

CRITICAL EVALUATION OF THE WEIGHTED SET COVERING GAME: A VACCINE PRICING MODEL FOR PEDIATRIC IMMUNIZATION

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ABSTRACT

This paper presents a game theory and optimization based approach to setting the prices of pediatric vaccines in the USA. The contribution of this paper is to determine the most cost-effective way to purchase vaccines while reducing uncertainty around the profits of manufacturers in an oligopoly market. An extension on the work in this paper is presented that allows for the study of affordability, profitability, immunization coverage, pricing strategies, and selling strategies in an oligopoly market.

1. INTRODUCTION

The problem of how to price vaccines for children is addressed by [Robbins] using a generalized Bertrand pricing model. This model sets prices of multiple pediatric vaccines in the USA public sector. The model is meant to be used by the USA pediatric vaccine manufacturers to give them insight on how to price their vaccines given that they are interchangeable with vaccines from other manufacturers. The model is meant to be used by a single buyer, such as a government organization, to decide the most cost-effective way to buy vaccines for children.

This paper is appropriate for the journal in which it appears, Institute for Operations Research and the Management Sciences (INFORMS). INFORMS is one of the largest professional societies in the world for experts in the fields of operations research, management science, and analytics. Game theory and optimization are the two methodologies applied in this paper, and these two fields provide many decision models that can be used at the industrial scale.

The problem area of vaccination is of interest around the globe. The decade of vaccines (2010 – 2020) has the following five goals according to the Global Vaccine Action Plan (GVAP):

1. *Achieve a world free of poliomyelitis*
2. *Meet global and regional elimination targets*
3. *Meet vaccination coverage targets in every region, country and community*
4. *Develop and introduce new and improved vaccines and technologies*
5. *Exceed the Millennium Development Goal 4 target for reducing child mortality and integration indicators [GVAP]*

The third and fourth goals highlight the focus of this paper under review. USA organizations such as the Center for Disease Control (CDC) and the National Vaccine Advisory Committee (NVAC) desire children immunization coverage; USA manufacturers such as Glaxo Smith Kline and Merck desire profitability for the research and development of their vaccines. These goals result from the issues and trade-offs in the vaccine market that decision models are expected to address. Two issues that prevent countries from achieving their national immunization requirements are inadequate financing and difficult access to vaccine supply [GVAP]. Financing issues speak to the difficulty of pricing vaccines such that they are affordable and acceptable for the buyer, and a fair profit can be sustained by the manufacturers [Moon]. Difficult access to supply results from supply disruption, where the factors of disruption include a decrease in the number of manufacturers for certain vaccines, increases in the number of vaccines

recommended, increasing scrutiny in the certification of manufacturing facilities and quality assurance, complex production processes, and unanticipated changes in demand [Shrestha]. These issues for buyers and manufacturers work against the goals for the decade of vaccines. For example, there is a shrinking proportion of countries that have immunized at least 90% of their children according to their national immunization schedule. The annual secretariat reports for GVAP show that the proportion of member states world-wide which have met this goal of 90% national coverage has been decreasing from 48%, 45%, 43%, to 40% for 2012, 2013, 2014, and 2015 respectively. Cost-effective regional pricing of vaccines is the problem of this paper, which means that this study could benefit regions across the globe that wish to establish their own vaccine supply for routine immunization. Some of the reasons why another region would want to establish their own vaccine supply are *supply security, control over production scheduling and sustainability, control of costs, socio-economic development, and rapid response to local epidemics including emerging infectious diseases* [Plotkin].

This paper answers the following three research questions:

1. How should manufacturers price their set of bundles in an oligopolistic market?
2. Are there equilibrium prices for a game modeling such a market?
3. What if tacit collusion is present?

By answering these three questions, this study gives its field a means to study the interdependent nature of firms in a vaccine oligopoly market. This study offers a mechanism for the CDC to set their annual contract vaccine prices. This study also gives vaccine manufacturers a way to consider the benefits of cooperation in their market.

This paper is written well enough to understand the technical content; but there is an online supplement which must read to fully understand the methodology. This online supplement is a separate 13-page document that is referred to in this paper for validating the author's application of his methodology. The issue with this supplement is that it is not written as a paper for the reader, but as a reference document for the author. This creates difficulty in interpreting the core mechanics of the methodology and restricts reproducibility of the final results for the USA case study presented in the paper.

2. LITERATURE REVIEW

The major areas that this work is based on are game theory and optimization. In this section we will cover vaccine terminology and concepts of game theory that were used in this paper. Then the integer program chosen by the author will be explained as it is a popular model in the field of optimization. Finally, we will compare the work in this paper to other work in the literature to conclude on the support for it.

An antigen is a protein in vaccines that stimulates the immune system to provide the immunization of a preventable disease. These antigens can be combined such that a single vaccine provides multiple immunizations. This combination of antigens is what is known as a bundle, also known as a combination vaccine, and even an antigen alone is considered a bundle, also known as a monovalent vaccine. This distinction between antigens and bundles is important for understanding the perspectives of the buyer and manufacturers respectively. The buyer desires antigens and does not care about the vaccines they are buying given an affordable vaccine coverage is achieved. The manufacturer sells bundles as it is in their interest to sell the vaccines that will be profitable to advance vaccine research and development.

In game theory, one type of game is the static game. First off, a game is made up of multiple decision-making units (DMUs) that want to provide a product or a service to a population. The relationship between these DMUs is that they all provide a similar set of products or services that can satisfy the demand of the population. This similarity between products means that some or all products are

homogeneous. Homogeneous products to a buyer are interchangeable, so the buyer purchases one product over the other based on cost. This type of market means that the success of one DMU results in lost opportunity for others, so every decision made by a DMU affects the other. When a game is labeled as static is because it is assumed that when each DMU makes their decision, they all decide at the same time.

A game is considered symmetric if all DMUs have identical strategies to choose from when making their decisions. So, a game is asymmetric if any DMU has a different set of strategies to choose from when making their decision. Two examples of strategies are: the feasible range of prices that can be set for products, and the feasible range of quantities that could be produced for products. These two strategies of price setting and quantity setting are common strategies explored in game theory. *There are two basic ways of modelling how firms compete in the market. The first takes the view that the firm's strategic variable is its output and originates in Cournot (1838). The second takes the view that the firm's basic strategic variable is price and, originates in the work of Bertrand (1883) [Dixon].* When the strategy of a DMU is: how to set prices for homogeneous products, then this game is known as the Bertrand Competition. In Bertrand Competition an optimal solution would be the Nash equilibrium price for each homogeneous product. *A Nash equilibrium occurs when each firm is choosing its strategy optimally, given the strategies of the other firms [Dixon].* A feasible vector of equilibrium prices would result in a system where rational DMUs know there is no pay-off for deviating from their set of Nash equilibrium prices; this is when the system becomes stable. The Bertrand Competition comes with a set of three core assumptions as outlined by [Robbins]

1. *Bertrand competition assumes that any firm can fully satisfy market demand*
2. *Firms engage in competition only once*
3. *The firms' products are assumed to be perfectly interchangeable (homogeneous products)*

The Bertrand Competition has an important characteristic where the DMUs are in an oligopoly market. An oligopoly is a market structure in which a small number of DMUs hold most of the market share, which is true for the USA pediatric vaccine market with five manufactures as of 2017. Given that there are a few number of competitors in a market with homogeneous products, it creates the opportunity for a volatile event, a price war. A price war is when DMUs continually undercut the price point of each other's homogenous product to capture more demand and be the first to bankrupt their competitors. The consequence of manufacturers going bankrupt in the USA pediatric market is that the overall supply of vaccines will decrease, and if the supply does decrease to a point where it cannot satisfy demand, then the spread of preventable diseases can only increase in the USA. The desire to avoid such an event calls for coordination in the market, which can be done with tacit collusion.

Tacit collusion in Bertrand Competition means that price changes are established by a dominant DMU, and then the other DMUs change their prices accordingly to reduce uncertainty around their profits. The dominance of a DMU tends to be established by their marginal cost of production such that a DMU with the smallest marginal cost for a homogenous product is the dominant DMU for that product. Tacit collusion allows for DMUs to mutually agree on their standing in the market as far as market share and profit is concerned for each product. This type of agreement allows for each DMU to stay in business and grow at their established rates. This agreement creates stability in the system, which means the existence of Nash equilibrium is more likely than if no collusion was present.

A game can be repeated infinitely many times to realize profit in the long run. This means that a game has infinitely many pay-off streams, so *a common assumption is that the player wants to maximize a weighted sum of her per-period payoffs, where she weights later periods less than earlier periods.* [Ratliff] This allows everyone to make their profit eventually, but requires a parameter that discounts the value of money over time. This discount factor can be understood as *How much more does she value a dollar*

today than a dollar received later? [Ratliff] The equilibrium prices that result from a repeated game represent a conditional profit, conditioned on the assumption that the game lasts that long.

The final aspect of this papers methodology is the integer program, the Weighted Set Covering Problem (WSCP). This formulation is well-established in literature [Vazirani] and answers the question: What set of subgroups cover the entire group at minimal weight? The formulation of the WSCP in terms of vaccine purchasing would be as follows:

Antigens ~ a set of antigens that are demanded (Known)

Bundles ~ a set of bundles that can be supplied (Known)

m_{ba} ~ binary value indicating if Bundle b supplies Antigen a (Known)

c_b ~ unit price of Bundle b (Known)

d_a ~ total demand for Antigen a (Known)

x_b ~ purchase Bundle b a discrete number of times to satisfy Antigen demand (Unknown)

$$\begin{aligned} \text{minimize Cost: } & \sum_{b \text{ in Bundles}} c_b x_b \\ \text{s. t. Demand: } & \sum_{b \text{ in Bundles}} m_{ba} x_b \geq d_a; \forall a \text{ in Antigens} \\ \text{s. t. Integrality: } & x_b \in \mathbb{Z}^+ \forall b \text{ in Bundles} \end{aligned}$$

The objective function of this program is to minimize the total purchasing cost for the buyer must meet the condition that enough bundles are purchased such that the desired immunization coverage is met. This formulation is from the buyer perspective; it doesn't consider the profitability or availability of bundles.

The work in the literature regarding general pricing strategies resulting from a retrospective analysis of vaccine pricing methods under different market conditions [Lee], [Moon]. These retrospective approaches provide supporting evidence for the framework of this papers methodology. The market conditions for when one pricing method would be preferred over another is outlined by [Lee]. The methods relevant to the USA pediatric vaccine market that were identified by [Lee] are shown in Table 1 below. The Cost-Plus method is not applied in this paper, but profit is desired by the manufactures when they set their prices. In the event of a price war the Price-Undercutting strategy would be at work, whereas if tacit collusion was present then the Price-Matching strategy would be used by the manufacturers. The Bundle strategy is being applied in this market because children need different kinds of antigens throughout their infancy years to ensure safety from preventable diseases. Another method in vaccine pricing is tiered pricing where a fair profit is earned by manufactures while charging lower prices to lower income countries [Moon]. Tiered pricing is not relevant to the scope of the USA case study in this paper, but the prices at which vaccines are sold to the USA by manufactures that also supply other regions, will affect the prices sold in those regions.

Table 1: Pricing Strategies

Strategy:	Cost-Plus	Price-Matching	Price-Undercutting	Bundle
Formulation:	Cost + Desired Profit Margin	Price = f(competitors)	Price << competitors	price[bundle] ≠ sum(price of several products)
Rational:	Guarantees profit; Inelastic demand, little competition	Big target population; Desire to main status quo	Elastic demand; Maximize quantity sold	Similar customers demand similar products

Similar work in literature is summarized by Table 2 below. The key differences between this paper and other work is that the scope of model in this paper is concerned with the USA whereas other work [Ruben] has a world-wide scope allowing for the analysis of the dependency between regions. Also, this paper relies on differentiable demand functions to solve for equilibrium prices while other approaches use optimization which doesn't require smooth functions to solve for stationary points that yield optimality.

Table 2: Vaccine Models in Literature

Paper Name	Authors	Year	Scope	Approach	Objective
Pediatric vaccine procurement policy: The monopsonist's problem	Matthew J. Robbins, Sheldon H. Jacobson	2011	USA	Optimization (MINLP)	Satisfy the annual immunization requirements of USA infants and meet a minimum provider profit at minimum total immunization cost.
Making combination vaccines more accessible to low-income countries: The antigen bundle pricing problem	Ruben A. Proano, Sheldon H. Jacobson, Wenbo Zhang	2011	Global	Optimization (MINLP)	Satisfy the annual immunization requirements of all infants and meet a minimum provider profit at maximum total social surplus.
A bilevel formulation of the pediatric vaccine pricing problem	Matthew J. Robbins, Brian J. Lunday	2015	USA	Optimization (MINLP)	Satisfy the annual immunization requirements of USA infants while maximizing price for providers and minimizing price for buyers.
A symmetric capacity-constrained differentiated oligopoly model for the United States pediatric vaccine market with linear demand	Matthew J. Robbins, Brian J. Lunday	2016	USA	Game Theory	Provide a range for the monovalent vaccine equilibrium prices in the USA public sector, based on the capacity of the manufacturers.
Pricing of new vaccines	Bruce Y. Lee, Sarah M. McGlone	2010	USA	Retrospective	Analyze vaccine pricing strategies to determine the metrics and market conditions that point to a set of solution methodologies.
A win-win solution?: A critical analysis of tiered pricing to improve access to medicines in developing countries	Suerie Moon, Elodie Jambert, Michelle Childs, Tido von Schoen-Angerer	2011	Global	Retrospective	Analyze pricing strategies across case studies to find market conditions that point to competitive or tiered pricing as the preferred pricing method.

The supporting evidence for the work in this paper can be found in 2 out of the 5 conclusions from the IOM Study on USA immunization (2003):

4. Current government strategies for purchasing and ensuring access to recommended vaccines have not addressed the relationships between the financing of vaccine purchases and the stability of the U.S. vaccine supply. Financial incentives are necessary to protect the existing supply of vaccine products, as well as to encourage the development of new vaccine products.

5. The vaccine recommendation process does not adequately incorporate consideration of a vaccine's price and societal benefits. [IOM]

Further supporting evidence comes from the National Vaccine Advisory Committee (NVAC) recommendations in response to the IOM Study on USA immunization (2003):

- *Expanding VFC (Vaccines for Children) coverage for underinsured children*
- *Expanded and stable funding through Section 317 (Immunization Program) for immunization program infrastructure and operations, as well as for vaccine purchase, within existing guidelines*
- *Rapid appropriation of new funds through Section 317 when new vaccines are recommended for universal use.*
- *Further exploration of regulatory and other factors impeding vaccine research and development to alleviate barriers. [NVAC]*

The set of recommendations from IOM and NVAC point to the need for government funding, a single buyer, to provide sufficient immunization coverage to children. Furthermore, it is recognized that the research and development of vaccines is pivotal for the safety from advanced diseases, so profitability is also necessary for manufacturers to research and develop new vaccines. These recommendations from both organizations support the model design of an oligopoly market with a single buyer.

3. METHODOLOGY REVIEW

The method in this paper appears valid, but there are two weaknesses regarding selling quantities and selling prices. The concern with selling quantity is that this model does not provide information on the selling quantity of each bundle, just the total number of bundles sold. The total number of bundles sold is formulated as a polynomial function of the total purchasing cost, $D(z(w)) = d - n(z(w))^y$. Where $z(w)$ is the total purchasing cost, d is the total number of children that need to be vaccinated, n and y are polynomial parameters that must be estimated, and $D(z(w))$ is the total number of bundles purchased. The polynomial parameters are assigned a value of 1 yielding, $D(z(w)) = d - z(w)$. Historically this relationship between total bundles sold and total cost may be true for the vaccine market, but that is not addressed which raises the concern of $D(z(w))$ being a knowable statistic. The concern with selling prices is that the Nash equilibrium prices are unrealistic. The resulting Nash equilibrium is that the lowest-cost manufacturer of a bundle with positive demand is guaranteed profit and the highest-cost manufacture of a bundle is guaranteed zero profit regardless of demand. This outcome is common in competitive markets where a lower-cost producer can set their selling price equal to the marginal cost of any higher-cost competitor, guaranteeing that they make a profit while their competitor makes none. The reason these price points are unrealistic is because manufactures will leave a market that makes them no profit; therefore, this solution undesirable in the USA pediatric vaccine market.

The model in this paper consists of the following five components:

- (1) The set of players (firms)
- (2) The appended game structure (i.e., the weighted set-covering optimization problem)
- (3) The manner in which players interact with the appended game structure
- (4) The set of strategies (prices) available to each player
- (5) The manner in which the players' payoffs (profits) depend on the strategies chosen.

The set of players (1) is defined as the manufacturers. The weighted set covering problem (2) is formulated as the following:

Antigens ~ a set of antigens that are demanded (Known)

Bundles ~ a set of bundles that can be supplied (Known)

History ~ a set of number indicating the order of previous optimal solutions (Known)

Covers ~ a set of the bundles that were chosen for each solution in History (Known)
m_{ba} ~ binary value indicating if Bundle *b* supplies Antigen *a* (Known)
p_b ~ the preperation cost of Bundle *b* (Known)
I ~ the cost to inject a vaccine (Known)
c_b ~ unit price of Bundle *b* (Known)
x_b ~ purchase Bundle *b* or not (Unknown)

$$\begin{aligned}
 &\text{minimize Cost: } \sum_{b \text{ in Bundles}} ((c_b + p_b + I)x_b) \\
 &\text{s. t. Demand: } \sum_{b \text{ in Bundles}} m_{ba}x_b \geq 1; \forall a \text{ in Antigens} \\
 &\text{s. t. Alternative: } \sum_{b \text{ in Covers}[s]} x_b - \sum_{i \text{ in (Bundles - Covers}[s])} x_i \leq |\text{Covers}[s]| - 1; \forall s \text{ in History} \\
 &\text{s. t. Binary: } x_b \in \mathbb{Z}^1 \forall b \text{ in Bundles}
 \end{aligned}$$

The key difference between this formulation and the one presented in the literature review is that this a binary program looking to assign bundles to antigens as opposed to produce bundles to provide all antigens for all children. The above formulation also has two additional sets, History and Covers. These sets are initially empty and by solving the WSCP, the chosen bundles in the WSCP solution becomes the data that is fed into the History and Covers sets with the desire of finding an alternative optimal solution when solving the WSCP again.

The way players interact with the WSCP (3) is covered in the online supplement. According to the online supplement, by using the set of alternative optimal solutions in the sets History and Covers from the WSCP, a set of veto firms and veto bundles can be identified. Once the set of veto firms is identified, then a pivot firm can also be found. When this pivot firm is identified it is the only firm that can increase its profit. This result of finding the firm that can increase profit suggests that this process of finding veto firms and veto bundles is a means to identify the dominant manufacturer of a bundle based on marginal cost of production. This process of identifying dominant manufacturers of bundles is then repeated until there is no longer a manufacturer that can realize larger profits.

The set of strategies available to each player (4) is a continuum of selling prices for each of the bundles a manufacturer offers. This range of prices is defined by the marginal cost and the CDC contract price as lower and upper bounds respectively. Given that the marginal costs and contract prices of bundles are different for different manufacturers, this makes for an asymmetric game.

The way the players make profit (5) is defined in the online supplement where a player's profit is a polynomial function of the same form as $D(z(w)) = d - n(z(w))^y$ but instead of using $z(w)$ as the measure of cost, just the total cost corresponding to the manufacturers veto bundles are considered. This again raises concerns for $D(z(w))$ and a player's profit being knowable statistics given that the of polynomial coefficients, n and y , are required to be estimated yet no process for estimating them is outlined.

Additional assumptions in this model that weren't covered in the literature review include that manufacturers are intelligent, meaning that they know everything there is to know when making decisions in this game. This means that all manufacturers are aware of each other portfolio of bundles, their marginal costs, their CDC contract prices, and the selling prices they strategically choose. Another assumption is that manufactures are rational, meaning they behave in a consistent manner to maximize their own profits. There is a no bankruptcy assumption, so the selling price of a bundle can never be below the manufacturers marginal cost or else the manufacturer would go out of business.

The approach I took to reproduce the USA case study results required assumptions on initializing part of the data set for the WSCP. I made these assumptions because there was no distinction on how to prepare the data for every WSCP parameter in the USA case study. The assumptions required to complete the data for the WSCP are in Table 3 below. The support for my assumptions come from the authors description of how firms increase their price, *as part of the process, (a bundle with the lowest unit cost) is bounded in price by the cost of the bundle with the second lowest unit cost. Therefore, a firm will only increase the price of a particular bundle up to the next best price of an equivalent competing bundle* [Robbins].

Table 3: WSCP Parameter Assumptions

Parameter	Value	Rational
Price	Maximum marginal cost across all firms	Price is indexed by bundles and this ensures no bankruptcy
Preparation	Maximum cost across all firms	Price is indexed by bundles and this will ensure profit for lower-cost firms

After making necessary assumptions, the next part of the WSCP was to split up the problem into three sub-problems. First off, the *five scheduled immunizations of interest in this study are (1) birth, (2) 2 months, (3) 4 months, (4) 6 months, and (5) 12–18 months* [Robbins]. These five schedules shown in Figure 1 below are what become aggregated into three sub-problems. The only antigens of interest in this study are DTaP, Hib, HepB, and IPV.

Vaccine ▼	Age ►	Birth	1 month	2 months	4 months	6 months	12 months	15 months	18 months	19–23 months	2–3 years	4–6 years
Hepatitis B	HepB		HepB			HepB						
Rotavirus			RV	RV	RV							
Diphtheria, Tetanus, Pertussis			DTaP	DTaP	DTaP		DTaP					DTaP
<i>Haemophilus influenzae</i> type b			Hib	Hib	Hib	Hib						
Pneumococcal			PCV	PCV	PCV	PCV					PPSV	
Inactivated Poliovirus			IPV	IPV		IPV						IPV
Influenza							Influenza (yearly)					
Measles, Mumps, Rubella							MMR					MMR
Varicella							Varicella					Varicella
Hepatitis A							HepA (2 doses)				HepA series	
Meningococcal											MCV	

Figure 1: Figure 1 2011 U.S. Recommended Childhood Immunization Schedule (Through Age 6) [Robbins]

Figure 2 below shows how the schedule was split into three sub-problems where only certain antigens are needed during any of the five immunization schedules for children, so only certain manufacturers and bundles have demand through the immunization schedule. Costs in Figure 2 to be aware of are: (6) Total cost ~ *marginal cost*, (7) Diff. cost ~ *preparation cost*, and (8) Max price ~ *CDC contract price*.

Table 3 Γ Instance 1 Information

(1) Firm	(2) Vaccine	(3) Available periods	(4) Prod. cost	(5) Federal excise tax	(6) Total cost	(7) Diff. cost	(8) Max price
GlaxoSmithKline	HepB	ENGRIX B®	1	\$0.70	\$0.75	\$1.45	\$0.25
Merck	HepB	RECOMBIVAX HB®	1	\$0.70	\$0.75	\$1.45	\$0.75

Table 4 Γ Instance 2 Information

(1) Firm	(2) Vaccine	(3) Available periods	(4) Prod. cost	(5) Federal excise tax	(6) Total cost	(7) Diff. cost	(8) Max price
GlaxoSmithKline	DTaP	Infanrix®	2, 3, 4	\$0.90	\$2.25	\$3.15	\$0.25
	Hib	Hiberix®	2, 3, 4	\$0.70	\$0.75	\$1.45	\$0.75
	HepB	ENGRIX B®	2, 4	\$0.70	\$0.75	\$1.45	\$0.25
	DTaP-HepB-IPV	Pediarix®	2, 3, 4	\$1.30	\$3.75	\$5.05	\$0.25
Merck	Hib	PedvaxHIB®	2, 3, 4	\$0.70	\$0.75	\$1.45	\$0.75
	HepB	RECOMBIVAX HB®	2, 4	\$0.70	\$0.75	\$1.45	\$0.75
	Hib-HepB	COMVAX®	2, 3, 4	\$0.80	\$1.50	\$2.30	\$0.75
Sanofi Pasteur	DTaP	Tripedia®	2, 3, 4	\$0.90	\$2.25	\$3.15	\$0.75
	Hib	ActHIB®	2, 3, 4	\$0.70	\$0.75	\$1.45	\$0.75
	IPV	IPOL®	2, 3, 4	\$0.70	\$0.75	\$1.45	\$0.25
	DTaP-IPV/Hib	Pentacel®	2, 3, 4	\$1.30	\$3.75	\$5.05	\$0.75

Table 5 Γ Instance 3 Information

(1) Firm	(2) Vaccine	(3) Available periods	(4) Prod. cost	(5) Federal excise tax	(6) Total cost	(7) Diff. cost	(8) Max price
GlaxoSmithKline	DTaP	Infanrix®	5	\$0.90	\$2.25	\$3.15	\$0.25
	Hib	Hiberix®	5	\$0.70	\$0.75	\$1.45	\$0.75
Merck	Hib	PedvaxHIB®	5	\$0.70	\$0.75	\$1.45	\$0.75
Sanofi Pasteur	DTaP	Tripedia®	5	\$0.90	\$2.25	\$3.15	\$0.75
	Hib	ActHIB®	5	\$0.70	\$0.75	\$1.45	\$0.75
	DTaP/Hib	TriHIBit®	5	\$1.00	\$3.00	\$4.00	\$0.75

Figure 2: WSCP Problem Instances [Robbins]

The results I achieved when reproducing the results of the WSCP are shown below in Tables 4 – 5. Table 4 shows what the WSCP chose for bundles to create the lowest cost cover of bundles that satisfy the antigens of interest. Table 4 shows that during period 1 the buyer will purchase an equal number of HepB vaccines from GlaxoSmithKline and Merck at \$12.20 per unit (includes preparation and injection costs). During periods 2 – 4 the buyer will purchase an equal number of Hib vaccines from GlaxoSmithKline and Merck at \$12.20 per unit, and purchase DTaP-HepB-IPV from GlaxoSmithKline at \$15.30 per unit. During period 5 the buyer will purchase DTaP-Hib from Sanofi Pasteur at \$14.75 per unit.

Table 4: Minimum Cost Cover

Periods	Bundle	Firm	Consumer Price
1	HepB	GlaxoSmithKline	\$12.20
1	HepB	Merck	\$12.20
2, 3, 4	DTaP-HepB-IPV	GlaxoSmithKline	\$15.30
2, 3, 4	Hib	GlaxoSmithKline	\$12.20
2, 3, 4	Hib	Merck	\$12.20
2, 3, 4	Hib	SanofiPasteur	\$12.20
5	DTaP-Hib	SanofiPasteur	\$14.75

The selling price that each manufacturer chose can be found by subtracting the injection and preparation costs from the consumer price in Table 4. The vaccine prices for the bundles that the buyer wants is

shown in Table 5 below, where my prices are compared to the authors. The only price differences between my results and the authors regard the two vaccines, DTaP-HepB-IPV and DTaP-Hib. Both vaccines were priced higher by the author while my prices were set based on the highest cost manufacturer, meaning that the price point from Robbins exceeds the marginal cost of the highest costing manufacturer for those vaccines. This means that the way manufactures interact with the WSCP allows them to raise prices above the marginal cost of the highest cost manufacturer, again the exact execution of this for the USA case study is unclear from the online supplement but directly impacts the difference in our results. A reason why a manufacturer of a combination vaccine would be able to raise their selling price above the largest marginal cost of that vaccine is because there is a relatively large injection cost of \$10 for any vaccine. So, to individually inject multiple monovalent vaccines will become costlier to the consumer than injecting a single combination vaccine whose price point is less than the additional injection costs of the monovalent vaccines.

Table 5: Vaccine Price Comparison

Periods	Bundle	Firm	Robbins	Morris
1	HepB	GlaxoSmithKline	\$1.95	\$1.95
1	HepB	Merck	\$1.45	\$1.45
2, 3, 4	DTaP-HepB-IPV	GlaxoSmithKline	\$5.55	\$5.05
2, 3, 4	Hib	GlaxoSmithKline	\$1.45	\$1.45
2, 3, 4	Hib	Merck	\$1.45	\$1.45
2, 3, 4	Hib	SanofiPasteur	\$1.45	\$1.45
5	DTaP-Hib	SanofiPasteur	\$14.85	\$4.00

Overall, the author does not present research which is reproducible. I was able to build the WSCP, but there is no outline for how to initialize the USA data for the WSCP. Furthermore, the way firms decide to change their selling price in response to the WSCP is not described sufficiently. So, the reviewer has no certainty that they are reproducing results correctly for this case study. Another concern regarding the execution of the research is that some of the equilibrium prices for vaccines are \$0 meaning that the manufacturer will give them away and the consumer will only have to deal with the preparation and injection costs. These free price points are shown in Figure 3 below, under the Scenario Nash equilibrium column. These free values mean that the no bankruptcy assumption for some bundles and manufacturers was violated. Given the difficulty in reproducing questionable results, this is where I end my review to begin my extension on the static game. Therefore, the repeated game will not be covered in this review for the same reproducibility issues as in the static game.

Table 6 Equilibrium Prices for the Γ Instances

Firm	Vaccine	Price			
		Current	Inst. 1 Nash equilibrium	Inst. 2 Nash equilibrium	Inst. 3 Nash equilibrium
GlaxoSmithKline	DTaP	\$13.75	Free	Free	\$3.15
	Hib	\$8.66	Free	\$1.45	\$1.45
	HepB	\$9.75	\$1.95	\$1.95	Free
	DTaP-HepB-IPV	\$48.75	Free	\$5.55	Free
Merck	Hib	\$11.29	Free	\$1.45	Free
	HepB	\$10.00	\$1.45	\$1.45	Free
	Hib-HepB	\$28.80	Free	Free	Free
Sanofi Pasteur	DTaP	\$13.25	Free	Free	Free
	Hib	\$8.66	Free	\$1.45	Free
	IPV	\$11.51	Free	Free	Free
	DTaP/Hib	\$27.31	Free	Free	\$14.85
	DTaP-IPV/Hib	\$51.49	Free	\$5.05	Free

Note. Prices from the Centers for Disease Control and Prevention (vaccine price list ending in March 2010).

Figure 3: Table 6 Equilibrium Prices for the WSCP Instances [Robbins]

4. EXTENSION

In this section, we will walk through the process of building my vaccine pricing model as an alternative for the static game presented in the paper. Then we will present an experiment regarding the effect of collusion. The research questions that I have after reviewing this paper are the following:

1. How to create a direct characterization of demand?
2. How to formulate collusion in a model?
3. How to incorporate profitability and affordability in a model?

The reason I want a direct characterization of demand in my model is because this allows me to know what vaccines will be supplied to how many children. The reason I want to formulate collusion in my model is because this is known in literature to be helpful in an oligopoly market, so this is something I could verify empirically. The reason I want to formulate profitability and affordability in my model is so the users, vaccine buyers/producers, can tune the model to meet their goals of profit and immunization. Furthermore, a model like this would allow for profitability and affordability to be explored by machine learning and negotiated between firms and a buyer.

The hypotheses I have regarding each of my research questions are the following:

1. Demand of an antigen during a period is computed as Number of Children (times) Doses needed during the period. If multiple periods share a common dose requirement then the periods are given an equal fraction the dose requirement.
2. If any pair of firms provide the same bundle, then they agree to split the total demand equally.
3. If bundle profit margins and antigen coverage rates are known, then this allows firms and the buyer to tune profitability and coverage.

Let's start with the WSCP model from the literature review and update it to yield a model that applies my hypotheses. Here is a WSCP formulation for vaccine purchasing from the literature review.

Antigens ~ a set of antigens that are demanded (Known)

Bundles ~ a set of bundles that can be supplied (Known)

m_{ba} ~ binary value indicating if Bundle b supplies Antigen a (Known)

c_b ~ unit price of Bundle b (Known)

d_a ~ total demand for Antigen a (Known)

x_b ~ purchase Bundle b a discrete number of times to satisfy Antigen demand (Unknown)

$$\begin{aligned}
 &\text{minimize Cost: } \sum_{b \text{ in Bundles}} c_b x_b \\
 &\text{s. t. Demand: } \sum_{b \text{ in Bundles}} m_{ba} x_b \geq d_a; \forall a \text{ in Antigens} \\
 &\text{s. t. Integrality: } x_b \in \mathbb{Z}^+ \forall b \text{ in Bundles}
 \end{aligned}$$

The first step is that we will redefine the sets in this model. Sets are the categories containing all the information there is to know in the system you are modeling. The sets for my model include the following:

Antigens ~ a set of antigens that are demanded

Bundles ~ a set of bundles that can be supplied

Periods ~ a set of time periods when Bundles can supply Antigens

Firms ~ a set of firms that can supply Bundles

Then we redefine the parameters. Parameters are what define the relationships between the sets and contain numeric values that represent the problem in the system. The parameters for my model include the following:

$dosage_{it}$ ~ the number of doses of Antigen i required in Period t
 $prep_{jf}$ ~ the preparation cost of Bundle j produced by Firm f
 $inject$ ~ the cost to inject a vaccine
 map_{ij} ~ a binary value indicating if Antigen i is/isn't in Bundle j
 MC_{jf} ~ the marginal cost for Firm f to produce Bundle j
 $coverage_{it}$ ~ the desired immunization coverage of Antigen i during Period t
 $demand_t$ ~ the number of children to vaccinate during Period t (ie. annual births)
 $margin_{jtf}$ ~ the minimum allowable profit margin for Bundle j during Period t for Firm f
 $limit_{jf}$ ~ the upper limit on the unit price for Bundle j from Firm f
 $supply_{jf}$ ~ a binary value indicating if Firm f does/doesn't supply Bundle j

Next, we redefine the variables. Variables are what define the solution to the problem in the system that we want to solve. Variables contain the values we don't know and define the complexity of the optimization problem in terms of it being a linear, integer, or nonlinear program or any mixture of the three. The variables for my model include the following:

$price_{jtf}$ ~ the unit price of Bundle j during Period t from Firm f
 $quantity_{jtf}$ ~ the selling quantity for Bundle j during Period t from Firm f

Now we will define the objective function to be a minimization of the total budget required by the buyer to ensure the desired profitability and coverage goals. The formulation of the budget required is shown below. Every time a vaccine is purchased it faces the firm's unit price, it must be prepared based on its packaging, and it must be injected to provide the immunization. The buyer decides if the Budget is affordable and acceptable as a solution.

$$\text{minimize Budget: } \sum_{j \text{ in Bundles}} \sum_{t \text{ in Periods}} \sum_{f \text{ in Firms}} ((price_{jtf} + prep_{jf} + inject) * quantity_{jtf})$$

The first constraint regards the direct characterization of demand and immunization coverage. The parameter map_{ij} is what transforms demand to be in terms of bundles or antigens, which allows for direct communication between supply and demand. Immunization coverage is ensured by the parameter $coverage_{it}$ which is controlled by the buyer.

$$\text{s. t. Demand: } \sum_{j \text{ in Bundles}} \sum_{f \text{ in Firms}} supply_{jf} * map_{ij} * quantity_{jtf} \geq coverage_{it} * dosage_{it} * demand_t$$

$\forall i \text{ in Antigens}, t \text{ in Periods}$

The second constraint regards profitability. The parameter $margin_{jtf}$ ensures that prices are profitable, and this is controlled by the firms.

$$\text{s. t. Profit: } price_{jtf} \geq (1 + margin_{jtf}) * MC_{jf}$$

$\forall j \text{ in Bundles}, t \text{ in Periods}, f \text{ in Firms}$

The third constraint regards the CDC contract prices. The CDC establishes contract prices that are fixed for a full year, and if a firm places a price above the contract price then a government organization will not buy it. The parameter $limit_{jf}$ is controlled by the CDC to ensure prices are fair.

s. t. Limit: price_{jtf} ≤ limit_{jf}
∀ j in Bundles, t in Periods, f in Firms

The last constraint regards tacit collusion. Equilibrium prices are expected to yield balanced market share, and the buyer will always buy the cheapest bundle, so the final unit cost experienced by the buyer must be equivalent for interchangeable bundles. When the final price points of homogenous products are equivalent then the buyer is indifferent to purchasing from either firm. This indifference allows the firms to control how to split the demand in a coordinated market. So, in my formulation I'm assuming an equal share of the market demand for interchangeable bundles sold by different firms. Although, any type of agreed upon split between firms can be modeled and still be considered collusion.

s. t. Collusion: $\sum_{t \text{ in Periods}} \text{quantity}_{jtf} = \sum_{t \text{ in Periods}} \text{quantity}_{jtk}$
∀ j in Bundles, f in Firms, k in Firms : supply_{jf} + supply_{jk} > 1

The last two constraints state that price values must be real nonnegative numbers and quantity values must be discrete nonnegative numbers.

s. t. Nonnegativity: price_{jtf} ∈ ℝ⁺ ∀ j in Bundles, t in Periods, f in Firms
s. t. Integrality: quantity_{jtf} ∈ ℤ⁺ ∀ j in Bundles, t in Periods, f in Firms

My formulation is a Nonlinear Mixed Integer Program (NLMIP) because the variables are reals and integers and the *Budget* objective function contains the multiplication of real and integer variables in its formulation. The *Budget* objective function is of the same form as in the WSCP, but variables have been updated making the function nonlinear. The constraints *Demand*, *Profit*, *Limit* are linear equations representing practical limitations on variable values. The *Collusion* constraint is inspired by the expected market behavior of balanced market share when equilibrium prices are in place. So, by enforcing this condition of equal market share the expectation is that the program will solve for values of price that are like if not equilibrium prices. In the interest to see the effect of the *Collusion* constraint, an experiment will be conducted using all the data available in the paper under review to see how the vaccine pricing and procurement schedule is solved for the USA annual pediatric immunization schedule in the public market sector with and without *Collusion*.

The experiment was built in the modeling language AMPL (A Mathematical Programming Language) and solved on the NEOS Server (Network-Enabled Optimization System) by using the KNITRO solver (Nonlinear Interior point Trust Region Optimization) for NLMIPs. Data for the experiment that was assumed was *coverage_{it}* and *margin_{jtf}* at values of 90% and 10% respectively across all indices. The basis for 90% coverage is that the goal of the GVAP is to have at least 90% coverage for regional immunization requirements. The basis for 10% profit margin is to ensure profit unlike the static game model of this paper where some higher costing manufacturers face zero economic profit when producing a vaccine product. The exact periods of interest in this experiment are the first eight columns in Figure 1, which covers the immunization requirements from Birth – 18 months. The value for *demand_t* is the statistic given in this paper that there were 4.3 million births in the US in 2006. The other parameters are given values from Figures 1 – 2 in this review, and those exact values can be found in the appendix of this review. Also, the AMPL formulation is included in the appendix of this review.

In Figure 4 below you'll find a summary on the average pricing of vaccines across the 8 periods of interest. The price points in orange indicate the *Collusion* constraint being present and blue if not. The table under the graph gives the exact price of vaccines from manufacturers. If you compare prices between manufacturers of the same vaccine, then you'll see the difference in average pricing is always smaller when *Collusion* is present. In Figure 5 below you'll find a summary on the variability of pricing

vaccines across the 8 periods of interest. You can see that every vaccine for each manufacturer has a smaller standard deviation around the vaccine price when *Collusion* is present. Both statistics show that *Collusion* is stabilizing the prices of vaccines.

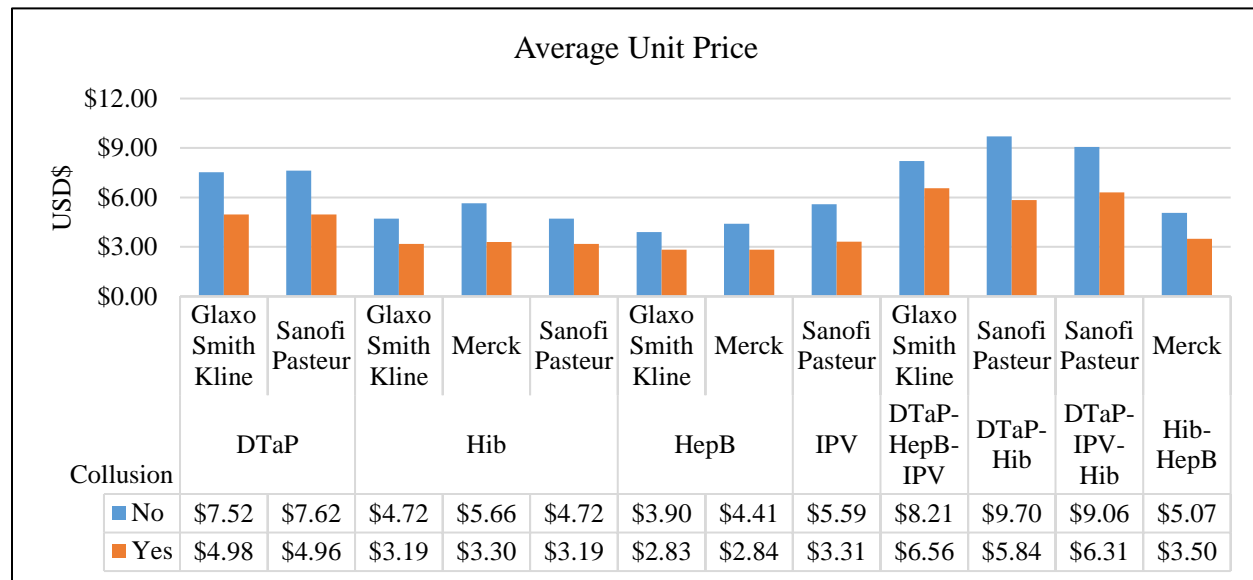


Figure 4: Average Vaccine Price under Collusion

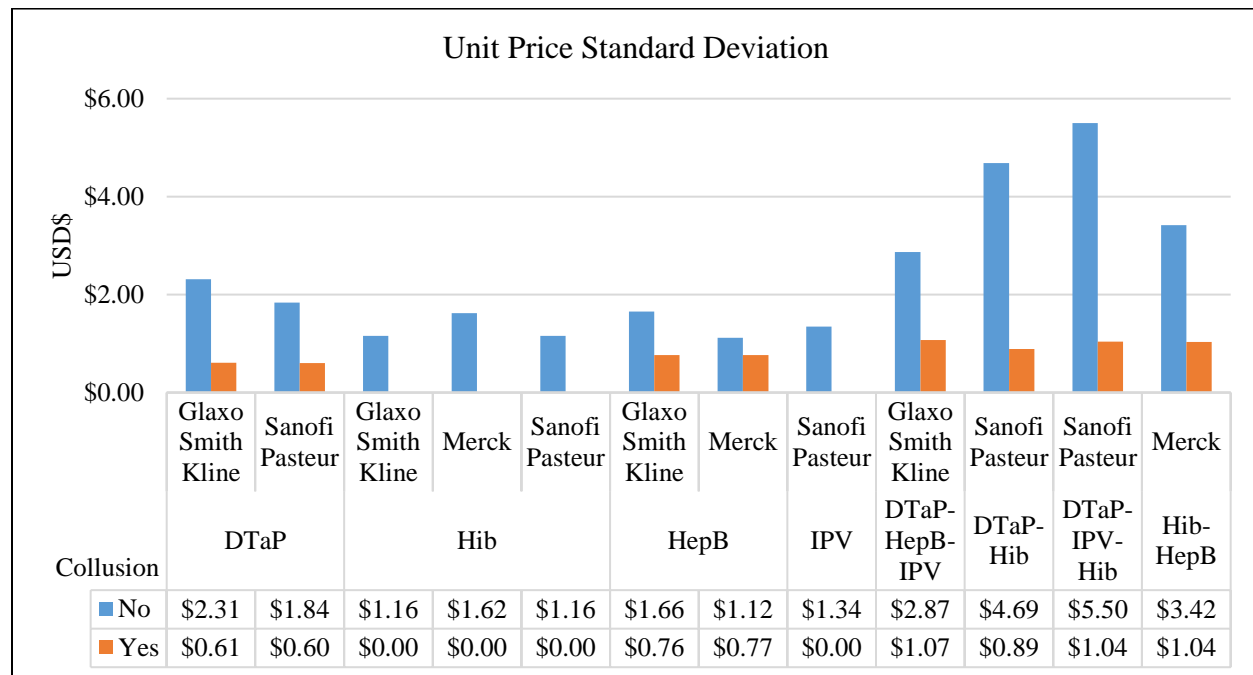


Figure 5: Vaccine Price Variability under Collusion

The total profit earned by each of the manufacturers, with and without *Collusion*, is given in Table 7 below. The total amount of profit earned by all manufacturers is the same regardless of *Collusion* but the distribution of profit is spread more evenly when *Collusion* is present. The total budget required to meet the profit margins and coverage rates of 10% and 90% does not change much with respect to Cost per Child. Cost per Child shows the total amount of money spent per child to fully or partially immunize

them, in this example its 90% coverage. These results show that *Collusion* does not significantly hurt profitability or affordability, but does help spread the profit in a more balanced way across the manufacturers.

Table 6: Profitability & Affordability

Firm	Without Collusion	With Collusion
GlaxoSmithKline	\$2,612,250.00	\$2,039,006.25
Merck	\$667,575.00	\$1,088,437.36
Sanofi Pasteur	\$5,945,287.50	\$6,097,668.75
Total Profit:	\$9,225,112.50	\$9,225,112.36
Budget Required:	\$367,055,000.00	\$368,748,000.00
Budget / Births = Cost per Child:	\$85.36	\$85.76

The next step that should be taken with this extension is to explore the implications on profitability and affordability as profit margin and coverage rates vary. One approach for doing this would be to design a full factorial experiment with two factors: coverage and margin, and the number of levels for each factor should be at least three to see if nonlinear relationships exist between coverage, margin, profitability and affordability. When this experiment is designed then each scenario in the experiment will require the model to be solved and then the solution values for the variables and objective function can be extracted to study profitability and affordability.

5. CONCLUSION

This paper contributed value to the literature. The model in this paper provides an oligopolistic solution to the pediatric vaccine market by combining game theory and optimization methods into one model that iteratively converges onto equilibrium prices in a limiting sense. By limiting sense, I refer to the tendency of a zero-economic profit solution for some vaccines. An example of this papers value to literature is it proposes that the buyer be the decision-maker of a weighted set covering problem (WSCP). This is a realistic representation of the buyer because there are organizations that build a budget for purchasing large quantities of monovalent and/or combination vaccines to immunize children. In addition to the concept of a single decision-maker, the demand is in terms of antigens and the supply is in terms of bundles which defines the sets of the WSCP. In my review, I've learned the fundamentals of game theory in oligopoly markets and how it can complement optimization models. This learning inspired a constraint that enforces market share to incentivize equilibrium prices, which I found to be plausible given the outcome of my USA experiment in the extension. I'm currently studying the implications of budget uncertainty on affordability and profitability in the global vaccine market for the Bill and Melinda Gates Foundation. This paper would be a useful baseline for the modeling of regions in a hierarchal model of the global vaccine market.

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APPENDIX

AMPL Model File

```
# ---- proposed model ----

# this model determines the minimum budget required to meet a desired antigen
immunization coverage and bundle profit margin

# ---- define set(s) ----

set antigens; # a set of antigens
set bundles; # a set of bundles
set periods; # a set of time periods
set firms; # a set of firms

# ---- define parameter(s) ----

param dosage{i in antigens, t in periods} default 0; # the number of doses of an
antigen required in a period
param prep{j in bundles, f in firms} default 0; # the preparation cost of a bundle
(this differs according to the type of packaging used by a firm)
param inject; # the cost to inject a vaccine
param map{i in antigens, j in bundles} default 0; # a binary parameter indicating if
an antigen is/isn't in a bundle (this maps antigens to bundles)
param MC{j in bundles, f in firms} default 0; # the marginal cost to produce a
bundle (this differs according to the type of production system used by a firm)
param coverage{i in antigens, t in periods} default 0.9; # the immunization coverage
for each antigen
param demand{t in periods} default 4.3e6; # the number of annual births
param margin{j in bundles, t in periods, f in firms} default 0.1; # the minimum
allowable profit margin for each bundle during a period for a firm
param limit{j in bundles, f in firms} default 0; # the upper limit on the unit price
for a bundle from a firm
param supply{j in bundles, f in firms} default 0; # a binary parameter indicating if
a firm does/doesn't supply a bundle

# ---- define variable(s) ----

var price{j in bundles, t in periods, f in firms} >= 0; # the unit price of a bundle
during a period for a firm
var quantity{j in bundles, t in periods, f in firms} integer >= 0; # the number of
bundles to purchase during a time period from a firm

# ---- define model formulation ----

minimize BUDGET: sum{j in bundles, t in periods, f in firms}((price[j,t,f] +
prep[j,f] + inject) * quantity[j,t,f]); # minimize total cost (ie. the required
budget)
s.t. DEMAND{i in antigens, t in periods}: sum{j in bundles, f in firms}(supply[j,f] *
map[i,j] * quantity[j,t,f]) >= coverage[i,t] * dosage[i,t] * demand[t]; # ensure the
desired antigen coverage is met
s.t. PROFIT{j in bundles, t in periods, f in firms}: price[j,t,f] >= (1 +
margin[j,t,f]) * MC[j,f]; # ensure the unit price yeilds the desired profit
```

```

s.t. LIMIT{j in bundles, t in periods, f in firms}: price[j,t,f] <= limit[j,f]; #
ensure the unit price does not exceed the maximum allowable price
s.t. COLLUSION{j in bundles, f1 in firms, f2 in firms: supply[j,f1] + supply[j,f2] >
1}: sum{t in periods}(quantity[j,t,f1]) = sum{t in periods}(quantity[j,t,f2]); #
ensure that firms supplying the same bundle sell the same amount to share the market
equally

```

AMPL Data File

```

# ---- define set(s) ----

# a set of antigens
set antigens :=
DTaP
Hib
HepB
IPV
;

# a set of bundles
set bundles :=
DTaP
Hib
HepB
IPV
DTaP-Hib
Hib-HepB
DTaP-HepB-IPV
DTaP-IPV-Hib
;

# a set of time periods
set periods := 1 2 3 4 5 6 7 8;

# a set of firms
set firms :=
GlaxoSmithKline
Merck
SanofiPasteur
;

# ---- define parameter(s) ----

# the number of doses of an antigen required in a period
param dosage :=
HepB 1 1
HepB 2 0.5
DTaP 3 1
Hib 3 1
HepB 3 0.5
IPV 3 1
DTaP 4 1
Hib 4 1

```

```

IPV 4 1
DTaP 5 1
Hib 5 1
HepB 5 0.25
IPV 5 0.25
Hib 6 0.5
HepB 6 0.25
IPV 6 0.25
DTaP 7 0.5
Hib 7 0.5
HepB 7 0.25
IPV 7 0.25
DTaP 8 0.5
HepB 8 0.25
IPV 8 0.25
;

# the preparation cost of a bundle (this differs according to the type of packaging
used by a firm)
param prep :=
DTaP GlaxoSmithKline 0.25
Hib GlaxoSmithKline 0.75
HepB GlaxoSmithKline 0.25
DTaP-HepB-IPV GlaxoSmithKline 0.25
Hib Merck 0.75
HepB Merck 0.75
Hib-HepB Merck 0.75
DTaP SanofiPasteur 0.75
Hib SanofiPasteur 0.75
IPV SanofiPasteur 0.25
DTaP-Hib SanofiPasteur 0.75
DTaP-IPV-Hib SanofiPasteur 0.75
;

# the cost to inject a vaccine
param inject := 10;

# a binary parameter indicating if an antigen is/isn't in a bundle (this maps
antigens to bundles)
param map :=
DTaP DTaP 1
Hib Hib 1
HepB HepB 1
IPV IPV 1
Hib Hib-HepB 1
HepB Hib-HepB 1
DTaP DTaP-HepB-IPV 1
HepB DTaP-HepB-IPV 1
IPV DTaP-HepB-IPV 1
DTaP DTaP-IPV-Hib 1
Hib DTaP-IPV-Hib 1
IPV DTaP-IPV-Hib 1
DTaP DTaP-Hib 1
Hib DTaP-Hib 1
;

```

```

# the marginal cost to produce a bundle (this differs according to the type of
production system used by a firm)
param MC :=
DTaP GlaxoSmithKline 3.15
Hib GlaxoSmithKline 1.45
HepB GlaxoSmithKline 1.45
DTaP-HepB-IPV GlaxoSmithKline 5.05
Hib Merck 1.45
HepB Merck 1.45
Hib-HepB Merck 2.3
DTaP SanofiPasteur 3.15
Hib SanofiPasteur 1.45
IPV SanofiPasteur 1.45
DTaP-Hib SanofiPasteur 4
DTaP-IPV-Hib SanofiPasteur 5.05
;

# the upper limit on the unit price for a bundle from a firm
param limit :=
DTaP GlaxoSmithKline 13.75
Hib GlaxoSmithKline 8.66
HepB GlaxoSmithKline 9.75
DTaP-HepB-IPV GlaxoSmithKline 48.75
Hib Merck 11.29
HepB Merck 10
Hib-HepB Merck 28.8
DTaP SanofiPasteur 13.25
Hib SanofiPasteur 8.66
IPV SanofiPasteur 11.51
DTaP-Hib SanofiPasteur 27.31
DTaP-IPV-Hib SanofiPasteur 51.49
;

# a binary parameter indicating if a firm does/doesn't supply a bundle
param supply :=
DTaP GlaxoSmithKline 1
Hib GlaxoSmithKline 1
HepB GlaxoSmithKline 1
DTaP-HepB-IPV GlaxoSmithKline 1
Hib Merck 1
HepB Merck 1
Hib-HepB Merck 1
DTaP SanofiPasteur 1
Hib SanofiPasteur 1
IPV SanofiPasteur 1
DTaP-Hib SanofiPasteur 1
DTaP-IPV-Hib SanofiPasteur 1
;

```

AMPL Run File

```
#####
```

```

#### if you're using NEOS then keep the following 4 lines of code commented out
#### if you're solving this problem with an ampl license on your machine then
uncomment these 4 lines of code
#####

# reset; # clear the work space
# model WSCP-Collusion.mod; # load the model file
# data WSCP-Collusion.dat; # load the data file
# option solver knitro; # choose a solver

option knitro_options 'ms_enable=1 ma_terminate=0 mip_integral_gap_rel=1e-5'; # in
the order it appears (comma seperated): enable multi start, let the solver decide how
many multiple start points to use based on problem size, pick the precision
(significant figures) for which a number (any integer variable you want to solve for)
is considered an integer
option display_eps 1.0e-09; # machine precision on all variables (ie. the smallest
magnitude in the decimal place for which a number is considered different from zero)
option solution_round 6; # round the displayed variable values to a certain number
of decimal places
solve; # solve the model instance
option omit_zero_rows 1; # if a variable took a value of zero then dont display it
option display_1col 1000; # force the display to be in long format
display sum{j in bundles, t in periods, f in firms}((price[j,t,f] + prep[j,f] +
inject) * quantity[j,t,f]); # display the value of the objective function
display price, quantity; # display the values of the variables of interest

```