Physics: Applied Electromagnetics, Geometric Optics and Semiconducting materials

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Slides Chap 1: **Coordinate System,**

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Introduction

- Coordinate system play a crucial role in describing a system.
- A good choice of the coordinate system contribute on simplifying the problem.
- In order to describe simply the motion of a body A material point is used.
- A material body is assimilated to a material point when
  - It is not rolling on itself
  - Its dimensions characteristics are much less than the distances it is travelling

Locating a point in space

- The motion of an object is relative i.e. it depends on a **reference R**.
- A reference is defined by closing a dimensionally stable solid.
- The location of a point in the reference is made by choosing a direct orthonormal frame of reference i.e. a particular point (usually the center of mass of the solid) and three orthogonal axes.
- A good diagram helps solving a physics problem clearly.

The Cartesian coordinate system

- We consider (O;X,Y,Z)-a reference frame and its basis $(\vec{u}_x, \vec{u}_y, \vec{u}_z)$. The vectors $\vec{u}_x, \vec{u}_y, \vec{u}_z)$ are unit vectors respective to X,Y,Z axes
- *The basis* $(\vec{u}_x, \vec{u}_y, \vec{u}_z)$ *is direct orthonormal when* $||\vec{u}_x|| = ||\vec{u}_y|| = ||\vec{u}_z|| = 1$, *The three vectors are pairly orthogonal and* $[\vec{u}_x, \vec{u}_y] = \vec{u}_z$
- The position of a point M is defined by a position vector defined by $\vec{r} = \overrightarrow{OM}$.
- The **cartesian coordinates** x,y, and z are defined by.

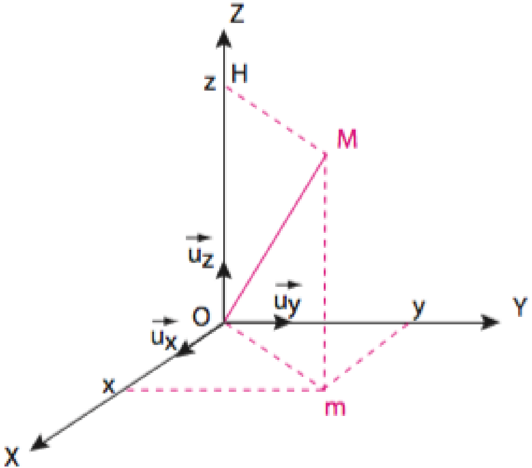
$$\vec{r} = \overrightarrow{OM} = x\vec{u}_x + y\vec{u}_y + z\vec{u}_z \tag{1}$$

The Cartesian coordinate system

- To represent this coordinate system, we first mark the material point M.
- Then we project this point M on the axis OZ: we then obtain the point H of coordinate z on the axis OZ.
- We project M orthogonally in the plane (OX, OY) by drawing a parallel to the OZ axis passing through M: we then obtain the point m.
- We then draw the lines passing through m and parallel to the axes OX and OY: the intersections of these lines with the axes OX and OY give the x and y coordinates of the point M (Figure 1).

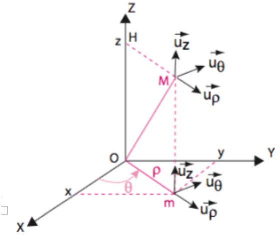
The Cartesian coordinate system

- a graph showing the cartesian coordinates



The cylindrical coordinate system

- The position of point M is here defined in a frame $(O, \vec{u}_\rho, \vec{u}_\theta, \vec{u}_z)$. We introduce here the direct orthonormal basis $(\vec{u}_\rho, \vec{u}_\theta, \vec{u}_z)$, associated with the cylindrical coordinates (ρ, θ, z).



The cylindrical coordinate system

- The cylindrical frame of reference is associated with point M, it is therefore a mobile local frame of reference. In this frame of reference, the position vector of point M is written:

$$\vec{r} = \overrightarrow{OM} = \rho\vec{u}_\rho + z\vec{u}_z \tag{2}$$

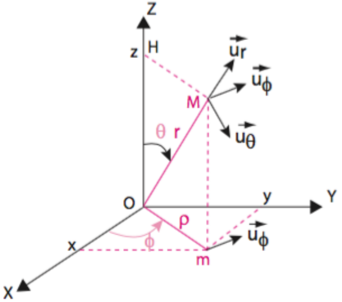
- The relationship between the Cartesian coordinate system and cylindrical coordinate system is as follows

$$x = \rho\cos(\theta), y = \rho\sin(\theta), z = z. \tag{3}$$

and $\rho = \sqrt{x^2 + y^2}$

The spherical coordinate system

- The position of point M is here defined in a frame $(O, \vec{u}_r, \vec{u}_\theta, \vec{u}_\phi)$. Here we introduce the basis $(\vec{u}_r, \vec{u}_\theta, \vec{u}_\phi)$ direct orthonormal basis, associated with spherical coordinates (r, θ, ϕ).



The spherical coordinate system

- The position vector is $\vec{r} = \overrightarrow{OM} = r\vec{u}_r$
- The relationship between the Cartesian coordinate system and spherical coordinate system is as follows

$$x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta. \tag{4}$$

Geographic coordinate systems

- As we said earlier, coordinate system is a frame of reference in which elements can be represented in space. This system makes it possible to be located on the whole of the terrestrial globe thanks to a couple of geographical coordinates. For historical, technical and usage reasons, there are a large number of coordinate systems.
- The EPSG - European Petroleum Survey Group - has defined a list of georeferenced coordinate systems. Codes have been associated with these systems to identify them.
- In 2005, the group became the Surveying and Positioning Committee of the International Association of Oil and Gas Producers (OGP). These codes are used today as a reference. The EPSG Geodetic Parameter Register (http://www.epsg-registry.org/) allows you to find the coordinate systems of a territory.

Geographic coordinate systems

- Cartesian coordinates are independent of the shape of the earth, so it is difficult to use them for practical orientation.
- This is why it is interesting to locate yourself by referring to a reference surface: in this case, an ellipsoid which is the geometric figure that best approximates the shape of the earth.
- To define this ellipsoid, it is necessary to define a center for it (it is confused with the origin O of a geodetic reference frame and therefore of a Cartesian coordinate system), two lengths (the semi-major axis which measures approximately 6370 km and the semi-minor axis which measures approximately 6350 km) and an orientation (the minor axis coincides with the axis $(O; \vec{u}_z)$).

Geographic coordinate systems

- By defining a meridian of origin which will fix the plane $(O; \vec{u}_x; \vec{u}_z)$, all Points of the ellipsoid can be located using two coordinates: λ (the longitude) and ϕ (the latitude).
- All points on the earth can then be located using a third coordinate: h (the height from the ellipsoid). We are talking about geographic coordinate systems.

Geographic coordinate systems

- The geographic coordinate system are (λ, ϕ, h) as shown in the Figure .

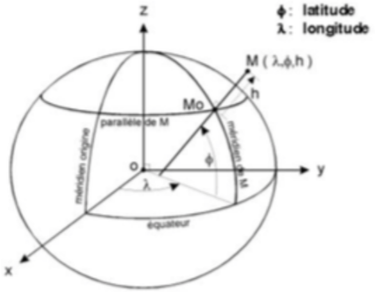


Figure: Geographical Coordinates

Geographic coordinate systems

- Longitudes and latitudes can be expressed in different units and notations (like all angles). Nowadays, degrees are the most common unit of angle and the most common notations are: DMS (Degree Minute Second) ($49^o 30' 00'' - 123^o 30' 00''$); DM (Degree Minute) ($49^o 30.0' - 123^o 30.0'$); DD (Decimal Degree) ($49.5000^o - 123.5000^o$). Geographical coordinates are very often given in DMS.
- However, computer scientists generally find the sexagesimal system (DMS) impractical to manipulate and therefore prefer to convert minutes and seconds to decimal fractions of a degree (decimal degrees).

  The general formula is then the following:

  $$Latitude(decimaldegrees) = degrees + (minutes/60) + (seconds/3600)$$

Geographic coordinate systems

- For example, for a latitude of $45^o54^{'}36^{''}(45 \textit{degrees},\ 54\ \textit{minutes and}\ 36\ \textit{seconds})$, obtain: $45 + (54/60) + (36/3600) = 45.91^o$. Conversely, the conversation takes place in an iterative process.
- Let a longitude of 121.136^o: the number before the decimal point indicates degrees $\Longrightarrow\ 121^o$. Multiply the number after the decimal point by $60 \implies 0.136 \times 60 = 8.16$ The number before the decimal point indicates the minutes (8'). Multiply the number after the decimal by $60 \implies 0.16 \times 60 = 9.6$ The result indicates the seconds (9.6"). The longitude is therefore $121^o 8' 9.6''$

Geographic coordinate systems

- To finish with the units of angle measurement, we will present the grades. The grade is a unit of angle measurement quite used in France. The grade is the consequence of the meter, in that dividing the entire earth's circumference (40,000 km) by 400 grades, we get that 1 grade = 100 km and 1 meter = $10^{-5} gon$. In France, the grade is the legal unit of angle measurement for all topographic (surveying, civil engineering) and geodetic (IGN) work carried out in France. To convert from degrees to degrees (decimal degrees) and vice versa, a rule of three is sufficient.

Geographic coordinate systems

- Since it is not possible to directly visualize the world as a single plane using the two coordinate systems presented, it has been necessary to use projections to make maps. We then speak of plane coordinates (or of a plane coordinate system) composed of two coordinates: E and N (or X and Y). These two distances to the origin can be expressed in different units of length (meters, kilometers, miles ...). Converting geographic positions from a curved surface to a flat surface requires the use of a mathematical formula called a map projection. This "flattening" process causes alterations in the shape and surface of the mapped elements, but also in the distances and directions between these elements.

Geographic coordinate systems

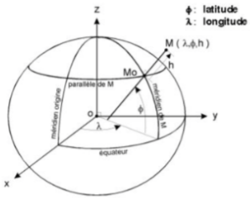
- The variety of mathematical methods makes it possible to obtain projections which more or less distort surfaces, angles or distances. In short, there are three major coordinate systems that require different reference systems.

system of coordinates	reference system
Cartesian (X,Y,Z)	Reference Frame
Geographical (λ, ϕ, h)	Reference frame +ellipsoid
Planes (E,N)	referential+Ellipsoid+projection

Table: The three great geographical system of coordinates

Geographic coordinate systems

- To switch from geographic coordinates to cartesian coordinates, the operation is quite simple as shown in the Figure:



$X = (N + h)\cos\phi\cos\lambda$
$Y = (N + h)\cos\phi\sin\lambda$
$Z = (N \times (1 - e^2) + h)\sin\phi$
$e^2 = \frac{a^2 - b^2}{a^2}$
$W = \sqrt{1 - e^2\, sin^2\phi}$
$N = \frac{a}{W}$
$\rho = \frac{a\,(1 - e^2)}{W^3}$
$r = N\cos\phi$

Geographic coordinate systems

- Conversely, to switch from Cartesian coordinates to geographic coordinates, the formulas below should be used as shown in the Figure:

$$f = 1 - \sqrt{1 - e^2}$$

$$R = \sqrt{X^2 + Y^2 + Z^2}$$

$$\lambda = arctg\left[\frac{Y}{X}\right]$$

$$\mu = arctg\left[\frac{Z}{\sqrt{X^2+Y^2}} \times \left((1-f) + \left(\frac{e^2 a}{R}\right)\right)\right]$$

$$\phi = arctg\left[\frac{Z(1-f)+e^2 a\,(\sin\mu)^3}{(1-f)\left(\sqrt{X^2+Y^2}-e^2 a\,(\cos\mu)^3\right)}\right]$$

$$h = \left(\sqrt{X^2+Y^2} \times \cos\phi\right) + (Z\sin\phi) - \left(a\sqrt{1-e^2(\sin\phi)^2}\right)$$

exercises

- Given point P(-2, 6, 3) and vector $\mathbf{A} = y\mathbf{a}_x + (x+z)\mathbf{a}_y$,
  i) represent graphically the point P,
  ii) express P and A in cylindrical and spherical coordinates
  iii) Evaluate A at P in the Cartesian, cylindrical, and spherical systems
- (a) Convert points P(1, 3, 5), T(0, -4, 3), and S(-3, -4, -10) from Cartesian to cylindrical and spherical coordinates. (b) Transform vector

$$Q = \frac{\sqrt{x^2+y^2}\mathbf{a}_x}{\sqrt{x^2+y^2+z^2}} - \frac{yz\mathbf{a}_z}{\sqrt{x^2+y^2+z^2}} \tag{5}$$

  to cylindrical and spherical coordinates. (c) Evaluate Q at T in the three coordinate systems