Assignment1

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

https://github.com/N−Manaswini23/assignment1/blob/main/assignment1%20(2).py

QUESTION 3.3:

Suppose X has a binomial distribution . Show that X = 3 is the most likely outcome.(Hint : P(X = 3) is the maximum among all P(x_i), (x_i= 0,1,2,3,4,5,6).Assume p=0.5

SOLUTION

:

Let X be a binomial random variable which has probability p=0.5

$$pX = 0.5 \tag{0.0.1}$$

Given number of times event(X) is performed(n)=6

$$n(x) = 6 \tag{0.0.2}$$

Given probability of event(p)=0.5
Probability that event(X) does not occur is (1-p)=1-0.5=0.5

$$1 - p(x) = 0.5 \tag{0.0.3}$$

We know that binomial probability

$$\Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{0.0.4}$$

For $\Pr(X = k)$ to be most likely outcome(highest probability), $\Pr(X = k)$ should be maximum ,where k={0,1,2,3,4,5,6}
To find maximum of $\Pr(X = k)$,let us apply logarithm on both sides for equation (0.0.4) and then diffenrentiate it with respect to p.

$$\log \Pr(X = k) = \log \binom{n}{k} \times p^k \times (1-p)^{n-k} \tag{0.0.5}$$

$$= \log[\left[\frac{n!}{(n-k)\times !k!}\right]] + k \times \log p + (n-k) \times log(1-p) \tag{0.0.6}$$

Differentaiate eq (0.0.6) with respect to p

$$\frac{\mathrm{d} \log \Pr(X = k)}{\mathrm{d}p} = \frac{\mathrm{d} \log \left[\frac{n!}{(n-k!)\times k!}\right]}{\mathrm{d}p} + k \times \frac{\mathrm{d} \log p}{\mathrm{d}p} \tag{0.0.7}$$

$$+ (n-k) \times \frac{\mathrm{d} \log(1-p)}{\mathrm{d}p}$$

$$= 0 + \frac{k}{p} - \frac{n-k}{1-p} \tag{0.0.8}$$

To find maximum ,substitute $\frac{\mathrm{d} \log \Pr(X=k)}{\mathrm{d}p} = 0$ in (0.0.8)

$$\frac{\mathrm{k}}{\mathrm{p}} = \frac{\mathrm{n} - k}{1-p} \tag{0.0.9}$$

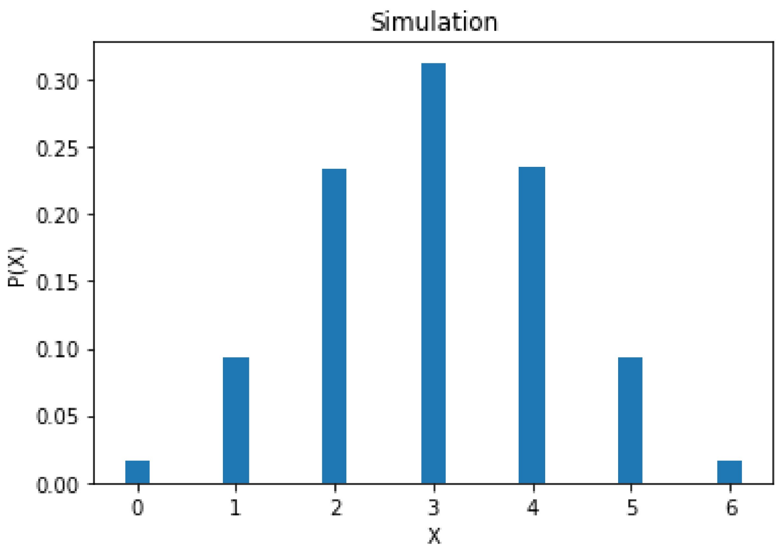
$$\frac{\mathrm{n} - k}{\mathrm{k}} = \frac{1-p}{\mathrm{p}} \tag{0.0.10}$$

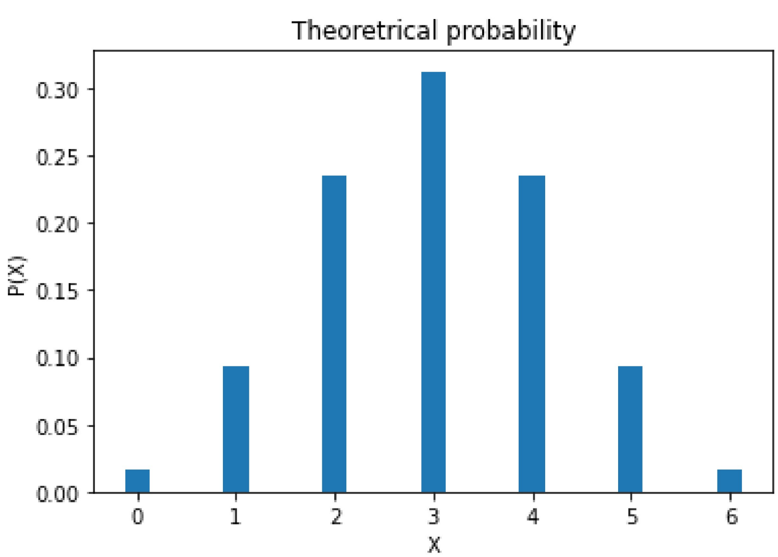
$$\frac{\mathrm{n}}{\mathrm{k}} - 1 = \frac{1}{\mathrm{p}} - 1 \tag{0.0.11}$$

$$\frac{\mathrm{n}}{\mathrm{k}} = \frac{1}{\mathrm{p}} \tag{0.0.12}$$

$$k = n \times p \tag{0.0.13}$$

substituting n=6,p=1-p=$\frac{1}{2}$ in (0.0.13)
We get $k = 6 \times 0.5 = 3$ $\Pr(X = 3)$ is maximum,
$\therefore \Pr(X = 3)$ is most likely
outcome.
Hence proved.


Simulation
0.30
0.25
0.20
0.15
0.10
0.05
0.00
P(X)
0
1
2
3
4
5
6
X


Theoretrical probability
0.30
0.25
0.20
0.15
0.10
0.05
0.00
P(X)
0
1
2
3
4
5
6
X