

# Problem: 10.10.2.3

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## 1 PROBLEM

If the tangents PA and PB from a point **P** to a circle with center **O** are inclined to each other at  $80^\circ$ , find  $\angle POA$ .

## 2 SOLUTION

$$\angle APB = 80^\circ \quad (2.0.1)$$

$$\angle APO = \frac{1}{2} \angle APB \quad (2.0.2)$$

$$= 40^\circ = \theta \text{ (say)} \quad (2.0.3)$$

Therefore, it can be said that **P** lies on the line

$$(-\sin \theta \quad \cos \theta) \mathbf{x} = 0 \quad (2.0.4)$$

Let the circle be  $\|\mathbf{x}\|^2 = r^2$  and **A** be  $\begin{pmatrix} 0 \\ r \end{pmatrix}$ .

Therefore, the tangent that **P** lies on is given by:

$$(0 \quad 1) \mathbf{x} = r \quad (2.0.5)$$

The point **P** is given by:

$$\begin{pmatrix} -\cos \theta & \sin \theta \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ r \end{pmatrix} \quad (2.0.6)$$

Augmented Matrix:

$$\left( \begin{array}{cc|c} -\sin \theta & \cos \theta & 0 \\ 0 & 1 & r \end{array} \right) \quad (2.0.7)$$

$$\xrightarrow{R_1 \leftarrow \frac{R_1}{-\sin \theta} + \cot \theta R_2} \quad (2.0.8)$$

$$\left( \begin{array}{cc|c} 1 & 0 & r \cot \theta \\ 0 & 1 & r \end{array} \right) \quad (2.0.9)$$

$$\mathbf{P} = \begin{pmatrix} r \cot \theta \\ r \end{pmatrix} \quad (2.0.10)$$

let  $\angle POA = \phi$

$$\cos \phi = \frac{(\mathbf{P} - \mathbf{O})^\top (\mathbf{A} - \mathbf{O})}{\|\mathbf{P} - \mathbf{O}\| \|\mathbf{A} - \mathbf{O}\|} \quad (2.0.11)$$

$$= \frac{\begin{pmatrix} r \cot \theta & r \end{pmatrix} \begin{pmatrix} 0 \\ r \end{pmatrix}}{r^2 \operatorname{cosec} \theta} \quad (2.0.12)$$

$$\cos \phi = \sin \theta \quad (2.0.13)$$

$$\Rightarrow \phi = 90^\circ - \theta \quad (2.0.14)$$

$$\phi = 50^\circ \quad (2.0.15)$$

<b>O</b>	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Center of the given circle
<b>A</b>	$\begin{pmatrix} 0 \\ 5 \end{pmatrix}$	Point where tangent is taken
$r$	5	radius of given circle
$\angle APB$	$80^\circ$	Angle between tangents

TABLE 0: Table 1

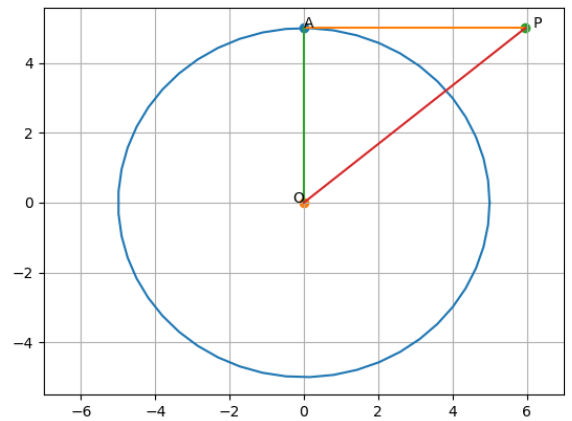


Fig. 0: Figure 1