Assignment2

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

https://github.com/N−Manaswini23/Assignment−2/blob/main/assignment2%20(2).py

GATE QUESTION 63

Let the random variable X have the distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \leq x < 1 \\ \frac{3}{5} & 1 \leq x < 2 \\ \frac{1}{2} + \frac{x}{8} & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases} \quad (0.0.1)$$

Then $P(2 \leq X < 4)$ is equal to

SOLUTION

Let X be a binomial random variable.
Cumulative distribution function F(x) is given in (0.0.1),it is represented in tabular form (0)

S.No	x(range)	F(x)
1	$x < 0$	0
2	$0 \leq x < 1$	$\frac{x}{2}$
3	$1 \leq x <2$	$\frac{3}{5}$
4	$2 \leq x < 3$	$\frac{1}{2} + \frac{x}{8}$
5	3 ≤ x	1

TABLE 0: This is table 1

We know that

$$F_X(r) = \Pr(X \leq r) \quad (0.0.2)$$

Substituting $x = \{0, 1, 2, 3\}$ in the piecewise cumulative distribution function.

$$x = 0 \implies F(0) = \frac{x}{2} = 0 \quad (0.0.3)$$

$$x = 1 \implies F(1) = \frac{3}{5} \quad (0.0.4)$$

$$x = 2 \implies F(2) = \frac{1}{2} + \frac{x}{8} = \frac{1}{2} + \frac{2}{8} = \frac{3}{4} \quad (0.0.5)$$

$$x = 3 \implies F(3) = 1 \quad (0.0.6)$$

$$x = 4 \implies F(4) = 1 \quad (0.0.7)$$

we get $F(x)$ values at x as respresented in table (0)

S.No	$F(x)$	value of $valueofF(X)$
1	$F(0)$	0
2	$F(1)$	$\frac{3}{5}$
3	$F(2))$	$\frac{3}{4}$
4	$F(3)$	1
5	$F(4)$	1

TABLE 0: This is table2

$P(2 \leq X < 4)$ can be written as follows:

$$P(2 \leq X < 4) = P(0 \leq X < 4) - P(0 \leq X < 2) \quad (0.0.8)$$

$$= P(3 < X < 4) + P(0 \leq X \leq 3) - P(0 \leq X < 2) \quad (0.0.9)$$

$$= P(3 < X < 4) + F(3) - P(0 \leq X < 2) \quad (0.0.10)$$

$$= P(3 < X < 4) + 1 - P(0 \leq X < 2) \quad (0.0.11)$$

we know that

$$P(X > 3) = 1 - P(X \leq 3) \quad (0.0.12)$$

$$= 1 - F(3) \quad (0.0.13)$$

$$= 0 \quad (0.0.14)$$

$$(0.0.15)$$

$$\Pr(0 \le X < 1) = \frac{x}{2} = \frac{1}{2} \quad (0.0.16)$$
$$\Pr(0 \le X < 2) = \frac{3}{5} \quad (0.0.17)$$
$$\Pr(0 \le X < 3) = \frac{x}{8} + \frac{1}{2} = \frac{3}{8} + \frac{1}{2} = \frac{7}{8} \quad (0.0.18)$$

S.No	Pr(X)	value of Pr(X)
1	$\Pr(X < 0)$	0
2	$\Pr(0 \le X < 1)$	$\frac{1}{2}$
3	$\Pr(0 \le X < 2)$	$\frac{3}{5}$
4	$\Pr(0 \le X < 3)$	$\frac{7}{8}$
5	$\Pr(0 \le X \le 3)$	1
6	$\Pr(X > 3)$	0

TABLE 0: This is table3

Substituting (0.0.14) and (0.0.17) in (0.0.11)

$$P(2 \le X < 4) = P(3 < X < 4) + 1 - P(0 \le X < 2) \quad (0.0.19)$$
$$= 0 + 1 - \frac{3}{5} \quad (0.0.20)$$
$$= \frac{2}{5} \quad (0.0.21)$$
$$\therefore P(2 \le X < 4) = \frac{2}{5} \quad (0.0.22)$$

VERIFICATION IF $P(2 \le X < 4) = F(4) - F(2)$

Here we take $F(4) = 1, F(2) = P(0 \le X < 2) = \frac{3}{5}$ because we need to find $P(2 \le X < 4)$, here $X = 2$ is included.If we use $F(2) = \frac{3}{4}$ then we obtain $P(2 < X < 4)$.

$$F(4) - F(2) = 1 - \frac{3}{5} \quad (0.0.23)$$
$$= \frac{2}{5} \quad (0.0.24)$$
$$= P(2 \le X < 4) \quad (0.0.25)$$
$$\therefore P(2 \le X < 4) = F(4) - F(2) \quad (0.0.26)$$

PROOF

$$P(2 \le X < 4) = P(0 \le X < 4) - P(0 \le X < 2) \quad (0.0.27)$$
$$= F(4) - F(2) \quad (0.0.28)$$

Hence proved.