Que: 10.10.1.3

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1 Problem

A tangent $\mathbf{P}-\mathbf{Q}$ at a point $\mathbf{P}$ of a circle of radius $r = 5cm$ meets a line through the center $\mathbf{O}$ at a point $\mathbf{Q}$ so that $\mathbf{O}-\mathbf{Q} = 12cm$. Find length $\mathbf{P}-\mathbf{Q}$.

2 Solution

Let the circle be

$$\|\mathbf{x}\|^2 = 25 \tag{2.0.1}$$

and the point $\mathbf{P} = \begin{pmatrix}0\\5\end{pmatrix}$

The tangent at $\mathbf{P}$, that $\mathbf{Q}$ lies on, is given by

$$\mathbf{x} = \begin{pmatrix}0\\5\end{pmatrix} + \lambda\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.2}$$

$$= \begin{pmatrix}\lambda\\5\end{pmatrix} \tag{2.0.3}$$

also,

$$\|\mathbf{Q}-\mathbf{O}\| = 12 \tag{2.0.4}$$

$$\|\mathbf{Q}\| = 12 \tag{2.0.5}$$

$$\|\mathbf{Q}\|^2 = 144 \tag{2.0.6}$$

for $\mathbf{Q}$

$$\lambda^2 + 25 = 144 \tag{2.0.7}$$

$$\lambda^2 = 119 \tag{2.0.8}$$

$$\lambda = \pm\sqrt{119} \tag{2.0.9}$$

$$\mathbf{P}-\mathbf{Q} = \begin{pmatrix}0\\5\end{pmatrix} - \begin{pmatrix}\sqrt{119}\\5\end{pmatrix} \tag{2.0.10}$$

$$= \begin{pmatrix}\sqrt{119}\\0\end{pmatrix} \tag{2.0.11}$$

$$\|\mathbf{P}-\mathbf{Q}\| = \sqrt{119} \tag{2.0.12}$$

$\mathbf{O}$	$\begin{pmatrix}0\\0\end{pmatrix}$
$\mathbf{P}$	$\begin{pmatrix}0\\5\end{pmatrix}$
r	5
$\|\mathbf{O}-\mathbf{Q}\|$	12

TABLE 0: Table1

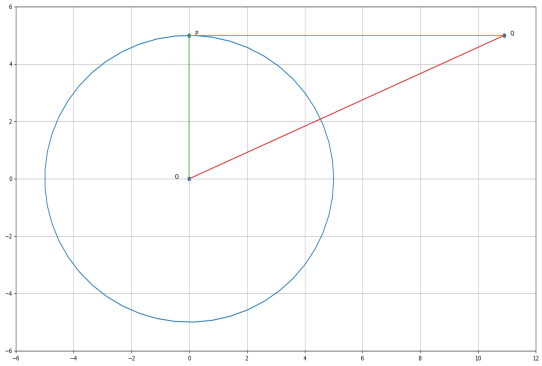

Fig. 0: Figure 1