

# Assignment1

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

[https://github.com/N-Manaswini23/assignment1/blob/main/assignment1%20\(2\).py](https://github.com/N-Manaswini23/assignment1/blob/main/assignment1%20(2).py)

## QUESTION 3.3:

Suppose  $X$  has a binomial distribution. Show that  $X = 3$  is the most likely outcome. (Hint :  $P(X = 3)$  is the maximum among all  $P(x_i)$ , ( $x_i = 0, 1, 2, 3, 4, 5, 6$ ). Assume  $p=0.5$ )

## SOLUTION

:

Let  $X$  be a binomial random variable which has probability  $p=0.5$

$$pX = 0.5 \quad (0.0.1)$$

Given number of times event( $X$ ) is performed( $n$ )=6

$$n(x) = 6 \quad (0.0.2)$$

Given probability of event( $p$ )=0.5  
Probability that event( $X$ ) does not occur is  $(1-p)=1-0.5=0.5$

$$1 - p(x) = 0.5 \quad (0.0.3)$$

We know that binomial probability

$$\Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \quad (0.0.4)$$

For  $\Pr(X = k)$  to be most likely outcome(highest probability),  $\Pr(X = k)$  should be maximum, where  $k=\{0, 1, 2, 3, 4, 5, 6\}$

To find maximum of  $\Pr(X = k)$ , let us apply

logarithm on both sides for equation eqref4 and then differentiate it with respect to  $p$ .

$$\log \Pr(X = k) = \log \left( \binom{n}{k} \times p^k \times (1 - p)^{n-k} \right) \quad (0.0.5)$$

$$= \log \left[ \frac{n!}{(n-k)! \times k!} \right] + k \times \log p + (n-k) \times \log(1-p) \quad (0.0.6)$$

Differentiate eq (??) with respect to  $p$

$$\frac{d \log \Pr(X = k)}{dp} = \frac{d \log \left[ \frac{n!}{(n-k)! \times k!} \right]}{dp} + k \times \frac{d \log p}{dp} \quad (0.0.7)$$

$$+ (n-k) \times \frac{d \log(1-p)}{dp} = 0 + \frac{k}{p} - \frac{n-k}{1-p} \quad (0.0.8)$$

To find maximum, substitute  $\frac{d \log \Pr(X=k)}{dp} = 0$  in (0.0.6)

$$\frac{k}{p} = \frac{n-k}{1-p} \quad (0.0.9)$$

$$\frac{n-k}{k} = \frac{1-p}{p} \quad (0.0.10)$$

$$\frac{n}{k} - 1 = \frac{1}{p} - 1 \quad (0.0.11)$$

$$\frac{n}{k} = \frac{1}{p} \quad (0.0.12)$$

$$k = n \times p \quad (0.0.13)$$

substituting  $n=6, p=1-p=\frac{1}{2}$  in (0.0.11)

We get  $k = 6 \times 0.5 = 3$   $\Pr(X = 3)$  is maximum,  $\therefore \Pr(X = 3)$  is most likely outcome.

Hence proved.

