1. $w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\quad b = -4 \quad$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$w^T x + b = 0$

$x_1 + 2x_2 + 3x_3 - 4 = 0$

rref = $\begin{array}{c} x_1 \\ [\,1, 2, 3 \mid -4\,] \end{array}$ $\quad x_1$ is pivot var

x_2 & x_3 are free variables

$x_1 = -2x_2 - 3x_3 + 4$

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_2 - 3x_3 + 4 \\ x_2 \\ x_3 \end{bmatrix}$

$= x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} + 4 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$x_2, x_3 \in R$

(5) Dist of w from plane =

$$= \frac{V_1^T w}{\|w\|} = \frac{(x^{(1)} - \text{vec to point on plane})^T w}{\|w\|}$$

$$= \frac{(x^{(1)T} - (\text{vector to point on plane})^T) w}{\|w\|}$$

$$= \frac{x^{(1)T} w - (\text{vector to point on plane})^T w}{\|w\|}$$

$\Rightarrow$ Dist of $x^{(1)}$ to the hyperplane $w^T x + b = 0$ is equal to $\frac{w^T x^{(1)} - (-b)}{\|w\|} = \frac{w^T x^{(1)} + b}{\|w\|}$

Suppose we had n sample $x^{(1)}, x^{(2)}, \ldots x^{(n)}$

3 feat scatterplot

Distance of sample $x^{(i)}$ (ith sample) to hyperplane is equal to $\frac{w^T x^{(i)} + b}{\|w\|}$

Date :

6. maximize $\left(\text{minimum of } \frac{|w^T x^{(i)} + b|}{\|w\|}\right)$

$= \text{maximize}\left(\frac{\underbrace{\text{minimum of } |w^T x^{(i)} + b|}_{=1}}{\|w\|}\right)$

$= \text{minimize } \|w\|^2/2$

$y^{(i)}(w^T x^{(i)} + b) \geq 1$ for $i = 1 \ldots n$

8. $X = [x^{(1)} \; x^{(2)} \; x^{(3)} \; x^{(4)} \; x^{(5)}]$

$= \begin{bmatrix} 1 & -1 & 0 & 2 & -2 \\ -1 & 1 & 4 & -3 & -2 \end{bmatrix}$

Consider hyperplane $3x_1 - 4x_2 + 1 = 0$

(a) Calc unit vector normal to hyperplane

$-4x_2 = -3x_1 - 1$

$x_2 = \frac{3x_1 + 1}{4}$ slope $= m = \frac{3}{4}$

normal vector will have slope $= -\frac{1}{m} = \frac{-4}{3}$

unit normal vector $= \frac{1}{5}\begin{bmatrix} 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 3/5 \\ -4/5 \end{bmatrix}$

(b) full margin $= \frac{2}{\|w\|}$

$\|w\| = \sqrt{3^2 + (-4)^2} = 5$

full margin width $= 2/5$

(c) $d = \frac{Ax_0 + By_0 + c}{\sqrt{A^2 + B^2}}$

$d_1 = \frac{3 + 4 + 1}{5} = \frac{8}{5}$ $\quad d_3 = \frac{0 - 16 + 1}{5} = -\frac{15}{5} = -3$

$d_2 = \frac{-3 - 4 + 1}{5} = \frac{-6}{5}$ $\quad d_4 = \frac{6 + 12 + 1}{5} = \frac{19}{5}$

$d_5 = \frac{-6 + 8 + 1}{5} = \frac{3}{5}$ $\quad$ smallest $= x^{(3)}$

largest $= x^{(4)}$

Date :

MLPA - Problem Set - 1

1. $w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $b = -4$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $w^T x + b = 0$

$x_1 + 2x_2 + 3x_3 - 4 = 0$

x_1 ↓ pivot var, x_2 & x_3 are free variables

hence $x_1 = -2x_2 - 3x_3 + 4$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_2 - 3x_3 + 4 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$x_2, x_3 \in R$

(7) $x_2 = -2x_1 + 4$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

→ $y = -2x + 4$

→ $2x_1 + x_2 - 4 = 0$

$$\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -4 \end{bmatrix} = 0$$

$2x_1 + x_2 = 4$

Solve this equation

$$\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 4$$

Augmented matrix $\begin{bmatrix} 2 & 1 & | & 4 \end{bmatrix}$

$$\text{rref} = \begin{array}{cccc} x_1 & x_2 & & \\ \left[1 \right. & 1/2 & | & \left. 2 \right] \\ \text{pivot} & \text{free} & & \end{array}$$

$x_1 + x_2/2 = 2$

$x_1 = 2 - x_2/2$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1/2\, x_2 + 2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

→ Solution vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$