Numerical Analysis, Exam 2, Spring 2020 *Due May 12th*

Name ______________________________

1. Consider the following integral $\int_1^{1.5} x^2 \ln x \, dx$. There are different methods to numerically approximate definite integrals, for example, using the Trapezoidal rule and the Simpson's rule.

   a) Which makes you happier, the Trapezoidal rule or the Simpson's rule? Provide the reasons for your choice.

   b) Using your program for the rule that makes you happier (Trapezoidal vs. Simpson's), prepare a table of approximated values for the given integral and the errors for $n = 2, 4, 8, 16, 32, 64$.

   c) Find the ratios by which the errors decrease. Is there any theoretical explanation to your results?

   d) Apply the error estimate formula for the chosen rule and compare the results to the actual errors.

   e) What is the advantage of using powers of 2 for n , i.e., $n = 2, 4, 8, 16, 32, 64$, instead of traditional consecutive numbers $n = 1,2,3,4, \ldots$

2. a) Find an approximated solutions to the following initial value problem using Euler's method with stepsizes $h = 0.1, 0.3, 0.5$.

   $$y'(x) = \frac{y}{x} - \left(\frac{y}{x}\right)^2, \quad 1 \le x \le 2, \quad y(1) = 1$$

   True answer: $y(x) = \frac{x}{1+\ln x}$

   b) Plot the original and all three approximated functions in the same axis.

   c) For each case compute the error and relative error at $x = 2$ using the true answer $y(x)$.

   d) Which h would you pick to approximate the given integral, 0.1, 0.3, 0.5 or another number? Why?

2. Consider the following linear system

$$\begin{cases} x + \dfrac{1}{2}y + \dfrac{1}{3}z = 2 \\ \dfrac{1}{2}x + \dfrac{1}{3}y + \dfrac{1}{4}z = -1 \\ \dfrac{1}{3}x + \dfrac{1}{4}y + \dfrac{1}{5}z = 0 \end{cases}$$

with the actual solution $(54, -264, 240)$.

a) Solve this linear system using Gaussian elimination with three-digit rounding arithmetic. Calculate the error |approximated solution – actual solution|.

b) Solve this linear system using Gaussian elimination with partial pivoting. Calculate the error |approximated solution – actual solution|.

c) Based on the results in *a)* and *b)* what can you say about the stability of this linear system?

d) Calculate the condition number of the coefficient matrix (the size of the matrix coefficient is 3 x 3).

e) How the calculated condition number supports your answer in *c)*?

4. Refer to the folder Covid-19 in the Moodle. Follow the instructions for the Experiment 6. Herd Immunity. Experiment with different parameters as instructed. As you analyze your results, focus on the questions in #11 and #12. Provide your analysis.

Upload your neatly written solutions and answers, as well as your Mathematica files to the Exam 2 folder on Moodle no later than May 12th.