Name ______________________________

PID ______________________________

Practice Exam 1

MTH 320, Friday February 15, 2019

Instructions: This exam is closed books, no calculators and no electronic devices of any kind. You are allowed one sheet of notes, front side only. There are four problems worth 25 points each. If a problem has multiple parts, it may be possible to solve a later part without solving the previous parts. Solutions should be written neatly and in a logically organized manner. Partial credit will be given if the student demonstrates an understanding of the problem and presents some steps leading to the solution. Correct answers with *no work* will be given *no credit*. The back sheets may be used as scratch paper but will not be graded for credit.

1	2	3	4	Total

Problem 1. (25 pts) For each statement, circle T if it is true and F if it is false. If true, give a brief explanation (a complete proof is not required), and if false, give a counterexample.

a. (T / F) The set $\mathbb{R} \setminus \mathbb{Q}$ is countable.

b. (T / F) If $S \subset \mathbb{R}$ is nonempty and bounded below then S has a least upper bound.

c. (T / F) If the sequence (a_n) converges and the sequence $(a_n + b_n)$ converges then the sequence (b_n) converges.

d. (T / F) If (a_n) is a monotone sequence then (a_n) converges.

e. (T / F) If (a_n) converges to a and (b_n) converges to b then the sequence $(a_{2n}b_{2n+1})$ converges to ab.

Problem 2. In the following, you will prove that

$$\bigcap_{n=1}^{\infty} \left(1 - \frac{1}{n}, 2 + \frac{1}{n}\right) = [1, 2].$$

a. (5 pts) Show that $[1, 2] \subset \displaystyle\bigcap_{n=1}^{\infty} \left(1 - \frac{1}{n}, 2 + \frac{1}{n}\right)$.

b. (10 pts) Show that if $x > 2$ then $x \notin \displaystyle\bigcap_{n=1}^{\infty} \left(1 - \frac{1}{n}, 2 + \frac{1}{n}\right)$.

c. (10 pts) Show that if $x < 1$ then $x \notin \displaystyle\bigcap_{n=1}^{\infty} \left(1 - \frac{1}{n}, 2 + \frac{1}{n}\right)$.

Problem 3. Recall that $n! = 1 \cdot 2 \cdots n$ is the product of the first n natural numbers.

a. (10 pts) Prove by induction that for $n \in \mathbb{N}$, if $n \geq 4$ then $2^n < 3(n-1)!$.

b. (15 pts) Use a., or any other method, to show that $\lim_{n \to \infty} \dfrac{2^n}{n!} = 0$.

Problem 4. Define a sequence recursively by $x_1 = 1$ and

$$x_{n+1} = \sqrt{6 + x_n} \qquad \text{for} \qquad n \in \mathbb{N}.$$

a. (10 pts) Show that the sequence is increasing.

b. (10 pts) Show that the sequence is bounded and $0 < x_n \leq 3$ for all $n \in \mathbb{N}$.

c. (5 pts) Compute the limit: $x = \lim_{n\to\infty} x_n$.