16-720 Computer Vision
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Homework 3

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1 Lucas-Kanade Tracking

Q1.1

For a warp function:

$$\mathcal{W}(\mathbf{x};\mathbf{p}) = \begin{bmatrix}\mathcal{W}_x\\ \mathcal{W}_y\end{bmatrix} = \begin{bmatrix}1 & 0 & p_1\\ 0 & 1 & p_2\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix} \tag{1}$$

$$= \begin{bmatrix}x+p_1\\ y+p_2\end{bmatrix} \tag{2}$$

Therefore:

$$\frac{\partial \mathcal{W}(\mathbf{x};\mathbf{p})}{\partial \mathbf{p}^T} = \begin{bmatrix}\frac{\partial \mathcal{W}_x(\mathbf{x},\mathbf{p})}{\partial \mathbf{p}_1^T} & \frac{\partial \mathcal{W}_x(\mathbf{x},\mathbf{p})}{\partial \mathbf{p}_2^T}\\ \frac{\partial \mathcal{W}_y(\mathbf{x},\mathbf{p})}{\partial \mathbf{p}_1^T} & \frac{\partial \mathcal{W}_y(\mathbf{x},\mathbf{p})}{\partial \mathbf{p}_2^T}\end{bmatrix} \tag{3}$$

$$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \tag{4}$$

Since we are trying to obtain:

$$\arg\min_{\Delta p} \sum_{\mathbf{x}\in\mathbb{N}} \left\|\mathcal{I}_{t+1}\left(\mathbf{x}'+\Delta\mathbf{p}\right) - \mathcal{I}_t(\mathbf{x})\right\|_2^2 \tag{5}$$

$$= \arg\min_{\Delta p} \sum_{\mathbf{x}\in\mathbb{N}} \left\|\mathcal{I}_{t+1}\left(\mathbf{x}'\right) + \frac{\partial \mathcal{I}_{t+1}(\mathbf{x}')}{\partial \mathbf{x}'^T}\frac{\partial \mathcal{W}(\mathbf{x};\mathbf{p})}{\partial \mathbf{p}^T}\Delta\mathbf{p} - \mathcal{I}_t(\mathbf{x})\right\|_2^2 \tag{6}$$

$$= \arg\min_{\Delta p} \sum_{\mathbf{x}\in\mathbb{N}} \left\|\frac{\partial \mathcal{I}_{t+1}(\mathbf{x}')}{\partial \mathbf{x}'^T}\frac{\partial \mathcal{W}(\mathbf{x};\mathbf{p})}{\partial \mathbf{p}^T}\Delta\mathbf{p} - \left[\mathcal{I}_t(\mathbf{x}) - \mathcal{I}_{t+1}\left(\mathbf{x}'\right)\right]\right\|_2^2 \tag{7}$$

Where $\mathbf{x}' = \mathbf{x} + \mathbf{p}$.

Therefore, we have:

$$\mathbf{A} = \frac{\partial \mathcal{I}_{t+1}(\mathbf{x}')}{\partial \mathbf{x}'^T}\frac{\partial \mathcal{W}(\mathbf{x};\mathbf{p})}{\partial \mathbf{p}^T} \tag{8}$$

$$= \begin{bmatrix}\frac{\partial \mathcal{I}_{t+1}(\mathbf{x}'_\mathbf{1})}{\partial \mathbf{x}'^T_\mathbf{1}} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \frac{\partial \mathcal{I}_{t+1}(\mathbf{x}'_\mathbf{D})}{\partial \mathbf{x}'^T_\mathbf{D}}\end{bmatrix}\begin{bmatrix}\frac{\partial \mathcal{W}(\mathbf{x_1};\mathbf{p})}{\partial \mathbf{p}^T}\\ \vdots\\ \frac{\partial \mathcal{W}(\mathbf{x_D};\mathbf{p})}{\partial \mathbf{p}^T}\end{bmatrix} \tag{9}$$

$$\mathbf{b} = \mathcal{I}_t\left(\mathbf{x}\right) - \mathcal{I}_{t+1}\left(\mathbf{x}'\right) \tag{10}$$

$$= \begin{bmatrix} \mathcal{I}_t\left(\mathbf{x_1}\right) - \mathcal{I}_{t+1}\left(\mathbf{x}'_\mathbf{1}\right) \\ \vdots \\ \mathcal{I}_t\left(\mathbf{x_D}\right) - \mathcal{I}_{t+1}\left(\mathbf{x}'_\mathbf{D}\right) \end{bmatrix} \tag{11}$$

To guarantee a unique solution for $\Delta\mathbf{p}$, $\mathbf{A^T A}$ needs to be full rank, in other words, $det\left(\mathbf{A^T A}\right) \neq 0$

Q1.3

Result pictures are shown below:

(a) (b) (c) (d) (e)

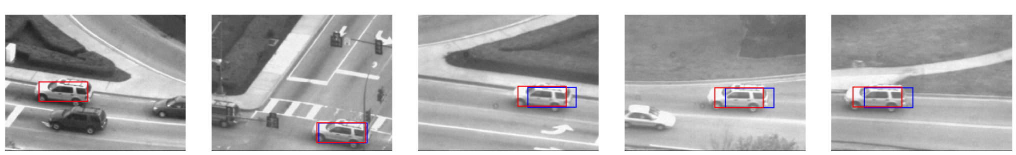
Figure 1: Car Sequence with One Single Template

(a) (b) (c) (d) (e)

Figure 2: Girl Sequence with One Single Template

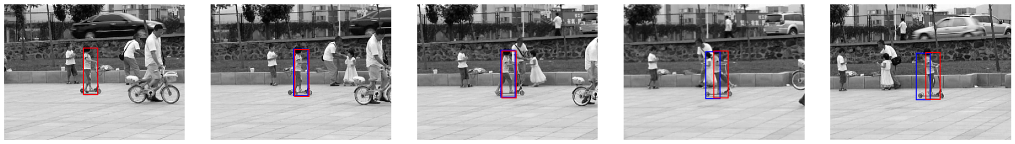
Q1.4

Result pictures are shown below, in which red rectangles are created by baseline tracker with correction and blue rectangles are those created in **Q1.3**:



(a) (b) (c) (d) (e)

Figure 3: Car Sequence with Template Correction



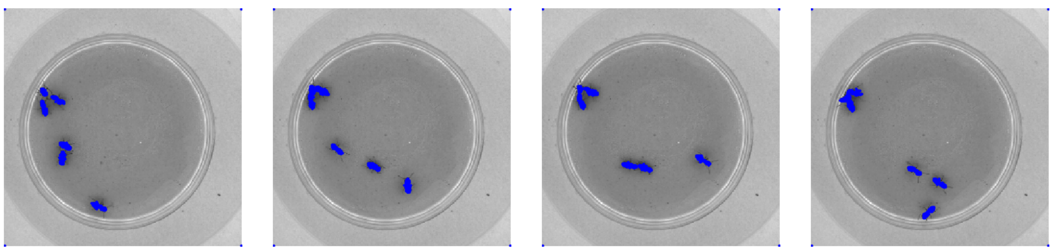
(a) (b) (c) (d) (e)

Figure 4: Girl Sequence with Template Correction

2 Affine Motion Subtraction

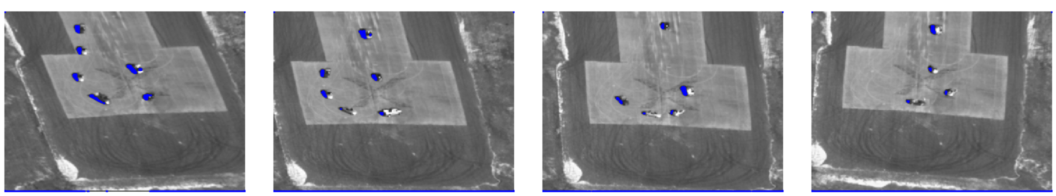
Q2.3

Result images are shown below:



(a) (b) (c) (d)

Figure 5: Motion Detection of Ant Sequence



(a) (b) (c) (d)

Figure 6: Motion Detection of Aerial Sequence

3 Efficient Tracking

Q3.1

Because some time-consuming tasks like Hessian, steepest descent images could be pre-computed, so we do not need to compute it repeatedly in optimization gradient descent iterations.