$$

The resonant condition may be achieved by adjusting L, C or **ω**

Variable ω

Resonance: Series *RLC*

At resonance:

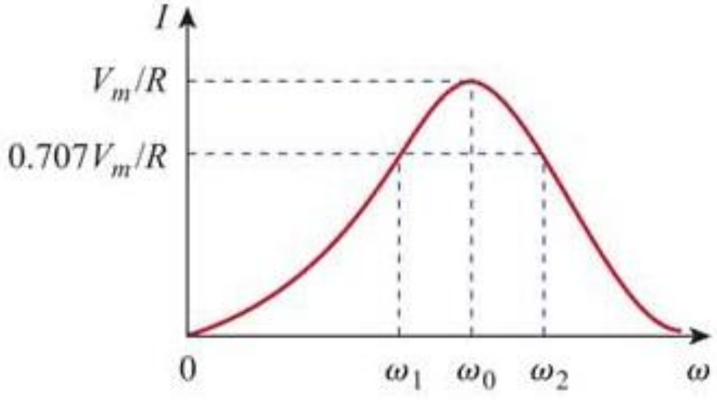
Impedance is purely resistive, **Z** = R

The voltage $\mathbf{V_s}$ and the current **I** are in phase

The magnitude of the transfer function $H(\omega)$ = Z(ω) is minimum

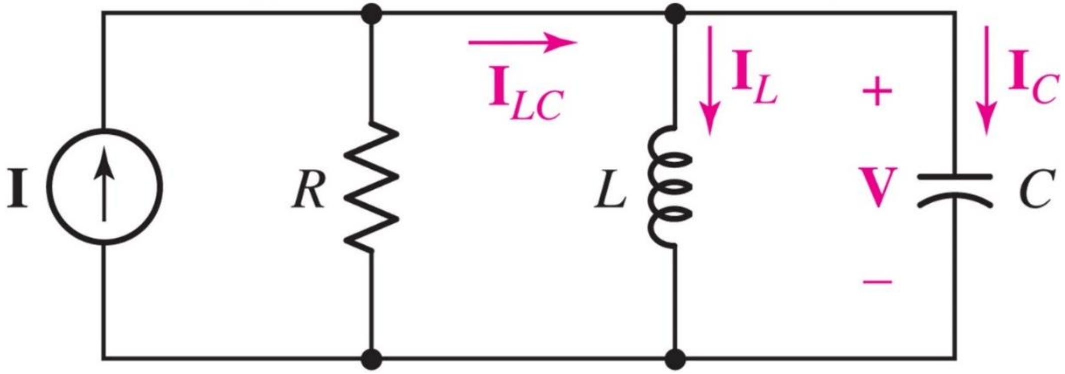
The frequency response of the circuit's current magnitude

$$I = |I| = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



Resonance: Parallel *RLC*

A network is in *resonance* when the voltage and current at the network input terminals are in phase.



$$Y = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

The resonant frequency is $$\omega_o = \frac{1}{\sqrt{LC}}$$

The resonant condition may be achieved by adjusting L, C or **ω**

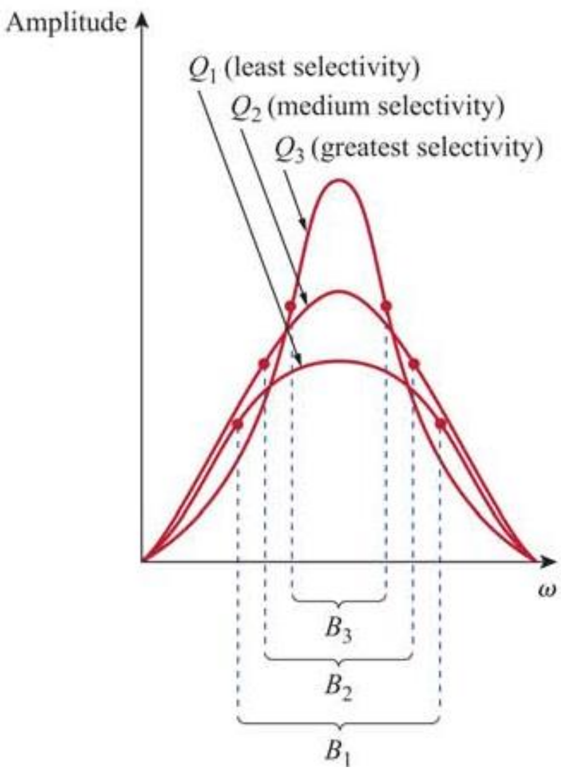
Variable ω

The Quality Factor *Q*

Height of the response curve – depends on R for a constant amplitude excitation

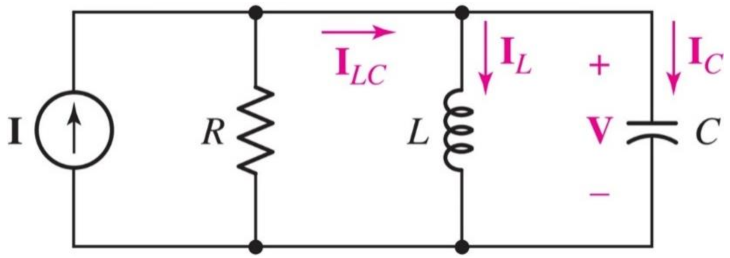
Width of the curve and steepness of the sides - depends on L and C

Sharpness – determined by the maximum amount of the energy that can be stored in the circuit, compared with the energy that is lost during one complete period



$$Q = \text{ quality factor } \equiv 2\pi \frac{\text{maximum energy stored}}{\text{total energy lost per period}}$$

For the parallel RLC circuit, the quality factor at resonance is



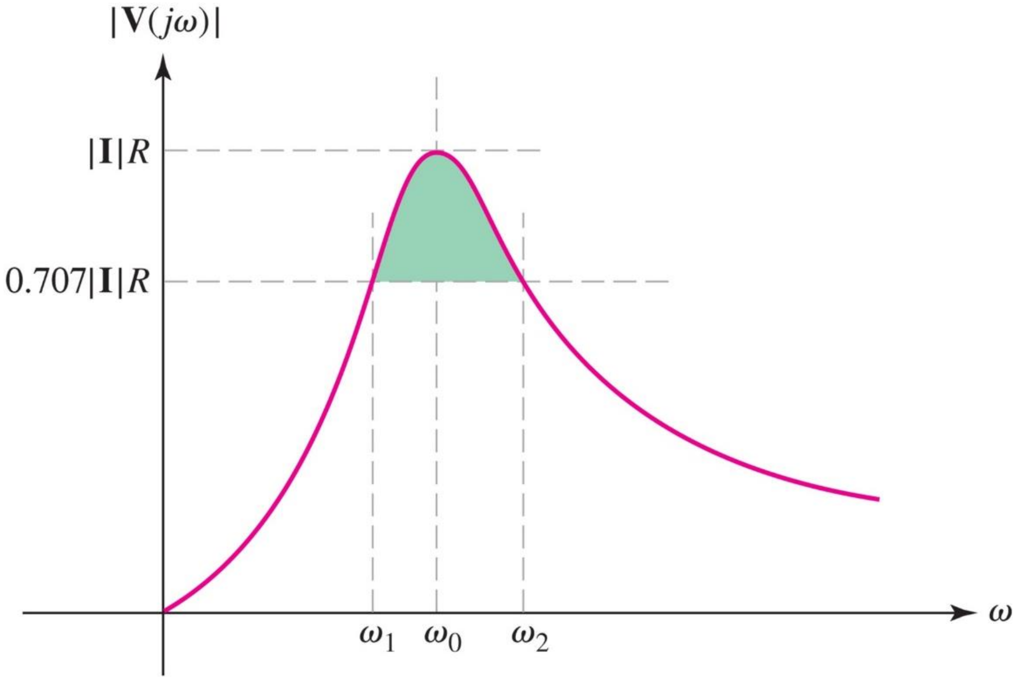
$$Q_0 = 2\pi f_0 RC = \omega_0 RC$$

Bandwidth

Bandwidth - Width of the curve - depends on L and C

ω_1: the lower half-power frequency

ω_2: the upper half-power frequency.



The (half-power) bandwidth is defined as the difference of these two half-power frequencies:

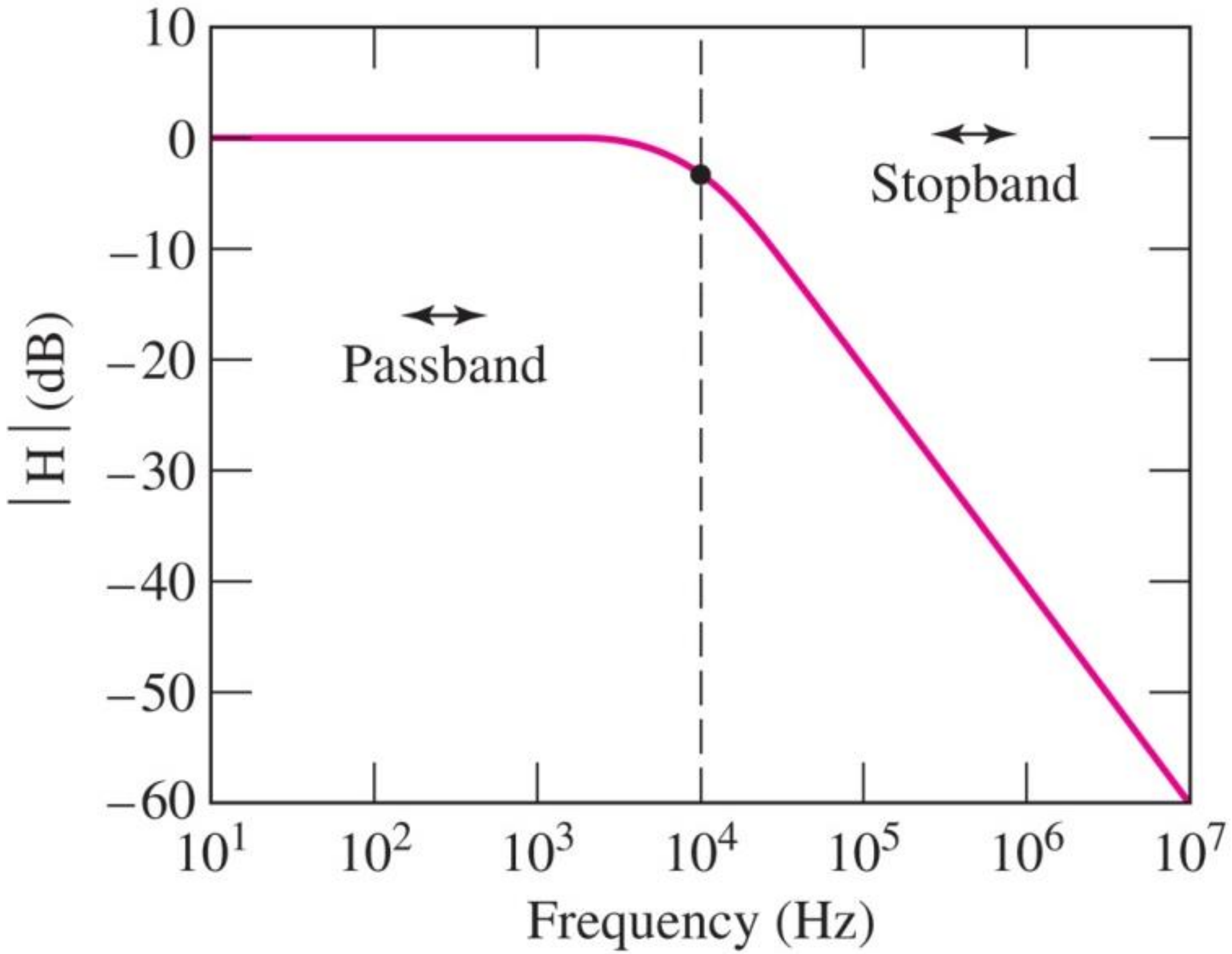
$$B = \omega_2 - \omega_1$$

Filters

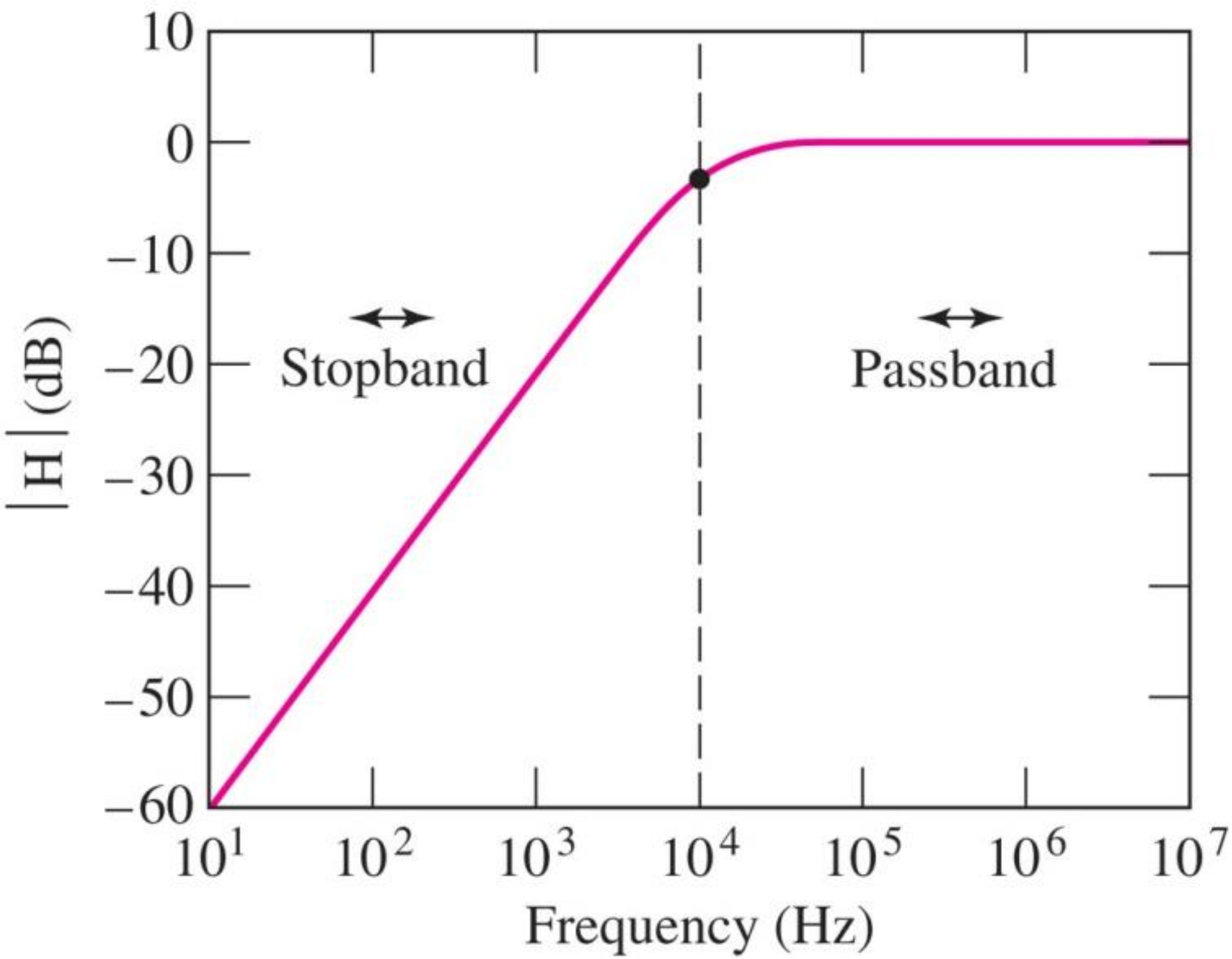
A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others

A filter is a passive filter if it consists of only passive elements R, L, and C

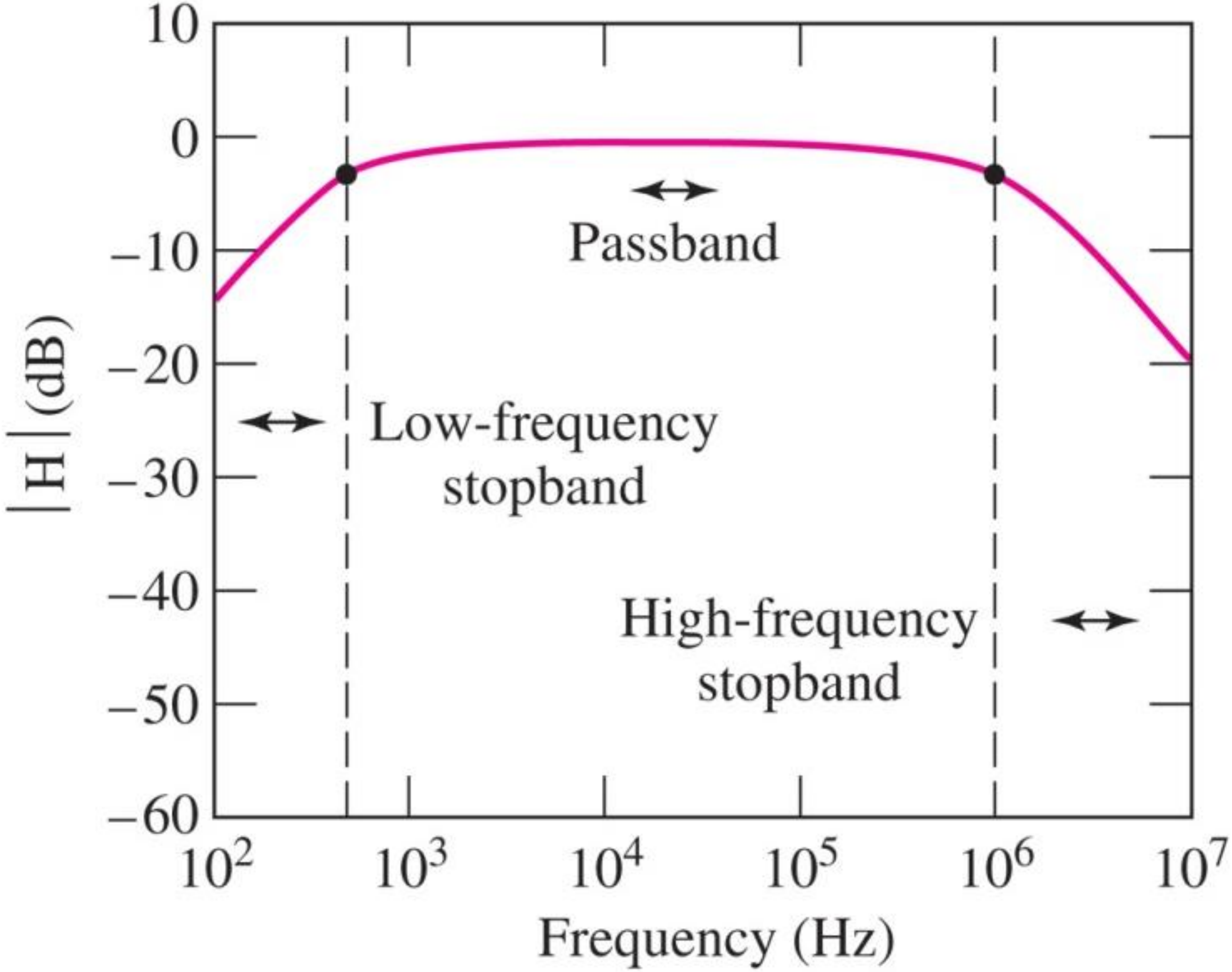
Filters: The Lowpass filter



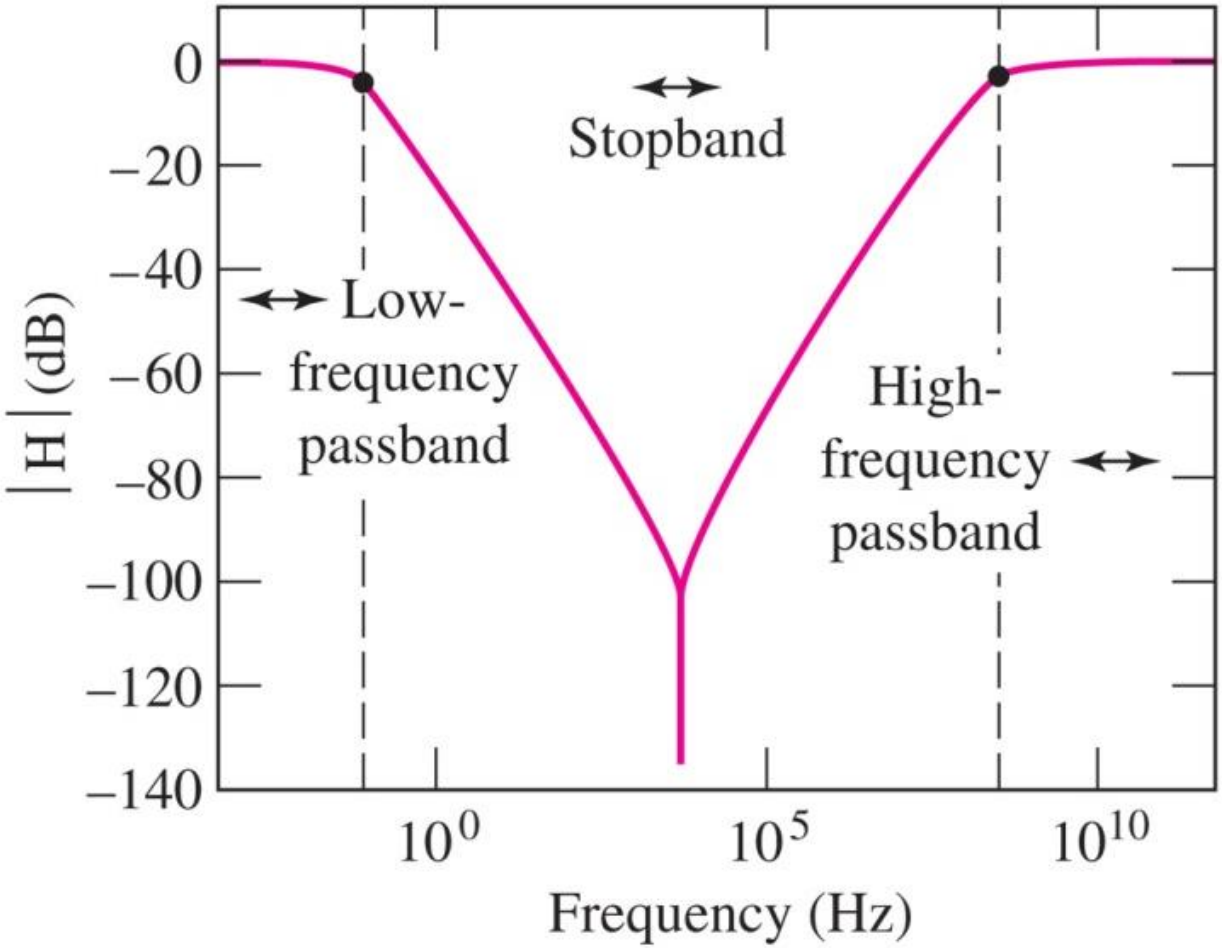
Filters: The Highpass Filter



Filters: The Bandpass Filter



Filters: The Bandstop Filter



Filters

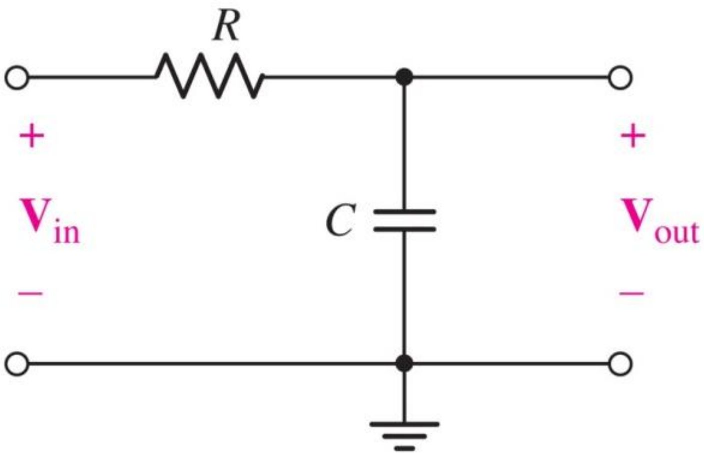
Summary of the characteristics of ideal filters.

Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

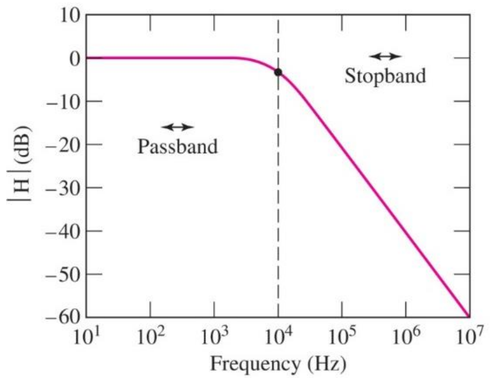
Passive Lowpass Filter

The transfer function is $\mathrm{H}(j\omega) = \dfrac{1}{1 + j\omega CR}$ and the corner frequency is $\omega_0 = 1/RC$.

The low pass filters are designed to pass frequencies from DC up to the cutoff ω_0 frequency



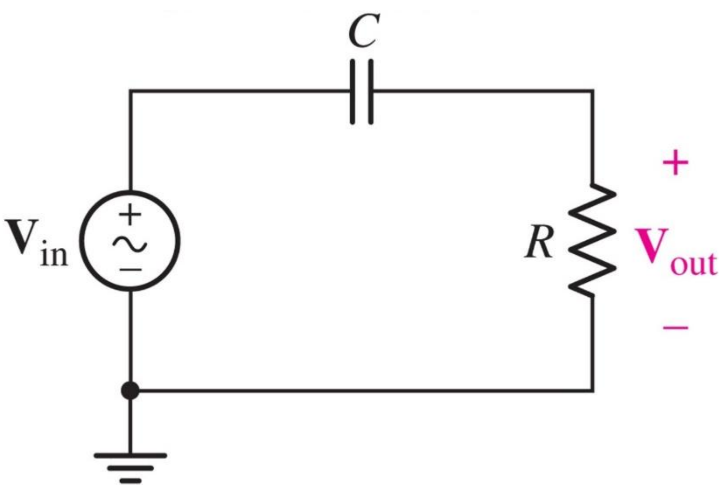
$$H = \frac{1}{\sqrt{1 + (\omega / \omega_0)^2}}$$

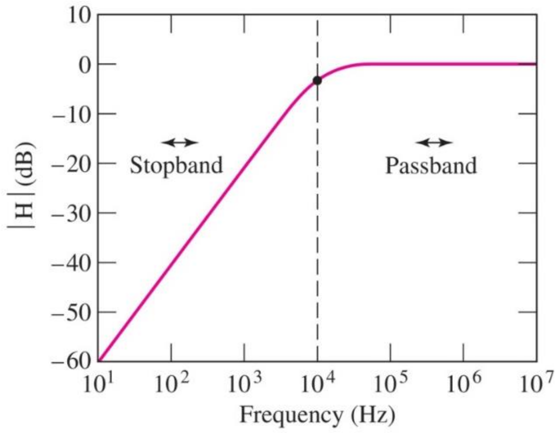


Passive Highpass Filter

The transfer function is $H(j\omega) = j\omega RC/(1+j\omega RC)$ and the corner frequency is $\omega_0 = 1/RC$.

A high pass filter is designed to pass all frequencies above its cutoff frequency, ω_0

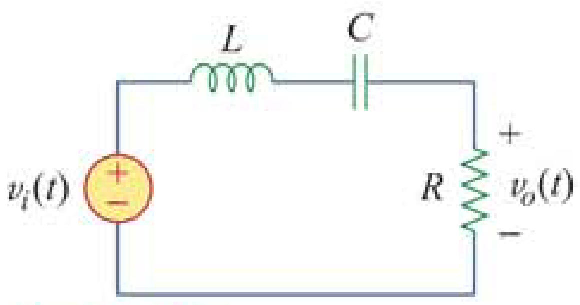


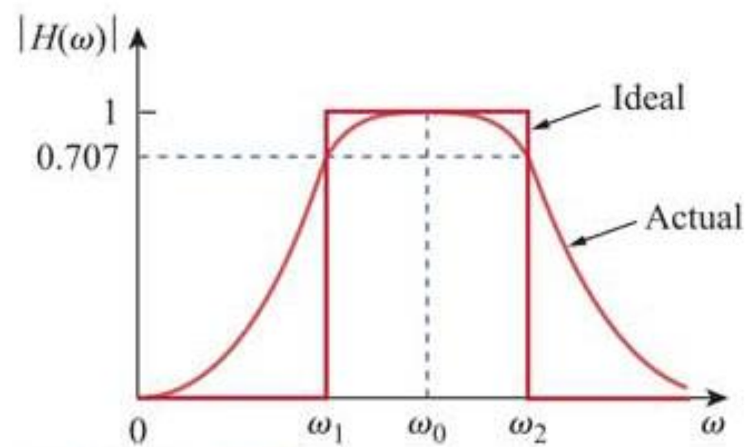


Passive Bandpass Filter

The transfer function is $H(j\omega) = \dfrac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$ and the corner frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$.

A high pass filter is designed to pass all frequencies above its cutoff frequency, ω_0

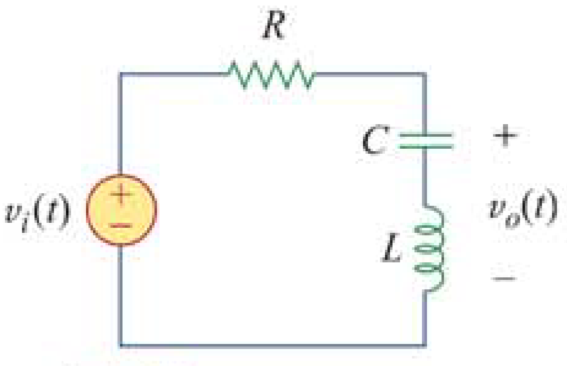


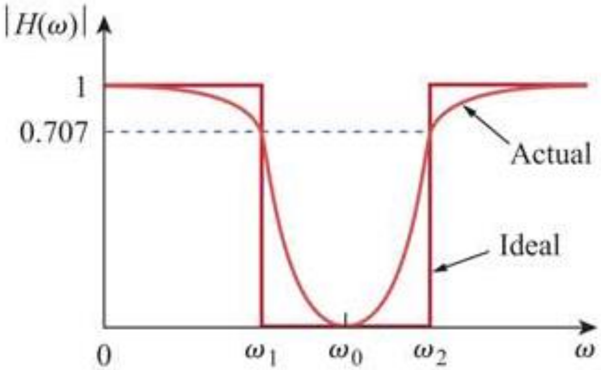


Passive BandStop Filter

The transfer function is $\mathrm{H}(j\omega) = \dfrac{j\left(\omega L - \frac{1}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$ and the corner frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$.

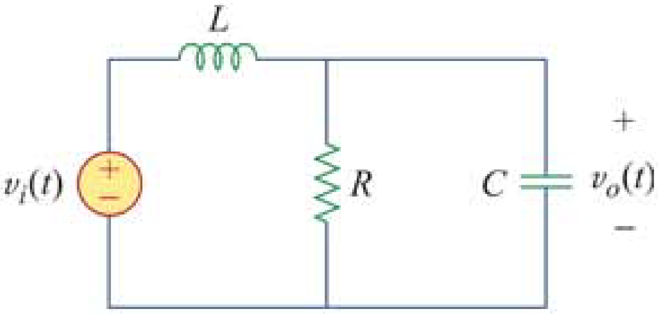
A high pass filter is designed to pass all frequencies above its cutoff frequency, ω_0





Filter

Determine what type of filter is shown in the Figure below. Calculate the corner or cutoff frequency. Consider R = 2 kΩ, L = 2H, C = 2μF



Filter