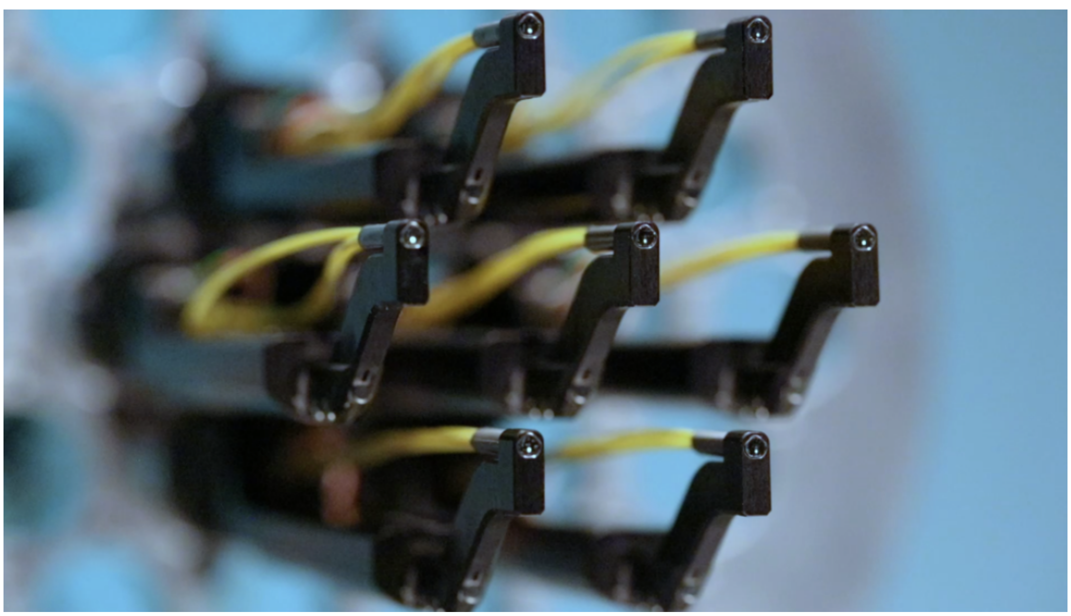
Robotics practicals

Lab : Micro robots for astrophysics surveys

March 24, 2025



Homework done by:

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1 SCARA theory

1.1 Workspace definition

1.1.1 Regarding the above image, draw the workspace of such positioner (i.e. the area where the fiber optic can be placed)

As a reference the nominal lengths of the robot's arms are:

- $l_{\alpha nom} = 7.4mm$
- $l_{\beta nom} = 14.3458mm$

For those given lengths, the workspace looks like a doughnut. (See Figure : 1)

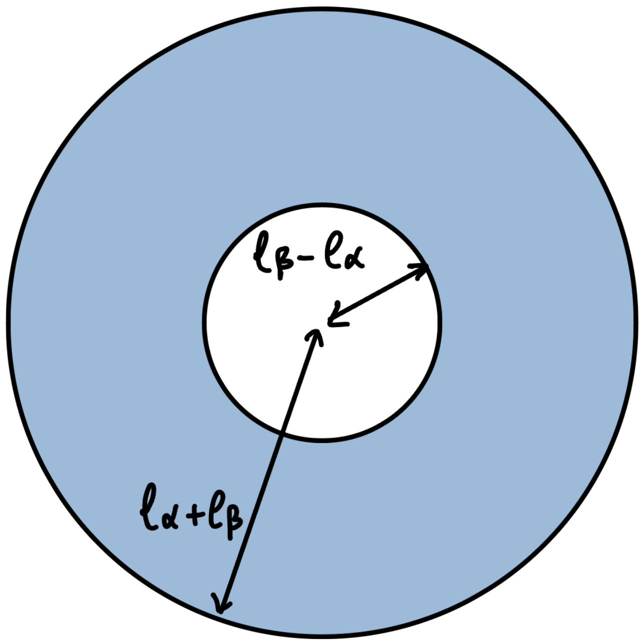


Figure 1: SCARA robot workspace

1.1.2 What happens to the workspace when both arm lengths are identical? (draw again)

The minimal distance for the hollow ring $l_\alpha - 1_\beta = 0$, so the workspace area becomes a circle. (See Figure : 2)

1.1.3 Take the case 1.1.2 again and let's define p as the distance between the center of 2 robots. Draw the combined workspace of 7 robots of which the centers are placed at the vertices & center of a regular hexagon if $p = l_{alpha} + l_{beta}$

(See Figure 3)

1.2 SCARA mathematical models

1.2.1 Given the following configuration, provide the Direct Geometric Model (DGM) of a SCARA robot in matrix expression

The DGM for a SCARA robot arm, whose resting position (both angles 0) is vertical, is :

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_\alpha * sin(\alpha) + l_\beta * sin(\alpha + \beta) \\ l_\alpha * cos(\alpha) + l_\beta * cos(\alpha + \beta) \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

with (x_0, y_0) being the coordinates of alpha's rotation point (the "shoulder").

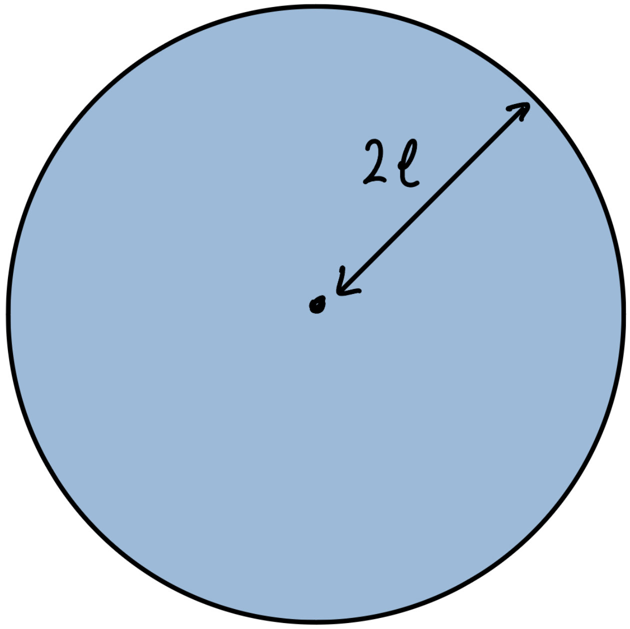


Figure 2: SCARA robot workspace for equal arm lengths

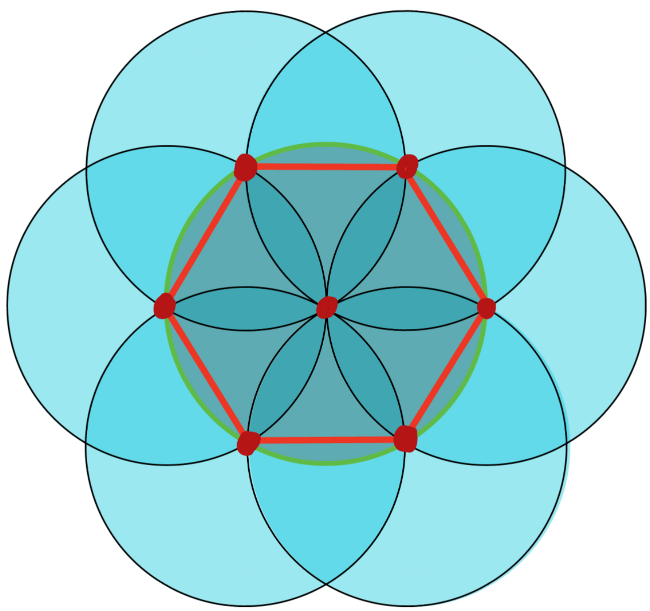


Figure 3: 7 robot configuration

1.2.2 Derive the Jacobian matrix of the SCARA configuration

$$J = \begin{bmatrix} \frac{\delta x}{\delta \alpha} & \frac{\delta x}{\delta \beta} \\ \frac{\delta y}{\delta \alpha} & \frac{\delta y}{\delta \beta} \end{bmatrix} = \begin{bmatrix} l_\alpha * cos(\alpha) + l_\beta * cos(\alpha + \beta) & l_\beta * cos(\alpha + \beta) \\ -l_\alpha * sin(\alpha) - l_\beta * sin(\alpha + \beta) & -l_\beta * sin(\alpha + \beta) \end{bmatrix} \quad (1)$$

1.2.3 Provide finally the analytical expression of error in cartesian position in terms of angular error & comment

The error in cartesian position (x,y) can be approximated using the the Jacobian matrix as follows :

$$\Delta_{pos} = \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix} \approx J * \begin{bmatrix} \Delta_\alpha \\ \Delta_\beta \end{bmatrix} \tag{2}$$

2 Experimental part

2.2 Measuring arms length

2.2.1 Knowing the kinematics of the robot and the available python function (goto, get centroid, fit circle), propose 2 methods for determining the length of both arms

The first naive method would be to go to the (0,0), (0,180) and (180,180) angular positions and measure the distances which would add up to $2l_\beta$ and $2(l_\alpha + l_\beta)$. But this assumes that we have perfect control over the angular position to align them in a perfect straight line, which is not the case. The second and better method is to measure several points fixing one angle, and fit a circle to them. The output radius is the length we are looking for.

2.2.2 Using the method you find the fittest measure the actual length of each arm

We will use the second method, because it is (almost) independent of the angular precision. Indeed if the robot doesn't hit precisely 180 deg, we can't know the lengths accurately with the first method. With the second, we make the end-effector move to four positions, note their positions (x,y) given by the camera and fit them to a circle using cam.fit_circle(). This doesn't depend on the angular precision of the robot anymore but on the precision of the camera, which we assume to be largely better. We use the second method to measure the beta-arm (keep $\alpha = 0°$ and move β, radius is length of the arm) and the alpha-arm (keep $\beta = 0°$ and move α, radius is sum of both arms). (See Table 1) It must be noted however that although the measure of the beta-arm is purely dependent on the precision of the camera, when we measure the alpha-arm we assume the beta-arm to be at exactly 0 degrees, such that the SCARA assumes an outstretched position. Since the angular control of the robot isn't perfect, there may be a small angle meaning the arms aren't fully aligned. This adds some error to the measurement.

Angle α [°]	Angle β [°]	x [mm]	y [mm]
0	0	94.193	83.726
0	90	117.058	63.312
0	180	96.636	40.453
0	270	73.786	60.847
0	0	94.191	83.727
90	0	109.873	70.860
180	0	97.003	55.184
270	0	81.325	68.015
Radius, center x & y (first 4 points)	**14.342**	95.597	69.456
Radius, center x & y (last 4 points)	**21.671**	95.421	62.090

Table 1: Measured lengths of robot arms l_α and l_β

The radius obtained from the first four measures directly corresponds to the length of the beta-arm.

$$l_\beta = 14.342[mm] \tag{3}$$

The second radius corresponds to length the outstrechted SCARA robot. To find the length of the alpha arm, we must simply substract the previously obtained beta-arm from the radius.

$$l_\alpha = radius - l_\beta = 7.329[mm] \tag{4}$$

We thus see that the robot arms after assembly are respectively 0.0038[mm] and 0.071 [mm] shorter than their theoretical lengths.

2.3 Accuracy and precision measurement

2.3.1 What is the difference between accuracy and precision (or repeatability) ?

Accuracy measures how close the robot's placements are to the true target position. If a robot is accurate, the average position over multiple tries will be close to the desired target, even if individual placements vary.

Precision or repeatability describes the consistency of the robot's placements. A highly precise robot will place the fiber in the same spot every time, even if that spot is slightly off from the desired position.

2.3.2 How do you quantify them?

Accuracy is quantified by measuring the deviation of each position from the desired target and computing a metric like the error (RMSE or other).

Precision is quantified using the standard deviation or variance of the repeated placements. A lower standard deviation indicates higher repeatability.

2.3.3 Practical measurements

We measured the position of the end-effector for the movement between angles ($\alpha_1 = 0°$, $\beta_1 = 0°$) and (($\alpha_2 = 0°$, $\beta_2 = 180°$) to measure the beta-arm's accuracy. Then we did the measures for the alpha-arm with two different angle pairs ($\alpha_3 = 100°$, $\beta_3 = 0°$) and ($\alpha_4 = 10°$, $\beta_4 = 0°$). NB : the choice of different angles between first and second set of measures might seem uncalled for, but our hardware setup broke down in the middle of the measures. The distraction from trying to solve the problem explains that we weren't thorough in keeping the same values. That being said the precision shouldn't be affected by the width of the angular movement, so whether we choose an 180° turn or only a quarter-turn does not impact our measurements.

(See Tables 2 and 3)

(See graphs 4a and 4d for the overall plots of the position alpha and beta arms) (See Graphs 4b,4c,4e,4f with mean and standard deviation for each position)

Measurement		x1 [mm]	y1 [mm]		x2 [mm]	y2 [mm]
1		94.187	83.718		97.010	55.171
2		94.193	83.721		97.011	55.170
3		94.193	83.721		97.012	55.171
4		94.194	83.721		97.012	55.170
5		94.194	83.721		97.011	55.170
6		94.195	83.721		97.012	55.170
7		94.195	83.721		97.011	55.170
8		94.195	83.721		97.012	55.170
9		94.195	83.722		97.012	55.171
10		94.195	83.721		97.012	55.171
Average		**94.194**	**83.721**		**97.012**	**55.170**

Table 2: Measurements of (x,y) coordinates corresponding to angles (0,0) and (0,180)

2.3.4 Accuracy and repeatability determination

The analysis of the SCARA robot's arms based on the provided mean and standard deviation values for the angles α (theta1) and β (theta2) leads to the following conclusions:

- **Alpha Arm 1**:
  - Mean: $(\alpha_1, \beta_1) = (116.95, 59.54)$
  - Std: $(\sigma_{\alpha_1}, \sigma_{\beta_1}) = (0.00048, 0.00185)$
  - **Precision**: High precision due to very low standard deviation values.
- **Alpha Arm 2**:
  - Mean: $(\alpha_2, \beta_2) = (97.97, 83.61)$

Measurement		x3 [mm]	y3 [mm]		x4 [mm]	y4 [mm]
1		116.952	59.529		97.966	83.605
2		116.951	59.535		97.965	83.605
3		116.951	59.535		97.966	83.605
4		116.951	59.535		97.965	83.606
5		116.951	59.536		97.965	83.606
6		116.951	59.536		97.966	83.605
7		116.951	59.537		97.965	83.606
8		116.951	59.536		97.965	83.606
9		116.951	59.536		97.965	83.606
10		116.951	59.535		97.965	83.606
Average		**116.951**	**59.535**		**97.965**	**83.606**

Table 3: Measurements of (x,y) coordinates corresponding to angles (100,0) and (10,0)

  - Std: $(\sigma_{\alpha_2}, \sigma_{\beta_2}) = (0.00031, 0.00043)$
  - **Precision**: Very high precision with extremely low standard deviation values.

- **Beta Arm 1**:
  - Mean: $(\alpha_1, \beta_1) = (94.19, 83.72)$
  - Std: $(\sigma_{\alpha_1}, \sigma_{\beta_1}) = (0.00199, 0.000796)$
  - **Precision**: High precision, though slightly less than Alpha Arm 2.
  - **Accuracy**: The mean values are close to the initial position $(94.194, 83.721)$, indicating good accuracy.

- **Beta Arm 2**:
  - Mean: $(\alpha_2, \beta_2) = (97.01, 55.17)$
  - Std: $(\sigma_{\alpha_2}, \sigma_{\beta_2}) = (0.000466, 0.000233)$
  - **Precision**: Very high precision.

Overall, the SCARA robot's arms demonstrate high precision, indicating reliable and consistent performance in positioning tasks. As we do not consider the theoretical Cartesian position in this section, we cannot discuss about the accuracy, which is how close the value is to the expected one. We will discuss it later on Part 2.5 .

As the accuracy can be calibrate, for applications such as fiber optic positioning, where precise alignment is critical, the observed excellent precision suggests that the system is well-suited for these tasks. The consistency in achieving the expected positions across different measurements reinforces the system's reliability for high-precision applications.

2.4 Backlash determination

2.4.1 What is backlash in a rotary system ? What can be a source of it ?

Backlash is the amount of clearance in a mechanism due by the gaps between parts. This clearance means we cannot reach precisely a given position since the small amount of play will allow the end-effector to wiggle slightly around its desired position. In rotary systems backlash is typically due to gears : the tooth of gear A has a tiny gap between touching the previous tooth and the next tooth of gear B, and can move between these two.

2.4.2 Why is it a problem in the case of positioning a fiber optic ?

Since we want the fiber optics to be positioned exactly on the spot where the star's light will be focused, even a very small deviation from the position can reduce significantly the amount of light collected. Thus backlash is a problem.

2.4.3 How can you get rid of backlash ?

Backlash can be reduced in different ways, the most common being pre-constraining : we use a torsion spring on the gears to force the gear A's teeth to always be in contact with the next (for example) teeth of gear B. Thus the position is fully determined. There are also more specialized ways to reduce backlash, such as split gears or even harmonic/strain wave gears. Also, you can quantify it and account for it when turning in the direction where it occurs (in the preconstrained case).

2.4.4 Measure the angular backlash of the positioner using the provided python functions

We were told the robots have a pre-constraining mechanism in the gearboxes of their motors to reduce backlash. It applies torque on the gears in one rotational direction, such that the teeth of gear A are always forced in contact with the next teeth of gear B. This solves the accuracy problem because it mostly eliminates the play between two gears which allows for inconsistent positionning of the end-effector. But it also induces a different behaviour depending on the rotation direction. If the arm is turning in the same direction as pre-constraining, when it tries to stop the gears will be pushed by the pre-constraining mechanism until they are stopped by the next tooth of the other gear, making the arm move just a bit further than its intended target. In the other direction, the mechanism will push the gears back until they are in contact with the previous tooth of the other gear, making the arm move back a tiny amount at the end of the movement.

If we approach the same position clockwise or counter-clockwise, we should thus see an offset in position due to the preconstarining-mechanism.

To measure backlash of alpha-arm's gearbox, we made the alpha-arm move between three positions (beta-arm was fixed at 0°) : 0°, 45° and 90° in a loop $(0, 45, 90, 45, 0$ and repeat). This allowed us to approach the position 45° from both clockwise and counterclockwise rotation direction.

In the following tables, we noted the (x,y) coordinates that the end-effector attains for the middle position (45°) while coming from 90° position for Table 4 and from 0° for Table 5. We repeated the exact same method for the beta-arm's gearbox by fixing $\alpha = 0°$ and moving the beta-arm, see Tables 6 and 7

See also Figures 5 and 6 for a visual representation of the same data.

Measurement	Position x [mm]	y [mm]
1	109.853	78.250
2	109.853	78.250
3	109.853	78.250
4	109.853	78.250
5	109.853	78.250
6	109.854	78.251
7	109.854	78.251
8	109.854	78.251
9	109.854	78.251
10	109.854	78.251
Average	**109.8541**	**78.2509**

Table 4: (x,y) position of the end-effector for a counter-clock wise movement ending at $\alpha = 45°$

2.5 Theory VS practice

2.5.1 Calculate the theoretical position that you should obtain with the angular combination chosen in 2.3.3

The DGM of the SCARA robot defined in point 1.2.1 and plugging in the actual lengths of the arms that we measured (section 2.2.1) allows us to calculate the positions we should obtain for a movement between angles (0,0) and (0,180) and angles (100,0) and (10,0).

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_\alpha * sin(\alpha) + l_\beta * sin(\alpha + \beta) \\ l_\alpha * cos(\alpha) + l_\beta * cos(\alpha + \beta) \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \tag{5}$$

where (x_0, y_0) are the image coordinates of the attach point of the SCARA arm (its "shoulder" so to speak).

We have found this "shoulder" in section 2.2.2 (the center points associated to arm alpha).

Measurement	Position x [mm]	y [mm]
1	109.854	78.250
2	109.854	78.249
3	109.854	78.250
4	109.854	78.249
5	109.855	78.250
6	109.855	78.250
7	109.854	78.250
8	109.855	78.250
9	109.855	78.250
10	109.855	78.250
Average	**109.8550**	**78.2502**

Table 5: (x,y) coordinates of the end-effector for a clockwise movement ending at $\alpha = 45°$

Measurement	Position x [mm]	y [mm]
1	104.698	80.532
2	104.698	80.532
3	104.698	80.533
4	104.698	80.533
5	104.698	80.533
6	104.698	80.533
7	104.699	80.533
8	104.699	80.533
9	104.699	80.534
10	104.699	80.534
Average	**104.6987**	**80.5333**

Table 6: (x,y) position of the end-effector for a counter-clockwise movement ending at $\beta = 45°$

Measurement	Position x [mm]	y [mm]
1	104.712	80.534
2	104.711	80.535
3	104.711	80.535
4	104.711	80.535
5	104.711	80.535
6	104.711	80.535
7	104.711	80.536
8	104.711	80.536
9	104.711	80.536
10	104.711	80.536
Average	**104.7113**	**80.5353**

Table 7: (x,y) coordinates of the end-effector for a clockwise movement ending at $\beta = 45°$

Sadly, we did not measure the exact position of this "shoulder", so we have to approximate it. In the (0,0) configuration, the end-effector should be directly above the shoulder by a height of $L_\alpha + L_\beta = 21.671[mm]$. This allows us to determine $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 95.421 \\ 62.090 \end{bmatrix}$.

We can now determine the theoretical values of the angle pairs (α_1, β_1) to (α_4, β_4)

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} l_\alpha * sin(\alpha_1) + l_\beta * sin(\alpha_1 + \beta_1) \\ l_\alpha * cos(\alpha_1) + l_\beta * cos(\alpha_1 + \beta_1) \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 95.421 \\ 83.761 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} l_\alpha * sin(\alpha_2) + l_\beta * sin(\alpha_2 + \beta_2) \\ l_\alpha * cos(\alpha_2) + l_\beta * cos(\alpha_2 + \beta_2) \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 95.421 \\ 55.078 \end{bmatrix} \tag{7}$$

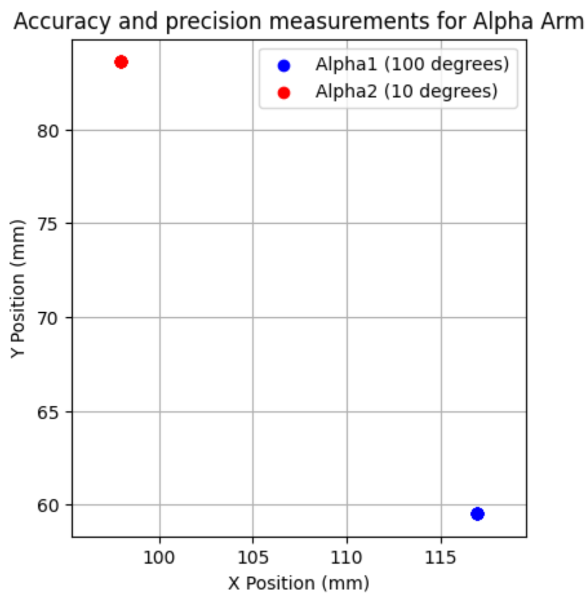
$$\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} 116.763 \\ 58.326 \end{bmatrix} \qquad \begin{bmatrix} x_4 \\ y_4 \end{bmatrix} = \begin{bmatrix} 99.184 \\ 83.432 \end{bmatrix} \tag{8}$$

2.5.2 Compare the theoretical value with the measured one and comment

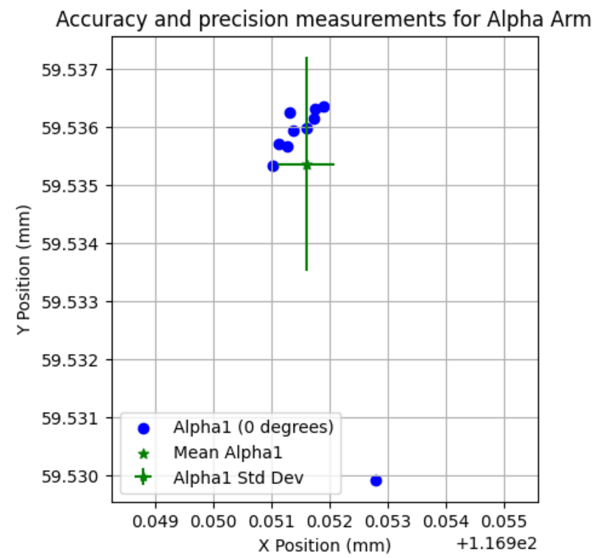
Angle pair [°]		$x_{theoretical}$	$y_{theoretical}$		$x_{measured}$	$y_{measured}$
(0,0)		95.421	83.761		94.194	83.721
(0,180)		95.421	55.078		97.012	55.170
(100,0)		116.763	58.326		116.951	59.535
(10,0)		99.184	83.432		97.965	83.606

Table 8: Theoretical vs measured values comparison

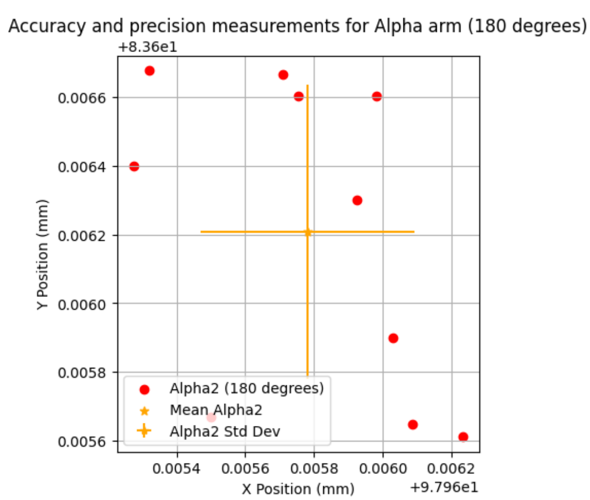
We can observe that the predicted values are off by one or two millimeters. This is again due to the fact that we do not have very precise control over the angular position of the motors. The predicted values are pretty close, but I suppose they would not be good enough for the level of precision needed by the astrobots. So the camera feedback is essential to position the optical fibers well enough in order to collect starlight.



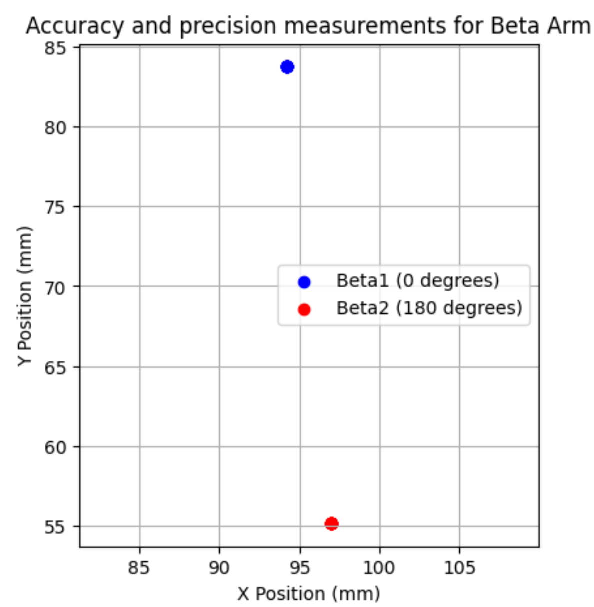
(a) Alpha Arm Measurements



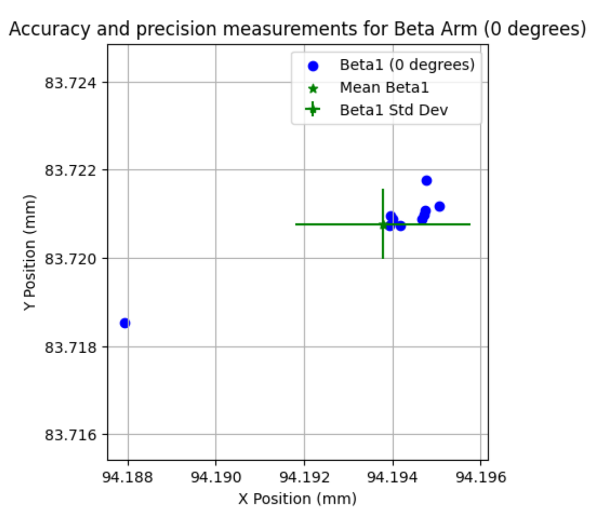
(b) Alpha Arm 1 Measurements



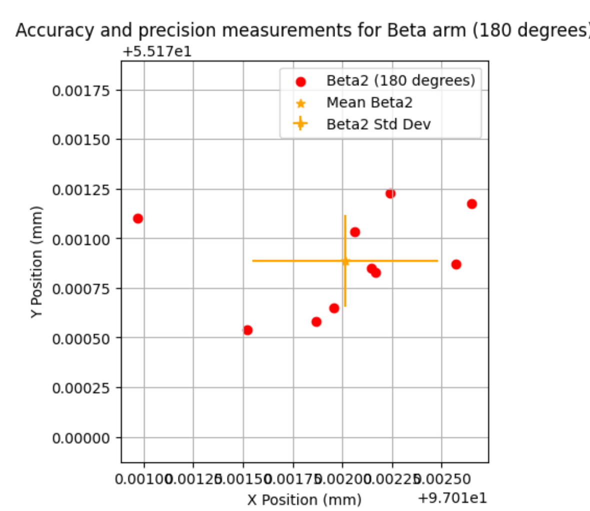
(c) Alpha Arm 2 Measurements



(d) Beta Arm Measurements



(e) Beta Arm 1 Measurements



(f) Beta Arm 2 Measurements

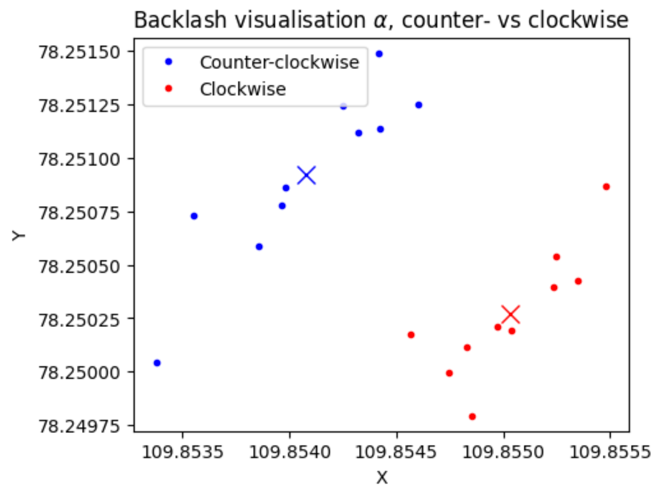


Figure 5: Backlash visualization, α

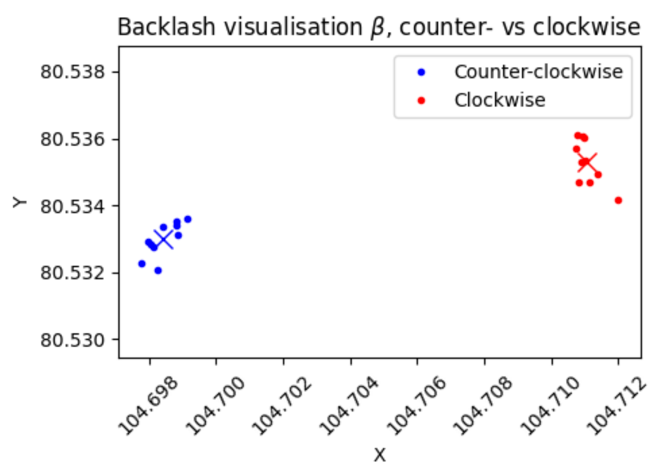


Figure 6: Backlash visualization, β