Queueing Theory: Psychiatrists doing intakes
EBB074A05

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1 General info

This file contains the code and the results that go with this youtube movie: `https://youtu.be/bCU3oP6r-00`.

There are some exercises for you to make for your assignments.

1.1 TODO Set theme and font size

Set the theme and font size so that it is easier to read on youbute

2 Base situation

5 psychiatrists do intakes. See my queueing book for further background.

2.1 Load standard modules

```python
import numpy as np
import matplotlib.pylab as plt
from matplotlib import style

style.use('ggplot')

np.random.seed(3)
```

2.2 Simulate queue length

```python
def computeQ(a, c, Q0=0):  # initial queue length is 0
    N = len(a)
    Q = np.empty(N)  # make a list to store the values of  Q
    Q[0] = Q0
    for n in range(1, N):
        d = min(Q[n - 1], c[n])
        Q[n] = Q[n - 1] + a[n] - d
    return Q
```

2.3 Arrivals

We start with run length 10 for demo purpose.

```python
a = np.random.poisson(11.8, 10)
print(a)
```

```
[12  9  7 13 14  9  9 11 12 10]
```

2.4 Service capacity

```python
def unbalanced(a):
    p = np.empty([5, len(a)])
    p[0, :] = 1.0 * np.ones_like(a)
    p[1, :] = 1.0 * np.ones_like(a)
    p[2, :] = 1.0 * np.ones_like(a)
    p[3, :] = 3.0 * np.ones_like(a)
    p[4, :] = 9.0 * np.ones_like(a)
    return p

p = unbalanced(a)
print(p)
```

```
[[1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
 [1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
 [1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
 [3. 3. 3. 3. 3. 3. 3. 3. 3. 3.]
 [9. 9. 9. 9. 9. 9. 9. 9. 9. 9.]]
```

2.5 Include holidays

```python
def spread_holidays(p):
    for j in range(len(a)):
        psych = j % 5
        p[psych, j] = 0

spread_holidays(p)
print(p)
```

```
[[0. 1. 1. 1. 1. 0. 1. 1. 1. 1.]
 [1. 0. 1. 1. 1. 1. 0. 1. 1. 1.]
 [1. 1. 0. 1. 1. 1. 1. 0. 1. 1.]
 [3. 3. 3. 0. 3. 3. 3. 3. 0. 3.]
 [9. 9. 9. 9. 0. 9. 9. 9. 9. 0.]]
```
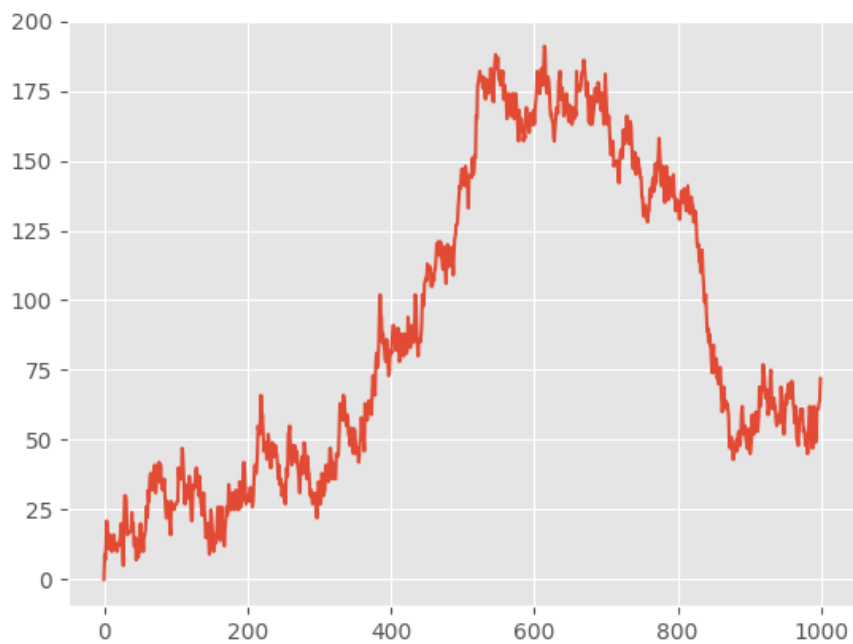
2.6 Total weekly service capacity

```python
s = np.sum(p, axis=0)
print(s)
```

```
[14. 14. 14. 12.  6. 14. 14. 14. 12.  6.]
```

2.7 Simulate the queue length process

```
1  np.random.seed(3)
2
3  a = np.random.poisson(11.8, 1000)
4  p = unbalanced(a)
5  spread_holidays(p)
6  s = np.sum(p, axis=0)
7
8  Q1 = computeQ(a, s)
9
10 plt.clf()
11 plt.plot(Q1)
12 plt.savefig("psych1.png")
13 "psych1.png"
```


3 Evaluation of better (?) plans

3.1 Balance the capacity more evenly over the psychiatrists

I set the seed to enforce a start with the same arrival pattern.

```
1  def balanced(a):
2      p = np.empty([5, len(a)])
3      p[0, :] = 2.0 * np.ones_like(a)
4      p[1, :] = 2.0 * np.ones_like(a)
5      p[2, :] = 3.0 * np.ones_like(a)
```

3

```
6      p[3, :] = 4.0 * np.ones_like(a)
7      p[4, :] = 4.0 * np.ones_like(a)
8      return p
9
10  np.random.seed(3)
11  a = np.random.poisson(11.8, 1000)
12
13
14  p = balanced(a)
15  spread_holidays(p)
16  s = np.sum(p, axis=0)
17  Q2 = computeQ(a, s)
18
19  plt.plot(Q2)
20  plt.savefig("psych2.png")
21  "psych2.png"
```
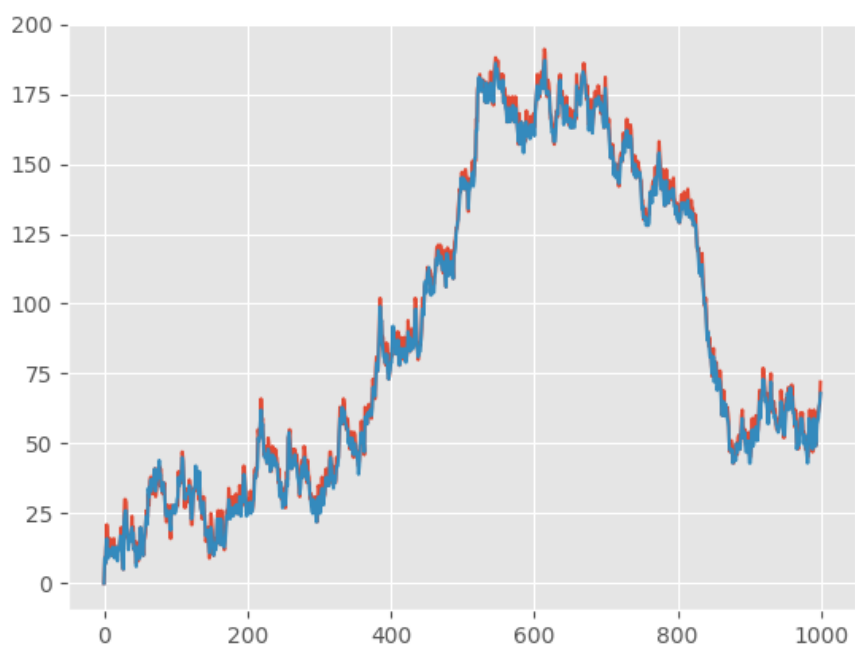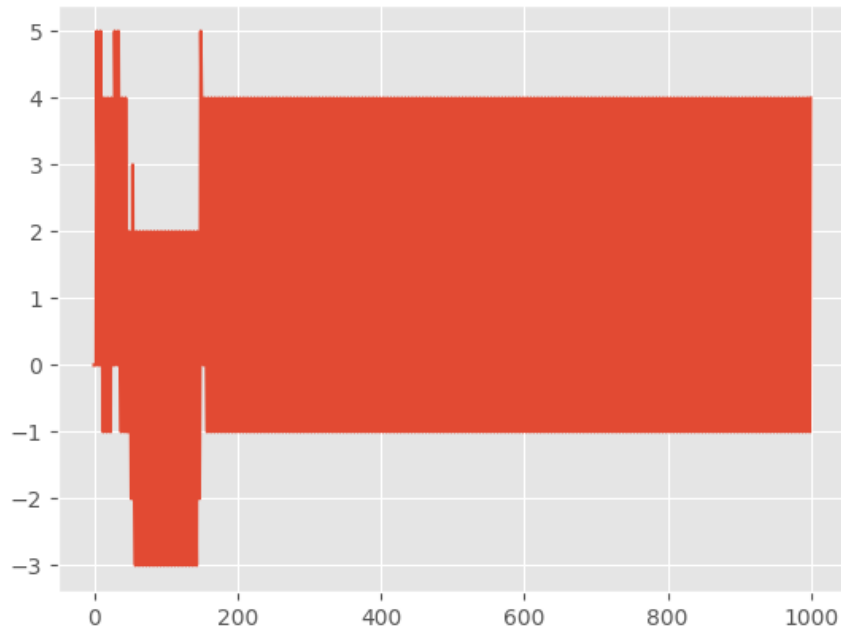

What is the effect?

```
1  plt.clf()
2  plt.plot(Q1-Q2)
3  plt.savefig("psych22.png")
4  "psych22.png"
```

The effect of balancing capacity is totally uninteresting.

3.2 Synchronize holidays

What is the effect of all psychiatrists taking holidays in the same week?

```
1  a = np.random.poisson(11.8, 10)
2
3
4  def synchronize_holidays(p):
5      for j in range(int(len(a) / 5)):
6          p[:, 5 * j] = 0
7
8  p = unbalanced(a)
9  synchronize_holidays(p)
10 print(p)
```

```
[[0. 1. 1. 1. 1. 0. 1. 1. 1. 1.]
 [0. 1. 1. 1. 1. 0. 1. 1. 1. 1.]
 [0. 1. 1. 1. 1. 0. 1. 1. 1. 1.]
 [0. 3. 3. 3. 3. 0. 3. 3. 3. 3.]
 [0. 9. 9. 9. 9. 0. 9. 9. 9. 9.]]
```
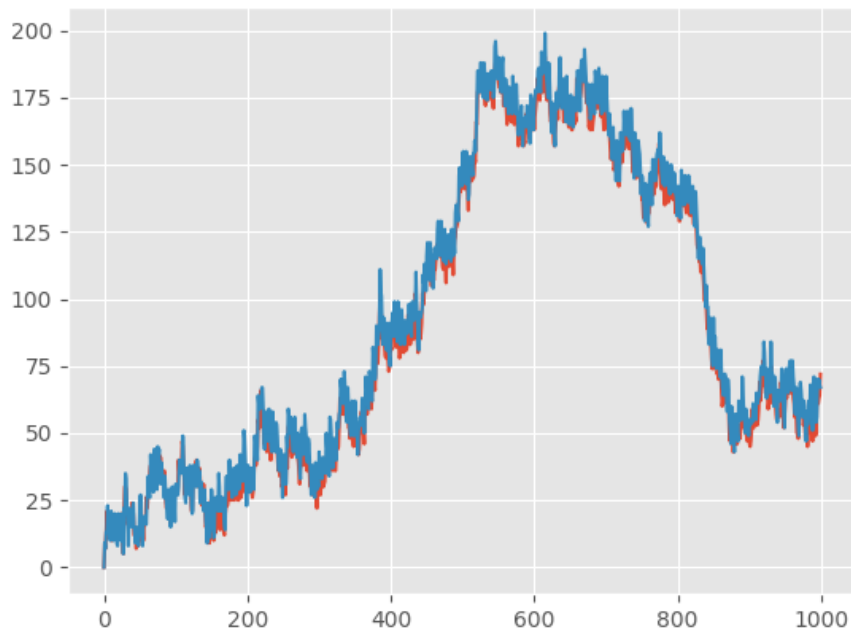
```
1  np.random.seed(3)
2
3  a = np.random.poisson(11.8, 1000)
4  p = unbalanced(a)
5  spread_holidays(p)
```

```
6   s = np.sum(p, axis=0)
7   Q3 = computeQ(a, s)
8
9   plt.clf()
10  plt.plot(Q3)
11
12  p = balanced(a)
13  synchronize_holidays(p)
14  s = np.sum(p, axis=0)
15  Q4 = computeQ(a, s)
16
17  plt.plot(Q4)
18  plt.savefig("psych3.png")
19  "psych3.png"
```


All these proposals will not solve the problem. We need something smarter. For this, we steal an idea from supermarkets: dynamic control.

Ex 3.1. Just to improve your coding skills (and your creativity), formulate another vacation plan. Implement this idea in code, and test its success/failure. Make a graph to show its effect on the dynamics of the queue length. (I don't mind whether your proposal works or not; as long as you 'play' and investigate, all goes.) Include your code (and if you ported all this code to R , then include your R code), and comment on difficult points.

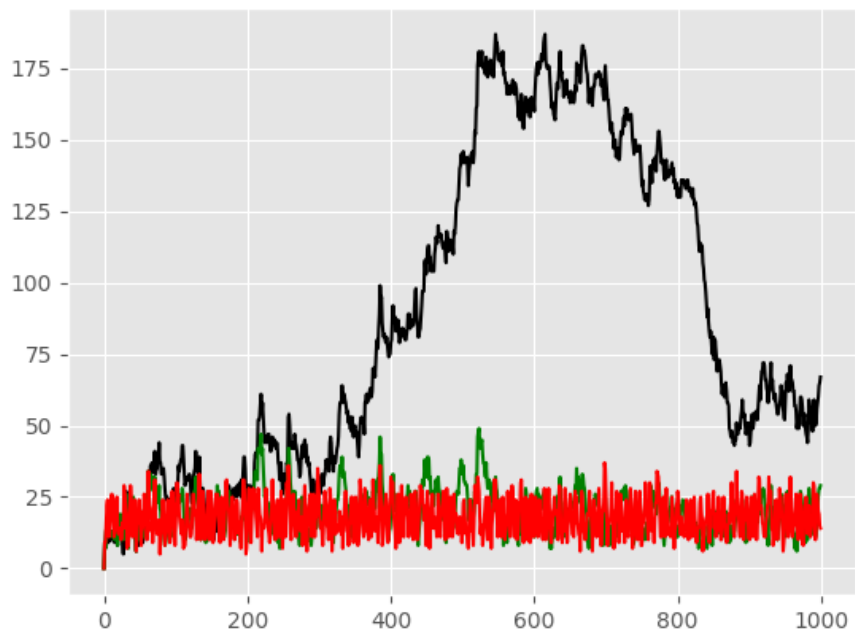# 4   Control capacity as a function of queue length

```
1   lower_thres = 12
2   upper_thres = 24
```

```python
3
4  def computeQExtra(a, c, e, Q0=0):  #  initial queue length is 0
5      N = len(a)
6      Q = [0] * N  # make a list to store the values of  Q
7      Q[0] = Q0
8      for n in range(1, N):
9          if Q[n - 1] < lower_thres:
10             C = c - e
11         elif Q[n-1] >= upper_thres:
12             C = c + e
13         d = min(Q[n-1], C)
14         Q[n] = Q[n-1] + a[n] -d
15     return Q
16
17
18  np.random.seed(3)
19  a = np.random.poisson(11.8, 1000)
20  c = 12
21  Q = computeQ(a, c * np.ones_like(a))
22  Qe1 = computeQExtra(a, c, 1)
23  Qe5 = computeQExtra(a, c, 5)
24
25  plt.clf()
26  plt.plot(Q, label="Q", color='black')
27  plt.plot(Qe1, label="Qe1", color='green')
28  plt.plot(Qe5, label="Qe5", color='red')
29  plt.savefig("psychfinal.png")
30  "psychfinal.png"
```

We see, dynamically controlling the service capacity (as a function of queue length) is a much better plan.

Ex 4.1. Use simulation to show that the psychiatrists don't have more work.

Ex 4.2. In the real case the psychiatrists hired an extra person to do intakes when the queue became very long, 100 or higher, and then they hired this person for one month. Suppose this person can do 5 intakes a day and works for 4 days a week. Does this extra capacity suffice or not? Make a plot of how the queue behaves under this rule. Make the horizon of plot sufficiently long (this may need some experimenting). Comment on this policy of hiring an extra person. In other words, if you were a consultant, what would you advice the psychiatrists?