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Assignment5

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

https://github.com/N-Manaswini23/assignment5/blob/main/assignment5.py

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GATE 2021 ST QUESTION 44

Let (X, Y) have a bivariate normal distribution with the joint probability density function

$$f_{X,Y}(x,y) = \frac{1}{\pi} e^{(\frac{3}{2}xy - \frac{25}{32}x^2 - 2y^2)}$$
 (0.0.1)

$$-\infty < x, y < \infty \tag{0.0.2}$$

Then E(XY) equals

SOLUTION

Given probability density function for (X, Y)

$$f_{X,Y}(x,y) = \frac{1}{\pi} e^{(\frac{3}{2}xy - \frac{25}{32}x^2 - 2y^2)}$$
 (0.0.3)

$$-\infty < x, y < \infty \tag{0.0.4}$$

Joint pdf of bivariate normal distribution $N(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$ is

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-p^2}} \times e^{\frac{-1}{2(1-p^2)}\left[\left[\frac{(x-\mu_x)}{\sigma_x}\right]^2 + \left[\frac{(y-\mu_y)}{\sigma_y}\right]^2 - 2\rho\left[\frac{(x-\mu_x)}{\sigma_x}\frac{(y-\mu_y)}{\sigma_y}\right]\right]}$$
(0.0.5)

Comparing (0.0.5) and (0.0.3) we get

μ_{x}	μ_{y}	σ_{x}	$\sigma_{\rm y}$	ρ
0	0	1	<u>5</u>	<u>3</u> 5

TABLE 0: Table 1

We need to find E(XY)

$$E(XY) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} xy f_{X,Y}(x, y) dy dx \qquad (0.0.6)$$

Substiting (0.0.5) in (0.0.6) and simplifing ig, we get

$$E(XY) = \rho \sigma_x \sigma_y + \mu_x \mu_y \qquad (0.0.7)$$

Substituting table(0) in (0.0.7) we get

$$E(XY) = \frac{3}{8} \tag{0.0.8}$$

$$\therefore 8E(XY) = 3$$
 (0.0.9)