Assignment8

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

https://github.com/N−Manaswini23/Assignmemt8/tree/main/python%20codes

Download latex code from

https://github.com/N−Manaswini23/Assignmemt8/blob/main/assignment8.tex

CSIR UGC JUNE 2013 QUESTION 70

Let X and Y be independent random variables each following a uniform distribution on$(0,1)$.Let $W = XI_{\{Y \leq X^2\}}$,where I_A denotes the indicator function of set A.Then which of the following statements are true?

1) The cumulative distribution function of W is given by
$$F_W(t) = t^2 I_{\{0 \leq t \leq 1\}} + I_{\{t>1\}} \tag{0.0.1}$$
2) $P[W > 0] = \frac{1}{3}$
3) The cumulative distribution function of W is continuous
4) The cumulative distribution function of W is given by
$$F_W(t) = \left(\frac{2+t^3}{3}\right) I_{\{0 \leq t \leq 1\}} + I_{\{t>1\}} \tag{0.0.2}$$

SOLUTION

Given X and Y be two independent random variables.
Given $W = XI_{\{Y \leq X^2\}}$
$X \in (0,1)$,$Y \in (0,1)$,$W \in [0,1)$
The PDF for X is

$$p_X(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & otherwise \end{cases} \tag{0.0.3}$$

The PDF for Y is

$$p_Y(y) = \begin{cases} 1 & 0 < y < 1 \\ 0 & otherwise \end{cases} \tag{0.0.4}$$

$$I_{\{Y \leq X^2\}} = \begin{cases} 1 & Y \leq X^2 \\ 0 & otherwise \end{cases} \tag{0.0.5}$$

$$W = \begin{cases} X & Y \leq X^2 \\ 0 & otherwise \end{cases} \tag{0.0.6}$$

$$p_W(t) = \int_{-\infty}^{\infty} p_X(t) I_{\{Y \leq X^2\}} \text{dy} \tag{0.0.7}$$

$$0 < y < 1 \tag{0.0.8}$$
$$0 < y \leq x^2 \tag{0.0.9}$$

For $0 < t < 1$,

$$p_W(t) = \int_0^{t^2} p_X(t) I_{\{Y \leq X^2\}} \text{dy} \tag{0.0.10}$$
$$= t^2 \tag{0.0.11}$$

For $w = 0$,

$$p_W(W=0) = 1 - \Pr(W > 0) \tag{0.0.12}$$
$$= 1 - \int_0^1 t^2 \text{dt} \tag{0.0.13}$$
$$= \frac{2}{3} \tag{0.0.14}$$

$\therefore$ PDF of W is as follows

$$p_W(t) = \begin{cases} \frac{2}{3} & t = 0 \\ t^2 & 0 < t < 1 \\ 0 & otherwise \end{cases} \tag{0.0.15}$$

$$F_W(t) = \Pr(W \leq t) \tag{0.0.16}$$

The CDF of W is as follows:

$$F_W(t) = \begin{cases} 0 & t < 0 \\ \frac{2+t^3}{3} & 0 \leq t \leq 1 \\ 1 & otherwise \end{cases} \tag{0.0.17}$$

The CDF above can be written as

$$F_W(t) = \left(\frac{2+t^3}{3}\right) I_{\{0 \leq t \leq 1\}} + I_{\{t>1\}} \tag{0.0.18}$$

$$\Pr(W > 0) = 1 - F_W(0) \tag{0.0.19}$$
$$= \frac{1}{3} \tag{0.0.20}$$
$$\therefore \Pr(W > 0) = \frac{1}{3} \tag{0.0.21}$$

$\therefore$ option 2 and 4 are correct.