

Que: 10.10.1.3

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1 PROBLEM

A tangent $\mathbf{P}-\mathbf{Q}$ at a point \mathbf{P} of a circle of radius $r = 5\text{cm}$ meets a line through the center \mathbf{O} at a point \mathbf{Q} so that $OQ = 12\text{cm}$. Find length PQ .

2 SOLUTION

Let the circle be

$$\|\mathbf{x}\|^2 = 25 \quad (2.0.1)$$

$$(2.0.2)$$

here,

and the point $\mathbf{P} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

The tangent at \mathbf{P} , that \mathbf{Q} lies on, is given by

$$\mathbf{x} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.3)$$

$$i.e. \mathbf{m} = (1//0), \mathbf{h} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \quad (2.0.4)$$

Let,

$$\mathbf{Q} = \mathbf{h} + \mu_i \mathbf{m} \quad (2.0.5)$$

also,

$$\|\mathbf{Q} - \mathbf{O}\| = 12 \quad (2.0.6)$$

$$(2.0.7)$$

Hence, \mathbf{Q} lies on circle with center \mathbf{O} and radius 12cm. Equation of this circle is:

$$\|\mathbf{x}\|^2 - 144 = 0 \quad (2.0.8)$$

$$i.e. \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -144 \quad (2.0.9)$$

For circle, $\mathbf{V} = \mathbf{I}$

$$\mu_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{(\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}))^2 - g} \right) \quad (2.0.10)$$

where,

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V} \mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f \quad (2.0.11)$$

\mathbf{O}	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
\mathbf{P}	$\begin{pmatrix} 0 \\ 5 \end{pmatrix}$
r	5
$\ \mathbf{O} - \mathbf{Q}\ $	12

TABLE 0: Table1

$$\mu_i = \pm \sqrt{119} \quad (2.0.12)$$

Therefore,

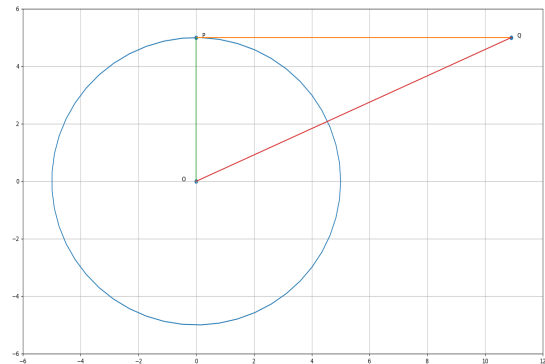
$$\mathbf{Q} = \mathbf{h} + \mu_i \mathbf{m} \quad (2.0.13)$$

$$= \begin{pmatrix} \sqrt{119} \\ 5 \end{pmatrix} \quad (2.0.14)$$

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} - \begin{pmatrix} \sqrt{119} \\ 5 \end{pmatrix} \quad (2.0.15)$$

$$= \begin{pmatrix} \sqrt{119} \\ 0 \end{pmatrix} \quad (2.0.16)$$

$$PQ = \|\mathbf{P} - \mathbf{Q}\| = \sqrt{119} \quad (2.0.17)$$



(2.0.10)

Fig. 0: Figure 1