Assignment1

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

https://github.com/N−Manaswini23/assignment1/blob/main/assignment1%20(2).py

GATE QUESTION 63

Let the random variable X have the distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \le x < 1 \\ \frac{3}{5} & 1 \le x < 2 \\ \frac{1}{2} + \frac{x}{8} & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases} \tag{0.0.1}$$

Then $P(2 \le X \le 4)$ is equal to

SOLUTION

The given function is cumulative didtribution function(cdf) but not probability density function ,because integration over given intervals exceeds 1.
Let X be a binomial random variable.
Cumulative distribution function F(x) is given in (0.0.1)
We know that

$$F_X(r) = \Pr(X \le r) \tag{0.0.2}$$

S.No	x(range)	F(x)
1	$x < 0$	0
2	$0 \le x < 1$	$\frac{x}{2}$
3	$1 \le x <2$	$\frac{3}{5}$
4	$2 \le x < 3$	$\frac{1}{2} + \frac{x}{8}$
5	$3 \le$ x	1

TABLE 0: This is table 1

We need to find $P(2 \le X < 4)$,we know that

$$P(a \le X < b) = P(0 \le X < b) - P(0 \le X < a) \tag{0.0.3}$$

$$= P(0 \le X \le b) - P(b) - [P(0 \le X \le a) - P(a)] \tag{0.0.4}$$

$$= F(b) - F(a) - P(b) + P(a) \tag{0.0.5}$$

$$\therefore P(2 \le X < 4) = F(4) - F(2) - P(4) + P(2) \tag{0.0.6}$$

According to piecewise function given in the question:

$$4 > 3 \tag{0.0.7}$$

$$\therefore F(4) = 1 \tag{0.0.8}$$

$$F(2) = \frac{1}{2} + \frac{x}{8} \tag{0.0.9}$$

$$= \frac{1}{2} + \frac{2}{8} \tag{0.0.10}$$

$$= \frac{3}{4} \tag{0.0.11}$$

$$P(4) = P(X \le 4) - P(X < 4) \tag{0.0.12}$$

$$= F(4) - P(X < 4) \tag{0.0.13}$$

$$= 1 - 1 \tag{0.0.14}$$

$$= 0 \tag{0.0.15}$$

$$P(2) = P(X \le 2) - P(X < 2) \tag{0.0.16}$$

$$= F(2) - \frac{3}{5} \tag{0.0.17}$$

$$= \frac{3}{4} - \frac{3}{5} \tag{0.0.18}$$

$$= \frac{3}{20} \tag{0.0.19}$$

Substituting (0.0.11),(0.0.18),(0.0.14) and (0.0.8) in

(0.0.6)

$$P(2 \leq X < 4) = F(4) - F(2) + P(2) - P(4) \tag{0.0.20}$$

$$= 1 - \frac{3}{4} - 0 + \frac{3}{20} \tag{0.0.21}$$

$$= \frac{2}{5} \tag{0.0.22}$$

$$\therefore P(2 \leq X < 4) = \frac{2}{5} \tag{0.0.23}$$

$$\tag{0.0.24}$$