MTH 320: HW 6

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Due: February 27, 2019

Problems from Abbott's book (2nd ed.) are labeled by **Abbott chpt.sec.#.**

1. **(Abbott 3.2.2)** Let

$$A = \left\{ (-1)^n + \frac{2}{n} : n \in \mathbb{N} \right\} \qquad \text{and} \qquad B = \{x \in \mathbb{Q} : 0 < x < 1\}.$$

   Answer the following questions for each set:

   (a) What are the limit points?

   (b) Is the set open? Is it closed?

   (c) Does the set contain any isolated points?

   (d) Find the closure of the set.

2. **(Abbott 3.2.4)** Let A be nonempty and bounded above so that $s = \sup A$ exists.

   (a) Show that $s \in \overline{A}$.

   (b) Can an open set contain its supremeum?

3. **(Abbott 3.2.6)** Decide whether the following statements are true or false. Supply proofs for those that are true and counterexamples for those that are false.

   (a) As open set that contains every rational must necessarily be all of $\mathbb{R}$.

   (b) The Nested Interval Property remains true if the term "closed interval“ is replaced by "closed set“.

   (c) Every nonempty open set contains a rational.

   (d) The Cantor set is closed.

4. **(Abbott 3.2.7)** Let $A \subset \mathbb{R}$ and let L be the set of all limit points of A. Show that the set L is closed.

5. **(Abbott 3.3.1)** Show that if K is compact then $\sup K \in K$.

6. **(Abbott 3.3.8)** Let K and L be nonempty compact sets and define

$$d = \inf\{|x - y| : x \in K \text{ and } y \in L\}.$$

   (a) If K and L are disjoint, show that $d > 0$ and that $d = |x_0 - y_0|$ for some $x_0 \in K$ and $y_0 \in L$.

   (b) Show that it's possible to have $d = 0$ if we assume only that the disjoint sets K and L are closed.