

Que: 11.10.3.17

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1 PROBLEM

In triangle ABC with vertices $\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, Find the equation and length of altitude from vertex A

2 SOLUTION

1) Direction vector of side BC

$$\mathbf{m} = \mathbf{B} - \mathbf{C} \quad (2.0.1)$$

$$= \begin{pmatrix} 3 \\ -3 \end{pmatrix} \quad (2.0.2)$$

Direction vector of side BC is normal of altitude from A

2) Equation of the altitude

$$\mathbf{m}^T (\mathbf{x} - \mathbf{A}) = 0 \quad (2.0.3)$$

$$\begin{pmatrix} 3 & -3 \end{pmatrix} \mathbf{x} = -3 \quad (2.0.4)$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -1 \quad (2.0.5)$$

3) Equation of line BC

$$\mathbf{x} = \mathbf{B} + \mu \mathbf{m} \quad (2.0.6)$$

$$\mathbf{x} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -3 \end{pmatrix} \quad (2.0.7)$$

$$i.e. \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 3 \quad (2.0.8)$$

4) Optimization problem

The length of altitude can be expressed as a optimization problem,

$$\min \|\mathbf{A} - \mathbf{x}\|^2 \quad (2.0.9)$$

such that

$$\mathbf{n}^T \mathbf{x} = c \quad (2.0.10)$$

where,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.11)$$

$$c = 3 \quad (2.0.12)$$

(2.0.9) is solved using Gradient Descent in codes/11.10.3.17.py.

5) Update equation for gradient descent
Here, function to be minimized is

$$g(\mathbf{x}) = \|\mathbf{A} - \mathbf{x}\|^2 \quad (2.0.13)$$

$$i.e. f(\mu) = \|\mathbf{A} - \mathbf{B} - \mu \mathbf{m}\|^2 \quad (2.0.14)$$

$$= \left\| \begin{pmatrix} -2 \\ 4 \end{pmatrix} - \mu \begin{pmatrix} 3 \\ -3 \end{pmatrix} \right\|^2 \quad (2.0.15)$$

The parameter μ is updated using the following equation,

$$\mu = \mu - \alpha \frac{\partial f(\mu)}{\partial \mu} \quad (2.0.16)$$

$$= \mu - \alpha (2\mu \|\mathbf{m}\|^2 - 2\mathbf{m}^T (\mathbf{A} - \mathbf{B})) \quad (2.0.17)$$

$$\mu = \mu - \alpha (18\mu + 36) \quad (2.0.18)$$

Where α is the learning rate.

The results obtained are using 0.01 as learning rate, iterated until the difference between cost in consecutive iterations is lesser than 5×10^{-7} .

The results obtained are:

$$\mu_{min} = -0.99986708 \quad (2.0.19)$$

$$\min \|\mathbf{A} - \mathbf{x}\|^2 = 2.000000318 \quad (2.0.20)$$

Therefore, the length of altitude is given by,

$$l = \sqrt{2} \approx 1.414213562 \quad (2.0.21)$$

Parameter	Value	Description
A	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$	Vertex 'A' of the triangle
B	$\begin{pmatrix} 4 \\ -1 \end{pmatrix}$	Vertex 'B' of triangle
C	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$	Vertex 'C' of triangle
learnRate	0.01	Learning rate of the model

TABLE 5: Table

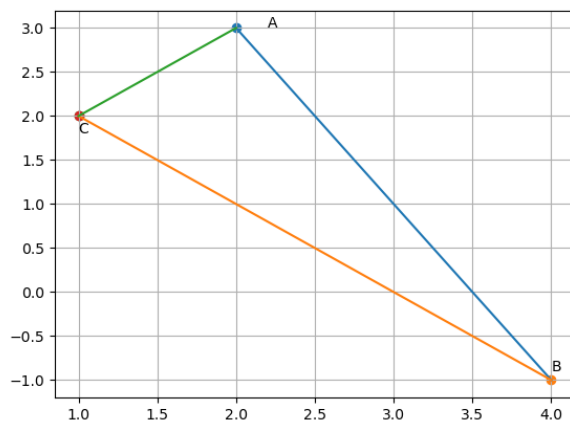


Fig. 5: Figure