Question: 9.10.5.11

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1 Problem

ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

2 Solution

Consider $\triangle ABC$ such that

$$\mathbf{A} = \begin{pmatrix} -a \\ 0 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{2.0.2}$$

$$\tag{2.0.3}$$

Both the triangles ($\triangle ABC$ and $\triangle ADC$) are right angled.
Hence, it can be said that **B** and **D** lie on circle having **A** and **C** as its diametric end-points.
Equation of this circle is given by:

$$\left\| \mathbf{x} - \left(\frac{\mathbf{A} + \mathbf{C}}{2} \right) \right\|^2 = \left\| \frac{\mathbf{A} - \mathbf{C}}{2} \right\|^2 \tag{2.0.4}$$

$$\|\mathbf{x}\|^2 = a^2 \tag{2.0.5}$$

Points on this circle can be represented as

$$\mathbf{x} = a \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2.0.6}$$

Let

$$\mathbf{B} = a \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix} \tag{2.0.7}$$

$$\mathbf{D} = a \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} \tag{2.0.8}$$

$$\cos\angle CAD = \frac{(\mathbf{C} - \mathbf{A})^\top (\mathbf{D} - \mathbf{A})}{\|\mathbf{C} - \mathbf{A}\| \|\mathbf{D} - \mathbf{A}\|} \tag{2.0.9}$$

$$= \sqrt{1 + \cos\theta_1} \tag{2.0.10}$$

$$\cos\angle CBD = \frac{(\mathbf{C} - \mathbf{B})^\top (\mathbf{D} - \mathbf{B})}{\|\mathbf{C} - \mathbf{B}\| \|\mathbf{D} - \mathbf{B}\|} \tag{2.0.11}$$

$$= \sqrt{1 + \cos\theta_1} \tag{2.0.12}$$

$$\implies \angle CAD = \angle CBD \tag{2.0.13}$$

Parameter	Value	Description
A	$\begin{pmatrix} -5 \\ 0 \end{pmatrix}$	End of common hypotenuse
C	$\begin{pmatrix} 5 \\ 0 \end{pmatrix}$	Other end of common hypotenuse
θ_1	$\frac{3\pi}{5}$	parameter θ for **B**
θ_2	$-\frac{9\pi}{20}$	parameter θ for **D**
a	1	parameter for **A**, **C**

TABLE 0: Table1

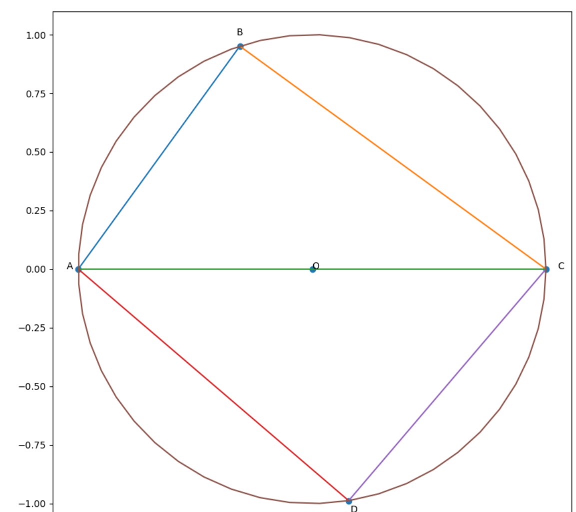

Fig. 0: Figure 1