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MLPA assignment - 01

1) $w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\quad b = -4$ $\quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$w^T x + b = 0.$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + (-4) = 0$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_2 - 3x_3 + 4 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} + 4 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$x_2, x_3 \in R.$

5) Distance of sample $x^{(i)}$ from the plane $w^T x + b = 0$ is

$$\frac{|w^T x^{(i)} + b|}{\|w\|}.$$

Scalar projection $= \frac{V \cdot W}{\|W\|}$

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$V \rightarrow V_1$; $w = [1, 2, 3]$

$V.w \Rightarrow w^T x^{(i)}$.

Scalar projection $= \frac{w^T x^{(i)}}{\|w\|}$.

Distance $= \left| \frac{w^T x^{(i)}}{\|w\|} \right| = \frac{|w^T x^{(i)}|}{\|w\|}$

$\therefore$ Distance of sample $x^{(i)}$ from plane is $\frac{|w^T x^{(i)} + b|}{\|w\|}$.

6. maximize $\left(\min \text{ of } \left| \frac{w^T x^{(i)} + b}{\|w\|} \right| \right)$

$= \max \left(\frac{\min \text{ of } (|w^T x^{(i)} + b|) = 1}{\|w\|} \right)$

$= \text{minimize } \frac{\|w\|^2}{2}$

$y^{(i)} (w^T x^{(i)} + b) \geq 1$ for $i = 1 \ldots n$.

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7. $x_2 = -2x_1 + 4$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ -2x + 4 \end{bmatrix}.$$

$$= x_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Sum of scaled direction vector & a point on the line $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

8. $X = \begin{bmatrix} x^{(1)} & x^{(2)} & x^{(3)} & x^{(4)} & x^{(5)} \end{bmatrix}$

$$= \begin{bmatrix} 1 & -1 & 0 & 2 & -2 \\ -1 & 1 & 4 & -3 & -2 \end{bmatrix}$$

$= 3x_1 - 4x_2 + 1 = 0.$

a) $x = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

$$u = \frac{x}{\|x\|} = \frac{\begin{bmatrix} -3 \\ 4 \end{bmatrix}}{\sqrt{3^2 + (-4)^2}}.$$

$$u = \frac{1}{5} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

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b) $M = \frac{2}{\|w\|}$

$= \frac{2}{\sqrt{3^2 + (-4)^2}} = \frac{2}{5}$

c) $3x_1 - 4x_2 + 1 = 0.$

$x = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 4 \\ 2 & -3 \\ -2 & -2 \end{bmatrix}$

$d_i = \frac{w \cdot x_i + b}{\|w\|}$, $b = 1.$

$d_1 = \frac{3(1) + (-4) \times (-1) + 1.}{\sqrt{3^2 + (-4)^2}} = 2$

$d_2 = \frac{3(-1) + (-4)(1) + 1}{\sqrt{25}} = -1.$

$d_3 = \frac{3(0) + (-4)4 + 1}{\sqrt{25}} = -3$