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«О проверке полиномиальности функций по модулю
степени простого числа»

Выпускная квалификационная работа

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1 Введение

В наше время функций k -значных логик применяются, например в криптографии. В частности, теория о полиномиальности k -значных функций используется для улучшения алгоритма гомоморфного шифрования [1].

Функция k -значной логики называется полиномиальной функцией, если ее можно представить полиномом по модулю k . В общем случае для произвольного k установлено, что при простом k все функции k -значной логики представимы полиномами по модулю k . При составном k , полиномы образуют собственный замкнутый подкласс $Pol_k \subseteq P_k$, не являющийся предполным в P_k [2]. Другими словами при составном k найдутся функции которые невозможно представить полиномом по модулю k . Более того разные полиномы могут представлять одну функцию, что влечет с собой не однозначность. Для одноместных функций предложены канонические виды полиномов при $k = p^m$, где p — простое число, $m \geq 1$ [3]. При помощи этих видов посчитано количество одноместных полиномиальных функций в P_k [3].

Для функций многих переменных при составных k , предложен канонический вид полиномиальных функций [4]. Этот канонический вид полиномов обобщает [3] на функций многих переменных. В [5] установлен ряд критериев полиномиальности одноместных и двуместных функций при $k = p^n$. В [6] предложен алгоритм полиномиальности для функций одной переменной при составных k . В [7] предложен критерий полиномиальности функций для функций многих переменных при $k = p^n$ и произвольного k . В настоящей работе, разработаны алгоритмы проверки полиномиальности одноместных функций при $k = p^n$, и построение полинома, реализующего данную функцию. Первый алгоритм опирается на [5], а второй алгоритм опирается на [6]. Оценивается сложность алгоритмов. Результат работы алгоритмов представлены в виде таблиц и графиков, где оцениваются разные параметры вывода программы. Для качественного анализа алгоритма и однозначности входных параметров используется теория канонического вида полиномов. Данный вид позволяет значительно оптимизировать анализ алгоритма.

2 Основные определения

Пусть $k \geq 2$, $E_k = \{0, 1, \dots, k-1\}$. Функция $f : E_k \rightarrow E_k$, называется одноместной функцией k -значной логики. Множество всех одноместных k -значных функций обозначим $P_k^{(1)}$. Пусть $N = \{0, 1, 2, \dots\}$ — множество натуральных чисел с нулем. Мономом называется: $\underbrace{x \cdot \dots \cdot x}_s$, если $s > 0$, или константа 1, если $s = 0$. Полиномом называется: $\sum_{i=1}^l c_i x^{s_i}$, где $c_i \in E_k$, x^{s_i} — различные мономы, $i = 0, 1, \dots, l$, $l \geq 1$, либо константа 0. Если $f(x)$ — полином по модулю k , то его длиной $l(f)$ называем число слагаемых с ненулевыми коэффициентами, и его степенью $d(f)$ называем максимальную степень его слагаемых с ненулевыми коэффициентами. Каждый полином будем рассматривать как полином по модулю k , приводя коэффициенты по модулю k , а также рассматривая операции сложения и умножения по модулю k . Если в каком-то полиноме все коэффициенты равны нулю, то это пустой полином, который представляет функцию k -значной логики, тождественно равную нулю. Константу 0 будем называть пустым полиномом. Функция k -значной логики называется полиномиальной, если ее можно представить полиномом по модулю k . Множество всех полиномиальных функций k -значной логики обозначим Pol_k . Равенство $P_k = Pol_k$ верно тогда и только тогда, когда k простое число [2].

3 Постановка задачи

1. Изучить и реализовать алгоритм проверки полиномиальности по модулю степени простого числа, предложенный в литературе.
2. Программно выполнить алгоритм для различных значений степени простого числа p и различных простых чисел в основании. Зафиксировать результаты работы программы и оформить их в виде таблиц. Провести анализ быстродействия и других характеристик алгоритма.
3. Изучить и реализовать другой предложенный алгоритм проверки полиномиальности по модулю простого числа. Провести сравнительный анализ двух алгоритмов по их быстродействию, эффективности использования памяти и другим характеристикам.

4 Полученные результаты

4.1 Вспомогательные определения

Определение 1. Если $t \in N$, то $t!$ обозначает факториал числа t , т.е. $t! = t \cdot (t-1) \dots 1$ при $s \geq 1$ и $0! = 1$.

Определение 2. Если $t, i \in N$, где $t \geq 1$, то C_t^i означает биномиальный коэффициент из t по i , т.е. $C_s^i = \frac{s!}{i!(s-i)!}$.

4.2 Алгоритм 1

Укажем основной результат [5].

Теорема 1. Функций $f(x) \in Pol_k$ тогда и только тогда, когда для любых $r \in E_{p^n}$

$$\sum_{s=0}^r (-1)^{r-s} C_r^s f(s) \equiv 0 \pmod{p^{\nu(r)}}$$

где $\nu(t) = \min\{n, \mu(t)\}$, $\mu(t)$ — показатель наибольшей степени p , делящей $t!$.

Следствие 1. Из теоремы 1, следует что если функция $f(x) \in Pol_{p^n}^{(1)}$, то его полином при $k = p^n$ можно представить в виде $F(x) = \sum_{j=0}^N \frac{1}{j!} \Delta^j f(0) x(x-1) \dots (x-j+1)$.

Теоремы 1 и следствие 1 является основой для алгоритма 1 проверки полиномиальности одноместных функций при $k = p^n$.

Алгоритм 1. Проверка полиномиальности функций из $P_{p^n}^{(1)}$, где p — простое число. На вход подается число p — простое число, число $n \geq 1$ и функция $f(x) \in P_{p^n}^{(1)}$.

На выходе получаем «да» и полином представляющий данную функцию при $k = p^n$, если $f(x) \in Pol_{p^n}^{(1)}$, либо получаем «нет» если $f(x) \notin Pol_{p^n}^{(1)}$.

Описание алгоритма:

1. Положим $k = p^n$.
2. Создаем список P из k элементов, где будем хранить коэффициенты полиномов;
3. Цикл для всех $r = 0, 1, \dots, k-1$ выполним 2.1-2.5:

3.1. положим $\nu(r) = \min\{n, \mu(r)\}$;

3.2. положим $s_r = \sum_{s=0}^r (-1)^{r-s} C_r^s f(s)$;

3.3. если $s_r \neq 0 \pmod{p^{\nu(r)}}$, то алгоритм останавливается и выдает $f(x) \notin Pol_k^{(1)}$.

3.4. создаем список a ;

3.5. положим $a_0 = 1$;

3.6. цикл для всех $i = 1, \dots, r - 1$ выполним 3.6.1-3.6.2:

3.6.1. добавляем в конец списка a нулевой элемент;

3.6.2. цикл для всех $j = i, \dots, 1$ положим $a_j = a_j - i \cdot a_{j-1}$;

3.7. если $r > 0$, то добавляем в конец списка a нулевой элемент;

3.8. создаем список z из $k - l(a)$ элементов, где $l(a)$ длина списка a и заполняем весь список нулевыми элементами;

3.9. обновляем список a , выполняя операцию конкатенаций списков z и a ;

3.10. положим $P_r = \frac{s_r}{j!} \cdot a$

4. алгоритм останавливается и выдает $f(x) \in Pol_k^{(1)}$ и $f(x) = \sum_{r=0}^{k-1} x^{k-1-r} \sum_{i=0}^{k-1} P_{ir}$.

4.3 Алгоритм 2

Укажем результаты из [6].

В [6] получено следующий алгоритм.

Алгоритм 2. Проверка полиномиальности функций из $P_{p^m}^{(1)}$, где p — простое число.

Вход: число $m \in \mathbb{N}$, $m \geq 1$, функция $f \in P_{p^m}^{(1)}$.

Выход: «да» и представление $f(x) = \sum_{s=0}^{s_p(m)} a_s x^s$, если $f \in Pol_{p^m}^{(1)}$, и «нет», если $f \notin Pol_{p^m}^{(1)}$.

Описание алгоритма.

1. Положим $f_{s_p(m)}(x) := f(x)$.

2. Положим $t_m := s_p(m) + 1$.

3. Цикл: для всех $t = s_p(m), s_p(m) - 1, \dots, 1$ выполним 3.1-3.5:

3.1. $d_t := (\Delta^{(t)} f_t)(0) = \sum_{i=0}^t (-1)^i C_t^i f_t(t-i) = \sum_{i=0}^t (-1)^i C_t^i \left(f(t-i) - \sum_{j=t+1}^{t_m-1} a_j (t-i)^j \right) \in E_{p^m}$;

3.2. рассмотрим в кольце Z_p^m уравнение $d_t = t! \cdot z$ (1) относительно неизвестной z ;

- 3.3. если уравнение (1) не имеет решений в кольце Z_{p^m} , то алгоритм останавливается и выдает $f \notin Pol_{p^m}^{(1)}$;
- 3.4. если уравнение (4) имеет решения в кольце Z_{p^m} , то найдем такое его единственное решение $a_t \in E_{p^m}$, что $a_t < p^{m-c_{p,m}(t)}$;
- 3.5. положим $f_{t-1}(x) := f_t(x) - a_t \cdot x^t$.
4. Цикл: для всех $b \in E_{p^m}$ выполним 4.1-4.2:
- 4.1. положим $c_b := f(b) - \sum_{s=1}^{s_p(m)} a_s b^s \in E_{p^m}$;
- 4.2. если $c_b \neq f(0)$, то алгоритм останавливается и выдает $f \notin Pol_{p^m}^{(1)}$;
5. Положим $a_0 := f(0) \in E_{p^m}$, алгоритм останавливается и выдает $f \in Pol_{p^m}^{(1)}$ и $f(x) = \sum_{s=0}^{s_p(m)} a_s x^s$. Окончание описание алгоритма.

4.4 Входные параметры

4.4.1 Составные модули

Для анализа алгоритма полиномиальности функций в P_k , рассмотрены следующие составные модули $k = p^n$:

1. $k_1 = 2^2$
2. $k_2 = 2^3$
3. $k_3 = 2^4$
4. $k_4 = 2^5$
5. $k_5 = 3^2$
6. $k_6 = 3^3$

4.4.2 Канонические виды полиномов

Канонические виды полиномов:

1. $k_1 : ax^3 + bx^2 + cx + d$, где $a, b \in E_2, c, d \in E_4$ - всего 2^6 функции.
2. $k_2 : ax^3 + bx^2 + cx + d$, где $a, b \in E_4, c, d \in E_8$ - всего 2^{10} функции.
3. $k_3 : ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, где $a, b \in E_2, c, d \in E_8, e, f \in E_{16}$ - всего 2^{16} функции.
4. $k_4 : ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + h$, где $a, b \in E_2, c, d \in E_4, e, f \in E_{16}, g, h \in E_{32}$ - всего 2^{24} функции.
5. $k_5 : ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, где $a, b, c \in E_3, d, e, f \in E_9$ - всего 3^{12} функции.
6. $k_6 : ax^8 + bx^7 + cx^6 + dx^5 + ex^4 + fx^3 + gx^2 + hx + i$, где $a, b, c \in E_3, d, e, f \in E_9, g, h, i \in E_{27}$ - всего 3^{39} функции.

4.4.3 Список функций на вход программы

Аттрибуты.

Таблицы списков функций имеют следующие атрибуты: *name*, *function*.

name - название функций;

function - вектор значений функций, т.е. $\text{function} = [f(0), f(1), \dots, f(p^n - 1)]$;

1. Список функций для $p=2$, $n=2$.

Таблица 1: Polynomial

name	function
<i>F1</i>	[0, 3, 0, 3]
<i>F2</i>	[1, 1, 3, 1]
<i>F3</i>	[0, 1, 2, 1]
<i>F4</i>	[0, 2, 0, 2]
<i>F5</i>	[1, 0, 3, 0]
<i>F6</i>	[0, 0, 2, 2]
<i>F7</i>	[0, 0, 2, 0]
<i>F8</i>	[0, 3, 2, 3]
<i>F9</i>	[0, 0, 2, 0]
<i>F10</i>	[1, 2, 3, 0]
<i>F11</i>	[1, 2, 3, 2]
<i>F12</i>	[0, 2, 0, 2]
<i>F13</i>	[0, 2, 0, 0]
<i>F14</i>	[0, 2, 0, 0]
<i>F15</i>	[0, 3, 0, 1]
<i>F16</i>	[0, 1, 0, 3]
<i>F17</i>	[1, 1, 3, 1]
<i>F18</i>	[0, 0, 0, 2]
<i>F19</i>	[0, 3, 2, 3]
<i>F20</i>	[0, 0, 2, 2]
<i>F21</i>	[1, 2, 3, 0]
<i>F22</i>	[1, 2, 3, 0]
<i>F23</i>	[0, 1, 0, 3]
<i>F24</i>	[0, 3, 0, 1]
<i>F25</i>	[1, 0, 3, 2]
<i>F26</i>	[1, 0, 3, 2]
<i>F27</i>	[1, 0, 1, 2]
<i>F28</i>	[0, 1, 0, 3]
<i>F29</i>	[1, 0, 3, 0]
<i>F30</i>	[1, 2, 3, 0]

Таблица 2: Unpolynomial

name	function
<i>F1</i>	[3, 0, 0, 3]
<i>F2</i>	[1, 0, 2, 3]
<i>F3</i>	[0, 0, 2, 3]
<i>F4</i>	[2, 3, 1, 2]
<i>F5</i>	[3, 3, 0, 2]
<i>F6</i>	[0, 0, 1, 1]
<i>F7</i>	[3, 0, 0, 1]
<i>F8</i>	[2, 3, 1, 3]
<i>F9</i>	[1, 3, 2, 1]
<i>F10</i>	[1, 1, 2, 2]
<i>F11</i>	[2, 1, 3, 1]
<i>F12</i>	[1, 3, 0, 0]
<i>F13</i>	[0, 2, 2, 3]
<i>F14</i>	[0, 3, 3, 2]
<i>F15</i>	[2, 0, 1, 1]
<i>F16</i>	[0, 2, 1, 0]
<i>F17</i>	[3, 3, 2, 1]
<i>F18</i>	[0, 1, 1, 3]
<i>F19</i>	[3, 0, 0, 2]
<i>F20</i>	[0, 2, 1, 1]
<i>F21</i>	[2, 2, 3, 2]
<i>F22</i>	[1, 1, 3, 2]
<i>F23</i>	[1, 1, 3, 0]
<i>F24</i>	[0, 3, 0, 2]
<i>F25</i>	[3, 2, 1, 1]
<i>F26</i>	[2, 1, 3, 2]
<i>F27</i>	[3, 1, 1, 2]
<i>F28</i>	[2, 0, 2, 3]
<i>F29</i>	[2, 0, 1, 2]
<i>F30</i>	[1, 2, 1, 1]

2. Список функций для $p=2$, $n=3$.

Таблица 3: Polynomial

name	function
$F1$	[2, 4, 6, 0, 2, 4, 6, 0]
$F2$	[2, 2, 6, 6, 2, 2, 6, 6]
$F3$	[3, 1, 5, 7, 7, 5, 1, 3]
$F4$	[1, 2, 5, 6, 1, 2, 5, 6]
$F5$	[1, 4, 1, 0, 1, 4, 1, 0]
$F6$	[3, 1, 7, 7, 3, 5, 7, 3]
$F7$	[0, 0, 0, 4, 0, 0, 0, 4]
$F8$	[3, 7, 3, 3, 3, 7, 3, 3]
$F9$	[3, 2, 1, 6, 7, 2, 5, 6]
$F10$	[3, 7, 7, 5, 3, 3, 7, 1]
$F11$	[0, 1, 6, 1, 4, 1, 2, 1]
$F12$	[2, 5, 0, 7, 6, 1, 4, 3]
$F13$	[1, 4, 3, 4, 5, 4, 7, 4]
$F14$	[2, 1, 0, 3, 6, 5, 4, 7]
$F15$	[2, 1, 2, 3, 2, 5, 2, 7]
$F16$	[0, 6, 6, 2, 4, 6, 2, 2]
$F17$	[3, 5, 3, 7, 3, 1, 3, 3]
$F18$	[0, 7, 0, 7, 0, 7, 0, 7]
$F19$	[1, 3, 1, 7, 1, 3, 1, 7]
$F20$	[3, 7, 1, 7, 7, 7, 5, 7]
$F21$	[0, 1, 4, 5, 0, 1, 4, 5]
$F22$	[2, 6, 2, 0, 2, 2, 2, 4]
$F23$	[1, 0, 3, 2, 5, 4, 7, 6]
$F24$	[2, 0, 0, 2, 6, 4, 4, 6]
$F25$	[2, 6, 4, 6, 6, 6, 0, 6]
$F26$	[2, 4, 0, 0, 6, 4, 4, 0]
$F27$	[3, 2, 3, 4, 3, 6, 3, 0]
$F28$	[2, 6, 4, 4, 6, 2, 0, 0]
$F29$	[3, 5, 7, 5, 3, 5, 7, 5]
$F30$	[3, 7, 5, 7, 7, 7, 1, 7]

Таблица 4: Unpolynomial

name	function
$F1$	[0, 2, 3, 5, 4, 1, 0, 4]
$F2$	[6, 2, 0, 1, 0, 7, 1, 7]
$F3$	[5, 4, 3, 3, 1, 0, 6, 0]
$F4$	[5, 5, 4, 1, 4, 3, 7, 5]
$F5$	[3, 7, 0, 3, 0, 6, 1, 6]
$F6$	[7, 2, 6, 5, 5, 1, 5, 5]
$F7$	[4, 6, 6, 7, 3, 3, 1, 6]
$F8$	[1, 0, 2, 3, 6, 6, 5, 7]
$F9$	[0, 1, 4, 1, 2, 6, 1, 0]
$F10$	[5, 7, 0, 5, 3, 6, 6, 2]
$F11$	[5, 6, 3, 5, 1, 4, 0, 2]
$F12$	[1, 7, 6, 4, 5, 2, 3, 0]
$F13$	[1, 2, 5, 5, 5, 6, 5, 5]
$F14$	[3, 4, 7, 6, 4, 6, 6, 6]
$F15$	[5, 3, 0, 0, 7, 1, 1, 1]
$F16$	[0, 3, 7, 3, 2, 6, 6, 0]
$F17$	[3, 0, 0, 1, 3, 0, 7, 0]
$F18$	[2, 0, 4, 6, 4, 2, 4, 1]
$F19$	[7, 4, 6, 6, 6, 2, 3, 5]
$F20$	[6, 3, 4, 1, 2, 7, 6, 1]
$F21$	[3, 0, 0, 5, 4, 4, 6, 7]
$F22$	[0, 6, 7, 2, 1, 5, 3, 4]
$F23$	[7, 2, 4, 0, 7, 4, 0, 4]
$F24$	[1, 7, 0, 3, 1, 5, 3, 0]
$F25$	[3, 4, 1, 2, 2, 7, 2, 4]
$F26$	[6, 2, 5, 6, 2, 0, 3, 5]
$F27$	[0, 4, 4, 5, 2, 1, 0, 0]
$F28$	[1, 3, 6, 0, 1, 1, 6, 2]
$F29$	[3, 5, 7, 5, 0, 2, 3, 3]
$F30$	[7, 1, 6, 6, 0, 2, 4, 1]

3. Список функций для $p=2$, $n=4$.

Таблица 5: Polynomial

name	function
$F1$	[0, 10, 14, 14, 4, 2, 2, 6, 8, 10, 6, 14, 12, 2, 10, 6]
$F2$	[1, 15, 13, 1, 1, 3, 13, 5, 1, 7, 13, 9, 1, 11, 13, 13]
$F3$	[1, 6, 13, 8, 1, 2, 13, 4, 1, 14, 13, 0, 1, 10, 13, 12]
$F4$	[0, 4, 6, 0, 4, 12, 10, 8, 8, 4, 14, 0, 12, 12, 2, 8]
$F5$	[0, 12, 0, 0, 0, 12, 0, 0, 0, 12, 0, 0, 0, 12, 0, 0]
$F6$	[0, 3, 8, 3, 0, 11, 8, 11, 0, 3, 8, 3, 0, 11, 8, 11]
$F7$	[1, 12, 11, 6, 5, 8, 15, 2, 9, 4, 3, 14, 13, 0, 7, 10]
$F8$	[1, 11, 9, 3, 1, 11, 9, 3, 1, 11, 9, 3, 1, 11, 9, 3]
$F9$	[0, 10, 12, 12, 0, 14, 12, 0, 0, 2, 12, 4, 0, 6, 12, 8]
$F10$	[1, 14, 9, 2, 1, 14, 9, 2, 1, 14, 9, 2, 1, 14, 9, 2]
$F11$	[1, 11, 7, 3, 5, 3, 11, 11, 9, 11, 15, 3, 13, 3, 3, 11]
$F12$	[0, 9, 10, 1, 4, 1, 14, 9, 8, 9, 2, 1, 12, 1, 6, 9]
$F13$	[1, 0, 13, 2, 1, 4, 13, 6, 1, 8, 13, 10, 1, 12, 13, 14]
$F14$	[0, 1, 4, 1, 0, 9, 4, 9, 0, 1, 4, 1, 0, 9, 4, 9]
$F15$	[1, 15, 1, 5, 1, 11, 1, 1, 1, 7, 1, 13, 1, 3, 1, 9]
$F16$	[0, 2, 4, 0, 0, 6, 4, 4, 0, 10, 4, 8, 0, 14, 4, 12]
$F17$	[0, 3, 8, 5, 0, 7, 8, 9, 0, 11, 8, 13, 0, 15, 8, 1]
$F18$	[0, 8, 8, 4, 0, 0, 8, 12, 0, 8, 8, 4, 0, 0, 8, 12]
$F19$	[1, 1, 9, 13, 1, 9, 9, 5, 1, 1, 9, 13, 1, 9, 9, 5]
$F20$	[0, 4, 12, 8, 0, 4, 12, 8, 0, 4, 12, 8, 0, 4, 12, 8]
$F21$	[0, 13, 2, 1, 4, 5, 6, 9, 8, 13, 10, 1, 12, 5, 14, 9]
$F22$	[0, 13, 14, 11, 4, 1, 2, 15, 8, 5, 6, 3, 12, 9, 10, 7]
$F23$	[0, 14, 8, 10, 0, 14, 8, 10, 0, 14, 8, 10, 0, 14, 8, 10]
$F24$	[0, 14, 10, 0, 4, 2, 14, 4, 8, 6, 2, 8, 12, 10, 6, 12]
$F25$	[0, 11, 14, 7, 4, 11, 2, 7, 8, 11, 6, 7, 12, 11, 10, 7]
$F26$	[1, 12, 13, 14, 1, 0, 13, 2, 1, 4, 13, 6, 1, 8, 13, 10]
$F27$	[1, 11, 1, 1, 1, 15, 1, 5, 1, 3, 1, 9, 1, 7, 1, 13]
$F28$	[0, 4, 8, 10, 0, 8, 8, 14, 0, 12, 8, 2, 0, 0, 8, 6]
$F29$	[0, 5, 2, 1, 4, 13, 6, 9, 8, 5, 10, 1, 12, 13, 14, 9]
$F30$	[0, 11, 12, 11, 0, 3, 12, 3, 0, 11, 12, 11, 0, 3, 12, 3]

Таблица 6: Unpolynomial

name	function
$F1$	[10, 7, 2, 6, 5, 5, 0, 1, 3, 3, 12, 5, 10, 6, 8, 2]
$F2$	[14, 2, 2, 8, 5, 13, 7, 2, 7, 7, 2, 4, 4, 8, 10, 13]
$F3$	[10, 5, 3, 14, 9, 15, 2, 8, 15, 10, 10, 9, 7, 1, 5, 9]
$F4$	[2, 2, 3, 4, 9, 6, 2, 1, 1, 1, 9, 8, 5, 0, 13, 12]
$F5$	[0, 5, 7, 15, 7, 13, 12, 9, 11, 1, 3, 1, 12, 6, 0, 7]
$F6$	[8, 1, 11, 15, 6, 14, 8, 3, 10, 15, 11, 13, 0, 11, 4, 11]
$F7$	[10, 3, 8, 4, 11, 12, 14, 13, 0, 4, 9, 1, 5, 8, 8, 9]
$F8$	[10, 10, 14, 14, 13, 14, 3, 11, 5, 2, 4, 0, 12, 3, 15, 12]
$F9$	[15, 3, 15, 12, 6, 2, 3, 2, 11, 15, 1, 5, 10, 13, 9, 0]
$F10$	[5, 6, 12, 1, 14, 3, 15, 14, 5, 9, 13, 10, 2, 14, 14, 11]
$F11$	[6, 7, 1, 14, 7, 4, 9, 0, 0, 0, 3, 0, 4, 13, 1, 0]
$F12$	[3, 13, 6, 6, 4, 15, 1, 6, 6, 3, 14, 10, 1, 9, 9, 13]
$F13$	[10, 14, 7, 7, 2, 3, 4, 14, 13, 7, 6, 14, 10, 13, 11, 9]
$F14$	[13, 11, 13, 14, 14, 13, 15, 12, 8, 10, 14, 6, 5, 1, 10, 1]
$F15$	[0, 11, 4, 7, 15, 2, 12, 14, 1, 6, 10, 2, 11, 6, 5, 13]
$F16$	[0, 4, 1, 15, 3, 0, 13, 15, 3, 11, 10, 9, 4, 3, 3, 6]
$F17$	[1, 13, 4, 7, 14, 8, 13, 4, 12, 2, 1, 13, 4, 12, 14, 10]
$F18$	[3, 14, 4, 1, 7, 0, 15, 8, 8, 13, 2, 15, 2, 4, 5, 3]
$F19$	[0, 6, 1, 1, 5, 5, 1, 7, 15, 6, 6, 7, 15, 3, 8, 7]
$F20$	[2, 4, 6, 15, 10, 0, 8, 15, 9, 15, 14, 11, 4, 3, 10, 6]
$F21$	[9, 1, 8, 0, 3, 8, 13, 7, 12, 15, 3, 5, 9, 13, 14, 4]
$F22$	[15, 6, 4, 12, 15, 2, 2, 1, 1, 2, 5, 0, 14, 7, 6, 6]
$F23$	[13, 2, 13, 3, 14, 6, 5, 4, 11, 3, 15, 6, 15, 5, 7, 2]
$F24$	[13, 9, 7, 7, 14, 9, 13, 8, 5, 0, 3, 7, 5, 12, 1, 7]
$F25$	[4, 10, 13, 15, 13, 12, 13, 6, 4, 14, 10, 5, 9, 6, 10, 15]
$F26$	[5, 7, 13, 9, 9, 0, 3, 5, 10, 12, 11, 10, 9, 5, 11, 0]
$F27$	[10, 9, 3, 13, 4, 0, 10, 1, 13, 14, 3, 12, 11, 7, 7, 14]
$F28$	[1, 8, 5, 9, 10, 11, 8, 3, 15, 0, 10, 13, 6, 6, 10, 0]
$F29$	[2, 13, 13, 5, 15, 2, 8, 8, 7, 4, 14, 1, 1, 5, 4, 13]
$F30$	[14, 9, 5, 0, 8, 1, 6, 0, 14, 5, 0, 7, 5, 4, 13, 2]

4. Список функций для $p=2$, $n=5$.

Таблица 7: Polynomial

name	function
F_1	[0, 10, 12, 10, 16, 26, 12, 26, 0, 10, 12, 10, 16, 26, 12, 26, 0, 10, 12, 10, 16, 26, 12, 26, 0, 10, 12, 10, 16, 26, 12, 26]
F_2	[1, 28, 29, 14, 17, 8, 13, 10, 1, 20, 29, 6, 17, 0, 13, 2, 1, 12, 29, 30, 17, 24, 13, 26, 1, 4, 29, 22, 17, 16, 13, 18]
F_3	[0, 1, 14, 27, 20, 13, 2, 23, 8, 25, 22, 19, 28, 5, 10, 15, 16, 17, 30, 11, 4, 29, 18, 7, 24, 9, 6, 3, 12, 21, 26, 31]
F_4	[1, 15, 11, 23, 5, 23, 15, 15, 9, 31, 19, 7, 13, 7, 23, 31, 17, 15, 27, 23, 21, 23, 31, 15, 25, 31, 3, 7, 29, 7, 7, 31]
F_5	[1, 26, 15, 4, 21, 6, 19, 16, 9, 18, 23, 28, 29, 30, 27, 8, 17, 10, 31, 20, 5, 22, 3, 0, 25, 2, 7, 12, 13, 14, 11, 24]
F_6	[1, 14, 13, 30, 17, 14, 29, 14, 1, 14, 13, 30, 17, 14, 29, 14, 1, 14, 13, 30, 17, 14, 29, 14, 1, 14, 13, 30, 17, 14, 29, 14]
F_7	[0, 29, 14, 3, 20, 1, 2, 23, 8, 5, 22, 11, 28, 9, 10, 31, 16, 13, 30, 19, 4, 17, 18, 7, 24, 21, 6, 27, 12, 25, 26, 15]
F_8	[1, 27, 3, 13, 5, 31, 23, 17, 9, 3, 11, 21, 13, 7, 31, 25, 17, 11, 19, 29, 21, 15, 7, 1, 25, 19, 27, 5, 29, 23, 15, 9]
F_9	[0, 18, 26, 12, 4, 6, 14, 0, 8, 26, 2, 20, 12, 14, 22, 8, 16, 2, 10, 28, 20, 22, 30, 16, 24, 10, 18, 4, 28, 30, 6, 24]
F_{10}	[1, 2, 31, 6, 21, 2, 3, 22, 9, 2, 7, 6, 29, 2, 11, 22, 17, 2, 15, 6, 5, 2, 19, 22, 25, 2, 23, 6, 13, 2, 27, 22]
F_{11}	[0, 6, 10, 18, 4, 30, 14, 26, 8, 22, 18, 2, 12, 14, 22, 10, 16, 6, 26, 18, 20, 30, 30, 26, 24, 22, 2, 2, 28, 14, 6, 10]
F_{12}	[1, 20, 19, 22, 5, 24, 7, 10, 9, 28, 27, 30, 13, 0, 15, 18, 17, 4, 3, 6, 21, 8, 23, 26, 25, 12, 11, 14, 29, 16, 31, 2]
F_{13}	[0, 12, 22, 20, 20, 12, 26, 20, 8, 12, 30, 20, 28, 12, 2, 20, 16, 12, 6, 20, 4, 12, 10, 20, 24, 12, 14, 20, 12, 12, 18, 20]
F_{14}	[0, 24, 16, 6, 0, 20, 16, 2, 0, 16, 16, 30, 0, 12, 16, 26, 0, 8, 16, 22, 0, 4, 16, 18, 0, 0, 16, 14, 0, 28, 16, 10]
F_{15}	[0, 20, 18, 28, 4, 20, 22, 12, 8, 20, 26, 28, 12, 20, 30, 12, 16, 20, 2, 28, 20, 20, 6, 12, 24, 20, 10, 28, 28, 20, 14, 12]
F_{16}	[1, 7, 15, 21, 21, 3, 19, 1, 9, 31, 23, 13, 29, 27, 27, 25, 17, 23, 31, 5, 5, 19, 3, 17, 25, 15, 7, 29, 13, 11, 11, 9]
F_{17}	[1, 9, 31, 1, 21, 1, 19, 9, 9, 25, 7, 17, 29, 17, 27, 25, 17, 9, 15, 1, 5, 1, 3, 9, 25, 25, 23, 17, 13, 17, 11, 25]
F_{18}	[1, 28, 19, 22, 5, 8, 7, 18, 9, 20, 27, 14, 13, 0, 15, 10, 17, 12, 3, 6, 21, 24, 23, 2, 25, 4, 11, 30, 29, 16, 31, 26]
F_{19}	[1, 21, 17, 27, 1, 17, 1, 7, 1, 13, 17, 19, 1, 9, 1, 31, 1, 5, 17, 11, 1, 1, 1, 23, 1, 29, 17, 3, 1, 25, 1, 15]
F_{20}	[0, 5, 8, 29, 0, 21, 8, 13, 0, 5, 8, 29, 0, 21, 8, 13, 0, 5, 8, 29, 0, 21, 8, 13, 0, 5, 8, 29, 0, 21, 8, 13]
F_{21}	[0, 22, 20, 28, 16, 26, 4, 0, 0, 30, 20, 4, 16, 2, 4, 8, 0, 6, 20, 12, 16, 10, 4, 16, 0, 14, 20, 20, 16, 18, 4, 24]
F_{22}	[0, 16, 26, 2, 4, 28, 14, 30, 8, 8, 2, 26, 12, 20, 22, 22, 16, 0, 10, 18, 20, 12, 30, 14, 24, 24, 18, 10, 28, 4, 6, 6]
F_{23}	[0, 7, 2, 19, 4, 23, 22, 19, 8, 7, 10, 19, 12, 23, 30, 19, 16, 7, 18, 19, 20, 23, 6, 19, 24, 7, 26, 19, 28, 23, 14, 19]
F_{24}	[0, 24, 28, 22, 16, 12, 28, 26, 0, 0, 28, 30, 16, 20, 28, 2, 0, 8, 28, 6, 16, 28, 28, 10, 0, 16, 28, 14, 16, 4, 28, 18]
F_{25}	[1, 8, 7, 16, 21, 0, 11, 8, 9, 24, 15, 0, 29, 16, 19, 24, 17, 8, 23, 16, 5, 0, 27, 8, 25, 24, 31, 0, 13, 16, 3, 24]
F_{26}	[0, 4, 8, 30, 0, 24, 24, 18, 0, 12, 8, 6, 0, 0, 24, 26, 0, 20, 8, 14, 0, 8, 24, 2, 0, 28, 8, 22, 0, 16, 24, 10]
F_{27}	[1, 4, 17, 8, 1, 4, 17, 24, 1, 4, 17, 8, 1, 4, 17, 24, 1, 4, 17, 8, 1, 4, 17, 24, 1, 4, 17, 8, 1, 4, 17, 24]
F_{28}	[0, 18, 2, 22, 4, 26, 22, 30, 8, 2, 10, 6, 12, 10, 30, 14, 16, 18, 18, 22, 20, 26, 6, 30, 24, 2, 26, 6, 28, 10, 14, 14]
F_{29}	[0, 29, 22, 7, 20, 17, 26, 27, 8, 5, 30, 15, 28, 25, 2, 3, 16, 13, 6, 23, 4, 1, 10, 11, 24, 21, 14, 31, 12, 9, 18, 19]
F_{30}	[0, 30, 6, 26, 20, 14, 10, 26, 8, 30, 14, 26, 28, 14, 18, 26, 16, 30, 22, 26, 4, 14, 26, 26, 24, 30, 30, 26, 12, 14, 2, 26]

Таблица 8: Unpolynomial

name	function
F_1	[27, 19, 8, 26, 18, 5, 20, 22, 26, 26, 22, 30, 15, 27, 24, 10, 9, 14, 15, 17, 16, 10, 1, 10, 7, 9, 29, 15, 17, 10, 21, 27]
F_2	[25, 20, 19, 18, 3, 11, 16, 6, 19, 16, 2, 23, 2, 12, 12, 24, 28, 24, 20, 22, 23, 15, 20, 17, 27, 8, 10, 21, 13, 24, 7, 31]
F_3	[7, 6, 16, 16, 28, 12, 12, 0, 31, 4, 17, 9, 18, 29, 12, 7, 7, 14, 16, 29, 3, 30, 12, 3, 25, 20, 26, 17, 24, 2, 22, 29]
F_4	[1, 1, 16, 7, 26, 30, 20, 31, 17, 28, 0, 18, 20, 24, 8, 8, 1, 16, 22, 6, 9, 28, 11, 24, 24, 21, 22, 27, 29, 5, 2, 17]
F_5	[15, 11, 4, 0, 12, 18, 20, 25, 18, 24, 17, 11, 16, 5, 17, 20, 5, 16, 29, 27, 27, 29, 2, 23, 24, 19, 22, 30, 7, 15, 12, 19]
F_6	[17, 24, 15, 3, 18, 19, 18, 6, 27, 5, 31, 14, 2, 24, 20, 23, 3, 30, 27, 23, 23, 6, 25, 25, 26, 19, 4, 3, 17, 13, 29, 23]
F_7	[27, 26, 12, 21, 16, 3, 6, 21, 14, 8, 17, 14, 16, 20, 28, 24, 6, 20, 5, 25, 13, 16, 20, 18, 23, 12, 7, 3, 0, 26, 7, 22]
F_8	[28, 2, 19, 0, 16, 18, 17, 31, 12, 2, 9, 23, 4, 20, 28, 5, 23, 6, 1, 15, 13, 5, 15, 1, 2, 21, 13, 12, 31, 26, 4, 12]
F_9	[9, 6, 6, 13, 12, 9, 24, 17, 26, 8, 10, 31, 19, 26, 14, 6, 9, 30, 19, 28, 28, 10, 2, 26, 16, 30, 17, 28, 10, 7, 5, 26]
F_{10}	[11, 14, 5, 12, 14, 12, 10, 31, 5, 12, 3, 17, 30, 5, 17, 17, 26, 8, 25, 13, 8, 4, 26, 22, 31, 0, 23, 7, 13, 26, 7, 3]
F_{11}	[19, 14, 31, 19, 26, 21, 6, 28, 25, 19, 4, 11, 24, 12, 31, 16, 29, 28, 30, 20, 17, 22, 26, 7, 25, 28, 6, 28, 25, 8, 5, 11]
F_{12}	[6, 28, 8, 12, 24, 8, 7, 29, 14, 26, 3, 31, 5, 3, 26, 25, 28, 20, 28, 29, 10, 29, 26, 14, 0, 17, 7, 29, 23, 5, 8, 15]
F_{13}	[31, 21, 15, 25, 17, 9, 8, 21, 9, 24, 18, 27, 9, 23, 25, 23, 1, 25, 3, 3, 4, 27, 13, 11, 29, 4, 30, 18, 28, 4, 10, 29]
F_{14}	[20, 9, 23, 22, 8, 28, 27, 11, 20, 5, 4, 2, 19, 3, 16, 27, 16, 6, 2, 16, 5, 9, 16, 10, 2, 24, 4, 0, 3, 31, 4, 2]
F_{15}	[3, 26, 22, 27, 11, 19, 27, 3, 29, 6, 5, 10, 10, 16, 22, 4, 21, 21, 24, 8, 26, 28, 0, 30, 6, 29, 28, 28, 2, 12, 15, 19]
F_{16}	[12, 22, 10, 1, 20, 16, 13, 2, 1, 27, 9, 4, 22, 20, 2, 24, 20, 0, 10, 24, 18, 27, 1, 24, 3, 3, 3, 16, 7, 2, 3, 4]
F_{17}	[11, 26, 18, 21, 6, 26, 3, 17, 28, 1, 3, 20, 30, 18, 12, 9, 26, 10, 10, 19, 16, 27, 16, 31, 29, 20, 11, 18, 11, 5, 30, 18]
F_{18}	[13, 19, 18, 19, 10, 11, 12, 6, 17, 25, 5, 28, 19, 26, 3, 20, 3, 15, 22, 18, 16, 11, 28, 17, 10, 7, 21, 22, 18, 30, 26, 17]
F_{19}	[19, 22, 18, 22, 17, 12, 15, 1, 9, 12, 19, 4, 22, 0, 28, 8, 13, 14, 3, 19, 14, 11, 4, 13, 2, 12, 27, 0, 7, 8, 2, 5]
F_{20}	[24, 17, 21, 25, 11, 1, 12, 8, 29, 24, 16, 7, 15, 15, 16, 21, 23, 26, 24, 18, 5, 3, 18, 23, 24, 19, 24, 21, 23, 12, 8, 12]
F_{21}	[28, 24, 12, 28, 23, 17, 12, 4, 14, 21, 29, 24, 14, 31, 23, 20, 13, 29, 13, 12, 19, 5, 11, 25, 26, 15, 29, 13, 4, 23, 28, 4]
F_{22}	[29, 23, 10, 8, 10, 17, 17, 1, 25, 28, 1, 20, 23, 10, 9, 13, 5, 29, 11, 30, 23, 22, 20, 1, 31, 13, 20, 21, 25, 12, 23, 23]
F_{23}	[22, 26, 22, 23, 1, 10, 6, 12, 22, 2, 16, 29, 14, 17, 9, 1, 12, 23, 31, 26, 17, 2, 5, 29, 23, 18, 23, 31, 21, 13, 1, 3]
F_{24}	[29, 4, 12, 29, 26, 1, 22, 10, 5, 14, 17, 19, 28, 4, 12, 30, 16, 1, 19, 0, 30, 28, 20, 12, 11, 12, 31, 6, 7, 13, 30, 23]
F_{25}	[25, 4, 18, 29, 6, 16, 20, 0, 1, 17, 18, 10, 10, 29, 25, 19, 9, 19, 20, 14, 6, 11, 27, 4, 4, 8, 18, 9, 29, 9, 28, 7]
F_{26}	[25, 2, 18, 3, 13, 28, 13, 24, 14, 29, 10, 15, 16, 15, 17, 23, 19, 27, 17, 17, 6, 18, 29, 27, 3, 30, 0, 22, 14, 19, 15, 24]
F_{27}	[5, 2, 14, 14, 31, 17, 15, 2, 29, 12, 20, 10, 12, 19, 5, 21, 28, 21, 26, 5, 18, 10, 6, 16, 3, 17, 23, 25, 19, 7, 9, 7]
F_{28}	[23, 9, 22, 20, 13, 8, 16, 19, 20, 4, 10, 10, 5, 17, 1, 11, 12, 16, 1, 10, 21, 18, 1, 24, 1, 24, 28, 24, 24, 22, 23, 23]
F_{29}	[21, 26, 21, 2, 27, 5, 4, 19, 17, 27, 25, 26, 1, 20, 7, 12, 26, 23, 2, 14, 28, 17, 21, 0, 14, 5, 16, 31, 25, 28, 31, 24]
F_{30}	[7, 10, 10, 29, 30, 19, 23, 11, 28, 0, 12, 31, 20, 7, 9, 19, 14, 31, 8, 10, 18, 6, 9, 0, 9, 19, 15, 9, 12, 21, 29, 3]

5. Список функций для $p=3$, $n=2$.

Таблица 9: Polynomial

name	function
$F1$	[0, 1, 7, 3, 7, 7, 6, 4, 7]
$F2$	[0, 1, 3, 0, 1, 0, 0, 1, 6]
$F3$	[1, 2, 7, 7, 5, 7, 4, 8, 7]
$F4$	[1, 4, 2, 1, 4, 8, 1, 4, 5]
$F5$	[0, 6, 4, 0, 3, 1, 0, 0, 7]
$F6$	[2, 4, 4, 5, 4, 7, 8, 4, 1]
$F7$	[1, 4, 5, 7, 7, 8, 4, 1, 2]
$F8$	[2, 0, 6, 5, 6, 0, 8, 3, 3]
$F9$	[2, 5, 6, 2, 2, 6, 2, 8, 6]
$F10$	[1, 5, 5, 7, 2, 5, 4, 8, 5]
$F11$	[1, 7, 3, 4, 1, 0, 7, 4, 6]
$F12$	[1, 7, 2, 4, 7, 2, 7, 7, 2]
$F13$	[1, 4, 1, 7, 1, 4, 4, 7, 7]
$F14$	[0, 7, 2, 0, 1, 2, 0, 4, 2]
$F15$	[2, 3, 5, 5, 6, 5, 8, 0, 5]
$F16$	[1, 7, 3, 7, 4, 6, 4, 1, 0]
$F17$	[1, 4, 2, 7, 1, 8, 4, 7, 5]
$F18$	[1, 3, 1, 4, 3, 4, 7, 3, 7]
$F19$	[1, 4, 2, 1, 1, 2, 1, 7, 2]
$F20$	[1, 7, 4, 4, 1, 1, 7, 4, 7]
$F21$	[0, 0, 1, 6, 0, 1, 3, 0, 1]
$F22$	[0, 2, 8, 6, 2, 8, 3, 2, 8]
$F23$	[2, 3, 0, 5, 6, 0, 8, 0, 0]
$F24$	[1, 3, 4, 4, 3, 4, 7, 3, 4]
$F25$	[1, 2, 1, 7, 2, 4, 4, 2, 7]
$F26$	[1, 0, 7, 1, 0, 4, 1, 0, 1]
$F27$	[2, 8, 3, 5, 8, 6, 8, 8, 0]
$F28$	[2, 2, 7, 2, 5, 4, 2, 8, 1]
$F29$	[2, 5, 6, 5, 2, 0, 8, 8, 3]
$F30$	[2, 1, 5, 2, 4, 2, 2, 7, 8]

Таблица 10: Unpolynomial

name	function
$F1$	[5, 0, 7, 0, 3, 2, 0, 4, 6]
$F2$	[5, 7, 0, 5, 3, 3, 7, 1, 5]
$F3$	[4, 0, 7, 8, 7, 2, 6, 2, 5]
$F4$	[6, 0, 4, 0, 5, 3, 6, 6, 0]
$F5$	[6, 5, 0, 3, 4, 0, 5, 3, 1]
$F6$	[5, 7, 1, 4, 0, 5, 7, 1, 4]
$F7$	[0, 8, 3, 1, 5, 7, 6, 4, 0]
$F8$	[5, 7, 7, 7, 3, 1, 3, 1, 2]
$F9$	[2, 0, 0, 0, 5, 0, 4, 7, 2]
$F10$	[8, 6, 1, 4, 8, 1, 7, 4, 1]
$F11$	[0, 1, 4, 4, 3, 5, 1, 4, 3]
$F12$	[7, 4, 4, 2, 5, 0, 3, 5, 3]
$F13$	[0, 4, 6, 7, 5, 1, 3, 4, 7]
$F14$	[1, 7, 4, 3, 8, 7, 2, 4, 0]
$F15$	[8, 4, 1, 3, 8, 3, 1, 7, 8]
$F16$	[5, 4, 1, 8, 2, 6, 3, 1, 0]
$F17$	[3, 4, 7, 3, 0, 2, 2, 8, 2]
$F18$	[0, 2, 8, 6, 2, 8, 7, 1, 4]
$F19$	[2, 2, 4, 3, 1, 7, 6, 7, 1]
$F20$	[4, 8, 1, 2, 6, 4, 8, 1, 0]
$F21$	[0, 8, 7, 5, 3, 2, 3, 0, 8]
$F22$	[1, 4, 7, 3, 7, 4, 4, 1, 1]
$F23$	[3, 2, 3, 0, 2, 6, 4, 0, 7]
$F24$	[2, 8, 2, 6, 4, 4, 2, 4, 2]
$F25$	[5, 4, 7, 1, 0, 0, 1, 7, 6]
$F26$	[1, 2, 1, 3, 3, 6, 2, 3, 6]
$F27$	[5, 3, 3, 7, 8, 0, 4, 1, 5]
$F28$	[5, 7, 5, 4, 0, 4, 8, 0, 0]
$F29$	[4, 4, 6, 5, 4, 1, 4, 6, 4]
$F30$	[5, 0, 6, 8, 1, 8, 0, 8, 6]

5. Список функций для $p=3$, $n=3$.

Таблица 11: Polynomial

name	function
$F1$	[0, 9, 6, 12, 24, 18, 15, 21, 3, 9, 0, 15, 21, 15, 0, 24, 12, 12, 18, 18, 24, 3, 6, 9, 6, 3, 21]
$F2$	[1, 26, 19, 16, 23, 10, 22, 2, 10, 19, 17, 19, 7, 14, 10, 13, 20, 10, 10, 8, 19, 25, 5, 10, 4, 11, 10]
$F3$	[1, 24, 6, 16, 18, 21, 22, 21, 9, 19, 6, 24, 7, 0, 12, 13, 3, 0, 10, 15, 15, 25, 9, 3, 4, 12, 18]
$F4$	[1, 24, 7, 7, 21, 22, 13, 0, 10, 19, 15, 25, 25, 12, 13, 4, 18, 1, 10, 6, 16, 16, 3, 4, 22, 9, 19]
$F5$	[1, 13, 10, 10, 19, 10, 10, 7, 1, 1, 4, 10, 10, 10, 10, 25, 1, 1, 22, 10, 10, 1, 10, 10, 16, 1]
$F6$	[1, 13, 23, 13, 16, 23, 16, 10, 5, 10, 22, 23, 22, 25, 23, 25, 19, 5, 19, 4, 23, 4, 7, 23, 7, 1, 5]
$F7$	[1, 10, 1, 10, 25, 10, 10, 13, 10, 1, 1, 1, 10, 16, 10, 10, 4, 10, 1, 19, 1, 10, 7, 10, 10, 22, 10]
$F8$	[1, 26, 9, 25, 20, 18, 4, 14, 0, 19, 8, 9, 16, 2, 18, 22, 23, 0, 10, 17, 9, 7, 11, 18, 13, 5, 0]
$F9$	[2, 10, 9, 26, 19, 21, 5, 1, 24, 20, 10, 18, 17, 19, 3, 23, 1, 6, 11, 10, 0, 8, 19, 12, 14, 1, 15]
$F10$	[0, 7, 6, 15, 22, 12, 21, 1, 18, 18, 25, 24, 6, 13, 3, 12, 19, 9, 9, 16, 15, 24, 4, 21, 3, 10, 0]
$F11$	[1, 6, 25, 16, 24, 13, 22, 6, 19, 19, 6, 16, 7, 24, 4, 13, 6, 10, 10, 6, 7, 25, 24, 22, 4, 6, 1]
$F12$	[2, 1, 25, 11, 25, 7, 11, 13, 16, 2, 19, 25, 11, 16, 7, 11, 4, 16, 2, 10, 25, 11, 7, 7, 11, 22, 16]
$F13$	[1, 17, 13, 10, 2, 16, 10, 5, 1, 1, 26, 22, 10, 11, 25, 10, 14, 10, 1, 8, 4, 10, 20, 7, 10, 23, 19]
$F14$	[2, 9, 15, 14, 24, 9, 17, 21, 21, 11, 0, 24, 23, 15, 18, 26, 12, 3, 20, 18, 6, 5, 6, 0, 8, 3, 12]
$F15$	[1, 0, 15, 25, 12, 12, 4, 15, 18, 19, 9, 6, 16, 21, 3, 22, 24, 9, 10, 18, 24, 7, 3, 21, 13, 6, 0]
$F16$	[0, 11, 16, 18, 26, 4, 18, 5, 19, 0, 2, 7, 18, 17, 22, 18, 23, 10, 0, 20, 25, 18, 8, 13, 18, 14, 1]
$F17$	[1, 12, 16, 7, 15, 16, 13, 9, 7, 19, 21, 16, 25, 24, 16, 4, 18, 7, 10, 3, 16, 16, 6, 16, 22, 0, 7]
$F18$	[0, 23, 18, 24, 5, 6, 3, 14, 3, 18, 23, 9, 15, 5, 24, 21, 14, 21, 9, 23, 0, 6, 5, 15, 12, 14, 12]
$F19$	[2, 23, 1, 8, 11, 13, 14, 17, 7, 20, 14, 10, 26, 2, 22, 5, 8, 16, 11, 5, 19, 17, 20, 4, 23, 26, 25]
$F20$	[0, 4, 22, 24, 13, 13, 3, 22, 13, 18, 4, 22, 15, 13, 13, 21, 22, 13, 9, 4, 22, 6, 13, 13, 12, 22, 13]
$F21$	[1, 14, 14, 19, 17, 14, 19, 11, 23, 1, 23, 14, 19, 26, 14, 19, 20, 23, 1, 5, 14, 19, 8, 14, 19, 2, 23]
$F22$	[0, 3, 14, 12, 18, 20, 15, 15, 26, 9, 21, 5, 21, 9, 11, 24, 6, 17, 18, 12, 23, 3, 0, 2, 6, 24, 8]
$F23$	[0, 10, 6, 21, 19, 12, 24, 10, 18, 9, 10, 24, 3, 19, 3, 6, 10, 9, 18, 10, 15, 12, 19, 21, 15, 10, 0]
$F24$	[0, 24, 23, 6, 21, 20, 12, 0, 17, 18, 15, 14, 24, 12, 11, 3, 18, 8, 9, 6, 5, 15, 3, 2, 21, 9, 26]
$F25$	[2, 21, 9, 14, 9, 18, 17, 24, 0, 11, 12, 9, 23, 0, 18, 26, 15, 0, 20, 3, 9, 5, 18, 18, 8, 6, 0]
$F26$	[1, 9, 25, 1, 3, 1, 1, 24, 13, 1, 18, 7, 1, 12, 10, 1, 6, 22, 1, 0, 16, 1, 21, 19, 1, 15, 4]
$F27$	[0, 12, 8, 12, 24, 5, 15, 0, 2, 9, 21, 26, 21, 6, 23, 24, 9, 20, 18, 3, 17, 3, 15, 14, 6, 18, 11]
$F28$	[1, 11, 11, 4, 11, 2, 7, 20, 20, 10, 11, 11, 13, 11, 2, 16, 20, 20, 19, 11, 11, 22, 11, 2, 25, 20, 20]
$F29$	[2, 19, 10, 14, 16, 22, 17, 22, 16, 11, 10, 19, 23, 7, 4, 26, 13, 25, 20, 1, 1, 5, 25, 13, 8, 4, 7]
$F30$	[0, 4, 22, 18, 4, 25, 18, 13, 19, 0, 4, 4, 18, 4, 7, 18, 13, 1, 0, 4, 13, 18, 4, 16, 18, 13, 10]

Таблица 12: Unpolynomial

name	function
$F1$	[20, 16, 16, 16, 5, 1, 10, 5, 5, 3, 8, 12, 26, 1, 4, 10, 18, 23, 20, 20, 6, 20, 26, 13, 10, 10, 3]
$F2$	[12, 19, 19, 17, 22, 20, 0, 19, 26, 14, 12, 20, 16, 23, 18, 3, 15, 23, 6, 1, 13, 3, 11, 15, 21, 23, 12]
$F3$	[18, 13, 3, 21, 6, 24, 12, 26, 9, 19, 12, 24, 20, 11, 15, 3, 11, 13, 21, 23, 20, 14, 0, 5, 23, 15, 21]
$F4$	[4, 24, 1, 24, 21, 0, 10, 17, 13, 15, 20, 17, 4, 1, 7, 24, 3, 8, 4, 10, 13, 22, 11, 8, 19, 26, 7]
$F5$	[14, 18, 20, 25, 8, 24, 16, 16, 1, 9, 2, 8, 7, 15, 24, 3, 7, 8, 23, 4, 6, 3, 6, 10, 7, 21, 11]
$F6$	[5, 26, 0, 17, 26, 1, 1, 12, 13, 20, 18, 25, 8, 16, 20, 7, 26, 25, 3, 5, 4, 3, 18, 18, 13, 25, 24]
$F7$	[11, 7, 9, 14, 0, 20, 8, 14, 4, 17, 18, 6, 16, 12, 15, 26, 8, 17, 15, 14, 14, 8, 24, 1, 4, 23, 14]
$F8$	[8, 9, 19, 13, 22, 4, 3, 14, 17, 15, 20, 8, 9, 16, 24, 22, 9, 0, 11, 25, 23, 23, 7, 18, 12, 10, 10]
$F9$	[0, 17, 4, 1, 10, 4, 2, 2, 17, 21, 11, 2, 14, 2, 25, 13, 14, 9, 1, 8, 12, 5, 14, 5, 25, 16, 0]
$F10$	[19, 14, 25, 6, 15, 15, 3, 24, 7, 4, 21, 2, 10, 19, 2, 13, 23, 1, 11, 16, 15, 5, 21, 17, 19, 23, 22]
$F11$	[12, 19, 16, 25, 11, 14, 12, 25, 20, 15, 25, 13, 25, 9, 2, 5, 2, 20, 22, 22, 3, 4, 3, 15, 17, 9, 1]
$F12$	[20, 0, 8, 12, 14, 11, 13, 18, 8, 0, 2, 4, 9, 5, 4, 7, 15, 19, 10, 23, 5, 24, 9, 22, 4, 8, 9]
$F13$	[9, 26, 21, 3, 13, 26, 3, 7, 26, 1, 26, 6, 5, 11, 17, 19, 19, 2, 12, 23, 15, 1, 9, 7, 8, 8, 2]
$F14$	[20, 19, 9, 15, 19, 23, 2, 21, 17, 19, 15, 26, 11, 2, 1, 23, 8, 10, 18, 25, 5, 17, 12, 23, 19, 3, 11]
$F15$	[19, 10, 8, 17, 15, 15, 13, 5, 3, 26, 5, 26, 20, 10, 10, 20, 3, 11, 26, 8, 1, 16, 5, 2, 23, 15, 23]
$F16$	[18, 24, 1, 19, 5, 15, 12, 19, 0, 2, 2, 16, 24, 25, 13, 16, 14, 22, 11, 14, 26, 12, 17, 12, 17, 12, 12]
$F17$	[16, 5, 4, 14, 22, 8, 2, 17, 26, 12, 21, 19, 10, 7, 4, 13, 23, 18, 8, 18, 17, 14, 26, 22, 15, 8, 24]
$F18$	[15, 3, 18, 15, 22, 23, 0, 22, 15, 21, 12, 22, 14, 8, 15, 16, 11, 25, 0, 12, 13, 17, 19, 13, 7, 15, 23]
$F19$	[0, 2, 3, 9, 18, 1, 24, 4, 20, 21, 25, 9, 7, 13, 17, 23, 9, 23, 8, 12, 17, 1, 5, 7, 21, 16, 0]
$F20$	[17, 8, 3, 12, 22, 16, 26, 22, 9, 23, 20, 1, 5, 22, 6, 13, 12, 18, 3, 17, 9, 1, 13, 19, 7, 23, 5]
$F21$	[19, 11, 3, 5, 25, 22, 5, 17, 8, 17, 5, 9, 2, 2, 3, 7, 4, 18, 16, 15, 24, 1, 6, 8, 1, 5, 6]
$F22$	[6, 17, 4, 12, 12, 13, 25, 24, 5, 15, 6, 4, 3, 0, 16, 25, 16, 1, 20, 2, 19, 22, 18, 14, 21, 3, 2]
$F23$	[22, 2, 13, 19, 10, 18, 18, 2, 20, 7, 21, 0, 1, 12, 12, 24, 8, 23, 26, 16, 6, 23, 14, 2, 19, 4, 3]
$F24$	[16, 15, 5, 20, 11, 24, 0, 25, 1, 18, 21, 10, 1, 10, 11, 5, 23, 4, 21, 18, 4, 20, 14, 19, 2, 3, 17]
$F25$	[22, 6, 23, 11, 13, 11, 4, 22, 7, 17, 19, 19, 13, 20, 8, 19, 9, 21, 8, 17, 10, 13, 18, 13, 13, 15, 8]
$F26$	[4, 6, 1, 10, 17, 17, 1, 17, 3, 21, 11, 5, 13, 10, 5, 14, 26, 16, 14, 6, 16, 11, 14, 3, 7, 4, 6]
$F27$	[7, 0, 5, 1, 17, 0, 5, 4, 5, 2, 9, 23, 9, 17, 20, 23, 21, 7, 12, 23, 5, 18, 8, 6, 4, 16, 2]
$F28$	[1, 8, 4, 23, 4, 1, 17, 16, 2, 13, 20, 23, 22, 16, 25, 25, 12, 14, 17, 16, 26, 23, 10, 16, 25, 9, 10]
$F29$	[19, 15, 23, 18, 22, 13, 23, 2, 24, 20, 3, 17, 1, 25, 7, 7, 20, 10, 16, 15, 17, 14, 8, 3, 9, 23, 5]
$F30$	[24, 15, 26, 22, 4, 0, 18, 2, 22, 21, 3, 13, 13, 17, 20, 21, 16, 8, 3, 17, 1, 15, 24, 9, 9, 20, 23]

4.5 Результат работы программы в виде таблиц

Аттрибуты.

Таблицы результатов имеют следующие атрибуты: $F(x)$, $time_1$, $time_2$, $F(x) \in Pol$, $coef_{A1}$, $coef_{A2}$, $d_{A1}(F)$, $d_{A2}(F)$.

$F(x)$ - название функций;

$time_1$ - Время выполнения алгоритма 1;

$time_2$ - Время выполнения алгоритма 2;

$F(x) \in Pol$ - полиномиальность функций;

$coef_{A1}$ - Коэффициенты полинома алгоритма 1;

$coef_{A2}$ - Коэффициенты полинома алгоритма 2;

$d_{A1}(F)$ - Длина полинома алгоритма 1;

$d_{A2}(F)$ - Длина полинома алгоритма 2;

Программная реализация:

Язык программирования: Программа была написана на языке Python. Хранение функций и полиномов: Функции и полиномы хранились в памяти в виде списков (list), что позволяло легко манипулировать данными и проводить необходимые вычисления. Коэффициенты полиномов также хранились в виде списков, обеспечивая эффективный доступ и изменение данных.

Использованные библиотеки: Для работы с полиномами были использованы пользовательские модули PolynomialA1 и PolynomialA2, а для анализа данных и создания таблиц применялись библиотеки pandas и timeit.

Обработка данных: Реализована программа которая генерировала случайные функции. Созданы отдельные списки полиномиальных функций, и отдельные списки неполиномиальных функций. Реализованы программы на основе алгоритма 1 и алгоритма 2. Программы строили полиномы на основе входных параметров и проверяла их полиномиальность. Результаты работы каждого алгоритма сохранялись в таблицы и файлы формата LaTeX для дальнейшего анализа и представления.

Производительность и ресурсы: Анализ работы алгоритмов включал измерение времени выполнения и оценки использования ресурсов, что позволило выявить наиболее эффективный алгоритм в различных сценариях.

Таблица 13: $p = 2, n = 2$. Polynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.000034	0.000030	True	[0, 2, 3, 2]	[0, 2, 1, 0]	3	2
$F2$	0.000019	0.000020	True	[1, 1, 0, 3]	[1, 3, 0, 1]	3	3
$F3$	0.000016	0.000016	True	[0, 3, 1, 1]	[0, 3, 1, 1]	3	3
$F4$	0.000014	0.000014	True	[0, 0, 2, 0]	[0, 2, 0, 0]	1	1
$F5$	0.000015	0.000014	True	[1, 3, 3, 1]	[1, 1, 1, 1]	4	4
$F6$	0.000013	0.000014	True	[0, 3, 3, 2]	[0, 3, 1, 0]	3	2
$F7$	0.000013	0.000014	True	[0, 1, 0, 3]	[0, 3, 0, 1]	2	2
$F8$	0.000015	0.000014	True	[0, 3, 3, 1]	[0, 1, 1, 1]	3	3
$F9$	0.000013	0.000014	True	[0, 1, 0, 3]	[0, 3, 0, 1]	2	2
$F10$	0.000014	0.000013	True	[1, 1, 2, 2]	[1, 1, 0, 0]	4	2
$F11$	0.000013	0.000013	True	[1, 3, 1, 1]	[1, 3, 1, 1]	4	4
$F12$	0.000013	0.000013	True	[0, 0, 2, 0]	[0, 2, 0, 0]	1	1
$F13$	0.000015	0.000013	True	[0, 2, 3, 1]	[0, 0, 1, 1]	3	2
$F14$	0.000013	0.000013	True	[0, 2, 3, 1]	[0, 0, 1, 1]	3	2
$F15$	0.000014	0.000013	True	[0, 0, 0, 3]	[0, 2, 0, 1]	1	2
$F16$	0.000015	0.000013	True	[0, 0, 0, 1]	[0, 0, 0, 1]	1	1
$F17$	0.000014	0.000013	True	[1, 1, 0, 3]	[1, 3, 0, 1]	3	3
$F18$	0.000011	0.000013	True	[0, 2, 3, 3]	[0, 2, 1, 1]	3	3
$F19$	0.000013	0.000013	True	[0, 3, 3, 1]	[0, 1, 1, 1]	3	3
$F20$	0.000013	0.000013	True	[0, 3, 3, 2]	[0, 3, 1, 0]	3	2
$F21$	0.000013	0.000012	True	[1, 1, 2, 2]	[1, 1, 0, 0]	4	2
$F22$	0.000014	0.000014	True	[1, 1, 2, 2]	[1, 1, 0, 0]	4	2
$F23$	0.000014	0.000013	True	[0, 0, 0, 1]	[0, 0, 0, 1]	1	1
$F24$	0.000014	0.000013	True	[0, 0, 0, 3]	[0, 2, 0, 1]	1	2
$F25$	0.000013	0.000013	True	[1, 1, 2, 0]	[1, 3, 0, 0]	3	2
$F26$	0.000013	0.000013	True	[1, 1, 2, 0]	[1, 3, 0, 0]	3	2
$F27$	0.000015	0.000013	True	[1, 0, 2, 1]	[1, 2, 0, 1]	3	3
$F28$	0.000013	0.000013	True	[0, 0, 0, 1]	[0, 0, 0, 1]	1	1
$F29$	0.000014	0.000013	True	[1, 3, 3, 1]	[1, 1, 1, 1]	4	4
$F30$	0.000014	0.000013	True	[1, 1, 2, 2]	[1, 1, 0, 0]	4	2

Вывод по таблице:

1. Общее время выполнения A1: 0.0004356000863481313
2. Общее время выполнения A2: 0.00042109997593797743
3. Отношение время выполнения A1 и A2: 1.0344338903792518
4. Отношение длин полиномов A1 и A2: 1.1911764705882353

Таблица 14: $p = 2, n = 2$. Unpolynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.000013	0.000012	False	NaN	NaN	NaN	NaN
$F2$	0.000009	0.000010	False	NaN	NaN	NaN	NaN
$F3$	0.000011	0.000008	False	NaN	NaN	NaN	NaN
$F4$	0.000013	0.000012	False	NaN	NaN	NaN	NaN
$F5$	0.000008	0.000010	False	NaN	NaN	NaN	NaN
$F6$	0.000006	0.000010	False	NaN	NaN	NaN	NaN
$F7$	0.000008	0.000010	False	NaN	NaN	NaN	NaN
$F8$	0.000007	0.000007	False	NaN	NaN	NaN	NaN
$F9$	0.000007	0.000007	False	NaN	NaN	NaN	NaN
$F10$	0.000006	0.000010	False	NaN	NaN	NaN	NaN
$F11$	0.000009	0.000008	False	NaN	NaN	NaN	NaN
$F12$	0.000007	0.000010	False	NaN	NaN	NaN	NaN
$F13$	0.000012	0.000008	False	NaN	NaN	NaN	NaN
$F14$	0.000008	0.000011	False	NaN	NaN	NaN	NaN
$F15$	0.000007	0.000010	False	NaN	NaN	NaN	NaN
$F16$	0.000007	0.000007	False	NaN	NaN	NaN	NaN
$F17$	0.000007	0.000007	False	NaN	NaN	NaN	NaN
$F18$	0.000006	0.000007	False	NaN	NaN	NaN	NaN
$F19$	0.000007	0.000006	False	NaN	NaN	NaN	NaN
$F20$	0.000006	0.000009	False	NaN	NaN	NaN	NaN
$F21$	0.000008	0.000011	False	NaN	NaN	NaN	NaN
$F22$	0.000017	0.000007	False	NaN	NaN	NaN	NaN
$F23$	0.000010	0.000007	False	NaN	NaN	NaN	NaN
$F24$	0.000011	0.000007	False	NaN	NaN	NaN	NaN
$F25$	0.000012	0.000007	False	NaN	NaN	NaN	NaN
$F26$	0.000007	0.000009	False	NaN	NaN	NaN	NaN
$F27$	0.000011	0.000007	False	NaN	NaN	NaN	NaN
$F28$	0.000010	0.000007	False	NaN	NaN	NaN	NaN
$F29$	0.000006	0.000007	False	NaN	NaN	NaN	NaN
$F30$	0.000010	0.000007	False	NaN	NaN	NaN	NaN

Вывод по таблице:

1. Общее время выполнения A1: 0.0003108999808318913
2. Общее время выполнения A2: 0.0002933000505436212
3. Отношение время выполнения A1 и A2: 1.0600065709352906

Таблица 15: $p = 2, n = 3$. Polynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.000074	0.000032	True	[2, 4, 5, 7, 2, 7, 1, 0]	[2, 2, 0, 0]	7	2
$F2$	0.000056	0.000021	True	[2, 4, 5, 7, 0, 5, 5, 6]	[2, 6, 2, 0]	7	3
$F3$	0.000052	0.000018	True	[3, 1, 6, 5, 1, 1, 0, 0]	[3, 3, 3, 0]	6	3
$F4$	0.000054	0.000018	True	[1, 4, 1, 3, 3, 5, 7, 2]	[1, 4, 3, 2]	8	4
$F5$	0.000057	0.000017	True	[1, 0, 4, 0, 7, 1, 6, 1]	[1, 2, 1, 0]	6	3
$F6$	0.000057	0.000018	True	[3, 4, 5, 7, 4, 3, 6, 1]	[3, 0, 3, 3]	8	3
$F7$	0.000053	0.000017	True	[0, 0, 0, 4, 2, 7, 0, 3]	[0, 4, 2, 2]	4	3
$F8$	0.000058	0.000018	True	[3, 0, 0, 4, 2, 1, 0, 5]	[3, 0, 2, 2]	5	3
$F9$	0.000054	0.000017	True	[3, 5, 7, 1, 1, 6, 1, 2]	[3, 5, 1, 1]	8	4
$F10$	0.000046	0.000016	True	[3, 4, 1, 3, 5, 3, 7, 5]	[3, 0, 1, 3]	8	3
$F11$	0.000050	0.000016	True	[0, 5, 3, 0, 1, 5, 5, 6]	[0, 5, 1, 3]	6	3
$F12$	0.000050	0.000017	True	[2, 5, 3, 1, 0, 3, 3, 4]	[2, 7, 2, 2]	7	4
$F13$	0.000049	0.000017	True	[1, 1, 4, 0, 1, 5, 6, 2]	[1, 7, 3, 1]	7	4
$F14$	0.000048	0.000016	True	[2, 5, 3, 5, 4, 2, 7, 5]	[2, 3, 2, 2]	8	4
$F15$	0.000050	0.000017	True	[2, 0, 0, 4, 2, 3, 4, 2]	[2, 4, 2, 1]	6	4
$F16$	0.000052	0.000017	True	[0, 5, 7, 0, 6, 6, 7, 7]	[0, 3, 0, 3]	6	2
$F17$	0.000053	0.000017	True	[3, 0, 4, 4, 3, 4, 6, 5]	[3, 6, 1, 3]	7	4
$F18$	0.000052	0.000016	True	[0, 0, 4, 4, 5, 0, 6, 4]	[0, 2, 3, 2]	5	3
$F19$	0.000052	0.000016	True	[1, 0, 0, 4, 0, 7, 4, 3]	[1, 0, 0, 2]	5	2
$F20$	0.000050	0.000016	True	[3, 5, 3, 1, 6, 7, 3, 3]	[3, 3, 0, 1]	8	3
$F21$	0.000050	0.000017	True	[0, 4, 1, 3, 3, 5, 7, 2]	[0, 4, 3, 2]	7	3
$F22$	0.000053	0.000018	True	[2, 0, 4, 4, 5, 6, 2, 7]	[2, 6, 3, 3]	7	4
$F23$	0.000051	0.000016	True	[1, 1, 0, 0, 2, 0, 4, 0]	[1, 5, 2, 0]	4	3
$F24$	0.000050	0.000016	True	[2, 5, 7, 1, 1, 2, 5, 1]	[2, 5, 1, 0]	8	3
$F25$	0.000053	0.000016	True	[2, 1, 2, 1, 4, 7, 6, 7]	[2, 1, 0, 3]	8	3
$F26$	0.000053	0.000017	True	[2, 5, 3, 5, 6, 0, 7, 0]	[2, 7, 0, 3]	6	3
$F27$	0.000051	0.000016	True	[3, 0, 4, 4, 7, 6, 3, 7]	[3, 4, 2, 1]	7	4
$F28$	0.000051	0.000016	True	[2, 1, 6, 1, 7, 4, 0, 1]	[2, 3, 1, 0]	7	3
$F29$	0.000049	0.000017	True	[3, 4, 5, 3, 4, 6, 1, 3]	[3, 6, 2, 2]	8	4
$F30$	0.000054	0.000016	True	[3, 1, 2, 1, 4, 7, 6, 7]	[3, 1, 0, 3]	8	3

Вывод по таблице:

1. Общее время выполнения A1: 0.0015826998860575259
2. Общее время выполнения A2: 0.0005179000145290047
3. Отношение время выполнения A1 и A2: 3.055995060160184
4. Отношение длин полиномов A1 и A2: 2.082474226804124

Таблица 16: $p = 2, n = 3$. Unpolynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.000012	0.000014	False	NaN	NaN	NaN	NaN
$F2$	0.000016	0.000009	False	NaN	NaN	NaN	NaN
$F3$	0.000012	0.000009	False	NaN	NaN	NaN	NaN
$F4$	0.000007	0.000008	False	NaN	NaN	NaN	NaN
$F5$	0.000008	0.000008	False	NaN	NaN	NaN	NaN
$F6$	0.000009	0.000012	False	NaN	NaN	NaN	NaN
$F7$	0.000013	0.000008	False	NaN	NaN	NaN	NaN
$F8$	0.000008	0.000011	False	NaN	NaN	NaN	NaN
$F9$	0.000018	0.000014	False	NaN	NaN	NaN	NaN
$F10$	0.000008	0.000008	False	NaN	NaN	NaN	NaN
$F11$	0.000012	0.000008	False	NaN	NaN	NaN	NaN
$F12$	0.000008	0.000010	False	NaN	NaN	NaN	NaN
$F13$	0.000012	0.000008	False	NaN	NaN	NaN	NaN
$F14$	0.000020	0.000015	False	NaN	NaN	NaN	NaN
$F15$	0.000008	0.000011	False	NaN	NaN	NaN	NaN
$F16$	0.000007	0.000008	False	NaN	NaN	NaN	NaN
$F17$	0.000009	0.000010	False	NaN	NaN	NaN	NaN
$F18$	0.000018	0.000014	False	NaN	NaN	NaN	NaN
$F19$	0.000008	0.000008	False	NaN	NaN	NaN	NaN
$F20$	0.000037	0.000016	False	NaN	NaN	NaN	NaN
$F21$	0.000008	0.000011	False	NaN	NaN	NaN	NaN
$F22$	0.000007	0.000008	False	NaN	NaN	NaN	NaN
$F23$	0.000008	0.000008	False	NaN	NaN	NaN	NaN
$F24$	0.000008	0.000008	False	NaN	NaN	NaN	NaN
$F25$	0.000018	0.000014	False	NaN	NaN	NaN	NaN
$F26$	0.000008	0.000008	False	NaN	NaN	NaN	NaN
$F27$	0.000011	0.000008	False	NaN	NaN	NaN	NaN
$F28$	0.000007	0.000010	False	NaN	NaN	NaN	NaN
$F29$	0.000017	0.000015	False	NaN	NaN	NaN	NaN
$F30$	0.000007	0.000010	False	NaN	NaN	NaN	NaN

Вывод по таблице:

1. Общее время выполнения A1: 0.00035080002271570265
2. Общее время выполнения A2: 0.00031010000384412706
3. Отношение время выполнения A1 и A2: 1.1312480437505368

Таблица 17: $p = 2, n = 4$. Polynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.000298	0.000055	True	[0, 13, 5, 2, 9, 6, 9, 11, 0, 12, 14, 12, 5, 11, 6, 7]	[0, 13, 7, 5, 1, 0]	14	4
$F2$	0.000266	0.000037	True	[1, 4, 3, 1, 1, 1, 12, 9, 0, 13, 9, 7, 13, 10, 15, 12]	[1, 8, 5, 1, 0, 0]	15	4
$F3$	0.000264	0.000036	True	[1, 4, 3, 5, 5, 0, 4, 10, 5, 2, 3, 4, 4, 0, 0, 4]	[1, 8, 7, 4, 1, 1]	13	6
$F4$	0.000260	0.000035	True	[0, 13, 9, 7, 13, 2, 8, 4, 3, 11, 8, 2, 10, 4, 11, 11]	[0, 13, 5, 1, 1, 0]	15	4
$F5$	0.000264	0.000035	True	[0, 0, 0, 8, 12, 2, 15, 0, 3, 9, 0, 8, 5, 14, 3, 13]	[0, 0, 6, 5, 0, 1]	11	3
$F6$	0.000283	0.000037	True	[0, 8, 6, 14, 5, 7, 13, 10, 0, 14, 12, 14, 4, 10, 7, 7]	[0, 4, 6, 7, 1, 1]	14	5
$F7$	0.000283	0.000036	True	[1, 9, 6, 2, 13, 14, 0, 7, 13, 5, 15, 0, 8, 14, 3, 14]	[1, 5, 2, 3, 0, 1]	14	5
$F8$	0.000279	0.000058	True	[1, 8, 6, 10, 9, 8, 7, 3, 11, 1, 7, 1, 6, 5, 8, 0]	[1, 0, 6, 4, 0, 0]	15	3
$F9$	0.000420	0.000045	True	[0, 4, 3, 9, 13, 3, 15, 12, 9, 1, 13, 7, 0, 8, 12, 13]	[0, 8, 1, 1, 0, 0]	14	3
$F10$	0.000285	0.000038	True	[1, 8, 6, 6, 5, 1, 11, 8, 11, 4, 12, 8, 3, 14, 7, 5]	[1, 4, 6, 1, 1, 1]	16	6
$F11$	0.000259	0.000036	True	[1, 13, 9, 7, 1, 6, 3, 3, 14, 9, 3, 4, 6, 14, 14, 0]	[1, 1, 1, 6, 1, 1]	15	6
$F12$	0.000254	0.000035	True	[0, 9, 6, 2, 9, 14, 1, 1, 14, 12, 14, 13, 3, 5, 2, 0]	[0, 1, 0, 7, 1, 0]	14	3
$F13$	0.000289	0.000036	True	[1, 4, 3, 5, 13, 12, 10, 15, 1, 14, 9, 0, 10, 11, 0, 4]	[1, 8, 5, 1, 1, 0]	14	5
$F14$	0.000268	0.000036	True	[0, 12, 9, 11, 1, 9, 12, 7, 0, 14, 12, 14, 4, 10, 7, 7]	[0, 8, 5, 4, 0, 0]	14	3
$F15$	0.000407	0.000061	True	[1, 0, 0, 4, 12, 7, 10, 1, 5, 7, 1, 1, 8, 13, 7, 2]	[1, 4, 2, 6, 1, 1]	14	6
$F16$	0.000316	0.000037	True	[0, 12, 9, 7, 5, 6, 2, 13, 8, 9, 11, 13, 8, 11, 4, 12]	[0, 4, 7, 6, 0, 1]	15	4
$F17$	0.000281	0.000035	True	[0, 8, 6, 2, 9, 14, 9, 12, 3, 14, 1, 0, 14, 11, 8, 4]	[0, 0, 2, 0, 0, 1]	14	2
$F18$	0.000257	0.000035	True	[0, 8, 6, 6, 5, 1, 3, 8, 9, 1, 13, 12, 3, 11, 11, 7]	[0, 0, 2, 6, 0, 0]	15	2
$F19$	0.000267	0.000036	True	[1, 8, 6, 2, 9, 14, 1, 0, 4, 1, 12, 6, 2, 14, 8, 9]	[1, 8, 2, 6, 0, 0]	15	4
$F20$	0.000248	0.000036	True	[0, 4, 3, 9, 9, 15, 12, 7, 4, 12, 4, 11, 3, 4, 3, 0]	[0, 12, 5, 2, 1, 0]	14	4
$F21$	0.000262	0.000035	True	[0, 1, 0, 12, 12, 9, 4, 1, 10, 9, 7, 4, 8, 14, 10, 8]	[0, 5, 2, 4, 1, 1]	14	5
$F22$	0.000270	0.000035	True	[0, 13, 5, 14, 1, 7, 2, 5, 1, 10, 1, 13, 5, 9, 11, 12]	[0, 5, 1, 6, 1, 0]	15	4
$F23$	0.000237	0.000035	True	[0, 8, 6, 2, 5, 10, 6, 7, 7, 14, 10, 4, 10, 5, 0, 0]	[0, 8, 4, 1, 0, 1]	13	4
$F24$	0.000267	0.000035	True	[0, 9, 6, 14, 13, 11, 7, 13, 3, 7, 3, 1, 6, 5, 15, 13]	[0, 5, 4, 4, 1, 0]	15	4
$F25$	0.000280	0.000036	True	[0, 13, 5, 2, 5, 14, 14, 0, 10, 4, 1, 13, 4, 6, 2, 14]	[0, 5, 1, 4, 0, 1]	14	4
$F26$	0.000255	0.000035	True	[1, 4, 3, 9, 1, 3, 14, 14, 12, 5, 4, 11, 3, 11, 13, 0]	[1, 8, 1, 1, 1, 0]	15	5
$F27$	0.000269	0.000037	True	[1, 0, 0, 8, 4, 6, 1, 8, 9, 7, 0, 6, 6, 3, 15, 1]	[1, 0, 2, 7, 1, 0]	13	4
$F28$	0.000281	0.000037	True	[0, 8, 6, 6, 5, 1, 3, 0, 0, 3, 9, 7, 12, 14, 2, 8]	[0, 8, 4, 7, 1, 0]	13	4
$F29$	0.000259	0.000036	True	[0, 1, 0, 8, 0, 6, 10, 14, 1, 3, 1, 15, 12, 4, 7, 3]	[0, 13, 6, 0, 1, 1]	13	4
$F30$	0.000255	0.000035	True	[0, 4, 3, 5, 13, 12, 2, 11, 0, 11, 9, 14, 8, 10, 12, 9]	[0, 0, 7, 4, 0, 0]	14	2

Вывод по таблице 1:

1. Общее время выполнения A1: 0.008387899928493425
2. Общее время выполнения A2: 0.0011523999855853617
3. Отношение время выполнения A1 и A2: 7.278635919309553
4. Отношение длин полиномов A1 и A2: 3.4754098360655736

Таблица 18: $p = 2, n = 4$. Unpolynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.000021	0.000017	False	NaN	NaN	NaN	NaN
$F2$	0.000023	0.000015	False	NaN	NaN	NaN	NaN
$F3$	0.000010	0.000015	False	NaN	NaN	NaN	NaN
$F4$	0.000008	0.000015	False	NaN	NaN	NaN	NaN
$F5$	0.000008	0.000015	False	NaN	NaN	NaN	NaN
$F6$	0.000010	0.000015	False	NaN	NaN	NaN	NaN
$F7$	0.000014	0.000014	False	NaN	NaN	NaN	NaN
$F8$	0.000019	0.000015	False	NaN	NaN	NaN	NaN
$F9$	0.000023	0.000021	False	NaN	NaN	NaN	NaN
$F10$	0.000013	0.000031	False	NaN	NaN	NaN	NaN
$F11$	0.000016	0.000031	False	NaN	NaN	NaN	NaN
$F12$	0.000014	0.000024	False	NaN	NaN	NaN	NaN
$F13$	0.000010	0.000015	False	NaN	NaN	NaN	NaN
$F14$	0.000014	0.000014	False	NaN	NaN	NaN	NaN
$F15$	0.000019	0.000014	False	NaN	NaN	NaN	NaN
$F16$	0.000007	0.000014	False	NaN	NaN	NaN	NaN
$F17$	0.000008	0.000019	False	NaN	NaN	NaN	NaN
$F18$	0.000009	0.000014	False	NaN	NaN	NaN	NaN
$F19$	0.000007	0.000014	False	NaN	NaN	NaN	NaN
$F20$	0.000012	0.000014	False	NaN	NaN	NaN	NaN
$F21$	0.000008	0.000014	False	NaN	NaN	NaN	NaN
$F22$	0.000008	0.000019	False	NaN	NaN	NaN	NaN
$F23$	0.000018	0.000014	False	NaN	NaN	NaN	NaN
$F24$	0.000018	0.000014	False	NaN	NaN	NaN	NaN
$F25$	0.000008	0.000014	False	NaN	NaN	NaN	NaN
$F26$	0.000019	0.000014	False	NaN	NaN	NaN	NaN
$F27$	0.000008	0.000014	False	NaN	NaN	NaN	NaN
$F28$	0.000013	0.000019	False	NaN	NaN	NaN	NaN
$F29$	0.000008	0.000015	False	NaN	NaN	NaN	NaN
$F30$	0.000009	0.000014	False	NaN	NaN	NaN	NaN

Вывод по таблице:

1. Общее время выполнения A1: 0.0003831998910754919
2. Общее время выполнения A2: 0.0005014000053051859
3. Отношение время выполнения A1 и A2: 0.7642598464717817

Таблица 19: $p = 2, n = 5$. Polynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.002272	0.000098	True	[0, 8, 31, 16, 12, 12, 13, 30, 19, 26, 19, 21, 7, 16, 21, 27, 7, 5, 10, 14, 3, 27, 29, 19, 18, 21, 28, 11, 24, 27, 17, 16]	[0, 24, 15, 0, 2, 0, 1, 0]	31	4
$F2$	0.001958	0.000080	True	[1, 16, 17, 13, 25, 30, 9, 24, 14, 6, 9, 17, 31, 27, 30, 14, 29, 16, 15, 19, 31, 21, 21, 31, 23, 1, 21, 6, 28, 2, 23, 2]	[1, 8, 5, 9, 1, 3, 0, 1]	32	7
$F3$	0.002083	0.000153	True	[0, 17, 17, 1, 16, 29, 23, 21, 28, 18, 24, 7, 17, 26, 13, 14, 3, 19, 25, 3, 20, 20, 12, 16, 1, 7, 17, 8, 14, 19, 26, 0]	[0, 1, 13, 13, 3, 2, 0, 1]	30	6
$F4$	0.002145	0.000080	True	[1, 9, 22, 26, 17, 0, 25, 8, 2, 20, 23, 28, 27, 5, 28, 23, 7, 12, 8, 13, 3, 1, 17, 28, 5, 28, 26, 26, 16, 13, 4, 24]	[1, 1, 2, 6, 3, 1, 1, 0]	31	7
$F5$	0.001984	0.000077	True	[1, 9, 31, 8, 12, 30, 7, 16, 20, 3, 19, 9, 9, 0, 5, 22, 23, 11, 14, 26, 3, 18, 16, 7, 20, 31, 1, 26, 27, 18, 9, 23]	[1, 9, 7, 4, 0, 3, 1, 1]	31	7
$F6$	0.002052	0.000080	True	[1, 16, 25, 7, 21, 6, 6, 3, 10, 20, 6, 13, 17, 24, 28, 31, 13, 25, 8, 4, 11, 8, 25, 29, 3, 24, 7, 21, 1, 8, 8, 1]	[1, 24, 13, 5, 0, 3, 0, 0]	32	5
$F7$	0.001977	0.000095	True	[0, 17, 17, 25, 12, 23, 14, 25, 8, 7, 29, 26, 28, 2, 3, 14, 4, 0, 7, 28, 1, 23, 17, 21, 14, 9, 31, 28, 16, 24, 13, 23]	[0, 17, 5, 5, 1, 0, 0, 1]	30	5
$F8$	0.002243	0.000087	True	[1, 9, 10, 11, 5, 17, 3, 15, 30, 22, 11, 26, 1, 13, 3, 2, 19, 30, 0, 17, 11, 30, 7, 27, 15, 12, 25, 17, 22, 30, 15, 19]	[1, 25, 14, 13, 3, 2, 0, 1]	31	7
$F9$	0.001980	0.000077	True	[0, 1, 28, 31, 5, 19, 17, 6, 31, 15, 4, 4, 19, 16, 18, 2, 3, 7, 24, 11, 29, 2, 9, 13, 8, 3, 28, 30, 12, 8, 10, 21]	[0, 1, 4, 11, 1, 0, 0, 1]	31	5
$F10$	0.002263	0.000086	True	[1, 9, 23, 22, 28, 12, 5, 5, 5, 4, 26, 3, 25, 17, 3, 11, 26, 7, 2, 3, 8, 31, 13, 19, 17, 24, 6, 25, 17, 26, 31, 28]	[1, 17, 11, 2, 0, 3, 0, 0]	32	5
$F11$	0.002054	0.000081	True	[0, 9, 22, 2, 13, 6, 12, 28, 18, 10, 7, 8, 24, 22, 15, 0, 0, 20, 14, 17, 13, 22, 19, 5, 24, 19, 14, 31, 31, 29, 6, 26]	[0, 25, 6, 2, 1, 2, 1, 1]	29	7
$F12$	0.002115	0.000081	True	[1, 9, 18, 13, 13, 2, 20, 27, 28, 30, 29, 7, 29, 25, 16, 29, 24, 14, 23, 6, 4, 22, 21, 26, 4, 22, 25, 13, 19, 30, 9, 6]	[1, 25, 6, 15, 3, 1, 1, 0]	32	7
$F13$	0.002005	0.000080	True	[0, 17, 21, 10, 9, 24, 7, 21, 6, 11, 2, 16, 20, 24, 2, 17, 19, 6, 30, 25, 4, 11, 5, 8, 30, 0, 4, 24, 28, 23, 21, 15]	[0, 17, 13, 8, 2, 3, 1, 0]	30	6
$F14$	0.002034	0.000080	True	[0, 0, 0, 12, 29, 17, 9, 9, 23, 7, 19, 19, 30, 31, 14, 5, 7, 21, 24, 31, 6, 23, 25, 10, 9, 26, 19, 26, 13, 24, 14, 2]	[0, 16, 0, 4, 1, 3, 0, 0]	29	4
$F15$	0.001995	0.000077	True	[0, 1, 0, 4, 25, 15, 16, 18, 25, 15, 7, 17, 22, 27, 31, 21, 23, 12, 29, 25, 31, 5, 11, 0, 2, 28, 21, 25, 11, 4, 2, 27]	[0, 1, 12, 2, 3, 0, 1, 1]	29	6
$F16$	0.001971	0.000076	True	[1, 9, 31, 16, 24, 16, 30, 17, 2, 0, 25, 13, 9, 18, 14, 3, 9, 5, 1, 6, 4, 4, 27, 10, 11, 23, 3, 21, 31, 31, 26, 15]	[1, 25, 7, 4, 0, 1, 0, 1]	31	6
$F17$	0.002059	0.000082	True	[1, 17, 1, 9, 29, 3, 27, 8, 11, 6, 29, 25, 19, 17, 19, 12, 20, 30, 31, 16, 10, 30, 6, 27, 18, 18, 30, 26, 20, 25, 2, 11]	[1, 9, 13, 13, 3, 1, 0, 1]	32	7
$F18$	0.001999	0.000079	True	[1, 9, 18, 13, 5, 14, 2, 4, 9, 13, 23, 6, 16, 31, 16, 19, 16, 24, 8, 26, 3, 11, 16, 30, 10, 26, 13, 12, 0, 19, 15, 16]	[1, 1, 6, 13, 3, 2, 1, 1]	31	8
$F19$	0.001988	0.000080	True	[1, 8, 2, 9, 13, 11, 7, 26, 1, 25, 1, 28, 23, 20, 14, 18, 30, 21, 28, 16, 31, 20, 1, 2, 16, 6, 3, 11, 8, 7, 15, 15]	[1, 24, 14, 9, 2, 2, 1, 0]	32	7
$F20$	0.002212	0.000082	True	[0, 8, 6, 6, 29, 5, 5, 31, 11, 6, 5, 9, 30, 18, 15, 31, 28, 31, 16, 5, 1, 30, 12, 11, 28, 30, 17, 0, 24, 12, 22, 3]	[0, 16, 6, 12, 3, 0, 0, 0]	30	4
$F21$	0.002001	0.000078	True	[0, 8, 27, 27, 24, 16, 14, 27, 30, 0, 15, 19, 4, 15, 13, 2, 3, 20, 2, 30, 8, 7, 27, 28, 7, 0, 6, 30, 19, 26, 16, 0]	[0, 0, 3, 15, 3, 0, 1, 0]	28	4
$F22$	0.002013	0.000076	True	[0, 1, 12, 27, 25, 16, 15, 22, 8, 23, 25, 25, 26, 14, 23, 4, 13, 20, 24, 14, 24, 23, 8, 25, 28, 19, 27, 26, 18, 29, 23, 5]	[0, 1, 8, 3, 2, 0, 1, 1]	31	6
$F23$	0.002094	0.000082	True	[0, 9, 26, 31, 5, 20, 20, 12, 3, 9, 16, 16, 26, 13, 27, 19, 8, 3, 13, 6, 14, 6, 31, 31, 16, 8, 6, 22, 24, 18, 18, 11]	[0, 9, 10, 13, 2, 3, 1, 1]	31	7
$F24$	0.001969	0.000079	True	[0, 8, 31, 28, 17, 17, 0, 26, 9, 9, 12, 14, 6, 1, 31, 24, 16, 28, 16, 23, 26, 3, 11, 3, 24, 19, 17, 13, 18, 22, 27, 5]	[0, 16, 3, 2, 2, 1, 0, 0]	30	5
$F25$	0.001993	0.000078	True	[1, 17, 21, 22, 24, 29, 1, 9, 17, 14, 28, 1, 19, 31, 10, 14, 10, 28, 0, 18, 19, 16, 5, 23, 9, 4, 10, 1, 13, 18, 5, 19]	[1, 17, 13, 4, 1, 2, 1, 1]	31	8
$F26$	0.003506	0.000091	True	[0, 0, 4, 25, 9, 19, 16, 13, 28, 25, 19, 25, 0, 8, 12, 27, 11, 12, 9, 12, 4, 11, 28, 29, 3, 18, 21, 7, 10, 28, 9, 10]	[0, 16, 4, 9, 3, 3, 0, 1]	29	6
$F27$	0.001989	0.000081	True	[1, 0, 0, 12, 9, 29, 22, 4, 9, 23, 11, 28, 23, 22, 25, 11, 7, 16, 8, 13, 23, 20, 18, 21, 2, 15, 7, 27, 13, 23, 28, 14]	[1, 16, 12, 4, 0, 2, 1, 0]	30	6
$F28$	0.002092	0.000083	True	[0, 9, 10, 11, 5, 1, 27, 27, 20, 20, 27, 9, 10, 1, 15, 18, 19, 28, 1, 2, 29, 27, 4, 19, 4, 7, 28, 0, 27, 21, 14, 26]	[0, 17, 14, 13, 2, 3, 0, 1]	30	6
$F29$	0.001971	0.000077	True	[0, 17, 21, 18, 1, 10, 31, 12, 5, 29, 10, 14, 23, 10, 21, 31, 5, 28, 31, 27, 0, 20, 10, 20, 8, 10, 11, 14, 10, 3, 13, 14]	[0, 17, 5, 4, 2, 0, 1, 0]	30	5
$F30$	0.002009	0.000078	True	[0, 17, 21, 30, 28, 23, 14, 24, 16, 26, 18, 1, 13, 16, 20, 4, 8, 19, 8, 5, 16, 7, 30, 9, 18, 0, 31, 13, 20, 22, 7, 26]	[0, 1, 9, 14, 3, 3, 0, 0]	30	5

Вывод по таблице:

1. Общее время выполнения A1: 0.06302580004557967
2. Общее время выполнения A2: 0.0025132999871857464
3. Отношение время выполнения A1 и A2: 25.076910980353148
4. Отношение длин полиномов A1 и A2: 5.146067415730337

Таблица 20: $p = 2, n = 5$. Unpolynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.000017	0.000032	False	NaN	NaN	NaN	NaN
$F2$	0.000028	0.000031	False	NaN	NaN	NaN	NaN
$F3$	0.000012	0.000031	False	NaN	NaN	NaN	NaN
$F4$	0.000008	0.000030	False	NaN	NaN	NaN	NaN
$F5$	0.000010	0.000035	False	NaN	NaN	NaN	NaN
$F6$	0.000027	0.000049	False	NaN	NaN	NaN	NaN
$F7$	0.000020	0.000047	False	NaN	NaN	NaN	NaN
$F8$	0.000020	0.000046	False	NaN	NaN	NaN	NaN
$F9$	0.000012	0.000030	False	NaN	NaN	NaN	NaN
$F10$	0.000026	0.000029	False	NaN	NaN	NaN	NaN
$F11$	0.000017	0.000029	False	NaN	NaN	NaN	NaN
$F12$	0.000024	0.000029	False	NaN	NaN	NaN	NaN
$F13$	0.000024	0.000029	False	NaN	NaN	NaN	NaN
$F14$	0.000011	0.000029	False	NaN	NaN	NaN	NaN
$F15$	0.000010	0.000029	False	NaN	NaN	NaN	NaN
$F16$	0.000016	0.000029	False	NaN	NaN	NaN	NaN
$F17$	0.000010	0.000028	False	NaN	NaN	NaN	NaN
$F18$	0.000010	0.000030	False	NaN	NaN	NaN	NaN
$F19$	0.000010	0.000037	False	NaN	NaN	NaN	NaN
$F20$	0.000011	0.000029	False	NaN	NaN	NaN	NaN
$F21$	0.000025	0.000030	False	NaN	NaN	NaN	NaN
$F22$	0.000012	0.000029	False	NaN	NaN	NaN	NaN
$F23$	0.000016	0.000029	False	NaN	NaN	NaN	NaN
$F24$	0.000010	0.000028	False	NaN	NaN	NaN	NaN
$F25$	0.000010	0.000028	False	NaN	NaN	NaN	NaN
$F26$	0.000010	0.000029	False	NaN	NaN	NaN	NaN
$F27$	0.000010	0.000029	False	NaN	NaN	NaN	NaN
$F28$	0.000010	0.000028	False	NaN	NaN	NaN	NaN
$F29$	0.000024	0.000029	False	NaN	NaN	NaN	NaN
$F30$	0.000010	0.000029	False	NaN	NaN	NaN	NaN

Вывод по таблице 1:

1. Общее время выполнения A1: 0.00045979995047673583
2. Общее время выполнения A2: 0.0009480000007897615
3. Отношение время выполнения A1 и A2: 0.48502104440262117

Таблица 21: $p = 3, n = 2$. Polynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.000090	0.000049	True	[0, 1, 0, 7, 0, 2, 5, 5, 8]	[0, 4, 3, 2, 1, 0]	6	4
$F2$	0.000070	0.000034	True	[0, 0, 0, 6, 0, 2, 7, 3, 1]	[0, 0, 6, 0, 2, 2]	5	3
$F3$	0.000068	0.000032	True	[1, 8, 4, 6, 2, 0, 0, 6, 2]	[1, 2, 6, 0, 2, 0]	7	4
$F4$	0.000063	0.000031	True	[1, 0, 0, 1, 4, 7, 2, 5, 2]	[1, 0, 6, 2, 2, 2]	7	5
$F5$	0.000065	0.000030	True	[0, 0, 0, 7, 7, 5, 3, 1, 1]	[0, 3, 1, 0, 1, 1]	6	4
$F6$	0.000067	0.000031	True	[2, 1, 0, 8, 6, 4, 6, 2, 2]	[2, 4, 5, 0, 0, 2]	8	4
$F7$	0.000068	0.000030	True	[1, 8, 4, 3, 4, 3, 0, 5, 3]	[1, 8, 1, 1, 1, 1]	8	6
$F8$	0.000066	0.000030	True	[2, 1, 0, 7, 8, 1, 2, 3, 3]	[2, 1, 4, 1, 0, 1]	8	5
$F9$	0.000066	0.000031	True	[2, 0, 0, 1, 2, 7, 5, 5, 1]	[2, 0, 8, 2, 0, 2]	7	4
$F10$	0.000065	0.000030	True	[1, 8, 4, 0, 2, 7, 4, 3, 3]	[1, 8, 3, 0, 1, 1]	8	5
$F11$	0.000065	0.000029	True	[1, 1, 0, 2, 0, 6, 8, 5, 2]	[1, 4, 0, 0, 1, 1]	7	4
$F12$	0.000065	0.000031	True	[1, 1, 0, 4, 3, 0, 8, 2, 6]	[1, 4, 7, 2, 1, 1]	7	6
$F13$	0.000060	0.000031	True	[1, 8, 4, 4, 3, 7, 2, 2, 0]	[1, 8, 8, 2, 1, 2]	8	6
$F14$	0.000062	0.000030	True	[0, 0, 0, 3, 6, 7, 8, 6, 4]	[0, 6, 8, 0, 1, 1]	6	4
$F15$	0.000063	0.000030	True	[2, 1, 0, 8, 4, 5, 5, 3, 2]	[2, 4, 0, 2, 2, 2]	8	5
$F16$	0.000063	0.000030	True	[1, 8, 4, 6, 4, 1, 4, 2, 4]	[1, 5, 7, 1, 0, 2]	9	5
$F17$	0.000062	0.000036	True	[1, 8, 4, 1, 0, 5, 2, 8, 2]	[1, 2, 7, 2, 1, 0]	8	5
$F18$	0.000102	0.000033	True	[1, 1, 0, 5, 4, 0, 6, 1, 3]	[1, 4, 3, 1, 1, 2]	7	6
$F19$	0.000065	0.000030	True	[1, 0, 0, 7, 2, 7, 8, 5, 1]	[1, 6, 2, 2, 0, 2]	7	5
$F20$	0.000064	0.000031	True	[1, 1, 0, 6, 3, 3, 4, 8, 8]	[1, 7, 4, 1, 2, 1]	8	6
$F21$	0.000063	0.000032	True	[0, 8, 4, 7, 0, 3, 8, 4, 2]	[0, 2, 4, 0, 1, 2]	7	4
$F22$	0.000088	0.000032	True	[0, 8, 4, 0, 0, 6, 8, 1, 2]	[0, 2, 4, 2, 1, 2]	6	5
$F23$	0.000075	0.000031	True	[2, 1, 0, 6, 7, 2, 7, 6, 8]	[2, 4, 4, 0, 0, 2]	8	4
$F24$	0.000065	0.000031	True	[1, 1, 0, 1, 6, 0, 7, 2, 3]	[1, 1, 5, 2, 2, 1]	7	6
$F25$	0.000074	0.000030	True	[1, 8, 4, 6, 7, 0, 4, 6, 2]	[1, 2, 8, 0, 0, 0]	8	3
$F26$	0.000072	0.000047	True	[1, 0, 0, 3, 8, 6, 2, 4, 3]	[1, 0, 4, 2, 0, 2]	7	4
$F27$	0.000085	0.000032	True	[2, 1, 0, 6, 6, 1, 3, 5, 2]	[2, 1, 2, 1, 0, 2]	8	5
$F28$	0.000064	0.000031	True	[2, 0, 0, 5, 1, 3, 0, 6, 3]	[2, 3, 3, 2, 1, 0]	6	5
$F29$	0.000059	0.000030	True	[2, 1, 0, 7, 1, 0, 5, 2, 5]	[2, 7, 0, 2, 2, 1]	7	5
$F30$	0.000061	0.000029	True	[2, 0, 0, 6, 5, 5, 4, 8, 7]	[2, 0, 6, 1, 1, 0]	7	4

Вывод по таблице:

1. Общее время выполнения A1: 0.0020645999757107347
2. Общее время выполнения A2: 0.0009642999502830207
3. Отношение время выполнения A1 и A2: 2.1410350328285066
4. Отношение длин полиномов A1 и A2: 1.5319148936170213

Таблица 22: $p = 3, n = 2$. Unpolynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.000021	0.000015	False	NaN	NaN	NaN	NaN
$F2$	0.000020	0.000013	False	NaN	NaN	NaN	NaN
$F3$	0.000013	0.000022	False	NaN	NaN	NaN	NaN
$F4$	0.000020	0.000013	False	NaN	NaN	NaN	NaN
$F5$	0.000019	0.000012	False	NaN	NaN	NaN	NaN
$F6$	0.000012	0.000012	False	NaN	NaN	NaN	NaN
$F7$	0.000011	0.000012	False	NaN	NaN	NaN	NaN
$F8$	0.000012	0.000012	False	NaN	NaN	NaN	NaN
$F9$	0.000011	0.000016	False	NaN	NaN	NaN	NaN
$F10$	0.000012	0.000012	False	NaN	NaN	NaN	NaN
$F11$	0.000013	0.000012	False	NaN	NaN	NaN	NaN
$F12$	0.000012	0.000012	False	NaN	NaN	NaN	NaN
$F13$	0.000011	0.000021	False	NaN	NaN	NaN	NaN
$F14$	0.000011	0.000017	False	NaN	NaN	NaN	NaN
$F15$	0.000013	0.000013	False	NaN	NaN	NaN	NaN
$F16$	0.000019	0.000017	False	NaN	NaN	NaN	NaN
$F17$	0.000018	0.000017	False	NaN	NaN	NaN	NaN
$F18$	0.000035	0.000030	False	NaN	NaN	NaN	NaN
$F19$	0.000011	0.000017	False	NaN	NaN	NaN	NaN
$F20$	0.000012	0.000012	False	NaN	NaN	NaN	NaN
$F21$	0.000009	0.000012	False	NaN	NaN	NaN	NaN
$F22$	0.000010	0.000012	False	NaN	NaN	NaN	NaN
$F23$	0.000032	0.000030	False	NaN	NaN	NaN	NaN
$F24$	0.000012	0.000012	False	NaN	NaN	NaN	NaN
$F25$	0.000012	0.000020	False	NaN	NaN	NaN	NaN
$F26$	0.000012	0.000012	False	NaN	NaN	NaN	NaN
$F27$	0.000012	0.000012	False	NaN	NaN	NaN	NaN
$F28$	0.000012	0.000021	False	NaN	NaN	NaN	NaN
$F29$	0.000011	0.000012	False	NaN	NaN	NaN	NaN
$F30$	0.000018	0.000016	False	NaN	NaN	NaN	NaN

Вывод по таблице 1:

1. Общее время выполнения A1: 0.00044540007365867496
2. Общее время выполнения A2: 0.00046510001993738115
3. Отношение время выполнения A1 и A2: 0.9576436348436225

Таблица 23: $p = 3, n = 3$. Polynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.001133	0.000104	True	[0, 19, 13, 0, 21, 0, 14, 2, 23, 3, 9, 6, 12, 11, 16, 26, 6, 24, 12, 16, 0, 26, 6, 1, 9, 22, 9]	[0, 10, 13, 7, 0, 4, 2, 0, 0]	23	5
$F2$	0.001100	0.000092	True	[1, 20, 13, 9, 12, 8, 11, 12, 17, 20, 11, 4, 13, 15, 18, 14, 26, 16, 15, 17, 13, 3, 14, 11, 0, 21, 16]	[1, 20, 10, 7, 6, 6, 0, 2, 1]	26	8
$F3$	0.001112	0.000089	True	[1, 20, 4, 18, 19, 11, 12, 2, 10, 18, 2, 6, 6, 16, 24, 16, 11, 17, 2, 17, 13, 24, 9, 3, 3, 24, 13]	[1, 20, 16, 4, 2, 4, 2, 2, 0]	27	8
$F4$	0.001091	0.000086	True	[1, 2, 18, 18, 1, 14, 23, 8, 20, 3, 9, 21, 0, 4, 14, 20, 22, 25, 15, 18, 10, 13, 5, 7, 18, 25, 14]	[1, 2, 3, 6, 1, 7, 2, 1, 1]	26	9
$F5$	0.001109	0.000089	True	[1, 18, 4, 0, 7, 25, 19, 20, 7, 6, 3, 23, 13, 19, 20, 14, 8, 11, 24, 1, 17, 14, 1, 24, 6, 20, 12]	[1, 18, 1, 7, 1, 8, 2, 0, 2]	26	8
$F6$	0.001108	0.000093	True	[1, 19, 22, 0, 11, 5, 12, 21, 0, 0, 26, 10, 18, 25, 19, 23, 26, 23, 20, 2, 8, 7, 14, 14, 1, 17, 20]	[1, 19, 25, 6, 8, 4, 0, 2, 2]	24	8
$F7$	0.001114	0.000086	True	[1, 18, 13, 9, 21, 14, 12, 6, 7, 8, 9, 19, 17, 15, 20, 11, 0, 10, 5, 12, 14, 17, 18, 9, 6, 14, 2]	[1, 18, 7, 0, 0, 7, 1, 2, 1]	26	7
$F8$	0.001115	0.000087	True	[1, 11, 11, 23, 4, 0, 2, 24, 20, 26, 16, 16, 20, 10, 4, 17, 21, 8, 24, 7, 22, 11, 6, 0, 0, 22, 24]	[1, 2, 8, 8, 2, 3, 1, 0, 1]	24	8
$F9$	0.001810	0.000137	True	[2, 11, 20, 5, 24, 10, 19, 4, 11, 10, 5, 1, 3, 14, 4, 11, 18, 3, 3, 20, 0, 11, 22, 8, 6, 11, 24]	[2, 2, 20, 7, 3, 1, 1, 0]	26	8
$F10$	0.001913	0.000093	True	[0, 20, 13, 9, 3, 26, 24, 21, 18, 23, 24, 2, 23, 2, 11, 0, 25, 0, 16, 2, 9, 24, 7, 19, 16, 4, 17]	[0, 2, 10, 8, 5, 5, 1, 2, 1]	24	8
$F11$	0.001243	0.000087	True	[1, 20, 4, 0, 22, 22, 16, 6, 13, 1, 8, 23, 7, 12, 18, 23, 1, 15, 7, 13, 0, 6, 5, 0, 21, 10, 2]	[1, 11, 10, 0, 6, 3, 0, 2, 0]	24	6
$F12$	0.001108	0.000084	True	[2, 18, 4, 0, 4, 22, 2, 18, 18, 24, 22, 22, 3, 22, 9, 14, 1, 23, 15, 2, 24, 11, 0, 13, 7, 6, 19]	[2, 0, 19, 4, 0, 2, 0, 0, 1]	25	5
$F13$	0.001500	0.000097	True	[1, 18, 13, 18, 18, 13, 9, 13, 26, 14, 24, 18, 7, 6, 26, 26, 8, 18, 12, 8, 5, 25, 10, 18, 13, 20, 8]	[1, 9, 13, 8, 0, 7, 2, 2, 2]	27	8
$F14$	0.001533	0.000096	True	[2, 19, 13, 0, 21, 21, 6, 19, 19, 25, 26, 16, 7, 1, 12, 22, 11, 8, 20, 8, 8, 15, 7, 2, 23, 18, 11]	[2, 1, 16, 5, 2, 5, 2, 1, 2]	26	9
$F15$	0.001183	0.000093	True	[1, 11, 11, 23, 22, 6, 9, 26, 26, 21, 1, 8, 4, 13, 16, 12, 8, 24, 15, 26, 9, 2, 6, 5, 12, 12, 22]	[1, 11, 23, 7, 1, 7, 1, 2, 1]	27	9
$F16$	0.001051	0.000092	True	[0, 9, 26, 18, 21, 22, 5, 5, 12, 20, 1, 11, 18, 13, 9, 21, 19, 9, 22, 0, 16, 21, 15, 2, 20, 16, 11]	[0, 18, 26, 5, 5, 8, 2, 1, 0]	25	7
$F17$	0.001099	0.000086	True	[1, 2, 18, 18, 4, 23, 9, 14, 3, 16, 11, 20, 15, 5, 14, 6, 1, 24, 20, 14, 6, 7, 26, 20, 22, 9, 8]	[1, 11, 15, 5, 6, 0, 0, 0, 1]	27	6
$F18$	0.001102	0.000089	True	[0, 11, 20, 14, 0, 0, 22, 12, 3, 2, 7, 15, 1, 10, 25, 4, 10, 9, 23, 14, 15, 24, 1, 14, 5, 25, 7]	[0, 11, 26, 2, 3, 5, 0, 1, 2]	24	7
$F19$	0.001480	0.000092	True	[2, 2, 0, 18, 3, 22, 21, 9, 18, 22, 20, 10, 8, 1, 18, 15, 6, 23, 10, 6, 21, 11, 6, 21, 11, 1, 15]	[2, 20, 15, 3, 5, 1, 2, 2, 0]	26	8
$F20$	0.001126	0.000088	True	[0, 11, 20, 23, 12, 8, 11, 21, 18, 1, 8, 11, 12, 7, 2, 23, 25, 24, 1, 13, 0, 0, 7, 4, 26, 25, 15]	[0, 2, 17, 5, 1, 2, 2, 0, 2]	24	7
$F21$	0.001226	0.000089	True	[1, 9, 8, 18, 8, 21, 17, 7, 4, 16, 25, 22, 2, 15, 19, 11, 24, 4, 26, 22, 22, 23, 2, 14, 8, 16, 1]	[1, 0, 26, 5, 2, 4, 2, 0, 1]	27	7
$F22$	0.001097	0.000086	True	[0, 19, 22, 18, 17, 12, 0, 13, 4, 5, 23, 1, 21, 4, 22, 15, 11, 5, 26, 16, 19, 2, 16, 19, 24, 17, 3]	[0, 1, 16, 7, 1, 1, 2, 2, 0]	25	7
$F23$	0.001171	0.000170	True	[0, 10, 26, 18, 24, 10, 5, 13, 4, 16, 3, 0, 2, 2, 1, 3, 4, 12, 4, 0, 23, 25, 14, 24, 26, 7, 4]	[0, 19, 23, 5, 6, 7, 1, 1, 2]	24	8
$F24$	0.001189	0.000092	True	[0, 2, 0, 9, 12, 17, 5, 13, 2, 25, 23, 16, 18, 16, 12, 6, 5, 5, 8, 7, 26, 21, 0, 26, 1, 25, 21]	[0, 20, 15, 1, 7, 4, 2, 1, 1]	24	8
$F25$	0.001116	0.000090	True	[2, 19, 22, 9, 23, 10, 21, 0, 17, 20, 26, 0, 0, 19, 9, 7, 26, 21, 23, 14, 4, 14, 7, 11, 25, 15, 8]	[2, 19, 16, 2, 4, 3, 0, 0, 2]	24	7
$F26$	0.001112	0.000093	True	[1, 0, 18, 18, 4, 23, 15, 7, 9, 8, 16, 4, 23, 14, 2, 22, 14, 9, 8, 12, 25, 11, 15, 2, 7, 21, 25]	[1, 9, 9, 2, 7, 5, 2, 0, 1]	26	8
$F27$	0.001095	0.000092	True	[0, 19, 22, 18, 11, 12, 12, 11, 17, 0, 8, 16, 15, 7, 9, 3, 0, 23, 15, 8, 3, 10, 26, 17, 20, 5, 2]	[0, 1, 22, 7, 0, 5, 2, 1, 1]	24	7
$F28$	0.001169	0.000121	True	[1, 1, 0, 18, 12, 19, 15, 14, 6, 24, 19, 24, 2, 26, 6, 16, 13, 12, 20, 20, 21, 3, 8, 15, 1, 6, 13]	[1, 1, 18, 7, 7, 1, 2, 0, 1]	26	8
$F29$	0.001101	0.000085	True	[2, 19, 13, 18, 12, 16, 26, 15, 12, 19, 26, 23, 9, 0, 20, 24, 18, 13, 15, 8, 3, 5, 5, 8, 21, 0, 20]	[2, 1, 4, 0, 4, 3, 2, 2, 1]	25	8
$F30$	0.001084	0.000085	True	[0, 9, 26, 0, 9, 6, 22, 19, 14, 22, 16, 11, 24, 5, 6, 12, 11, 2, 17, 23, 7, 11, 25, 4, 9, 8, 10]	[0, 9, 2, 6, 4, 8, 0, 1, 1]	25	7

Вывод по таблице 1:

1. Общее время выполнения A1: 0.036389100016094744
2. Общее время выполнения A2: 0.002863000030629337
3. Отношение время выполнения A1 и A2: 12.710129104712511
4. Отношение длин полиномов A1 и A2: 3.3794642857142856

Таблица 24: $p = 3, n = 3$. Unpolynomial

$F(x)$	$time_1$	$time_2$	$F(x) \in Pol$	$coef_{A1}$	$coef_{A2}$	$d_{A1}(F)$	$d_{A2}(F)$
$F1$	0.000021	0.000031	False	NaN	NaN	NaN	NaN
$F2$	0.000017	0.000029	False	NaN	NaN	NaN	NaN
$F3$	0.000023	0.000029	False	NaN	NaN	NaN	NaN
$F4$	0.000016	0.000029	False	NaN	NaN	NaN	NaN
$F5$	0.000015	0.000028	False	NaN	NaN	NaN	NaN
$F6$	0.000030	0.000029	False	NaN	NaN	NaN	NaN
$F7$	0.000022	0.000028	False	NaN	NaN	NaN	NaN
$F8$	0.000015	0.000028	False	NaN	NaN	NaN	NaN
$F9$	0.000013	0.000028	False	NaN	NaN	NaN	NaN
$F10$	0.000015	0.000029	False	NaN	NaN	NaN	NaN
$F11$	0.000015	0.000028	False	NaN	NaN	NaN	NaN
$F12$	0.000015	0.000028	False	NaN	NaN	NaN	NaN
$F13$	0.000021	0.000028	False	NaN	NaN	NaN	NaN
$F14$	0.000014	0.000028	False	NaN	NaN	NaN	NaN
$F15$	0.000014	0.000038	False	NaN	NaN	NaN	NaN
$F16$	0.000015	0.000028	False	NaN	NaN	NaN	NaN
$F17$	0.000014	0.000038	False	NaN	NaN	NaN	NaN
$F18$	0.000019	0.000028	False	NaN	NaN	NaN	NaN
$F19$	0.000019	0.000028	False	NaN	NaN	NaN	NaN
$F20$	0.000015	0.000028	False	NaN	NaN	NaN	NaN
$F21$	0.000012	0.000028	False	NaN	NaN	NaN	NaN
$F22$	0.000021	0.000028	False	NaN	NaN	NaN	NaN
$F23$	0.000021	0.000028	False	NaN	NaN	NaN	NaN
$F24$	0.000014	0.000028	False	NaN	NaN	NaN	NaN
$F25$	0.000015	0.000029	False	NaN	NaN	NaN	NaN
$F26$	0.000022	0.000029	False	NaN	NaN	NaN	NaN
$F27$	0.000024	0.000029	False	NaN	NaN	NaN	NaN
$F28$	0.000014	0.000029	False	NaN	NaN	NaN	NaN
$F29$	0.000014	0.000051	False	NaN	NaN	NaN	NaN
$F30$	0.000015	0.000028	False	NaN	NaN	NaN	NaN

Вывод по таблице 1:

1. Общее время выполнения A1: 0.0005205999477766454
2. Общее время выполнения A2: 0.0008956999226938933
3. Отношение время выполнения A1 и A2: 0.5812213829503268

Вывод по анализу работы алгоритмов 1 и 2.

Вывод по анализу работы алгоритмов 1 и 2:

Сравнение времени выполнения: Алгоритм 1 и алгоритм 2 были проанализированы с точки зрения времени выполнения на наборе тестовых данных. Время выполнения каждого алгоритма было измерено и сравнено. По результатам таблиц, алгоритм 1 для не полиномиальных функций работает быстрее, чем алгоритм 2, то есть в среднем определяет не полиномиальность не полиномиальных функций быстрее, чем алгоритм 2. В другом случае, когда функция является полиномиальной алгоритм 2, работает значительно быстрее, так как требует меньше итераций для полной проверки функций.

Размер полиномов: Был проведен анализ размера полиномов, которые были построены каждым из алгоритмов. По результатам таблиц проверки алгоритмов, алгоритм 2 значительно превосходит алгоритм 1 по качеству длины полинома, соответственно превосходит по эффективности использования ресурсов. Алгоритм 2 строит полиномы на основе канонического вида полиномов, алгоритм 1 не учитывает этого и строит какой-то полином, реализующий данную функцию.

5 Заключение

В ходе исследования был проведен обзор существующих методов проверки полиномиальности функций по модулю составного числа. Полученные данные позволили более глубоко понять проблематику и основные подходы к решению данной задачи.

Был разобран и реализован предложенный алгоритм проверки полиномиальности по модулю степени простого числа. Программа была успешно выполнена для различных значений степени простого числа и различных простых чисел в основании.

Полученные результаты работы программы были зафиксированы и представлены в виде таблиц. Был проведен анализ быстродействия алгоритма, его использования памяти и других характеристик. В рамках выполненной работы был проведен сравнительный анализ двух алгоритмов проверки полиномиальности по модулю степени простого числа. Результаты сравнения позволили выявить преимущества и недостатки каждого из алгоритмов.

6 Литература

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7 Приложение

7.1 Алгоритм 1.

```
class PolynomialA1:
```

```
    def __init__(self, prime_number: int, power: int, function: list[int]) -> None:
        self.p = prime_number
        self.n = power
        self.function = function
```

```
    def factorial(self, number: int) -> int:
        if number == 0 or number == 1:
            return 1
        else:
            return number * self.factorial(number - 1)
```

```
    def find_mod(self, r_value: int) -> int:
        if r_value < self.p:
            return 0
        factorial_number = self.factorial(r_value)
        for i in range(2, self.n + 1):
            if factorial_number % (self.p ** i) != 0:
                return i - 1
        return self.n
```

```
    def evaluate_sum(self, r_value: int) -> bool:
        sum_result = 0
        for s in range(0, r_value + 1):
            sum_result += (((-1)**(r_value - s)) * (self.factorial(r_value) //
                (self.factorial(s) * self.factorial(r_value - s)))) * self.function[s]
        return sum_result
```

```
    def check_polynomial(self) -> bool:
        if self.n == 1:
            return True
        coefficients = []
        polynomial = [0 for _ in range(self.p**self.n)]
```



```

for r in range(self.p ** self.n):
    mod = self.find_mod(r)
    coefficients.append(self.evaluate_sum(r))
    if coefficients[r] % (self.p**mod) != 0:
        return False

    if (exp:=self.divide_by_mod(coefficients[r],(self.factorial(r)))):
        coefficients_polynom = self.calculate_coefficients(r-1)
        for i in range(self.p**self.n - 1, -1, -1):
            polynom[i] += exp*coefficients_polynom[i]
polynom = [elem % (self.p**self.n) for elem in polynom]
polynom.reverse()
return True, polynom

def calculate_coefficients(self, n: int) -> list[int]:
    if n >= 0:
        coefficients = [1]
        for i in range(1, n + 1):
            coefficients.append(0)
            for j in range(i, 0, -1):
                coefficients[j] = coefficients[j] - i * coefficients[j-1]
            coefficients.append(0)
        return [0] * (self.p**self.n - len(coefficients)) + coefficients
    return [0] * (self.p**self.n - 1) + [1]

def modular_division(self, a: int, b: int, m: int) -> int:
    if b == 0:
        raise ValueError("Divide_by_zero")
    inverse_b = pow(b, -1, m)
    if inverse_b is None:
        raise ValueError("Do_not_exist")
    result = (a * inverse_b) % m

    return result

def divide_by_mod(self, delta: int, fact_j: int) -> int:
    while fact_j % self.p == 0:
        fact_j //= self.p
        delta //= self.p

```

```
fact_j %= self.p**self.n
delta %= self.p**self.n

return self.modular_division(delta, fact_j, self.p ** self.n)
```

7.2 Алгоритм 2.

```
class PolynomialA2:
    def __init__(self, prime_number: int, power: int, function: list[int]) -> None:
        self.p = prime_number
        self.n = power
        self.function = function

    def factorial(self, number: int) -> int:
        if number == 0 or number == 1:
            return 1
        else:
            return number * self.factorial(number - 1)

    def find_mod(self, r_value: int) -> int:
        factorial_number = self.factorial(r_value)
        for i in range(2, self.n + 1):
            if factorial_number % (self.p ** i) != 0:
                return i - 1
        return self.n

    def s_p(self, m: int) -> int:
        if m == 1:
            return 1
        for i in range(m * self.p + 1):
            if self.find_mod(i) >= m:
                return i - 1

    def evaluate_internal_sum(self, t: int, i: int, t_n: int, a: int) -> int:
        res = 0
        for j in range(t+1, t_n):
            res += a[j] * (t - i)**j
        return res

    def evaluate_sum(self, t: int, t_n: int, a: list) -> int:
        result = 0
        for i in range(t+1):
            result += (-1)**i * (self.factorial(t) //
```

```

        (self.factorial(i) * self.factorial(t - i))) * (self.function[t-i] -
        self.evaluate_internal_sum(t, i, t_n, a))
    return result % (self.p ** self.n)

def find_solution(self, d: int, t: int):
    for i in range(self.p ** self.n):
        if (self.factorial(t) * i) % (self.p ** self.n) == d:
            return i
    return -1

def check_sum(self, s_p: int, a: int, b: int) -> int:
    res = 0
    for s in range(1, s_p + 1):
        res += a[s] * b**s
    return res

def check_polynomial(self) -> bool:
    if self.n == 1:
        return True
    s_p = self.s_p(self.n)
    t_list = [i for i in range(s_p, 0, -1)]
    d_list = [0] * (len(t_list) + 1)
    a_list = [0] * (len(t_list) + 1)
    t_n = t_list[0] + 1
    for t in t_list:
        d_list[t] = self.evaluate_sum(t, t_n, a_list)
        a_list[t] = self.find_solution(d_list[t], t)
        if a_list[t] == -1:
            return False
    for b in range(self.p ** self.n):
        if ((self.function[b] -
            self.check_sum(t_list[0], a_list, b)) %
            (self.p ** self.n)) != self.function[0]:
            return False
    a_list[0] = self.function[0]
    return (True, a_list)

```