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Assignment1

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

https://github.com/N-Manaswini23/assignment1/blob/main/assignment1%20(2).py

QUESTION 3.3:

Suppose X has a binomial distribution . Show that X = 3 is the most likely outcome. (Hint: P(X = 3) is the maximum among all $P(x_i)$, $(x_i = 0,1,2,3,4,5,6)$. Assume p=0.5

SOLUTION

:

Let X be a binomial random variable which has probability p=0.5

$$pX = 0.5$$
 (0.0.1)

Given number of times event(X) is performed(n)=6

$$n(x) = 6 \tag{0.0.2}$$

Given probability of event(p)=0.5Probability that event(X) does not occur is (1-p)=1-0.5=0.5

$$1 - p(x) = 0.5 \tag{0.0.3}$$

We know that binomial probability

$$\Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \tag{0.0.4}$$

For Pr(X = k) to be most likely outcome(highest probability), Pr(X = k) should be maximum ,where $k=\{0,1,2,3,4,5,6\}$

To find maximum of Pr(X = k), let us apply

logarithm on both sides for equation (0.0.4) and then differentiate it with respect to p.

$$\log \Pr(X = k) = \log \binom{n}{k} \times p^k \times (1 - p)^{n-k} \quad (0.0.5)$$

$$= \log \left[\frac{n!}{(n-k) \times !k!} \right] + k \times \log p$$

$$+ (n-k) \times \log(1-p) \quad (0.0.6)$$

Differentaiate eq (0.0.6) with respect to p

$$\frac{\mathrm{d}\log\Pr\left(X=k\right)}{\mathrm{d}p} = \frac{\mathrm{d}\log\left[\frac{n!}{(n-k!)\times k!}\right]}{\mathrm{d}p} + k \times \frac{\mathrm{d}\log p}{\mathrm{d}p}$$

$$+ (n-k) \times \frac{\mathrm{d}\log(1-p)}{\mathrm{d}p}$$

$$= 0 + \frac{k}{p} - \frac{n-k}{1-p} \qquad (0.0.8)$$

To find maximum ,substitute $\frac{d \log Pr(X=k)}{dp} = 0$ in (0.0.8)

$$\frac{\mathbf{k}}{\mathbf{p}} = \frac{\mathbf{n} - \mathbf{k}}{1 - p} \tag{0.0.9}$$

$$\frac{n-k}{k} = \frac{1-p}{p}$$
 (0.0.10)

$$\frac{n}{k} - 1 = \frac{1}{p} - 1 \tag{0.0.11}$$

$$\frac{n}{k} = \frac{1}{p} \tag{0.0.12}$$

$$k = n \times p \tag{0.0.13}$$

substituting $n=6, p=1-p=\frac{1}{2}$ in (0.0.13)We get $k=6\times0.5=3$ Pr (X=3) is maximum, \therefore Pr (X=3) is most likely outcome.

Hence proved.



