Que: 11.10.3.17

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1 Problem

In triangle ABC with vertices $\mathbf{A} = \begin{pmatrix}2\\3\end{pmatrix}$, $\mathbf{B} = \begin{pmatrix}4\\-1\end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix}1\\2\end{pmatrix}$, FInd the equation and length of altitude from vertex $\mathbf{A}$

2 Solution

1) Direction vector of side BC

$$\mathbf{m} = \mathbf{B} - \mathbf{C} \tag{2.0.1}$$
$$= \begin{pmatrix}3\\-3\end{pmatrix} \tag{2.0.2}$$

Direction vector of side BC is normal of altitude from $\mathbf{A}$

2) Equation of the altitude

$$\mathbf{m}^\top (\mathbf{x} - \mathbf{A}) = 0 \tag{2.0.3}$$
$$\begin{pmatrix}3 & -3\end{pmatrix}\mathbf{x} = -3 \tag{2.0.4}$$
$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = -1 \tag{2.0.5}$$

3) Equation of line BC

$$\mathbf{x} = \mathbf{B} + \mu\mathbf{m} \tag{2.0.6}$$
$$\tag{2.0.7}$$

4) Length of altitude
Distance between vertex $\mathbf{A}$ and any point on line BC is given by

$$d = \|\mathbf{A} - (\mathbf{B} + \mu\mathbf{m})\| \tag{2.0.8}$$
$$\tag{2.0.9}$$

The length of altitude can be finding the minma of d.

$$f(\mu) = \|\mathbf{A} - (\mathbf{B} + \mu\mathbf{m})\|^2 \tag{2.0.10}$$
$$= \|\mathbf{A} - \mathbf{B}\|^2 - 2\mu\mathbf{m}^\top (\mathbf{A} - \mathbf{B}) + \mu^2 \|\mathbf{m}\|^2 \tag{2.0.11}$$

for extremas,

$$\frac{\partial f}{\partial \mu} = -2\mathbf{m}^\top (\mathbf{A} - \mathbf{B}) + 2\mu \|\mathbf{m}\|^2 = 0 \tag{2.0.12}$$
$$\mu = \frac{\mathbf{m}^\top (\mathbf{A} - \mathbf{B})}{\|\mathbf{m}\|^2} \tag{2.0.13}$$
$$= \frac{\begin{pmatrix}3 & -3\end{pmatrix}\begin{pmatrix}-2\\4\end{pmatrix}}{18} \tag{2.0.14}$$
$$= \frac{-18}{18} \tag{2.0.15}$$
$$= -1 \tag{2.0.16}$$

also for $\mu = -1$,

$$\frac{\partial^2 f}{\partial \mu^2} = 2\|\mathbf{m}\|^2 > 0 \tag{2.0.17}$$

so $\mu = -1$ is the minima.

$$d_{min} = \|\mathbf{A} - (\mathbf{B} + \mu\mathbf{m})\| \tag{2.0.18}$$
$$= \sqrt{2} \tag{2.0.19}$$

Therefore, the length of alititude is $\sqrt{2}$

Parameter	Value	Desription
$\mathbf{A}$	$\begin{pmatrix}2\\3\end{pmatrix}$	Vertex 'A' of the triangle
$\mathbf{B}$	$\begin{pmatrix}4\\-1\end{pmatrix}$	Vertex 'B' of triangle
$\mathbf{C}$	$\begin{pmatrix}1\\2\end{pmatrix}$	Vertex 'C' of triangle

TABLE 4: Table

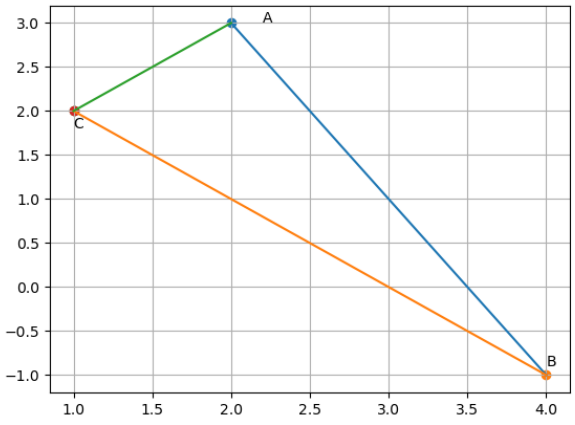

Fig. 4: Figure