

# Assignment6

CS20Btech11035 -NYALAPOGULA MANASWINI

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<https://github.com/N-Manaswini23/assignment6/blob/main/python%20codes/assignment6.py>

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## GATE 2019 ME SET-2 QUESTION 28

The variable  $x$  takes a value between 0 and 10 with uniform probability distribution. The variable  $y$  takes a value between 0 and 20 with uniform probability distribution. The probability that sum of variables  $(x + y)$  being greater than 20 is

### SOLUTION

Let  $X$  and  $Y$  be two independent random variables. Let  $Z$  be another random variable where  $Z = X + Y$   
 $X \in [0, 10], Y \in [0, 20], Z \in [0, 30]$

$$\int_0^{10} p_X(x) dx = 1 \quad (0.0.1)$$

$$\therefore p_X(x) = \frac{1}{10} \quad (0.0.2)$$

The PDF for  $X$  is

$$p_X(x) = \begin{cases} \frac{1}{10} & 0 \leq x \leq 10 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.3)$$

Similarly, PDF for  $Y$  is

$$p_Y(y) = \begin{cases} \frac{1}{20} & 0 \leq y \leq 20 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.4)$$

$x = z - y$ . Using this pdf of  $X$  can be written as

$$p_X(z - y) = \begin{cases} \frac{1}{10} & 0 \leq z - y \leq 10 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.5)$$

$$z - 10 \leq y \leq z \quad (0.0.6)$$

$$0 \leq y \leq 20 \quad (0.0.7)$$

pdf of  $Z$  by convolution can be written as

$$p_Z(z) = \int_{-\infty}^{\infty} p_X(z - y) p_Y(y) dy \quad (0.0.8)$$

From 0.0.4 and 0.0.5

$$p_Z(z) = \frac{1}{200} \int_{-\infty}^{\infty} dy \quad (0.0.9)$$

For  $0 \leq z \leq 10$

$$p_Z(z) = \frac{1}{200} \int_0^z dy \quad (0.0.10)$$

$$= \frac{z}{200} \quad (0.0.11)$$

For  $10 < z \leq 20$ ,

$$p_Z(z) = \frac{1}{200} \int_{z-10}^z dy \quad (0.0.12)$$

$$= \frac{1}{20} \quad (0.0.13)$$

For  $20 < z \leq 30$ ,

$$p_Z(z) = \frac{1}{200} \int_{z-10}^{20} dy \quad (0.0.14)$$

$$= \frac{30 - z}{200} \quad (0.0.15)$$

$\therefore$  PDF of  $Z$  is as follows

$$p_Z(z) = \begin{cases} \frac{z}{200} & 0 \leq z \leq 10 \\ \frac{1}{20} & 10 < z \leq 20 \\ \frac{30-z}{200} & 20 < z \leq 30 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.16)$$

$$F_Z(z) = \Pr(Z \leq z) \quad (0.0.17)$$

The CDF of  $Z$  is as follows:

$$F_Z(z) = \begin{cases} 0 & z < 0 \\ \frac{z^2}{400} & z \leq 10 \\ \frac{z-5}{20} & z \leq 20 \\ \frac{60z-500-z^2}{400} & z \leq 30 \\ 1 & z > 30 \end{cases} \quad (0.0.18)$$

$$\Pr(z > 20) = 1 - F_Z(20) \quad (0.0.19)$$

$$= \frac{1}{4} \quad (0.0.20)$$

$$\therefore \Pr(x + y > 20) = \frac{1}{4} = 0.25 \quad (0.0.21)$$

