Assignment1

CS20Btech11035 -NYALAPOGULA MANASWINI

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QUESTION 3.3:

Suppose X has a binomial distribution . Show that $X = 3$ is the most likely outcome.(Hint : $P(X = 3)$ is the maximum among all $P(x_i)$, ($x_i = 0,1,2,3,4,5,6$).Assume $p = \frac{1}{2}$

SOLUTION

:
Let X be a binomial random variable which has probability $p = \frac{1}{2}$

$$p(X) = \frac{1}{2} \qquad (0.0.1)$$

Given number of times event(X) is performed(n) = 6

$$n(x) = 6 \qquad (0.0.2)$$

Given probability of event(p) = $\frac{1}{2}$
Probability that event(X) does not occur is $(1 - p) = 1 - \frac{1}{2} = \frac{1}{2}$

$$1 - p(x) = \frac{1}{2} \qquad (0.0.3)$$

We know that binomial probability

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \qquad (0.0.4)$$

For $\Pr(X = k)$ to be most likely outcome(highest probability), $\Pr(X = k)$ should be maximum ,where $k = \{0, 1, 2, 3, 4, 5, 6\}$
To find maximum of $\Pr(X = k)$,let us apply logarithm on both sides for equation (0.0.4) and then diffenrentiate it with respect to p.

$$\log \Pr(X = k) = \log {}^nC_k \times p^k \times (1 - p)^{n-k} \qquad (0.0.5)$$

$$= \log {}^nC_k + k \times \log p$$
$$+ (n - k) \times log(1 - p) \qquad (0.0.6)$$

Differentaiate eq (0.0.6) with respect to p

$$\frac{d \log \Pr(X = k)}{dp} = \frac{d \log {}^nC_k}{dp} + k \times \frac{d \log p}{dp} \qquad (0.0.7)$$
$$+ (n - k) \times \frac{d \log(1 - p)}{dp}$$

$$= 0 + \frac{k}{p} - \frac{n-k}{1-p} \qquad (0.0.8)$$

To find maximum ,substitute $\frac{d \log \Pr(X=k)}{dp} = 0$ in (0.0.8)

$$\frac{k}{p} = \frac{n-k}{1-p} \qquad (0.0.9)$$

$$\frac{n-k}{k} = \frac{1-p}{p} \qquad (0.0.10)$$

$$\frac{n}{k} - 1 = \frac{1}{p} - 1 \qquad (0.0.11)$$

$$\frac{n}{k} = \frac{1}{p} \qquad (0.0.12)$$

$$k = n \times p \qquad (0.0.13)$$

substituting $n = 6, p = 1 - p = \frac{1}{2}$ in (0.0.13)
We get $k = 6 \times \frac{1}{2} = 3$ $\Pr(X = 3)$ is maximum,
$\therefore \Pr(X = 3)$ is most likely outcome.
Hence proved.
P.T.O

Simulation

Theoretrical probability