Sets:

N nodes

A arcs

T time

G antigens

B bundles (vaccines)

P producers (origin nodes)

M markets (destination nodes)

W warehouses (transshipment nodes)

Parameters:

e_{nb} set up cost to process bundle $b \in B$ at node $n \in N$

c_{nb} unit cost to process bundle $b \in B$ at node $n \in N$

r_{ab} unit cost to deliver bundle $b \in B$ on arc $a \in A$

L_{ab} lead time to deliver bundle $b \in B$ on arc $a \in A$

d_{mgt} demand for antigen $g \in G$ at node $m \in M$ during time $t \in T$

u_{nb} inventory capacity to hold bundle $b \in B$ at node $n \in N$

q_{nb} production capacity to process bundle $b \in B$ at node $n \in N$

h_{nb} holding cost for bundle $b \in B$ at node $n \in N$

s_{mg} shortage cost for antigen $g \in G$ at node $m \in M$

I_{nb0} initial inventory of bundle $b \in B$ at node $n \in N$

map_{bg} boolean indicating if bundle $b \in B$ supplies antigen $g \in G$

o_g number of doses of antigen $g \in G$ required per demand unit

f_n profit margin (%) desired at node $n \in N$

v_{mt} vaccine budget at node $m \in M$ during time $t \in T$

Variables:

x_{abt} units of bundle $b \in B$ sent on arc $a \in A$ during time $t \in T$

y_{abt} unit price of bundle $b \in B$ sent on arc $a \in A$ during time $t \in T$

z_{nbt} units of bundle $b \in B$ made at node $n \in N$ during time $t \in T$

I_{nbt} units of bundle $b \in B$ held in inventory of node $n \in N$ during time $t \in T$

Program:

- Minimize holding costs for MFG's and WH's, and shortage costs for Markets

$$\text{Minimize: } \text{Cost} = \sum_{t \in T} \sum_{b \in B} \sum_{j \in \{P \cup W\}} \left(h_{jb} I_{jbt}\right) + \sum_{t \in T} \sum_{g \in G} \sum_{j \in M} \left(s_{jg} \left(d_{jgt} o_g - \sum_{b \in B} \left(I_{jbt} map_{bg}\right) \right) \right)$$

- Inventory levels depend on previous inventory, production, and transportation

$$I_{jbt} = I_{jb(t-1)} + z_{jbt} + \sum_{i \in N} \left(x_{(i,j)b\left(t - L_{(i,j)b}\right)} \right) - \sum_{k \in N} \left(x_{(j,k)bt} \right) \qquad i, j, k \in N, b \in B, t \in T$$

- Prices are non-decreasing when transporting bundles downstream to Markets

$$y_{(j,k)bt} \geq \left(1 + f_j\right) \left(y_{(i,j)b\left(t - L_{(i,j)b}\right)} + \frac{e_{jb}}{u_{jb}} + c_{jb} + r_{(j,k)b} \right) \qquad i, j, k \in N{:}\ k > i, b \in B, t \in T$$

- Tiered pricing for Markets is enforced based on their income

$$y_{(i,j)bt} \geq y_{(i,k)bt} \qquad j, k \in M{:}\ k > j, i \in N, b \in B, t \in T$$

- Market budget is not exceeded **(non-linear)**

$$\sum_{i \in N} \sum_{b \in B} \left(y_{(i,j)b\left(t - L_{(i,j)b}\right)} x_{(i,j)b\left(t - L_{(i,j)b}\right)} \right) \leq v_{jt} \qquad j \in M, t \in T$$

- Inventory capacity is not exceeded, Inventory is non-negative

$$0 \leq I_{nbt} \leq u_{nb} \qquad b \in B, n \in N, t \in T$$

- Production capacity is not exceeded, Production is non-negative

$$0 \leq z_{nbt} \leq q_{nb} \qquad b \in B, n \in N, t \in T$$

- Transportation is non-negative

$$x_{abt} \geq 0 \qquad a \in A, b \in B, t \in T$$