Que: 11.10.3.17

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1 Problem

In triangle ABC with vertices $\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, Find the equation and length of altitude from vertex \mathbf{A}

2 Solution

1) Direction vector of side BC

$$\mathbf{m} = \mathbf{B} - \mathbf{C} \tag{2.0.1}$$

$$= \begin{pmatrix} 3 \\ -3 \end{pmatrix} \tag{2.0.2}$$

Direction vector of side BC is normal of altitude from \mathbf{A}

2) Equation of the altitude

$$\mathbf{m}^{\mathsf{T}} \left(\mathbf{x} - \mathbf{A} \right) = 0 \tag{2.0.3}$$

$$(3 \quad -3)\mathbf{x} = -3$$
 (2.0.4)

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -1 \tag{2.0.5}$$

3) Normal vector of line BC

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.6}$$

4) Equation of line BC

$$\mathbf{n}^{\mathsf{T}} \left(\mathbf{x} - \mathbf{B} \right) = 0 \tag{2.0.7}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 3 \tag{2.0.8}$$

(2.0.9)

On comparing with $\mathbf{n}^{\mathsf{T}}\mathbf{x} = c$,

$$c = 3$$
 (2.0.10)

5) Length of altitude

$$length = \frac{\left|\mathbf{n}^{\mathsf{T}}\mathbf{A} - c\right|}{\|\mathbf{n}\|} \tag{2.0.11}$$

$$=\sqrt{2} \tag{2.0.12}$$

Parameter	Value	Desription
A	$\binom{2}{3}$	Vertex 'A' of the triangle
В	$\begin{pmatrix} 4 \\ -1 \end{pmatrix}$	Vertex 'B' of triangle
С	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$	Vertex 'C' of triangle

TABLE 5: Table

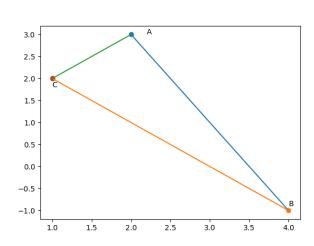


Fig. 5: Figure