Assignment5

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

https://github.com/N−Manaswini23/assignment4/tree/main/python%20codes

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https://github.com/N−Manaswini23/assignment4/blob/main/assignment4.tex

GATE 2021 XE-A QUESTION 7(PG:9)

Let (X, Y) have a bivariate normal distribution with the joint probability density function

$$f_{X,Y}(x,y) = \frac{1}{\pi} e^{(\frac{3}{2}xy - \frac{25}{32}x^2 - 2y^2)} \quad (0.0.1)$$

$$-\infty < x, y < \infty \quad (0.0.2)$$

Then $E(XY)$ equals

SOLUTION

Given probability density function for (X, Y)

$$f_{X,Y}(x,y) = \frac{1}{\pi} e^{(\frac{3}{2}xy - \frac{25}{32}x^2 - 2y^2)} \quad (0.0.3)$$

$$-\infty < x, y < \infty \quad (0.0.4)$$

$$\int_{-\infty}^{+\infty} e^{-(ax^2+bx+c)} = \sqrt{\frac{\pi}{a}} e^{\frac{b^2-4ac}{4a}} \quad (0.0.5)$$

$$2\int_{-\infty}^{+\infty} x^2 e^{-x^2} = \int_{-\infty}^{+\infty} e^{-x^2} = \sqrt{\pi} \quad (0.0.6)$$

We need to find $E(XY)$

$$E(XY) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} xy f_{X,Y}(x,y) \mathrm{d}y \mathrm{d}x \quad (0.0.7)$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} xy \frac{1}{\pi} e^{(\frac{3}{2}xy - \frac{25}{32}x^2 - 2y^2)} \mathrm{d}y \mathrm{d}x \quad (0.0.8)$$

$$= \frac{1}{\pi} \int_{-\infty}^{+\infty} x e^{-\frac{25}{32}x^2} \int_{-\infty}^{+\infty} y e^{(\frac{3}{2}xy - 2y^2)} \mathrm{d}y \mathrm{d}x \quad (0.0.9)$$

let

$$I_1 = \int_{-\infty}^{+\infty} y e^{(\frac{3xy}{2} - 2y^2)} \mathrm{d}y \quad (0.0.10)$$

Solving first part of integration of (0.0.9)

$$I_1 = \frac{-1}{4}\left[\int_{-\infty}^{+\infty} (\frac{3x}{2} - 4y) e^{(\frac{3xy}{2} - 2y^2)} \mathrm{d}y\right] - \frac{3}{2}x\frac{(-1)}{4}\left[\int_{-\infty}^{+\infty} e^{(\frac{3}{2}xy - 2y^2)} \mathrm{d}y\right] \quad (0.0.11)$$

On solving this using (0.0.5) and integration rules we get

$$\int_{-\infty}^{+\infty} y e^{(\frac{3xy}{2} - 2y^2)} \mathrm{d}y = \frac{3x}{8}\sqrt{\frac{\pi}{2}} e^{\frac{9x^2}{32}} \quad (0.0.12)$$

Substituting (0.0.11) in (0.0.9) we get

$$E(XY) = \frac{1}{\sqrt{2\pi}} \frac{3}{8} \int_{-\infty}^{+\infty} x^2 e^{-\frac{x^2}{2}} \mathrm{d}x \quad (0.0.13)$$

Solving this using (0.0.6) we get

$$E(XY) = \frac{3}{8} \quad (0.0.14)$$

$$\therefore 8E(XY) = 3 \quad (0.0.15)$$