Research paper presentation

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Title

Performance Analysis of NOMA Assisted Underwater Visible Light Communication System

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Prerequisites

1. Meijer g function
2. fox H function
3. NOMA
4. error function and complementary error function

Non Orthogonal Multiple Access(NOMA)

NOMA was proposed as a candidate radio access technology for 5G cellular systems.Non-orthogonal multiple access (NOMA) has evolved as a spectrally efficient,multiple access scheme that can cater to a large number of devices.This presentation presents analytical investigation of NOMA assisted Underwater Visible Light Communication(UWVLC) system.

Meijer G function

It is defined as follows:

$$G^{m,n}_{p,q}\left[\begin{array}{c|c} a_1,..,a_p & z \\ b_1,..,b_q & \end{array}\right] = \frac{1}{2\pi\mathrm{i}}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j - s)\prod_{j=1}^{n}\Gamma(1-a_j+s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+s)\prod_{j=n+1}^{p}\Gamma(a_j-s)} z^s \mathrm{d}s \tag{1}$$

where,

1. $\Gamma()$ is gamma function
2. $0 \leq m \leq q$,$0 \leq n \leq p$ and m,n,p,q are integers
3. $a_k - b_j \neq 1, 2, 3...$ for $k = 1, 2, ..., n$,$j = 1, 2, ..., m$
4. $z \neq 0$

Fox H function

It is defined as follows:

$$H_{p,q}^{m,n}\left[\, z \,\middle|\, \begin{matrix} (a_1, A_1), .., (a_p, A_p) \\ (b_1, B_1), .., (b_q, B_q) \end{matrix} \,\right] = \frac{1}{2\pi\mathrm{i}} \int_L \frac{\Pi_{j=1}^{m} \Gamma(b_j + B_j s) \Pi_{j=1}^{n} \Gamma(1 - a_j - A_j s)}{\Pi_{j=m+1}^{q} \Gamma(1 - b_j - B_j s) \Pi_{j=n+1}^{p} \Gamma(a_j + A_j s)} z^{-s} \mathrm{d}s \tag{2}$$

Error function(erf) and complementary error function(erfc)

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} \mathrm{d}t \tag{3}$$

$$erfc(z) = 1 - erf(z) \tag{4}$$

NOMA users

The system model of NOMA UWVLC consists of the underwater NOMA users, i.e., near-user UV1 and far user (UV2) at different height or distance from the on-surface floating vehicle (FV).

Transmitted superposition coded signal($\hat{r}_{sc}$)

$$\hat{r}_{sc} = \sqrt{\psi_1} s_1 + \sqrt{\psi_2} s_2 \tag{5}$$

where,

1. $\psi_1 = \delta P_t$ is the ratio of power allocated to UV_1
2. $\psi_2 = (1 - \delta) P_t$ is the ratio of power allocated to UV_2
3. δ is the Power Allocation Coefficient
4. P_t is the power transmitted by the source
5. s_1, s_2 are on-off keying (OOK) modulated symbols for users UV_1 and UV_2 respectively.

Received signal

FV(Floating vehicle) transmits this signal to NOMA users(UV_1,UV_1) as $FV - UV_1$,$FV - UV_2$ respectively(assumed to be EGG (Exponential generalised gamma) distributed.

Received signal

$$y_{UV_i} = \eta I_i \hat{r}_s c + n_i \tag{6}$$

where,

1. $i = 1, 2$
2. η is responsivity(optical to electrical conversion coefficient)
3. I_i is real valued irradiance fluctuation.
4. n_i is the Additive White Gaussian Noice(AWGN) $\sim \mathcal{N}(0, \frac{N_0}{2})$
5. $\hat{r}_{sc}$ is transmitted Super coded signal

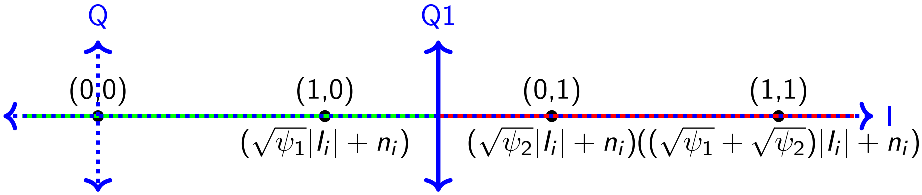
PDF of UWVLC channel

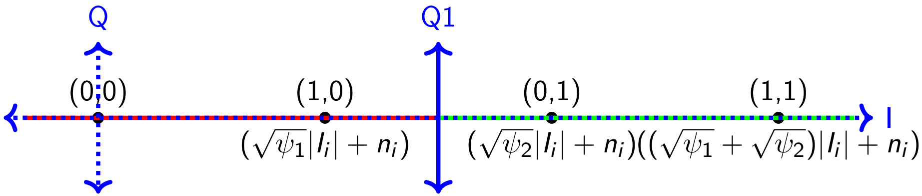
$$f_\rho(\rho) = \frac{w_0}{r\rho} G_{0,1}^{1,0}\left[\frac{1}{\lambda_0}\left(\frac{\rho}{\mu_r}\right)^{\frac{1}{r}} \,\middle|\, \begin{matrix} - \\ 1 \end{matrix} \right] + \frac{c_0(1-w_0)}{r\rho\Gamma(a_0)} G_{0,1}^{1,0}\left[\frac{1}{b_0^{c_0}}\left(\frac{\rho}{\mu_r}\right)^{\frac{c_0}{r}} \,\middle|\, \begin{matrix} - \\ a_0 \end{matrix} \right] \tag{7}$$

where,

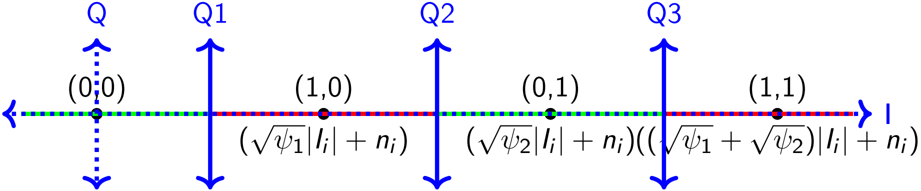
1. $\rho = \frac{(\eta I)^r}{N_0}$
2. N_0 is noise power
3. r depicts detection technique involved
    1. $r = 1$ represents heterodyne detection(HD)
    2. $r = 2$ represents intensity modulation/direct detection (IM/DD)
4. w_0 is mixture coefficient of GG(generalized gamma) distribution
5. λ_0 is scale parameter of Exponential distribution
6. a_0, b_0, c_0 are parameters of GG(generalized gamma) distribution
7. μ_r is function of average electrical SNR(signal to noise ratio)
8. $G, \Gamma()$ are meijer g function,gamma function respectively

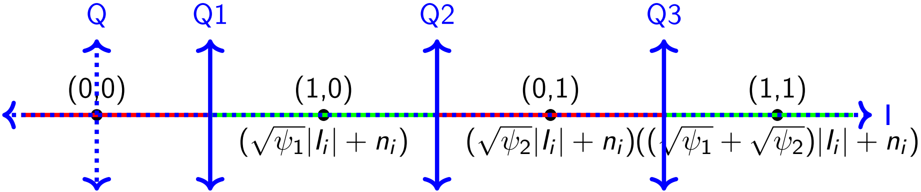
signal constellation of NOMA far user UV_2





signal constellation of NOMA near user UV_1





Average BER

Bit error rate

BER(Bit error rate) is the number of bit errors per unit time.These errors may occur due to noise, interference , distortion etc

Instantaneous BER of far user UV_2

The bit $s2 = 0$ of symbol $(0, 0)$ and $(1, 0)$ is erroneously detected as $s2 = 1$ The PDF of constellation point $(0, 0)$ can be expressed as

$$f_{0,0} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-0)^2}{2\sigma^2}} \tag{8}$$

The error probability of point (0, 0) is($\sigma^2 = \frac{N_0}{2}$)

$$P_{UV_{2|(0,0)}} = \frac{1}{4} \int_{\sqrt{\frac{\rho_1}{2}}}^{\infty} e^{-\frac{(x-0)^2}{N_0^2}} \mathrm{d}x = \frac{1}{4} Q(\sqrt{\rho_1}) \tag{9}$$

Average BER

Instantaneous BER of far user UV_2

Similarly,

$$P_{UV_{2|(1,0)}} = \frac{1}{4}\int_{\sqrt{\frac{\rho_2}{2}}}^{\infty} e^{-\frac{x^2}{N_0^2}}\mathrm{d}x = \frac{1}{4}Q(\sqrt{\rho_2}) \tag{10}$$

$$P_{UV_{2|(0,1)}} = \frac{1}{4}\int_{\sqrt{\frac{\rho_2}{2}}}^{\infty} e^{-\frac{x^2}{N_0^2}}\mathrm{d}x - \frac{1}{4}\int_{\sqrt{\frac{\rho_3}{2}}}^{\infty} e^{-\frac{x^2}{N_0^2}}\mathrm{d}x = \frac{1}{4}\left(Q(\sqrt{\rho_2}) - Q(\sqrt{\rho_3})\right) \tag{11}$$

$$P_{UV_{2|(1,1)}} = \frac{1}{4}\int_{\sqrt{\frac{\rho_1}{2}}}^{\infty} e^{-\frac{x^2}{N_0^2}}\mathrm{d}x - \frac{1}{4}\int_{\sqrt{\frac{\rho_4}{2}}}^{\infty} e^{-\frac{x^2}{N_0^2}}\mathrm{d}x = \frac{1}{4}\left(Q(\sqrt{\rho_1}) - Q(\sqrt{\rho_4})\right) \tag{12}$$

$$P_{UV_2} = P_{UV_{2|(0,0)}} + P_{UV_{2|(1,0)}} + P_{UV_{2|(0,1)}} + P_{UV_{2|(1,1)}} \tag{13}$$

$$P_{UV_2} = \frac{1}{4}\left(2Q(\sqrt{\rho_1}) + 2Q(\sqrt{\rho_2}) - Q(\sqrt{\rho_3}) - Q(\sqrt{\rho_4})\right) \tag{14}$$

Average BER

Instantaneous BER of far user UV_2

where,

1. Q(.) denotes Q-function
2. $\rho_1 = \frac{(\sqrt{\psi_1}+\sqrt{\psi_2})^2|I_2|^2}{2N_0}$
3. $\rho_2 = \frac{(\sqrt{\psi_2}-\sqrt{\psi_1})^2|I_2|^2}{2N_0}$
4. $\rho_3 = \frac{2\psi_2|I_2|^2}{N_0}$
5. $\rho_4 = \frac{2(\sqrt{\psi_1}+\sqrt{\psi_2})^2|I_2|^2}{N_0}$

Instantaneous BER of near user UV_1

$$P_{UV_{1|(0,0)\to(1,0)}} = \frac{1}{4}\left(Q(\sqrt{\rho_6}) - Q(\sqrt{\rho_{12}})\right) \tag{15}$$

$$P_{UV_{1|(0,0)\to(1,1)}} = \frac{1}{4}Q(\sqrt{\rho_{10}}) \tag{16}$$

$$P_{UV_{1|0,1)\to(1,0)}} = \frac{1}{4}\left(Q(\sqrt{\rho_8}) - Q(\sqrt{\rho_9})\right) \tag{17}$$

$$P_{UV_{1|(0,1)\to(1,1)}} = \frac{1}{4}Q(\sqrt{\rho_6}) \tag{18}$$

$$P_{UV_{1|(1,0)\to(0,0)}} = \frac{1}{4}\left(Q(\sqrt{\rho_5}) - Q(\sqrt{\rho_7})\right) \tag{19}$$

$$P_{UV_{1|(1,0)\to(0,1)}} = \frac{1}{4}\left(Q(\sqrt{\rho_8}) - Q(\sqrt{\rho_9})\right) \tag{20}$$

$$P_{UV_{1|(1,1)\to(0,1)}} = \frac{1}{4}\left(Q(\sqrt{\rho_6}) - Q(\sqrt{\rho_{12}})\right) \tag{21}$$

$$P_{UV_{1|(1,1)\to(0,0)}} = \frac{1}{4}\left(Q(\sqrt{\rho_{12}}) - Q(\sqrt{\rho_{11}})\right) \tag{22}$$

Average BER

Instantaneous BER of near user UV_1

$$
\begin{aligned}
P_{UV_1} &= P_{UV_{1|(0,0)\rightarrow(1,0)}} + P_{UV_{1|(0,0)\rightarrow(1,1)}} + P_{UV_{1|0,1)\rightarrow(1,0)}} \\
&\quad + P_{UV_{1|0,1)\rightarrow(1,1)}} + P_{UV_{1|(1,0)\rightarrow(0,0)}} + P_{UV_{1|(1,0)\rightarrow(0,1)}} \\
&\quad + P_{UV_{1|(1,1)\rightarrow(0,1)}} + P_{UV_{1|(1,1)\rightarrow(0,0)}} \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
P_{UV_1} &= \frac{1}{4}\left(Q(\sqrt{\rho_5}) + 3Q(\sqrt{\rho_6}) - Q(\sqrt{\rho_7}) + 2Q(\sqrt{\rho_8}) - 2Q(\sqrt{\rho_9})\right) \\
&\quad + \frac{1}{4}\left(Q(\sqrt{\rho_{10}}) - Q(\sqrt{\rho_{11}}) - Q(\sqrt{\rho_{12}})\right) \qquad (24)
\end{aligned}
$$

Average BER

Instantaneous BER of far user UV_1

where,

1. Q(.) denotes Q-function
2. $\rho_5 = \frac{(\sqrt{\psi_1}-\sqrt{\psi_2})^2|I_1|^2}{2N_0}$
3. $\rho_6 = \frac{\psi_1|I_1|^2}{2N_0}$
4. $\rho_7 = \frac{2\psi_1|I_1|^2}{N_0}$
5. $\rho_8 = \frac{(\sqrt{\psi_2}-\sqrt{\psi_1})^2|I_1|^2}{2N_0}$
6. $\rho_9 = \frac{(2\sqrt{\psi_2}-\sqrt{\psi_1})^2|I_1|^2}{2N_0}$
7. $\rho_{10} = \frac{(2\sqrt{\psi_2}+\sqrt{\psi_1})^2|I_1|^2}{2N_0}$
8. $\rho_{11} = \frac{2(\sqrt{\psi_2}-\sqrt{\psi_1})^2|I_1|^2}{2N_0}$
9. $\rho_{12} = \frac{(\sqrt{\psi_1}+\sqrt{\psi_2})^2|I_1|^2}{2N_0}$

Average BER

Average BER

$$\bar{P}_{UV_i} = \int_0^\infty P_{UV_i} f_\rho(\rho) \mathrm{d}\rho \tag{25}$$

$$Q(x) = \frac{1}{2} erfc(\frac{x}{\sqrt{2}}) \tag{26}$$

$$erfc(\sqrt{x}) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0}\left[x \,\middle|\, \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix} \right] \tag{27}$$

Average BER can be obtained by using (24),(14),(7),(27),(26),we get

$$\begin{aligned} \bar{P}_{UV_1} &= \frac{1}{4}\left(X(\bar{\rho_5}) + 3X(\bar{\rho_6}) - X(\bar{\rho_7}) + 2X(\bar{\rho_8}) - 2X(\bar{\rho_9})\right) \\ &+ \frac{1}{4}\left(X(\bar{\rho_{10}}) - X(\bar{\rho_{11}}) - X(\bar{\rho_{12}})\right) \end{aligned} \tag{28}$$

$$\bar{P}_{UV_2} = \frac{1}{4}\left(2X(\bar{\rho_1}) + 2X(\bar{\rho_2}) - X(\bar{\rho_3}) - X(\bar{\rho_4})\right) \tag{29}$$

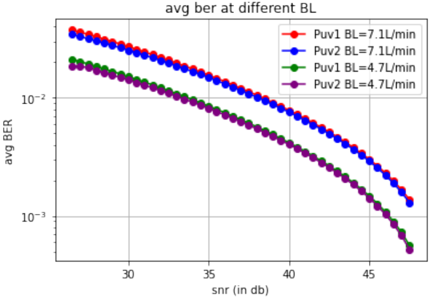
Average BER

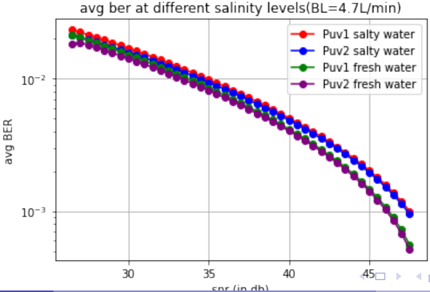
Average BER

where,

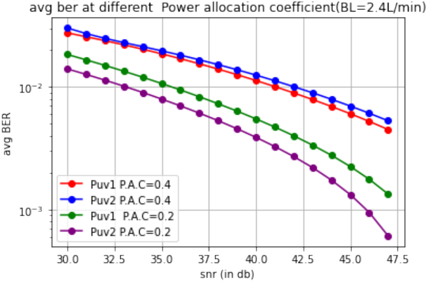
1. $\bar{\rho}_j = E(\rho_j)$ where $j = 1, 2, 3, 4$ for NOMA far user(UV_2) ,$j = 5, 6, 7, ...12$ for NOMA near user(UV_1)
2. $\mu_r = \frac{\bar{\rho}_j}{2w_0\lambda_0^2 + b_0^2(1-w_0)\frac{\Gamma(a_0+\frac{2}{c_0})}{\Gamma(a_0)}}$
3. $$X(\bar{\rho}_j) = \frac{w_0}{2\sqrt{\pi}} H_{2,2}^{1,2}\left[\frac{2}{\lambda_0^r\mu_r} \middle| \begin{matrix} (1,1),(\frac{1}{2},1) \\ (1,r),(0,1) \end{matrix}\right] + \frac{(1-w_0)}{2\sqrt{\pi}\Gamma(a_0)} H_{2,2}^{1,2}\left[\frac{2}{b_0^r\mu_r} \middle| \begin{matrix} (1,1),(\frac{1}{2},1) \\ (a_0,\frac{r}{c_0}),(0,1) \end{matrix}\right] \quad (30)$$
4. $r = 2$

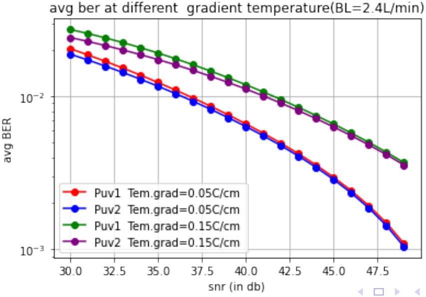
Average BER graphs





Average BER graphs





Ergodic Capacity

Ergodic Capacity

Sum rate of NOMA UWVLC (Ergodic Capacity) is given as

$$C_{sum} = \sum_{i=1}^{2} C_{UV_i} \tag{31}$$

$$C_{UV_i} = E[\ln(1 + \zeta \rho_{UV_i})] \tag{32}$$

$$\zeta = \frac{e}{2\pi} \tag{33}$$

Solving (32) we get

$$C_{UV_i} = w_0 H^{3,1}_{2,3}\left[\frac{1}{\lambda_0^r \zeta \mu_r} \middle| \begin{matrix} (0,1),(1,1) \\ (1,r),(0,1),(0,1) \end{matrix} \right] + \frac{(1-w_0)}{\Gamma(a_0)} H^{3,1}_{2,3}\left[\frac{1}{b_0^r \zeta \mu_r} \middle| \begin{matrix} (0,1),(1,1) \\ (a_0, \frac{r}{c_0}),(0,1),(0,1) \end{matrix} \right] \tag{34}$$

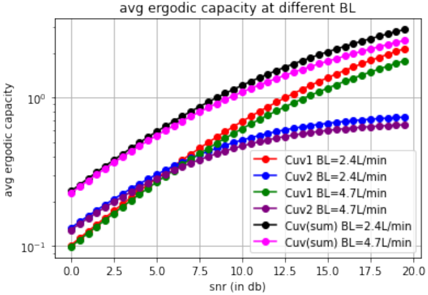
Ergodic Capacity

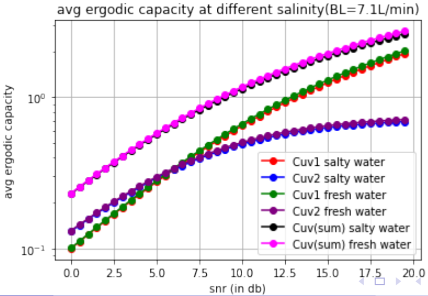
Ergodic Capacity

where,

1. $\bar{\rho}_{UV_1} = \frac{\delta P_t E[|I_1|^2]}{N_0}$
2. $\bar{\rho}_{UV_2} = \frac{(1-\delta) P_t E[|I_2|^2]}{\delta P_t E[|I_2|^2] + N_0}$
3. $\mu_r = \frac{\bar{\rho}_{UV_i}}{2 w_0 \lambda_0^2 + b_0^2 (1 - w_0) \frac{\Gamma(a_0 + \frac{2}{c_0})}{\Gamma(a_0)}}$ where $i = 1, 2$
4. $r = 2$
5. δ is power allocation coefficient

Ergodic capacity graphs





Ergodic capacity graphs