Meihui Zhang, Sihang Liu

Hao Xing

Algorithmic and High-Frequency Trading

01 May 2020

Trading in Volatile time

Summary

In order to analyze which strategy works the best, we use the most volatile data from Fer 18, 2020 to Apr 24,2020. In general, during the following four market situations, we would like to choose strategies accordingly.

1. When the market trend has a negative slope with a big mean-reverting level we may choose a long-short strategy.
2. When the market trend has a negative slope with a small mean-reverting level we may choose either a long-short strategy and ad-hoc band.
3. When the market trend has a positive slope with a big mean-reverting level we may choose an ad-hoc band strategy.
4. When the market trend has a positive slope with a small mean-reverting level we may choose a buy-hold strategy.

For the delay in execution problem, most of time the investors will be better off if they do not choose to delay the execution while the mean-reverting speed is large. If the mean-reverting speed is small, then people might be better off if choosing the one-day delay strategy.

Introduction

Because of the COVID-19 pandemic, the global stock market crash began in February, 2020. As of March 2020, the global stocks have seen a downturn of at least 25% during the crash. During this period, we can observe a downward trend for the US stock market. In this project, we want to construct a trading strategy trying to make profit during the volatile market environment.

Data preparation and illustration

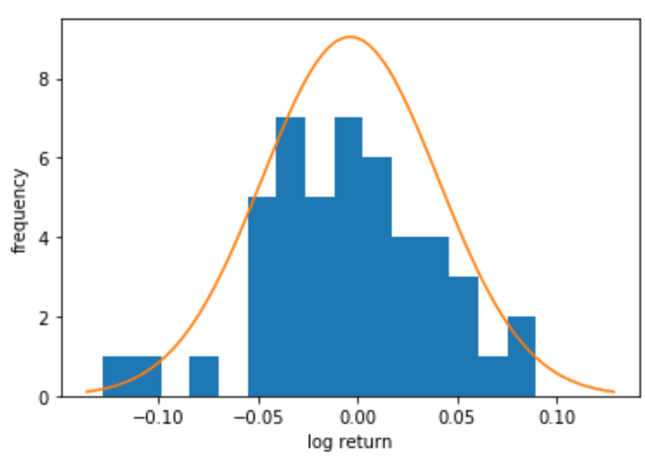
Given a time series data of the adjusted close prices of S&P500 index from Feb.20, 2020 until Apr.24, 2020, we first calculated the log return of the stock prices by using equation (1):

$$r_t = log \frac{S_t}{S_{t-1}} \qquad (1)$$

where S_t is the adjusted close price for the current day, and r_t is the log return for the same day.

After that, we drew a histogram of the log returns and fitted the data into a normal distribution. The fitted normal distribution is with mean -0.3667% and standard deviation 4.4112%. From Figure 1, we can observe that the empirical distribution is right skewed and has a slightly heavier tail than the fitted distribution as well.

Figure 1. Fitted Normal Distribution of the Log Returns



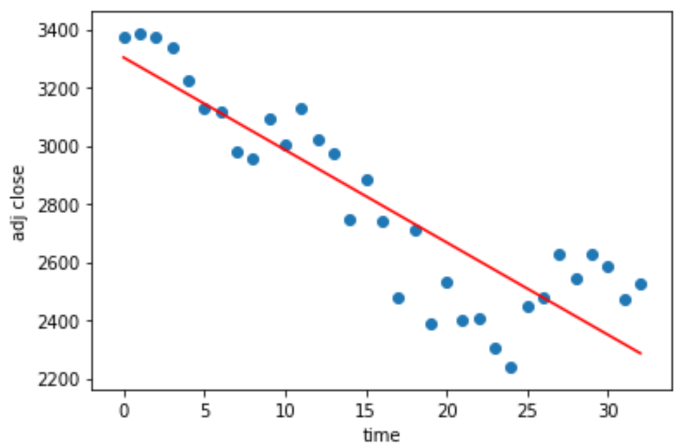
Mean-reverting behavior of the stock

We splitted the data into two parts. The first part is the formation period which contains the first 80% of data, and the rest part is the testing period. Then, we fitted the S&P500 prices to a linear function of time, which can be expressed as:

$$S^{ave}(t) = at + b \quad (2)$$

In our data formation period, we can obviously observe a downward linear trend from Figure 2 with $a = -31.7728$ and $b = 3302.9384$.

Figure 2. Downward Linear Trend of S&P 500 Index



After we got the S&P 500 price $S(t)$ and the linear trend of it, $S^{ave}(t)$, we can calculate the deviation of the S&P 500 prices from $S^{ave}(t)$, which we denote it as $Y(t)$:

$$Y(t) = S(t) - S^{ave}(t) \quad (3)$$

We can observe a mean-reverting behavior of the stock price deviation $Y(t)$. The dynamic process of $Y(t)$ is:

$$dY_t = \kappa(\theta - Y_t)dt + \sigma dW_t \quad (4)$$

where κ is the mean-reverting speed and θ is the mean-reverting level. In our sample data, We can also calculate $\sigma = 130.9916$ directly by using the sample volatility of Y(t) in the portfolio formation period through estimation of the realized variance:

$$\sigma^2 \approx \frac{1}{T}\sum_{t=1}^{T} \Delta {Y_t}^2 \quad (5)$$

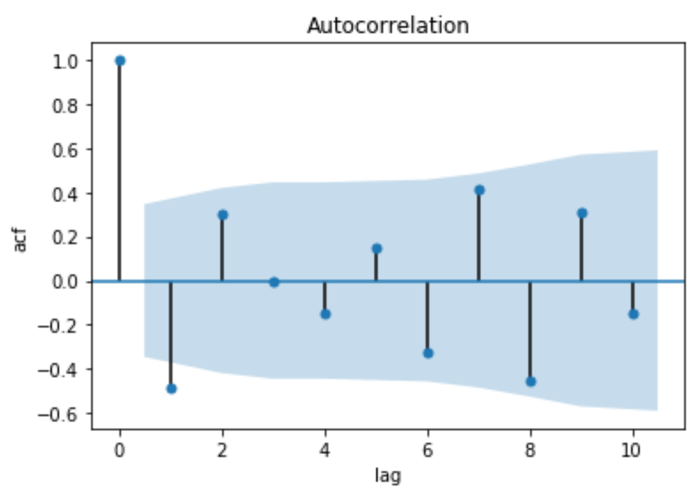
In order to find κ and θ in equation (4), we first need to fit an AR(1) model of ΔY:

$$\Delta Y(t) = Y(t) - Y(t-1) \quad (6)$$

$$\Delta Y(t) = A + B\Delta Y(t-1) \quad (7)$$

Figure 3 shows the auto-correlation of ΔY. There exists a significant negative auto-correlation of lag 1 with 95% confidence interval.

Figure 3. Auto-correlation of ΔY



From equation (4), we can get:

$$Y_{t+1} - Y_t = \kappa(\theta - Y_t)\Delta t + \sigma(W_{t+1} - W_t) \quad (8)$$

$$Y_t - Y_{t-1} = \kappa(\theta - Y_{t-1})\Delta t + \sigma(W_t - W_{t-1}) \quad (9)$$

(8) − (9):

$$\Delta Y_t = (1 - \kappa\Delta t)\Delta Y_{t-1} + \sigma\varepsilon_t \quad (10)$$

$$\varepsilon_t = W_{t+1} - 2W_t + W_{t-1} \quad (11)$$

Therefore, from equation (6) and (9), we can solve:

$$\begin{cases} \kappa = \dfrac{1-B}{\Delta t} \\ \theta = \kappa^{-1} A \Delta t \end{cases} \quad (12)$$

In our case, $\theta = 2.7960$ and $\kappa = 1.4769$.

The reason why we are using the deviation of the prices instead of directly using stock prices in our evaluation is we can observe the stock prices are decreasing very fast in the data window we chose. If we remove the downward trend, the remaining part of the price process would correspond to a mean-reverting model.

Trading strategy using ad-hoc bands

For the training set, we define the ad-hoc bands for $Y(t)$ as upper bound Y_u and lower bound Y_l:

$$\begin{aligned} Y_u &= \theta + n\sigma \\ Y_l &= \theta - n\sigma \end{aligned} \quad (13)$$

The next step is to convert the two bands of $Y(t)$, the deviation of prices, to the boundaries of the actual prices which we define as $S^u(t)$ and $S^l(t)$:

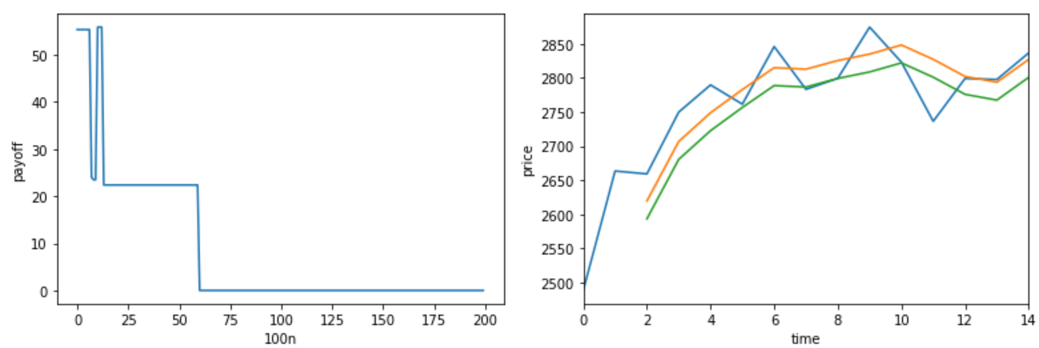
$$\begin{aligned} S^u(t) &= S^{ave}(t) + Y_u \\ S^l(t) &= S^{ave}(t) + Y_l \end{aligned} \quad (14)$$

The idea of this strategy is entering the long position when price reaches the lower bound S^l and closing the position when price reaches the upper bound S^u. This means when we are in a long position, the next move is to close it. We are not able to short the stock here. And on the last day of the training period, we are forced to close the position. And in the testing period, we use rolling mean with 3 days to get S^{ave}.

We can try many different numbers for n to find the best one with the highest payoff during the testing period.

Here we use n from 0.01 to 2 to find the best n with initial capital \$1,000 and it turns out that $n = 0.1$ can generate the highest payoff \$55.8778. Figure 4 shows the payoffs calculated with these 200ns.

Figure 4. Payoffs with different bands(l); Ad-hoc bands with $n = 0.1$(r)



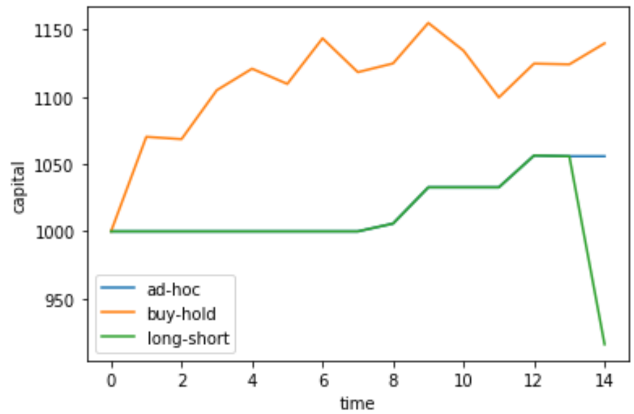
In order to evaluate the performance of the above strategy, we compared it with other two strategies, buy-and-hold strategy and long-short strategy.

The buy-and-hold strategy means we use all of our initial capital \$1,000 to buy the stock at the beginning day of the testing period and sell them out at the last day of the period. We do not need to do anything else during the whole process. The long-short strategy is that we first short \$1,000 value of the stock and use this amount of money to construct the strategy using the best ad-hoc bands we find before ($n = 0.1$). At the end of the testing period, buy back the same units of stock to close the position.

Figure 5 compares the capital processes of these three different strategies. Because during the testing period, the S&P 500 price bounced back a lot, when we use the long-short strategy which includes shorting the stock at the beginning, we would suffer a loss by \$83.9932. And also because of the increasing price process, the buy-hold strategy has the best payoff which is

\$139.8911. The original ad-hoc bands strategy is in the middle creating a profit of \$55.8778. Notice that the ad-hoc capital processes coincide with the long-short capital process until t = 13.

Figure 5. Capital processes of the three strategies



Trading strategy using optimal bands

Suppose we are already in a long position and we would like to find the best liquidation time before T, the last day of our formation period, we need to solve the optimal exiting problem:

$$H(t,S) = E_{t,S}\left[e^{-\rho(\tau-T)}(S_\tau - c)\right] \qquad (15)$$

where ρ is the risk-free rate, τ is the optimal stopping time, c is the transaction cost and T is the maturity of our investment horizon.

To further simplify the problem, we let the risk-free rate be zero in our evaluation and c is 0.01. Based on the current situation, the risk-free rate is indeed close to zero, so our assumption is reasonable. Then the problem becomes:

$$H(t,S) = E_{t,S}[(S_\tau - c)] \qquad (16)$$

$H(t,S)$ is the highest payoff we can get from time t. In the portfolio formation period, we have modeled for the S&P 500 prices to be:

$$S^{ave}(t) = at + b \qquad (2)$$

$$S(t) = Y(t) + S^{ave}(t) \quad (3)$$

And $Y(t)$, the deviation from the S&P 500 in the portfolio formation period is following the mean-reverting process, with the dynamic:

$$dY_t = \kappa(\theta - Y_t)dt + \sigma dW_t \quad (4)$$

Therefore, we can derive:

$$\begin{aligned} dS(t) &= d\,Y(t) + dS^{ave}(t) \\ &= (\kappa(\theta - Y_t) + a)dt + \sigma dW_t \\ &= (\kappa(\theta - S_t + at + b) + a)dt + \sigma dW_t \quad (17) \end{aligned}$$

Now, let's derive the free boundary equations for the optimal exiting problem.

$H(t,S)$ satisfies:

$$max\{(\partial_t + L)H(t,S), S - c - H(t,S)\} = 0 \quad (18)$$

with terminal condition:

$$H(T,S) = S - c \text{ for any } S$$

where:

$$L = (\kappa(\theta - S_t + at + b) + a)\partial_s + \frac{1}{2}\sigma^2\partial_{ss}^2 \quad (19)$$

The associated free-boundary problem is:

$$\begin{cases} (\partial_t + L)H(t,S) = 0 & S < S_u \\ H(t,S) = S - c & S \geq S_u \\ H'(t,S) = 1 & S = S_u \end{cases} \quad (20)$$

Once we have solved the optimal exciting problem, the next step is to find the optimal time to enter the position.

Similarly, we define:

$$G(t,S) = E_{t,S}\left[H(\eta, S_\eta) - S_\eta - c\right] \quad (21)$$

$G(t,S)$ satisfies:

$$max\{(\partial_t + L)H(t,S), H(t,S) - S - G(t,S) - c\} = 0 \quad (22)$$

with terminal condition:

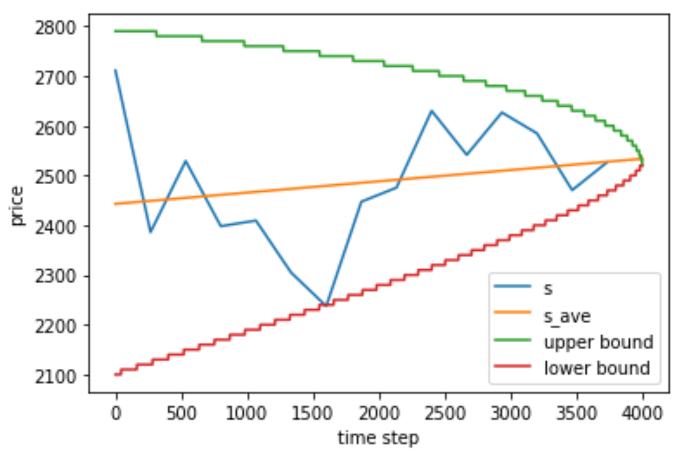
$$G(T,S) = H(T,S) - S - c \text{ for any } S \quad (23)$$

The associated free-boundary problem is:

$$\begin{cases} (\partial_t + L)G(t,S) = 0 & S > S_l \\ G(t,S) = H(S) - S - c & S \leq S_l \\ G'(t,S) - H'(t,S) = 1 & S < S_l \end{cases} \quad (24)$$

We solved equations (18) and (22) through the explicit scheme of finite difference method. And in order to better test the result, we used the same length of data in training and testing periods. Figure 6 shows the two boundaries in the training dataset.

Figure 6. Optimal bands in training period

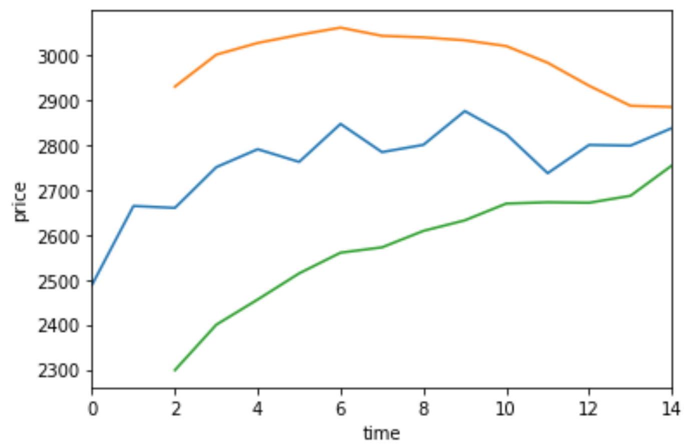


Using the two boundaries for S, we can calculate the associated optimal bands for Y using:

$$\begin{aligned} Y_l &= S_l - S^{ave} \\ Y_u &= S_u - S^{ave} \end{aligned} \quad (25)$$

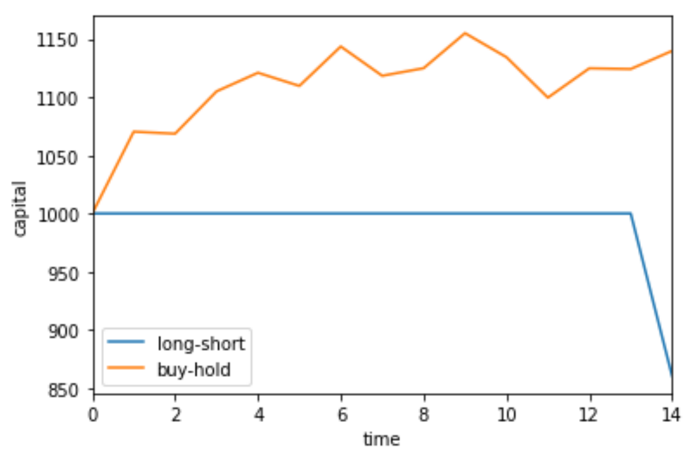
And we can calculate the similar bands for S in the testing period using rolling means of prices with 3 days from equation (25). Unfortunately, during the testing period we chose, the stock prices never hit the boundaries. We think this is because the price in the testing period is less volatile than the training period. We calculated the standard deviation of both periods. In training data, it is $123.1547 while in testing data it is only $93.1143, which proves our hypothesis.

Figure 7. Optimal bands in testing period



We also compared it with the other two strategies like what we did in ad-hoc bands strategy. Our result showed that the buy-hold strategy would earn the highest profit.

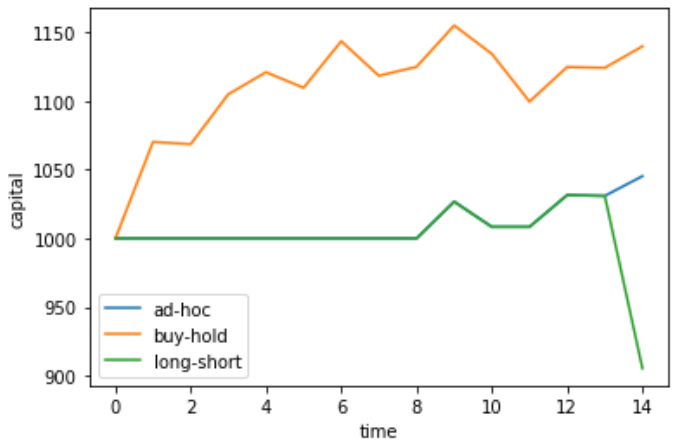
Figure 8. Long-short strategy and buy-hold strategy in testing period



Delay in execution

In the real world, execution (buy and sell) is sometimes delayed by 1day, typically when trading mutual funds. In this part, we consider the situation where we delay our transaction by 1 day.

Figure 9. Execution delayed 1 day of three strategies in testing period



Compared to figure 5, they are very similar to each other, but the one-day-delay strategy's payoff is slightly less than the former one.

Extension

In this section, we change the training and testing period to see if we can get a better result. To do this, we chose a time window where both testing and training data have a downward trend.

Figure 10. Optimal Bands in Training Period

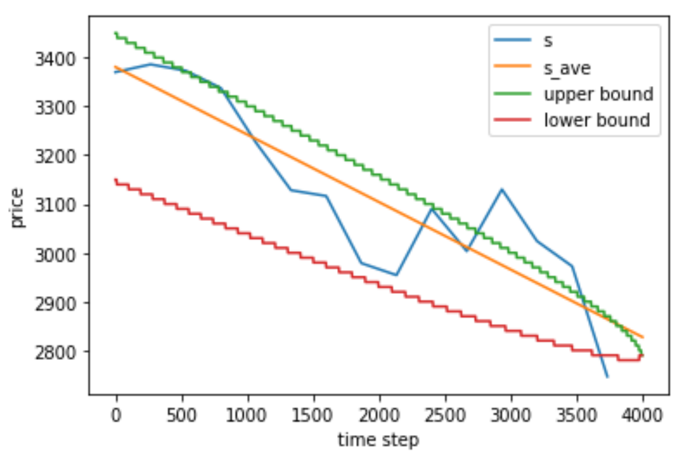


Figure 11. Optimal Bands in Testing Period

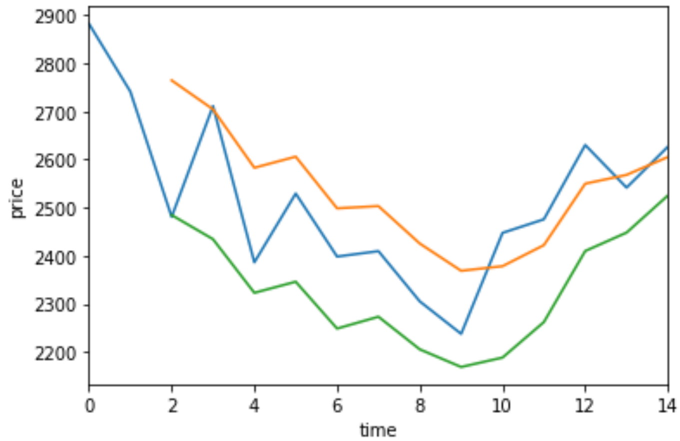
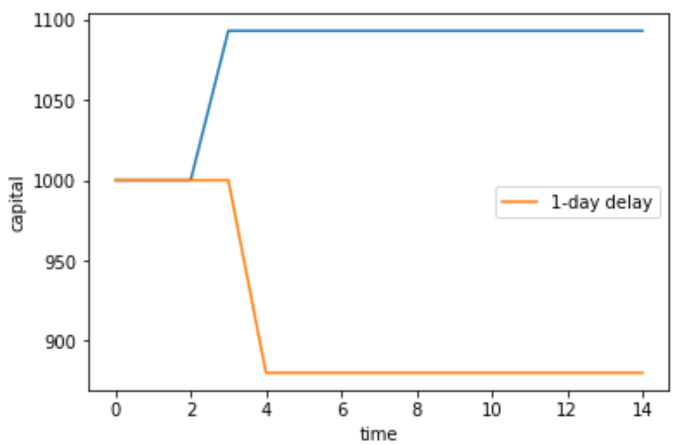


Figure 12. Compare payoff in testing period with 1 day delay case



In the above example, we buy at day 2 when stock price hits the lower bound and sell it at day 3 when price reaches the upper bound. The total payoff of the testing period is $92.8713. However, if we delay the transaction by one day, we would suffer a loss by $119.8406. We only trade once because this is a very short trading period.

Conclusion

From our analysis, we see the P&L of every strategy depends on the market trend and how volatile the market is. And it also depends on the correlation between the data from our training set and the test set.

To talk about the difference between the buy-and-hold, long-short and the ad-hoc band strategy, we need to divide the market into four situations. When the market trend is going down with a big volatility, investors would better off while using the long-short strategy. When the market trend is going down but the volatility is small, then investors would be better off if they use either a long-short or ad-hoc band strategy. If the market has an upward trend, and the volatility is large, then investors should use an ad-hoc band strategy. At last, if the market trend is up sloping and the volatility is small, then investors should use the buy-hold strategy.

In order to choose whether to delay the execution by one day or not depends on the volatility of the market and the mean-reverting level of the price deviation. When the mean-reverting level is relatively high, people should not delay their execution by one day. Since a delayed trading may result in a missed quote. For instance, when the stock price goes below the lower bound, with a high mean-reverting speed, people may buy back the stock at a much higher price the next day. This may cause a huge loss for investors.

In general, through our analysis, we can see that the stock market is very risky and investors must be really cautious when entering the market.

Prove of work

We confirmed that each person in the team contributes equally for this project. Meihui wrote the first half of the code and Sihang wrote the second half of the code.