Que: 11.10.3.17

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1 Problem

In triangle ABC with vertices $\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, FInd the equation and length of altitude from vertex $\mathbf{A}$

2 Solution

1) Direction vector of side BC

$$\mathbf{m} = \mathbf{B} - \mathbf{C} \tag{2.0.1}$$

$$= \begin{pmatrix} 3 \\ -3 \end{pmatrix} \tag{2.0.2}$$

Direction vector of side BC is normal of altitude from $\mathbf{A}$

2) Equation of the altitude

$$\mathbf{m}^\top (\mathbf{x} - \mathbf{A}) = 0 \tag{2.0.3}$$

$$\begin{pmatrix} 3 & -3 \end{pmatrix} \mathbf{x} = -3 \tag{2.0.4}$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -1 \tag{2.0.5}$$

3) Equation of line BC

$$\mathbf{x} = \mathbf{B} + \mu \mathbf{m} \tag{2.0.6}$$

$$\mathbf{x} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -3 \end{pmatrix} \tag{2.0.7}$$

$$i.e. \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 3 \tag{2.0.8}$$

4) Optimization problem
The length of altitude can be expressed as a optimization problem,

$$\min \|\mathbf{A} - \mathbf{x}\|^2 \tag{2.0.9}$$

such that

$$\mathbf{n}^\top \mathbf{x} = c \tag{2.0.10}$$

where,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.11}$$

$$c = 3 \tag{2.0.12}$$

Above equation is solved using `cvxpy` in `codes/11.10.3.17.py`. The results obtained are:

$$\mu_{min} = -1 \tag{2.0.13}$$

$$min \|\mathbf{A} - \mathbf{x}\|^2 = 2 \tag{2.0.14}$$

Therefore, the length of altitude is given by,

$$l = \sqrt{2} \tag{2.0.15}$$

Parameter	Value	Desription
A	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$	Vertex 'A' of the triangle
B	$\begin{pmatrix} 4 \\ -1 \end{pmatrix}$	Vertex 'B' of triangle
C	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$	Vertex 'C' of triangle

TABLE 4: Table

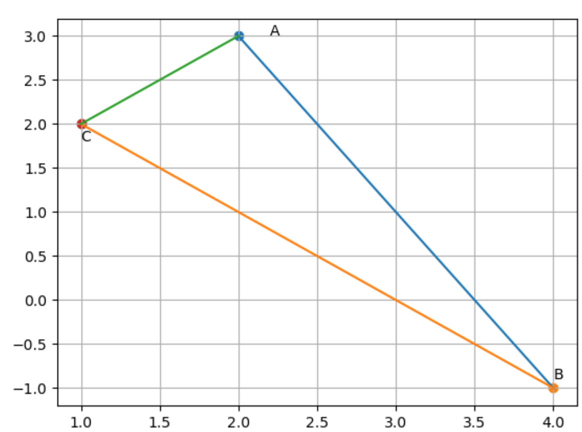

Fig. 4: Figure