Robot Devices, Kinematics, Dynamics, and Control (EN 530.646)
Useful Formula for Lab 3

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In writing `getXi.m`, the following will be useful. In fact the following formula is in the class and MLS (proposition 2.9). But I will summarize here.

Given $g = (R, \boldsymbol{p}) \in SE(3)$, we want to compute $\boldsymbol{\xi} = [\boldsymbol{v}^T, \boldsymbol{\omega}^T]^T$ which is a unit twist. The exponential twist is $\theta\widehat{\boldsymbol{\xi}}$. First compute

$$\theta = \text{acos}\left(\frac{\text{tr}(R) - 1}{2}\right)$$

which is a rotation angle about $\boldsymbol{\omega}$.

If $\theta = 0$, then it means the twist is a pure translation. In that case,

$$\boldsymbol{\omega} = [0,\, 0,\, 0]^T$$
$$\boldsymbol{v} = \boldsymbol{p}/\|\boldsymbol{p}\|$$

and

$$\theta = \|\boldsymbol{p}\|.$$

If $\theta \neq 0$, then you can compute

$$\boldsymbol{\omega} = \frac{1}{2\sin\theta}\left(R - R^T\right)^{\vee}.$$

The translation part is computed via solving

$$\left[(\mathbb{I} - e^{\theta\hat{\boldsymbol{\omega}}})\hat{\boldsymbol{\omega}} + \boldsymbol{\omega}\boldsymbol{\omega}^T\theta\right]\boldsymbol{v} = \boldsymbol{p}$$

or

$$\left[\mathbb{I}\theta + (1 - \cos\theta)\hat{\boldsymbol{\omega}} + (\theta - \sin\theta)\hat{\boldsymbol{\omega}}^2\right]\boldsymbol{v} = \boldsymbol{p}.$$