

Assignment2

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

[https://github.com/N-Manaswini23/Assignment-2/blob/main/assignment2%20\(3\).py](https://github.com/N-Manaswini23/Assignment-2/blob/main/assignment2%20(3).py)

GATE EC QUESTION 63

Suppose X is a real-valued random variable. Which of the following values CANNOT be attained by $E[X]$ and $E[X^2]$ respectively?

- (A) 0 and 1
- (B) $\frac{1}{2}$ and $\frac{1}{3}$
- (C) 2 and 3
- (D) 2 and 5

SOLUTION

For uniform distribution X,

$$E[X^k] = \int_{-\infty}^{\infty} x^k dF_X(x) \quad (0.0.1)$$

$$var(X) = \int_{-\infty}^{\infty} (X - E[X])^2 dF_X(x) \quad (0.0.2)$$

$$= \int_{-\infty}^{\infty} X^2 dF_X(x) - 2E[X] \int_{-\infty}^{\infty} X dF_X(x) + (E[X])^2 \int_{-\infty}^{\infty} dF_X(x) \quad (0.0.3)$$

$$= E[X^2] - 2(E[X])^2 + (E[X])^2 \quad (0.0.4)$$

$$var(X) = E[X^2] - (E[X])^2 \quad (0.0.5)$$

From (0.0.5) , we can conclude that

$$var(X) \geq 0 \quad (0.0.6)$$

From (0.0.6)

$$E[X^2] - (E[X])^2 \geq 0 \quad (0.0.7)$$

- (A) $E[X] = 0$ and $E[X^2] = 1$

$$E[X^2] - (E[X])^2 = 1 - 0 \quad (0.0.8)$$

$$= 1 \quad (0.0.9)$$

$$\therefore E[X^2] - (E[X])^2 \geq 0 \quad (0.0.10)$$

$\therefore E[X] = 0$ and $E[X^2] = 1$ can be attained

- (B) $E[X] = \frac{1}{2}$ and $E[X^2] = \frac{1}{3}$

$$E[X^2] - (E[X])^2 = \frac{1}{3} - \frac{1}{4} \quad (0.0.11)$$

$$= \frac{1}{12} \quad (0.0.12)$$

$$\therefore E[X^2] - (E[X])^2 \geq 0 \quad (0.0.13)$$

$\therefore E[X] = \frac{1}{2}$ and $E[X^2] = \frac{1}{3}$ can be attained

- (C) $E[X] = 2$ and $E[X^2] = 3$

$$E[X^2] - (E[X])^2 = 3 - 4 \quad (0.0.14)$$

$$= -1 \quad (0.0.15)$$

$$\therefore E[X^2] - (E[X])^2 \leq 0 \quad (0.0.16)$$

$\therefore E[X] = 2$ and $E[X^2] = 3$ cannot be attained

- (D) $E[X] = 2$ and $E[X^2] = 5$

$$E[X^2] - (E[X])^2 = 5 - 4 \quad (0.0.17)$$

$$= 1 \quad (0.0.18)$$

$$\therefore E[X^2] - (E[X])^2 \geq 0 \quad (0.0.19)$$

$\therefore E[X] = 2$ and $E[X^2] = 5$ can be attained

$\therefore E[X] = 2$ and $E[X^2] = 3$ cannot be attained