

Que: 11.10.3.17

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1 PROBLEM

In triangle ABC with vertices $\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, Find the equation and length of altitude from vertex A

2 SOLUTION

1) Direction vector of side BC

$$\mathbf{m} = \mathbf{B} - \mathbf{C} \quad (2.0.1)$$

$$= \begin{pmatrix} 3 \\ -3 \end{pmatrix} \quad (2.0.2)$$

Direction vector of side BC is normal of altitude from A

2) Equation of the altitude

$$\mathbf{m}^T (\mathbf{x} - \mathbf{A}) = 0 \quad (2.0.3)$$

$$\begin{pmatrix} 3 & -3 \end{pmatrix} \mathbf{x} = -3 \quad (2.0.4)$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -1 \quad (2.0.5)$$

3) Equation of line BC

$$\mathbf{x} = \mathbf{B} + \mu \mathbf{m} \quad (2.0.6)$$

$$\mathbf{x} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -3 \end{pmatrix} \quad (2.0.7)$$

$$i.e. \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 3 \quad (2.0.8)$$

4) Optimization problem

The length of altitude can be expressed as an optimization problem,

$$\min \|\mathbf{A} - \mathbf{x}\|^2 \quad (2.0.9)$$

such that

$$\mathbf{n}^T \mathbf{x} = c \quad (2.0.10)$$

where,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.11)$$

$$c = 3 \quad (2.0.12)$$

(2.0.9) is solved using Gradient Descent in codes/11.10.3.17.py. The results obtained are using 0.01 as learning rate, iterated until the difference between cost in consecutive iterations is lesser than 5×10^{-7} .

The results obtained are:

$$\mu_{min} = -0.99986708 \quad (2.0.13)$$

$$\min \|\mathbf{A} - \mathbf{x}\|^2 = 2.000000318 \quad (2.0.14)$$

Therefore, the length of altitude is given by,

$$l = \sqrt{2} \approx 1.414213562 \quad (2.0.15)$$

Parameter	Value	Description
A	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$	Vertex 'A' of the triangle
B	$\begin{pmatrix} 4 \\ -1 \end{pmatrix}$	Vertex 'B' of triangle
C	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$	Vertex 'C' of triangle
learnRate	0.01	Learning rate of the model

TABLE 4: Table

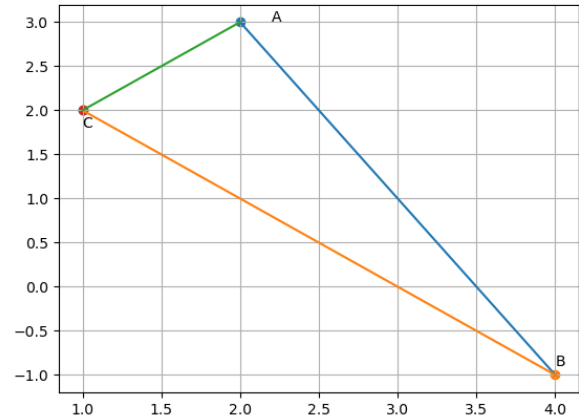


Fig. 4: Figure