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Problem: 10.10.2.3

Nikam Pratik Balasaheb (EE21BTECH11037)

1 Problem

If the tangents PA and PB from a point P to a circle with center O are inclined to each other at 80° , find $\angle POA$.

2 Solution

$$\angle APB = 80^{\circ} \tag{2.0.1}$$

$$\angle APO = \frac{1}{2} \angle APB \tag{2.0.2}$$

$$=40^{\circ} = \theta (say)$$
 (2.0.3)

Therefore, it can be said that **P** lies on the line

$$(-\sin\theta \cos\theta)\mathbf{x} = 0 \tag{2.0.4}$$

Let the circle be $\|\mathbf{x}\|^2 = r^2$ and \mathbf{A} be $\begin{pmatrix} 0 \\ r \end{pmatrix}$.

Therefore, the tangent that P lies on is given by:

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = r \tag{2.0.5}$$

The point **P** is given by:

$$\begin{pmatrix} -\cos\theta & \sin\theta \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ r \end{pmatrix} \tag{2.0.6}$$

Augmented Matrix:

$$\begin{pmatrix} -\sin\theta & \cos\theta & 0\\ 0 & 1 & r \end{pmatrix} \tag{2.0.7}$$

$$\stackrel{R_1 \leftarrow \frac{R_1}{-\sin\theta} + \cot\theta R_2}{\longleftrightarrow} \tag{2.0.8}$$

$$\begin{array}{ccc}
\stackrel{R_1 \leftarrow \frac{R_1}{-\sin\theta} + \cot\theta R_2}{\longleftrightarrow} & (2.0.8) \\
\begin{pmatrix}
1 & 0 & r \cot\theta \\
0 & 1 & r
\end{pmatrix} & (2.0.9)$$

$$\mathbf{P} = \begin{pmatrix} r \cot \theta \\ r \end{pmatrix} \tag{2.0.10}$$

let $\angle POA = \phi$

$$\cos \phi = \frac{(\mathbf{P} - \mathbf{O})^{\top} (\mathbf{A} - \mathbf{O})}{\|\mathbf{P} - \mathbf{O}\| \|\mathbf{A} - \mathbf{O}\|}$$
(2.0.11)

$$= \frac{\left(r\cot\theta \quad r\right)\binom{0}{r}}{r^2\csc\theta} \tag{2.0.12}$$

$$\cos \phi = \sin \theta \tag{2.0.13}$$

$$\Rightarrow \quad \phi = 90^{\circ} - \theta \qquad (2.0.14)$$

$$\phi = 50^{\circ} \qquad (2.0.15)$$

$$\phi = 50^{\circ} \tag{2.0.15}$$

0	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Center of the given circle
A	$\begin{pmatrix} 0 \\ 5 \end{pmatrix}$	Point where tangent is taken
r	5	radius of given circle
∠APB	80°	Angle between tangents

TABLE 0: Table 1

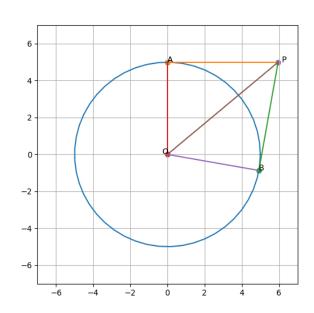


Fig. 0: Figure 1