

Assignment 1

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QUESTION 3.3:

Suppose X has a binomial distribution . Show that X = 3 is the most likely outcome.(Hint : P(X = 3) is the maximum among all P(x_i), (x_i= 0,1,2,3,4,5,6). Assume p=0.5

apply logarithm on both sides for equation(0-1) and then differentiate it with respect to p.

$$\log P = \log \left[\binom{n}{k} p^k (1-p)^{n-k} \right] \quad (0-1)$$

$$= \log \left[\frac{n!}{(n-k)!k!} \right] + k \log p + (n-k) \log(1-p) \quad (0-2)$$

SOLUTION:

Let X be a binomial random variable which has probability p=0.5
Given number of times event(X) is performed(n)=6
Given probability of event(p)=0.5
Therefore probability that event(X) does not occur is (1-p)=1-0.5=0.5
We know that binomial probability

Differentiate eq (0-2) with respect to p

$$\begin{aligned} \frac{d \log P}{dp} &= \frac{d \log \left[\frac{n!}{(n-k)!k!} \right]}{dp} + k \frac{d \log p}{dp} \\ &\quad + (n-k) \frac{d \log(1-p)}{dp} \\ &= 0 + \frac{k}{p} - \frac{n-k}{1-p} \end{aligned} \quad (0-3)$$

To find maximum ,substitute $\frac{d \log P}{dp} = 0$ in (0-3)

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \quad (0-1)$$

For P(X=k) to be most likely outcome(highest probability), P(X=k) should be maximum ,where k={0,1,2,3,4,5,6}

$$\begin{aligned} \frac{k}{p} &= \frac{n-k}{1-p} \\ \frac{n-k}{k} &= \frac{1-p}{p} \\ \frac{n}{k} - 1 &= \frac{1}{p} - 1 \\ \frac{n}{k} &= \frac{1}{p} \\ k &= np \end{aligned} \quad (0-4)$$

To find maximum of $P(X=k)$ let us

substituting $n=6, p=1-p=\frac{1}{2}$ in (0-4)

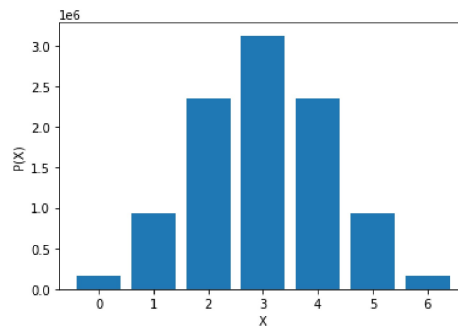
We get $k = 6 * 0.5 = 3$ Therefore

$P(X = 3)$ is maximum,

therefore $P(X=3)$ is most likely outcome.

Hence proved.

Number of times $(X=k)(k=0,1,2,3,4,5,6)$ has occurred out of 10000000 experiments



Submitted by Student unknown on .

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