Assignment2

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

https://github.com/N−Manaswini23/Assignment−2/blob/main/assignment2%20(3).py

Download latex code from

https://github.com/N−Manaswini23/Assignment−2/blob/main/assgnment2.tex

GATE EC QUESTION 63

Suppose X is a real-valued random variable.Which of the following values CANNOT be attained by$E[X]$ and $E[X^2]$ respectively?

(A) 0 and 1
(B) $\frac{1}{2}$ and $\frac{1}{3}$
(C) 2 and 3
(D) 2 and 5

SOLUTION

We know that

$$var(X) = E[(X - E[X])^2] \tag{0.0.1}$$
$$var(X) = E[X^2] - (E[X])^2 \tag{0.0.2}$$

For uniform distribution in the interval $[a, b]$

$$var(X) = \frac{(b-a)^2}{12} \tag{0.0.3}$$

From (0.0.1),(0.0.3)(for uniform distribution) , we can conclude that

$$var(X) \geq 0 \tag{0.0.4}$$

From (0.0.2)

$$E[X^2] - (E[X])^2 \geq 0 \tag{0.0.5}$$

(A) $E[X] = 0$ and $E[X^2] = 1$

$$E[X^2] - (E[X])^2 = 1 - 0 \tag{0.0.6}$$
$$= 1 \tag{0.0.7}$$
$$\therefore E[X^2] - (E[X])^2 \geq 0 \tag{0.0.8}$$

$\therefore E[X] = 0$ and $E[X^2] = 1$ can be attained

(B) $E[X] = \frac{1}{2}$ and $E[X^2] = \frac{1}{3}$

$$E[X^2] - (E[X])^2 = \frac{1}{3} - \frac{1}{4} \tag{0.0.9}$$
$$= \frac{1}{12} \tag{0.0.10}$$
$$\therefore E[X^2] - (E[X])^2 \geq 0 \tag{0.0.11}$$

$\therefore E[X] = \frac{1}{2}$ and $E[X^2] = \frac{1}{3}$ can be attained

(C) $E[X] = 2$ and $E[X^2] = 3$

$$E[X^2] - (E[X])^2 = 3 - 4 \tag{0.0.12}$$
$$= -1 \tag{0.0.13}$$
$$\therefore E[X^2] - (E[X])^2 \leq 0 \tag{0.0.14}$$

$\therefore E[X] = 2$ and $E[X^2] = 3$ cannot be attained

(D) $E[X] = 2$ and $E[X^2] = 5$

$$E[X^2] - (E[X])^2 = 5 - 4 \tag{0.0.15}$$
$$= 1 \tag{0.0.16}$$
$$\therefore E[X^2] - (E[X])^2 \geq 0 \tag{0.0.17}$$

$\therefore E[X] = 2$ and $E[X^2] = 5$ can be attained

$\therefore E[X] = 2$ and $E[X^2] = 3$ cannot be attained