Summary

Assignment 5_myusuf2

Q1

The Hope Valley Health Care Association owns and operates six nursing homes in adjoining states. An evaluation of their efficiency has been undertaken using two inputs and two outputs. The inputs are staffing labor (measured in average hours per day) and the cost of supplies (in thousands of dollars per day). The outputs are the number of patient-days reimbursed by third-party sources and the number of patient-days reimbursed privately

Q1a: Formulate and perform DEA analysis under all DEA assumptions of FDH, CRS, VRS, IRS, DRS, and FRH.

Solution 1a

DMU	Staff Hours per day	Supplies per Day	Reimbursed Patient-Days	Privately Paid Patient-Days
Facility 1	150	200	14,000	3,500
Facility 2	400	700	14,000	21,000
Facility 3	320	1,200	42,000	10,500
Facility 4	520	2,000	28,000	42,000
Facility 5	350	1,200	19,000	25,000
Facility 6	320	700	14,000	15,000

<u>Facility 1</u>

```
/* Objective function */
max: 14000 U1 +3500 U2;
/* Constraints */
14000 U1 +3500 U2 - 150 V1 - 200 V2 <= 0;
14000 U1 +21000 U2 - 400 V1 - 700 V2 <= 0;
42000 U1 +10500 U2 - 320 V1 - 1200 V2 <= 0;
28000 U1 +42000 U2 - 520 V1 - 2000 V2 <= 0;
19000 U1 +25000 U2 - 350 V1 - 1200 V2 <= 0;
14000 U1 +15000 U2 - 320 V1 - 700 V2 <= 0;
+ 150 V1 + 200 v2 = 1;
```

<u>Facility 2</u>

```
/* Objective function */
max: 14000 U1 +21000 U2;
/* Constraints */
14000 U1 +3500 U2 - 150 V1 - 200 V2 <= 0;
14000 U1 +21000 U2 - 400 V1 - 700 V2 <= 0;
42000 U1 +10500 U2 - 320 V1 - 1200 V2 <= 0;
28000 U1 +42000 U2 - 520 V1 - 2000 V2 <= 0;
```

```
19000 U1 +25000 U2 - 350 V1 - 1200 V2 <= 0;
14000 U1 +15000 U2 - 320 V1 - 700 V2 <= 0;
+ 400 V1 + 700 V2 = 1;
```

Facility 3

```
/* Objective function */
max: 42000 U1 +10500 U2;
/* Constraints */
14000 U1 +3500 U2 - 150 V1 - 200 V2 <= 0;
14000 U1 +21000 U2 - 400 V1 - 700 V2 <= 0;
42000 U1 +10500 U2 - 320 V1 - 1200 V2 <= 0;
28000 U1 +42000 U2 - 520 V1 - 2000 V2 <= 0;
19000 U1 +25000 U2 - 350 V1 - 1200 V2 <= 0;
14000 U1 +15000 U2 - 320 V1 - 700 V2 <= 0;
+ 320 V1 - 1200 V2= 1;
```

Facility 4

```
/* Objective function */
max: 28000 U1 +42000 U2;
/* Constraints */
14000 U1 +3500 U2 - 150 V1 - 200 V2 <= 0;
14000 U1 +21000 U2 - 400 V1 - 700 V2 <= 0;
42000 U1 +10500 U2 - 320 V1 - 1200 V2 <= 0;
28000 U1 +42000 U2 - 520 V1 - 2000 V2 <= 0;
19000 U1 +25000 U2 - 350 V1 - 1200 V2 <= 0;
14000 U1 +15000 U2 - 320 V1 - 700 V2 <= 0;
+ 520 V1 - 2000 V2= 1;
```

Facility 5

```
/* Objective function */
max: 19000 U1 +25000 U2;
/* Constraints */
14000 U1 +3500 U2 - 150 V1 - 200 V2 <= 0;
14000 U1 +21000 U2 - 400 V1 - 700 V2 <= 0;
42000 U1 +10500 U2 - 320 V1 - 1200 V2 <= 0;
28000 U1 +42000 U2 - 520 V1 - 2000 V2 <= 0;
19000 U1 +25000 U2 - 350 V1 - 1200 V2 <= 0;
14000 U1 +15000 U2 - 320 V1 - 700 V2 <= 0;
+ 350 V1 - 1200 V2= 1;
```

Facility 6

```
/* Objective function */
max: 14000 U1 +15000 U2;
/* Constraints */
14000 U1 +3500 U2 - 150 V1 - 200 V2 <= 0;
14000 U1 +21000 U2 - 400 V1 - 700 V2 <= 0;
42000 U1 +10500 U2 - 320 V1 - 1200 V2 <= 0;
28000 U1 +42000 U2 - 520 V1 - 2000 V2 <= 0;
19000 U1 +25000 U2 - 350 V1 - 1200 V2 <= 0;
```

14000 U1 +15000 U2 - 320 V1 - 700 V2 <= 0;
+ 320 V1 - 700 V2= 1;

DMU	Facility 1	Facility 2	Facility 3	Facility 4	Facility 5	Facility 6
Objective function	max: 14000 U1 +3500 U2	max: 14000 U1 +21000 U2	max: 42000 U1 +4000 U2	max: 28000 U1 +42000 U2	max: 19000 U1 +25000 U2	max: 14000 U1 +15000 U2
Solved	0	0	0	0	0	0
Max. efficiency(100%)	1	1	1	1	0.97	0.87
Variables (Weights)	7.142857e-05	0.000000e+00	2.380952e-05	0.000000e+00	0.0000115123	1.620029e-05
	0.000000e+00	4.761905e-05	0.000000e+00	2.380952e-05	0.0000303506	4.270987e-05
	5.172414e-03	1.376147e-03	1.724138e-03	6.880734e-04	0.0010989011	1.546392e-03
	1.120690e-03	6.422018e-04	3.735632e-04	3.211009e-04	0.0005128205	7.216495e-04

Q1b: Determine the Peers and Lambdas under each of the above assumptions

Solution 1b: The solution is knitted in pdf in R and posted to my Github as: “Assignment5QT1”

Q1c: Summarize your results in a tabular format

Solution 1c

CRS	DRS	IRS	VRS	FDH	ADD
[1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675	[1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675	[1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963	[1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963	[1] 1 1 1 1 1 1	[1] 1 1 1 1 1 1
peer1 peer2 peer3	peer1 peer2 peer3	peer1 peer2 peer3	peer1 peer2 peer3	peer1	peer1
[1,] 1 NA NA	[1,] 1 NA NA	[1,] 1 NA NA	[1,] 1 NA NA	[1,] 1	[1,] 1
[2,] 2 NA NA	[2,] 2 NA NA	[2,] 2 NA NA	[2,] 2 NA NA	[2,] 2	[2,] 2
[3,] 3 NA NA	[3,] 3 NA NA	[3,] 3 NA NA	[3,] 3 NA NA	[3,] 3	[3,] 3
[4,] 4 NA NA	[4,] 4 NA NA	[4,] 4 NA NA	[4,] 4 NA NA	[4,] 4	[4,] 4
[5,] 1 2 4	[5,] 1 2 4	[5,] 5 NA NA	[5,] 5 NA NA	[5,] 5	[5,] 5
[6,] 1 2 4	[6,] 1 2 4	[6,] 1 2 5	[6,] 1 2 5	[6,] 6	[6,] 6
L1 L2 L3 L4	L1 L2 L3 L4	L1 L2 L3 L4 L5	L1 L2 L3 L4 L5	L1 L2 L3 L4 L5 L6	L1 L2 L3 L4 L5 L6
[1,] 1.0000000 0.00000000 0 0.0000000	[1,] 1.0000000 0.00000000 0 0.0000000	[1,] 1.0000000 0.0000000 0 0 0.0000000	[1,] 1.0000000 0.0000000 0 0 0.0000000	[1,] 1 0 0 0 0 0	[1,] 1 0 0 0 0 0
[2,] 0.0000000 1.00000000 0 0.0000000	[2,] 0.0000000 1.00000000 0 0.0000000	[2,] 0.0000000 1.0000000 0 0 0.0000000	[2,] 0.0000000 1.0000000 0 0 0.0000000	[2,] 0 1 0 0 0 0	[2,] 0 1 0 0 0 0
[3,] 0.0000000 0.00000000 1 0.0000000	[3,] 0.0000000 0.00000000 1 0.0000000	[3,] 0.0000000 0.0000000 1 0 0.0000000	[3,] 0.0000000 0.0000000 1 0 0.0000000	[3,] 0 0 1 0 0 0	[3,] 0 0 1 0 0 0
[4,] 0.0000000 0.00000000 0 1.0000000	[4,] 0.0000000 0.00000000 0 1.0000000	[4,] 0.0000000 0.0000000 0 1 0.0000000	[4,] 0.0000000 0.0000000 0 1 0.0000000	[4,] 0 0 0 1 0 0	[4,] 0 0 0 1 0 0
[5,] 0.2000000 0.08048142 0 0.5383307	[5,] 0.2000000 0.08048142 0 0.5383307	[5,] 0.0000000 0.0000000 0 0 1.0000000	[5,] 0.0000000 0.0000000 0 0 1.0000000	[5,] 0 0 0 0 1 0	[5,] 0 0 0 0 1 0
[6,] 0.3428571 0.39499264 0 0.1310751	[6,] 0.3428571 0.39499264 0 0.1310751	[6,] 0.4014399 0.3422606 0 0 0.2562995	[6,] 0.4014399 0.3422606 0 0 0.2562995	[6,] 0 0 0 0 0 1	[6,] 0 0 0 0 0 1
The results indicate that DMUs 1, 2, 3, and 4 are efficient. DMUs 5 and 6 are not efficient	The results indicate that DMUs 1, 2, 3, and 4 are efficient. DMUs 5 and 6 are not efficient	The results indicate that DMUs 1, 2, 3, 4, and 5 are efficient. DMU 6 is not efficient	The results indicate that DMUs 1, 2, 3, 4,and 5 are efficient. DMU 6 is not efficient	The results indicate that DMUs 1, 2, 3, 4, 5, and 6 are efficient.	The results indicate that DMUs 1, 2, 3, 4,and 5 are efficient.

Q1d: Compare and contrast the above results

Solution 1d

Compare and Contrast:

a. CRS and DRS indicate that DMUs only 1, 2,3, and 4 are efficient while 5 and 6 are not
b. IRS, VRS and ADD indicate that DMUs 1, 2,3, 4 and 5 are most efficient while 6 is not efficient
c. FDH indicates that DMUs all the DMUs are efficient

Q2

The Research and Development Division of the Emax Corporation has developed three new products. A decision now needs to be made on which mix of these products should be produced. Management wants primary consideration given to three factors: total profit, stability in the workforce, and achieving an increase in the company's earnings next year from the $75 million achieved this year.

Q2a

Define y1+ and y1-, respectively, as the amount over (if any) and the amount under (if any) the employment level goal. Define y2+ and y2- in the same way for the goal regarding earnings next year. Define x1, x2, and x3 as the production rates of Products 1, 2, and 3, respectively. With these definitions, use the goal programming technique to express y1+, y1-, y2+, and y2- algebraically in terms of x1, x2, and x3. Also express P in terms of x1, x2, and x3.

Solution 2a.

	Unit Contribution Product:					
Factor	**1**	**2**	**3**	**Goal**	**Units**	**Penalty weight**
Total profit	20	15	25	Maximun P	$m	0
Employment level	6	4	5	= 50	Hundreds of employees	-6
Earnings next year	8	7	5	>=75	$m	-3

20x1 + 15x2 + 25x3 = P (for Profit Goal)
6x1 + 4x2 + 5x3 = 50 (for Employment Goal)
8x1 + 7x2 + 5x3 >= 75 (for Earnings next year Goal)

Let x1, x2, and x3 represent the production rates of the three products respectively.

The objective is to:
Maximize Z = P - 6C - 3D, where
P = total (discounted) profit over the life of the new products,
C = change (in either direction) in the current level of employment,
D = decrease (if any) in next year's earnings from the current year's level.
0 (amount of profit over the life of the new products) +
6(amount change in the current level of employment) +
3(decrease amount in next year's earnings) +

We now introduce some auxiliary variables, i.e., extra variables that help formulate the model. These variables are: y1, and y2 defined as follows:

Y1 = 6x1 + 4x2 + 5x3 - 50

Y2 = 8x1 + 7x2 + 5x3 - 75

We now structure our Auxillary Variables as:

$$y_i = y_i^+ - y_i^- \quad \forall i = 1, 2, 3$$

$$y_i^+ = \begin{cases} y_i & \text{if } y_i \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$y_i^- = \begin{cases} |y_i| & \text{if } y_i \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

Therefore: y1 = (y1P - y1M) and y2 = (y2P - y2M)

Model formulation of the overall objective:

Maximixe Z = $6y_1^- + 3y_2^-$

$Y_1^+ - y_1^-$ = 6x1 + 4x2 + 5x3 - 50

$Y_2^+ - y_2^-$ = 8x1 + 7x2 + 5x3 - 75

For P;

20x1 + 15x2 + 25x3 = maximum (Profit Goal)

Therefore, since y+ - y - = y, the above expression for y gives the above constraints

Final Formulation:

Therefore: y1 = (y1P - y1M) and y2 = (y2P - y2M)

Objective Function: Maximixe Z = $6y_1^- + 3y_2^-$

6x1 + 4x2 + 5x3 – ($Y_1^+ - y_1^-$) = 50

8x1 + 7x2 + 5x3 – ($Y_2^+ - y_2^-$) = 75

P = 20x1 + 15x2 + 25x3

Xj >= 0; y_i^+ >= 0; y_i^- >= 0

x1, x2, x3, y1P, y1M, y2P, y2M >= 0 non-negativity constraint

Q2b: Express management's objective function in terms of x1, x2, x3, y1+, y1- , y2+ and y2-.
Solution 2b.

Maximize Z = 20x1 + 15x2 + 25x3 - 6 y1P - 6 y1M - 3 y2M
This is because the employment level is in a dual direction

Q2c: Formulate and solve the linear programming model. What are your findings?
Solution 2c.
The solution is knitted in pdf in R and posted to my Github as "Assignment5QT2"

```
```{r}
library(lpSolveAPI)
gp <- read.lp("Assignment5QT.lp")
gp
```
```

```
Model name:
            x1    x2    x3   y1p   y1m   y2m
Maximize    20    15    25    -6    -6    -3
R1           6     4     5     1    -1     0  =  50
Kind       Std   Std   Std   Std   Std   Std
Type      Real  Real  Real  Real  Real  Real
Upper      Inf   Inf   Inf   Inf   Inf   Inf
Lower        0     0     0     0     0     0
```

```
## Solve
```{r}
solve(gp)
get.objective(gp)
get.variables(gp)
```
```

```
[1] 0
[1] 250
[1]  0  0 10  0  0  0
```

```
### Remarks

Applying the simplex method to this formulation yields an optimal solution of y1m = 0, y2m = 0, x1 = 0, x2 = 0, x3 = 10, y1p = 0,
y1m = 0. Note that the solution is given in the order in which the variables appear in the formulation. This implies that y1 = 0
and y2 = 0, so the first goals is fully satisfied.
***
```

Our final findings are: We can now establish that the total discounted profit would be 10m multiplied by 25m would be 250 million over the life of the new products.

```
## Formulation and Solution
```{r}
mx <- read.lp("Assignment5QT.lp")
mx
solve(mx)
get.objective(mx)
get.variables(mx)
```
```

```
Model name:
            x1    x2    x3   y1p   y1m   y2m   y2p
Maximize    20    15    25    -6    -6    -3     0
R1           6     4     5     1    -1     0     0  =  50
R2           8     7     5     0     0    -1     1  =  75
Kind       Std   Std   Std   Std   Std   Std   Std
Type      Real  Real  Real  Real  Real  Real  Real
Upper      Inf   Inf   Inf   Inf   Inf   Inf   Inf
Lower        0     0     0     0     0     0     0
[1] 0
[1] 250
[1]  0  0 10  0  0  0 25
```

```
***
The employment goal is superseded by 25 employees and the final penalty from exceeding the goal is in the amount of 250.
***

We can now establish that the total discounted profit  would be 10m multiply by 25m would be  250 million over the life of the new
products
```