KONGU ENGINEERING COLLEGE, PERUNDURAI, ERODE – 638 060

ODD SEMESTER 2017 – 2018

CYCLE TEST – I

Roll No............................

Programme : B.E Branch : CSE Semester : V	Date : 31.07.2017 Time : 09.15 a.m – 10.45 a.m
Course Code : 14CST52 Course Name : Theory of Computation	Duration : 1 ½ Hours Max. Marks : 50

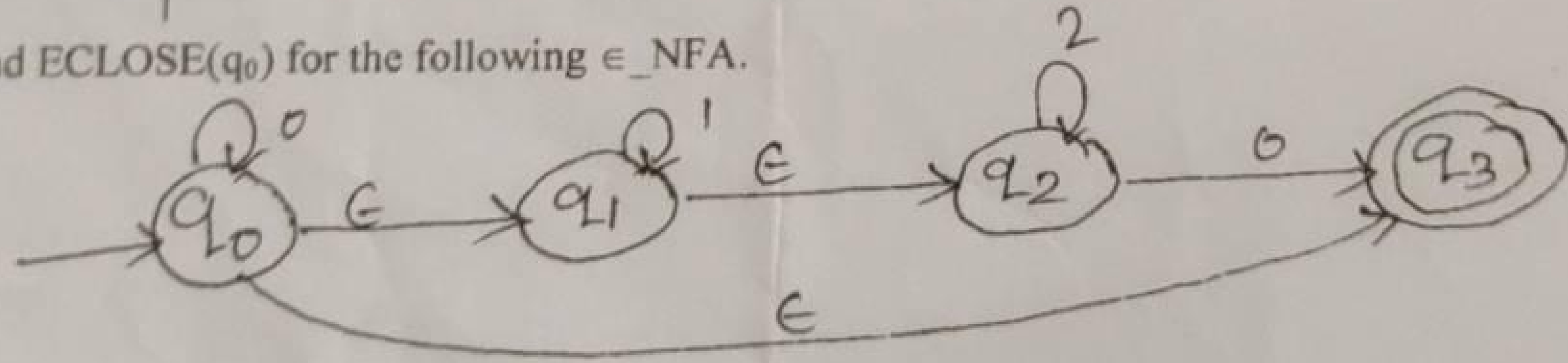
PART - A (10 X 2 = 20 Marks)

ANSWER ALL THE QUESTIONS

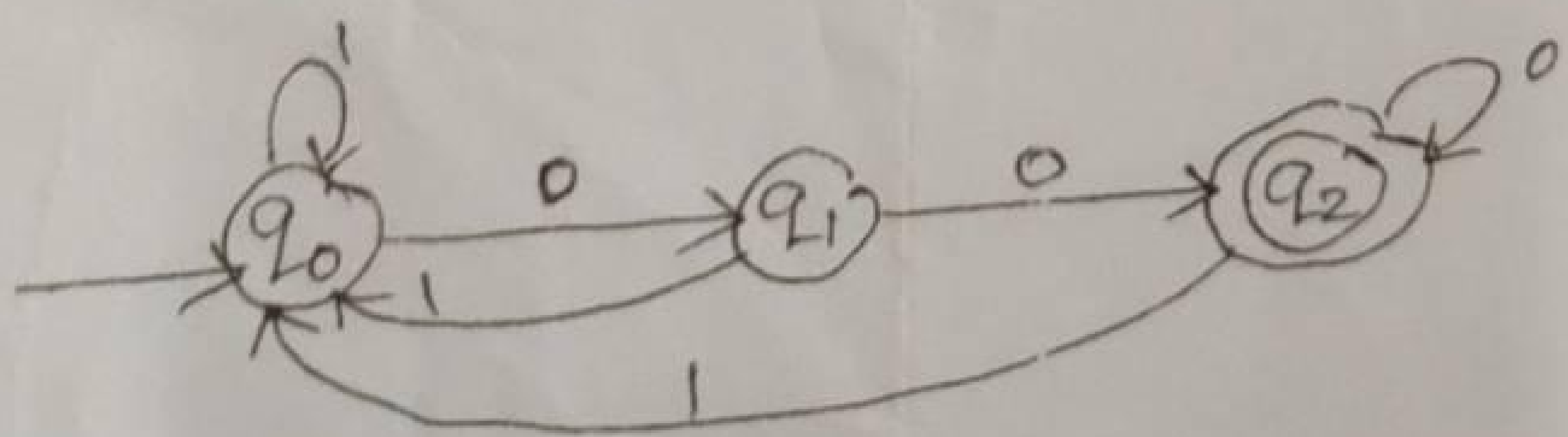
1. Define Inductive proof.
2. Let £={a,b,c}.Find $£^3$
3. Write the formal definition of DFA.
4. Construct DFA for the language which consists of set of all strings that begin with 01 and end with 11.
5. Consider the following NFA and check whether the string 101101 is accepted or not

δ	0	1
→ q_0	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\{q_1\}$	$\{q_2\}$
$^*q_2$	$\{q_1\}$	ϕ

6. Find ECLOSE(q_0) for the following ∈_NFA.



7. Identify the language accepted by the following finite automata.



8. Distinguish between equivalent states and distinguishable states.
9. Find the regular expression to describe the following languages:
   a) The set of all strings that begin with 110
   b) The set of all strings of even length.
10. Specify the operators and it precedence of regular expression operators.

PART - B (3 X 10 = 30 Marks)

ANSWER ANY THREE QUESTIONS

11. a) Prove that "Every expression has an equal number of left and right parenthesis". (5)

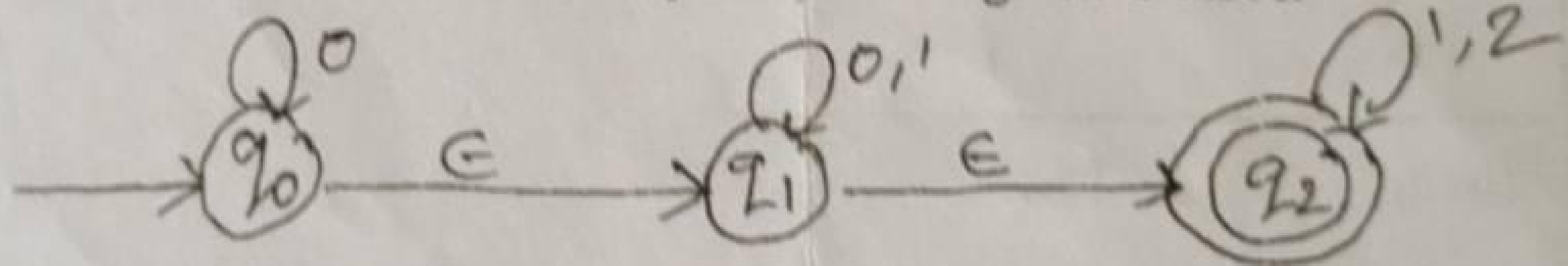
b) Using mathematical induction, prove that (5)

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

12. Convert the following NFA to DFA (10)

0	0	1
p	{p,q}	{p}
→ q	{r}	{r}
r	{s}	{ϕ}
*s	{s}	{s}

13. Find the equivalent NFA (without epsilon) for the given ∈-NFA: (10)



14. Construct an NFA for the following regular expression $(0+1)^* + 0^* 1$ (10)

14CST52 - Theory of computation
Answer key
Continuous Assessment Test - I

Part 1

1. Inductive proof : we have to prove the statement $S(n)$
   1. The basis where we show $S(i)$ for a particular integer i. usually $i=0$ or 1 or some higher value for some cases.
   2. Inductive step : Assume $n \geq i$ where i is the basis integer and we show that "If $S(n)$ then $S(n+1)$".

2. Let $\Sigma = \{a, b, c\}$

$$\Sigma^3 = \{aaa, aab, aac, aba, abb, abc, aca, acb, acc, baa, bab, bac \ldots \}$$

3. DFA $D = (Q, \Sigma, \delta, q_0, F)$
   where Q = finite set of states
   Σ = " " input symbols
   δ = set of transition function.
   It is of the form $\delta(p, a) = q$
   Current state $p \in Q$
   Input symbol $a \in \Sigma$
   Output state $q \in Q$.
   q_0 - Initial state
   F - set of final / accepting states. $F \subseteq Q$

4. DFA for the lang. consisting of strings sta
with 01 and end with 11.

5. $q_0 \xrightarrow{1} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_0 \xrightarrow{1} q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_0$

$\xrightarrow{0} q_1 \xrightarrow{1} q_2$ Accepte

$\xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{1} \phi$

NFA accepted the string 101101.

6. $ECLOSE(q_0) = \{q_0, q_1, q_2, q_3\}$

7. $L(D) = \{$set of all strings that end with 00$\}$

8. The states p and q are equivalent if for all input string w, $\hat{\delta}(p,w)$ is an accepting state if and only if $\hat{\delta}(q,w)$ is an accepting state.

If two states are not equivalent then are distinguishable.

9. Regular Expression for

a) set of all strings that begin with 110

RE : $110(0+1)^*$

b) Set of all strings of even length

$(00+01+10+11)^*$

10. RE operators precedence

closure $*$

Concatenation $\cdot$

Union $+$

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11.a. Theorem "Every expr has an equal no. of left and right paranthesis". 3 (5)

Proof: Let the statement S(G) about any expr. G.

Basis: If G is defined by the basis then G is a no. or variable (2)

These expr has 0 left and 0 right paranthesis.

Induction: (3)

There are three rules for constructing expr.

① $G = E+F$

② $G = E*F$

③ $G = (E)$

Assume S(E) and S(F) are true.

E has n number of left and right paranthesis.

F m " "

Then,

① If $G = E+F$ then G has m+n left and right paranthesis.

② If $G = E*F$ then G "

③ If $G = (E)$ then there are n+1 left and n+1 right paranthesis.

In each of the three case, we see that left and right paranthesis in G are the same.

11.b. Prove $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

Basis: Let $i=1$.

$LHS = \sum_{i=1}^{1} i^3 = 1.$

$RHS = \left[\frac{1(1+1)}{2}\right]^2 = 1$

Since LHS = RHS, Basis is true.

Induction:

Assume $S(n) = \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$ is true.

We have to prove that $S(n+1)$ is also true

i.e. $\sum_{i=1}^{n+1} i^3 = \left[\frac{(n+1)(n+1+1)}{2}\right]^2$ is also

LHS of $S(n+1)$.

$$\sum_{i=1}^{n+1} i^3 = (n+1)^3 + \sum_{i=1}^{n} i^3$$

$$= [n^3+3n^2+3n+1] + \left[\frac{n(n+1)}{2}\right]^2$$

$$= \frac{4n^3+12n^2+12n+4+n^4+2n^3+n^2}{4}$$

$$= \frac{n^4+6n^3+13n^2+12n+4}{4} \rightarrow (1)$$

RHS of $S(n+1)$: $\left[\frac{(n+1)(n+2)}{2}\right]^2 = \frac{((n+1)(n+2))^2}{4}$

$$= \frac{(n+1)^2 (n+2)^2}{4} = \frac{(n^2+2n+1)(n^2+4n+4)}{4}$$

$$= \frac{n^4+4n^3+4n^2+2n^3+8n^2+8n+n^2+4n+4}{4}$$

5

$$= \frac{n^4 + 6n^3 + 13n^2 + 12n + 4}{4} \rightarrow (2)$$

from (1) and (2), LHS = RHS. $S(n+1)$ is also true.

Hence Induction is also true.

12. Convert the following NFA to DFA.

δ	0	1
$\rightarrow p$	$\{p, q\}$	$\{p\}$
q	$\{r\}$	$\{r\}$
r	$\{s\}$	ϕ
$*s$	$\{s\}$	$\{s\}$

Transistion table — (7 marks)

δ	0	1
$\rightarrow$ [p]	[p, q]	[p]
[p, q]	[p, q, r]	[p, r]
[p, q, r]	[p, q, r, s]	[p, r]
[p, r]	[p, q, s]	[p]
* [p, q, s]	[p, q, r, s]	[p, r, s]
* [p, q, r, s]	[p, q, r, s]	[p, r, s]
* [p, r, s]	[p, q, s]	[p, s]
* [p, s]	[p, q, s]	[p, s]

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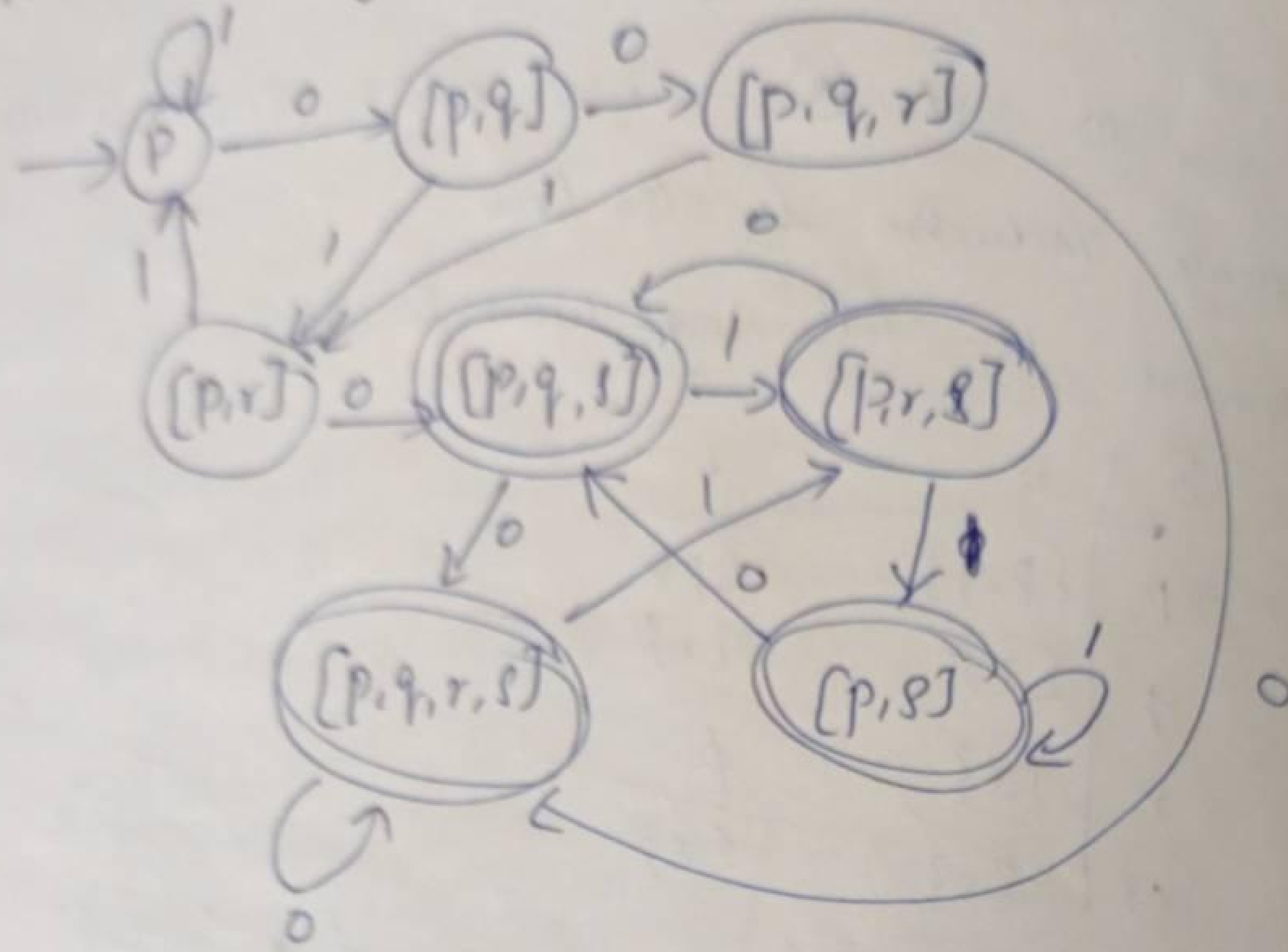
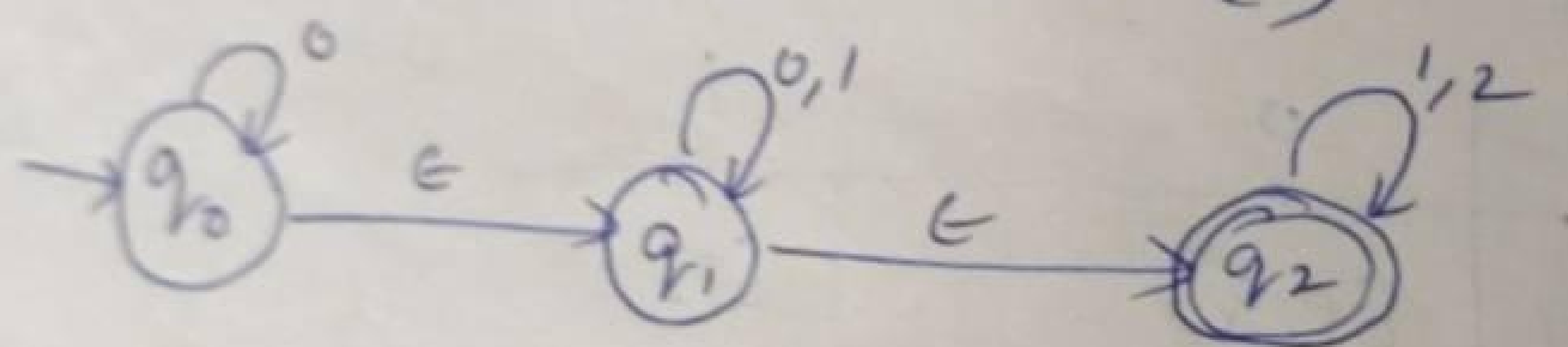
Transition diagram — 3 marks



13. ENFA to NFA (without ϵ)



For both ENFA and NFA Q, Σ, q_0 are same

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1, 2\}$

Initial state q_0.

Step 1: Find ECLOSE for all states (1 marks)

$ECLOSE(q_0) = \{q_0, q_1, q_2\}$

$ECLOSE(q_1) = \{q_1, q_2\}$

$ECLOSE(q_2) = \{q_2\}$

Step 2: For state q_0, apply all i/p symbols. (1)

$\hat{\delta}(q_0, 0) = ECLOSE(\delta(\hat{\delta}(q_0, \epsilon), 0))$

$= ECLOSE(\delta((q_0, q_1, q_2), 0))$

$= ECLOSE(q_0, q_1) = \{q_0, q_1, q_2\}$

$\hat{\delta}(q_0, 1) = ECLOSE(\delta(\hat{\delta}(q_0, \epsilon), 1))$

$= ECLOSE(\delta((q_0, q_1, q_2), 1))$

$= ECLOSE(q_1, q_2) = \{q_1, q_2\}$

$\hat{\delta}(q_0, 2) = ECLOSE(\delta(\hat{\delta}(q_0, \epsilon), 2))$

$= ECLOSE(\delta((q_0, q_1, q_2), 2))$

$= \{q_2\}$

step 3: For state q_1, apply all i/p symbols. (1)

$\hat{\delta}(q_1, 0) = ECLOSE(\delta(\hat{\delta}(q_1, \epsilon), 0))$

$= ECLOSE(\delta((q_1, q_2), 0))$

$= \{q_1, q_2\}$

$\hat{\delta}(q_1, 1) = \{q_1, q_2\}$

$\hat{\delta}(q_1, 1) = \{q_2\}$

step 4: For state q_2, apply all i/p symbols (1)

$\hat{\delta}(q_2, 0) = \phi$

$\hat{\delta}(q_2, 1) = \{q_2\}$

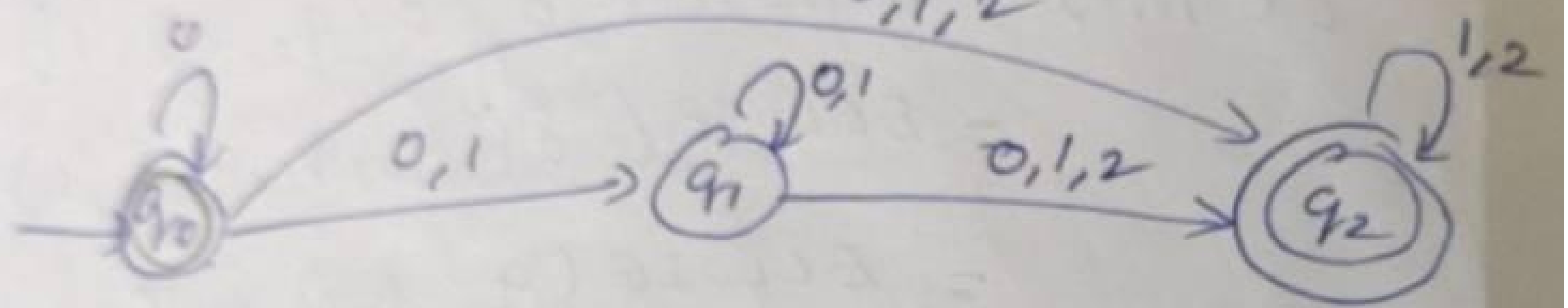
$\hat{\delta}(q_2, 2) = \{q_2\}$

Transition table: (3 marks)

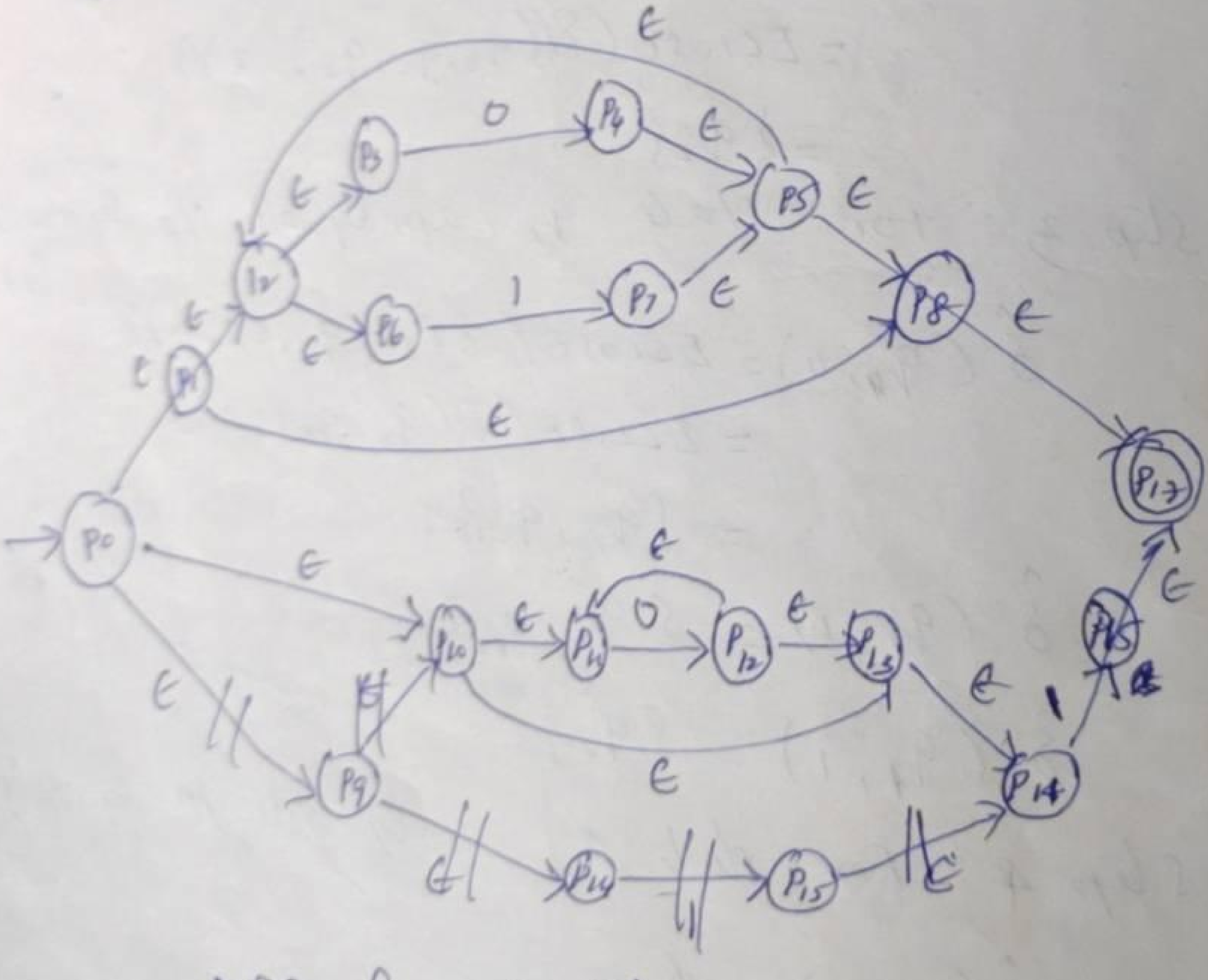
δ	0	1	2
$\rightarrow q_0$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
$*q_2$	ϕ	$\{q_2\}$	$\{q_2\}$

(3 marks)

Transition diagram



14. NFA for $(0+1)^* + 0^* 1$



NFA for $(0+1)^*$ $\longrightarrow$ 4 marks

" $0^* 1$ $\longrightarrow$ 4 marks

$(0+1)^* + 0^* 1$ $\longrightarrow$ 2 marks.

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KONGU ENGINEERING COLLEGE

PERUNDURAI ERODE - 638 060.

(Autonomous)

Estd : 1984

Name of the Student	B. PREETHA	Register No.	1 5 C S R 1 4 1
Programme	B.E	Branch & Semester	CSE V
Course Code and Name	14CS T52 Theorey of computation	Date	31.07.2017
		No. of Pages Used	14.

MARKS TO BE FILLED IN BY THE EXAMINER

PART - A		PART - B			Grand Total Max. Marks : 50
Question No.	Max Marks : 2	Question No.		Max Marks : 10	
1	2	11	i)	5	46½ / 50
2	2		ii)	5	
3	2	12	i)		
4	2		ii)		
5	2	13	i)	10	
6	2		ii)		
7	½	14	i)	10	
8	0		ii)		B. Preethy.
9	2				
10	2				
TOTAL	16½	TOTAL		30	

Total Marks in Words : Forty Six and half

INSTRUCTION TO THE CANDIDATE

1. Check the Question Paper, Programme, Course Code, Branch Name etc., before answering the questions.
2. Use both sides of the paper for answering questions.
3. POSSESSION OF ANY INCRIMINATING MATERIAL AND MALPRACTICE OF ANY NATURE IS PUNISHABLE AS PER RULES.

R.C. Suganthi

Name of the Examiner

[signature] 1/8/17

Signature of the Examiner with Date

PART - A

1. Inductive proof

Inductive proof is a type of Formal Proof.
For $i = 1, 2, \ldots$ or higher values of somes cases
Theorem contains some parameter.

Basics

$s(i) =$ true.

Inductive.

Assume that $s(n)$ is true
We have to proof that $s(n+1)$ is also true.

2. $\Sigma = \{a, b, c\}$ Σ^3

$\Sigma = \{a, b, c\}$

$\Sigma^3 = \{ \cancel{\epsilon}$, aaa, abc, acb, abb, acc, aca, acb, aab, aac, baa, bab, bac, bba, bbc, bcb, bcc, bbb, cab, cac, cbc, ccc, cba, cbb, a, b, c, ab, bc, ... $\}$.

3. Formal Definition of DFA

For each input symbol of one state it exactly transition to only one state.

DFA $D = (Q, \Sigma, \delta, q0, F)$

Q - Finite set of states

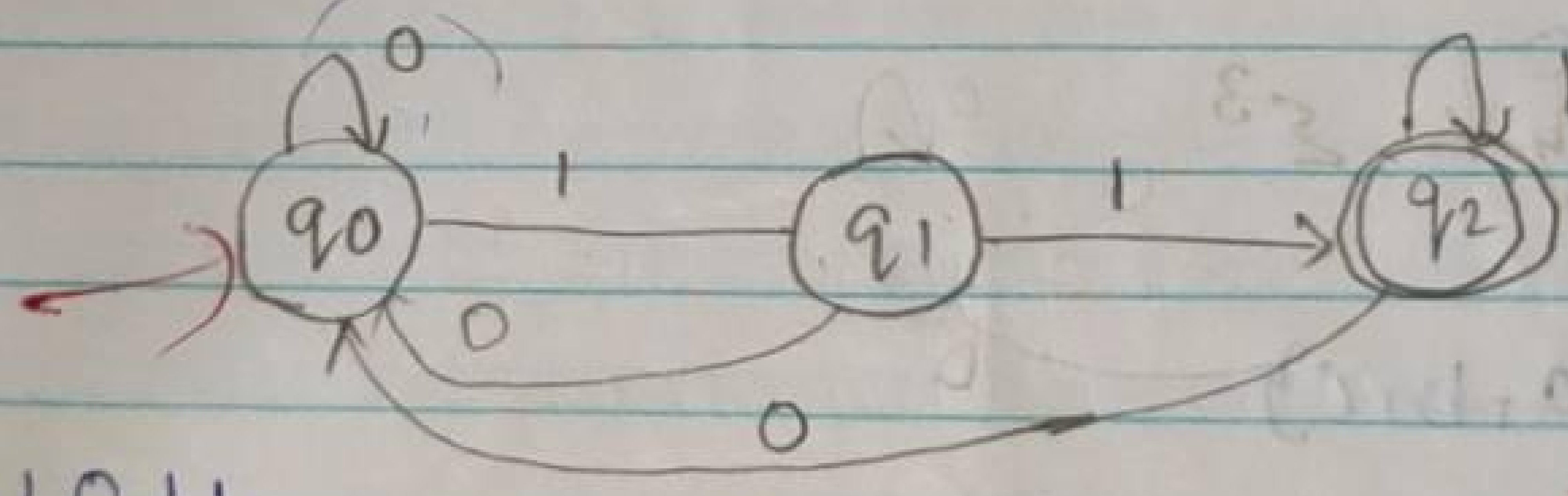
Σ - Finite set of Input Symbol

δ - Transition function

$q0 \rightarrow$ Initial state

$F \rightarrow$ Final state.

4. Begin with 01 and end with 11



(i) 01011

$q0 \xrightarrow{0} q0 \xrightarrow{1} q_1 \xrightarrow{0} q0 \xrightarrow{1} q1 \xrightarrow{1} q2$

$\therefore$ q2 is accepted state.

$\therefore$ The given string is accepted by DFA.

(ii) 10010

$q0 \xrightarrow{1} q_1 \xrightarrow{0} q0 \xrightarrow{0} q0 \xrightarrow{1} q1 \xrightarrow{0} q0$

$\therefore$ q0 is not accepted state.

$\therefore$ The given string is not accepted by DFA.

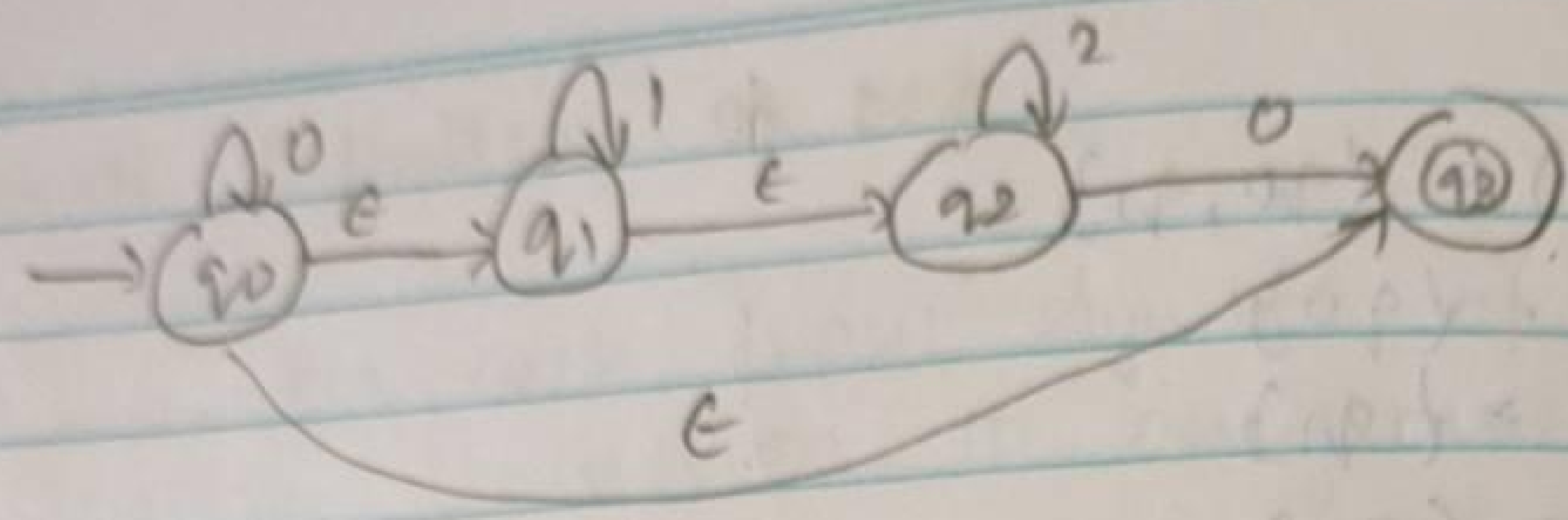
5.

$\delta(q_0, 0) = \{q_0, q_1\}$
$\delta(q_0, 1) = \{q_0\}$
$\delta(q_1, 0) = \{q_1\}$
$\delta(q_1, 1) = \{q_2\}$
$\delta(q_2, 0) = \{q_1\}$
$\delta(q_2, 1) = \{\phi\}$.

$\hat{\delta}(q_0, 101101) = \delta(\hat{\delta}(q_0, 10110, 1))$
$\hat{\delta}(q_0, 1) = \delta(q_0, 1) = \{q_0\}$
$\hat{\delta}(q_0, 10) = \delta(\hat{\delta}(q_0, 1), 0)$
$= \delta(\{q_0\}, 0) = \{q_0, q_1\}$
$\hat{\delta}(q_0, 101) = \delta(\hat{\delta}(q_0, 10), 1)$
$= \delta(\{q_0, q_1\}, 1) = \delta((q_0, 1) \cup \delta(q_1, 1))$.
$= \{q_0, q_2\}$.
$\hat{\delta}(q_0, 1011) = \delta(\hat{\delta}(q_0, 101, 1)$
$= \delta(\hat{\delta}\{q_0, q_2\}), 1) = \delta(q_0, 1) \cup \delta(q_2, 1)$
$= \{q_0, \phi\} = \{q_0\}$.
$\hat{\delta}(q_0, 10110) = \delta(\hat{\delta}(q_0, 1011, 0)$
$= \delta(\{q_0\}, 0)$
$= \{q_0, q_1\}$.
$\hat{\delta}(q_0, 101101) = \delta(\hat{\delta}(q_0, 10110, 1)$
$= \delta(\{q_0, q_1\}, 1)$
$= \delta(q_0, 1) \cup \delta(q_1, 1)$
$= \{q_0\} \cup \{q_2\}$
$= \{q_0, q_2\}$.

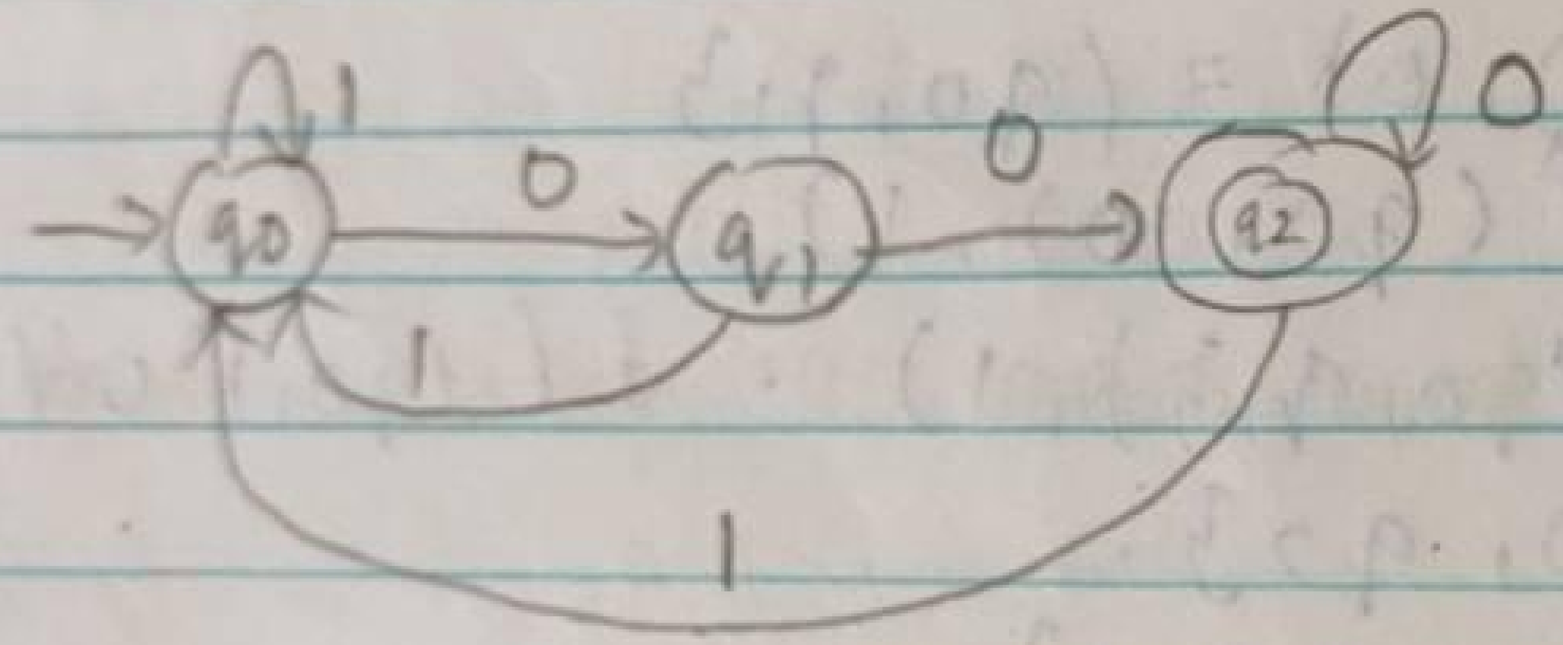
$\therefore$ The final state q_2 is present in it. So it is a accepted state.

6.



ECLOSE $(q_0) = \{q_0, q_1, q_2, q_3\}$
ECLOSE $(q_1) = \{q_1, q_2\}$
ECLOSE $(q_2) = \{q_2\}$
ECLOSE $(q_3) = \{q_3\}$.

7.



L = { set of all string begins with 10 and ends with 00 }.

8.

Equivalent state	Distinguisable states
Every DFA is in NFA, the class of language accepted by NFA is equal to class of language accepted by DFA included in it.	If the two states are not equal, then ~~the~~ state is called Distinguisable State.

$\hat{\delta}(q, w)$

For every NFA, there is a equivalent state. If two states are equal, then it is called equivalent state.	Two methods (i) Table filling algorithm (ii) Minimization of DFA.

9.

a) Set of all strings that begin with 110

$$110\,(0+1)^*$$

b) The set of strings of even length

$$(ab + aa + ba + bb)^*$$

10. Regular Expression operators.

Precedence are and operators is,

$*$, $\cdot$, $+$

$*$ → zero or more occurance

$+$ → one or more occurance

PART - B.

11. Theorem :-

a). Every expression has an equal number of left and right paraenthesis.

Basics :-

If G is an expression defined by basis then statement G is a number of variable.

It contains 0 number of equ, left and right paraenthis.

∴ In Basic, there is equal number of left and right parenthesis

∴ Hence proved.

Inductive :-

In Inductive, we have three rules to define a step (i.e)

G=E+F, G=E·F G=(E)

Assume S(E) and S(F) is a true for a pro and S(E) has equal number of left and right paranthesis.

(i) If G=E+F then the expression has m+n left and right paranthesis.

(ii) If $G = E * F$ then the expression has $m \cdot n$ left and right paranthesis.

(iii) If $G = (E)$ then the expression has $n+1$ left and right parenthesis.

From the above expression, the induction has equal number of left and right paranthesis
∴ Hence proved.

(b) $$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

<u>Basic</u>

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Put $n=1$

LHS:- $$\sum_{i=1}^{1} i^3 = 1.$$

RHS:- $$\left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 4/4 = 1.$$

∴ LHS = RHS.

∴ Basis is true and Proved

<u>Inductive</u>

Assume $s(n)$ is true

i.e $$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

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we here to prove that $S(n+1)$ is also the

LHS of $S(n+1) = \sum_{i=1}^{n+1} i^3$ $\quad (a+b)^3 \quad (a+b)(a+b)^2$

$$= \left[\frac{n(n+1)}{2}\right]^2 + (n+1)^3$$

$$= \frac{n^2(n+1)^2}{4} + (n+1)^3$$

$$= \frac{n^2(n^2+1+2n) + 4(n+1)^3}{4}$$

$$= \frac{n^4+n^2+2n^3+4(n+1)(n^2+2n+1)}{4}$$

$$= \frac{n^4+n^2+2n^3+4[n^3+2n^2+n+n^2+2n+1]}{4}$$

$$= \frac{n^4+n^2+2n^3+4n^3+8n^2+4n+4n^2+8n+4}{4}$$

$$= \frac{n^4+6n^3+13n^2+12n+4}{4}$$

RHS of $S(n+1) = \left[\frac{(n+1)(n+2)}{2}\right]^2$

$$= \frac{(n+1)^2(n+2)^2}{4}$$

$$= \frac{(n^2+2n+1)(n^2+4n+4)}{4}$$

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$$= \frac{(n^4 + 4n^3 + 4n^2 + 2n^3 + 8n^2 + 8n + n^2 + 4n + 4)}{4}$$

$$= \frac{n^4 + 6n^3 + 13n^2 + 12n + 4}{4}$$

$\therefore$ LHS = RHS

$\therefore$ Inductive is true and proved.

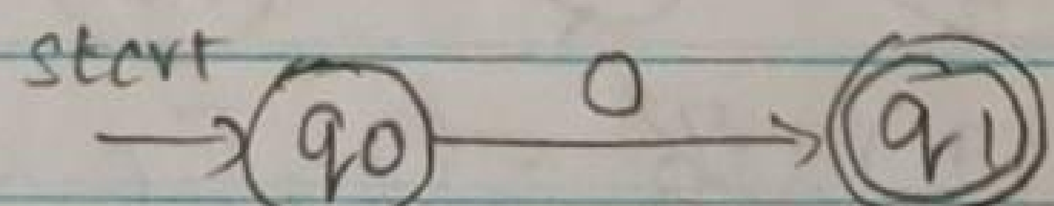
$$\therefore \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

14. $(0+1)^* + 0^* 1$

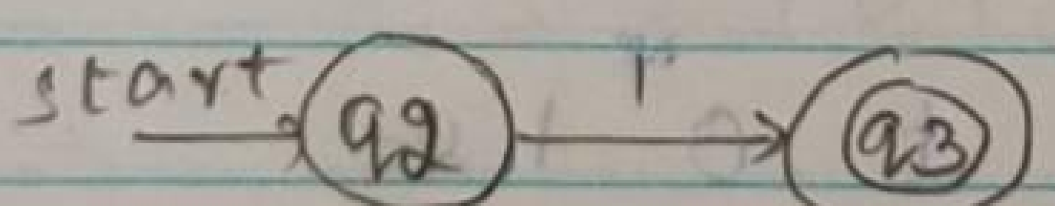
Put $r_1 = (0+1)^*$ $r_2 = 0^* 1$

Step 1 :- To find r_1

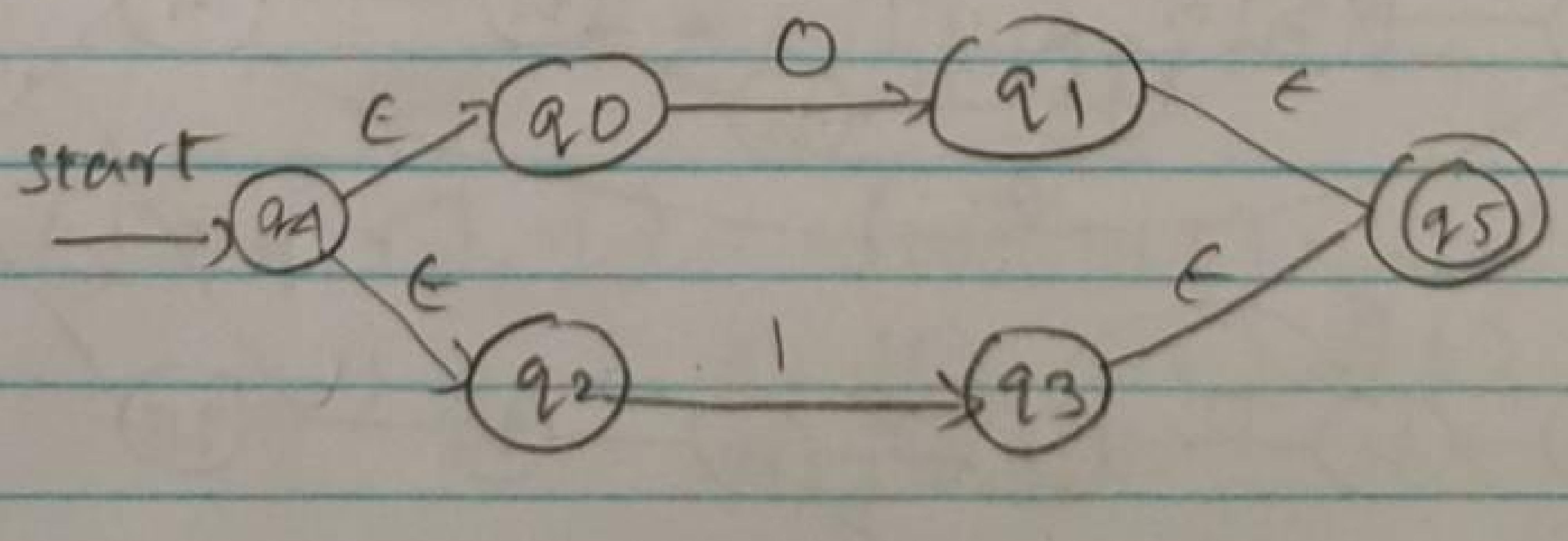
Find NFA for 0



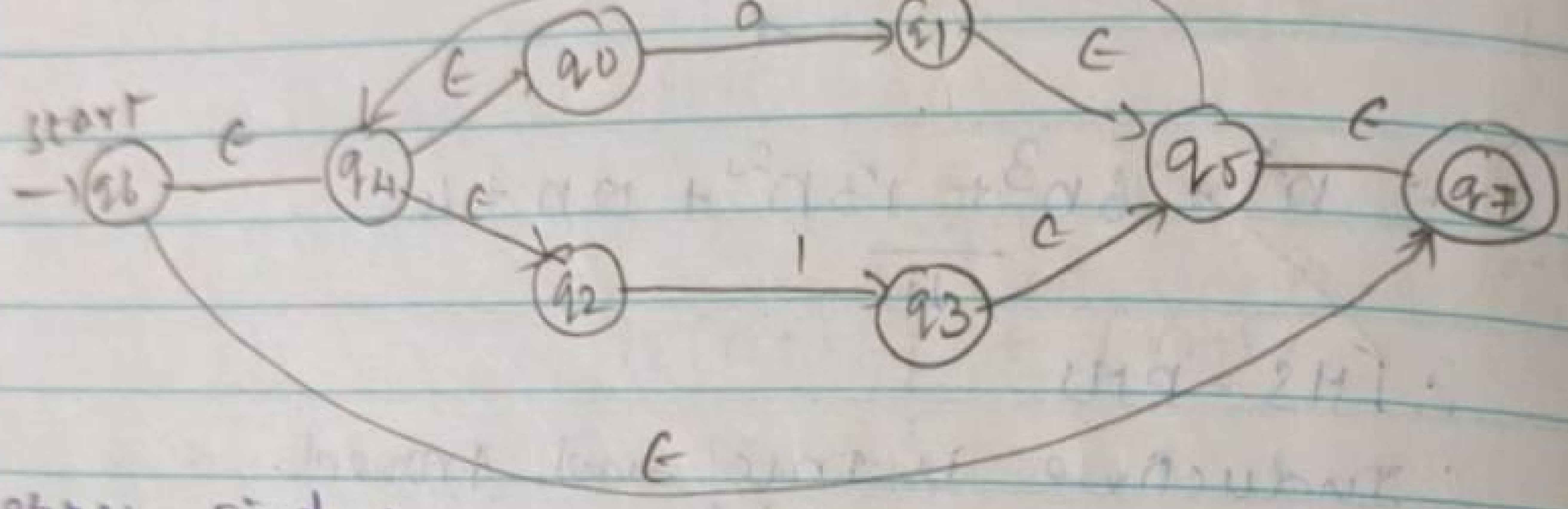
Find NFA for 1



Find NFA for (0+1)

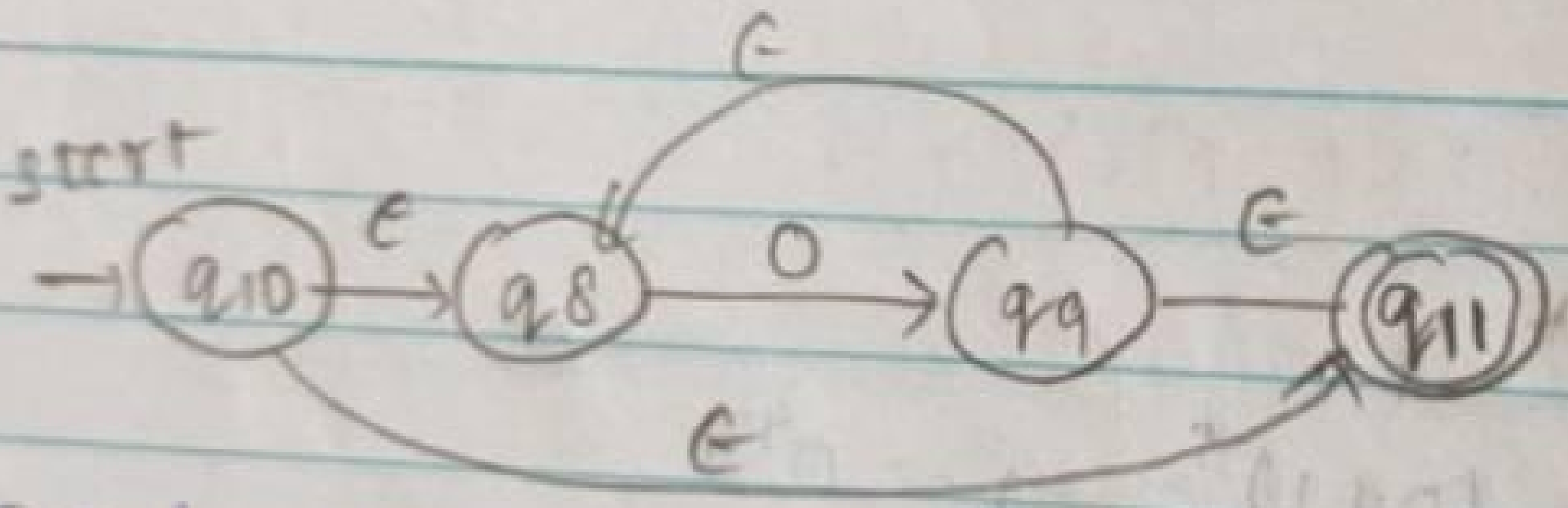


Find NFA for $(0+1)^*$

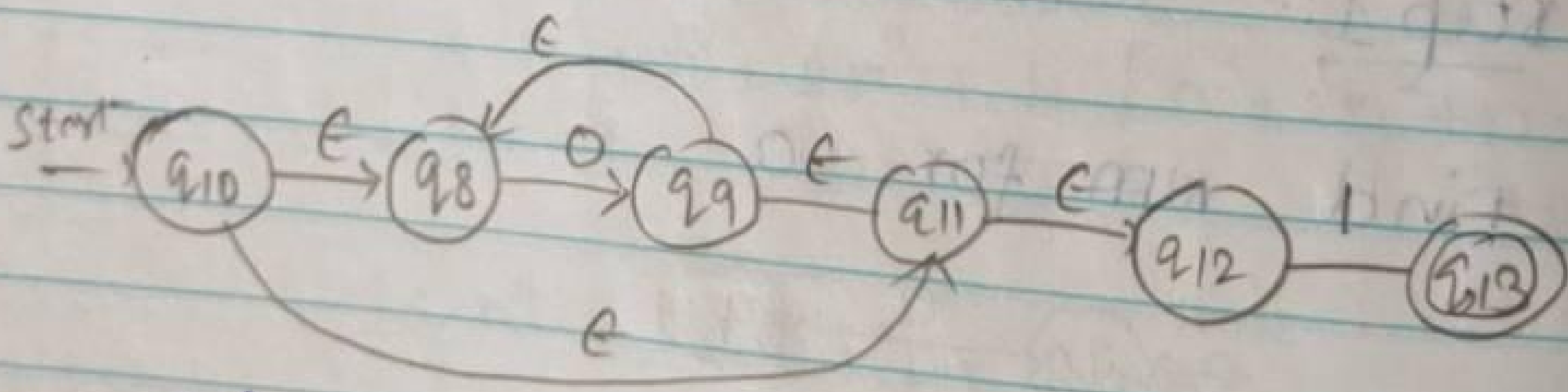


step2:- Find r_2:-

Find NFA for 0^*.

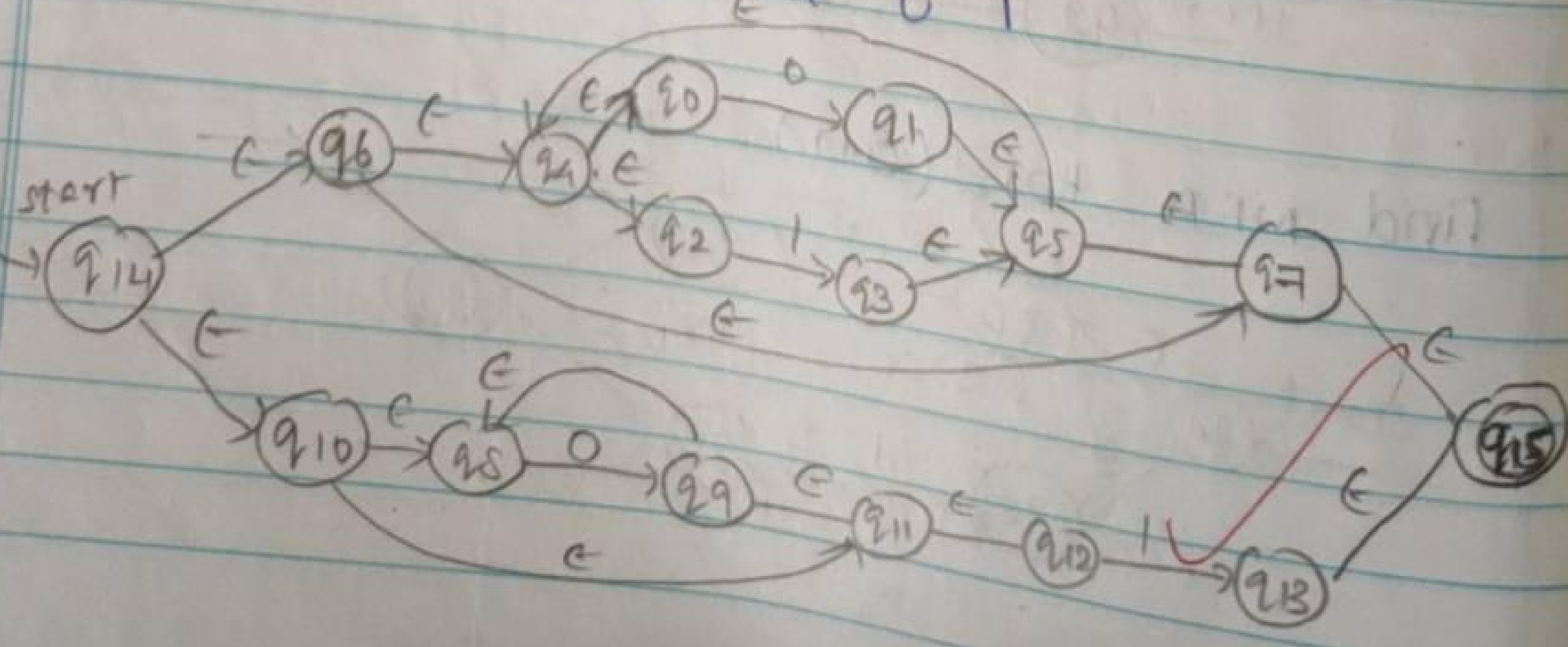


Find NFA for $0^* 1$



step3:- Find $r_1 + r_2$:-

Find NFA for $(0+1)^* + 0^* 1$



12.

δ	0	1
$\rightarrow p$	$\{p, q\}$	$\{p\}$
$* q$	$\{r\}$	$\{r\}$
r	$\{s\}$	$\{\phi\}$
$* s$	$\{s\}$	$\{s\}$

Here I take $0 \mapsto \delta$

Step 1 :-

$\delta_D(p, 0) = \delta_N(p, 0) = [p, q]$

$\delta_D(p, 1) = \delta_N(p, 1) = [p]$.

Step 2 :-

$\delta_D([p, q], 0) = \delta_N(p, 0) \cup \delta_N(q, 0)$
$= \{p, q\} \cup \{r\}$
$= [p, q, r]$

$\delta_D([p, q], 1) = \delta_N(p, 1) \cup \delta_N(q, 1)$
$= \{p\} \cup \{r\}$
$= [p, r]$.

Step 3 :-

$\delta_D([p, q, r], 0) = \delta_N(p, 0) \cup \delta_N(q, 0) \cup \delta_N(r, 0)$
$= \{p, q\} \cup \{r\} \cup \{s\}$
$= [p, q, r, s]$.

$\delta_D([p, q, r], 1) = \delta_N(p, 1) \cup \delta_N(q, 1) \cup \delta_N(r, 1)$
$= \{p\} \cup \{r\} \cup \{\phi\}$
$= [p, r]$.

Step 4 :-

$\delta_D([p,r],0) = \delta_N(p,0) \cup \delta_N(r,0)$
$= \{p,q\} \cup \{s\}$
$= [p,q,s]$.

$\delta_D([p,r],1) = \delta_N(p,1) \cup \delta_N(r,1)$
$= \{p\} \cup \{\phi\}$
$= [p]$.

Step 5 :-

$\delta_D([p,q,r,s],0) = \delta_N(p,0) \cup \delta_N(q,0) \cup \delta_N(r,0) \cup \delta_N(s,0)$
$= \{p,q\} \cup \{r\} \cup \{s\} \cup \{s\}$
$= [p,q,r,s]$.

$\delta_D([p,q,r,s],1) = \delta_N(p,1) \cup \delta_N(q,1) \cup \delta_N(r,1) \cup \delta_N(s,1)$
$= \{p\} \cup \{r\} \cup \{s\}$
$= [p,r,s]$.

Step 6 :-

$\delta_D([p,q,s],0) = \delta_N(p,0) \cup \delta_N(q,0) \cup \delta_N(s,0)$
$= \{p,q\} \cup \{r\} \cup \{s\}$
$= [p,q,r,s]$.

$\delta_D([p,q,s],1) = \delta_N(p,1) \cup \delta_N(q,1) \cup \delta_N(s,1)$
$= \{p\} \cup \{r\} \cup \{s\}$
$= [p,r,s]$.

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Step 7 :-

$\delta D([p,r,s],0) = \delta N(p,0) \cup \delta N(r,0) \cup \delta N(s,0)$
$= \{p,q\} \cup \{s\} \cup \{s\}$
$= [p,q,s]$

$\delta D([p,r,s],1) = \delta N(p,1) \cup \delta N(r,1) \cup \delta N(s,1)$
$= \{p\} \cup \{\phi\} \cup \{s\}$
$= [p,s]$.

Step 8 :-

$\delta D([p,s],0) = \delta N(p,0) \cup \delta N(s,0)$
$= \{p,q\} \cup \{s\}$
$= [p,q,s]$.

$\delta D([p,s],1) = \delta N(p,1) \cup \delta N(s,1)$
$= \{p\} \cup \{s\}$
$= [p,s]$.

Transition Table

δ	0	1
→ [p]	[p,q]	[p]
[p,q]	[p,q,r]	[p,r]
[p,q,r]	[p,q,r,s]	[p,r]
[p,r]	[p,q,s]	[p]
* [p,q,r,s]	[p,q,r,s]	[p,r,s]
* [p,q,s]	[p,q,r,s]	[p,r,s]
* [p,r,s]	[p,q,s]	[p,s]
* [p,s]	[p,q,s]	[p,s].

Transition Diagram.