Que: 10.10.1.3

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1 Problem

A tangent $\mathbf{P}-\mathbf{Q}$ at a point $\mathbf{P}$ of a circle of radius $r = 5cm$ meets a line through the center $\mathbf{O}$ at a point $\mathbf{Q}$ so that $\mathbf{O}-\mathbf{Q} = 12cm$. Find length $\mathbf{P}-\mathbf{Q}$.

2 Solution

Let the circle be

$$\|\mathbf{x}\|^2 = 25 \tag{2.0.1}$$

$$\tag{2.0.2}$$

and the point $\mathbf{P} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

The tangent at $\mathbf{P}$, that $\mathbf{Q}$ lies on, is given by

$$\mathbf{x} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.3}$$

$$i.e.\mathbf{m} = \begin{pmatrix} 1//0 \end{pmatrix} \ , \ \mathbf{h} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \tag{2.0.4}$$

Let,

$$\mathbf{Q} = \mathbf{h} + \mu_i \mathbf{m} \tag{2.0.5}$$

also,

$$\|\mathbf{Q} - \mathbf{O}\| = 12 \tag{2.0.6}$$

$$\tag{2.0.7}$$

Hence,Q lies on circle with center $\mathbf{O}$ and radius 12cm. Equation of this circle is:

$$\|x\|^2 - 144 = 0 \tag{2.0.8}$$

$$i.e.\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \ , \ f = -144 \tag{2.0.9}$$

For circle,$\mathbf{V} = \mathbf{I}$

$$\mu_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-m^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{(\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}))^2 - g(\mathbf{h}} \right. \tag{2.0.10}$$

where,

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V} \mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f \tag{2.0.11}$$

$\mathbf{O}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\mathbf{P}$	$\begin{pmatrix} 0 \\ 5 \end{pmatrix}$
r	5
$\|\mathbf{O}-\mathbf{Q}\|$	12

TABLE 0: Table1

here,

$$\mu_i = \pm\sqrt{119} \tag{2.0.12}$$

Therefore,

$$\mathbf{Q} = \mathbf{h} + \mu_i \mathbf{m} \tag{2.0.13}$$

$$= \begin{pmatrix} \sqrt{119} \\ 5 \end{pmatrix} \tag{2.0.14}$$

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} - \begin{pmatrix} \sqrt{119} \\ 5 \end{pmatrix} \tag{2.0.15}$$

$$= \begin{pmatrix} \sqrt{119} \\ 0 \end{pmatrix} \tag{2.0.16}$$

$$\|\mathbf{P} - \mathbf{Q}\| = \sqrt{119} \tag{2.0.17}$$

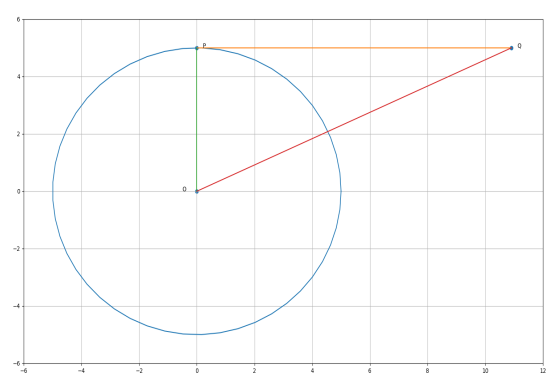

Fig. 0: Figure 1