

Assignment1

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

[https://github.com/N-Manaswini23/assignment1/blob/main/assignment1%20\(2\).py](https://github.com/N-Manaswini23/assignment1/blob/main/assignment1%20(2).py)

QUESTION 3.3:

Suppose X has a binomial distribution. Show that $X = 3$ is the most likely outcome. (Hint : $P(X = 3)$ is the maximum among all $P(x_i)$, ($x_i = 0, 1, 2, 3, 4, 5, 6$). Assume $p = \frac{1}{2}$)

SOLUTION

:

Let X be a binomial random variable which has probability $p = \frac{1}{2}$

$$p(X) = \frac{1}{2} \quad (0.0.1)$$

Given number of times event(X) is performed(n) = 6

$$n(x) = 6 \quad (0.0.2)$$

Given probability of event(p) = $\frac{1}{2}$
Probability that event(X) does not occur is $(1 - p) = 1 - \frac{1}{2} = \frac{1}{2}$

$$1 - p(x) = \frac{1}{2} \quad (0.0.3)$$

We know that binomial probability

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \quad (0.0.4)$$

For $\Pr(X = k)$ to be most likely outcome(highest probability), $\Pr(X = k)$ should be maximum, where $k = \{0, 1, 2, 3, 4, 5, 6\}$

To find maximum of $\Pr(X = k)$, let us apply logarithm on both sides for equation (0.0.4) and then differentiate it with respect to p .

$$\log \Pr(X = k) = \log {}^nC_k \times p^k \times (1 - p)^{n-k} \quad (0.0.5)$$

$$\begin{aligned} &= \log {}^nC_k + k \times \log p \\ &+ (n - k) \times \log(1 - p) \end{aligned} \quad (0.0.6)$$

Differentiate eq (0.0.6) with respect to p

$$\frac{d \log \Pr(X = k)}{dp} = \frac{d \log {}^nC_k}{dp} + k \times \frac{d \log p}{dp} \quad (0.0.7)$$

$$\begin{aligned} &+ (n - k) \times \frac{d \log(1 - p)}{dp} \\ &= 0 + \frac{k}{p} - \frac{n - k}{1 - p} \end{aligned} \quad (0.0.8)$$

To find maximum, substitute $\frac{d \log \Pr(X = k)}{dp} = 0$ in (0.0.8)

$$\frac{k}{p} = \frac{n - k}{1 - p} \quad (0.0.9)$$

$$\frac{n - k}{k} = \frac{1 - p}{p} \quad (0.0.10)$$

$$\frac{n}{k} - 1 = \frac{1}{p} - 1 \quad (0.0.11)$$

$$\frac{n}{k} = \frac{1}{p} \quad (0.0.12)$$

$$k = n \times p \quad (0.0.13)$$

substituting $n = 6, p = 1 - p = \frac{1}{2}$ in (0.0.13)

We get $k = 6 \times \frac{1}{2} = 3$ $\Pr(X = 3)$ is maximum,
 $\therefore \Pr(X = 3)$ is most likely outcome.

Hence proved.

P.T.O

