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Question: 9.10.5.11

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1 Problem

ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

2 Solution

let $\angle CBD = \theta$ and $\angle CAD = \phi$ Traingle ABC is right angled with right angle at **B**

Therefore, **B** lies on the circle centered at

$$\mathbf{O} = \frac{\mathbf{A} + \mathbf{C}}{2} \tag{2.0.1}$$

and radius $\frac{\|\mathbf{A}-\mathbf{C}\|}{2}$.

Similarly, $\hat{\mathbf{D}}$ lies on the same circle.

 $\angle CAD$ and $\angle CBD$ lie on the segment $\mathbf{C} - \mathbf{D}$ of the circle.

$$\therefore, \angle CAD = \angle CBD \tag{2.0.2}$$

For example, Consider right triangles such that,

$$\mathbf{A} = \begin{pmatrix} -1\\1 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{C} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.4}$$

$$\mathbf{B} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{D} = \begin{pmatrix} -1\\1 \end{pmatrix} \tag{2.0.6}$$

$$\cos \theta = \frac{(\mathbf{C} - \mathbf{B})^{\top} (\mathbf{D} - \mathbf{B})}{\|\mathbf{C} - \mathbf{B}\| \|\mathbf{D} - \mathbf{B}\|}$$

$$= \frac{1}{\sqrt{2}} \cos \phi$$

$$= \frac{(\mathbf{C} - \mathbf{A})^{\top} (\mathbf{D} - \mathbf{A})}{\|\mathbf{C} - \mathbf{A}\| \|\mathbf{D} - \mathbf{A}\|}$$
(2.0.8)

$$=\frac{1}{\sqrt{2}}$$
 (2.0.9)

$$\implies \theta = \phi \tag{2.0.10}$$

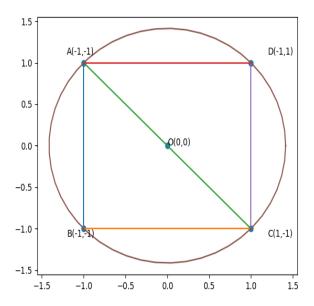


Fig. 0: Figure 1