

Assignment8

CS20Btech11035 -NYALAPOGULA MANASWINI

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CSIR UGC JUNE 2013 QUESTION 70

Let X and Y be independent random variables each following a uniform distribution on $(0, 1)$. Let $W = XI_{\{Y \leq X^2\}}$, where I_A denotes the indicator function of set A . Then which of the following statements are true?

- 1) The cumulative distribution function of W is given by

$$F_W(t) = t^2 I_{\{0 \leq t \leq 1\}} + I_{\{t > 1\}} \quad (0.0.1)$$

- 2) $P[W > 0] = \frac{1}{3}$
- 3) The cumulative distribution function of W is continuous
- 4) The cumulative distribution function of W is given by

$$F_W(t) = \left(\frac{2+t^3}{3}\right) I_{\{0 \leq t \leq 1\}} + I_{\{t > 1\}} \quad (0.0.2)$$

SOLUTION

Given X and Y be two independent random variables.

Given $W = XI_{\{Y \leq X^2\}}$

$X \in (0, 1), Y \in (0, 1), W \in [0, 1]$

The PDF for X is

$$p_X(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.3)$$

The PDF for Y is

$$p_Y(y) = \begin{cases} 1 & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.4)$$

$$I_{\{Y \leq X^2\}} = \begin{cases} 1 & Y \leq X^2 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.5)$$

$$W = \begin{cases} X & Y \leq X^2 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.6)$$

$$p_W(t) = \int_{-\infty}^{\infty} p_X(t) I_{\{Y \leq X^2\}} dy \quad (0.0.7)$$

$$0 < y < 1 \quad (0.0.8)$$

$$0 < y \leq x^2 \quad (0.0.9)$$

For $0 < t < 1$,

$$p_W(t) = \int_0^{t^2} p_X(t) I_{\{Y \leq X^2\}} dy \quad (0.0.10)$$

$$= t^2 \quad (0.0.11)$$

For $w = 0$,

$$p_W(W = 0) = 1 - \Pr(W > 0) \quad (0.0.12)$$

$$= 1 - \int_0^1 t^2 dt \quad (0.0.13)$$

$$= \frac{2}{3} \quad (0.0.14)$$

\therefore PDF of W is as follows

$$p_W(t) = \begin{cases} \frac{2}{3} & t = 0 \\ t^2 & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.15)$$

$$F_W(t) = \Pr(W \leq t) \quad (0.0.16)$$

The CDF of W is as follows:

$$F_W(t) = \begin{cases} 0 & t < 0 \\ \frac{2+t^3}{3} & 0 \leq t \leq 1 \\ 1 & \text{otherwise} \end{cases} \quad (0.0.17)$$

The CDF above can be written as

$$F_W(t) = \left(\frac{2+t^3}{3}\right) I_{\{0 \leq t \leq 1\}} + I_{\{t > 1\}} \quad (0.0.18)$$

$$\Pr(W > 0) = 1 - F_W(0) \quad (0.0.19)$$

$$= \frac{1}{3} \quad (0.0.20)$$

$$\therefore \Pr(W > 0) = \frac{1}{3} \quad (0.0.21)$$

\therefore option 1 and 4 are correct.

