Assignment7

CS20Btech11035 -NYALAPOGULA MANASWINI

Download python code from

https://github.com/N−Manaswini23/assignment7/tree/main/python%20codes

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https://github.com/N−Manaswini23/assignment7/blob/main/assignment7.tex

CSIR UGC NET EXAM(Dec 2016) Q-49

There are two boxes. Box-1 contains 2 red balls and 4 green balls. Box-2 contains 4 red balls and 2 green balls. A box is selected at random and a ball is chosen randomly from the selected box. If the ball turns out to be red, what is the probability that Box-1 had been selected?

SOLUTION

Box-1 has 2 red balls and 4 green balls.
Box-2 has 4 red balls and 2 green balls.
Let $B \in \{1, 2\}$ represent a random variable where 1 represents selecting box-1 and 2 represents selecting box-2.

Event	definition	value
$\Pr(B = 1)$	Probability of selecting Box-1	$\frac{1}{2}$
$\Pr(B = 2)$	Probability of selecting Box-2	$\frac{1}{2}$
$\Pr(R = 1\|B = 1)$	Probability of drawing red ball from Box-1	$\frac{1}{3}$
$\Pr(G = 1\|B = 1)$	Probability of drawing green ball from Box-1	$\frac{2}{3}$
$\Pr(R = 1\|B = 2)$	Probability of drawing red ball from Box-2	$\frac{2}{3}$
$\Pr(G = 1\|B = 2)$	Probability of drawing green ball from Box-2	$\frac{1}{3}$

TABLE 0: Table 1

From Baye's theorem

$$\Pr(R = 1) = \Pr(R = 1|B = 1) \times \Pr(B = 1) + \Pr(R = 1|B = 2) \times \Pr(B = 2) \quad (0.0.1)$$

Substiting values from table (0) in (0.0.1)

$$\Pr(R = 1) = \frac{1}{2} \quad (0.0.2)$$

$$\Pr((R = 1)(B = 1)) = \Pr(R = 1|B = 1) \times \Pr(B = 1) \quad (0.0.3)$$

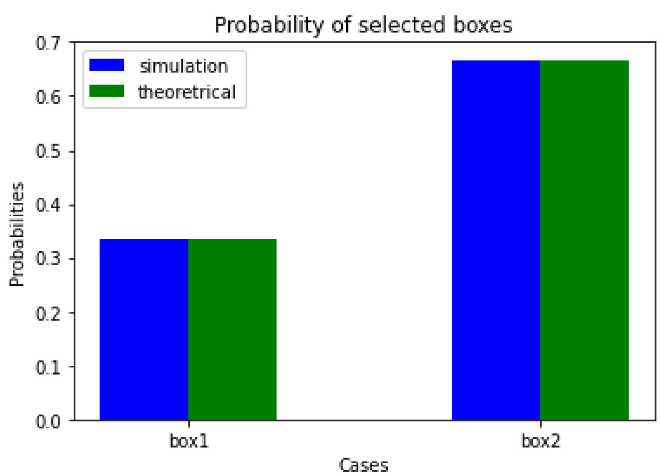
$$= \frac{1}{6} \quad (0.0.4)$$

We need to find $\Pr(B = 1|R = 1)$

$$\Pr(B = 1|R = 1) = \frac{\Pr((R = 1)(B = 1))}{\Pr(R = 1)} \quad (0.0.5)$$

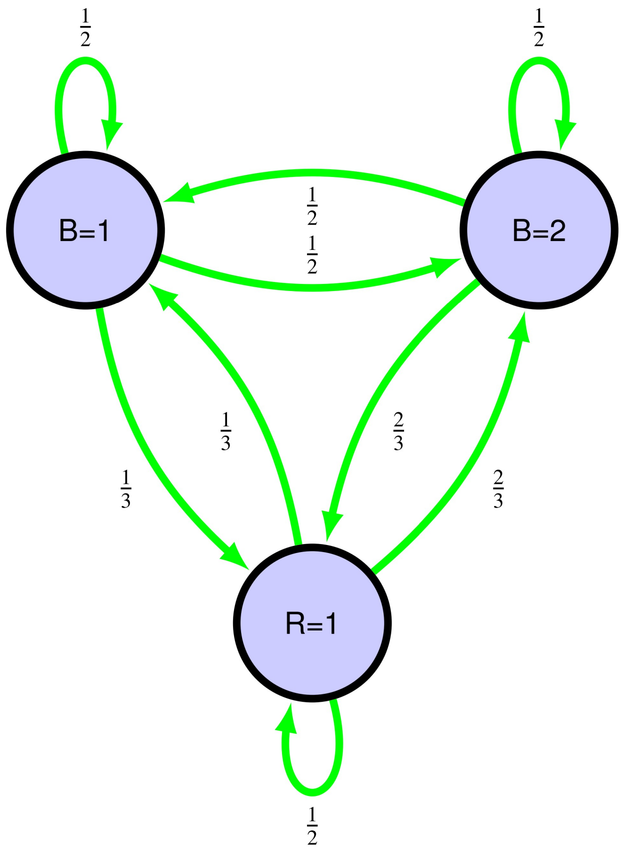
$$= \frac{1}{3} \quad (0.0.6)$$

$\therefore$ The desired probability that box-1 is selected $= \frac{1}{3}$



SOLUTION USING MARKOV CHAIN

markov chain



Let us represent the above markov chain diagram in a transition matrix P.
Let states $\{1, 2, 3\}$ correspond to events $\{B = 1, B = 2, R = 1\}$ respectively.
The value of P_{ij} is equal to probability of transition from state i to state j

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{2} \end{bmatrix} \tag{0.0.7}$$

We need to find $\Pr(B = 1|R = 1)$

$$\Pr(B = 1|R = 1) = P_{31} \tag{0.0.8}$$

$$\therefore \Pr(B = 1|R = 1) = \frac{1}{3} \tag{0.0.9}$$