Question: 9.10.5.11

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1 Problem

ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

2 Solution

let $\angle CBD = \theta$ and $\angle CAD = \phi$
Traingle ABC is right angled with right angle at $\mathbf{B}$
Therefore, $\mathbf{B}$ lies on the circle centered at

$$\mathbf{O} = \frac{\mathbf{A} + \mathbf{C}}{2} \tag{2.0.1}$$

and radius $\frac{\|\mathbf{A}-\mathbf{C}\|}{2}$.

Similarly, $\mathbf{D}$ lies on the same circle.

$\angle CAD$ and $\angle CBD$ lie on the segment $\mathbf{C} - \mathbf{D}$ of the circle.

$$\therefore, \angle CAD = \angle CBD \tag{2.0.2}$$

For example, Consider right triangles such that,

$$\mathbf{A} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{C} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.4}$$

$$\mathbf{B} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{D} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{2.0.6}$$

$$\cos\theta = \frac{(\mathbf{C} - \mathbf{B})^\top (\mathbf{D} - \mathbf{B})}{\|\mathbf{C} - \mathbf{B}\| \|\mathbf{D} - \mathbf{B}\|} \tag{2.0.7}$$

$$= \frac{1}{\sqrt{2}} \cos\phi \qquad = \frac{(\mathbf{C} - \mathbf{A})^\top (\mathbf{D} - \mathbf{A})}{\|\mathbf{C} - \mathbf{A}\| \|\mathbf{D} - \mathbf{A}\|} \tag{2.0.8}$$

$$= \frac{1}{\sqrt{2}} \tag{2.0.9}$$

$$\implies \theta = \phi \tag{2.0.10}$$

Parameter	Value	Description
$\mathbf{A}$	$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	End of common hypotenuse
$\mathbf{C}$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	Other end of common hypotenuse
B	$\begin{pmatrix} -1 \\ -1 \end{pmatrix}$	Non-common vertex of $\triangle ABC$
D	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	Non-comon vertx of$\triangle ADC$

TABLE 0: Table1

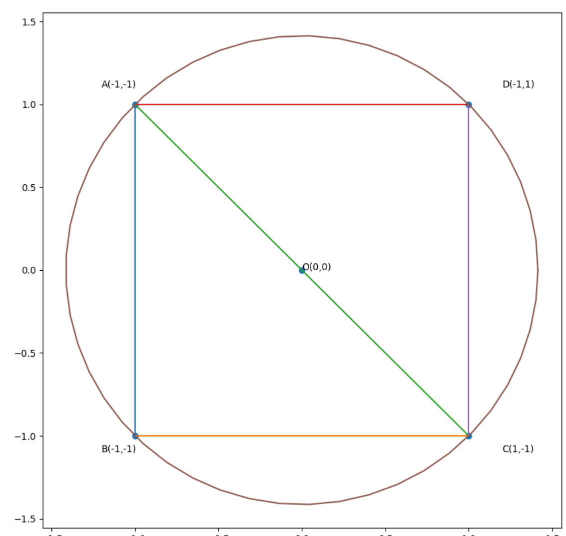

Fig. 0: Figure 1