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Safety-Critical Control With Control Barrier Function Based on Disturbance Observer

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***Abstract*—In this article, we investigate the continuous and sampled-data safety-critical control problems with control barrier functions in the presence of time-varying disturbances. To this end, a nonlinear disturbance observer is first designed to estimate the disturbance, and the continuous safe control design of the nominal systems is formulated as a quadratic program. We then design a continuous composite controller by integrating the disturbance compensation term and the state feedback term computed via solving the quadratic program, such that the undesirable influence of time-varying disturbances on both control performance and safety property can be effectively attenuated. It shows that under the proposed continuous control method, the robust safety property of dynamical systems can be strictly guaranteed in the presence of time-varying disturbances. Moreover, the results on the continuous safe control are extended into the sampled-data case, where the control input keeps the same in the inter-sample time intervals. A practical example of adaptive cruise control is introduced, and the simulation results are presented to verify the superiorities of the proposed control method.**

***Index Terms*—Safety-critical control, nonlinear disturbance observer, control barrier function, disturbance/uncertain estimation and attenuation.**

I. Introduction

In many practical applications, the real-time safety property of dynamical systems has been becoming more and more important [1]–[4]. For example, the autonomous vehicles in traffic needs to avoid the collision [2], and the robotic manipulator should avoid the obstacles [3]. Intuitively, the safety means that something bad never happens. The forward invariance of state set is usually used to encode the safety property of dynamical systems. The forward invariance means that the trajectories will stay inside a set forever if they start inside the set.

The control barrier function (CBF) can provide a framework for the control design to guarantee the safety requirement of dynamical systems [5]–[10]. Similar with control Lyapunov function (CLF), some control input constraints dependent on states are imposed by CBFs, such that the control invariance of the set can be guaranteed, and this property ensures that the trajectories of dynamics stay inside the safe region forever [11]–[15]. For the control-affine systems, the safe controller design can be transformed into an online quadratic program with the control input constraints. Therefore, it is of advantage to use the safe control design with CBFs for high-dimensional systems in real-time, which makes it more and more popular in many applications [7], [8].

It is well known that various disturbances including external disturbances and model uncertainties are inevitable in practice [16]–[20]. For safety-critical control with CBFs, the disturbances may lead to the unsafe or dangerous behaviors, since the CBFs heavily rely on the accurate model. In [21], [22], the authors considered the input-to-state safety using CBFs, which shows that due to the existence of disturbances, the proposed control methods only can guarantee the forward invariance of a larger set instead of the original safe set. Several robust CBFs in [23]–[27] were considered in order to ensure the safety for dynamical systems subject to disturbances and uncertainties. Most of the aforementioned results on robust safe control were developed based on the worst-case of disturbances/uncertainties. Such a control design leads to the conservativeness of admissible safe input set, even the infeasibility of the quadratic programs when the disturbances/uncertainties are large enough. To cope with this issue, in [28] the authors integrated the disturbance estimation into the quadratic program via CBF, while the developed quadratic program needs to know the upper bound of the disturbance estimation error. In [29], the authors considered a new disturbance observer-based CBF, which relies on the ultimate convergence bound of the disturbance estimation error. Hence, a careful selection of the bounds is quite important in [28], [29], otherwise, the CBF constraints will lead to some more conservative sets of the admissible safe inputs.

In practice, sampled-data controller via zero-order holder is easier to implement in digital computer [30]. However, the safety property of dynamical systems may be violated in the inter-sample time intervals when the continuous safe control methods proposed by [23], [26], [28], [29] are discretely implemented in digital computer. Hence, discretizing the continuous safe controller via zero-order holder can not solve the sampled-data control problems. To handle this issue, in [8], [11], the authors designed new discrete-time CBFs to guarantee the safety property of the continuous-time systems, while the proposed control methods can not effectively attenuate the influences of disturbances on both safety property and control performance.

In this article, we are concerned about the continuous and sampled-data safety-critical control problems for a class

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of nonlinear control-affine systems subject to time-varying disturbances. By using disturbance/uncertainty estimation and attenuation (DUEA) technique, we first develop a novel nonlinear disturbance observer-based safety-critical control (DOB-SCC) method via CBF in the continuous form. The proposed continuous control method can attenuate the influences of disturbances on both safety property and control performance, and can strictly guarantee that the trajectories disturbed by disturbances stay inside the safe set all the time. In addition, the continuous safe control design is extended into the sampled-data case, where the control input remains constant in the inter-sample time intervals.

The main contributions of this article are summarized as follows:

1) Most of existing robust safe control methods via CBFs (see Refs. [23]–[26]) were developed based on the worst-case of disturbances/uncertainties, which leads to the conservativeness of admissible safe input set. However, in this article we design robust safe control methods by actively attenuating the influence of disturbances, such a control design can greatly reduce the conservativeness of admissible safe input set.
2) In the existing results on continuous safe control with disturbance compensation [28], [29], the proposed control methods need to know the upper bounds of disturbances in CBF constraints, and the constraints of optimization problems include the dynamics of disturbance observers. However, in this paper we remove the aforementioned conditions, and the upper bounds of disturbances are not used in CBF constraints. In addition, the disturbance observer is removed from the optimization problem in the proposed control method, which makes the proposed safe control easier to compute.
3) In [28], [29], the authors proposed the continuous safe control methods with disturbance compensation, while they can not be directly implemented in digital computer, since the safety property of dynamical systems can not be guaranteed in the inter-sample time intervals. To cope with this gap, this article designs a discrete-time CBF constraint and proposes a novel sampled-data safe control method based on DUEA technique, which can strictly ensure the safety property of dynamical systems for all the time, despite in the presence of time-varying disturbances.

Notation: The set of the real numbers is denoted by $\mathbb{R}$. $\mathrm{Int}(\mathcal{C})$ and $\partial(\mathcal{C})$ stand for the interior and boundary of the set $\mathcal{C}$. $|\cdot|$ stands for the absolute value of a scalar. $||\cdot||$ denotes the Euclidean norm of a vector and the corresponding induced matrix norm. A continuous function $\alpha:[0,\infty)\to[0,\infty)$ is said to belong to *class* $\mathcal{K}_\infty$ if $\alpha(0)=0$, $\lim_{r\to\infty}\alpha(r)=\infty$, and it is strictly increasing. A continuous function $\alpha:\mathbb{R}\to\mathbb{R}$ is said to belong to *class* $\mathcal{K}_{\infty,e}$ if $\alpha(0)=0$, $\lim_{r\to\infty}\alpha(r)=\infty$, $\lim_{r\to-\infty}\alpha(r)=-\infty$, and it is strictly increasing.

II. Preliminaries

In this article, the safety-critical control problem is investigated for a class of nonlinear control-affine systems with time-varying disturbances:

$$\dot{x}=f(x)+g_1(x)u+g_2(x)d \tag{1}$$

with system state $x\in\mathbb{R}^n$ and control input $u\in U\subset\mathbb{R}$. $d\in D\subset\mathbb{R}$ is the non-vanishing time-varying disturbance. The functions $f(x):\mathbb{R}^n\to\mathbb{R}^n$, $g_1(x):\mathbb{R}^n\to\mathbb{R}^n$, and $g_2(x):\mathbb{R}^n\to\mathbb{R}^n$ are locally Lipschitz with respect to its arguments. The sets U and D are supposed to be compact. We assume that system (1) is forward complete, that is, system (1) has the unique solution in $t\in[0,\infty)$ for every measurable locally essentially bounded control input and every initial condition.

Some common concepts motivated by [5], [6] are introduced as follows.

Definition 1 *(Robust forward invariance & Robust safety): For a given set $\mathcal{S}\subset\mathbb{R}^n$, we call $\mathcal{S}$ robust forward invariant if for every $x_0\in\mathcal{S}$, it holds that $x(t)\in\mathcal{S}$, $\forall t\geq 0$ and $\forall d\in D$. We call that the robust safety of system (1) is guaranteed on the set $\mathcal{S}\subset\mathbb{R}^n$ if it is robust forward invariant.*

A safe set $\mathcal{C}\subset\mathbb{R}^n$ is first defined and given as follows:

$$\mathcal{C}=\{x\in\mathbb{R}^n:h(x)\geq 0\} \tag{2}$$

where the function $h:\mathbb{R}^n\to\mathbb{R}$ is continuously differentiable. Then, we define $\partial\mathcal{C}=\{x\in\mathbb{R}^n:h(x)=0\}$, and $\mathrm{Int}(\mathcal{C})=\{x\in\mathbb{R}^n:h(x)>0\}$. Suppose that $\mathrm{Int}(\mathcal{C})$ is non-empty and the set $\mathcal{C}$ does not have isolated points, that is, $\overline{\mathrm{Int}(\mathcal{C})}=\mathcal{C}$ and $\mathrm{Int}(\mathcal{C})\neq\emptyset$.

When the disturbance d is absent, i.e., considering the nominal version of system (1), we give the definition of CBF derived from [5], [6] as follows.

Definition 2 *(Control barrier function): For a given set $\mathcal{C}$ shown in (2), we call that the function $h(x)$ is a CBF on the set $\mathcal{C}$ for system (1) without disturbance d if*

$$\sup_{u\in U}\left\{L_fh(x)+L_{g_1}h(x)u\right\}\geq-\alpha_1(h(x)) \tag{3}$$

for all $x\in\mathbb{R}^n$, where $\alpha_1\in\mathcal{K}_{\infty,e}$.

For the nominal version of system (1), we can define a set of control values satisfying the constraint (3) as follows.

$$K_{cbf}(x)=\left\{u\in U:L_fh(x)+L_{g_1}h(x)u\geq-\alpha_1(h(x))\right\} \tag{4}$$

for all $x\in\mathcal{C}$, where $h(x)$ is a CBF on the set $\mathcal{C}$.

When the disturbance $d(t)=0$ in system (1), it can be concluded that any controller $u:\mathcal{C}\to U$ satisfying $u(x)\in K_{cbf}(x)$ can guarantee the forward invariance of the set $\mathcal{C}$, that is, the safety of system (1) can be ensured on the set $\mathcal{C}$. The safety-critical controller can be obtained by solving a quadratic program.

However, when the disturbance d is present, the robust safety property of system (1) may be destroyed and its control performance is also deteriorated. Hence, the safety-critical control satisfying $u(x)\in K_{cbf}(x)$ is not appropriate any longer. In this article, we assume that $L_{g_1}h(x)\neq 0$ and $L_{g_2}h(x)\neq 0$, i.e., the relative degrees for both the control input and the disturbance are equal to 1. When $L_{g_2(x)}h(x)=0$ for all $x\in\mathcal{C}$, it can be seen that the disturbance disappears in $\dot{h}(x)$, i.e., the relative degree of disturbance with respect to $h(x)$ is

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greater than 1. In this case, the robust safety-critical control problem can be solved when $u(x) \in K_{cbf}(x)$. Then, we take three examples to explain the different cases. Consider the following three systems:

$$\begin{aligned} &\text{Example 1: } \dot{x}_1 = x_2 + d,\ \dot{x}_2 = u \\ &\text{Example 2: } \dot{x}_1 = x_2,\ \dot{x}_2 = u + d \\ &\text{Example 3: } \dot{x}_1 = x_2,\ \dot{x}_2 = x_3 + u,\ \dot{x}_3 = -x_3 + d. \end{aligned} \tag{5}$$

The safe set is given as $\mathcal{C} = \{x_2 \in \mathbb{R} :\ h(x) = 2 - x_2 \geq 0\}$, i.e, the goal of the controller design is to guarantee that $x_2 \leq 2$ for all $t \geq 0$. For the three examples, we have that

$$\begin{aligned} &\text{Example 1: } \dot{h}(x) = -\dot{x}_2 = -u \\ &\text{Example 2: } \dot{h}(x) = -\dot{x}_2 = -u - d \\ &\text{Example 3: } \dot{h}(x) = -\dot{x}_2 = x_3 - u. \end{aligned} \tag{6}$$

It can be seen that d disappears in $\dot{h}(x)$ for examples 1 and 3, which means that disturbance does not affect the safe control designs, and we can design the safety-critical controllers when $u(x) \in K_{cbf}(x)$ [5]. However, the disturbance d exists in $\dot{h}(x)$ for example 2, which indicates that the effect of disturbance is inevitable on the safe control design.

The aim of this article is to design a new safe control method for system (1), such that the robust safety property of nonlinear system (1) can be strictly ensured on the set $\mathcal{C}$ in the presence of time-varying disturbances.

III. Continuous Safety-Critical Control via Disturbance Observer

In this article, by designing a nonlinear disturbance observer, we employ the DUEA technique in the continuous robust safety-critical control design, and can strictly guarantee that the set $\mathcal{C}$ is robust forward invariant for system (1).

A. Disturbance Observer Design

Assume that the model of the disturbance d in system (1) can be described by

$$\dot{\xi}(t) = A\xi(t),\ d(t) = C\xi(t) \tag{7}$$

where $\xi \in \mathbb{R}^q$, and A and C have the appropriate dimensions. From a practical point of view, it is reasonable to describe disturbances by using system (7), since many disturbance signals can be modeled as dynamics (7), such as constant signal, ramp signal, and harmonic signal or their combinations [16], [17]. In addition, we suppose that (7) is neutral stable.

Motivated by [16], we design the nonlinear disturbance observer as

$$\begin{aligned} \dot{z} &= (A - s(x)g_2(x)C)z + Ar(x) \\ &\quad - s(x)(g_2Cr(x) + f(x) + g_1(x)u) \\ \hat{\xi} &= z + r(x),\ \hat{d} = C\hat{\xi} \end{aligned} \tag{8}$$

where the internal state variable $z \in \mathbb{R}^q$ and the nonlinear function $r(x) \in \mathbb{R}^q$. $s(x) \in \mathbb{R}^{q\times n}$ is the nonlinear observer gain and designed as $s(x) = \frac{\partial r(x)}{\partial x}$.

Let $e_\xi = \xi - \hat{\xi}$ denote the estimation error. By taking the time derivative of e_ξ, one can obtain the error dynamics as follows:

$$\dot{e}_\xi = (A - s(x)g_2(x)C)e_\xi. \tag{9}$$

Derived from [16], the stability conclusion of dynamics (9) is presented as follows.

Lemma 1 *Consider the nonlinear system (1) with the disturbance model (7). If $s(x)$ is selected such that system (9) is globally exponentially stable regardless of x, then the disturbance estimation error $e = d - \hat{d}$ can exponentially converge to the original point as time goes to infinity, that is, let $V_e = \frac{1}{2}e^2$, there must exist a positive constant β_e such that $\dot{V}_e \leq -2\beta_e V_e$.*

In [16], the detailed selection guidelines of the nonlinear gain function $s(x)$ can be found to ensure the stability property of error dynamics (9), and the parameter β_e can be explicitly obtained.

B. Safety-Critical Controller Design

Based on DUEA technique, a composite control law is designed as

$$u(x, \hat{d}) = \bar{u}(x) + k_d(x)\hat{d} \tag{10}$$

where $\bar{u}(x) = k(x)$ and $k_d(x)\hat{d}$ are the state feedback and disturbance compensation terms to be determined, respectively. We assume that the control gain $k_d(x) : \mathbb{R}^n \to \mathbb{R}$ is selected to make $g_1(x)k_d(x) = -g_2(x)$, which can be achieved in many practical applications [16].

Remark 1 In the proposed composite control law (10), the first one $\bar{u}(x) = k(x)$ is the state feedback term, which is designed based on the nominal system (1) in the absence of disturbance. Due to the existence of disturbance, the trajectories of the control systems may go to the unsafe set when only $\bar{u}(x)$ is used. To cope with this issue, we design the second disturbance compensation term $k_d(x)\hat{d}$, which can be used to attenuate the influence of disturbances on the safety property.

Under the proposed composite control law (10), the closed-loop control system can be formulated as

$$\dot{x}(t) = f(x) + g_1(x)\bar{u}(x) + g_2(x)e. \tag{11}$$

In Lemma 1, we have define a Lyapunov function $V_e = \frac{1}{2}e^2$ to guarantee the global exponential stability of the disturbance estimation error e. Let $h_e(x,t) = h(x) - \beta V_e$, where $\beta > 0$. Define a new safe set $\mathcal{C}_e \subset \mathbb{R}^n$ as follows:

$$\mathcal{C}_e = \{x \in \mathbb{R}^n : h_e(x,t) \geq 0\}. \tag{12}$$

It can be seen from (12) that $\mathcal{C}_e$ is a subset of $\mathcal{C}$, and $\mathcal{C}$ is the safe set for system (1) when $\mathcal{C}_e$ is the safe set for system (1). In addition, $\mathcal{C}_e$ asymptotically converges to $\mathcal{C}$ as time tends to infinity.

Theorem 1 *Consider the nonlinear system (1) with the disturbance model (7). We design the continuous composite controller $u = \bar{u}(x) + k_d(x)\hat{d}$ given in (10) with nonlinear*

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disturbance observer (8). Assume that $h_e(x(0),0) > 0$, $\frac{\partial h_e}{\partial x} \neq 0$ for all $x \in \partial\mathcal{C}_e$, and $\bar{u}$ satisfies

$$L_f h(x) + L_{g_1} h(x)\bar{u} \geq -\beta_c h(x) + \frac{(L_{g_2} h(x))^2}{2\beta(2\beta_e - \beta_c)} \tag{13}$$

where $2\beta_e > \beta_c > 0$, then the trajectories of system (1) are robust safe on the set $\mathcal{C}$ for all $t \geq 0$.

Proof. According to Lemma 1, we have that $\dot{V}_e \leq -2\beta_e V_e$, where β_e is a positive constant. Taking the time derivative of $h_e(x,t)$, we have

$$\begin{aligned}
\dot{h}_e(x,t) &= \dot{h}(x) - \beta\dot{V}_e \\
&\geq L_f h(x) + L_{g_1} h(x)u + L_{g_2} h(x)d + \beta\beta_e e^2 \\
&= L_f h(x) + L_{g_1} h(x)\bar{u} + L_{g_2} h(x)e \\
&\quad + \left(\beta\beta_e - \frac{\beta\beta_c}{2}\right)e^2 + \frac{\beta\beta_c}{2}e^2 \\
&= L_f h(x) + L_{g_1} h(x)\bar{u} - \frac{(L_{g_2} h(x))^2}{2\beta(2\beta_e - \beta_c)} \\
&\quad + \left(\sqrt{\left(\beta\beta_e - \frac{\beta\beta_c}{2}\right)}e + \frac{L_{g_2} h(x)}{\sqrt{2\beta(2\beta_e - \beta_c)}}\right)^2 + \frac{\beta\beta_c}{2}e^2 \\
&\geq L_f h(x) + L_{g_1} h(x)\bar{u} - \frac{(L_{g_2} h(x))^2}{2\beta(2\beta_e - \beta_c)} + \frac{\beta\beta_c}{2}e^2.
\end{aligned} \tag{14}$$

With the CBF constraint condition (13) in mind, one has from (14) that

$$\dot{h}_e(x,t) \geq -\beta_c h(x) + \frac{\beta\beta_c}{2}e^2 = -\beta_c h_e(x,t). \tag{15}$$

By (15), we can conclude that h_e is a CBF on the set $\mathcal{C}_e$ for system (1). Finally, it can be observed that $h_e \geq 0$ for all $t \geq 0$ when $h_e(x(0),0) > 0$ according to Theorem 2 in [31]. Hence, we have that $h \geq 0$ for all $t \geq 0$, which means that system (1) is safe on the set $\mathcal{C}$. □

Remark 2 In Theorem 1, we assume that $h_e(x(0),0) > 0$, where $h_e(x,t) = h(x) - \beta V_e$ and $V_e = \frac{1}{2}e^2$ with $\beta > 0$. At the beginning, the estimation error e is relatively large, which leads to the conservativeness of the proposed control method. However, when the estimation error e exponentially converges to zero as time tends to infinity, the influence of disturbance d on safety disappears, and the conservativeness of the proposed control method becomes smaller. Nevertheless, in [28], [29], the proposed control methods need to know the upper bounds of disturbances, and the CBF constraints of optimization problems need to use the upper bound information of disturbances. Hence, a careful selection of the bounds is quite important in [28], [29], otherwise, the CBF constraints will lead to some more conservative sets of the admissible safe inputs.

Remark 3 In [21], [22], the input-to-state safe control problems were considered in the presence of disturbances, and two kinds of control methods were proposed. The results in [22] show that only the robust forward invariance of a larger set $\mathcal{C}_w \supset \mathcal{C}$ can be guaranteed under the proposed control method, and the closeness between $\mathcal{C}$ and $\mathcal{C}_w$ is bounded by the magnitude of the disturbances. As an extension of [22], the reference [21] considered the safe control with tunable input-to-state safety barrier function, which can regulate the size of the larger invariant set $\mathcal{C}_w$ to approximate the safe set $\mathcal{C}$. However, both the safe control methods proposed in [21], [22] are the functions of the current state, and can not effectively attenuate the influence of disturbances on the control performance and safety property, since the disturbance compensate is not considered in the control design.

Remark 4 In [28], [29], the authors considered the disturbance observer-based safe control methods, where the CBF constraints rely on the upper bound information of disturbance and the disturbance estimates are integrated into the quadratic programs. Different from [28], [29], in this article the composite safe controller (10) includes the state feedback control term $\bar{u}(x)$ and the disturbance compensation term $k_d(x)\hat{d}$. The state feedback term $\bar{u}(x)$ can be computed by solving an optimization problem and should satisfy the following CBF constraint (13). It can be seen from (13) that the disturbance d does influence the CBF constraint. Hence, the disturbance compensation term $k_d(x)\hat{d}$ can be viewed as a pitch of the state feedback term $\bar{u}(x)$, which makes the proposed DOBSCC method easy to implement.

Remark 5 In [29], the developed quadratic program needs to know the ultimate convergence bound of the disturbance estimation error. The results in [29] stated that $\lim_{t\to\infty} \mathcal{C}_e = \mathcal{C}$ only when $\dot{d} = 0$, otherwise, the set $\mathcal{C}_e$ can only be regulated arbitrarily close to the original safe set $\mathcal{C}$, which will lead to a conservative safe control set. However, this article can effectively handle the robust safety-critical control problem in the presence of time-varying disturbances, i.e., $\dot{d} = 0$ is not required. In the proposed DOBSCC method, we do not need to access the bounds of disturbance and disturbance estimation error, and can guarantee that the set $\mathcal{C}_e$ asymptotically converges to the original safe set $\mathcal{C}$ since $\lim_{t\to\infty} e(t) = 0$. Hence, the proposed control method has less conservativeness.

C. Optimization-Based Control

According to Theorem 1, the composite controller (10) satisfying the CBF constraint (13) can only guarantee the robust safety property of system (1), but not its stability. Similar with the CBF constraint, the CLF constraint needs to be employed in the control design to ensure the stability of the closed-loop systems. The input-to-state stability of system (11) is introduced by using input-to-state stable CLF (see Ref. [28]), which is defined as follows.

Definition 3 *(Input-to-state stable control Lyapunov function): A continuously differential and positive definite function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is an input-to-state stable CLF for system (11), if there exist functions $\alpha_2 \in \mathcal{K}_\infty$ and $\alpha_3 \in \mathcal{K}_\infty$ such that for all x and e,*

$$\inf_{u \in U} \left\{ L_f V(x) + L_{g_1} V(x)\bar{u} + L_{g_2} V(x)e \right\} \leq -\alpha_2(V(x)) + \alpha_3(||e||). \tag{16}$$

Let $V(x)$ denote an input-to-state stable CLF for control system (11). Define $\sigma > 0$ as the convergence rate of the CLF. Any nominal control input $\bar{u}$ satisfying the constraint $L_f V(x) +$

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$L_{g_1}V(x)\bar{u} \leq -\sigma V(x)$ can guarantee the input-to-state stability of system (11) in the presence of estimation error e, which means that the composite controller (10) can exponentially stabilize the nonlinear system (1) since the estimation error e is exponentially stable.

Hence, we can design the nominal term $\bar{u}$ in the composite controller (10) for system (1), such that both the stability and safety properties can be guaranteed by solving a quadratic program as follows:

$$\begin{aligned} \min_{\bar{u},\delta} \quad & (\bar{u}-u_{ref})^T H(\bar{u}-u_{ref}) + \mu\delta^2 \\ \text{s.t.} \quad & L_f h(x) + L_{g_1}h(x)\bar{u} \geq -\beta_c h(x) + \frac{(L_{g_2}h(x))^2}{2\beta(2\beta_e-\beta_c)} \\ & L_f V(x) + L_{g_1}V(x)\bar{u} \leq -\sigma V(x) + \delta \\ & \bar{u} \in \Omega \end{aligned} \tag{17}$$

where μ is a positive penalty factor, and δ is a slack parameter. H is a positive definite matrix. Ω is the input constraint set. u_{ref} is the reference control input. The aim of adding the slack parameter is to increase the feasibility of the quadratic program, since it easily becomes infeasible when the constraints of control, stability, and safety are conflicting.

Remark 6 It should be stated that the composite control input u is the actual input. Hence, in (17), the control constraint set Ω is only for the nominal control $\bar{u}$, not for the actual control input u. For a given control constraint set Ω_0, it is easy to make the composite control input u satisfy the constraint set Ω_0. Noting that $u(x,\hat{d}) = \bar{u}(x) + k_d(x)\hat{d}$, we can define a new constraint set Ω for the nominal control input $\bar{u}$, such that the composite control input u satisfies the constraint set Ω_0 if and only if $\bar{u} \in \Omega$.

IV. Sampled-Data Safety-Critical Control via Disturbance Observer

In the previous section, the proposed safe control method has the continuous form, and can not be directly implemented in digital computer, since the robust safety property of dynamical systems can not be guaranteed in the inter-sample time intervals. Hence, the sampled-data DOBSCC method via zero-order holder needs to be considered. In this section, we extend the results on the continuous safe control method into the sampled-data case, which can guarantee the robust safety property of systems in the inter-sample time intervals when the discrete-time CBF constraints are satisfied.

Let T denote the sampling period. By (10), the sampled-data composite control law via zero-order holder is designed as

$$\begin{aligned} u(t) = \; & u(kT) = \bar{u}(x(kT)) + k_d(x(kT))\hat{d}(kT), \\ & t \in [kT,(k+1)T), k = 0,1,\cdots \end{aligned} \tag{18}$$

where the control gain $k_d(x)$ is selected as that in (10), the disturbance estimate $\hat{d}(kT)$ can be obtained by the nonlinear observer (8), and $\bar{u}(x(kT))$ is computed based on a discrete-time quadratic program to be designed.

For the sake of simplicity, let x_k, u_k, $\bar{u}_k$, and e_k denote the shorthands of $x(kT)$, $u(kT)$, $\bar{u}(x(kT))$, and $e(kT)$, respectively. Different from the continuous-time CBF (13), we design a discrete-time CBF constraint as follows:

$$L_f h(x_k) + L_{g_1}h(x_k)\bar{u}_k \geq -\beta_c h(x_k) + \frac{(L_{g_2}h(x_k))^2}{2\beta(2\beta_e-\beta_c)} + \phi(x_k,T) \tag{19}$$

when the margin function $\phi(x_k,T): \mathbb{R}_{>0} \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ satisfies $\phi(x_k,0) = 0$. The margin function $\phi(x_k,T)$ is induced by the sampled-data control, and can be used to guarantee the robust safety of system (1) in the inter-sample time intervals. The aim of the sampled-data safety-critical control is to find the margin function, such that the robust safety property of system (1) can be ensured for all $t \geq 0$ when the discrete-time CBF (19) is satisfied.

For system (1), we define $\mathcal{R}(x_k,T)$ as the set of states that can be researched from x_k in the time interval $[kT, kT+T)$. Since the sets U and D are compact, and the functions $f(x)$, $g_1(x)$, and $g_2(x)$ are locally Lipschitz, we can define

$$\Pi(x_k) = \sup_{z\in\mathcal{R}(x_k,T), u\in U, d\in D} ||f(z) + g_1(z)u + g_2(z)d||. \tag{20}$$

By (20), it can be easily deduced that $||x(t)-x_k|| \leq T\Pi(x_k)$, $t \in [kT,(k+1)T)$, $k = 0,1,\cdots$.

Theorem 2 *Consider the nonlinear system (1) with the disturbance model (7). Let $l_1(x_k)$, $l_2(x_k)$, $l_3(x_k)$, and $l_4(x_k)$ denote the Lipschitz constants of $L_f h(x)$, $L_{g_1}h(x)$, $\beta_c h(x)$, and $L_{g_1}h(x)k_d(x)d$ on the set $\mathcal{R}(x_k,T)$. Under the proposed sampled-data controller (18) with nonlinear disturbance observer (8), assume $h_e(x(0),0) > 0$, $\frac{\partial h_e}{\partial x} \neq 0$ for all $x \in \partial\mathcal{C}_e$, and $\bar{u}$ satisfies*

$$L_f h(x_k) + L_{g_1}h(x_k)\bar{u}_k \geq -\beta_c h(x_k) + \frac{(L_{g_2}h(x_k))^2}{2\beta(2\beta_e-\beta_c)} + \phi(x_k,T) \tag{21}$$

where $2\beta_e > \beta_c > 0$, $\phi(x_k,T) = (l_1(x_k) + l_2(x_k)u_{max} + l_3(x_k) + l_4(x_k))\Pi(x_k)T + l_5 T$, $u_{max} = \max_{u\in U}|u|$, $e_{max} = \max_{t\geq 0}|e(t)|$, $l_5 = (2\beta\beta_e^2 - \beta\beta_c\beta_e)\, e_{max}^2$. Then, we can guarantee that the trajectories of system (1) are robust safe on the set $\mathcal{C}$ for all $t \geq 0$,

Proof. One has that $\dot{V}_e \leq -2\beta_e V_e$ by Lemma 1. By taking the time derivative of $h_e(x,t)$, in the time interval $t \in [kT,(k+1)T)$, it yields

$$\begin{aligned} & \dot{h}_e(x,t) = \dot{h}(x) - \beta\dot{V}_e \\ & \geq L_f h(x) + L_{g_1}h(x)u_k + L_{g_2}h(x)d + \beta\beta_e e^2 \\ & = L_f h(x_k) + L_{g_1}h(x_k)\bar{u}_k + L_{g_2}h(x_k)e_k + \left(\beta\beta_e - \frac{\beta\beta_c}{2}\right)e_k^2 \\ & \quad + \beta_c h(x_k) + G(t) - \beta_c h(x) + \frac{\beta\beta_c}{2}e^2 \end{aligned} \tag{22}$$

where

$$\begin{aligned} G(t) = & L_f h(x) - L_f h(x_k) + L_{g_1}h(x)u_k - L_{g_1}h(x_k)\bar{u}_k \\ & + \beta_c h(x) - \beta_c h(x_k) + \left(\beta\beta_e - \frac{\beta\beta_c}{2}\right)(e^2 - e_k^2) \\ & + L_{g_2}h(x)d - L_{g_2}h(x_k)e_k. \end{aligned} \tag{23}$$

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Noting that $g_1(x)k_d(x) = -g_2(x)$ and $u_k = \bar{u}_k + k_d(x_k)\hat{d}_k$, we have from (23) that

$$\begin{aligned} G(t) =& L_f h(x) - L_f h(x_k) + (L_{g_1}h(x) - L_{g_1}h(x_k))u_k \\ &+ \beta_c h(x) - \beta_c h(x_k) + \left(\beta\beta_e - \frac{\beta\beta_c}{2}\right)(e^2 - e_k^2) \\ &- (L_{g_1}h(x)k_d(x)d - L_{g_1}h(x_k)k_d(x_k)d_k). \end{aligned} \tag{24}$$

According to Lemma 1, we have that the disturbance estimation error e is globally exponentially stable. In addition, since the sets U and D are compact, we can define that $u_{max} = \max_{u\in U}|u|$ and $e_{max} = \max_{t\geq 0}|e(t)|$. Noting that $V_e = \frac{1}{2}e^2$ and $\dot{V}_e \leq -2\beta_e V_e$, one has

$$\left(\beta\beta_e - \frac{\beta\beta_c}{2}\right)(e^2 - e_k^2) \leq l_5 T \tag{25}$$

where $l_5 = \left(2\beta\beta_e^2 - \beta\beta_c\beta_e\right)e_{max}^2$.

Recall that $l_1(x_k)$, $l_2(x_k)$, $l_3(x_k)$, and $l_4(x_k)$ are the Lipschitz constants of $L_f h(x)$, $L_{g_1}h(x)$, $\beta_c h(x)$, and $L_{g_1}h(x)k_d(x)d$ on the set $\mathcal{R}(x_k, T)$. Collecting (24) and (25), in the time interval $t \in [kT, (k+1)T)$ one gets

$$\begin{aligned} |G(t)| \leq& (l_1(x_k) + l_2(x_k)u_{max} + l_3(x_k) + l_4(x_k))||x(t) - x_k|| + l_5 T \\ \leq& (l_1(x_k) + l_2(x_k)u_{max} + l_3(x_k) + l_4(x_k))\Pi(x_k)T + l_5 T \\ =& \phi(x_k, T) \end{aligned} \tag{26}$$

where $\phi(T, x_k)$ has been defined in (21).

By the proof given in Theorem 1, we can easily deduce the following inequality from (22) and (26).

$$\begin{aligned} \dot{h}_e(x,t) \geq& L_f h(x_k) + L_{g_1}h(x_k)\bar{u}_k + \beta_c h(x_k) - \frac{(L_{g_2}h(x_k))^2}{2\beta(2\beta_e - \beta_c)} \\ &- |G(t)| - \beta_c h(x) + \frac{\beta\beta_c}{2}e^2 \\ \geq& L_f h(x_k) + L_{g_1}h(x_k)\bar{u}_k + \beta_c h(x_k) - \frac{(L_{g_2}h(x_k))^2}{2\beta(2\beta_e - \beta_c)} \\ &- \phi(x_k, T) - \beta_c h(x) + \frac{\beta\beta_c}{2}e^2. \end{aligned} \tag{27}$$

When the condition (21) holds, from (27) one can get that

$$\dot{h}_e(x,t) \geq -\beta_c h(x) + \frac{\beta\beta_c}{2}e^2 = -\beta_c h_e(x,t). \tag{28}$$

According to Theorem 2 in [31], it can be observed from (28) that $h_e \geq 0$ for all $t \geq 0$ when $h_e(x(0), 0) > 0$. Hence, we can conclude that $h \geq 0$ for all $t \geq 0$ when the CBF constraint (21) holds, that is, system (1) is safe on the set $\mathcal{C}$ for all $t \geq 0$. □

Assume that for a given sampling period T, the exponential stability of nominal system (1) can be guaranteed when the discrete-time CLF constraint $V(x_{k+1}) - V(x_k) \leq -\sigma V(x_k)$ holds where $0 < \sigma < 1$. Similar with the optimal problem (17), by Theorem 2, the nominal term $\bar{u}$ in the proposed sampled-data controller (18) can be computed by solving the following optimization problem:

$$\begin{aligned} \min_{\bar{u},\delta} \quad & (\bar{u} - u_{ref})^T H(\bar{u} - u_{ref}) + \mu\delta^2 \\ \text{s.t.} \quad & L_f h(x_k) + L_{g_1}h(x_k)\bar{u}_k \\ & \geq -\beta_c h(x_k) + \frac{(L_{g_2}h(x_k))^2}{2\beta(2\beta_e - \beta_c)} + \phi(x_k, T) \\ & V(x_{k+1}) - V(x_k) \leq -\sigma V(x_k) + \delta \\ & \bar{u} \in \Omega \end{aligned} \tag{29}$$

where $\phi(x_k, T)$ has been given in the statement of Theorem 2.

Hence, it can be concluded that the sampled-data composite controller (18) can guarantee the safety and stability of system (1) if the nominal term can be obtained by solving the discrete-time optimization problem (29).

Remark 7 It can be seen from (21) that the margin $\phi(x_k, T)$ can be regulated to be arbitrarily small by selecting an appropriate sampling period T. A small margin $\phi(x_k, T)$ can reduce the conservativeness of the allowable control input set at a cost of increasing the computation burden. In [11], the sampled-data safe control was investigated for continuous-time system without considering disturbances, such a control design can not guarantee the safety property of systems when disturbances are present. In [8], even though the authors considered the sampled-data safe control problem in the presence of disturbances, the disturbances are handled by dominating its influence on safety property, which leads to a conservative set of the allowable safe control inputs. To handle this issue, we propose a composite safe controller to handle the safe control problem in the presence of disturbances.

Remark 8 In Theorem 2, it can be obtained that the Lipschitz constants $l_1(x)$, $l_2(x)$, $l_3(x)$, $l_4(x)$ can be computed for system (1), and we can estimate the upper bound of $\phi(x_k, T) = (l_1(x_k) + l_2(x_k)u_{max} + l_3(x_k) + l_4(x_k))\Pi(x_k)T + l_5 T$, since all the functions used are known and locally Lipschitz. Actually, many practical systems satisfy the Lipschitz conditions, such as adaptive cruise control system (see Ref. [4]) and so on.

V. Simulation Results

This section gives the simulation results of adaptive cruise control to demonstrate the superiorities of the proposed DOB-SCC methods. To save the limited space, we only give the simulation results of the proposed control method in the continuous form. In addition, to demonstrate the superiorities of the proposed DOBSCC method, we present the simulation results of the traditional safety-critical control without using the disturbance compensation. In this simulation, let SCC refer to the traditional safety-critical control. In the adaptive cruise control task (see Refs. [4], [28]), there are two vehicles including the leader and follower. We model the leader and follower as point-masses, which move on the straight road. The follower is driven by adaptive cruise control module, as shown in figure 1. The safety constraint of the follower is to keep a safe distance between the leader and follower.

The position and velocity of the following vehicle are denoted as p and v, respectively. z is the distance between

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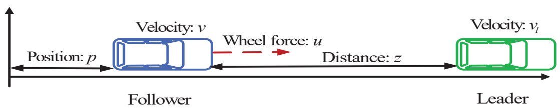

Fig. 1. Adaptive cruise control.

the leader and the follower. Letting $x=[p,v,z]^T$, we can give the dynamics of the adaptive cruise control system as

$$\dot{p}=v,\ \dot{v}=-\frac{F_r}{m}+\frac{1}{m}u+\frac{1}{m}g\Delta\theta,\ \dot{z}=v_l-v \tag{30}$$

where u is the control input, g is the gravitational constant, m denotes the mass of the following vehicle, v_l is the velocity of the lead vehicle, $\Delta\theta$ stands for the disturbances to $\dot{v}$ and is used to reflect aerodynamic force and unmodeled road grad. $F_r=f_0+f_1v+f_2v^2$ refers to the aerodynamics drag term, where f_0, f_1, and f_2 can be empirically determined.

The goal of the adaptive cruise control module is to make the following vehicle to achieve a reference speed v_d, i.e., $(v-v_d)\longrightarrow 0$. In addition, the input constraints of the following car should be satisfied and are given as $-c_dmg\leq u\leq c_amg$, where the two constants c_a and c_d are positive.

The CBF is selected as $h(x)=z-T_hv-\frac{1}{2}\frac{(v-v_l)^2}{c_dg}$, where T_h is the lookahead time. The CLF is set as $V(x)=(v-v_d)^2$. The input reference is $u_{ref}=F_r$. In the quadratic program (17), we set the rates of both CBF and CLF as $\beta_c=\sigma=5$. The positive definite matrix $H=\frac{2}{m^2}$, the positive penalty factor $\mu=2\mathrm{e}^{-2}$. The parameters are given as follows. $g=9.81m/s^2$, $m=1650kg$, $f_0=0.1N$, $f_1=5Ns/m$, $f_2=0.25Ns^2/m$, $c_a=c_d=1$, $T_h=1.8$ second, $v_l=14m/s$, and $v_d=24m/s$.

Let $\Delta\theta=200\cos(2\pi t)$. Then, we have that the disturbance $\Delta\theta$ can be modeled as

$$\dot{\xi}(t)=A\xi(t),\ \Delta\theta(t)=C\xi(t) \tag{31}$$

where $A=[0,1;-4\pi^2,0]$ and $C=[1,0]$.

For the proposed disturbance observer (8), let $s(x)=[0,3363.9,0;0,16819.5,0]$ to make $(A-s(x)g_2(x)C)$ Hurwitz, the function $r(x)$ is set as $r(x)=[3363.9v;16819.5v]$. We select $\beta_e=10$ and $\beta=\frac{1}{300}$.

Figure 2 gives the trajectories of the velocity v of the following vehicle, the distance z to the lead vehicle, the wheel force u, and the slack variable δ under the two control methods. From figure 2, it can be observed that the following vehicle tends to reach the reference velocity $v_d=24m/s$ at the beginning. However the following vehicle starts to decelerate due to the CBF constraint when it is closer to the lead vehicle. Finally, the following vehicle keeps the same velocity $v=v_l=14m/s$ as the lead vehicle, and the distance between them is unchanged. In addition, the control input constraints are satisfied under the both control methods, while under the proposed DOBSCC method, the velocity of the following vehicle is more smooth in contrast to that under SCC method.

In figures 3 and 4, it depicts the trajectories of the CBF and CLF under the two control methods, and the disturbance estimation error under the proposed nonlinear disturbance observer. It can be observed from figure 3 that the robust safety property can not be guaranteed under the traditional SSC method since it does not effectively attenuate the influence of the time-varying disturbance. However, under the proposed DOBSSC method, the CBF $h(x)$ is always nonnegative, which means that the robust safety property can be ensured.

It can be concluded that compared with the traditional safety-critical control with CBF, the proposed DOBSCC method can effectively enhance the control performance and strictly ensure the safety property of the dynamical systems with time-varying disturbances.

VI. Conclusions

This article has considered the continuous and sampled-data robust safety-critical control problem of dynamical systems in the presence of disturbances. To handle this, a new continuous safety-critical control method with control barrier function has been developed based on nonlinear disturbance observer. It shows that under the proposed continuous DOBSSC method, the robust safety property of dynamical systems can be strictly guaranteed in the presence of time-varying disturbances. The results on the continuous safe control method have been extended into the sampled-data case, which can guarantee the robust safety property of systems in the inter-sample time intervals when the discrete-time CBF constraints are satisfied. The simulation results of adaptive cruise control are presented to verify the superiorities of the proposed control method. In the future, we consider the safe control of nonlinear systems with multi-input and multi-disturbance via exponential CBFs.

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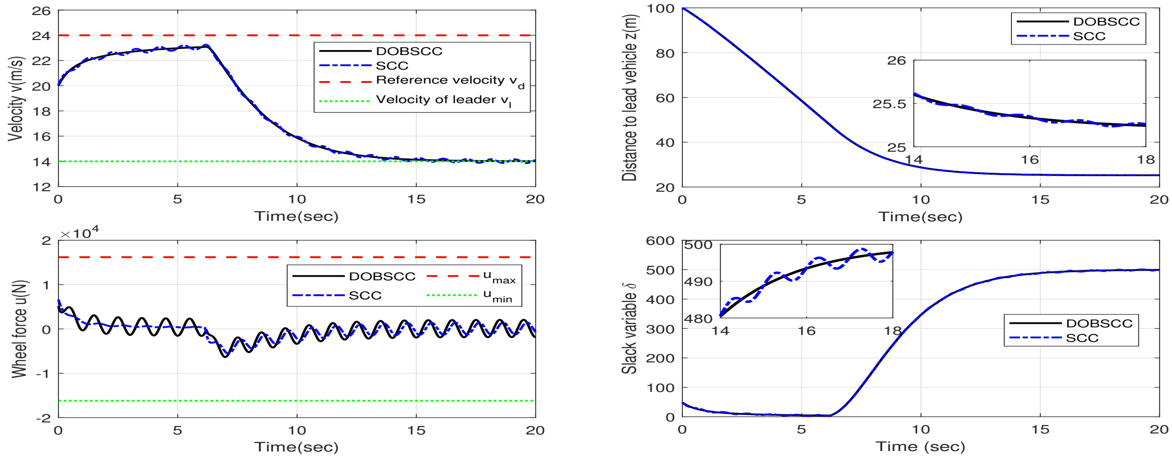

Fig. 2. The trajectories of the velocity v of the following vehicle, the distance z to the lead vehicle, the wheel force u, and the slack variable δ under the proposed DOBSSC method and the traditional SSC method.

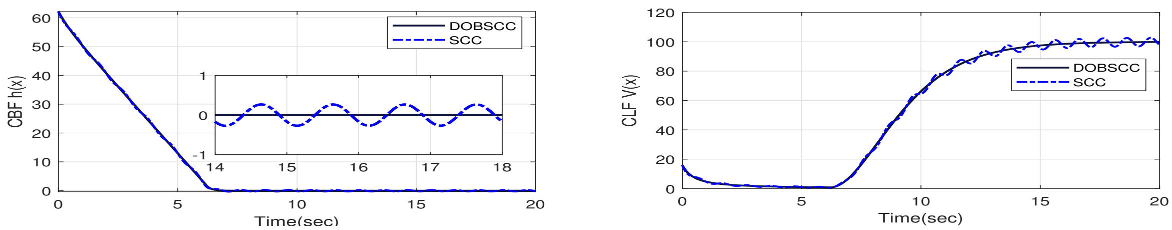

Fig. 3. The trajectories of the CBF and CLF under the proposed DOBSSC method and the traditional SSC method.

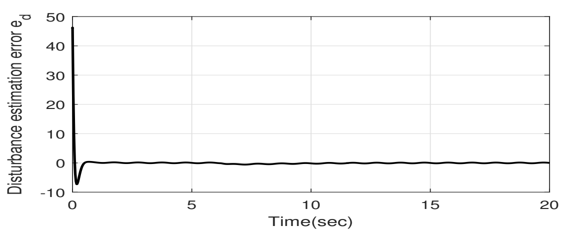

Fig. 4. The trajectories of the disturbance estimation error under the proposed nonlinear disturbance observer.

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