

PCM \rightarrow prediction \rightarrow residual \rightarrow Rice coding \rightarrow FLAC bits

Pulse Code Modulation (Raw audio file)

Step 1: Lets take 8 PCM samples $8 \times 16 = 128$ bits

$$x = [1000, 1010, 1025, 1030, 1020, 1015, 1018, 1022]$$

Step 2: $\bar{x}[n] = 2 \cdot x[n-1] - x[n-2]$

Predictor (order-2):
we store $x[0]$ & $x[1]$ raw [initial samples]

Step 3: For $n \geq 2$ \therefore (as we need atleast 2 samples to predict)

$\rightarrow n=2$

$$\bar{x}[2] = 2 \cdot (1010) - 1000 = 1020$$

residual

$$e[2] = x[2] - \bar{x}[2] = 1025 - 1020 = +5$$

$\rightarrow n=3$

$$\bar{x}[3] = 2 \cdot (1025) - 1010 = 1040$$

$$e[3] = -10$$

$\rightarrow n=4$

$$\bar{x}[4] = 2 \cdot (1030) - 1025 = 1035$$

$$e[4] = -15$$

→ $n=5$:

$$\bar{x}[5] = 2(1020) - 1030 = 1010$$

$$e[5] = +5$$

→ $n=6$:

$$\bar{x}[6] = 2(1015) - 1020 = 1010$$

$$e[6] = +8$$

→ $n=7$

$$\bar{x}[7] = 2(1018) - 1015 = 1021$$

$$\text{Residual } e[7] = 1022 - 1021 = +1$$

$$\therefore e = [+5, -10, -15, +5, +8, +1]$$

Step 4:

signed → unsigned mapping (standard Rize map)

$$u = \begin{cases} 2e & e > 0 \\ -2e - 1 & e < 0 \end{cases}$$

unsigned integers

(i.e., it maps $0 \rightarrow 0$; $-1 \rightarrow 1$; $+1 \rightarrow 2$; $-2 \rightarrow 3$; ...)

$$e = +5 \Rightarrow u = 2(5) = 10$$

$$e = -10 \Rightarrow u = -2(-10) - 1 = 19$$

$$e = -15 \Rightarrow u = 30 - 1 = 29$$

$$e = +5 \Rightarrow u = 10$$

$$e = +8 \Rightarrow u = 16$$

$$e = +1 \Rightarrow u = 2$$

$$u = [10, 19, 29, 10, 16, 2]$$

Step 5:

Rice coding with parameter $k=2$

quotient $q = \lfloor u/2^k \rfloor$ (written in unary: q zeros followed by 1)

remainder $r = u \bmod 2^k$ (written in binary)

with $k=2$, $2^k=4$

$\rightarrow u=10; q = \lfloor 10/4 \rfloor = 2; r = 10 \bmod 4 = 2$

unary for $q=2 \rightarrow 001$ (3 bits)

Remainder $\rightarrow 10$ (2 bits)

Rice code bits $\rightarrow 00110 \rightarrow$ total 5 bits

$\rightarrow u=19; q = \lfloor 19/4 \rfloor = 4; r=3$

unary for 4 $\rightarrow 00001$ (5 bits)

Remainder $\rightarrow 11$ (2 bits)

Rice bits $\rightarrow 00001101 \rightarrow 7$ bits

$\rightarrow u=29; q = \lfloor 29/4 \rfloor = 7; r=1$

unary for 7 $\rightarrow 00000001$ (8 bits)

Remainder 01 (2 bits)

Rice bits: $0000000101 \rightarrow 10$ bits

$\rightarrow u=10$ again \rightarrow same as (i): 5 bits (00110)

$\rightarrow u=16; q = \lfloor 16/4 \rfloor = 4; r=0$

00001 (5) + remainder 00 (2)
7 bits

$$\rightarrow u=2; \quad q = \lfloor 2/4 \rfloor = 0; \quad r=2$$

unary for $0 \rightarrow 1$ (Just a 1) = 1 bit

Remainder 10 (2 bits)

Rice bits: 1 10 \rightarrow 3 bits

$$\Rightarrow \text{Total rice bits for residuals} \\ = 5 + 7 + 10 + 5 + 7 + 3 = 37 \text{ bits}$$

Step 6

Raw initial samples: $2 \times 16 = 32$ bits

Rice - coded residuals: 37 bits

$$\text{total bits stored} = 32 + 37 = 69 \text{ bits}$$

Compare with raw PCM: $8 \times 16 = 128$ bits

$$\rightarrow \text{compression ratio (raw/encoded)} = 128/69 \approx 1.855$$

$$\rightarrow \text{Space reduction} = (1 - 69/128) \times 100 \rightarrow 46.1\% \text{ smaller}$$

Step 7 Decoding

$$\rightarrow x[0] = 1000; \quad x[1] = 1010$$

$$\rightarrow x[2] = 2 \cdot x[1] - x[0] = 2 \cdot 1010 - 1000 = 1020$$

\rightarrow From bitstream decoder reads Rice code for 1st

residual: unary: 00 1 then

remainder: 10 $\rightarrow q=2; r=2$

Recover:

$$u = q \cdot 4 + r = 2 \cdot 4 + 2 = 10$$

$$\bullet \text{ even} \rightarrow e = u/2$$

$$\text{Reconstruct: } x[2] = \bar{x}[2] + e \quad \bullet \text{ Odd} \rightarrow e = -\left(\frac{u+1}{2}\right)$$

$$= 1020 + 5$$

$= 1025$ // matches originally