Question: 12.11.2.15

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1 Problem

Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

2 Solution

Line equations are

$$\mathbf{x} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ -6 \\ 1 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{x} = \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \tag{2.0.2}$$

Comparing with $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$,

$$\mathbf{a}_1 = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} \quad \mathbf{b}_1 = \begin{pmatrix} 7 \\ -6 \\ 1 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{a}_2 = \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix} \quad \mathbf{b}_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \tag{2.0.4}$$

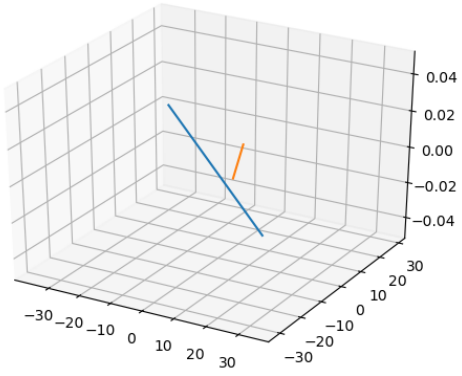

Fig. 0: given lines

Distance between the lines is given by

$$d = \left| \frac{(\mathbf{b}_1 \times \mathbf{b}_2)^\top (\mathbf{a}_2 - \mathbf{a}_1)}{\|\mathbf{b}_1 \times \mathbf{b}_2\|} \right| \tag{2.0.5}$$

$$\mathbf{b}_1 \times \mathbf{b}_2 = \begin{pmatrix} -4 \\ -6 \\ -8 \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{a}_2 - \mathbf{a}_1 = \begin{pmatrix} 4 \\ 6 \\ 8 \end{pmatrix} \tag{2.0.7}$$

$$d = \left| \frac{(\mathbf{b}_1 \times \mathbf{b}_2)^\top (\mathbf{a}_2 - \mathbf{a}_1)}{\|\mathbf{b}_1 \times \mathbf{b}_2\|} \right| \tag{2.0.8}$$

$$= \frac{116}{2\sqrt{29}} \tag{2.0.9}$$

$$= 2\sqrt{29} \tag{2.0.10}$$