Assignment3

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

https://github.com/N−Manaswini23/assignment3/blob/main/assignment3.py

GATE EC QUESTION 63

Suppose X is a real-valued random variable.Which of the following values CANNOT be attained by$E[X]$ and $E[X^2]$ respectively?

(A) 0 and 1
(B) $\frac{1}{2}$ and $\frac{1}{3}$
(C) 2 and 3
(D) 2 and 5

SOLUTION

We know that

$$var(X) = E\left[X - E(X)\right]^2 \tag{0.0.1}$$

From (0.0.1) , we can conclude that

$$var(X) \geq 0 \tag{0.0.2}$$

We know that

$$var(X) = E[X^2] - (E[X])^2 \tag{0.0.3}$$

From (**??**)

$$E[X^2] - (E[X])^2 \geq 0 \tag{0.0.4}$$

(A) $E[X] = 0$ and $E[X^2] = 1$

$$E[X^2] - (E[X])^2 = 1 - 0 \tag{0.0.5}$$
$$= 1 \tag{0.0.6}$$
$$\therefore E[X^2] - (E[X])^2 \geq 0 \tag{0.0.7}$$

$\therefore E[X] = 0$ and $E[X^2] = 1$ can be attained

(B) $E[X] = \frac{1}{2}$ and $E[X^2] = \frac{1}{3}$

$$E[X^2] - (E[X])^2 = \frac{1}{3} - \frac{1}{4} \tag{0.0.8}$$
$$= \frac{1}{12} \tag{0.0.9}$$
$$\therefore E[X^2] - (E[X])^2 \geq 0 \tag{0.0.10}$$

$\therefore E[X] = \frac{1}{2}$ and $E[X^2] = \frac{1}{3}$ can be attained

(C) $E[X] = 2$ and $E[X^2] = 3$

$$E[X^2] - (E[X])^2 = 3 - 4 \tag{0.0.11}$$
$$= -1 \tag{0.0.12}$$
$$\therefore E[X^2] - (E[X])^2 \leq 0 \tag{0.0.13}$$

$\therefore E[X] = 2$ and $E[X^2] = 3$ cannot be attained

(D) $E[X] = 2$ and $E[X^2] = 5$

$$E[X^2] - (E[X])^2 = 5 - 4 \tag{0.0.14}$$
$$= 1 \tag{0.0.15}$$
$$\therefore E[X^2] - (E[X])^2 \geq 0 \tag{0.0.16}$$

$\therefore E[X] = 2$ and $E[X^2] = 5$ can be attained
$\therefore E[X] = 2$ and $E[X^2] = 3$ cannot be attained