

Question : 11.10.3.18

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1 PROBLEM

If p is the length of perpendicular from origin to the line whose intercepts on the axes are a and b , then show that

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} \quad (1.0.1)$$

2 SOLUTION

The x-intercept of the line is $\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix}$ and the y-intercept is $\mathbf{B} = \begin{pmatrix} 0 \\ b \end{pmatrix}$. The direction vector of the line is given by,

$$\mathbf{m} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ b \end{pmatrix} \quad (2.0.1)$$

$$= \begin{pmatrix} a \\ -b \end{pmatrix} \quad (2.0.2)$$

The normal vector is,

$$\mathbf{n} = \begin{pmatrix} b \\ a \end{pmatrix} \quad (2.0.3)$$

The perpendicular distance between origin $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and the line,

$$p = \frac{|\mathbf{n}^\top (\mathbf{O} - \mathbf{A})|}{\|\mathbf{n}\|} \quad (2.0.4)$$

$$= \frac{ab}{\sqrt{a^2 + b^2}} \quad (2.0.5)$$

$$\Rightarrow \frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2} \quad (2.0.6)$$

$$= \frac{1}{a^2} + \frac{1}{b^2} \quad (2.0.7)$$

Hence proved.

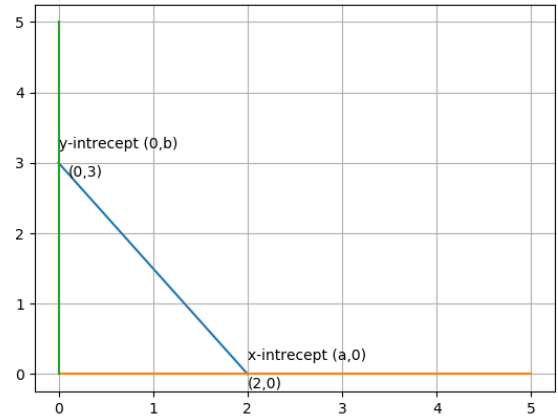


Fig. 0: Line having intercepts a and b