

Question: 9.10.5.11

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1 PROBLEM

ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

2 SOLUTION

let $\angle CBD = \theta$ and $\angle CAD = \phi$ Triangle ABC is right angled with right angle at **B**

Therefore, **B** lies on the circle centered at

$$\mathbf{O} = \frac{\mathbf{A} + \mathbf{C}}{2} \quad (2.0.1)$$

and radius $\frac{\|\mathbf{A} - \mathbf{C}\|}{2}$.

Similarly, **D** lies on the same circle.

$\angle CAD$ and $\angle CBD$ lie on the segment **C – D** of the circle.

$$\therefore, \angle CAD = \angle CBD \quad (2.0.2)$$

For example, Consider right triangles such that,

$$\mathbf{A} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{C} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.4)$$

$$\mathbf{B} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad (2.0.5)$$

$$\mathbf{D} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (2.0.6)$$

$$\cos \theta = \frac{(\mathbf{C} - \mathbf{B})^\top (\mathbf{D} - \mathbf{B})}{\|\mathbf{C} - \mathbf{B}\| \|\mathbf{D} - \mathbf{B}\|} \quad (2.0.7)$$

$$= \frac{1}{\sqrt{2}} \cos \phi = \frac{(\mathbf{C} - \mathbf{A})^\top (\mathbf{D} - \mathbf{A})}{\|\mathbf{C} - \mathbf{A}\| \|\mathbf{D} - \mathbf{A}\|} \quad (2.0.8)$$

$$= \frac{1}{\sqrt{2}} \quad (2.0.9)$$

$$\Rightarrow \theta = \phi \quad (2.0.10)$$

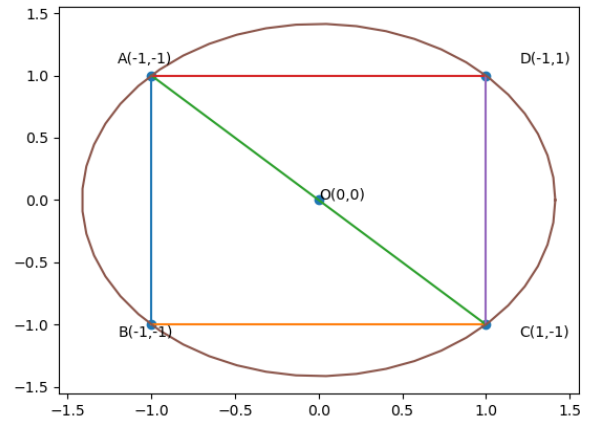


Fig. 0: Figure 1