

Assignment 1

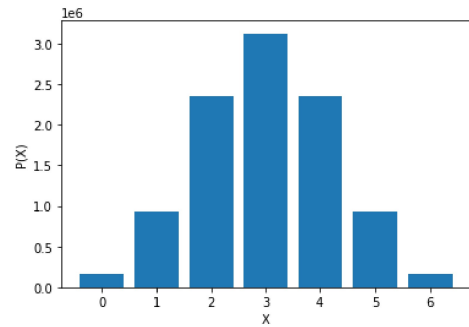
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March 14, 2021

QUESTION:

Suppose X has a binomial distribution . Show that $X = 3$ is the most likely outcome.(Hint : $P(X = 3)$ is the maximum among all $P(x_i)$, ($x_i = 0,1,2,3,4,5,6$). Assume $p=0.5$

Number of times ($X=k$)($k=0,1,2,3,4,5,6$) has occurred out of 10000000 experiments



SOLUTION:

Given number of times event is performed(n)=6

Given probability of event(p)=0.5

Therefore probability that event does not occur is $(1-p)=1-0.5=0.5$

We know that binomial probability

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

substituting $n=6, p=1-p=\frac{1}{2}$

$$\begin{aligned} P(X=k) &= \binom{6}{k} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{6-k} \\ &= \binom{6}{k} \left(\frac{1}{2}\right)^{6-k+k} \\ &= \binom{6}{k} \left(\frac{1}{2}\right)^6 \end{aligned}$$

For $P(X=k)$ to be most likely

outcome(highest probability), $\binom{6}{k}$

should be maximum ,where

$k=\{0,1,2,3,4,5,6\}$ since all

$P(X=k)$ have same power of $\frac{1}{2}$ irrespective of k .

Values of $\binom{6}{k}$ are

$$\binom{6}{0} = 1,$$

$$\binom{6}{1} = 6,$$

$$\binom{6}{2} = 15,$$

$$\binom{6}{3} = 20,$$

$$\binom{6}{4} = 15,$$

$$\binom{6}{5} = 6,$$

$$\binom{6}{6} = 1$$

Therefore $\binom{6}{3}$ is maximum,

therefore $P(X=3)$ is most likely outcome.

Hence proved.