MATH 401 STOCHASTIC PROCESSES
HOMEWORK # 6
Due: Wednesday, April 12

1. The state of process changes daily according to a two-state Markov chain. If the process is in state i during one day, then it is in state j the following day with probabilty p_{ij} where, $p_{00} = 0.4, p_{01} = 0.6, p_{10} = 0.2, p_{11} = 0.8$. Every day a message is sent. If the state of the Markov chain that day is i, then, the message sent is "good" with probabilty p_i and is "bad" with probability $q_i = 1 - p_i, i = 0, 1$.

(a) If the process is in state 0 on Monday, what is the probabilty that a good message is sent on Tuesday?

(b) In the long run, what proportion of messages are good?

(c) Let Y_n be 1 if a good message is sent on day n and let it equal 2 otherwise. Is $\{Y_n, n \geq 1\}$ a Markov chain? If so, give its transition probability matrix. If not, explain why not.

2. Suppose a rocket is launched and, as it is tracked, a sequence of course correction signals is sent to the rocket. Suppose the system has four states that are labeled as : State 0 : on-course; no further correction needed; State 1: Minor deviation; State 2: Major deviation; State 3 : Abort; off-course; a self-destruct signal is sent. Let X_n represent the state of the system after the nth course correction and assume that the sequence of signlas is modeled by a Markov chain with the transition probability matrix

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0.25 & 0.25 & 0 \\ 0 & 0.5 & 0.25 & 0.25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Suppose that upon launch, the rocket starts in state 2. Find the probability that it will eventually get on-course.

(b) Suppose that upon launch, the rocket starts in state 2. Find the probability that it will eventually be destroyed.

(c) When the rocket is launched, 50,000 lb. of fuel are used. Every time a minor correction is made, 1000 lb. of fuel are used; every time a major correction is made 5000 lb. of fuel are used. Assuming that the rocket started in state 2, determine the expected fuel needed for the mission.

3. An individual either drives his car or walks in going from his home to his office in the morning, and from his office to his home in the afternoon. He uses the following strategy: if it is raining in the morning, then he drives the car, provided it is at home to be taken. Similarly, if it is raining in the afternoon and his car is at the office, then the drives the car home. He walks on any morning or afternoon that it is not raining or the car is not where he is. Assume that, independent of the past, it rains during successifve mornings and afternoons with constant probability p. In the long run, on what fraction of *days* does our man walk in the rain?

4. Classify, with proper explanation and justification, the states of the Markov chain with the following transition probabilities:

$$P_1 = \begin{bmatrix} 0 & \frac{1}{/}2 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}; \qquad P_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}; \qquad P_3 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \qquad P_4 = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5. A Markov chain on states $0, 1, \ldots,$ has transition probabilities

$$p_{ij} = \frac{1}{i+2}, \text{ for } j = 0, 1, \ldots, i, i+1.$$

Find and describe the stationary distribution.