104intersection-bootstrap

B-MAT-100

Exercise 1

- Create a function that takes three numbers a, b and c and returns the discriminant of the quadratic equation $ax^2 + bx + c = 0$

$$\Delta = b^2 - 4ac$$

Exercise 2

- Create a function that takes three numbers a, b and c and returns the number of solutions to the quadratic equation $ax^2 + bx + c = 0$

Exercise 3

- Create a function that takes three numbers a, b and c and returns the solutions to the quadratic equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \text{ , } if\ \Delta > 0$$

Exercise 4

- Create a function that takes the definition of a line L (a point P and a vector V) and a coefficient t and returns the coordinates of the point $L(t)$

$$L(t): \begin{cases} x(t) = x_p + tx_v \\ y(t) = y_p + ty_v \\ z(t) = z_p + tz_v \end{cases}$$

Formulas

- Equation of a sphere

$$x^2 + y^2 + z^2 = R^2$$

- Equation of a cylinder

$$x^2 + y^2 = R^2$$

- Equation of a cone

$$x^2 + y^2 = \tan^2(\theta)\, z^2$$

- Square development

$$(u + v)^2 = u^2 + 2uv + v^2$$

Example: Intersection of a line with a sphere

$$L(t): \begin{cases} x(t) = x_p + tx_v \\ y(t) = y_p + ty_v \\ z(t) = z_p + tz_v \end{cases}$$

$$x^2 + y^2 + z^2 = R^2$$

$$\left(x_p + tx_v\right)^2 + \left(y_p + ty_v\right)^2 + \left(z_p + tz_v\right)^2 = R^2$$

...

$$(x_v^2 + y_v^2 + z_v^2)t^2 + 2\left(x_p x_v + y_p y_v + z_p z_v\right)t + \left(x_p^2 + y_p^2 + z_p^2\right) = R^2$$

Exercise 5

- Create a function that takes the definition of a line (a point P and a vector V) and a radius R and returns the coefficients a, b and c of the quadratic equation for the intersection of the line and the sphere

Exercise 6

- Create a function that takes the definition of a line (a point P and a vector V) and a radius R and returns the coefficients a, b and c of the quadratic equation for the intersection of the line and the cylinder

Exercise 7

- Create a function that takes the definition of a line (a point P and a vector V) and an angle θ and returns the coefficients a, b and c of the quadratic equation for the intersection of the line and the cone