The graph in figure 1 uses nodes to represent 10 customer locations (i.e., customer regions), with an annual product demand (in Tons) given by $D = \{80, 100, 120, 75, 90, 110, 100, 85, 90, 105\}$. Nodes in $J = \{3, 5, 6, 7, 8\}$ are locations where potential production facilities could be opened. Setting up a facility at a given location requires a one time investment (in thousands of dollars) given by $F = (f_3, f_5, f_6, f_7, f_8) = (4000, 3000, 5000, 2000, 2500)$.

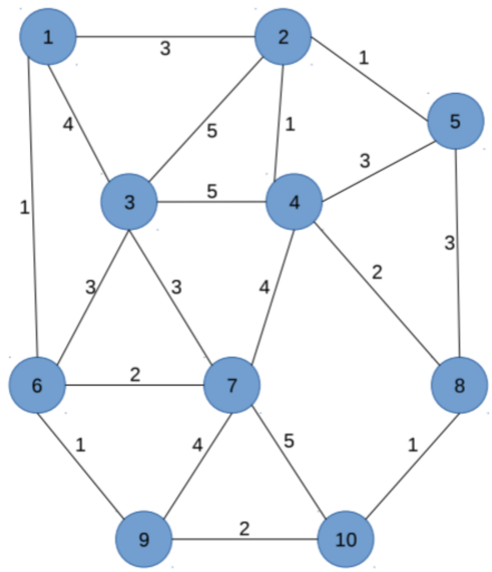

Figure 1:

The capacity of each facility is also a function of where it could be opened and is given by $C = (C_3, C_5, C_6, C_7, C_8) = (650, 450, 800, 350, 500)$. The arcs between any two nodes depict the shortest distance between them, and their lengths in hundreds of Kms are shown in each arc. The amount of product that can be sent through a given arc cannot exceed 150 Tons. Additionally, the cost associated with sending a ton of product from node i to node j, along arc (i, j) is $\$15d_{ij}$, where d_{ij} is the distance between nodes i and j in Km. In addition there is a fixed cost of $\$(500 + 0.30d_{ij})$ for sending any amount of product between nodes i and j.

In some cases, the waste generated by the production of material at a facility must be disposed of at special waste disposal locations. We need to identify which waste facilities to open, given a capacity for each waste unit and per-unit transportation costs from shipping waste from a plant facility to a waste facility. In figure 1 nodes $W = \{4, 9, 10\}$ are potential locations for the waste disposal facilities. The amount of waste material produced by a facility in J is proportional to the amount of goods it produces; for example, if a plant in location 5 produces 500 units, then it also produces $500\alpha_5$ units (Tons) of waste. The proportionality constants for the waste are given by $\alpha = \{\alpha_3, \alpha_5, \alpha_6, \alpha_7, \alpha_8\} = \{0.1, 0.15, 0.2, 0.125, 0.175\}$.

The one-time fixed cost associated with opening a waste disposal facility (in thousands of dollars) is given by $F^W = (f_4^w, f_9^w, f_{10}^w) = (5000, 7000, 6000)$ and their capacities of processing waste are $(C_4^w, C_9^w, C_{10}^w) = (100, 140, 120)$ in Tons. The cost of shipping one ton of waste from a plant to a disposal facility is given by \$10 per kilometer traveled. Assuming that customers can satisfy their demand with product from multiple plants and that waste from a plant can be shipped to multiple disposal locations, propose an optimization model to determine the optimal way to satisfy customer demand and waste removal requirements. Where should the plants and waste disposal facilities be located?