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KONGU ENGINEERING COLLEGE, PERUNDURAI 638 060
 ODD SEMESTER 2017-2018
 CONTINUOUS ASSESSMENT TEST 3 - OCTOBER 2017
 (Regulations 2014)

| | |
|-------------------------------------|---------------------------|
| Programme : BE | Date : 25.10.2017 |
| Branch : CSE | Time : 9.15 am – 10.45 am |
| Semester : V | |
| Course Code : 14CST52 | Duration : 1 ½ Hours |
| Course Name : Theory of Computation | Max. Marks : 50 |

PART - A (10 × 2 = 20 Marks)

ANSWER ALL THE QUESTIONS

| | | | |
|-----|--|-----|------|
| 1. | Mention the language accepted by PDA | C03 | [K2] |
| 2. | Write the formal definition of Turing machine. | C04 | [K3] |
| 3. | Design a turing machine that accepts string that contains exactly 2b's over {a,b}. | C04 | [K3] |
| 4. | Define ID for PDA. | C04 | [K1] |
| 5. | Consider a turning machine that computer the function $f(x,y) = y$ | C04 | [K3] |
| 6. | When a language is said to be recursively enumerable? | C04 | [K2] |
| 7. | Show that the complement of recursive language is recursive. | C04 | [K1] |
| 8. | Define PCP with an example. | C04 | [K1] |
| 9. | What is DPDA? Write its properties | C04 | [K1] |
| 10. | State the rules for converting PDA to CFG. | C03 | [K2] |

Part - B (3 × 10 = 30 Marks)

ANSWER ANY THREE QUESTIONS

| | | | | |
|-----|--|------|-----|------|
| 11. | Find the CFG for the language whose PDA is given as $M = \{(q_0, q_1), \{a, b\}, \delta, q_0, z_0, \phi\}$ and δ is defined as i) $\delta(q_0, a, z_0) = \{(q_0, qz_0)\}$ ii) $\delta(q_0, a, a) = \{q_0, aa\}$ iii) $\delta(q_0, b, a) = \{(q_0, a)\}$ iv) $\delta(q_0, a, a) = \{(q_1, \epsilon)\}$ v) $\delta(q_1, \epsilon, z_0) = \{(q_1, \epsilon)\}$ | (10) | C03 | [K3] |
| 12 | i) Construct a PDA that accept the language $L = \{wcw^R / w \in \{a, b\}^+\}$ | (5) | C03 | [K3] |
| | ii) Design a turing machine to perform addition of two integers. | (5) | C05 | [K4] |
| 13 | Design a Turing machine that computes $f(m, n) = mxn$ | (10) | C05 | [K4] |
| 14. | Write the code for the turing machine $M = \{(q_1, q_2, q_3), (0, 1), (01, B), \delta(q_1, B, (q_2, 3))\}$ when δ as $\delta(q_1, 1) = (q_3, 0, R)$ $\delta(q_3, 0) = (q_1, 1, R)$ $\delta(q_3, 1) = (q_2, 0, R)$ $\delta(q_3, B) = (q_3, 1, L)$ | (10) | C04 | [K2] |

| Bloom's Taxonomy Level | Remembering (K1) | Understanding (K2) | Applying (K3) | Analysing (K4) | Evaluating (K5) | Creating (K6) |
|------------------------|------------------|--------------------|---------------|----------------|-----------------|---------------|
| Percentage | 26.67 | 33.33 | 26.17 | 13.33 | - | - |

Part A

1. Language Accepted by PDA

1. empty stack
2. Final state

2. TM $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Q - Finite set of states

Σ - " i/p symbol

Γ - Tape Symbol

δ - Transition function

$$\delta(q, x) = (p, y, D)$$

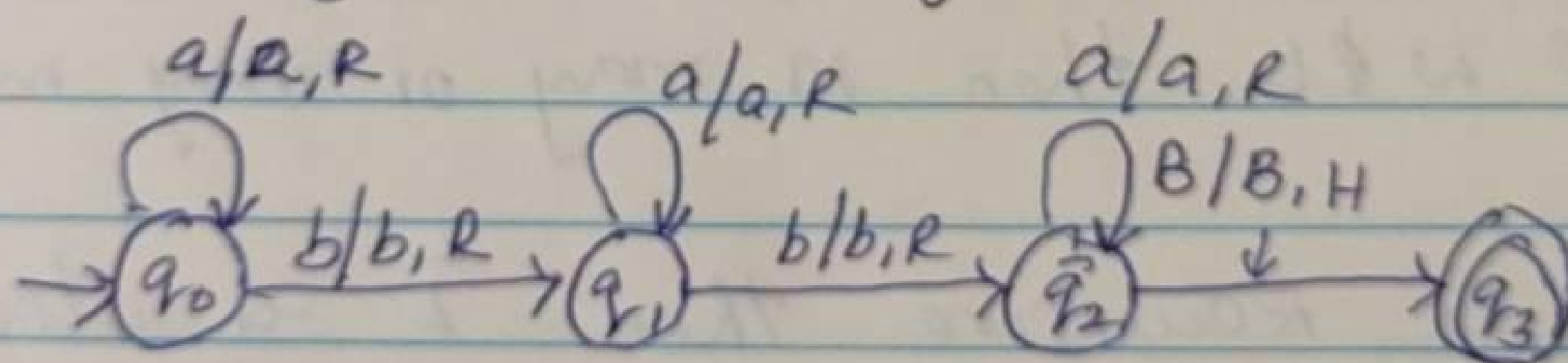
D may be L or R.

q_0 - start state

B - Blank symbol

F - Set of accepting states

3. TM for accepting string that contains exactly 2 b's over $\{a, b\}$



4. ID for PDA.

Execution status of PDA.

Represented by (q, w, γ)

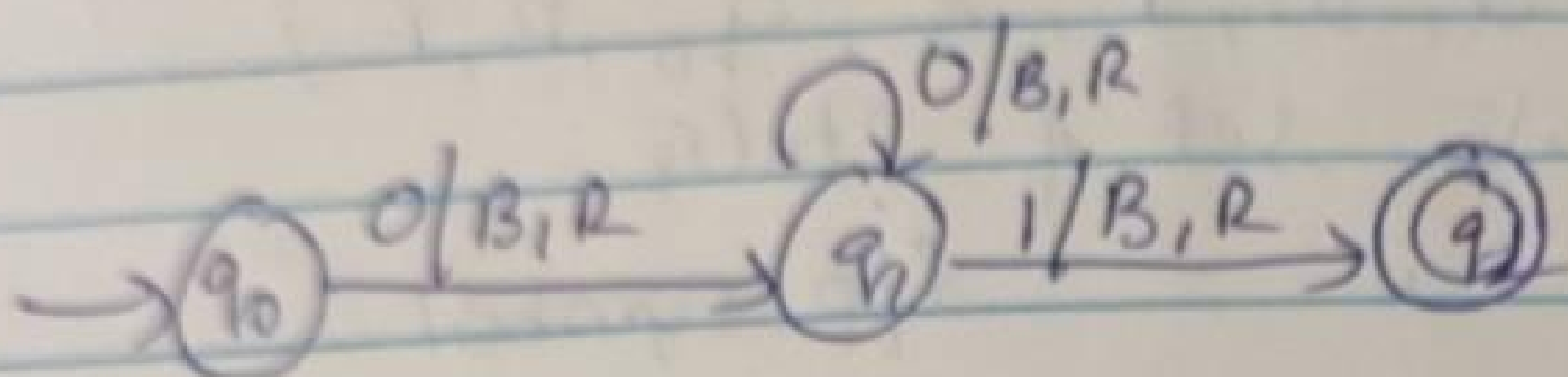
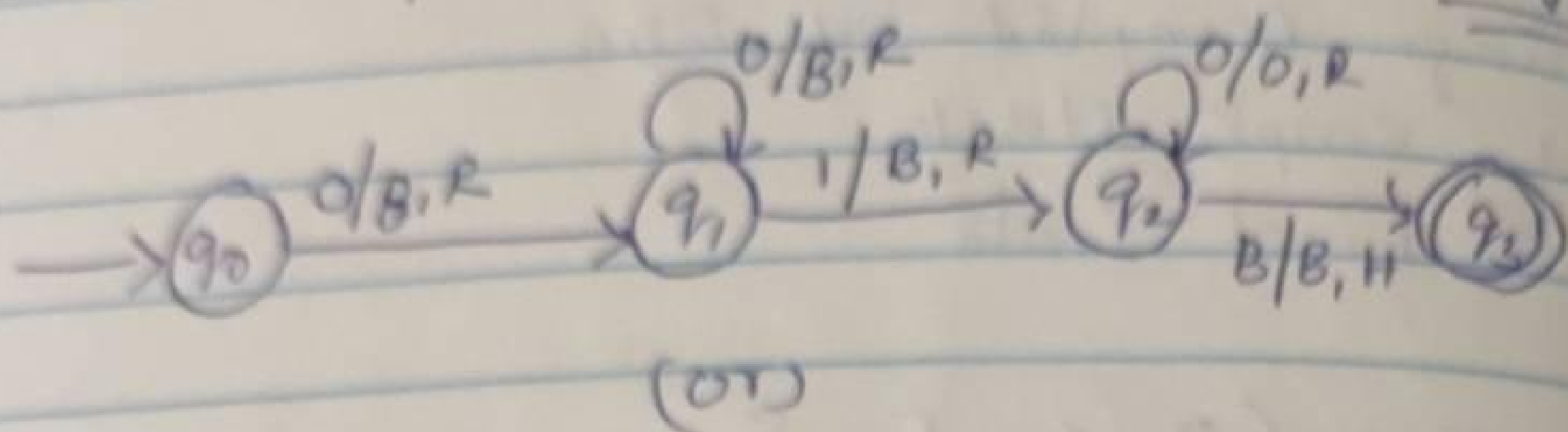
q - state

w - remaining i/p

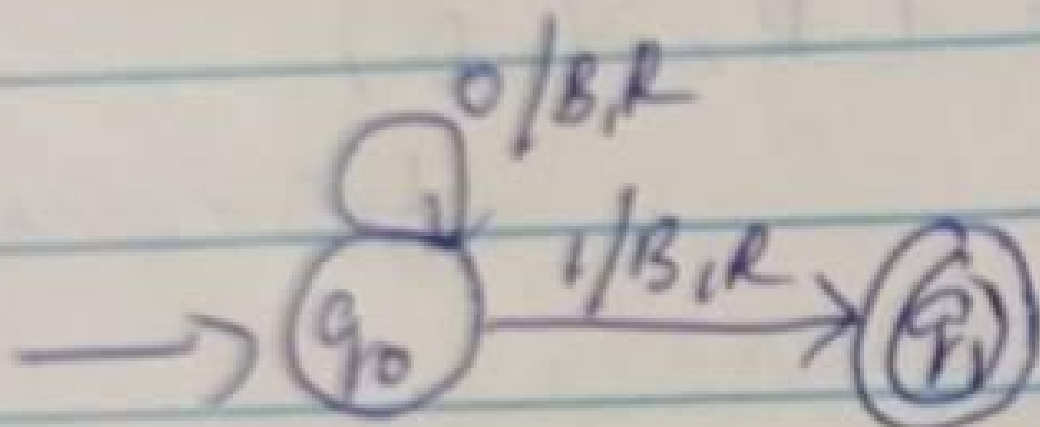
γ - stack contents

5. TM to compute $f(x, y) = y$

$x, y \geq 0$



If $x, y \geq 0$



6. Recursively Enumerable

A Lang L is RE if $L = L(M)$ for some TM M .

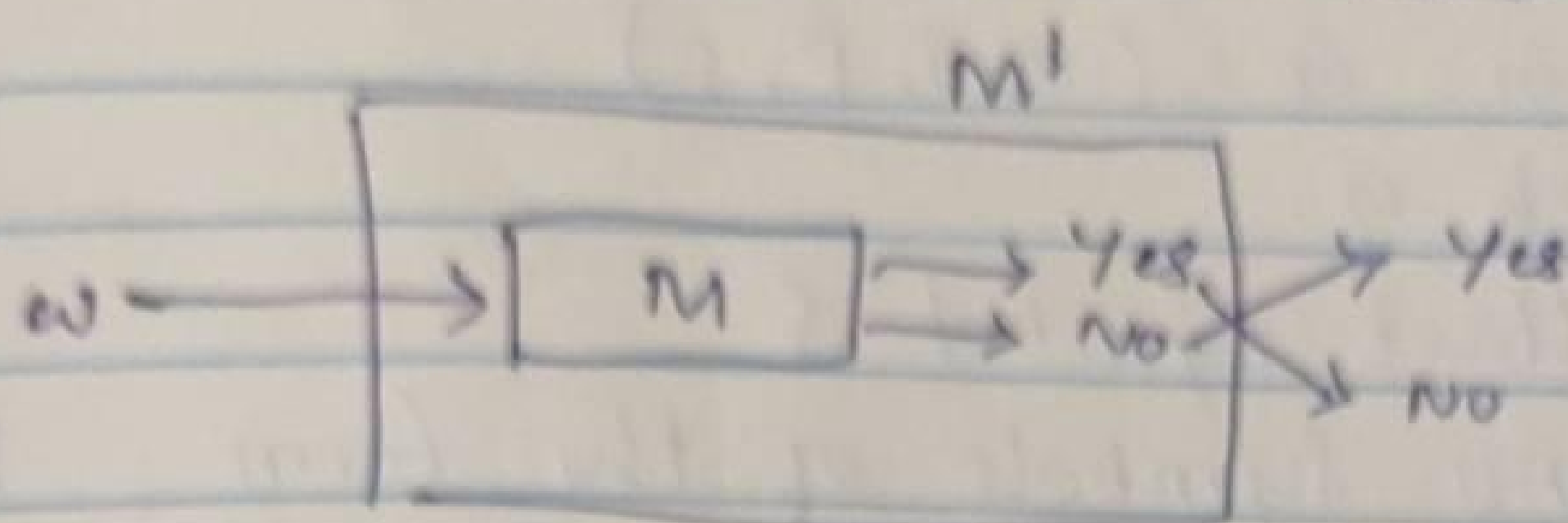
If $w \in L$ then M accepts and halts

If $w \notin L$ then M may or may not be

T. If L is Recursive then \bar{L} is also rec.

If L is Recursive then there is a TM which is halted when the string is accepted or not. We can design a TM for the Lang. \bar{L} using TM M , which

also halted for the string whether it is accepted or not. So \bar{L} is also Recursive.



8. PCP

An instance of PCP consists of two lists of strings over Σ :

$$A = w_1, w_2, \dots, w_k$$

$$B = x_1, x_2, \dots, x_k \text{ for some integer } k.$$

The instance of PCP has a solution if there is any sequence of integers i_1, i_2, \dots, i_m with $m \geq 1$ such that

$$w_{i_1} w_{i_2} \dots w_{i_m} = x_{i_1} x_{i_2} \dots x_{i_m}$$

is a solution to this instance of PCP.

9. DPDA

BPDA $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, f)$ is DPDA iff

- ① $\delta(q, a, x)$ has at most one member (transition).
- ② If $\delta(q, a, x)$ is non empty then $\delta(q, \epsilon, x)$ must be empty.

10. Rules for converting PDA to CFG:
Let PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0)$ then the
CFG $G = (V, T, P, S)$

V consists of:

1) Start symbol S

2) All symbols of the form $[p x q]$
where $p, q \in Q$ and $x \in \Gamma$

Terminals $T = \Sigma$

Productions of G :

a) For all states p

$$S \rightarrow [q_0 z_0 p]$$

b) Let $\delta(q, a, x) = (r, y_1 y_2 \dots y_k)$

where $a \in \Sigma$ or $a = \epsilon$ and k

$$[q x r_k] = a [r y_1 r_1] [r_1 y_2 r_2] \dots [r_{k-1} y_k r_k]$$

Part B

$$V = \{ S, [q_0 z_0 q_0], [q_0 z_0 q_1], [q_1 z_0 q_0], [q_1 z_0 q_1], [q_0 a q_0], [q_0 a q_1], [q_1 a q_0], [q_1 a q_1], [q_0 b q_0], [q_0 b q_1], [q_1 b q_0], [q_1 b q_1] \}$$

$$T = \{ a, b \}$$

S - start symbol.

Productions P :

① For start symbol S , productions are:

$$S \rightarrow [q_0 z_0 q_0] / [q_0 z_0 q_1]$$

② For every transition function, productions are:

i) For $\delta(q_0, a, z_0) = (q_0, a z_0)$

$$[q_0 z_0 q_0] \rightarrow a [q_0 a q_0] [q_0 z_0 q_0] / a [q_0 a q_1] [q_1 z_0 q_0]$$

$$[q_0 z_0 q_1] \rightarrow a [q_0 a q_0] [q_0 z_0 q_1] / a [q_0 a q_1] [q_1 z_0 q_1] \checkmark$$

ii) for $\delta(q_0, a, a) = (q_0, aa)$

$$[q_0 a q_0] \rightarrow a [q_0 a q_0] [q_0 a q_0] / a [q_0 a q_1] [q_1 a q_0] \quad \} \times$$

$$[q_0 a q_1] \rightarrow a [q_0 a q_0] [q_0 a q_1] / a [q_0 a q_1] [q_1 a q_1] \quad \} \times$$

iii) for $\delta(q_0, b, a) = (q_0, a)$

$$[q_0 a q_0] \rightarrow b [q_0 a q_0]$$

$$[q_0 a q_1] \rightarrow b [q_0 a q_1]$$

iv) for $\delta(q_0, a, a) = (q_1, \epsilon)$

$$[q_0 a q_1] \rightarrow a$$

v) $\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$$[q_1 z_0 q_1] \rightarrow \epsilon$$

Resultant Grammar after simplification

$$S \rightarrow [q_0 z_0 q_1]$$

$$[q_0 z_0 q_1] \rightarrow a [q_0 a q_1] [q_1 z_0 q_1] = a [q_0$$

$$[q_0 a q_1] \rightarrow b [q_0 a q_1] / a$$

$$\cancel{[q_1 z_0 q_1] \rightarrow \epsilon}$$

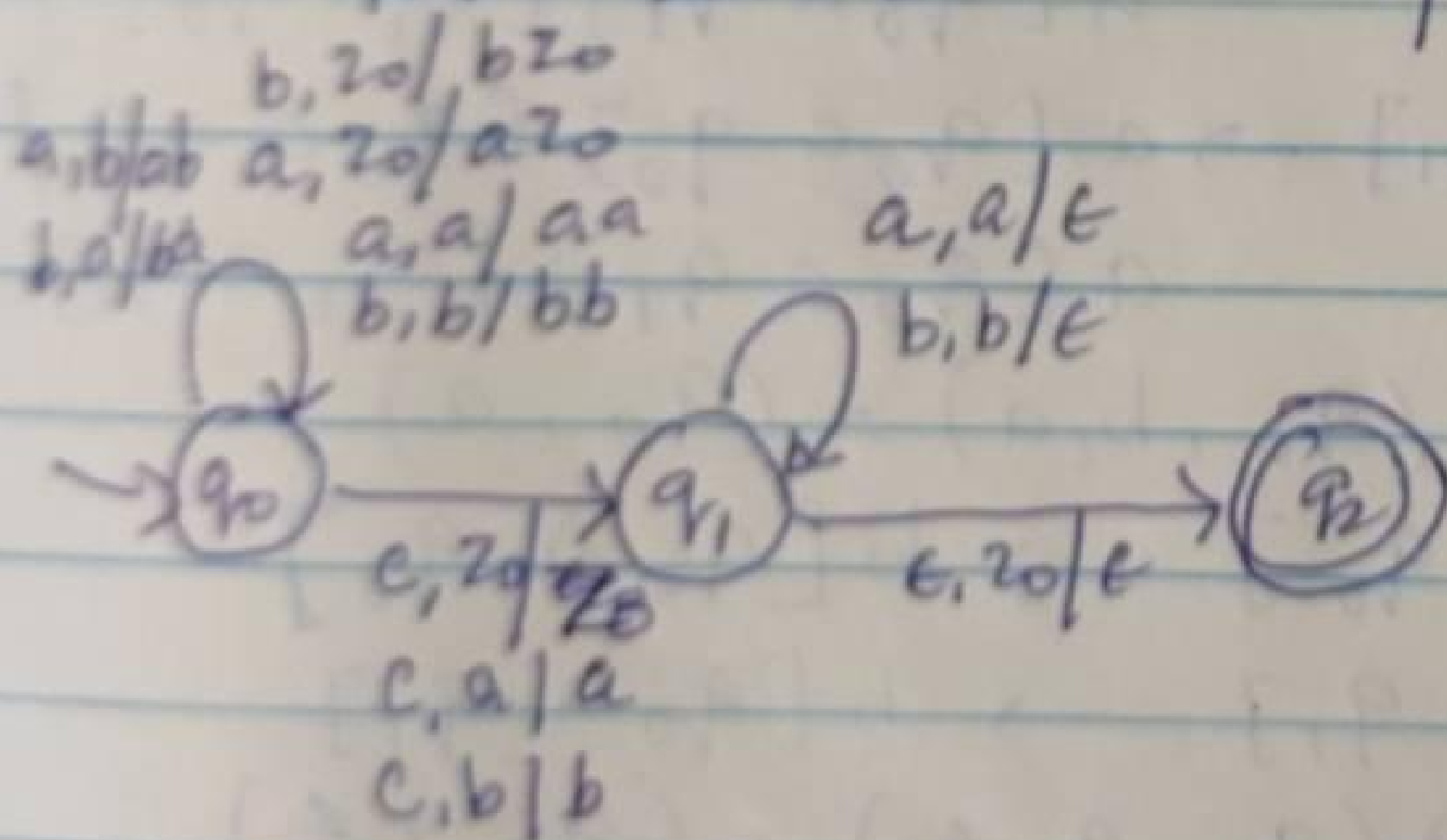
Final Grammar by Sub $A = [q_0 z_0 q_1]$
 $B = [q_0 a q_1]$

$$\begin{array}{l} S \rightarrow A \\ A \rightarrow a B \\ B \rightarrow b B / a \end{array}$$

\Rightarrow
(or)

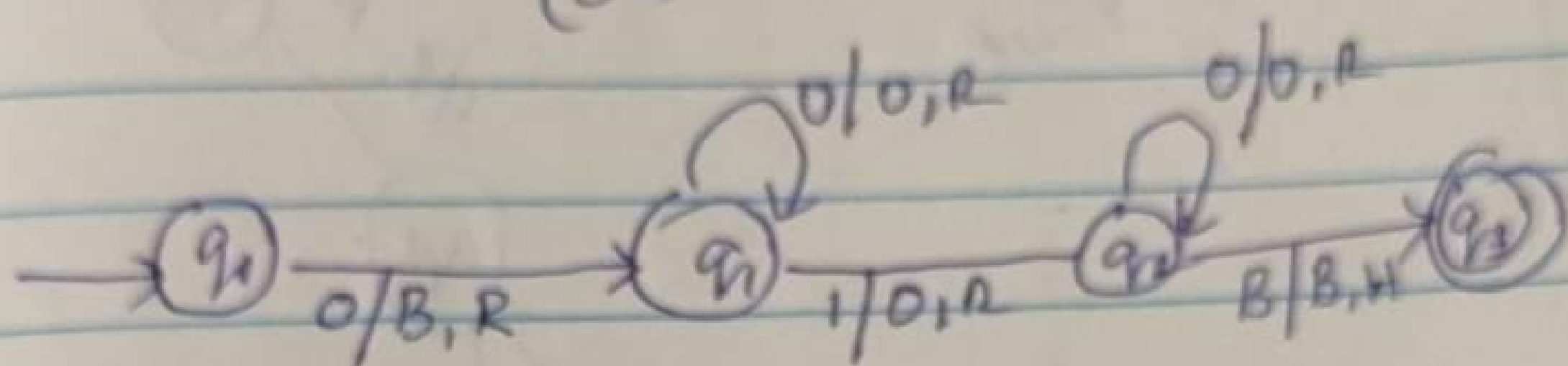
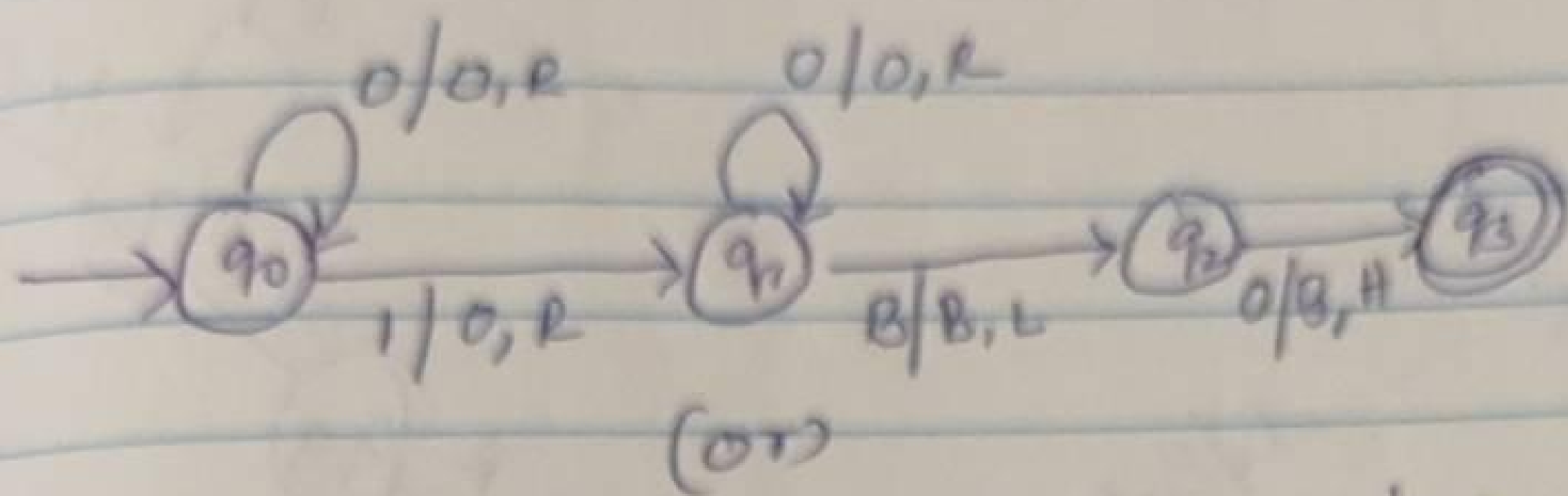
$$\begin{array}{l} S \rightarrow a B \\ B \rightarrow b B / a \end{array}$$

Q. i PDA for $L = \{w c w^R \mid w \in \{a, b\}^+\}$

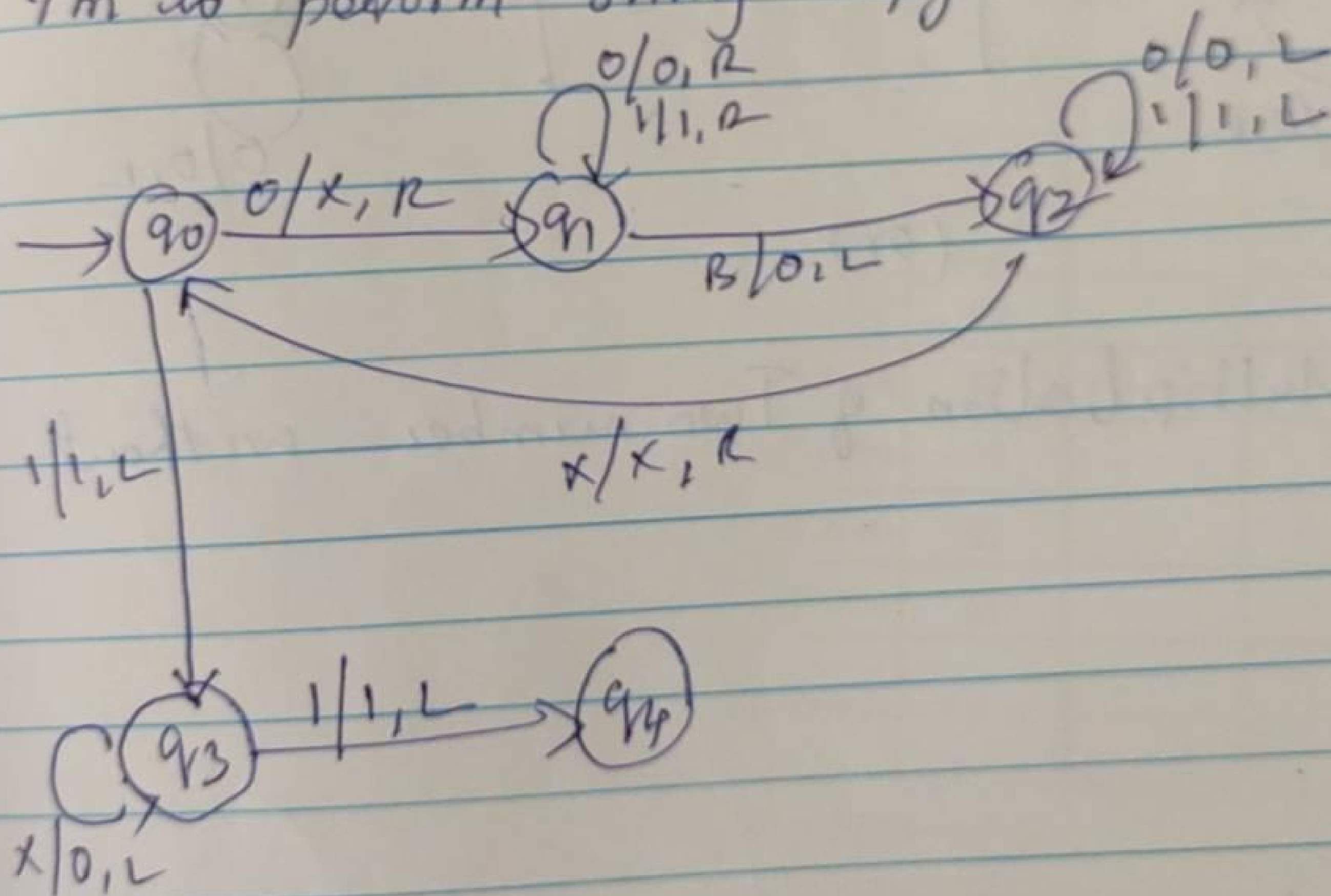


(5)

ii) TM to perform addition two numbers.
 Input $0^x 1 0^y$ if to add $x+y$.

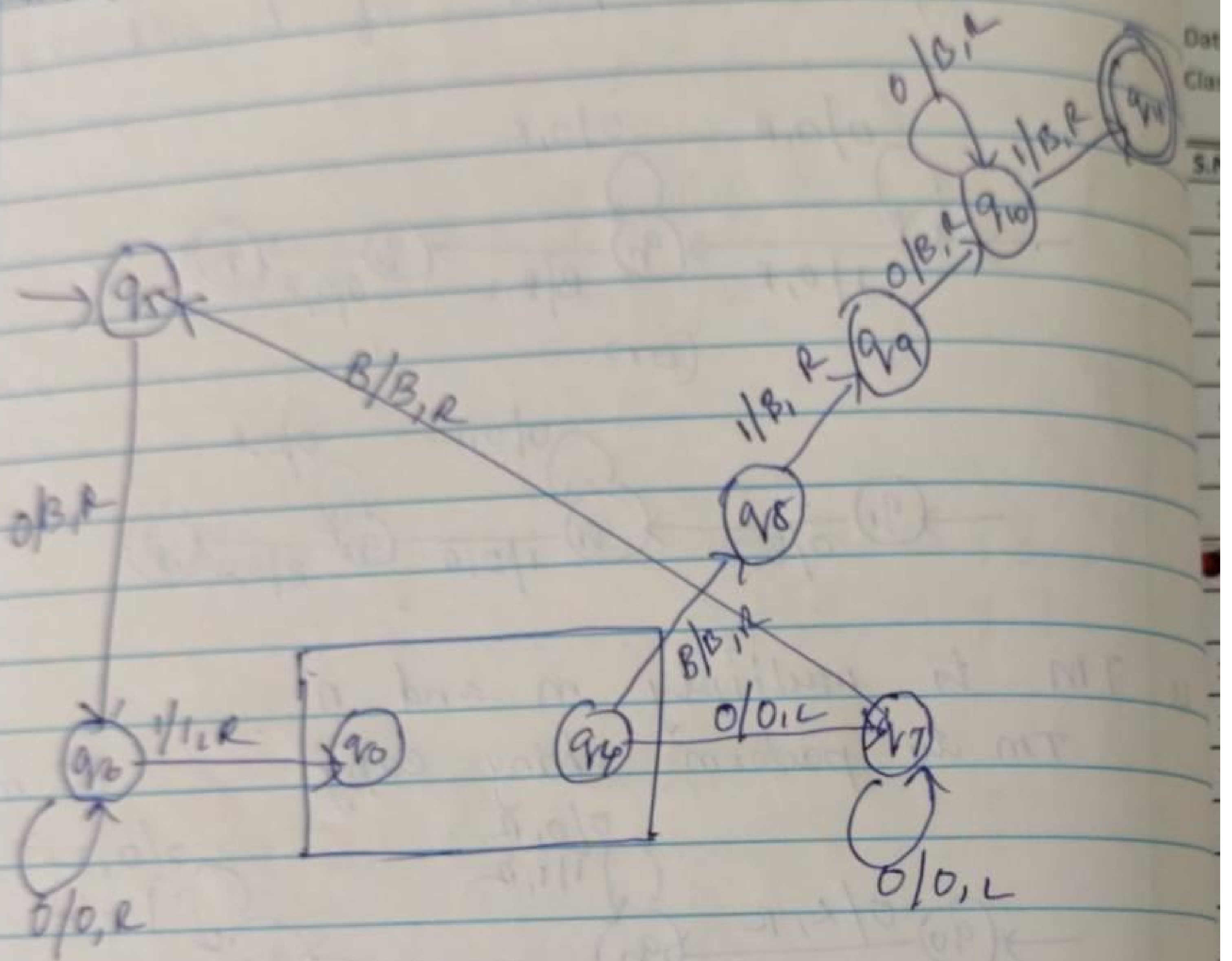


13. TM to multiply m and n .
 TM to perform string copy: (5 marks)



TM for Main function :

15 marks



(or)

Multiplication of Two number without Subtraction

14.

Code for States - (1)
 Code for Tape Symbol - (1)
 Code for Direction - (1)

TM code = $0^1 1 0^2 1 0^3 1 0 1 0^2 1 1 0^3 1 0 1 0^1 1 0^2 1 0^2 1 1 0^3 1 0^2 1 0^2 1 0 1 0^2 1 1$
 $0^3 1 0^3 1 0^3 1 0^2 1 0$ (7)



KONGU ENGINEERING COLLEGE

PERUNDURAI ERODE - 638 060.

(Autonomous)



Vani Rajasekar
Name and Signature of Hall Supdt. with Date
25/10/17

| | | | |
|----------------------|---------------------------------|-------------------|-----------|
| Name of the Student | M. RAJUVARSHINI | Register No. | 15CSRI60 |
| Programme | BE | Branch & Semester | CSE - 'C' |
| Course Code and Name | 14CS32 THEORY OF COMPUTATION | Date | 25-10-17 |
| | | Nd. of Pages Used | 9 |

MARKS TO BE FILLED IN BY THE EXAMINER

| PART - A | | PART - B | | Grand Total Max. Marks : 50 |
|------------------------------------|---------------|--------------|----------------|--|
| Question No. | Max Marks : 2 | Question No. | Max Marks : 10 | |
| 1 | 2 | 11 i) | 10 | <div style="border: 1px solid red; border-radius: 50%; padding: 10px; display: inline-block;"> 48/50 </div> V. good |
| 2 | 2 | 11 ii) | | |
| 3 | 2 | 12 i) | | |
| 4 | 2 | 12 ii) | | |
| 5 | 2 | 13 i) | 10 | |
| 6 | 2 | 13 ii) | | |
| 7 | 2 | 14 i) | 10 | |
| 8 | 2 | 14 ii) | | |
| 9 | 2 | TOTAL | 30 | |
| 10 | 2 | | | |
| TOTAL | 18 | | | |
| Total Marks in Words : Eighty Nine | | | | |

INSTRUCTION TO THE CANDIDATE

1. Check the Question Paper, Programme, Course Code, Branch Name etc., before answering the questions.
2. Use both sides of the paper for answering questions.
3. POSSESSION OF ANY INCRIMINATING MATERIAL AND MALPRACTICE OF ANY NATURE IS PUNISHABLE AS PER RULES.

R. C. Sugam
Name of the Examiner

Signature of the Examiner
with Date
26/10/17

Part A

1. Language accepted by PDA
1. Acceptance by empty stack
2. Acceptance by final state

2. Definition of TM

TM, $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

where Q - finite of states

Σ - finite set of input symbols

Γ - finite set of stack symbols

δ - transition function

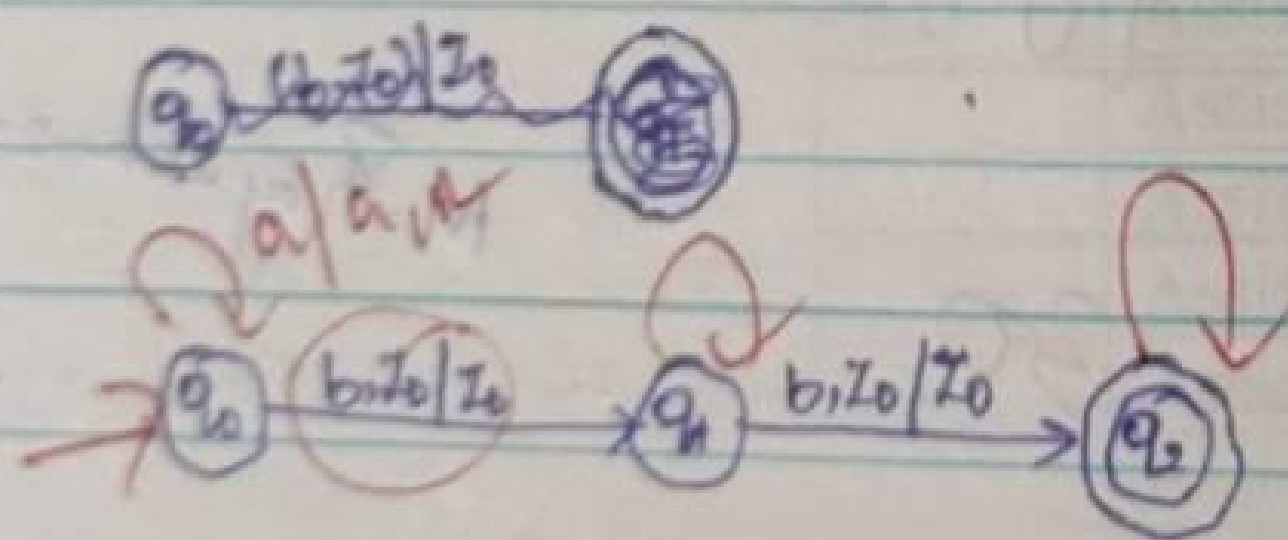
q_0 - start symbol

B - Blank space

F - final state

3. $M = \{ \text{string contains exactly 2b's over } \{a, b\} \}$

$w = bb$



string processing

δ a b

q_0 ϕ $\{q_1, Z_0\}$

q_1 ϕ $\{q_0, Z_0\}$

q_2 ϕ ϕ

$q_0bb \vdash q_1Z_0b$

[]

4. Instantaneous Description of TM

Execution state of TM

$\delta(q, a, x) = (p, y, d)$

1. q - states in TM

2. a - tape head

3. d - direction of movement

$x_1, x_2, \dots, x_{i-1}, x_i, x_{i+1}, \dots, x_n$

$\delta(q, a, x) = (p, y, R)$

$x_1, x_2, \dots, x_{i-1}, p, x_i, x_{i+1}, \dots, x_n$

5. $f(x, y) = y$

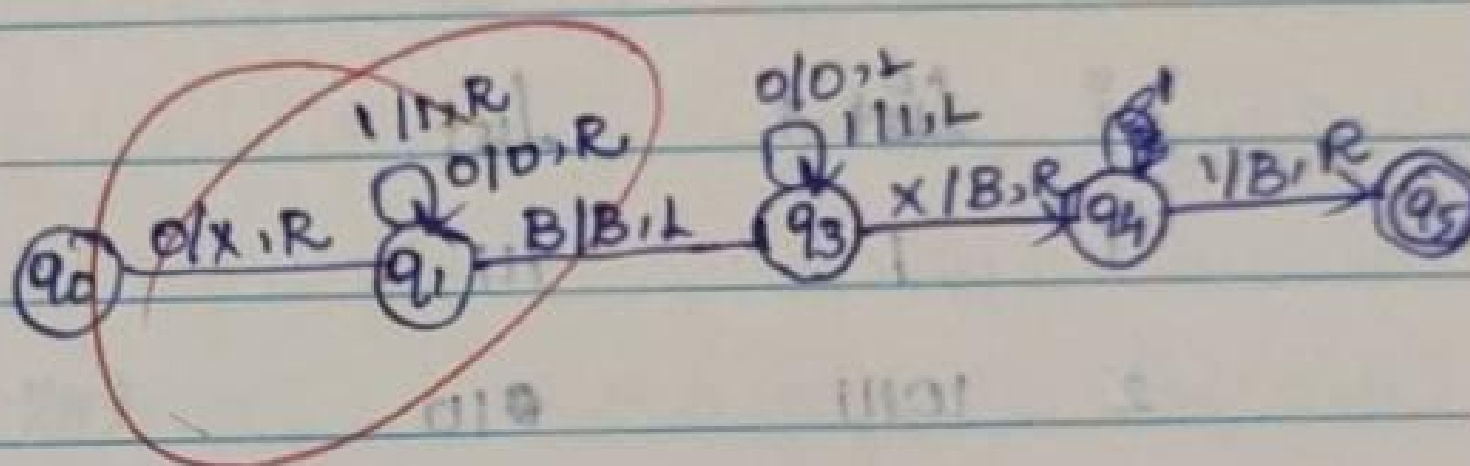
$f(1, 2) = 2$

B0100B

↓

B0100B

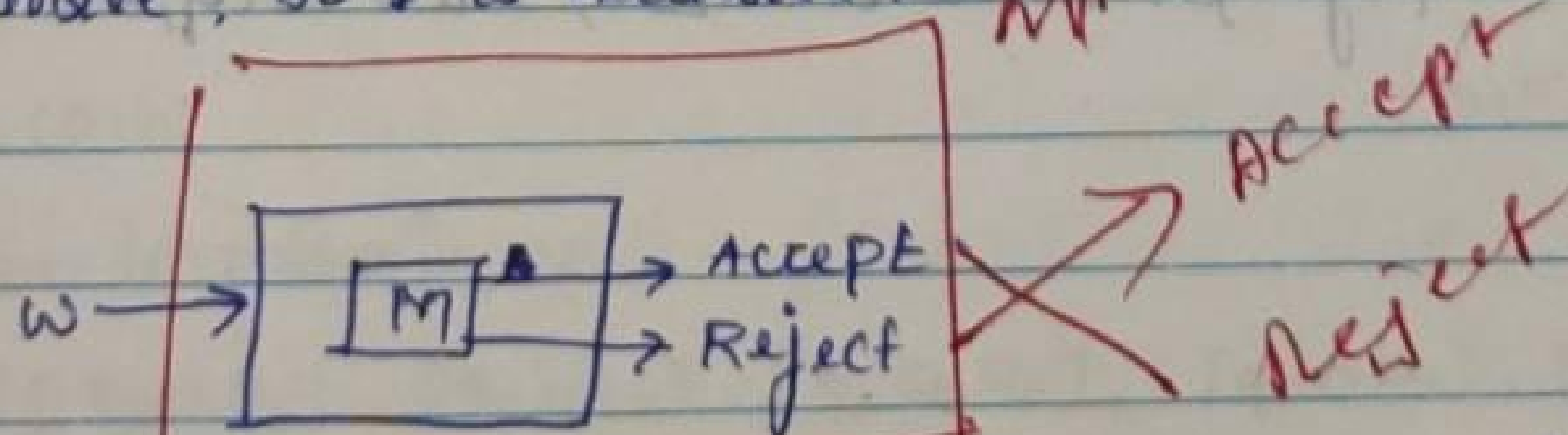
BBB00B



6. A language L is said to be recursively enumerable for some $T = T(M)$ TM halts whether or not accept the input.

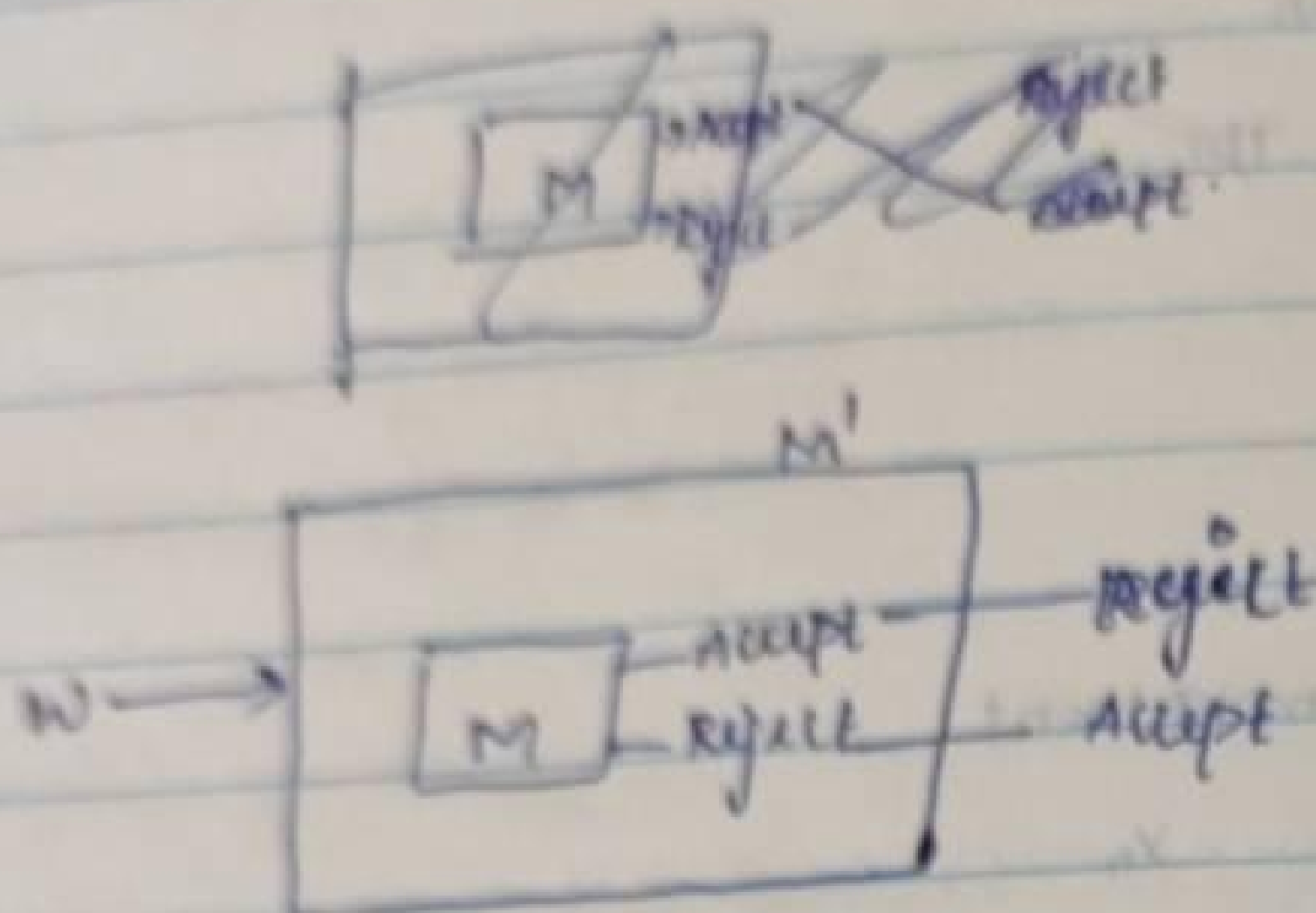
Theorem:

7. L is recursive, so L is recursive.

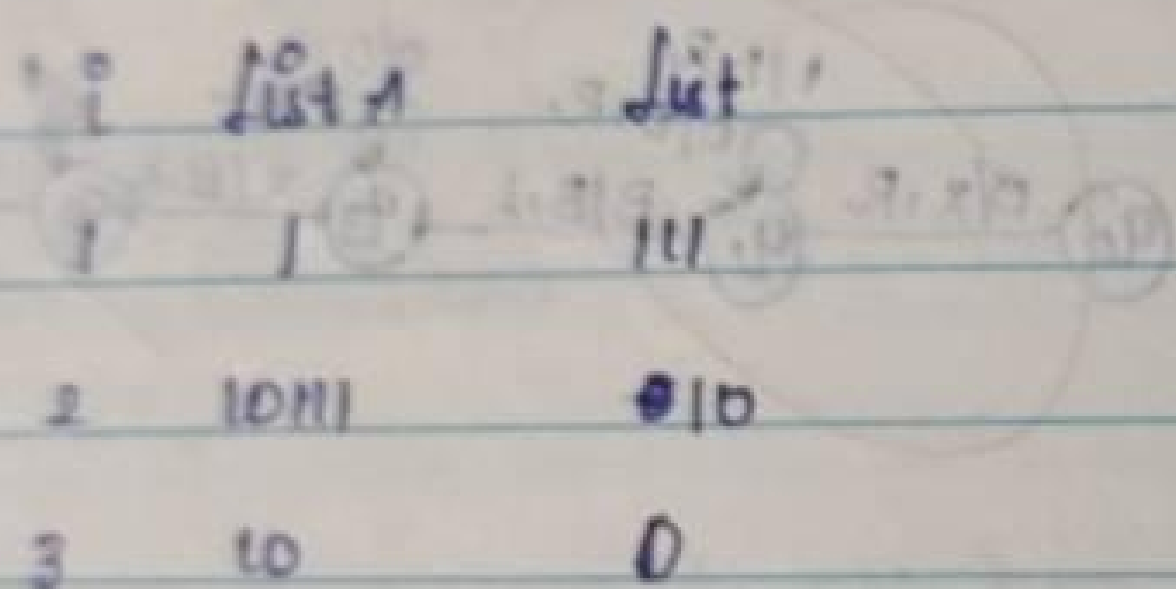


If w is in L , then M accepts w

If w is not in L , then \bar{M} accepts w .



8. Post Correspondence Problem is used to solve a problem about string is undecidable.



$$w_2 w_1 w_3 = x_2 x_1 x_3$$

9. A PDA is said to be deterministic, there is never choice of move in any situation.

PDA is DPDA iff

1) $\delta(q, a, x)$ has at most 1 number pair for any $q \in Q$, $a \in \Sigma$ or $a = \epsilon$
 2) $\delta(q, a, x)$ is non-empty for some $a \in \Sigma$, then $\delta(q, \epsilon, x)$ is non-empty

10. Rules for converting pda to cfa

1) PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0)$ for some $L \in L(P)$ then CFG $G = (V, T, P, S)$

1) start symbol $= S$

$S \rightarrow [q_0 Z_0 P]$

P - input states.

2) T - set of terminals

3) V - start symbol and all forms of states $[pxq]$

where $p, q \in Q, x \in \Gamma$

V - set of non-terminals

4) P - production

$\delta(q, a, \gamma) = (r, c)$

$[syt] \rightarrow ra$

$\delta(q, a, \gamma) = (r, c, \gamma_1 \gamma_2 \gamma_3 \dots \gamma_k)$

$[sy_k] \rightarrow a[t_1, t_2] [r, \gamma_1 \gamma_2 \gamma_3 \dots \gamma_k]$

Part-B

11. $M = \{ (q_0, q_1), \{a, b\}, \delta, q_0, Z_0, \phi \}$

CFG, $G = (V, T, P, S)$

1) Set of terminals $= T$

$T = \Sigma = \{a, b\}$

2) Set of non terminals $= V$

$V = \{ S, [q_0 a q_0], [q_0 b q_0], [q_0 Z_0 q_0], [q_0 a q_1], [q_0 b q_1], [q_0 Z_0 q_1], [q_1 a q_0], [q_1 b q_0], [q_1 Z_0 q_0], [q_1 a q_1], [q_1 b q_1], [q_1 Z_0 q_1] \}$

$$b) S \rightarrow [q_0 z_0 q_0] / [q_0 z_0 q_1]$$

12. A) Production:

$$\delta(q_0, a, a) = (q_1, \epsilon)$$

$$[q_0 a q_1] \rightarrow a \rightarrow \textcircled{1}$$

$$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$$

$$[q_1 z_0 q_1] \rightarrow \epsilon \rightarrow \textcircled{2}$$

$$\delta(q_0, a, z_0) = (q_0, a z_0)$$

$$[q_0 z_0 q_0] \rightarrow a [q_0 a q_0] [q_0 z_0 q_0] / a [q_0 a q_1] [q_1 z_0 q_0]$$

$$[q_0 z_0 q_1] \rightarrow a [q_0 a q_0] [q_0 z_0 q_1] / a [q_0 a q_1] [q_1 z_0 q_1]$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$[q_0 a q_0] \rightarrow a [q_0 a q_0] [q_0 a q_0] / a [q_0 a q_1] [q_1 a q_0]$$

$$[q_0 a q_1] \rightarrow a [q_0 a q_0] [q_0 a q_1] / a [q_1 a q_1] [q_1 a q_1]$$

$$\delta(q_0, b, a) = (q_0, a)$$

$$\text{or } \delta(q_0, b, a) = (q_0, ba)$$

$$[q_0 a q_0] \rightarrow b [q_0 a q_0]$$

$$[q_0 a q_0] \rightarrow b [q_0 b q_0] [q_0 a q_0] / b [q_0 b q_1] [q_1 a q_0]$$

$$[q_0 a q_1] \rightarrow b [q_0 a q_1]$$

$$[q_0 a q_1] \rightarrow b [q_0 b q_0] [q_0 a q_1] / b [q_1 b q_1] [q_1 a q_1]$$

Rules:

$$1) S \rightarrow [q_0 z_0 q_0] / [q_0 z_0 q_1]$$

$$2) [q_0 a q_1] \rightarrow a$$

$$3) [q_1 z_0 q_1] \rightarrow \epsilon$$

$$4) [q_0 z_0 q_0] \rightarrow a [q_0 a q_0] [q_0 z_0 q_0] / a [q_0 a q_1] [q_1 z_0 q_0]$$

$$5) [q_0 z_0 q_1] \rightarrow a [q_0 a q_0] [q_0 z_0 q_1] / a [q_1 a q_1] [q_1 z_0 q_1]$$

$$6) [q_0 a q_0] \rightarrow a [q_0 a q_0] [q_0 a q_0] / a [q_0 a q_1] [q_1 a q_0]$$

$$7) [q_0 a q_1] \rightarrow a [q_0 a q_0] [q_0 a q_1] / a [q_1 a q_1] [q_1 a q_1]$$

$$8) [q_0 a q_0] \rightarrow b [q_0 a q_0] \quad \text{or } [q_0 a q_0] \rightarrow b [q_0 b q_0] [q_0 a q_0] / b [q_0 b q_1] [q_1 a q_0]$$

$$9) [q_0 a q_1] \rightarrow b [q_0 a q_1] \quad [q_0 a q_1] \rightarrow b [q_0 b q_0] [q_0 a q_1] / b [q_1 b q_1] [q_1 a q_1]$$

Resultant productions:

$[q_0 a q_1] \rightarrow a$

$A \rightarrow [q_0 a q_1]$

$A \rightarrow a$

13. $f(m, n) = \max$

$m = 2 \quad n = 3$

subroutine: string copy.

Input: B 00 1 000 1 B B.

string copy:

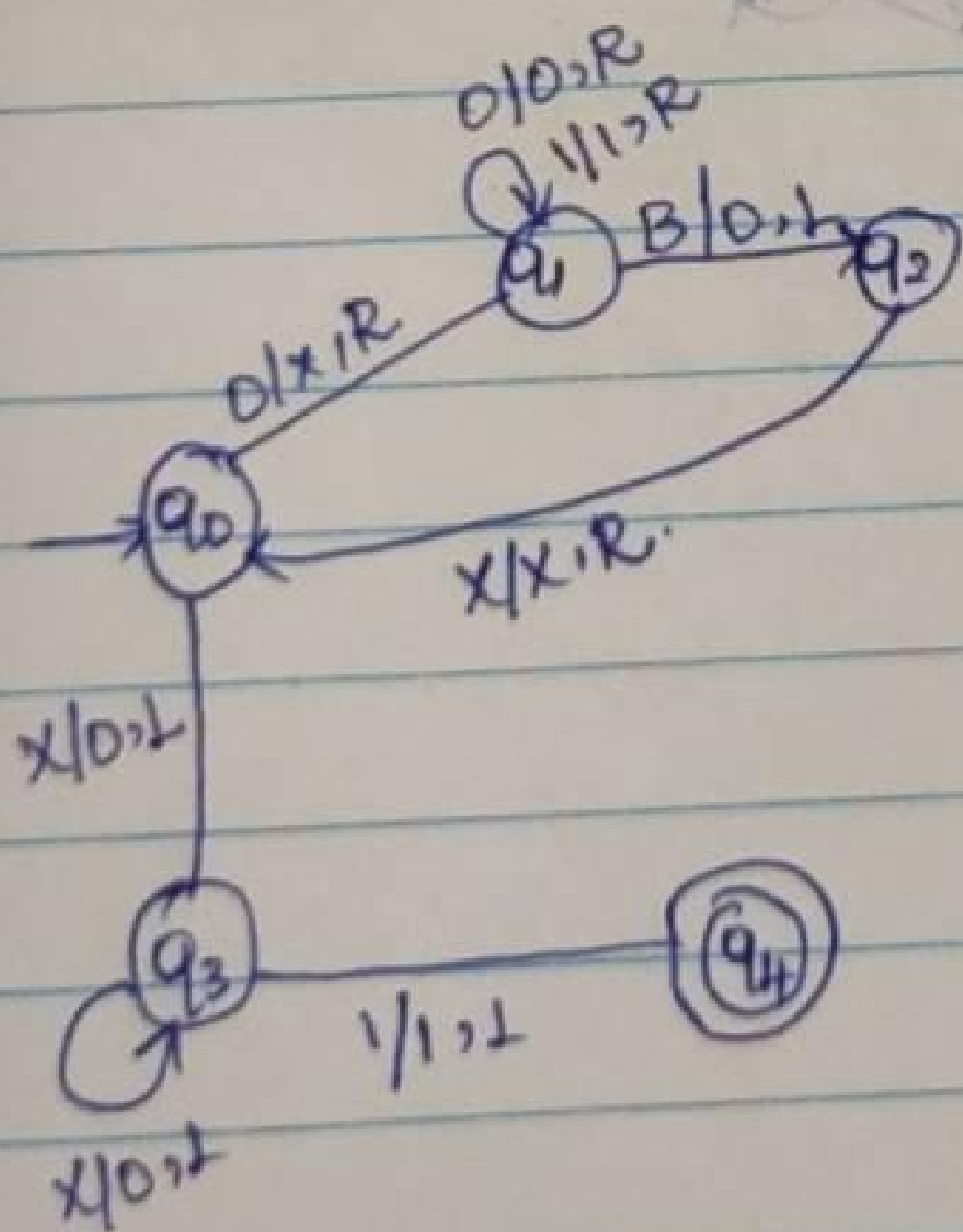
1 000 1 B B B.

1 x 00 1 0 B B B

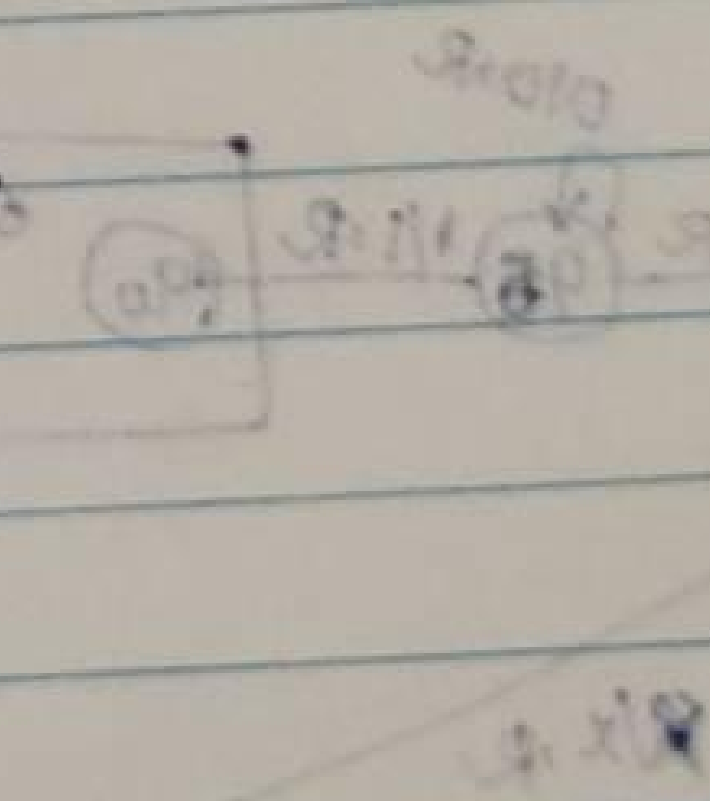
1 x x 0 1 0 0 B B B

1 x x x 1 0 0 0 B B B

1 000 1 000 B



Transition table



Transition table.

| S | 0 | 1 | x | B |
|-----------------|-------------------------|--|--------------------------|---------------------------|
| q ₅ | (q ₆ , x, R) | φ | φ | φ |
| q ₆ | (q ₆ , 0, R) | (q ₆ , 1, R) | x | x |
| q ₀ | (q ₁ , x, R) | (q₀, 0, L) (q ₃ , 0, L) | φ | φ |
| q ₁ | (q ₁ , 0, R) | (q ₁ , 1, R) | φ | (q ₂ , 0, L) |
| q ₂ | φ | φ | (q ₀ , x, R) | φ |
| q ₃ | φ | φ | (q ₃ , 0, L) | φ |
| q ₄ | φ | (q ₇ , φ, L) | φ | φ (q ₈ , B, L) |
| q ₇ | (q ₇ , 0, L) | φ | (q ₅ , x, R) | φ |
| q ₈ | φ | (q ₉ , B, L) | φ | φ |
| q ₉ | φ | (q ₁₀ , B, L) | (q ₉ , B, L) | φ |
| q ₁₀ | φ | φ | (q ₁₀ , B, L) | φ |

String processing:

$$M = \{(q_1, q_2, q_3), (0, 1), (0, B), \delta(q, B), (q_0, 3)\}$$

$$\delta(q_1, 1) = (q_3, 0, R)$$

$$C_1 = 00|00|000|0|00 = 0^2|0^2|0^3|0^1|0^2$$

$$\delta(q_3, 0) = (q_1, 1, R)$$

$$C_2 = 000|0|0|00|00 = 0^3|0^1|0^1|0^2|0^2$$

$$\delta(q_3, 1) = (q_2, 0, R)$$

$$C_3 = 000|00|00|0|00 = 0^3|0^2|0^2|0^1|0^2$$

$$\delta(q_3, B) = (q_3, 1, 1)$$

$$C_4 = 000|000|000|00|0 = 0^3|0^3|0^3|0^2|0^1$$

$$\text{code for TM, } M = \{C_1 || C_2 || C_3 || C_4\}$$

$$M = 00|00|000|0|00||000|0|0|00|00||000|00|00|0|00||000|000|000|00|$$