MATH 401 STOCHASTIC PROCESSES
HOMEWORK # 7
Due: 10 a.m. on Friday, April 21 Corrected

Note: Define all the variables clearly and completely. show all the work to get credit. Answers without proper and properly written explanations and supporting arguments will not get any credit. Work independently.

1. Consider a Markov chain with state space $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and transition matrix

$$\begin{bmatrix} 0.1 & 0.1 & 0.2 & 0.2 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0 & 0.1 & 0.1 & 0.1 & 0 & 0.3 & 0.2 & 0.2 \\ 0.6 & 0 & 0 & 0.1 & 0.1 & 0.1 & 0.1 & 0 \\ 0 & 0 & 0 & 0.3 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 & 0.4 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0.9 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0.2 & 0 \end{bmatrix}$$

Determine the limiting behavior of the Markov chain by computing the limiting matrix P^∞, the matrix whose entries are given by $\lim_{n\to\infty} p_{ij}^n$. Answers using brute-force computation and raising P to successive powers to find the pattern will not be acceptable.[Hint: Think of what should be $\lim_{n\to\infty} p_{ij}^{(n)}$.]

2. For the Markov chain with the transition probability matrix given in Problem # 1, find the expected number of visits

(a) from state 6 to state 1;

(b) from **state 1 to state 6**;

(c) from **state 0 to state 2**;

(d) from **state 2 to state 1**;

3. For the Markov chain with transition probability matrix given in Problem #1, find the probability of ever visiting

(a) states **0, 1 and 2 from 0, 1, and 2**.

(b) **state 6 from state 5**;

(c) **state 4 from state 2**.

4. Let $\{X_t, t \geq 0\}$ be a Poisson process with rate μ. Determine and describe the conditional distribution of X_t given that $X_{t+s} = n$, where $s, t > 0$.

5. Calls come to a phone bank as a Poisson process with rate $\lambda = 4$ per minute. Suppose the operator goes on a break the duration of which is a random variable T that is uniformly distributed between 0 and 1, so that X_T is the number of calls that arrive during the break.

(a) Determine the conditional moments $E(X_T|T = t)$ and $E(X_T^2|T = t)$.

(b) Determine the (unconditional) mean $E(X_T)$, the average number of arrivals during a break, and $\text{Var}(X_T)$.