Assignment2

CS20Btech11035 -NYALAPOGULA MANASWINI

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GATE EC QUESTION 63

Suppose X is a real-valued random variable.Which of the following values CANNOT be attained by$E[X]$ and $E[X^2]$ respectively?

1) 0 and 1
2) $\frac{1}{2}$ and $\frac{1}{3}$
3) 2 and 3
4) 2 and 5

SOLUTION

We know that

$$var(X) = E[(X - E[X])^2] \tag{0.0.1}$$

$$var(X) = E[X^2] - (E[X])^2 \tag{0.0.2}$$

For uniform distribution in the interval $[a, b]$

$$var(X) = \frac{(b-a)^2}{12} \tag{0.0.3}$$

For uniform distribution,$(b-a)^2 \geq 0$
By definition of variance,it is average value of $(X - E[X])^2$.
Since $(X - E[X])^2 \geq 0$,average $E[(X - E[X])^2] \geq 0$.

$$\therefore var(X) \geq 0 \tag{0.0.4}$$

$$\therefore E[X^2] - (E[X])^2 \geq 0 \tag{0.0.5}$$

1) $E[X] = 0$ and $E[X^2] = 1$

$$E[X^2] - (E[X])^2 = 1 - 0 \tag{0.0.6}$$

$$= 1 \tag{0.0.7}$$

$$\therefore E[X^2] - (E[X])^2 \geq 0 \tag{0.0.8}$$

$\therefore E[X] = 0$ and $E[X^2] = 1$ can be attained

2) $E[X] = \frac{1}{2}$ and $E[X^2] = \frac{1}{3}$

$$E[X^2] - (E[X])^2 = \frac{1}{3} - \frac{1}{4} \tag{0.0.9}$$

$$= \frac{1}{12} \tag{0.0.10}$$

$$\therefore E[X^2] - (E[X])^2 \geq 0 \tag{0.0.11}$$

$\therefore E[X] = \frac{1}{2}$ and $E[X^2] = \frac{1}{3}$ can be attained

3) $E[X] = 2$ and $E[X^2] = 3$

$$E[X^2] - (E[X])^2 = 3 - 4 \tag{0.0.12}$$

$$= -1 \tag{0.0.13}$$

$$\therefore E[X^2] - (E[X])^2 \leq 0 \tag{0.0.14}$$

$\therefore E[X] = 2$ and $E[X^2] = 3$ cannot be attained

4) $E[X] = 2$ and $E[X^2] = 5$

$$E[X^2] - (E[X])^2 = 5 - 4 \tag{0.0.15}$$

$$= 1 \tag{0.0.16}$$

$$\therefore E[X^2] - (E[X])^2 \geq 0 \tag{0.0.17}$$

$\therefore E[X] = 2$ and $E[X^2] = 5$ can be attained

$\therefore E[X] = 2$ and $E[X^2] = 3$ cannot be attained