

PCM \rightarrow prediction \rightarrow residual \rightarrow Rice coding \rightarrow FLAC bits

Pulse Code Modulation (Raw audio file)

Step 1: Lets take 8 PCM samples $8 \times 16 = 128$ bits

$$x = [1000, 1010, 1025, 1030, 1020, 1015, 1018, 1022]$$

Step 2: $\bar{x}[n] = 2 \cdot x[n-1] - x[n-2]$

Predictor (order -2):

we store

$x[0]$ & $x[1]$ raw [initial samples]
for decoding

Step 3: For $n \geq 2$ \because (as we need atleast 2 samples to predict)

$$\rightarrow n=2$$

$$\bar{x}[2] = 2 \cdot (1010) - 1000 = 1020$$

residual

$$[e[2] = x[2] - \bar{x}[2]] = 1025 - 1020 = +5$$

$$\rightarrow n=3$$

$$\bar{x}[3] = 2 \cdot (1025) - 1010 = 1040$$

$$e[3] = -10$$

$$\rightarrow n=4$$

$$\bar{x}[4] = 2 \cdot (1030) - 1025 = 1035$$

$$e[4] = -15$$

$\rightarrow n=5:$

$$\bar{x}[5] = 2(1020) - 1030 = 1010$$

$$e[5] = +5$$

$\rightarrow n=6:$

$$\bar{x}[6] = 2(1015) - 1020 = 1010$$

$$e[6] = +8$$

$\rightarrow n=7$

$$\bar{x}[7] = 2(1018) - 1015 = 1021$$

$$\text{Residual } e[7] = 1022 - 1021 = +1$$

$$\therefore e = [+5, -10, -15, +5, +8, +1]$$

Step 4:

Signed \rightarrow Unsigned mapping (standard Rice map)

$$\frac{u}{v} = \begin{cases} 2e & e > 0 \\ -2e-1 & e < 0 \end{cases}$$

unsigned integers

(i.e., it maps $0 \rightarrow 0; -1 \rightarrow 1; +1 \rightarrow 2; -2 \rightarrow 3; \dots$)

$$e = +5 \Rightarrow u = 2(5) = 10$$

$$e = -10 \Rightarrow u = -2(-10) - 1 \Rightarrow 19$$

$$e = -15 \Rightarrow u = 30 - 1 = 29$$

$$e = +5 \Rightarrow u = 10$$

$$e = +8 \Rightarrow u = 16$$

$$e = +1 \Rightarrow u = 2$$

$$u = [10, 19, 29, 10, 16, 2]$$

Step 5:

Rice coding with parameter $k=2$

quotient $q = \lceil u/2^k \rceil$ (written in unary: q zeros followed by 1)

remainder $r = u \bmod 2^k$ (written in binary)

with $k=2, 2^k=4$

$$\rightarrow u=10; q = \lceil 10/4 \rceil = 2; r = 10 \bmod 4 = 2$$

unary for $q=2 \rightarrow 001$ (3 bits)

Remainder $\rightarrow 10$ (2 bits)

Rice code. bits $\rightarrow 00110$ \rightarrow total 5 bits

$$\rightarrow u=19; q = \lceil 19/4 \rceil = 4; r=3$$

unary for 4 $\rightarrow 00001$ (5 bits)

Remainder $\rightarrow 11$ (2 bits)

Rice bits $\rightarrow 00001101 \rightarrow 7$ bits

$$\rightarrow u=29; q = \lceil 29/4 \rceil = 7; r=1$$

unary for 7 $\rightarrow 00000001 = 8$ bits

Remainder 01 (2 bits)

Rice bits: $0000000101 \rightarrow 10$ bits

$\rightarrow u=10$ again \rightarrow same as (i): 5 bits (00110)

$$\rightarrow u=16; q = \lceil 16/4 \rceil = 4; r=0$$

00001 (5) + remainder 00 (2)
7 bits

$\rightarrow u = 2;$; $q = \lceil 2/4 \rceil = 0$; $r = 2$
 unary for $0 \rightarrow 1$ (Just a t) = 1 bit
 remainder 10 (2 bits)
 Rice bits: 1 10 \rightarrow 3 bits

\Rightarrow total rice bits for residuals
 $= 5 + 7 + 10 + 5 + 7 + 3 = 37$ bits

Step 6

Raw initial samples: $2 \times 16 = 32$ bits

Rice-coded residuals: 37 bits

total bits stored = $32 + 37 = 69$ bits

Compare with raw PCM: $8 \times 16 = 128$ bits

\rightarrow compression ratio (raw/encoded) = $128/69 \approx 1.855$

\rightarrow Space reduction = $(1 - 69/128) \times 100 \rightarrow 46.1\%$ smaller

Step 7

Decoding

$\rightarrow x[0] = 1000$; $x[1] = 1010$

$\rightarrow x[2] = 2 \cdot x[1] - x[0] = 2 \cdot 1010 - 1000 = 1020$

\rightarrow From bitstream decoder reads Rice code for 1st

residual: unary: 00 1 then
remainder: 10 $\rightarrow q = 2$; $r = 2$

Recover:

$$u = q \cdot 4 + r = 2 \cdot 4 + 2 = 10 \quad \begin{aligned} \text{even} \rightarrow e &= u/2 \\ &= 10/2 = 5 \end{aligned}$$

$$\text{Reconstruct: } x[2] = \bar{x}[2] + e \quad \begin{aligned} \text{odd} \rightarrow e &= -(u+1)/2 \\ &= -10/2 = -5 \end{aligned}$$

$= 1020 + 5 = 1025 //$ matches originally