Que: 10.10.1.3

Nikam Pratik Balasaheb (EE21BTECH11037)

1 PROBLEM

A tangent $\mathbf{P} - \mathbf{Q}$ at a point $\mathbf{P}$ of a circle of radius $r = 5cm$ meets a line through the center $\mathbf{O}$ at a point $\mathbf{Q}$ so that $\mathbf{O} - \mathbf{Q} = 12cm$. Find length $\mathbf{P} - \mathbf{Q}$.

2 SOLUTION

Let the circle be

$$\|\mathbf{x}\|^2 = 25 \qquad (2.0.1)$$

and the point $\mathbf{P} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

The tangent at $\mathbf{P}$, that $\mathbf{Q}$ lies on, is given by

$$\mathbf{x} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (2.0.2)$$

$$= \begin{pmatrix} \lambda \\ 5 \end{pmatrix} \qquad (2.0.3)$$

also,

$$\|\mathbf{Q} - \mathbf{O}\| = 12 \qquad (2.0.4)$$

$$\|\mathbf{Q}\| = 12 \qquad (2.0.5)$$

$$\|\mathbf{Q}\|^2 = 144 \qquad (2.0.6)$$

for $\mathbf{Q}$

$$\lambda^2 + 25 = 144 \qquad (2.0.7)$$

$$\lambda^2 = 119 \qquad (2.0.8)$$

$$\lambda = \pm \sqrt{119} \qquad (2.0.9)$$

Fig. 0: Figure 1

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} - \begin{pmatrix} \sqrt{119} \\ 5 \end{pmatrix} \qquad (2.0.10)$$

$$= \begin{pmatrix} \sqrt{119} \\ 0 \end{pmatrix} \qquad (2.0.11)$$

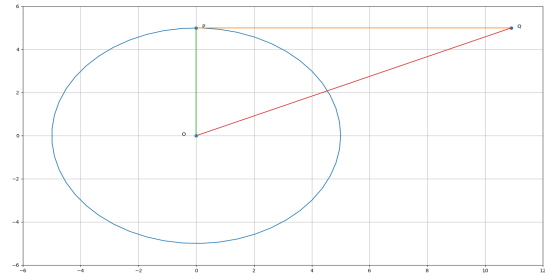$$\|\mathbf{P} - \mathbf{Q}\| = \sqrt{119} \qquad (2.0.12)$$