Question: 12.11.1.5

Nikam Pratik Balasaheb (EE21BTECH11037)

1 Problem

Find the direction cosines of the sides of a triangle whose vertices are $\begin{pmatrix}3\\5\\-4\end{pmatrix}$, $\begin{pmatrix}-1\\1\\2\end{pmatrix}$ and $\begin{pmatrix}-5\\-5\\-2\end{pmatrix}$.

2 Solution

Vertices are given by

$$\mathbf{A} = \begin{pmatrix}3\\5\\-4\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix}-1\\1\\2\end{pmatrix} \tag{2.0.2}$$

$$\mathbf{C} = \begin{pmatrix}-5\\-5\\-2\end{pmatrix} \tag{2.0.3}$$

The sides are,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix}4\\4\\-6\end{pmatrix} = \mathbf{m_1} \tag{2.0.4}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix}4\\6\\4\end{pmatrix} = \mathbf{m_2} \tag{2.0.5}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix}-8\\-10\\2\end{pmatrix} = \mathbf{m_3} \tag{2.0.6}$$

$$\tag{2.0.7}$$

The axes are,

$$\mathbf{X} = \begin{pmatrix}1\\0\\0\end{pmatrix} \tag{2.0.8}$$

$$\mathbf{Y} = \begin{pmatrix}0\\1\\0\end{pmatrix} \tag{2.0.9}$$

$$\mathbf{Z} = \begin{pmatrix}0\\0\\1\end{pmatrix} \tag{2.0.10}$$

Direction cosines of side $\mathbf{m_1}$,

$$\cos\theta_1 = \frac{\mathbf{m_1}^\top \mathbf{X}}{\|\mathbf{m_1}\| \, \|\mathbf{X}\|} \tag{2.0.11}$$

$$= \frac{2}{\sqrt{17}} \tag{2.0.12}$$

$$\cos\theta_2 = \frac{\mathbf{m_1}^\top \mathbf{Y}}{\|\mathbf{m_1}\| \, \|\mathbf{Y}\|} \tag{2.0.13}$$

$$= \frac{2}{\sqrt{17}} \tag{2.0.14}$$

$$\cos\theta_1 = \frac{\mathbf{m_1}^\top \mathbf{Z}}{\|\mathbf{m_1}\| \, \|\mathbf{Z}\|} \tag{2.0.15}$$

$$= \frac{-3}{\sqrt{17}} \tag{2.0.16}$$

$$\tag{2.0.17}$$

Direction cosines of side $\mathbf{m_2}$,

$$\cos\theta_1 = \frac{\mathbf{m_2}^\top \mathbf{X}}{\|\mathbf{m_2}\| \, \|\mathbf{X}\|} \tag{2.0.18}$$

$$= \frac{2}{\sqrt{17}} \tag{2.0.19}$$

$$\cos\theta_2 = \frac{\mathbf{m_2}^\top \mathbf{Y}}{\|\mathbf{m_2}\| \, \|\mathbf{Y}\|} \tag{2.0.20}$$

$$= \frac{3}{\sqrt{17}} \tag{2.0.21}$$

$$\cos\theta_1 = \frac{\mathbf{m_2}^\top \mathbf{Z}}{\|\mathbf{m_2}\| \, \|\mathbf{Z}\|} \tag{2.0.22}$$

$$= \frac{2}{\sqrt{17}} \tag{2.0.23}$$

$$\tag{2.0.24}$$

Direction cosines of side $\mathbf{m_3}$,

$$\cos\theta_1 = \frac{\mathbf{m_3}^\top \mathbf{X}}{\|\mathbf{m_3}\|\,\|\mathbf{X}\|} \tag{2.0.25}$$
$$= \frac{-4}{\sqrt{42}} \tag{2.0.26}$$
$$\cos\theta_2 = \frac{\mathbf{m_3}^\top \mathbf{Y}}{\|\mathbf{m_3}\|\,\|\mathbf{Y}\|} \tag{2.0.27}$$
$$= \frac{-5}{\sqrt{42}} \tag{2.0.28}$$
$$\cos\theta_1 = \frac{\mathbf{m_3}^\top \mathbf{Z}}{\|\mathbf{m_3}\|\,\|\mathbf{Z}\|} \tag{2.0.29}$$
$$= \frac{1}{\sqrt{42}} \tag{2.0.30}$$
$$\tag{2.0.31}$$

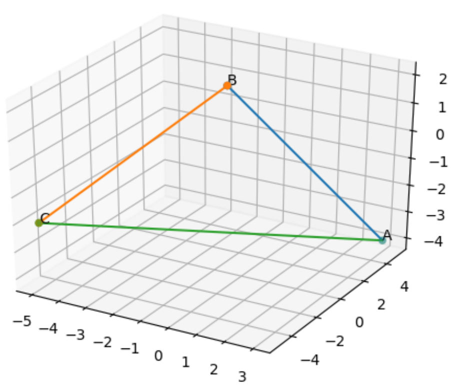

Fig. 0: Triangle ABC