In [ ]:
```python
## Load libraries
import pandas as pd
import numpy as np
import sys
import matplotlib.pyplot as plt
import matplotlib.cm as cm
from keras.datasets import mnist
plt.style.use('dark_background')
%matplotlib inline
```

```
WARNING:tensorflow:From c:\Users\SA RAVI\anaconda3\envs\aimlsem1\lib\site-packages\keras\src\losses.py:2976: The name tf.losses.sparse_softmax_cross_entropy is deprecated. Please use tf.compat.v1.loss
es.sparse_softmax_cross_entropy instead.
```

In [ ]:
```python
np.set_printoptions(precision=2)
```

In [ ]:
```python
import tensorflow as tf
```

In [ ]:
```python
tf.__version__
```

Out[ ]:
```
'2.15.0'
```

Load MNIST Data

In [ ]:
```python
## Load MNIST data
(X_train, y_train), (X_test, y_test) = mnist.load_data()
X_train = X_train.transpose(1, 2, 0)
X_test = X_test.transpose(1, 2, 0)
X_train = X_train.reshape(X_train.shape[0]*X_train.shape[1], X_train.shape[2])
X_test = X_test.reshape(X_test.shape[0]*X_test.shape[1], X_test.shape[2])

num_labels = len(np.unique(y_train))
num_features = X_train.shape[0]
num_samples = X_train.shape[1]

# One-hot encode class labels
Y_train = tf.keras.utils.to_categorical(y_train).T
Y_test = tf.keras.utils.to_categorical(y_test).T


# Normalize the samples (images)
xmax = np.amax(X_train)
xmin = np.amin(X_train)
X_train = (X_train - xmin) / (xmax - xmin) # all train features turn into a number between 0 and 1
X_test = (X_test - xmin)/(xmax - xmin)

print('MNIST set')
print('---------------------')
print('Number of training samples = %d'%(num_samples))
print('Number of features = %d'%(num_features))
print('Number of output labels = %d'%(num_labels))
```

```
MNIST set
---------------------
Number of training samples = 60000
Number of features = 784
Number of output labels = 10
```

A generic layer class with forward and backward methods

In [ ]:
```python
class Layer:
  def __init__(self):
    self.input = None
    self.output = None

  def forward(self, input):
    pass

  def backward(self, output_gradient, learning_rate):
    pass
```

The softmax classifier steps for a batch of comprising b samples represented as the $725 \times b$-matrix (724 pixel values plus the bias feature absorbed as its last row)

$$\mathbf{X} = \left[\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(b-1)}\right]$$

with one-hot encoded true labels represented as the $10 \times b$-matrix (10 possible categories)

$$\mathbf{Y} = \left[\mathbf{y}^{(0)} \quad \ldots \quad \mathbf{y}^{(b-1)}\right]$$

using a randomly initialized 10×725-weights matrix $\mathbf{W}$:

1. Calculate $10 \times b$-raw scores matrix :

$$\begin{aligned} \left[\mathbf{z}^{(0)} \quad \ldots \quad \mathbf{z}^{(b-1)} \ldots\right] &= \mathbf{W}\left[\mathbf{z}^{(0)} \quad \ldots \quad \mathbf{z}^{(b-1)} \ldots\right] \\ &= \left[\mathbf{W}\mathbf{z}^{(0)} \quad \ldots \quad \mathbf{W}\mathbf{z}^{(b-1)}\right] \\ \Rightarrow \mathbf{Z} &= \mathbf{W}\mathbf{X}. \end{aligned}$$

2. Calculate $10 \times b$-softmax predicted probabilities matrix:

$$\begin{aligned} \left[\mathbf{a}^{(0)} \quad \ldots \quad \mathbf{a}^{(b-1)}\right] &= \left[\mathrm{softmax}\left(\mathbf{z}^{(0)}\right) \quad \ldots \quad \mathrm{softmax}\left(\mathbf{z}^{(b-1)}\right)\right] \\ \Rightarrow \mathbf{A} &= \mathrm{softmax}(\mathbf{Z}). \end{aligned}$$

3. Predicted probability matrix get a new name: $\hat{\mathbf{Y}} = \mathbf{A}$.
4. The crossentropy (CCE) loss for the ith sample is

$$L_i = \sum_{k=0}^{9} -y^{(i)} \log\left(\hat{y}_k^{(i)}\right) = -{\mathbf{y}^{(i)}}^{\mathrm{T}} \log\left(\mathbf{y}^{(i)}\right)$$

which leads to the average crossentropy (CCE) batch loss for the batch as:

$$\begin{aligned} L &= \frac{1}{b}\left[L_0 + \cdots + L_{b-1}\right] \\ &= \frac{1}{b}\left[-{\mathbf{y}^{(0)}}^{\mathrm{T}} \log\left(\hat{\mathbf{y}}^{(0)}\right) + \cdots + -{\mathbf{y}^{(b-1)}}^{\mathrm{T}} \log\left(\hat{\mathbf{y}}^{(b-1)}\right)\right]. \end{aligned}$$

5. The computational graph for the samples in the batch are presented below:

$$\begin{array}{ccc} L_0 & & L_{b-1} \\ \downarrow & & \downarrow \\ \hat{\mathbf{y}}^{(0)} = \mathbf{a}^{(0)} & & \hat{\mathbf{y}}^{(b-1)} = \mathbf{a}^{(b-1)} \\ \downarrow & \cdots & \downarrow \\ \mathbf{z}^{(0)} & & \mathbf{z}^{(b-1)} \\ \downarrow & & \downarrow \\ \mathbf{W} & & \mathbf{W} \end{array}$$

6. Calculate the gradient of the average batch loss w.r.t. weights as:

$$
\begin{aligned}
\Rightarrow \nabla_{\mathbf{W}}(L) &= \frac{1}{b}\left(\underbrace{\left[\nabla_{\mathbf{W}}\left(\mathbf{z}^{(0)}\right) \times \nabla_{\mathbf{z}^{(0)}}\left(\hat{\mathbf{y}}^{(0)}\right) \times \nabla_{\hat{\mathbf{y}}^{(0)}}(L_0)\right]}_{\text{sample }0} + \cdots + \underbrace{\left[\nabla_{\mathbf{W}}\left(\mathbf{z}^{(b-1)}\right) \times \nabla_{\mathbf{z}^{(b-1)}}\left(\hat{\mathbf{y}}^{(b-1)}\right) \times \nabla_{\hat{\mathbf{y}}^{(b-1)}}(L_{b-1})\right]}_{\text{sample }b-1}\right)\\
&= \frac{1}{b}\left(\underbrace{\left[\nabla_{\mathbf{W}}\left(\mathbf{z}^{(0)}\right) \times \nabla_{\mathbf{z}^{(0)}}\left(\mathbf{a}^{(0)}\right) \times \nabla_{\hat{\mathbf{y}}^{(0)}}(L_0)\right]}_{\text{sample }0} + \cdots + \underbrace{\left[\nabla_{\mathbf{W}}\left(\mathbf{z}^{(b-1)}\right) \times \nabla_{\mathbf{z}^{(b-1)}}\left(\hat{\mathbf{y}}^{(b-1)}\right) \times \nabla_{\hat{\mathbf{y}}^{(b-1)}}(L_{b-1})\right]}_{\text{sample }b-1}\right).
\end{aligned}
$$

10. The full gradient can be written as $\nabla_{\mathbf{W}}(L) =$

$$
\nabla_{\mathbf{W}}(L) = \begin{cases}
\displaystyle\sum_{i=0}^{b-1} \begin{bmatrix} \mathbf{x}^{(i)} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{x}^{(i)} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{x}^{(i)} & \mathbf{0} & \cdots & \mathbf{0}\\ & \ddots & \ddots & \ddots & \cdots & \cdots\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{x}^{(i)} \end{bmatrix} \begin{bmatrix} a_0^{(i)}\left(1-a_0^{(i)}\right) & -a_1^{(i)}a_0^{(i)} & -a_2^{(i)}a_0^{(i)} & \cdots & -a_9^{(i)}a_0^{(i)}\\ -a_0^{(i)}a_1^{(i)} & a_1^{(i)}(1-a_1) & -a_2^{(i)}a_1^{(i)} & \cdots & -a_9^{(i)}a_1^{(i)}\\ a_0^{(i)}a_2^{(i)} & -a_1^{(i)}a_2^{(i)} & a_2^{(i)}\left(1-a_2^{(i)}\right) & \cdots & -a_9^{(i)}a_2^{(i)}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ -a_0^{(i)}a_9^{(i)} & -a_1^{(i)}a_9^{(i)} & a_2^{(i)}a_9^{(i)} & \cdots & -a_9^{(i)}\left(1-a_9^{(i)}\right) \end{bmatrix} \times \begin{bmatrix} -y_0^{(i)}/\hat{y}_0^{(i)}\\ -y_1^{(i)}/\hat{y}_1^{(i)}\\ -y_2^{(i)}/\hat{y}_2^{(i)}\\ \vdots\\ -y_9^{(i)}/\hat{y}_9^{(i)} \end{bmatrix}\\
\displaystyle\sum_{i=0}^{b-1} \begin{bmatrix} a_0^{(i)}\left(1-a_0^{(i)}\right) & -a_1^{(i)}a_0^{(i)} & -a_2^{(i)}a_0^{(i)} & \cdots & -a_9^{(i)}a_0^{(i)}\\ -a_0^{(i)}a_1^{(i)} & a_1^{(i)}(1-a_1) & -a_2^{(i)}a_1^{(i)} & \cdots & -a_9^{(i)}a_1^{(i)}\\ a_0^{(i)}a_2^{(i)} & -a_1^{(i)}a_2^{(i)} & a_2^{(i)}\left(1-a_2^{(i)}\right) & \cdots & -a_9^{(i)}a_2^{(i)}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ -a_0^{(i)}a_9^{(i)} & -a_1^{(i)}a_9^{(i)} & a_2^{(i)}a_9^{(i)} & \cdots & a_9^{(i)}\left(1-a_9^{(i)}\right) \end{bmatrix} \times \begin{bmatrix} -y_0^{(i)}/\hat{y}_0^{(i)}\\ -y_1^{(i)}/\hat{y}_1^{(i)}\\ -y_2^{(i)}/\hat{y}_2^{(i)}\\ \vdots\\ -y_9^{(i)}/\hat{y}_9^{(i)} \end{bmatrix} \mathbf{x}^{(i)\mathrm{T}}
\end{cases}
$$

CCE loss and its gradient for the batch samples:

$$
\begin{aligned}
L &= \frac{1}{b}[L_0 + \cdots + L_{b-1}]\\
&= \frac{1}{b}\left[-\mathbf{y}^{(0)\mathrm{T}}\log\left(\hat{\mathbf{y}}^{(0)}\right) + \cdots + -\mathbf{y}^{(b-1)\mathrm{T}}\log\left(\hat{\mathbf{y}}^{(b-1)}\right)\right].
\end{aligned}
$$

$$
\begin{bmatrix} \nabla_{\hat{\mathbf{y}}^{(0)}}(L_0) & \ldots & \nabla_{\hat{\mathbf{y}}^{(b-1)}}(L_{b-1}) \end{bmatrix} = \begin{bmatrix} -y_0^{(0)}/\hat{y}_0^{(0)} & \cdots & -y_0^{(0)}/\hat{y}_0^{(b-1)}\\ -y_1^{(0)}/\hat{y}_1^{(0)} & \cdots & -y_1^{(b-1)}/\hat{y}_1^{(b-1)}\\ -y_2^{(0)}/\hat{y}_2^{(0)} & \cdots & -y_2^{(b-1)}/\hat{y}_2^{(b-1)}\\ \vdots & & \\ -y_9^{(0)}/\hat{y}_9^{(0)} & \cdots & -y_9^{(b-1)}/\hat{y}_9^{(b-1)} \end{bmatrix}
$$

In [ ]:
```python
## Define the loss function and its gradient
def cce(Y, Yhat):
  return(np.mean(np.sum(-Y*np.log(Yhat), axis = 0)))

def cce_gradient(Y, Yhat):
  return(-Y/Yhat)

# TensorFlow in-built function for categorical crossentropy loss
#cce = tf.keras.losses.CategoricalCrossentropy()
```

Softmax activation layer class:

Forward:

$$
\begin{aligned}
\begin{bmatrix} \mathbf{a}^{(0)} & \ldots & \mathbf{a}^{(b-1)} \end{bmatrix} &= \begin{bmatrix} \mathrm{softmax}\left(\mathbf{z}^{(0)}\right) & \ldots & \mathrm{softmax}\left(\mathbf{z}^{(b-1)}\right) \end{bmatrix}\\
\Rightarrow \mathbf{A} &= \mathrm{softmax}(\mathbf{Z}).
\end{aligned}
$$

Backward:

$$
\begin{bmatrix} \nabla_{\mathbf{z}^{(0)}}(L_0) & \ldots & \nabla_{\mathbf{z}^{(b-1)}}(L_{b-1}) \end{bmatrix} = \begin{bmatrix} \nabla_{\mathbf{z}^{(0)}}\left(\mathbf{a}^{(0)}\right) \times \nabla_{\mathbf{a}^{(0)}}(L_0) & \cdots & \nabla_{\mathbf{z}^{(b-1)}}\left(\mathbf{a}^{(b-1)}\right) \times \nabla_{\mathbf{a}^{(b-1)}}(L_{b-1}) \end{bmatrix}
$$

$$
= \left[\begin{bmatrix} a_0^{(0)}\left(1-a_0^{(0)}\right) & -a_1^{(0)}a_0^{(0)} & -a_2^{(0)}a_0^{(0)} & \cdots & -a_9^{(0)}a_0^{(0)}\\ -a_0^{(0)}a_1^{(0)} & a_1^{(0)}(1-a_1) & -a_2^{(0)}a_1^{(0)} & \cdots & -a_9^{(0)}a_1^{(0)}\\ a_0^{(0)}a_2^{(0)} & -a_1^{(0)}a_2^{(0)} & a_2^{(0)}\left(1-a_2^{(0)}\right) & \cdots & -a_9^{(0)}a_2^{(0)}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ -a_0^{(0)}a_9^{(0)} & -a_1^{(0)}a_9^{(0)} & a_2^{(0)}a_9^{(0)} & \cdots & -a_9^{(0)}\left(1-a_9^{(0)}\right) \end{bmatrix} \times \begin{bmatrix} -y_0^{(0)}/\hat{y}_0^{(0)}\\ -y_1^{(0)}/\hat{y}_1^{(0)}\\ -y_2^{(0)}/\hat{y}_2^{(0)}\\ \vdots\\ -y_9^{(0)}/\hat{y}_9^{(0)} \end{bmatrix} \;\ldots\ldots\; \begin{bmatrix} a_0^{(b-1)}\left(1-a_0^{(b-1)}\right) & -a_1^{(b-1)}a_0^{(b-1)} & -a_2^{(b-1)}a_0^{(b-1)} & \cdots & -a_9^{(b-1)}a_0^{(b-1)}\\ -a_0^{(b-1)}a_1^{(b-1)} & a_1^{(b-1)}(1-a_1) & -a_2^{(b-1)}a_1^{(b-1)} & \cdots & -a_9^{(b-1)}a_1^{(b-1)}\\ a_0^{(b-1)}a_2^{(b-1)} & -a_1^{(b-1)}a_2^{(b-1)} & a_2^{(b-1)}\left(1-a_2^{(b-1)}\right) & \cdots & -a_9^{(b-1)}a_2^{(b-1)}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ -a_0^{(b-1)}a_9^{(b-1)} & -a_1^{(b-1)}a_9^{(b-1)} & a_2^{(b-1)}a_9^{(b-1)} & \cdots & -a_9^{(b-1)}\left(1-a_9^{(b-1)}\right) \end{bmatrix} \times \begin{bmatrix} -y_0^{(b-1)}/\hat{y}_0^{(b-1)}\\ -y_1^{(b-1)}/\hat{y}_1^{(b-1)}\\ -y_2^{(b-1)}/\hat{y}_2^{(b-1)}\\ \vdots\\ -y_9^{(b-1)}/\hat{y}_9^{(b-1)} \end{bmatrix}\right]
$$

In [ ]:
```python
'''Z = np.array([[1., 2., 3.], [4., 5., 6.]])
print(Z)
tf.nn.softmax(Z, axis = 0).numpy()'''
```

Out[ ]: 'Z = np.array([[1., 2., 3.], [4., 5., 6.]])\nprint(Z)\ntf.nn.softmax(Z, axis = 0).numpy()'

In [ ]:
```python
## Softmax activation layer class
class Softmax(Layer):
  def forward(self, input):
    self.input = input
    self.output = tf.nn.softmax(self.input, axis = 0).numpy()

  def backward(self, output_gradient, learning_rate = None):
    ## Following is the inefficient way of calculating the backward gradient
    softmax_gradient = np.empty((self.input.shape[0], output_gradient.shape[1]), dtype = np.float64)
    for b in range(softmax_gradient.shape[1]):
      softmax_gradient[:, b] = np.dot((np.identity(self.output.shape[0])-self.output[:, b].reshape(-1,1).T) * self.output[:, b].reshape(-1,1), output_gradient[:, b])
    return(softmax_gradient)
    ## Following is the efficient of calculating the backward gradient
    #T = (np.transpose(np.identity(self.output.shape[0]) - np.atleast_2d(self.output).T[:, np.newaxis, :], (1, 2, 0)) * np.atleast_2d(self.output))
    #return(np.einsum('ijk, ik -> jk', T, output_gradient))
```

Dense layer class:

Forward:

$$
\begin{aligned}
\begin{bmatrix} \mathbf{z}^{(0)} & \ldots & \mathbf{z}^{(b-1)} \ldots \end{bmatrix} &= \mathbf{W}\begin{bmatrix} \mathbf{z}^{(0)} & \ldots & \mathbf{z}^{(b-1)} \ldots \end{bmatrix}\\
&= \begin{bmatrix} \mathbf{W}\mathbf{z}^{(0)} & \ldots & \mathbf{W}\mathbf{z}^{(b-1)} \end{bmatrix}\\
\Rightarrow \mathbf{Z} &= \mathbf{W}\mathbf{X}.
\end{aligned}
$$

Backward:

$$
\begin{aligned}
\nabla_{\mathbf{W}}(L) &= \frac{1}{b}\left[\nabla_{\mathbf{W}}(\mathbf{z}^{(0)}) \times \nabla_{\mathbf{z}^{(0)}}(L) + \cdots + \nabla_{\mathbf{W}}(\mathbf{z}^{(b-1)}) \times \nabla_{\mathbf{z}^{(b-1)}}(L)\right]\\
&= \frac{1}{b}\left[\nabla_{\mathbf{z}^{(0)}}(L)\mathbf{x}^{(0)\mathrm{T}} + \cdots + \nabla_{\mathbf{z}^{(b-1)}}(L)\mathbf{x}^{(b-1)\mathrm{T}}\right].
\end{aligned}
$$

In [ ]:
```python
## Dense layer class
class Dense(Layer):
    def __init__(self, input_size, output_size):
        self.weights = np.empty((output_size, input_size+1)) # bias trick
        self.weights[:, :-1] = 0.01*np.random.randn(output_size, input_size)
        self.weights[:, -1] = 0.01 # set all bias values to the same nonzero constant

    def forward(self, input):
        self.input = np.vstack([input, np.ones((1, input.shape[1]))]) # bias trick
        self.output= np.dot(self.weights, self.input)

    def backward(self, output_gradient, learning_rate):
```

```
        ## Following is the inefficient way of calculating the backward gradient
        dense_gradient = np.zeros((self.output.shape[0], self.input.shape[0]), dtype = np.float64)

        for b in range(output_gradient.shape[1]):
          dense_gradient += np.dot(output_gradient[:, b].reshape(-1, 1), self.input[:, b].reshape(-1, 1).T)

        dense_gradient = (1/output_gradient.shape[1])*dense_gradient
        #dense_gradient = np.mean(dense_gradient, axis = 1)
        ## Following is the efficient way of calculating the backward gradient
        #dense_gradient = (1/output_gradient.shape[1])*np.dot(np.atleast_2d(output_gradient), np.atleast_2d(self.input).T)
        self.weights = self.weights + learning_rate * (-dense_gradient)
```

Function to generate sample indices for batch processing according to batch size

In [ ]:
```
## Function to generate sample indices for batch processing according to batch size
def generate_batch_indices(num_samples, batch_size):
  # Reorder sample indices
  reordered_sample_indices = np.random.choice(num_samples, num_samples, replace = False)
  # Generate batch indices for batch processing
  batch_indices = np.split(reordered_sample_indices, np.arange(batch_size, len(reordered_sample_indices), batch_size))
  return(batch_indices)
```

Example generation of batch indices

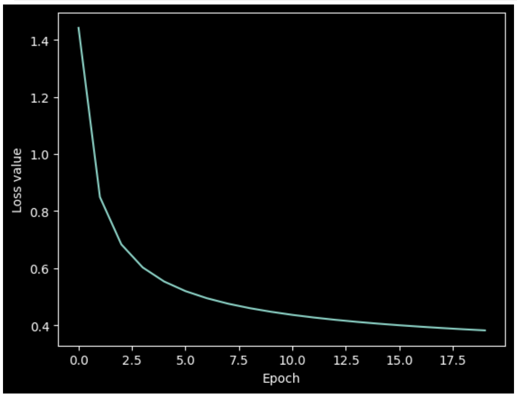
In [ ]:
```
## Example generation of batch indices
#num_samples = 64
batch_size = 8
batch_indices = generate_batch_indices(num_samples, batch_size)
print(batch_indices)
```

Train the 0-layer neural network using batch training with batch size = 16

In [ ]:
```
## Train the 0-layer neural network using batch training with batch size = 16
learning_rate = 0.01 # learning rate
batch_size = 200 # batch size
nepochs = 20 # number of epochs
loss_epoch = np.empty(nepochs, dtype = np.float32) # create empty array to store losses over each epoch

# Neural network architecture
dlayer = Dense(num_features, num_labels) # define dense layer
softmax = Softmax() # define softmax activation layer

# Steps: run over each sample in the batch, calculate loss, gradient of loss,
# and update weights.

epoch = 0
while epoch < nepochs:
  batch_indices = generate_batch_indices(num_samples, batch_size)
  loss = 0
  for b in range(len(batch_indices)):
    dlayer.forward(X_train[:,batch_indices[b]]) # forward prop
    softmax.forward(dlayer.output) # Softmax activate
    loss += cce(Y_train[:,batch_indices[b]], softmax.output) # calculate loss
    # Backward prop starts here
    grad = cce_gradient(Y_train[:,batch_indices[b]], softmax.output)
    grad = softmax.backward(grad)
    grad = dlayer.backward(grad, learning_rate)
  loss_epoch[epoch] = loss/len(batch_indices)
  print('Epoch %d: loss = %f'%(epoch+1, loss_epoch[epoch]))
  epoch = epoch + 1
```

```
Epoch 1: loss = 1.442402
Epoch 2: loss = 0.849283
Epoch 3: loss = 0.683426
Epoch 4: loss = 0.602557
Epoch 5: loss = 0.553313
Epoch 6: loss = 0.519562
Epoch 7: loss = 0.494687
Epoch 8: loss = 0.475379
Epoch 9: loss = 0.459935
Epoch 10: loss = 0.447142
Epoch 11: loss = 0.436330
Epoch 12: loss = 0.427059
Epoch 13: loss = 0.419007
Epoch 14: loss = 0.411885
Epoch 15: loss = 0.405593
Epoch 16: loss = 0.399913
Epoch 17: loss = 0.394804
Epoch 18: loss = 0.390118
Epoch 19: loss = 0.385812
Epoch 20: loss = 0.381864
```

In [ ]:
```
## Plot training loss as a function of epoch:
plt.plot(loss_epoch)
plt.xlabel('Epoch')
plt.ylabel('Loss value')
plt.show()
```



In [ ]:
```
## Accuracy on test set
dlayer.forward(X_test)
softmax.forward(dlayer.output)
ypred = np.argmax(softmax.output.T, axis = 1)
print(ypred)
ytrue = np.argmax(Y_test.T, axis = 1)
print(ytrue)
np.mean(ytrue == ypred)
```

```
[7 2 1 ... 4 5 6]
[7 2 1 ... 4 5 6]
```

Out[ ]: 0.9035

In [ ]:
```
def plot_misclassified_images(X_test, ytrue, ypred, num_images=5):
    misclassified_indices = np.where(ytrue != ypred)[0]
    random_misclassified_indices = np.random.choice(misclassified_indices, num_images, replace=False)

    fig, axes = plt.subplots(1, num_images, figsize=(20, 5))
    for i, index in enumerate(random_misclassified_indices):
        axes[i].imshow(X_test[:, index].reshape(28, 28), cmap='gray')
        axes[i].set_title(f"True: {ytrue[index]}, Predicted: {ypred[index]}")
        axes[i].axis('off')
    plt.tight_layout()
    plt.show()

plot_misclassified_images(X_test, ytrue, ypred)
```