EE2802: Assignment4

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1 Problem

A line passes through $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\begin{pmatrix} h \\ k \end{pmatrix}$. If the slope of the line is m, show that
$k - y_1 = m(h - x_1)$

2 Solution

let the line joining $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} h \\ k \end{pmatrix}$ be $\mathbf{x} = \mathbf{x_0} + \lambda\mathbf{m_1}$

$$\mathbf{A} - \mathbf{B} = (\mathbf{x_0} + \lambda_1\mathbf{m_1}) - (\mathbf{x_0} + \lambda_2\mathbf{m_1}) = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} - \begin{pmatrix} h \\ k \end{pmatrix} \tag{2.0.1}$$

$$\implies (\lambda_1 - \lambda_2)\,\mathbf{m_1} = \begin{pmatrix} x_1 - h \\ y_1 - k \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{m_1} = k\begin{pmatrix} x_1 - h \\ y_1 - k \end{pmatrix} \tag{2.0.3}$$

slope m is given by $m = tan(\theta)$, θ being angle between line and x axis i.e. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \mathbf{x} = 0$

$$cos\theta = \frac{\mathbf{m_1}^T\mathbf{m_2}}{\|\mathbf{m_1}\|\|\mathbf{m_2}\|} \tag{2.0.4}$$

$$= \frac{x_1 - h}{\sqrt{(x_1 - h)^2 + (y_1 - k)^2}} \tag{2.0.5}$$

$$m = tan\theta = \frac{\sqrt{1 - cos^2\theta}}{cos\theta} \tag{2.0.6}$$

$$m = \frac{y_1 - k}{x_1 - h} \tag{2.0.7}$$

$$\implies k - y_1 = m(h - x_1) \tag{2.0.8}$$

Hence Proved.