Recommendations for Applichem

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1) Introduction

1.1 Abstract

The purpose of this project is to utilize Linear Programming as a tool to aid in the decision making process for Applichem. The Applichem Company, a specialty chemicals manufacturer, was facing an issue where their manufacturing plants were suspected to be running inefficiently. Thus, they decided to close one of their four plants located around the globe. Data Envelopment Analysis (DEA) was used to perform analysis for the purpose of this case. Data used to perform analysis was given in Exhibits 2-5 in the Applichem (A) (Abridged) document. An AMPL implementation of the DEA method was used to generate the efficiencies based on the data provided. Multiple graphs were used in the analysis of the data as well. Suggestions regarding decision making and improvement of the manufacturing plants are given using the results from the AMPL implementation and data provided. The goal of the project was to determine the efficient and inefficient plants and whether or not to close any plants.

1.2 Procedure

First, Linear Programming, Data Envelopment Analysis (DEA) methodology was studied and applied for use in data analysis. Then, a flow chart was created to describe the methodological process used to address the efficiencies of Applichem's manufacturing plants. Next, multiple AMPL implementation script files were created with the ability to run all the optimization problems necessary for determining the efficiency of Applichem's plants. In order to visualize the efficiencies of each plant, multiple graphs were plotted using Excel. The Excel results were then used to provide insight on what each Applichem plant should do to increase efficiency. Duality and sensitivity of linear programming were also applied to confirm findings.

2) Data Envelopment Analysis (DEA)

2.1 Brief Description

Data Envelopment Analysis (DEA) is a linear programming methodology used in operations research and economics to estimate the "efficiency" of multiple decision making units (DMUs). DEA is able to determine the relative efficiency of Decision-making Units (DMUs ie. Mexico, Frankfurt, Gary, Sunchem manufacturing plants in this case) and are characterized in terms of a structure of multiple inputs and outputs. It uses costs such as labor costs, material costs, and utility costs as reflections of "inputs", whereas the production revenue reflects the "outputs". Using inputs and outputs, DEA searches for the points with the lowest costs for any given outputs, an efficiency frontier is then formed with the connection of these points. A numerical coefficient is assigned to each of the manufacturing plants, representing each plants' relative efficiency respectively. The value of efficiency is within the range of 0 and 1, as a decimal to represent a percentage. The closer the value is to 1, means the more efficient the manufacturing plant is operating.

2.2 DEA Formulation

The model derived from DEA starts off with three sets being Inputs, Outputs, and Facilities. There are two variables being the value of a unit of an input (W_I) and the value of a unit of an output (T_O). There are three parameters being the cost of an input for a facility ($C_{I,F}$), price of an output for a facility ($P_{O,F}$), and a decision making unit (Z). The decision making unit is a symbolic parameter that doesn't take a specific value but takes on the identity of any object in a set. The DMU was used to represent each Facility for each calculation of their efficiencies.

The efficiency was maximized in our objective function by summing all of the outputs for a specified facility and dividing by the summation of all it's inputs, but the summation of the inputs was set equal to 1 to maintain a linear program and that can be seen in the Inputs constraint. The constraints of the model are the Inputs constraint as described before and Output Limit constraints which prevents all facilities from having a total output value be higher than their total input value. This is a generalized model with a data file that used to calculate and compare the efficiencies of Mexico, Frankfurt, Gary, and Sunchem for the purpose of further decision-making regarding the shutdown or improvement of any of these facilities.The following is the generalized DEA formulation:

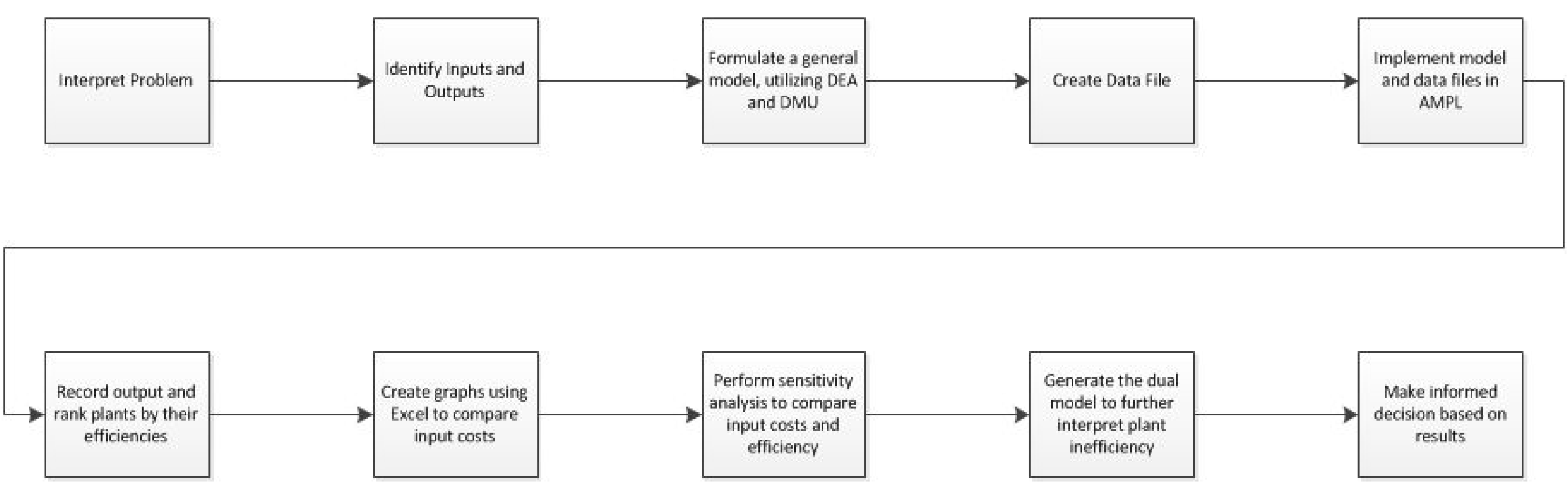
$$\text{Max} \sum_{\{O \,€\, \text{Outputs}\}} (P_{O,Z} * T_O)$$

s.t.

$$\sum_{\{I \,€\, \text{Intputs}\}} (C_{I,F} * W_I) \;-\; \sum_{\{O \,€\, \text{Outputs}\}} (P_{O,F} * T_O) \;\geq\; 0 \quad \text{for every } F \,€\, \text{Facilities}$$

$$\sum_{\{I \,€\, \text{Intputs}\}} (C_{I,Z} * W_I) = 1$$

2.2 DEA Flow Chart



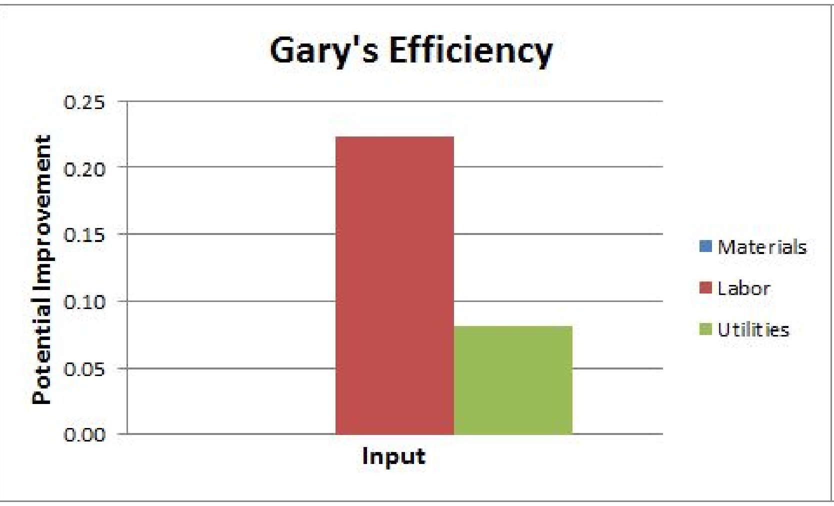
3) Applichem’s Situation and Suggestions

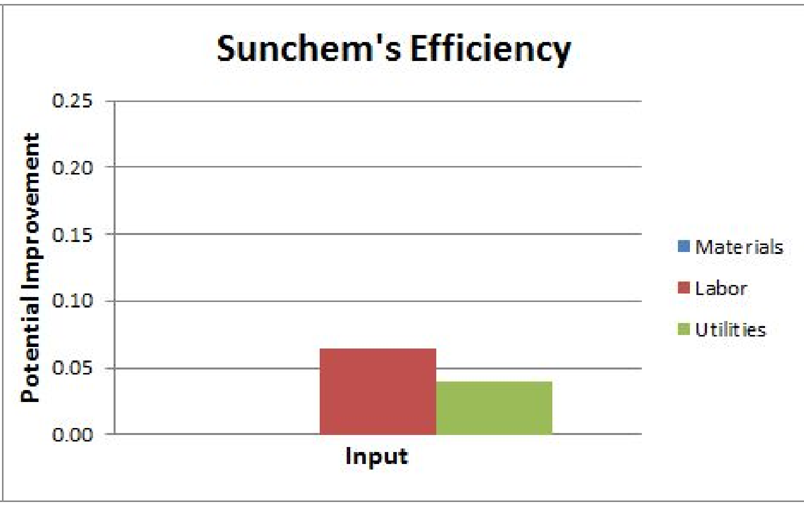
3.1 Solution Output Analysis

Mexico		Frankfurt	
Labor	2.44	Labor	0.1886
Materials	0	Materials	0.0291
Utiltities	0	Utiltities	0
Production	0.0058	Production	0.0026
Efficiency	1	Efficiency	1
Gary		**Sunchem**	
Labor	0	Labor	0
Materials	0.1174	Materials	0.2722
Utiltities	0	Utiltities	0
Production	0.0062	Production	0.0143
Efficiency	0.8713	Efficiency	0.5770

Table 1.

The Labor, Materials, and Utilities are the values of the variable representing these inputs. For instance, Labor for Mexico is 2.44 while the other input variables are practically zero, this shows that the weighted value for Labor Costs is a large factor going into the calculation of Mexico’s efficiency. The Production is the value of the variable representing this output. The most valuable number in this solution is the efficiency of each plant. It is clear that Mexico and Frankfurt are 100% efficient, Gary is approximately 87% efficient, and Sunchem is the most inefficient plant with an efficiency of approximately 58%.





3.2 Decision Making Suggestion

There are two facilities that went under review based on their DEA results. Gary and Sunchem had an efficiency lower than 100%, Gary with an efficiency of 87% and Sunchem with an efficiency of 58%. These plants went through further sensitivity analysis using an AMPL script to calculate reduced efficiency and see where each plant has the potential for improvement. The following graphs visually represent the reduced efficiency output:Gary has the potential to increase it's efficiency by 22% if the plant can reduced its total annual labor costs by one unit ($1,000,000 US). Now, a large decrease of this magnitude may seem impractical, but it shows us that if Gary focuses on maintaining the same rate of production while decreasing Labor costs it will significantly increase the overall efficiency. Sunchem on the other hand has small potential for improvement based on the sensitivity report. Sunchem can increase efficiency by 6% if this plant decreases its total annual Labor costs by one unit. A 6% improvement for Sunchem would bring it to an overall efficiency of 64% which does not make this plant noticeably more efficient. Our interpretation of the reduced efficiency from the sensitivity report leads us to conclude that the facility to close is Sunchem and the facility to improve is Gary.

3.3 Improvement Suggestions

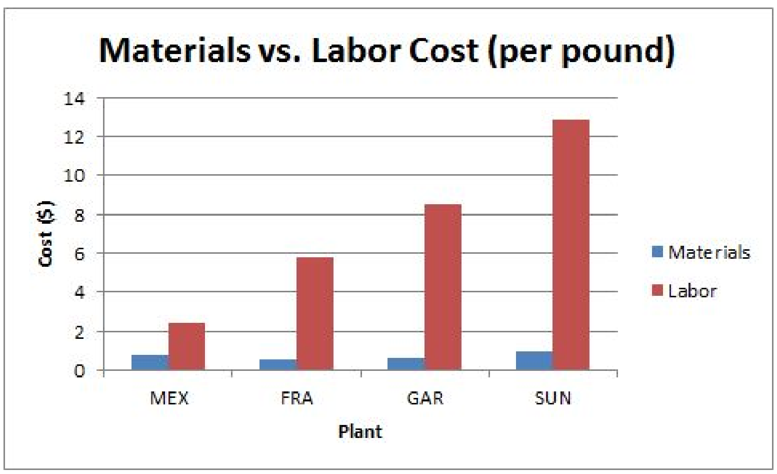
Gary is the only plant that is in need of improvement because Mexico and Frankfurt are 100% efficient, and Sunchem is too inefficient without any potential for improvement. Gary has three courses of action to choose from for improvement, the first is to focus on reducing total annual labor costs, second is to focus on reducing the total annual utilities costs, and the third course of action that we suggest is a combination of reducing both total annual costs of utilities and labor. Gary has a total potential of increasing overall efficiency by 30% theoretically and 13% actually, based on both the sensitivity report of reduced efficiency and the DEA output. Gary is already 87% efficient and can be over 90% efficient with some focus on reducing labor and utilities annual costs. The potential for improving efficiency by reducing labor is 22% and by reducing utilities is 8%. By focusing on reducing both of these annual costs, Gary can find the simplest way to become 100% efficient.

3.4 Duality Interpretation

Plant	Dual Variable Value	Efficiency Change	Current Efficiency
MEX	-1	0.00	1
FRA	-1	0.00	1
GAR	-0.3684	0.50	0.871280618
SUN	-0.1053	0.47	0.576964947

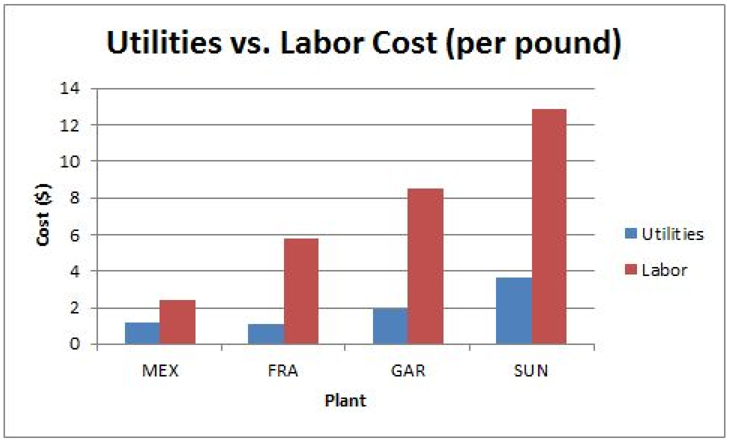
The previous chart represents the output of the dual model. The dual variable values represent the increase of one unit ($1,000,000 US) on the right hand side of the Output Limit constraint in the primal model for each facility. The one unit increase on the righthand side of this constraint means that each plant has to spend at least $1,000,000 more on total annual input costs in comparison to what they make from sales on total annual output revenue. The current efficiency represents the dual objective solution of minimizing inefficiency which is the same exact solution as the primal objective of maximizing efficiency. The efficiency change is the resultant efficiency of each facility from the increase of one unit on the righthand side of the constraint of interest. Notice that Mexico and Frankfurt would have a large reduction in efficiency, this is because the difference between their total input cost and total output revenue is so small that an increase of one unit would ruin these efficient plants. Sunchem would have the smallest reduction in efficiency from this unit increase because the difference between its total input cost and total output revenue is large enough that meeting the new requirements of this constraint would be easier for this inefficient plant.

4) Graphs



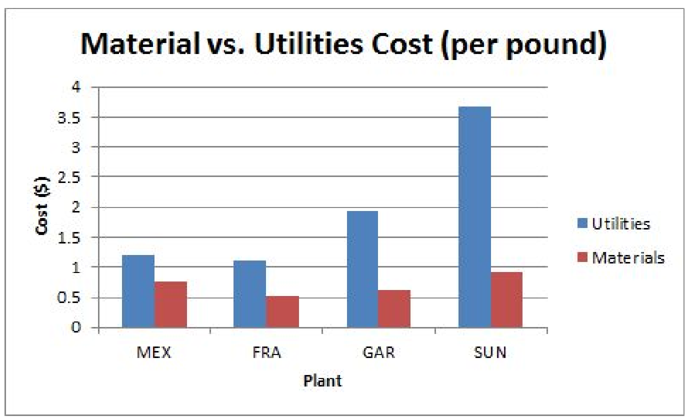
Graph 1. This graph represents the Labor vs. Utility cost per pound for the four different facilities.

The labor cost for all of the facilities was significantly higher than the material cost for each facility. The facilities with the lower labor costs were also the facilities with the higher efficiency.



Graph 2. This graph represents the Utility vs. Labor cost per pound for the four different facilities.

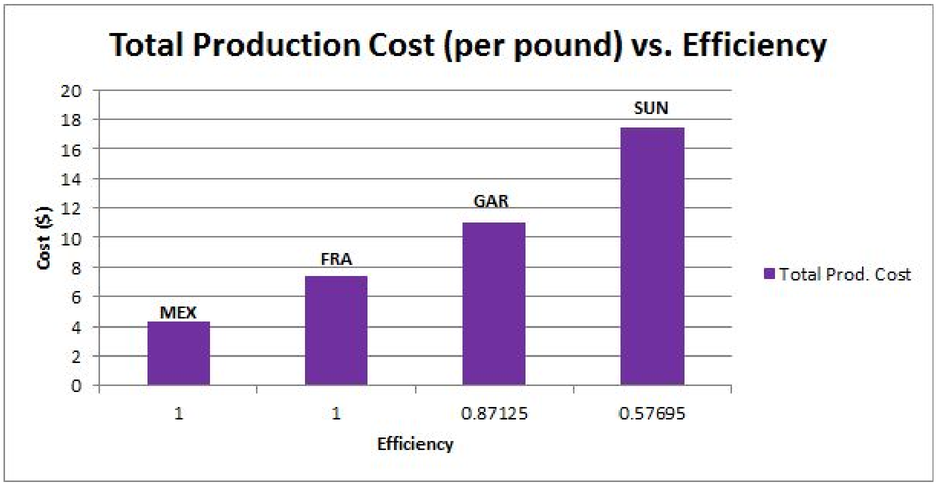
The labor cost for all of the facilities was significantly higher than the material cost for each facility.



Graph 3. This graph represents the Utilities vs. Materials cost per pound for the four different facilities.

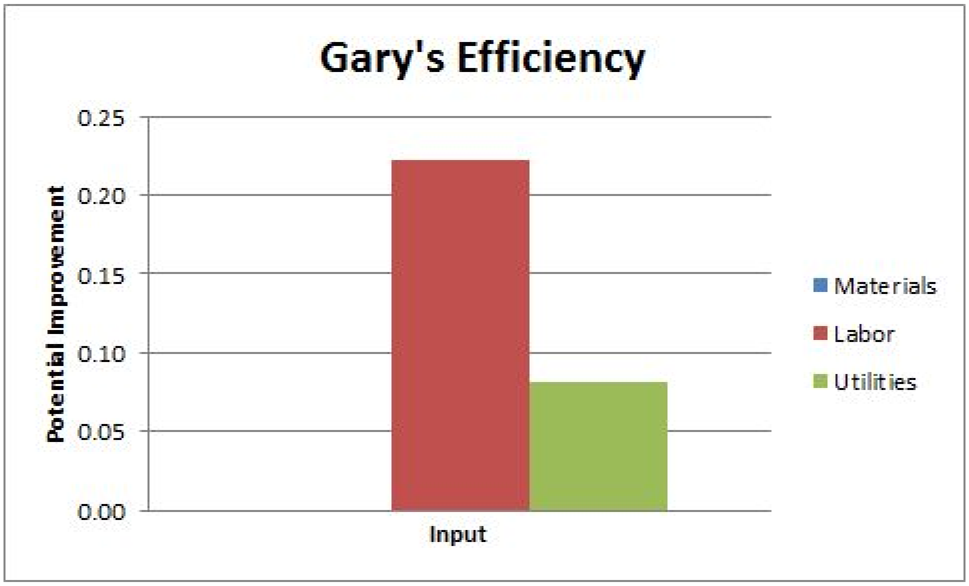
The utilities cost was higher than the materials cost for each of the four facilities.

For each of the three graphs shown above it is expected that the facilities with the higher cost for each of the three inputs are also going to be the facilities with the lower efficiencies.



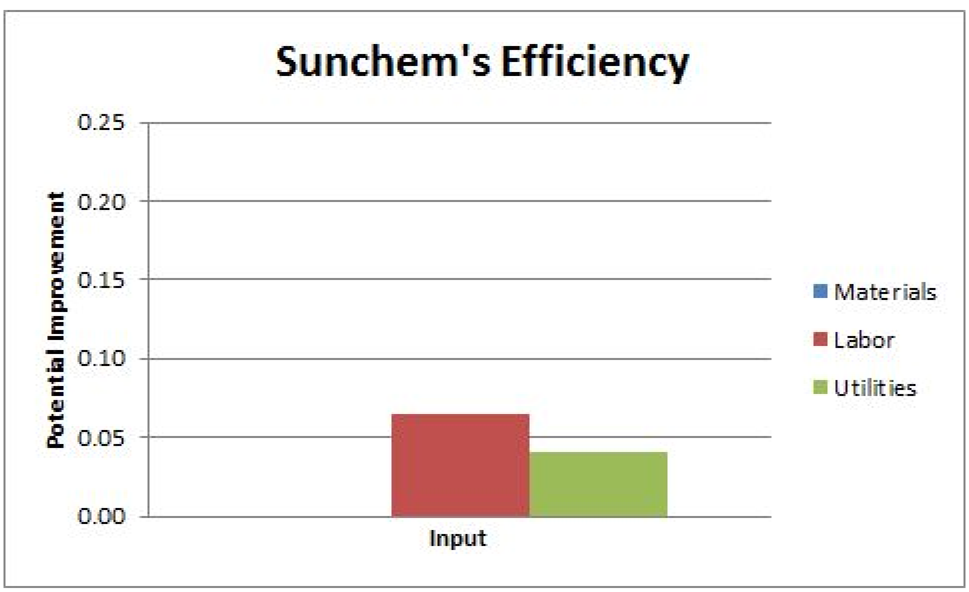
Graph 4. This graph represents the Total Production Cost per pound vs. the Efficiency of each of the four different facilities.

Graph 4 shows that the facilities that had the higher total production cost were also the facilities with the lower efficiency ratings. This is expected since all of the facilities sell their product for the same price therefor the facilities that require more money to produce the product will also be the facilities that are the least efficient. The plant's overall efficiency is a direct correlation to the total production cost to a certain point in the cost. The Mexico and Frankfurt facilities ended up having the same efficiency even though the total production cost for the Frankfurt facility was significantly higher.



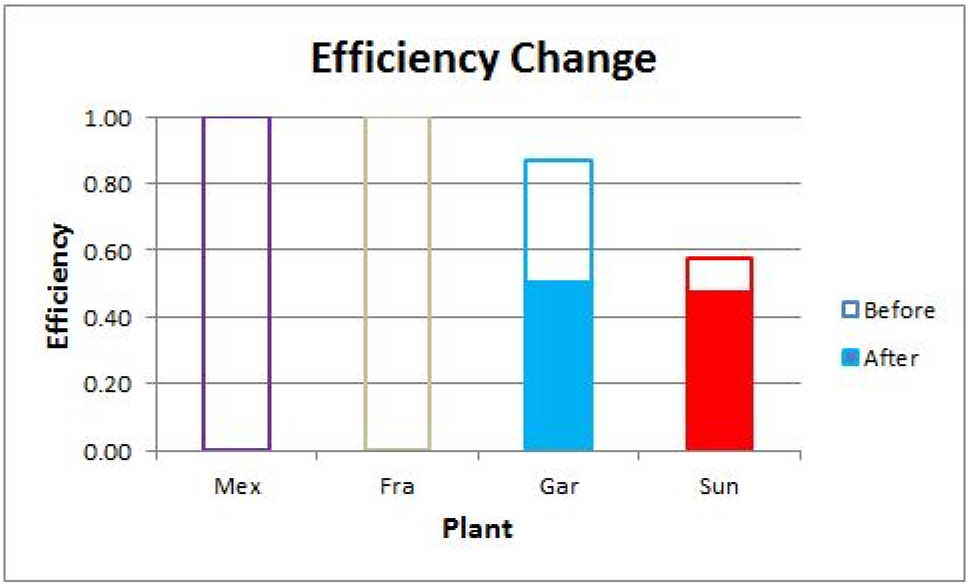
Graph 5. This graph represents the change in efficiency of Gary for each input when it is changed by one unit ($1,000,000).

Gary is the second least efficient of the four Applichem facilities. As seen in graph 5, labor has the largest impact on the efficiency of the facility. If the overall labor cost were to be decreased by a $1,000,000 or 1 unit then the overall efficiency of Gary would increase by 22 percent. Labor has the greatest effect on the efficiency therefore it should be focal point to make the plant more efficient.

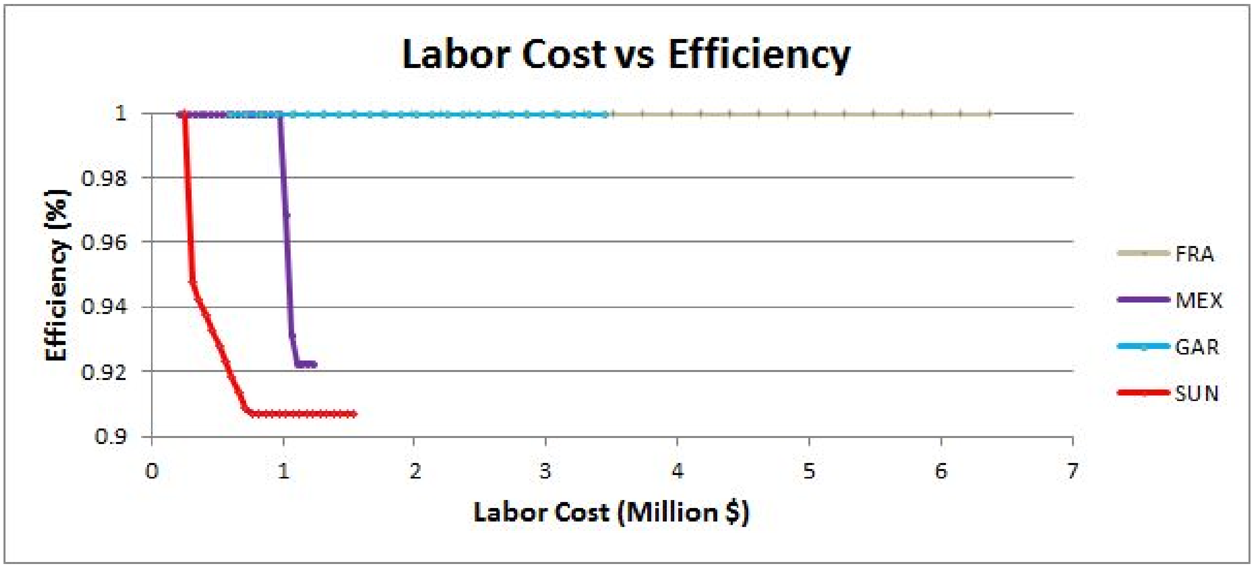


Graph 6. This graph represents the change in efficiency of Sunchem for each input when it is changed by one unit ($1,000,000).

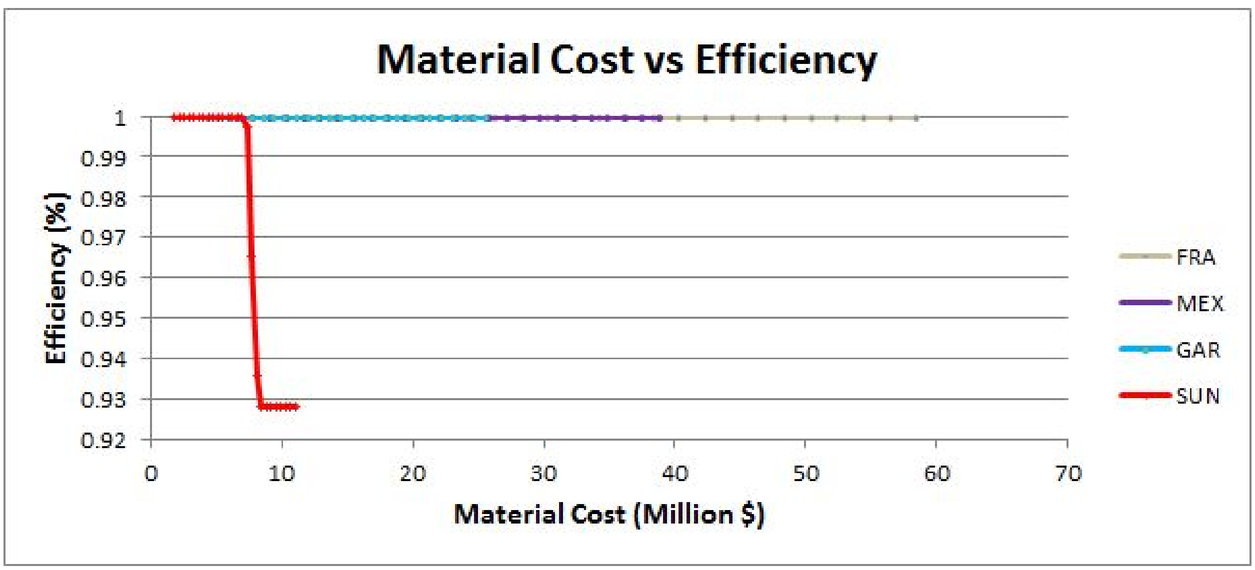
Sunchem is the most inefficient facility out of all of the facilities. As seen in graph 4, labor has the largest impact on the efficiency of the facility. If the overall labor cost were to be decreased by a $1,000,000 or 1 unit then the overall efficiency of Sunchem would increase by 6 percent. Labor has the greatest effect on the efficiency therefore it should be focal point to make the plant more efficient.



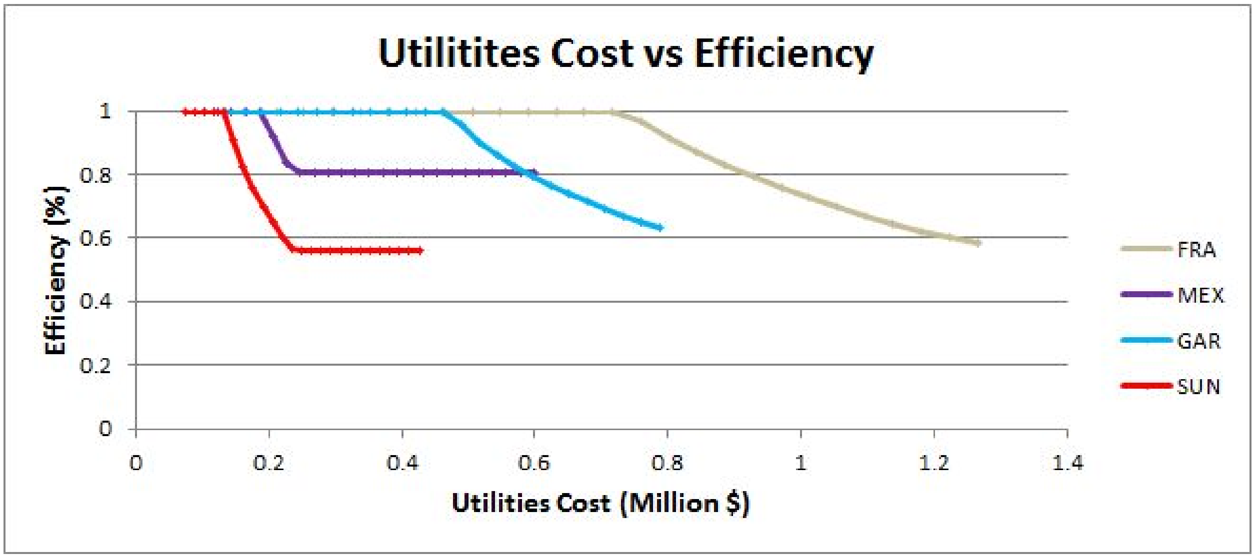
Graph 7. This graph represents the change in efficiency for a one unit increase on the right hand side of the Output Limit Constraint. This shows the decrease in efficiency for each plant if the total value of all inputs increases by one unit ($1,000,000 US) more than the total value of all outputs for every plant.



Graph 8. This graph represents the change in efficiency, for the four plants, as the annual labor cost increases from 50% - 300% of original labor costs. Notice that Sunchem's efficiency drops-off fairly quickly, followed by Mexico's which shows that their efficiency is impacted by labor costs while Gary's and Frankfurt's efficiency aren't affected.



Graph 9. This graph represents the change in efficiency, for the four plants, as the annual material cost increases from 50% - 300% of original material costs. Sunchem is the only plant whose efficiency is affected by an increase in material costs



Graph 10. This graph represents the change in efficiency, for the four plants, as the annual utility cost increases from 50% - 300% of original utilities cost. Notice that all plants are impacted by increasing utilities costs at different rates.

Appendix

A) AMPL Files

Model File

```
set I;             #set of inputs
set O;             #set of outputs
set F;             #set of facilities

param c{I,F};             #total cost of an input for a facility
param p{O,F};                    #total price of an output for a facility
param z symbolic;                #DMUs
var w{I} >= 0;                   #the value of a unit of an input for a facility
var t{O} >= 0;                   #the value of a unit of an output for a facility

maximize Efficiency: sum{o in O}p[o,z]*t[o];
s.t. OutputLimit{f in F}: sum{i in I}c[i,f]*w[i] - sum{o in O}p[o,f]*t[o] >= 0;
s.t. Inputs: sum{i in I}c[i,z]*w[i] = 1;
```

}

Data File

```
set I:= Materials Labor Utilities;

set O:= Production;

set F:= MEX FRA GAR SUN;


param c:      MEX         FRA         GAR         SUN :=

Materials     12.90860    20.14000    8.51620     3.67440

Labor         .40936      2.19640     1.18440     .51280

Utilities     .20640      .42180      .27160      .14680;


param p:      MEX         FRA         GAR         SUN :=

Production    173.72000   383.80000   141.40000   40.40000;

        #The parameters represent total money in US$ for the 1982 year in terms of $1,000,000
```

Run File

```
reset;

model Project.mod;

data Project.dat;

option solver cplex;

for {k in F}

{

        let z := k;

        solve;

        display w, t;

}
```

Changing Costs File

```
reset;

model Project.mod;

data Project.dat;

option solver cplex;

for {k in F}

{

let z := k;

        for {j in I}

        {

        print "F", "I", "Efficiency" > ("Project_Costs" & j & ".txt");

                for { g in .5*c[j,k]..3*c[j,k] by .1*c[j,k] }

                {

                        let c[j,k]:=g;

                        solve;

                        print k, j, g, Efficiency >> Project_Costs.txt;

                }

        }

}
```

Reduced Efficiency File

```
reset;
model Project.mod;
data Project.dat;
option solver cplex;
option cplex_options 'sensitivity';
for {k in F}
{
	let z := k;
	for {j in I}
	{
		solve;
		display _varname, _var.rc;
		print k, w.rc[j], Efficiency >> Project_rc.txt;
	}
}
```

Dual Constraint File

```
reset;

model Project.mod;

data Project.dat;

option solver cplex;

option cplex_options 'sensitivity';

for {k in F}

{

	let z := k;

	for {j in F}

	{

		solve;

		display OutputLimit.dual;

		print k, OutputLimit.dual[j], Efficiency >> Project_dual.txt;

	}

}
```

Dual Model File

```
set I;          #set of inputs
set O;          #set of outputs
set F;          #set of facilities

param c{I,F} default 0;         #total cost of an input for a facility
param p{O,F};                   #total price of an output for a facility
param z symbolic;               #DMUs
var x{F} <= 0;                  #the shadow efficiency for a facility
var y;                          #the efficiency of a facility

minimize Inefficiency: y;
s.t. InputsD{i in I}: sum{f in F}c[i,f]*x[f] + c[i,z]*y >= 0;
s.t. OutputsD{o in O}: -sum{f in F}p[o,f]*x[f] >= p[o,z];
```

Dual Run File

```
reset;

model ProjectD.mod;

data Project.dat;

option solver cplex;

for {k in F}

{

	let z := k;

	for {j in F}

	{

		solve;

		print k, x[j], Inefficiency >> ProjectD_var.txt;

	}

}
```

B) Data Tables

Mexico		Frankfurt	
Labor	2.44	Labor	0.1886
Materials	0	Materials	0.0291
Utiltities	0	Utiltities	0
Production	0.0058	Production	0.0026
Efficiency	1	Efficiency	1
Gary		**Sunchem**	
Labor	0	Labor	0
Materials	0.1174	Materials	0.2722
Utiltities	0	Utiltities	0
Production	0.0062	Production	0.0143
Efficiency	0.8713	Efficiency	0.5770

Plant	Dual Variable Value	Efficiency Change	Current Efficiency
MEX	-1	0.00	1
FRA	-1	0.00	1
GAR	-0.3684	0.50	0.871280618
SUN	-0.1053	0.47	0.576964947

Shadow Efficiency				
Plant	Input	Reduced Efficiency	Improved Efficiency	Current Efficiency
MEX	Materials	1.78E-15	0.00	1
	Labor	5.55E-17	0.00	
	Utilities	2.78E-17	0.00	
FRA	Materials	0	0.00	1
	Labor	0	0.00	
	Utilities	0	0.00	
GAR	Materials	0	0.00	0.8713
	Labor	-0.2227	0.22	
	Utilities	-0.0812	0.08	
SUN	Materials	0	0.00	0.5770
	Labor	-0.0647	0.06	
	Utilities	-0.0403	0.04	

Table 1. This table represents the shadow efficiency of each facility and how their efficiency could improve based on a one unit ($1,000,000) change in input cost.(possibly put in appendix)

Mexico								
Input	Cost	Efficiency	Input	Cost	Efficiency	Input	Cost	Efficiency
Materials	6.4543	1	Labor	0.20468	1	Utilities	0.1032	1
Materials	7.74516	1	Labor	0.245616	1	Utilities	0.12384	1
Materials	9.03602	1	Labor	0.286552	1	Utilities	0.14448	1
Materials	10.32688	1	Labor	0.327488	1	Utilities	0.16512	1
Materials	11.61774	1	Labor	0.368424	1	Utilities	0.18576	1
Materials	12.9086	1	Labor	0.40936	1	Utilities	0.2064	0.922315282
Materials	14.19946	1	Labor	0.450296	1	Utilities	0.22704	0.838560349
Materials	15.49032	1	Labor	0.491232	1	Utilities	0.24768	0.807299506
Materials	16.78118	1	Labor	0.532168	1	Utilities	0.26832	0.807297835
Materials	18.07204	1	Labor	0.573104	1	Utilities	0.28896	0.807296164
Materials	19.3629	1	Labor	0.61404	1	Utilities	0.3096	0.807294493
Materials	20.65376	1	Labor	0.654976	1	Utilities	0.33024	0.807292822
Materials	21.94462	1	Labor	0.695912	1	Utilities	0.35088	0.807291151
Materials	23.23548	1	Labor	0.736848	1	Utilities	0.37152	0.80728948
Materials	24.52634	1	Labor	0.777784	1	Utilities	0.39216	0.80728781
Materials	25.8172	1	Labor	0.81872	1	Utilities	0.4128	0.807286139
Materials	27.10806	1	Labor	0.859656	1	Utilities	0.43344	0.807284468
Materials	28.39892	1	Labor	0.900592	1	Utilities	0.45408	0.807282797
Materials	29.68978	1	Labor	0.941528	1	Utilities	0.47472	0.807281126
Materials	30.98064	1	Labor	0.982464	1	Utilities	0.49536	0.807279455
Materials	32.2715	1	Labor	1.0234	0.968577278	Utilities	0.516	0.807277784
Materials	33.56236	1	Labor	1.064336	0.931360102	Utilities	0.53664	0.807276114
Materials	34.85322	1	Labor	1.105272	0.922326642	Utilities	0.55728	0.807274443
Materials	36.14408	1	Labor	1.146208	0.922322855	Utilities	0.57792	0.807272772
Materials	37.43494	1	Labor	1.187144	0.922319069	Utilities	0.59856	0.807271101
Materials	38.7258	1	Labor	1.22808	0.922315282			

Frankfurt								
Input	**Cost**	**Efficiency**	**Input**	**Cost**	**Efficiency**	**Input**	**Cost**	**Efficiency**
Materials	10.07	1	Labor	1.0982	1	Utilities	0.2109	1
Materials	12.084	1	Labor	1.31784	1	Utilities	0.25308	1
Materials	14.098	1	Labor	1.53748	1	Utilities	0.29526	1
Materials	16.112	1	Labor	1.75712	1	Utilities	0.33744	1
Materials	18.126	1	Labor	1.97676	1	Utilities	0.37962	1
Materials	20.14	1	Labor	2.1964	1	Utilities	0.4218	1
Materials	22.154	1	Labor	2.41604	1	Utilities	0.46398	1
Materials	24.168	1	Labor	2.63568	1	Utilities	0.50616	1
Materials	26.182	1	Labor	2.85532	1	Utilities	0.54834	1
Materials	28.196	1	Labor	3.07496	1	Utilities	0.59052	1
Materials	30.21	1	Labor	3.2946	1	Utilities	0.6327	1
Materials	32.224	1	Labor	3.51424	1	Utilities	0.67488	1
Materials	34.238	1	Labor	3.73388	1	Utilities	0.71706	1
Materials	36.252	1	Labor	3.95352	1	Utilities	0.75924	0.967314472
Materials	38.266	1	Labor	4.17316	1	Utilities	0.80142	0.916541764
Materials	40.28	1	Labor	4.3928	1	Utilities	0.8436	0.870846327
Materials	42.294	1	Labor	4.61244	1	Utilities	0.88578	0.829502836
Materials	44.308	1	Labor	4.83208	1	Utilities	0.92796	0.791917844
Materials	46.322	1	Labor	5.05172	1	Utilities	0.97014	0.757601113
Materials	48.336	1	Labor	5.27136	1	Utilities	1.01232	0.726144109
Materials	50.35	1	Labor	5.491	1	Utilities	1.0545	0.697203666
Materials	52.364	1	Labor	5.71064	1	Utilities	1.09668	0.67048941
Materials	54.378	1	Labor	5.93028	1	Utilities	1.13886	0.645753988
Materials	56.392	1	Labor	6.14992	1	Utilities	1.18104	0.622785382
Materials	58.406	1	Labor	6.36956	1	Utilities	1.22322	0.601400818
						Utilities	1.2654	0.581441891

Gary								
Input	Cost	Efficiency	Input	Cost	Efficiency	Input	Cost	Efficiency
Materials	4.2581	1	Labor	0.5922	1	Utilities	0.1358	1
Materials	5.10972	1	Labor	0.71064	1	Utilities	0.16296	1
Materials	5.96134	1	Labor	0.82908	1	Utilities	0.19012	1
Materials	6.81296	1	Labor	0.94752	1	Utilities	0.21728	1
Materials	7.66458	1	Labor	1.06596	1	Utilities	0.24444	1
Materials	8.5162	1	Labor	1.1844	1	Utilities	0.2716	1
Materials	9.36782	1	Labor	1.30284	1	Utilities	0.29876	1
Materials	10.21944	1	Labor	1.42128	1	Utilities	0.32592	1
Materials	11.07106	1	Labor	1.53972	1	Utilities	0.35308	1
Materials	11.92268	1	Labor	1.65816	1	Utilities	0.38024	1
Materials	12.7743	1	Labor	1.7766	1	Utilities	0.4074	1
Materials	13.62592	1	Labor	1.89504	1	Utilities	0.43456	1
Materials	14.47754	1	Labor	2.01348	1	Utilities	0.46172	1
Materials	15.32916	1	Labor	2.13192	1	Utilities	0.48888	0.953230838
Materials	16.18078	1	Labor	2.25036	1	Utilities	0.51604	0.903186398
Materials	17.0324	1	Labor	2.3688	1	Utilities	0.5432	0.858146401
Materials	17.88402	1	Labor	2.48724	1	Utilities	0.57036	0.82223999
Materials	18.73564	1	Labor	2.60568	1	Utilities	0.59752	0.79198286
Materials	19.58726	1	Labor	2.72412	1	Utilities	0.62468	0.76387377
Materials	20.43888	1	Labor	2.84256	1	Utilities	0.65184	0.737691817
Materials	21.2905	1	Labor	2.961	1	Utilities	0.679	0.713245386
Materials	22.14212	1	Labor	3.07944	1	Utilities	0.70616	0.690367448
Materials	22.99374	1	Labor	3.19788	1	Utilities	0.73332	0.668911742
Materials	23.84536	1	Labor	3.31632	1	Utilities	0.76048	0.648749646
Materials	24.69698	1	Labor	3.43476	1	Utilities	0.78764	0.629767591
Materials	25.5486	1						

Sunchem								
Input	**Cost**	**Efficiency**	**Input**	**Cost**	**Efficiency**	**Input**	**Cost**	**Efficiency**
Materials	1.8372	1	Labor	0.2564	1	Utilities	0.0734	1
Materials	2.20464	1	Labor	0.30768	0.947447967	Utilities	0.08808	1
Materials	2.57208	1	Labor	0.35896	0.942447915	Utilities	0.10276	1
Materials	2.93952	1	Labor	0.41024	0.937500046	Utilities	0.11744	1
Materials	3.30696	1	Labor	0.46152	0.932603548	Utilities	0.13212	1
Materials	3.6744	1	Labor	0.5128	0.927757625	Utilities	0.1468	0.906899011
Materials	4.04184	1	Labor	0.56408	0.922961497	Utilities	0.16148	0.824515632
Materials	4.40928	1	Labor	0.61536	0.918214401	Utilities	0.17616	0.755862817
Materials	4.77672	1	Labor	0.66664	0.913515589	Utilities	0.19084	0.697771973
Materials	5.14416	1	Labor	0.71792	0.908864327	Utilities	0.20552	0.647979821
Materials	5.5116	1	Labor	0.7692	0.906968805	Utilities	0.2202	0.604826623
Materials	5.87904	1	Labor	0.82048	0.906964152	Utilities	0.23488	0.567067575
Materials	6.24648	1	Labor	0.87176	0.906959499	Utilities	0.24956	0.557713429
Materials	6.61392	1	Labor	0.92304	0.906954846	Utilities	0.26424	0.557712611
Materials	6.98136	1	Labor	0.97432	0.906950193	Utilities	0.27892	0.557711792
Materials	7.3488	0.997317	Labor	1.0256	0.90694554	Utilities	0.2936	0.557710973
Materials	7.71624	0.965519	Labor	1.07688	0.906940887	Utilities	0.30828	0.557710154
Materials	8.08368	0.935686	Labor	1.12816	0.906936234	Utilities	0.32296	0.557709336
Materials	8.45112	0.927996	Labor	1.17944	0.906931581	Utilities	0.33764	0.557708517
Materials	8.81856	0.927962	Labor	1.23072	0.906926928	Utilities	0.35232	0.557707698
Materials	9.186	0.927928	Labor	1.282	0.906922275	Utilities	0.367	0.557706879
Materials	9.55344	0.927894	Labor	1.33328	0.906917622	Utilities	0.38168	0.557706061
Materials	9.92088	0.92786	Labor	1.38456	0.90691297	Utilities	0.39636	0.557705242
Materials	10.28832	0.927826	Labor	1.43584	0.906908317	Utilities	0.41104	0.557704423
Materials	10.65576	0.927792	Labor	1.48712	0.906903664	Utilities	0.42572	0.557703604
Materials	11.0232	0.927758	Labor	1.5384	0.906899011			

C) References

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