Assignment5

CS20Btech11035 -NYALAPOGULA MANASWINI

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GATE 2021 XE-A QUESTION 7(PG:9)

Let (X, Y) have a bivariate normal distribution with the joint probability density function

$$f_{X,Y}(x, y) = \frac{1}{\pi}e^{(\frac{3}{2}xy-\frac{25}{32}x^2-2y^2)} \tag{0.0.1}$$

$$-\infty < x, y < \infty \tag{0.0.2}$$

Then $E(XY)$ equals

SOLUTION

Given probability density function for (X, Y)

$$f_{X,Y}(x, y) = \frac{1}{\pi}e^{(\frac{3}{2}xy-\frac{25}{32}x^2-2y^2)} \tag{0.0.3}$$

$$-\infty < x, y < \infty \tag{0.0.4}$$

$$\int_{-\infty}^{+\infty} e^{-(ax^2+bx+c)} = \sqrt{\frac{\pi}{a}}e^{\frac{b^2-4ac}{4a}} \tag{0.0.5}$$

$$2\int_{-\infty}^{+\infty} x^2e^{-x^2} = \int_{-\infty}^{+\infty} e^{-x^2} = \sqrt{\pi} \tag{0.0.6}$$

We need to find $E(XY)$

$$E(XY) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xyf_{X,Y}(x, y)\mathrm{d}y\mathrm{d}x \tag{0.0.7}$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xy\frac{1}{\pi}e^{(\frac{3}{2}xy-\frac{25}{32}x^2-2y^2)}\mathrm{d}y\mathrm{d}x \tag{0.0.8}$$

$$= \frac{1}{\pi}\int_{-\infty}^{+\infty} xe^{-\frac{25}{32}x^2}\int_{-\infty}^{+\infty} ye^{(\frac{3}{2}xy-2y^2)}\mathrm{d}y\mathrm{d}x \tag{0.0.9}$$

let

$$I_1 = \int_{-\infty}^{+\infty} ye^{(\frac{3xy}{2}-2y^2)}\mathrm{d}y \tag{0.0.10}$$

Solving first part of integration of (0.0.9)

$$I_1 = \frac{-1}{4}\left[\int_{-\infty}^{+\infty}(\frac{3x}{2}-4y)e^{(\frac{3xy}{2}-2y^2)}\mathrm{d}y\right] - \frac{3}{2}x\frac{(-1)}{4}\left[\int_{-\infty}^{+\infty} e^{(\frac{3}{2}xy-2y^2)}\mathrm{d}y\right] \tag{0.0.11}$$

On solving this using (0.0.5) and integration rules we get

$$\int_{-\infty}^{+\infty} ye^{(\frac{3xy}{2}-2y^2)}\mathrm{d}y = \frac{3x}{8}\sqrt{\frac{\pi}{2}}e^{\frac{9x^2}{32}} \tag{0.0.12}$$

Substituting (0.0.12) in (0.0.9) we get

$$E(XY) = \frac{1}{\sqrt{2\pi}}\frac{3}{8}\int_{-\infty}^{+\infty} x^2e^{-\frac{x^2}{2}}\mathrm{d}x \tag{0.0.13}$$

Solving this using (0.0.6) we get

$$E(XY) = \frac{3}{8} \tag{0.0.14}$$

$$\therefore 8E(XY) = 3 \tag{0.0.15}$$