

Assignment2

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

[https://github.com/N-Manaswini23/Assignment-2/blob/main/assignment2%20\(2\).py](https://github.com/N-Manaswini23/Assignment-2/blob/main/assignment2%20(2).py)

GATE QUESTION 63

Let the random variable X have the distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \leq x < 1 \\ \frac{3}{5} & 1 \leq x < 2 \\ \frac{1}{2} + \frac{x}{8} & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases} \quad (0.0.1)$$

Then $P(2 \leq X \leq 4)$ is equal to

SOLUTION

The given function is cumulative distribution function(cdf) but not probability density function ,because integration over given intervals exceeds 1.

Let X be a binomial random variable.

Cumulative distribution function F(x) is given in (0.0.1)

We know that

$$F_X(r) = \Pr(X \leq r) \quad (0.0.2)$$

S.No	x(range)	F(x)
1	$x < 0$	0
2	$0 \leq x < 1$	$\frac{x}{2}$
3	$1 \leq x < 2$	$\frac{3}{5}$
4	$2 \leq x < 3$	$\frac{1}{2} + \frac{x}{8}$
5	$3 \leq x$	1

TABLE 0: This is table 1

We need to find $P(2 \leq X < 4)$, we know that

$$P(a \leq X < b) = P(0 \leq X < b) - P(0 \leq X < a) \quad (0.0.3)$$

$$= P(0 \leq X \leq b) - P(b) - [P(0 \leq X \leq a) - P(a)] \quad (0.0.4)$$

$$= F(b) - F(a) - P(b) + P(a) \quad (0.0.5)$$

$$\therefore P(2 \leq X < 4) = F(4) - F(2) - P(4) + P(2) \quad (0.0.6)$$

According to piecewise function given in the question:

$$4 > 3 \quad (0.0.7)$$

$$\therefore F(4) = 1 \quad (0.0.8)$$

$$F(2) = \frac{1}{2} + \frac{x}{8} \quad (0.0.9)$$

$$= \frac{1}{2} + \frac{2}{8} \quad (0.0.10)$$

$$= \frac{3}{4} \quad (0.0.11)$$

$$P(4) = P(X \leq 4) - P(X < 4) \quad (0.0.12)$$

$$= F(4) - P(X < 4) \quad (0.0.13)$$

$$= 1 - 1 \quad (0.0.14)$$

$$= 0 \quad (0.0.15)$$

$$P(2) = P(X \leq 2) - P(X < 2) \quad (0.0.16)$$

$$= F(2) - \frac{3}{5} \quad (0.0.17)$$

$$= \frac{3}{4} - \frac{3}{5} \quad (0.0.18)$$

$$= \frac{3}{20} \quad (0.0.19)$$

Substituting (0.0.11),(0.0.19),(0.0.15) and (0.0.8) in

(0.0.6)

$$P(2 \leq X < 4) = F(4) - F(2) + P(2) - P(4) \quad (0.0.20)$$

$$= 1 - \frac{3}{4} - 0 + \frac{3}{20} \quad (0.0.21)$$

$$= \frac{2}{5} \quad (0.0.22)$$

$$\therefore P(2 \leq X < 4) = \frac{2}{5} \quad (0.0.23)$$

$$(0.0.24)$$

