Assignment6

CS20Btech11035 -NYALAPOGULA MANASWINI

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GATE 2019 ME SET-2 QUESTION 28

The variable x takes a value between 0 and 10 with uniform probability distribution. The variable y takes a value between 0 and 20 with uniform probability distribution.The probability that sum of variables $(x + y)$ being greater than 20 is

SOLUTION

Let X and Y be two independent random variables. Let Z be another random variable where $Z = X + Y$ $X \in [0, 10]$,$Y \in [0, 20]$,$Z \in [0, 30]$

$$\int_0^{10} p_X(x)\text{dx} = 1 \tag{0.0.1}$$

$$\therefore p_X(x) = \frac{1}{10} \tag{0.0.2}$$

The PDF for X is

$$p_X(x) = \begin{cases} \frac{1}{10} & 0 \le x \le 10 \\ 0 & otherwise \end{cases} \tag{0.0.3}$$

Similarly,PDF for Y is

$$p_Y(y) = \begin{cases} \frac{1}{20} & 0 \le y \le 20 \\ 0 & otherwise \end{cases} \tag{0.0.4}$$

$x = z - y$.Using this pdf of X can be written as

$$p_X(z-y) = \begin{cases} \frac{1}{10} & 0 \le z-y \le 10 \\ 0 & otherwise \end{cases} \tag{0.0.5}$$

$$z - 10 \le y \le z \tag{0.0.6}$$

$$0 \le y \le 20 \tag{0.0.7}$$

pdf of Z by convolution can be written as

$$p_Z(z) = \int_{-\infty}^{\infty} p_X(z-y)p_Y(y)\text{dy} \tag{0.0.8}$$

From 0.0.4 and 0.0.5

$$p_Z(z) = \frac{1}{200}\int_{-\infty}^{\infty} \text{dy} \tag{0.0.9}$$

For $0 \le z \le 10$

$$p_Z(z) = \frac{1}{200}\int_0^{z} \text{dy} \tag{0.0.10}$$

$$= \frac{z}{200} \tag{0.0.11}$$

For $10 < z \le 20$,

$$p_Z(z) = \frac{1}{200}\int_{z-10}^{z} \text{dy} \tag{0.0.12}$$

$$= \frac{1}{20} \tag{0.0.13}$$

For $20 < z \le 30$,

$$p_Z(z) = \frac{1}{200}\int_{z-10}^{20} \text{dy} \tag{0.0.14}$$

$$= \frac{30-z}{200} \tag{0.0.15}$$

$\therefore$ PDF of Z is as follows

$$p_Z(z) = \begin{cases} \frac{z}{200} & 0 \le z \le 10 \\ \frac{1}{20} & 10 < z \le 20 \\ \frac{30-z}{200} & 20 < z \le 30 \\ 0 & otherwise \end{cases} \tag{0.0.16}$$

$$F_Z(z) = \Pr(Z \le z) \tag{0.0.17}$$

The CDF of Z is as follows:

$$F_Z(z) = \begin{cases} 0 & z < 0 \\ \frac{z^2}{400} & 0 \le z \le 10 \\ \frac{z-5}{20} & 10 < z \le 20 \\ \frac{60z-500-z^2}{400} & 20 < z \le 30 \\ 1 & z > 30 \end{cases} \tag{0.0.18}$$

$$\Pr(z > 20) = 1 - F_Z(20) \tag{0.0.19}$$
$$= \frac{1}{4} \tag{0.0.20}$$
$$\therefore \Pr(x + y > 20) = \frac{1}{4} = 0.25 \tag{0.0.21}$$