MLPA - Assignment - 01

1 → $W = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $b = -4$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$w^T x + b = 0$$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + (-4) = 0$$

$$x_1 + 2x_2 + 3x_3 - 4 = 0$$

$$x = \begin{bmatrix} 4 - 2x_2 - 3x_3 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

2 → Distance of Sample $x^{(1)}$ from the plane $w^T x + b = 0$

is $\dfrac{|w^T x^{(1)} + b|}{\|w\|}$

Scalar projection $= \dfrac{v \cdot w}{\|w\|}$

$v \to v_1$; $w = [1, 2, 3]$

$v \cdot w \Rightarrow w^T x^{(1)}$

Scalar projection $= \dfrac{w^T x^{(1)}}{\| w}$

$\therefore$ Distance of $x^{(1)}$ from the plane is the magnitude of its scalar projection.

Distance $= \left| \dfrac{w^T x^{(1)}}{\|w\|} \right| = \dfrac{|w^T x^{(1)}|}{\|w\|}$

$\therefore$ Distance of the sample $x^{(1)}$ from the plane is $w^T x + b = 0$

$$\frac{|w^T x^{(1)} + b|}{\|w\|}$$

6) Samples $\to x^{(1)}, x^{(2)}, \dots, x^{(1)}$
Output label $\to y^{(1)}, y^{(2)}, \dots, y^{(n)}$

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6) → $$\text{maximize}\left(\text{minimize of } \frac{|w^T x^{(i)} + b|}{\|w\|}\right) =$$

$$\text{Maximize}\left(\text{minimize of } \overbrace{\frac{|w^T x^{(i)} + b|}{\|w\|}}^{=V}\right)$$

$$\Rightarrow \text{Maximize}\left(\text{minimize of } \frac{w^T x^{(i)} + b}{\|w\|}\right) = \left(\frac{V}{\|w\|}\right)$$

$$y^{(i)}\left(w^T x^{(i)} + b\right) \geq 1$$

7) → $x_2 = -2x_1 + 4$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ -2x + 4 \end{bmatrix}$$

$$= x_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

The straight line can ~~be visualized~~ as the sum of a Scaled direction vector $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and a Specific point on the line $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$.

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8) $X = \begin{bmatrix} x^{(1)} & x^{(2)} & x^{(3)} & x^{(4)} & x^{(5)} \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 0 & 2 & -2 \\ -1 & 1 & 4 & -3 & -2 \end{bmatrix}$

$\Rightarrow \quad 3x_1 - 4x_2 + 1 = 0$

a) The co-efficients of x_1 & x_2 are 3 and -4

$x = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$.

unit normal vector, $u = \dfrac{x}{\|x\|} = \dfrac{\begin{bmatrix} 3 \\ -4 \end{bmatrix}}{\sqrt{3^2 + (-4)^2}}$

$u = \dfrac{1}{5}\begin{bmatrix} 3 \\ -4 \end{bmatrix}$.

b) The full margin width (M)

$M = \dfrac{2}{\|x\|}$

$$M = \frac{2}{\sqrt{3^2 + (-4)^2}}$$

$$= \frac{2}{5}$$

c) $3x_1 - 4x_2 + 1 = 0$ $\quad \cancel{X}^1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 4 \\ 2 & -3 \\ -2 & -2 \end{bmatrix}$

directed distance $d_i = \frac{x \cdot X_i + b}{\|x\|}$, $b = 1$

$$d_1 = \frac{3 \times 1 + (-4) \times (-1) + 1}{\sqrt{3^2 + (-4)^2}} = \frac{10}{5} = 2$$

$$d_2 = \frac{3 \times (-1) + (-4) \times 1 + 1}{\sqrt{25}} = \frac{-5}{5} = -1$$

$$d_3 = \frac{3 \times 0 + (-4) \times 4 + 1}{\sqrt{25}} = \frac{-15}{5} = -3$$

$$d_4 = \frac{3\times 2 + (-4)\times(-3)+1}{\sqrt{25}} = \frac{17}{5}$$

$$d_5 = \frac{3\times(-2)+(-4)\times(-2)+1}{\sqrt{25}} = \frac{-3}{5}$$

$\therefore$ The smallest margin is x_3, $d_3 = -3$

The largest margin is X_4, $d_4 = \frac{17}{5}$