106bombyx

B-MAT-200

Sequences

- Sequence of numbers $(x_0, x_1, x_2, x_3, \dots)$
- Or more generally $(x_i), i \geq 0$
- Can also be defined as a function $f: \mathbb{N} \rightarrow \mathbb{R}$

$$f(i) = x_i$$

- Examples
  - Factorial: $(1,2,6,24,120, \dots)$
  - Prime numbers: $(2,3,5,7,11,13,17,19,23 \dots)$

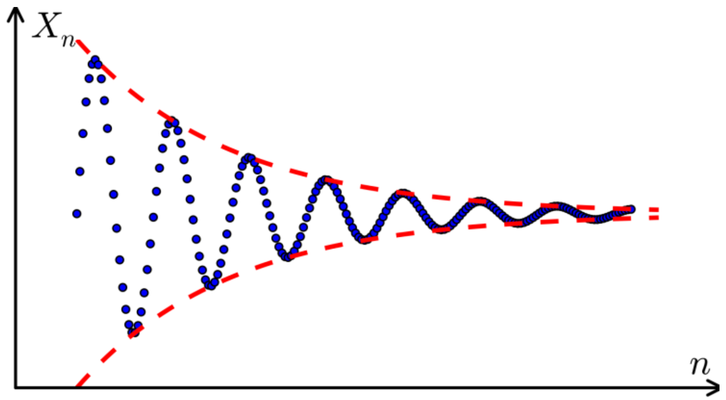
Definition by recursion

- An element is defined by the previous elements in the sequence:
  - Initial value: x_0
  - Recursion: $x_n = f(x_{n-1}, x_{n-2}, \ldots, x_0)$
- Examples:
  - Fibonacci: $x_{i+1} = x_i + x_{i-1}$ which yields: $(0,1,1,2,3,5,8,\ldots)$
  - Bombyx evolution:

$$x_{i+1} = kx_i \frac{(1000 - x_i)}{1000}$$

Sequence properties

- A sequence can be bounded if:
  - There exists M (upper bound) such as $x_i \leq M$, for every i
  - There exists m (lower bound) such as $x_i \geq m$, for every i
- A sequence can have a limit when $i \to \infty$
- If a sequence has a finite limit, the sequence is *convergent*
- If it does not converge (infinite limit or no limit), it is *divergent*



Subsequences and adherent values

- A subsequence is formed from another sequence by removing some elements
  - Example: $(2,4,6,8,\dots)$ is a subsequence of $(1,2,3,4,\dots)$
- A sequence has an adherent value a if it has a subsequence that converges to a
  - Example: $(1,2,1,2,1,2,\dots)$ has two adherent values: 1 and 2

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- Goal: plot the evolution of butterflies by computing the logistic map

$$x_{i+1} = k x_i \frac{1000 - x_i}{1000}$$

- Inputs:
  - n: number of butterflies in the first generation
  - k: growth rate between 1 and 4
  - i0, i1: initial and final generations
- Inputs must be (n, k) OR (n, i0, i1)

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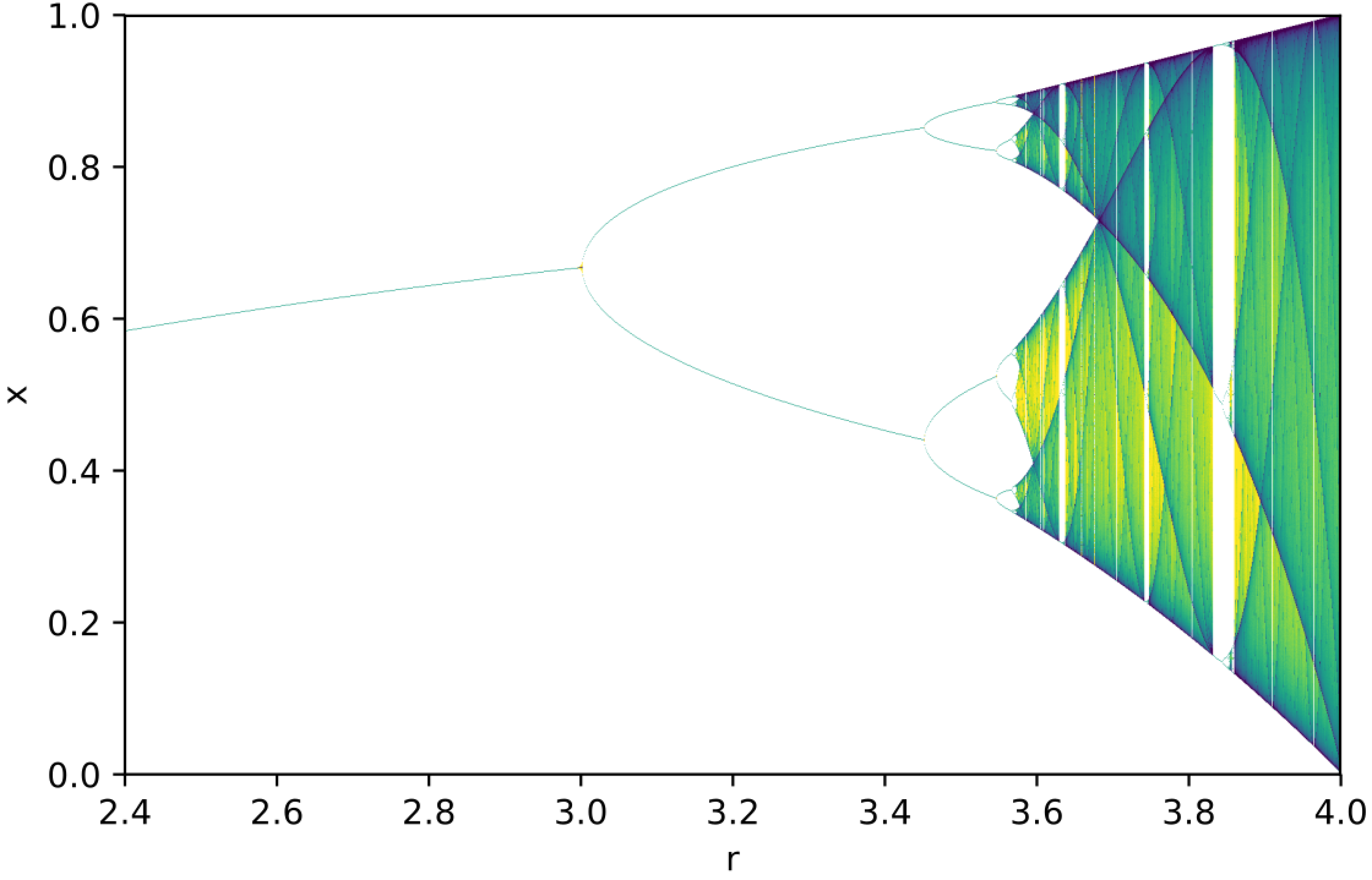
- Output if n and k are given as parameters:
  - Logistic map
  - Display the first 100 points of the curve: (i, x_i)
- Output if n, i0 and i1 are given as parameters:
  - “Bifurcation diagram”
  - For every k between 1 and 4, with a step of 0.01, display every points of the sequence between i0 and i1 (included): (k, x_i)

Suggested bonus

- Plot the curves with gnuplot
- Learn about fractals and plot:
  - Julia set
  - Mandelbrot set


1.0
0.8
0.6
0.4
0.2
0.0
x
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
4.0
r