

Roll No.

| | |
|-------------------------------------|-------------------------|
| Programme : B.E | Date : 28.09.2017 |
| Branch : CSE | Time : 9.15am - 10.45am |
| Semester : V | |
| Course Code : 14CST52 | Duration : 1 ½ Hours |
| Course Name : Theory of Computation | Max. Marks : 50 |

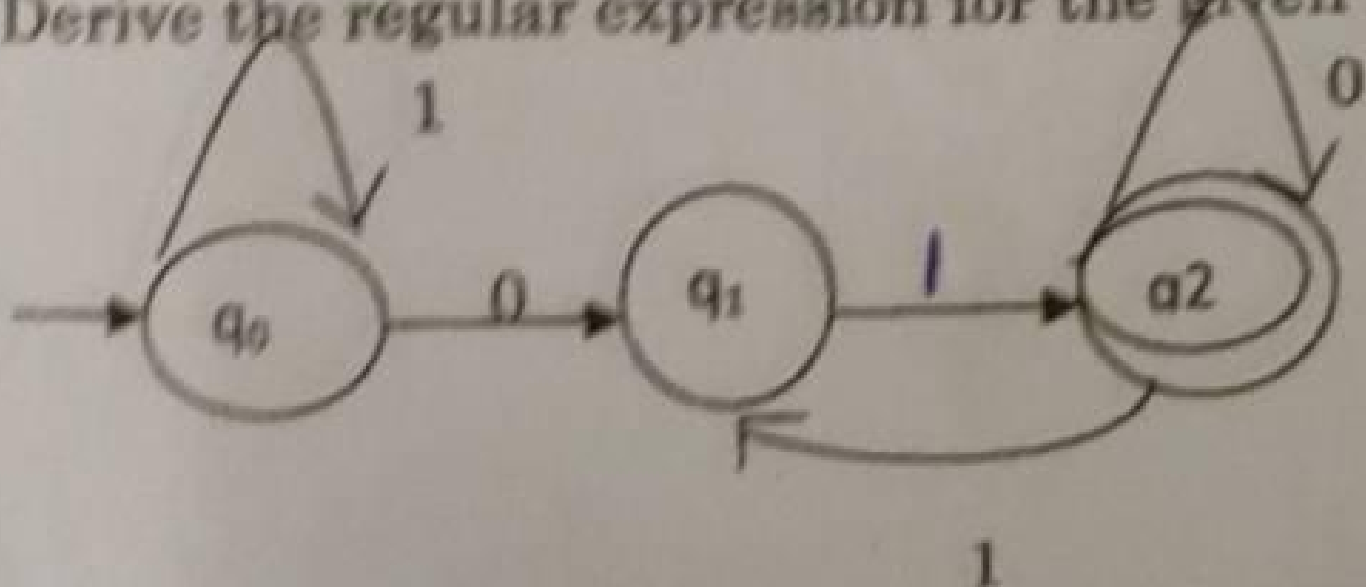
PART - A (10 X 2 = 20 Marks)

ANSWER ALL THE QUESTIONS

| | | | |
|-----|---|----------|----|
| 1. | Frame the regular expression for the DFA with 'n' states and has i as initial state, 'j' as final state and 'k' as intermediate states. | Co2, Co3 | K2 |
| 2. | Mention any four closure properties of regular language. | Co2 | K1 |
| 3. | State pumping lemma with its properties. | Co2 | K1 |
| 4. | Write the formal definition of CFG. | Co2 | K1 |
| 5. | Determine the CFL for the CFG $S \rightarrow OS1 / 1S0 / SS / 01 / 10$ | Co2 | K3 |
| 6. | Identify the CFG for the CFL $L = \{a^i b^j \mid i \geq j\}$ | Co2 | K2 |
| 7. | Draw the parse tree for the derivation of string id+id*id from grammar $E \rightarrow E+E / E * E / id$ | Co2 | K2 |
| 8. | Show that $S \rightarrow aB/ab$ $A \rightarrow aAb/a$ $B \rightarrow ABb/b$ is ambiguous. | Co2 | K2 |
| 9. | State the two ways of acceptance by PDA and define them. | Co3 | K1 |
| 10. | Identify the useless symbols and simplify the given grammar $S \rightarrow aAa / aBC$ $A \rightarrow aS/bD$ $B \rightarrow aBa / b$ $C \rightarrow abb/DD$ $D \rightarrow aDa$ | Co2 | K2 |

PART - B (3 X 10 = 30 Marks)

ANSWER ANY THREE QUESTIONS

| | | | | |
|-----|---|------|-----|----|
| 11. | Derive the regular expression for the given DFA  | (10) | Co3 | K3 |
| 12. | a) Show that $L = \{a^{n^2} \mid n \geq 1\}$ is not a regular language. | (10) | Co1 | K3 |

| | | | | | |
|-----|----|--|-------|-----|----|
| | b) | Design a PDA for $L = \{ ww^R / w \in \{0,1\}^* \}$ | (6) | Co3 | K3 |
| 13. | | Convert the following CFG to CNF $S \rightarrow 0A0/1B1/BB$ $A \rightarrow C$ $B \rightarrow S/A$ $C \rightarrow S/\epsilon$ | (4/6) | Co2 | K2 |
| 14. | | Convert the following CFG to GNF $S \rightarrow AB$ $A \rightarrow BS/b$ $B \rightarrow SA/a$ | (10) | Co2 | K3 |

| Bloom's Taxonomy Level | Remembering (K1) | Understanding (K2) | Applying (K3) | Analysing (K4) | Evaluating (K5) | Creating (K6) |
|------------------------|---------------------|-----------------------|------------------|-------------------|--------------------|------------------|
| Percentage | 26.67 | 40 | 33.33 | . | . | . |

Module Test - II

14CST52 - Theory of Computation

Answer key

Part A (10 x 2 = 20 marks)

$$R_{ij}^k = R_{ij}^{k-1} + R_{ik}^{k-1} (R_{kk})^* R_{kj}^{k-1}$$

closure properties (any four)

- (1) Union of two RL is regular
- (2) Intersection of " " "
- (3) The complement of " " "
- (4) Difference of two RL is regular
- (5) Reverse of a regular language is regular
- (6) Closure of a regular language is regular
- (7) Concatenation of " "
- (8) Homomorphism of a " " "
- (9) Inverse homomorphism " " "

$G = (N, T, P, S)$

$N \rightarrow$ Non-Terminals

$T \rightarrow$ Terminals

$P \rightarrow$ Production

$S \rightarrow$ Start State Symbol

g. Pumping lemma

- a tool to test certain languages are regular or not

for every string w split as xyz where

(1) $y \neq \epsilon$, $|y| \leq 1$

(2) $|xy| \leq n$

(3) for all $k \geq 0$, the string xy^kz should be in L where $|w| \leq n$

5. Set of all strings with equal number of a 's and b 's

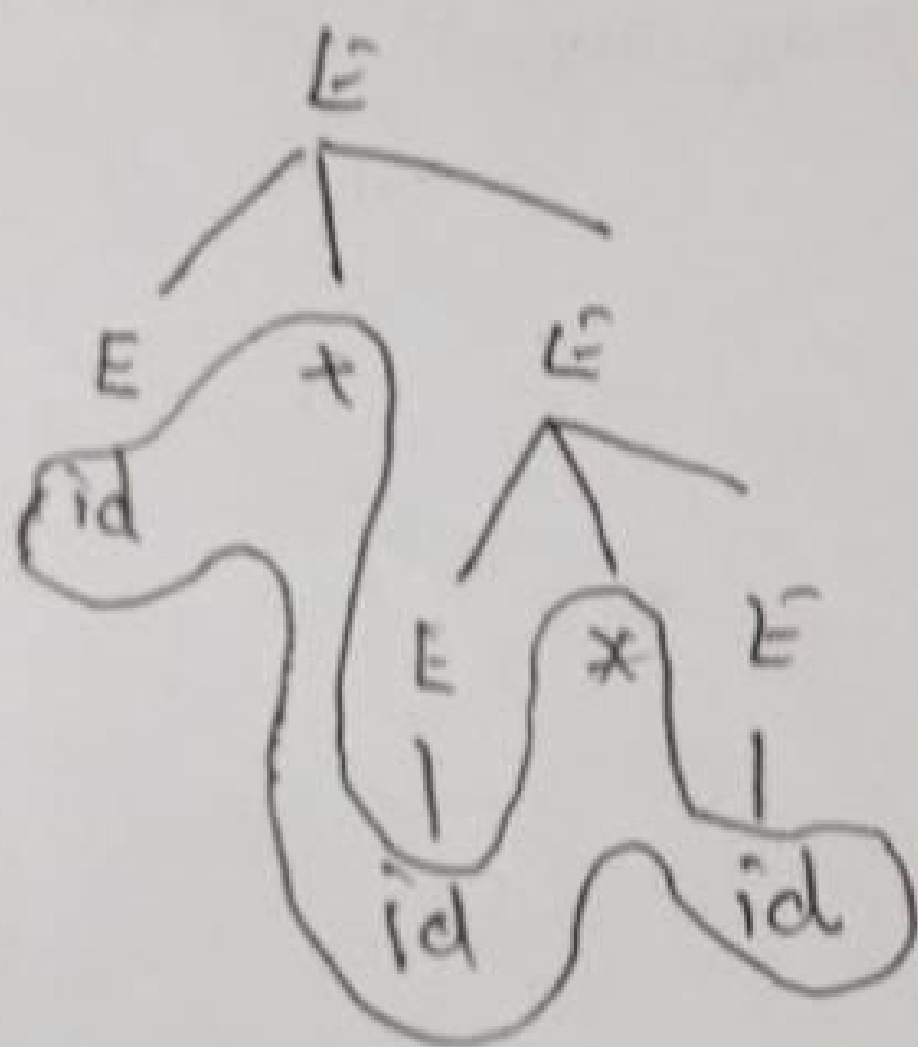
6. CFG for $L = \{a^i b^j \mid i \geq j\}$

~~$S \rightarrow ab$~~
 ~~$S \rightarrow asb$~~

$S \rightarrow asb$

$S \rightarrow \epsilon$

7. Parse tree



8. $S \rightarrow aB \mid ab$

$A \rightarrow aAb \mid a$

$B \rightarrow ABb \mid b$ is ambiguous

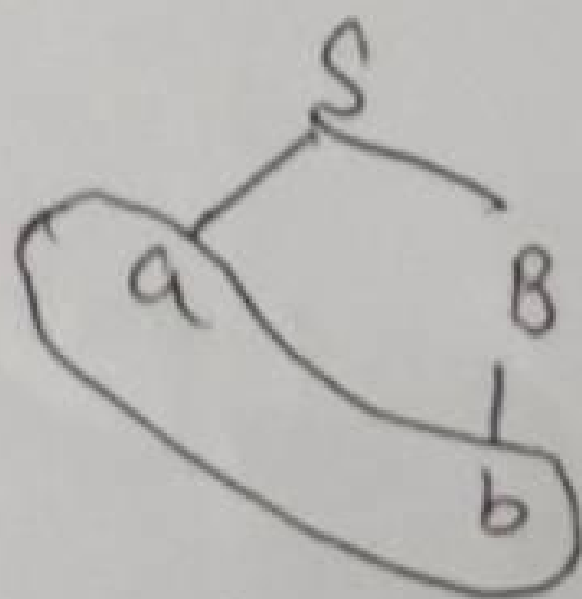
(1)

$S \rightarrow ab$

(2)

$S \rightarrow aB$

$S \rightarrow ab$



Since two different parse tree exists for single string

Two ways of acceptance by PDA

(i) Acceptance by final state

(ii) Acceptance by empty stack

Useless symbols

Useless symbol is D because it is not generating

Eliminate D .

$S \rightarrow aAa \mid aBC$

$A \rightarrow aS \mid \epsilon$

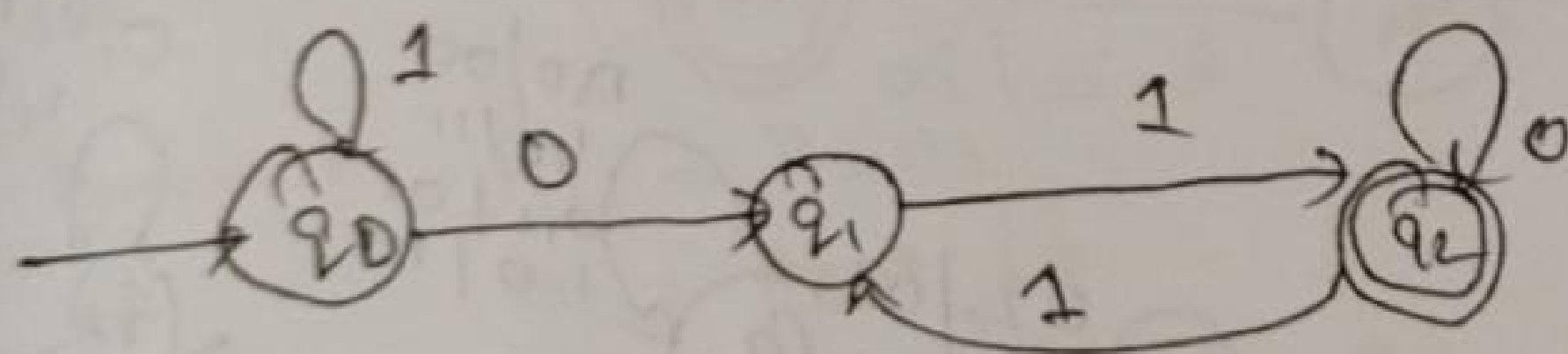
$B \rightarrow aBa \mid b$

$C \rightarrow abb$

Part B (3x10 = 30 marks)

(i) Obtain RE from DFA

or State elimination method.



$$R_{ij}^k = R_{ij}^{k-1} + R_{ik}^{k-1} (R_{kk}^{k-1})^* R_{kj}^{k-1} \quad \text{--- (1)}$$

$$R_{13}^3 = R_{13}^2 + R_{13}^2 (R_{33}^2)^* R_{33}^2 \quad \text{--- (1)}$$

Finaling

$$R_{13}^2 + 1^* 01 \rightarrow \text{--- (2)}$$

$$R_{33}^2 + (0 + \epsilon + 11) \quad \text{--- (2)}$$

(4) x

Final calculation of R_{13}^3 is result $1^* 01 (0 + 11)^*$

11.10.1. $\{a^n \mid n \geq 1\}$ is not a R.L.

$$w = a^{n^2} \quad |w| = n^2 > n \quad \text{--- (1)}$$

$$w = xyz = a^{n^2} \quad |xyz| = n^2 \quad \text{--- (2)}$$

$$|xyyz| = |xyz| + |y| \leq n^2 + n$$

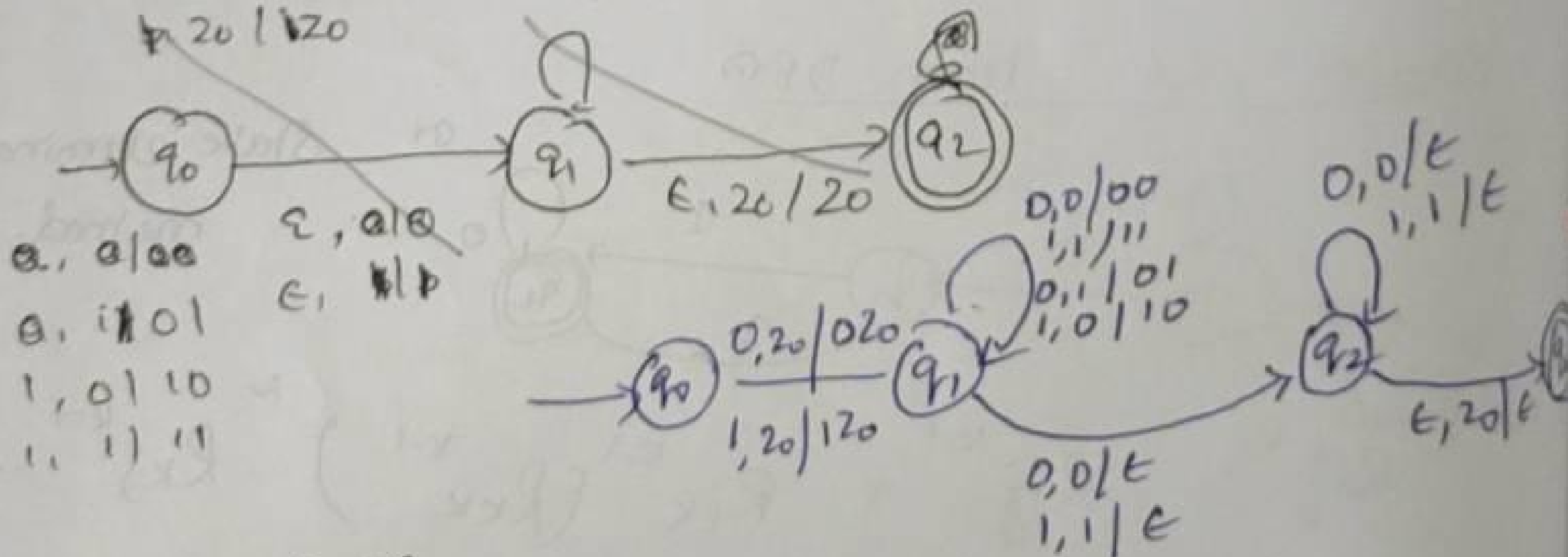
$$|xy^2z| < n^2 + n + (n+1)$$

$$|xy^2z| < n^2 + 2n + 1 < (n+1)^2$$

$$|xyz| = n^2 \quad |xy^2z| \neq n^2 \text{ but } < (n+1)^2$$

Perfect square cannot be between n^2 & $(n+1)^2$

5) PDA $w = a^n$
 $0, 20 \mid 020$
 $1, 20 \mid 120$



6) CFG to CNF

after simplification

$S \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid \bar{B}$

CNF \rightarrow

$$\begin{array}{l} S \rightarrow D_1 D_2 \mid D_2 D_2 \\ D_1 \rightarrow 0 \\ D_2 \rightarrow 1 \end{array}$$

14) CFG to GNF

$$\begin{array}{l} S \rightarrow AB \\ A \rightarrow AB \mid b \\ B \rightarrow SA \mid a \end{array}$$

other lemma 1

$$\begin{array}{l} A_1 \rightarrow A_2 A_3 \\ A_2 \rightarrow A_3 A_1 \mid b \\ A_3 \rightarrow A_1 A_2 \mid a \end{array}$$

$$A_3 \rightarrow A_3 A_1 A_3 A_2 \mid b A_3 A_2 \mid a$$

lemma 2

$$\begin{array}{l} A_1 \rightarrow A_2 A_3 \\ A_2 \rightarrow A_3 A_1 \mid b \\ B_1 \rightarrow A_1 A_3 A_2 \mid A_1 A_3 A_2 B_1 \\ A_3 \rightarrow b A_3 A_2 \mid a \mid b A_3 A_2 B_1 \mid a B_1 \end{array}$$

$$A_1 \rightarrow b A_3 A_2 A_1 A_3 \mid a A_1 A_3 \mid b A_3 A_2 B_1 A_1 A_3 \mid a B_1 A_3 A_2 A_1 A_3 \mid b A_3$$

$$A_2 \rightarrow b A_3 A_2 A_1 \mid a A_1 \mid b A_3 A_2 B_1 A_1 \mid a B_1 A_1 \mid b$$

$$B_1 \rightarrow b A_3 A_2 A_1 A_3 A_2 A_3 \mid a A_1 A_3 A_2 A_3 \mid b A_3 A_2 B_1 A_1 A_3 A_2 A_3 \mid a B_1 A_3 A_2 A_3 \mid b A_3 A_2 A_3$$

$$(b A_3 A_2 A_1 A_3 A_2 A_3 \mid a A_1 A_3 A_2 A_3 \mid b A_3 A_2 B_1 A_1 A_3 A_2 A_3 \mid a B_1 A_3 A_2 A_3 \mid b A_3 A_2 A_3)$$

$$A_3 \rightarrow b A_3 A_2 A_1 \mid a B_1 A_1 \mid a B_1 A_1 A_3 A_2 A_3 \mid b A_3 A_2 B_1 A_1 A_3 A_2 A_3 \mid b A_3 A_2 B_1$$



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Name and Signature of Hall Supdt. with Date

| | | | |
|----------------------|----------------------------------|-------------------|------------|
| Name of the Student | G. Raxuen | Register No. | 15 CSR 169 |
| Programme | B.E | Branch & Semester | CSE-C V |
| Course Code and Name | 14CST52 Theory Of Computation | Date | 28.9.17 |
| | | No. of Pages Used | 15 |

MARKS TO BE FILLED IN BY THE EXAMINER

| PART - A | | PART - B | | Grand Total Max. Marks : 50 | |
|--------------|---------------|--------------|----------------|---|--|
| Question No. | Max Marks : 2 | Question No. | Max Marks : 10 | | |
| 1 | 2 | 11 | i) 10 | <div style="border: 2px solid red; border-radius: 50%; padding: 20px; display: inline-block;"> 46.5 50 </div> v. good G. Raxuen | |
| 2 | 2 | | ii) | | |
| 3 | 0 | 12 | i) | | |
| 4 | 2 | | ii) | | |
| 5 | 2 | 13 | i) 10 | | |
| 6 | 2 | | ii) | | |
| 7 | 2 | 14 | i) | | |
| 8 | 1 | | ii) | | |
| 9 | 2 | TOTAL | 30 | | |
| 10 | 2 | | | | |
| TOTAL | 16.5 | | | | |

Total Marks in Words : Forty six and half

INSTRUCTION TO THE CANDIDATE

1. Check the Question Paper, Programme, Course Code, Branch Name etc., before answering the questions.
2. Use both sides of the paper for answering questions.
3. POSSESSION OF ANY INCRIMINATING MATERIAL AND MALPRACTICE OF ANY NATURE IS PUNISHABLE AS PER RULES.

Name of the Examiner

Signature of the Examiner with Date

1. Regular Expression:-
Regular Expression for DFA with n states and k intermediate with i as initial state and j as final state is given as,

$$R_{ij}^k = R_{ij}^{k-1} + (R_{ik})^{k-1} (R_{kk})^{k-1} (R_{kj})^{k-1}$$

2. Closure properties:-

Union of two regular language is also regular.

Intersection of two regular language is also regular.

Difference of two regular language is also regular.

Complement of a regular language is also regular.

Homomorphism of a regular language is also regular.

3. Pumping lemma:-

It is used to state that the language is not accepted by the DFA.

Properties:-

power is always $\geq n$

xy^kz where $k=0,1,2,\dots$

4. CFG:-
Context Free Grammar is defined as,

$$G = \{V, T, P, S\}$$

V - finite set of nonterminals

T - finite set of Terminals

P - Production

S - starting state.

5

$$S \rightarrow 0S1 \mid 1S0 \mid SS \mid 01 \mid 10.$$

Language for the Grammar is

$L = \{ \text{Set of all strings with equal number of 0's and equal number of 1's} \}$
i.e., no. of 0's = no. of 1's

6.

$$L = \{ a^i b^j \mid i \geq 0 \}$$

CFG is,

$$G = \{V, T, P, S\}$$

P:

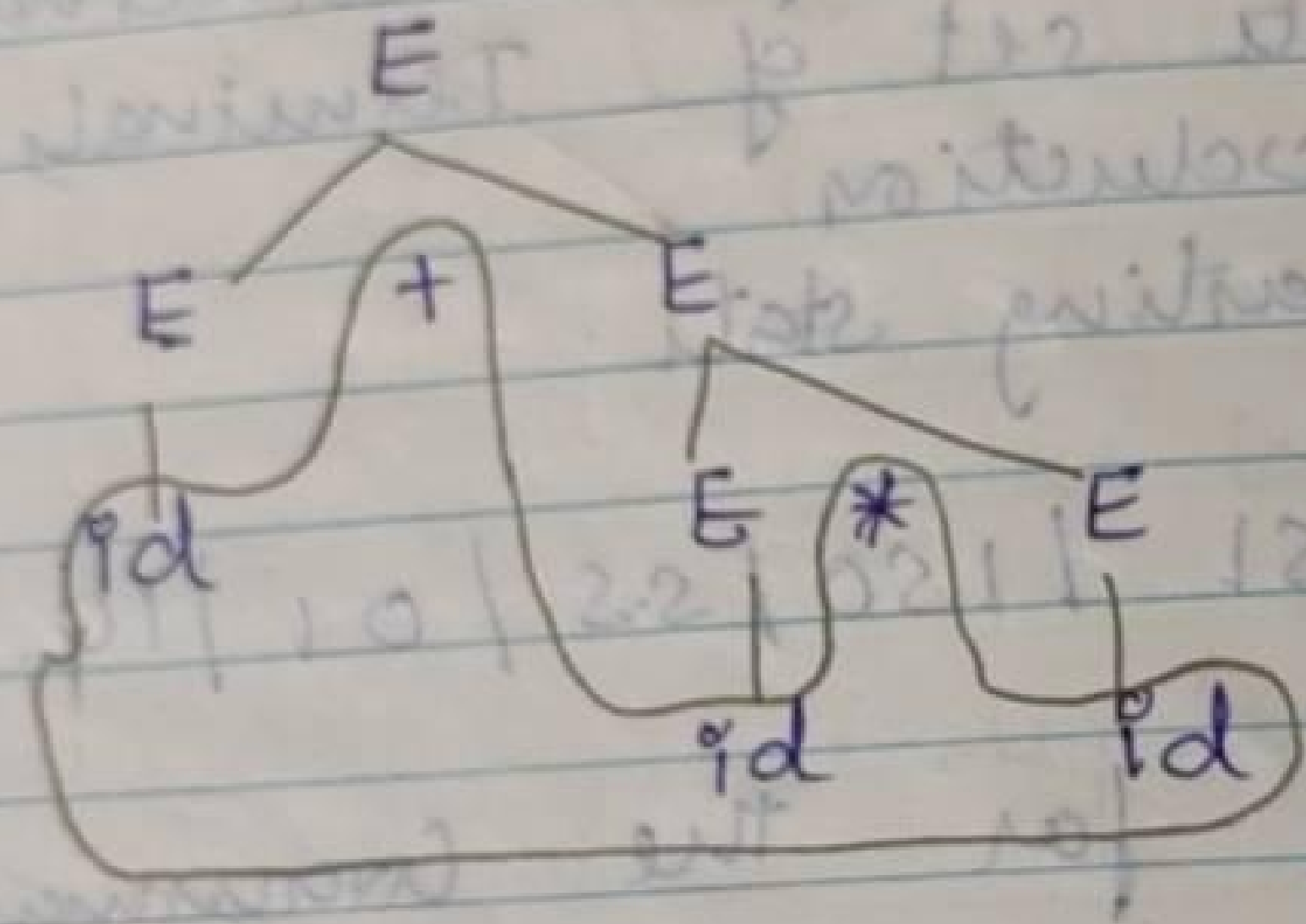
$$S \rightarrow asb \mid \epsilon$$

$$G = \{ \{S\}, \{a, b\}, P, S \}$$

$E \rightarrow E + E$
 $E \rightarrow E * E$
 $E \rightarrow id$

$\{2, 9, T, V\} = P$

String $id + id * id$.
Parse Tree



String: $id + id * id$ prints 110 for $aABb$

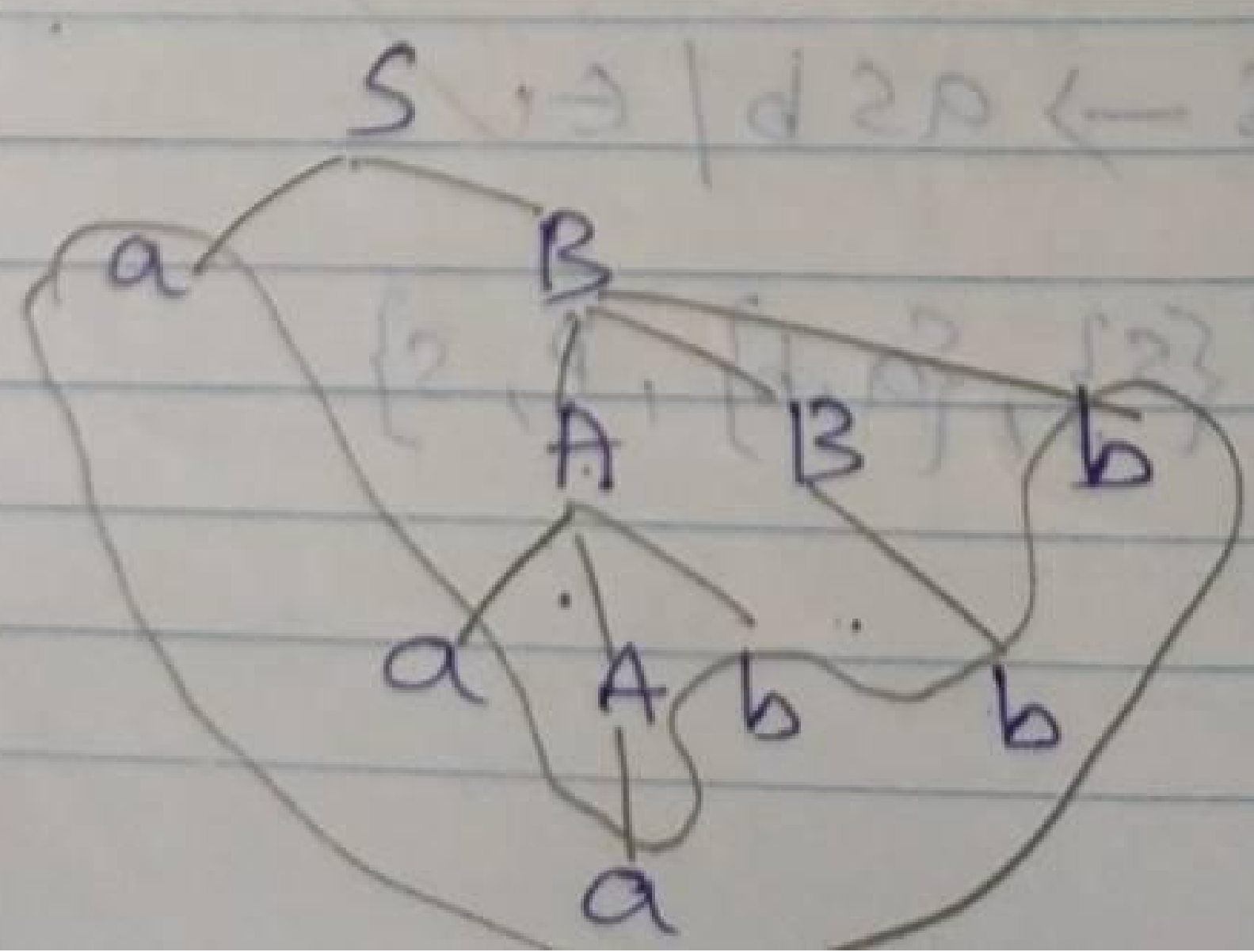
2.

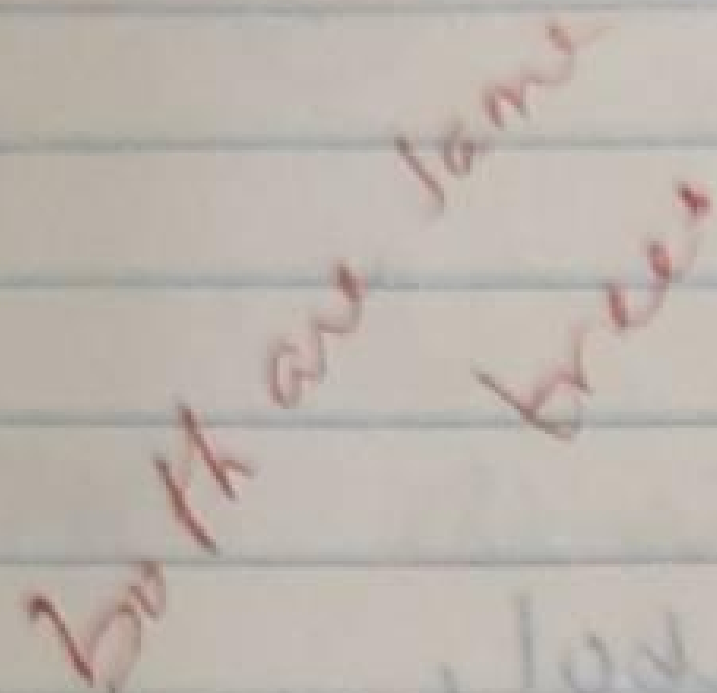
$S \rightarrow aB | ab$
 $A \rightarrow aAb | a$
 $B \rightarrow ABb | b$

is ambiguous.

Let us consider the string, $aaabbb$.

Parse Tree





Both the parse tree gives same string.

9. Two ways of acceptance by PDA

Accepting by final state

Accepting by empty stack.

Accepting by final state :-

Here the PDA has the final state as,

$$E, Z_0 \mid e.$$

20 is the final element in the stack. So if 20 is reached the string is accepting.

Accepting by empty stack:-

the entire stack is removed including 20.

$S \rightarrow aAa|aBC$

$A \rightarrow aS|aB|bD$

$B \rightarrow aBa|b$

$C \rightarrow abb|DD$

$D \rightarrow aDa$

To identify useless symbol,

Reachable symbol = $\{S, A, B, C, a, b, D\}$

Generating symbol = $\{a, b, S, B, C, A\}$

The useless symbol is \boxed{D}
eliminating all the production for D , we get

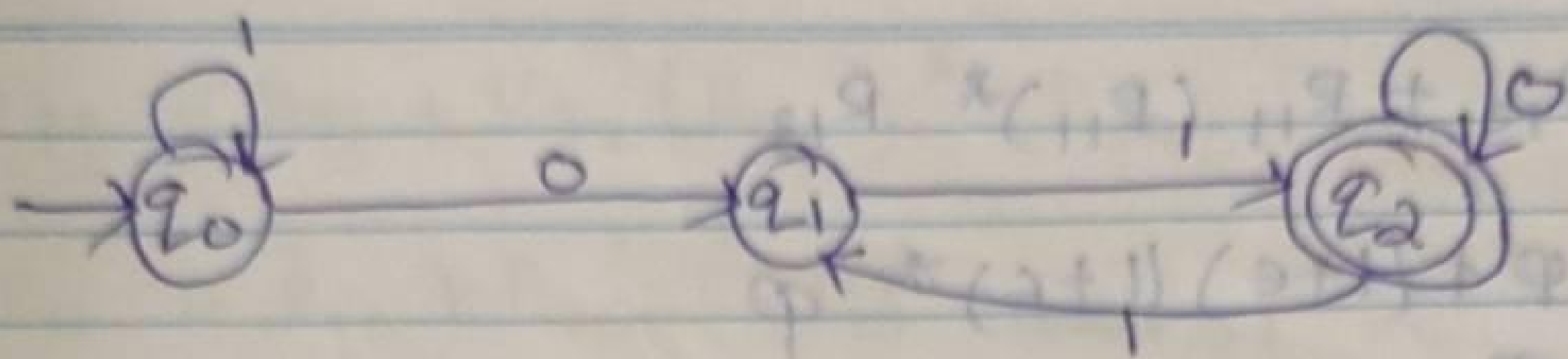
$S \rightarrow aAa|aBC$

$A \rightarrow aS$

$B \rightarrow aBa|b$

$C \rightarrow abb$

11



Starts elimination method:

$$R_{ij}^k = R_{ij}^{k-1} + R_{ik}^{k-1} (R_{kk}^{k-1})^* R_{kj}^{k-1}$$

$k=0$.

$$R_{11} = 1 + \epsilon$$

$$R_{21} = \varnothing$$

$$R_{31} = \varnothing$$

$$R_{12} = 0$$

$$R_{22} = \epsilon$$

$$R_{32} = 1 + \epsilon$$

$$R_{13} = \varnothing$$

$$R_{23} = 1$$

$$R_{33} = 0 + \epsilon$$

$k=1$.

$$R_{ij} = R_{ij} + R_{i1} (R_{11})^* R_{1j}$$

$$R_{11} = R_{11} + R_{11} (R_{11})^* R_{11}$$

$$= 1 + \epsilon + (1 + \epsilon) (1 + \epsilon)^* (1 + \epsilon) + \varnothing =$$

$$= 1 + \epsilon + (1 + \epsilon)^*$$

$$R_{11} = 1 + \epsilon$$

$$R_{12} = R_{12} + R_{11} (R_{11})^* R_{12}$$

$$= 0 + (1 + \epsilon) (1 + \epsilon)^* \cdot 0$$

$$= 0 + (1 + \epsilon)^* \cdot 0$$

$$= 0 + 1^* \cdot 0$$

$$= 1^* \cdot 0$$

$$R_{13} = R_{13} + R_{11} (R_{11})^* R_{13}$$

$$= \varnothing + (1+\epsilon)(1+\epsilon)^* \varnothing$$

$$R_{13} = \varnothing$$

$$R_{21} = R_{21} + R_{21} (R_{11})^* R_{12}$$

$$= \varnothing + \varnothing (1+\epsilon)^* \varnothing (1+\epsilon)$$

$$= \varnothing$$

$$R_{22} = R_{22} + R_{21} (R_{11})^* R_{12}$$

$$= \epsilon + \varnothing (1+\epsilon)^* \varnothing$$

$$= \epsilon$$

$$R_{23} = R_{23} + R_{21} (R_{11})^* R_{13}$$

$$= 1 + \varnothing (1+\epsilon)^* \varnothing$$

$$R_{23} = 1$$

$$R_{31} = R_{31} + R_{31} (R_{11})^* R_{11}$$

$$= \varnothing + \varnothing (1+\epsilon)^* (1+\epsilon)$$

$$= \varnothing$$

$$R_{32} = R_{32} + R_{31} (R_{11})^* R_{12}$$

$$= 1 + \varnothing (1+\epsilon)^* \varnothing$$

$$= 1$$

$$R_{33} = R_{33} + R_{31} (R_{11})^* R_{13}$$

$$= 0 + \epsilon + \varnothing (1+\epsilon)^* \varnothing$$

$$= 0 + \epsilon$$

R_{22} .

$$R_{1j} = R_{1j} + R_{12}(R_{22})^* R_{2j}$$

$$R_{11} = R_{11} + R_{12}(R_{22})^* R_{21}$$

$$= (1 + \epsilon) + 0 \cdot \epsilon^* \cdot (1 + \epsilon) \cdot \varnothing$$

$$= 1 + \epsilon + 0 \cdot \epsilon^* \cdot (1 + \epsilon) = (1 + \epsilon) \cdot (1 + \epsilon) = 1 + \epsilon$$

$$R_{12} = R_{12} + R_{12}(R_{22})^* R_{22}$$

$$= 0 + 0 \cdot \epsilon^* \cdot \epsilon$$

$$= 0 + 0 \cdot \epsilon$$

$$= 0 \cdot (\epsilon)$$

$$R_{13} = R_{13} + R_{12}(R_{22})^* R_{23}$$

$$= \varnothing + 0 \cdot \epsilon^* \cdot 1$$

$$= 0 \in 1$$

$$R_{21} = R_{21} + R_{22}(R_{22})^* R_{21}$$

$$= \varnothing + \epsilon(\epsilon^*) \varnothing = \varnothing$$

$$R_{22} = R_{22} + R_{22}(R_{22})^* R_{22}$$

$$= \epsilon + \epsilon \epsilon^* \cdot \epsilon = \epsilon$$

$$R_{23} = R_{23} + R_{22}(R_{22})^* R_{23}$$

$$= 1 + \epsilon \cdot \epsilon^* \cdot 1$$

$$= 1 + \epsilon \cdot 1$$

$$R_{31} = R_{31} + R_{32}(R_{22})^* R_{21}$$

$$= \varnothing + 1 \cdot \epsilon^* \cdot \varnothing = \varnothing$$

$$R_{32} = R_{32} + R_{32}(R_{22})^* R_{22}$$

$$= 1 + 1 \cdot \epsilon^* \cdot \epsilon = 1 + 1$$

$$R_{33} = R_{33} + R_{32}(R_{22})^* R_{23}$$

$$= 0 + \epsilon + 1 \cdot \epsilon^* \cdot 1 = 0 + \epsilon + 1 \cdot \epsilon \cdot 1$$

$$K=3.$$

$$R_{ij}^{(0)} = R_{ij}^{(1)} + R_{i3}^{(1)} (R_{33}^{(1)})^* R_{3j}^{(1)}$$

$$R_{13} = R_{13}^{(1)} + R_{13}^{(1)} (R_{33}^{(1)})^* R_{33}^{(1)} = 0 + 0 + 0 = 0$$

$$= 0 + 0(0+0)(0+0)$$

the Regular Expression. RE \mathcal{L}_1 ,

$$RE = 1^* \cdot 0 (10^* 1)^* 10^*$$

$$\text{---}$$

$$10^* (10^* 1)^* 10^* = 10^*$$

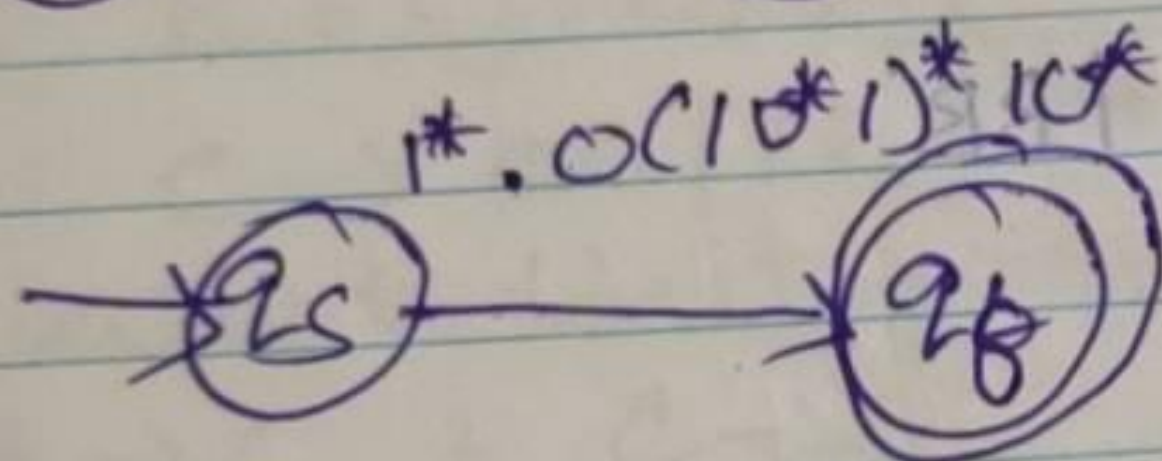
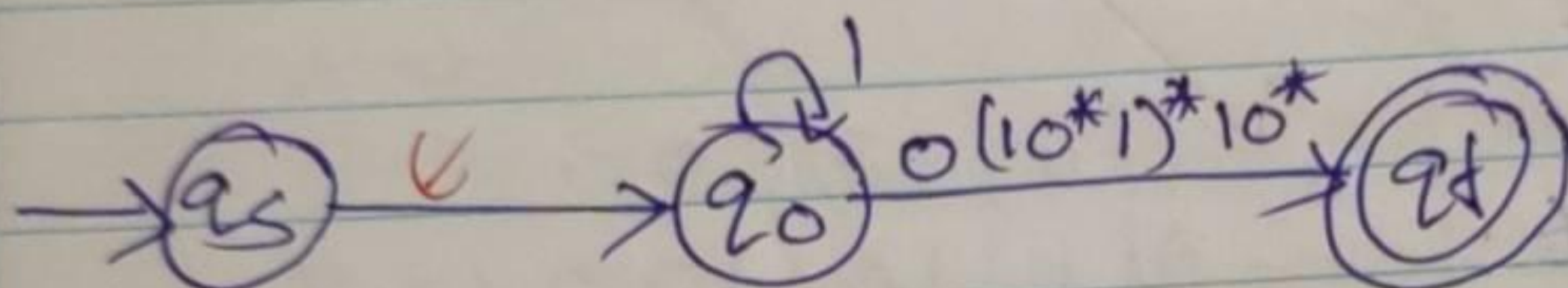
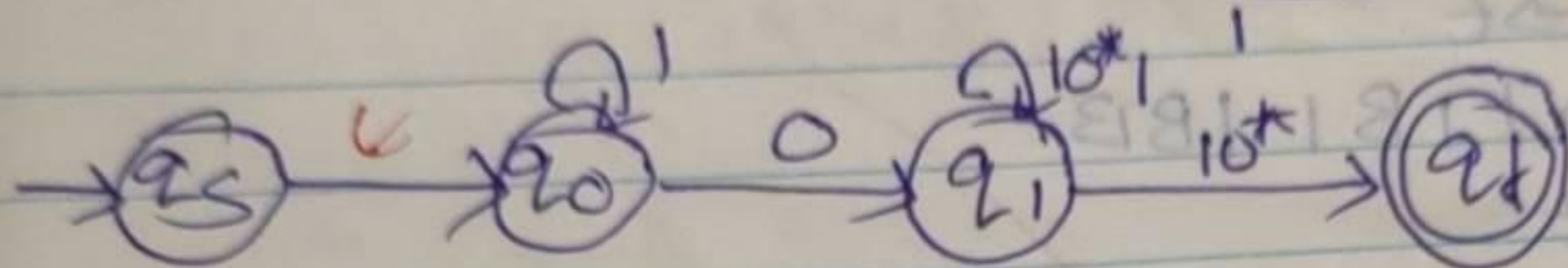
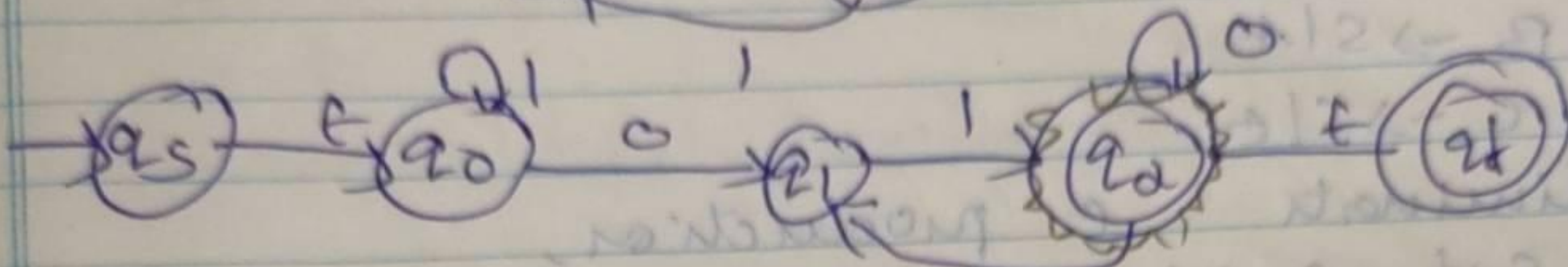
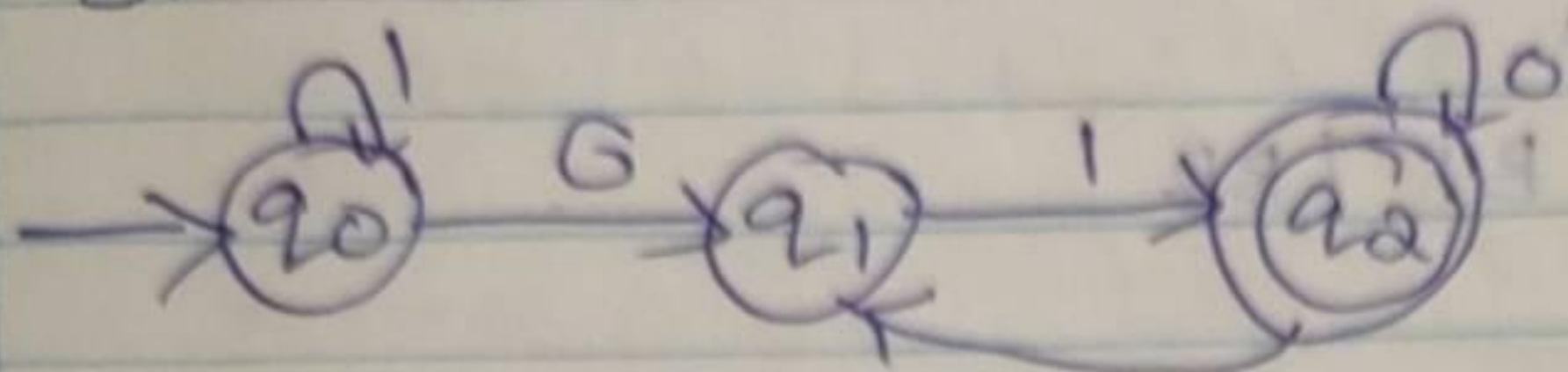
$$\phi = \phi(1^*) + \phi$$

$$10^* (10^* 1)^* 10^* = 10^*$$

$$10^* (10^* 1)^* 10^* = 10^*$$

$$10^* (10^* 1)^* 10^* = 10^*$$

State Elimination method.



$$R.E = 1^* \cdot 0(10^*1)^*10^*+$$

13.

CFG to CNF

$S \rightarrow OAO | IBI | BB$

$A \rightarrow \epsilon C$

$B \rightarrow S | A$

$C \rightarrow S | \epsilon$

Eliminate ϵ production,

Sub $C \rightarrow \epsilon$

$S \rightarrow OAO | IBI | BB$

$A \rightarrow C | \epsilon$

$B \rightarrow S | A$

$C \rightarrow S$

Sub $A \rightarrow \epsilon$

$S \rightarrow OAO | O O | IBI | BB$

$A \rightarrow C$

$B \rightarrow S | A$

$C \rightarrow S$

Sub $B \rightarrow \epsilon$

$S \rightarrow OAO | O O | IBI | I | B | \epsilon$

$A \rightarrow C$

$B \rightarrow S | A$

$C \rightarrow S$

Sub $S \rightarrow \epsilon$

$S \rightarrow OAO | O O | IBI | I | B$

Sub $A \rightarrow \epsilon$

$B \rightarrow S | A | \epsilon$

$C \rightarrow S$

Sub $B \rightarrow \epsilon$

$S \rightarrow OAO | O O | IBI | I | B | \epsilon$

$C \rightarrow \epsilon$

Sub B → C

$S \rightarrow 00A0 \mid 00 \mid 1B \mid 11 \mid B$

$A \rightarrow C$

$B \rightarrow 0S \mid A$

$C \rightarrow S$

Eliminate unit production

$S \rightarrow 00 \mid 1B \mid 11 \mid B$ The CNF form.

$B \rightarrow 0S$

$A \rightarrow C$

$A \rightarrow S$

$A \rightarrow 00 \mid 1B \mid 11 \mid B$

$B \rightarrow 00 \mid 1B \mid 11 \mid B$

$C \rightarrow 00 \mid 1B \mid 11 \mid B$

Eliminate Unit Production.

$A \rightarrow C$ $C \rightarrow S$

$S \rightarrow 0S0 \mid 00 \mid 1B \mid BB$

$A \rightarrow 0S0 \mid 00 \mid 1B \mid BB$

$B \rightarrow 0S0 \mid 00 \mid 1B \mid BB$

$C \rightarrow 0S0 \mid 00 \mid 1B \mid BB$

Useless Symbol.

Reachable symbol = $\{S, 0, 1, B\}$

Generating symbol = $\{0, 1, S, A, B, C\}$

Useless symbol are A and C.

$S \rightarrow 0S0 \mid 00 \mid 1B \mid BB$

$B \rightarrow 0S0 \mid 00 \mid 1B \mid BB$

$C_1 \rightarrow 0$

$C_2 \rightarrow 1$

$C_3 \rightarrow C_1S$

$C_4 \rightarrow C_2B$

$S \rightarrow C_1SC_1 \mid C_1C_1 \mid C_2BC_2 \mid BB$

$S \rightarrow C_3C_1 \mid C_1C_1 \mid C_4C_2 \mid BB$

14.

CFG to GNF

$S \rightarrow AB$

$A \rightarrow BS \mid b$

$B \rightarrow SA \mid a$

The given production is in CNF

Sub $S = A_1, A = A_2, B = A_3$

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_1 \mid b$

$A_3 \rightarrow A_1 A_2 \mid a$

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_1 \mid b$

$A_3 \rightarrow A_1 A_2 \mid a$

Sub A_1 in A_3

$A_3 \rightarrow A_2 A_3 A_2 \mid a$

Sub A_2 in A_3

$A_3 \rightarrow A_3 A_1 A_3 A_2 \mid b A_3 A_1 \mid a$

Sub. Introduce B_3

$B_3 \rightarrow A_1 A_3 A_2 \mid A_1 A_3 A_2 B_3$

Sub B_3 in A_3

$A_3 \rightarrow A_3 B_3 \mid b A_3 A_2 \mid a$

Sub other rule of A_3 in A_3

$A_3 \rightarrow bA_3A_2A_1A_3A_2 \mid bA_3A_2 \mid aA_3A_2$

Sub A_3 in A_2

$A_2 \rightarrow bA_3A_2A_1A_3A_2A_1 \mid bA_3A_2A_1 \mid aA_1 \mid b$

Sub A_2 in A_1

$A_1 \rightarrow bA_3A_2A_1A_3A_2A_1A_3A_2A_1 \mid bA_3A_2A_1A_3A_2A_1 \mid aA_1A_3A_2A_1 \mid aA_1A_3A_2A_1$

$A_1 \rightarrow bA_3A_2A_1A_3A_2A_1 \mid bA_3A_2A_1A_3A_2A_1 \mid aA_1A_3A_2A_1 \mid aA_1A_3A_2A_1$

Sub other rule of A_3 in A_3

$A_3 \rightarrow bA_3A_2B_3 \mid aB_3 \mid bA_3A_2 \mid a$

Sub A_3 in A_2

$A_2 \rightarrow bA_3A_2B_3A_1 \mid aB_3A_1 \mid bA_3A_2A_1 \mid aA_1 \mid b$

Sub A_2 in A_1

$A_1 \rightarrow bA_3A_2B_3A_1A_3 \mid aB_3A_1A_3 \mid bA_3A_2A_1A_3 \mid aA_1A_3 \mid b$

Sub A_1 in B_3

$B_3 \rightarrow bA_3A_2B_3A_1A_3A_3A_2 \mid aB_3A_1A_3A_3A_2 \mid b$

$bA_3A_2A_1A_3A_3A_2 \mid aA_1A_3A_3A_2 \mid bA_3A_3A_2 \mid$

$bA_3A_2B_3A_1A_3A_3A_2B_3 \mid aB_3A_1A_3A_3A_2B_3 \mid bA_3A_2A_1A_3A_3A_2B_3 \mid$

$aA_1A_3A_3A_2B_3 \mid bA_3A_3A_2B_3$

The Grammar in GNF is

$$A_1 \rightarrow bA_3A_2B_3A_1A_3 \mid aB_3A_1A_3 \mid bA_3A_2A_1A_3 \mid aA_1A_3 \mid bA_3$$

$$A_2 \rightarrow bA_3A_2B_3A_1 \mid aB_3A_1 \mid bA_3A_2A_1 \mid aA_1 \mid b$$

$$A_3 \rightarrow bA_3A_2B_3 \mid aB_3 \mid bA_3A_2 \mid a$$

$$B_3 \rightarrow bA_3A_2B_3A_1A_3A_3A_2 \mid aB_3A_1A_3A_3A_2 \mid bA_3A_2A_1A_3A_3A_2 \mid aA_1A_3A_3A_2 \mid bA_3A_3A_2$$

$$bA_3A_2B_3A_1A_3A_3A_2B_3 \mid aB_3A_1A_3A_3A_2B_3$$

$$bA_3A_2A_1A_3A_3A_2B_3 \mid aA_1A_3A_3A_2B_3 \mid bA_3A_3A_2B_3$$

$$A_1 \rightarrow bA_3A_2B_3A_1A_3A_3A_2 \mid aB_3A_1A_3A_3A_2 \mid bA_3A_2A_1A_3A_3A_2 \mid aA_1A_3A_3A_2 \mid bA_3A_3A_2$$

$$A_2 \rightarrow bA_3A_2B_3A_1 \mid aB_3A_1 \mid bA_3A_2A_1 \mid aA_1 \mid b$$

$$A_3 \rightarrow bA_3A_2B_3 \mid aB_3 \mid bA_3A_2 \mid a$$