Assignment 1

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QUESTION 3.3:
Suppose X has a binomial distribution . Show that X = 3 is the most likely outcome.(Hint : P(X = 3) is the maximum among all P(x_i), (x_i= 0,1,2,3,4,5,6).Assume p=0.5

SOLUTION:
Let X be a binomial random variable which has probability p=0.5

$$p(X) = 0.5 \tag{0-1}$$

Given number of times event(X) is performed(n)=6

$$n(x) = 6 \tag{0-2}$$

Given probability of event(p)=0.5
Therefore probability that event(X) does not occur is (1-p)=1-0.5=0.5

$$1 - p(x) = 0.5 \tag{0-3}$$

We know that binomial probability

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{0-4}$$

For P(X=k) to be most likely outcome(highest probability), P(X=k) should be maximum ,where k={0,1,2,3,4,5,6}
To find maximum of P(X=k) ,let us apply logarithm on both sides for *equation*(0-4) and then diffenrentiate it with respect to p.

$$\log P = \log[\binom{n}{k} \times p^k \times (1-p)^{n-k}] \tag{0-5}$$

$$= \log[\frac{n!}{(n-k)\times!k!}] + k \times \log p$$
$$+ (n-k) \times log(1-p) \tag{0-6}$$

Differentaiate eq (0-6) with respect to p

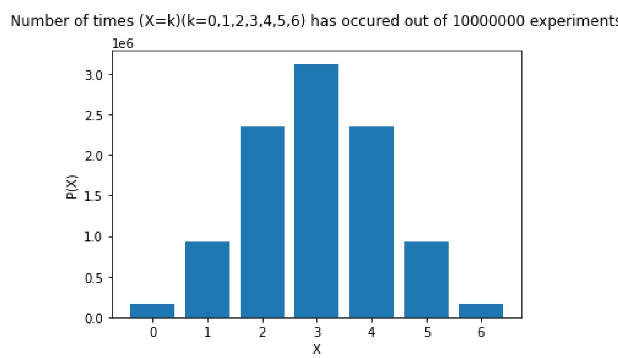
$$\frac{\mathrm{d}\log P}{\mathrm{d}p} = \frac{\mathrm{d}\log[\frac{n!}{(n-k!)\times k!}]}{\mathrm{d}p} + k \times \frac{\mathrm{d}\log p}{\mathrm{d}p}$$
$$+ (n-k) \times \frac{\mathrm{d}\log(1-p)}{\mathrm{d}p}$$
$$= 0 + \frac{k}{p} - \frac{n-k}{1-p} \tag{0-7}$$

To find maximum ,substitute $\frac{\mathrm{d}\log P}{\mathrm{d}p} = 0$ in (0-7)0

$$\frac{k}{p} = \frac{n-k}{1-p}$$
$$\frac{n-k}{k} = \frac{1-p}{p}$$
$$\frac{n}{k} - 1 = \frac{1}{p} - 1$$
$$\frac{n}{k} = \frac{1}{p}$$
$$k = n \times p \tag{0-8}$$

substituting n=6,p=1-p=$\frac{1}{2}$ in (0-8)
We get $k = 6 \times 0.5 = 3$ Therefore $P(X = 3)$ is maximum,
therefore P(X=3) is most likely outcome.
Hence proved.

Submitted by Student unknown *on* .


Number of times (X=k)(k=0,1,2,3,4,5,6) has occured out of 10000000 experiments
1e6
3.0
2.5
2.0
1.5
1.0
0.5
0.0
P(X)
0
1
2
3
4
5
6
X