16-720 Computer Vision
Spring 2020

Homework 4

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1 Theory

Q1.1

For given point $\mathbf{x}$, we have $\tilde{x_1} = \tilde{x_2} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{\mathbf{T}}$. Therefore, we have:

$$\tilde{x_2}^{\mathbf{T}} F \tilde{x_1} = 0 \tag{1}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \tag{2}$$

$$F_{33} = 0 \tag{3}$$

Q1.2

For pure translation parallel to x-axis:

$$t = \begin{bmatrix} t_1 \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$E = tR \tag{6}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \tag{7}$$

Suppose that $\tilde{x_1}^T = \begin{bmatrix} a_1 & a_2 & 1 \end{bmatrix}$ and $\tilde{x_2}^T = \begin{bmatrix} b_1 & b_2 & 1 \end{bmatrix}$, we have:

$$l_1^T = \tilde{x_2}^T E \tag{8}$$

$$= \begin{bmatrix} b_1 & b_2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \tag{9}$$

$$= \begin{bmatrix} 0 & t_1 & -b_2 t_1 \end{bmatrix} \tag{10}$$

$$l_2^T = \tilde{x_1}^T E^T \tag{11}$$

$$= \begin{bmatrix} a_1 & a_2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_1 \\ 0 & -t_1 & 0 \end{bmatrix} \tag{12}$$

$$= \begin{bmatrix} 0 & -t_1 & a_2 t_1 \end{bmatrix} \tag{13}$$

Thus, we could acquire epipolar lines in the two cameras: $t_1 y_1 - b_2 t_1 = 0, -t_1 y_2 + a_2 t_1 = 0$. Both do not contain x component, so they are parallel to the x-axis.

Q1.3

Assume that the coordinate of the object in 3D world is $\begin{bmatrix} u & v & w \end{bmatrix}^T$, and let $\begin{bmatrix} x_i & y_i \end{bmatrix}^T$ be the position at time i. Thus we have:

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K \left(R_1 \begin{bmatrix} u \\ v \\ w \end{bmatrix} + t_1 \right) \tag{14}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = R_1^{-1} \left(K^{-1} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} - t_1 \right) \tag{15}$$

$$= R_1^T K^{-1} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} - R_1^T t_1 \tag{16}$$

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = K \left(R_2 \begin{bmatrix} u \\ v \\ w \end{bmatrix} + t_2 \right) \tag{17}$$

$$= K \left(R_2 \left(R_1^T K^{-1} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} - R_1^T t_1 \right) + t_2 \right) \tag{18}$$

$$= K R_2 R_1^T K^{-1} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} - K R_2 R_1^T t_1 + K t_2 \tag{19}$$

Therefore:

$$R_{rel} = K R_2 R_1^T K^{-1} \tag{20}$$

$$t_{rel} = -K R_2 R_1^T t_1 + K t_2 \tag{21}$$

$$E = t_{rel} \times R_{rel} \tag{22}$$

$$F = \left(K^{-1}\right)^T E K^{-1} \tag{23}$$

$$= \left(K^{-1}\right)^T (t_{rel} \times R_{rel}) K^{-1} \tag{24}$$

Q1.4

Assume that the distance between object and mirror is d, thus the distance between two images is $2d$ and there is pure translation:

$$R_{rel} = \mathbf{I} \tag{25}$$

$$t_{rel} = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix} \tag{26}$$

$$F = \left(K^{-1}\right)^T (t_{rel} \times R_{rel}) K^{-1} \tag{27}$$

$$= \left(K^{-1}\right)^T \begin{bmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix} K^{-1} \tag{28}$$

Thus, fundamental matrix $\mathbf{F}$ is a skew-symmetric matrix.

2 Fundamental Matrix Estimation

Q2.1

The recovered matrix $\mathbf{F}$ is:

$$\mathbf{F} = \begin{bmatrix} 9.78833286e-10 & -1.32135929e-07 & 1.12585666e-03 \\ -5.73843315e-08 & 2.96800276e-09 & -1.17611996e-05 \\ -1.08269003e-03 & 3.04846703e-05 & -4.47032655e-03 \end{bmatrix} \tag{29}$$

and example output image is shown below:

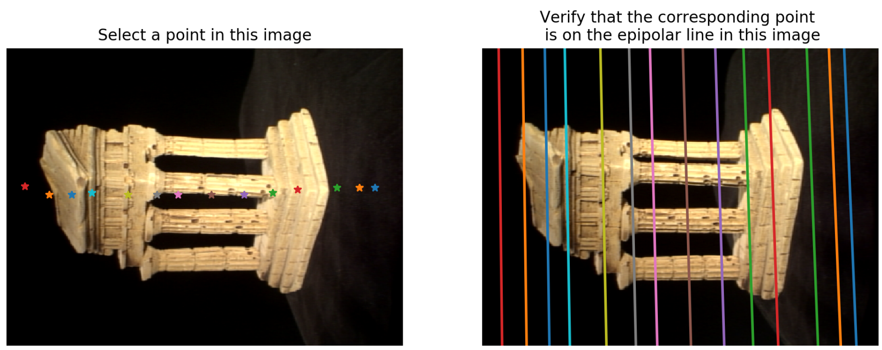


Figure 1: Visualized Epipolar Lines

3 Metric Reconstruction

Q3.1

$\mathbf{E}$ can be calculated by $\mathbf{K_2^T F K_1}$. Using the eight-point algorithm, the estimated $\mathbf{E}$ is:

$$\mathbf{E} = \begin{bmatrix} 2.26268684e-03 & -3.06552495e-01 & 1.66260633e+00 \\ -1.33130407e-01 & 6.91061098e-03 & -4.33003420e-02 \\ -1.66721070e+00 & -1.33210351e-02 & -6.72186431e-04 \end{bmatrix} \tag{30}$$

Q3.2

Suppose $\mathbf{C}_{1i}$ and $\mathbf{C}_{2i}$ is the ith row for $\mathbf{C}_1$ and $\mathbf{C}_2$. If $\tilde{\mathbf{w}}_i$ is a 4×1 vector of the 3D coordinate in the homogeneous form, we have:

$$\mathbf{C}_1 \tilde{\mathbf{w}}_i = \widehat{\mathbf{x}_{1i}} \tag{31}$$

$$\begin{bmatrix} \mathbf{C}_{11} \\ \mathbf{C}_{12} \\ \mathbf{C}_{13} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \\ 1 \end{bmatrix} = \begin{bmatrix} x_{1i} \\ y_{1i} \\ 1 \end{bmatrix} \tag{32}$$

$$\mathbf{C}_2 \tilde{\mathbf{w}}_i = \widehat{\mathbf{x}_{2i}} \tag{33}$$

$$\begin{bmatrix} \mathbf{C}_{21} \\ \mathbf{C}_{22} \\ \mathbf{C}_{23} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \\ 1 \end{bmatrix} = \begin{bmatrix} x_{2i} \\ y_{2i} \\ 1 \end{bmatrix} \tag{34}$$

Therefore, we have:

$$\mathbf{A}_i = \begin{bmatrix} x_{1i}\mathbf{C}_{13} - \mathbf{C}_{11} \\ y_{1i}\mathbf{C}_{13} - \mathbf{C}_{12} \\ x_{2i}\mathbf{C}_{23} - \mathbf{C}_{21} \\ y_{2i}\mathbf{C}_{23} - \mathbf{C}_{22} \end{bmatrix} \tag{35}$$

4 3D Visualization

Q4.1

Choose window size 20, the matched result is shown below:

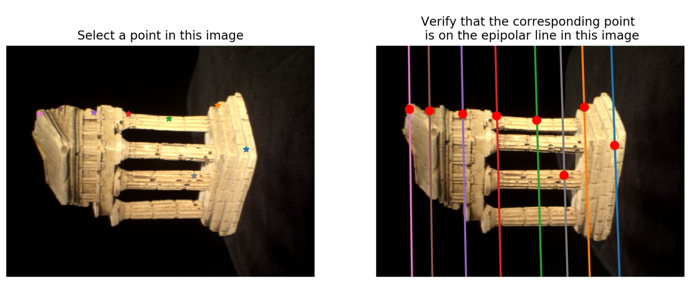
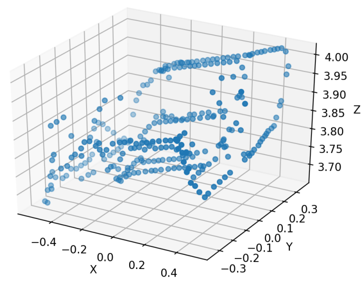
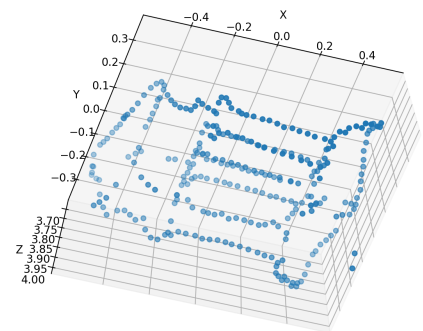


Figure 2: Corresponding Points

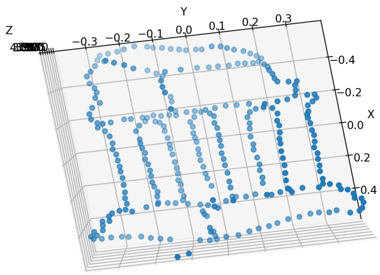
Q4.2 The results for $3D$ visualization is shown below:



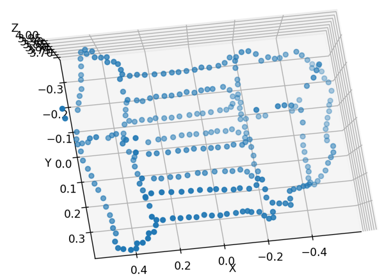
(a)



(b)



(c)



(d)

Figure 3: 3D Visualization

5 Bundle Adjustment

Q5.1

Using the noisy correspondences, without RANSAC, the visualization of epipolar lines is like:

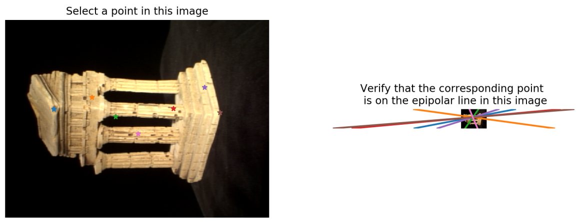


Figure 4: Visualization without RANSAC

With RANSAC implemented, the result looks much better:

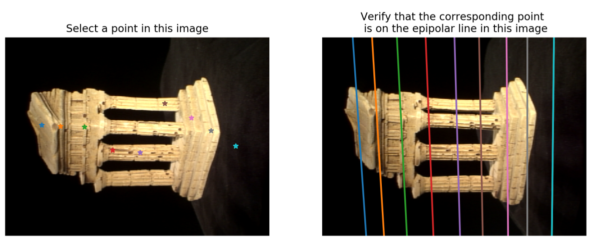


Figure 5: Visualization with RANSAC

Obtained fundamental matrix $\mathbf{F}$ is:

$$\mathbf{F} = \begin{bmatrix} 1.44845968e-08 & -3.12242026e-07 & 1.07078108e-03 \\ 1.67046292e-07 & 1.00324202e-08 & -8.51505084e-05 \\ -1.04288634e-03 & 9.23538949e-05 & -2.07767875e-03 \end{bmatrix} \tag{36}$$

For fundamental matrix, we have:

$$\tilde{\mathbf{x}_2}^T \mathbf{F} \tilde{\mathbf{x}_1} = 0 \tag{37}$$

So the error metrics used to determine if point i is an inlier is:

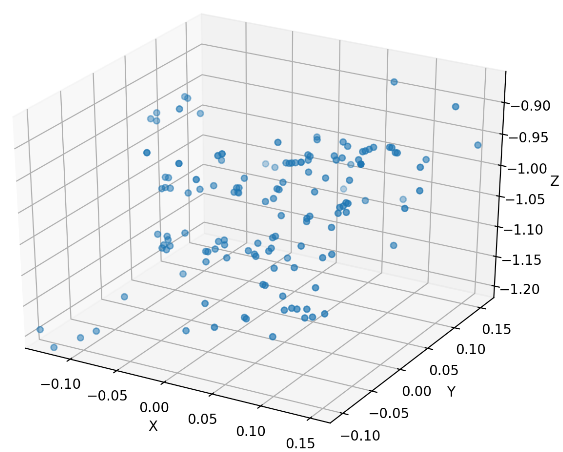
$$err = abs\left(\tilde{\mathbf{x}_{2i}}^T \mathbf{F} \tilde{\mathbf{x}_{1i}}\right) \tag{38}$$

Set tolerance as 0.8 and after 100 iterations, the **ransacF** function was able to find an ideal enough matrix $\mathbf{F}$.

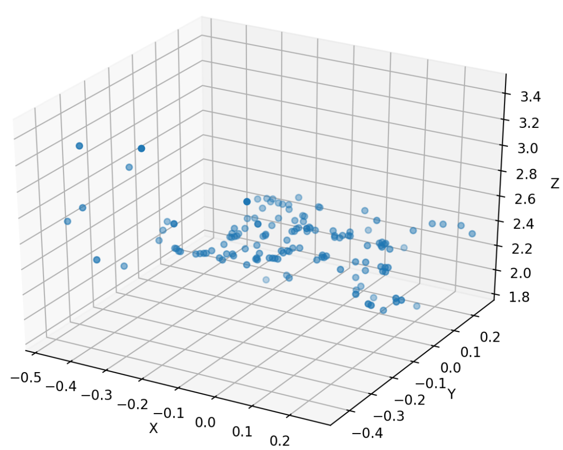
While tuning the parameters, turning the tolerance to a smaller number would decrease the inlier number, which would cause lower accuracy for RANSAC, and with more iterations, RANSAC would be able to find a better solution.

Q5.3

The resulting images are shown below:



(a) without bundle adjustment



(b) with bundle adjustment

Figure 6: 3D Reconstruction with Noisy Data

Without bundle adjustment, the reprojection error is 51.484053051875, while with bundle adjustment the error is 32.15606321697, which significantly decreased.

6 Multi View Keypoint Reconstruction

Q6.1

In this case, I used the triangulate function I've written before to calculate 3 sets of $\begin{bmatrix}\mathbf{w} & err\end{bmatrix}$, and compared the errors to decide the one $\mathbf{w}$ with the smallest error, the chose this $\mathbf{w}$ as the one used in reconstruction. An example resulting image is shown below:

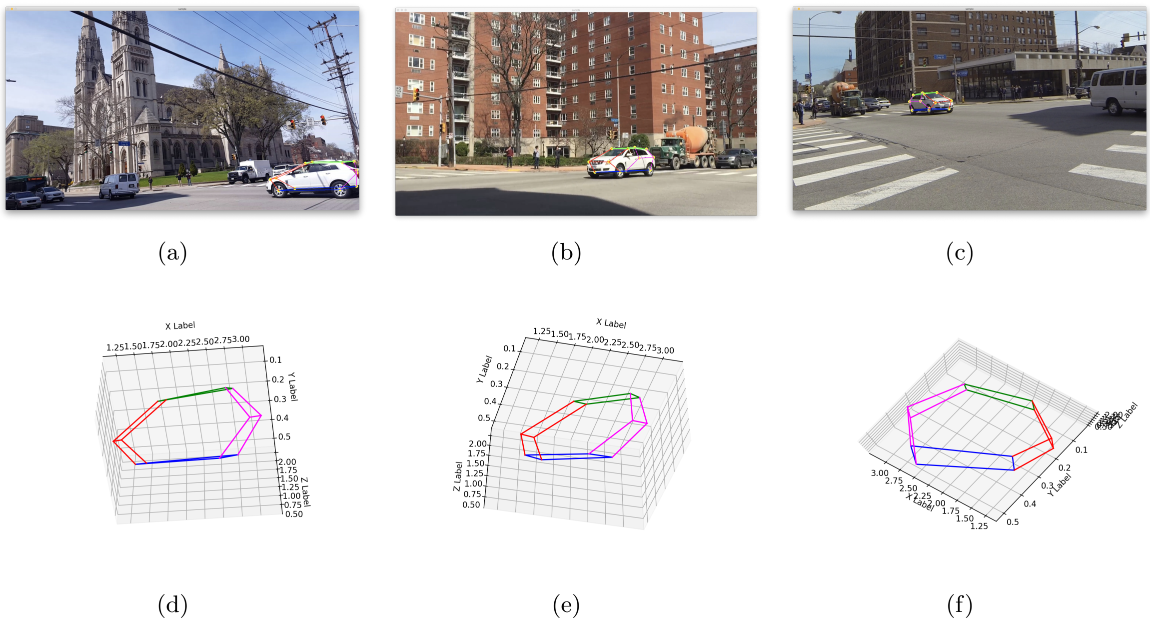


Figure 7: Detections and the Reconstructions from Multiple Views

Tuning the parameter threshold would influence the accuracy in keypoints detection, with lower threshold would lead to more accurate detection and reconstruction. The reconstruction error is 724.8793276.

Q6.2

The reconstruction result is shown below:

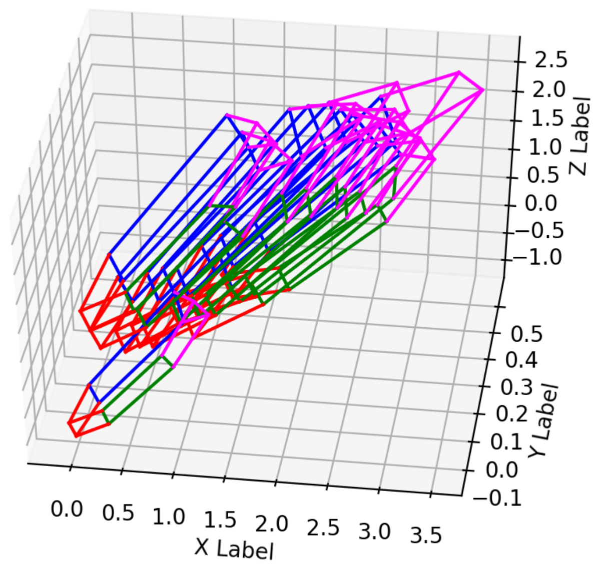


Figure 8: Spatiotemporal Reconstruction