EE2802: Assignment4

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1 Problem

A line passes through $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\begin{pmatrix} h \\ k \end{pmatrix}$. If the slope of the line is m, show that

$$k - y_1 = m(h - x_1) \tag{1.0.1}$$

2 Solution

let the line joining $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} h \\ k \end{pmatrix}$ be $\mathbf{x} = \mathbf{x_0} + \lambda \mathbf{m_1}$

$$\mathbf{A} - \mathbf{B} = (\mathbf{x_0} + \lambda_1 \mathbf{m_1}) - (\mathbf{x_0} + \lambda_2 \mathbf{m_1}) = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} - \begin{pmatrix} h \\ k \end{pmatrix}$$
(2.0.1)

$$\implies (\lambda_1 - \lambda_2) \mathbf{m_1} = \begin{pmatrix} x_1 - h \\ y_1 - k \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{m_1} = k \begin{pmatrix} x_1 - h \\ y_1 - k \end{pmatrix} \tag{2.0.3}$$

slope m is given by $m = tan(\theta)$, θ being angle between line and x axis i.e. $(1 \ 0)\mathbf{x} = 0$

$$\cos \theta = \frac{\mathbf{m_1}^T \mathbf{m_2}}{\|\mathbf{m_1}\| \|\mathbf{m_2}\|}$$
 (2.0.4)

$$=\frac{x_1-h}{\sqrt{(x_1-h)^2+(y_1-k)^2}}$$
 (2.0.5)

$$\cos \theta = \frac{\mathbf{m_1}^T \mathbf{m_2}}{\|\mathbf{m_1}\| \|\mathbf{m_2}\|}$$

$$= \frac{x_1 - h}{\sqrt{(x_1 - h)^2 + (y_1 - k)^2}}$$

$$m = \tan \theta = \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta}$$

$$v_1 - k$$
(2.0.4)
(2.0.5)

$$m = \frac{y_1 - k}{x_1 - h} \tag{2.0.7}$$

$$\implies k - y_1 = m(h - x_1) \tag{2.0.8}$$

Hence Proved.

