Que: 11.10.3.17

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1 Problem

In triangle ABC with vertices $\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, FInd the equation and length of altitude from vertex $\mathbf{A}$

2 Solution

1) Direction vector of side BC

$$\mathbf{m} = \mathbf{B} - \mathbf{C} \tag{2.0.1}$$
$$= \begin{pmatrix} 3 \\ -3 \end{pmatrix} \tag{2.0.2}$$

Direction vector of side BC is normal of altitude from $\mathbf{A}$

2) Equation of the altitude

$$\mathbf{m}^\top (\mathbf{x} - \mathbf{A}) = 0 \tag{2.0.3}$$
$$\begin{pmatrix} 3 & -3 \end{pmatrix} \mathbf{x} = -3 \tag{2.0.4}$$
$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -1 \tag{2.0.5}$$

3) Equation of line BC

$$\mathbf{x} = \mathbf{B} + \mu \mathbf{m} \tag{2.0.6}$$
$$\mathbf{x} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -3 \end{pmatrix} \tag{2.0.7}$$
$$i.e. \quad \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 3 \tag{2.0.8}$$

4) Optimization problem

The length of altitude can be expressed as a optimization problem,

$$\min \|\mathbf{A} - \mathbf{x}\|^2 \tag{2.0.9}$$

such that

$$\mathbf{n}^\top \mathbf{x} = c \tag{2.0.10}$$

where,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.11}$$
$$c = 3 \tag{2.0.12}$$

(2.0.9) is solved using Gradient Descent in `codes/11.10.3.17.py`.

5) Update equation for gradient descent

Here, function to be minimized is

$$f(\mathbf{x}) = \|\mathbf{A} - \mathbf{x}\|^2 \tag{2.0.13}$$
$$\tag{2.0.14}$$

Gradient of this function is

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = 2(\mathbf{x} - \mathbf{A}) \tag{2.0.15}$$

But due to the constraint, only the component orthogonal to $\mathbf{n}$ is considered. Hence, the gradient is updated as

$$\text{grad} = -2\left((\mathbf{x} - \mathbf{A}) - \frac{\mathbf{n}^\top (\mathbf{x} - \mathbf{A})}{\|\mathbf{n}\|}\mathbf{n}\right) \tag{2.0.16}$$

The parameter $\mathbf{x}$ is updated using the following equation,

$$\mu = \mu - \alpha \times \text{grad} \tag{2.0.17}$$

Where α is the learning rate.

The results obtained are using 0.01 as learning rate, iterated until the diffrence between cost in consecutive iterations is lesser than 5×10^{-7}.

The results obtained are:

$$\mathbf{x}_{min} = \begin{pmatrix} 1.00039877 \\ 1.99960123 \end{pmatrix} \tag{2.0.18}$$
$$\min \|\mathbf{A} - \mathbf{x}\|^2 = 2.000000318 \tag{2.0.19}$$

Therefore, the length of altitude is given by,

$$l = \sqrt{2} \approx 1.414213562 \tag{2.0.20}$$

Parameter	Value	Desription
A	$\begin{pmatrix}2\\3\end{pmatrix}$	Vertex 'A' of the triangle
B	$\begin{pmatrix}4\\-1\end{pmatrix}$	Vertex 'B' of triangle
C	$\begin{pmatrix}1\\2\end{pmatrix}$	Vertex 'C' of triangle
learnRate	0.01	Learning rate of the model

TABLE 5: Table

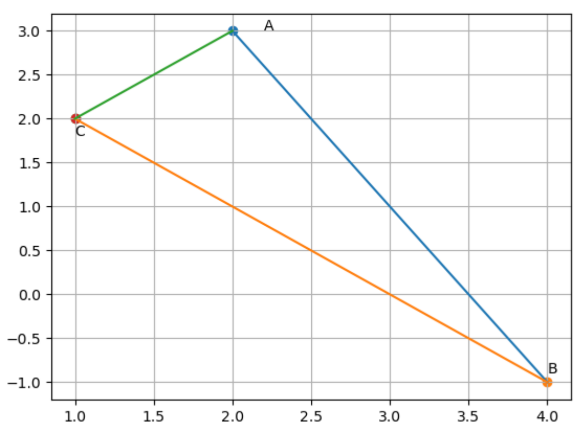

Fig. 5: Figure