ENGI-1500
Physics -2

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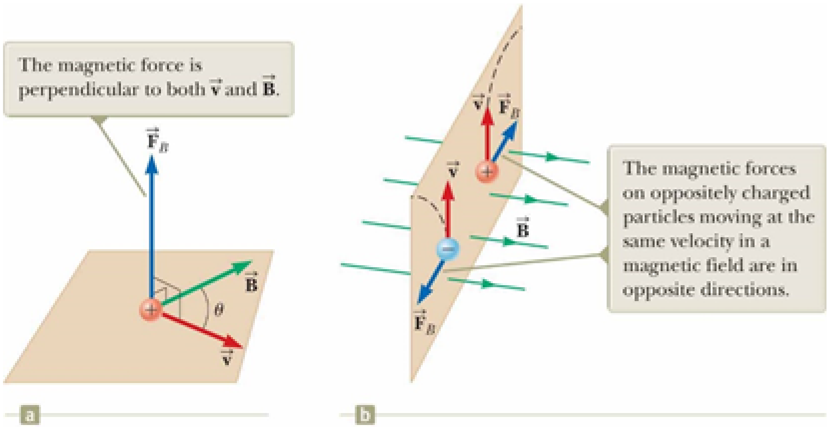
Faculty of Applied Sciences & Technology
Humber Institute of Technology and Advanced Learning
Winter 2023

HUMBER

Reminder of the previous week

Particle in a magnetic field



$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$1\ \mathrm{T} = 1\frac{\mathrm{N}}{\mathrm{C}\cdot\mathrm{m/s}}$$

$$1\ \mathrm{T} = 1\frac{\mathrm{N}}{\mathrm{A}\cdot\mathrm{m}}$$

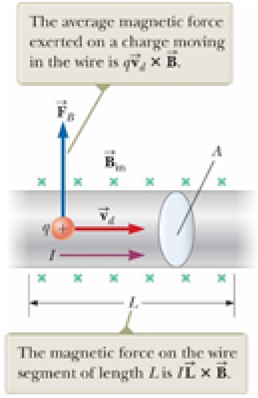
$$1\ \mathrm{T} = 10^4\ \mathrm{G}$$

Magnetic force on current

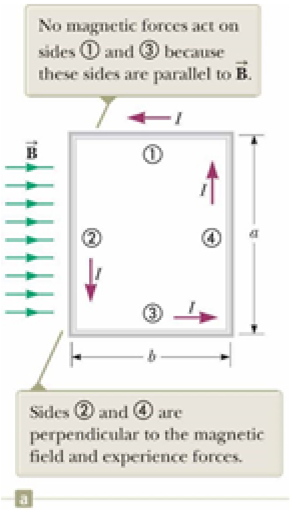
$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$

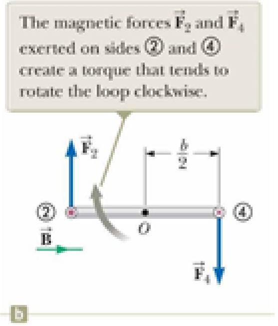
$$d\vec{\mathbf{F}}_B = I d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

$$\vec{\mathbf{F}}_B = I\int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$



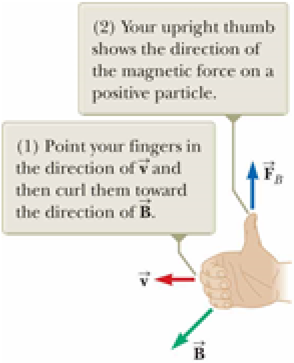
Torque on a current loop





$$\tau_{\max} = F_2\frac{b}{2} + F_4\frac{b}{2}$$
$$= (IaB)\frac{b}{2} + (IaB)\frac{b}{2}$$
$$= IabB$$

Right Hand Rule



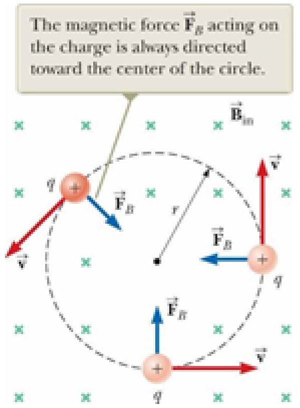
Motion of a charged particle

$$\sum F = F_B = ma$$

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB} \qquad \omega = \frac{v}{r} = \frac{qB}{m}$$

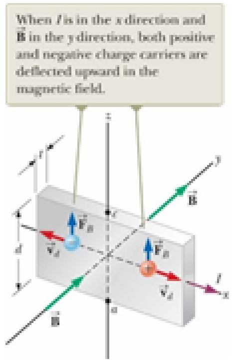
$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$



The Hall Effect

$$\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$$

$$R_H = \frac{1}{nq} \text{ (Hall coefficient)}$$

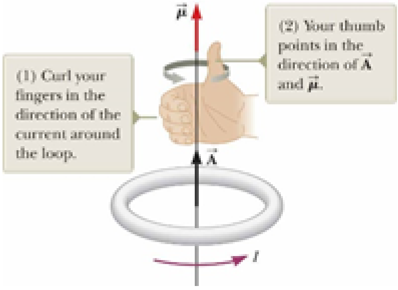


$$\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

$$\vec{\boldsymbol{\mu}} \equiv I\vec{\mathbf{A}}$$

$$\vec{\boldsymbol{\mu}}_{\text{coil}} = NI\vec{\mathbf{A}}$$

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$$



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Week 6 / Class 6

Sources of Magnetic Fields (Ch. 29)
Review

Outline of Week 6 / Class 6

- Reminder of the previous week

- Sources of Magnetic Fields (Ch. 29)
    - The Biot – Savart Law
    - The Magnetic Force Between Two Parallel Conductors
    - Ampère’s Law
    - The Magnetic Field of a Solenoid
    - Gauss’s Law in Magnetism
    - Magnetism in Matter [Reading from Textbook]

- Review

- Examples
- Next week’s topic

Sources of Magnetic Fields (Ch. 29)

The Biot – Savart Law

The Biot – Savart Law

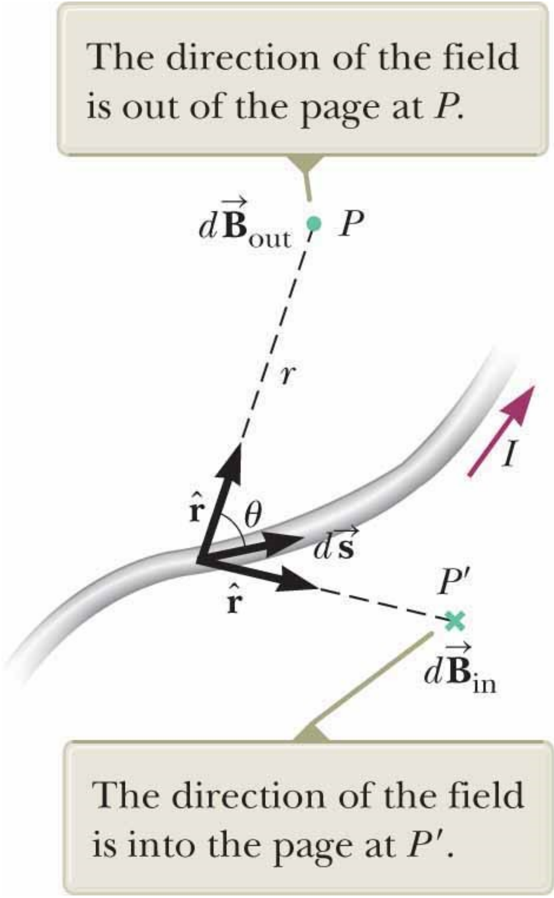
- ***Hans Christian Ørsted*** discovered that electric currents create magnetic fields (1819 - 1820).
  - Oersted found that electric current in a wire changed the orientation of a nearby compass needle.

- ***Jean-Baptiste Biot*** and ***Félix Savart*** performed quantitative experiments on the force exerted by an electric current on a nearby magnet.

- Biot and Savart arrived at a mathematical expression that gives the magnetic field in terms of the current that produces the field.

- Findings of their experiments were as follows (for magnetic field ***dB*** at point ***P*** associated with a length element of ***ds*** on a wire carrying steady current ***I***):

  - ***dB*** is perpendicular to both ***ds*** (points in the direction of the current) and the unit vector ***r*** (directed from ***ds*** toward ***P***).
  - The magnitude of ***dB*** is inversely proportional to $\mathbf{r^2}$ (r = distance from ***ds*** to ***P***).
  - The magnitude of ***dB*** is proportional to the current ***I*** and the magnitude ***ds*** of length element ***ds.***
  - The magnitude of ***dB*** is proportional to ***sinΘ*** (Θ = angle between vectors ***ds*** and ***r***)



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The Biot – Savart Law

- The experimental observations are summarized in the mathematical expression known today as ***the Biot–Savart law*** (for a small derivative element of *ds*):

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

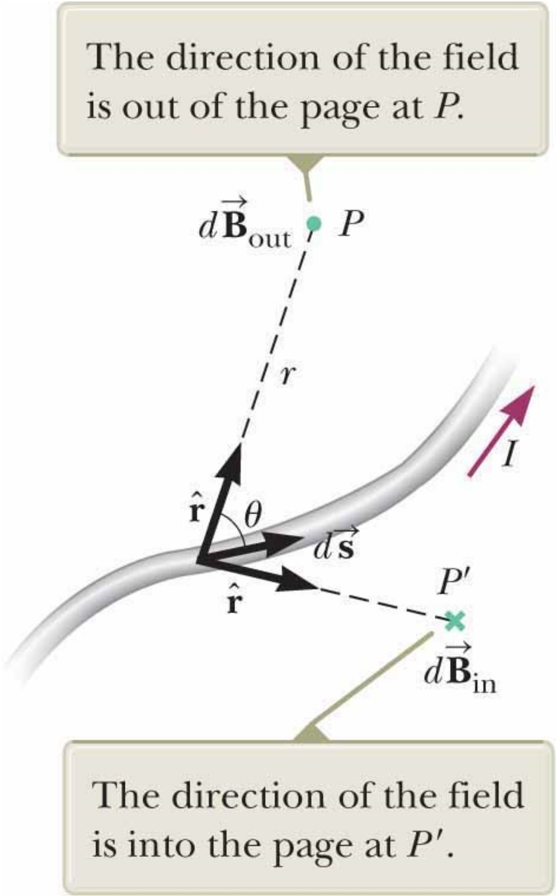
- μ_0 is a constant called the permeability of free space;

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

- To find the total magnetic field B created at some point by a current of finite size, we must sum up the contributions from all current elements I*ds* that make up the current. That is, we must take the integral of the expression over the entire current distribution;

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

The integrand is a cross product and therefore a vector quantity.



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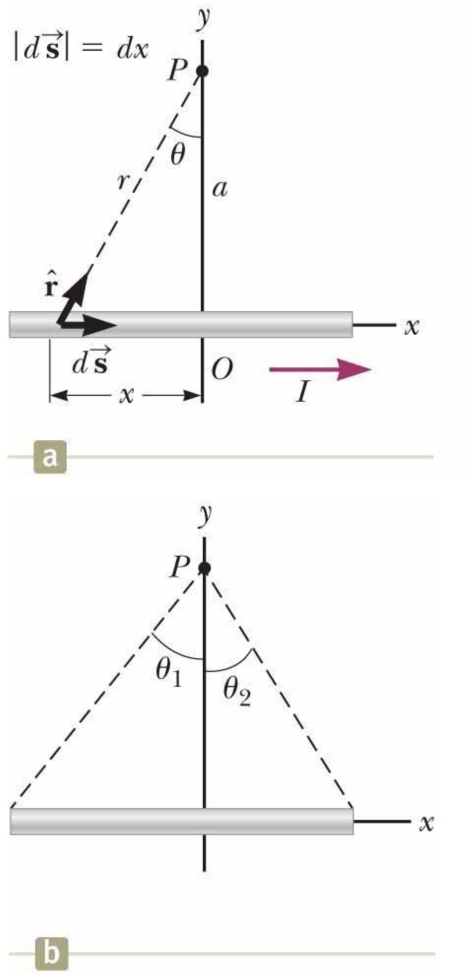
Magnetic field surrounding a thin, straight conductor

Example 29.1

Consider a thin, straight wire of finite length carrying a constant current ***I*** and placed along the ***x axis*** as shown in the figures. Determine the ***magnitude and direction of the magnetic field*** at point ***P*** due to this current.

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2} \qquad \vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$



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Solution

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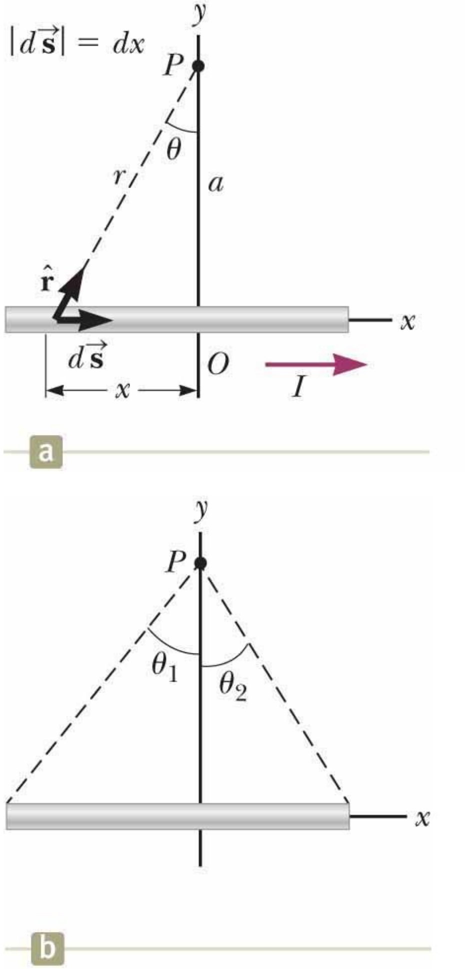
Conceptualize

Using the Biot – Savart law, we can find the magnetic field contribution from a small element of current and then integrate over the current distribution.



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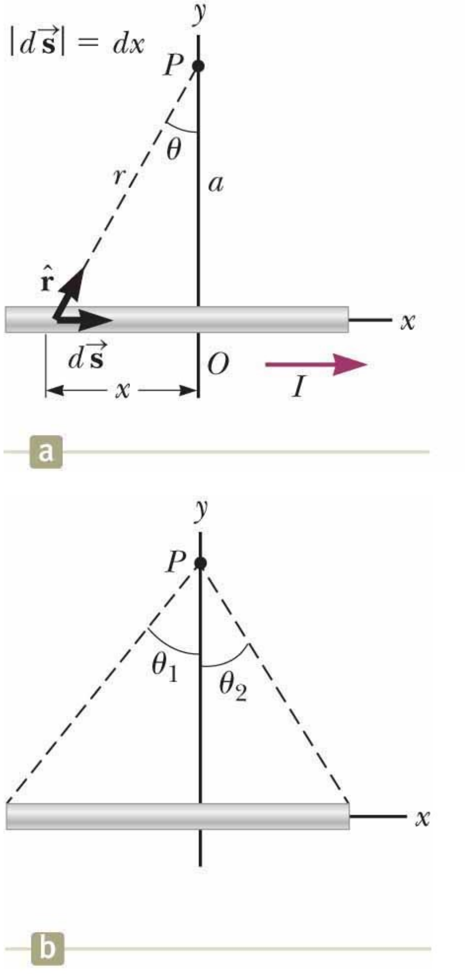

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Solution

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2} \qquad \vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

Analyze

Let's use the length element *ds* in the figure. The direction of the magnetic field at point ***P*** is out of the page.

Let's evaluate the cross product in the Biot – Savart law:

$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = |d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|\,\hat{\mathbf{k}} = \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right]\hat{\mathbf{k}} = (dx\cos\theta)\hat{\mathbf{k}}$$

Substituting into *dB* equation:

$$(1) \quad d\vec{\mathbf{B}} = (dB)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{\mathbf{k}}$$

We need to express *dx* and ***r*** in terms of ***ϴ***

r in terms of ***ϴ***:

$$(2) \quad r = \frac{a}{\cos\theta}$$

x in terms of ***ϴ***:

$$x = -a\tan\theta$$

Negative sign is necessary because *ds* is located at a negative value of x.

Magnetic field surrounding a thin, straight conductor

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Solution

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2} \qquad \vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

Analyze

Let's use the length element *ds* in the figure. The direction of the magnetic fi

$$d\vec{\mathbf{B}} = (dB)\,\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\,\hat{\mathbf{k}}$$

$$x = -a\tan\theta$$

Now we can write *dx* in terms of ***Θ***:

$$(3)\quad dx = -a\sec^2\theta\,d\theta = -\frac{a\,d\theta}{\cos^2\theta}$$

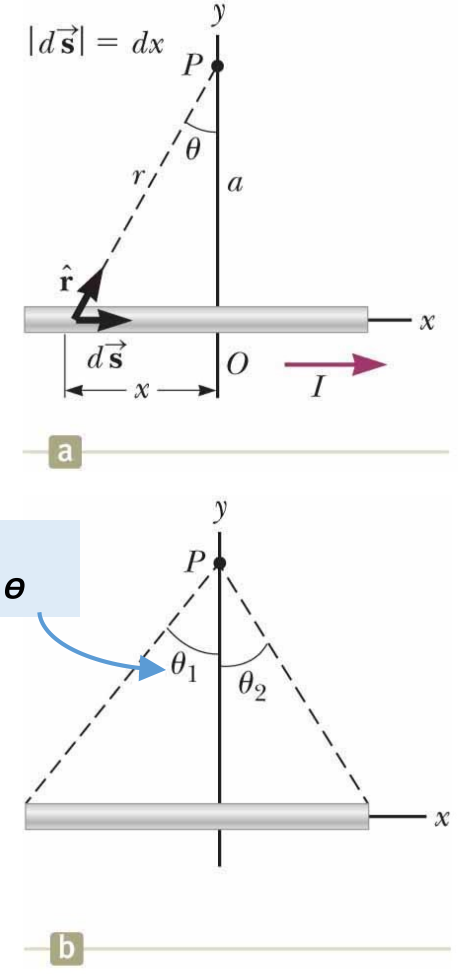
Substituting everything into the ***dB*** equation:

$$(4)\quad dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\,d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta = \boxed{-\frac{\mu_0 I}{4\pi a}\cos\theta\,d\theta}$$

Now we can integrate over ***Θ***

Integrating over ***Θ*** (from Θ_1 to Θ_2):

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$



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Solution

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2} \qquad \vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

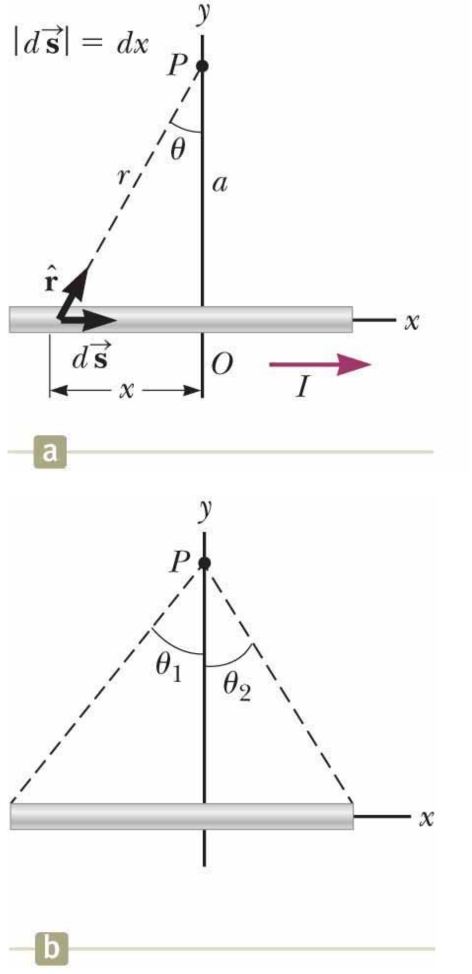
Finalize

We can use the final result to find the magnetic field of an infinitely long, straight wire by taking the angle ϴ to the limits: $\Theta_1=\pi/2$ (for $x=-\infty$) and $\Theta_1=-\pi/2$ (for $x=+\infty$).

$$\frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

$$(\sin\Theta_1 - \sin\Theta_2) = [\sin(\pi/2) - \sin(-\pi/2)] = 2$$

$$B = \frac{\mu_0 I}{2\pi a}$$



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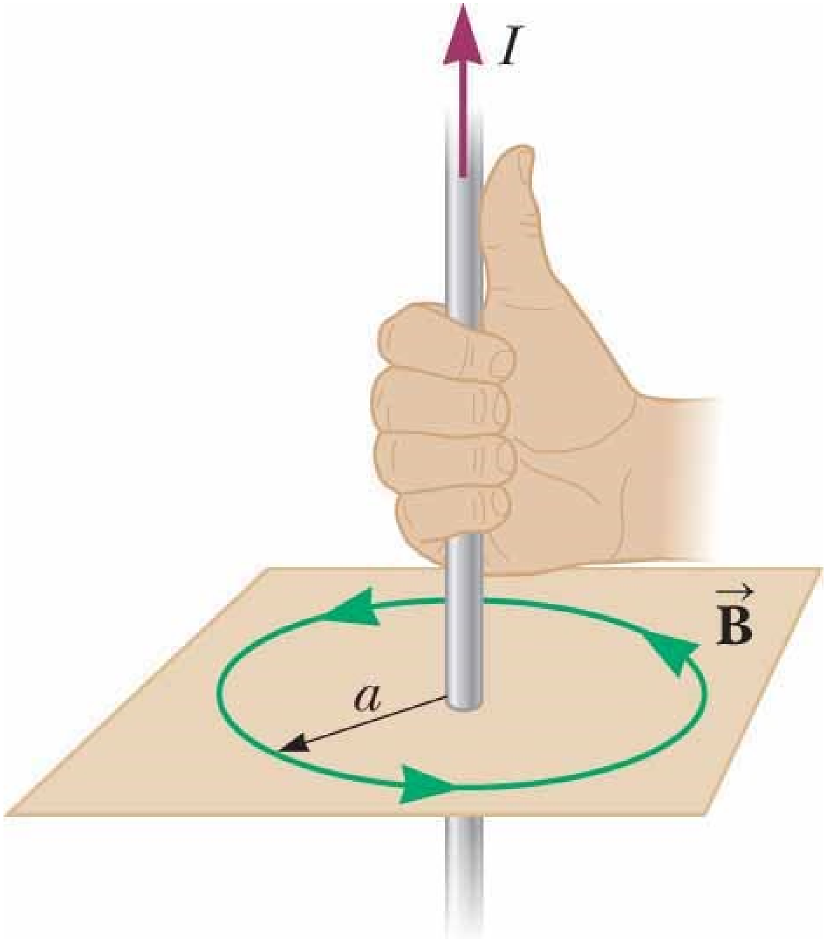
Right Hand Rule

- The magnetic field surrounding a long ,straight, current-carrying wire can be found with the ***right hand rule***:
  - As shown in the figure, circle the wire with the right hand by making sure the thumb is positioned along the direction of the current.
  - The fingers wrap in the direction of the magnetic field.
- The Magnetic field line circles are concentric with the wire and they lie in ***planes perpendicular to the wire***
- The magnitude of ***B*** is constant on any circle of radius ***a***
- The magnetic field lines form a closed loop.



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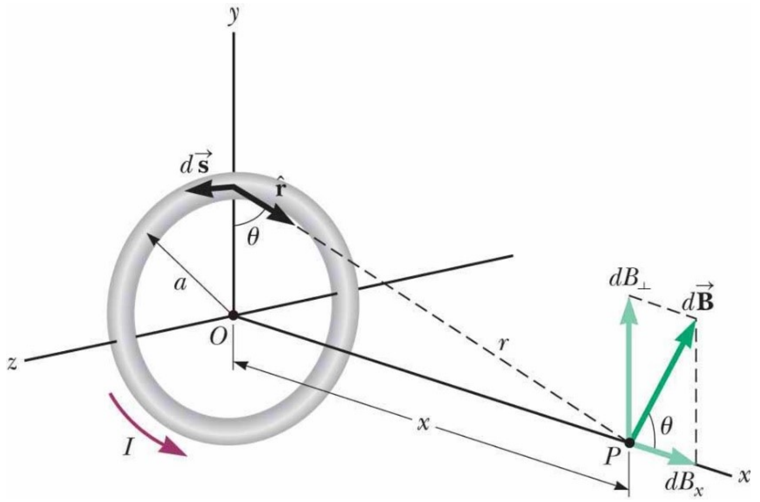
Magnetic field on the axis of a circular current loop

Example 29.3

Consider a circular wire loop of radius ***a*** located in the ***yz plane*** and carrying a steady current ***I*** as in the figure. Calculate the magnetic field at an axial point ***P*** a distance ***x*** from the center of the loop.

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2} \qquad \vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$



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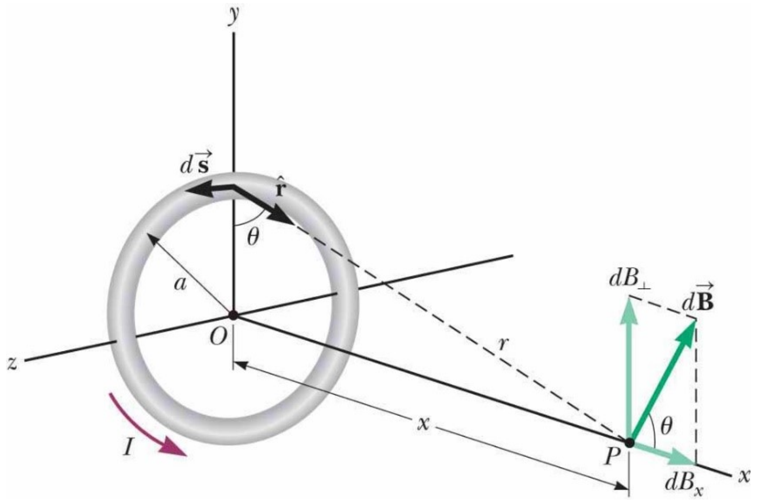
Solution

Every $d\vec{s}$ element is perpendicular to the vector $\hat{\mathbf{r}}$ at the location of the element. Therefore, $d\vec{s} \times \hat{\mathbf{r}} = (ds)(1)(\sin 90°) = \boldsymbol{ds}$

All length elements around the loop are at the same distance r from P: $\mathbf{r^2 = a^2 + x^2}$



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Solution

Use Equation 30.1 to find the magnitude of $d\vec{\mathbf{B}}$ due to the current in any length element $d\vec{\mathbf{s}}$:

$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{\mathbf{s}}\times\hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{(a^2+x^2)}$$

Find the x component of the field element:

$$dB_x = \frac{\mu_0 I}{4\pi}\frac{ds}{(a^2+x^2)}\cos\theta$$

Integrate over the entire loop:

$$B_x = \oint dB_x = \frac{\mu_0 I}{4\pi}\oint\frac{ds\cos\theta}{a^2+x^2}$$

From the geometry, evaluate $\cos\theta$:

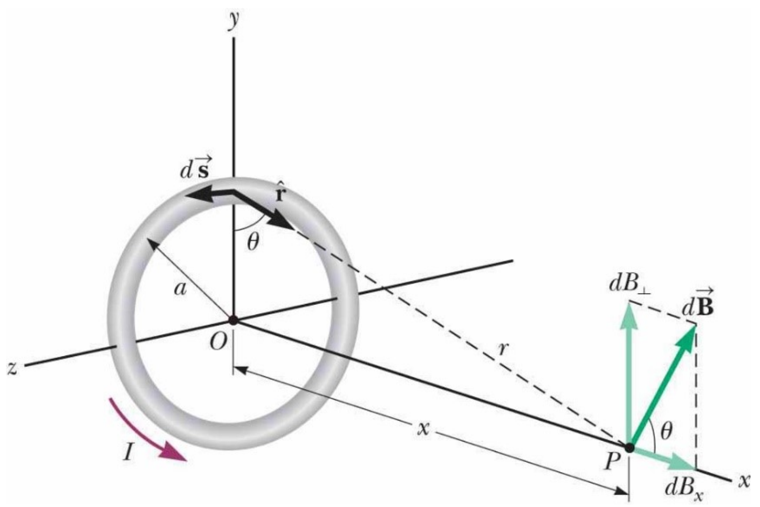
$$\cos\theta = \frac{a}{(a^2+x^2)^{1/2}}$$

Substitute this expression for $\cos\theta$ into the integral and note that x and a are both constant:

$$B_x = \frac{\mu_0 I}{4\pi}\oint\frac{ds}{a^2+x^2}\left[\frac{a}{(a^2+x^2)^{1/2}}\right] = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2+x^2)^{3/2}}\oint ds$$

Integrate around the loop:

$$B_x = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2+x^2)^{3/2}}(2\pi a) = \boxed{\frac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}}}$$



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Integrate around the loop:

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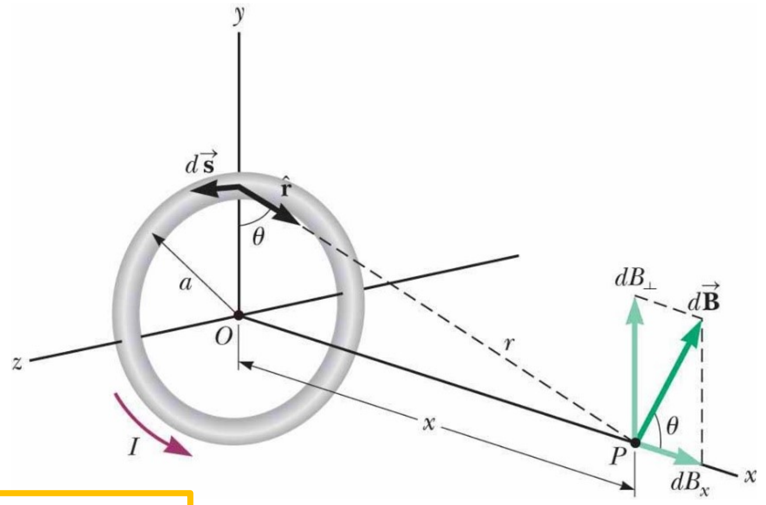

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Sources of Magnetic Fields (Ch. 29)

The Magnetic Force Between Two Parallel Conductors

The Magnetic Force Between Two Parallel Conductors

- Consider two long, straight, parallel wires separated by a distance ***a*** and carrying currents $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$ in the same direction.

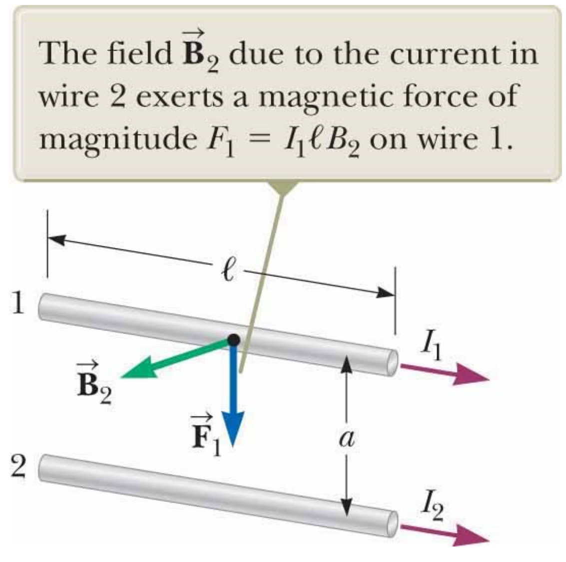
- Let's determine the force exerted on one wire due to the magnetic field set up by the other wire.

- Wire 2, which carries a current $\boldsymbol{I_2}$ and is identified arbitrarily as the source wire, creates a magnetic field $\boldsymbol{B_2}$ at the location of wire 1, the test wire.
  - The magnitude of this magnetic field is the same at all points on wire 1.
  - The direction of B_2 is perpendicular to wire 1 as shown in the figure.

- The magnetic force on a length ***l***, of wire 1 is:

$$\vec{\mathbf{F}}_1 = I_1 \vec{\ell} \times \vec{\mathbf{B}}_2 \xrightarrow{\boldsymbol{l} \text{ is perpendicular to } \boldsymbol{B_2}} F_1 = I_1 \ell B_2$$

$$F_1 = I_1 \ell B_2 = I_1 \ell \left(\frac{\mu_0 I_2}{2\pi a} \right) = \frac{\mu_0 I_1 I_2}{2\pi a} \ell$$



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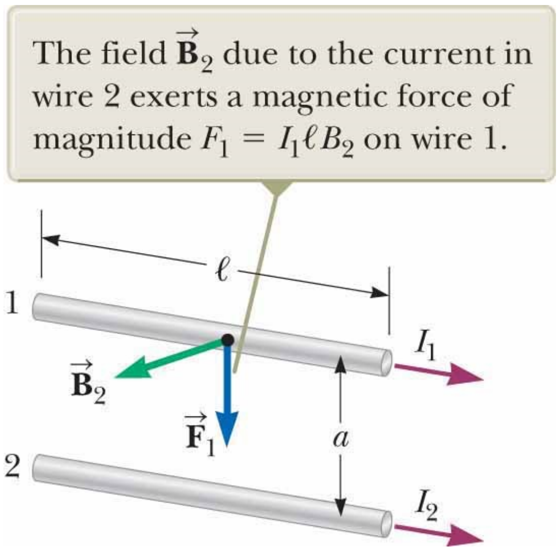
The Magnetic Force Between Two Parallel Conductors

- When the field set up at wire 2 by wire 1 is calculated, the force $\boldsymbol{F_2}$ acting on wire 2 is found to be equal in magnitude and opposite in direction to $\boldsymbol{F_1}$.
- When the currents are in opposite directions (that is, when one of the currents is reversed in the figure), the forces are reversed and the wires repel each other.
- Hence, parallel conductors carrying currents in the ***same*** direction ***attract*** each other, and parallel conductors carrying currents in ***opposite*** directions ***repel*** each other.
- Because the magnitudes of the forces are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply $\boldsymbol{F_B}$.
- This force can be expressed in terms of per unit length as follows:



$$F_1 = I_1 \ell B_2 = I_1 \ell \left(\frac{\mu_0 I_2}{2\pi a} \right) = \frac{\mu_0 I_1 I_2}{2\pi a} \ell \;\longrightarrow\; \frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Magnetic Force Between Two Parallel Conductors

Definition of the unit Ampere

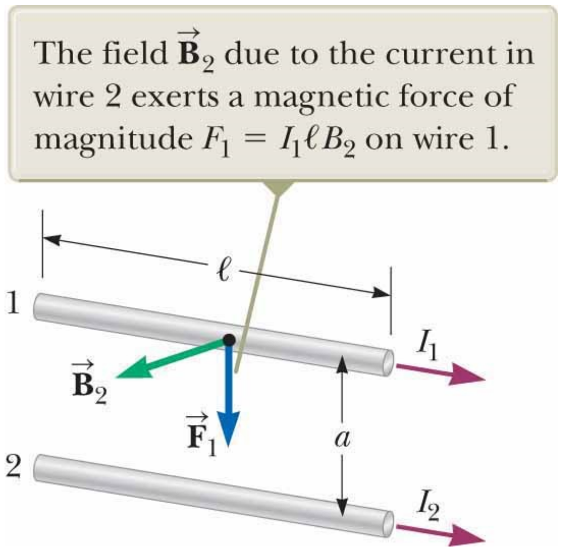
- When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by ***1m*** is ***2x10⁻⁷ N/m***, the current in each wire is defined to be ***1A***.

- The value $2\text{x}10^{-7}$ N/m is obtained from the magnetic force equation with $I_1 = I_2 = 1A$ and $a = 1m$.

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

- Because this definition is based on a force, a mechanical measurement can be used to standardize the ***ampere***.

- The National Institute of Standards and Technology uses an instrument called a ***current balance*** for primary current measurements. The results are then used to standardize other, more conventional instruments such as ammeters.



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Sources of Magnetic Fields (Ch. 29)

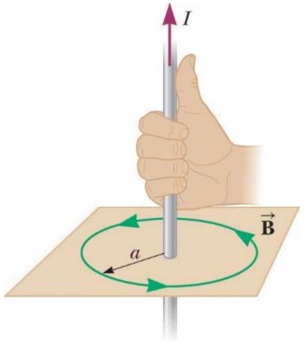
Ampere’s Law

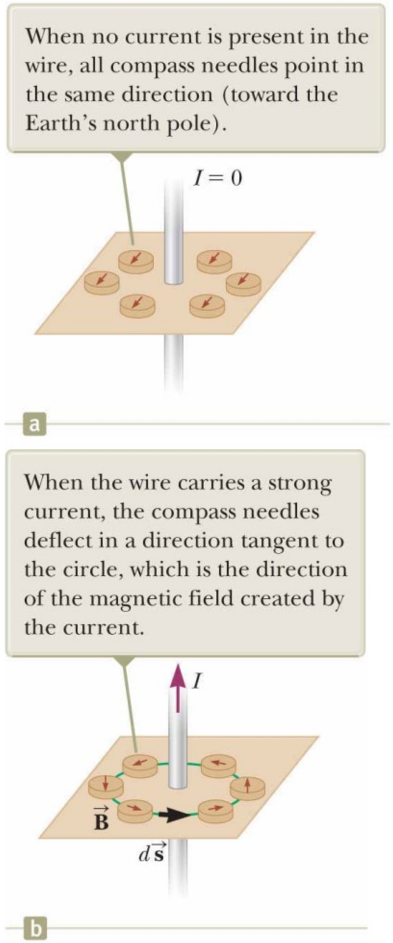
Ampere’s Law

- Earlier we discussed that the direction of the magnetic field produced by the current in the wire is consistent with the right-hand rule.
  - The magnetic field line has no beginning and no end. Rather, it forms a closed loop around the current carrying wire.

- Now let’s evaluate the product $\vec{B}\cdot d\vec{s}$ for a small length element $d\vec{s}$ on the circular path defined by the compass needles and sum the products for all elements over the closed circular path.
  - Along this path, the vectors $d\vec{s}$ and $\vec{B}$ are parallel at each point, so $\vec{B}\cdot d\vec{S} = Bds$
  - The magnitude of $\vec{B}$ is constant on this circle

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$





Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Ampere’s Law

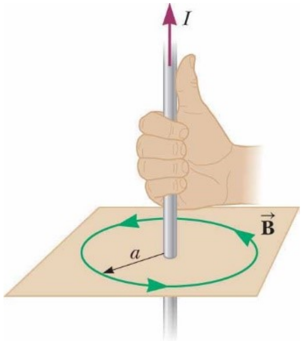
- Earlier we discussed that the direction of the magnetic field produced by the current in the wire is consistent with the right-hand rule.
  - The magnetic field line has no beginning and no end. Rather, it forms a closed loop around the current carrying wire.

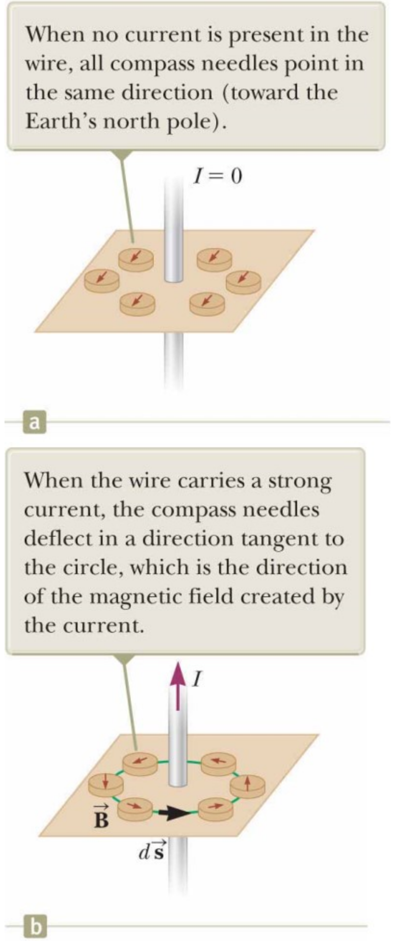
- Now let’s evaluate the product $\vec{B}\cdot\vec{ds}$ for a small length element $\vec{ds}$ on the circular path defined by the compass needles and sum the products for all elements over the closed circular path.
  - Along this path, the vectors $\vec{ds}$ and $\vec{B}$ are parallel at each point, so $\vec{B}\cdot\vec{dS} = Bds$
  - The magnitude of $\vec{B}$ is constant on this circle

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

The line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$$



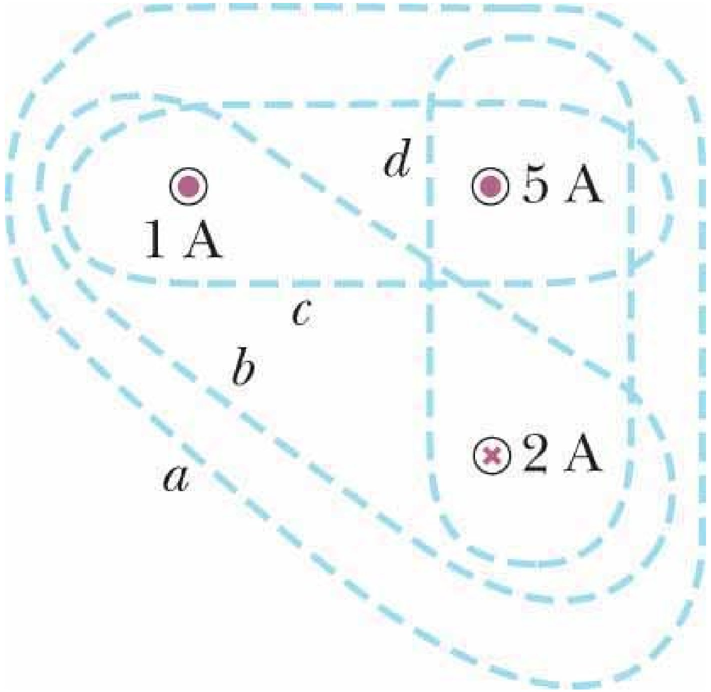


Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Ampere's Law

Quick Quiz 29.3

Rank the magnitudes of $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for the closed paths a through d in the figure from greatest to least.



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Ampere's Law

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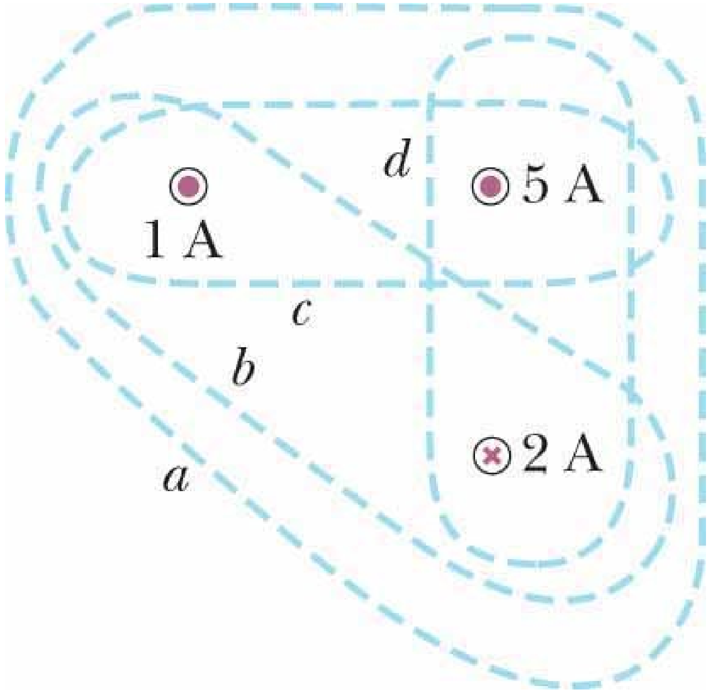


Solution

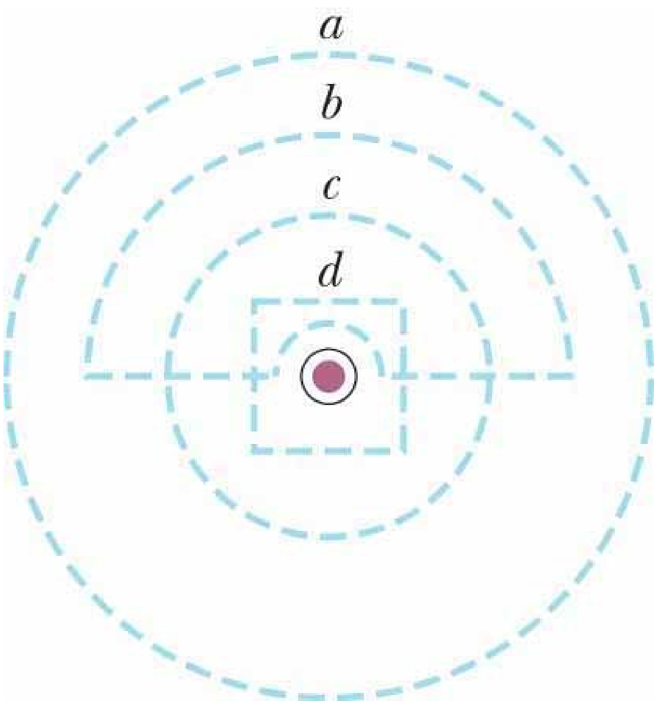
$c > a > d > b$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Ampere’s Law

Quick Quiz 29.4

Rank the magnitudes of $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for the closed paths a through d in the figure from greatest to least.

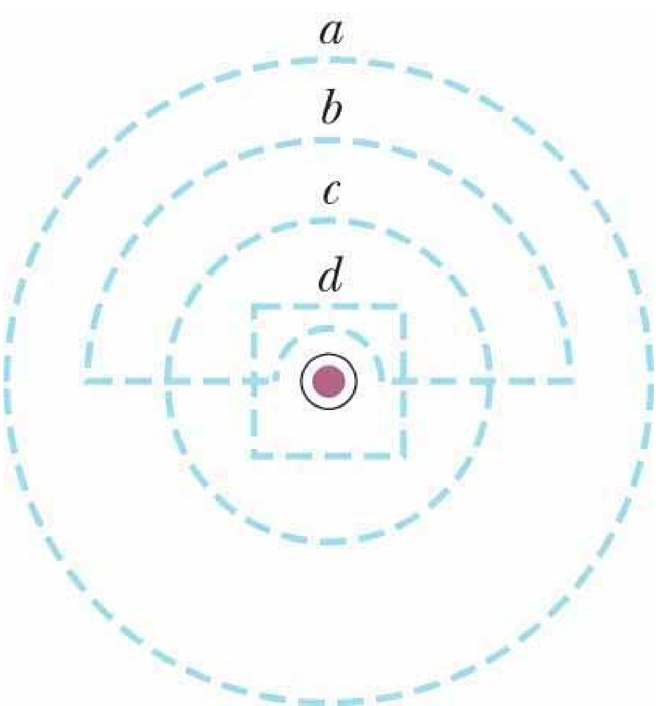


Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Ampere's Law

Quick Quiz 29.4

Rank the magnitudes of $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for the closed paths a through d in the figure from greatest to least.



Solution

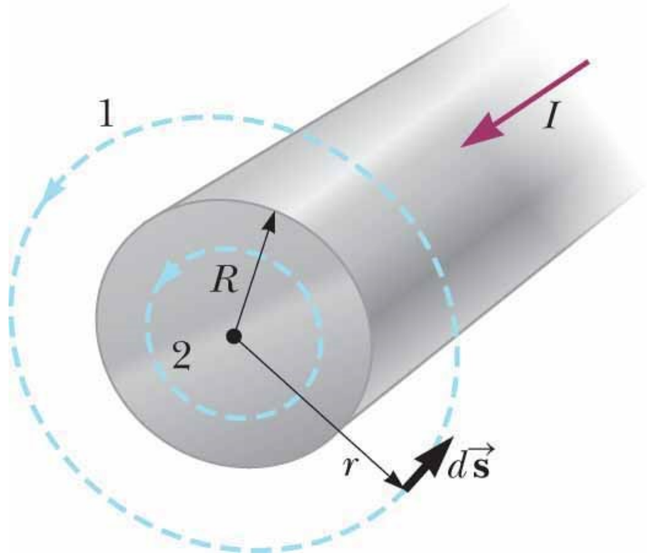
$$\boldsymbol{a = c = d > b = 0}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Magnetic Field Created by a Long Current-Carrying Wire

Example 29.5

A long, straight wire of radius $\boldsymbol{R}$ carries a steady current $\boldsymbol{I}$ that is uniformly distributed through the cross section of the wire. Calculate the magnetic field a distance $\boldsymbol{r}$ from the center of the wire in the regions $\boldsymbol{r \geq R}$ and $\boldsymbol{r < R}$.



<u>Source:</u> Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Magnetic Field Created by a Long Current-Carrying Wire

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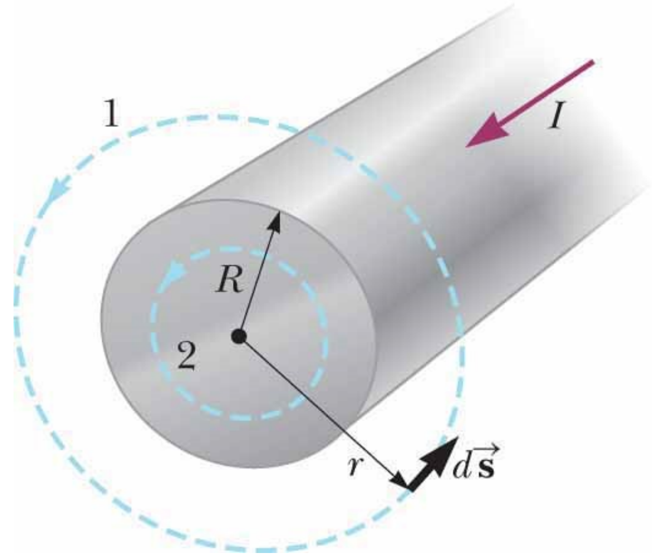
A long, straight wire of radius ***R*** carries a steady current ***I*** that is uniformly distributed through the cross section of the wire. Calculate the magnetic field a distance ***r*** from the center of the wire in the regions $r \geq R$ and $r < R$.

Solution

Conceptualize

Study the figure to understand the structure of the wire and the current in the wire. The current creates magnetic fields everywhere, both inside and outside the wire. Based on our discussions about long, straight wires, we expect the magnetic field lines to be circles centered on the central axis of the wire.



Categorize

Because the wire has a high degree of symmetry, we categorize this example as an Ampère's law problem. For the $r \geq R$ case, we should arrive at the same result as was obtained in the previous example, where we applied the Biot–Savart law to the same situation.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Magnetic Field Created by a Long Current-Carrying Wire

Example 29.5

A long, straight wire of radius ***R*** carries a steady current ***I*** that is uniformly distributed through the cross section of the wire. Calculate the magnetic field a distance ***r*** from the center of the wire in the regions ***r ≥ R*** and ***r < R***.

Solution

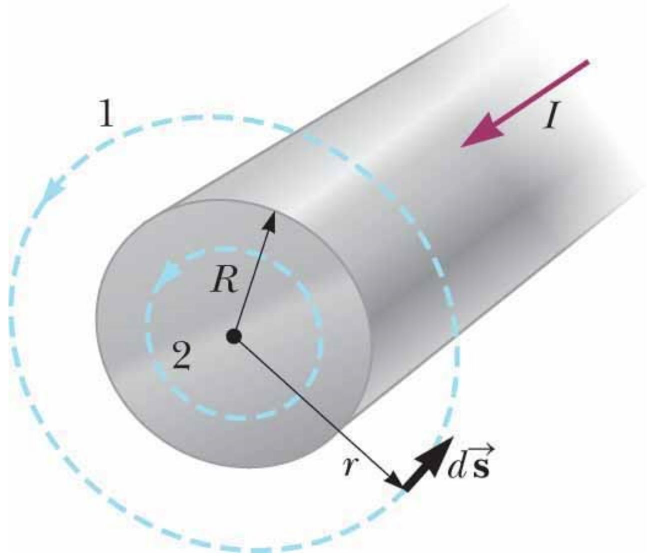
For the exterior of the wire

For the magnetic field exterior to the wire, let us choose for our path of integration ***circle 1*** in the figure. From symmetry, ***B*** must be constant in magnitude and parallel to ***ds*** at every point on this circle.

The total current passing through the plane of the circle is ***I*** and we can apply the Ampère's law:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = B(2\pi r) = \mu_0 I$$

$$B = \boxed{\frac{\mu_0 I}{2\pi r}} \quad (\text{for } r \geq R)$$



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

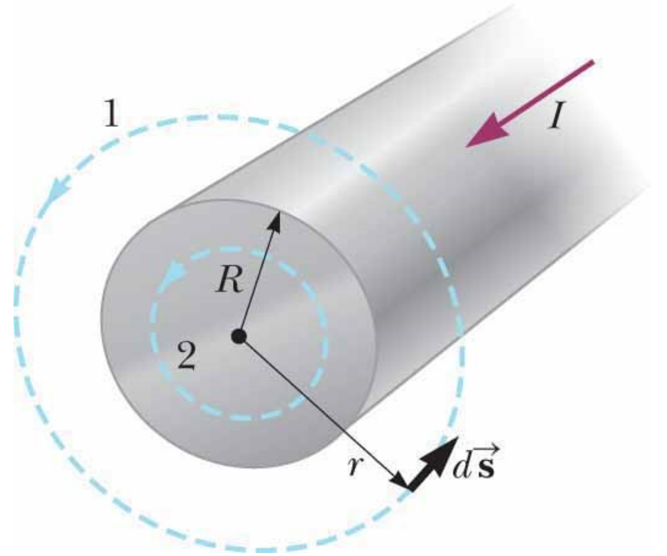
The Magnetic Field Created by a Long Current-Carrying Wire

Example 29.5

A long, straight wire of radius ***R*** carries a steady current ***I*** that is uniformly distributed through the cross section of the wire. Calculate the magnetic field a distance ***r*** from the center of the wire in the regions ***r ≥ R*** and ***r < R***.

Solution

For the interior of the wire



Now for the interior of the wire (r<R), the current ***I'*** passing through the plane of circle 2 is less than the total current ***I***. Because the current is uniformly distributed across the cross section of the wire, the current density ***J*** is constant in the interior of the wire. Therefore, for any area ***A*** of the interior perpendicular to the length of the wire, the current passing through that area is ***I' = JA***

$$\frac{I'}{I} = \frac{JA'}{JA} = \frac{\pi r^2}{\pi R^2}$$

$$\Rightarrow I' = \frac{r^2}{R^2} I$$

Now we can apply Ampere's law to circle 2:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I' = \mu_0 \left(\frac{r^2}{R^2} I \right) \longrightarrow B = \boxed{\left(\frac{\mu_0 I}{2\pi R^2} \right) r} \quad (\text{for } r < R)$$

<u>Source:</u> Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

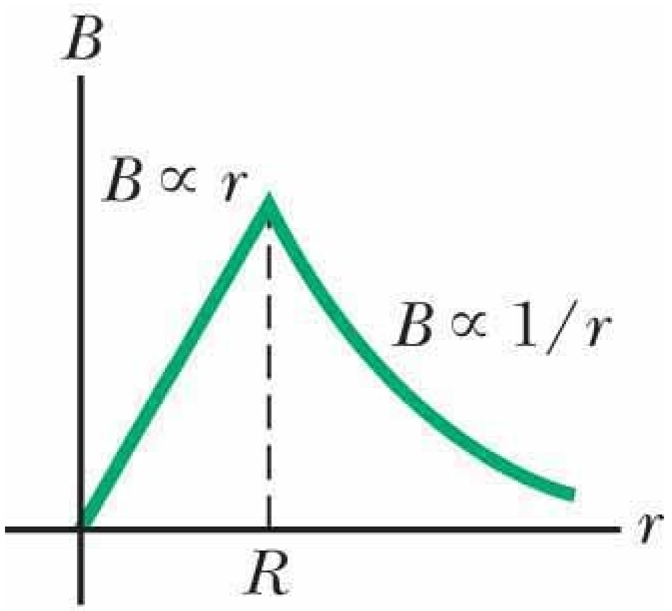
The Magnetic Field Created by a Long Current-Carrying Wire

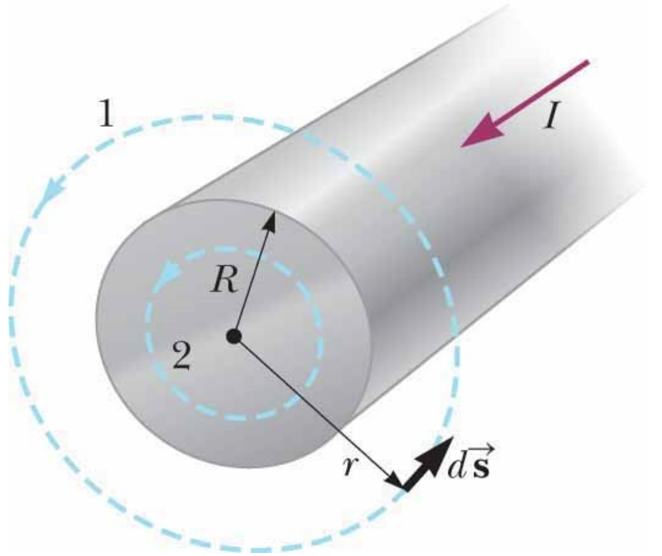
Example 29.5

A long, straight wire of radius ***R*** carries a steady current ***I*** that is uniformly distributed through the cross section of the wire. Calculate the magnetic field a distance ***r*** from the center of the wire in the regions ***r* ≥ *R*** and ***r* < *R***.

Solution

$$\left.\begin{aligned} B &= \frac{\mu_0 I}{2\pi r} \quad (\text{for } r \geq R) \\ B &= \left(\frac{\mu_0 I}{2\pi R^2}\right) r \quad (\text{for } r < R) \end{aligned}\right\} B = \frac{\mu_0 I}{2\pi R}$$





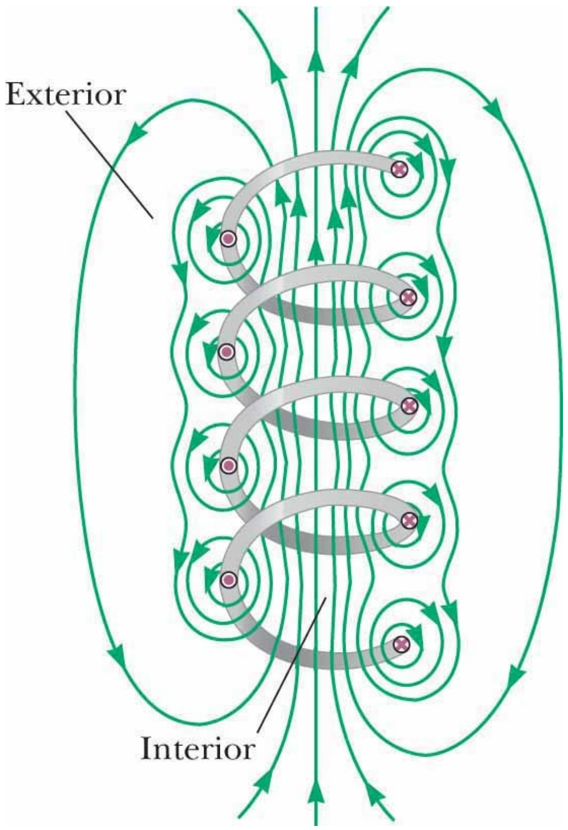
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Sources of Magnetic Fields (Ch. 29)

The Magnetic Field of a Solenoid

The Magnetic Field of a Solenoid

- A ***solenoid*** is a long wire wound in the form of a helix.
- With this configuration, a ***reasonably uniform magnetic field*** can be produced in the space surrounded by the turns of wire—which we shall call the interior of the solenoid—when the solenoid carries a current.
- When the turns are closely spaced →
  - Each loop can be approximated as circular loop
  - Net magnetic field = vector sum of fields resulting from all turns
- Figure shows the magnetic field lines surrounding a loosely wound solenoid



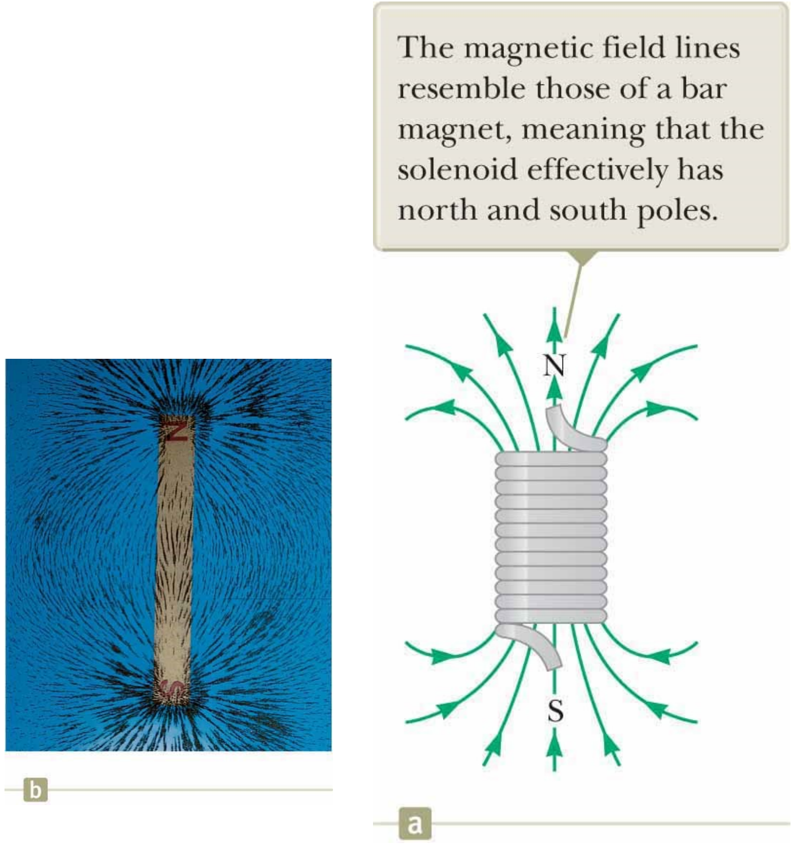
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Magnetic Field of a Solenoid

- If turns are closely spaced and the solenoid is of finite length → the external magnetic field is similar to that of a bar magnet

- As length of the solenoid increases →
  - Interior field becomes more uniform
  - Exterior field becomes weaker

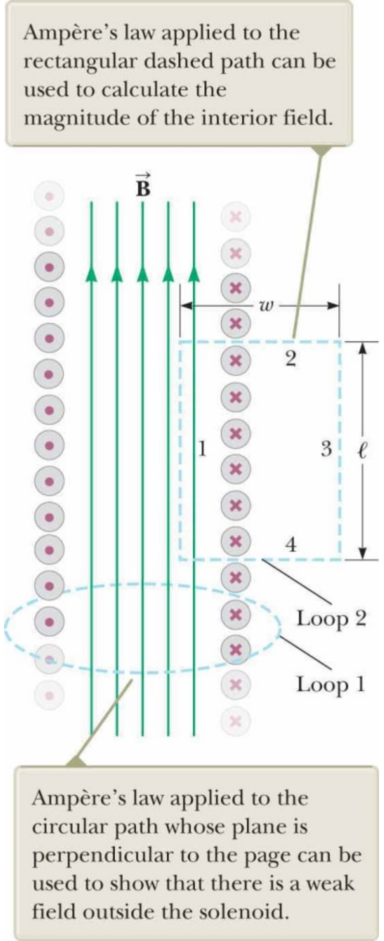
- ***Ideal solenoid***:
  - when the turns are closely spaced and
  - The length is much greater than the radius of turns



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Magnetic Field of a Solenoid

- Let’s try to find the magnetic field of a solenoid.
- Let’s focus on the external first.
- Consider the cross section of a solenoid in the figure. Let’s take the amperian loop (loop 1) perpendicular to the page, surrounding the ideal solenoid.
  - This loop encloses a small current as the charges in the wire move coil by coil along the length of the solenoid.
  - Therefore, there is a nonzero magnetic field outside the solenoid but it’s weak.
  - For an ideal solenoid, this weak field is the only field external to the solenoid.



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Magnetic Field of a Solenoid

- Now let's focus on the interior magnetic field.

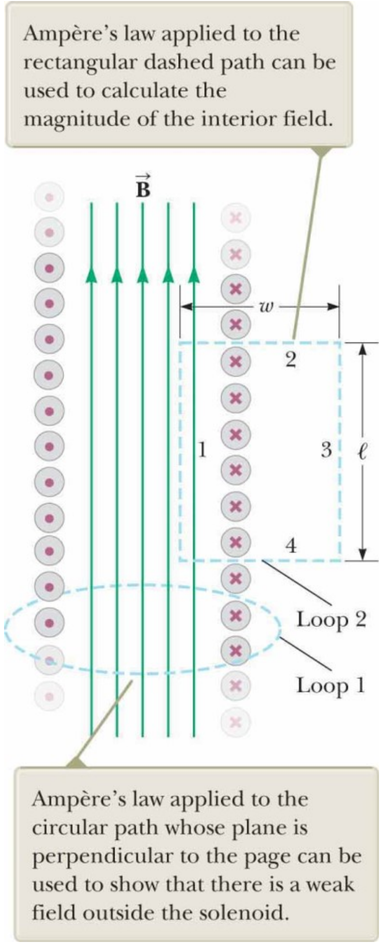
- We can use Ampère's law to obtain a quantitative expression for the interior magnetic field in an ideal solenoid. Because the solenoid is ideal, ***B*** in the interior space is uniform and parallel to the axis and the magnetic field lines in the exterior space form circles around the solenoid.

- Consider the rectangular path (loop 2) of length $\boldsymbol{\ell}$ and width ***w*** shown in the figure.

- Let's apply Ampère's law to this path by evaluating the integral of $\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{s}}$ over each side of the rectangle:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_{\text{path 1}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \int_{\text{path 1}} ds = B\ell$$

- Can we see that the contributions from path 2, 3 and 4 are all zero?



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

The Magnetic Field of a Solenoid

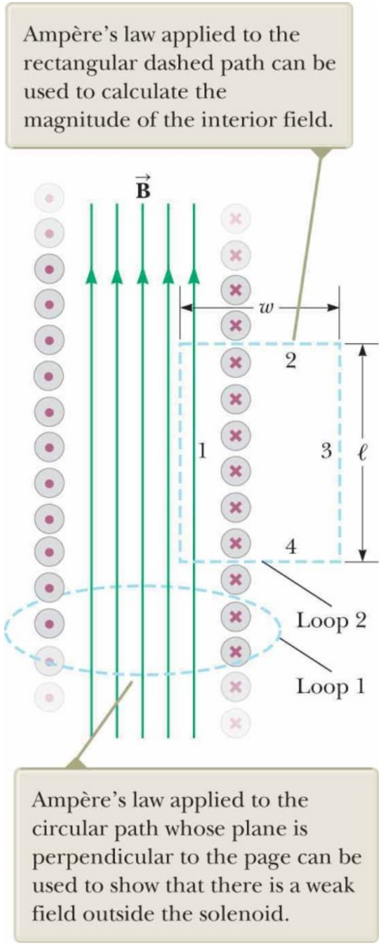
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_{\text{path 1}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \int_{\text{path 1}} ds = B\ell$$

- The right side of Ampère's law involves the total current ***I*** through the area bounded by the path of integration. In this case, the total current through the rectangular path equals the current through each turn multiplied by the number of turns. If ***N*** is the number of turns in the length, the total current through the rectangle is ***NI***:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_0 NI$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 nI$$

where ***n = N/ℓ***, is the number of turns per unit length.



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Sources of Magnetic Fields (Ch. 29)

Gauss's Law in Magnetism

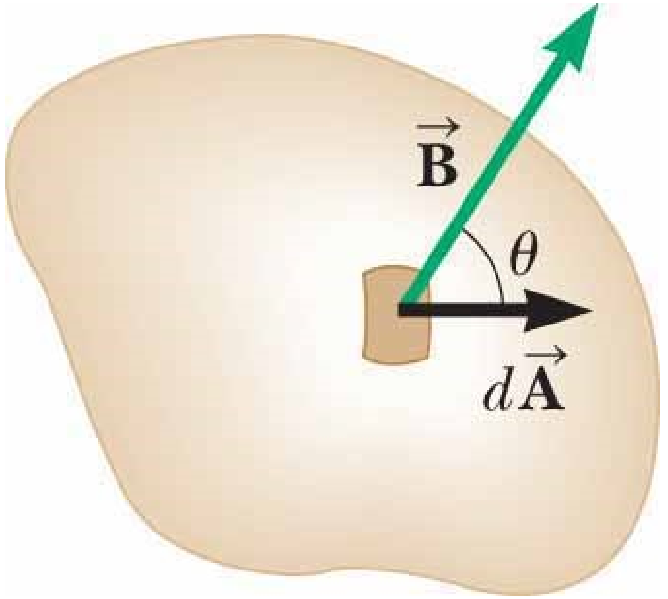
Gauss's Law in Magnetism

- We can define the magnetic flux similar to the definition of electric flux earlier.
- Consider an element of area ***dA*** on an arbitrarily shaped surface (figure)
- If the magnetic field at this element is ***B***:
  - Magnetic flux through this element is ***B·dA***
  - ***dA*** = vector perpendicular to the surface
- Total magnetic flux ***ΦB*** through the surface is:

$$\Phi_B \equiv \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

- Special case: plane of area ***A*** in a uniform field ***B*** that makes an angle ***ϴ*** with ***dA***
  - Magnetic flux through plane is:

$$\Phi_B = BA\cos\theta$$



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

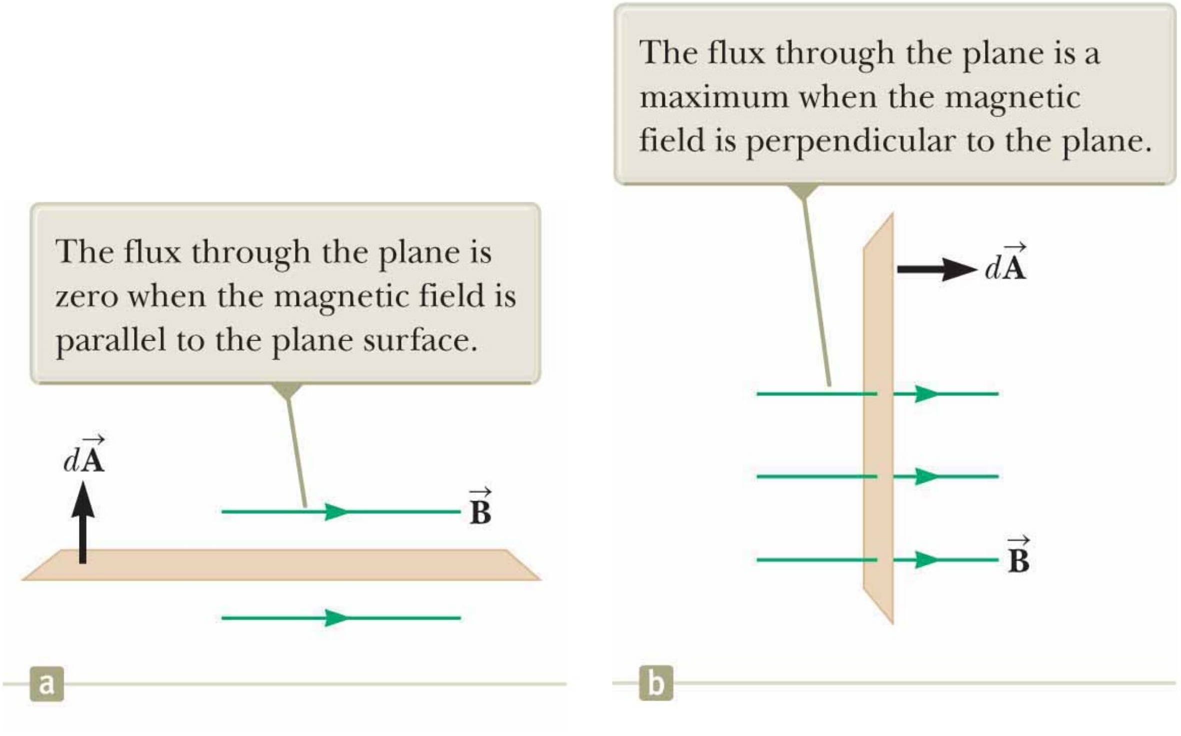
Gauss's Law in Magnetism

- If the magnetic field is parallel to the plane (figure (a)):
  - $\theta = 90°$
  - Dot product is zero → the flux through the plane = 0

- If the field is perpendicular to the plane (figure (b)):
  - $\theta = 0°$
  - Dot product is max → the flux through the plane = BA (the maximum value)

- Unit of magnetic flux is $T{\cdot}m^2$ → defined as ***weber (Wb)***

$$1\ \text{Wb} = 1\ \text{T} \cdot \text{m}^2$$

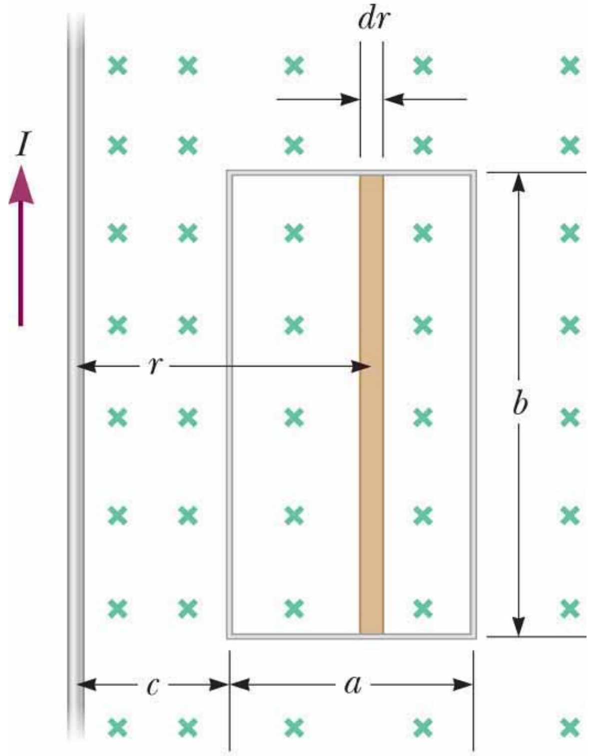


Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Magnetic Flux Through a Rectangular Loop

Example 29.7

A rectangular loop of width ***a*** and length ***b*** is located near a long wire carrying a current ***I***. The distance between the wire and the closest side of the loop is ***c***. The wire is parallel to the long side of the loop.
Find the ***total magnetic flux*** through the loop due to the current in the wire.



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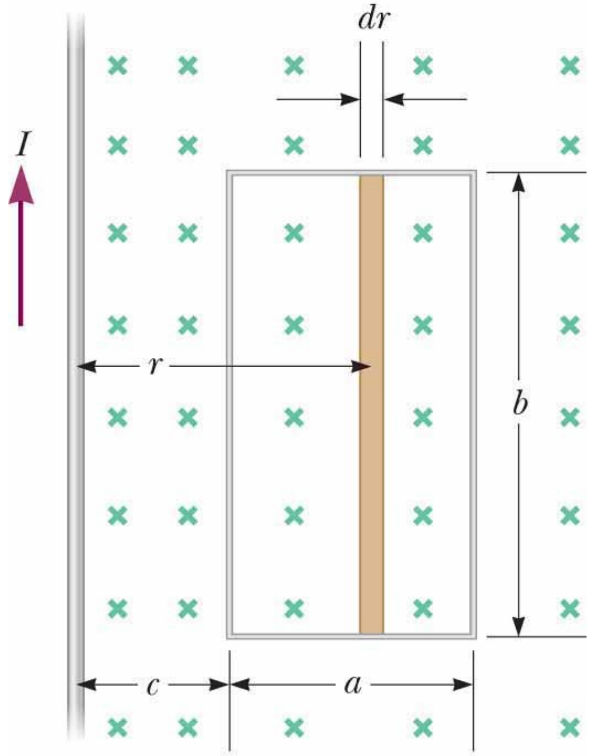

Solution

$$B = \boxed{\frac{\mu_0 I}{2\pi r}}$$

Conceptualize
The magnetic field lines due to the wire will be circles, many of which will pass through the rectangular loop. We know that the magnitude of the magnetic field is a function of distance ***r*** from a long wire. Therefore, the magnetic field varies over the area of the rectangular loop.

Categorize
Because the magnetic field varies over the area of the loop, we must ***integrate*** over this area to find the total flux. That identifies this as an analysis problem.

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Magnetic Flux Through a Rectangular Loop

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A rectangular loop of width ***a*** and length ***b*** is located near a long wire carrying a current ***I***. The distance between the wire and the closest side of the loop is ***c***. The wire is parallel to the long side of the loop.
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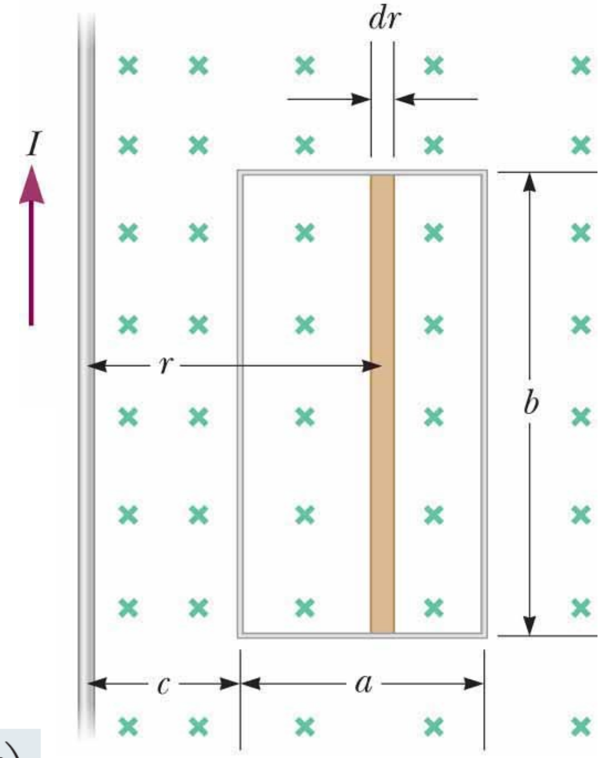


Solution

Noting that ***B*** is parallel to *d**A*** at any point within the loop, we can find the magnetic flux through the rectangular area:

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA = \int \frac{\mu_0 I}{2\pi r}\,dA$$

We can now express the area element (the rectangular strip in the figure) as a *d**A** = b dr* **:**

$$\Phi_B = \int \frac{\mu_0 I}{2\pi r}\,b\,dr = \frac{\mu_0 I b}{2\pi}\int \frac{dr}{r}$$

Integrating from r=c to r=a+c :

$$\Phi_B = \frac{\mu_0 I b}{2\pi}\int_c^{a+c} \frac{dr}{r} = \frac{\mu_0 I b}{2\pi}\ln r\Big|_c^{a+c}$$

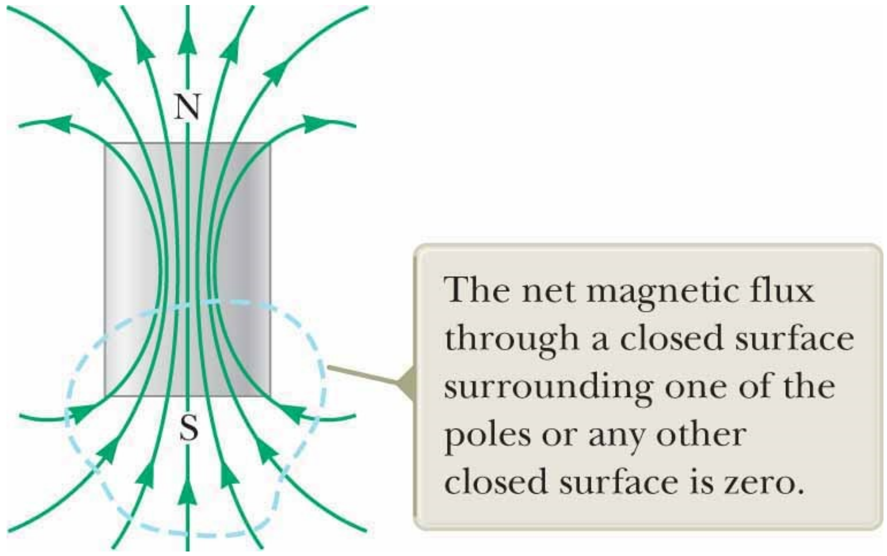
$$= \frac{\mu_0 I b}{2\pi}\ln\left(\frac{a+c}{c}\right) = \boxed{\frac{\mu_0 I b}{2\pi}\ln\left(1+\frac{a}{c}\right)}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Gauss’s Law in Magnetism

The net magnetic flux through any closed surface is always zero: $$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

- We learned that the magnetic fields are continuous and form closed loops
- Figure: magnetic field lines do not begin or end at any point
- For any closed surface :
    - Number of lines entering surface = number leaving surface
    - Net magnetic flux = 0



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Sources of Magnetic Fields (Ch. 29)

Magnetism in Matter [Reading from Textbook]

Magnetism in Matter

Optional

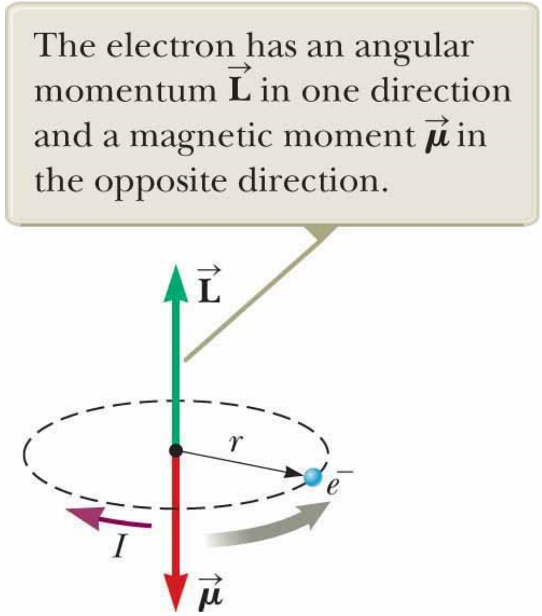


TABLE 29.1 Magnetic Moments of Some Atoms and Ions

Atom or Ion	Magnetic Moment (10^{-24} J/T)
H	9.27
He	0
Ne	0
Ce^{3+}	19.8
Yb^{3+}	37.1

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24}\ \text{J/T}$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Faraday’s Law (Ch. 30)

- Faraday’s Law of Induction [Covered Previous Week]
- Motional EMF
- Lenz’s Law
- The General Form of Faraday’s Law
- Generators and Motors
- Eddy Currents

→ **Review (Ch. 22 to 29)**

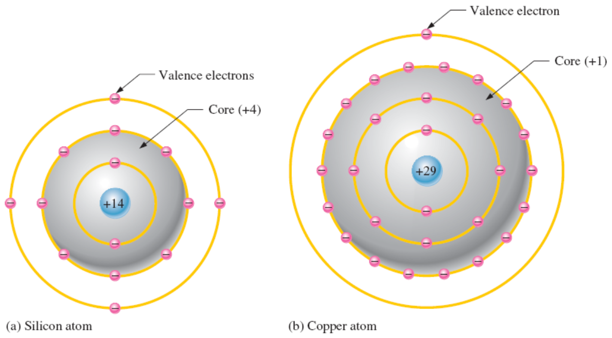
Review

Wk-1 (Ch. 22)

- Electrical charges
  - ***Positive*** and ***Negative*** charges
  - Same signs repel each other
  - Opposite signs attract each other

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

- The atom - Bohr model and valence electrons



Source: Floyd, Thomas L. Electronic devices: conventional current version. Pearson, 2012.

- Coulomb's Law

Electric force between charges

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

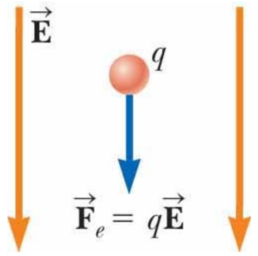
$$k_e = 8.9876 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$$

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

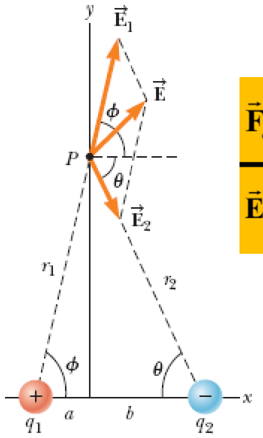
$$\varepsilon_0 = 8.8542 \times 10^{-12} \ \mathrm{C^2/N \cdot m^2}$$

$$e = 1.60218 \times 10^{-19} \ \mathrm{C}$$

- Electric Fields



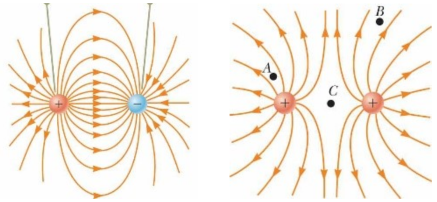
Electric field due to charges

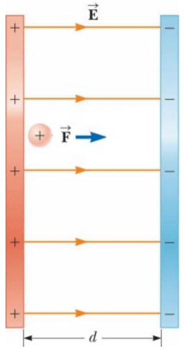


$$\vec{\mathbf{F}}_e = k_e \frac{q q_0}{r^2} \hat{\mathbf{r}}$$

$$\vec{\mathbf{E}} = k_e \sum_i \frac{q}{r^2} \hat{\mathbf{r}}$$

- Electric Field Lines



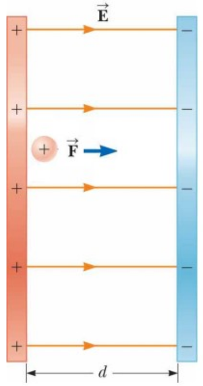


Charges in a Uniform Field

$$\sum \vec{\mathbf{F}} = q\vec{\mathbf{E}} = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} = \frac{q\vec{\mathbf{E}}}{m}$$

- Motion of a charged particle in uniform E field

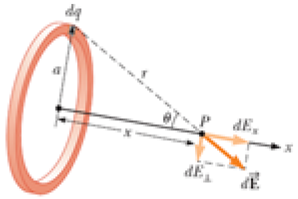


Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Wk-2 (Ch. 23 & Ch. 24)

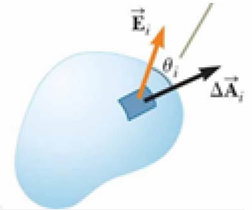
Continuous Charge Distributions

- E field of cont. charge dist.



$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

- Electric flux

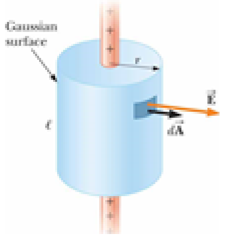


$$\Phi_E \equiv \int_{\text{surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

- Gauss's Law

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$$

- Net flux through any closed surface surrounding a point charge $\boldsymbol{q}$ is given by q/ε_0.
- Net electric flux through closed surface that surrounds no charge = 0



$$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2k_e \frac{\lambda}{r}$$

Electric Potential

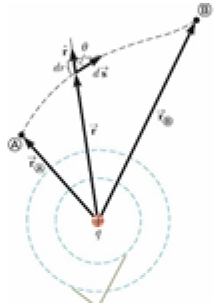
- Electric Potential Diff.

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- Electric potential due to a point charge

$$V = k_e \frac{q}{r}$$



- E field from E potential

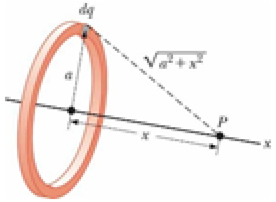
$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

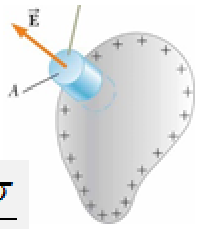
- E potential of cont. charge dist.

$$V = k_e \int \frac{dq}{r}$$

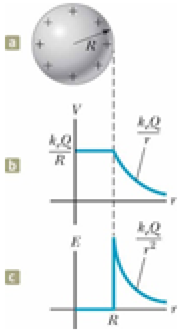


$$V = \frac{k_e}{\sqrt{a^2 + x^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{a^2 + x^2}}}$$

- E field of charged conductors



$$E = \frac{\sigma}{\varepsilon_0}$$



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

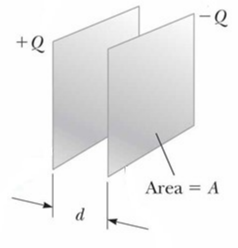
Wk-3 (Ch. 25 & Ch. 26)

Capacitance and Dielectrics

- Definition

$$Q = C\Delta V$$

$$C = \frac{\varepsilon_0 A}{d}$$



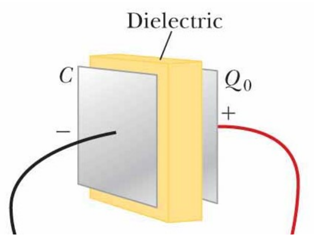
$$U_E = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

- With Dielectric Slab

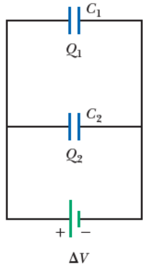
$$C = \kappa C_0$$

$$C = \varepsilon_r C_0$$

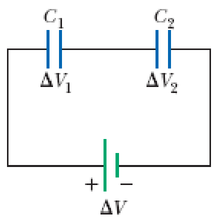
$$C = \kappa\frac{\varepsilon_0 A}{d}$$



- Combination of Capacitors



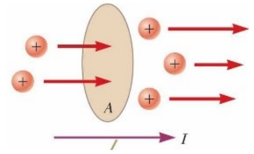
$$C_{eq} = C_1 + C_2 + C_3 + \cdots \quad \text{(parallel combination)}$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \quad \text{(series combination)}$$

Current and Resistance

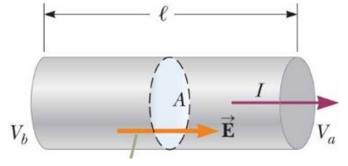
- Electric current



$$I \equiv \frac{dQ}{dt}$$

$$1\ \text{A} = 1\ \text{C/s}$$

- Resistance



$$R = \rho\frac{\ell}{A}$$

$$R \equiv \frac{\Delta V}{I}$$

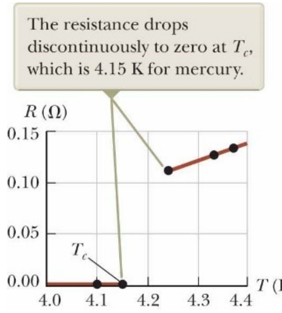
- Resistivities

TABLE 26.2 Resistivities and Temperature Coefficients of Resistivity for Various Materials

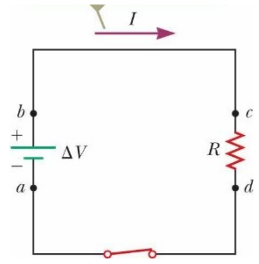
Material	Resistivity[a] ($\Omega \cdot$ m)	Temperature Coefficient[b] α [(°C)$^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 26.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and 1.50×10^{-6} $\Omega \cdot$ m.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

- Superconductors

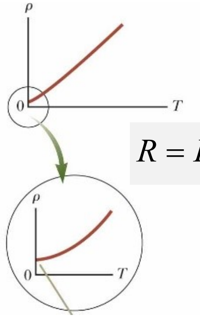


- Electrical Power



$$P = IV \qquad P = I^2R \qquad P = \frac{V^2}{R}$$

- Resistance and Temperature

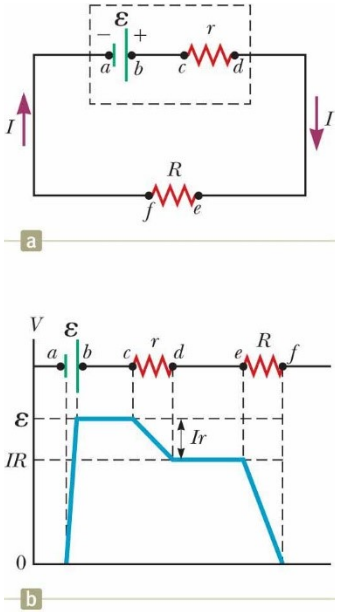


$$R = R_0\left[1 + \alpha\left(T - T_0\right)\right]$$

Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Wk-4 (Ch. 27)

Electromotive Force (EMF)



$$\Delta V = \varepsilon - Ir$$

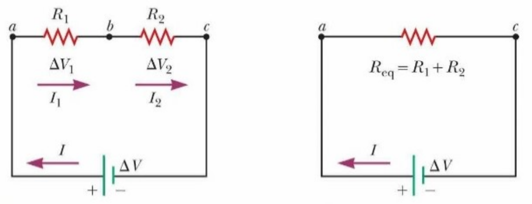
$$\varepsilon = IR + Ir$$

$$I = \frac{\varepsilon}{R + r}$$

$$I\varepsilon = I^2 R + I^2 r$$

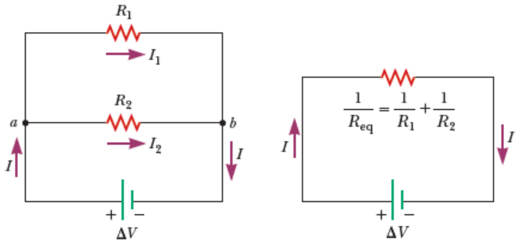
Resistor Combinations

Series Combination



$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n$$

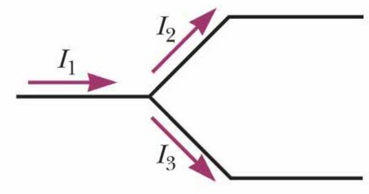
Parallel Combination



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Kirchhoff's Rules

Kirchhoff's Current (Junction) Rule

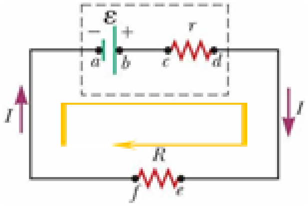


$$\sum_{\text{junction}} I = 0$$

$$I_1 - I_2 - I_3 = 0$$

$$I_1 = I_2 + I_3$$

Kirchhoff's Voltage (Loop) Rule



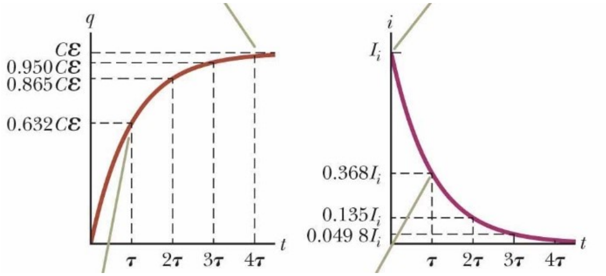
$$\sum_{\text{closed loop}} \Delta V = 0$$

RC Circuits

$$q(t) = C\varepsilon\left(1 - e^{-t/RC}\right)$$
$$= Q_{\max}\left(1 - e^{-t/RC}\right)$$

$$i(t) = \frac{\varepsilon}{R} e^{-t/RC}$$

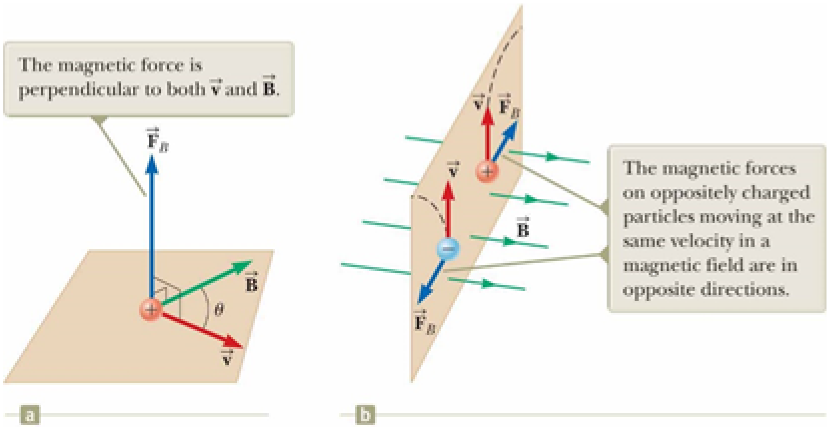
$$\tau = RC$$



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Wk-5 (Ch. 28)

Particle in a magnetic field



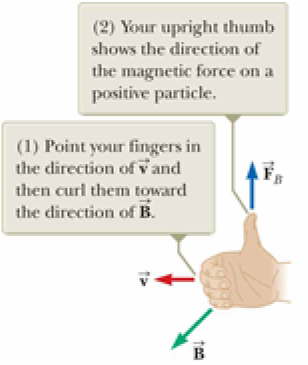
$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$1\ \mathrm{T} = 1\frac{\mathrm{N}}{\mathrm{C}\cdot\mathrm{m/s}}$$

$$1\ \mathrm{T} = 1\frac{\mathrm{N}}{\mathrm{A}\cdot\mathrm{m}}$$

$$1\ \mathrm{T} = 10^4\ \mathrm{G}$$

Right Hand Rule



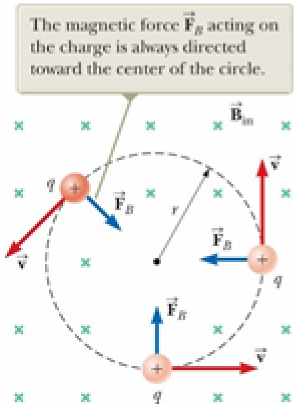
Motion of a charged particle

$$\sum F = F_B = ma$$

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB} \qquad \omega = \frac{v}{r} = \frac{qB}{m}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

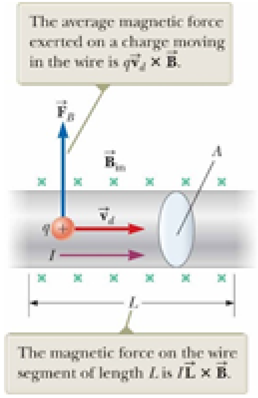


Magnetic force on current

$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$

$$d\vec{\mathbf{F}}_B = I\,d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

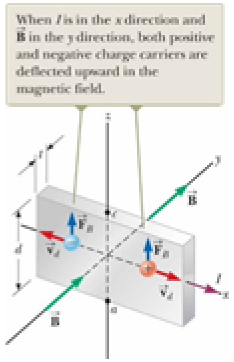
$$\vec{\mathbf{F}}_B = I\int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$



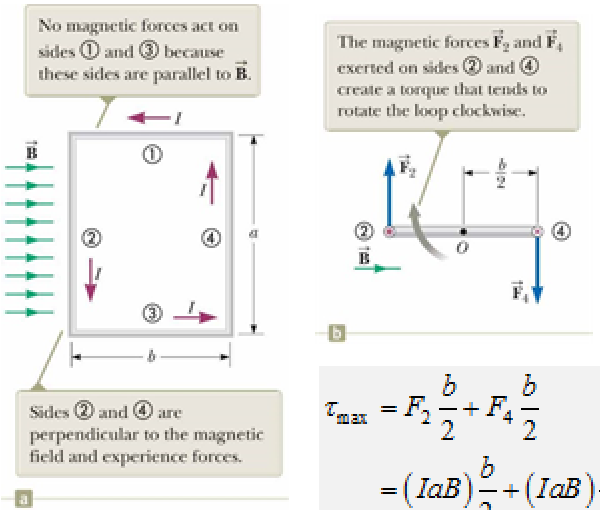
The Hall Effect

$$\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$$

$$R_H = \frac{1}{nq}\ \text{(Hall coefficient)}$$



Torque on a current loop



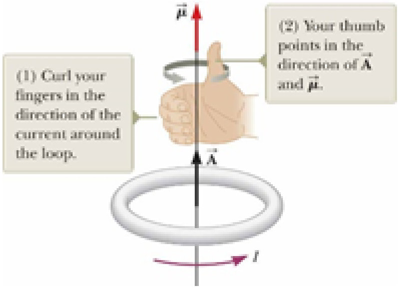
$$\tau_{max} = F_2\frac{b}{2} + F_4\frac{b}{2} = (IaB)\frac{b}{2} + (IaB)\frac{b}{2} = IabB$$

$$\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

$$\vec{\boldsymbol{\mu}} \equiv I\vec{\mathbf{A}}$$

$$\vec{\boldsymbol{\mu}}_{coil} = NI\vec{\mathbf{A}}$$

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$$



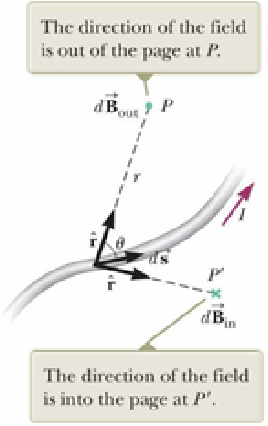
Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Wk-6 (Ch. 29)

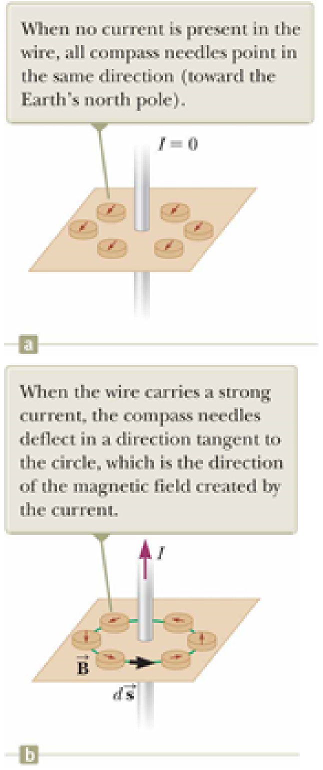
The Biot – Savart Law

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

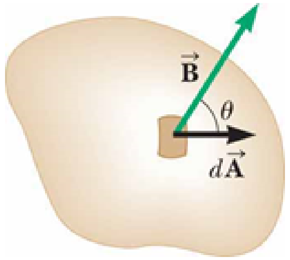


Ampère's Law



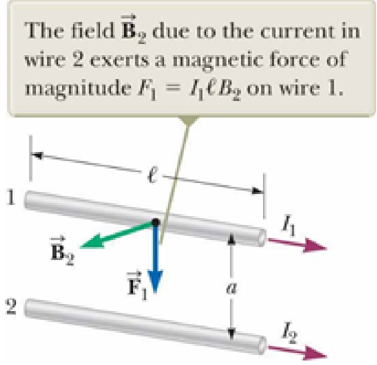
$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

Gauss's Law in Magnetism



$$\Phi_B \equiv \int \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}}$$

The Magnetic Force Between Two Parallel Conductors

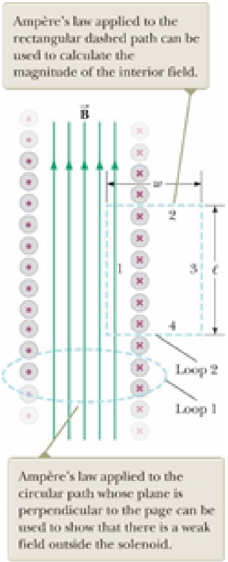


$$F_1 = I_1 \ell B_2 = I_1 \ell\left(\frac{\mu_0 I_2}{2\pi a}\right) = \frac{\mu_0 I_1 I_2}{2\pi a}\ell$$

The Magnetic Field of a Solenoid

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = B\ell = \mu_0 N I$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 n I$$



Source: Serway, Raymond A., and John W. Jewett. *Physics for scientists and engineers*. 10th Edition. Cengage learning, 2018.

Midterm Exam Overview

Physics-2, ENGI-1500, W23 Midterm Exam

- Date: Tuesday, February 21, 2022 | regular class time [8:55am - 11:35am]

8:55 - 9:00	Seating and exam distribution
9:00 - 11:30	**Exam time (150 minutes)**
11:30 - 11:35	Collection of papers

- Place: BCTI-310
- Exam will be <u>in-person</u>. Please be present on time.
- Exam will consist of classical questions and multiple-choice questions.
- Formula page will be provided. Calculators are allowed.
- Office hour before midterm exam (optional): on Thursday, February 16, 2023:
  - 1pm to 2pm
  - NX building, 3rd floor [please email so I can let you in]
  - If you have questions but can't join the office hour, please feel free to email me.
- If you have questions, comments, concerns about the exam please let me know.

Midterm Exam First Page

Midterm Exam | Group A
ENGI-1500, Winter 2023 | February 21, 2023

ENGI-1500 Physics-2
Midterm Exam - Group A

Student Name: ____________________

Student ID: ____________________

Place: BCTI-310 [in-person, synchronous exam]
Date: Tuesday, February 21, 2023 – 8:55am – 11:35am
8:55 - 9:00 Seating and exam distribution
9:00 - 11:30 Exam time (150 minutes)
11:30 - 11:35 Collection of all papers

Your Marks			Version
			A

Instructions:
Please do not start the exam until you read the following instructions carefully and sign at the bottom.

- Ensure your full name and student number is entered correctly at the top of the first page.
- Exam time is 2 hours and 30 minutes (150 minutes).
- No group work, or any form of collaboration between students is permitted in the exam room.
- This is a closed book exam. Formula sheet will be provided as appendix.
- No extra blank papers or workbooks are allowed. If necessary, you may write on the back of the pages or ask for additional pages from your instructor.
- All personal electronic devices (phones, computers, tablets, smart watches, earphones, etc.) must be put away. No digital aids are permitted, except for a calculator. Please note; cell phones are not permitted as calculators.
- This exam consists of classical questions and multiple-choice questions.
- All multiple-choice questions are to be answered on the exam sheet, please mark your choice clearly.
- Each multiple choice question has only one correct answer.
- Please show your work and solution steps clearly on the question sheets. You are encouraged to show your solution steps for the classical questions to be considered for partial credits.

The exam policy explained above is in immediate effect at the start of the exam.

Student Signature		Date	

Please do not turn this page until instructed to do so.

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Midterm Exam Appendix

Appendix-A: Formulas

Coulomb's law (electric force):

$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{12}$

Electric field:

$\vec{\mathbf{E}} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$ $\quad \vec{\mathbf{E}} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$ $\quad \vec{\mathbf{E}} = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}}$

Continuous Charge Distributions:

ρ=Q/V → dq=ρdV ; σ=Q/A → dq=σdA ; λ=Q/l → dq=λdl

Electric force and field relationship:

$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$

Electric flux through a surface:

$\Phi_E = EA\cos\theta$ $\quad \Phi_E = \int_{\text{surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$

Gauss's law:

$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\epsilon_0}$

Electric potential difference:

$\Delta V = \frac{\Delta U}{q} = -\int_{A}^{B} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ $\quad \Delta V = -Ed$

Electric potential:

$V = k_e \frac{q}{r}$ $\quad V = k_e \int \frac{dq}{r}$

Electric potential energy:

$U = k_e \frac{q_1 q_2}{r_{12}}$

Electric field in terms of electric potential:

$E_x = -\frac{dV}{dx}$

Capacitance, charge and potential difference:

$C = \frac{Q}{\Delta V}$

Equivalent capacitance (parallel comb.):

$C_{eq} = C_1 + C_2 + C_3 + \cdots$

Equivalent capacitance (series comb.):

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$

Capacitor with a dielectric constant:

$C = \kappa C_0$

Energy stored in a capacitor:

$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$

Electric current:

$I = \frac{dQ}{dt}$ $\quad I_{avg} = nqv_d A$

Electric current density:

$J = \frac{I}{A}$ $\quad J = \sigma E$

Ohm's law, resistance and resistivity:

$R \equiv \frac{\Delta V}{I}$ $\quad R = \rho \frac{\ell}{A}$ $\quad \rho = \rho_0[1 + \alpha(T - T_0)]$

Drift velocity:

$\vec{\mathbf{v}}_d = \frac{q\vec{\mathbf{E}}}{m_e} \tau$

Electrical power:

$P = I\Delta V$ $\quad P = I^2 R = \frac{(\Delta V)^2}{R}$

Equivalent resistance (series comb.):

$R_{eq} = R_1 + R_2 + R_3 + \cdots$

Equivalent resistance (parallel comb.):

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

RC circuits as a function of time (charging):

$q(t) = Q_{max}(1 - e^{-t/RC})$ $\quad i(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$

RC circuits as a function of time (discharging):

$q(t) = Q_i e^{-t/RC}$ $\quad i(t) = -\frac{Q_i}{RC} e^{-t/RC}$

$\tau = RC$

Circuit analysis – Kirchhoff's rules:

$\sum_{\text{junction}} I = 0$ $\quad \sum_{\text{closed loop}} \Delta V = 0$

Magnetic force on a moving charged particle:

$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ $\quad F_B = |q| vB\sin\theta$

Magnetic dipole moment:

$\vec{\boldsymbol{\mu}} \equiv I\vec{\mathbf{A}}$

Charged particle moving in a uniform magnetic field (circular path):

$r = \frac{mv}{qB}$ $\quad \omega = \frac{qB}{m}$

Magnetic force on a current carrying conductor:

$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$ $\quad d\vec{\mathbf{F}}_B = I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$

Torque on a current loop in a uniform magnetic field:

$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$

Newton's 2nd law:

$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$

Appendix-B: Constants

Description	Symbol	Value	Unit
Avogadro's number	N_A	$6.022\,141\,79 \times 10^{23}$	particles / mol
Coulomb constant	$k_e = 1/(4\pi\varepsilon_0)$	$8.987\,551\,788 \times 10^{9}$	$N \cdot m^2/C^2$
Electron mass	m_e	$9.109\,382\,15 \times 10^{-31}$	kg
Proton mass	m_p	$1.672\,621\,637 \times 10^{-27}$	kg
Neutron mass	m_n	$1.674\,927\,211 \times 10^{-27}$	kg
Elementary (electron) charge	e	$-1.602\,176\,487 \times 10^{-19}$	C
Elementary (proton) charge	p	$+1.602\,176\,487 \times 10^{-19}$	C
Gravitational constant	G	$6.674\,28 \times 10^{-11}$	$N \cdot m^2/kg^2$
Permittivity of free space	ε_0	$8.854\,187\,817 \times 10^{-12}$	$C^2/N \cdot m^2$
Permeability of free space	μ_0	$4\pi \times 10^{-7}$	$T \cdot m/A$
Speed of light in vacuum	c	$2.997\,924\,58 \times 10^{8}$	m/s

Midterm Exam Appendix

Midterm Exam Appendix — Group A
ENGI-1500, Winter 2023 — February 21, 2023

Appendix-C: Derivatives & Integrals

$$\frac{d}{dx}k = 0 \tag{1}$$
$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x) \tag{2}$$
$$\frac{d}{dx}\left[k \cdot f(x)\right] = k \cdot f'(x) \tag{3}$$
$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x) \tag{4}$$
$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2} \tag{5}$$
$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x) \tag{6}$$
$$\frac{d}{dx}x^n = nx^{n-1} \tag{7}$$
$$\frac{d}{dx}\sin x = \cos x \tag{8}$$
$$\frac{d}{dx}\cos x = -\sin x \tag{9}$$
$$\frac{d}{dx}\tan x = \sec^2 x \tag{10}$$
$$\frac{d}{dx}\cot x = -\csc^2 x \tag{11}$$
$$\frac{d}{dx}\sec x = \sec x \tan x \tag{12}$$
$$\frac{d}{dx}\csc x = -\csc x \cot x \tag{13}$$
$$\frac{d}{dx}e^x = e^x \tag{14}$$
$$\frac{d}{dx}a^x = a^x \ln a \tag{15}$$
$$\frac{d}{dx}\ln|x| = \frac{1}{x} \tag{16}$$
$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \tag{17}$$
$$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} \tag{18}$$
$$\frac{d}{dx}\tan^{-1} x = \frac{1}{x^2+1} \tag{19}$$
$$\frac{d}{dx}\cot^{-1} x = \frac{-1}{x^2+1} \tag{20}$$
$$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}} \tag{21}$$
$$\frac{d}{dx}\csc^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}} \tag{22}$$

$$\int dx = x + C \tag{1}$$
$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \tag{2}$$
$$\int \frac{dx}{x} = \ln|x| + C \tag{3}$$
$$\int e^x\,dx = e^x + C \tag{4}$$
$$\int a^x\,dx = \frac{1}{\ln a}a^x + C \tag{5}$$
$$\int \ln x\,dx = x\ln x - x + C \tag{6}$$
$$\int \sin x\,dx = -\cos x + C \tag{7}$$
$$\int \cos x\,dx = \sin x + C \tag{8}$$
$$\int \tan x\,dx = -\ln|\cos x| + C \tag{9}$$
$$\int \cot x\,dx = \ln|\sin x| + C \tag{10}$$
$$\int \sec x\,dx = \ln|\sec x + \tan x| + C \tag{11}$$
$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C \tag{12}$$
$$\int \sec^2 x\,dx = \tan x + C \tag{13}$$
$$\int \csc^2 x\,dx = -\cot x + C \tag{14}$$
$$\int \sec x \tan x\,dx = \sec x + C \tag{15}$$
$$\int \csc x \cot x\,dx = -\csc x + C \tag{16}$$
$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C \tag{17}$$
$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \tag{18}$$
$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{|x|}{a} + C \tag{19}$$

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Review Questions & Solutions

Questions

Review Questions

1 Electric Fields [Ch. 22]

1.1 [Textbook 22.5]

A 7.50-nC point charge is located 1.80 m from a 4.20-nC point charge.

(a) Find the magnitude of the electric force that one particle exerts on the other.

(b) Is the force attractive or repulsive?

1.2 [Textbook 22.10]

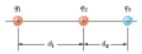
Three point charges lie along a straight line as shown in Figure P22.10, where $q_1 = 6.00\ \mu C$, $q_2 = 1.50\ \mu C$, and $q_3 = -2.00\ \mu C$. The separation distances are $d_1 = 3.00$ cm and $d_2 = 2.00$ cm. Calculate the magnitude and direction of the net electric force on

(a) q_1,

(b) q_2, and

(c) q_3.

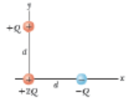
Figure P22.10



1.3 [Textbook 22.11]

S A point charge $+2Q$ is at the origin and a point charge $-Q$ is located along the x axis at $x = d$ as in Figure P22.11. Find a symbolic expression for the net force on a third point charge $+Q$ located along the y axis at $y = d$.

Figure P22.11



Solutions

Midterm Review Questions with Solutions
ENGI-1500 Physics-2, Winter 2023

Review Questions

1 Electric Fields [Ch. 22]

1.1 [Textbook 22.5]

A 7.50-nC point charge is located 1.80 m from a 4.20-nC point charge.

(a) Find the magnitude of the electric force that one particle exerts on the other.

(b) Is the force attractive or repulsive?

Solution:

(a) $|F| = \dfrac{k_e |q_1||q_2|}{r^2}$

$$F = \frac{k_e e^2}{r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.50\times10^{-9}\ \mathrm{C})(4.20\times10^{-9}\ \mathrm{C})}{(1.80\ \mathrm{m})^2}$$

$$= \boxed{8.74\times10^{-6}\ \mathrm{N}}$$

(b) The charges are like charges. $\boxed{\text{The force is repulsive.}}$

Reading / Preparation for Next Week

Next week:

- Midterm exam on Tuesday, February 21, 2023