Assignment1

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

```
https://github.com/N−Manaswini23/assignment1/
    blob/main/assignment1%20(2).py
```

QUESTION 3.3:

Suppose X has a binomial distribution . Show that $X = 3$ is the most likely outcome.(Hint : $P(X = 3)$ is the maximum among all $P(x_i)$, (x_i= 0,1,2,3,4,5,6).Assume $p = \frac{1}{2}$

SOLUTION

:

Let X be a binomial random variable which has probability $p = \frac{1}{2}$

$$p(X) = \frac{1}{2} \tag{0.0.1}$$

Given number of times event(X) is performed(n) = 6

$$n(x) = 6 \tag{0.0.2}$$

Given probability of event(p) = $\frac{1}{2}$
Probability that event(X) does not occur is $(1 - p) = 1 - \frac{1}{2} = \frac{1}{2}$

$$1 - p(x) = \frac{1}{2} \tag{0.0.3}$$

We know that binomial probability

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \tag{0.0.4}$$

For $\Pr(X = k)$ to be most likely outcome(highest probability), $\Pr(X = k)$ should be maximum ,where $k = \{0, 1, 2, 3, 4, 5, 6\}$
To find maximum of $\Pr(X = k)$,let us apply logarithm on both sides for equation (0.0.4) and then diffenrentiate it with respect to p.

$$\log \Pr(X = k) = \log {}^nC_k \times p^k \times (1 - p)^{n-k} \tag{0.0.5}$$

$$= \log {}^nC_k + k \times \log p + (n - k) \times log(1 - p) \tag{0.0.6}$$

Differentaiate eq (0.0.6) with respect to p

$$\frac{d \log \Pr(X = k)}{dp} = \frac{d \log {}^nC_k}{dp} + k \times \frac{d \log p}{dp} + (n - k) \times \frac{d \log(1 - p)}{dp} \tag{0.0.7}$$

$$= 0 + \frac{k}{p} - \frac{n - k}{1 - p} \tag{0.0.8}$$

To find maximum ,substitute $\frac{d \log \Pr(X=k)}{dp} = 0$ in (0.0.8)

$$\frac{k}{p} = \frac{n - k}{1 - p} \tag{0.0.9}$$

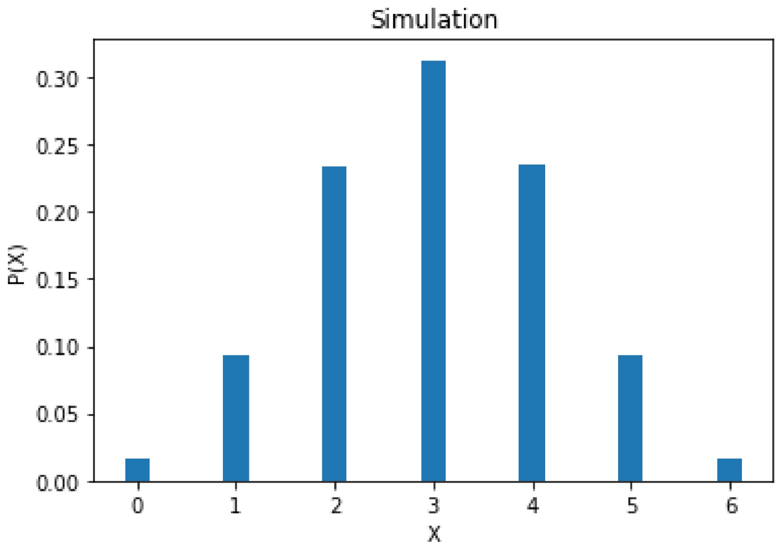
$$\frac{n - k}{k} = \frac{1 - p}{p} \tag{0.0.10}$$

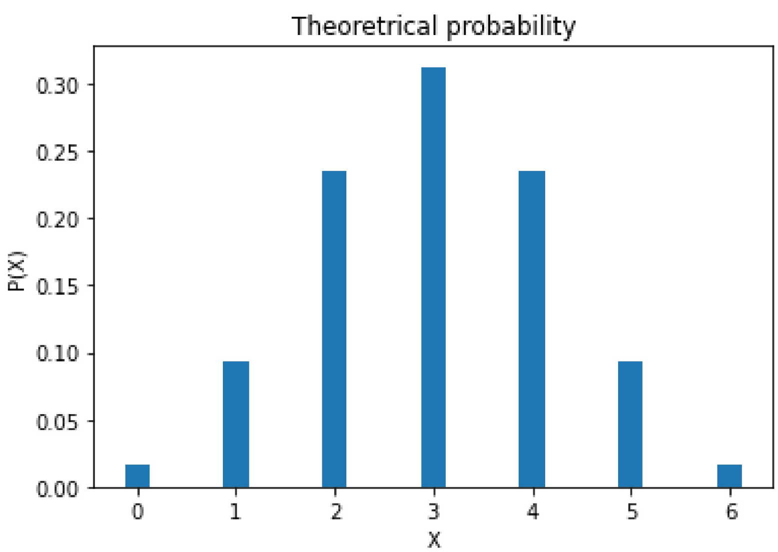
$$\frac{n}{k} - 1 = \frac{1}{p} - 1 \tag{0.0.11}$$

$$\frac{n}{k} = \frac{1}{tp} \tag{0.0.12}$$

$$k = n \times p \tag{0.0.13}$$

substituting $n = 6, p = 1 - p = \frac{1}{2}$ in (0.0.13)
We get $k = 6 \times \frac{1}{2} = 3$ $\Pr(X = 3)$ is maximum,
$\therefore \Pr(X = 3)$ is most likely
outcome.
Hence proved.
P.T.O


Simulation
0.30
0.25
0.20
0.15
0.10
0.05
0.00
P(X)
0
1
2
3
4
5
6
X


Theoretical probability
0.30
0.25
0.20
0.15
0.10
0.05
0.00
P(X)
0
1
2
3
4
5
6
X