Assignment5

CS20Btech11035 -NYALAPOGULA MANASWINI

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GATE 2021 ST QUESTION 44

Let (X, Y) have a bivariate normal distribution with the joint probability density function

$$f_{X,Y}(x,y) = \frac{1}{\pi} e^{(\frac{3}{2}xy - \frac{25}{32}x^2 - 2y^2)} \tag{0.0.1}$$

$$-\infty < x, y < \infty \tag{0.0.2}$$

Then $E(XY)$ equals

SOLUTION

Given probability density function for (X, Y)

$$f_{X,Y}(x,y) = \frac{1}{\pi} e^{(\frac{3}{2}xy - \frac{25}{32}x^2 - 2y^2)} \tag{0.0.3}$$

$$-\infty < x, y < \infty \tag{0.0.4}$$

Joint pdf of bivariate normal distribution $N(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$ is

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-p^2}} \times e^{\frac{-1}{2(1-p^2)}\left[\left[\frac{(x-\mu_x)}{\sigma_x}\right]^2 + \left[\frac{(y-\mu_y)}{\sigma_y}\right]^2 - 2\rho\left[\frac{(x-\mu_x)}{\sigma_x}\frac{(y-\mu_y)}{\sigma_y}\right]\right]} \tag{0.0.5}$$

Comparing (0.0.5) and (0.0.3) we get

$$\mu_x = \mu_y = 0 \tag{0.0.6}$$

$$\sigma_x = 1 \tag{0.0.7}$$

$$\sigma_y = \frac{5}{8} \tag{0.0.8}$$

$$\rho = \frac{3}{5} \tag{0.0.9}$$

We need to find $E(XY)$

$$E(XY) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xy f_{X,Y}(x,y)\mathrm{d}y\mathrm{d}x \tag{0.0.10}$$

Substiting (0.0.5) in (0.0.10) and simplifing ig,we get

$$E(XY) = \rho\sigma_x\sigma_y + \mu_x\mu_y \tag{0.0.11}$$

Substituting (0.0.6),(0.0.7),(0.0.8),(0.0.9) in (0.0.11) we get

$$E(XY) = \frac{3}{8} \tag{0.0.12}$$

$$\therefore 8E(XY) = 3 \tag{0.0.13}$$