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Analyzing the Black-Litterman Model and its Applications

1. Abstract

The Black-Litterman model is a widely used asset allocation model throughout the investment management industry. Even though the BL model's familiarity, it is still a challenge to exactly explain how it works, the pros and cons of this model. In this paper, we uncover the intuition behind the Black-Litterman asset allocation model and develop an extension for improvement. We begin our analysis by introducing the traditional mean-variance approach and the deficit of this approach. The result of the mean-variance optimal portfolio is counterintuitive. The M-V model has been criticized with many - drawbacks. As an alternative remedy, we implement the Black-Litterman method to improve the traditional mean-variance method. Additionally, we point out the property and deficit of the Black-Litterman model focusing on the investor's degree of confidence (tau and omega). To address this problem, we incorporate the Investor Confidence Calibration (ICC) as an additional layer of confidence for the investor to update their views to their portfolio.

2. Traditional Mean-Variance Approach

The mean-variance model was introduced by Markowitz in 1952. The model aims to construct portfolios with higher returns and lower risks. When given a target return, the model attempts to minimize the portfolio's standard deviation or alternatively, if the model is given a certain level of risk it will find the appropriate weights to maximize the total return of the portfolio.

The mean-variance model can be expressed in the following way: in the first situation, there is no risk-free asset to invest in. We can only put money in n securities which have the returns as follows:

$$\mathbf{R} \sim \mathrm{MVN}_n(\mu, \Sigma)$$

The function we need to solve is:

$$\min_{\mathrm{w}} \frac{1}{2} \mathrm{w}' \Sigma \mathrm{w}$$

$$s.t\ \mathrm{w}'\mu = p\ and\ \mathrm{w}'\mathbf{1} = 1$$

where w is the portfolio's targeted weight, p is the expected return investors choose. p depends on investors' risk aversion level. Different investors will have different expected returns. Since we do not have a risk-free asset here, all the capital should be invested in the n securities, the total weight should sum to 1.

In the real world, people can choose to either save money or borrow money from banks. In this circumstance, the risk-free asset can be added to the model. Then, the goal of the function is changed to:

$$\min_{\mathrm{w}} \frac{1}{2} \mathrm{w}'\Sigma \mathrm{w}$$

$$s.t \left(1 - \sum_{i=1}^{n} \omega_i\right) r_f + \mathrm{w}'\mu = p$$

where r_f is the risk-free rate and ω_i is the percentage invested in i^{th} security. **Figure 1(a)** shows the efficient frontier with and without a risk-free asset.

The solution to the model with the risk-free asset is the following:

$$\mathrm{w} = \xi \Sigma^{-1}\left(\mu - r_f \mathbf{1}\right)$$

where $\xi = \sigma_{min}^2/(p - r_f)$, σ_{min}^2 is the minimized variance.

There are several drawbacks to the mean-variance model.

1. The mean-variance model is used to analyze the whole market.
2. The mean-variance model treats losses and gains equally. However, investors are often risk and loss averse. People tend to find that the utility from gains does not compensate for the pains from investment losses.
3. The mean-variance model assumes a constant risk aversion parameter. However, investors' risk aversion level varies when circumstance changes.
4. The weight an investor obtains from the mean-variance model is extremely sensitive to any change in the asset expected returns. Σ contains the securities' volatilities which are normally less than 20%, so Σ^{-1} would include large numbers. Small changes to expected returns multiplied by Σ^{-1} would generate significant changes.
5. The mean-variance model allocates investor assets heavily in the least volatile assets like gold and cash (**Figure 1(b)**). Both assets have the low standard deviations. The model sacrifices much more profit than people expected to avoid risk.

3. Black-Litterman Model

The Black-Litterman model is a widely used asset allocation model developed by Fischer Black and Robert Litterman at Goldman Saches in 1990. This model aims to provide a way to mitigate problems associated with mean-variance optimization (MVO) (Litterman 2002). As explained above, there are several drawbacks to the Mean-Variance model, which makes it of little practical value.

The Black-Litterman (BL) model avoids these problems by using the Bayesian approach and subjective views from analysts. The final estimate of expected returns is combined with an investor's views of the expected returns of assets and the market equilibrium vector of expected returns. The market equilibrium vector of expected returns derivates from a benchmark portfolio used in unconstrained MVO(Simonian 2011). The model can offer an appropriate outcome for the investors since the optimal results provide a neutral reference and obviates the need to solve each asset's expected return individually. Then the BL model improves the mean-variance process by one more step, by including neutral weights for the analysts that can be adjusted to express their observation to the future financial-industrial.

These "views" are formed as expected returns for portfolios of assets, but not as expected returns for individual assets. The investor's views often come from the personal or institutional views which are subjective and could be affected by the degree of confidence. Hence, the BL model blended (1) the equilibrium expected returns which benchmark weights for the entire asset's allocation without any active views, and (2) proportional weights of the portfolios about which investors' views. Hence, the resulting of the portfolio could be seen as optimal since it reflects the observation of investment from analysts, both passive and positive, and also their behavior when tradeoff between risk and return.

3.1 The process of Black-Litterman Model

The formulation of the Black-Litterman model uses the Markowitz framework to contain the portfolio managers' views about the future, so the optimal portfolio is intuitive. The Black-Litterman model incorporates the Modern Portfolio Theory, the Bayesian approach, and the Capital Asset Pricing Model (CAPM). Starting from the CAPM equilibrium distribution, the BL model uses the perspective of Bayesian statistics to develop a probability distribution for a new expected return. Then the BL model attempts to estimate the new expected returns by combining the investor's views about the future and some implied returns from CAPM as prior knowledge. Finally, a posterior distribution can be calculated (Idzorek 2007).

The Black–Litterman model posterior expectation is as follows:

$$E[R|q] = \pi + (\tau\Sigma)P^T[P^T(\tau\Sigma)^{-1}P + \Omega\,]^{-1}[Q - P\pi]$$

The posterior covariance is given by:

$$Var[R|q] = (Var[R|q])$$

The optimal weights in the BL model is

$$w = \xi Var[R|q]^{-1}\,E[R|q]$$

In these formulas, τ is a positive parameter which reflects a subjective level of confidence. Σ is a N*N matrix which reflects excess return of covariances. P is a K*N matrix of portfolio weights, specifically with K investor's observations of the future. Ω is a K*K diagonal matrix which measures the degree of confidence of a view. π is an N*1 vector of CAPM equilibrium expected returns and Q is the K*1 vector of the expected returns for the investor's views. By calculating the posterior expectation and the posterior covariance, we obtain the optimal weights to build the portfolio (**Figure 2**). Finally, the BL model provides more intuitive and reasonable changes in the investor's asset allocation.

3.2 Drawback of BL Model

A drawback of the Black-Litterman model is that when adding views to calculate the distribution of the posterior, if the views are true, the BL model is optimal. However, it is possible that the views turn out to be partially true or even totally wrong. Since no one could exactly forecast the future market, these investors' observations are all subjective. Then the BL model may become suboptimal or lead to extreme losses.

Another problem is the uncertainty and confusion of how to set an appropriate value for tau and omega. Both tau and omega are subjective parameters that mostly correspond to the investor's confidence in passive versus positive investment views. This is an indefinite work for portfolio managers to determine such an abstract investor's level of confidence in exact numbers. Moreover, people's behaviors could change according to market behavior. When the stock index performed well, they are more confident about the future, on the contrary, when the stock index performed badly, they are more risk-averse (O'Toole 2017). Usually, higher tau reflects a higher risk level, and a manager can adjust tau to allow the BL model to obtain an optimal portfolio. But if the investor has great confidence for the market, tau will go to zero and the BL model needs to consider constraint on the maximum amount of risk that can be taken.

4. Influence on Weight from Black-Litterman Views

Before we apply the Black-Litterman model to the data, we want to first take a look of our dataset. We collected data from Bloomberg between 12/31/2010 – 03/31/2020. **Figure 3** shows the cumulative returns of all assets collected. In this figure, the benchmark is cash and we set the benchmark return at 1. In this case, we have Russia, Brazil and Gold underperforming during this period, since they are below the cash level. All other assets are above cash and are considered outperformers.

4.1 Different Levels of Positive Views

In our analysis, we first generalize the weight through the mean-variance method which shown in **Figure 4**. We observe that the NKY, the Japan index, has the highest weight at 44%. The second-highest weight is the DAX, the German index, at 24%; the third highest weight is GC1, the Gold Commodity index with a weight of 26%. Next, SHSZ300, the China index weight 6%. At last, the IBOV, the brazil index weights less than 1%.

After we observe the result from the Mean-Variance analysis, we want to see the weight representation from the Black-Litterman model to the same data set as well. From **Figure 5**, we observe the Black-Litterman model has a different weight representation compared to the Mean-Variance model. To compare how changes in one view can influence the weight from the Black-Litterman Model, we tested for three views: IBOV increases by 1%, 30%, and 50%. We want to apply the view to increase IBOV because we observe IBOV has the smallest weight in the Mean-Variance model. Also, we want to know how would the weight change if we add some positive views to the Black-Litterman model.

We can observe from **Figure 5(a)**, the BL model puts the largest weight on SPTSX which is not shown in the Mean-Variance model. We see the IBOV now weighs 7%, which is larger than the weight from Mean-Variance analysis at less than 1%. However, it is still not clear whether this influence comes from our view or from the prior belief of the expected return.

Next, we apply the second view that IBOV will increase by 30%. From **Figure 5(b)**, IBOV has a higher weight compared to Figure 5(a), but not as big as our view changed about IBOV. All the other assets only have a mild adjustment. At last, we changed the view to IBOV would increase by 50%. As we can see in **Figure 5(c)**, IBOV weighs 16%. It is larger than what we got from Figure 5(b), but still not a huge improvement. We observe that Black-Litterman model is not too volatile depending on an investor's views.

We can compare **Tables 1(a)**, **1(b)** and **1(c)**, and see they have the same prior belief of the expected returns because all use the same data set as a base. As we increase the level of increment of IBOV, even to an extremely high level, we still observe only a mild increase in the Black-Litterman blended expected return from IBOV.

4.2 Different Levels of Negative Views

We have shown how a positive view might influence the weight in the BL model. Now we want to see if it works in a similar fashion when we apply a negative view to it. We apply the negative view on SPTSX since it has the largest weight from the Black-Litterman analysis. We want to know how the weight changes with our negative. We apply 3 views: the SPTSX will decrease by 1%, 30%, and 50%.

As shown in **Figure 6**, when we think the SPTSX Index will decrease by 1%, the weight is 61%. When we think it will decrease by 30% the weight becomes 60%. And finally, when we think it will decrease by 50%, the weight only changes to 58%. No matter how we change the view for SPTSX here, the BL model would always put the highest weight on it. And the weight remains around 60%.

In **Table 2**, it shows the Black-Litterman blended expected return for SPTSX decrease for the three levels. When it decreases by 1%, we get the expected return as7.5%; when it decreases by 30% the expected return decreases to 7.2%. The difference is only about 0.3%. When we change the view to SPTSX will decrease by 50%, the expected return decreases to 7.1%, still not a huge difference.

4.3 Influence of Number of Views

Figure 7(a) shows what happens in the Black-Litterman result when we assume the Chinese Index, SHSZ30, will increase by 6%. Compared to the previous pie charts, it is the first time we see a non-zero weight for SHSZ30 by the BL model. Because of our positive view about SHSZ30, now the weight becomes approximately 1%. However, as we introduced the second view that AS51 would increase by 30%, the section for the Shenzhen stock price index disappeared. All other assets look similar.

In **Figure 7(c)**, we introduce a third view, that the UKX is larger than the Shenzhen index by 5%. Intuitively, we want to see the weight for UKX becomes larger. We get the same result as our intuition and it's also the first time we see a non-zero weight for the UKX in the Black-Litterman model. After our analysis, we conclude that the Black-Litterman asset allocation is fairly consistent and is not as volatile as the Mean-Variance approach.

5. Replicating Goldman Sachs Black-Litterman Models

Our team analyzed and replicated the results from the Goldman Sachs research paper: "*The Intuition Behind Black-Litterman Model Portfolios.*" The paper focuses on the difference between the Black-Litterman Model and the traditional Mean-Variance approach, as we have done in the previous sections of this paper. The authors, He and Litterman, successfully demonstrated the benefits of the Black-Litterman Model over the Mean Variance Model and correspond with our conclusions stated in earlier sections of the paper.

In order to get a better understanding of the intricacies of the Black-Litterman model, our group replicated the scenarios proposed within the Goldman Sachs' research paper, by coding our own Black-Litterman Model. There are two scenarios our group successfully replicated. The first scenario proposed was for Germany to outperform France and the UK by 5% overall; or Germany would outperform by 2.5% and France and the UK would underperform by 2.5%. Our team was able to successfully replicate these results in our model by adjusting the P Matrix to match. The charts in **Figure 8** show both Goldman's results (graph on the left) and our results (graph on the right).

The second scenario presented in the paper incorporated an additional viewpoint along with Germany outperforming France and the UK, now Canada is expected to outperform the US by 3%. The charts in **Figure 9** show the results of these two investor views using the Black-Litterman Model and our replication model. Goldman's graph is on the left and our results are on the right. As seen in the graphs, again we were able to successfully replicate both scenarios.

5.1 Black-Litterman Models with Real-Data

After replicating the Goldman Sachs' results, our group decided to test our model using updated, real data, while also increasing the number of securities to 13 to make the model more complex. For our original weights we used weighted-averages based on country GDP. Then we added Gold and Cash as two potential assets to invest in and allocated approximately 5% and 1% to these assets, respectively. This weighted-average portfolio produced an expected return of 5.77%, a standard deviation of 19.17%, and a Sharpe ratio of 0.27.

Using this new model, we implemented three scenarios and compared our Black-Litterman Model with the original weighted-average model. The first scenario we implemented mirrored Goldman's first view in its paper, where Germany was expected to outperform France and the UK by 5%. Using our own Black-Litterman model, the graphs in **Figure 10** show the changes in the expected returns by asset and the change in weights by asset. The portfolio's expected return was 6.01%, the portfolio standard deviation was 19.29%, and a Sharpe ratio of 0.29, a slightly better result than the weighted average portfolio.

Next, we introduced a second view: first, Germany would outperform France and the UK by 5%, and second, the US and China would outperform all other assets. In **Figure 11**, we see the model reacts by allocating a greater percentage of assets to the US, China, and Germany, while reducing the weights across all other assets. This portfolio's expected return is 6.69%, with a standard deviation of 19.64%, and a Sharpe ratio of 0.32.

For our final scenario, we incorporated the view that Gold would outperform Germany by 1%. It should be stated that Germany had the third highest-expected return of 5.9%, while Gold had an expected return of 2.4%. Introducing this new should cause a drastic change in the model's optimal allocation. Clearly, we see in **Figure 12,** this view had a big effect on the model's optimal allocation because now we're of the belief that Gold should be the third-highest allocation. The end result was a portfolio that had an expected return of 6.42%, a standard deviation of 19.27%, and a Sharpe ratio of 0.31.

6. Contribution to the Black-Litterman Model – The Investor Confidence Calibration (ICC)

After reviewing multiple papers about the Black-Litterman Model, our group concluded that an addendum could be made to the overall model in order to give an investor more flexibility when constructing his or her portfolio, specifically regarding individual securities within each of these investment views. We introduced the Investor Confidence Calibration ("ICC") to the model.

This calibration method was added to the posterior expectation and allows investors to add a second level of confidence to their analysis by placing a confidence level around individual securities within each viewpoint. The ICC acts as a supplement to the P Matrix. The ICC differs from tau and omega by focusing on individual assets from each viewpoint. Incorporating the ICC into the Black-Litterman model is an expedited way for the investor to update their views to the portfolio rather than adjusting the entire portfolio (tau), recalibrating their original subjective viewpoint (P Matrix), or altering the entire viewpoint's error (omega). The ICC adds an additional layer of confidence for the investor and we demonstrate its abilities below.

First, we discuss the granularities between ICC and tau, the P Matrix, and omega. The tau parameter allows an investor to adjust the level of confidence for the overall portfolio, but does not allow the investor to change his or her views on an individual securities basis. We keep tau within the overall model as it holds great value in signaling the investor's confidence in the market equilibrium.

Second, the ICC is similar to the P Matrix in that both the ICC and the P Matrix represent the investor's subjective views. However, in the original Black-Litterman model, the investor adjusts the P Matrix in order to express a change in his or her opinion forcing the investor to lose their original insight. Instead, we propose supplementing the P Matrix with the ICC. This allows the investor to maintain their original views while permitting the investor to analyze potential changes to the overall allocation model by changing the confidence in individual securities. We discuss the steps in detail as to how we incorporate the ICC into our model.

Finally, omega allows the investor to express his or her confidence in each viewpoint. However, a change in omega affects the entire investor's viewpoint and does not drill down to the individual securities. Our group also determined that omega is of significant importance as it acts as a "belief error" matrix, meaning the investor recognizes that the view is prone to forecasting errors. However, omega has limitations in that it does not allow the investor to change the view on an individual security level, just the overall viewpoint.

To implement the ICC, we conduct the following steps:

1. Calculate the Black-Litterman Expectation

2. Define our ICC matrix for each security within the portfolio, within the bounds [0, 1]. If there are no changes to our portfolio, we set the ICC weight equal to 0. If the ICC is set to 0, the model produces the original Black-Litterman optimal allocation.

3. The model takes the ICC matrix and inputs the deviation from the original Black-Litterman Optimal allocation model:

$$\text{Deviation} = (W_{fc} - W_{mkt}) * C\%$$

- W_{fc} = Full Investor Conviction (100% belief in idea)
- W_{mkt} = Original BL Optimal Allocation
- C% = Confidence in individual security weight

4. The model calculates the new weights by incorporating the ICC adjustments:

$$W^* = W_{mkt} + \text{Deviation}$$

5. The model solves for $\Omega_{k,k}$ so the squared difference between the ICC and BL weights are minimized

Proposed ICC weight formula in the Black-Litterman Model:

$$\omega_k = [\lambda\Sigma]^{-1}[(\tau\Sigma)^{-1} + P^T_k \Omega_{k,k}^{-1} P_k]^{-1} [(\tau\Sigma)^{-1} \pi + P^T_k \Omega_{k,k}^{-1} Q_k]$$

Let's use an example for further explanation of the ICC. Suppose after the investor sets his or her portfolio to the expectation Germany will outperform France and the UK by 5%, a new piece of economic news comes out that may have an impact on the investor's view of Germany. The investor does not want to alter his or her entire view but wants to analyze how the portfolio might change vs. the original expectation by only changing the German weight component. Rather than changing our P Matrix, the investor only changes the ICC matrix and examines the change in confidence. The new portfolio reflects the investor's change in confidence in the Germany's future performance and constructs a new optimal portfolio.

Our group demonstrates the benefit of the ICC by adding this component to our Black-Litterman model and repeating the second scenarios we presented above, where Germany outperforms France and the UK and the US and China are expected to outperform all other assets. We change the expectation of Germany to 80% (0.8 in the ICC matrix). Again this only changes the German component, leaving the France and UK components untouched. Additionally, lets change the confidence level of the US and China to 60% from our initial viewpoint. The investor can now analyze the difference in the portfolios.

We conduct this analysis two times: one without short positions and the second with short positions. In the no-short positions model, we can see in **Figure 13** the change in weights from the original Black-Litterman model. The graph on the left is the original optimal weights using the Black-

Litterman model. The graph on the right represents the refined model that incorporates the ICC matrix. We see clearly that both the US and China weights are adjusted downward; however, both still affect the entire portfolio and the investor is able to maintain their viewpoint that these two countries will outperform.

We see drastic differences in the short-position model, **Figure 14**, which allows the entire portfolio to normalize to a 100% weighting. This analysis is not as effective as the no-short model in observing the affect the ICC has on the original Black-Litterman model because the original Black-Litterman model does not propose any short postions. However, it is helpful to see that the ICC maintains the relative positioning from the two viewpoints that the investor wishes to achieve.

7. Conclusion

In our research we have studied and analyzed the difference between the Mean-Variance model and the Black-Litterman Model, concluding the Black-Litterman model offers an optimal allocation over the Mean-Variance model. We have demonstrated that the Black-Litterman model is less volatile and more consistent than the Mean-Variance model in our first demonstration.

Our group successfully constructed our own Black-Litterman model and tested its' abilities by effectively replicating the Goldman Sachs' Black-Litterman scenarios presented in the paper. Our model matched both the one- and two-scenario outcomes. Next, to add another level of complexity, we incorporated more securities in our model and used real data collected from Bloomberg. We then ran three additional scenarios to observe the effects this would have on our optimal allocation.

Finally, we discussed the drawbacks in the Black-Litterman model, specifically the model is dependent on an investor's subjective views. If the investor's views are incorrect, the Black-Litterman model is subject to produce sub-optimal weights. To account for constantly changing viewpoints and market news, we created the ICC, a supplement to the P Matrix. The ICC allows investors to change their viewpoints on individual securities within a viewpoint. We believe the ICC provides the investor with another tool to achieve optimal asset allocation. Our team provides one final table, **Table 3**, which shows the difference in weights for the various models we have described in our paper.

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Appendix

Figure 1(a): Efficient Frontier with and without a Risk-free asset

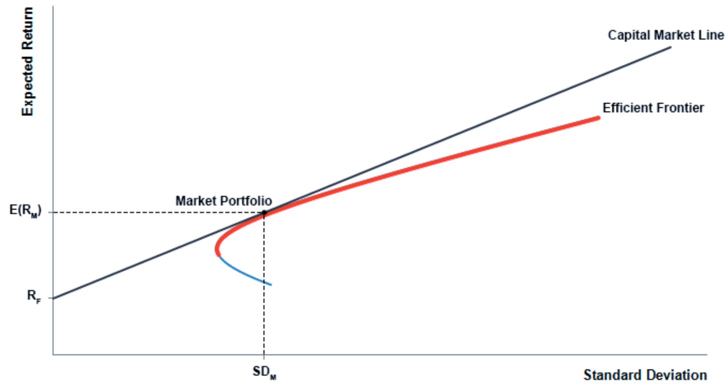

Figure 1(b): Mean-Variance Result Using Real Data

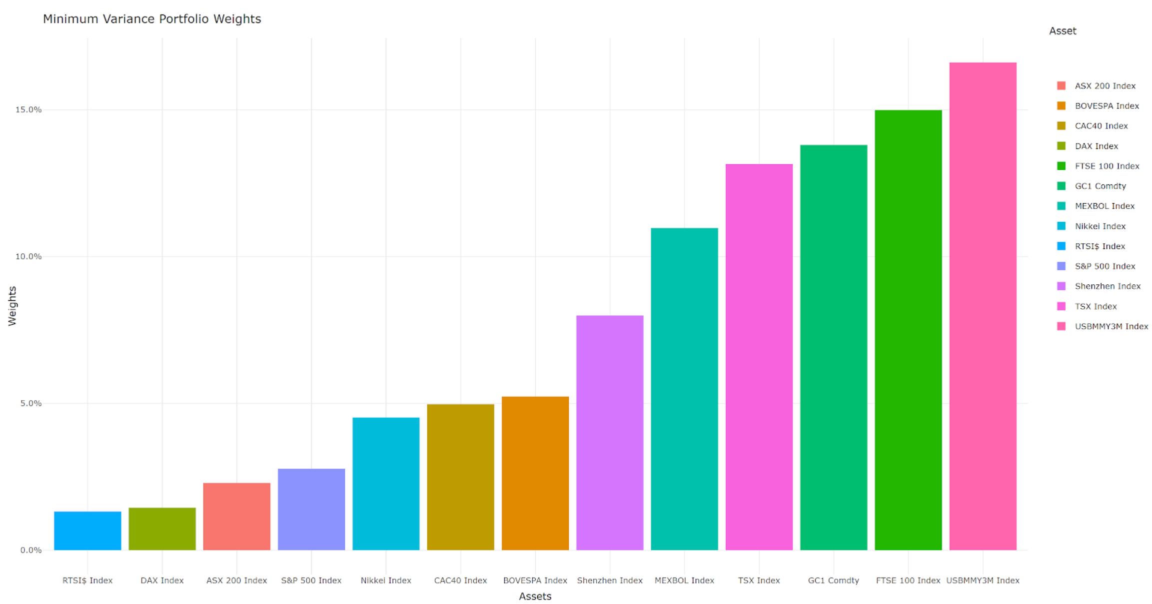

Figure 2: Process of Black-Litterman model

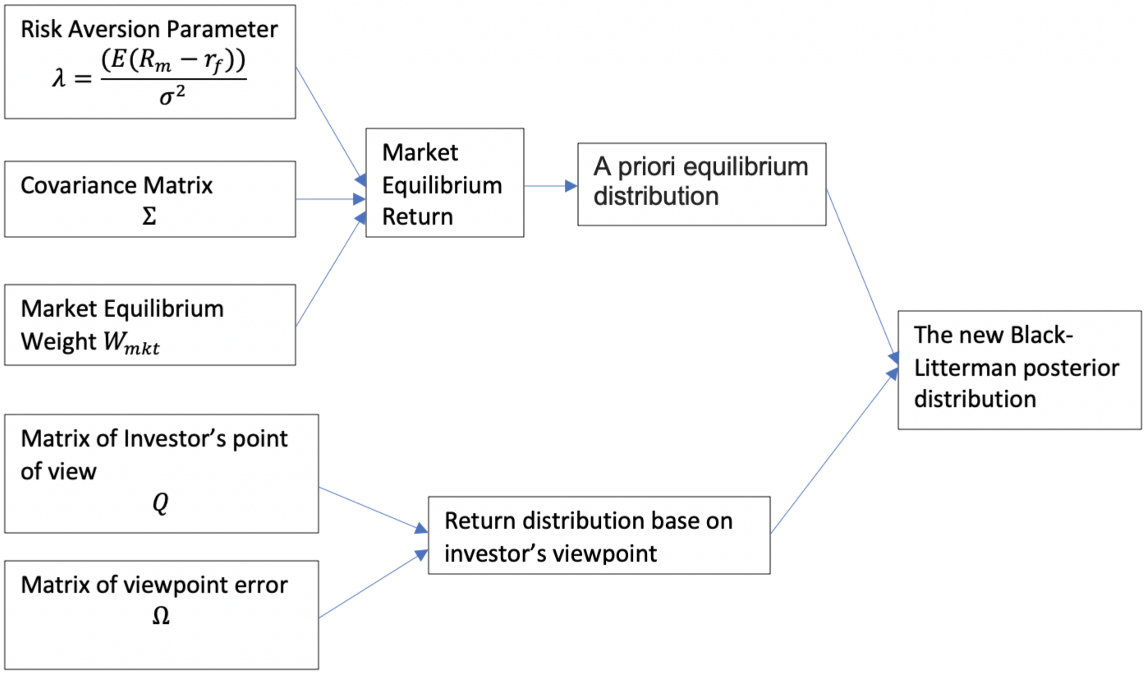

Figure 3: Asset Cumulative Returns from Data

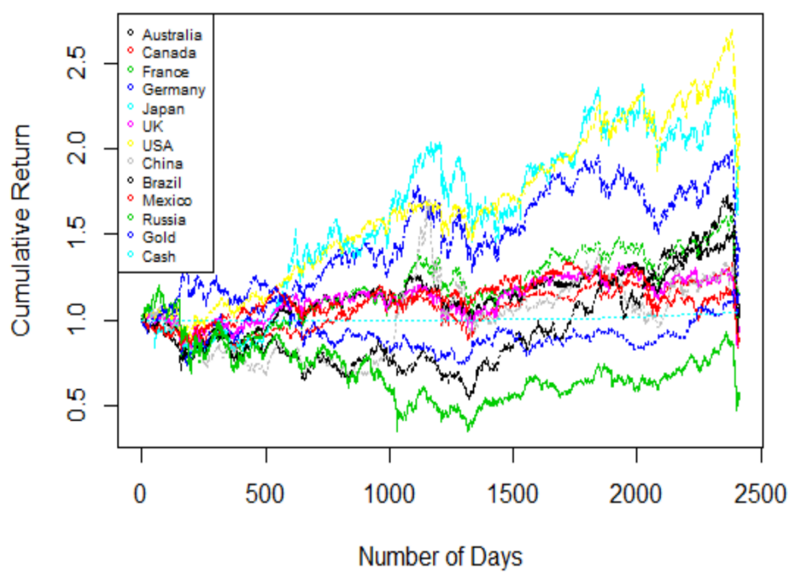

Figure 4: Mean-Variance Weight

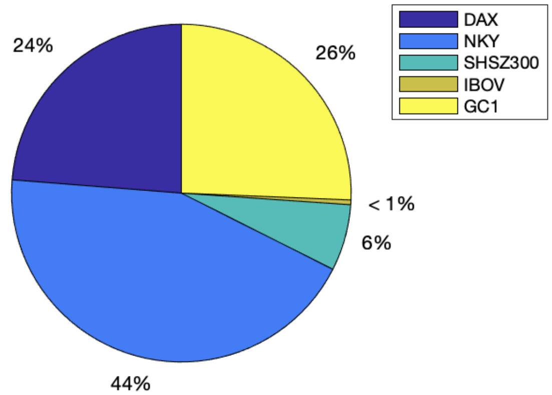

Figure 5: BL Weight for IBOV Increase by 1%, 30%, 50% Views

5(a) 5(b) 5(c)

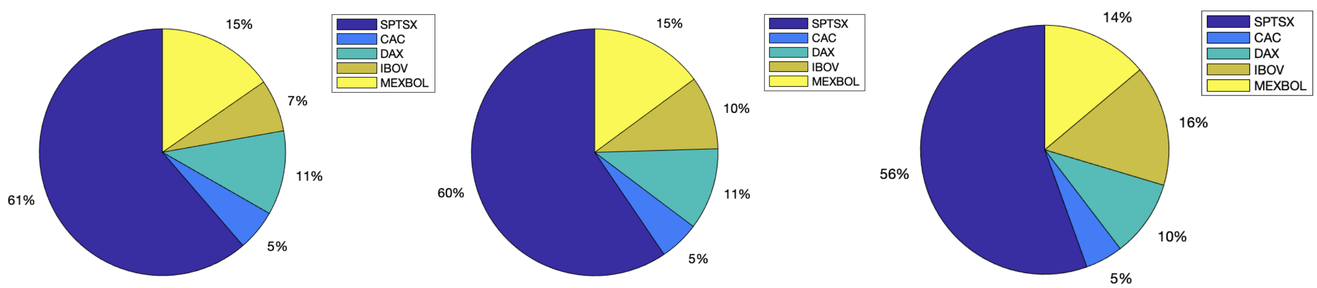

Figure 6: BL Weight for SPTSX decrease by 1%, 30%, 50% Views

6(a) 6(b) 6(c)

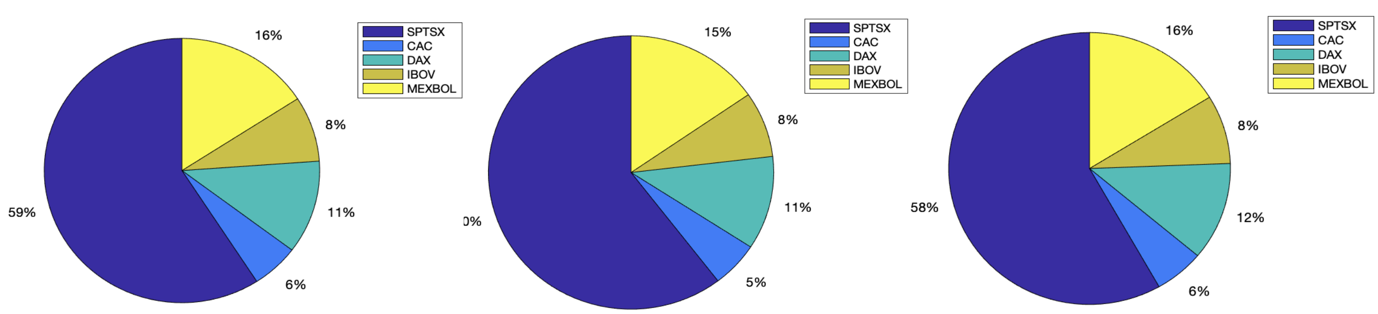

Figure 7: BL Weight for Number of Views Changed

7(a) 7(b) 7(c)

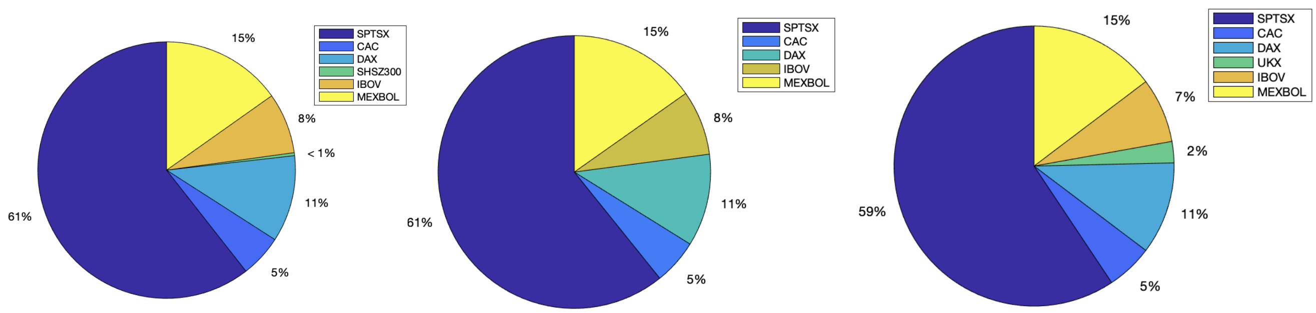

Figure 8: Goldman Sachs Black-Litterman Replication – One View

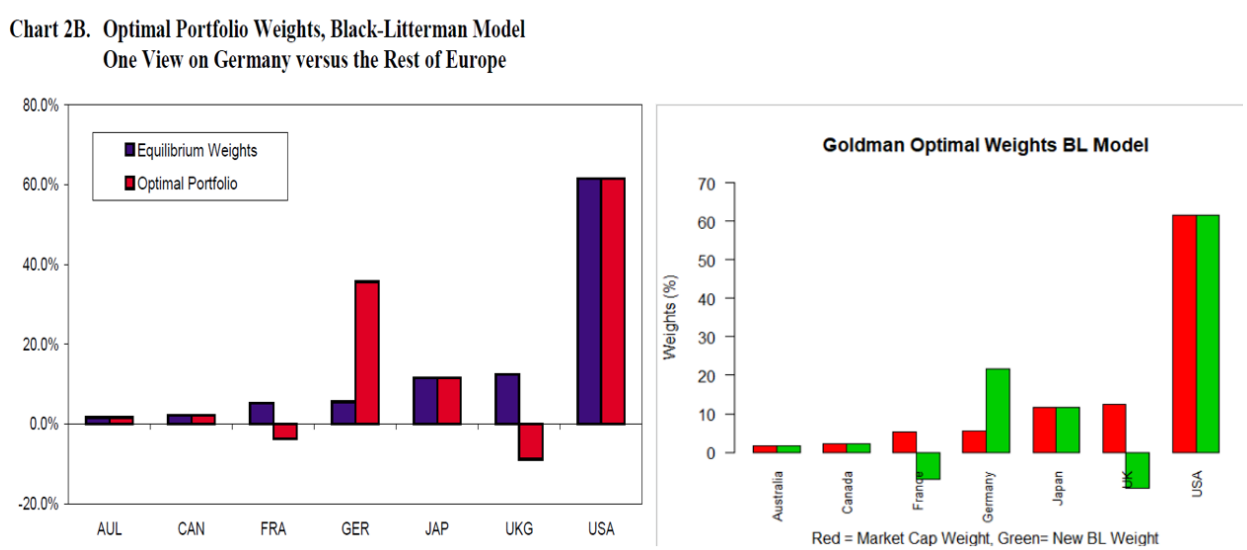

Figure 9: Goldman Sachs Black-Litterman Replication – Two Views

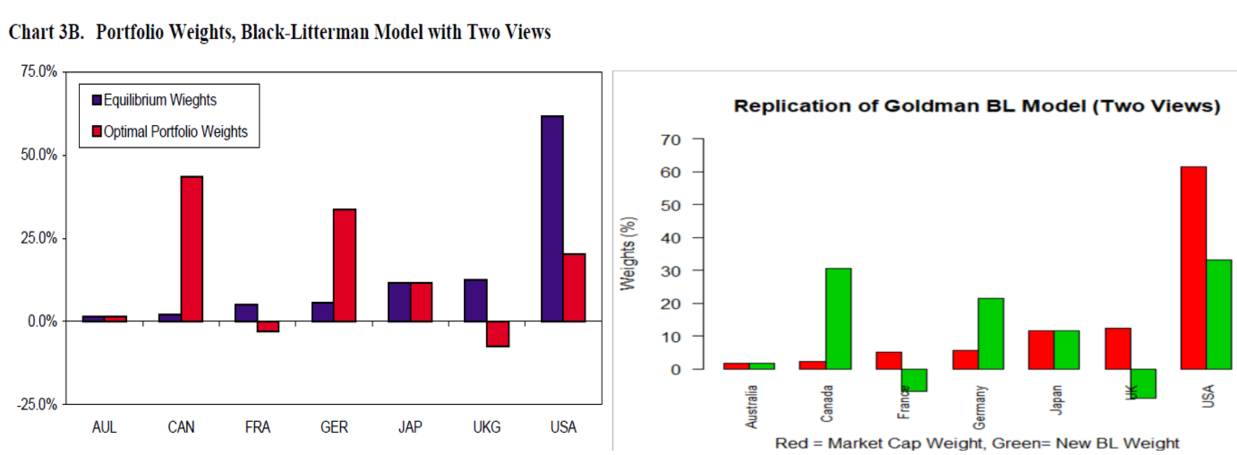

Figure 10: Implementing Black-Litterman Model with Real Data – One View

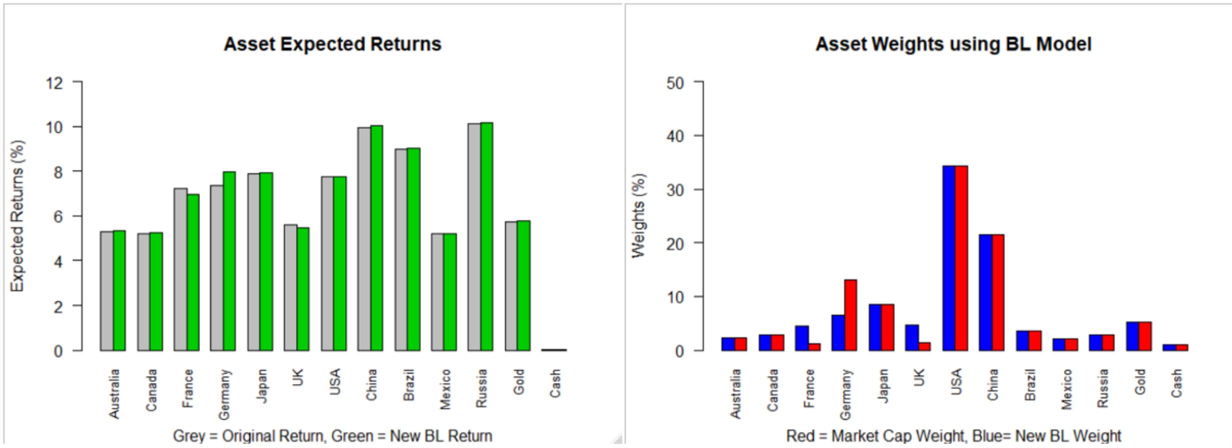

Figure 11: Implementing Black-Litterman Model with Real Data – Two Views

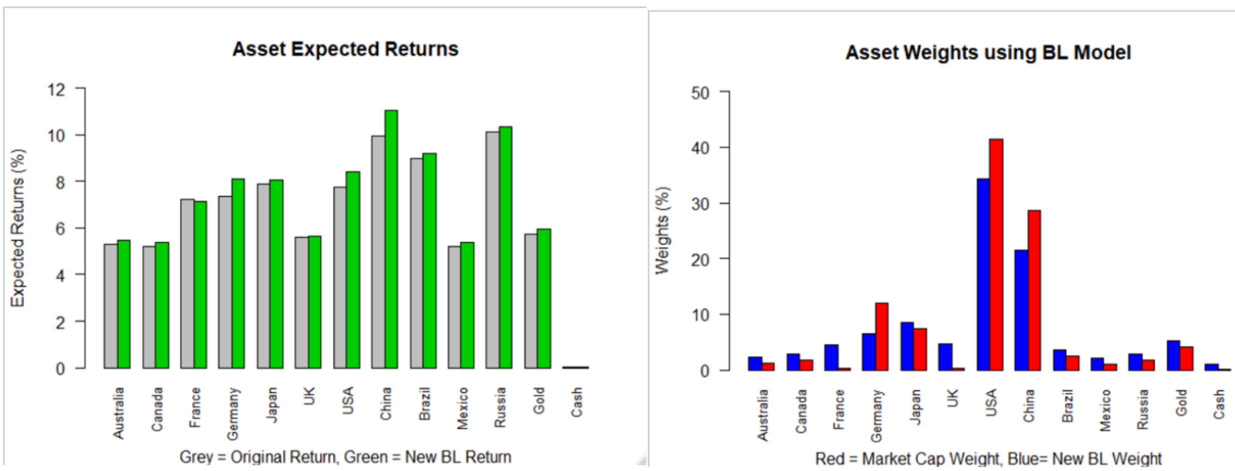

Figure 12: Implementing Black-Litterman Model with Real Data – Three Views

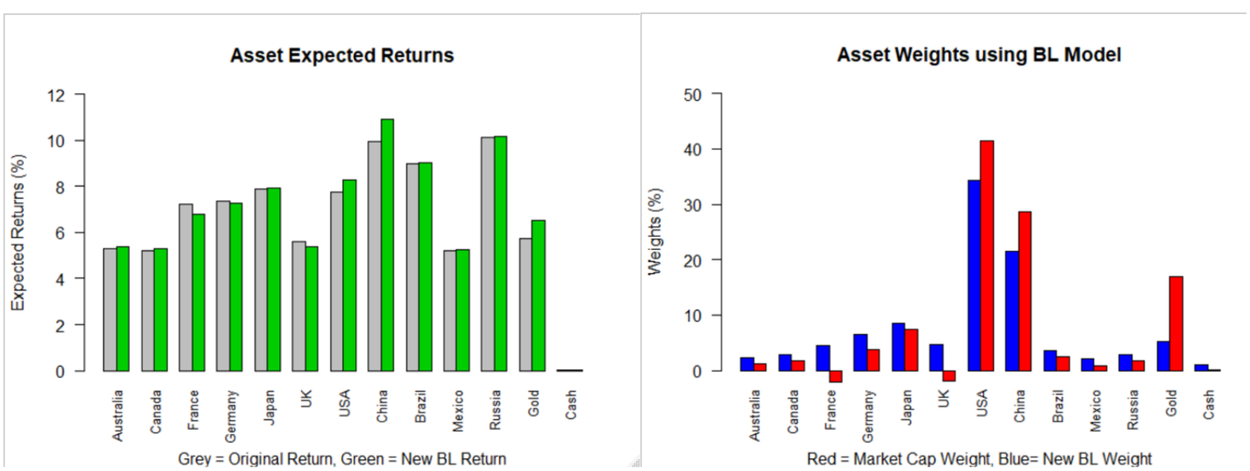

Figure 13: Refined BL Model with ICC Component (No Short Positions)

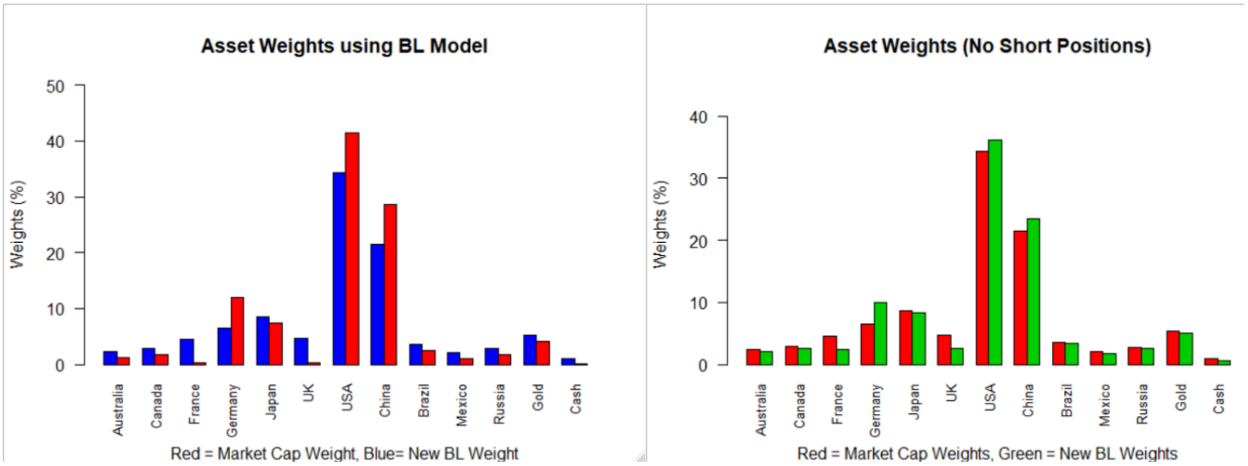

Figure 14: Refined BL Model with ICC Component (With Short Positions)

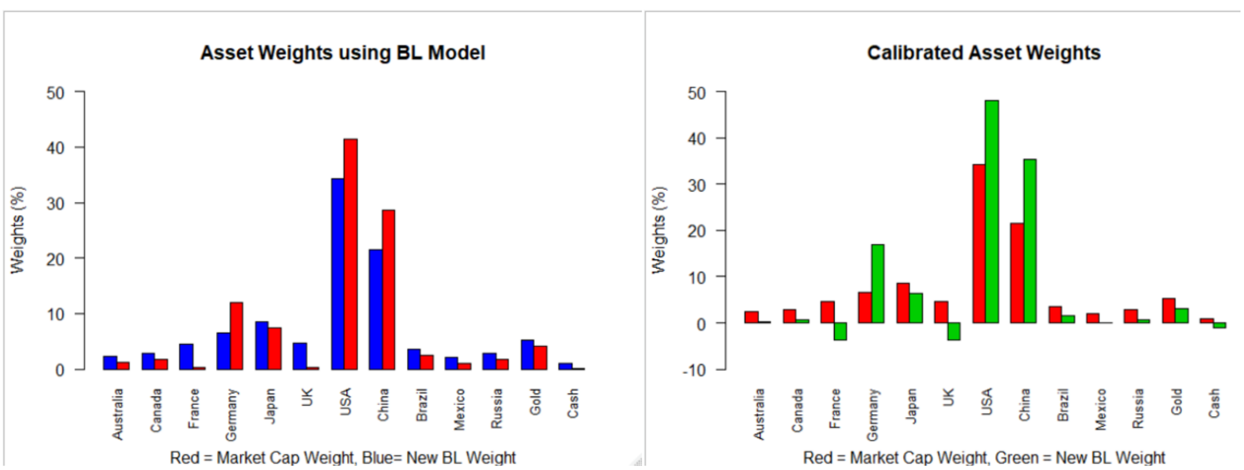

Table 1(a): IBOV increase by 1%

Asset_Name	Prior_Belief_of_Expected_Return	Black_Litterman_Blended_Expected_Return
"AS51"	0.036215	0.03581
"SPTSX"	0.075747	0.074979
"CAC"	0.081269	0.080493
"DAX"	0.079249	0.078532
"NKY"	0.026816	0.026589
"UKX"	0.064005	0.063357
"SHSZ300"	0.024187	0.023916
"IBOV"	0.095011	0.092901
"MEXBOL"	0.055156	0.054499
"RTSI"	0.079187	0.078271
"GC1"	0.0084967	0.0083985

Table 1(b): IBOV increase by 30%

Asset_Name	Prior_Belief_of_Expected_Return	Black_Litterman_Blended_Expected_Return
"AS51"	0.036215	0.037192
"SPTSX"	0.075747	0.077598
"CAC"	0.081269	0.08314
"DAX"	0.079249	0.080979
"NKY"	0.026816	0.027361
"UKX"	0.064005	0.065569
"SHSZ300"	0.024187	0.024841
"IBOV"	0.095011	0.1001
"MEXBOL"	0.055156	0.056741
"RTSI"	0.079187	0.081398
"GC1"	0.0084967	0.0087334

Table 1(c): IBOV increase by 50%

Asset_Name	Prior_Belief_of_Expected_Return	Black_Litterman_Blended_Expected_Return
"AS51"	0.036215	0.038144
"SPTSX"	0.075747	0.079404
"CAC"	0.081269	0.084965
"DAX"	0.079249	0.082666
"NKY"	0.026816	0.027894
"UKX"	0.064005	0.067095
"SHSZ300"	0.024187	0.025478
"IBOV"	0.095011	0.10506
"MEXBOL"	0.055156	0.058287
"RTSI"	0.079187	0.083554
"GC1"	0.0084967	0.0089644

Table 2: Black Litterman Blended Expected Return for SPTSX Decrease

Asset Name	1%	30%	50%
"AS51"	0.035877	0.034731	0.033941
"SPTSX"	0.074976	0.072367	0.070568
"CAC"	0.080631	0.078473	0.076985
"DAX"	0.078638	0.076571	0.075145
"NKY"	0.026595	0.02585	0.025337
"UKX"	0.063477	0.061689	0.060456
"SHSZ300"	0.023979	0.023274	0.022789
"IBOV"	0.094224	0.091562	0.089727
"MEXBOL"	0.05474	0.053333	0.052363
"RTSI"	0.078537	0.076336	0.074819
"GC1"	0.0083808	0.0079889	0.0077186

Table 3: Difference in Model Weights

Asset	Original Mkt Cap Weights	Minimum Variance Weights	Black-Litterman Weights	New Calibrated Weights	New Calibrated Weights (No Shorts)
Australia	2.33%	14.53%	1.21%	0.19%	1.57%
Canada	2.90%	1.38%	1.77%	0.74%	2.13%
France	4.55%	3.22%	0.21%	-3.67%	2.28%
Germany	6.50%	0.90%	11.63%	16.18%	8.64%
Japan	8.58%	0.98%	7.32%	6.19%	7.73%
UK	4.65%	12.98%	0.30%	-3.58%	2.37%
USA	34.31%	0.88%	40.42%	46.19%	38.53%
China	21.54%	9.48%	27.96%	33.95%	25.95%
Brazil	3.62%	1.50%	2.52%	1.43%	2.84%
Mexico	2.03%	10.95%	0.92%	-0.10%	1.27%
Russia	2.78%	3.30%	1.65%	0.62%	2.01%
Gold	5.28%	19.35%	4.10%	3.02%	4.48%
Cash	0.93%	20.53%	0.00%	-1.15%	0.19%