Question : 11.10.3.18

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1 Problem

If p is the length of perpendicular from origin to the line whose intercepts on the axes area and b, then show that

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} \tag{1.0.1}$$

2 Solution

The x-intercept of the line is $\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix}$ and the y-intercept is $\mathbf{B} = \begin{pmatrix} 0 \\ b \end{pmatrix}$ The direction vector of the line is given by,

$$\mathbf{m} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ b \end{pmatrix} \tag{2.0.1}$$

$$= \begin{pmatrix} a \\ -b \end{pmatrix} \tag{2.0.2}$$

The normal vector is,

$$\mathbf{n} = \begin{pmatrix} b \\ a \end{pmatrix} \tag{2.0.3}$$

The line equation is,

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \tag{2.0.4}$$

$$\begin{pmatrix} b & a \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} a \\ 0 \end{pmatrix} \right) = 0 \tag{2.0.5}$$

$$\begin{pmatrix} b & a \end{pmatrix} \mathbf{x} = ab \tag{2.0.6}$$

comparing the line equation with

$$\mathbf{n}^\top \mathbf{x} = c, \tag{2.0.7}$$

$$c = ab \tag{2.0.8}$$

The perpendicular distance between origin $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and the line,

$$p = \frac{|n^\top \mathbf{O} - c|}{\|\mathbf{n}\|} \tag{2.0.9}$$

$$= \frac{ab}{\sqrt{a^2 + b^2}} \tag{2.0.10}$$

$$\implies \frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2} \tag{2.0.11}$$

$$= \frac{1}{a^2} + \frac{1}{b^2} \tag{2.0.12}$$

Hence proved.

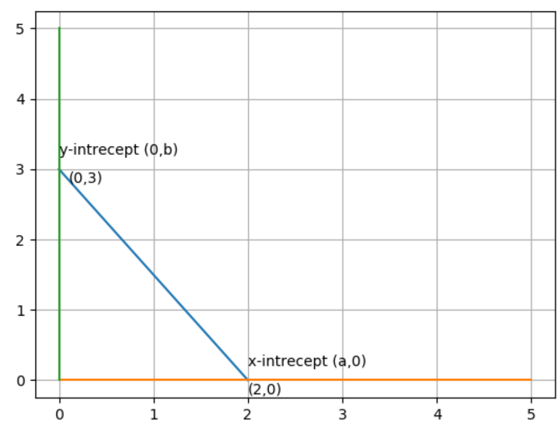

Fig. 0: Line having intercepts a and b