Assignment2

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

https://github.com/N−Manaswini23/Assignment−2/blob/main/assignment2%20(2).py

GATE QUESTION 63

Let the random variable X have the distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \le x < 1 \\ \frac{3}{5} & 1 \le x < 2 \\ \frac{1}{2} + \frac{x}{8} & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases} \tag{0.0.1}$$

Then $P(2 \le X \le 4)$ is equal to

SOLUTION

Let X be a binomial random variable.
Cumulative distribution function F(x) is given in (0.0.1)
CDF(cumulative distribution function) of a random variable X is defined as follows:

$$F_X(r) = \Pr(X \le r) = \int_{-\infty}^{r} f_x(t).dt \tag{0.0.2}$$

where f_x is probability density function.

S.No	x(range)	F(x)
1	$x < 0$	0
2	$0 \le x < 1$	$\frac{x}{2}$
3	$1 \le x < 2$	$\frac{3}{5}$
4	$2 \le x < 3$	$\frac{1}{2} + \frac{x}{8}$
5	3 ≤ x	1

TABLE 0: This is table 1

We need to find $P(2 \le X < 4)$,we know that

$$F(4) - F(2) = P(X \le 4) - P(X \le 2) \tag{0.0.3}$$

$$= \int_{-\infty}^{4} f_x(t).dt - \int_{-\infty}^{2} f_x(t).dt \tag{0.0.4}$$

$$= \int_{2}^{4} f_x(t).dt \tag{0.0.5}$$

$$= P(2 < x < 4) \tag{0.0.6}$$

But we need to find $P(2 \le x < 4)$

$$P(2 \le x < 4) = P(2 < x < 4) + P(X = 2) \tag{0.0.7}$$

$$P(2 \le x < 4) = F(4) - F(2) + P(X = 2) \tag{0.0.8}$$

$$P(X = 2) = \int_{2}^{2} f_x(t).dt \tag{0.0.9}$$

$$= 0 \tag{0.0.10}$$

$$\therefore P(2 \le x < 4) = F(4) - F(2) \tag{0.0.11}$$

According to piecewise function given in the question:

$$4 > 3 \tag{0.0.12}$$

$$\therefore F(4) = 1 \tag{0.0.13}$$

$$F(2) = \frac{1}{2} + \frac{x}{8} \tag{0.0.14}$$

$$= \frac{1}{2} + \frac{2}{8} \tag{0.0.15}$$

$$= \frac{3}{4} \tag{0.0.16}$$

Substituting (0.0.16),(0.0.10) and (0.0.13) in

(0.0.11)

$$P(2 \leq X < 4) = F(4) - F(2) \quad (0.0.17)$$

$$= 1 - \frac{3}{4} \quad (0.0.18)$$

$$= \frac{1}{4} \quad (0.0.19)$$

$$\therefore P(2 \leq X < 4) = \frac{1}{4} \quad (0.0.20)$$

$$(0.0.21)$$