Que: 11.11.4.9

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1 Problem

Find the equations of hyperbola having Vertices $\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$ and Foci $\begin{pmatrix} 0 \\ \pm 5 \end{pmatrix}$

2 Solution

Transverse axis:

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.1}$$

Center of hyperbola:

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.2}$$

$\mathbf{V_1}$ and $\mathbf{F_1}$ beign vertex and focus on same side of center,

$$\mathbf{V_1} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{F_1} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \tag{2.0.4}$$

Eccentricity,

$$e = \frac{\|\mathbf{F_1} - \mathbf{O}\|}{\|\mathbf{V_1} - \mathbf{O}\|} \tag{2.0.5}$$

$$= \frac{5}{3} \tag{2.0.6}$$

The distance between center and directrix,

$$\frac{\|\mathbf{V_1} - \mathbf{O}\|}{e} = \frac{9}{5} \tag{2.0.7}$$

Also, the directrix is perpendicular to the transverse axis.
Hence, the equation of directrix is,

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 0 \\ \frac{9}{5} \end{pmatrix} \right) = 0 \tag{2.0.8}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = \frac{9}{5} \tag{2.0.9}$$

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n}\mathbf{n}^\top \tag{2.0.10}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{25}{9} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.11}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -\frac{16}{9} \end{pmatrix} \tag{2.0.12}$$

$$\mathbf{u} = ce^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \tag{2.0.13}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.14}$$

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 \tag{2.0.15}$$

$$= 16 \tag{2.0.16}$$

Equation of the hyperbola:

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2.0.17}$$

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & -\frac{16}{9} \end{pmatrix} \mathbf{x} + 16 = 0 \tag{2.0.18}$$

$$\tag{2.0.19}$$

$\mathbf{F}_1$	$\begin{pmatrix} 0 \\ 5 \end{pmatrix}$	Focus
$\mathbf{F}_2$	$\begin{pmatrix} 0 \\ -5 \end{pmatrix}$	Focus
$\mathbf{V}_1$	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	Vertex
$\mathbf{V}_2$	$\begin{pmatrix} 0 \\ -3 \end{pmatrix}$	Vertex

TABLE 0: Table1

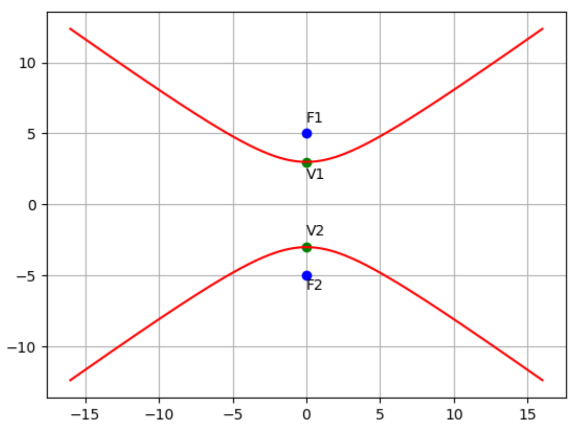

Fig. 0: Figure 1