Trading in a volatile time

Due to the uncertainty introduced by the coronavirus, US equity market has entered a period of extreme volatile time after nearly 10 years of bull market. Equity market indice dropped significantly. However, there were big gain of the market due to long term investors bought under-priced stocks, Fed interventions, or expectation of government rescue package. This project aims to construct a trading strategy making use of the volatile market environment.

Data: You are provided a time series data of SP500 index from Feb. 20, 2020 until Mar. 24, 2020 (https://finance.yahoo.com/quote/%5EGSPC/history/). Before the due date of the project, you can download new data from the previous link to train and test your trading model.

Outline of the project: The following is an outline of what you need to do with your teammates for this project. Some of the description is intentionally vogue. You are encouraged to go beyond of the following outline. Interesting extensions will be awarded with additional marks.

1. Data preparation and illustration
   Calculate the log return of SP500 index using the "adj close" price. Draw the histogram of the log return, fit to a normal distribution, compare the tail of the fitted distribution with the empirical distribution.

2. AR(1) model
   a) Split the data into a portfolio formation period and a portfolio testing period. You can download more data during your project.
   b) In the portfolio formation period, fit the SP500 price as a linear function of time, call this linear function $\mathrm{S}^{\mathrm{ave}}(\mathrm{t})$ and think it as the linear trend of SP500 in the portfolio formation period.
   c) In the portfolio formation period, calculate $Y(t) = S(t) - \mathrm{S}^{\mathrm{ave}}(t)$, where S(t) is the SP500 price at time t. You can think Y(t) is the deviation from the SP500 trend. Fit a AR(1) model on $\Delta\mathrm{Y}(\mathrm{t}) = \mathrm{Y}(\mathrm{t})\text{-}\mathrm{Y}(\mathrm{t}\text{-}1)$ and see if there is a significant negative auto-correlation of lag 1.
   d) Extract a continuous time mean reverting model for Y using the AR(1) model for $\Delta\mathrm{Y}(\mathrm{t})$. Use the sample volatility of Y(t) in the portfolio formation period as the volatility parameter in your model.

3. Trading strategy using ad-hoc bands
   Design trading strategies by choose several ad-hoc bands using the volatility of Y(t) in the portfolio formation period. Call the lower bound Y_{l} and the upper bound Y_{u}. Test these trading strategies using data in the portfolio testing period. In the testing period, you can calculate $\mathrm{S}^{\mathrm{ave}}(\mathrm{t})$ in a rolling window (for example, average of SP500 prices in the last three days). When S(t) reaches $\mathrm{S}^{\mathrm{ave}}(\mathrm{t}) + \mathrm{Y}_{\mathrm{l}}$, buy SP500; when $\mathrm{S}^{\mathrm{ave}}(\mathrm{t}) + \mathrm{Y}_{\mathrm{u}}$ is reached, sell SP500.

The initial capital is assumed to be \$1,000. For different choice of ad-hoc bands, Calculate the cumulative P&L of your strategy at the end of the portfolio testing period. When the end of the portfolio testing period is reached, you are forced to close your position. Compare the performance of your strategies and also against the buy and hold strategy in the same period of time using mean, variance and Sharpe ratio of P&Ls.

For the best ad-hoc bands that you identify, short \$1000 value of SP500 (say X units of SP500) at the beginning of the portfolio testing period and use the \$1000 to construct your strategy, buy back X units of SP500 at the end of the portfolio testing period. What is the performance of your long-short strategy?

4. Trading strategy using optimal bands
   a) In the portfolio formation period, consider $S(t) = Y(t) + S^{ave}(t)$ as a model for the SP500 price. Here $S^{ave}(t)$ comes from 2 (b) and Y(t) is obtained from 2 (d). Take the end of the portfolio formation period as the investment horizon. Assume the discounting factor is zero. Write down the free boundary equations for the exiting and entering problems. Both these free boundary equations for functions with respect to t and S(t).
   b) Solve these two free boundary PDEs using finite difference methods (this is similar to American option pricing problem on a finite horizon).
   c) Find the optimal entering boundary $S_l(t)$ and the optimal exiting boundary $S_u(t)$. Since we work with a finite horizon problem, both boundaries are time dependent. Using these two boundaries, we can calculate boundaries for Y(t) via $Y_l(t) = S_l(t) - S^{ave}(t)$ and $Y_u(t) = S_u(t) - S^{ave}(t)$.
   d) In the portfolio testing period, calculate $S^{ave}(t)$ in a rolling window as in #3. When S(t) reaches $Y_l(t) + S^{ave}(t)$ buy SP500 and when S(t) reaches $Y_d(t) + S^{ave}(t)$ sell SP500. Form a long-short strategy like #3, what is the performance of this strategy? P&L is calculated by the end of the portfolio testing period.

5. Delay in execution
   For the strategies constructed in #3 and #4, if execution (buy and sell) is delayed by 1 day (typical case when trading mutual funds), what is the performance of these long-short strategies?

Note: In the portfolio formation period, we "look ahead" to calculate $S^{ave}(t)$. It would be better to calculate $S^{ave}(t)$ in a rolling window. However, this would complicate #4, because we need to first find the dynamic of $S^{ave}(t)$ and write down free boundary equations which depend on two states $S^{ave}(t)$ and Y(t).

Group: Form groups on your own with 3 people in each group. A group of 2 people or less will be compensated slightly in marking.

Report: A report summarizing your findings is needed to be submitted. A report should contain following components:

1. An executive summary on what you have found.

2. A more detailed description of your methods and results.
3. A short paragraph confirming that each person in the team contribute equally for this project. It is **required** that each person in a group to write part of codes.

The report is limited to 15 A4 pages with fonts at least 11.

Submission: Deadline May 1st (Friday) 12:00pm. Submit a PDF file of the report and a zip file containing all R codes and data to Questromtool. Only one person each group needs to submit to Questromtool.

Marking: Due to impact of the coronavirus, school recommended to adjust the evaluation of each course. As a result, the project is adjusted to **50% of the final mark**. If the report confirms that each member contributes equally, everyone in the same team will get the same mark.

Warning:

- Group discussion and collaboration should be done remotely online! Keep social distancing!
- This is extreme volatile time the market is dangerous. Please do not try your strategies using your real money!